MAGNETOHYDRODYNAMICS AND ENERGY CONVERSION

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XV. PLASMA MAGNETOHYDRODYNAMICS AND ENERGY CONVERSION[*]

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RESEARCH OBJECTIVES

1. Plasma Magnetohydrodynamics

Our group is pursuing the intermediate objectives of improving the reliability and capability of the existing magnetically driven shock tube, for the purpose of increasing the reproducibility of the resulting data, enlarging the amount of plasma generated by the shock wave, increasing the resulting shock-front velocity, and making possible examination of unusual shock transitions and properties of the shock layer.

The reliability and reproducibility seem to be related to the breakdown behavior of the drive current, so experiments are being conducted which should make this breakdown more uniform in spatial distribution and give the drive current a more advantageous time distribution.

The capability is now limited by failure of shocks to form at low initial pressures (less than $50\ \mu$ Hg of hydrogen) and by leakage of plasma past the drive current sheet through the boundary layer. Removal of these barriers is the key to achieving the experimental goals outlined above.

Simultaneously, efforts are being made to devise or improve methods for taking experimental data because existing methods are not adequate for the task of obtaining detailed flow information, and are often very costly to implement.

A. H. Shapiro, W. H. Heiser

2. Energy Conversion

(a) Magnetohydrodynamic Energy Conversion

(i) The objective of our research in magnetohydrodynamic energy conversion is to study theoretically and experimentally the properties of parametric and wave-type power generators to determine their ranges of applicability. Our

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interest is primarily in large-signal operation of the type that might be suitable for large-scale power generation.

During the past year, progress has been made in devising a mathematical model to describe the small-signal behavior of the parametric machine. This model was verified with a solid-conductor analog, and the model was used to predict the small-signal behavior of parametric machines with gaseous conductors.[1] Small-signal experiments were performed with a magnetically driven shock tube to verify the analysis of generator operation with a gaseous conductor. In these experiments net power was generated.[2]

During the past year, our research on wave-type power generators has been directed toward achieving in a modified homopolar experiment the plasma conditions (flow velocity and conductivity) necessary for studying the amplifying interaction between magneto-acoustic waves and distributed electric circuits. Thus far, we have obtained a uniform, flowing plasma, but the conductivity is too low for wave experiments. Modifications are being made to improve the plasma conductivity in the experiment.

H. H. Woodson

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(ii) We are concerned with electrohydrodynamic surface interactions. Previous investigations have clarified the fundamental aspects of wave propagation and instability,[1,2] with some consideration given to the nonlinear aspects of the problem.[3,4] Our investigations are now devoted to understanding the ways in which growing field-coupled surface waves can be used to provide electromechanical energy conversion. We have been shown that the mechanical medium can be an ionized gas[5] or a fluid.[6] Our present concern is with the effects of external electrical coupling and internal mechanical losses.

J. R. Melcher

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(b) Thermionic Energy Conversion

The present research objectives of our group are oriented toward a better understanding of the mechanisms controlling the performance characteristics of cesium thermionic converters. Toward the achievement of this goal, the following areas are under investigation.

(i) Theoretical and experimental studies of the effect of crystallographic orientation on the electron emission properties of a material both in vacuum and in cesium.

(ii) Theoretical and experimental studies of the transport properties of the interelectrode gap in cesium thermionic converters.

(iii) Measurements of the thermal conductivity of the cesium vapor over a wide range of temperatures.

(iv) Experimental studies of the oscillations observed at low cesium pressures.

E. N. Carabateas

3. Alkali-Metal Magnetohydrodynamic Generators

The over-all objective of our program is to determine the electrical properties of both superheated and wet flowing alkali-metal vapors at temperatures near 2000°K. A small potassium boiler, superheater, and condenser are now being constructed.

During the coming year, we aim to complete and check out the plasma facility, and to measure the electrical conductivity of the plasma both with and without a magnetic field. Theoretical studies of the conductivity of the wet vapor will also be completed.

J. L. Kerrebrock, M. A. Hoffman, G. C. Oates

4. Magneto-Fluid Dynamics

This group is principally concerned with interactions between electromagnetic fields and those electrically conducting fluids that can be treated on a continuum basis. Our work includes theoretical and experimental aspects and involves both the investigation of magnetohydrodynamic phenomena and their utilization for engineering applications, particularly for electrical power generation. The following problems are receiving attention.

(a) Magnetohydrodynamic Wave Phenomena

One of our experiments is concerned with the excitation of Alfvén waves in a liquid metal (NaK alloy). Electrical excitation by means of a current sheet has proved to be markedly superior to the mechanical methods used in experiments reported previously. Efforts are, at present, directed toward a systematic study of the excitation, transmission, attenuation, and reflection of these waves in the frequency range up to approximately 10 kc.

A second waveguide study is concerned with MHD wave propagation in nonuniform plasmas. A high-density cesium glow discharge is being set up for the experimental part of this investigation.

W. D. Jackson, J. P. Penhune, G. B. Kliman, N. Gothard, C. W. Rook

(b) Moving Space-Charge Waves in a Plasma

The nature of certain macroscopic instabilities observed in the plasma of glow discharge tubes is being examined. Traveling waves of electron and ion density have been observed spontaneously arising within several noble gas plasmas. These waves have also been generated with sounding probes and with external excitation. A linearized theory has been developed and its predictions are now being correlated with our experimental observations.

