

UNCLASSIFIED

AD 296 452

*Reproduced
by the*

**ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA**



UNCLASSIFIED

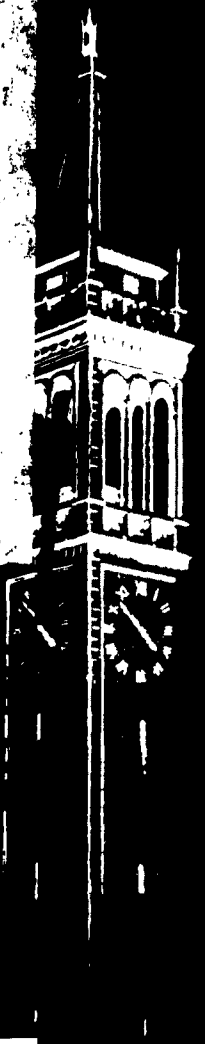
NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

63-2-4

296 452

CATALOGED BY UJIA

296 452



Waves in Drifting and Accelerating Electron Streams In Radial Flow

by

W. B. Bridges

C. K. Birdsall

Series No. 60, Issue No. 459

Contract No. AF 49(638)-102

June 29, 1962

ELECTRONICS RESEARCH LABORATORY

UNIVERSITY OF CALIFORNIA

BERKELEY, CALIFORNIA

**"Qualified requestors may obtain
copies of this report from ASTIA"**

**Electronics Research Laboratory
University of California
Berkeley, California**

**WAVES IN DRIFTING AND ACCELERATING ELECTRON STREAMS
IN RADIAL FLOW**

by

**W. B. Bridges
C. K. Birdsall**

**Institute of Engineering Research
Series No. 60, Issue No. 459**

**Physics Division
Air Force Office of Scientific Research
Contract No. AF 49(638)-102
Division File No. 13-25-J**

June 29, 1962

TABLE OF CONTENTS

	Page
I. INTRODUCTION	1
II. BASIC PROBLEM.	1
III. GOVERNING EQUATIONS, NEW VARIABLES.	1
IV. KINETIC POWER; KINETIC ENERGY; CONSERVATION THEOREM	7
V. CIRCUIT EQUATION; OPEN-CIRCUIT.	8
VI. OPEN-CIRCUIT SOLUTIONS FOR $v_o(r) \sim r^m$	9
VII. REDUCTION TO PLANAR DRIFTING FLOW; $n=0$, $m=0$	13
VIII. FASTER AND SLOWER SPACE-CHARGE WAVE AMPLITUDES.	14
IX. SOLUTIONS FOR $3m+n=2$	17
X. EXAMPLE OF CYLINDRICAL DRIFTING FLOW $n=1, m=0$	21
REFERENCES	28

I. INTRODUCTION

The solutions for space-charge waves obtained by Hahn (1939) and Ramo (1939) for a rectilinear, drifting stream are quite well known. Less well known are the somewhat more mathematically complicated solutions obtained when the stream flow is allowed to be along radial lines and when the stream is accelerating. First-order current for radial drifting flow has been obtained by Feenberg (1946). Current and velocity in accelerating parallel flow have been obtained by Llewellyn (1941), by Smullin (1951), Tien (1952), Hutter (1954), Müller (1955), Higuchi (1956), and Eichenbaum and Peter (1959). Accelerating radial flow was looked into by Quate (1952). The present work attempts to unify all of these cases and a few more in one general solution.

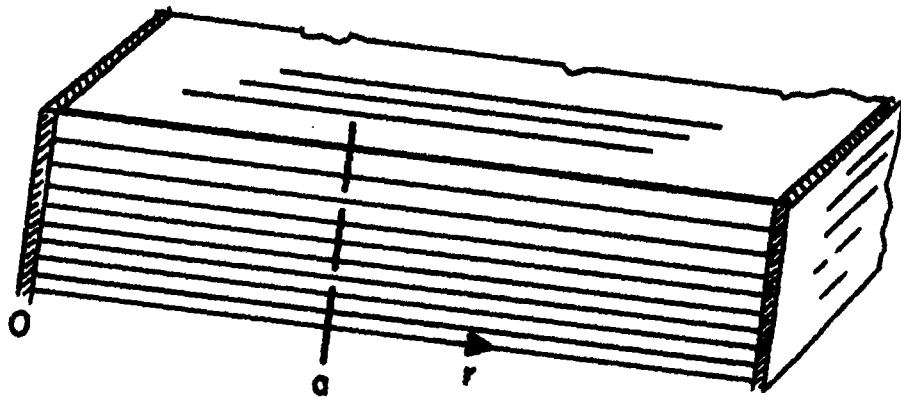
II. BASIC PROBLEM

As with parallel flow, it is desirable to know how an initial modulation is carried by the stream from one point to another. The models to be analyzed are shown in Fig. 1. The mode with motion along radius r only and with no variations of any variable normal to r will be studied. The solutions will be for $i_1(r, t)$, $v_1(r, t)$, $\rho_1(r, t)$, and $E_1(r, t)$ for all r and t .

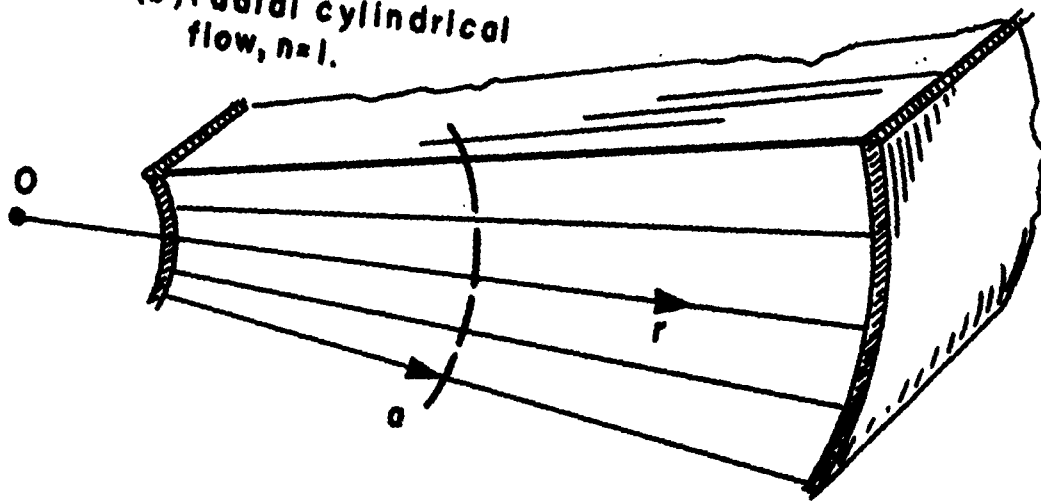
III. GOVERNING EQUATIONS, NEW VARIABLES

The linearization used in parallel flow is applicable. Eulerian hydrodynamical equations will be used. Assuming a driving signal sinusoidal with time, the zero-order equation (dc) and the first-order (ac) equations may be separated. The dc potential V_0 , and hence the dc velocity v_0 , is allowed to vary with r . The velocity variation $v_0(r)$

(a) radial planar flow, $n=0$.



(b) radial cylindrical flow, $n=1$.



(c) radial spherical flow, $n=2$.

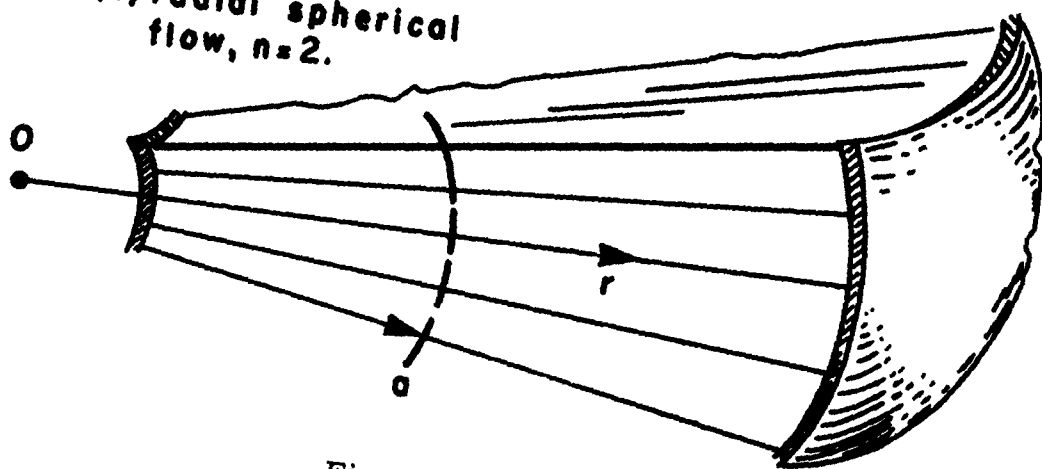


Figure 1

will be left arbitrary, to be specified later to suit specific problems. The state of the background is hidden in v_0 in that the presence of ions or electrodes, or just average electron space charge is implied by the choice of $v_0(r)$. Because the flow expands (or contracts) i_0 and ρ_0 also depend on r .

