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NRL Report 5837

**A DIGITAL COMPUTER PROGRAM
FOR CALCULATING
SPHEROIDAL WAVE FUNCTION EIGENVALUES
AND EXPANSION CONSTANTS**

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ABSTRACT

Using the well-known procedures of Bouwkamp ("Theoretische en numerieke behandeling van de buiging door een ronde opening," Diss. Groningen, Groningen Batavia, 1941) for finding the eigenvalues of differential equations solvable by application of three-term recursion formulas, the authors have developed a digital computer program in symbolic language for obtaining the eigenvalues of the separated steady-state wave equation in spheroidal coordinates, i.e., for the equation

$$\frac{d}{d\eta} \left[(1 - \eta^2) \frac{dS_{m\ell}}{d\eta} \right] + \left[A_{m\ell} - h^2 \eta^2 - \frac{m^2}{1 - \eta^2} \right] S_{m\ell} = 0$$

where the parameters ℓ , m , and h are as defined by Stratton et al. in "Spheroidal Wave Functions" (New York: Wiley, 1956). The symbolic language used is universal (e.g., add a to b , multiply c by d , etc.) and may be applied to any digital computer by simple conversion to machine language. The program developed requires at least 3400 working cells. It computes the eigenvalue $A_{m\ell}$ to a maximum of 13 significant places for any integer values of ℓ and m ($\ell \geq m$). The value of h is left arbitrary. The program also computes all the spheroidal wave function expansion coefficients (d_p) up to $p \leq 200$.

PROBLEM STATUS

This is an interim report on one phase of the problem; work is continuing.

AUTHORIZATION

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A DIGITAL COMPUTER PROGRAM FOR CALCULATING SPHEROIDAL WAVE FUNCTION EXPANSION CONSTANTS

INTRODUCTION

The solution of the Helmholtz wave equation in spheroidal coordinates leads to spheroidal wave functions of the angle and radial types. It is customary to expand these wave functions in two infinite series consisting of associated Legendre functions and spherical Bessel functions, respectively. Tables of expansion coefficients have appeared in the literature at various times. Unfortunately, these tables have not been unique since the entries depended upon the definitions of the functions as conceived by their authors and upon the author's choice of normalization. In the decade 1950-1960 there appeared, among others, two new books of tables, one by Stratton, Morse, et al. (1) and the other by Flammer (2). The first collection lists the expansion coefficients $d_n(h|m\ell)$ for $h = 0.1$ (0.1) 1, $h = 1$ (0.2) 8, $m = 0$ (1) 8, and $\ell = 0$ (1) 8. The second book is a miscellany from many sources covering some of the ranges in the first but normalized by an entirely different scheme. Flammer's book, in addition, contains a cogent account of wave function theory. The extent of these existing tables has been found to be insufficient for many computation needs in the field of applied acoustics. The range of the eccentricity parameter h is too restrictive, and interpolation for h values not listed is impractical.

The authors of the present report have therefore turned to the difficult task of writing a digital computer program which is universal enough to supply spheroidal wave function expansion constants in any quantity for any values of m , n , and h . Our efforts have been considerably lightened by the lucid explanation in Flammer's book, for which we acknowledge a debt of gratitude. Sections of Flammer's work appear in the following brief explanation of the procedure we used to formulate the computer program.

ANGLE SPHEROIDAL WAVE FUNCTIONS

Prolate spheroidal angle functions $S(h, \eta)$ are defined to be functions which satisfy the ordinary differential equation

$$\frac{d}{d\eta} \left[(1 - \eta^2) \frac{dS}{d\eta} \right] + \left[A - h^2 \eta^2 - \frac{m^2}{1 - \eta^2} \right] S = 0. \quad (1)$$

The parameters A and m are separation constants. We specify here that m is an integer and that $A(h) \rightarrow \ell(\ell + 1)$ as $h \rightarrow 0$, where ℓ is the order (an integer) of the associated Legendre function $P_\ell^m(\eta)$. For each value of the integers m and ℓ there corresponds a function $S_{m\ell}$ which can be expanded in an infinite series of associated Legendre functions. The function $S_{m\ell}$ is

$$S_{m\ell}(h, \eta) = \sum_n' d_n(h|m\ell) P_{m+n}^m(\eta). \quad (2)$$

If one substitutes Eq. (2) into Eq. (1), one obtains the following three-term recursion formula in $d_n(h|m\ell)$;

$$\rho_{m,n} d_{n+2} + \sigma_{m,n} d_{n-2} + (\tau_{m,n} - A_{m\ell}) d_n = 0 \quad (3a)$$

where

$$\rho_{m,n} = \frac{(n+2m+1)(n+2m+2)}{(2n+2m+3)(2n+2m+5)} h^2, \quad (3b)$$

$$\sigma_{m,n} = \frac{n(n-1)}{(2m+2n-1)(2m+2n-3)}, \quad (3c)$$

and

$$\tau_{m,n} = \left[\frac{2(n+m)(n+m-1) - 2m^2 - 1}{(2n+2m+3)(2n+2m-1)} \right] h^2 + (n+m)(n+m+1). \quad (3d)$$

A simple rearrangement of Eq. (3a) gives

$$\frac{dn}{d_{n-2}} = \frac{-\sigma_{m,n}}{\tau_{m,n} - A_m \ell + \rho_{m,n} \frac{d_{n+2}}{dn}}. \quad (4a)$$

If one multiplies both sides of Eq. (4a) by $\rho_{m,n-2}$ one obtains

$$N_n^m = \frac{-\beta_n^m}{\tau_n^m - A_m \ell + N_{n+2}^m} \quad (4b)$$

or

$$[N_{n+2}^m]_1 = \frac{-\beta_{n+2}^m}{\tau_{n+2}^m - A_m \ell + N_{n+4}^m} \quad (4c)$$

in which

$$N_n^m = \rho_{m,n-2} \frac{d_n}{d_{n-2}}, \quad N_{n+2}^m = \rho_{m,n} \frac{d_{n+2}}{d_n},$$

and

$$\beta_n^m = \rho_{m,n-2} \sigma_{m,n}.$$

One can also solve Eq. (4b) for N_{n+2}^m and obtain

$$[N_{n+2}^m]_2 = \frac{-\beta_n^m}{N_n^m} + A_m \ell - \tau_n^m. \quad (4d)$$

Equations (4c) and (4d) are continued fractions. If one subtracts these equations for a given $A_m \ell$ and the result is zero, the number $A_m \ell$ is a characteristic value of Eq. (1). In symbols, to determine $A_m \ell$ we attempt to make the following calculation

$$[N_{n+2}^m]_1 - [N_{n+2}^m]_2 \cong 0. \quad (4e)$$

The first term on the left-hand side is computed by initially taking n so large that N_{n+4}^m is negligible, and then finding the continued fraction $[N_{n+2}^m]_1$ for successively smaller values of n . The second term on the left-hand side is computed by assuming N_{n+2}^m is zero for $n+2 < 0$ and then computing $[N_{n+2}^m]_2$ for successively larger values of n . It is evident from this procedure that one can obtain $A_m \ell$ only by successive approximation. A very effective method of hastening the computation of $A_m \ell$ has been devised by Bouwkamp (3).

In this method one first computes $A_m \ell$ by allowing $A_m \ell = \ell(\ell + 1)$. Then one computes a correction term $\delta(A_m \ell)$ by taking the variation of Eq. (4e) due to a variation $\delta(A_m \ell)$; in symbols this is

$$[N_{n+2}^m (A_m \ell + \delta(A_m \ell))]_1 - [N_{n+2}^m (A_m \ell + \delta(A_m \ell))]_2 = 0. \quad (5a)$$

Now

$$N_{n+2}^m (A_m \ell + \delta(A_m \ell)) \approx N_{n+2}^m (A_m \ell) + \delta(N_{n+2}^m). \quad (5b)$$

Hence, Eq. (5a) reads

$$[N_{n+2}^m (A_m \ell)]_1 - [N_{n+2}^m (A_m \ell)]_2 + \delta[N_{n+2}^m]_1 + \delta[N_{n+2}^m]_2 = 0. \quad (5c)$$

From Eq. (4c) one finds

$$\begin{aligned} \delta[N_{n+2}^m]_1 &= \frac{\beta_{n+2}^m [-\delta(A_m \ell) + \delta(N_{n+4}^m)]}{(\tau_{n+2}^m - A_m \ell + N_{n+4}^m)^2} \\ &= -\delta(A_m \ell) \left[\frac{(N_{n+2}^m)^2}{\beta_{n+2}^m} + \frac{(N_{n+2}^m)^2 (N_{n+4}^m)^2}{\beta_{n+2}^m \beta_{n+4}^m} + \dots \right]. \end{aligned} \quad (6a)$$

Similarly, from Eq. (4d)

$$\begin{aligned} \delta[N_{n+2}^m]_2 &= \frac{\beta_n^m}{(N_n^m)^2} \delta(N_n^m) + \delta(A_m \ell) \\ &= \delta(A_m \ell) \left[1 + \frac{\beta_n^m}{(N_n^m)^2} + \frac{\beta_{n-2}^m \beta_n^m}{(N_n^m)^2 (N_{n-2}^m)^2} + \dots \right]. \end{aligned} \quad (6b)$$

Since n is an integer it is convenient to let $n = \ell - m$. Substituting Eqs. (6a) and (6b) into Eq. (5c) and solving for $\delta(A_m \ell)$ one obtains

$$\delta(A_m \ell) = \frac{[N_{\ell-m+2}^m]_1 - [N_{\ell-m+2}^m]_2}{\left[1 + \frac{\beta_{\ell-m}^m}{(N_{\ell-m}^m)^2} + \frac{\beta_{\ell-m-2}^m \beta_{\ell-m}^m}{(N_{\ell-m-2}^m)^2 (N_{\ell-m}^m)^2} + \dots \right] + \left[\frac{(N_{\ell-m+2}^m)^2}{\beta_{\ell-m+2}^m} + \frac{(N_{\ell-m+2}^m)^2 (N_{\ell-m+4}^m)^2}{\beta_{\ell-m+2}^m \beta_{\ell-m+4}^m} + \dots \right]}. \quad (6c)$$

Repeated corrections by Eqs. (6c) and (5a) lead to very accurate values of the characteristic number $A_m \ell$. Now

$$\frac{d_n}{d_{n-2}} = \frac{N_n^m}{\rho_{m,n-2}}. \quad (7a)$$

The number N_n^m is calculated from Eq. (4b) using the corrected value of $A_m \ell$. The ratio d_n/d_{n-2} is therefore accurately calculated. From Eq. (7a) one deduces that

$$d_n = d_0 \left[\frac{d_n}{d_{n-2}} \frac{d_{n-2}}{d_{n-4}} \dots \frac{d_2}{d_0} \right], \quad n \geq 2. \quad (7b)$$

The expansion constants d_n can be normalized in several different ways. We adopt here the normalization of Morse and Feshbach (4),

$$\sum_{n=0,1}^{\infty} \frac{(n+2m)!}{n!} d_n = \frac{(\ell+m)!}{(\ell-m)!}. \quad (7c)$$

Hence, the magnitude of d_0 is fixed to be

$$d_0 = \frac{(\ell+m)!}{(\ell-m)! \left\{ \sum_{n=2}^{\infty} \frac{(n+2m)!}{n!} \left[\frac{d_n}{d_{n-2}} \frac{d_{n-2}}{d_{n-4}} \dots \frac{d_2}{d_0} \right] + (2m)! \right\}} \quad \text{for } n \text{ even.} \quad (7d)$$

Similarly we find d_1 to be

$$d_1 = \frac{(\ell+m)!}{(\ell-m)! \left\{ \sum_{n=3}^{\infty} \frac{(n+2m)!}{n!} \left[\frac{d_n}{d_{n-2}} \frac{d_{n-2}}{d_{n-4}} \dots \frac{d_3}{d_1} \right] + (1+2m)! \right\}} \quad \text{for } n \text{ odd.} \quad (7e)$$

All remaining d_n are found by substituting Eqs. (7d) and (7e) into Eq. (7a).

The prolate spheroidal angle functions $S(h, \eta)$ can be converted into oblate spheroidal angle functions by making the transformation $h \rightarrow -jg$. The prolate expansion coefficients $d_n(h|m\ell)$ become $d_n(-jg|m\ell)$ by this same transformation. Thus to find $d_n(-jg|m\ell)$ one replaces h^2 in Eq. (3a), (3b), (3c), and (3d) by $-g^2$. The normalization procedure is identical for both prolate and oblate cases.

THE COMPUTER PROGRAM

The computer program is divided into two parts. In the first part the program computes the eigenvalue $A_{m\ell}$ to 13 significant places, or less depending on the parameters m , ℓ , h . In the second part the program computes any number of spheroidal wave function expansion constants $d_n(h|m\ell)$ where n may take on integer values up to $n = 400$. These constants are computed to 13 significant places or less depending on the parameters m , ℓ , h . The precision of $A_{m\ell}$ controls the precision of $d_n(h|m\ell)$.

