AFCRL-62-748

QUANTUM ELECTRONICS LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

SCIENTIFIC REPORT NO. 2

A MODIFIED FABRY-PEROT INTERFEROMETER AS A DISCRIMINATION FILTER AND A MODULATOR FOR LONGITUDINAL MODES

Helmut K. V. Lotsch

CONTRACT NO. AF 19(604)-8052
ELECTRONICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

1 SEPTEMBER 1962

AFCRL-62-748

CALIFORNIA INSTITUTE OF TECHNOLOGY

Quantum Electronics Laboratory

A MODIFIED FABRY-PEROT INTERFEROMETER AS A DISCRIMINATION FILTER AND A MODULATOR FOR LONGITUDINAL MODES*

Helmut K. V. Lotsch

Contract No. AF19(604)-8052

Project 4600

Task 460006

1 September 1962

Scientific Report No. 2

Prepared for

Electronics Research Directorate
Air Force Cambridge Research Laboratories
Office of Aerospace Research
United States Air Force
Bedford, Massachusetts

*This research was sponsored in part under Contract No. AF19(604)-8052, and in part by a NATO Fellowship.

Requests for additional copies by Agencies of the Department of Defense, their contractors, and other Government agencies should be directed to the:

ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

Department of Defense contractors must be established for ASTIA services or have their "need-to-know" certified by the cognizant military agency of their project or contract.

All other persons and organizations should apply to the:

U.S. DEPARTMENT OF COMMERCE
OFFICE OF TECHNICAL SERVICES
WASHINGTON 25, D. C.

A Modified Fabry-Perot Interferometer as a Discrimination Filter and a Modulator for Longitudinal Modes

Helmut K. V. Lotsch

Summary

Kleinman and Kisliuk [1] have proposed a symmetric modified Fabry-Perot interferometer in which, by providing an extra reflecting surface outside and parallel to the maser plates, certain longitudinal Fabry-Perot orders become more lossy than others. By that is meant modes with lower losses oscillate preferentially. In this paper starting from [1] an unsymmetric modified interferometer is investigated; the space between the active medium and the extra reflecting surface is assumed to be filled by an arbitrary, lossless dielectric medium. When the ratio of this dielectric constant to that of the active medium is modulated, the analysis shows that longitudinal modes are modulated in their amplitude and frequency. In particular, the modulation of this ratio about the value 1 appears to be very interesting, since the modes with the lowest losses, e.g., for ratios larger than 1, possess the largest losses for ratios smaller than 1, and vice versa.

1. Introduction

The Fabry-Perot interferometer plays a significant role in the development of devices, such as the maser, which operate in the millimeter through optical wavelength range. Schawlow and Townes [2], Prokhorov [3], and Dicke [4] have suggested its application as a resonator for the infrared and optical masers. Maiman [5] and Collins et al [6] have experimentally demonstrated the feasibility of stimulated radiation in ruby using a Fabry-Perot resonator.

The Fabry-Perot resonator has to be considered as a multimode resonant cavity [7] and [8]; the modes may be specified by three numbers as in microwave techniques. Fox and Li [9] have theoretically investigated a resonant cavity with parallel mirrors; Connes [10], Fox and Li [9] and Boyd and Gordon [11] and [12] a resonant cavity with concave mirrors. Experimentally a fine structure which is due to various orders of the Fabry-Perot resonator [13] has been seen in the output of both the gaseous [14] to [16] and the ruby [17] and [18] optical maser.

Ultimately the usefulness of the optical maser may be limited by the excitation of many undesirable modes. Therefore Kleinman and Kisliuk [1] and a number of others mentioned in [1] have proposed that it would be useful to discriminate against many of the Fabry-Perot orders which can occur in the output, by increasing their losses relative to other "preferred" modes. Kogelnik and Rigrod [14] have experimentally separated some of the transversal modes and Murtry and Siegman [19] some of the longitudinal modes.

2. Modified Interferometer and Its Analysis

Kleinman and Kisliuk [1] have proposed that another reflecting surface outside and parallel to the maser plates with a short and adjustable separation should be provided. Taking only the transmission losses into account they have shown that certain longitudinal modes become more lossy than others, and that in this case the extra modes expected due to the increased over-all length of the resonator cavity are negligible because they are very lossy compared with the "preferred" modes. As already mentioned [1], the calculation for an unsymmetric modified interferometer with only one extra reflecting surface yields similar results; but they failed to mention that in contrast only odd modes occur, since the even modes are prevented by non-symmetry. Therefore the unsymmetric structure of Fig. 1 is subsequently investigated. Unlike [1] the region between the maser and the extra reflecting surface is filled by a lossless dielectric medium with an arbitrary and possibly time-dependent dielectric constant.

The problem under consideration is shown schematically in Fig. 1 It is convenient for the following analysis to consider a one-dimensional structure. The region $0 \leq z \leq a$ is occupied by a medium with the real relative dielectric constant $\epsilon_1 \geqq 1$ and real conductivity σ_1 ; the region $a \leq z \leq b$ by a lossless $(\sigma_2 \equiv 0)$ medium with the real relative dielectric constant $\epsilon_2 = \epsilon_2(t)$. However, this time-dependence must be restricted to periods much longer than the decrement time of the waves within the cavity resonator.

We shall now consider steady-state solutions of the electric field in which time enters as the factor $e^{i\omega t}$. According to [20; §13.15(2)] and [21; IV.3(8a,b)] the propagation constants for both regions are given by

$$k_v = \omega\sqrt{\epsilon_v \mu_v} = \omega/c$$

$$k_1 = k_v\sqrt{\epsilon_1}\ \left(1 - i\frac{\sigma_1}{\epsilon_1\omega}\right)^{1/2} \approx k_v\sqrt{\epsilon_1} - i\frac{\sigma_1}{2c\epsilon_v\sqrt{\epsilon_1}} + \cdots$$

$$k_2 = k_v\sqrt{\epsilon_2}\ ; \qquad \sigma_2 \cong 0 \quad . \tag{1}$$

Reflection losses result from absorption in the reflectors and from the transmission through them. At optical frequencies a multi-layered dielectric reflector [22] can have a 99.5 percent reflection coefficient. Therefore we assume that the surface at $z = 0$ is perfectly reflecting. We only take into account the losses due to imperfect reflection and transmission at the surface $z = b$ by a suitable reflection coefficient

$$r_b = e^{-2f} \quad . \tag{2}$$

Unlike the usual definition this reflection coefficient for the electric field is unity for a perfect conductor, i.e., a totally reflecting surface; the negative sign is already accounted for in the solution of the wave equation, Fig. 1. Its phase angle is immaterial since it can be corrected by a very small spatial adjustment of the reflecting plane; it has been assumed zero. For these boundary conditions the electric field is indicated in Fig. 1. Perfect reflection at the surface $z = 0$ is not an essential restriction, as shown in the Appendix. These reflection losses have only to be small so that the odd solution clearly dominates. Thus we neglect possible coupling effects, as known in microwave techniques.

It is convenient for the later derivation to define now

$$T = \tanh f = \frac{1 - r_b}{1 + r_b} \quad . \tag{3}$$

The reflectivity of the surface at $z = a$ is given according to [20; §13.16] or [21; IV,2], neglecting the small conductivity σ_1 by

$$r_a = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \quad . \tag{4}$$

A physical discussion of (3) and (4) indicates that there are no restrictions on the reflectivities of the surfaces at $z = a$ and b .