W. D. Jackson, R. S. Cooper

(c) Magnetohydrodynamic Channel Flow and Turbulence

The flow characteristics of electrically conducting fluids in channels or ducts are of interest in connection with many engineering applications of magnetohydrodynamics. While these include both liquid and ionized gas flows, the use of liquid metals has advantages for a considerable range of laboratory investigations.

The major experimental effort at the present time involves the construction of an NaK flow loop that will be used for studies of pressure-drop versus flow-rate relations (including those for MHD power-conversion devices) and the characteristics of turbulence in the presence of magnetic fields.

The theoretical treatment of turbulent-flow problems, thus far, has had only partial success and the need for a rational method has long been evident. An attempt is being made to develop Weiner's "Calculus of Random Functionals" and to apply it to both sustained and decaying turbulent-flow situations.

W. D. Jackson, J. M. Reynolds III, J. R. Ellis, Jr.,
F. W. Fraim IV, H. D. Jordan

(d) Mathematical Methods in Continuum Magnetohydrodynamics

This research is concerned with mathematical methods for the analysis of the interactions occurring in magnetohydrodynamics. The present work has grown out of the investigation of MHD channel flows; particularly out of the analytical and numerical techniques that were used to obtain solutions to the nonlinear differential equation governing the interaction of a traveling ac magnetic field with an MHD channel flow.

Our research concerns the application of techniques, such as perturbation expansions and iterational and variational methods, to a variety of nonlinear continuum MHD problems. The purpose of this effort is twofold: First, to produce solutions to specific problems that are of practical interest; second, to obtain a better understanding of the broad classes of problems to which these techniques are applicable.

J. P. Penhune

(e) Local Fluid-Velocity Measurement in an Incompressible Magnetohydrodynamic Flow

The behavior of several different types of probes is being investigated to develop devices for measuring the local fluid veloctiy in an MHD flow for the case in which the applied magnetic field is perpendicular to the fluid velocity. The development of such probes will be important for experimental investigation of MHD flows, particularly those associated with MHD power-generation devices.

Three types of probe are being investigated experimentally. The first is the standard Pitot tube in which the $\bar{J} \times \bar{B}$ force raises the fluid pressure at the stagnation point above the usual stagnation pressure of the fluid. The velocity-pressure relation is being determined as a function of the magnetic field. A second approach is the investigation of a two-dimensional aerofoil aligned parallel to the magnetic field. The last probe is a miniaturized electromagnetic flowmeter for which the calibration depends on the fluid Reynolds and Hartmann numbers, as well as on the local velocity.

A. H. Shapiro, W. D. Jackson, D. A. East, J. H. Olsen

(f) Liquid-Metal Magnetohydrodynamic Power Systems

The generation of electrical power on space vehicles offers a potential application for MHD generators to operate on a closed-cycle system in which a nuclear reactor is the termal-energy source. An important feature of an MHD scheme is the absence of rotating parts and, to utilize this, a working fluid is required with a sufficiently high electric conductivity at the temperatures involved.

A scheme in which a liquid metal is used as the working fluid is under investigation. Kinetic energy is imparted to this flow by driving it with its own vapor in a condensing-ejector system. The operation of this system on alkali metals is being studied, and the relative merits of ac and dc generators for this application are being examined.

W. D. Jackson, G. A. Brown

(g) Magnetohydrodynamic Induction Generator

The MHD induction machine utilizes the interaction between a traveling magnetic field (such as that produced by a polyphase winding) and a channeled, flowing fluid that may be either a plasma or a liquid metal.

The theoretical analysis of this machine has been completed for the case of slug flow of an incompressible fluid. It is now being extended to include entrance and exit effects, velocity profiles, and compressible fluids.

A two-phase, linear traveling-field coil system is in the course of construction. It will be used to study the operating characteristics of the induction generator on both plasma and liquid-metal flows.

W. D. Jackson, E. S. Pierson, M. H. Reid

(h) A-C Properties of Superconductors

Recent intensive efforts to fabricate hard superconductors have opened up a wide range of possibilities for utilizing these materials in the production of high dc fields, particularly in situations for which these are required in large volumes. The advantages associated with reducing field-power dissipation also apply to the production of ac fields, but there is an additional problem in that reactive power has to be circulated. This problem implies essentially zero-loss capacitive energy-storage elements, in addition to essentially infinite Q inductors. It is thus of interest to investigate the behavior of superconducting materials carrying ac currents in the presence of ac magnetic fields. As well as establishing the merits of superconductivity materials in inductor and capacitor fabrication, such investigations provide an additional method of gaining insight into the mechanism of superconductivity.

Present investigations deal with superconducting materials in the form of wire or ribbon, and two experimental techniques are being pursued.

(i) The current-carrying capacities of short, straight lengths of superconducting wire or ribbon are being determined as a function of frequency in the range up to 10 kc.

(ii) A-C solenoids, fabricated to avoid electric eddy currents and insulated to accommodate electrical fields arising from $\partial B/\partial T$ effects, are being tested. In both cases, the ac current required for transition to normal conductivity is obtained and, in the case of solenoids, the measurement of Q is being attempted.

