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(SELECTED ARTICLES)

BY: L. I. Kudryashev and L. I. Zhemkov

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COLLECTION OF SCIENTIFIC PAPERS  
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By

L. I. Kudryashev and L. I. Zhemkov

GENERALIZATION OF THE THEORY OF REGULAR THERMAL REGIME  
FOR THE CASE OF VARIABLE COEFFICIENTS OF HEAT  
CONDUCTIVITY AND SPECIFIC HEAT

Prof. L. I. Kudryashev and L. I. Zhemkov

Theoretical Section

The regular heat regime method has for some time past been more and more practically used in various heat calculations. Its simplicity and the great accuracy of the results explains this.

Boussinesq first directed our attention to the property of regularity. He noted that parabolic linear differential equations possess the quality of regularity consisting in the relative rate of change in the variable on the left side of the equation remaining a constant value after the completion of a certain change in the independent variable. G. M. Kondratyev's profound analysis of this property applicable to Fourier's heat conductivity equation also laid the foundation of the theory of regular heat regime.

At present the theory of regular heat regime is used without exception in all the problems of heat physics. The First Inter-University Conference on the Regular Heat Regime in Leningrad in March 1958

demonstrated this convincingly.

Nevertheless the theory of regular heat regime continues to be based on the analysis of a Fourier differential equation with constant coefficients, i.e., specific heat and the coefficient of heat conductivity are considered constants.

This artificial procedure simplifying mathematical analysis of the phenomena at the same time substantially narrows the circle of questions which the regular regime theory may solve.

In actual fact, the thermophysical properties of substances change with temperature and consequently a Fourier heat conductivity equation will represent a non-linear equation to which is not applicable the Fourier solution according to which the desired solution appears in the form of a product of two functions independent of each other—the time function and the coordinate function.

Therefore, having set the task of generalizing the theory of regular heat regime for the case of variable coefficients of thermal conductivity and specific heat it is first necessary to analyze and solve the problem of linearizing the non-linear differential equation of thermal conductivity. This task is the central one.

Numerous experimental data indicate that the rate of change of temperature in time when  $\alpha = \text{const}$  does not remain constant. The cause of these deviations is the dependence of thermophysical properties of the calorimeter material on temperature.

This indicates that the temperature regularity cannot be taken as the base if one analyzes the phenomena with regard to the variability of  $\lambda$  and  $C_p$ . In this case the temperature no longer exhausts the nature of the process. Thus one must direct his attention to searching for a new thermal function, a new thermodynamic potential, which will describe the heat process with a parabolic linear equation.

A frequent case of regularity of such a function (when  $\lambda$  and  $C_p = \text{const}$ ) will be the temperature regularity which is the subject of G. M. Kondrat'yev's research and that of all the other authors.

The fact that treating the experimental data with respect to excess heat content  $i_{\text{exc.}} = i_{\text{body}} - i_{\text{surr. med.}}$ , and not with respect to excess temperature, gives a more constant rate in time than the rate with respect to temperature.

This indicates that the desired potential must necessarily correspond in its constitution to the dependence of  $\lambda$  and  $C_p$  on the temperature and in the end must characterize as heat both that accumulated by the body and that transferred by thermal conductivity to the heat exchanging surface.

In addition, for completely rigid linearization, as will be demonstrated below, there will also be needed a transformation of the time variable.

Designations:  $t$  - temperature,  $\tau$  - time,  $\lambda$  - coefficient of heat conductivity,  $C_p$  - specific heat,  $\gamma$  - specific gravity,  $i$  - enthalpy,  $\alpha$  - temperature conductivity,  $V$  - volume of the body,  $F$  - body surface participating in heat exchange,  $q_v$  - intensity of internal source of heat in a unit of volume  $q_n$  - vector of heat flow,  $v = t_{\text{body}} - t_{\text{medium}}$  - excess temperature,  $\alpha$  - coefficient of heat delivery,  $\Psi$  - coefficient of non-uniformity of the temperature field,  $m$  - rate of change of function of the state,  $d$  - characteristic size of the calorimeter.

Subscripts:  $w$  - value on the surface or average value over the surface,  $v$  - average value throughout the volume,  $o$  - initial value,  $m_v : m_f : m_f'$  - rates of change of the different functions.

Passing to the mathematical formulation of the problem, let us make the two following remarks.

In the overwhelming majority of textbooks the heat conductivity

equation is derived without regard to the dependence of the coefficients of heat conductivity and specific heat on the temperature, since no problems with respect to these relations were posed. Such an approach is wrong, since from the very beginning the analysis is carried out based on a particular case of the Fourier equation. Only in G. M. Kondrat'ev's last monograph is a conclusion from this equation adduced with regard to the variability in the thermophysical properties of the material.

Further, in our opinion the problem of non-steady heat conductivity is more elegantly and strictly formulated by adducing the equation of the conservation of mass. Here the formulation of the problem turns out to be of the most general form, from which all particular cases follow.

Let us examine the non-steady process of cooling or heating a body of volume  $V$  and with a surface of heat exchange  $F$  situated in some medium or other.

If we regard the process of heating or cooling as isobaric, the equation of the balance of the body's energy in the integral form is

$$\int_V q_v dV = \frac{\partial}{\partial \tau} \int_V \gamma i dV + \int_F q_n dF. \quad (1)$$

Thus the amount of heat given off by internal sources is expended in changing the body's enthalpy and transferring heat by thermal conductivity to the surface of the body where heat exchange with the surrounding medium occurs according to one law or another.

That the problem may be a closed one, let us add to Eq. (1) the equation of the conservation of mass in the form

$$\frac{\partial}{\partial \tau} \int_V \gamma dV = 0. \quad (2)$$

The first integral on the right side of Eq. 1 can be written in explicit form thus

$$\frac{\partial}{\partial \tau} \int_V \gamma i dV = \int_V \gamma \frac{\partial i}{\partial \tau} dV + \int_V i \frac{\partial}{\partial \tau} (\gamma dV)$$

or with regard to Eq. (2):

$$\frac{\partial}{\partial \tau} \int_V \gamma i dV = \int_V \gamma \frac{\partial i}{\partial \tau} dV. \quad (3)$$

According to Ostrogradskiy's theorem

$$\int_V q_n dF = \int_V \operatorname{div} q_n dV. \quad (4)$$

In view of the Fourier hypothesis

$$q_n = -\lambda \operatorname{grad} t, \quad (5)$$

we have

$$\int_V q_n dF = - \int_V \operatorname{div} (\lambda \operatorname{grad} t) dV. \quad (6)$$

Substituting (3) and (6) in Eq. (1) and by passing to the limit, we obtain the differential equation

$$\gamma \frac{\partial i}{\partial \tau} = \operatorname{div} (\lambda \operatorname{grad} t) + q_V. \quad (7)$$