The electric and magnetic fields have to be continuous across the plane $z = a$. It follows from Maxwell's second equation that the condition on the magnetic field can be expressed by the continuity of the first derivative of the electric field. Thus, we obtain the relation

$$k_2 \tan(k_1 a) = - k_1 \tan[k_2(b - a) - if] \quad , \tag{5}$$

which gives, in general, complex eigenvalues for the angular frequency ω . However, considering the physical situation we require that ω be real and allow σ_1 to assume an appropriate negative value. The negative value of σ_1 corresponds to supplying sufficient gain to maintain a steady-state oscillation at this frequency $(\omega/2\pi)$ and describes simultaneously the losses of the mode in question.

Similar to [1,(8)] we choose the dimensions so that

$$n(b - a) = ma \sqrt{\frac{\epsilon_1}{\epsilon_2}} \tag{6}$$

where m and n are positive integers; i.e., $m \geq 1$ and $n \gg 1$. With this definition we can write

$$k_2(b - a) = m\pi + \frac{m}{n}\varphi \tag{7a}$$

$$k_2 a \sqrt{\frac{\epsilon_1}{\epsilon_2}} = n\pi + \varphi \quad . \tag{7b}$$

The conductivity to be determined is contained in the parameter

$$\kappa = \tanh\left(\sigma_1 a/2c\epsilon_v \sqrt{\epsilon_1}\right) \quad . \tag{8}$$

We obtain by substituting (7) into (5) and simultaneously considering the definition (3) and (8) for the real part

$$\tan\varphi = -\left(\tan\frac{m}{n}\varphi\right)\frac{\sqrt{\epsilon_1/\epsilon_2} + \kappa T}{1 + \sqrt{\epsilon_1/\epsilon_2}\,\kappa T} \tag{9}$$

and for the imaginary part

$$\kappa = -T \frac{\sqrt{\epsilon_1/\epsilon_2} - \tan\varphi \tan\frac{m}{n}\varphi}{1 - \sqrt{\epsilon_1/\epsilon_2}\tan\varphi\tan\frac{m}{n}\varphi} \tag{10}$$

if we take the ratio k_1/k_2 real and according to (1) approximately $\sqrt{\epsilon_1/\epsilon_2}$. With this assumption we consider σ_1 to be small and thus κ according to (8) small, too. Therefore, since $T \leq 1$ according to (3) we obtain for (9) the approximation by neglecting the product κT

$$\tan\varphi = -\sqrt{\epsilon_1/\epsilon_2}\,\tan\frac{m}{n}\varphi \ , \tag{11}$$

Indeed, by this approximation we neglect the small but finite value, determined by T, of the mode in question at the surface $z = b$. Therefore the roots of (11) appear to belong to perfectly conducting surfaces both at $z = 0$ and $z = b$. When $\tan\varphi$ is eliminated between (10) and (11) we obtain approximately

$$\kappa = -T\sqrt{\frac{\epsilon_1}{\epsilon_2}}\,\frac{1 + \tan^2\frac{m}{n}\varphi}{1 + \frac{\epsilon_1}{\epsilon_2}\tan^2\frac{m}{n}\varphi} \ . \tag{12}$$

But when the above assumption is not valid, or we are looking for a better approximation, we have to eliminate $\tan\varphi$ between (9) and (10) which leads to a quadratic equation for κ.

κ is a function of ϵ_1 and ϵ_2. When we remove the discontinuity at $z = a$, i.e., $\epsilon_1 = \epsilon_2$, (12) reduces to the physically expected relation $\kappa = -T$, since we have taken into account only the losses of the reflecting surface at $z = b$. κ depends upon φ through $\tan^2(m/n)\varphi$. The smallest values belong to the "preferred" modes having the lowest losses. (12) yields for the two principal roots of (11) $\tan^2(m/n)\varphi$ equal infinite and zero, i.e., $\varphi = n\pi/2m$ and $\varphi = 0$, respectively,

$$\kappa_{min} = - T \sqrt{\frac{\epsilon_2}{\epsilon_1}} \qquad \epsilon_1 > \epsilon_2$$

$$\kappa_{min} = - T \sqrt{\frac{\epsilon_1}{\epsilon_2}} \qquad \epsilon_1 < \epsilon_2 \qquad (13)$$

For the "original" interferometer having no surface at $z = b$ we have to put $T = 1$ according to (3) and the lowest losses are determined by the square root of the ratio ϵ_2/ϵ_1 and ϵ_1/ϵ_2 respectively. κ vanishes in the limit $\epsilon_2 \to \infty$, i.e., total reflection at the discontinuity $z = a$ according to (4), since we approximately neglected all the different losses besides the losses in the plane $z = b$.

We obtain the largest values by considerations similar to those leading to (13).

$$\kappa_{max} = - T \sqrt{\frac{\epsilon_1}{\epsilon_2}} \qquad \epsilon_1 > \epsilon_2 \qquad (14a)$$

$$\kappa_{max} = - T \sqrt{\frac{\epsilon_2}{\epsilon_1}} \qquad \epsilon_1 < \epsilon_2 \qquad (14b)$$

At first view the expression (14b) for the case $\epsilon_1 < \epsilon_2$ seems somewhat strange, when considering the limit $\epsilon_2 \to \infty$. But in the following chapter it will be shown that this result is completely consistent with the physical situation, since the special mode in question $\varphi = (n\pi/2m)$ would possess a finite value at the "perfectly conducting" $(\epsilon_2 \to \infty)$ surface $z = a$.

We define now the discrimination ratio

$$R = \kappa/\kappa_{min} \qquad (15)$$

and obtain with (12) and (13)

$$R = \frac{\epsilon_1}{\epsilon_2} \frac{1 + \tan^2 \frac{m}{n} \varphi}{1 + \frac{\epsilon_1}{\epsilon_2} \tan^2 \frac{m}{n} \varphi} \qquad \epsilon_1 > \epsilon_2$$

and (16)

$$R = \frac{1 + \tan^2 \frac{m}{n} \varphi}{1 + \frac{\epsilon_1}{\epsilon_2} \tan^2 \frac{m}{n} \varphi} \qquad \epsilon_1 < \epsilon_2 \; .$$

When we remove the discontinuity at $z = a$, i.e. $\epsilon_1 = \epsilon_2$ and $r_a = 0$ according to (4), the discrimination ratio yields clearly one. Its maximum values are (ϵ_1/ϵ_2) for $\epsilon_1 > \epsilon_2$ and (ϵ_2/ϵ_1) for $\epsilon_1 < \epsilon_2$ respectively.

