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DISTRIBUTED EQUIVALENTS OF BRUNE SECTIONS

by

Akio Matsumoto

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FOREWORD

The work reported herein was sponsored by the Rome Air Development Center, Air Force Systems Command under contract No. AF-30(602)-2213.

ABSTRACT

~~This report presents~~ six new descriptive Brune sections ~~which~~ in many instances are preferable to those suggested by Ikeno and Saito. Included is a complete design technique.

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DISTRIBUTED EQUIVALENTS OF BRUNE SECTIONS

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I. Introduction A rational positive real function $Z(p)$ may be synthesized, according to Darlington procedure, as a cascade of Brune, type C and D sections, a Foster function (reactance) and a resistance. If there are transmission zeros at $p = 0$ or (and) $p = \infty$, ladders will also be necessary.

A so-called Brune section is made up of a pair of close-coupled coils (L_1, L_2) and a capacitor (C_o), and has a pair of transmission zeros (see Fig. 1(a))

$$\left.\begin{aligned} \pm p_o &= \pm j\omega_o = \pm \frac{j}{\sqrt{MC_o}} \\ M &= \sqrt{L_1 L_2} \end{aligned}\right\} \quad (1)$$

on the imaginary axis of the $p(= j\omega)$ plane. The section may also be represented in π - structure, with an inductance L and three capacitances K, K_1 and K_2. The equivalence of the two structures holds if

$$\left.\begin{aligned} L &= L_1 + L_2 - 2M \\ K &= \frac{MC_o}{L_1 + L_2 - 2M} \\ K_1 &= \frac{(L_2 - M)C_o}{L_1 + L_2 - 2M} \\ K_2 &= \frac{(L_1 - M)C_o}{L_1 + L_2 - 2M} \end{aligned}\right\} \quad (2)$$

There is a relation

$$K(K_1 + K_2) + K_1 K_2 = 0 \tag{2a}$$

corresponding to the close-coupling between L_1 and L_2. At a high frequency, these sections have a voltage transformation ratio

$$\frac{V_2}{V_1} = \frac{L_2}{M} = \frac{K}{K+K_2} \equiv n \tag{3}$$

The value of n may be greater or smaller than unity according as $L_2 \gtrless M$, $K_2 \lessgtr 0$.

No distributed network is known that is a direct equivalent of a Brune section itself, but certain networks are known which are equivalent to a Brune section accompanied by another element, such as a line or a reactance.

Ikeno[1] proposed a loop of 4 lines (Fig. 2(a)), that will be equivalent to a cascade of a Brune section and a line (Fig. 2(b), (c)), but there must be certain conditions among the given parameters (L_1, L_2, C_o and Z) in order to give positive characteristic impedances (Z_o, Z_{o1}, Z_{o2} and Z_{o3}) of the lines in the loop. If these conditions are not satisfied, his loop cannot be realized.

Saito[2] made another proposal. His section consists of a two-wire line and an open-circuited coaxial line. But the condition of realizability of his network is just the same as Ikeno's.

A possibility remains, to realize the network by extracting a sufficiently large number of cascaded lines, if only its input impedance is given. But this procedure may not be a good practice because of the complication of the network.

In this article is described a group of 6 networks, made of a shielded 3-wire line connected in different ways, which have a network representation of a Brune section in cascade with a line. They have certain restrictions coming from the hyper-dominancy of the capacitance matrix of the line, but still have a large adaptability arising from the abundance of the parameters. In fact, one has 6 parameters (C_{11}, C_{22}, C_{33}, C_{12}, C_{13} and C_{23}) in a 3-wire line, whereas its network representation contains only 4 parameters (C_o, L_1, L_2 and Z). If the line is given, its network representation is uniquely determined. This is the standpoint of analysis. On the other hand, from the standpoint of synthesis, the line cannot be determined uniquely that will have a given network representation. There remains some freedom of selection of parameters which is left to the designer. This freedom may be made use of in obtaining appropriate geometrical dimensions of the line.

II. Restrictions on the realizability of Ikeno loop.

The elements of Ikeno loop (Fig. 2(a)) are given as follows:

$$
\left.
\begin{aligned}
Z_o &= \frac{L_1 + L_2 - 2M + Z(1 + L_1 C_o)}{1 + M C_o} \\
Z_{o2} &= \frac{L_1 + L_2 - 2M + Z(1 + L_1 C_o)}{C_o \left\{ Z - (M - L_2) \right\}} \\
Z_{o2} &= \frac{L_1 + L_2 - 2M + Z(1 + L_1 C_o)}{C_o (L_1 - M + L_1 C_o Z)} \\
Z_{o3} &= \frac{Z \left\{ L_1 + L_2 - 2M + Z(1 + L_1 C_o) \right\}}{L_1 + L_2 - 2M + (L_1 - M) C_o Z}
\end{aligned}
\right\} \quad (4)
$$

The parameters L_1, L_2, M, C_o and Z refer to Fig. 2(b). L_1 and L_2 are closely coupled.

The numerators of the right-hand sides of Eq. (4) are all positive. To have positive values of the elements of the loop, the denominators must all be positive:

$$Z - (M - L_2) > 0 \quad (5a)$$

$$L_1 - M + L_1 C_o Z > 0 \quad (5b)$$

$$L_1 + L_2 - 2M + (L_1 - M) C_o Z > 0 \quad (5c)$$

The case $L_1 = L_2$ is trivial, because that means the Brune section is merely a shunt Foster.

First, consider the case $L_1 < L_2$. Then Eq. (5b) requires

$$C_o Z > \sqrt{\frac{L_2}{L_1}} - 1 \tag{6}$$

while Eq. (5c) requires the contrary, so that the case $L_1 < L_2$ cannot exist.

Next, take the case $L_1 > L_2$. Here Eqs. (5b) and (5c) are satisfied, and (5a) is the only necessary. That is

$$L_1 > L_2 \text{ and } Z > M - L_2 \tag{7}$$

are the necessary (and also sufficient) conditions for the realizability of Ikeno loop. In other words, the coil facing outside should be of a greater inductance than that facing inside, and moreover the inductance $M-L_2$(positive) should be smaller than the characteristic impedance of the line behind the Brune section.

If either of these two conditions is not satisfied, Ikeno loop cannot be realized.

To make Ikeno loop correspond to the network representation Fig. 2(c), one has only to interchange $L_1 \longleftrightarrow L_2$ and $Z_{o1} \longleftrightarrow Z_{o3}$. Here the necessary and sufficient conditions of realizability are

$$L_2 > L_1 \text{ and } Z > M - L_1 \tag{7a}$$

III. Notations of line constants

A uniform multiwire line[3] can be treated with the use of its distributed capacitances:

$$[C] = \begin{bmatrix} C_{11} & -C_{12} & \cdots & -C_{1n} \\ -C_{12} & C_{22} & \cdots & -C_{2n} \\ \multicolumn{4}{c}{\text{------------------------}} \\ -C_{1n} & -C_{2n} & \cdots & C_{nn} \end{bmatrix} \tag{8}$$

Its distributed inductances $[L]$ are related to the distributed capacitances $[C]$ in such a way that

$$p^2 \; [L] \; [C] = \gamma^2 \; 1_n , \quad p \equiv j\omega \tag{9}$$

where γ is the propagation constant of any transverse mode in the (non-dissipative) line.

$p/\gamma = v$, the velocity of propagation, is independent on the frequency.

Define $[\eta]$ and $[\xi]$ by

$$[\eta] = \frac{p}{\gamma} \; [C]$$

$$= \begin{bmatrix} \eta_{11} & -\eta_{12} & \cdots\cdots & -\eta_{1n} \\ \eta_{12} & \eta_{22} & & -\eta_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ -\eta_{1n} & -\eta_{2n} & \cdots\cdots & \eta_{nn} \end{bmatrix} , \tag{10}$$

$$[\zeta] = \frac{p}{\gamma} \; [L]$$

$$= \begin{bmatrix} \zeta_{11} & \zeta_{12} & \cdots\cdots & \zeta_{1n} \\ \zeta_{12} & \zeta_{22} & \cdots\cdots & \zeta_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \zeta_{1n} & \zeta_{2n} & \cdots\cdots & \zeta_{nn} \end{bmatrix} , \tag{11}$$

then $[\eta]$ and $[\zeta]$ may be called the characteristic admittance matrix and the characteristic impedance matrix respectively. They are reciprocal to each other,

$$[\eta] \, [\zeta] = [\zeta] \, [\eta] = 1_n \tag{12}$$

Now the major frequency variable λ in the distributed domain, is defined by

$$\lambda = \tanh \gamma \ell \tag{13}$$

where ℓ is the recurrent length of the elements.

