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(b) Section IV, Paragraph 4.2.2 of AFBM Exhibit 58-1
(c) Paragraph 1.2.1.2 of AFSSD Exhibit 61-27A

In accordance with the requirements of references (a),
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<u>Title</u>	<u>Number and Date</u>
Interstation Data	WDL-TR2017
Transmission Study	28 February 1963
Final Report	

PHILCO CORPORATION
Western Development Laboratories

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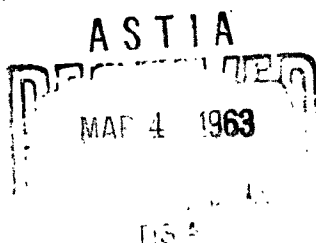
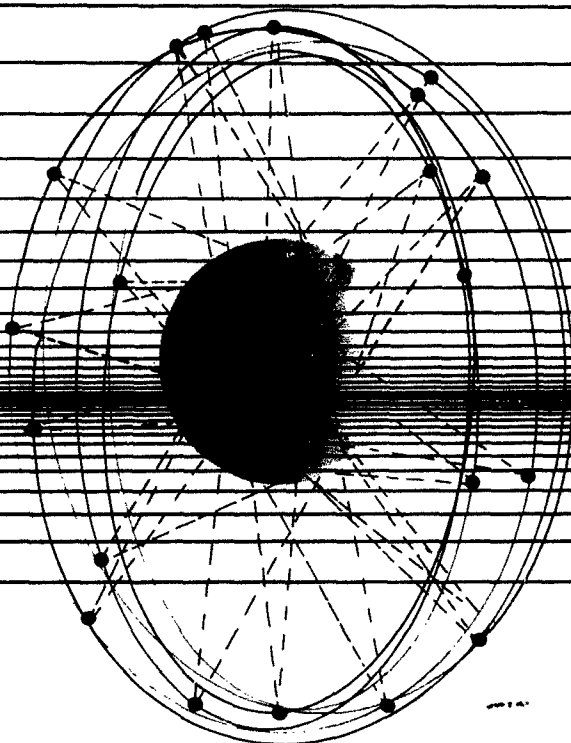
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WESTERN DEVELOPMENT LABORATORIES

TECHNICAL DOCUMENTARY REPORT

WDL-TR2017

28 FEBRUARY 1963



**INTERSTATION DATA
TRANSMISSION STUDY
FINAL REPORT**

PREPARED FOR:

**AIR FORCE SPACE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
INGLEWOOD, CALIFORNIA**

CONTRACT AF04(695) -113

PHILCO

A SUBSIDIARY OF Ford Motor Company

**WESTERN DEVELOPMENT LABORATORIES
PALO ALTO, CALIFORNIA**

TECHNICAL DOCUMENTARY REPORT

INTERSTATION DATA
TRANSMISSION STUDY
FINAL REPORT

Prepared by

PHILCO CORPORATION
Western Development Laboratories
Palo Alto, California

Contract AF04(695)-113

Prepared for

SPACE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
Inglewood, California

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WESTERN DEVELOPMENT LABORATORIES

ABSTRACT

PHILCO WDL-TR2017
INTERSTATION DATA
TRANSMISSION STUDY-
FINAL REPORT

UNCLASSIFIED

104 pp including illusts.
A/C AF04(695)-113

This report summarizes the data transmission capabilities between ground stations. The topics that are reviewed are:

1. Characteristics of typical equipment
2. Characteristics of transmission media
3. Comparative evaluation of performance
4. Improvement through coding

Results are presented in chart form, wherever practical, in order to facilitate use of this information.

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FOREWORD

This Technical Documentary Report on Definitive Contract AF04(695)-113 has been prepared in accordance with Exhibit "A" of that contract and Paragraph 4.2.2 of AFBM Exhibit 58-1, "Contractor Reports Exhibit," dated 1 October 1959, as revised and amended.

This report was prepared by Philco Western Development Laboratories in fulfilling the requirements of Paragraph 1.2.1.2 of AFSSD Exhibit 61-27A, "Satellite Control Subsystem Work Statement," dated 15 February 1962, as revised and amended.

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SUMMARY

Summaries of the various sections of this report are given in the following paragraphs:

Characteristics of Typical Equipment

In this section, the operational characteristics of equipment designed to transmit data through a single voice channel are summarized. The additional equipment necessary to transmit a number of voice channels of data over radio networks is not considered. Ten different equipment models are reviewed. Some of the characteristics that are summarized include: type of modulation, requirements placed on transmission medium, transmission rates, equalization provided, size, weight, prime power requirement, etc. The summary is in chart form.

Characteristics of Transmission Media

The typical characteristics of the media through which data is transmitted are considered in this section. Some of the media that are considered include: wire lines, HF radio, microwave line-of-sight, and the various scatter propagation techniques. Since the characteristics of these media are usually expressed in different terms, a chart approach was not feasible; instead, each of the media are discussed briefly. This section is important since the requirements of some of the equipment summarized in the preceding section may be incompatible with the characteristics of certain of the media.

Comparative Evaluation of Performance

The selection of one type of equipment out of the various types available is a complicated procedure in which the specific requirements of the customer play a key role. In this section, the various types of equipment are rated in terms of maximum data transmission rate, reliability, error rate, and availability. These criteria can then be weighted in order to compare the overall performance with the customer's needs. An example of this type of approach is given.

Improvement through Coding

When the allowable error rate is very small, coding is essential. In this section, error detection and error correction for both white noise and burst error interference are considered. It is concluded that long word length codes (100-500 bits) with error detection is the most promising technique, but that a feedback path is required.

SECTION 1

INTRODUCTION

1.1 GENERAL

This report consists of a summary of the capabilities of data transmission systems used between ground stations. Since this is a summary, most of the information was obtained through a search of the existing literature. In order to present this information in a concise form that can be used by the systems engineer, wherever possible the results are presented in a large chart so that a quick comparison of systems can be made.

SECTION 2

DIGITAL DATA TRANSMISSION EQUIPMENT

2.1 GENERAL

The characteristic and specifications of ten data transmission systems are summarized in Table 2-1. This table is arranged to provide easy comparison of particular characteristics or specifications. The categories are self-explanatory. Only systems designed to operate from conventional voice channels (about 300 to 3000 cps) have been considered.

The data used have been taken only from manufacturer's literature and from other reports (as referenced), so there are some gaps in the table, and some of the data may have been revised by the time the information in this table is used.

The systems shown in the table are designed for transmission over wire lines, although most of the systems can also be used over radio links with addition of suitable radio equipment. For example, Lenkurt supplies a second "Duobinary" system, the HF2X, for use over radio links. It is similar to the 26B, but it accepts, or converts, serial data into 16 parallel channels for transmission. Diversity reception is used to increase communication reliability.