The relative "Q" of the mode in question is given by the relation

$$\text{"Q"} = 1/R \tag{17}$$

and with (16)

$$\text{"Q"} = \frac{\epsilon_2}{\epsilon_1} \frac{1 + \frac{\epsilon_1}{\epsilon_2} \tan^2 \frac{m}{n} \varphi}{1 + \tan^2 \frac{m}{n} \varphi} \qquad \epsilon_1 > \epsilon_2$$

and (18)

$$\text{"Q"} = \frac{1 + \frac{\epsilon_1}{\epsilon_2} \tan^2 \frac{m}{n} \varphi}{1 + \tan^2 \frac{m}{n} \varphi} \qquad \epsilon_1 < \epsilon_2$$

3. Discussion

The results of the theoretical analysis are best discussed with the aid of various examples; the different values are given in Table I. The calculated reflection coefficients r_a and r_b according to (4) and (3) are summarized in Table II, together with the smallest values κ_{min} for the losses according to (13).

Fig. 2 represents the graphical solutionsof (11) for the various cases and the roots are marked by small circles. The relative "Q" is calculated with these values from (16) and the results are summarized in Fig. 3; the length of the different lines is proportional to the relative "Q". This mode spectrum is periodic with $(n/m)\pi$ and Fig. 2 is drawn for the roots determining the modes of smallest order.

When in Fig. 1 the complete structure is mirrored on the line $z = 0$, thus region I is extended from $z = -a$ to $z = +a$, and the extra reflecting surfaces are at $z = \pm b$, then we obtain the symmetric structure as investigated in [1] . In this case even modes also occur. They alternate with the odd modes so that we obtain about twice as many longitudinal modes with about half the separation for a symmetric structure.

It is convenient to discuss the three different cases of the ratio $(\epsilon_1/\epsilon_2) \gtreqless 1$ separately:

α) $\epsilon_1 = \epsilon_2$, (example D)

The discontinuity at $z = a$ is removed, $r_a = 0$, and all possible longitudinal modes possess the same relative "Q" . The first six principal orders are sketched in Fig. 4 for the chosen ratio $(m/n) = (1/5)$. Both terminal planes at $z = 0$ and $z = b$ are considered as perfectly reflecting because of the approximation leading to (11).

(6) represents a function of the quantities a, b, m and n . In our "academic" examples we fixed the ratio m/n for convenience, so that only a or b is freely eligible. In Fig. 4 we have drawn the principal orders for an equally given distance b . With the correspondent φ-values according to (11) or from Fig. 2. The quantity a has been determined from (7). For the present case the plane $z = a$ possesses no significant meaning. But we conclude from Fig. 4 that in the "imagined" plane, $z = a$ to each value of field strength, at least one order can be associated. Therefore in a practical case in which a and b have to be considered as fixed, n and φ are determined from (7b) and with them m from (7a) for each field strength in question in the plane $z = a$. Thus the mentioned character of a multimode resonator becomes quite obvious.

β) $\epsilon_1 > \epsilon_2$,(examples A,B and C)

We see from Fig. 3 that with increasing ratio (ϵ_1/ϵ_2) , the orders $\varphi_c, (\varphi_c + 5\pi), (\varphi_c + 10\pi), \cdots$, dominate, while the relative "Q" of the other orders decreases more or less, in particular of the order φ_f, $(\varphi_f + 5\pi)$, $(\varphi_f + 10\pi)$, $\cdots$. Comparing the different principal orders of Fig. 4 we recognize that the mode φ_c has a maximum in the plane $z = a$, while the mode φ_f has a node. In order to explain this phenomenon physically, we construct a "Gedanken-Experiment". We think of the region $0 \leq z \leq a$ as filled by a medium with the reduced dielectric constant $\epsilon_1^* = (\epsilon_1/\epsilon_2)$, its small conductivity being neglected. The region $a \leq z \leq b$ shall be described correspondingly with $\epsilon_2^* = 1$. In our "Gedanken-Experiment" this second region can be considered as a selective receiver, in particular for φ_c and φ_f according to Fig. 4, the resonant condition of which is not influenced by a change of ϵ_1^* . When an electric wave coming from the left side hits the discontinuity at $z = a$, it is partially reflected in phase. Therefore those modes having an even character with respect to the plane $z = a$ are preferred, while the odd modes are less likely to be excited as shown by a decreasing "Q" with increasing ϵ_1^* . In the limit $\epsilon_1^* \to \infty$, i.e., $\epsilon_2 \to 0$, only these orders φ_c , $(\varphi_c + 5\pi)$, $(\varphi_c + 10\pi)$, $\cdots$, can be excited, being completely consistent with (12) to (14) . All those orders φ_a, φ_b, φ_d, φ_e having in the plane $z = a$ a value of field strength between zero and maximum depend upon a change of ϵ_1^* . Because of the in phase reflection at the discontinuity $z = a$, the slope of the mode across this plane decreases. Thus the frequency of the order in question decreases compared with that for the case $\epsilon_1^* = 1$.

γ) $\epsilon_1 < \epsilon_2$, (examples E and F)

We see from Fig. 3 that with a decreasing ratio ϵ_1/ϵ_2 the orders φ_f, $(\varphi_f + 5\pi)$, $(\varphi_f + 10\pi)$, $\cdots$, dominate, while the relative "Q" of the other orders decreases more or less, in particular the orders φ_c, $(\varphi_c + 5\pi)$, $(\varphi_c + 10\pi)$ ... . From Fig. 4 we know that in the plane $z = a$, φ_f possesses its node and φ_c its extreme value. Applying again our "Gedanken-Experiment" we find that an electric wave traveling from the left side toward the discontinuity at $z = a$ becomes reflected with opposite phase. Therefore these modes are preferred having an odd character with respect to the plane $z = a$, while now the "Q" of the even ones decreases with decreasing ϵ_1^* . In the limits $\epsilon_2 \to \infty$, we can consider the discontinuity as a perfectly conducting plane. Therefore, each mode, having a finite field strength in the plane $z = a$ requires an infinite amount of power, i.e. it cannot be excited. This result is again completely consistent with (12) to (14). Thus in this limit only orders φ_f, $(\varphi_f + 5\pi)$, $(\varphi_f + 10\pi)$ $\cdots$, are possible, the excitation of which requires, because of the total reflection, a vanishingly small amount of power, (13), according to the assumptions. All the other orders depend upon a change of ϵ_1^* in this case too. Now the slope across the plane $z = a$ increases because of the reflection with opposite phase and thus the frequency of the mode in question increases too.

4. Conclusion

The preceding one-dimensional analysis is based on the solution of the wave equation, where a negative conductivity is used to represent the active maser. A negative value of σ_1 corresponds to supplying sufficient gain to maintain a steady-state oscillation of the mode in question. When this assumption and the related approximations are valid, we obtain several quite interesting properties of the modified Fabry-Perot interferometer under consideration. The purpose of the extra surface is to provide discrimination between the longitudinal Fabry-Perot orders of the original maser by making some orders very lossy compared to other orders. This phenomenon is best illustrated by the various examples, Fig. 3. The amplitudes of the different modes depend upon the ratio ϵ_1/ϵ_2 . Therefore by changing or modulating this ratio, the modes are changed or modulated in their amplitudes. Because the dielectric constants are determining factors for the frequency of the modes, we find that the freely oscillating modes are additionally modulated in their eigenfrequency. If the ratio ϵ_1/ϵ_2 is modulated about the value 1 , we see from Fig. 3 that for a value larger than 1 the orders φ_c , etc. are preferred and for a value smaller than 1 the orders φ_f , etc. i.e., the selection of the "preferred" modes is modulated. But since the "preferred" modes in one case are identical to the modes with largest losses in the other case, this modulation of the "selection" seems to be very effective. --It is not essential for σ_2 to be zero, it can have a small but finite value. In this case a modulation of σ_2 would lead to similar effects but would cause increased losses.