Let us consider, for the moment, a very low frequency, such that

$$\lambda = \tanh \gamma\ell = \gamma\ell , \tag{14}$$

then one has

$$\begin{aligned} \lambda \left[\eta\right] &= p \left[C\right] \ell \\ \lambda \left[\zeta\right] &= p \left[L\right] \ell \end{aligned} \tag{15}$$

Thus, $\left[\eta\right]$ and $\left[\zeta\right]$ may also be comprehended, in a sense, to be the total capacitance matrix $\left[C\right] \ell$ and the total inductance matrix $\left[L\right] \ell$ respectively.

$\left[\eta\right]$ must be hyperdominant:

$$\left.\begin{array}{l} \eta_{11} - \eta_{12} - \cdots - \eta_{1n} = \eta_{1e} \geq 0 \\ -\eta_{12} + \eta_{2n} - \cdots - \eta_{2n} = \eta_{2e} \geq 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ -\eta_{1n} - \eta_{2n} \cdots\cdots + \eta_{nn} = \eta_{ne} \geq 0 \end{array}\right\} \tag{16}$$

On the other hand, the entries of ζ have the properties

$$\zeta_{\mu\mu} \geq \zeta_{\mu\upsilon} , \quad \mu \neq \upsilon \tag{17}$$

A hyperdominant matrix η is realizable as the characteristic admittance matrix of a multiwire line.

The fundamental equations of transmission of a multiwire line may be written as:

$$\begin{aligned} [V] &= c\,[V]_{\ell} + s\,[\zeta]\,[I]_{\ell} \\ [I] &= c\,[I]_{\ell} + s\,[\eta]\,[V]_{\ell} \end{aligned} \qquad (18)$$

where $[V]$ and $[I]$ are the column matrices with entries V_1, V_2, ..., V_n and I_1, I_2, ..., I_n. They indicate voltages and currents of the wires 1, 2, ..., and n, at the near end. Those at the far end are signified with suffixes ℓ.
c and s stand for

$$c = \cosh \gamma\ell\,, \quad s = \sinh \gamma\ell \qquad (19)$$

IV. Saito section

N. Saito presented a section which would be equivalent to a Brune section plus a line to the left. His section consists of a 2 - wire line, along with a distributed capacitor (an open-circuited coaxial line), as shown in Fig. 3.

Actual calculation yield:

$$\left.\begin{aligned} Z &= \frac{1}{\eta_{11} + \eta_{22} - 2\eta_{12}} \\ C_o &= C_a \\ \Omega_o^2 &= \frac{1}{C_a(\zeta_{22} - \zeta_{12})} \\ \frac{L_1}{M} = n_1 &= \frac{\eta_{11} - \eta_{12}}{\eta_{11} + \eta_{22} - 2\eta_{12}} \end{aligned}\right\} \qquad (20)$$

Evidently n_1 must be smaller than 1, from the hyperdominant property of the line.

Eqs. (20) give

$$\left.\begin{aligned} \eta_{12} &= \frac{n_1}{M} - \frac{n_1(1-n_1)}{Z} \\ \eta_{11} &= \frac{n_1}{M} + \frac{n_1^2}{Z} \\ \eta_{22} &= \frac{n_1}{M} + \frac{(1-n_1)^2}{Z} \end{aligned}\right\} \qquad (21)$$

η_{12} cannot be negative, and therefore it is necessary that

$$Z > (1-n_1)M = M - L_1 \qquad (22)$$

Compare this with Eq. (7a). The condition of realizability of Saito section is just the same as that of Ikeno's. The only advantage of Saito's is that his section can be built with less number of conductors (3 inner and 2 outer) as compared with Ikeno's (4 inner and 4 outer).

V. Network (I)

The network (I) shown in Fig. 4 has been treated already[4] as an equivalent to a type C section plus a line in the case $\zeta_{13} > \zeta_{23}$ in a previous report. In the case $\zeta_{13} < \zeta_{23}$, on the other hand, the network will go into a Brune section plus a line.

Parameters of the equivalent network are given as:

$$\left.\begin{aligned}
\frac{1}{Z} &= Y = \frac{\eta_{11}\eta_{22} - \eta_{12}^2}{\eta_{22}} \\
C_o &= \frac{(\eta_{22} - \eta_{12})^2}{\eta_{22}} \\
M &= \frac{\eta_{22}\eta_{23}(\zeta_{23} - \zeta_{13})}{(\eta_{22} - \eta_{12})^2} > 0 \\
L_2 &= \frac{\eta_{22}^2(\zeta_{23} - \zeta_{13})^2}{\zeta_{33}(\eta_{22} - \eta_{12})^2} \\
\Omega_o^2 &= \frac{1}{\eta_{23}(\zeta_{23} - \zeta_{13})}
\end{aligned}\right\} \tag{23}$$

In this network, the terms ratio

$$\frac{L_2}{M} = n = 1 - \frac{(\eta_{22} - \eta_{12})\zeta_{13}}{\eta_{23}\,\zeta_{33}} \tag{24}$$

can only be smaller than unity.

Let the values Z, C_o, Ω_o and n be given, and entries of η are to be found. Here the problem is, " what conditions are necessary and sufficient on the values Z, C_o, Ω_o and n, in order that the network (I) would correspond to a hyperdominant η ?"

It is already known that n must be smaller than unity. Are there any other conditions necessary? To simplify the analysis, first take $\eta_{12} = 0$. Then Eq. (23) reduces to

$$\eta_{11} = \frac{1}{Z}, \quad \eta_{22} = C_o$$

$$\Omega_o^2 = \frac{\eta_{11}\eta_{22}\eta_{33} - \eta_{13}^2\eta_{22} - \eta_{23}^2\eta_{11}}{\eta_{23}(\eta_{23}\eta_{11} - \eta_{13}\eta_{22})} \tag{25}$$

$$n = 1 - \frac{\eta_{13}\eta_{22}}{\eta_{11}\eta_{23}}$$

Eliminate η_{11} and η_{22} from these equations. Then one obtains

$$\eta_{23} = \frac{(C_o\eta_{33})^{1/2}}{\left(1 + n\Omega_o^2 + \frac{(1-n)^2}{C_o Z}\right)^{1/2}}$$

$$\eta_{13} = \left(\frac{\eta_{33}}{C_o}\right)^{1/2} \frac{1-n}{Z\left(1 + n\Omega_o^2 + \frac{(1-n)^2}{C_o Z}\right)^{1/2}} \tag{26}$$

Hyperdominancy of η requires

$$\eta_{23} \leq \eta_{22},$$

$$\eta_{13} \leq \eta_{11}, \tag{27}$$

$$\eta_{13} + \eta_{23} \leq \eta_{33}$$

Therefore it is necessary that

$$\eta_{33} \leq C_o\left(1 + n\,\Omega_o^{\,2} + \frac{(1-n)^2}{C_o Z}\right)$$

$$\eta_{33} \leq \frac{C_o\left(1 + n\,\Omega_o^{\,2} + \frac{(1-n)^2}{C_o Z}\right)}{(1-n)^2} \tag{28}$$

$$\eta_{33} \geq \frac{C_o\left\{1 + \frac{1-n}{C_o Z}\right\}^2}{1 + n\,\Omega_o^{\,2} + \frac{(1-n)^2}{C_o Z}}$$

The second of these inequalities is naturally satisfied if the first is satisfied. η can be obtained as hyperdominant, if a value of η_{33} can be found that satisfies the inequality.

$$\frac{C_o\left\{1 + \frac{1-n}{C_o Z}\right\}^2}{1 + n\,\Omega_o^{\,2} + \frac{(1-n)^2}{C_o Z}} \leq \eta_{33} \leq C_o\left(1 + n\,\Omega_o^{\,2} + \frac{(1-n)^2}{C_o Z}\right) \tag{29}$$

This is always possible if

$$1 + \frac{1-n}{C_o Z} < 1 + n\,\Omega_o^{\,2} + \frac{(1-n)^2}{C_o Z} \tag{30a}$$

but impossible if

$$1 + \frac{1-n}{C_o Z} > 1 + n\,\Omega_o^{\,2} + \frac{(1-n)^2}{C_o Z} \tag{30b}$$

The criterion comes down to

$$\Omega_o^{\,2} \begin{matrix} > \\ < \end{matrix} \frac{1-n}{C_o Z} \qquad \begin{matrix} \text{possible} \\ \text{impossible} \end{matrix} \tag{30c}$$

Substitute $\Omega_o^2 C_o$ by M^{-1} and nM by L_2, then one will have the same criterion as that in Ikeno's. Network (I) is no better than Ikeno loop, so far as the restrictions are concerned. The advantage of Network (I) may lie in its compactness, with 3 inner conductors and 1 outer conductor, whereas Ikeno loop needs 4 inner and 4 outer.