The computation of eigenvalues is accomplished by Bouwkamp's procedure (3), the essential step of which is found in Eq. (6c). We select a starting value of $A_{m\ell}$ according to the given magnitude of h . A short discussion of the method of selecting a good starting value for $A_{m\ell}$ is presented in Appendix E of this report. If this starting value is poor the program may converge to an erroneous eigenvalue. After the initial value of $A_{m\ell}$ is set, the value is improved by application of Eq. (6c). Repeated use of Eq. (6c) eventually leads to very accurate values of $A_{m\ell}$.

When the computation of the eigenvalue is completed the computation of the constants d_n is accomplished by using the relations found in Eqs. (7a) to (7e).

The computer program finally assembled is reproduced in Appendix B of this report. The meanings of the various instructions used in the program may be found in the publication Narec Bulletin No. 27 (July 27, 1961) available from the Research Computation Center at NRL. We note that the language of this program is entirely symbolic (i.e., add a to b, multiply c by d, etc.). The language is therefore universal. It may be converted to the machine language of a specific computer by replacing each symbolic command by the equivalent machine language command.

A set of input instructions for the particular version of this symbolic program in the NIP language of the Narec-NRL is found in Appendix A.

ADDITIONAL REMARKS

The authors of this report are in the process (to date) of preparing digital computer programs for computing radial and angle spheroidal wave functions of any order and any argument, using the eigenvalues and expansion coefficients found by the program described here.

ACKNOWLEDGMENT

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APPENDIX A

INSTRUCTIONS FOR USE OF NAREC-NRL SPHEROIDAL WAVE FUNCTION ROUTINE

The Narec-NRL spheroidal wave function routine is found on Narec Tape No. 4593

1. Prepare a machine-language Operator Instruction Sheet (see Fig. A1).

2. Prepare a Data Input Sheet as follows:

a. Insert the prolate spheroidal values of m , ℓ , and h in successive two-word entries on the sheet, reserving two twelve-digit Narec floating point words (i.e., fractional part and exponent) for each of the three numbers. These values will take up six lines.

b. Determine the highest subscript (XXX) in the d_{XXX} constants desired. If $\ell-m$ is even, the subscript will be even; if $\ell-m$ is odd, the subscript will be odd. Insert the value of XXX as a two-line Narec floating point number in the two lines following the six noted above. This makes eight lines.

c. Count the actual number of d's to be typed out. If $\ell-m$ is even, the count is $(XXX + 2)/2$. If $\ell-m$ is odd, the count is $(XXX + 1)/2$. Convert this count to a hex number and pre-fix zeros to make up a "4-digit hex number" (say YYYY). Insert this hex number in the standard word appearing in the ninth line. This word is

YYYY de 064a e5.

d. If $\ell-m$ is even, convert $XXX + 2$ into a four-digit hex number, add eight zeros, and insert in line 10.

If $\ell-m$ is odd, convert $XXX + 1$ into a four-digit hex number, add eight zeros, and insert in line 10.

When this procedure is complete the data tape contains 10 lines, forming one block of data input. At the head of the block write "nnnn" and at the tail write "mmmm."

Any number of blocks may be inserted, provided the separation symbols (nnnn and mmmm) are used. When the data input sheet is completed, it is handed to Narec Research Computation Center for further processing.

3. Manual Changes

a. If oblate spheroidal constants are desired, write on the blue machine-language Instruction Sheet

Manual Changes: line 0413 - 0587 e7 0587 eb

07b3 - 0800 00 0000 00

07b4 - 0001 00 0000 00

OPERATOR INSTRUCTION SHEET Date _____

RCC Problem Number _____ Estimated Computation Time 15 MIN

Problem Title _____ Programmer _____

Run _____ Tel. Ext. _____

OUTPUT FORMAT: Copies _____ original _____ carbons.

A: Hexadecimal, Decimal: 2 cols. B: Hexadecimal, Decimal: --- cols.

INPUT TAPE INFORMATION

	Tape No.	Locations		Check Sum (if known)		Tape No.	Locations		Check Sum (if known)
		From	Thru				From	Thru	
1	4593	0400	0864	1275 A0 1202 40	6				
2	2700	0000	0344		7				
3	Put data tape on F-1212A				8				
4					9				
5					10				

SPECIAL INPUT INSTRUCTIONS, MANUAL CHANGES, AND STARTING ORDER

hand input:

line 0413	0587 C7	0517 E4
0743	020000	000000
0744	001000	000000

PRINT-OUTS

Estimated time to First Print-out _____ Total Number of Output Lines _____

Time Per Line _____ Location of Final Stop Order _____

RESTART ORDERS

Read in Tapes _____

Manual Changes _____

Starting Order LO RO _____

DUMP: Output Format A B Other _____

Read in Tapes _____

Manual Changes _____

Starting Order LO RO _____

SPECIAL INSTRUCTIONS, ADDITIONAL INFORMATION, ETC.

Fig. A1 - Sample of Operator Instruction Sheet

b. If the tolerance on $A_{m\ell}$ is to be changed, compute the new tolerance as a two-line Narec floating point number and write this number as

Manual Change: line 0610 _____
 0611 _____

The tolerance on $A_{m\ell}$ is fixed at 10^{-11} in the program.

c. If the tolerance used in comparing $N_{\ell-m+2}^m$ with $-\gamma_n^m + A_{m\ell}$ is to be changed, compute a new tolerance as a two-line Narec floating point number and write

Manual Change: line 05fe: _____
 05ff: _____

This tolerance is started at 10^{-2} in the program.

d. If the eigenvalue tolerance ($= \ell$ tol) comparing the change computed eigenvalue for increasing N's is to be changed, compute a new tolerance as a two-line Narec floating point number and write

Manual Change: line 06d0: _____
 06d1: _____

This tolerance is fixed at 10^{-11} in the program.

e. If the n th ratio tolerance ($= n$ tol) comparing the change in $N_{\ell-m+4}^n$ as the number of N's increase is to be changed, compute a new tolerance as a two-line Narec floating point number and write

Manual Change: line 06C3: _____
 06C4: _____

This tolerance is fixed at 10^{-11} in the program.

f. Starting values for $A_{m\ell}$ depend upon whether h is considered "small" or "large." A short discussion of the method of making a proper selection of the starting value for $A_{m\ell}$ is found in Appendix E.

As an example of the use of the instructions outlined above, we select the following as an appropriate example of the preparation of a data input sheet.

Example: Let $m = 1$, $\ell = 7$, and $h = 10$ for the oblate case. We desire the largest expansion constant to be d_{60} (note that the subscript must be an even integer). The data input sheet reads as follows.

n n n n n
080000 000000
000100 000000
070000 000000
000400 000000
0A0000 000000
000400 000000
0f0000 000000
000600 000000
001fde 064Ae5
003E00 000000

m m m m m

This sheet is sent to Narec Research Computation Center for further processing.

Since the spheroid is oblate, make the following "manual changes" on the standard blue-colored Narec Operator Instruction Sheet:

line 0413 → 0587e7 0587eb
07b3 → 080000 000000
07b4 → 000100 000000

APPENDIX B

REPRODUCTION OF COMPUTER PROGRAM IN SYMBOLIC FORM

NOTE: Any group of letters and numbers not specifically appearing in the list of commands of Narec Bulletin # 27 is to be considered a machine address. For example, spro, k1, k2, etc. are addresses, while p, pd, mp are standard commands. \ Method of reading: at address spro in the computer memory place the command "go to left half of address bgn etc.

spro	lo bgn		j22 p
	0		mp j22
k1	k1 a		p j24
	ro 512		j3 p
k2	j3 p		mp j10
	-j2 pd		p j25
	p j9		j11 p
	j3 p		mp j10
	j5 pd		-j14 pd
	p j10		-j5 pd
	j3 p		p j26
	mp j6		j12 p
	p j11		mp j13
	-j5 pd		p j27
	p j12		j26 p
	j8 pd		p/ j27
	p j13		mp j22
	j2 p		j25 pd
	mp j2	k3	kfdg pd
	mp j6		p j28
	p j14	k4	fl yadl
	mp j6		j9 p
	p j15		p j100
	j2 p	sn7	j100 p
	j5 pd		cmp j5
	p j16		fl k5
	j2 p	sn8	fl k6
	mp j6		fl k7
	p j17	k7	j100 p
	-j5 pd		-j6 pd
	p j18		p j100
	j6 pd		fl k4
	p j19	k5	jq20 p
	-j8 pd		p jq30
	p j20		p jq31
	j8 pd		fl k12
	j6 pd	k6	j5 p
	p j21		p jq30
	j4 p		p jq31
	mp j4		fl k12
	p j22	k8	j2 p
	p/ j6		jq31 pd
	p j23		p j32
			j5 pd
			mp j32
			p j33
			j15 p

	-j5 pd	k16	k16 a
	p j34		lo 512
	jq31 p	k17	fl k10
	mp j6		jl01 p
	p j35	k18	p j400
	jl8 p		adl 2
	j35 pd		k18
	p j36		jq31 p
	j8 pd		j6 pd
	mp j36		p jq31
	p j37	k19	fl k13
	j34 p		jq30 p
	p/ j37		p jq31
	p j38		p jl04
	j5 p	kg19	jl04 p
	-j38 pd		j6 pd
	mp j23	sn2	p jl04
	j33 pd		cmp jl02
	p j39	snb	fl qql
k9	fr 0		fl kg24
k10	jq31 p	snc	fl kg24
	-j5 pd	qql	nl k20
	p j40	k20	k22 a
	jl7 p		lo k21
	jq31 pd	k21	lal k9
	p j41		il k9
	-j5 pd	k22	k22 a
	p j42		lo 512
	jq31 pd		fl k8
	p j45	k23	j39 p
	j6 pd		p j300
	p j44		adl 2
	-j8 pd		k23
	p j43		jq31 p
	jq31 p		j6 pd
	mp j40	kg24	p jq31
	mp j41		fl kg19
	mp j42		jq30 p
	p/ j45		p jq31
	p/ j45	k24	p jl05
	p/ j43		jl05 p
	p/ j44		j6 pd
	mp j24	sn3	p jl05
	p jl01		cmp jl02
k11	fr 0	snd	fl k25
k12	j5 p	gk24	fl hv10
	p j900		fl hv10
	j9 p	k25	-j400 p
	j8 pd	k26	p/ j900
khb	p jl02	k27	-j300 pd
	jq30 p		j28 pd
	p jl03	k28	p j901
	p jq31		p jl06
k13	jl03 p		adl 2
	j6 pd		k25
sn1	p jl03		adl 2
	cmp jl02		k26
sn9	fl kg14		adl 2
	fl k19		k27
sna	fl k19		adl 2
kg14	nl k14		k28
k14	k16 a	hv10	fl k24
	lo k15		j9 p
k15	lal k11		j6 pd
	il k11		p jl20

	p j123		j124 p
	-j6 pd		-j6 pd
	p j124		p j124
	jq30 p		fl hv22
	p j121	j124	0
hvl1	j121 p		0
	j6 pd		0
sn4	p j121		0
	cmp j123	hv28	lsr hv24
sne	fl hv12		hv29
	fl hv15	hv29	j5 p
snf	fl hv15		p 0
hvl2	j401 p	hv30	wl00 p
hvl3	p/ j901		cmp j5
wbl0	p/ j901	snj	fl hv31
hvl4	p v100		fl hv33
	adl 2	snk	fl hv31
	hvl2	hv31	yz12 p
	adl 2	hv32	wl00 pd
	hvl3		p yz12
	adl 2		adl 2
	hvl4		hv30
	adl 2		adl 2
	wbl0		hv32
	fl hv11		fl hv30
hvl5	ls1 hv14	hv33	yz12 p
	hvl8		j5 pd
hvl7	j5 p		p yz12
hvl8	p 0	k29	j9 p
hv22	v100 p		p j107
hw21	cmp j5	k30	j107 p
	fl hw22		j6 pd
sng	fl hv28		p j107
	fl hw22		p j111
hw22	ls1 hv22		p j112
	hv23		p jq31
hu22	ls1 hv22		nl k31
	hw24	k31	k33 a
hw23	adl 2		lo k32
	hv23	k32	lal k11
hw24	0 p		il k11
	p w100	k33	k33 a
	jq30 p		lo 512
	p j122	k34	fl k10
hw25	wl00 p		nl k35
hv23	mp 0	k35	k37 a
hv24	p w100		lo k36
hv25	j122 p	k36	lal k9
	j6 pd		il k9
sn5	p j122	k37	k37 a
	cmp j124		lo 512
snh	fl hv26	k38	fl k8
	fl hv27		j39 p
sni	fl hv27		-j28 pd
hv26	adl 2		p j108
	hv23		mp wz10
	fl hw25		/p/
hv27	adl 2		p j109
	hv22		-j101 p
	adl 2	sn6	p/ j108
	hv24		/p/
	adr 2	snm	cmp j109
	hw24		fl kg39
	adl 2	snn	fl kg39
	hw25		fl k30