While this analysis emphasizes the mode selectivity features of an oscillating laser device, it is also clear that they apply to a passive modified interferometer or a laser device operating below threshold, e.g. to control the gain of an amplifier.

The derivation reveals that the ratio ϵ_1/ϵ_2 represents the determining parameter, therefore it is indeed immaterial whether ϵ_1 or ϵ_2 is changed. However, this fact indicates also that a possible modulation of ϵ_1 originated by an a.c. pumping will cause an interference in the output.

Fig. 3 reveals that the separation of the longitudinal modes is a function of the ratio ϵ_1/ϵ_2 which may prove an interesting phenomenon for longitudinal mode separation.

If these theoretical results can be verified experimentally, e.g., by a photo-mixing experiment, this described modified Fabry-Perot interferometer is an interesting tool to apply to optical masers and their basic research.

When comparing this work with Kleinman's and Kisliuk's papers [1] it must be kept in mind that in contrast their analysis is based upon the magnetic field, even though they did not emphasize it.

Acknowledgement

The author acknowledges with thanks the helpful discussions with Professor N. George, Professor W. R. Smythe, Mr. K. S. H. Lee, and Mr. A. G. Lieberman. In particular he would like to express his grateful appreciation to the German Academic Exchange Service, Bonn, Germany which has enabled these studies by awarding him a NATO Fellowship.

Appendix

When the surface at $z = 0$ is not perfectly reflecting, but the reflection coefficient

$$r_o = e^{-2g} \tag{19}$$

is small so that the odd solution will still clearly dominate, then we obtain the starting equation indicated in Fig. 5. The continuity of the electric and magnetic field at $z = a$ gives the condition

$$k_2 \tan(k_1 a - ig) = - k_1 \tan[k_2(b-a) - if] \; . \tag{20}$$

With the definition

$$G = \tanh g = \frac{1 - r_o}{1 + r_o} \tag{21}$$

we obtain for the real part

$$\tan\varphi = -(\tan\frac{m}{n}\varphi)\,\frac{\sqrt{\epsilon_1/\epsilon_2} + \kappa T + G(\kappa\sqrt{\epsilon_1/\epsilon_2} + T)}{1 + \sqrt{\epsilon_1/\epsilon_2}\,\kappa T + G(T\sqrt{\epsilon_1/\epsilon_2} + \kappa)} \; . \tag{22}$$

and for the imaginary part

$$\kappa = -T\,\frac{\sqrt{\epsilon_1/\epsilon_2} - \tan\varphi\tan\frac{m}{n}\varphi + \frac{G}{T}(1 - \sqrt{\epsilon_1/\epsilon_2}\tan\varphi\tan\frac{m}{n}\varphi)}{1 - \sqrt{\epsilon_1/\epsilon_2}\tan\varphi\tan\frac{m}{n}\varphi + GT(\sqrt{\epsilon_1/\epsilon_2} - \tan\varphi\tan\frac{m}{n}\varphi)} \; . \tag{23}$$

These equations reduce to the correspondent equations (9) and (10) in the limit $G \to 0$, i.e. $g \to 0$. Therefore in the first approximation we can neglect the influence of a slightly not perfectly reflecting surface at $z = 0$, since the losses are supposed to be very small.

REFERENCES

[1] Kleinman, D.A. and Kisliuk, P.P., Discrimination against unwanted orders in the Fabry-Perot resonator, The Bell System Technical Journal 41, March 1962, pp. 453-462.

[2] Schawlow, A.L. and Townes, C.H., Infrared and optical masers, Phys. Rev. 112, Dec. 15, 1958, pp. 1940-1949.

[3] Prokhorov, A.M., Molecular amplifier and generator for submillimeter waves, J.E.T.P. 34, 1958, p. 1658.

[4] Dicke, R.H., U.S. Patent 2,851,652, September 9, 1958.

[5] Maiman, T.H., Stimulated optical radiation in ruby, Nature 187, 1960, pp. 493-494.

[6] Collins, R.J., Nelson, D.F., Schawlow, A.L., Bond, W., Garrett, C.G.B. and Kaiser, W., Coherence, narrowing, directionality and relaxation oscillation in the light emission from ruby, Phys. Rev. Letters 5, 1960, p. 303.

[7] Meissner, K.W., Interference spectroscopy, J. Opt. Soc. Am. 31, 1941, pp. 405-427; 32, 1942, pp. 185-211.

[8] Born, M. and Wolf, E., Principles of optics, Pergamon Press (1959), London, New York, Paris, Los Angeles, §7.6.2 - 7.6.4 .

[9] Fox, A.G. and Li, T., Resonant modes in a maser interferometer, The Bell System Technical Jour 40, March 1961, pp. 453-488.
And, Resonant modes in a maser interferometer, Advances in Quantum Electronics, edited by J. R. Singer,(1961) Columbia University Press, New York and London, pp. 308-317.

[10] Connes, P., Increase of the product of luminosity x resolution with interferometers by employing path differences independent of the incidence, Revue d'Optique 35, 1956, pp. 37-43; L'étalon de Fabry-Perot sphérique, J. Phys. Radium 19, 1958, pp. 262-269.

[11] Boyd, G.D. and Gordon, J.P., Confocal multimode resonator for millimeter through optical wavelength maser, The Bell System Tech. Jour. 40, March 1961, pp. 489-508; The confocal resonator for millimeter through optical wavelength masers, Advances in Quantum Electronics, edited by J. R. Singer,(1961) Columbia University Press, New York and London, pp. 318-327.

[12] Boyd, G.D. and Kogelnik, K.H., Generalized confocal resonator theory, The Bell System Tech. Jour. XLI, July 1962, pp. 1347-1369.

[13] Fox, A.G., Li, T., Kleinman, D.A. and Kisliuk, P.P., Comments on 'Discrimination against unwanted orders in the Fabry-Perot resonator', The Bell System Tech. Jour. XLI, July 1962, pp. 1475 - 1476.

[14] Kogelnik, H. and Rigrod, W.W., Visual display of isolated optical-resonator modes, Proc. IRE 50, February 1962, p. 220.

[15] Rigrod, W.W., Kogelnik, H., Brangaccio, D.J. and Herriott, D.R., Gaseous optical maser with external concave mirrors, J. Appl. Phys. 33, February 1962, pp. 743-744.

[16] Gloge, D., Gesichtspunkte zum Bau von He-Ne-Lasern für die Nachrichtentechnik, Frequenz 16, May 1962, p. 196.

[17] Lipsett, M.S. and Strandberg, M.W., Mode control in ruby optical masers of elastic deformation, Appl. Optics 1, May 1962, pp. 343-357.

[18] Evtuhov, V. and Neeland J.K., Observations relating to the transverse and longitudinal modes of a ruby laser, Appl. Optics 1, July 1962, pp. 517-520.

[19] McMurtry, B.J. and Siegman, A.E., Photomixing experiments with a ruby optical maser and a traveling-wave microwave phototube, Appl. Optics 1, January 1962, pp. 51-53.