A third investigation is planned to obtain data on the behavior of superconductors in an externally applied ac field. These will be derived either from a rotating magnet system or from a separate copper-conductor ac solenoid.

The work is, at present, experimental in character, but future theoretical studies are envisaged.

W. D. Jackson, A. N. Chandra, C. R. Phipps, Jr.

(i) Blood-Flow Studies*

Another activity is concerned with medical engineering, and is being carried on in association with the Cardiovascular Laboratories of the Peter Bent Brigham Hospital, Boston. Since my own interest is primarily in the use of magnetohydrodynamic methods for blood-flow measurement, there is, in fact, a close relation to the field of magnetofluid dynamics. The scope of the group, however, is rather broader than this topic alone would imply.

The aim of our work is to apply engineering methods to the study of the cardiovascular system, and we are engaged in this jointly with Dr. Dexter and his associates at the Peter Bent Brigham Hospital. At present, we are evaluating flow-measuring devices (electromagnetic or magnetohydrodynamic, ultrasonic and thermistor) as a preliminary approach to the study of the characteristics of blood flow. A second aspect of our work is the identification and analysis of the mechanisms responsible for the regulation of cardiovascular functions.

W. D. Jackson

A. WORK COMPLETED

1. THE EFFECT OF SWIRL ON THE ELECTROMAGNETIC FLOWMETER

This research has been completed by H. D. Meyer and the results have been accepted by the Department of Mechanical Engineering, M.I.T., as a thesis in partial fulfillment of the requirements for the degrees of Master of Science and Bachelor of Science, September 1962.

W. D. Jackson, J. M. Reynolds III

2. EXPERIMENTAL CHARACTERISTICS OF A PLASMA JET

This research has been completed by M. D. Leis and the results have been accepted by the Department of Electrical Engineering, M.I.T., as a thesis in partial fulfillment of the requirements for the degree of Bachelor of Science, June 1962.

W. D. Jackson

3. BLOOD-FLOW STUDIES*

The present phase of this work has been completed and the results have been accepted as theses by the Department of Electrical Engineering, M.I.T., in partial fulfillment of the requirements for the degrees indicated.

P. G. Katona,"Analysis of Blood-Pressure Regulation Using Correlation Techniques," S.M. Thesis, June 1962.

J. R. Ellis, Jr., "Relative Merits of Certain Channel Sections in Electromagnetic Flowmeters," S. B. Thesis, June 1962.

J. E. Thompson, "Thermistors as Blood Flow-Rate Transducers," S.B. Thesis, June 1962.

J. D. Cervenka, "Flow Measurement Using Point-Contact Electrodes," S.B. Thesis, June 1962.

W. D. Jackson

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B. EXPERIMENTAL MEASUREMENT OF THE THERMAL CONDUCTIVITY OF CESIUM VAPOR

An experimental measurement of the thermal conductivity of cesium vapor was made by a variation of the hot-wire cell method.[1] The hot-wire cell was operated at 600°K. At this temperature level the heat transfer resulting from thermal radiation was considerably greater than that resulting from thermal conduction. Because of this unfavorable ratio of heat fluxes, slight uncertainties in the thermal emissivity of nickel covered with a surface layer of cesium can result in large errors in the observed value of the thermal conductivity of the cesium vapor. Two cells were therefore used to measure simultaneously the thermal conductivity of cesium vapor and the thermal emissivity of nickel in a moderate-temperature cesium gas. The two cells were constructed so that the heat fluxes resulting from radiation were identical. End corrections for the finite-length tubes were made initially by measuring the thermal conductivities of air at room temperature with each cell individually and with both cells simultaneously, as was done at higher temperatures with cesium.

The thermal conductivity cells consisted of nickel wires, which were used as both resistance heaters and resistance thermometers, placed along the axis of copper tubes. Nickel and copper were used because of their stability in a cesium atmosphere and their availability.

The heat transfer[2] by radiation between a wire and a surrounding tube is

$$\frac{q}{A_w} = \left(\frac{1}{\left(\frac{1}{\epsilon_w} - 1\right) + \frac{D_w}{D_t}\left(\frac{1}{\epsilon_t} - 1\right) + \frac{1}{\bar{F}_{tw}}} \right) \sigma \left(T_w^4 - T_t^4\right). \tag{1}$$

The wire and tube emissivities were anticipated to be approximately 0.1 (later verified). For a wire diameter of 0.010 inch and tube diameters of 3/8 inch and 1/2 inch

$$\frac{1}{\epsilon_w} - 1 \approx 9$$

$$\frac{D_w}{D_t}\left(\frac{1}{\epsilon_t} - 1\right) \approx 0.18 \text{ and } 0.24$$

and

$$\bar{F}_{tw} = 1.00.$$

Thus the difference in radiative heat transfer in the two apparatus operating at the same tube and wall temperatures was 0.6 per cent.