The zero-order equations are: the r^{th} component of the equations of motion,

$$\frac{dv_0}{dt} = v_0 \frac{dv_0(r)}{dr} = \eta E_0(r); \quad (1)$$

the definition of current

$$i_0 = \rho_0 v_0; \quad (2)$$

and the continuity equation, for no variations normal to r ,

$$\nabla \cdot i_0 = \frac{1}{r^n} \frac{d}{dr} (r^n i_0) = 0. \quad (3)$$

Here $n=0,1,2$ for planar, cylindrical and spherical flow respectively.

The continuity equation may be integrated to obtain

$$i_0(r) = \left(\frac{a}{r}\right)^n i_0(a) = \left(\frac{a}{r}\right)^n i_{0a}, \quad (4)$$

where a is some conveniently chosen radius, such as the radius of the input. Quantities at radius a will be denoted by the additional subscript a . Similarly

$$\rho_0(r) = \frac{i_{0a}}{v_0(r)} \left(\frac{a}{r}\right)^n \quad (5)$$

The first-order equation of motion is just the r component equation,

$$\frac{dv_1}{dt} = \eta E_1 \quad (6)$$

The definition of current is

$$i_1 = \rho_0 v_1 + \rho_1 v_0 \quad (7)$$

The continuity equation is

$$\nabla \cdot \vec{i}_1 + \frac{\partial \rho_1}{\partial t} = 0, \quad (8)$$

which is simply

$$\frac{1}{r^n} \frac{\partial}{\partial r} (r^n i_1) + \frac{\partial \rho_1}{\partial t} = 0 \quad (9)$$

Several simplifications may be made at this stage in order to reduce the algebraic work. The reader will recall that in planar flow the stream (or mean) phase,

$$\bar{\phi}_e = \omega \left(t - \frac{r}{v_0} \right) \quad (10)$$

occurs in the solutions for i_1 , v_1 , ρ_1 , and E_1 . Thus, we anticipate that it will also occur here, and recognize $\bar{\phi}_e$ as

$$\bar{\phi}_e = \omega (t - T) \quad (11)$$

where T is the zero-order transit time from the source to the point r . T is $T(r)$ where v_0 changes with distance, as

$$T(r) = \int_{\text{source}}^r \frac{dr}{v_0(r)}, \quad (12)$$

and

$$\frac{dT}{dr} = \frac{1}{v_0(r)}. \quad (13)$$

Inserting the phase and a judicious choice on first-order quantities, the new variables are chosen to be:

$$i(r, t) = i_0(r) + i_1(r, t) = i_0(r) + r^{-n} \hat{i}_1(r) e^{j\omega [t - T(r)]}, \quad (14)$$

$$v(r, t) = v_0(r) + v_1(r, t) = v_0(r) + \hat{v}_1(r) e^{j\omega [t - T(r)]}, \quad (15)$$

$$\rho(r, t) = \rho_0(r) + \rho_1(r, t) = \rho_0(r) + r^{-n} \hat{\rho}_1(r) e^{j\omega [t - T(r)]}, \quad (16)$$

$$E(r, t) = E_o(r) + E_1(r, t) = E_o(r) + r^{-n} \hat{E}_1(r) e^{j\omega [t - T(r)]}, \quad (17)$$

The units of the new variables are not fixed; for example, \hat{i}_1 , has the units of amps/m² for n=0, amps/m for n=1 (to be interpreted as amps per meter length/2 π along z, the cylindrical axis) and amps for n=2 (to be interpreted as the total current/(4 π)).

Using the new variables in the first-order equations, one obtains, with $\exp j\omega(t-T)$ understood,

$$v_o \frac{d\hat{v}_1}{dr} + \hat{v}_1 \frac{dv_o}{dr} = \eta r^{-n} \hat{E}_1 \quad (18)$$

Note that the $\partial v_1/\partial t$ term was cancelled by part of the $(v_o + v_1)d(v_o + v_1)/dr$ term, the dT/dr part, a saving afforded by insertion of the mean phase.

The remaining equations are

$$\frac{d\hat{i}_1}{dr} - j \frac{\omega}{v_o} \hat{i}_1 + j\omega \hat{\rho}_1 = 0, \quad (19)$$

$$\hat{i}_1 = r^n \rho_o \hat{v}_1 + v_o \hat{\rho}_1. \quad (20)$$

Next, the equation in \hat{i}_1 and \hat{E}_1 is to be obtained. Insert $\hat{\rho}_1$ from (20) into (19) and insert ρ_o from (5) to obtain

$$\hat{v}_1 = \frac{v_o^2}{j\omega a^n i_{oa}} \frac{d\hat{i}_1}{dr} \quad (21)$$

Differentiate this to obtain

$$\frac{d\hat{v}_1}{dr} = \frac{1}{j\omega a^n i_{oa}} \left[\left(2v_o \frac{dv_o}{dr} \frac{d\hat{i}_1}{dr} \right) + v_o^2 \frac{d^2 \hat{i}_1}{dr^2} \right] \quad (22)$$

Insert \hat{v}_1 from (19) and $\frac{d\hat{v}_1}{dr}$ from (22) into (18) to obtain the desired equation in \hat{i}_1 and \hat{E}_1

$$\frac{d^2 \hat{i}_1}{dr^2} + \left(\frac{3}{v_o} \frac{dv_o}{dr} \right) \frac{d\hat{i}_1}{dr} = \frac{\eta i_{oa}}{\epsilon_o v_o^3} \left(\frac{a}{r} \right)^n j\omega \epsilon_o E_1 \quad (23)$$

The right hand side can be simplified by recognizing that the multiplier of the displacement current is the local plasma wave number (squared) as,

$$\frac{\eta_{oa}^i}{\epsilon_o v_o^3} \left(\frac{a}{r}\right)^n = \left(\frac{\eta \rho_o(r)}{\epsilon_o}\right) \frac{1}{v_o^2(r)} = \frac{\omega_p^2(r)}{v_o^2(r)} = \beta_p^2(r) \quad (24)$$

Hence (23) is

$$\frac{d^2 \hat{i}_1}{dr^2} + \left(\frac{3}{v_o} \frac{dv_o}{dr}\right) \frac{d\hat{i}_1}{dr} = \beta_p^2(r) (j\omega \epsilon_o \hat{E}_1) \quad (25)$$

or, if one likes, it is

$$\frac{d^2 \hat{i}_1}{dT^2} + \left(\frac{3}{v_o} \frac{dv_o}{dT}\right) \frac{d\hat{i}_1}{dT} = \omega_p^2(r) (j\omega \epsilon_o \hat{E}_1) \quad (26)$$

The last of the trio of field-driven equations to be obtained is the equation relating $\hat{\rho}_1$ and \hat{E}_1 . The steps in derivation are, roughly: take v_o^2 times the definition of current, take d/dr of this, expand the derivative of $\hat{i}_1 v_o^2$, insert $d\hat{i}_1/dr$ from earlier work and \hat{i}_1 from the definition of current; insert this result into the $d(\hat{\rho}_1 v_o^3)/dr$ equation, factor out $\hat{v}_1 v_o$ and take d/dr of this, then insert $\eta r^{-n} \hat{E}_1$ for this derivative. The result is

$$\begin{aligned} & \frac{d^2(\hat{\rho}_1 v_o)}{dr^2} + \frac{d(\hat{\rho}_1 v_o)}{dr} \left\{ \frac{2}{v_o} \frac{dv_o}{dr} + i_{oa} a^n \left[\frac{1}{v_o^2} \frac{d^2 v_o}{dr^2} \left(\frac{j\omega}{v_o} + \frac{2}{v_o} \frac{dv_o}{dr} \right) \right] \right\} \\ & = \frac{\eta}{r^n} \hat{E}_1 \left[\frac{n}{r} i_{oa} a^n + \frac{1}{v_o} \frac{dv_o}{dr} + \left(\frac{-\omega^2}{v_o^2} + 2j \frac{\omega}{v_o} \frac{dv_o}{dr} + \frac{2}{v_o} \frac{d^2 v_o}{dr^2} \right) \left(\frac{j\omega}{v_o} + \frac{2}{v_o} \frac{dv_o}{dr} \right) \right] \\ & \quad - \eta i_{oa} \left(\frac{a}{r}\right)^n \frac{d\hat{E}_1}{dr} \quad (27) \end{aligned}$$

Equations (18), (19), and (27) may be used with any form of driving field.