TABLE 2-1 DIGITAL DATA

	RIXON "SEBIT" 24 B DATA MODEM	LENKURT "DUOBINARY-DATATEL" 26 B DATA TRANSMISSION SYSTEM	ROBERTSHAW-FULTON PHASE "MULTILOK"	STROMBERG CARLSON HIGH SPEED BINARY DATA TRANSCIVER MODEL S-C-301	
Theory of Operation (Modulation & Demodulation)	Two-level vestigial sideband AM system with its carrier at 2500 cps and utilizing the lower sideband. Receiver uses amplitude detection, followed by synchronous sampling and regeneration of the received data in both time and amplitude.	FM system using three-level pulses. The 3-level pulses are obtained from the original binary data through the Duobinary coding process. The coded signal occupies only half the bandwidth of the original binary signal, and permits error detection without redundancy.	Uses non-synchronous phase-shift keying and coherent detection. The information is contained in the amount of subcarrier phase shift. An extra phase position is required to eliminate ambiguities, so 3 phase positions are required for simplex operation and 5 positions for duplex operation. Frequency-division multiplexing can be used, with each channel (subcarrier) operating at 75 bauds (simplex) or 150 bauds (duplex), or a single high-speed channel can be used.	Vestigial sideband AM and envelope detection, using 3-level pulses.	freq bin rat rec pro
Operational Modes	Full duplex-4 wire; or half duplex with manual or automatic control on transmit-receive on 2 wire basis. "Sepath" is available for HF radio or other media applications.	This is for wire communication applications. HF2X is available for HF radio or other media applications.	Non-synchronous (simplex) operation at half-channel capacity, synchronous (duplex) at full channel capacity.		use bas Ala tion (co bps bps
Transmission Media Requirements	"Voice grade" wire communication facilities: Attenuation: 3 db down at 900 cps, 6 db down at 600 cps; 3 db down at 2300 cps, 6 db down at 2500 cps (2200 cps carrier). Delay: 1 ms or less at 800 cps, 1 ms or less at 2350 cps, 2 ms or less at 2700 cps with respect to 1500 to 1700 cps minimum delay frequency.	Frequency response: with equalizers, 900-2300 cps, + 3 db; 400-2700 cps, + 6 db; without equalization, 900-2500 cps, + 1 db. Envelope delay: with equalizers, within 1 ms from 800-2350 cps, and within 1.7 ms from 600-2700 cps; without equalization, within 0.3 ms from 900-2500 cps. Line impedance 600 ohms nominal at 1000 cps. Line S/N (rms signal power-to-rms white noise power) ratio for bit error rate of 1×10^{-5} : 16 db at 2400 bps, and 10 db at 1200 bps.		Standard telephone facilities.	
Transmission Rates	Serial: 600, 1200, 2400 baud with internal clock (others available, special); 500-2500 baud with external clock.	600, 1200, or 2400 bps synchronous, or up to 1200 bps asynchronous. Serial transmission.	Max: 1500 bps non-synchronous, 3000 bps synchronous. (With 20 simplex and 40 duplex channels, parallel transmission.)	2400 $\pm$ 50 bps, serial synchronous transmission. Other rates can be used by substituting plug-in components.	less up t miss
Size, Weight	8-3/4" high, 24" deep, in standard 19" rack; 117 lbs.	For 1 duplex modem, and power supply: standard 19" wide, 15-3/4" high, 13-1/2" deep.	Approx: Transmitter - 10" Hx2"Hx 10"L; receiver - 6"Hx2"Hx10"L (two inputs each). Cabinet with 20 non-synchronous (50 sync) channels and power supply - 21"Hx24"Hx60" high.	Cabinet: 11-1/2" high, 21-1/2" wide, 18-1/2" deep. Rack mounting available, 50 lb. with power supply and cabinet.	deep
Primary Input Requirements	120V AC $\pm$ 10% at 45-65 cps, 115 watts minimum	117 $\pm$ 10% volts AC, 50 to 60 cps transmitter 20 watts receiver 20 watts		117V AC $\pm$ 10% at 60 cps 30 watts if used with external power supply (+12V and -12V nom.)	115 20 1
Equalization	Envelope delay and amplitude equalizers included.	Equalizers available as modular plug-in components (amplitude and phase).		Delay equalizer not included.	Par on 1 amp sta
Data Input and Level	+5 to +50V mark, -30 to +1/2V space, digital information at the bit rate.	Binary signal amplitude EIA standard $\pm$ 3 to +20 volts (other options available).		Serial mark impulses (+5V min. and 10 us max rise time) or polar (either polarity, between -12V and +12V) binary input. Max. spacing period is 16 bits. No synchronization signals required.	2 Z and 10 us max rise time) or polar (either polarity, between -12V and +12V) binary input. Max. spacing period is 16 bits. No synchroni- zation signals required
Input Impedance	Greater than 5000 ohms.	Approximately 5000 ohms.		10,000 ohms (nom.)	22
Output Level	-20 to +3 dbm.	-24 to +6 dbm, adjustable.	-20 to +10 dbm.	0 dbm max. (50% modulation).	-6
Output Impedance	600 ohms.	600 ohms (balanced).	600 ohms.	600 ohms balanced.	600
Input Level	-35 to +5 dbm (AGC).	-40 to 0 dbm adjustable. Short term variations 10 db above and 20 db below adjusted level.		36 to -14 dbm and -16 to +6 dbm in two ranges.	
Input Impedance		600 ohms, balanced		600 ohms, balanced	
Output Level	+5V mark, 0V space	$\pm$ 10 volts (other options available).		Mark impulse output: approx. 20 us rise time, 24 us width at base, 11 volts positive going with -11 volts as reference.	S
Output Impedance	600 ohms	3000 ohms (other options available)		820 ohms.	800
Carrier Frequencies	2500 or 2200 cps for 24 or 1200 bps (lower SB transmission); 2200 or 1600 cps for 600 bps (double SB transmission).	1100 to 2300 cps.		2750 $\pm$ 100 cps.	1100
Clock Input	Level: minimum 5V peak-to-peak square wave (500 to 2500 cps); impedance: greater than 5000 ohms.	Square wave, positive going edges coincide with bit transitions, at data bit rate. Amplitude $\pm$ 3 to $\pm$ 20 volts; impedance approx. 5000 ohms (other options available).			
Clock Output	Level: 5V peak-to-peak square wave at bit rate; impedance: 600 ohms.	Square wave at data bit rate, positive going edges coincide with bit transitions. $\pm$ 10 volts into 3000 ohms. (other options available).		Approx. 0.40 us rise pulse width at base; pulse going, with -11V reference; 50 us max. fall impedance.	
Synchronization		Receiver synchronized to transmit clock. 15 seconds max. time required for automatic bit synchronization.		Clock synchronization between 2350 and 2450 cps.	
Environmental Conditions		Ambient temperature - 0° to 50°C.		Temp. range: 50-100°F.	

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TABLE 2-1 DIGITAL DATA TRANSMISSION EQUIPMENT CHARACTERISTICS

26 B DATA TRANS-	ROBERTSHAW-FULTON PHASE "MULTILOK"	STROMBERG CARLSON HIGH SPEED BINARY DATA TRANS- CEIVER MODEL S-C-301	AT&T "DATA PHONE" DIGITAL SUBSET	HALLICRAFTERS CTDS-2400/M2C	MIT LINCOLN LABORATORY "COBY" DATA TRANSMISSION MODEM	AT&T (BELL) A-1 (DDT AND DDR)
three-level pulses are (original binary coding signal occupies 1/3 of the channel, and permits out redundancy.	Uses non-synchronous phase-shift keying and coherent detection. The information is contained in the amount of subcarrier phase shift. An extra phase position is required to eliminate ambiguities, so 3 phase positions are required for simplex operation and 5 positions for duplex operation. Frequency-division multiplexing can be used, with each channel (subcarrier) operating at 75 bauds (simplex) or 150 bauds (duplex), or a single high-speed channel can be used.	Vestigial sideband AM and envelope detection, using 3-level pulses.	FSK system, available on lease from commercial telephone companies. Binary data is accepted at various rates, and is regenerated in the receiver. No synchronization is provided.	Uses code translation (5 bits to 9 bits) and differentially coherent phase-reversal modulation techniques, together with synchronous detection. The code translation provides error detection, and a means of recovering timing from the received signal. Designed by Lincoln Laboratory.	A more recent and improved version of the CTDS, using 4 bit to 5 bit code translation.	Vestigial sideband 3 system using envelope detection. Available on lease from commercial telephone companies.
communication is available for applications.	Non-synchronous (simplex) operation at half-channel capacity, synchronous (duplex) at full channel capacity.		Two-wire dial-up service, for use with switched plant on a toll basis-alternate 2-way operation. Also, 4-wire leased facility operation possible. Modes: 1-teletype (to 100 wpm); 2-low speed, to 180 bps; 3-high speed, to about 1000 bps.	Simplex or duplex operation. Simplex operation possible while still retaining a 2-way test and monitoring capability. Capable of transmitting multiplexed outputs of 20 to 40 teletypewriters on a single communication channel.	Four-wire circuits between user terminals.	
se: with cps, ± 3 db; without equalizers, ± 1 db. squalizers, 2350 cps, and 3-2700 cps; within 0.3 ms line impedance 200 cps. Line to-rms white bit error db at 2400 bps.		Standard telephone facilities.			Frequency response: 800-2400 cps, ± 2 db. Envelope delay: 800-2400 cps, total difference in delay between any two frequencies shall not exceed 500 microseconds. Net loss: the 1000 cps net loss of the circuit shall be not greater than 20 db nor less than 10 db. Circuits designed to work into terminal impedances of 600 ohms resistive, balanced above ground.	
10 bps synchronous asynchronous.	Max: 1500 bps non-synchronous, 3000 bps synchronous. (With 20 simplex and 40 duplex channels, parallel transmission.)	2400 $\pm$ 50 bps, serial synchronous transmission. Other rates can be used by substituting plug-in components.	Max. about 1200 bps over leased lines and 600 bps over dial-up toll service. Serial transmission.	1200 and 2400 bps, serial transmission.	1200 bps (at a 1500 baud rate). Serial transmission.	Max 1600 bps, serial transmission.
m, and power wide, 15-3/4"	Approx: Transmitter - 10" Hx2"Wx 10"L; receiver - 6"Hx2"Wx10"L (two inputs each). Cabinet with 20 non-synchronous (40 sync) channels and power supply - 24"x24"x60" high.	Cabinet: 11-1/2" high, 21-1/2" wide 18-1/2" deep. Rack mounting available. 50 lb. with power supply and cabinet.	10-1/2" high, 19" wide, 12" deep. 50 lb.	5-1/4" high x 19" wide x 20" deep (nom.)		6 transmitters and 6 receivers in one 19" x 6' open rack.
0 to 40 cps		117V AC $\pm 10\%$ at 60 cps 30 watts if used with external power supply (+12V and -12V nom.)	115 V AC at 60 cps 20 watts	115V AC $\pm 10\%$, 45 to 65 cps 50 watts (nom.)		
as modular nplitude and		Delay equalizer not included.	Required for rates over 750 cps on private line channels. Plug-in amplitude equalizer available, and standard 200-cps delay equalizer	Not included.		
le EIA ts (other		Serial mark impulses (+5V min. and 10 us max rise time) or polar (either polarity, between -12V and +12V) binary input. Max. spacing period is 16 bits. No synchronization signals required.	20 ma. for space, and 0 ma. for mark, for two lower speed modes (designed for use with teletypewriter). High-speed mode: 0 volt mark, -12 volt space or +6 volt mark, -6 volt space.	0 ma. = -6 volts, zero = 0 volts (nom). Other options available.	Dipulse inputs 2.2 volts peak-to-peak (0 dbm). Adjustable over range 1 to 5 volts peak-to-peak. 1200 cps dipulses indicate mark, no signal indicates a space.	
s.		10,000 ohms (nom.)	2200 ohms.	10,000 ohms (nom.)	600 ohms.	
ble.	to ± 10 dbm.	0 dbm max. (50% modulation).	-6 to 0 dbm.	-15 to +5 dbm in 2 db steps.	0 dbm (0.78 volts rms into 600 ohms resistive) adjustable -20 to +6 dbm.	
	600 ohms.	600 ohms balanced.	600 ohms.	600 ohms, balanced or unbalanced.	600 ohms.	
e. Short above and level.		36 to -14 dbm and -16 to +6 dbm in two ranges.		-35 to +3 dbm.	-15 db nominal (-25 db to -5db)	
		600 ohms, balanced		600 ohms, balanced or unbalanced.	600 ohms.	
ns available.		Mark impulse output: approx. 20 us rise time, 24 us width at base, 11 volts positive going with -11 volts as reference.	Same as transmitter input.	Same as transmitter input.	Same as transmitter data input.	
ns available)		820 ohms.	800 ohms.	150 ohms (nom.)	600 ohms.	
		2750 $\pm$ 100 cps.	1100 to 2100 cps.		1500 cps.	
ding transitions, itude ± 3 to approx. 5000 (leble).					Transmitter accepts 1200 cps sine wave.	
rate, ncide with lts into s avail-		Approx. 0.40 us rise time; 10us pulse width at base; 11 volts positive going, with -11 volts as reference; 50 us max. jitter; 820 ohms impedance.		Level: same as transmitter data input level. Impedance; 150 ohms (nom.).	Receiver provides 1200 cps sine wave (recovered from 180° phase changes).	
trans- ax. time it		Clock synchronization automatic between 2350 and 2450 pps.	Not provided by subset.	Timing coherently generated from master clock (recovered from encoded data for receiver).	Sync bits may occur at multiples of every four data bits. Timing recovered from data.	
0 to 50°C. max.		Temp. range: 50-100°F.				