kg39	jq30 p		p yzl0
	p jq31		adl 2
k39	j9 p		k58
	-j6 pd		adl 2
	p j110		k60
k40	j111 p		fl k58
	-j6 pd	kv63	adl 2
	p j111		hv57
k41	j110 p		ls1 hv57
	-j111 pd		k58
	fc1 k55		ls1 hv57
k42	j112 p		k60
	p jq31		adl 2
	nl k43		k61
k43	k45 a		fl hv56
	lo k44	hv64	lsr k61
k44	lal k11		kv65
	il k11	kv65	j5 p
k45	k45 a		p 0
	lo 512	k64	yl00 p
k46	fl k10		cmp j5
	nl k47	sns	fl k65
k47	k49 a		fl k67
	lo k48	snt	fl k65
k48	lal k9	k65	yz11 p
	il k9	k66	yl00 pd
k49	k49 a		p yz11
	lo 512		adl 2
k50	fl k8		k64
	j39 p		adl 2
	-j28 pd		k66
	j113 pd		fl k64
	p j114	k67	yz12 p
	-j101 p		yz11 pd
	p/ j114		p yz13
	p j113		j113 p
k51	j113 p		-j106 pd
	mp j113		p/ yz13
	p/ j101		p yz14
k52	p k300		j28 pd
k53	j112 p	snu	p j28
	-j6 pd		yz14 p
	p j112		/p/
k54	adl 2		-j126 pd
	k52	k62	fc1 k79
	fl k40		fl ltcl
k55	lsr k52	k70	pt 11
	k56		fl hov 4
k56	j5 p	k68	ro k68
	p 0		lo dj8
hv56	j5 p	k69	j151 p
	p yzl0		p j103
hv57	k300 p		p j104
	cmp j5		p j105
sno	fl k58		p j121
	fl hv64		p j122
snp	fl k58		p j123
k58	k300 p		p yz11
	cmp j5		p yz12
snq	fl k59		p j107
	fl kv63		p j111
snr	fl k59		p j112
k59	yz10 p		p j113
k60	mp k300		nl apne
k61	p y100	dj8	dj8 a

	ro 512		vh14
dj9	j5 p		fl vh10
	p j900	vh16	lsr vh14
dj10	dj50 p		vh17
	ls1 dj10	vh17	j5 p
	gk24		p 0
	j9 p	vh19	cx10 p
	j8 pd		j5 pd
	p j102		p cx10
	j151 p	vh20	bl00 p
	p j103		/p/
	p j104		cmp j5
	p j105		fl vh21
	ls1 w18		fl vh24
	k18		fl vh21
	ls1 w23	vh21	cx10 p
	k23	vh22	mp bl00
	ls1 w18	vh23	p cl00
	k25		p cx10
	ls1 w9		ad1 2
	k26		vh20
	ls1 w23		ad1 2
	k27		vh22
	ls1 w91		ad1 2
	k28		vh23
	fr khb		fl vh20
dj11	lsr djf6	vh24	lsr vh23
	dj12		vh25
dj12	j5 p	vh25	j5 p
	p 0		p 0
	jq30 p	vh27	jq30 p
	p wy10		p cq10
vh10	wy10 p	vh28	cq10 p
	j6 pd		j6 pd
	p wy10		p cq10
vh11	j901 p		cq10 p
	/p/		jl7 pd
hov6	cmp j5		p qc10
	fl vh12		p qc12
	fl vh16	vh29	cl00 p
	fl vh12		/p/
vh12	wyl0 p		cmp j5
	mp j6		fl vh30
	jl7 pd		fl vh35
	j5 pd		fl vh30
	p wyl1	vh30	cq10 p
	-j6 pd		/p/
	mp wyl1		cmp qc10
	p wyl2		fl vh31
	wyl0 p		fl vh32
	jl7 pd		fl vh32
	p wyl3	vh31	qc10 p
	-j5 pd		-j5 pd
	mp wyl3		p qc11
	p wyl4		mp qc12
	wyl2 p		p qc12
	p/ wyl4		qc11 p
	p/ j22		p qc10
vh13	mp j901		fl vh30
vh14	p bl00	vh32	qc12 p
vh15	ad1 2		p/cq10
	vh11	vh33	mp cl00
	ad1 2	vh34	p vx10
	vh13		ad1 2
	ad1 2		vh29

	adl 2	/p/
	vh33	cmp j5
	adl 2	fl vh46
	vh34	fl vh48
	fl vh28	fl vh46
vh35	lsr vh34	vh46
	vh36	cl00 p
vh36	j5 p	mp qq3
	p 0	vh47
vh38	vx10 p	adl 2
	/p/	vh45
	cmp j5	adl 2
	fl vh39	vh46
	fl vh80	adl 2
	fl vh39	vh47
vh39	qq2 p	fl vh45
vh40	vx10 pd	vh48
	p qq2	rep 6
	adl 2	nr eop
	vh38	vh49
	adl 2	0
	vh40	0
jhb	fl vh38	vh50
	j9 p	vvxl p
	cmp jl51	vh51
	fr ersp	pt 11
	fr vh44	adl 2
	p bcl0	vh50
vh41	-j5 pd	fr vh49
	p bcl1	j2
	cmp jl51	080000
	fl vh42	0
	fl vh42	000100
	bcl0 p	0
	mp bcl1	j3
	p bcl0	080000
	bcl1 p	0
	fl vh41	000200
vh42	j3 p	0
	j2 pd	j4
	cmp jl51	080000
	fr ersp	0
	fl vh54	000100
vh43	p bcl2	0
	-j5 pd	j150
	p bcl3	0c0000
	cmp jl51	0
	fl vh55	4 zz
	fl vh55	0
	bcl2 p	j5
	mp bcl3	080000
	p bcl2	0
	bcl3 p	1 zz
vh44	fr vh43	0
	j5 p	j6
	p bcl0	080000
	fl vh42	0
vh54	j5 p	2 zz
	p bcl2	0
vh55	bcl2 p	j7
	p/bcl0	0c0000
	p/qq2	0
	p qq3	2 zz
	p vvxl	0
vh45	cl00 p	j8
		080000
		0
		3 zz
		0
		j9
		0
		0
		0
		0
		j10
		0
		0
		0
		j11
		0

	0		0
	0		0
	0		0
j12	0	j28	0
	0		0
	0		0
	0		0
j13	0	j100	0
	0		0
	0		0
j14	0	jq20	0
	0		0
	0		0
j15	0	jq30	0
	0		0
	0		0
j16	0	jq31	0
	0		0
	0		0
j17	0	j32	0
	0		0
	0		0
j18	0	j33	0
	0		0
	0		0
j19	0	j34	0
	0		0
	0		0
j20	0	j35	0
	0		0
	0		0
j21	0	j36	0
	0		0
	0		0
j22	0	j37	0
	0		0
	0		0
j23	0	j38	0
	0		0
	0		0
j24	0	j39	0
	0		0
	0		0
j25	0	j40	0
	0		0
	0		0
j26	0	j41	0
	0		0
	0		0
j27	0	j42	0
	0		0

j45	0	j112	0
	0		0
	0		0
	0		0
j44	0	j108	0
	0		0
	0		0
	0		0
j43	0	wz10	0a3d70
	0		a3d70a
	0		fffa00
	0	j109	0
j101	0		0
	0		0
	0	j110	0
j102	0		0
	0		0
	0	j113	0
j103	0		0
	0		0
	0	j114	0
j104	0		0
	0		0
	0	yz10	0
j105	0		0
	0		0
	0	yz11	0
j106	0		0
	0		0
	0	yz13	0
j120	0		0
	0		0
	0	yz14	0
j121	0		0
	0		0
	0	j126	0afec0
j122	0		0
	0		ffdc00
	0	wy10	0
j123	0		0
	0		0
	0	wy11	0
yz12	0		0
	0		0
	0	wy12	0
j107	0		0
	0		0
	0	wy13	0
j111	0		0
	0		0
	0		0

wyl4	0			vh48 1
	0			040083
	0			240030
	0			lo kl
cx10	0		psm	0
	0			0
	0			0
	0			0
cq10	0			0
	0			0
	0			0
	0			0
qc10	0			0
	0			0
	0			0
	0			0
qc12	0			0
	0			0
	0			0
	0			0
qc11	0		prep	0
	0			0
	0		wj71	0
	0			0
qq2	0		apne	240081
	0			0400fd
	0		apnf	apnf a
	0			ro 512
bc13	0			jq30 p
	0			p jq31
	0		eop	fl kl2
	0			stop
bc10	0		nrun	240083
	0			040030
	0			lo bgn
	0			0
bc12	0		fset	pc 37
	0			pc 37
	0			pc 7
	0			pc 4
bc11	0			pc 4
	0			j2 p
	0			0300f1
	0			pc 37
qq3	0			pc 6
	0			pc 4
	0			pc 4
	0			j3 p
j151	0			0300f1
	0			pc 37
	0			pc 8
	0			pc 4
w18	j400 zz			pc 4
	0			j4 p
w23	j300 zz			0303f1
	0			pc 37
w9	j900 zz			pc 37
	0			pc 37
w91	j901 zz			pc 16
	0			pc 16
bgn	rd psm			pc 16
	lo i400			pc 16
400	t psm			pc 41
	j2 8			j28 p
	t prep			fl k70

vh80	j17 p		lo dj60
	cmp j151		l11 k9
	fr ersp		il k9
	fr vh83		
vh81	fl vh84		dj61 dj61 a
	-j5 pd		lo 512
	p hall		dj62 fl k8
	cmp j151		j39 p
	fr vh82		-j28 pd
	fr vh82		j113 pd
	hal0 p		p j114
	mp hall		-j101 p
	p hal0		p/j114
	hall p		p j113
vh82	fr vh81		dj63 p gg10
	qq2 p		p nlst
	hal0 pd		adl 4098
	p qq2		dj63
ersp	fr jhb		j112 p
	nl vh83		-j6 pd
vh83	Obad82		p j112
	j5 p		fl dj52
	p hal0		dj64 lsl wj65
	fr vh82		dj40
hal0	0		j112 p
	0		j6 pd
	0		p j111
	0		jq30 p
hall	0		p j110
	0		fl dj52
	0		wj65 dj65 zz
	0		0
vh84	j17 p		dj65 adl 2
	jq30 pd		dj63
	p hal0		lsl dj63
	fr vh81		dj70
dj50	jq30 p		dj70 0 p
	p jq31		cmp j5
	j150 p		fl djf6
	p j111		fl dj72
dj42	p j112		djf6 p j901
	j151 p		skip
	p j113		adl 2
	j5 p		djf6
dj41	p gg11		adl 2
dj51	j9 p		dj70
	p j110		fl dj70
dj52	j111 p		0
	-j6 pd		dj72 lsl wj71
	p j111		dj71
dj53	j110 p		lsr w9
	-j111 pd		dj73
dj40	fcl dj74		dj71 adr 0
dj54	j112 p		dj73
	p jq31		dj73 j5 p
	nl dj55		p 0
dj55	dj57 a		fl dj11
	lo dj56		k79 j160 p
dj56	lal k11		j5 pd
	il k11		p j160
dj57	dj57 a		cmp j7
	lo 512		fl k69
dj58	fl k10		fl k69
	nl dj59		fl kv70
dj59	dj61 a		j160 0
			0