VI. Network (II)

Network (II), the second of Fig. 4, has also been shown to be equivalent to a type C section plus a line if $\eta_{13} < \eta_{2e}$, as given in a previous report.[4] In case $\eta_{13} > \eta_{2e}$, one has transmission zeros on the imaginary axis of λ :

$$\pm \lambda_o = \pm j\Omega_o = \frac{\pm j}{\left\{(\eta_{13} - \eta_{2e})(\zeta_{22} - \zeta_{23} - \zeta_{12} + \zeta_{13})\right\}^{1/2}} \tag{31}$$

Other parameters of the equivalent network are given as:

$$\left.\begin{aligned}
\frac{1}{Z} &= \eta_{33} - \frac{(\eta_{13} + \eta_{23})^2}{\eta_{11} + \eta_{22} - 2\eta_{12}} \\
C_o &= \frac{(\eta_{11} + \eta_{22} - 2\eta_{12} - \eta_{13} - \eta_{23})^2}{\eta_{11} + \eta_{22} - 2\eta_{12}} \\
\frac{L_2}{M} &= \frac{\eta_{1e}\,\eta_{3e} + \eta_{13}(\eta_{13} + \eta_{2e} + \eta_{3e})\ (\eta_{11} + \eta_{22} - 2\eta_{12})}{(\eta_{13} - \eta_{2e})\left\{\eta_{3e}(\eta_{3e}(\eta_{1e} + \eta_{2e}) + (\eta_{13} + \eta_{23})(\eta_{1e} + \eta_{2e} + \eta_{3e})\right\}}
\end{aligned}\right\} \tag{32}$$

Again the problem is to determine $[\eta]$ for a given set of values of Z, C_o, Ω_o and $n = L_2/M$. Let $\eta_{12} = \eta_{23} = 0$, then one has

$$\begin{aligned}
\eta_{11} + \eta_{22} &= \frac{C_o}{2} + \eta_{13} + \sqrt{\frac{1}{4} C_o^2 + C_o\,\eta_{13}} \\
\eta_{33} &= \frac{1}{Z} + \frac{C_o}{z} + \eta_{13} - \sqrt{\frac{1}{4} C_o^2 + C_o\,\eta_{13}}
\end{aligned} \tag{33}$$

The condition $\eta_{33} \geq \eta_{13}$ requires

$$\eta_{13} < \frac{1}{Z} + \frac{1}{C_o Z^2} \equiv m \qquad (34)$$

For the moment, let aside the equation on n, and consider the remaining 3 equations on Ω_o, Z and C_o. These equations contain η_{11}, η_{22}, η_{33} and η_{13}. η_{22} and η_{33} can be expressed in terms of η_{11} and η_{13} by the use of Eq. (33), and upon putting those expressions in Eq. (31), an equation, in η_{11} and η_{13}, will be obtained, from which curves η_{11} versus η_{13} may be plotted. Fig. 5-7 show examples, for $C_o = \frac{1}{2}$, Z = 1, C_o = 1, Z = 1, and C_o = 2, Z = 1. In these figures, all curves η_{11} versus η_{13}, drawn for different values of Ω_o^2, pass the 3 points:

i) the origin (0, 0)

ii) point P $(0, C_o)$

iii) point S $\left(\sqrt[3]{\frac{C_o}{Z^2}} + \sqrt[3]{\frac{C_o^2}{Z}},\quad C_o + \sqrt[3]{\frac{C_o^2}{Z}}\right)$

The curve for $\Omega_o^2 = 0$ passes Q:

$$(\eta_{13}, \eta_{11})_Q = (m, m) = \left(\frac{1}{Z} + \frac{1}{C_o Z^2},\ \frac{1}{Z} + \frac{1}{C_o Z^2}\right) \qquad (35)$$

The curves cross over the abscissa at T:

$$(\eta_{13}, \eta_{11})_T = (C_o \Omega_o^2, 0) \qquad (36)$$

The curves for $\Omega_o^2 = \infty$ pass R and U,

$$(\eta_{13}, \eta_{11})_R = (2C_o, 2C_o) \qquad (37)$$

$$(\eta_{13}, \eta_{11})_U = \left(\frac{1}{Z} + \frac{1}{C_o Z^2},\ C_o + \frac{1}{Z}\right) \qquad (38)$$

Points Q, R, S and U will be identical if $C_o Z = 1$.

At the neighborhood of the point P, η_{13} and η_{2e} are very small, but η_{11} and η_{33} are equal to C_o and $1/Z$ respectively; therefore n must be very large there.

The shaded areas show the permissible areas of a point (η_{13}, η_{11}). The permissible region of a curve η_{11} vs. η_{13} begins at P and ends on the line OQ (as in Fig. 5) or on the line $\eta_{13} = m$ (as in Fig. 6 and 7). The value of n changes with η_{13} and must have a certain lower limit because η_{13} has a certain upper limit. TABLE I shows the range of n thus determined.

TABLE I

Permissible values of n for Network II, $\eta_{12} = \eta_{23} = 0$
($n = L_2/M$)

$C_o Z$ \ Ω_o^2	0	0.5	1	2	∞	
2	$3 < n$	$4.85 < n$	$7.20 < n$	$12.5 < n$	∞ only	
1	$2 < n$	$3.67 < n$	$5.50 < n$	$9.0 < n$	∞ only	
0.5	$3 < n$	$5.60 < n$	$7.39 < n$	$11.4 < n$	∞ only	

In the above, it was assumed $\eta_{12} = \eta_{23} = 0$. Take off this assumption, then one may have a broader range of permissible values of n.

From the three relations in Eq. (32), one obtains

$$\frac{L_2}{M} = \frac{\eta_{13}(1 + C_o Z) + \eta_{1e}\,\eta_{3e}\,Z}{\eta_{13} - \eta_{2e}} \qquad \text{(a)}$$

The smallest possible value of L_2/M is

$$n_{min} = 1 + C_o Z \qquad \text{(b)}$$

which occurs when $\eta_{2e} = \eta_{3e} = 0$. Let us examine if any value of Ω_o is permissible or not, under this assumption, that is, $\eta_{2e} = \eta_{3e} = 0$, $n = n_{min}$. Here one has

$$\eta_{1e} = \frac{1}{Z} + C_o, \quad \eta_{13} + \eta_{23} = \frac{1 + C_o Z}{C_o Z^2} \qquad \text{(c)}$$

$$\eta_{13}^2\Omega_o^2 = \eta_{13}\eta_{23} + \eta_{12}\left(\frac{1+C_oZ}{C_oZ^2}\right) \qquad \text{(d)}$$

The relations (c), along with $\eta_{2e} = \eta_{3e} = 0$, satisfy Eq. (32), and the equation (d) is the only condition to be examined. η_{13} and η_{23} are bound by (c),

$$0 \leq \eta_{13} \leq \frac{1+C_o\,Z}{C_o\,Z^2} \qquad \text{(e)}$$

$$0 \leq \eta_{23} \leq \frac{1+C_o\,Z}{C_o\,Z^2} \qquad \text{(f)}$$

but η_{12} is arbitrary ($\eta_{12} \geq 0$). Therefore Ω_o can be made to take any value from 0 to ∞.

For example, $\Omega_o = 0$ when $\eta_{23} = \eta_{12} = 0$; $\Omega_o = \infty$ when $\eta_{12} = \infty$ or $\eta_{13} = 0$.