IV. KINETIC POWER; KINETIC ENERGY; CONSERVATION THEOREM

A simple power theorem is readily derived for the case of radial flow. The equation of motion is

$$\frac{\partial v_1}{\partial t} + v_o \frac{\partial v_1}{\partial r} + v_1 \frac{\partial v_o}{\partial r} = \eta E_1$$

or

$$\frac{\partial v_1}{\partial t} + \frac{\partial}{\partial r} (v_1 v_o) = \eta E_1$$

Multiply both sides by $(r^{n_{i_1}})/\eta$ and add and subtract $\frac{v_1 v_o}{\eta} \frac{\partial}{\partial r} (r^{n_{i_1}})$ to give

$$\frac{r^{n_{i_1}}}{\eta} \frac{\partial v_1}{\partial t} + \frac{r^{n_{i_1}}}{\eta} \frac{\partial (v_1 v_o)}{\partial r} + \frac{v_1 v_o}{\eta} \frac{\partial}{\partial r} (r^{n_{i_1}}) - \frac{v_1 v_o}{\eta} \frac{\partial}{\partial r} (r^{n_{i_1}}) - r^{n_{i_1}} E_1 = 0$$

The second and third terms combine; the fourth term can be given in terms of $\partial \rho_1 / \partial t$ from the equation of continuity and then combined with the $\rho_1 v_o$ part of the first term; i_1 in the last term should be replaced by $J_1(t) - \epsilon_o \partial E_1 / \partial t$ from the total current equation. Divide the result by r^n to obtain

$$\frac{1}{r^n} \frac{\partial}{\partial r} \left[r^n \left(\frac{v_o v_1 i_1}{\eta} \right) \right] + \frac{\partial}{\partial t} \left(\frac{\epsilon_o}{2} E_1^2 + \frac{\rho_o}{2\eta} v_1^2 + \frac{v_o}{\eta} \rho_1 v_1 \right) = J_1 E_1 \quad (1)$$

The terms here may be recognized as the kinetic power density, P_k , the electric energy density, W_E , and the kinetic energy density, W_k , so that (1) may be written as,

$$\nabla_r \cdot P_k + \frac{\partial}{\partial t} (W_E + W_k) = J_1 E_1 \quad (2)$$

The use of the divergence here emphasizes the meaning of conservation, that any net power density increase through a surface must be balanced by a decrease in energy density (for $J_1 = 0$) or be supplied by a source ($J_1 \neq 0$).

V. CIRCUIT EQUATION; OPEN-CIRCUIT

Where there is no ac return path the total current density must vanish, $J_1(t) = 0$. Hence, the open-circuit equation is

$$i_1(r, t) + \frac{\partial E_1(r, t)}{\partial t} = 0 \quad (1)$$

or

$$\hat{i}_1(r) + j\omega \epsilon_0 \hat{E}_1(r) = 0$$

Inserting this value of \hat{E}_1 into the current equation, (III-25) produces the homogeneous equation in \hat{i}_1

$$\frac{d^2 \hat{i}_1}{dr^2} + \left(\frac{3}{v_0} \frac{dv_0}{dr} \right) \frac{d\hat{i}_1}{dr} + \beta_p^2(r) \hat{i}_1 = 0 \quad (2)$$

This is a pleasing result from the point of view of the physics displayed; if the acceleration is mild, so that the middle term is relatively small, then \hat{i}_1 behaves just as in a drifting stream with simple harmonic motion at the local plasma frequency. It is interesting to note that this equation is formally identical with that obtained by Smullin (1951) and Tien (1952) for the case of parallel (planar) flow. The effects of radial flow are buried inside $\beta_p^2(r)$ which depends on n . It is necessary only to solve the equation in \hat{i}_1 because \hat{v}_1 and $\hat{\rho}_1$ are easily obtained from \hat{i}_1 using the earlier equations. It is worthwhile to give the homogeneous equation in velocity,

$$\frac{d^2 \hat{v}_1}{dr^2} + \left(\frac{2}{v_0} \frac{dv_0}{dr} + \frac{n}{r} \right) \frac{d\hat{v}_1}{dr} + \left(\beta_p^2(r) + \frac{n}{rv_0} \frac{dv_0}{dr} + \frac{1}{v_0} \frac{d^2 v_0}{dr^2} \right) \hat{v}_1 = 0, \quad (3)$$

if only to show that the symmetry among the i_1 , v_1 , and ρ_1 equations obtained in planar drifting flow no longer holds.

VI. OPEN CIRCUIT SOLUTIONS FOR $v_o(r) \sim r^m$

The variation of $v_o(r)$ with r must be given in order to solve (V-2) or (V-3). Among the many possible variations of interest, a common one is that for v_o varying as some power of the radius,

$$v_o(r) = \left(\frac{r}{a}\right)^m v_{oa} , \quad (1)$$

with corresponding dc potential

$$V_o(r) = -\frac{v_o^2(r)}{2\eta} = -\frac{1}{2\eta} \left(\frac{r}{a}\right)^{2m} v_{oa}^2 = \left(\frac{r}{a}\right)^{2m} V_{oa} . \quad (2)$$

These variations have been used to solve the planar flow model for $2m = 1$ (linear potential rise) by Tien (1952), and more generally, by Muller (1955). The extension to radial flow is relatively straightforward.

With v_o as given,

$$\frac{1}{v_o} \frac{dv_o}{dr} = \frac{m}{r} ,$$

$$\frac{1}{v_o} \frac{d^2 v_o}{dr^2} = \frac{m(m-1)}{r^2} ,$$

$$\beta_p^2(r) = \beta_{pa}^2 \left(\frac{a}{r}\right)^{3m+n} .$$

(V-2) and (V-3) then become

$$\frac{d^2 \hat{i}_1}{dr^2} + \frac{3m}{r} \frac{d\hat{i}_1}{dr} + \beta_{pa}^2 \left(\frac{a}{r}\right)^{3m+n} \hat{i}_1 = 0, \quad (3)$$

$$\frac{d^2 \hat{v}_1}{dr^2} + \frac{2m+n}{r} \frac{d\hat{v}_1}{dr} + \left[\frac{m(m+n-1)}{r^2} + \beta_{pa}^2 \left(\frac{a}{r}\right)^{3m+n} \right] \hat{v}_1 = 0. \quad (4)$$

These are Bessel equations of a standard form [see, for example, Jahnke and Emde (1945)] and have the solutions

$$\hat{i}_1 = r^{\frac{1-3m}{2}} Z_{\pm b} \left[\pm k \left(\frac{r}{a}\right)^d \right], \quad (5)$$

$$\hat{v}_1 = r^{\frac{1-(2m+n)}{2}} Z_{\pm(b+1)} \left[\pm k \left(\frac{r}{a}\right)^d \right], \quad (6)$$

where Z is a Bessel function of the first or second kind, J or N. The quantities b, d, k are given in terms of m and n by

$$b \equiv \frac{3m-1}{2-(3m+n)} \quad (7)$$

$$d \equiv \frac{2-(3m+n)}{2} \quad (8)$$

$$k \equiv \frac{2\beta_{pa} a}{2-(3m+n)} \quad (9)$$

\hat{i}_1 and \hat{v}_1 of this form satisfy (III-21) provided that like signs are chosen for both \hat{i}_1 and \hat{v}_1 . The (+) sign in the argument should be taken if the stream moves in the positive r-direction (expanding stream flow) and the (-) sign if the stream moves in the negative r-direction (converging stream flow). For given values of m and n, the choice of sign of the order can be made in order to give a positive value for convenience.