End corrections for axial conduction along the wires to the ends were the same for the two cells. The cells were operated at pressures low enough so that the effect of natural convection on the heat transfer rate was less than 0.1 per cent of that caused by conduction. Convection amounts to 1 per cent of conduction at a Grashoff number of 1570, and 25 per cent at a Grashoff number of 2200.[3] The Grashoff number in the present apparatus was approximately 12. The energy transferred by thermionically emitted electrons in the cell was computed to be approximately 2×10^{-10} watt, well below the range of accuracy of the other measurements. Errors attributed to misalignment of the heated wire were calculated to be less than 0.1 per cent.

The difference in heat transfer in the two cells was attributed only to the difference in the gas conduction. Thus,

$$\frac{\Delta q}{\Delta T} = 2\pi k\, L\left(\frac{1}{\ln\frac{r_{o1}}{r_i}} - \frac{1}{\ln\frac{r_{o2}}{r_i}}\right). \tag{2}$$

Two measurements of the thermal conductivity of cesium vapor were made with the apparatus. The values obtained were 0.003046 Btu/hr ft°F at 1085°R (603°K), and 0.002946 at 1080°R (600°K). The average of these values is 0.002996 ± 1.6 per cent (experimental scatter). For a Prandtl number of 0.7 and a specific heat that includes the change in dimerization with temperature, the viscosity of cesium vapor is calculated to be 0.0547 lbm/hr ft (4.72×10^{-6} $\mathrm{lb_f}$ sec/ft^2).

S. I. Freedman, J. H. Sununu

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C. WORK FUNCTION OF A CONDUCTOR

Finding many descriptions of the electron emission process very unsatisfactory, the author presents a report that he hopes will resolve most of the problems, give a new insight into the emission process, and help in predicting emission properties of materials. Following is the author's concept of an electron emitting surface, which he is now attempting to correlate with the large amount of existing experimental data.

It is assumed that the total potential energy change in removing an electron from a crystal is made up of three parts that add in a linear fashion: monopole work, dipole work, and work against external electrostatic fields.

Monopole work is defined as the contribution to the potential energy change of an electron (which is removed from a conductor) arising from interactions with induced surface charges that are induced by the escaping electron itself and not by external sources. The monopole work is simply an extension of the concept of "image-force work" into the surface electron cloud and into the crystal. Monopole work is that contribution to the total potential energy change which depends on the square of the electronic charge. The name "monopole work" is given because these forces vary as the inverse square of the distance from the surface.

Dipole work is defined as the contribution to the potential energy change of an electron (which is removed from a conductor) arising from interactions with permanent surface electrostatic fields. To be precise, dipole work equals the charge on an electron times the difference in electrostatic potential between the average potential inside the conductor and the potential at infinity of a conductor of uniform surface characteristics. The name "dipole work" is given because these forces are caused by a surface dipole or double layer.

External electrostatic fields are those fields caused by external sources and by surface charges induced on the conductor in order to maintain a uniform average potential inside the conductor.

It is well established that monopole work contributes substantially to the total potential energy difference between an electron inside a conductor and the same electron outside the conductor (Schottky effect). The existence of dipole work is a subject of considerable debate and speculation, which the author hopes to resolve in part. Electrostatic fields are generally considered to exist around the conductor so as to make the electrostatic potential at the surface of the conductor vary as the work function divided by the electronic charge. This point will be proved rigorously in the proof of Theorem 1.

Work function, for the purpose of this report, means the difference in energy between an electron at the Fermi energy and an electron at rest just outside the conductor. High fields at the surface of the conductor will not be considered in this treatment so that questions of Schottky effect and the meaning of "just outside the conductor" are not difficult to answer.

THEOREM 1: The electrostatic fields that exist around an isolated conductor are such that the difference in electrostatic potential between any two points near the surface of the conductor equals the difference in work function between the two corresponding regions of the surface, divided by the charge on the electron.

PROOF 1: Consider a process in which an electron at the Fermi energy is removed from the interior of a conductor through a surface of work function ϕ_1, moved through an external electrostatic potential difference $\Delta\Psi_{12}$, and then returned to the metal at the

Fermi energy through a surface work function ϕ_2. The change in energy for this process is

$$\Delta E = \phi_1 + e\Delta\Psi_{12} - \phi_2.$$

But, if the Fermi energy is uniform throughout the metal, the energy change must equal zero. Therefore,

$$\Delta\Psi_{12} = \frac{\phi_2 - \phi_1}{e}.$$

Since the process can be carried out through any two regions of the surface, the proof is completed.

This result is not surprising if one remembers the thermodynamic definition of work function (which, incidently, agrees with the definition stated above when the definition of μ is chosen to agree with the statistical mechanical definition),

$$\phi = - e\Psi - \mu.$$

Here, Ψ is the electrostatic potential that exists just outside the surface in question, and μ is the chemical potential of the electrons in the conductor.

THEOREM 2: The variation of work function with the nature of a surface must be caused by a change in the surface double layer. The monopole work associated with any surface of a conducting material of uniform composition is independent of the details of the surface.

PROOF 2: In a conductor of uniform composition the average electrostatic potential in the interior is constant. Therefore, the net field from all external sources and surface charges, taken together, must be zero inside the conductor. However, in the proof of Theorem 1 it was shown that the electrostatic potential varies as the work function just outside the conductor. In order to satisfy Maxwell's equations, there must be a surface double layer whose strength varies as the work function. That is, the dipole work associated with a surface varies as the work function.