Thus it has been shown that $n = 1 + C_o\,Z$ is always realizable regardless of the value of Ω_o. It is more complicated to show a value of n greater than $1 + C_oZ$ is always realizable or not, but one has the affirmative answer, since Ω_o can be changed by η_{12}, without affecting other parameters (C_o, Z and n). If n is sufficiently large to satisfy the permissible range given in TABLE I, it is realizable with a line of $\eta_{12} = \eta_{23} = 0$.

Of course a line with $\eta_{\mu\upsilon} = 0$ or $\eta_{\mu e} = 0$ requires an extreme geometircal structure, so that in practice it is desirable to have all positive values for the partial capacitances ($C_{\mu\upsilon}$ and $C_{\mu e}$). This statement applies to all networks described in Chap. V - X.

VII. Network (III)

This network, Fig. 4(III), has a pair of transmission zeros at

$$\pm \lambda_o = \frac{\pm j}{\left\{(\eta_{33} - \eta_{23})(\zeta_{22} - \zeta_{23} - \zeta_{12} + \zeta_{13})\right\}^{1/2}} \tag{39}$$

λ_o is always imaginary because $\eta_{33} \geq \eta_{23}$ and $\zeta_{22} - \zeta_{23} - \zeta_{12} + \zeta_{13} \geq 0$.

Other network parameters are:

$$Z^{-1} = \eta_{11} - \frac{(\eta_{12} + \eta_{13})^2}{\eta_{22} + \eta_{33} - 2\eta_{23}} \tag{40a}$$

$$C_o = \frac{(\eta_{22} + \eta_{33} - 2\eta_{23} - \eta_{12} - \eta_{13})^2}{\eta_{22} + \eta_{33} - 2\eta_{23}} \tag{40b}$$

$$n = \frac{(\eta_{22} + \eta_{33} - 2\eta_{23})\ \eta_{1e}\,\eta_{3e} + \eta_{13}(\eta_{1e} + \eta_{2e} + \eta_{3e})}{(\eta_{33} - \eta_{23})\left\{\eta_{1e}(\eta_{2e} + \eta_{3e}) + (\eta_{12} + \eta_{13})(\eta_{1e} + \eta_{2e} + \eta_{3e})\right\}} \tag{40c}$$

The expression for n may be simplified with the use of those for Z^{-1} and C_o:

$$n = Z\ \frac{\eta_{1e}\,\eta_{3e} + \eta_{13}(\eta_{1e} + \eta_{2e} + \eta_{3e})}{\eta_{3e} + \eta_{13}} \tag{40c'}$$

Thus for any values of η_{1e} and $\eta_{2e} + \eta_{3e}$, n should be within the region

$$\eta_{1e} Z \leq n \leq (\eta_{1e} + \eta_{2e} + \eta_{3e})\, Z \tag{41}$$

or

$$0 \leq n \leq 1 + C_o Z \tag{41'}$$

On the other hand, one can easily see that n is greater or smaller than unity according as $\eta_{13}\,\eta_{2e}$ is greater or smaller than $\eta_{12}\,\eta_3$.

First the case $n = n_{max} = 1 + C_o Z$, $\eta_{3e} = 0$ should be examined, whether it is realizable or not for any value of Ω_o. Here one has

$$\begin{aligned}
&\eta_{13}^2\ (\eta_{1e} + \eta_{2e} + \eta_{3e})(1 + \Omega_o^2) \\
&- \eta_{13}\ \left\{\eta_{1e}(\eta_{2e} + \eta_{3e}) + (\eta_{12} + \eta_{13})(\eta_{1e} + \eta_{2e} + \eta_{3e})\right\} \\
&- \eta_{23}\,\eta_{11}(\eta_{22} + \eta_{33} - 2\eta_{23}) = 0
\end{aligned} \tag{42}$$

For a given set of values of Z and C_o, one has

$$\eta_{11} = \frac{1}{2} + (\eta_{12} + \eta_{13}) + \frac{C_o}{2} - \sqrt{\frac{C_o^2}{4} + (\eta_{12} + \eta_{13})\, C_o} \tag{43a}$$

$$\eta_{1e} = \frac{1}{2} + \frac{C_o}{2} - \sqrt{\frac{C_o^2}{4} + (\eta_{12} + \eta_{13})\, C_o} \tag{43b}$$

$$\eta_{22} + \eta_{33} - 2\eta_{23} = \eta_{12} + \eta_{13} + \frac{C_o}{2} + \sqrt{\frac{C_o^2}{4} + (\eta_{12} + \eta_{13})\, C_o}$$

$$\eta_{2e} + \eta_{3e} = \frac{C_o}{2} + \sqrt{\frac{C_o^2}{4} + (\eta_{12} + \eta_{13})\, C_o} \tag{43d}$$

Therefore the above equation in η_{13} may be considered to be a quadriatic one, with $(\eta_{12} + \eta_{13})$ and η_{23} as parameters. η_{13} has a minimum positive root when $\eta_{23} = 0$,

$$(\eta_{13})_{min} = \frac{\eta_{1e}(\eta_{2e} + \eta_{3e}) + (\eta_{12} + \eta_{13})(\eta_{1e} + \eta_{2e} + \eta_{3e})}{(\eta_{1e} + \eta_{2e} + \eta_{3e})(1 + \Omega_o^2)} \tag{44}$$

This $(\eta_{13})_{min}$ cannot be greater than $\eta_{12} + \eta_{13}$;

$$\eta_{12} + \eta_{13} \geq \frac{\eta_1 \ (\eta_2 \ + \eta_3 \ ) + (\eta_{12} + \eta_{13})(\eta_{1e} + \eta_{2e} + \eta_{3e})}{(\eta_{1e} + \eta_{2e} + \eta_{3e})(1 + \Omega_o^2)} \tag{45}$$

which is equivalent to

$$\Omega_o^2 \geq \frac{\eta_{1e}(\eta_{2e} + \eta_{3e})}{(\eta_{12} + \eta_{13})(\eta_{1e} + \eta_{2e} + \eta_{3e})} \tag{45'}$$

$\eta_{12} + \eta_{13}$ can be made small or large so far as η_{1e} remains positive, and consequently η_{1e} can be made as small as required. It turns out that the above inequality can hold if one chooses a proper value of $\eta_{12} + \eta_{13}$, and consequently some positive values of η_{23} are permissible to fit the assumptions.

Thus one has no restriction on Ω_o, so long as the given data are any Z, C_o, and $n = n_{max} = 1 + C_o Z$.

The criterion of realizability for any n ($0 < n < n_{max}$) needs more complicated examinations. But C_o, Z and n specifies η_{1e}, $\eta_{2e} + \eta_{3e}$, η_{13}/η_{3e} in terms of $\eta_{12} + \eta_{13}$, whereas Ω_o may be changed with the choice of η_{23}. From this fact one can conclude that the only restriction is $n < n_{max} = 1 + C_o Z$.

If one restricts himself to the case $\eta_{13} = \eta_{23} = 0$, then he will come to the same restrictions ($L_1 > L_2$, $Z \geq M - L_2$) as in Ikeno loop. Positive values of η_{13} and η_{23} make it possible to take off these restrictions, but still the restriction, $n \leq 1 + C_o Z$, remains necessary, to have a hyperdominant $[\eta]$.

VIII. Network (IV).

The network is shown in Fig. 4 (IV). The input and output currents are made up of two components respectively:

$$I_{in} = I_1 + I_2 \qquad (46)$$
$$I_{out} = I_{1\ell} + I_{3\ell}$$

One might better obtain the admittance matrix rather than the cascade matrix:

$$\left.\begin{aligned}
[Y] &= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix} \\
Y_{11} &= \frac{1+\lambda^2\{\eta_{22}(\zeta_{11}+\zeta_{22}-2\zeta_{12})-1\}}{\lambda\zeta_{11}+\lambda^3\eta_{23}^2|\zeta|} \\
Y_{22} &= \frac{1+\lambda^2\{\eta_{33}(\zeta_{11}+\zeta_{33}-2\zeta_{13})-1\}}{\lambda\zeta_{11}+\lambda^3\eta_{23}^2|\zeta|} \\
-Y_{12} &= \frac{1+\lambda^2\eta_{23}(\zeta_{11}-\zeta_{12}+\zeta_{23}-\zeta_{13})}{c\{\lambda\zeta_{11}+\lambda^3\eta_{23}^2|\zeta|\}}
\end{aligned}\right\} \qquad (47)$$