It will be convenient to use a normalized radius,

$$x \equiv \frac{r}{a} \quad (10)$$

and a characteristic stream conductance,

$$g(x) \equiv \frac{\omega}{\omega_p(x)} \rho_0(x) \equiv g_a x^{-\left(\frac{n+m}{2}\right)} \quad (11)$$

with the dimensions of current density divided by velocity. The general solution for \hat{i}_1 is a linear combination of functions of the first and second kind as,

$$\hat{i}_1(x) = x^{-bd} \left[AJ_b(kx^d) + BN_b(kx^d) \right] \quad (12)$$

and

$$\frac{d\hat{i}_1}{dx} = -\frac{kd}{a} x^{-bd+d-1} \left[AJ_{b+1}(kx^d) + BN_{b+1}(kx^d) \right] \quad (13)$$

where A and B are arbitrary constants. From (III-21) and (1), (11), and (13) above, one obtains

$$v_1(x) = \frac{x^{\left[\frac{1-(2m+n)}{2}\right]}}{g_a a^n} \left[AJ_{b+1}(kx^d) + BN_{b+1}(kx^d) \right] \quad (14)$$

There are, of course, a variety of ways to specify boundary conditions in order to fix A and B. One way is to specify the current and velocity fluctuation at the initial plane, $i_1(a)$ and $v_1(a)$. This specification gives the solution in the form of a transmission (or "ABCD") matrix, which may be multiplied with similar matrices, allowing a cascaded solution for the more complicated dc potential distributions. One obtains

$$\hat{i}_1(x) = \frac{\pi k}{2} x^{-bd} \left\{ \left[J_{b+1}(k)N_b(kx^d) - N_{b+1}(k)J_b(kx^d) \right] \hat{i}_{1a} + j \left[J_b(k)N_{b+1}(kx^d) - N_b(k)J_{b+1}(kx^d) \right] a^n g_a \hat{v}_{1a} \right\}, \quad (15)$$

$$\hat{v}_1(x) a^n g(x) = \frac{\pi k}{2} x^{-(bd+n)} \left\{ j \left[J_{b+1}(k) N_{b+1}(kx^d) - N_{b+1}(k) J_{b+1}(kx^d) \right] \hat{i}_{1a} \right. \\ \left. - \left[J_b(k) N_{b+1}(kx^d) - N_b(k) J_{b+1}(kx^d) \right] a^n g_a \hat{v}_{1a} \right\}. \quad (16)$$

The bracketed terms of these equations are in the form of the biradial Bessel functions, defined as

$$H_{\mu, \nu}(\xi, \zeta) = \frac{\pi}{2} \left[J_{\mu}(\xi) N_{\nu}(\zeta) - N_{\mu}(\xi) J_{\nu}(\zeta) \right] \quad (17)$$

These and similar functions have been defined and tabulated by various authors and are useful not only as shorthand notation but for their properties of differentiation, integration, recursion, etc. [see Kino, (1955)]. Using the biradial functions in (15) and (16) and transforming back to the laboratory coordinate system, one has

$$\begin{pmatrix} i_1(x, t) \\ g(x) v_1(x, t) \end{pmatrix} = \begin{pmatrix} H_{b+1, b}(k, kx^d) & jH_{b, b}(k, kx^d) \\ jH_{b+1, b+1}(k, kx^d) & -H_{b, b+1}(k, kx^d) \end{pmatrix} \begin{pmatrix} i_{1a} \\ g_a v_{1a} \end{pmatrix} \\ \cdot kx^{d-1/2} \exp j \left[\omega t - \frac{\omega a}{v_{oa}} \frac{x}{1-m} - 1 \right] \quad (18)$$

This equation is most extraordinary in that (for $v_o \sim r^m$) it covers planar, cylindrical, and spherical streaming, drifting or accelerating flow!

VII. REDUCTION TO PLANAR DRIFTING FLOW; $n=0$, $m=0$

For $m=0$, $n=0$, (VI-3) and (VI-4) become the planar space-charge wave equations which have solutions $\cos \beta_p r$ and $\sin \beta_p r$. It is instructive to note that (VI-18) also produces these answers. For $m=0$ and $n=0$,

$$\begin{aligned} b &= -1/2 \\ d &= 1 \\ k &= \beta_{pa} a \\ g(x) &= g_a \end{aligned}$$

The biradial Bessel functions now have half integer orders, which are particularly simple, as

$$\begin{aligned} J_{1/2}(\xi) &= \sqrt{\frac{2}{\pi}} \frac{\sin \xi}{\sqrt{\xi}} = N_{-1/2}(\xi), \\ J_{-1/2}(\xi) &= \sqrt{\frac{2}{\pi}} \frac{\cos \xi}{\sqrt{\xi}} = -N_{1/2}(\xi). \end{aligned}$$

Using these relations, (VI-18) reduces to

$$\begin{pmatrix} i_1(r, t) \\ g_a v_1(r, t) \end{pmatrix} = \begin{pmatrix} \cos \beta_{pa}(r-a) & j \sin \beta_{pa}(r-a) \\ j \sin \beta_{pa}(r-a) & \cos \beta_{pa}(r-a) \end{pmatrix} \begin{pmatrix} i_{1a} \\ g_a v_{1a} \end{pmatrix} \exp j\omega \left[t - \frac{r-a}{v_{oa}} \right] \quad (1)$$

VIII. FASTER AND SLOWER SPACE-CHARGE WAVE AMPLITUDES

The first space-charge wave analyses were made by Hahn (1939) and Ramo (1939) for a planar drifting stream and many of their concepts have formed a foundation for our thinking about waves in electron streams. One of these concepts is that of "faster" and "slower" space-charge waves; that is, the fluctuation amplitude in space is thought of as the interference between two constant amplitude traveling waves, traveling faster and slower than the average stream velocity. One would like to see whether this concept can be extended to the radially flowing stream and what parts of the extension are useful.

The solution for the planar drift space was given in the preceding section as

$$i_1(r, t) = \left\{ \cos \left[\beta_{pa} (r-a) \right] i_{1a} + j \sin \left[\beta_{pa} (r-a) \right] g_a v_{1a} \right\} \cdot \exp \left\{ j \left[\omega t - \beta_e (r-a) \right] \right\} \quad (1)$$

Writing the cosine and sine in terms of the exponentials gives

$$i_1(r, t) = \left\{ \frac{1}{2} (i_{1a} + v_{1a} g_a) \exp \left[j \beta_{pa} (r-a) \right] + \frac{1}{2} (i_{1a} - v_{1a} g_a) \exp \left[-j \beta_{pa} (r-a) \right] \right\} \exp j \omega t - \beta_e (r-a) , \quad (2)$$

or

$$i_1(r, t) = \frac{1}{2} (i_{1a} + v_{1a} g_a) \exp j \left[\omega t - (\beta_e - \beta_{pa}) (r-a) \right] + \frac{1}{2} (i_{1a} - v_{1a} g_a) \exp j \left[\omega t - (\beta_e + \beta_{pa}) (r-a) \right] . \quad (3)$$

The first and second terms of (3) are called the **faster and slower space-charge waves**, respectively, with **space-independent amplitudes and wave-numbers**:

$$\begin{aligned} |i_{1f}(a)| &= \frac{1}{2} (i_{1a} + v_{1a} g_a), \quad \beta_f = \beta_e - \beta_{pa} \\ |i_{1s}(a)| &= \frac{1}{2} (i_{1a} - v_{1a} g_a), \quad \beta_s = \beta_e + \beta_{pa} \end{aligned} \quad (4)$$

The solution for radial flow with $v_o \sim r^m$ as obtained earlier is repeated here:

$$i_1(x) = (H_{b+1, b} i_{1a} + jH_{b, b} g_a v_{1a}) kx^{d-1/2} \exp \left\{ j \left[\omega t - \int_1^x \beta_e(x) dx \right] \right\} \quad (5)$$

The argument of the biradial functions is (k, kx^d) and is omitted for simplicity. This equation can be rearranged to have a form similar to (2):

$$i_1(x) = \left[\frac{1}{2} (i_{1a} + v_{1a} g_a) (H_{b+1, b} + jH_{b, b}) + \frac{1}{2} (i_{1a} - v_{1a} g_a) (H_{b+1, b} - jH_{b, b}) \right] kx^{d-1/2} \cdot \exp \left\{ j \left[\omega t - \int_1^x \beta_e(x) dx \right] \right\} \quad (6)$$

The combinations of biradial functions occurring in (6) become exponentials for large argument. For all arguments, these combinations may be written as exponentials times a varying magnitude, as

$$H_{b+1, b} + jH_{b, b} = M^i(x) \exp \left[\pm j\phi^i(x) \right] \quad (7)$$

where

$$M^i(x)^2 \equiv H_{b+1, b}^2 + H_{b, b}^2$$

$$\phi^i(x) \equiv \tan^{-1} \left(\frac{H_{b, b}}{H_{b+1, b}} \right) \quad (8)$$

From the phase an effective local plasma wave number may be defined, for current, as

$$\left[\beta_p^i(x) \right]_{eff} \equiv \frac{1}{a} \frac{d}{dx} \left[\tan^{-1} \left(\frac{H_{b, b}}{H_{b+1, b}} \right) \right]$$

$$= \beta_p(x) \cdot \left[\frac{1}{(k x^d) (H_{b, b}^2 + H_{b+1, b}^2)} \right] \quad (9)$$

The bracketed term in (9) is unity for sufficiently large arguments.