	0		fl dj42
	0	ntol	0afec0
kv70	wzl0 p		0
	p/kten		ffdc00
	p wzl0		0
	j151 p	ltc1	j28 p
	p j160		/p/
	flhov5		p j29
kten	080000		lsr mbl
	0		k62
	2 zz		fl k79
	0	mb1	ltc2 zz
	0		0
	0	ltc2	j28 p
	0		/p/
	0		-j29 pd
dj74	nlst p		/p/
	/p/		-ltol pd
	p n'su		fcl il00
	ls1 wj74		fl fset
	dj40		0
	ls1 wj41	100	j28 p
	dj41		/p/
	ls1 wj63		p j29
	dj63		fl k79
	j150 p	ltol	0afec0
	kten pd		0
	p w150		ffdc00
	p j111		0
	fl dj42	j29	0
wj74	dj75 zz		0
	0		0
wj41	ggl1 zz		0
	0	kfdg	080000
wj63	ggl0 zz		0000f0
	0		1 zz
w150	0		0
	0	yad1	nr1b p
	0		-j5 pd
	0		fcl zin
nlst	0		fl vav
	0	vav	j4 p
	0		/p/
	0		cmp m992
nlsu	0		fl k3
	0		fl gnul
	0		fl gnul
	0	zin	j4 p
	0		/p/
dj75	-nlsu p		cmp m992
	/nlst/pd		fl k3
	/p/		fl gnu5
	cmp ntol		fl gnu5
	fl dj64	gnul	j3 p
	fl dj64		-j2 pd
	nlst p		mp j6
	/p/		j5 pd
	p nlsu		p m9r2
	ls1 wj41		mp m9r2
	dj41		p m9r3
	ls1 wj63		mp m9r2
	dj63		p m9r4
	w150 p		mp m9r2
	kten pd		p m9r5
	p w150		mp m9r2
	p j111		p m9r6

gnu2

mp m9r2
 p m9r7
 mp m9r2
 p m9r8
 j4 p
 mp j4
 p m9ra
 mp j4
 p m9rb
 mp j4
 p m9rc
 mp j4
 p m9rd
 j2 p
 mp j2
 p m9re
 mp m9re
 p mare
 j4 p
 mp m9r2
 p m9rf
 m9r3 p
 m8r0 pd
 p/ m8r1
 p m9rg
 m9re p
 mp m8r3
 p m9rh
 -m9rh p
 m9r3 pd
 m8r2 pd
 mp m9r2
 p/ m8r4
 p/ j4
 p m9rj
 m9r3 p
 mp m8r5
 m9r5 pd
 m8r6 pd
 mp m8r0
 p m9rk
 m9r3 p
 j5 pd
 mp m9re
 mp m8r8
 p m9rh
 -m9rh p
 m9rk pd
 p/ m8r7
 p/ m9ra
 p m9rn
 m9r6 p
 mp m8ra
 p m9rh
 m9r4 p
 mp m8rb
 p m9rk
 m9r2 p
 mp m8rc
 m9rk pd
 m9rh pd
 p/ m8r9
 p/ m8r9
 p m9rq
 m9r2 p

mp m8re
 p m9rh
 m9r4 p
 mp m8rd
 m9rh pd
 mp m9re
 p/ m8r9
 p m9rt
 m9r2 p
 mp mare
 p/ m8r1
 -m9rt pd
 m9rq pd
 p/ m9rb
 p m9ru
 m9r7 p
 mp m8rg
 p m9rh
 m9r5 p
 mp m88f
 p m9rk
 m9r3 p
 mp clal
 m8rh pd
 m9rk pd
 m9rh pd
 p/ m8rf
 p/ m8rf
 p m9rt
 m9r5 p
 mp m8rn
 p m9rh
 m9r3 p
 mp m8rj
 p m9rk
 m8rp pd
 m9rh pd
 mp m9re
 p/ m8rk
 p m9rq
 m9r3 p
 j5 pd
 mp mare
 mp m9r1
 p/ m8r1
 -m9rq pd
 m9rt pd
 p/ m9rc
 p m9rw
 m9r8 p
 mp m8rq
 p m9rh
 m9r6 p
 mp m8rs
 p m9rk
 m9r4 p
 mp m8rt
 p m9rq
 m9r2 p
 mp m8ru
 m9rq pd
 m9rk pd
 m9rh pd
 p/ m8r7
 p/ m8r7

	p m9rh		p g3f7
	m9r6 p		j2 p
	mp m8rv		mp j2
	p m9rk		p g3f8
	m9r4 p		mp g3f8
	mp m8rw		p g3f9
	p m9rq		mp g3f8
	m9r2 p		p g4f0
	mp m8ry	gnu6	g3f4 p
	m9rq pd		mp j4
	m9rk pd		mp j6
	mp m8re		p g4f1
	p/ m8r3		g3f5 p
	p/ m8r7		-g3f8 pd
	p m9rk		j5 pd
	m8rz p		p/ j6
	mp m9r4		p g4f2
	p m9rq		p/ g2ck
	m9r2 p		p/ j4
	mp m9r0		mp g3f4
	m9rq pd		p g4f3
	mp mare		g3f6 p
	p/ m8rk		mp m8r0
	p m9rq		p g5a0
	m9r2 p		g3f5 p
	mp m9re		mp m992
	p/ m9rl		p g5al
	p m9rt		g3f5 p
	-m9rt p		mp m991
	m9rq pd		j5 pd
	-m9rk pd		mp g3f8
	m9rh pd		mp j6
	p/ m9rd		p g5a2
	p m9rx		-g5a2 p
gnu3	fl hov 1		g3f9 pd
	m9rf p		j5 pd
	m9re pd		g5al pd
	-m9rg pd		g5a0 pd
	-m9rj pd		p/ m8r4
	-m9rn pd		p/ g3f0
	-m9ru pd		p g4f4
	-m9rw pd		g3f6 p
	-m9rx pd		mp m8ra
gnu4	p j28		p g5a0
	fl k3		g3f5 p
gnu5	j4 p		mp g2c2
	mp j4		p g5al
	p g3f0		g3f5 p
	mp j4		mp g2c1
	p g3f1		g2c4 pd
	mp j4		mp g3f8
	p g3f2		mp j6
	mp j4		p g5a2
	p g3f3		g3f9 p
	j3 p		mp g2c5
	-j2 pd		-g5a2 pd
	mp j6		m8rd pd
	j2 pd		g5al pd
	j5 pd		g5a0 pd
	p g3f4		mp g3f4
	mp g3f4		p/ m8rk
	p g3f5		p/ g3f1
	mp g3f5		p g4f5
	p g3f6		g3f7 p
	mp g3f5		mp m8rg

p g5a0		g2cb pd
g3f6 p		g5a2 pd
mp g2c3		g5a1 pd
p g5a1		g5a0 pd
g3f5 p		mp g3f4
mp g2c6		p/ g2cj
p g5a2		p/ g3f3
g3f6 p		fl hov3
mp m992	gnu7	-g3f0 p
p g5a3		g4f1 pd
g3f5 p		-g4f2 pd
mp g2c1		-g4f3 pd
m991 pd		-g4f4 pd
g5a3 pd		-g4f5 pd
mp g3f8		-g4f6 pd
mp m992		-g4f7 pd
p g5a3	gnu8	p j28
g3f5 p		fl k3
mp g2c5	m8r0	0a0000
g2c8 pd		0
mp g3f9		3 zz
mp m991		0
p g5a4	m8r1	080000
g4f0 p		0
mp j6		4 zz
p g5a5		0
-g5a5 p	m8r2	Gb0000
g5a4 pd		0
-g5a3 pd		4 zz
g2c7 pd		0
g5a2 pd	m8r3	080000
g5a1 pd		0
g5a0 pd		6 zz
p/ m8r7		0
p/ g3f2	m8r4	080000
p g4f6		0
g3f7 p		7 zz
mp m8rq		0
p g5a0	m8r5	0d0000
g3f6 p		0
mp g2c9		5 zz
p g5a1		0
g3f5 p	m8r6	0a8000
mp g2ca		0
p g5a2		5 zz
g3f6 p		0
mp g2cc	m8r7	080000
p g5a3		0
g3f5 p		11 zz
mp g2cd		0
g2ce pd	m8r8	0c0000
g5a3 pd		0
mp g3f8		9 zz
p g5a3		0
g3f5 p	m8r9	080000
mp g2cf		0
g2cg pd		8 zz
mp g3f0		0
p g5a4	m8ra	084000
g4f0 p		0
mp g2ch		6 zz
p g5a5		0
-g5a5 p	m8rb	0c7400
g5a4 pd		0
-g5a3 pd		11 zz

m8rc	0	m9r3	0 zz
	0afa80		0
	0		0
	13 zz		0
m8rd	0	m88f	09a600
	094000		0
	0		13 zz
	6 zz		0
m8re	0	clal	0a93f0
	0a7000		0
	0		16 zz
	8 zz		0
m8rf	0	m8rh	0af8c0
	080000		0
	0		15 zz
	9 zz		0
m8rg	0	m8rj	0a3c00
	0fc000		0
	0		11 zz
	6 zz		0
nrlb	0	m8rk	080000
	0 zz		0
	0		10 zz
	0		0
m8rt	0	m8rn	0e6000
	0fedf9		0
	0		7 zz
	20 zz		0
m8ru	0	m8rp	0b7c00
	088d0f		0
	c00000		10 zz
	22 zz		0
m8rv	0	m8rq	083c00
	0b3580		0
	0		10 zz
	13 zz		0
m8rw	0	m8rs	0f0590
	0f91f0		0
	0		16 zz
	17 zz		0
m8ry	0	m902	0a0000
	091f8e		0
	0		4 zz
	19 zz		0
m8rz	0	m9r4	0 zz
	0b1800		0
	0		0
	9 zz		0
m9r0	0	m9r5	0 zz
	0bc200		0
	0		0
	15 zz		0
m9r1	0	m9r6	0 zz
	080000		0
	0		0
	5 zz		0
m991	0	m9r7	0 zz
	0c0000		0
	0		0
	2 zz		0
m9r2	0	m9r8	0 zz
	0 zz		0
	0		0
	0		0
	0		0

m9r9	0 zz		0
	0		0
	0		0
m9ra	0 zz	m9rx	0 zz
	0		0
	0		0
m9rb	0 zz		0
	0	g2c1	0b8000
	0		0
	0		5 zz
m9rc	0 zz		0
	0	g2c2	0e4000
	0		0
	0		7 zz
m9rd	0 zz		0
	0	g2c3	0aa000
	0		0
	0		9 zz
m9re	0 zz		0
	0	g2c4	0c8000
	0		0
	0		5 zz
m9rf	0 zz		0
	0	g2c5	0d0000
	0		0
	0		4 zz
m9rg	0 zz		0
	0	g2c6	0ef000
	0		0
	0		8 zz
m9rh	0 zz		0
	0	g2c7	0e0000
	0		0
	0		4 zz
m9rj	0 zz		0
	0	g2c8	0c0000
	0		0
	0		3 zz
m9rk	0 zz		0
	0	g2c9	081580
	0		0
	0		13 zz
m9rn	0 zz		0
	0	g2ca	0a3280
	0		0
	0		17 zz
m9rq	0 zz		0
	0	g2cb	0fc400
	0		0
	0		10 zz
m9rt	0 zz		0
	0	g2cc	0eac00
	0		0
	0		10 zz
mare	0 zz		0
	0	g2cd	0ea600
	0		0
	0		12 zz
m9ru	0 zz		0
	0	g2ce	0c6e00
	0		0
	0		11 zz
m9rw	0 zz		0
		g2cf	0e8800

	0		C
	9 zz		C
	C		C
g2cg	09ec00	g4f1	C zz
	0		C
	10 zz		C
	C		0
g2ch	Cd4000	g4f2	C zz
	0		C
	6 zz		C
	0		C
g2cj	080000	g4f3	C zz
	0		C
	14 zz		0
	0		C
g2ck	080000	g4f4	C zz
	0		C
	3 zz		C
	C		C
g3f0	C zz	g4f5	0 zz
	C		C
	0		0
	0		C
g3f1	0 zz	g4f6	C zz
	0		0
	0		C
	C		0
g3f2	0 zz	g4f7	0 zz
	C		0
	0		C
	C		0
g3f3	C zz	g5a0	C zz
	C		0
	0		C
	0		C
g3f4	C zz	g5a1	C zz
	0		0
	0		C
	C		0
g3f5	C zz	g5a2	C zz
	0		0
	C		C
	0		0
g3f6	0 zz	g5a3	0 zz
	C		0
	0		0
	0		C
g3f7	0 zz	g5a4	0 zz
	0		0
	0		0
	0		0
g3f8	0 zz	g5a5	0 zz
	0		0
	C		C
	0		0
g3f9	C zz	m9rf	p
	0	p	
	0	p	g3f0
	0	m9rx	p
g4f0	0 zz	p	

	p	g3f1
	g3f0	p
	mp	hov2
	-g3f1	pd
	fc1	gnu3
	fl	k3
hov2	0a3d70	
	a3d70a	
	fff900	
	000000	
hov3	p	g4f7
	g3f0	p
	p	
	p	m9rf
	g4f7	p
	p	
	p	m9rg
	m9rf	p
	mp	hov2
	-m9rg	pd
	fc1	gnu7
	fl	k3
hov4	pc	37
	pc	16
	pc	16
	pc	16
	pc	41
	j126	p
	pt	11
	nl	k68
hov5	j126	p
	mp	m8ro
	mp	j6
	p	j126
	p	ttol
	lsr	hov6
	hov7	
	fl	k69

NOTE: Reserve in the memory 200 consecutive floating point number locations for each of the starting addresses listed below. There will then be in the memory 2200 floating point number reservations. For example, beginning with address b100 there will follow b101, b102, etc., each of which is reserved for storage.

b100

c100

j300

j400

j900

k300

vx10

vvx1

v100

w100

y100

* * *

APPENDIX C

REPRODUCTION OF PROGRAM IN MACHINE LANGUAGE FORM

The following list of commands is a Narec "machine language" translation of the symbolic program appearing in Appendix B. It is to be noted that list contains both fixed-point and floating-point commands. Two columns of numbers appear in the list. The left hand column is extraneous to the actual computation schedule. It is used only for visual identification of the lines in the right hand column which constitutes the true read-in. The left-hand column is not read into the computer. However the lines in the program must be read in consecutively beginning at address 0400.