The work function equals the monopole work M, plus the dipole work D, minus the electron degeneracy energy ζ.

$$\phi_1 = M_1 + D_1 - \zeta$$

$$\phi_2 = M_2 + D_2 - \zeta.$$

Since the dipole work varies as the work function and the electron degeneracy energy is constant, the monopole work must be constant and therefore independent of the details of the surface, which was to be proved.

When an electron has first entered the electron cloud of the surface, and before it

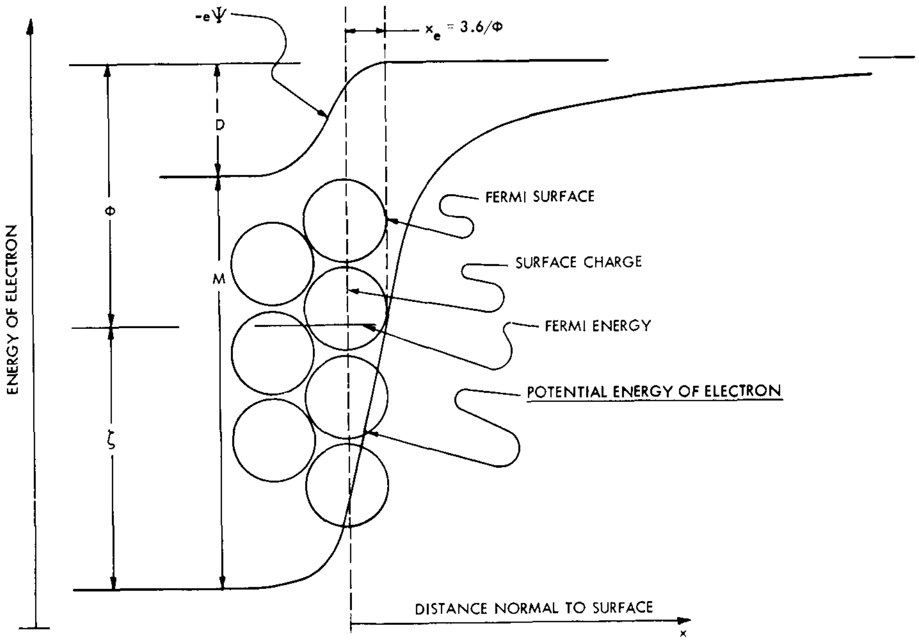

Fig. XV-1. Energy diagram of a surface.

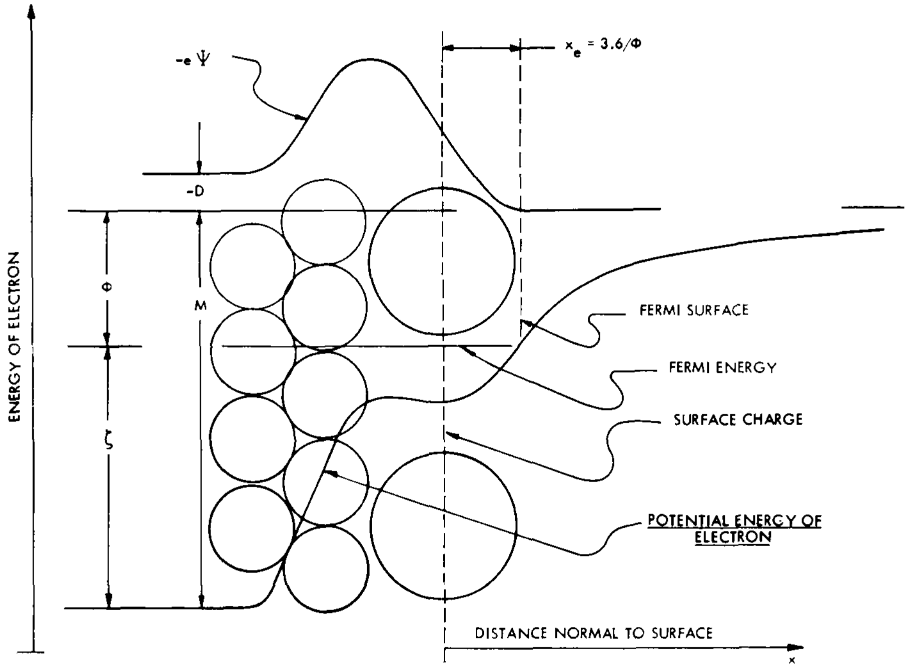

Fig. XV-2. Energy diagram of a surface with an adsorbed ionizible layer.

has reached the first plane of nuclei, it is repelled by the electron cloud and is attracted by the nuclei. This is the region of the surface dipole layer. The field of an electron is given by

$$\bar{E} = -\frac{14.4}{r^2}\,r,$$

where E is in volts per angstrom and r is in angstroms. Electronic separations are of the order of angstroms, as is the thickness of the dipole layer. This is adequate to provide the several electron volts of dipole work that is necessary to explain a work-function variation of an electron volt from one crystallographic surface to another.

Since electric fields exist outside the conductor, but not inside, there also must be a surface charge equal to the external field times the permittivity of free space. The surface charge is an induced charge and is dependent on the distribution of the work function over the surface of the conductor, as well as the presence of externally applied fields, whereas the double layer is permanent and determines the work function.