The faster and slower waves thus defined have non-constant amplitude:

$$\begin{aligned}
|i_{1f}(x)| &= \frac{1}{2}(i_{1a} + g_a v_{1a})M^i(x), \quad \beta_f(x) = \beta_e(x) - \left[\beta_p^i(x) \right]_{\text{eff}} \\
|i_{1s}(x)| &= \frac{1}{2}(i_{1a} - g_a v_{1a})M^i(x), \quad \beta_s(x) = \beta_e(x) + \left[\beta_p^i(x) \right]_{\text{eff}}
\end{aligned}
\tag{10}$$

The same method applied to the velocity equation yields a different effective local plasma wave number:

$$\begin{aligned}
\left[\beta_p^v(x) \right]_{\text{eff}} &= \frac{1}{a} \frac{d}{dx} \left[\tan^{-1} \left(\frac{H_{b+1, b+1}}{-H_{b, b+1}} \right) \right] \\
&= \beta_p(x) \left[\frac{1}{(k x)^2 \left(H_{b+1, b+1}^2 + H_{b, b+1}^2 \right)} \right]
\end{aligned}
\tag{11}$$

Again the bracketed term is unity for sufficiently large arguments.

Using the kinetic power density as calculated earlier,

$$\bar{P}_k \equiv \text{Re} \left[\frac{v_o(x) i_1(x) v_1^*(x)}{\eta} \right]
\tag{12}$$

it can be shown that

$$x^n \bar{P}_k(x) = \bar{P}_{ka} = x^n \bar{P}_{kf}(x) + x^n \bar{P}_{ks}(x) = \bar{P}_{kfa} + \bar{P}_{ksa}
\tag{13}$$

where

$$\bar{P}_{kf}(x) = \text{Re} \left[\frac{v_o(x) i_{1f}(x) v_{1f}^*(x)}{\eta} \right]
\tag{14}$$

and similarly for the slower wave. Not only is the total kinetic power conserved in radial flow, but the kinetic power in the faster and slower waves defined by (10) is conserved separately, similar to the results for planar flow.

IX. SOLUTIONS FOR $3m + n = 2$

Using $3m + n = 2$ causes $b \rightarrow \infty$, $d \rightarrow 0$, $k \rightarrow \infty$, making awkward the use of the Bessel function solutions. Such combinations of m and n include:

- (i) Planar space-charge limited flow, $m = 2/3$, $n = 0$;
- (ii) Cylindrical flow $v_o \sim r^{1/3}$, $m = 1/3$, $n = 1$;
- (iii) Spherical, drifting stream, $m = 0$, $n = 2$.

Although some limiting process might be used to adapt the Bessel function solutions to fit these situations, a direct solution of the current equation (VI-3) can be obtained. Inserting $2-n$ for $3m$, one obtains,

$$\frac{d^2 \hat{i}_1}{dr^2} + \frac{2-n}{r} \frac{d \hat{i}_1}{dr} + \frac{\beta_{pa}^2 a^2}{r^2} \hat{i}_1 = 0 \quad (1)$$

The solution to this equation is of the form

$$\hat{i}_1(r) \sim r^p$$

with the determinantal equation

$$p^2 + (1-n)p + (\beta_{pa} a)^2 = 0 \quad (2)$$

This equation has two solutions,

$$p = \frac{n-1}{2} \pm \sqrt{\left(\frac{n-1}{2}\right)^2 - (\beta_{pa} a)^2} = \frac{n-1}{2} \pm s, \quad (3)$$

where the quantity s has been defined in an obvious way. Then one has

$$i_1(r) = A r^{\left(\frac{n-1}{2} + s\right)} + B r^{\left(\frac{n-1}{2} - s\right)} \quad (4)$$

$$\hat{v}_1(r) = \frac{v_o^2(r)}{j\omega i_{oa} a^n} \left[A' \left(\frac{n-1}{2} + s \right) r^{\left(\frac{n-3}{2} + s \right)} + B' \left(\frac{n-1}{2} - s \right) r^{\left(\frac{n-3}{2} - s \right)} \right] \quad (5)$$

Imposing the initial conditions at radius a and transforming back to laboratory coordinate system gives the solution

$$\begin{pmatrix} i_1(x, t) \\ g(x) v_1(x, t) \end{pmatrix} = \begin{pmatrix} \left[\frac{x^s + x^{-s}}{2} + \frac{1-n}{2s} \frac{x^s - x^{-s}}{2} \right] & j \left[\sqrt{\frac{1-n}{2s} - 1} \frac{x^s - x^{-s}}{2} \right] \\ j \left[\sqrt{\frac{1-n}{2s} - 1} \frac{x^s - x^{-s}}{2} \right] & \left[\frac{x^s + x^{-s}}{2} - \frac{1-n}{2s} \frac{x^s - x^{-s}}{2} \right] \end{pmatrix} \cdot \begin{pmatrix} i_{1a} \\ g_a v_{1a} \end{pmatrix} x^{-\frac{(n+1)}{2}} \exp j \left[\omega t - j \frac{\omega a}{v_{oa}} \frac{3 \left(\frac{1+n}{3} - 1 \right)}{1+n} \right] \quad (6)$$

This is obviously not an easy form to use. Other forms can be given that are simpler and also display the physical behavior more clearly.

The transmission matrix can be made simpler by looking at the nature of s. s will be real or imaginary depending on the magnitude of $(\beta_{pa} a)^2$, that is, on the current.

For planar flow, n=0, m is 2/3, which is fully space-charge limited accelerated flow. This flow requires that

$$\frac{-J_a a^2}{v_a^{3/2}} = \frac{4}{9} \epsilon_o \sqrt{-2\eta} \quad (7)$$

The quantity $(\beta_{pa} a)^2$ is given by

$$(\beta_{pa} a)^2 = \frac{\eta}{\epsilon_o} \frac{J_a}{v_o^3(a)} a^2 = \left(\frac{J_a a^2}{v_a^{3/2}} \right) \left(\frac{1}{2\epsilon_o \sqrt{-2\eta}} \right) \quad (8)$$

Inserting (7) into (8) leads to

$$(\beta_{pa} a)^2 = 2/9 \quad (9)$$

so that

$$s = 1/6, \quad p = -1/2 \pm 1/6 = -2/3, -1/3 \quad (10)$$

With s real, the transmission matrix elements A, B, C, D , are

$$A = \cosh (s \ln x) + \frac{(1-n)}{2} \frac{\sinh (s \ln x)}{s} ; \quad (11a)$$

$$B = j\beta_{pa} a \frac{\sinh (s \ln x)}{s} ; \quad (11b)$$

$$C = B ; \quad (11c)$$

$$D = \cosh (s \ln x) - \frac{(1-n)}{2} \frac{\sinh (s \ln x)}{s} \quad (11d)$$

It is seen that these elements tend to increase or decrease monotonically with distance x which is quite different from the periodic elements of ordinary space-charge wave flow.

For cylindrical flow, $n=1$, m is $1/3$ and the flow is accelerating. $n-1$ is zero so that s is $j\beta_{pa} a$, always imaginary. With s imaginary, the transmission matrix elements A, B, C, D , are using $s=j\zeta$

$$A = \cos(\zeta \ln x) - \frac{(1-n)}{2} \frac{\sin(\zeta \ln x)}{\zeta} ; \quad (12a)$$

$$B = -j\beta_{pa} a \frac{\sin(\zeta \ln x)}{\zeta} ; \quad (12b)$$

$$C = B ; \quad (12c)$$

$$D = \cos (\zeta \ln x) + \frac{(1-n)}{2} \sin \frac{(\zeta \ln x)}{\zeta} .$$

These elements, with the trigonometric functions, are periodic with x ; however, because of the logarithmic dependence the zeros of the trigonometric functions may be very far apart, especially for small ζ , small currents.