Enthalpy i is a function of the temperature, i.e.,

$$i = i(t)$$

and consequently we can write

$$\frac{\partial i}{\partial \tau} = \frac{\partial i}{\partial t} \cdot \frac{\partial t}{\partial \tau}.$$

But according to the definition of specific heat

$$\frac{\partial i}{\partial t} = C_p. \quad (8)$$

Consequently,

$$\frac{\partial i}{\partial \tau} = C_p \cdot \frac{\partial t}{\partial \tau}. \quad (9)$$

Substituting (9) in (7) we have

$$\gamma C_p \frac{\partial t}{\partial \tau} = \operatorname{div} (\lambda \operatorname{grad} t) + q_v. \quad (10)$$

For a full mathematical formulation of the non-steady heat conductivity problem we must add to Eq. (10) the relationships of the thermophysical properties of the material to the temperature:

$$\begin{aligned} \lambda &= \lambda(t); \\ C_p &= C_p(t), \end{aligned} \quad (11)$$

and also the initial conditions in the form:

$$\text{when } \tau = 0 \quad t = t_0(x, y, z) \quad (12a)$$

and the bounding conditions, assuming that heat exchange on the surface of the body takes place according to Newton's law:

$$-\lambda_w \left( \frac{\partial t}{\partial n} \right)_{w} = \alpha_w \cdot t_w. \quad (12b)$$

The system of differential equations (10), (11), (12) is a system of non-linear equations relative to temperature and therefore it is impossible to present the sought-for solution as a product of two functions, the function of time and that of the coordinates. Equation (10) has no property of regularity.

We shall demonstrate that introducing the new function  $\Phi = \int_0^t \frac{\lambda}{C_p} dt$  in place of temperature  $t$  and the integral argument  $\xi = \int_0^t \frac{\lambda}{C_p} dt$  in place of time  $\tau$  enables the indicated system of non-linear differential equations to be transformed into a system of linear differential equations.

First we shall prove the two following correlations:

$$a) \operatorname{grad} \xi = C_p \cdot \operatorname{grad} t \quad (13)$$

$$b) \operatorname{div}(\lambda \operatorname{grad} t) = \nabla^2 \phi. \quad (14)$$

The first correlation is proved simply if we take into account that

$$i = i(t)$$

In this case the expression for  $\operatorname{grad} i$  may be written in the explicit form

$$\begin{aligned} \operatorname{grad} i &= \frac{\partial i}{\partial x} i + \frac{\partial i}{\partial y} j + \frac{\partial i}{\partial z} \kappa = \frac{\partial i}{\partial t} \cdot \frac{dt}{\partial x} i + \frac{\partial i}{\partial t} \cdot \frac{dt}{\partial y} j + \frac{\partial i}{\partial t} \cdot \frac{dt}{\partial z} \kappa = \\ &= \frac{di}{dt} \left( \frac{dt}{\partial x} i + \frac{dt}{\partial y} j + \frac{dt}{\partial z} \kappa \right) = C_p \cdot \operatorname{grad} t. \end{aligned}$$

2. On the basis of the first correlation proved the second may also be proved:

$$\operatorname{div}(\lambda \operatorname{grad} t) = \nabla^2 \phi.$$

First of all, it is obvious that

$$\operatorname{div}(\lambda \operatorname{grad} t) = \operatorname{div} \left( \frac{\lambda}{C_p} \operatorname{grad} i \right). \quad (15)$$

Further considering (11) we may write

$$\frac{\lambda}{C_p} = f(t) = f_1(i). \quad (16)$$

This gives reason for writing (15) in the explicit form

$$\begin{aligned} \operatorname{div}(\lambda \operatorname{grad} t) &= \frac{\lambda}{C_p} \left[ \frac{\partial^2 i}{\partial x^2} + \frac{\partial^2 i}{\partial y^2} + \frac{\partial^2 i}{\partial z^2} \right] + \frac{d}{dt} \left( \frac{\lambda}{C_p} \right) \left[ \left( \frac{\partial i}{\partial x} \right)^2 + \right. \\ &\quad \left. + \left( \frac{\partial i}{\partial y} \right)^2 + \left( \frac{\partial i}{\partial z} \right)^2 \right]. \end{aligned} \quad (17)$$

Now introducing the new integral function

$$\Phi = \int_0^i \frac{\lambda}{C_p} di. \quad (18)$$

one can write

$$\begin{aligned} d\Phi &= \frac{\lambda}{C_p} di = \frac{\partial \Phi}{\partial z} dz + \frac{\partial \Phi}{\partial x} dx + \frac{\partial \Phi}{\partial y} dy + \frac{\partial \Phi}{\partial z} dz = \\ &= \frac{\lambda}{C_p} \left( \frac{\partial i}{\partial z} dz + \frac{\partial i}{\partial x} dx + \frac{\partial i}{\partial y} dy + \frac{\partial i}{\partial z} dz \right). \end{aligned}$$

From the last expression it is seen that

$$\left. \begin{aligned} \frac{\partial \Phi}{\partial \tau} &= \frac{\lambda}{C_p} \cdot \frac{\partial t}{\partial \tau} \\ \frac{\partial \Phi}{\partial x} &= \frac{\lambda}{C_p} \cdot \frac{\partial t}{\partial x} \\ \dots \dots \dots \end{aligned} \right\} \quad (19)$$

With regard to (16) we find that

$$\left. \begin{aligned} \frac{\partial^2 \Phi}{\partial x^2} &= \frac{\lambda}{C_p} \cdot \frac{\partial^2 t}{\partial x^2} + \frac{d}{dt} \left( \frac{\lambda}{C_p} \right) \cdot \left( \frac{\partial t}{\partial x} \right)^2 \\ \dots \dots \dots \end{aligned} \right\} \quad (20)$$

Adding the right and left sides of the equations in (20) we will have

$$\begin{aligned} \nabla^2 \Phi &= \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{\lambda}{C_p} \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) + \\ &+ \frac{d}{dt} \left( \frac{\lambda}{C_p} \right) \left[ \left( \frac{\partial t}{\partial x} \right)^2 + \left( \frac{\partial t}{\partial y} \right)^2 + \left( \frac{\partial t}{\partial z} \right)^2 \right] = \text{div} (\lambda \text{grad } t). \end{aligned} \quad (21)$$

Now, using the proven relations (13) and (14), it is easy to reduce Eq. (10) to the form:

$$\frac{C_p \cdot \gamma}{\lambda} \cdot \frac{\partial \Phi}{\partial \tau} = \nabla^2 \Phi + q_v.$$

or

$$\frac{\partial \Phi}{\partial \tau} = a \nabla^2 \Phi + a q_v. \quad (22)$$

The coefficient of temperature conductivity  $a = \frac{\lambda}{C_p \cdot \gamma}$  is a function of the temperature.