ION EQUIPMENT CHARACTERISTICS

WDL-TR2017

TAL SUBSET	HALLICRAFTERS CTDS-2400/M2C	MIT LINCOLN LABORATORY "COBI" DATA TRANSMISSION MODEM	AT&T (BELL) A-1 (DDT AND DDR)	COLLINS "KINEPLEX" AN/GCS-4	COLLINS "KINEPLEX" TE-210D-2
Available on lease from telephone companies. Available at various rates in the organization is	Uses code translation (5 bits-to-9 bits) and differentially coherent phase-reversal modulation techniques, together with synchronous detection. The code translation provides error detection, and a means of recovering timing from the received signal. Designed by Lincoln Laboratory.	A more recent and improved version of the CTDS, using 4 bit-to-5 bit code translation.	Vestigial sideband 3 level AM system using envelope detection. Available on lease from commercial telephone companies.	Differentially coherent, multiple level phase-shift-keyed system, using frequency multiplexed channels (subcarriers). Each phase position carries 3 bits of information, so each subcarrier handles 3 synchronous channels. Resonators and synchronous sampling are used to provide the differentially coherent phase references for detection (predicted wave detection; kinematic filtering).	Differentially coherent, multiple level phase-shift-keyed system, using frequency multiplexed channels (subcarriers). Each phase position carries 2 bits of information, so each subcarrier handles 2 synchronous channels. Resonators and synchronous sampling are used to provide the differentially coherent phase references for detection (predicted wave detection; kinematic filtering).
Up service, for instant on a toll-free operation. Facility operates: 1-teletype speed, to 180 to about 1000	Simplex or duplex operation. Simplex operation possible while still retaining a 2-way test and monitoring capability. Capable of transmitting multiplexed outputs of 20 to 40 teletypewriters on a single communication channel.	Four-wire circuits between user terminals.		Serial or parallel full duplex, continuous, unattended, at transmission rates below.	Serial or parallel full duplex, continuous, unattended, at 600, 1200, or 2400 bps rates.
		Frequency response: 800-2400 cps, ± 2 db. Envelope delay: 800-2400 cps, total difference in delay between any two frequencies shall not exceed 500 microseconds. Net loss: the 1000 cps net loss of the circuit shall be not greater than 20 db nor less than 10 db. Circuits designed to work into terminal impedances of 600 ohms resistive, balanced above ground.		Line requirements (for max data rates): Freq. response, ± 3 db, 900-3200 cps; ± 6 db, 550 to 3550 cps; differential delay, ± 1 ms max, 800-3300 cps, ± 1.4 ms max, 550-3550 cps; short term net loss variations ± 5 db max, net loss-long term, 20 db; impedance-600 or 1135 ohms nom. at 1000 cps; impulse noise threshold-12 db below rms level of composite signal; freq. error, ± 2.5 cps; broadband (Gaussian) noise - 28 db below rms level of composite signal, measured in 3 kc band.	Telephone facilities having the characteristics specified under FCC Tariff 237, Schedule 4A, maintained in accordance with standard telephone system practices.
30 bps over 30 bps over dial. Serial transmission.	1200 and 2400 bps, serial transmission.	1200 bps (at a 1500 baud rate). Serial transmission.	Max 1600 bps, serial transmission.	Max 18 channels, each 300 bps. Rates - 600, 1200, 2400, 3600, 4800, or 5400 bps (for synchronous serial or parallel digital data transmission).	2, 4, or 8 synchronous parallel 300 bps binary channels; or single synchronous binary channel at 600, 1200, or 2400 bps.
19" wide, 12"	5-1/4" high x 19" wide x 20" deep (nom.)		6 transmitters and 6 receivers in one 19" x 6' open rack.	4 drawers, each in case 19" wide x 8-3/4" high x 27" deep. Cabinet or rack mount.	2 drawers mounted in cabinet 19" wide, 22-3/4" high, and 28-3/4" deep, free standing or rack mounted. 120 lbs.
	115V AC $\pm 10\%$, 45 to 65 cps 50 watts (nom.)			120V AC $\pm 10\%$, 60 cps $\pm 10\%$ single phase. 350 watts max.	120V AC $\pm 10\%$, 47 to 450 cps, single phase. 200 watts max.
Is over 750 cps channels. Plug-in or available, and delay equalizer	Not included.			Phase delay compensator included	Phase delay compensator included.
5 and 0 ma for speed modes with teletype mode: 0 volt pace or +6 volt ce.	"One" = -0 volts, "Zero" = 0 volts (nom). Other options available.	Bipulse inputs 2.2 volts peak-to-peak (0 dbm). Adjustable over range 1 to 5 volts peak-to-peak. 1200 cps dipulses indicate mark, no signal indicates a space.		Synchronous single channel serial (600, 1200, 2400, 3600, 4800, or 5400 bps), or 2, 4, 8, 12, 16, or 18-300 bps parallel synchronous channels. 3 switch selected levels available - negative, polar, and positive.	Serial: single channel synchronous at 600, 1200 or 2400 bps, -1 to +0.25 volt for binary zero, and +3.6 to +6.0 volts for binary one. Parallel: 2, 4, or 8 parallel 300 bps synch. channels, -4.0 to +6.5 volts for binary zero and +1 to -1 volt for binary one.
	10,000 ohms (nom.)	600 ohms.		5000 ohms nom., serial input. 10,000 ohms nom., parallel input.	5000 ohms nom., serial input. 10,000 ohms nom., parallel input.
	-15 to +5 dbm in 2 db steps.	0 dbm (0.78 volts rms into 600 ohms resistive) adjustable -20 to +6 dbm.		-20 to +6 dbm (variable).	-20 to +4 dbm (variable).
	600 ohms, balanced or unbalanced.	600 ohms.		600 ohms or 1135 ohms at 1000 cps	600 ohms $\pm 20\%$, 900 to 2300 cps.
	-35 to +3 dbm.	-15 db nominal (-25 db to -5db)		-20 to 0 dbm (variable).	-35 to +5 dbm (variable).
	600 ohms, balanced or unbalanced.	600 ohms.		600 or 1135 ohms at 1000 cps.	600 ohms $\pm 20\%$, 900 to 2300 cps.
er input.	Same as transmitter input.	Same as transmitter data input.		Same as input data.	Similar to data input.
	150 ohms (nom.)	600 ohms.		600 ohms max.	600 ohms max.
		1500 cps.		935, 1375, 1815, 2255, 2695, and 3135 cps, depending on transmission rate.	935, 1375, 1815 and 2255 cps depending on transmission rate.
		Transmitter accepts 1200 cps sine wave.		Bipolar square waves, 3.25 to 6.5 volts peak-to-peak at the data rate; 5000 ohms nom. impedance. External clock (optional) input rate 100 kc.	Bipolar square waves, 2.9 to 6.8 volts peak-to-peak at the data rate; 5000 ohms nom. impedance. External clock (optional) input 100 kc, freq. stability $\pm 10^5$ per day, or greater, 5V rms min., 5000 ohms min.
	Level: same as transmitter data input level. Impedance; 150 ohms (nom.).	Receiver provides 1200 cps sine wave (recovered from 180° phase changes).		Same as input, but 600 ohms max. impedance.	Bipolar square waves, 3.6 to 6.0 volts peak-to-peak at data rate, 600 ohms impedance max.
subset.	Timing coherently generated from master clock (recovered from encoded data for receiver).	Sync bits may occur at multiples of every four data bits. Timing recovered from data.			