Each line of command consists of twelve hexadecimal digits. They are interpreted by the computer logic from left to right as follows:

digits 1 through 4 represent one address in memory

digits 5 and 6 represent a fixed-point or floating point order

digits 7 through 10 represent one address in memory

digits 11 and 12 represent a fixed-point or floating-point order.

Note that the program starts in fixed-point. To enter floating-point as a particular address one uses the standard form

(address) 50 0200 11

To exit floating point into fixed point one uses the symbols E4 or E5.

The symbol || indicates the division of a Narec word between the left order and the right order.

The symbol * means "skip this order."

APPENDIX D
TYPICAL OUTPUT SHEET

A typical output sheet is reproduced below with added description of the meaning of the numbers. A number without a decimal point is interpreted as fraction followed by an exponent. For example

is

+65511109498	+02
+65.511109 498	

nnnn

0000

m +0.000000000 -2
 l +6.
 h +6.600

0000	+65511109498	+02	Eigenvalue
0001			
0000	+10000000827	-07	Precision of Eigenvalue
0002	+10936175364	-01	
0003	+96013725118	-01	
0004	+38634237703	+00	
0005	+83439671453	+00	
0006	-39362130016	+00	
0007	+73217232273	-01	
0008	-78111671012	-02	
0009	+55342208234	-03	
0010	-28240348044	-04	
0011	+10937705008	-05	
0012	-33366809971	-07	
0013	+82430513625	-09	
0014	-16852080723	-10	
0015	+29012288326	-12	
0016	-42668929896	-14	
0017	+54260160987	-16	
0018	-60277682314	-18	
0019	+59019532925	-20	
0020	-51329455278	-22	
0021	+39924268608	-24	
0022	-27941066789	-26	
0023	+17690620699	-28	
0024	-10182634883	-30	
0025	+53519977323	-33	
0026	-25790419362	-35	
0027	+11436285148	-37	
0028	-46823211649	-40	
0029	+17755655572	-42	
0030	-62540105973	-45	
0031	+20515535018	-47	
0032	-62832443777	-50	
0033	+18007915429	-52	
0034	-48401306663	-55	
0035	+12224784786	-57	
0036	-29069549227	-60	
0037	+65196116625	-63	
0038	-13814023500	-65	
0039	+27696278773	-68	
0040	-52622999510	-71	
0041	+94885173204	-74	
0042	-16258237917	-76	
0043	+26506679241	-79	
0044	-41169066792	-82	
0045	+60984915784	-85	
0046	-86255720730	-88	
0047	+11660682339	-90	
0048	-15082269790	-93	
0049	+18682422465	-96	
0050	-22183121071	-99	

Prolate Spheroidal Wave
Expansion Constants

APPENDIX E NOTES

The computation results shown in Appendix D requires 116 seconds for completion. This time depends largely on the precision of the eigenvalue deemed acceptable. The precision is controlled in turn by the number of times the corrections δA_{m1} is applied to the eigenvalue A_{m1} . In this program the number of corrections is set at four. If greater precision is demanded, one enters the program on the right side of location 069C and replaces the existing address (=058F) containing the existing number of corrections (minue one) with another address containing a larger number. For example if it is desired to apply 9 corrections to A_{m1} instead of 4 corrections one scans through the program to find the fixed number "8." This is seen to be at address 0793. Hence location 069C is changed to read

069F E8 0793 DC

The computation of A_{m1} is highly sensitive to the starting value inserted in address 05B9. When h is large (i. e. >10) we use a formula by Meixner (5) to help fix a good approximation for A_{m1} . The Meixner approximation has eight terms (prolate) or seven terms (oblate). In the program the first term in either case is multiplied by a small number ($10^{-2}/2$) fixed in location 086B and 086C. The last term is then compared with the resultant multiplication. If the last term is $1/100$ or less of the first term then the Meixner approximation is used. Otherwise the standard approximation (i.e. approximation $h \rightarrow 0$) is used.

0400	063810000000	Go to left order of 0638 *
0401	040150020011	Enter floating point
0402	0585e60583ea	Compute "Starting" $A_{m\ell} =$ $\ell(\ell+1) + h^2 \left[\frac{2\ell(\ell+1) - 2m^2 - 1}{(2\ell-1)(2\ell+3)} \right] + 1.0000\dots050$
0403	0593e80585e6	
0404	058be90595e8	
0405	0585e6058deb	
0406	0597e8058bea	
0407	0599e80591e9	
0408	059be80583e6	
0409	0583eb058deb	
040a	059de8058deb	
040b	059fe80583e6	
040c	058be905a1e8	
040d	0583e6058deb	
040e	05a3e8058bea	
040f	05a5e8058de9	
0410	05a7e80591ea	
0411	05a9e80591e9	
0412	058de905abe8	
0413	0587e60587eb	
0414	05ade8058dec	
0415	05afe805ade6	
0416	05adeb05b1e8	
0417	0585e60595eb	
0418	05b3e80597e6	
0419	0595eb059dea	
041a	058bea05b5e8	
041b	0599e6059beb	
041c	05b7e805b5e6	
041d	05b7ec05adeb	
041e	05b3e906d4e9	Store $A_{m\ell}$ in 05B9 Go to left order 06D6 Is $\ell - m$ an odd number or an even number?
041f	05b9e806d6e2	
0420	0593e605bbe8	
0421	05bbe6058bdc	

0422	0426e20428e2	
0423	0424e20000e0	
0424	05bbe6058dea	
0425	05bbe80421e2	
0426	05bde605bfe8	(l - m is even) bring up zero Store zero in 05BF
0427	05cle80445e2	Store zero in 05C1 Go to 0445
0428	058be605bfe8	(l - m is odd) Bring up "1" Store "1" in 05BF
0429	05cle80445e2	Store "1" in 05C1 Go to 0445
042a	0583e605cle9	
042b	05c3e8058be9	
042c	05c3eb05c5e8	
042d	059fe6058bea	
042e	05c7e805cle6	
042f	058deb05c9e8	
0430	05a5e605c9e9	Compute $\tau_n^m = (m+n)(m+n+1)$
0431	05cbe80591e9	$+ \frac{h^2}{2} \left[1 - \frac{4m^2 - 1}{(2m + 2n - 1)(2m + 2n + 3)} \right]$
0432	05cbeb05cde8	
0433	05c7e605cdec	
0434	05cfe8058be6	
0435	05cfea05afeb	
0436	05c5e905d1e8	
0437	0000e30000e0	
0438	05cle6058bea	
0439	05d3e805a3e6	
043a	05cle905d5e8	
043b	058bea05d7e8	
043c	05cle905d9e8	
043d	058de905dbe8	
043e	0591ea05dee8	
043f	05cle605d3eb	
0440	05d5eb05d7eb	
0441	05d9ec05d9ec	
0442	05deec05dbec	
0443	05bleb05e0e8	
0444	0000e30000e0	

0445 058bc61900e8
 0446 0593e60591e9
 0447 05e2e805bfe6
 0448 05e4e805cle8
 0449 05e4e6058de9
 044a 05e4e805e2dc
 044b 044de20456e2
 044c 0456e20000e0
 044d 044ee40000e0
 044e 045050044f10
 044f 044420044422
 0450 045050020010
 0451 0438e205e0e6
 0452 0890e80000e0
 0453 0002d0045200
 0454 05cle6058de9
 0455 05cle80449e2
 0456 05bfe605cle8
 0457 05e6e80000e0
 0458 05e6e6058de9
 0459 05e6e805e2dc
 045a 045ce20465e2
 045b 0465e20000e0
 045c 045de40000e0
 045d 045f50045e10
 045e 043720043722
 045f 045f50020010
 0460 042ae205dle6
 0461 0a20e80000e0
 0462 0002d0046100
 0463 05cle6058de9
 0464 05cle80458e2
 0465 05bfe605cle8
 0466 05e8e80000e0
 0467 05e8e6058de9

 0468 05e8e805e2dc
 0469 046be20474e2
 046a 0474e20000e0
 046b 0890e70000e0
 046c 1900ec0000e0
 046d 0a20ea05b9e9
 046e 1902e805eae8
 046f 0002d0046b00
 0470 0002d0046c00
 0471 0002d0046d00
 0472 0002d0046e00
 0473 087FE20000E0

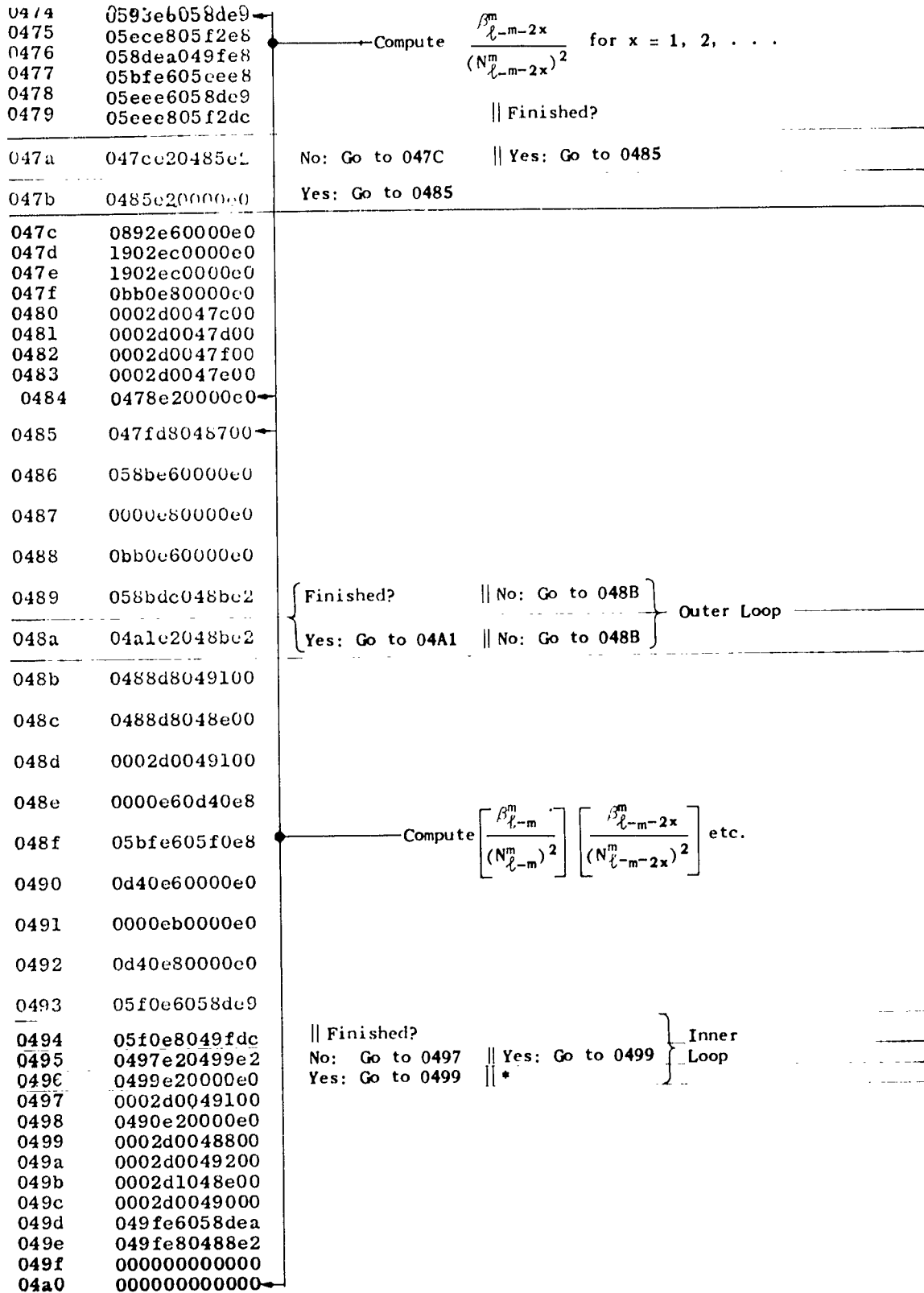
Compute $[N_{n+2}^m]_2 = \frac{-\beta_n^m}{N_n^m} + A_m \ell - \tau_n^m$

Finished?