Equation (22) is quasi-linear for function  $\Phi$ , since the coefficient of temperature conductivity  $a(t)$ , not depending explicitly on  $\Phi$ , nevertheless is connected with  $\Phi$  via the temperature. Therefore a supplementary scrutiny of the behavior of function  $a(t)$  is demanded.

It may, however, be shown that linearization of Eq. (22) can be concluded by passing to the new integral argument:

$$\xi = \int_0^\tau \frac{\lambda}{C_p \cdot \gamma} d\tau \quad (23)$$

in place of time  $\tau$ .

With the introduction of this argument Eq. (22) passes into a strictly linear differential equation for function  $\Phi$ .

Actually, introducing argument  $\xi$  into (22), we will get

$$\frac{\partial \Phi}{\partial \xi} = \nabla^2 \Phi + q_v. \quad (24)$$

Equation (24) is a rigorously differential equation of the parabolic type for function  $\Phi$ . This equation, as is easy to see, is analogous to the Fourier differential equation for heat conductivity for the temperature, if we replace temperature  $t$  with function  $\Phi$ , time  $\tau$  with argument  $\xi$ , and set  $a = 1$ .

So the task of linearizing the non-linear differential equation of heat conductivity is done.

The new function  $\Phi(\xi, x, y, z)$  describes the thermal process in the body as a linear differential equation of the parabolic type, to which we can in full measure apply the Fourier method of solution.

We may now proceed to the examination of other questions, one of which is the generalization of the theory of regular heat regime for the case of variable coefficients of heat conductivity and specific heat.

Let us examine the case wide-spread in practice, where the internal sources of heat are lacking,  $q_v = 0$ .

Equation (24) will take on a simpler form

$$\frac{\partial \Phi}{\partial \xi} = \nabla^2 \Phi. \quad (24a)$$

To linear equation (24a) will apply in full measure the Fourier method of solution, i.e.,

$$\Phi(\xi, x, y, z) = \varphi(\xi) \psi(x, y, z). \quad (25)$$

This affords reason for writing Eq. (14a) in the form

$$\frac{\tau'(\xi)}{\tau(\xi)} = \frac{\tau^2 \psi(x, y, z)}{\psi(x, y, z)}. \quad (26)$$

In other words Eq. (26) satisfies the Fourier condition for function  $\Phi$ .

The above presentation enables us to formulate the regularity condition in the form of the following supposition: if into the non-linear differential equation is introduced a new integral function  $\Phi = \int_0^t \frac{\lambda}{C_p} di$  instead of temperature  $t$  and the new integral argument  $\xi = \int_0^t ad\tau$  instead of time  $\tau$ , the non-linear differential equation of heat conductivity becomes a linear differential equation and possesses the property of regularity with respect to the new function.

We will now carry out an analysis of the variation factor in a temperature field  $\psi$ , which occupies one of the central places in the theory of regular temperature regime equally with the concept of rate of temperature change  $m_v$ . The actual temperature field in the body with the variables  $\lambda$  and  $C_p$  will not coincide with the temperature field when it is assumed that  $\lambda$  and  $C_p = \text{const.}$

As we will be able to see, it is not difficult to allow for this deformation of the temperature field, if one operates with the field of function  $\Phi$  in the body, i.e., conducting the analysis of the basis of Eq. (24a). The necessary transformations are simple and obvious.

By virtue of (14) we have  $\nabla^2 \Phi = \text{div}(\lambda \text{ grad } t)$ , but at the same time, from (24a):  $\tau^2 \Phi = \frac{\partial \Phi}{\partial \xi}$ .

Consequently,

$$\frac{\partial \Phi}{\partial \xi} = \text{div}(\lambda \text{ grad } t).$$

Integrating both sides of this equation over the volume and using Ostrogradskiy's theorem, we get

$$\int_V \frac{\partial \Phi}{\partial \xi} dv = \int_V dt v (\lambda \operatorname{grad} t) dv = - \int_F q_n dF. \quad (27)$$

The heat transmitted by heat exchange may be expressed according to Newton's law as follows:

$$Q = \int_F q_n dF = \alpha_w \cdot t_w \cdot F. \quad (28)$$

Substituting (28) in (27) and averaging the left side over the volume, we get

$$\left( \frac{\partial \Phi}{\partial \xi} \right)_V \cdot V = - \alpha_w \cdot t_w \cdot F. \quad (29)$$

Dividing both sides of relation (29) by the value of function  $\Phi_V$  averaged over the volume, we get

$$\frac{1}{\Phi_V} \cdot \left( \frac{\partial \Phi}{\partial \xi} \right)_V = - \frac{\alpha_w \cdot t_w \cdot F}{V \cdot \Phi_V}. \quad (30)$$

By virtue of (25) it is easy to show that the left side of the equality (30) is a constant value.

We will call this constant value the rate of change of function  $\Phi$  with respect to the argument  $\xi$  —  $m_\Phi$ .

Consequently,

$$\frac{1}{\Phi} \cdot \frac{\partial \Phi}{\partial \xi} = - \frac{\alpha_w \cdot t_w \cdot F}{V \cdot \Phi_V} = - m_\Phi \quad (31)$$

Integration of Eq. (31) leads to an expression for determining rate  $m_V$  for the excess temperature according to G. M. Kondrat'yev's theory.

After integrating:

$$\ln \phi = -m_{\phi} \xi + \text{const.} \quad (32)$$

And further for two successive values of argument  $\xi$ :

$$\ln \phi_1 - \ln \phi_2 = -m_{\phi}(\xi_2 - \xi_1). \quad (33)$$

Consequently,

$$m_{\phi} = \frac{\ln \phi_1 - \ln \phi_2}{\xi_2 - \xi_1}. \quad (34)$$

The calculations are interesting and valuable because they enable us to find a connection between the rate of change of  $\phi$  with respect to argument  $\xi - m_{\phi}$  and the rate of change of the function  $\phi$  with respect to time  $\tau - m_{\phi}$ . This connection makes it possible to evaluate the "degree" of linearization of the heat conductivity equation when only function  $\phi$  is introduced.

This is practically valuable because it enables us to estimate whether complete, rigorous linearization is necessary or whether it can be avoided by partial linearization, depending on the nature of the material's properties.

To the expression

$$\xi_2 - \xi_1 = \int_0^{\tau_2} \frac{\lambda}{c_p \gamma} d\tau - \int_0^{\tau_1} \frac{\lambda}{c_p \gamma} d\tau$$

let us apply the theorem of the average, which is completely legal, since  $a = \frac{\lambda}{c_p \gamma}$  is a continuous function without any exceptions at all in the whole investigated temperature range for isotropic solid bodies (the temperature ranges are examined without changes in the aggregate state).