For spherical flow $n=2$, m is zero meaning that the stream is drifting at constant velocity. Two possibilities arise here:

- (i) if $\beta_{pa} < 0.5$ (small currents) then s is real and the monotonically-increasing matrix elements of (11) are to be used;
- (ii) if $\beta_{pa} > 0.5$ (large currents) then s is imaginary and the periodic matrix elements of (12) are to be used.

The turning point is at β_{pa} of 0.5 which corresponds to a critical current. At less than critical current, the behavior is similar to that of planar drifting flow in which space-charge forces have been neglected; the apparent reason is that the flow expands sufficiently rapidly so that no debunching takes place. The critical current occurs where the space-charge wavelength, λ_{pa} , is twice the circumference of the a sphere. At larger currents the space-charge forces have become sufficiently strong to cause debunching before the stream has expanded appreciably. The critical current can also be associated with a perveance, as one can write $(\beta_{pa})^2$ in terms of the total average current I_o as,

$$(\beta_{pa})^2 = \frac{-I_o}{V_a^{3/2}} \frac{1}{8\pi\epsilon_o \sqrt{2}\eta} \quad (13)$$

Using the charge-to-mass ratio of electrons, the critical perveance is given by

$$\frac{I_o}{V_a^{3/2}} = -33 \times 10^{-6} \quad (\text{amps/volts}^{3/2}) \quad (14)$$

(For example, this perveance corresponds to $33\mu\text{A}$ at 1 volt, or 33 mA at 100 volts or 33A at 10 kilovolts.) For a model in which only a sector of spherical flow is used, the critical current and perveance will be correspondingly smaller by the ratio of the area used to that of a complete sphere.

In the limit of large radius of curvature, with small spacing between input and output and at larger than critical perveance, it can be shown that the transmission matrix elements are the same as in parallel flow, ordinary space-charge waves.

X. EXAMPLE OF CYLINDRICAL DRIFTING FLOW $n=1, m=0$

The example of a cylindrical klystron is chosen to illustrate both the physical and mathematical aspects of the rather involved solutions obtained. A sketch of such a device is shown in Fig. 2. For this example, $n=1, m=0$, which fixes

$$b = -1;$$

$$d = 1/2;$$

$$k = 2\beta_{pa} a$$

$$g(x) = g_a / \sqrt{x}.$$

For the sake of convenience, as there is no critical current or perveance, $\beta_{pa} a$ may be chosen arbitrarily; one easy choice is to make $\beta_{pa} a = 1/2$ and thus, $k=1$. The relation between input and output quantities from (VI-18) is,

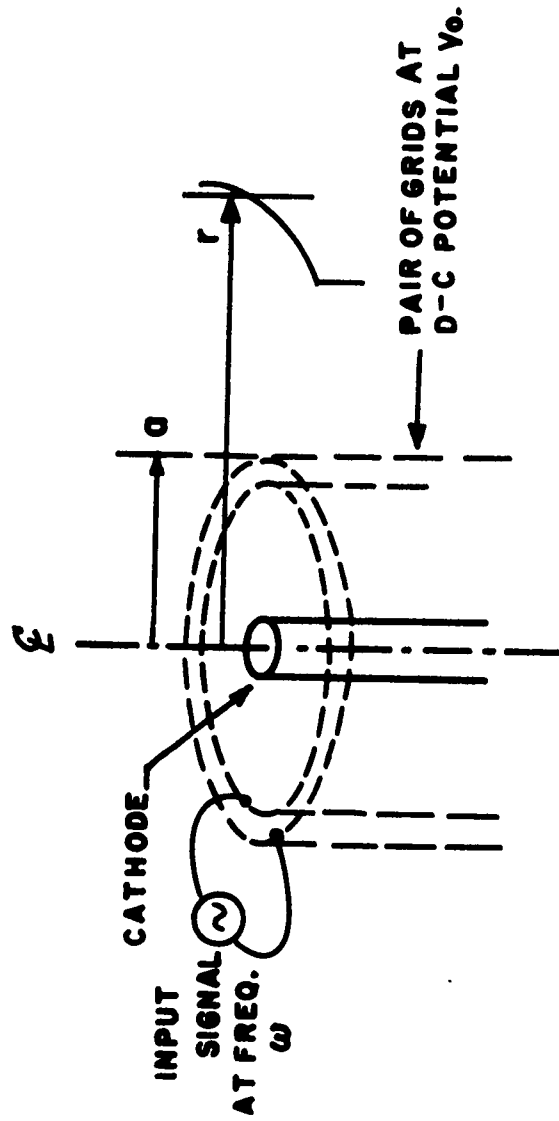


Figure 2

$$\begin{pmatrix} i_1(x) \\ g(x)v_1(x) \end{pmatrix} = \begin{pmatrix} -H_{0,1}(1, \sqrt{x}) & jH_{1,1}(1, \sqrt{x}) \\ jH_{0,0}(1, \sqrt{x}) & H_{1,0}(1, \sqrt{x}) \end{pmatrix} \begin{pmatrix} i_{1a} \\ g_a v_{1a} \end{pmatrix} \exp[-j(x-1)] \quad (1)$$

(Use has been made of the recursion relations among the $H_{\mu, \nu}$ functions.)

For velocity modulation only at the a (initial) plane ($i_{1a}=0$), one has

$$\frac{v_1(x)}{v_{1a}} \frac{g(x)}{g_a} = \frac{1}{\sqrt{x}} \frac{v_1(x)}{v_{1a}} = jH_{1,0}(1, \sqrt{x}), \quad (2)$$

and

$$\frac{i_1(x)}{g_a v_{1a}} = H_{1,1}(1, \sqrt{x}). \quad (3)$$

The pertinent biradial functions are shown in Fig. 3 as a function of x . Note that the velocity and current density are periodic as with planar drifting flow, but here the velocity maxima and current minima are not quite aligned, except at large distances. The bunching and debunching is much the same as in ordinary parallel-flow klystrons. However, the total ac current maxima will not stay constant but will grow with the distance as can be shown using the asymptotic formula for $H_{\mu, \nu}$. For $\xi, \zeta \rightarrow \infty$

$$H_{\mu, \nu}(\xi, \zeta) \approx \frac{\sin(\zeta - \xi) - \frac{\mu - \nu}{2} \pi}{\sqrt{\xi \zeta}}; \quad (4)$$

Thus, for large values of k and $k\sqrt{x}$, the current ($\sim i_1(x) x$) is,

$$\frac{i_1(x) x}{g_a v_{1a}} = \frac{\sin(k\sqrt{x} - k)}{\sqrt[4]{x}} x \quad (5)$$

so that the current maxima growth as $x^{3/4}$ for expanding flow.

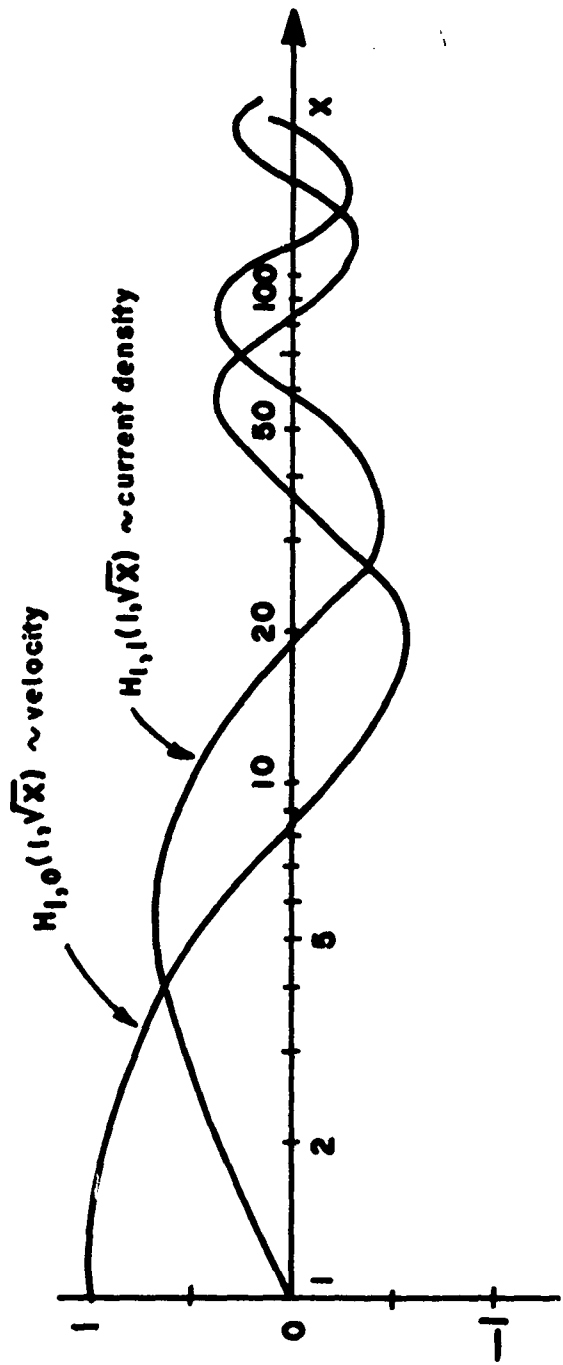


Figure 3

For current modulation only at the a (initial) plane

($v_{ia} = 0$), one has

$$v_1(x) \frac{g(x)}{i_{1a}} = \frac{v_1(x)}{\sqrt{x}} \frac{g_a}{v_{1a}} = jH_{0,0}(1, \sqrt{x}), \quad (6)$$

and

$$\frac{i_1(x)}{i_{1a}} = -H_{0,1}(1, \sqrt{x}). \quad (7)$$

The pertinent biradial functions are shown in Fig. 4. The same comments apply as in the case of velocity modulation.