The transmission zeros of this network are the zeros of Y_{12}. It has three zeros, one at $\lambda = 1$, corresponding to $c = \infty$, and the other two at

$$\pm\lambda_o = \frac{\pm j}{\{\eta_{23}(\zeta_{11}-\zeta_{12}+\zeta_{23}-\zeta_{13})\}^{\frac{1}{2}}}$$

$$= \frac{\pm j}{\left[\eta_{23}|\zeta|\{\eta_{2e}\eta_{3e}+\eta_{23}(\eta_{1e}+\eta_{2e}+\eta_{3e})\}\right]^{\frac{1}{2}}} \qquad (48)$$

λ_o is always imaginary. The entries of the cascade matrix of the network are:

$$A = -\frac{Y_{22}}{Y_{12}} = c\,\frac{1+\lambda^2\{\eta_{33}(\zeta_{11}+\zeta_{33}-2\zeta_{13})-1\}}{1+\lambda^2\eta_{23}(\zeta_{11}-\zeta_{12}+\zeta_{23}-\zeta_{13})} \qquad (49a)$$

$$B = -\frac{1}{Y_{12}} = c\,\frac{\lambda\zeta_{11}+\lambda^3|\zeta|\eta_{23}^2}{1+\lambda^2\eta_{23}(\zeta_{11}-\zeta_{12}+\zeta_{23}-\zeta_{13})} \qquad (49b)$$

Since this network has a transmission zero at $\lambda = 1$, one can extract a line, from the right-hand side of the network, with the characteristic impedance

$$Z = \frac{1}{\left[Y_{22}\right]_{\lambda = 1}}$$

$$= \frac{\eta_{22}}{\eta_{22}\eta_{33} + \eta_{11}\eta_{22} - 2\eta_{13}\eta_{22} - (\eta_{12} + \eta_{13})^2} \quad (50)$$

One can also obtain this value of Z from $[\eta]$ and the end connection at the right-hand end of the network:

$$\frac{1}{Z} = \eta_{1e} + \eta_{2e} + \frac{\eta_{2e}(\eta_{12} + \eta_{23})}{\eta_{22}}$$

$$= \frac{\eta_{22}\eta_{33} + \eta_{11}\eta_{22} - 2\eta_{13}\eta_{22} - (\eta_{12} + \eta_{23})^2}{\eta_{22}} \quad (50')$$

After extraction of a line with characteristic impedance Z, there will remain a network, whose cascade matrix is $\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$:

$$A_1 = \frac{1 + \lambda^2 \eta_{23}^2 (\zeta_{11} + \zeta_{33} - 2\zeta_{13}) / \eta_{22}}{1 + \lambda^2 \eta_{23}(\zeta_{11} - \zeta_{12} + \zeta_{23} - \zeta_{13})} \quad (51a)$$

$$B_1 = \frac{\lambda \dfrac{\zeta_{11}(\zeta_{11} + \zeta_{33} - 2\zeta_{13}) - \eta_{22}|\zeta|}{\zeta_{11} + \zeta_{33} - 2\zeta_{13}}}{1 + \lambda^2 \eta_{23}(\zeta_{11} - \zeta_{12} + \zeta_{23} - \zeta_{13})} \quad (51b)$$

The whole network may be represented as a cascade of π-type Brune section plus a line where

$$L = \zeta_{11} - \frac{\eta_{22}|\zeta|}{\zeta_{11} + \zeta_{33} - 2\zeta_{13}} \quad (52a)$$

$$K = \frac{1}{L}\,(\zeta_{11} - \zeta_{12} + \zeta_{23} - \zeta_{13})\,\eta_{23} \tag{52b}$$

$$\Omega_o^{\,2} = \frac{1}{\eta_{23}(\zeta_{11} - \zeta_{12} + \zeta_{23} - \zeta_{13})} \tag{52c}$$

$$\frac{K + K_2}{K} = \frac{1}{n} = \left[A_1\right]_{\lambda = \infty}$$

$$= \frac{\eta_{23}\,(\zeta_{11} + \zeta_{33} - 2\zeta_{13})}{\eta_{22}\,(\zeta_{11} - \zeta_{12} + \zeta_{23} - \zeta_{13})} \tag{52d}$$

It can be easily shown that n is here greater than unity, and therefore K_2 is negative.

Let us see what values of L, K, K, and K_2 make it possible to be realized by the network (IV). First put

$$\eta_{12} = \eta_{13} = 0 \tag{53}$$

Then one has also

$$\zeta_{12} = \zeta_{13} = 0 \tag{53'}$$

and consequently

$$\zeta_{11} = L + Z = \frac{1}{\eta_{11}} \tag{54a}$$

$$\zeta_{33} = Z + \frac{Z^2}{L} = \frac{\eta_{22}}{\eta_{22}\eta_{33} - \eta_{23}^{\,2}} \tag{54b}$$

$$\eta_{22}\,\eta_{33} = \eta_{23}^{\,2} + \frac{\Omega_o^2\,\eta_{23}^{\,2}\,\eta_{11}}{\eta_{11} - \Omega_o^2\,\eta_{23}} \tag{54c}$$

$$\frac{1}{n} = \frac{\eta_{23}(\eta_{22}\eta_{33} - \eta_{23}^{\,2} + \eta_{11}\eta_{22})}{\eta_{22}(\eta_{22}\eta_{33} - \eta_{23}^{\,2} + \eta_{23}\,\eta_{11})} \tag{54d}$$

Specify L, Z and Ω_o and vary η_{23}. Then η_{22} and η_{33} will follow η_{23}. Any positive values of η_{23} are permissible so long as $\eta_{23} \leq \eta_{22}$ and $\eta_{23} \leq \eta_{33}$.

Evidently η_{23} cannot be greater than $\left[\Omega_o^2 (L+Z)\right]^{-1}$ from Eq. (54c). The permissible range of η_{23} corresponds to the permissible range of **n**.

Above equations (54a), (54b) and (54c) yield:

$$\eta_{22}\eta_{33} - \eta_{23}^2 = \frac{\Omega_o^2 \eta_{23}^2 \eta_{11}}{\eta_{11} - \Omega_o^2 \eta_{23}} \tag{55a}$$

$$\eta_{22} = (Z + \frac{Z^2}{L}) \frac{\Omega_o^2 \eta_{23}^2 \eta_{11}}{\eta_{11} - \Omega_o^2 \eta_{23}} \tag{55b}$$

$$\eta_{33} = \frac{L}{Z(L+Z)} \left\{ \frac{1+\Omega_o^2}{\Omega_o^2} - \frac{\eta_{23}}{\eta_{11}} \right\} \tag{55c}$$

The conditions $\eta_{22} \geq \eta_{23}$, $\eta_{33} \geq \eta_{23}$ write

$$\eta_{23} \geq \frac{L}{\Omega_o^2 (L+Z)^2} \tag{56a}$$

$$\eta_{23} \leq \frac{1+\Omega_o^2}{\Omega_o^2} \frac{L}{(L+Z)^2} \tag{56b}$$

One has two cases for the upper limit of η_{23}, according as

$$\frac{1+\Omega_o^2}{\Omega_o^2} \frac{L}{(L+Z)^2} \lessgtr \frac{1}{\Omega_o^2 (L+Z)} \tag{57}$$

or, that is the same,

$$\frac{\Omega_o^2 L}{Z} < 1 \tag{57'}$$

Case 1. $\Omega_o^2 L/Z < 1$:

$$\frac{L}{\Omega_o^2 (L+Z)^2} \leq \eta_{23} \leq \frac{1+\Omega_o^2}{\Omega_o^2} \frac{L}{(L+Z)^2} \tag{58a}$$

Case 2.

$$\Omega_o^2 L/Z > 1:$$

$$\frac{L}{\Omega_o^2 (L+Z)^2} \leq \eta_{23} \leq \frac{1}{\Omega_o^2 (L+Z)} \tag{58b}$$

In either case, at the minimum permissible value of η_{23}, one has $\eta_{22} = \eta_{23}$ and $n=1$. At the maximum permissible value of η_{23}, one has

Case 1:

$$\eta_{23} = \eta_{33}, \; n = \frac{1}{1 - \frac{\Omega_o^2 L}{Z}} \tag{59a}$$

Case 2:

$$\eta_{22} = \infty, \; n = \infty \tag{59b}$$

Thus one will see that any value of n greater than 1 is realizable in Case 2, but there is an upper limit of n to be realizable in Case 1.