Consequently

$$\xi_2 - \xi_1 = \left( \frac{\lambda}{C_p \gamma} \right)_m (\tau_2 - \tau_1), \quad (35)$$

where  $\left( \frac{\lambda}{C_p \gamma} \right)_m$  is the average value in the investigated interval of time (temperature range).

Substituting (35) in the expression for  $m_\phi$ , we get

$$m_\phi = \frac{\ln \phi_1 - \ln \phi_2}{\left( \frac{\lambda}{C_p \gamma} \right)_m (\tau_2 - \tau_1)}. \quad (36)$$

Or

$$m'_\phi = \frac{\ln \phi_1 - \ln \phi_2}{\tau_2 - \tau_1}, \quad (37)$$

here  $m'_\phi$  is the rate of change of  $\phi$  in time, hence the connection between  $m_\phi$  and  $m'_\phi$ :

$$m'_\phi = \left( \frac{\lambda}{C_p \gamma} \right)_m \cdot m_\phi. \quad (38)$$

Taking (38) into account we will write expression (31) in the form

$$m'_\phi = \frac{c_p \cdot t_w \cdot R}{\phi_1 \cdot V} \cdot \left( \frac{\lambda}{C_p \gamma} \right)_m. \quad (39)$$

From comparison with the ordinary form of notation for rate  $m_v$  we get an expression for coefficient  $\Psi$ .

$$\Psi = \frac{t_w t_r}{\phi_1}. \quad (40)$$

Coefficient  $\Psi$  by analogy with the variation factor of the temperature field  $\Psi = \frac{t_r}{t_v}$  according to Kondrat'yev characterizes this field, but allows for the variability of thermophysical properties.

In distinction to the variation factor of the temperature field in the theory of regular temperature regime this coefficient, as is

seen from (40), represents an already complicated complex into which  $\lambda$  and  $C_p$  enter as functions of the temperature.

The results obtained are of interest from both the theoretical and the practical point of view.

That the structure of the very simple and practically convenient formulas for the regular temperature regime be preserved is, above all, highly valuable. But now these formulas have a qualitatively new sense, since there is taken into account in them the change in the thermophysical properties of the material during the non-steady process of heat exchange.

The congruity of the expressions for rates and variable factors of the temperature field indicates that the theory of regular temperature regime is a particular case in the generalized theory of regular heat regime. Actually, taking  $\lambda$  and  $C_p = \text{const}$ , as is done in the theory of temperature regularity, we immediately obtain from formulas (36) and (40) expressions for the rate and variation factor used in Kondrat'yev's theory.

The rate for the temperature will be a constant value only in case  $\lambda$  and  $C_p = \text{const}$  (when there is a constant value  $\alpha$ ), but actually there is no such situation, and hence on studying non-steady thermal processes there appear various requirements in relation to the material of the calorimeter, its dimensions and shape, temperature range, etc., which complicate the work. These requirements are not always compatible with each other.

In the generalized theory of regular temperature regime, as we were able to satisfy ourselves, no simplifying assumptions were made in the theoretical section in respect to the thermophysical properties of the material.

Therefore it is to be expected that this makes it possible to extend the limits of use of the theory of thermal regularity in regard to the kinds of materials, temperature ranges, calorimeter dimensions and shapes, etc.

Actually, experimental verification corroborated the theses of the generalized theory.

### Experimental Section

The set of experiments was carried out on the basis of the theory of simulation. A number of features of the tests resulted from the propositions of the theory. These exceptions consisted in corroborating the theoretical propositions with the example of a material with very greatly changing thermophysical properties.

In solving the problem formulated by the system of equations (10), (11), and (12) there appear the invariants  $F_0$  and  $Bi$ .

In the experiments it was consequently necessary to fulfil the requirement

$$\left. \begin{array}{l} F_0 = idem \\ Bi = idem \end{array} \right\} \quad (48)$$

or

$$\left. \begin{array}{l} \frac{\alpha z}{d^2} = idem \\ \frac{\alpha d}{t} = idem \end{array} \right\}$$

Starting from the conditions (48) and also allowing for the theoretical propositions, in the experiments we used as the material for the calorimeter graphite, which has a low valued coefficient of heat conductivity and whose thermophysical properties are strongly dependent on temperature.

An experimental specimen was made in the form of a graphite cylinder 50 mm in diameter and 400 mm in length.

In one of the cross-sections of the specimen were embedded four thermocouples (copper-constantan) which permitted the approximate evaluation of the temperature field.

The specimen was placed in an aerodynamic tunnel and after heating with a special furnace was blasted with a stream of air. Temperature changes were read off with a PP type potentiometer, since the rate of cooling in all experiments enabled us to make measurements from all four thermocouples.

In the experiments the air velocity was varied from 0 to 25 m/sec (which corresponded with Reynolds numbers up to 50,000); the temperature to which the cylinder was heated, and the position of the thermocouples in respect to the air current were also changed.

To process the experimental data the curve  $\phi = \phi(t)$  is plotted. This curve (or table) is suitable for any bodies of a given material, since  $\phi$  is a function of the state.

As in the theory of regular temperature regime, where excess values of the temperature are used to find rate  $m_v$ , in the generalized theory it is also necessary to calculate the excess values

$$\phi = \phi_{\text{body}} - \phi_{\text{surr. medium.}}$$

In processing the experimental data for many materials cooled in air there occurs an interesting detail which facilitates treatment of the experiments.

It turns out that in this case  $\phi$  of the body is much greater than  $\phi_{\text{air}}$  and therefore it is possible without perceptible error to use not the excess but the absolute values of  $\phi$  of the body to find rate  $m_\phi$ .

To illustrate—at 25°C

$$\phi_{\text{air}} = 0.1 \frac{\text{kcal}}{\text{m}^2/\text{hr}}$$

and

$$\Phi_{\text{graphite}} = 390 \frac{\text{kcal}}{\text{m}^2/\text{hr}}.$$

Of course, in every case one must compare  $\Phi_{\text{body}}$  and  $\Phi$  of the surrounding medium.

For example, when cooling in liquids  $\Phi$  of the liquid may prove to be of the same order as  $\Phi$  of the cooled body.

The indicated feature discovered when processing the experimental data, showed that introduction of the function  $\Phi$  allows for the variability on the thermophysical properties not only in the cooled body but also in the cooling medium.

Thus in calculating the difference  $\Phi_{\text{body}} - \Phi_{\text{surr. med.}}$  allowance is made for the variability in thermophysical properties even of the system "body--surrounding medium."