Theorem 2 rests heavily on the assumption that the work function is made up of two parts, that is, monopole work and dipole work. At the present time, it is not at all clear whether or not this assumption is warrented. Other terms may be important. However, any other terms that do prove to be important can be formally combined with the monopole work, and the important result of Theorem 2 is preserved: the variation of work function with the nature of a surface must be caused by a change in the surface double layer, alone.

It is generally recognized that a close-packed surface has a larger proportion of electrons outside the first plane of nuclei than a less densely packed surface, and thereby presumably has a stronger double layer (and therefore higher work function) than a less densely packed surface. Also, a close-packed surface has more nuclei per unit area to produce a stronger double layer. This prediction is in qualitative agreement with many experimental observations. Calculations are now being made to relate these ideas to the experimental data in a quantitative fashion.

The determination of the electron distribution near the surface of a conductor covered with an ionizable adsorbed film is a difficult problem. Some guess must be about the nature of the adsorbed particles. Are they atoms, ions or something in between? Rasor[1] assumes that surface particles are either atoms or ions in a number given by their thermal excitation probabilities. In effect, Rasor calculates the modification of the dipole work by the adsorbed particles. It is the author's belief that this is the reason for Rasor's striking success in computing the change in the work function as a function of surface coverage. The change in the dipole work equals the change in the work function.

Figures XV-1 and XV-2 show schematically the thermionic emitting surface described in this paper. Figure XV-1 shows a bare metal surface; Fig. XV-2 shows a surface partially covered with ionizable atoms (positive ions shown). A few surface atoms are shown; the conductor is to the left, free space to the right. The potential energy of an electron is plotted against the distance normal to the surface x, measured from the effective plane of the surface charge mentioned in the Schottky model. The potential energy, shown smooth inside the conductor, is the average potential energy used in the free-electron gas approximation. The electrostatic potential times the electronic charge is also shown plotted against the distance normal to the surface.

The Fermi energy is shown, as is the Fermi surface, that is, the surface on which Fermi electrons have their classical turning point. A physical interpretation is given of the Schottky cutoff distance x_e,

$$x_e = \frac{1}{4\pi\epsilon_o} \cdot \frac{e^2}{4\phi} = 3.6/\phi,$$

where x_e is in angstroms, and ϕ is in electron volts. The distance, x_e, is simply the distance between the effective plane of the surface charge and the Fermi surface.

The electron degeneracy energy ζ, and the monopole work M, both being functions of the substrate compostion alone, are shown. The work function ϕ and the dipole work D, which determines the work function, are also shown.

M. F. Koskinen

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D. POWER FLOW IN THE MAGNETOHYDRODYNAMIC INDUCTION MACHINE

The magnetohydrodynamic (MHD) induction machine, shown schematically in Fig.XV-3 for the flat linear version, utilizes the interaction between a traveling magnetic field and a channeled electrically conducting fluid to convert energy between mechanical and electrical forms. In principle, this type of machine is analogous to conventional rotating induction machines, and accordingly the well-established features of asynchronous operation apply to it. This report presents a generalization of previous work[1, 2] to a magnet core of arbitrary permeability and conductivity.

1. The Model

The model to be analyzed is shown in Fig. XV-4. The fluid flows in the x direction between two parallel exciting plates of infinite extent in the x and z directions, a distance 2a apart. The fluid velocity is assumed to be constant and in the x direction (slug flow) to uncouple the electromagnetic and fluid equations and thus allow an analytical solution to be obtained. The region outside the plates is filled with a core of permeability μ_c and conductivity σ_c. The exciting plates, separated from the fluid and core by insulators of infinitesimal thickness to prevent current flow in the y direction, are assumed thin so that they can be replaced by current sheets with a surface conductivity $\sigma_s = \sigma_e b$, where b is the plate thickness and σ_e the material conductivity. The plates are driven by a current source that gives a symmetric surface current density

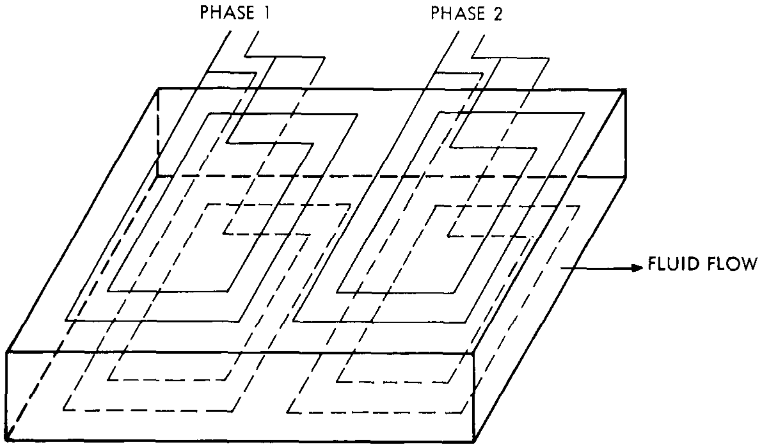

Fig. XV-3. The MHD induction machine.