This network is suitable for the case $n>1$, $\Omega_o^2 L/Z > 1$, because it is then always realizable.

If one replaces the π-type Brune section by a T-type one, the condition $\Omega_o^2 L/Z \lesssim 1$ should be replaced by

$$\frac{(n-1)^2}{n} \lessgtr C_o Z \tag{60}$$

and Eq. (59a) by

$$n_{max} = \frac{1}{1 - \frac{(n-1)^2}{n C_o Z}} \tag{61}$$

If n_{max} is greater than n, the value of n is permissible, but if the contrary, not. Thus only those values of n that satisfy

$$n > 1 + C_o Z \tag{62}$$

are permissible.

The above reasoning is based on the assumption $\eta_{12} = \eta_{13} = 0$, and consequently the restrictions mentioned may be changed by introducing positive values of η_{12} and η_{13}.

IX. Network (V)

This network is shown in Fig. 4 (V). Its equivalent network has parameters of the values:

$$Z = \frac{\eta_{33}}{\eta_{11}\eta_{33} - \eta_{13}^2} \tag{63a}$$

$$C_o = \frac{\eta_{13}^2}{\eta_{33}} \tag{63b}$$

$$\Omega_o^2 = \frac{1}{\eta_{23}\zeta_{23}} \tag{63c}$$

$$\frac{L_2}{M} = \frac{\eta_{33}(\eta_{23}\eta_{11} + \eta_{12}\eta_{13})}{\eta_{23}(\eta_{11}\eta_{33} - \eta_{13}^2)} \tag{63d}$$

From the above relations one has

$$\left.\begin{aligned} \eta_{11} &= \frac{1+C_oZ}{Z}, \quad \eta_{33} = \frac{\eta_{13}^2}{C_o} \\ \frac{\eta_{12}\,\eta_{13}}{\eta_{23}} &= \frac{n-1-C_oZ}{Z} \end{aligned}\right\} \tag{64}$$

It is necessary that

$$n \geq 1 + C_oZ \tag{65}$$

in order to give positive values of η_{12}, η_{13} and η_{23}.

The equations (63a), (63b) and (63d) are satisfied by Eq. (64) and the only equation is Eq. (63c) left for examination. Since

$$\Omega_o^{-2} = \eta_{23}\zeta_{23} = \frac{\eta_{23}^2 \cdot n}{Z|\eta|}, \tag{66}$$

it follows that Ω_o may be made arbitrarily small by choosing a small value of η_{23}, or may be made arbirtarily large by choosing a small value of $|\eta|$, that is, $\eta_{1e} \doteqdot \eta_{2e} \doteqdot \eta_{3e} \doteqdot 0$. Thus the only restriction is the inequality (65). The location of transmission zeros has no concern on the realizability of the network. This network is sutiable for comparatively large values of n.

X. Network (VI)

This network has the form Fig. 4 (VI). The values of the parameters of the equivalent network are:

$$Y = \frac{1}{Z} = \eta_{11} - \frac{\eta_{12}^2}{\eta_{22}} \tag{67a}$$

$$C_o = \frac{\eta_{12}^2}{\eta_{22}} \tag{67b}$$

$$\Omega_o^2 = \frac{1}{(\eta_{22} - \eta_{23})(\zeta_{33} - \zeta_{23})} \tag{67c}$$

$$\frac{L_2}{M} = \frac{\eta_{22}(\zeta_{33} - \zeta_{23})}{\zeta_{33}(\eta_{22} - \eta_{23})} \tag{67d}$$

Put the first and the second of these relations into the last, it may be easily seen that

$$\frac{L_2}{M} = n < 1 \tag{68}$$

For a given value of n, Ω_o can be made smallest with $\eta_{1e} = 0$ and $\eta_{23} = 0$,

$$(\Omega_o^2)_{min} = \frac{1-n}{1+C_oZ} \tag{69}$$

On the other hand, Ω_o^2 can be made as large as required, regardless of other requirements. Thus

$$\infty > \Omega_o^2 \geq \frac{1-n}{1+C_oZ} \tag{70}$$

or,

$$M - L_2 \leq Z + \frac{1}{C_o} \tag{70'}$$

One may choose η_{1e} and η_{23} so small that the inequality (70') is quite near to the equality. Then $M - L_2$ can be made greater than Z, but cannot, of course, be made greater than $Z + \frac{1}{C_o}$.

Ikeno loop has the restriction $M - L_2 < Z$ so that this network (VI) has a bit broader range of realizsbility than Ikeno's.

XI. Concluding Remarks

Selection of the networks (I)-(VI)

Chapters V-X makes it possible to adopt one of them, according as $n = L_2/M$ is greater or smaller than $1 + C_o Z$.

(i) $n > 1 + C_o \cdot Z$

Here the networks (II), (IV) and (V) may be of use.

(ii) $n < 1 + C_o Z$.

Here network (III) fits the best. If $n<1$, networks (I) and (VI) may also be of use; network (VI) has the restriction $M - L_2 \leq Z + \frac{1}{C_o}$, network (I) has the restriction $M - L_2 \leq Z$; the latter restriction is the same as those for Ikeno's and Saito's sections

Thus it is always possible to find a distributed equivalent of any Brune section accompanied by any line in cascade. The realized network may be one of the six networks described in chapters V-X. They are made of a three-wire line in a shield, with input and output terminals on opposite ends of the line. This is good for making cascade connections.

The writer is grateful to Prof. D. C. Youla of his suggestion that there may exist distributed sections that may be equivalent to Brune or type C sections, even with same connections.

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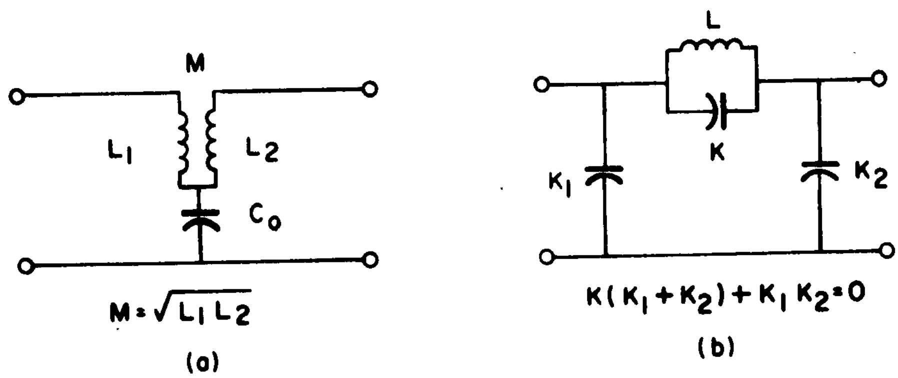

Fig. 1. Brune Section in T or π.

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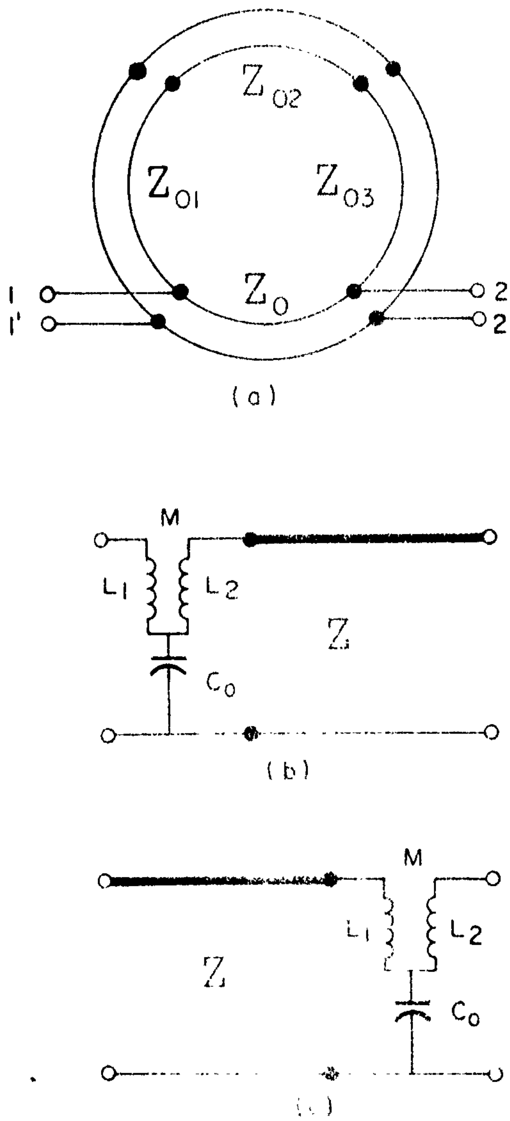