No: Go to 046B || Yes: Go to 0474

Yes: Go to 0474 || *

|| Store $[N_{\ell-m+2}^m]_2$ in 05EA



04a1	0492d904a200	Compute $\left[1 + \frac{\beta_{\ell-m}^m}{(N_{\ell-m}^m)^2} + \frac{\beta_{\ell-m}^m \beta_{\ell-m-2}^m}{(N_{\ell-m}^m)^2 (N_{\ell-m-2}^m)^2} + \dots \right]$
04a2	058bc60000e8	
04a3	0d40e6058bdc	
04a4	04a6e204abe2	
04a5	04a6e20000e0	
04a6	05f4e60000e0	
04a7	0d40e905f4e8	Store computation in 05F4*
04a8	0002d004a300	
04a9	0002d004a700	
04aa	04a3c20000e0	
04ab	05f4e6058be9	
04ac	05f4e80000e0	
04ad	0593e605f6e8	Store max. (n+2x) in 05F8
04ae	05f6e6058de9	
04af	05f6e805f8e8	
04b0	05fae805cle8	
04b1	04b2e40000e0	
04b2	04b45004b310	
04b3	044420044422	Compute $[N_{n+2x}^m]_1$ for determining (n+2x) max.
04b4	04b450020010	
04b5	0438e204b6e4	
04b6	04b85004b710	
04b7	043720043722	
04b8	04b850020010	
04b9	042ae205dle6	Finished? Yes: Go to 04C0 Yes: Go to 04C0 No: Go to 04AE
04ba	05b9ea05fce8	
04bb	05fecb0000f9	
04bc	0600e805e0e7	
04bd	05fcec0000f9	
04be	0600dc04c0e2	
04bf	04c0e204ace2	

04c0	05bfe605cle8	
04c1	0593e6058dea	
04c2	0602e80000e0	
04c3	05f8e6058dea	
04c4	05f8e80000e0	
04c5	0602e605f8ea	
04c6	04dbf40000e0	If finished go to 04DB *
04c7	05fae605cle8	
04c8	04c9e40000e0	
04c9	04cb5004ca10	
04ca	044420044422	
04cb	04cb50020010	
04cc	0438e204cde4	
04cd	04cf5004ce10	• Compute $\frac{[N_{\ell-m+2x}^m]_1^2}{\beta_{\ell+m+2x}^m}$ for $x = 1, 2, \dots$
04ce	043720043722	
04cf	04cf50020010	
04d0	042ae205dle6	
04d1	05b9ea0604c9	
04d2	0606e805e0e7	
04d3	0606ec0604e8	Store $[N_{\ell-m+2}^m]_1$ in 0604
04d4	0604e60604eb	
04d5	05e0ec0000e0	
04d6	0ed0e80000e0	Store $(N_{\ell-m+2x}^m)^2 / \beta_{\ell-m+2x}^m$ in OEDO *
04d7	05fae6058dea	
04d8	05fae80000e0	
04d9	0002d004d600	
04da	04c3e20000e0	

04db 04d6d904dc00
 04dc 058be60000e8
 04dd 058be60608e8
 04de 0ed0e6058bdc
 04df 04e1e204efe2
 04e0 04e1e20000e0
 04e1 0ed0e6058bdc
 04e2 04e1e204eae2
 04e3 04e1e20000e0
 04e4 0608e60000e0
 04e5 0ed0eb0000e0
 04e6 1060e80c08e8
 04e7 0002d004e100
 04e8 0002d004e500
 04e9 04e1e20000e0
 04ea 0002d004dc00
 04eb 04ded804e100
 04ec 04ded804e500
 04ed 0002d004e600
 04ee 04dde20000e0
 04ef 04e6d904f000
 04f0 058be60000e8
 04f1 1060e6058bdc
 04f2 04f4e204f9e2
 04f3 04f4e20000e0
 04f4 060ae60000e0
 04f5 1060e9060ae8
 04f6 0002d004f100
 04f7 0002d004f500
 04f8 04f1e20000e0

Compute $\frac{(N_{\ell-m+2}^m)^2}{\beta_{\ell-m+2}^m} + \frac{(N_{\ell-m+2}^m)^2}{\beta_{\ell-m+2}} \cdot \frac{(N_{\ell-m+4}^m)}{\beta_{\ell-m+4}} + \dots$

|| Finished?

No: Go to 04F4 || Yes: Go to 04F9

No: Go to 04F4

|| Store $[(N_{\ell-m+2}^m)^2 / \beta_{\ell-m+2}^m] [(N_{\ell-m+4}^m)^2 / \beta_{\ell-m+4}^m]$
 etc. in 060A*

04f9	05f4e6060ae9	Bring up $1 + 2 \beta_{\ell-m}^m / (N_{\ell-m}^m)^2$ etc Add $2 (N_{\ell-m+2}^m)^2 / \beta_{\ell-m+2}^m$
04fa	060ce80604e0	Store in 060C Bring up $[N_{\ell-m+2}^m]_1$
04fb	05eaca060cee	Subtract $[N_{\ell-m+2}^m]_2$ Divide by 060C to form $\delta A_{m\ell}$
04fc	060ce805b9e9	Store $\delta A_{m\ell}$ in 060E Add $A_{m\ell}$
04fd	05b9e8060ee6	Store $\delta A_{m\ell} + A_{m\ell}$ in 05B9 Bring up $\delta A_{m\ell}$
04fe	0000190610ea	Form abs value Subtract tolerance in 0610
04ff	069bf406e5e2	Does $A_{m\ell}$ meet precision? No: Go to 0693 Yes: Go to 06C5
0500	000bf00873e2	Output: Print to 11 significant digits Go to 0873
0501	050111050910	Go to right side of this line Go to left order of 0509
0502	0632e605e4e8	Bring up zero from 0632 Store zero in 05E4
0503	05e6e805e8e8	~ in 05E6 ~ in 05E8
0504	05eee805f0e8	~ in 05EE ~ in 05F0
0505	05f2e8060ae8	~ in 05F2 ~ in 060A
0506	05f4e805f6e8	~ in 05F4 ~ in 05F6
0507	05f8e805fae8	~ in 05F8 ~ in 05FA
0508	0604e80647e4	~ in 0604 ~ Enter fixed point at 0647
0509	050950020611	Enter floating point
050a	058be61900e8	Bring up "1" Store in 1900
050b	066ee60600e0	Dummy command
050c	050bd8046a00	Change exit from 046A to 066E
050d	0593e60591e6	Bring up $n-m$ Add "4"
050e	05e2e80632e6	Store $n-m+4$ in 05E2 Bring up zero
050f	05e4e805e6e8	Store zero in 0524 Store zero in 05E6
0510	05e8e80000e0	Store zero in 05E8 *
0511	0634d8045200	
0512	0635d8046100	
0513	0634d8046b00	Compute all $[N_{\ell-m}^m]$
0514	0636d8046c00	
0515	0635d8046d00	
0516	0637d8046e00	
0517	0447e30000e0	

0518	0692d9051900	Transfer 19xx [=N _{xxx}] to right of 0519
0519	058beC0000e8	Bring up "1" Store "1" in 1902
051a	05bfeG0612e8	Bring up zero (even) or 1 (odd) Store in 0612
051b	0612e6058de9	Bring up zero or "1" Add 2
051c	0612e80006e0	Store 0 (or 1) + 2x in 0612 (= counter) *
051d	1902eG0000f9	Bring up 1902 + 2x Form abs value
051e	058bde0520e2	Finished? No: Go to 0520
051f	052fe20520e2	Yes: Go to 052F No: Go to 0520
0520	0612eG058deb	Bring up 0 (or 1) + 2x Mult. by 2
0521	05a3e9058be9	Add 2m add 1
0522	0614e8058dea	Store in 0614 subtract 2
0523	0614eb0616e8	Multiply by 0614 Store in 0616
0524	0612eG05a3e9	Bring up 0 (or 1) 12 x add 2m
0525	0618e8058bea	Store in 0618 Subtract 1
0526	0618eb061ae8	Multiply by 0618 Store in 061A
0527	0616e6061aec	Bring up 0616 Divide by 061A
0528	05adec0000e0	
0529	1902eb0000e0	
052a	11f0e80000e0	
052b	0002d0051d00	
052c	0002d0032900	Compute all $\left[\frac{d_n}{d_{n-2}} \right]$
052d	0002d0032a00	
052e	051be20000e0	

052f 052ad9053000
 0530 058be60000e8
 0531 061ce6058be9
 0532 061ce80000e0
 0533 11f0e60000f9
 0534 058bdc0536e2
 0535 053de20536e2
 0536 061ce60000e0
 0537 11f0eb0000e0
 0538 1380e8061ce8
 0539 0002d0053300
 053a 0002d0053700
 053b 0002d0053800
 053c 0533e20000e0

Compute $\frac{d_2}{d_0}, \frac{d_s}{d_0}, \frac{d_6}{d_0}$ etc.

Finished? || No: Go to 0536

Yes: Go to 053D || No: Goto 0536

053d 0538d0032e00
 053e 058be60000e8
 053f 05bfe6061ee8
 0540 061ee6058de9
 0541 061ee8061ee6
 0542 05a3e90620e8
 0543 0622e80000e0
 0544 1380e60000f9
 0545 058bdc0547e2
 0546 0555e20547e2
 0547 061ee60000f9
 0548 0620dc054ae2
 0549 054ee2054ee2
 054a 0620e6058bea
 054b 0624e80622eb
 054c 0622e80624e6
 054d 0620e80547e2
 054e 0622e6061eec
 054f 1380eb0000e0
 0550 1510e80000e0
 0551 0002d0054400
 0552 0002d0054f00
 0553 0002d0055000
 0554 0540e20000e0

Compute $\frac{(n+2m)!}{n!} \frac{d_n}{d_0}$

Finished? Go to 0555

0555	0550d9055600	Compute $\sum_n \frac{(n+2m)!}{n!} \frac{d_n}{d_0}$
0556	058be60000e8	
0557	1510e60000f9	
0558	058bdc055ae2	
0559	065ce2055ae2	Finished? No: Go to 055A
055a	0626e60000e0	Yes: Go to 065C No: Go to 055A
055b	1510e90626e8	
055c	0002d0055700	
055d	0002d0055b00	
055e	0557e20593e6	Return to 0557 Bring up " $\ell-m$ "
055f	0632dc0665e3	Compare with zero $(\ell-m) < 0$. Error: Go to right of 0665
0560	056ee3062ae8	$(\ell-m) = 0$. Go to right of 056E $(\ell-m) > 0$. Store $(\ell-m)$ in 062A
0561	058bea062ee8	Subtract "1" Store in 062E
0562	0632dc0566e2	Compare with zero $(\ell-m-1) < 0$. Go to left side of 0566
0563	0566e2062ae6	$(\ell-m-1) = 0$. Go to left side of 0566 $(\ell-m-1) > 0$. Bring up 062A
0564	062eeb062ae8	Multiply by 062E Store in 062A
0565	062ee60561e2	Bring up 062E Go to left side of 0561
0566	0585e60583e9	Bring up " ℓ " add $\Lambda_m \ell$
0567	0632dc0665e3	Compare with zero $(\ell-m) < 0$. Error: Go to right 0665
0568	0570e20000e0	Compute d_0 or d_1 $(\ell+m) = 0$. Go to left of 0570 *
0569	062ce8058bea	$(\ell+m) > 0$. Store $(\ell+m)$ in 062C Subtract "1"
056a	0628e80632dc	Store $(\ell+m-1)$ in 0628 Compare with zero
056b	0571e20571e2	$(\ell+m-1) < 0$. Go to left of 0571 $(\ell+m-1) = 0$. Go to left of 0571
056c	062ce60628eb	$(\ell+m-1) > 0$. Bring up 062C Multiply by 0628
056d	062ce80628e6	Store result in 062C Bring up 0628
056e	0569e3058be6	Go to right side of 0569 Bring up 1
056f	062ae80566e2	Store in 062A Go to left of 0566
0570	058be6062ce8	Bring up "1" Store in 062C
0571	062ce6062ae6	Bring up 062C Divide by 062A
0572	0626ec0630e8	Divide by 0626 Store $d_{0,1}$ in 0630
0573	16a0e80000e0	Store $d_{0,1}$ in 16A0 *

0574	1380e60000f9	
0575	058bdc0577e2	• Compute $d_n, d_{n+2}, d_{n+4},$ etc ($n = 0, 1$ to start)
0576	057de20577e2	When finished go to 057D
0577	1380e60630eb	
0578	16a2e80000e0	
0579	0002d0057400	
057a	0002d0057700	
057b	0002d0057800	
057c	0574e20000e0	
057d	0006dc064ae5	Finished? Yes: Go to right of 064A
057e	000000000000	
057f	16a0e60000e0	• Print out d_n, d_{n+2}, d_{n+4} etc.
0580	0000e00000e0	
0581	0002d0057f00	
0582	057ee30000e0	
0583	080000000000	} m
0584	000100000000	
0585	080000000000	} l
0586	000200000000	
0587	080000000000	} h
0588	000100000000	
0589	0e0000000000	} xxx in d_{xxx}
058a	000400000000	
058b	080000000000	
058c	000100000000	
058d	080000000000	} "2"
058e	000200000000	
058f	0e0000000000	
0590	000200000000	
0591	080000000000	
0592	000300000000	