An interesting singularity in the charge density occurs under certain conditions. Recall from planar drifting flow, with only the faster wave excited, that the ac charge density vanishes when excited at $\omega = \omega_p$. In the cylindrical drifting flow with the stream expanding, the local plasma frequency decreases with increasing x . Thus, by modulating at $r = a$ with a frequency $\omega < \omega_{pa}$ and letting the flow expand, there will be a radius at which $\omega = \omega_p(r)$. The question is whether the ac charge density can be made to vanish at this radius. For the model at hand, with $n = 1$, $m = 0$, the charge density is given by

$$\frac{\hat{\rho}_1(x)}{\rho_{oa}} = \frac{\hat{i}_1(x)}{i_{oa}} - \frac{\hat{v}_1(x)}{v_{oa}} \quad xa \quad (8)$$

Suppose that at some radius x_c

$$i_1(x_c) = v_1(x_c)g(x_c) \quad (9)$$

which says that the velocity and current are in phase at this point, and implies the existence of a "faster wave" only at this point, Using this

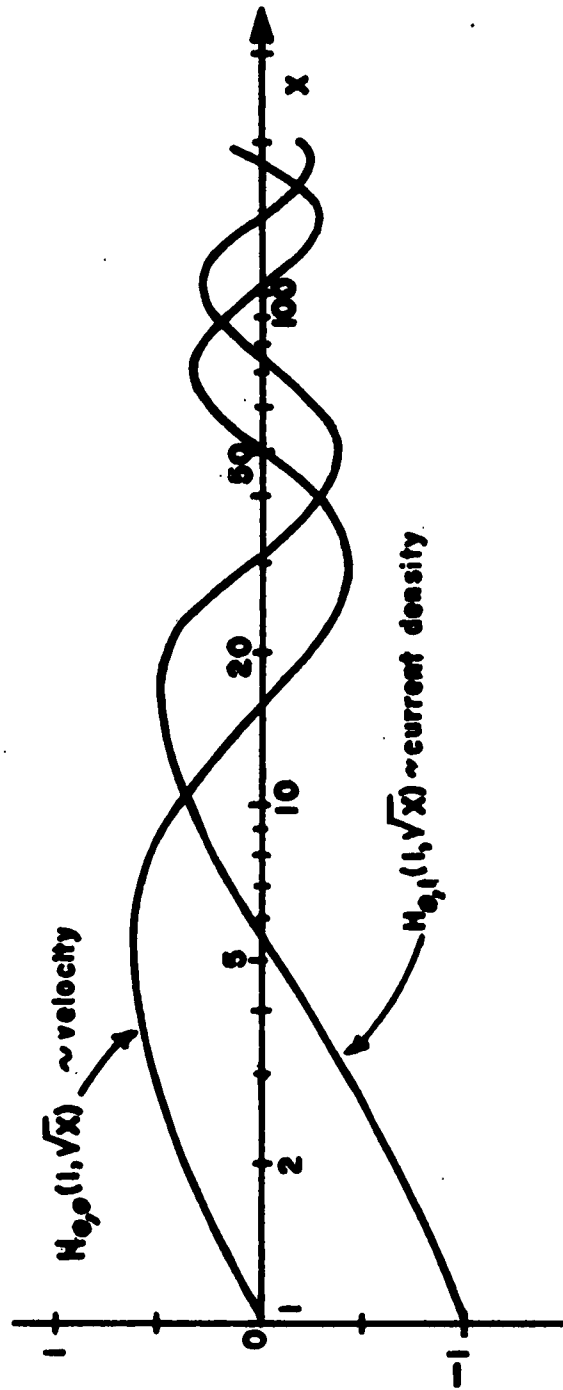


Figure 4

value for the velocity, the charge density becomes, at x_c ,

$$\frac{\hat{\rho}_1(x_c)}{\rho_{oa}} = \frac{\hat{i}_1(x_c)}{i_{oa}} \left(1 - \frac{\omega_p(x_c)}{\omega} x_c\right) \quad (10)$$

Hence, the charge density can be made to vanish at the point x_c by choosing

$$\omega = \omega_p(x_c) x_c \quad (11)$$

This zero in charge density comes about not by wave interference but by a local physical requirement of zero charge density. In order for the modulation at x_c to be that specified by (9), the modulation at the a (initial) plane must be

$$i_{la} = v_{la} g_{la} \left[\frac{-H_{1,o}(1, \sqrt{x_c}) + jH_{1,1}(1, \sqrt{x_c})}{H_{o,1}(1, \sqrt{x_c}) + jH_{o,o}(1, \sqrt{x_c})} \right] \quad (12)$$

The bracketed term is unity both for $x=1$, and large values of x and tends to have magnitude near unity for values in between; thus, requiring the "faster wave" only at $x=x_c$ implies that something like the "faster wave" exists for all other x .

REFERENCES

- 1939 Hahn, W. C. , "Small Signal Theory of Velocity-Modulated Electron Beams," Gen. Elec. Rev., Vol. 42 (June), pp. 258-270.
- Hahn, W. C. , "Wave Energy and Transconductance of Velocity-Modulated Electron Beams," Gen. Elec. Rev., Vol. 42 (November), pp. 497-502.
- Ramo, S. , "The Electronic-Wave Theory of Velocity Modulation Tubes," Proc. IRE, Vol. 27 (December), pp. 757-763.
- 1941 Llewellyn, F. B. , Electron Inertia Effects, Cambridge, England: Cambridge University Press.
- 1945 Jahnke, E. and Emde, F. , Tables of Functions, New York: Dover Publications (reprint).
- 1946 Feenberg, E. , "Elementary Treatment of Longitudinal Debunching in a Velocity Modulation System," Jour. Appl. Phys. , Vol. 17 (October), pp. 852-855.
- 1951 Smullin, L. D. , "Propagation of Disturbances in One-Dimensional Accelerated Electron Streams," Jour. Appl. Phys. , Vol. 22 (December), pp. 1496-1498.
- 1952 Quate, C. F. , "The Effects of Diverging Beams for the Purpose of Reducing Noise Space-Charge Waves," Conf. on Electron Tube Res. , June 17, Ottawa, Canada. (See patent no. 2, 792, 518, May 14, 1957 for details.)
- Tien, P. K. , and Field, L. M. , "Space-Charge Waves in an Accelerated Electron Stream for Amplification of Microwave Signals," Proc. IRE, Vol. 40 (June), pp. 688-695.
- 1954 Hutter, R. G. E. , "Traveling-Wave Tubes," Advances in Electronics and Electron Physics, Vol. VI, pp. 371-461.

- 1955 "Müller, R., "Space-Charge Waves in Accelerated and Decelerated One-Dimensional Electron Streams," Arch. Elek. Übertr., Vol. 9 pp. 505-512.
- Kino, G. S., "Normal Mode Theory in Perturbed Transmission Systems," T.R. 84 Electronics Res. Lab., Stanford Univ., May 2.
- 1956 Higuchi, H., "Propagation of Space-Charge Waves in an Accelerated Electron Beam," Report of the Microwave Communication Research Committee in Japan, December, pp. 87-89.
- 1959 Eichenbaum, A. L. . and Peter, R. W., "The Exponential Gun -- A Low Noise Gun for Traveling-Wave-Tube Amplifiers," RCA Rev., Vol. 20 (March), pp. 18-56.