Fig. 2. Ikeno Loop and its Equivalents

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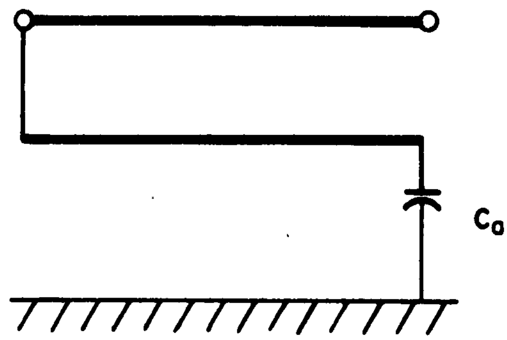

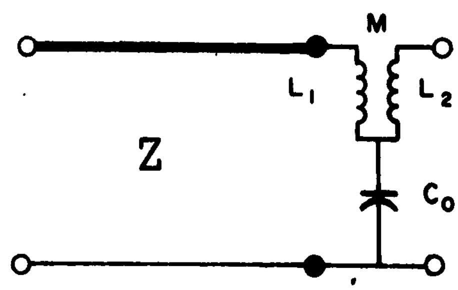

Fig. 3. Saito Section and its Equivalent

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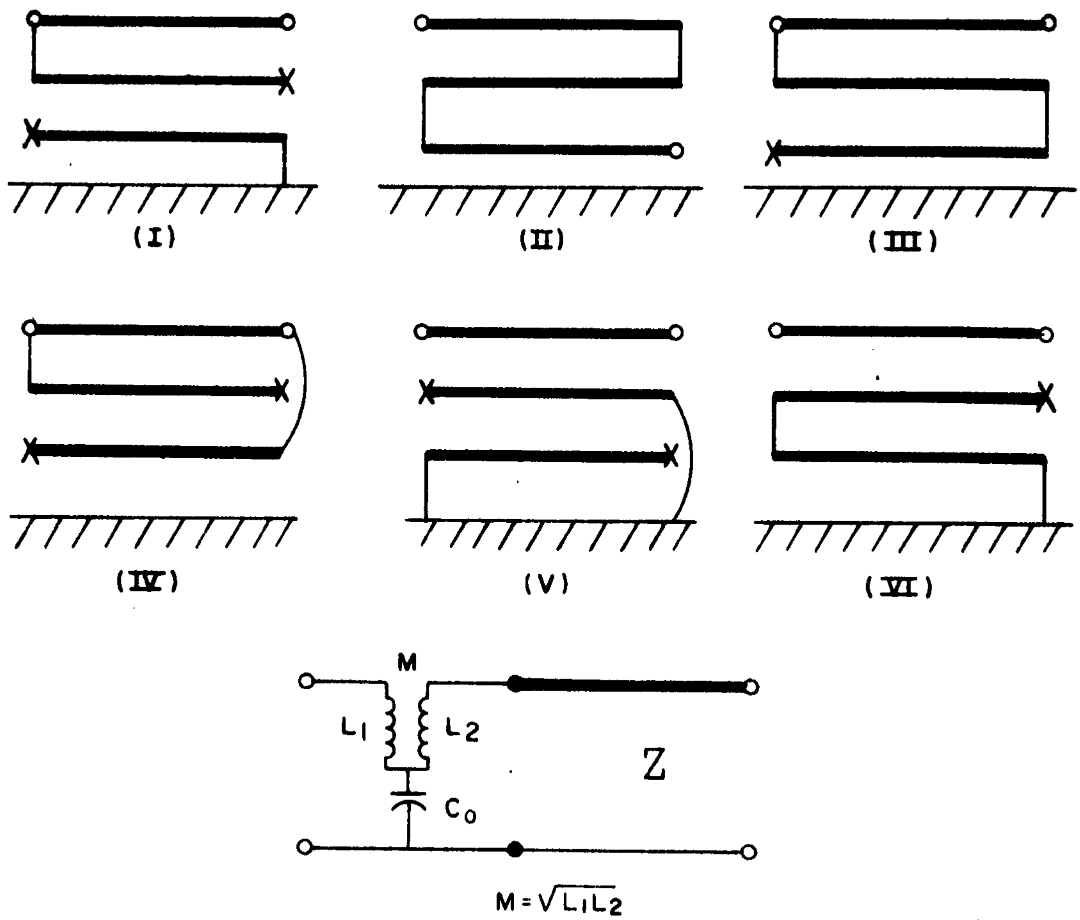