3

	Standard 19-1/2" deep.	high, 19-1/2" deep.	(Two inputs each). Cabinet with 20 non-synchronous (40 sync) channels and power supply - 24"x24"x60" high.	Available 50 lb. with power supply and cabinet.
Primary Input Requirements	120V AC + 10% at 45-65 cps, 115 watts minimum	117 + 10% volts AC, 50 to 60 cps transmitter 20 watts receiver 20 watts		117V AC +10% at 60 cps 30 watts if used with external power supply (+12V and -12V nom.)
Equalization	Envelope delay and amplitude equalizers included.	Equalizers available as modular plug-in components (amplitude and phase).		Delay equalizer not included.
Data Input and Level	+5 to +50V mark, -30 to +12V space, digital information at the bit rate.	Binary signal amplitude EIA standard ± 3 to +20 volts (other options available).		Serial mark impulses (+5V min. and 10 us max rise time) or polar (either polarity, between -12V and +12V) binary input. Max. spacing period is 16 bits. No synchronization signals required.
Input Impedance	Greater than 5000 ohms.	Approximately 5000 ohms.		10,000 ohms (nom.)
Output Level	-20 to +3 dbm.	-24 to +6 dbm, adjustable.	to +10 dbm.	0 dbm max. (50% modulation).
Output Impedance	600 ohms.	600 ohms (balanced).	600 ohms.	600 ohms balanced.
Input Level	-35 to +5 dbm (AGC).	-40 to 0 dbm adjustable. Short term variations 10 db above and 20 db below adjusted level.		36 to -14 dbm and -16 to +6 dbm in two ranges.
Input Impedance		600 ohms, balanced		600 ohms, balanced
Output Level	+5V mark, 0V space	+10 volts (other options available).		Mark impulse output: approx. 20 us rise time, 24 us width at base, 11 volts positive going with -11 volts as reference.
Output Impedance	600 ohms	3000 ohms (other options available)		820 ohms.
Carrier Frequencies	2500 or 2200 cps for 24 or 1200 bps (lower SB transmission); 2200 or 1600 cps for 600 bps (double SB transmission).	1100 to 2300 cps.		2750 +100 cps.
Clock Input	Level: minimum 5V peak-to-peak square wave (500 to 2500 cps); impedance: greater than 5000 ohms.	Square wave, positive going edges coincide with bit transitions, at data bit rate. Amplitude ± 3 to ± 20 volts; impedance approx. 5000 ohms (other options available).		
Clock Output	Level: 5V peak-to-peak square wave at bit rate; impedance: 600 ohms.	Square wave at data bit rate, positive going edges coincide with bit transitions, ± 10 volts into 3000 ohms, (other options available).		Approx. 0.40 us rise time; 10us pulse width at base; 11 volts positive going, with -11 volts as reference; 50 us max. jitter; 820 ohms impedance.
Synch onizat on		Receiver unsynchronized to transmit clock. 15 seconds max. time required for automatic bit synchronization.		Clock synchronization automatic between 2350 and 2450 pps.
Environmental Conditions		Ambient temperature - 0° to 50°C. Relative humidity: 90% max. Altitude: to 10,000 ft.		Temp. range: 50-100°F.
Special Features	1. Voice communication over circuit allowed at operator's discretion. 2. Front panel adjustment of equalization, with scope indication. 3. Quick-look status monitor. 4. Simple check-out procedure, integral scope. 5. Clock slaving provided to maintain a remote station clock in exact synchronism with a master clock at a central location, automatically. 6. Auxiliary equip. available for superimposing a 2 nd data channel for word or message frame synchronization (usually needed for communication in computer format). 7. Random test data signal generator, and complete performance monitor included.	1. Requires no receiver clock. 2. Error detection provided by coding, without introducing data redundancy, therefore no loss of transmission efficiency. 3. Built-in pattern generator for complete system alignment, and adjustment of amplitude and delay equalizer with use of an oscilloscope. 4. Data may be recorded directly from transmission line and played back through receiver, with any good tape recorder. 5. For 1200 bps, binary operation is used giving optimum system bandwidth again. This improves transmission over low-quality data channels.	Reference phase at receiver is derived from incoming carrier. A third phase is used to prevent ambiguity if a signal is momentarily lost. High SNR and very narrow bandwidth are claimed. Can operate on either a time or a frequency multiplex basis (synchronous or non-synchronous). In non-synchronous operation the channels can operate independently.	No synchronizing signals are required. The receiver has an AFC system for automatic synchronization.
Complexity-Reliability-Construction	Uses replaceable sub-assemblies, printed wiring, plug-in connectors, 300 internal test jacks, with normal oscilloscope patterns provided for many. Uses Mil Spec or approved equivalent components throughout. Testing has shown a mean-time-between failure record in excess of 7500 hours. Transistorized construction.	Transistor. Plug-in modular packaging. High quality Mil and industrial components. Simplicity of alignment and servicing due to simple functioning basic modules (a binary system, and its associated simplicity).	Tube-type transmitter, transistor receiver. Built to high quality commercial standards, for installation & operation under commercial conditions. Can be adapted to Mil Spec model. Uses plug-in printed circuit cards.	Solid-state circuitry. Simple plug-in printed circuit modules. A binary system and its associated simplicity.
Error Rates - Tolerance to Noise (See Section 4.2)	Bit error rate 10^{-5} for 16.5 db SNR, white noise only; 10^{-5} for 19 db SNR, combined severe delay and amplitude distortion and 4 db cyclic amplitude variation.	Bit error rate 10^{-6} over schedule 4A private line telephone channel or equivalent. FN provides immunity to short term subcarrier level variations. Line S/N ratio (rms signal power to rms white noise power) for bit error rate of 1×10^{-5} : 16 db at 2400 cps, and 10 db at 1200 cps.	Requires high quality channels to approach theoretical error rate. Signal-to-noise performance is degraded by using high speed ("diplex") operation.	
Remarks	Price: \$7500.00 fob, Silver Spring Md. Performance improved from earlier models 24A and 25. Auxiliary units available for converting facsimile signals to synchronous digital form for improved reliability and secure transmission, for code conversion for special data formats, and for inserting error control coding in the data format. Operational.	Supercedes earlier quaternary system. Optional equip.: 1 - optional rcvr clock available, with automatic sync (no manual adjustment needed regardless of inactive transmission time). 2 - line failure alarm. 3 - order wire complete with 2-way signalling. 4 - universal equalizers for correcting phase and amplitude distortion available as modular plug-in components. 5 - error detector. In production and available off-the shelf.		Experimental

12" x 12" x 12" cabinet with 20 two inputs each. Cabinet with 20 non-synchronous (40 sync) channels and power supply - 2 1/2" x 24" x 60" high.	available 30-15 with power supply and cabinet.					Cabinet
117V AC +10% at 60 cps 30 watts if used with external power supply (+12V and -12V nom.)	115 V AC at 60 cps 20 Watts	115V AC +10%, 45 to 65 cps 50 Watts (nom.)				120V single 350 wa
Delay equalizer not included.	Required for rates over 750 cps on private line channels. Plug-in amplitude equalizer available, and standard 200-type delay equalizer	Not included.				Phase
Serial mark impulses (+5V min. and 10 us max rise time) or polar (either polarity, between -12V and +12V) binary input. Max. spacing period is 16 bits. No synchronization signals required.	20 ma. for space, and 0 ma. for mark, for two lower speed modes (designed for use with teletype-writer). High-speed mode: 0 volt mark, -12 volt space or +6 volt mark, +6 volt space.	One -6 volts, zero = 0 volts (nom). Other options available.	Dipulse inputs 2.2 volts peak-to-peak (0 dbm). Adjustable over range 1 to 5 volts peak-to-peak. 1200 cps dipulses indicate mark, no signal indicates a space.			Synch (600, 5400 b; 18-300 channel availal positiv
10,000 ohms (nom.)	2200 ohms.	10,000 ohms (nom.)	600 ohms.			5000, 10,000
0 dbm max. (50% modulation).	-6 to 0 dbm.	-15 to +5 dbm in 2 db steps.	0 dbm (0.78 volts rms into 600 ohms resistive) adjustable -20 to +6 dbm.			-20 t
600 ohms.	600 ohms balanced.	600 ohms.	600 ohms, balanced or unbalanced.			600 oh
36 to -14 dbm and -16 to +6 dbm in two ranges.		-35 to +3 dbm.	-15 db nominal (-25 db to -5db)			-20 to
600 ohms, balanced		600 ohms, balanced or unbalanced.	600 ohms.			600 or
Mark impulse output: approx. 20 us rise time, 24 us width at base, 11 volts positive going with -11 volts as reference.	Same as transmitter input.	Same as transmitter input.	Same as transmitter data input.			Same as
820 ohms.	800 ohms.	150 ohms (nom.)	600 ohms.			600 ohm
2750 +100 cps.	1100 to 2100 cps.		1500 cps.			935, 13 3135 cps rate.
			Transmitter accepts 1200 cps sine wave.			Bipolar 7.5 volts rate; 50 External rate 100
Approx. 0.40 us rise time; 10us pulse width at base; 11 volts positive going, with -11 volts as reference; 50 us max. jitter; 820 ohms impedance.		Level: same as transmitter data input level. Impedance; 150 ohms (nom.).	Receiver provides 1200 cps sine wave (recovered from 180° phase changes).			Same as impedance
Clock synchronization automatic between 2350 and 2450 pps.	Not provided by subset.	Timing coherently generated from master clock (recovered from encoded data for receiver).	Sync bits may occur at multiples of every four data bits. Timing recovered from data.			
Temp. range: 50-100°F.						Temp.: Humidity: Altitude:
Reference phase at receiver is derived from incoming carrier. A third phase is used to prevent ambiguity if a signal is momentarily lost. High SNR and very narrow bandwidth are claimed. Can operate on either a time or a frequency multiplex basis (synchronous or non-synchronous). In non-synchronous operation the channels can operate independently.	No synchronizing signals are required. The receiver has an AFC system for automatic synchronization.	Designed to operate through switched, (not leased) telephone connections at 600 bps. 1000 bps requires equalized private line. System can operate unattended. A telephone is provided for alternate voice operation.	Parallel operation is available.			Integral
Tube-type transmitter, transistor receiver. Built to high quality commercial standards, for installation & operation under commercial conditions. Can be adapted to M11 Spec model. Uses plug-in printed circuit cards.	Solid-state circuitry. Simple plug-in printed circuit modules. A binary system and its associated simplicity.	Transistor. Simple and economical terminal equipment.	Transistor. Modular construction, printed wiring boards; removable power supply.	Transistor. Modular construction, printed wiring cards; removable power supply.	Tubes	Transistor than the Drawer unit contained it provides 1000 hours, 15 repair, 1 preventive oper
Requires high quality channels to approach theoretical error rate. Signal-to-noise performance is degraded by using high speed ("diplex") operation.		Very low error rates reported.	Uses amplitude limiting, band filtering, and center strobing to achieve noise immunity.	Specified bit error rate for Gaussian noise and 300 to 2500 or 3000 cps channel bandwidth: 1 in 10 ³ for 8 db signal-to-noise ratio 1 in 10 ⁴ for 9 db 1 in 10 ⁵ for 10 db	Estimated error rate: 10 ⁻⁵ signal uses only a bandwidth of 1000 to 2600 cps.	Error rate condition 20.3 to 1 bit error to 14 db due to tlstrated,
Experimental	Operational. Common-carrier supplied. Rents for \$40.00/mo., plus \$45.00 installation fee. Considerable jitter in the demodulated signal has been reported, primarily due to the FM deviation ratios and the demodulation method. Use at distances greater than 50 ft. from the customer digital equip. is prohibited, due to voice induction coil (which must be switched in and out for alternate voice), and relatively high impedance (10 K) on the digital signal lines.	Operational. Troposcatter substitute for Western Electric A1. Uses element-to-element comparison.		Operational. Used in the SAGE system.		Leased T1 with SAGE