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH II
 FLIGHT TUBE RESEARCH
 CONTRACT NO. AF 49(638)-102

DISTRIBUTION LIST

ADDRESSEE	NO. COPIES	ADDRESSEE	NO. COPIES	ADDRESSEE	NO. COPIES
AFOBR ATTN: Technical Library Washington 25, D. C.	2	High Speed Flight Station (NASA) ATTN: Technical Library Edwards AFB, California	1	The Ohio State University Antenna Laboratory 2024 Neil Avenue Columbus 10, Ohio ATTN: Security Officer	1
AFOBR (SRPH) Washington 25, D. C.	1	Langley Research Center (NASA) ATTN: Technical Library Langley AFB, California	1	Major General Casuarino Montenegro Filho General Edmundo de Azevedo (CIA) S/O Jardim Campus S/O Paulo, Brazil	1
ASD ATTN: Technical Library Wright Patterson AFB, Ohio	1	Lewis Research Center (NASA) ATTN: Technical Library 2100 Brookpark Road Cleveland 12, Ohio	1	Indiana University Lafayette, Indiana ATTN: Richard L. Funkhouser Engineering Librarian	1
AFCLL ATTN: Technical Library 1 G. Hanscom Field Bedford, Massachusetts	1	Goddard Space Flight Center (NASA) ATTN: Technical Library Greenbelt, Maryland	1	Professor J. Van Bladel Electrical Engineering Department College of Engineering University of Wisconsin Madison 6, Wisconsin	1
EOAR The Shell Building 4 Rue Conterzeum Brussels, Belgium	(Unclassified Reports)	George C. Marshall Space Flight Center (NASA) ATTN: Technical Library Redstone Arsenal, Alabama	1	Professor W. Lew Department of Physics The Hebrew University of Jerusalem Jerusalem, Israel	1
AHL ATTN: Technical Library Wright-Patterson AFB, Ohio	1	Wallops Station (NASA) ATTN: Technical Library Wallops Island, Virginia	1	University of Illinois Department of Electrical Engineering Urbana, Illinois ATTN: M. Von Foerster	1
ASTIA (TIPCR) Arlington Hall Station Arlington 12, Virginia	10	Institute of Aeronautical Sciences 2 East 64th Street New York 21, New York	(Unclassified Reports)	Ohio State University Department of Electrical Engineering Columbus, Ohio	1
Director of Research Headquarters, USAF ATTN: AFRDR Washington 25, D. C.	1	Applied Mechanics Reviews Southwest Research Institute 8500 Calders Road San Antonio 6, Texas	(Unclassified Reports)	The University of Michigan Department of Electrical Engineering Electron Physics Laboratory Ann Arbor, Michigan ATTN: Professor J. E. Kow	1
Office of Naval Research Department of the Navy ATTN: Code 420 Washington 25, D. C.	1	Linda Hall Library ATTN: Document Division 4100 Cherry Street Kansas City 10, Missouri	(Unclassified Reports)	Stanford University Electronic Research Laboratory Stanford, California	1
Naval Research Laboratory ATTN: Technical Library Washington 25, D. C.	1	AFOBR (SRAI) ATTN: Technical Library Holloman AFB, New Mexico	2	Massachusetts Institute of Technology Research Labs. of Electronics Room 20B-22, Document Office Cambridge 39, Massachusetts ATTN: J. H. Hewitt	1
Chief, Research and Development ATTN: Scientific Information Branch Department of the Army Washington 25, D. C.	1	AFSWC (SBOI) Kirtland AFB, New Mexico	1	Harvard University Cruft Laboratory Cambridge 38, Massachusetts ATTN: Technical Reports Collection	1
Chief, Physics Branch Division of Research, U. S. Atomic Energy Commission Washington 25, D. C.	1	Advanced Research Projects Agency Washington 25, D. C.	1	Technical Information Libraries Bell Telephone Laboratories, Inc. Whippany, New Jersey ATTN: Technical Reports Librarian	1
U. S. Atomic Energy Commission Technical Information Extension P. O. Box 62 Oak Ridge, Tennessee	1	Rand Corporation 1700 Main Street Santa Monica, California	1	California Institute of Technology Pasadena 4, California ATTN: R. Gould	1
National Bureau of Standards ATTN: Technical Library Room 205, Northwest Building Washington 25, D. C.	1	Chairman Canadian Joint Staff (DRR/DRS) 2430 Massachusetts Avenue, N.W. Washington 25, D. C.	(Unclassified Reports)	Sylvania Electric Company Mountain View, California ATTN: D. H. Goodman	1
Physics Program National Science Foundation Washington 25, D. C.	1	Office in Charge Office of Naval Research New York 100 First Post Office New York, New York	1	Professor William M. Surber Electrical Engineering Department Princeton University Princeton, New Jersey	1
Director, Army Research Office - Durham Box 261, Durham Station Durham, North Carolina	1	Office of Technical Services Department of Commerce Technical Reports Branch Washington 25, D. C.	1	General Electric Company Electron Tube Div. of the Research Lab. The Radio Schenectady, New York ATTN: E. D. McArthur	1
AFIC (LPHM) ATTN: Technical Library Arnold Air Force Station Tullahoma, Tennessee	1	Dr. D. Adcock, Head Aertron Systems Incorporated 1625 E. 18th Street Hawthorne, California	1	General Electric Company Missile and Space Vehicle Department 1218 Chestnut Street Philadelphia 4, Pennsylvania ATTN: Aerodynamics Engineering Operation R. F. Park, Manager	1
AFSTC (AFOTL) ATTN: Technical Library Edwards AFB, California	1	Dr. Harold Clever Office of Naval Research Washington 25, D. C.	1	Hughes Aircraft Company Fluorocel at Tullis Street Culver City, California ATTN: Documents Group Bldg. 6, Rm. X2015	1
AFMDC (MDF) Holloman AFB, New Mexico	1	Professor Charles Tompa Department of Physics Columbia University New York 27, New York	1	BCA Laboratories Princeton, New Jersey ATTN: Dr. W. M. Webster, Director Electronics Research Lab.	1
AFMDC (HUM) Holloman AFB, New Mexico	1	Professor Harvey Brooks Department of Physics Harvard University Cambridge 38, Massachusetts	1	Varian Associates 611 Mission Way Palo Alto, California ATTN: Technical Library	1
AFOMA (ORDSR-OTL) Redstone Arsenal, Alabama	1	Professor P. Kusch Department of Physics Columbia University New York 27, New York	1	Westinghouse Electric Corporation Electronic Tube Division P. O. Box 204 Elmsira, New York ATTN: Mr. Sheldon S. King, Librarian	1
Institute of Technology (AU) Library MGLI-LIB, Bldg. 125, Area B Wright-Patterson AFB, Ohio	1	Professor N. Blumberg Department of Physics Harvard University Cambridge 38, Massachusetts	1	Professor Zahrah Kaperian University of Southern California School of Engineering Department of Electrical Engineering University Park Los Angeles 7, California	1
AFSC (DCRB) Andrews AFB Washington 25, D. C.	1	Dr. Irving Runo Office of Naval Research 146 Broadway New York, New York	1	B. J. Mamm Dalmo-Victor Research Laboratories Belmont, California	1
Signal Corps Engineering Laboratory ATTN: (SIOFM/CL-RPO) Fort Monmouth, New Jersey	1	W. A. Kumplich, Manager Lockheed Aircraft Corporation Missiles and Space Division Technical Information Center 1651 Rosener Street Palo Alto, California	1	Headquarters Air Force Office of Scientific Research Office of Aerospace Research USAF Washington 25, D. C. ATTN: Marshall C. Harrington General Physics Division	1
Headquarters National Aeronautics and Space Administration ATTN: Technical Library Washington 25, D. C.	1	Mr. E. Oross Sentry Gyroscopic Company Electron Tube Division Mail Station 1 040 Great Neck, New York	1		
Ames Research Center (NASA) ATTN: Technical Library Moffett Field, California	1	DOPL (ORDTL 812) Washington 25, D. C.	1		
Technical Information Division Rd. 39, Room 301 Redstone Laboratory Tuskey, California	1	Prof. J. H. Milligan, Jr. Chairman, Department of Electrical Engineering New York University 25 Waverly Place New York, New York	1		
Mass Inst. Oxford University Oxford, England	1				
Miss Betty Doris Department of Archives 501 Library University of California Berkeley 4, California	1				