This permits the solution of the problem of cooling bodies in bounded volumes of cooling medium, cooling composite bodies, etc.

From the experiments result the data

$$v = v(\tau).$$

On graphically representing these relations it turned out that they are not exponents, since the rate  $m_v$  is a variable value. In all experiments a tendency on the part of rate  $m_v$  to increase with time was discovered. This fact is a result of the change in thermophysical properties, since the coefficient of heat emission does not change during the experiments (each experiment was conducted at a fixed value  $Re$ ).

With the growth of  $Re$  the rate increase during the experiment becomes sharper. For example, when  $Re = 50,000$ , rate  $m_v$  increases

threefold during the 10 minutes of the experiment.

The variability of the rate, especially clearly seen in the case of graphite, is also noticeable in experiments with steel, although steel possesses a much greater thermal conductivity.

Thus the experiments confirmed the proposition of the theory that with variable  $\lambda$  and  $C_p$  rate  $m_v$  is no longer stable during the experiments and, as a result of this, heat emission  $\alpha$  cannot be uniquely determined. With regular temperature regime we can determine only an approximate value of heat emission, and even that not always. To improve the results, as has already been indicated, there are usually recommendations as to dimensions, shape, and material of the calorimeter. This does not always satisfy the researcher, especially in experiments on natural objects. It is easy to bring about the transition to the new function  $\Phi = \Phi(\tau)$  based on the curves  $\Phi = \Phi(t)$ . The experiments have shown that these curves are not exponents either (they are derived from a quasi-linear equation).

Nevertheless the instability of rate  $m_\Phi$  is here considerably less than in rate  $m_v$ . This indicates that the determined degree of linearization has been accomplished.

Finally, to conduct a final verification of the theory it is necessary to plot  $\Phi = \Phi(\xi)$ . To do this we must replot  $\Phi$  along a  $\xi$  abscissa instead of a  $\tau$  one. There is a connection between  $\xi$  and  $\tau$ , and the values of  $\xi$  are easy to figure, since we know the values  $a = \frac{\lambda}{C_p \gamma} = a(t)$ , and  $(\tau)$ . All the experiments conducted with graphite, and in addition those with steel, showed that the rate of function  $\Phi$  with respect to argument  $\xi$  is a constant value in the whole range of values of argument  $\xi$  corresponding to the duration of the experiment.

Consequently  $\Phi = \Phi(\xi)$  is the exponent. The constancy of rate  $m_\Phi$  with respect to the argument  $\xi$  enables us to determine uniquely the

coefficient of heat emission  $\alpha$ .

To do this we can use any instant of the experiment since rate  $m_{\phi} = \text{const.}$  This corresponds to the physical picture of heat exchange occurring at a constant rate and temperature of the air bathing the calorimeter.

Thus the propositions of the theory are completely confirmed for all the experiments which were conducted at an Re from 1,000 to 50,000.

The theory was confirmed also by experiments with 3 Steel a cylinder ( $\phi$  50 mm, 400 mm in length). But for steel a severe instability in rate  $m_{\phi}$  is discovered only at high values of Re. i.e., at high values of  $\alpha$  as a result of high thermal conductivity and its lesser dependence on temperature than that of graphite.

In addition to this, the experiments confirmed one more proposition of the theory. In the theory of regular temperature regime it is demonstrated that the rate does not depend on the coordinate. This property of the rate is included in the generalized theory for rate  $m_{\phi}$ .

The experiments corroborate the uniformity of the rate at any point in the body. The changes registered by the four thermocouples, two on the surface and two inside the calorimeter, showed that rate  $m_{\phi}$  is the same at all points. This determines the average coefficient of heat emission with respect to the surface.

### Conclusions

1. By introducing a new thermodynamic potential  $\phi$  in place of temperature  $t$  and the integral argument  $\xi$  in place of time  $\tau$ , linearization of the differential equation of heat conductivity is accomplished.

2. Based on the linearized heat conductivity equation the generalization of the theory of regular heat regime is accomplished for

the case of variable coefficients of heat conductivity and specific heat.

3. Based on the analysis carried out an expression for rate  $m_0$  has been derived which makes allowances for the variability of the thermophysical properties of the material during a non-steady process.

4. The general concept of a variability factor of a temperature field with variable coefficients of heat conductivity and specific heat has been established.

5. The establishment of a unique relation between cooling rate  $m_0$  and coefficient of heat emission  $\alpha$  by the generalized variability factor of the temperature field has enabled us with the help of relations (31) and (39) to map out ways to use  $\alpha$ -calorimeters developed by G. M. Kondrat'yev with  $\lambda = \text{const}$  and  $C_p = \text{const}$ , in the general case where these coefficients are variable.

6. The experimental verification of the basic propositions of the theory has given a positive result. Experiments conducted on specimens of graphite and 3 Steel showed that the generalized theory of regular heat regime can be recommended as a reliable, method of calculation in the most varied problems of thermal physics.

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REGULAR HEAT REGIME IN BODIES WITH INTERNAL SOURCES OF  
ENERGY AND VARIABLE THERMOPHYSICAL PROPERTIES

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The subject of the present article is an examination of heat emission by a body inside of which a source of energy is active.

An extraordinary number of examples may be adduced since processes like this are very widely used in technology.

During work heat is given off by electrical conductors, cables, busbars, electric heating apparatus, various radio components, electrical devices, nuclear reactors, etc. It is highly important to study the heat regime of such systems, since it is directly connected with service life, reliability, and often, operational safety.

Thermal calculations of the work regimes of different electrical devices, measuring instruments, metal treating processes, etc., have lately been attracting more and more attention and they undoubtedly will occupy the place due them along with specific calculations, as, for example, those of strength, and electrical calculations.

A number of works, of which those of G. M. Kondrat'ev and G. N. Dul'nev [1, 2] are to be acknowledged the most complete, have been

devoted to the matter of investigating the heat regime of systems with internal sources of heat.

In these works, however, the analysis has been based on a Fourier equation with constant coefficients, i.e., as in the theory of regular temperature regime, it is considered that heat conductivity and specific heat are constants. It is understandable that quite a number of features associated with the dependence of  $\lambda$  and  $C_p$  on temperature cannot be explained here. With this equation as a basis we can derive regularity only in respect to temperature, which, as has already been noted by the authors in a previous article [3], cannot exhaustively characterize the process in the case of variable thermophysical properties.

An attempt to generalize the theory of heat regularity for the case of variable coefficients of heat conductivity and specific heat has not been made.

Experiments based on temperature regularity, as in the case of heat exchange without heat sources, show that rate  $m_v$  is a value variable in time and unable to be taken as a basis for determining the value of heat emission.