05b3	000000000000
05b4	000000000000
05b5	000000000000
05b6	000000000000
05b7	000000000000
05b8	000000000000
05b9	000000000000
05ba	000000000000
05bb	000000000000
05bc	000000000000
05bd	000000000000
05be	000000000000
05bf	000000000000
05c0	000000000000
05c1	000000000000
05c2	000000000000
05c3	000000000000
05c4	000000000000
05c5	000000000000
05c6	000000000000
05c7	000000000000
05c8	000000000000
05c9	000000000000
05ca	000000000000
05cb	000000000000
05cc	000000000000
05cd	000000000000
05ce	000000000000
05cf	000000000000
05d0	000000000000
05d1	000000000000
05d2	000000000000

05d3	000000000000
05d4	000000000000
05d5	000000000000
05d6	000000000000
05d7	000000000000
05d8	000000000000
05d9	000000000000
05da	000000000000
05db	000000000000
05dc	000000000000
05dd	00000005de10
05de	000000000000
05df	000000000000
05e0	000000000000
05e1	000000000000
05e2	000000000000
05e3	000000000000
05e4	000000000000
05e5	000000000000
05e6	000000000000
05e7	000000000000
05e8	000000000000
05e9	000000000000
05ea	000000000000
05eb	000000000000
05ec	000000000000
05ed	000000000000
05ee	000000000000
05ef	000000000000
05f0	000000000000
05f1	000000000000
05f2	000000000000

05f3	000000000000	
05f4	000000000000	
05f5	000000000000	
05f6	000000000000	
05f7	000000000000	
05f8	000000000000	
05f9	000000000000	
05fa	000000000000	
05fb	000000000000	
05fc	000000000000	
05fd	000000000000	
05fe	0a3d70a3d7a0	} 10 ⁻²
05ff	fffa00000000	
0600	000000000000	
0601	000000000000	
0602	000000000000	
0603	000000000000	
0604	000000000000	
0605	000000000000	
0606	000000000000	
0607	000000000000	
0608	000000000000	
0609	000000000000	
060a	000000000000	
060b	000000000000	
060c	000000000000	
060d	000000000000	
060e	000000000000	
060f	000000000000	
0610	0afec0000000	} 10 ⁻¹¹
0611	ffdc00000000	
0612	000000000000	

0613	000000000000
0614	000000000000
0615	000000000000
0616	000000000000
0617	000000000000
0618	000000000000
0619	000000000000
061a	000000000000
061b	000000000000
061c	000000000000
061d	000000000000
061e	000000000000
061f	000000000000
0620	000000000000
0621	000000000000
0622	000000000000
0623	000000000000
0624	000000000000
0625	000000000000
0626	000000000000
0627	000000000000
0628	000000000000
0629	000000000000
062a	000000000000
062b	000000000000
062c	000000000000
062d	000000000000
062e	000000000000
062f	000000000000
0630	000000000000
0631	000000000000

0632	000000000000	
0633	000000000000	
0634	089000000000	
0635	0a2000000000	
0636	190000000000	
0637	190200000000	
0638	063d32063910	Read m, l, h, etc. into 063D etc. Go to 0639
0639	063d81058308	Transfer 8 data lines from 063D etc into 0583 etc.
063a	064581057d01	Transfer 1 data line from 0645 into 057D
063b	040083240030	Transfer 768 commands from 0400 etc. to 2400 etc.
063c	040110063d10	Go to left order 0401 *
063d	000000000000	
063e	000000000000	
063f	000000000000	
0640	000000000000	
0641	000000000000	
0642	000000000000	
0643	000000000000	
0644	000000000000	
0645	000000000000	
0646	000000000000	
0647	2400810400fd	Transfer fresh program (256 lines) from 2400 to 400
0648	064850020011	Enter floating point
0649	05bfe605cle8	Bring up "odd-even" (= "one" or "zero") Store in 05C1
064a	0445e2000082	Go to 0445 COMPUTATION FINISHED: STOP
064b	240083040030	
064c	063810300000	
064d	0025f30025f3	Output: Carriage return (C.R.) C.R.
064e	0007f30004f3	Print character "m" Space (on typewriter)
064f	0004f30583e6	Space Bring up "m"

0650	0300f10025f3	Print value of m C.R.
0651	0006f30004f3	Print character "l" Space
0652	0004f30585e6	Space Bring up "l"
0653	0300f10025f3	Print value of "l" C.R.
0654	0008f30004f3	Print character "h" Space
0655	0004f30587e6	Space Bring up "h"
0656	0303f10025f3	Print value of "h" C.R.
0657	0025f30025f3	C.R. C.R.
0658	0010f30010f3	Print character "0" "0"
0659	0010f30010f3	"0" "0"
065a	0029f305b9e6	Tab (on typewriter) Bring up $A_{m\ell}$
065b	0500e20000e0	Go to 0500 *
065c	05a3e60632dc	Bring up 2m Compare 2m with zero
065d	0665e30666e3	2m is < zero. This is an error. Go to right 0665 2m = 0, Go to 0666
065e	066ce2058bea	2m > 0. Go to left of 066C Subtract "1"
065f	066ae80632dc	Store 066A Compare with zero
0660	0663e30663e3	Result is less than zero. Go to right of 0663 Result equals zero, go to right of 0663
0661	0668e6066aeb	Result > 0. Bring up 0668 Multiply by 066A
0662	0668e8066ae6	Store in 0668 Bring up 066A
0663	065ee30626e6	Go to right of 065E Bring up 0626 (=working cell)
0664	0668e90626e8	Add number in halo Store result in 0626
0665	055ee30666e4	Go to right of 055E Enter fixed point at 0666
0666	0bad82058be6	Error stop Bring up "1"
0667	0668e80663e3	Store "1" in 0668 Go to right side 0663
0668	000000000000	
0669	000000000000	
066a	000000000000	
066b	000000000000	
066c	05a3e605bfe9	Bring up 2m Add "0" or "1" from 05BF
066d	0668e8065ee3	Store in 0668 Go to right side of 065E

066e	05bfe605cle8	← Bring up "0" or "1" Store "0" or "1" in 05C1
066f	0589e603f8e8	Bring up upper limit of d_n constants Store in 05F8
0670	05fae80632e6	Store in 05FA Bring up zero
0671	0604e8058be6	Store "0" in 0604 Bring up "1"
0672	1E02E80593E6	Store "1" in 1E02 Bring up $\ell-m$
0673	058DEA0602e8	Subtract "2" Store $\ell-m-2$ in 0602
0674	05f8e6058dea	Bring up 05F8 Subtract 2
0675	05f8e80000e0	Store in limit cell (= 05F8) *
0676	0602e605f8ea	Bring up 0602 Subtract (limit - 2x)
0677	06a8f40000e0	If 0602 < (limit - 2x) Go to 0678 (left side): If 0602 ≥ (limit - 2x) Go to 06AB *
0678	05fae605cle8	Final calc. of $N_{\ell-m+4}^m$ to N_k^m (k = max. subscript) Calc. β
0679	067ae40000e0	
067a	067c50067b10	
067b	044420044422	
067c	067c50020010	Calc. τ
067d	0438e2067ee4	
067e	068050067f10	
067f	043720043722	
0680	068050020010	
0681	042ae205dle6	

0682	05b9ea0604e9	} Calc. $N_k^m = \frac{-f_k^m}{-A_{m\ell} + j_k^m + N_{k+2}^m}$, where $k = \text{limit} - 2x$
0683	0606e805e0e7	
0684	0606ec0604e8	
0685	1e00e306b5e8	Store N_k^m in 1E00 Store N_k^m in 06B5 (largest k to start)
0686	1002d0068500	
0687	05fae6058dea	Float left 0674
0688	05fae30674e2	Change exit of $[N_n^m]$ routine to read "go to 068E".
0689	068EE20000E0	Bring up $n = \ell - m + 2$ Add 2
068a	05fae6058de9	Store $\ell - m + 4$ in 05F8 Bring up "0" or "1"
068b	05f8e805bfe6	Final calc. of N_0^m to $N_{\ell-m+2}^m$
068c	0602e80674e2	Store "0" or "1" in 0602 Go to 0674
068d	068e00000000	
068e	0002d0068500	
068f	0685d8069000	
0690	0000e6058bdc	
0691	0692e20696e2	
0692	1902e80000e0	
0693	0002d0069200	
0694	0002d0069000	
0695	0690e2000000	
0696	0646d8069800	
0697	0636d9069900	
0698	0000d1069900	
0699	058be60000e8	
069a	0518e20000e0	Go to 0518 *

069b	069fe6058be9	Bring up counter base (= zero) Add one
069c	069fe8058fdc	Store count in 069F Is this number greater than 3?
069d	0502e20502e2	Number less than 3. Go to 0502 Number equal to 3: Go to 0502
069e	06a1e20000e0	Number greater than "3". Go to 06A1 *
069f	000000000000	
06a0	000000000000	
06a1	05fee606a4ec	Bring up $10^{-2}/x$ Divide by "2" (= 06A4)
06a2	05fee80632e6	Store $10^{-2}/x$ ($x = 1, 2, 4$, etc) in 05FE Bring up zero
06a3	069fe8087ae2	Store in 069F Go to 087A
06a4	080000000000	} ————— "2"
06a5	000200000000	
06a6	000000000000	
06a7	000000000000	
06a8	06b5e60000f9	← Increase "n" in $(d_n)_{\max}$ by 10. Then recompute all N_k^m ($k = \text{limit} - 2x$)
06a9	06b7e80000e0	
06aa	06b0d8067700	
06ab	06b1d8067200	
06ac	06b2d8068500	
06ad	0589e606a4e9	
06ae	06b3e805f8e8	
06af	0670e20000e0	←
06b0	06b900000000	
06b1	1e0200000000	
06b2	1e0000000000	
06b3	000000000000	
06b4	000000000000	
06b5	000000000000	
06b6	000000000000	
06b7	000000000000	
06b8	000000000000	

06b9	06b7e706b5fd	←
06ba	0000f906c3dc	
06bb	0689e20689e2	
06bc	06b5e60000f9	
06bd	06b7e80000e0	If $[N_{\ell-m+2}]_1$ for $n = \max + 10$ is not same (within tolerance) as $[N_{\ell-m+2}]_1$ for $n = \max$ recompute using a still larger n. If they agree to go to 0689
06be	06b1d8067200	
06bf	06b2d8068500	
06c0	06b3e606a4e9	
06c1	06b3e805f8e8	
06c2	0670e20000e0	←
06c3	0afec0000000	
06c4	ffdc00000000	
06c5	05b9e60000f9	Bring up $A_{m\ell}$ Form abs value
06c6	06d2e80000e0	Store in 06D2 *
06c7	06c9d904ff00	Transfer left address of 06C0 to replace right address of 04FF
06c8	069be20000e0	Go to 069B *
06c9	06ca00000000	
06ca	05b9e60000f9	Bring up $A_{m\ell}$ Form abs value
06cb	06d2ea0000f9	Subtract previous $ A_{m\ell} $ Form abs value
06cc	06d0ea06cef4	Subtract tolerance in 06D0 If $A_{m\ell}$ is not precise go to 06CE
06cd	064de2000000	If $A_{m\ell}$ meets precision go to 064D *
06ce	05b9e60000f9	Bring up latest $A_{m\ell}$ form abs value
06cf	06d2e8069be2	Store in 06D2 Go to 069B
06d0	0afec0000000	
06d1	ffdc00000000	
06d2	000000000000	
06d3	000000000000	
06d4	0800000000f0	
06d5	000100000000	

06d6	07b3e6058bea	Bring up number from 07B3 Subtract 1,000. . . .
06d7	06dbf406d8e2	If zero go to 06DB If negative go to 06D8
06d8	0587e60000f9	Bring up h Absolute value of h
06d9	07dddc0420e2	Is h larger than m992? If h is less than m992 go to 0420
06da	06dee206dee2	If h equals m992 or is greater than m992 go to 06DE
06db	0587e60000f9	Bring up h Form absolute value of h
06dc	07dddc0420e2	Is h larger than m992? If h is less than m992 go to 0420
06dd	073de2073de2	If h equals m992 or is greater than m992 go to 073D
06de	0585e60583ea	
06df	058deb058be9	
06e0	07c7e807c7eb	
06e1	07c9e807c7eb	
06e2	07dfe807c7eb	
06e3	07ele807c7eb	
06e4	07e3e807c7eb	
06e5	07e5e807c7eb	
06e6	07e7e80587e6	
06e7	0587eb07ece8	
06e8	0587eb07eee8	
06e9	0587eb07f0e8	
06ea	0587eb07f2e8	
06eb	0583e60583eb	
06ec	07f4e807f4eb	
06ed	0806e80000e0	lines 06DE to 0737
06ee	0587e607c7eb	compute all
06ef	07f6e807c9e6	terms in starting value of $A_{m\ell}$ for $h \rightarrow \infty$
06f0	0791e90793ec	Using Meixner (1) Eq. 10 (Prolate case)
06f1	07f8e807f4e6	
06f2	0797eb07fae8	
06f3	07fae707c9e9	

