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Research Translation

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A New Radiation Chart

HELGI NIILISK

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TRANSLATION OF

A NEW RADIATION CHART

(Novaia radiatsionnaia nomogramma)

by

Helgi Niilisk

Akademiia Nauk Estonskoi SSR. Izvestiia, Seriia
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A NEW RADIATION CHART

by

Helgi Niilisk

At present, graphic methods of computing the fluxes of the thermal radiation of the atmosphere with the aid of radiation charts are well known. The charts of F. Brooks [1], A. A. Dmitriev [2], R. Mugge and F. Moller [3, 4], G. Robinson [5, 6], F. N. Shekhter [7, 8], W. Elsasser [9], and G. Yamamoto [10] have been the most widely used. Unfortunately, all these charts contain certain deficiencies [11-14], viz., the use of insufficiently reliable characteristics of the absorption of long-wave radiation by atmospheric gases and the analysis of the integral function dependent only on the effective content of water vapor. The effect of carbon dioxide on absorption, moreover, is considered to be very approximate.¹⁾ The charts mentioned do not at all consider the absorption of thermal radiation by atmospheric ozone.

It has been established [13, 14] that one of the most important factors which determines the value of radiation fluxes, calculated from any chart, is the transmission function. Therefore, to make radiation charts more accurate, it is first of all necessary to study the available quantitative characteristics of long-wave radiation in the atmosphere, and on that basis to obtain the most reliable transmission function.

Proceeding from what has been said above, the purpose of our work is to construct the integral transmission function of the atmosphere by considering the mutual effect of water vapor, carbon dioxide, and ozone²⁾ on the absorption of thermal radiation in the atmosphere. On the basis of this new transmission function, it is possible to solve the problem of constructing a radiation chart which is sufficiently accurate and convenient from a practical point of view. This work completes a number of studies made by the author [14-18] which were devoted to the problem of calculating the fluxes of thermal radiation in the atmosphere.

Quantitative data on the absorption of long-wave radiation, used in the present work to determine the integral transmission function, are, for the most part, given for 300°K, and also for the usual room temperatures of the order of 285-295°K. Since at surface temperatures the integral transmission function is slightly dependent on temperature [10, 13], we can consider the transmission function obtained on the basis of the mentioned data to be

-
- 1) Consideration of the mutual influence of H₂O and CO₂ on the absorption of thermal radiation in the atmosphere is correct in principle only in G. Yamamoto's chart.
 - 2) As is known [13], the remaining atmospheric gases play an insignificant role in the absorption of long-wave radiation by the atmosphere.

sufficiently reliable for the temperature interval 270-310°K. It should be noted that the integral transmission function was not calculated for the entire spectrum (0-∞ μ), and for the 2.27-250 μ region (in parts of the spectrum, 0-2.27 and 250-∞ μ, the amount of energy of thermal radiation is negligible and, consequently, practically has no effect on the value of the transmission function at atmospheric temperatures).

The determination of the quantitative characteristics of radiation absorption by water vapor is the subject of numerous works [1, 12, 13, 20-42], the results of which were obtained either by theoretical calculations or on the basis of an experiment (under natural or laboratory conditions). Let us note that there are important discrepancies in the data of various authors for almost all the regions of the spectrum. The results of the most recent and complete works are used when possible to determine the integral transmission function in the present work.

The values of the absorption of radiation by water vapor in the near infrared region (spectral intervals: 2.27-2.99, 2.99-3.57 and 4.88-8.7 μ) are calculated on the basis of the corresponding formulas of D. Howard, D. Burch, and D. Williams [20]. There are no experimental data for the 3.57-4.88 μ region. It is only known that in this region of the spectrum, there are no bands of absorption by water vapor and the absorption is caused by the overlapping of this interval by the wings of the absorption bands situated in the neighboring regions of the spectrum. We can assume that the logarithmic coefficient of absorption k_w here does not exceed 0.5 cm²/g. Calculations have shown that a change in the absorption coefficient in the limits 0-0.5 cm²/g has almost no effect on the value of the transmission function 2.27-8.7 μ, and on the value of the integral transmission function. Using this as a basis, in the present work the transmission function P for the interval 3.57-4.88 μ was calculated from the formula

$$P = e^{-k_w w^*}, \quad (1)$$

using a certain average value $k_w = 0.2 \text{ cm}^2/\text{g}$. (here w^* - the effective content of water vapor).

To determine the transmission function in the 8.7-12 μ region of the spectrum, there are no satisfactory reasons to prefer the results of one or other author (works [12, 13, 21, 22, 26, 28, 35, 37-40]). Therefore, in the present work the transmission function for the mentioned interval of the spectrum was calculated from formula (1), using certain average values for the coefficients of absorption by water vapor (Table 1).