[20] Smythe, W.R., Static and dynamic electricity, McGraw-Hill Book Company, Inc. (1950).

[21] Simonyi, K., Theoretische Elektrotechnik, VEB Deutscher Verlag der Wissenschaften, Berlin (1956).

[22] Heavens, O.S., Optical properties of thin films, Butterworth, London (1955).

Table I - Numerical Values of the Various Examples A to F

	A	B	C	D	E	F
$\sqrt{\epsilon_1^*} = \sqrt{\epsilon_1/\epsilon_2}$	10	5	2	1	0.5	0.1
T	0.02	0.02	0.02	0.02	0.02	0.02
m/n	1/5	1/5	1/5	1/5	1/5	1/5

Table II - Calculated reflection coefficients r_a and r_b, and minimum values κ_{min} representing the losses of the mode in question

	A	B	C	D	E	F
r_a	0.82	0.667	0.334	0	-0.334	-0.82
r_b	0.96	0.96	0.96	0.96	0.96	0.96
κ_{min}	0.002	0.004	0.01	0.02	0.01	0.002

Figure Titles

Fig. 1. Schematic representation of the unsymmetric modified interferometer under consideration. The constant A does not enter into the derivation.

Fig. 2. Graphic solutions of (11) for the different examples A to F . The φ-scale is indicated for the first period determining the principal modes of smallest order.

Fig. 3. A period of the calculated mode spectrum for the different examples A to F . The length of each line is proportional to "Q" of the mode in question, and the numbers are the approximate φ-values for the principal modes.

Fig. 4. The principal modes of smallest order indicated for a modified interferometer having the fixed ratio (m/n) = (1/5) .

Fig. 5. Schematic representation of the unsymmetric, modified interferometer having a not perfectly reflecting surface at $z = 0$. The constant A does not enter into the derivation.