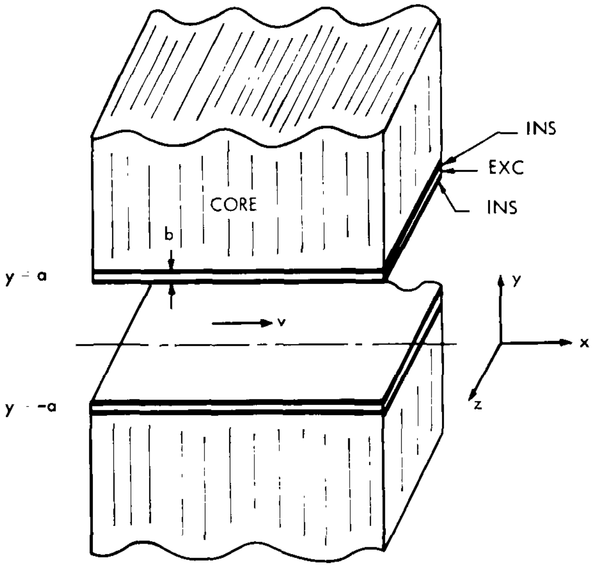

Fig. XV-4. The model.

$$\vec{K} = \vec{i}_z NI \cos(\omega t - kx), \tag{1}$$

which represents a traveling current wave of amplitude NI, frequency ω, wavelength $\lambda = 2\pi/k$, and velocity $v_s = \omega/k$. The surface current is considered to be produced by a balanced two-phase system with sinusoidally distributed windings of maximum turns density N and peak current I. Only two-phase excitation is considered, since, as shown by White and Woodson,[3] an n-phase system can be reduced to a two-phase equivalent.

2. Electromagnetic Fields

The electromagnetic fields are determined from Maxwell's equations with the usual magnetohydrodynamic approximation of neglecting displacement currents. Assuming constant velocity eliminates the need for the fluid equations. The analysis is simplified by the use of a vector potential $\vec{A}$ and a scalar potential ϕ defined by

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{2}$$

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}. \tag{3}$$

Noting that Ohm's law in a moving fluid is $\vec{J} = \sigma(\vec{E}+\vec{v}\times\vec{B})$ and substituting Eqs. 2 and 3 in Maxwell's equations gives

$$\nabla^2\vec{A} - \mu\sigma\frac{\partial \vec{A}}{\partial t} + \mu\sigma(\vec{v}\times\vec{\nabla}\times\vec{A}) = 0 \tag{4}$$

$$\nabla^2\phi - \mu\sigma\frac{\partial \phi}{\partial t} = 0. \tag{5}$$

Here,

$$\vec{\nabla}\cdot\vec{A} + \mu\sigma\phi = 0 \tag{6}$$

has been chosen to uncouple Eqs. 4 and 5. The x- and t-dependence of all quantities must be as $e^{j(\omega t - kx)}$ from the excitation and boundary conditions. This will not be indicated explicitly. $\vec{A}$ is due solely to currents, so that it, as well as $\vec{J}$, is in the z direction and independent of z. The inclusion of all components of $\vec{A}$ will not affect the fields obtained.

The vector potentials in the fluid and core, from Eq. 4 with the conditions that the normal magnetic field is continuous across the boundary and the tangential magnetic field is discontinuous by the surface current, are

$$\vec{A}_f = \vec{i}_z \frac{\mu_f NI \cosh \gamma y}{\gamma \sinh \gamma a + \alpha\delta \cosh \gamma a} \tag{7}$$

$$\vec{A}_c = \frac{\vec{i}_z \mu_f NI\, e^{-\delta(y-a)}}{\gamma \tanh \gamma a + \alpha\delta}\ ;\ y \geq a \tag{8}$$

where

$$\gamma^2 = k^2(1+jsR_M) \tag{9}$$

$$\delta^2 = k^2(1+jR_{Mc}) \tag{10}$$

$$\alpha = \frac{\mu_f}{\mu_c} \tag{11}$$

$$s = \frac{v_s - v}{v_s} \tag{12}$$

$$R_M = \frac{\mu_f \sigma_f v_s}{k} \tag{13}$$

$$R_{Mc} = \frac{\mu_c \sigma_c v_s}{k}. \tag{14}$$

The subscripts f, c, and e are used to denote fluid, core, and exciting plate quantities, respectively. s is the slip in terms of synchronous speed v_s, R_M and R_{Mc} the fluid and core magnetic Reynolds numbers. sR_M indicates the magnitude of the field-fluid interaction.