TABLE 1

The Absorption Coefficients of Water Vapor for the
8.7-12 μ Region of the Spectrum

Spectrum Region	$k_w, \text{ cm}^2/\text{g}$
8.7 - 9.0	0.15
9.0 - 11.5	0.10
11.5 - 12.0	0.20

Let us note that according to the most recent data [37-39], k_w about 9.6 and 11.1 μ , is approximately 0.1.

In the 9.0-10.3 μ interval, it is necessary to consider the effect of a strong absorption band of ozone. We previously found the transmission function for the 9.0-10.3 μ region [16].

In the 12-18 μ region, the gases which absorb the thermal radiation of the atmosphere are water vapor and carbon dioxide, since a strong absorption band of carbon dioxide¹⁾ is found in this interval. The transmission function for 12-18 μ region obtained in works [15, 18] on the basis of data [19] and [22] is used in the present work.

1) According to [13] we can disregard the effect of the remaining bands of absorption by carbon dioxide on the transfer of long-wave radiation in the atmosphere.

Data are given in works [13, 21, 22, 27-30, 36, 41, 42] on the absorption of long-wave radiation by water vapor in the far infrared region of the spectrum. A comparison of the results of these works shows that the results of the theoretical calculations of G. Yamamoto [22] coincide most closely with the most recent data of the laboratory measurements of C. Palmer [36]. Proceeding from this, the transmission function for the 18-250 μ interval in the present work is determined on the basis of the usage of generalized coefficients of absorption by water vapor according to the data of work [22].

Let us note that for all the examined intervals of the spectrum, the transmission function for diffuse radiation was determined from the following known relationship [13]:

$$P_F(w^*) = 2 \int_0^{\pi/2} P(w^* \sec \theta) \sin \theta \cos \theta d\theta, \quad (2)$$

where P_F is the transmission function for diffuse radiation, P is the transmission function for the directed radiation, θ is the zenith angle.

By using the results of the determination of the transmission functions for various parts of the spectrum, the integral transmission function for diffuse radiation was calculated from the following formula:

$$P_F = \frac{1}{f} \sum_{\Delta} f_{\Delta} P_{F\Delta}. \quad (3)$$

Here the index Δ designates the examined intervals of the spectrum, f is a fraction of the integral radiation of an absolutely black body arriving on the 2.27-250 μ part of the spectrum, f_{Δ} is a fraction of the radiation of an absolutely black body arriving at the interval of the spectrum Δ .

Tables 2 and 3 give the calculation results. The tables also give the values of the functions $\Delta P_1(w^*, u^*)$ and $\Delta P_2(w^*, m^*)$ with which the sought integral transmission function can be calculated from the relationship:

$$P_F(w^*, u^*, m^*) = 0.001 (\Delta P_1 + \Delta P_2). \quad (4)$$

where u^* is the effective content of carbon dioxide, m^* is the effective content of ozone.¹

Since during the compilation of Tables 2 and 3 approximate methods were also used (numerical integration, interpolation), we can consider the transmission function obtained reliable to an accuracy of 1/100.

Having the data of an aerological sounding, it is easy to determine the values of the fluxes of thermal radiation in the atmosphere by using Tables 2 and 3 and the usual graphic method based on the use of relationships (see [13]):

$$G = \int P_t dB, \quad (5)$$

where G is the flux of thermal radiation of the atmosphere, $B = \sigma T^4$ is the integral flux of radiation of an absolutely black body.

In other words, G is numerically equal to the area in the coordinate system (P_F , B).

Figure 1 gives the chart for the determination of G .

It should be noted that formula (5) was derived on the assumption of the independence of the integral transmission function on temperature [13]. But, as is known [2, 4, 9, 10, 13], the transmission function actually depends not only on the content of absorbing matter in the atmosphere, but also on pressure and temperature. The dependence P_F on pressure is usually calculated with the aid of an effective absorbing mass, (see [1-10, 13, 14]). However, the problem of calculating the temperature dependence has not yet been finally solved, and, to a considerable degree, this problem may be considered debatable [13].

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1. The values w^* , u^* , and m^* are expressed in "cm" (the thickness of the layer of the precipitated matter in centimeters under normal pressure and temperature).