				Cabinet or rack mount.	Deep, free-standing or rack-mounted, 120 lbs.
pa	115V AC +10%, 45 to 65 cps 50 watts (nom.)			120V AC +10%, 60 cps +10% single phase. 350 watts max.	120V AC +10%, 47 to 450 cps, single phase. 200 watts max.
tas over 750 cps channels. Plus; in zer available. and e delay equalizer	Not included.			Phase delay compensator included	Phase delay compensator included.
ce. And 0 ma for ower speed modes se with teletype- speed mode. 0 volt space or +6 volt pace.	One " = 0 volts, zero = 0 volts (nom). Other options avail- able.	Dipulse inputs 2.2 volts peak- to-peak (0 dbm). Adjustable over range 1 to 5 volts peak-to-peak. 1200 cps dipulses indicate mark, no signal indicates a space.		Synchronous single channel serial (600, 1200, 2400, 3600, 4800, or 5400 bps), or 2, 4, 8, 12, 16, or 18-300 bps parallel synchronous channels. 3 switch selected levels available - negative, polar, and positive.	Serial: single channel synchro- nous at 600, 1200 or 2400 bps, +1 to +0.25 volt for binary zero, and +3.6 to +6.0 volts for binary one. Parallel: 2, 4, or 8 parallel 300 bps synch. channels, -4.0 to -6.5 volts for binary zero and +1 to +1 volt for binary one.
	10,000 ohms (nom.)	600 ohms.		5000 ohms nom., serial input. 10,000 ohms nom., parallel input.	5000 ohms nom., serial input. 10,000 ohms nom., parallel input.
	-15 to +5 dbm in 2 db steps.	0 dbm (0.78 volts rms into 600 ohms resistive) adjustable -20 to +6 dbm.		-20 to +6 dbm (variable).	-20 to +4 dbm (variable).
	600 ohms, balanced or unbal- anced.	600 ohms.		600 ohms or 1135 ohms at 1000 cps	600 ohms +20%, 900 to 2300 cps.
	+35 to +3 dbm.	-15 db nominal (-25 db to -5db)		-20 to 0 dbm (variable).	-35 to +5 dbm (variable).
	600 ohms, balanced or unbalanced.	600 ohms.		600 or 1135 ohms at 1000 cps.	600 ohms +20%, 900 to 2300 cps.
ter input.	Same as transmitter input.	Same as transmitter data input.		Same as input data.	Similar to data input.
	150 ohms (nom.)	600 ohms.		600 ohms max.	600 ohms max.
ts.		1500 cps.		935, 1375, 1815, 2255, 2695, and 3135 cps, depending on transmission rate.	935, 1375, 1815 and 2255 cps depending on transmission rate.
		Transmitter accepts 1200 cps sine wave.		Bipolar square waves, 3.25 to 7.5 volts peak-to-peak at the data rate; 5000 ohms nom. impedance External clock (optional) input rate 100 kc.	Bipolar square waves, 2.9 to 6.8 volts peak-to-peak at the data rate; 5000 ohms nom. impedance. External clock (optional) input 100 kc. freq. stability 1x10 ⁻⁵ per day, or greater, 5V rms min., 5000 ohms min.
	Level: same as transmitter data input level. Impedance: 150 ohms (nom.).	Receiver provides 1200 cps sine wave (recovered from 180° phase changes).		Same as input, but 600 ohms max. impedance.	Bipolar square waves, 3.6 to 6.0 volts peak-to-peak at data rate. 600 ohms impedance max.
y subset.	Timing coherently generated from master clock (recovered from encoded data for receiver).	Sync bits may occur at multiples of every four data bits. Timing; recovered from data.			
				Temp.: -29° to +54°C with ovens. Humidity: 0 to 80% relative, with- out condensation. Altitude: sea level to 15,000 ft.	Temp.: 0° to 52°C. Humidity: 0 to 80% relative, with- out condensation. Altitude: 0 to 15,000 ft. Vibration: MIL-T-4807A
rate through (leased) telephone 600 bps. 1000 bps and private line. rate unattended. A provided for alter- ation.	Parallel operation is available.			Integral test facility included.	Transition alarms provide relay con- tact closure for loss of input serial data rate timing, serial input data, serial output data, and for improper transmit and receive levels. Inte- gral test facility included.
imple and economical ment.	Transistor. Modular construction, printed wiring boards; removable power supply.	Transistor. Modular construction, printed wiring cards; removable power supply.	Tubes	Transistor. Larger and more complex than the other, slower, systems. Drawer mounted plug-in modular cir- cuit card construction. Self-con- tained integral test facility provides visual error indication. 1000 hours mean-time-between fail- ures, 15 minutes mean-time-to- repair, less than 5 hours per month preventive maintenance for continu- ous operation.	Transistor. Drawer mounted plug-in etched wire circuit cards. Self- contained integral test facility provides visual error indication. Greater than 3000 hours mean-time- between failures, less than 15 min- utes mean-time to repair, and pre- ventive maint. less than 5 hours per month for continuous operation.
rates reported.	Uses amplitude limiting, band fil- tering, and center strobing to achieve noise immunity.	Specified bit error rate for Gaus- sian noise and 300 to 2500 or 3000 cps channel bandwidth: 1 in 10 ⁻³ for 8 db signal-to-noise ratio 1 in 10 ⁻⁴ for 9 db 1 in 10 ⁻⁵ for 10 db	Estimated error rate: 10 ⁻³ signal uses only a bandwidth of 1000 to 2600 cps.	Error rate < 10 ⁻³ under favorable conditions. Test results indicate 20.5 to 26.5 db required for 10 ⁻⁵ bit error rate at 5400 bps, and 12 to 14 db at 2400 bps. Degradation due to the AM sync has been demon- strated, especially at 5400 bps.	
Common-carrier sup- for \$40.00/mo., plus ation fee. Consider- the demodulated sig- ported, primarily eviation ratios and on method. Use at er than 50 ft. from igital equip. is pro- voice induction t be switched in and te voice), and rela- edance (10 K) on nal lines.	Operational. Troposcatter substi- tute for Western Electric A1. Uses element-to-element comparison.		Operational. Used in the SAGE system.	Leased line required. Compatible with SAGE data system.	A diversity receiver, and a modi- fication kit adapting the TE-2100-2 for communication with the TE-206 Modem, are available. Leased line probably required for max. rate.

REFERENCES

- 2.1 Manufacturers literature and reports
- 2.2 Data Line Techniques, Armour Research Foundation Technical Report, 1 April 1961; AD 263 144
- 2.3 Available Modulation Techniques for Digital Data Transmission, Beckman/Systems Division Technical Memorandum No. TM-145-A, 5 December 1960.
- 2.4 Survey of Digital Communications, Hughes Aircraft Company Communications Division Report No. TR-1, 15 April 1961; AD 277176.

SECTION 3

THE PROPAGATION MEDIA

3.1 GENERAL

This section summarizes the characteristics of typical media through which data is transmitted. Since this is strictly a summary, all of the material here has been published in the literature, as referenced.

No attempt has been made to recommend a particular transmission mode since this will be primarily determined by such factors as site location and availability of existing networks. Instead, the characteristics of these media are reviewed, since these characteristics may have an effect on the selection of equipment. For example, an amplitude sensitive system may not be appropriate if the medium imposes amplitude fluctuations.

In discussing the propagation media, we will divide the media into two major areas: wire communications and radio communications. These major areas will be further subdivided as follows:

1. Wire Communications

- a. Short-haul telephone calls
- b. Long-haul telephone calls
- c. Exchange telephone calls

2. Radio Communications

- a. Microwave telephone links
- b. Ionospheric reflection (HF) radio communication
- c. Ionospheric-scatter radio communication
- d. Meteor-scatter radio communication
- e. Tropospheric-scatter radio communication
- f. Satellite relay

3.2 WIRE COMMUNICATIONS

When discussing the characteristics of wire communications, telephone calls will be classified into the following categories: exchange, short-haul, and long-haul.* Plots of typical values for the various characteristics associated with each of the categories are presented.