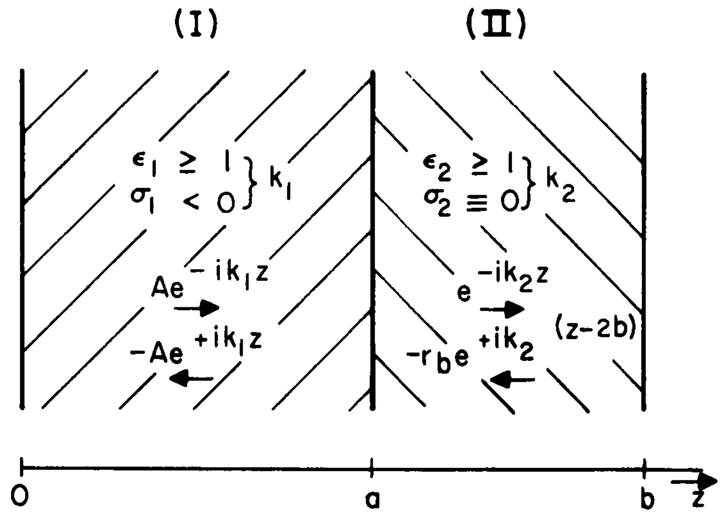


Fig. 1

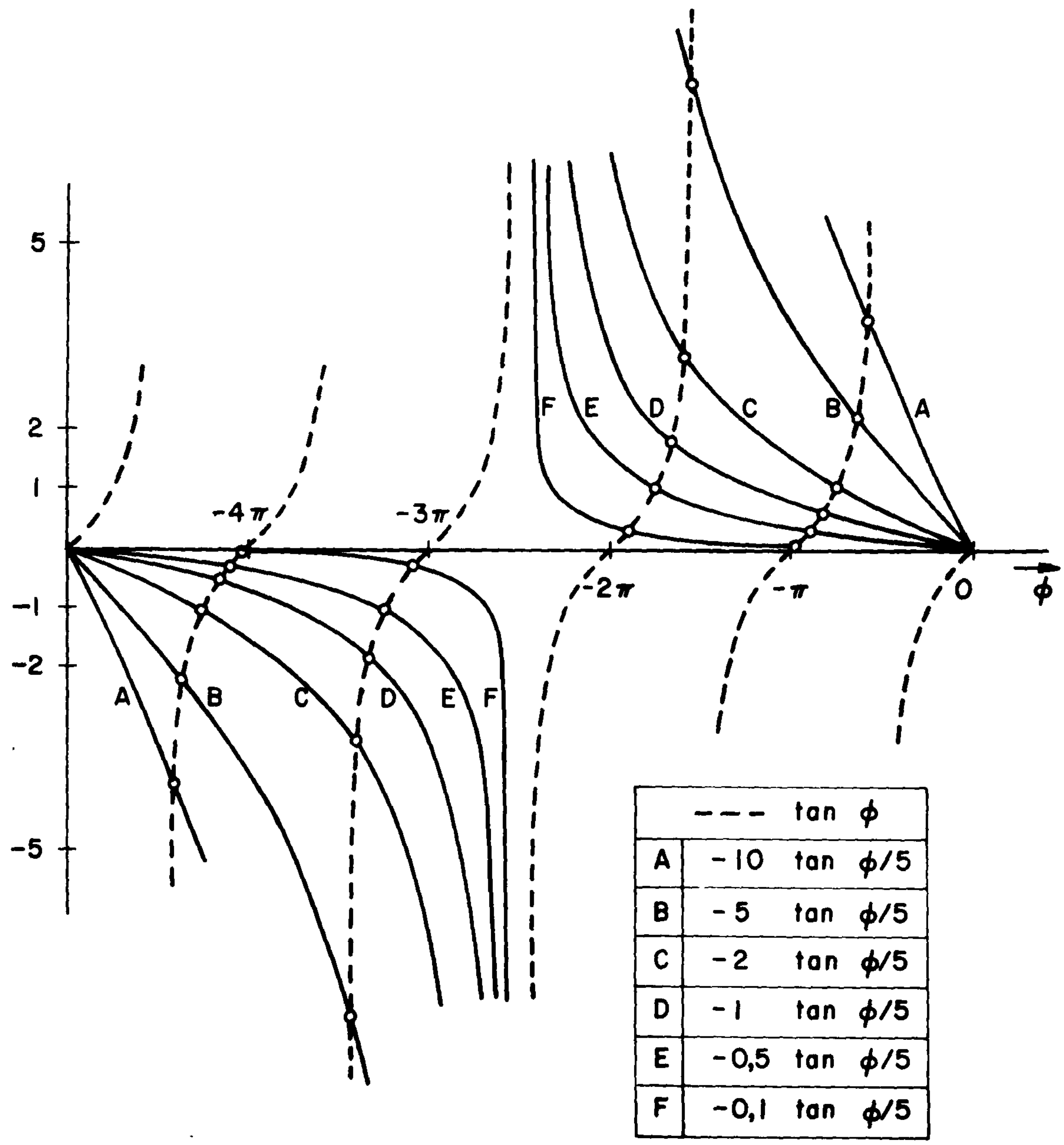
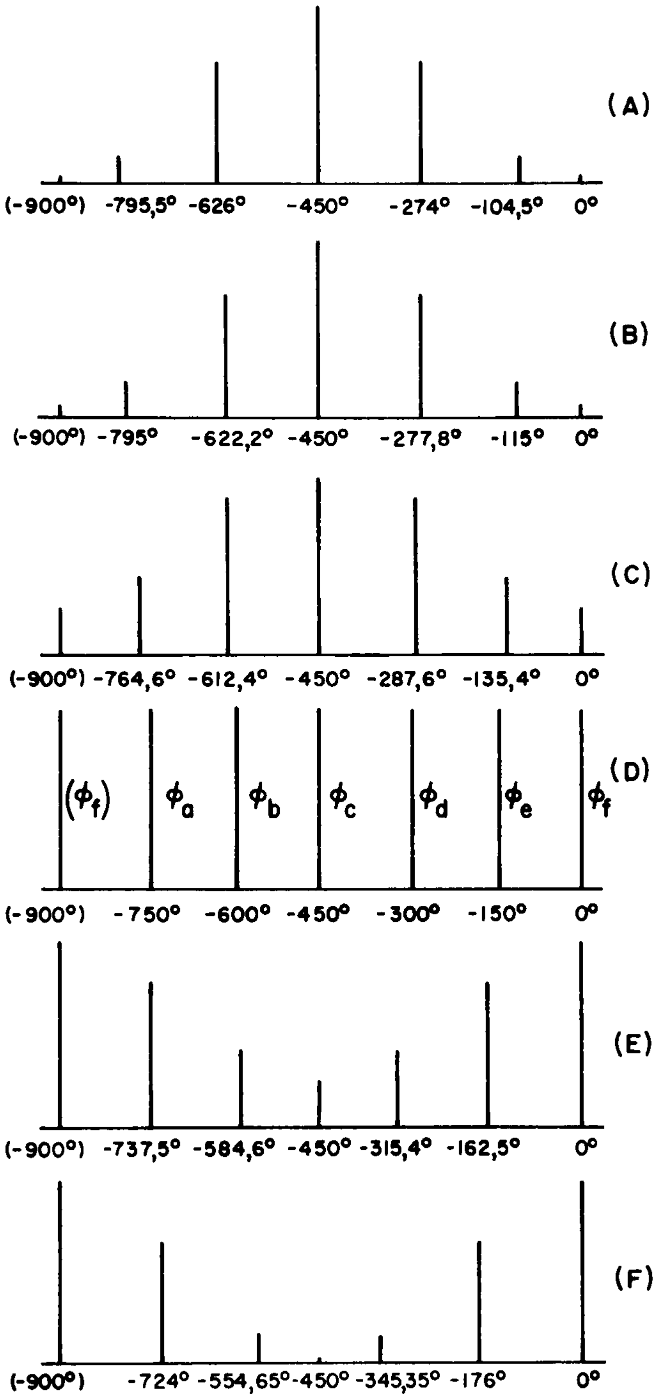
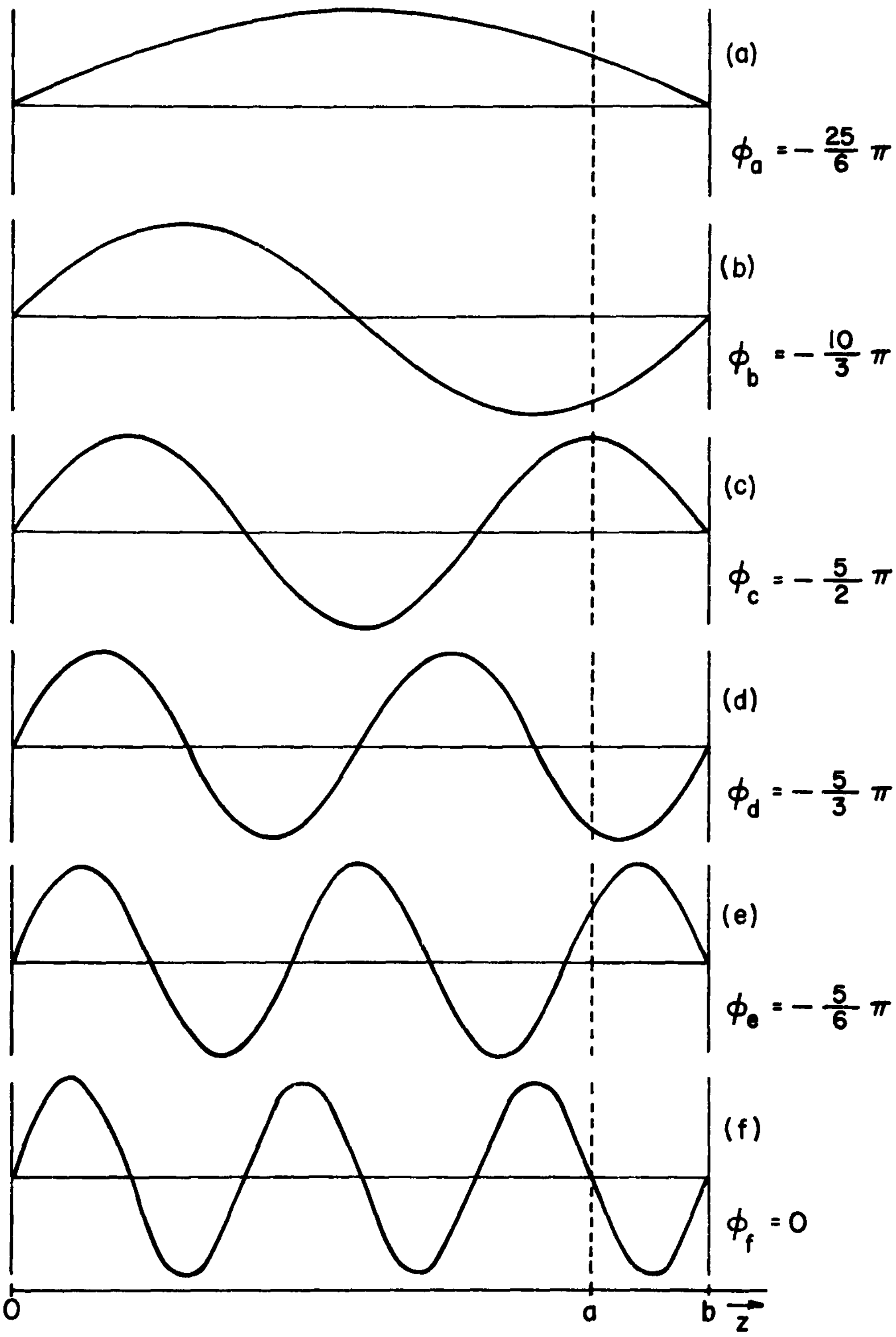

5
2
1
-1
-2
-5
-4π
-3π
-2π
-π
0
φ
A
B
C
D
E
F
--- tan φ
A -10 tan φ/5
B -5 tan φ/5
C -2 tan φ/5
D -1 tan φ/5
E -0,5 tan φ/5
F -0,1 tan φ/5

Fig.2



	A	B	C	D	E	F
$\sqrt{\epsilon_1/\epsilon_2}$	10	5	2	1	0.5	0.1

Fig 3



(a)
$\phi_a = -\frac{25}{6}\pi$
(b)
$\phi_b = -\frac{10}{3}\pi$
(c)
$\phi_c = -\frac{5}{2}\pi$
(d)
$\phi_d = -\frac{5}{3}\pi$
(e)
$\phi_e = -\frac{5}{6}\pi$
(f)
$\phi_f = 0$
0
a
b
z