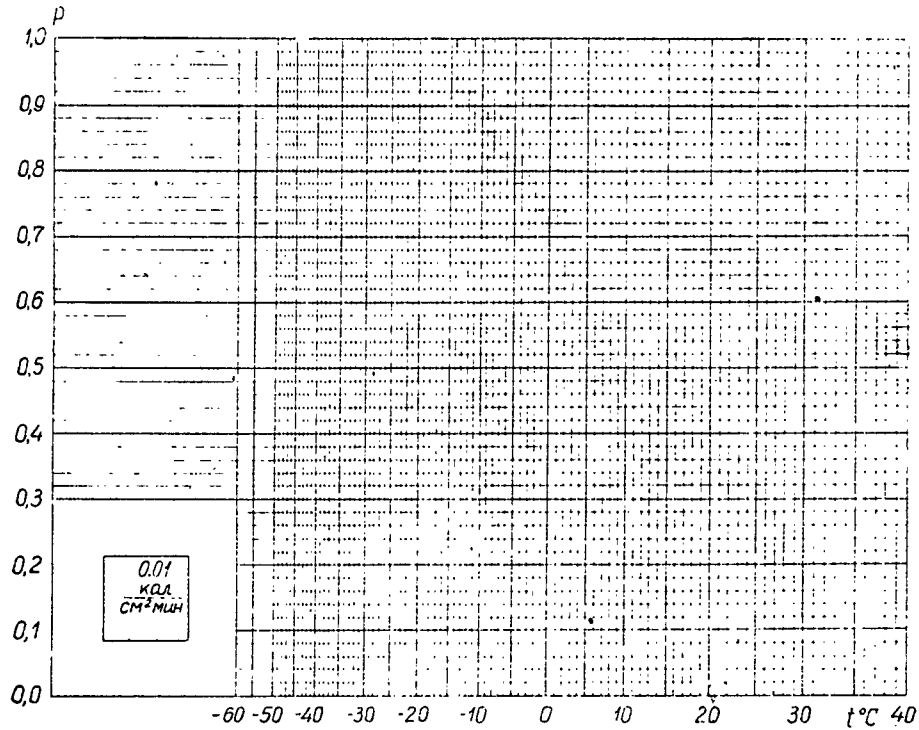


Fig. 1. Chart for the determination of fluxes of long-wave radiation in the atmosphere.

With a radiation chart, the present work attempts to evaluate the effect of the dependence $P_F = P_F(T)$ on the determination of the fluxes of the thermal radiation of the atmosphere.

By solving the general equations for the transfer of long-wave radiation in the atmosphere, we have the following expression for the intensity of the descending monochromatic radiation which propagates in the direction θ [13]:

$$I_{\lambda \downarrow}(z, \theta) = \int_0^{\infty} \frac{k_{\lambda}(T, p)}{\cos \theta} Q(\eta) E_{\lambda}(T) e^{-\frac{1}{\cos \theta} \int_z^{\infty} k_{\lambda}(T, p)(\xi) d\xi} d\eta. \quad (6)$$

Here $I_{\lambda \downarrow}(z, \theta)$ - is the intensity of the descending long-wave monochromatic radiation at the level z , $k_{\lambda}(T, p)$ is the absorption coefficient for the wave length λ , $T = T(\eta)$ is the temperature of the absorbing (radiating)

medium, $p = p(\eta)$ is the total pressure in the atmosphere, ρ is the density of the matter absorbing the radiation, $E_\lambda(T)$ is the intensity of the radiation of an absolutely black body for wave-length λ , θ is the zenith angle.

Let us express the absorption coefficient in the following form:

$$k_\lambda(T, p) = k_{0\lambda} F(T, p), \quad (7)$$

where $F(T, p)$ is a certain function of temperature and pressure ($k_{0\lambda}$ is considered independent of T and p).

With the calculation of (7), expression (6) will have the following form:

$$I_\lambda \downarrow = \int_z^\infty \frac{k_{0\lambda}}{\cos \theta} F(T, p) \varrho(\eta) E_\lambda(T) e^{-\frac{k_{0\lambda}}{\cos \theta} \int_z^\eta F(T, p) \varrho(\xi) d\xi} d\eta. \quad (8)$$

Let us introduce the effective absorbing mass w^* which is determined in the following fashion:

$$w^* = \int_z^\infty F(T, p) \varrho(\xi) d\xi. \quad (9)$$

In such a case

$$I_\lambda \downarrow = \int_0^{w^*} \frac{k_{0\lambda}}{\cos \theta} E_\lambda(T) e^{-\frac{k_{0\lambda}}{\cos \theta} w^*} dw^*. \quad (10)$$

Integrating (10) according to λ with the aid of the Ambartsumian-Lebedinskii method [13], we have

$$I \downarrow = \int_0^{w^*} E(T) dw^* \int_0^{k_0} \frac{k_0}{\cos \theta} f(k_0, T) e^{-\frac{k_0}{\cos \theta} w^*} dk_0. \quad (11)$$

It was assumed here that in regions of the spectrum, for which the inequality $k_0 < k_{0\lambda} < k_0 + dk_0$, the value of the absorption coefficient may in practice be considered as constant (k_0). The function $f(k_0, T)$ determines a