In the exciting plates $E_{ez} = NI/\sigma_s$. The scalar potential in the sheets, from Eq. 3, is

$$\phi_e = \frac{-NIz}{\sigma_s} - \frac{j\omega\mu_f NIz}{\gamma \tanh \gamma a + \alpha\delta} \tag{15}$$

The scalar potential is zero outside the exciting plates. Note that the tangential electric field is not continuous across the boundary. Its discontinuity, however, serves only to determine the dipole charge layer on the insulating strips.[4]

3. Electrical Impedance

The electrical characteristics of the MHD induction machine are conveniently expressed in terms of the impedance observed at the terminals of an exciting coil. The impedance may be obtained from the voltage measured between the coil terminals

$$V = 2\int_x^{x+\lambda} (\Delta V)\, N \cos kx dx, \tag{16}$$

where $\Delta V = \phi_e(z) - \phi_e(z+c)$ is the voltage across a single wire and the integral represents the sum of ΔV over a coil that is considered to be a wavelength long and of depth c in the z direction. V is twice the integral because each coil is made up of two sections located above and below the channel, as shown in Fig. XV-4. The equivalent resistance and inductance are

$$R = \frac{2\pi N^2 c}{k\sigma_s} - R_o \operatorname{Im}\left\{\frac{k}{\gamma \tanh \gamma a + \alpha\delta}\right\} \tag{17}$$

$$L = L_o \operatorname{Re}\left\{\frac{k}{\gamma \tanh \gamma a + \alpha\delta}\right\}, \tag{18}$$

where

$$R_o = \omega L_o = \frac{2\pi\mu_f N^2 \omega c}{k^2} \tag{19}$$

and Re and Im stand for the real and imaginary part of the quantity in the brackets. If Eqs. 9 and 10 are substituted in Eqs. 17 and 18, it is seen that, excluding the term depending on σ_s, the normalized ratios R/R_o and L/L_o depend on only five dimensionless parameters: s, R_M, R_{Mc}, α, and ak. Since δ appears everywhere multiplied by α, and since $\mu_c \gg \mu_o$ for a good core, the effect of core loss is small.

For a slit channel, $\gamma a \ll 1$, and by assuming a lossless core and exciting coils ($\sigma_c=0$, $\sigma_e=\infty$), the equations can be written approximately as

$$\frac{R}{R_o} = \left(\frac{1}{\alpha+ak}\right) \frac{sR_{M\alpha}}{1+s^2R_{M\alpha}^2} \tag{20}$$

$$\frac{L}{L_o} = \left(\frac{1}{\alpha+ak}\right) \frac{1}{1+s^2R_{M\alpha}^2} \tag{21}$$

where

$$R_{M\alpha} = \frac{\mu_f \sigma_f v_s a}{(\alpha+ak)} = \frac{R_M ak}{(\alpha+ak)} \tag{22}$$

is the pertinent magnetic Reynolds number. The functional dependance is the same as for the ideal core slit-channel case[2] except for the replacement of R_M by $R_{M\alpha}$. The difficulty with a nonideal core is that it is much harder to obtain a large $R_{M\alpha}$, $R_{M\alpha} \leq R_M$, and this results in poorer performance.

4. Power Flow

Relations are obtained for the time-average real power flow in an MHD induction machine of dimensions ℓ and c in the x and z directions, respectively. The power supplied by the exciting windings to the fluid, the real part of the surface integral of Poynting's vestor over the fluid, is

$$P_s = \frac{P_o \operatorname{Im}\{k\gamma \tanh \gamma a\}}{(\gamma \tanh \gamma a+\alpha\delta)(\gamma^* \tanh \gamma^* a+\alpha\delta^*)}, \tag{23}$$

where

$$P_o = \frac{\mu_f \omega N^2 I^2 c\ell}{k}, \tag{24}$$

The mechanical power output, P_m, and the power dissipated in the fluid, P_r, are

$$P_m = \int_{-a}^{a} \int_{x}^{x+\ell} \left\langle \frac{\partial p}{\partial x} \right\rangle v \, dxdy = (1-s)\, P_s \tag{25}$$

$$P_r = \oint_{Vol} \frac{\overline{J} \cdot \overline{J}^*}{2\sigma_f} dv = sP_s, \tag{26}$$

where $\left\langle \frac{\partial p}{\partial x} \right\rangle$ denotes the time-average pressure gradient. These are identical with the power relations obtained for rotating induction machines when P_s is identified as the "gap power".

The power dissipated in the core because of its finite conductivity, P_c, is found in the same manner as for P_s to be

$$P_c = \frac{P_o \operatorname{Im} \{\alpha\ k\delta\}}{(\gamma \tanh \gamma a + \alpha\delta)(\gamma^* \tanh \gamma^* a + \alpha\delta^*)}. \tag{27}$$

The power dissipated in the exciting coil, analogous to P_r, is

$$P_e = \int_{x}^{x+\ell} \frac{KK^*}{\sigma_s} c\, dx = \frac{N^2 I^2 c\ell}{\sigma_s} \tag{28}$$

Note that for $\ell = \lambda$, $P_o = I^2 R_o$ and $P_s + P_c + P_e = I^2 R$, as expected.

The efficiency of the induction generator, power out divided by power in, is $\frac{P_s + P_c + P_e}{P_m}$, where P_s and P_m are negative for generator operation ($s \leqslant 0$), while P_c and P_e, power losses, are always positive. For a lossless coil the efficiency is

$$\eta_g = \frac{1}{1-s} \left\{ 1 + \frac{\operatorname{Im}(\alpha\delta)}{\operatorname{Im}(\gamma \tanh \gamma a)} \right\}; \quad s \leqslant 0 \tag{29}$$

in which the term in brackets is less than or equal to 1. For a lossless core, δ real, this reduces to the efficiency found previously for a slit-channel machine with an ideal core.[2]

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