Fig 4

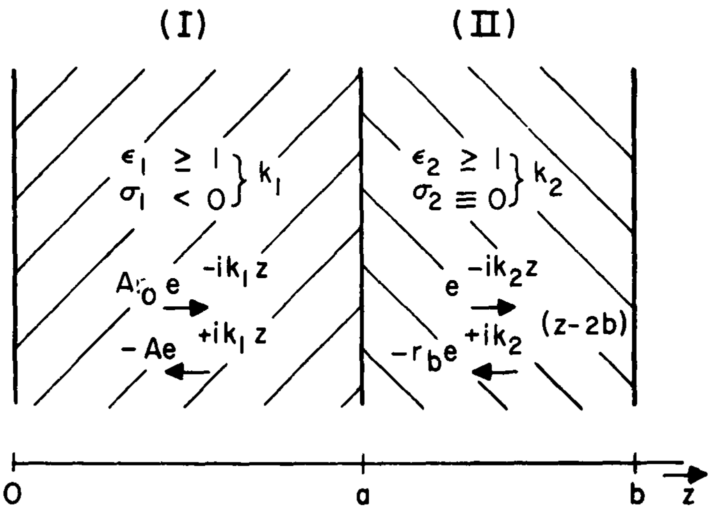

(I)
(II)
$\epsilon_1 \geq 1$
$\sigma_1 < 0$
k_1
$\epsilon_2 \geq 1$
$\sigma_2 \equiv 0$
k_2
$A_0 e^{-ik_1 z}$
$-Ae^{+ik_1 z}$
$e^{-ik_2 z}$
$-r_b e^{+ik_2 (z-2b)}$
0
a
b
z

Fig.5

DISTRIBUTION LIST

Contract AF19(604)-8052

APGC (PGAPI) (Code AF 29) 1
Eglin Air Force Base
Florida

Hq. USAF (AFOAC-S/E)(Code AF 166) 1
Communications-Electronics Dir.
Washington 25, D. C.

Director, Evans Signal Laboratory 1
Belmar, New Jersey
ATTN: Mr. O.C. Woodyard

Massachusetts Inst. of Technology 1
Signal Corps Liaison Officer
Cambridge 39, Mass.
ATTN: A.D.Bedrosian, Rm.26-131

Office of Chief Signal Officer 1
Engineering and Tech. Div.
Washington 25, D. C.
Attn: SIGNET-5

Library 2
National Bureau of Standards
Boulder Laboratories
Boulder, Colorado

AFCRL, Office of Aerospace Res. 10
CRIPA--Stop 39 (Code M 6)
L.G. Hanscom Field
Bedford, Massachusetts

AFCRL, Office of Aerospace Res. 3
Attn: Carlyle J. Slotton
L. G. Hanscom Field
Bedford, Massachusetts

Hq. AFCRL, OAR (CRIP,J.R.Marple) 1
L.G. Hanscom Field
Bedford, Massachusetts

Dr.J.I.Bohnert, Code 5210 1
U.S. Naval Research Laboratory
Washington 25, D. C.

Dr. William Brorwer 1
Diffraction Limited
Bedford, Massachusetts

Bell Telephone Laboratories 1
Murray Hill
New Jersey

Bendix Radio Division 1
Bendix Aviation Corporation
E. Joppa Road
Towson 4, Maryland
Attn: Dr. D.M. Allison Jr.

General Precision Laboratory 1
63 Bedford Road
Pleasantville, New York
Attn: Librarian

Melpar, Inc. 1
3000 Arlington Boulevard
Falls Church, Virginia
Attn: Eng. Technical Library

Microwave Associates, Inc. 1
South Avenue
Burlington, Massachusetts

National Research Council 1
Radio and E.E. Division
Ottawa, Ontario, Canada
Attn: Dr. G. A. Miller, Head
Microwave Section

Technical University 1
Oestervoldgade 10G
Copenhagen, Denmark
Attn: Prof. H. L. Knudsen

RADC (RAYLD) (Code AF 124) 1
Attn: Documents Library
Griffiss AFB, New York

WADD (WCLRSA, Mr. Portune) 1
Wright-Patterson AFB, Ohio
(Code AF 63)

Commanding General (Code Ar 5) 1
USASRDL
Ft. Monmouth, New Jersey
ATTN: Tech. Doc. Center
SIGRA/SL-ADT

Commanding General 1
USASRDL
Fort Monmouth, New Jersey
Attn: SIGFM/EL-AT

ASTIA (TIPAA) 10
Arlington Hall Station
Arlington 12, Virginia

National Bureau of Standards 1
U.S. Department of Commerce
Washington 25, D. C.
Attn: Mr. A.G. McNish

Defence Research Member 1
Canadian Joint Staff
2450 Massachusetts Ave., NW
Washington 8, D. C.

Hq. ESD (ESRDW, Maj. J.J.Hobson) 1
L. G. Hanscom Field
Bedford, Massachusetts

National Aeronautics and Sp.Adm. 1
Attn: Antenna Systems Branch
Goddard Space Flight Center
Greenbelt, Maryland

Office of Naval Research 6
Branch Office, London
Navy 100, Box 39
F.P.O. New York, New York

Dr. Edward O'Neill 1
Department of Physics
Boston University
Boston, Massachusetts

Convair, Div. of Gen.Dynamics 1
3165 Pacific Highway
San Diego 12, California
Attn: Mrs. Dora B. Burke, Lib.

Hughes Aircraft Company 1
Antenna Department
Building 12, MS 2714
Culver City, California
Attn: Dr. W. H. Kummer

ITT Laboratories 1
3700 East Pontiac Street
Fort Wayne 1, Indiana
Attn: Technical Library

The Mitre Corporation 1
244 Wood Street
Lexington 73, Massachusetts
Attn: Mrs. Claflin, Librarian

North American Aviation, Inc 1
Los Angeles International Airport
Los Angeles 45, California
Attn: Eng. Technical File

University of Toronto 1
Dept. of Electrical Eng.
Toronto, Canada
Attn: Prof. G. Sinclair

ASC (ASAPRD--Dist) 1
Code AF43
Wright-Patterson AFB, Ohio

Director, Electronics Div. 1
Air Technical Intelligence
Attn: AFCIN-4E1, Col. Gilbert
Wright-Patterson AFB, Ohio

Technical Information Office 1
European Office, Aerospace Res.
Shell Bldg., 47 Cantersteen
Brussels, Belgium

Department of the Army 1
Office of Chief Signal Officer
Washington 25, D. C.
Attn: SIGRD-4a-2

National Bureau of Standards 1
U.S. Department of Commerce
Washington 25, D. C.
Attn: Gustave Shapiro, Chief
Eng. Electronics Sect.

AFCRL, Office of Aerospace Res. 2
Attn: Contract Files
L.G. Hanscom Field
Bedford, Massachusetts

Electronic Systems Div.(AFSC) 1
Technical Information Div.
L.G. Hanscom Field
Bedford, Massachusetts (ESAT)

Director 1
U.S. Naval Research Laboratory
Washington 25, D. C.
Attn: Code 2027

Dr. George Parrent 1
Tech Ops, Inc.
Burlington, Massachusetts

AFSC Scientific and Technical
Liaison Office 1
c/o Department of the Navy
Room 2305, Munitions Bldg.
Washington 25, D. C.