TABLE 2

Values of the function $\Delta P_1(w^*, u^*)$

$\lg u^*$	$\lg w^*$	2.00	1.90	1.80	1.70	1.60	1.50	1.40	1.30	1.20	1.10	1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.00	$\bar{T}_{2.00}$	$\bar{T}_{1.90}$	$\bar{T}_{1.80}$	$\bar{T}_{1.70}$	$\bar{T}_{1.60}$	$\bar{T}_{1.50}$	$\bar{T}_{1.40}$	$\bar{T}_{1.30}$	$\bar{T}_{1.20}$	$\bar{T}_{1.10}$	$\bar{T}_{1.00}$	$\bar{T}_{0.90}$	$\bar{T}_{0.80}$	$\bar{T}_{0.70}$	$\bar{T}_{0.60}$	$\bar{T}_{0.50}$	$\bar{T}_{0.40}$	$\bar{T}_{0.30}$	$\bar{T}_{0.20}$	$\bar{T}_{0.10}$	$\bar{T}_{0.00}$																																																																																																																																																																																																																																																																																																																										
4.00	868	851	848	845	842	839	836	833	830	827	824	821	818	815	812	809	806	803	800	797	794	791	788	785	782	779	776	773	770	767	764	761	758	755	752	749	746	743	740	737	734	731	728	725	722	719	716	713	710	707	704	701	698	695	692	689	686	683	680	677	674	671	668	665	662	659	656	653	650	647	644	641	638	635	632	629	626	623	620	617	614	611	608	605	602	599	596	593	590	587	584	581	578	575	572	569	566	563	560	557	554	551	548	545	542	539	536	533	530	527	524	521	518	515	512	509	506	503	500	497	494	491	488	485	482	479	476	473	470	467	464	461	458	455	452	449	446	443	440	437	434	431	428	425	422	419	416	413	410	407	404	401	398	395	392	389	386	383	380	377	374	371	368	365	362	359	356	353	350	347	344	341	338	335	332	329	326	323	320	317	314	311	308	305	302	299	296	293	290	287	284	281	278	275	272	269	266	263	260	257	254	251	248	245	242	239	236	233	230	227	224	221	218	215	212	209	206	203	200	197	194	191	188	185	182	179	176	173	170	167	164	161	158	155	152	149	146	143	140	137	134	131	128	125	122	119	116	113	110	107	104	101	98	95	92	89	86	83	80	77	74	71	68	65	62	59	56	53	50	47	44	41	38	35	32	29	26	23	20	17	14	11	8	5	2	-1	-4	-7	-10	-13	-16	-19	-22	-25	-28	-31	-34	-37	-40	-43	-46	-49	-52	-55	-58	-61	-64	-67	-70	-73	-76	-79	-82	-85	-88	-91	-94	-97	-100																																						
4.10	901	898	895	892	889	886	883	880	877	874	871	868	865	862	859	856	853	850	847	844	841	838	835	832	829	826	823	820	817	814	811	808	805	802	799	796	793	790	787	784	781	778	775	772	769	766	763	760	757	754	751	748	745	742	739	736	733	730	727	724	721	718	715	712	709	706	703	700	697	694	691	688	685	682	679	676	673	670	667	664	661	658	655	652	649	646	643	640	637	634	631	628	625	622	619	616	613	610	607	604	601	598	595	592	589	586	583	580	577	574	571	568	565	562	559	556	553	550	547	544	541	538	535	532	529	526	523	520	517	514	511	508	505	502	499	496	493	490	487	484	481	478	475	472	469	466	463	460	457	454	451	448	445	442	439	436	433	430	427	424	421	418	415	412	409	406	403	400	397	394	391	388	385	382	379	376	373	370	367	364	361	358	355	352	349	346	343	340	337	334	331	328	325	322	319	316	313	310	307	304	301	298	295	292	289	286	283	280	277	274	271	268	265	262	259	256	253	250	247	244	241	238	235	232	229	226	223	220	217	214	211	208	205	202	199	196	193	190	187	184	181	178	175	172	169	166	163	160	157	154	151	148	145	142	139	136	133	130	127	124	121	118	115	112	109	106	103	100	97	94	91	88	85	82	79	76	73	70	67	64	61	58	55	52	49	46	43	40	37	34	31	28	25	22	19	16	13	10	7	4	1	-2	-5	-8	-11	-14	-17	-20	-23	-26	-29	-32	-35	-38	-41	-44	-47	-50	-53	-56	-59	-62	-65	-68	-71	-74	-77	-80	-83	-86	-89	-92	-95	-98	-101																						