3.2.1 Effective Channel Bandwidth

The effective bandwidth will be defined as the 20-db attenuation bandwidth. Figure 3-1 gives the statistical distribution of circuits having a given effective bandwidth.

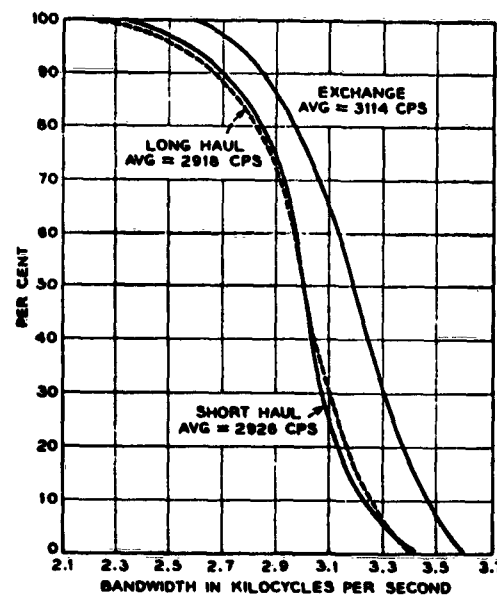


Fig. 3-1 Cumulative distributions of 20-db bandwidths, showing percentage of circuits having bandwidths greater than that shown on abscissa. (Ref. 3.1)

It is interesting to note that, of the 20 db attenuation points, 90 percent are less than approximately 2.7 kc for both long-haul and short haul calls, with exchange lines slightly better, (90 percent are less than 2.85 kc).

* These terms and most of the following curves are presented in an excellent article by Alexander, Gryb and Nast (Ref. 3.1).

Figure 3-2 gives the general shape of the relative attenuation curve which is divided into four portions. First, the low-frequency cutoff which rolls off at approximately 15 to 27 db per octave below frequency f_1 . The second portion, from frequency f_1 to f_2 is flat; the third portion, from f_2 to f_3 , is a linear attenuation rise.

This rise of attenuation between 1 kc and 2.6 kc vs. percentage of circuits is given in Fig. 3-3. The fourth portion of the curve is the high frequency roll-off. This rolls off at approximately 80 to 90 db per octave. Figure 3-4 gives the average attenuation characteristics for the different types of calls. The attenuation of any given circuit can be partially compensated for by use of attenuation equalization. Therefore, some of the harmful effects of attenuation distortion can be compensated.

3.2.2 Envelope Delay Distortion

Envelope delay may be defined as the rate-of-change of phase with frequency on a phase delay versus frequency plot. The envelope delay of a linear phase would be a constant. Figure 3-5 presents a graph of normalized envelope delay vs. frequency for the various calls. Exchange calls, it can be seen, again have better characteristics while the longer range circuits have more delay distortion.

Considering the frequency of minimum delay (FMD), this again varies from circuit to circuit. This frequency lies approximately in the center of the delay distortion curve. Figure 3-6 shows a plot of percentage of circuits having a FMD greater than the given frequency vs. that frequency. Figures 3-7 and 3-8 show the relationship of the percentage of circuits and the 0.5 millisecond and 1 millisecond delay bandwidths.

The ranges of delay distortion for 90 percent of the circuit characteristics are given in Fig. 3-9.

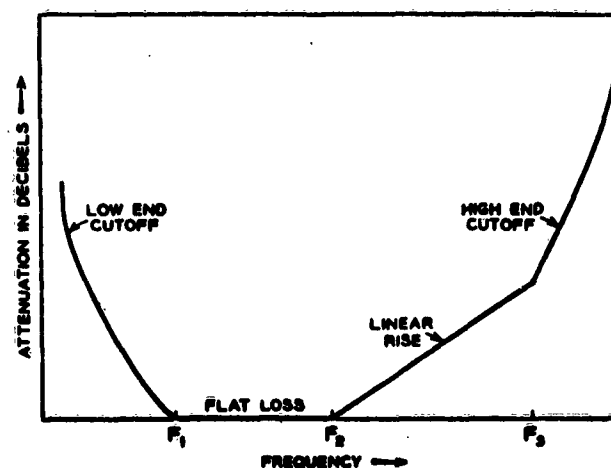


Fig. 3-2 Relative Attenuation Characteristic of Telephone Circuits (Ref. 3.1)

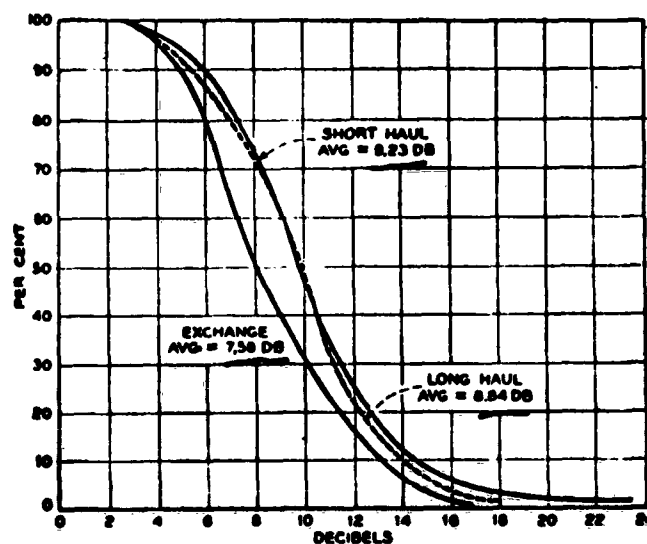


Fig. 3-3 Difference in Decibels Between 1 kc and 2.6 kc -- Percentage of Circuits Having Decibel Difference Greater Than That Shown on Abscissa (Ref. 3.1)

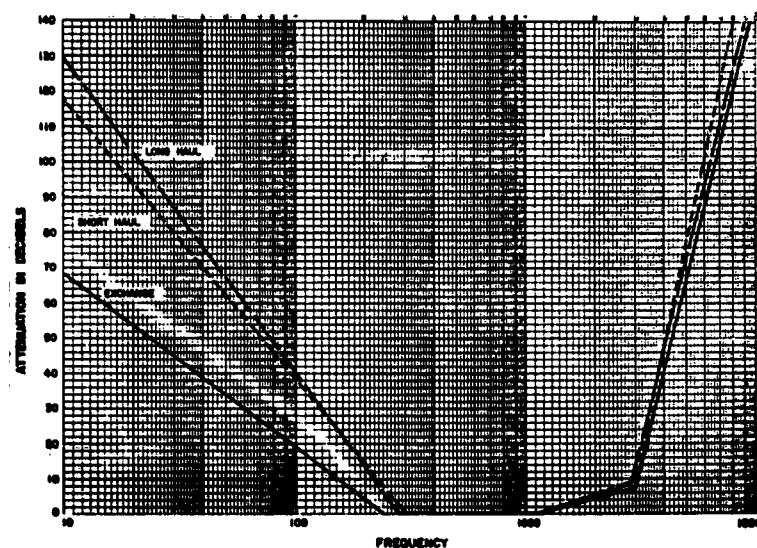


Fig. 3-4 Attenuation Characteristics

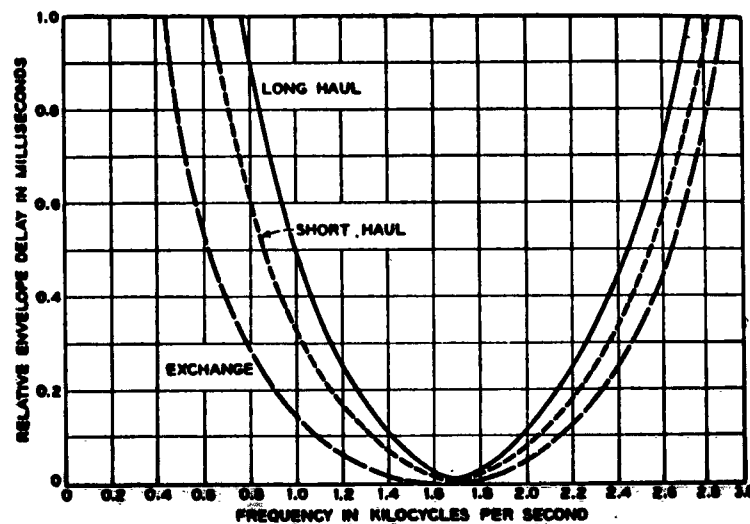


Fig. 3-5. Average Envelope Delay Characteristics (Ref. 3.1)

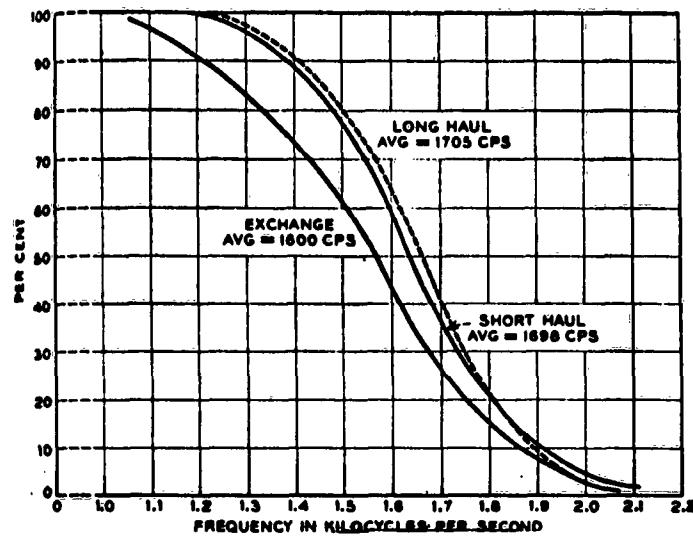


Fig. 3-6 Frequency of Minimum Envelope Delay -- Percentage of Circuits Having Frequency of Minimum Delay Greater than that Shown on Abscissa (Ref. 3.1).