ITT Federal Laboratories 1
Technical Library
500 Washington Avenue
Nutley 10, New Jersey

General Electric Company 1
Electronics Park
Syracuse, New York
Attn: Documents Library
B. Fletcher, 3-143A

Hughes Aircraft Company 1
Florence and Teale Streets
Culver City, California
Attn: Louis L. Bailin, Mgr.
Antenna Department

Motorola, Inc. 1
8201 East McDowell Road
Phoenix, Arizona
Attn: Dr. Thomas E. Tice

Philco Corporation 1
Research Division
Union Meeting Pond
Blue Bell, Pennsylvania
Attn: Research Librarian

University of Wisconsin 1
Dept. of Electrical Eng.
Madison, Wisconsin
Attn: Dr. Scheibe

Distribution List, Continued

RCA Laboratories 1
David Sarnoff Research Center
Princeton, New Jersey
Attn: Miss F. Cloak, Librarian

Director, USAF Project RAND 1
Via: A.F. Liaison Office
The Rand Corporation
1700 Main Street
Santa Monica, California

Raytheon Company 1
Wayland Laboratory
Wayland, Massachusetts
Attn: Alice G. Anderson, Lib.

Sylvania Electric Products Inc. 1
100 First Avenue
Waltham 54, Massachusetts
Attn: C.A. Thornhill, Librarian

Brown University 1
Department of Electrical Eng.
Providence, Rhode Island
Attn: Dr.C.M.Angulo

University of Southern California 1
University Park
Los Angeles 7, California
Attn: Dr. Raymond L. Chuan

Columbia University 1
Dept. of Electrical Engineering
Morningside Heights
New York, New York
Attn: Dr. Schlesinger

University of Florida 1
Dept. of Electrical Engineering
Gainesville, Florida
Attn: Prof. M.H.Latour, Library

Illinois Inst. of Technology 1
Technology Center
Dept. of Electrical Engineering
Chicago 16, Illinois
Attn: Paul C. Yuen
Electronics Res. Lab.

University of Kansas 1
Electrical Engineering Dept.
Lawrence, Kansas
Attn: Dr. H. Unz

Massachusetts Inst. of Technology 1
Research Lab. of Electronics
Bldg. 26, Room 327
Cambridge 39, Massachusetts
Attn: John H. Hewitt

University of Michigan 1
Engineering Research Institute
Willow Run Laboratories
Ypsilanti, Michigan
Attn: Librarian

Northwestern University 1
Microwave Laboratories
Evanston, Illinois
Attn: R. E. Beam

Polytechnic Institute Brooklyn 1
Microwave Research Institute
55 Johnson St., Brooklyn, N.Y.
Attn: Dr. Arthur A. Oliner

Polytechnic Institute Brooklyn 1
Microwave Research Institute
55 Johnson St., Brooklyn, N.Y.
Attn: Mr. A.E. Laemmel

Radio Corporation of America 1
Missile Control and Elec. Div.
Bedford Street
Burlington, Massachusetts
Attn: Librarian

The Rand Corporation 1
1700 Main Street
Santa Monica, California
Attn: Technical Library

Document Acquisitions Library 1
Space Technology Labs, Inc.
P.O. Box 95001
Los Angeles 45, California

TRG, Inc. 1
400 Border Street
East Boston, Massachusetts
Attn: Dr. Alan F. Kay

Space Sciences Laboratory 1
Leuschner Observatory
University of California
Berkeley 4, California
Attn: Dr. Samuel Silver

Case Institute of Technology 1
Electrical Engineering Dept.
10900 Euclid Avenue
Cleveland, Ohio
Attn: Prof. Robert Plonsey

University of Southern Calif. 1
University Park
Los Angeles 7, California
Attn: Z.A. Kaprielian, EE Dept.

Georgia Technology Research Inst. 1
Engineering Experiment Station
722 Cherry Street, NW
Atlanta, Georgia
Attn: Mrs. Crosland, Librarian

Illinois Institute of Technology 1
3301 S. Dearborn Street
Chicago 16, Illinois
Attn: Dr. George I. Cohn

Lowell Technological Institute 1
Research Foundation
P.O. Box 709
Lowell, Massachusetts
Attn: Dr. C. R. Mingins

Massachusetts Inst. of Technology 1
Lincoln Laboratory
P.O. Box 73
Lexington 73, Massachusetts
Attn: Mary A. Granese, Librarian

University of Minnesota 1
Minneapolis 14, Minnesota
Attn: R. H. Stumm, Library

Antenna Laboratory 1
Ohio State University
2024 Neil Avenue
Columbus 10, Ohio
Attn: Reports Librarian

University of Pennsylvania 1
Inst. of Cooperative Research
Philadelphia, Pennsylvania
Attn: Dept. of Electrical Eng.

Library 1
W.W. Hansen Lab. of Physics
Stanford University
Stanford, California

University of Washington 1
Seattle 5, Washington
Attn: D.K. Reynolds
Electrical Eng. Dept.

Radio Corp. of America 1
West Coast Division
8500 Balboa Boulevard
Van Nuys, California
Attn: L. R. Hund, Librarian

Rantec Corporation 1
23999 Ventura Boulevard
Calabasas, California
Attn: Grace Keener

Stanford Research Institute 1
Documents Center
Menlo Park, California
Attn: Acquisitions

Westinghouse Electric Corp. 1
Electronics Division Library
Friendship Int'l Airport 1897
Baltimore 3, Maryland

University of California 1
Electronics Research Lab.
332 Cory Hall,
Berkeley 4, California
Attn: J. R. Whinnery

Cornell University 1
School of Electrical Eng.
Ithaca, New York
Attn: Prof. G.C. Dalman

Harvard University 1
Technical Reports Collection
Gordon McKay Library
303 Pierce Hall, Oxford St.
Cambridge 38, Massachusetts

University of Illinois 1
College of Engineering
Urbana, Illinois
Attn: Dr. P. E. Mayes
Dept. of Elec. Eng.

The John Hopkins University 1
Applied Physics Laboratory
8621 Georgia Avenue
Silver Spring, Maryland
Attn: Mr. George L. Sielstad

McGill University 1
Dept. of Electrical Eng.
Montreal, Canada
Attn: Dr. T. Pavlasek

University of Michigan 1
Radiation Laboratory
912 N. Main Street
Ann Arbor, Michigan
Attn: Mr. Ralph E. Hiatt

Physical Science Laboratory 1
New Mexico State University
University Park, New Mexico
Attn: Mr. H.W. Haas

University of Oklahoma 1
Research Institute
Norman, Oklahoma
Attn: Prof. C. L. Farrar

Pennsylvania State University 1
Dept. of Electrical Eng.
University Park, Pennsylvania

Syracuse University Res.Inst. 1
Collendale Campus
Syracuse 10, New York
Attn: Dr. C. S. Grove, Jr.
Director, Engineering Res.

University of Texas 1
E.E. Research Laboratory
Austin 12, Texas
Attn: John R. Gerhardt