4.20	934	931	928	925	922	919	916	913	910	907	904	901	898	895	892	889	886	883	880	877	874	871	868	865	862	859	856	853	850	847	844	841	838	835	832	829	826	823	820	817	814	811	808	805	802	799	796	793	790	787	784	781	778	775	772	769	766	763	760	757	754	751	748	745	742	739	736	733	730	727	724	721	718	715	712	709	706	703	700	697	694	691	688	685	682	679	676	673	670	667	664	661	658	655	652	649	646	643	640	637	634	631	628	625	622	619	616	613	610	607	604	601	598	595	592	589	586	583	580	577	574	571	568	565	562	559	556	553	550	547	544	541	538	535	532	529	526	523	520	517	514	511	508	505	502	499	496	493	490	487	484	481	478	475	472	469	466	463	460	457	454	451	448	445	442	439	436	433	430	427	424	421	418	415	412	409	406	403	400	397	394	391	388	385	382	379	376	373	370	367	364	361	358	355	352	349	346	343	340	337	334	331	328	325	322	319	316	313	310	307	304	301	298	295	292	289	286	283	280	277	274	271	268	265	262	259	256	253	250	247	244	241	238	235	232	229	226	223	220	217	214	211	208	205	202	199	196	193	190	187	184	181	178	175	172	169	166	163	160	157	154	151	148	145	142	139	136	133	130	127	124	121	118	115	112	109	106	103	100	97	94	91	88	85	82	79	76	73	70	67	64	61	58	55	52	49	46	43	40	37	34	31	28	25	22	19	16	13	10	7	4	1	-2	-5	-8	-11	-14	-17	-20	-23	-26	-29	-32	-35	-38	-41	-44	-47	-50	-53	-56	-59	-62	-65	-68	-71	-74	-77	-80	-83	-86	-89	-92	-95	-98	-101											
4.30	967	964	961	958	955	952	949	946	943	940	937	934	931	928	925	922	919	916	913	910	907	904	901	898	895	892	889	886	883	880	877	874	871	868	865	862	859	856	853	850	847	844	841	838	835	832	829	826	823	820	817	814	811	808	805	802	799	796	793	790	787	784	781	778	775	772	769	766	763	760	757	754	751	748	745	742	739	736	733	730	727	724	721	718	715	712	709	706	703	700	697	694	691	688	685	682	679	676	673	670	667	664	661	658	655	652	649	646	643	640	637	634	631	628	625	622	619	616	613	610	607	604	601	598	595	592	589	586	583	580	577	574	571	568	565	562	559	556	553	550	547	544	541	538	535	532	529	526	523	520	517	514	511	508	505	502	499	496	493	490	487	484	481	478	475	472	469	466	463	460	457	454	451	448	445	442	439	436	433	430	427	424	421	418	415	412	409	406	403	400	397	394	391	388	385	382	379	376	373	370	367	364	361	358	355	352	349	346	343	340	337	334	331	328	325	322	319	316	313	310	307	304	301	298	295	292	289	286	283	280	277	274	271	268	265	262	259	256	253	250	247	244	241	238	235	232	229	226	223	220	217	214	211	208	205	202	199	196	193	190	187	184	181	178	175	172	169	166	163	160	157	154	151	148	145	142	139	136	133	130	127	124	121	118	115	112	109	106	103	100	97	94	91	88	85	82	79	76	73	70	67	64	61	58	55	52	49	46	43	40	37	34	31	28	25	22	19	16	13	10	7	4	1	-2	-5	-8	-11	-14	-17	-20	-23	-26	-29	-32	-35	-38	-41	-44	-47	-50	-53	-56	-59	-62	-65	-68	-71	-74	-77	-80	-83	-86	-89	-92	-95	-98	-101
4.40	1000	997	994	991	988	985	982	979	976	973	970	967	964	961	958	955	952	949	946	943	940	937	934	931	928	925	922	919	916	913	910	907	904	901	898	895	892	889	886	883	880	877	874	871	868	865	862	859	856	853	850	847	844	841	838	835	832	829	826	823	820	817	814	811	808	805	802	799	796	793	790	787	784	781	778	775	772	769	766	763	760	757	754	751	748	745	742	739	736	733	730	727	724	721	718	715	712	709	706	703	700	697	694	691	688	685	682	679	676	673	670	667	664	661	658	655	652	649	646	643	640	637	634	631	628	625	622	619	616	613	610																																																																																																																																																																																																																																		