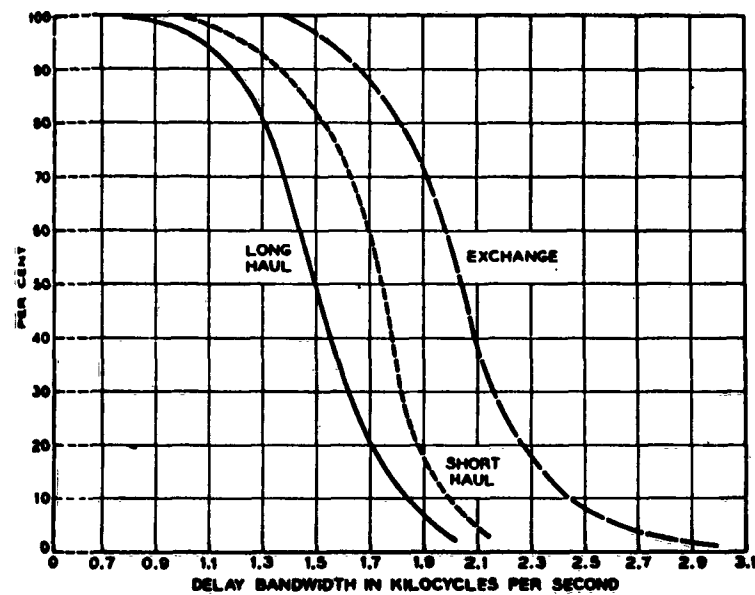


Fig. 3-7 Percentage of Circuits With 0.5 Millisecond Delay Bandwidth Greater than that Shown on Abscissa (Ref. 3.1).

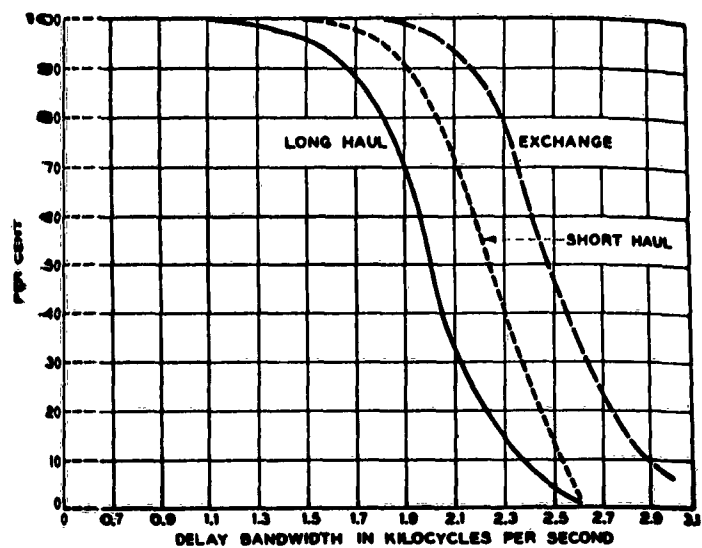


Fig. 3-8 Percentage of Circuits with 1-Millisecond Delay Bandwidth Greater than that Shown on Abscissa (Ref. 3.1)

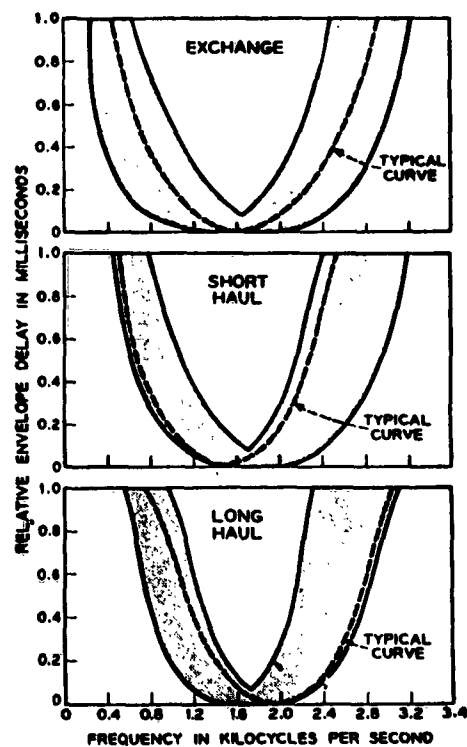


Fig. 3-9 Envelope Delay Distortion -- Locus of 90 Percent of Circuit Characteristics (Ref. 3.1)

3.2.3 Reflection

The presence of reflected energy develops ripples in their attenuation and delay characteristics. The existence of points of mismatch and hybrid unbalance are the sources of this energy reflection. Figure 3-10 gives the percentage of circuits having a maximum peak-to-peak amplitude ripple, in db, as a bar graph.

The ripple amplitude increases in the band edges where impedances deteriorate. The maximum ripple seems to occur in the frequency range of 2000 to 3000 cps.

3.2.4 End-to-End Frequency Shift

In some of the carrier systems, the shift in carrier frequency becomes a problem. The systems can be divided into three groups with respect to this problem. First are the systems where no particular effort was applied to frequency synchronization. An end-to-end shift of ± 20 cps can be expected in these systems. Second are the systems where some effort was exerted to synchronize circuits. The end-to-end shift may still be as much as ± 2 cps. Third are those systems where there is no shift at all. The presence of shift causes difficulty in identifying the signal output.

3.2.5 Circuit Net Loss

The loss in telephone transmission is usually measured at 1000 cps. Figure 3-11 gives the cumulative distribution of net circuit loss of the three types of calls.

3.2.6 Noise

The significant types of noise on telephone circuits can be classified into the following categories:

1. Baseline noise

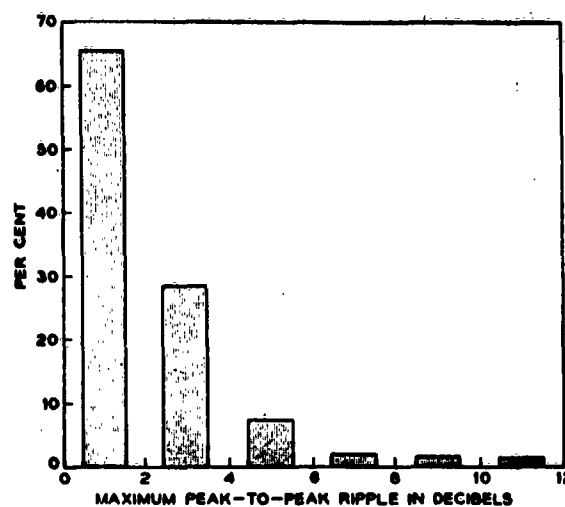


Fig. 3-10 Percentage of Circuits with Maximum Peak-to-Peak Amplitude Ripple (Ref. 3.1).

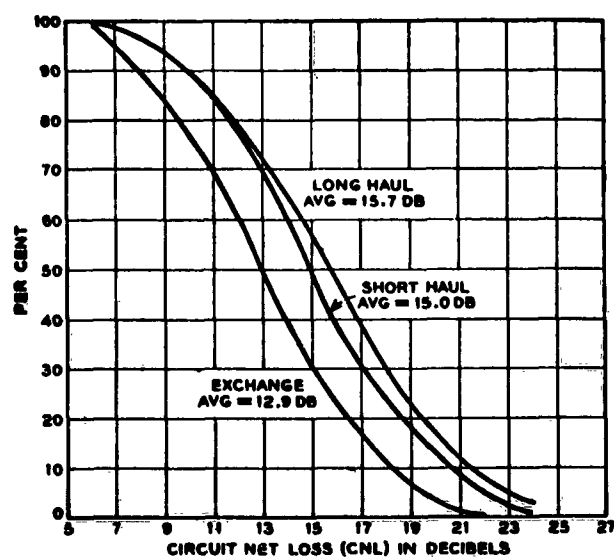


Fig. 3-11 Percentage of Circuits with 1000 Cps Net Loss Greater than that Shown on Abscissa. (Ref. 3.1)

2. High level steady noise

3. Impulse noise.

The level of baseline noise is sufficiently low that it has little effect upon data transmission. These levels are usually more than 10 db below the signal level and may be more than 30 db below

High level steady noise is usually associated with circuit outage. It is at a much higher level than the baseline noise and lasts on the order of 2 seconds.

The impulse noise is of an even higher level, often being as large as 10 db above the signal level, and may be found in the low baseband levels, also. Impulse noise usually occurs in burst on the order of 40 milliseconds. Figures 3-12 through 3-14 give typical plots of baseline and impulse noises. This type can be attributed to natural as well as man-made causes.

3.3 RADIO COMMUNICATIONS

In radio communications, parameters of the propagation media vary from one communication technique to another. If the same parameters (e.g., bandwidth, frequency of operation, range, attenuation) are examined, they will, of course, give different answers from method to method. Table 3-1 lists some of these parameters for the various media. An attempt was made to place an upper and lower bound on the parameters. In general, these bounds are based on information obtainable on present equipment.