06f4	0795e907c7eb
06f5	0799ec0587ec
06f6	07fce807c9e6
06f7	079beb07ele9
06f8	079de90791eb
06f9	07fee807c9e6
06fa	058be907f4eb
06fb	07aleb07fae8
06fc	07fae707fee9
06fd	079fec07ecec
06fe	0800e807e3e6
06ff	07a5eb07fae8
0700	07dfe607a7eb
0701	07fee807c7e6
0702	07a9eb07fee9
0703	07fae907a3ec
0704	07a3ec0802e8
0705	07c7e607adeb
0706	07fae807dfe6
0707	07abeb07fae9
0708	07f4eb07a3ec
0709	0804e807c7e6
070a	0806eb0793ec
070b	0804ea0802e9
070c	07eeec0808e8
070d	07e5e607bleb
070e	07fae807ele6
070f	07cbeb07fee8
0710	07c9e607cdeb
0711	07cfe907fee9

0712	07fae907afec
0713	07afec0804e8
0714	07e1e607d5eb
0715	07fae807c9e6
0716	07d1eb07fee8
0717	07d7e907fee9
0718	07fae907f4eb
0619	07d3ec0802e8
071a	07c9e6058be9
071b	0806eb07c5eb
071c	0793ec0802ea
071d	0804e907f0ec
071e	080ae807e7e6
071f	07d9eb07fae8
0720	07e3e607dbeb
0721	07fee807dfe6
0722	07b5eb0802e8
0723	07c7e607b7eb
0724	0802e907fee9
0725	07fae9079fec
0726	079fec07fae8
0727	07e3e607b9eb
0728	07fee807dfe6
0729	07bbbeb0802e8
072a	07c7e607bdeb
072b	0802e907fee9
072c	07f4ek0797ec
072d	079fec07fee8
072e	07bfe607dfeb
072f	0802e807c7e6

0730 07c1eb0802e9
 0731 0806eb07d3ec
 0732 0802e807c7e6
 0733 07f4eb07c3ec
 0734 0804e80804e7
 0735 0802e907feea
 0736 07fae907f2ec
 0737 080ce80865e2
 0738 07f6e607f4e9
 0739 07f8ea07fcea
 073a 0800ea0808ea
 073b 080aea080cea
 073c 08b9e80420e2
 073d 0887e60587eb
 073e 0835e80587eb
 073f 0837e80587eb
 0740 0839e80587eb
 0741 083be80585e6
 0742 0583ea058deb
 0743 0583e9058be9
 0744 083de8083deb
 0745 083fe8083feb
 0746 0841e8083feb
 0747 0843e80583e6
 0748 0583eb0845e8
 0749 0845eb0847e8
 074a 0845eb0849e8
 074b 083de60587eb
 074c 058deh084be8
 074d 083fe60845ea

|| Go to 0865

lines 0738 to 073C

compute starting $A_m \ell$ for $h \rightarrow \infty$ (Prolate case)

|| Go to 0420

Lines 073D to 078B

compute all

terms in starting value of $A_m \ell$ for $h \rightarrow \infty$
 Using Meixner (1) Eq. (19) (Oblate case)

074e	058be9058dec
074f	084de80833ec
0750	0587ec083deb
0751	084fe80841e6
0752	0791eb0859e8
0753	083fe607ddeb
0754	085be8083fe6
0755	07c5eb058be9
0756	0845eb058deb
0757	085de8085de7
0758	0847e9058be9
0759	085be90859e9
075a	0799ec0835ec
075b	0851e80841e6
075c	07a5eb0859e8
075d	083fe60811eb
075e	085be8083fe6
075f	080feb0815e9
0760	0845eb058deb
0761	085de80847e6
0762	0817eb085dea
0763	07abe9085be9
0764	0859e9083deb
0765	07d3ec0837ec
0766	0853e80843e6
0767	07b1eb0859e8
0768	0841e60813eb
0769	085be8083fe6
076a	0819eb085de8
076b	0841e607ddeb

076c	085fe8083fe6
076d	080feb07c5e9
076e	085fe90845eb
076f	07ddeb085fe8
0770	083fe60817eb
0771	081de90847eb
0772	07c5eb0861e8
0773	0849e6058deb
0774	0863e80863e7
0775	0861e9085fea
0776	081be9085de9
0777	085be90859e9
0778	079fec0839ec
0779	0855e80843e6
077a	07d9eb0859e8
077b	0841e6081feb
077c	085be8083fe6
077d	0821eb085de8
077e	0841e60825eb
077f	085fe8083fe6
0780	0827eb0829e9
0781	085fe90845eb
0782	085fe8083fe6
0783	082beb082de9
0784	0847eb0861e8
0785	0849e6082feb
0786	0863e80863e7
0787	0861e9085fea
0788	0823e9085de9
0789	085be90859e9

078a	083deb0831ec	
078b	083bec086de2	Go to 086D
078c	0835e7084be9	
078d	084dea084fea	lines 078C to 0790
078e	0851ea0853ea	compute starting value of $A_m \ell$ for $h \rightarrow \infty$ (Oblate case)
078f	0855ea0857ea	
0790	05b9e80420e2	Go to 0420
0791	0a0000000000	} _____ "5"
0792	000300000000	
0793	080000000000	
0794	000400000000	
0795	0b0000000000	
0796	000400000000	
0797	080000000000	
0798	000600000000	
0799	080000000000	
079a	000700000000	
079b	0d0000000000	
079c	000500000000	
079d	0a8000000000	
079e	000500000000	
079f	080000000000	
07a0	000b00000000	
07a1	0c0000000000	
07a2	000900000000	
07a3	080000000000	
07a4	000800000000	
07a5	084000000000	
07a6	000600000000	
07a7	0c7400000000	

07a8	000b00000000
07a9	0afa80000000
07aa	000d00000000
07ab	094000000000
07ac	000600000000
07ad	0a7000000000
07ae	000800000000
07af	080000000000
07b0	000900000000
07b1	0fc000000000
07b2	000600000000
07b3	000000000000
07b4	000000000000
07b5	0fedf9000000
07b6	001400000000
07b7	088d0fc00000
07b8	001600000000
07b9	0b3580000000
07ba	000d00000000
07bb	0f91f0000000
07bc	001100000000
07bd	091f8e000000
07be	001300000000
07bf	0b1800000000
07c0	000900000000
07c1	0bc200000000
07c2	000f00000000
07c3	080000000000
07c4	000500000000
07c5	0c0000000000

07c6	000200000000
07c7	000000000000
07c8	000000000000
07c9	000000000000
07ca	000000000000
07cb	09a600000000
07cc	000d00000000
07cd	0a93f0000000
07ce	001000000000
07cf	0af8c0000000
07d0	000f00000000
07d1	0a3c00000000
07d2	000b00000000
07d3	030000000000
07d4	000a00000000
07d5	0e6000000000
07d6	000700000000
07d7	0b7c00000000
07d8	000a00000000
07d9	083c00000000
07da	000a00000000
07db	0f0590000000
07dc	001000000000
07dd	000000000000
07de	000000000000
07df	000000000000
07e0	000000000000
07e1	000000000000
07e2	000000000000
07e3	000000000000

07e4	000000000000
07e5	000000000000
07e6	000000000000
07c7	000000000000
07e8	000000000000
07e9	00000007ea10
07ea	000000000000
07eb	000000000000
07ec	000000000000
07ed	000000000000
07ee	000000000000
07cf	000000000000
07f0	000000000000
07f1	000000000000
07f2	000000000000
07f3	000000000000
07f4	000000000000
07f5	000000000000
07f6	000000000000
07f7	000000000000
07f8	000000000000
07f9	000000000000
07fa	000000000000
07fb	000000000000
07fc	000000000000
07fd	000000000000
07fe	000000000000
07ff	000000000000
0800	000000000000
0801	000000000000

0802	000000000000
0803	000000000000
0804	000000000000
0805	000000000000
0806	000000000000
0807	000000000000
0808	000000000000
0809	000000000000
080a	000000000000
080b	000000000000
080c	000000000000
080d	000000000000
080e	000000080f10
080f	0b8000000000
0810	000500000000
0811	0e4000000000
0812	000700000000
0813	0aa000000000
0814	000900000000
0815	0c8000000000
0816	000500000000
0817	0d0000000000
0818	000400000000
0819	0ef000000000
081a	000800000000
081b	0e0000000000
081c	000400000000
081d	0c0000000000
081e	000300000000
081f	081580000000

0820	000d00000000
0821	0a3280000000
0822	001100000000
0823	0fc400000000
0824	000a00000000
0825	0eac00000000
0826	000a00000000
0827	0ea600000000
0828	000c00000000
0829	0c6e00000000
082a	000b00000000
082b	0e8800000000
082c	000900000000
082d	09ec00000000
082e	000a00000000
082f	0d4000000000
0830	000600000000
0831	080000000000
0832	000e00000000
0833	080000000000
0834	000300000000
0835	000000000000
0836	000000000000
0837	000000000000
0838	000000000000
0839	000000000000
083a	000000000000
083b	000000000000
083c	000000000000
083d	000000000000

083e	000000000000
083f	000000000000
0840	000000000000
0841	000000000000
0842	000000000000
0843	000000000000
0844	000000000000
0845	000000000000
0846	000000000000
0847	000000000000
0848	000000000000
0849	000000000000
084a	000000000000
084b	000000000000
084c	000000000000
084d	000000000000
084e	000000000000
084f	000000000000
0850	000000000000
0851	000000000000
0852	000000000000
0853	000000000000
0854	000000000000
0855	000000000000
0856	000000000000
0857	000000000000
0858	000000000000
0859	000000000000
085a	000000000000
085b	000000000000

085c	000000000000	
085d	000000000000	
085e	000000000000	
085f	000000000000	
0860	000000000000	
0861	000000000000	
0862	000000000000	
0863	000000000000	
0864	000000000000	
865	07f6c60000f9	Bring up 1 st term in Meixner Formula (Prolate case) Form abs value
0866	0835e8080ce6	Store in 0835 Bring up last term in Meixner Formula
0867	0000f90837e8	Form abs value Store in 0837
0868	08t5e6086beb	Bring up 0835 Multiply by $10^{-2}/2$
0869	0837ea0738f4	Subtract 0837 If positive go to 0738; If neg. go to 086A
086a	0420e20000e0	Go to 0420 *
086b	0a3d70a3d70a	} $10^{-2}/2$
086c	fff900000000	
086d	0857e80835e6	Store in 0857 Bring up 1 st term in Meixner Formula (Oblate case)
086e	0000f907f6e8	Form abs value Store in 07F6
086f	0857e60000f9	Bring up 0857 (= last term in Meixner Formula) Form abs value
0870	07f8e807f6e6	Store in 07F8 Bring up 07F6
0871	086beb07f8ea	Multiply by $10^{-2}/2$ Subtract 07F8
0872	078cf40420e2	If positive go to 078C; If negative go to right side Go to 0420
0873	0025f30010f3	Carriage return Space (in type out)
0874	0010f30010f3	Space Space
0875	0010f30029f3	Space
0876	0610e6000bf0	Bring up 0610 (= tol on eigenvalue) Print 11
0877	0501e40000e0	Enter fixed point at left side of 0501 *
0878	0000e00000e0	* *
0879	0000e00000e0	* *

087a	0610e60791eb	Bring up Tol of Eigenvalue Multiply by 5
087b	058deb0610e8	Multiply by 2 Store in 0610
087c	06d0c80000e0	Store in 06D0 *
087d	051ed9069c00	Place the left address of 051E into the right address of 069C
087e	0302e20000e0	Go to 0502 *
087f	046ed8069200	Place the left address of 046E into left address of 0692
0880	1002d0069200	Decrement left address of 0692 by 2
0881	0467c20000e0	Go to 0467 *

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$$\frac{d}{d\gamma} \left[(1 - \gamma^2) \frac{dS_m \ell}{d\gamma} \right] + \left[A_m \ell - h^2 \gamma^2 - \frac{m^2}{1 - \gamma^2} \right] S_m \ell = 0$$

where the parameters ℓ , m , and h are as defined by Stratton et al. in "Spheroidal Wave Functions" (New York: Wiley, 1956). The symbolic language used is universal (e.g., add a to b, multiply c by d, etc.) and may be applied to any digital computer by simple conversion to machine language. The program developed requires at least 3400 working cells. It computes the eigenvalue $A_m \ell$ to 13 significant places for any integer values of ℓ and m ($\ell \geq m$). The value of h is left arbitrary. The program also computes all the spheroidal wave function expansion coefficients (d_p) up to $p \leq 200$.

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