TABLE 3

Values of the function $\Delta P_1 (w^*, m^*)$

w^*	1.00	3.00	3.50	2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	0.00
1.00	87	87	86	82	82	81	80	78	76	73	71	68	65	62	58	55	51	48	45	42	40	38	36	34
1.05	87	87	86	82	82	81	80	78	76	73	71	68	65	62	58	55	51	48	45	42	40	38	36	34
1.10	86	85	84	81	81	80	79	78	76	73	71	68	65	62	58	55	50	47	44	41	39	37	35	32
1.15	84	84	83	80	80	79	78	77	75	72	70	67	64	61	57	54	50	47	44	41	39	37	35	32
1.20	82	82	82	79	79	78	77	76	75	73	71	68	66	63	59	56	52	49	46	43	40	38	36	34
1.25	80	80	80	77	77	76	75	74	73	71	69	66	63	61	57	54	51	48	45	42	39	37	35	34
1.30	77	77	77	75	75	74	73	71	69	67	65	63	61	59	55	52	49	46	44	41	38	36	34	31
1.35	76	76	75	74	74	73	72	71	69	67	65	63	61	59	55	52	49	46	44	41	38	36	34	31
1.40	76	75	75	74	73	72	71	70	68	66	64	62	60	58	54	51	48	45	43	40	37	35	33	30
1.45	75	74	74	73	72	71	70	69	67	65	63	61	59	57	53	50	47	44	42	39	37	35	33	30
1.50	74	73	73	72	71	70	69	68	66	64	62	60	58	56	52	49	46	43	41	38	36	34	32	29
1.55	73	72	72	71	70	69	68	67	65	63	61	59	57	55	51	48	45	42	40	37	35	33	31	28
1.60	72	71	71	70	69	68	67	66	64	62	60	58	56	54	50	47	44	41	39	37	35	33	31	28
1.65	70	70	69	68	67	66	65	64	62	60	58	56	54	52	48	45	42	39	37	35	33	31	29	27
1.70	69	68	68	67	66	65	64	63	61	59	57	55	53	51	47	44	41	39	37	35	33	31	29	27
1.75	67	67	66	65	64	63	62	61	59	57	55	53	51	49	45	42	39	37	35	33	31	29	27	25
1.80	66	66	65	64	63	62	61	60	58	56	54	52	50	48	44	41	39	37	35	33	31	29	27	25
1.85	64	64	63	62	61	60	59	58	56	54	52	50	48	46	42	39	37	35	33	31	29	27	25	23
1.90	62	62	61	60	59	58	57	56	54	52	50	48	46	44	40	37	35	33	31	29	27	25	23	21
1.95	60	60	59	58	57	56	55	54	52	50	48	46	44	42	38	35	33	31	29	27	25	23	21	19
2.00	58	58	57	56	55	54	53	52	50	48	46	44	42	40	36	33	31	29	27	25	23	21	19	17
2.05	56	56	55	54	53	52	51	50	48	46	44	42	40	38	34	31	29	27	25	23	21	19	17	15
2.10	54	54	53	52	51	50	49	48	46	44	42	40	38	36	32	29	27	25	23	21	19	17	15	13
2.15	52	52	51	50	49	48	47	46	44	42	40	38	36	34	30	27	25	23	21	19	17	15	13	11
2.20	50	50	49	48	47	46	45	44	42	40	38	36	34	32	28	25	23	21	19	17	15	13	11	9
2.25	48	48	47	46	45	44	43	42	40	38	36	34	32	30	26	23	21	19	17	15	13	11	9	7
2.30	46	46	45	44	43	42	41	40	38	36	34	32	30	28	24	21	19	17	15	13	11	9	7	5
2.35	44	44	43	42	41	40	39	38	36	34	32	30	28	26	22	19	17	15	13	11	9	7	5	3
2.40	42	42	41	40	39	38	37	36	34	32	30	28	26	24	20	17	15	13	11	9	7	5	3	1
2.45	40	40	39	38	37	36	35	34	32	30	28	26	24	22	18	15	13	11	9	7	5	3	1	0
2.50	38	38	37	36	35	34	33	32	30	28	26	24	22	20	16	13	11	9	7	5	3	1	0	0
2.55	36	36	35	34	33	32	31	30	28	26	24	22	20	18	14	11	9	7	5	3	1	0	0	0
2.60	34	34	33	32	31	30	29	28	26	24	22	20	18	16	12	9	7	5	3	1	0	0	0	0
2.65	32	32	31	30	29	28	27	26	24	22	20	18	16	14	10	7	5	3	1	0	0	0	0	0
2.70	30	30	29	28	27	26	25	24	22	20	18	16	14	12	8	5	3	1	0	0	0	0	0	0
2.75	28	28	27	26	25	24	23	22	20	18	16	14	12	10	6	3	1	0	0	0	0	0	0	0
2.80	26	26	25	24	23	22	21	20	18	16	14	12	10	8	4	1	0	0	0	0	0	0	0	0
2.85	24	24	23	22	21	20	19	18	16	14	12	10	8	6	2	0	0	0	0	0	0	0	0	0
2.90	22	22	21	20	19	18	17	16	14	12	10	8	6	4	1	0	0	0	0	0	0	0	0	0
2.95	20	20	19	18	17	16	15	14	12	10	8	6	4	2	0	0	0	0	0	0	0	0	0	0
3.00	19	19	18	17	16	15	14	13	11	9	7	5	3	1	0	0	0	0	0	0	0	0	0	0

fraction of the intensity of an absolutely black body arriving on the total number of regions of the spectrum to which the absorption coefficient k_0 corresponds. $E(T)$ in expression (11) designates the integral intensity of the radiation of an absolutely black body.

On the other hand, it is easy to show that by using an effective absorbing mass which is determined from formula (9), the integral transmission function of the atmosphere is expressed as follows:

$$P(\omega^*, T_1) = \int_0^{\infty} j(k_0, T_1) e^{-\frac{k_0}{\cos \theta} \omega^*} dk_0. \quad (12)$$

Here T_1 designates the temperature of the source of the radiation; and it is assumed that this source is an absolutely black body.