Fig. 4. Networks (I) ~ (VI), Equivalent to Brune Section in Cascade with a Line

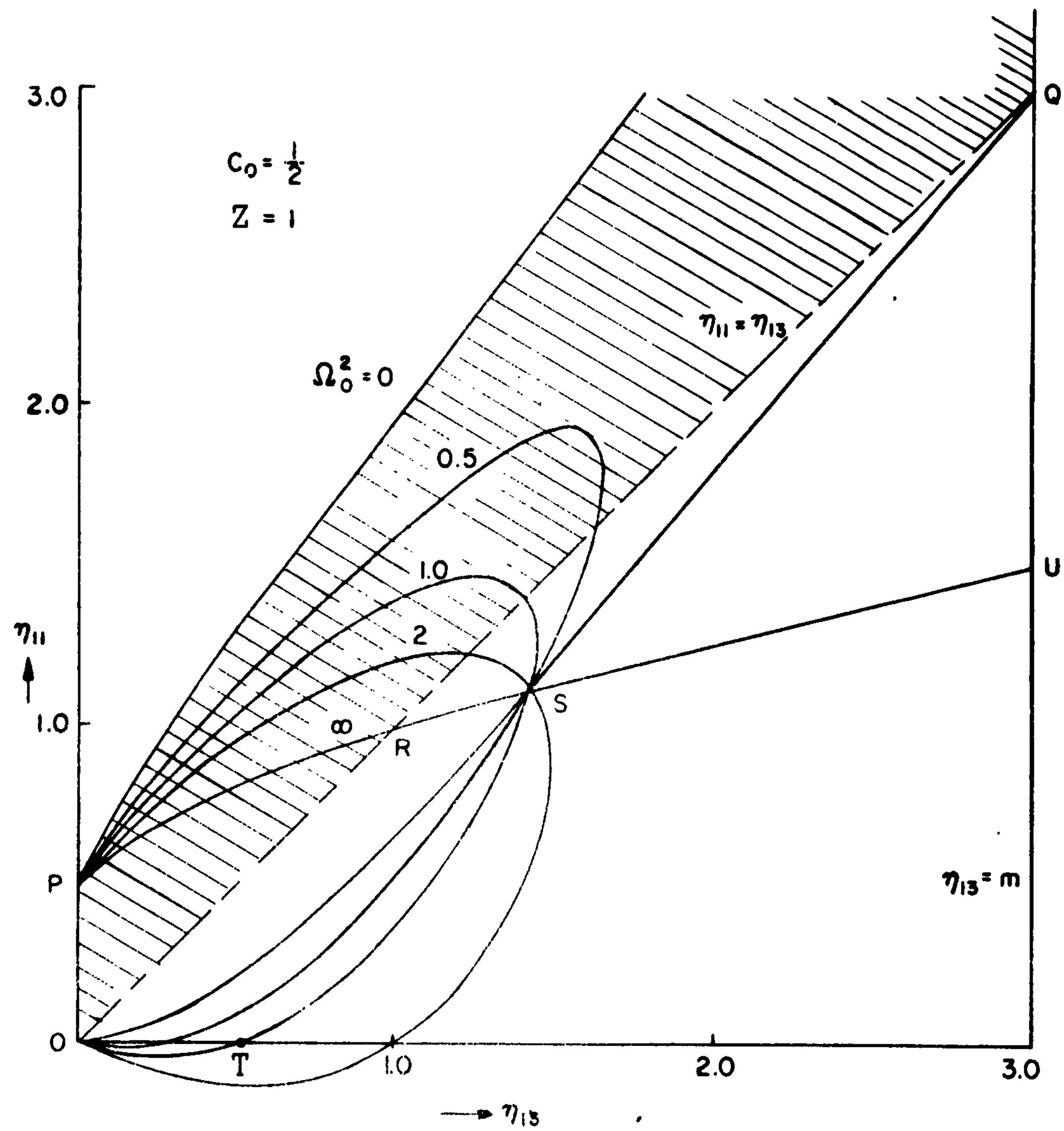

Fig. 5. Curves η_{11} vs. η_{13} for $C_0 = \frac{1}{2}$, $Z = 1$

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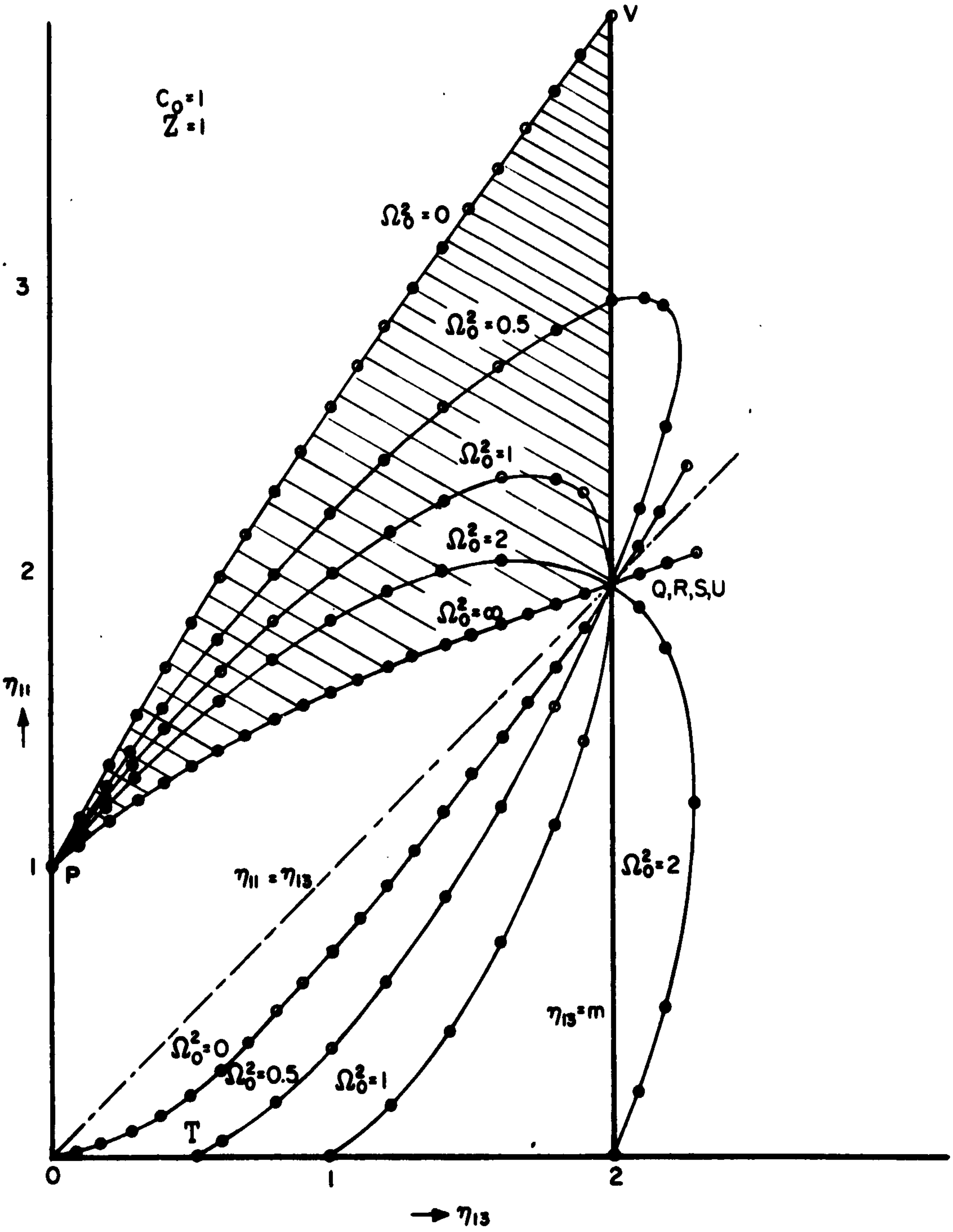

Fig. 6. Curves η_{11} vs η_{13} for $C_0 = 1$, $Z = 1$

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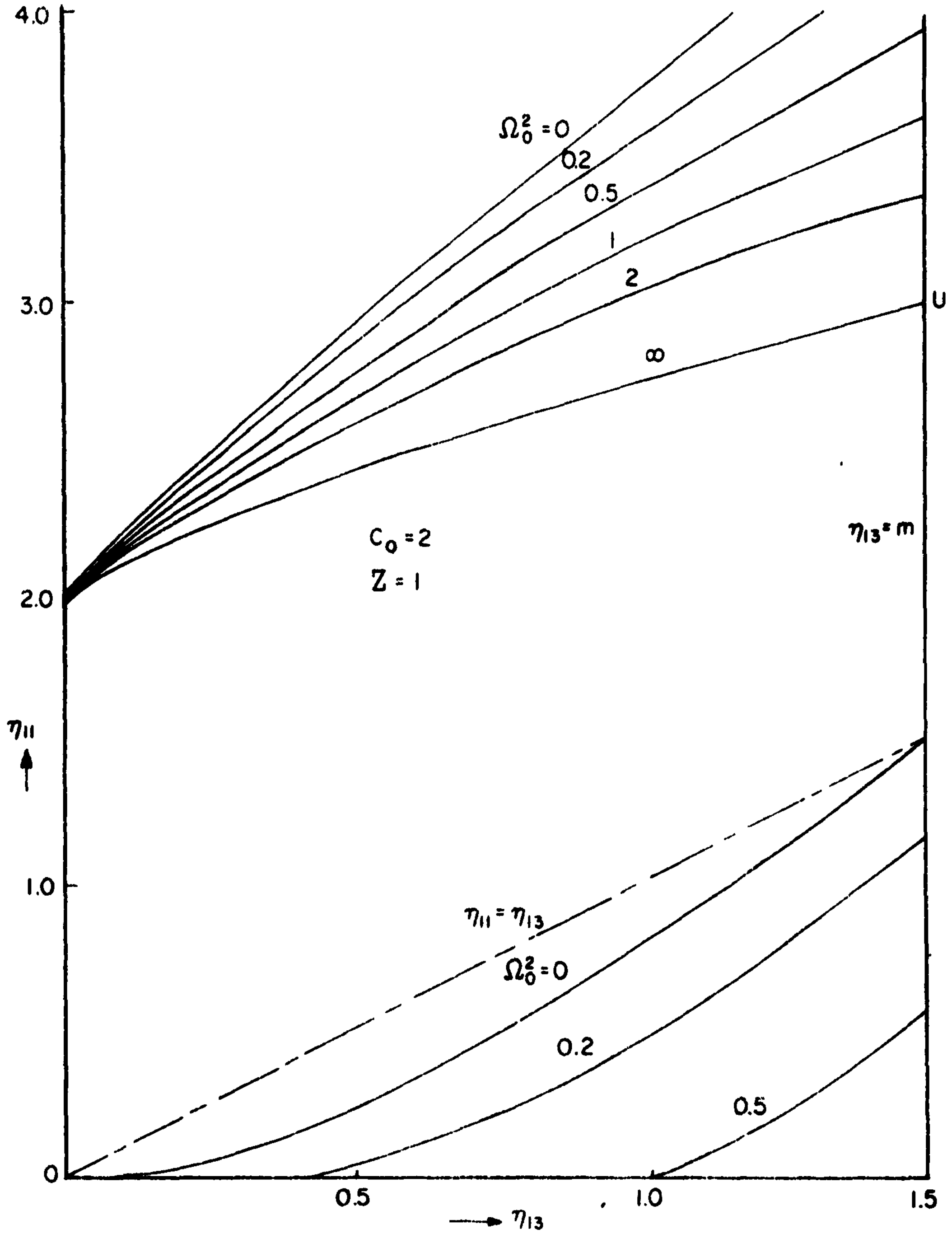

Fig. 7. Curves η_{11} vs. η_{13} for $C_0 = 2$, $Z = 1$

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