Proceeding from the authors' methods of linearizing a non-linear differential heat conductivity equation by introducing a new function  $\psi = \int_0^t \frac{\lambda}{C_p} dt$  in place of the temperature, and the argument  $\xi = \int_0^t adt$  in place of the time, we may propose a generalization of the theory of regular heat regime for that more complex case, too.

In a previous article the authors showed that, when function  $\psi$  and argument  $\xi$  are introduced, the heat conductivity equation becomes linear, transforming into the following:

$$\frac{\partial \psi}{\partial \xi} = \nabla^2 \psi + q_r, \quad (1)$$

where  $q_v$  is the internal source of heat. We can show that Eq. (1) possesses the property of regularity if the change in function  $\Phi$  follows a change in intensity of heat source  $q_v$ .

In other words, if this equality holds:

$$q_v = \beta \Phi_v, \quad (2)$$

then Eq. (1) is regular.

The coefficient of proportionality  $\beta$  is only a function of the coordinates and cannot include within itself the thermophysical constants which enter function  $\Phi$ . A similar connection holds also for temperature, but only in case the thermophysical properties are invariable. But in case of variable thermophysical properties the coefficient of proportionality will no longer be a constant but will depend on the temperature.

In this case a change in the temperature will not follow a change in heat source intensity according to law (2).

We thus see that the linearization of the heat conductivity equation which we have carried out has enabled us to realize relation (2) also for the case of variable thermophysical properties.

Let us divide Eq. (1) by  $\Phi_v$ .

We obtain

$$\frac{1}{\Phi_v} \left( \frac{\partial \Phi}{\partial \xi} \right)_v = -m_\Phi + \frac{q_v}{\Phi_v}. \quad (3)$$

Here an equation with average values has been examined.

Then integrating Eq. (3) we have

$$\ln \Phi_v = -m_\Phi \xi + \int \frac{q_v}{\Phi_v} d\xi + \text{const.} \quad (4)$$

If the condition (4 - 2) is fulfilled, then

$$\int \frac{q_v}{\Phi_v} d\xi = \beta \xi,$$

and therefore Eq. (4) takes on the form:

$$\ln \Phi_V = -m_\phi \xi + \beta \xi + \text{const.} \quad (5)$$

Letting

$$m_\phi - \beta = m', \quad (6)$$

we obtain

$$\ln \Phi = -m' \xi + \text{const.} \quad (7)$$

From Eq. (7) it is seen that to determine  $m'$  we use the usual procedure

$$m' = \frac{\ln \Phi_2 - \ln \Phi_1}{\xi_2 - \xi_1}. \quad (8)$$

Here again the rate with respect to time  $\tau$  and the rate with respect to argument  $\xi$  are connected just as the rates without heat sources (3).

The analysis which we have carried out, in our opinion, differs advantageously from the work of other authors in that it enables us to present very clearly the role of the heat source in the general thermal process.

This is easily demonstrable on the basis of the two expressions (6) and (7).

Actually, if  $\beta = 0$  the regular heat regime without a heat source holds true. If  $\beta = m$ , then that corresponds to a steady regime. There is no regularity and the temperature remains invariable, since the heat source completely covers the heat losses.

Intermediate cases where  $0 < \beta < m$  correspond to different regimes of regular cooling.

During the transition to  $\beta > m$  is accomplished the transition to a regular regime of heating the body.

We thus see that in respect to rates we use the principle of

superposition in full measure. This is entirely understandable, since the mechanism for transmitting heat within the body is the same both for the case with a heat source and the case without one.

We should particularly examine the case where  $m = 0$ . This corresponds to the state of the body in ideal insulation, i.e., when  $\alpha_w = 0$ .

In this case a regularity will prevail, but one of a special sort.

When  $\beta > 0$  heating will proceed, but when  $\beta < 0$  there will be cooling of the body. The rate of heating or cooling will be determined only by the intensity of the source (or of the outflow) and by the thermophysical properties of the body.

### Conclusions

1. By introducing the integral function  $\psi = \int_0^t \frac{1}{C_p} di$  and the integral argument  $\xi = \int_0^t ad\tau$  we obtain a linearized equation of heat conductivity with sources of heat.

2. We have demonstrated the property of regularity in the derived equation and derived an expression for the rate of function  $\Phi$  in respect to argument  $\xi$  when an internal source of heat is active.

3. The different cases or correlation of heat source and heat losses through the surface of the surface of the body have been clearly analyzed.

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GENERALIZATION OF G.M. KONDRAT'EV'S THEOREM  
FOR THE CASE OF VARIABLE COEFFICIENTS OF HEAT  
CONDUCTIVITY AND SPECIFIC HEAT AND THE USE OF  
THIS GENERALIZED THEOREM TO DETERMINE THERMO-  
PHYSICAL PROPERTIES OF MATERIALS

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The theorem of G. M. Kondrat'ev occupies a central position in the theory of regular temperature regime. This theory formulates the basic properties of rate of temperature change of a body in time while assuming the stability of its thermophysical properties, namely:

1. The rate of regular cooling of a homogeneous and isotropic body  $m_v$  with finite value of coefficient of heat emission  $\alpha_w$  is proportional to the surface of the body and inversely proportional to the specific heat of the body.

The coefficient of proportionality is the product of  $\alpha_w$  by the criterion  $\psi$ , monotonically decreasing as  $\alpha_w$  increases and appearing as a function of Biot's criterion or of the numbers  $\frac{L_0 \cdot \alpha_w}{\lambda}$ ;  $\frac{L_1 \cdot \alpha_w}{\lambda}$ , and of the shape of the body.

2. The limiting value of rate  $m_{v_\infty} = m_v|_{\alpha_w \rightarrow \infty}$  and the temperature conductivity  $a$  of the material are proportional:  $a = k \cdot m_{v_\infty}$ , the coefficient of proportionality  $k$  being a purely geometrical value, depending on the size and shape of the body [1].

G. M. Kondrat'ev's theorem opens the way for creating methods

of determining the coefficients of heat emission and of the thermophysical properties.

This theorem, however, has been proven only for the case  $\lambda = \text{const}$  and  $C_p = \text{const}$ . No attempts have been made to generalize Kondrat'ev's theorem for the case of variable thermophysical properties.

The authors have shown in previous works that the introduction of the new thermodynamic potential  $\Phi = \int_0^i \frac{\lambda}{C_p} di$  and of the new argument  $\xi = \int_0^i \frac{\lambda}{C_p \gamma} d\tau$  linearizes the non-linear Fourier differential equation of heat conductivity [2]. Based on this linearization the theory of heat regularity has been generalized for the case of variable thermophysical properties in the section dealing with the connection of the rate of function  $\Phi$  with the heat-emission conditions on the surface of the body. The relation between rate  $m_f$  and the intensity of heat exchange and a similar relation of  $m_v$  to  $\alpha_w$ , but already taking the variability of  $\lambda$  and  $C_p$  into consideration, were derived.