In addition, let us introduce a new function P^* which is determined by the relationship

$$P^*[\omega^*, T(\omega^*)] = P[\omega^*, T_1 = T(\omega^*)]. \quad (13)$$

In other words,

$$P^*[\omega^*, T(\omega^*)] = \int_0^{\infty} j(k_0, T(\omega^*)) e^{-\frac{k_0}{\cos \theta} \omega^*} dk_0. \quad (14)$$

Now let us find the derivatives P and P^* with respect to ω^* :

$$\frac{dP}{d\omega^*} = - \int_0^{\infty} \frac{k_0}{\cos \theta} j(k_0, T_1) e^{-\frac{k_0}{\cos \theta} \omega^*} dk_0. \quad (15)$$

Here it is assumed that temperature T_1 does not change.

$$\begin{aligned} \frac{dP^*}{d\omega^*} = & - \int_0^{\infty} \frac{k_0}{\cos \theta} j(k_0, T) e^{-\frac{k_0}{\cos \theta} \omega^*} dk_0 + \\ & + \frac{dT}{d\omega^*} \int_0^{\infty} \frac{\partial f(k_0, T)}{\partial T} e^{-\frac{k_0}{\cos \theta} \omega^*} dk_0. \end{aligned} \quad (16)$$

We see that the integral $S_1 = \int_0^{\infty} \frac{k_0}{\cos \theta} j(k_0, T) e^{-\frac{k_0}{\cos \theta} \omega^*} dk_0$ in expression (11) is equal to the first term of expression (16). Therefore, using $dP^*/d\omega^*$ and $dP/d\omega^*$ instead of S_1 , creates errors. In the first case we do not consider the second term of expression (16), and in the second case the "displacement effect" is not considered.

Let us try to make an approximate evaluation of the magnitude of these errors. Under average conditions in the troposphere¹, the relationship between T and w^* is well described by the formula

$$T = -8(w^*)^8 - 10w^* + 20, \quad (17)$$

where w^* is expressed in "cm" and T in $^{\circ}\text{C}$.

To simplify the calculations, let us use instead of expressions (12) and (14) the approximate formulas

$$P(w^*, T_1) = \sum_{j=1}^n f_j(T_1) e^{-k_j w^*}, \quad (18)$$

where T_1 is considered constant, and

$$P^*(w^*, T) = \sum_{j=1}^n f_j(T) e^{-k_j w^*}, \quad (19)$$

where $T = T(w^*)$.

Formulas (18) and (19) are taken from the monograph of K. Ia. Kondrat'ev [13].

This work also gives the absorption coefficients k_j and the connection between f_j and the temperature:

$$f_j = a_j + b_j T. \quad (20)$$

Here a_j and b_j are certain constants for each region j .

1. It is assumed that the vertical temperature gradient is equal to 6 deg/km, and the vapor density ρ_w decreases with height according to exponential law. At the earth's surface $T = 20^{\circ}\text{C}$ and $\rho_w = 7 \times 10^{-6} \text{ g/cm}^3$.

On the basis of the data given above, the following magnitudes were calculated in the present work: S_1 , $S_2 = dP^*/dw^*$, $S_3 = dP(T_1 = 260^\circ K)/dw^*$, $S_4 = dP(T_1 = 300^\circ K)/dw^*$. Figure 2 shows the results of these calculations. As this figure shows, the use of dP^*/dw^* instead of S_1 causes many great errors, especially in the upper layers of the troposphere. The differences between S_1 and S_3 , and also between S_1 and S_4 are rather small. Therefore, it is necessary to consider as completely substantiated the use of $dP(w^*, T_1)/dw^*$ instead of S_1 in formula (11) (assuming that T_1 is a certain average temperature in the atmosphere). In other words, we can disregard the influence of the displacement effect in the given case.

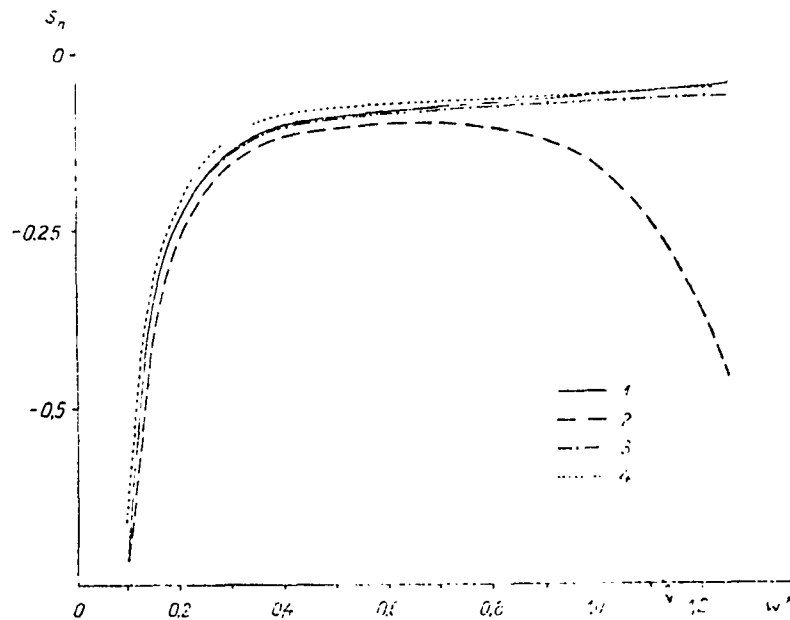


Fig. 2. A comparison of the functions S_n : 1- S_1 , 2- S_2 , 3- S_3 , 4- S_4 .

As is known [13], the dependence of the absorption coefficient on temperature and the "displacement effect" act in opposite directions. Therefore, it should be considered as proven that the relationship $k_\lambda = k_\lambda(T)$ is not considered. In such a case formula (9) has the following form:

$$\omega^* = \int F(p) Q(z) dz, \quad (21)$$

where $F(p)$ is a certain function of pressure.

On the basis of what has been said above, instead of (11) we will get

$$I_{\downarrow} = \int_{\omega_{\infty}^*}^0 E(T) d\omega^* \frac{dP}{d\omega^*} = \int_{P(\omega_{\infty}^*)}^1 E(T) dP. \quad (22)$$

Partially integrating (22), we get an expression analogous to formula (5):

$$I_{\leftarrow} = \oint P dE. \quad (23)$$

We can obtain an expression even for $I_{\uparrow}$ in this way.

We should note that the results given above are justified even for fluxes of long-wave radiation in the atmosphere. As is known [13], the radiation fluxes are calculated by integrating the corresponding intensities according to the solid angles corresponding to a semisphere. In such a case, we will have formula (5) instead of formula (23).

Summarizing the results given above, we can say that on the basis of the approximate formula $G = \int P_F(\omega^*) dB$, we can determine with sufficient accuracy the long-wave fluxes of the radiation of the atmosphere, but the use of the formula $G = \int P_F[\omega^*, T(\omega^*)] dB$ cannot be justified.

As was shown in works [16] and [43], atmospheric ozone plays only an insignificant role in the determination of the values of the integral fluxes of thermal radiation in the lower troposphere. Therefore, the effect of ozone on the absorption of long-wave radiation in the atmosphere can be disregarded when calculating descending fluxes ($G_{\downarrow}$) in the lower troposphere and ascending fluxes ($G_{\uparrow}$) to 15-20 km. In such a case, we can use only one table instead of Tables 2 and 3, in which the values of the integral transmission function $P_F(\omega^*, u^*)$ are given.

As an example, we calculated the descending flux, the ascending flux, and the effective radiation ($F = G \uparrow - G \downarrow$) at the levels 0.3 and 8 km for certain latitudinal zones of the earth. In addition, stratifications of temperature, humidity, and pressure, taken from works [10, 44, 45] were used. Data on the vertical distribution of ozone were taken from work [46]. The concentration of carbon dioxide in the atmosphere was taken equal to 0.03% (by volume) for all the zones, since there were no data on the latitudinal variation of CO_2 . When calculating the effective absorbing masses of water vapor, we applied the correction which is usually used $(p/p_0)^{0.5}$, and for carbon dioxide and ozone this correction was $(p/p_0)^{0.8}$ and $(p/p_0)^{0.2}$ respectively (see [15, 16, 18]). Table 4 gives the results of these calculations.

TABLE 4
Values of the Fluxes of Thermal Radiation in the
Atmosphere ($\text{cal/cm}^2 \text{ min}$)

latitu- dinal zone ($^{\circ}\text{N}$)	$z=0$		$z=3$		$z=8$	
	$G \downarrow$	F	$G \downarrow$	F	$G \downarrow$	F
0-10	0,574	0,093	0,392	0,194	0,164	0,296
10-20	0,545	0,101	0,348	0,209	0,132	0,317
20-30	0,507	0,114	0,312	0,220	0,117	0,313
30-40	0,429	0,117	0,275	0,208	0,097	0,293
40-50	0,359	0,122	0,240	0,198	0,082	0,270
50-60	0,296	0,125	0,201	0,187	0,076	0,246
60-70	0,236	0,129	0,169	0,172	0,077	0,218

Analogous calculations are carried out in works [14, 17] with the aid of the charts of F. Brooks, A. A. Dmitriev, R. Mugge and F. Moller, G. Robinson, F. N. Shekhter, V. Elsasser and G. Yamamoto. Comparing the results of the determination of $G \downarrow$, $G \uparrow$ and F which were obtained in the present work and in works [14, 17], we see that near the earth's surface and in the lower part of the troposphere, our results agree rather closely with the data obtained from the charts of F. Shekhter and F. Brooks. However,

when $z = 8$ km, there is a closer agreement with the results of the calculations from the charts of F. Møller and G. Robinson.

Unfortunately, at present there is no satisfactorily complete complex of experimental data, which makes it impossible to make a direct comparison of the results with the experiment.

Since the radiation nomogram chart in this work was constructed on the basis of a most careful analysis of contemporary data on the absorption of infrared radiation in the atmosphere, it may be considered most reliable.

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