This proved in essence the first part of the Kondrat'ev theorem for the case of variable thermophysical properties.

It is now necessary to generalize the second section of the Kondrat'ev theorem concerning the limiting value of the rate when the coefficient of heat emission on the surface of the body is infinite in value.

Let us examine the physical side of the process which goes on when  $\alpha_w \rightarrow \infty$ . As is known, in this limiting case the rate of cooling will be determined only by the rapidity of heat transmission in the body, i.e., by the magnitude of the coefficient of temperature conductivity  $a = \frac{\lambda}{C_p \gamma}$ .

The problem becomes a purely internal one. If  $\lambda$  and  $C_p$  are constant, then rate  $m_v|_{\alpha_w \rightarrow \infty}$  will also have a finite value determined

only by the magnitude of  $\underline{a}$  and the geometrical characteristics of the body. In the case where  $\lambda$  and  $C_p$  are also variable we should also expect that the sense of the second section of Kondrat'ev's theorem be maintained, since in this case, too, the rate will be determined only by the thermophysical properties of the material, which, notwithstanding the variability, are unable, nevertheless, to take on infinite values.

Let us turn to the heat-exchange equation in its linearized form:

$$-\left(\frac{\partial \phi}{\partial n}\right)_w = \zeta \cdot \phi_w. \quad (1)$$

Or

$$-\frac{\left(\frac{\partial \phi}{\partial n}\right)_w}{\phi_w} = \zeta. \quad (2)$$

Reasoning along the lines of Kondrat'ev's proof [1] we may notice that the magnitude  $\left(\frac{\partial \phi}{\partial n}\right)_w$ , numerically equal to the flow of heat through the surface, cannot equal zero or infinity when  $\alpha \rightarrow \infty$ .

When  $\left(\frac{\partial \phi}{\partial n}\right)_w = 0$  the very process of heat exchange between the body and the surrounding medium would be lacking, and this is incompatible with the examined case where  $\alpha_w \rightarrow \infty$ . An infinite value for  $\left(\frac{\partial \phi}{\partial n}\right)_w$  cannot occur either, for that would mean the presence of a step in function  $\phi$  near the surface of the body, which is impossible by virtue of the homogeneity of the body and the finite value of its thermophysical properties, which under no conditions take on infinite values, notwithstanding their variability in the process.

Consequently, we come anew to the conclusion that rate  $m_\phi$  has a finite value when  $\alpha_w \rightarrow \infty$ .

Further, for greater concreteness in exposition let us turn to the example of heat exchange of an unbounded wall.

Let us examine the solution for the linearized heat conductivity equation for this case:

$$\phi = \phi_w \cdot \frac{4}{\pi} \left\{ e^{-\left(\frac{\pi}{2}\right)^2 \frac{\xi}{N^2}} \cdot \cos\left(\frac{\pi}{2} \frac{x}{N}\right) - \frac{1}{3} e^{-\left(\frac{3}{2}\right)^2 \frac{\xi}{N^2}} \cdot \cos\left(\frac{3}{2} \pi \frac{x}{N}\right) + \dots \right\} \quad (3)$$

In the phase of the regular regime with respect to function  $\phi$  we have

$$\phi = \phi_w \cdot \frac{4}{\pi} e^{-\left(\frac{\pi}{2}\right)^2 \frac{\xi}{N^2}} \cdot \cos\left(\frac{\pi}{2} \frac{x}{N}\right). \quad (4)$$

From eq. (4) it is seen that the desired limiting rate is equal to

$$m_\phi|_{\alpha_w \rightarrow \infty} = \left(\frac{\pi}{2N}\right)^2 = \text{const.} \quad (5)$$

The equality (5) is highly interesting. The fact is that the value of  $\left(\frac{\pi}{2N}\right)^2 = \frac{1}{K}$ , where the factor  $K$  is the so-called coefficient of the shape of the body, first introduced by G. M. Kondrat'ev [1] when proving the theorem of the limiting value of rate  $m_w|_{\alpha_w \rightarrow \infty}$ .

Passing from the particular case of cooling a wall to the general law for any bodies, we obtain

$$m_\phi|_{\alpha_w \rightarrow \infty} \cdot k = 1. \quad (6)$$

Thus, in the case where  $\alpha_w \rightarrow \infty$  the value of the limiting rate of change of function  $\phi$  with respect to argument  $\xi$  is equal to a magnitude which is inverse to the coefficient of shape of the given body, independent of the properties of the material itself. This value is asymptotic for the rates of  $m_\phi = m_\phi(\alpha_w)$  for bodies of the given shape made of any material.

The expression (6) shows that the rate of  $m_\phi$  may change within the limits zero (steady state) to  $\frac{1}{K}$  for a given body shape. On the basis of the given property we may stipulate beforehand the expected

value of the rate and in the best way choose the apparatus for recording it, or, taking a body of another shape and size, we may pre-determine the value of the rate so as to attain the best results in the experiments.

The generalization of the second section of Kondrat'ev's theorem, which we have carried out for the case of variable thermophysical properties, has a very important practical signification. When determining the coefficient of temperature conductivity in accordance with Kondrat'ev's method  $a = k \cdot m_v |_{\alpha_w \rightarrow \infty}$ , the assumption is that  $\lambda = \text{const}$  and  $C_p = \text{const}$ . Since indeed this is incorrect, the value of  $a$  to be determined is some average value for the temperature range which is taken to determine the rate  $m_v |_{\alpha_w \rightarrow \infty}$ .

Generalizing the Kondrat'ev theorem enables us to show the new, practically important property of temperature change in time when  $\alpha_w \rightarrow \infty$ .

From the calculations in (6) we have

$$\frac{1}{\phi} \frac{\partial \phi}{\partial \xi} = -\frac{1}{K}. \quad (7)$$

Starting from the definition of function  $\phi$  and using the value theorem, we can write

$$\phi = \int_0^t \frac{\lambda}{C_p} di = \int_0^t \gamma a di = \gamma \bar{a} i, \quad (8)$$

where  $a = a(i)$ . Expression (7) may be written otherwise

$$\frac{d\phi}{\phi} = -\frac{1}{K} d\xi, \quad (9)$$

and from it, substituting (8), we may obtain

$$\frac{\partial \phi}{\phi} = \frac{\gamma \cdot a \cdot di}{\gamma \cdot \bar{a} \cdot i} = -\frac{1}{K} d\xi. \quad (10)$$

Since the average and the true values of  $a(i)$  change equally, we may write

$$\frac{di}{i} = -\frac{1}{K} d\xi. \quad (11)$$

Further, by virtue of defining  $\tau = \int_0^t a d\tau$ :

$$\frac{dl}{l} = -\frac{1}{K} a d\tau. \quad (12)$$

But, according to definition,  $l = C_p \cdot t$  and  $dl = C_p \cdot dt$ ; and therefore we will finally get

$$m_{v_{\alpha_w \rightarrow \infty}} = \frac{1}{l} \cdot \frac{\partial l}{\partial \tau} = -\frac{a}{K}. \quad (13)$$

Formula (13) thus shows that in the case of variable thermophysical properties the limiting rate of temperature change in time alters as the coefficient of temperature conductivity  $a = \frac{\lambda}{C_p \gamma}$ .

Detection of this very important property became possible only on the basis of the linearization and generalization of the Kondrat'ev theorem for the case of variable thermophysical properties.

The practical value of expression (13) is obvious since it turns out that law of change of  $m_{v_{\infty}}$  is the same as for the coefficient of temperature conductivity.

The method of conducting the experiments remains the same as in the theory of temperature regularity, i.e., a body of the material to be investigated with a known coefficient of shape  $K$  is cooled under conditions as close as possible to the condition of  $\alpha_w \rightarrow \infty$ , and the relation  $t = t(\tau)$  is recorded.

Now, however, on the basis of expression (13) the results of such an experiment enable us to determine even the course of the change in temperature conductivity  $a = a(t)$  and not the unique value of  $a$ , as in the theory of temperature regularity.

The property of rate  $m_{v_{\infty}}$  expressed in relation (13) substantially raises the value of the methods of the regular regime in problems of determining the thermophysical properties of materials.

The generalization of the Kondrat'ev theorem has made it possible

once more to become convinced that the assumption of  $\lambda = \text{const}$  in a number of cases not only leads to considerable quantitative errors in the calculations, but also hides important qualitative aspects of the processes.

It is easy to be convinced that the theorem proved by Kondrat'ev for the case where of  $\lambda = \text{const}$  and  $C_p = \text{const}$  is a particular case of the generalized theorem.

Really, taking  $\lambda = \text{const}$  and  $C_p = \text{const}$  in Formula (13), we obtain the relationship first derived by G. M. Kondrat'ev [1].

A considerable place is given in the theory of regular temperature regime to tests of materials according to calorimeter method "d". In this method, as is easy to see, the accuracy of the results depends to a certain degree on fulfilling the condition  $\alpha_w \rightarrow \infty$ . The question arises whether one cannot propose a method of determining the coefficient of temperature conductivity in a material based on the theory of heat regularity, but not necessarily demanding the creation of special conditions where  $\alpha \rightarrow \infty$ .

The theory of regular temperature regime gives no answer to this question.

It turns out that one can answer this question by basing himself on the generalized theory of heat regularity.

The problem consists in finding a connection between the rate of change of the excess temperature in time and the coefficient of temperature conductivity for any value of the coefficient of heat emission.

We will note the rate of function  $\Phi$  with respect to argument  $\xi$

$$\frac{1}{\Phi} \cdot \frac{\partial \Phi}{\partial \xi} = -m_\Phi.$$

As is known,  $m_\Phi = \text{const}$  for each fixed value of heat emission by virtue of the linearity of the equation

$$\frac{\partial \Phi}{\partial \xi} = \gamma^2 \Phi.$$

Further, by analogy with (10) we may write

$$\frac{d\Phi}{\Phi} = \frac{di}{i} = -m_{\Phi} \cdot d\tau. \quad (14)$$

By virtue of the definition:  $d\xi = a \cdot d\tau$ , and also  $i = C_p t$ ;  
 $di = C_p dt$ . Let us substitute these expressions in (14) and we will finally obtain

$$m_{\Phi} = -\frac{1}{t} \cdot \frac{\partial t}{\partial \tau} = m_{\Phi} a. \quad (15)$$

Here  $m_{\Phi}$  is a constant under the conditions of each experiment, so we see that a relation of type (13) is preserved also for the cases where  $\alpha_w \neq \infty$ .

Rate  $m_{\Phi}$ , appearing here as the coefficient of proportionality, changes, as has already been indicated, within the limits zero to  $\frac{1}{K}$ , depending on  $\alpha_w$ . This change for the different materials is represented by a family of curves with a common beginning ( $m_{\Phi} = \frac{1}{K}$  as  $\alpha_w \rightarrow \infty$ ).

In these two limiting states the internal thermophysical properties of the materials have no influence on the magnitude of rate  $m_{\Phi}$ , as was demonstrated above.

But in the range  $0 < \alpha_w < \infty$  each body has its own rate  $m_{\Phi}$ , determined by the thermophysical properties.

We should remember that from formulas (13) and (15) the coefficient of temperature conductivity is derived as a function of the time, so that for the same material, but for different  $\alpha_w$  (different  $m_{\Phi}$ ), different  $a(\tau)$  will be obtained. The transition itself to the relationship  $a = a(t)$  is accomplished by means of the curve  $t = t(\tau)$  derived from the experiment and must give the same result for the different experiments.

Such are the new qualitative aspects of the theorem of G. M. Kondrat'ev uncovered on generalizing the theory of regular heat regime

for the case of variable thermophysical properties of the materials.

### Conclusions

1. On the basis of the theory of heat regularity generalized for the case of variable thermophysical properties it has been demonstrated that in the case of variable coefficients of heat conductivity and specific heat the rate of function  $\Phi$  with respect to argument  $\xi$  when  $\alpha_w \rightarrow \infty$  has a finite value depending only on the shape of the body.

2. On the basis of G. M. Kondrat'ev's generalized theory it has been ascertained that when  $\lambda$  and  $C_p$  are variable the rate of excess temperature changes in time as the temperature conductivity of the calorimeter material  $a(\tau)$ .

3. The Kondrat'ev theory has been generalized not only for the case of  $\alpha_w \rightarrow \infty$  but also for the case of any value of the coefficient of heat emission.

Kondrat'ev's theorem, generalized for the case of variable coefficients of heat conductivity and specific heat, and also for the case of random  $\alpha_w$ , enables us more effectively than with the theory of temperature regularity to study the thermophysical properties of material, in particular, to determine the coefficient of temperature conductivity  $a(t)$ .

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| ARL (ARB)             | 1          | BSD (BSF)          | 1          |
|                       |            | AFFTC (FTY)        | 1          |
| OTHER AGENCIES        |            |                    |            |
| CIA                   | 1          |                    |            |
| NSA                   | 6          |                    |            |
| AID                   | 2          |                    |            |
| OTS                   | 2          |                    |            |
| AEC                   | 2          |                    |            |
| PWS                   | 1          |                    |            |
| NASA                  | 1          |                    |            |
| RAND                  | 1          |                    |            |