Some interference effects, such as noise and fast fading, are common to most radio systems. Fast fading, in most of the following media, is due to multipath transmission and follows a Rayleigh distribution. Figure 3-15a shows this distribution where 1.253330 corresponds to the average value or 0-db fade. The fading rate is

	MICROWAVE LINES	IONO-SCATTER	METEOR SCATTER	TROPO-SCATTER	H.F. IONOSPHERIC REFLECTION
OPERATING FREQUENCY RANGE	890 - 940 mc 2.45 - 2.5 mc 3.7 - 4.2 mc 5.925 - 6.425 mc 10.7 - 11.7 mc	Minimum 20 mc Maximum 100 mc Nominal 45 mc	Minimum 20 mc Maximum 150 mc Nominal 50 mc	Minimum 40 mc Maximum 10 mc (not clearly established) Nominal 400-6000 mc	Minimum 1 mc Maximum 70-80 mc Nominal 3-30 mc
BANDWIDTH	Minimum 5 mc Maximum 200 mc Nominal 30 mc	Minimum 10 kc Maximum 50 kc Nominal 30 kc	Minimum 10 kc Maximum 10 kc Nominal 1 mc	Minimum 200 kc Maximum 10-20 mc (not clearly established) Nominal 400 kc	Narrow bandwidth. < 4 kc
RANGE PER LINK	Nominal 30 mi.	Minimum 300 mi. Maximum 1500 mi. Nominal 540 mi.	Minimum 240 mi. Maximum 720 mi. Nominal 480 mi.	Minimum 30 mi. Maximum 450 mi. Nominal 60-300 mi.	Minimum 30 mi. Maximum 2500 mi. Nominal 1000 mi.
ATTENUATION	Approximately 140 db between 50 ft. Covers separa- ted by 30 miles may change with weather and locality.	Attenuated 80-100 db more than free space path transmission	See Figure 3-24	See Figure 3-29	See Figure 3-20
FADING	See Figures 3-17 and 3-18	Rayleigh Fading See Figures 3-15a and 3-15b.	Rayleigh Fading See Figures 3-15a and 3-15b	Rayleigh Fading See Figures 3-15a and 3-15b	1/10 to 10 fades per sec. Rayleigh Fading See Figures 3-15a and 3-15b
TIME DELAY DISTORTION FROM 1000 CPS-2500 CPS	200 sec delay between 1000 and 2500.	Depends essentially on the terminal equipment.	Depends essentially on the terminal equipment.	Depends essentially on the terminal equipment.	Depends essentially on the terminal equipment.
NOISE TYPE & RELATIVE POWER	Impulse Noise plus noise of the types in Figure 3-16.	See Figure 3-16	See Figure 3-16	See Figure 3-16	See Figure 3-16

Table 3-1 Parameters of Various Radio Links

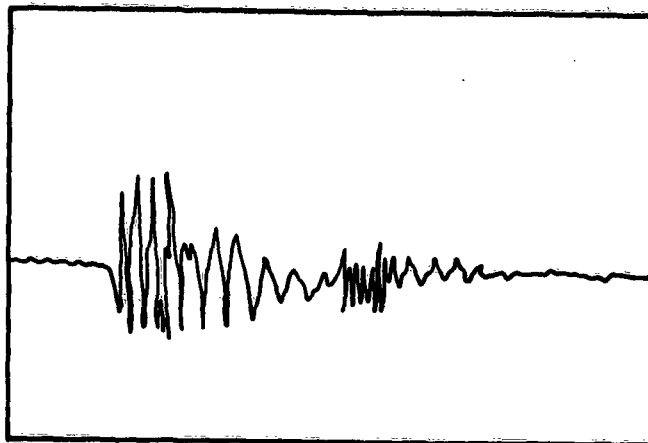


Fig. 3-12 H-44 Pulse (Impulse Noise) (Ref. 3.2)

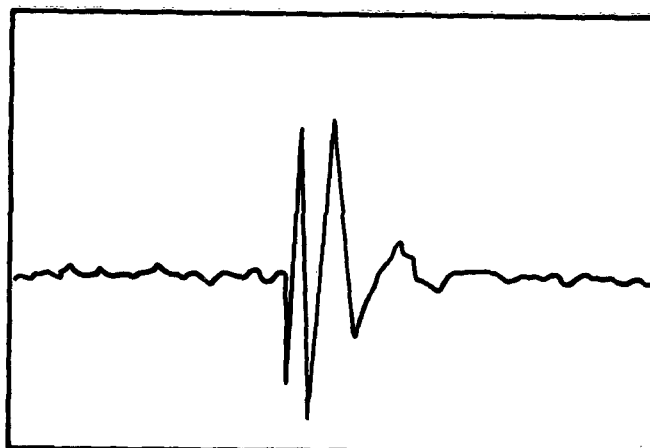


Fig. 3-13 H-44 Pulse (Impulse Noise) (Ref. 3.2)

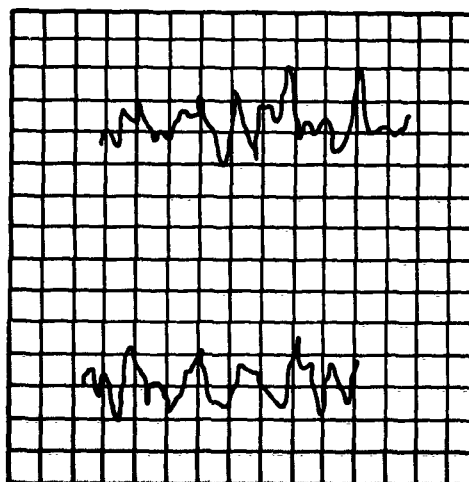


Fig. 3-14 Baseline Noise in Period of Impulse Noise Activity.

usually from 1/10 to 10 fades per second. Figure 3-15b shows the Rayleigh cumulative distribution plus distribution of fading using diversity (more than one means of transmitting the message.)

Noise is another parameter which will be discussed before entering discussion of the various propagation methods. The prevalent source of noise varies with the frequency band of interest. Figure 3-16 gives an idea of what noise is present in the particular band and its amplitude in microvolts per meter.

Noise due to solar storms becomes an important factor in ionospheric communications. When the solar storm activity is at its highest, ionospheric electron density becomes so great that radio waves are absorbed. Blackouts of radio communications from these storms can last from several hours to several days.

3.3.1 Microwave Line-of-Sight Communications

The microwave line-of-sight communication systems are composed of antennas spaced approximately 30 miles apart; microwave is beamed in relay fashion from one antenna to the next. In order to minimize the effect of ground reflections, the systems are placed sufficiently high on towers. The bandwidth capabilities of the microwave links are nominally 30 mc wide and are limited only by reflections. The terminal equipment dictates many of the parameters of operation such as time delay distortion and end-to-end frequencies shift. These values depend greatly on the type of equipment using the propagation media.

Propagation through microwave line-of-sight communication is subject to fades. Fading in this case is due mainly to weather conditions bending the beam between links. Fading may also be caused by multiple-patch interference. Figures 3-17 and 3-18 give the distribution of fades during different times of the year for two different paths.

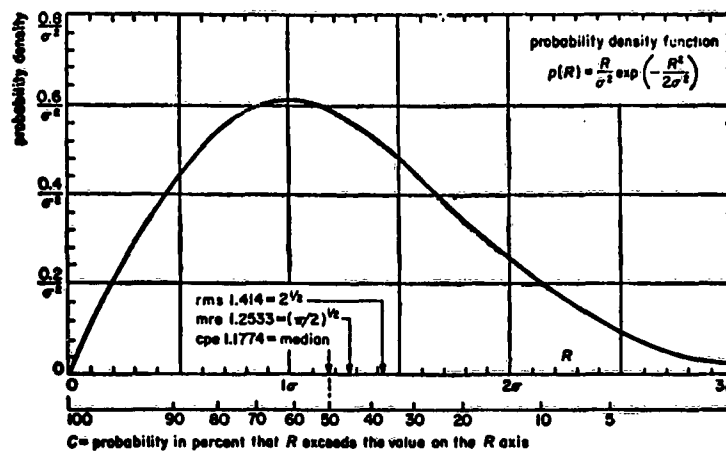


Fig. 3-15a Rayleigh Distribution. R is the Distance to the Origin in a Bivariate Normal Distribution. σ is the Standard Deviation for Either Component of the Normal Distribution (Ref. 3.4)

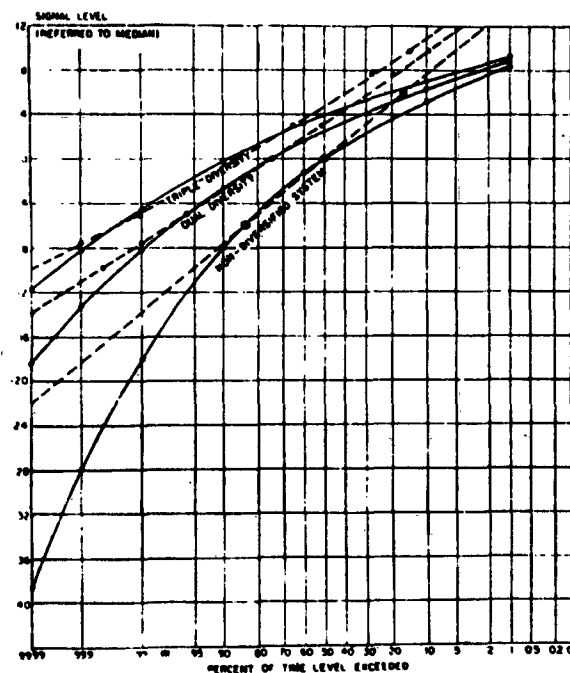


Fig. 3-15b Signal Level Distribution vs Order of Diversity (Ref. 3.5)

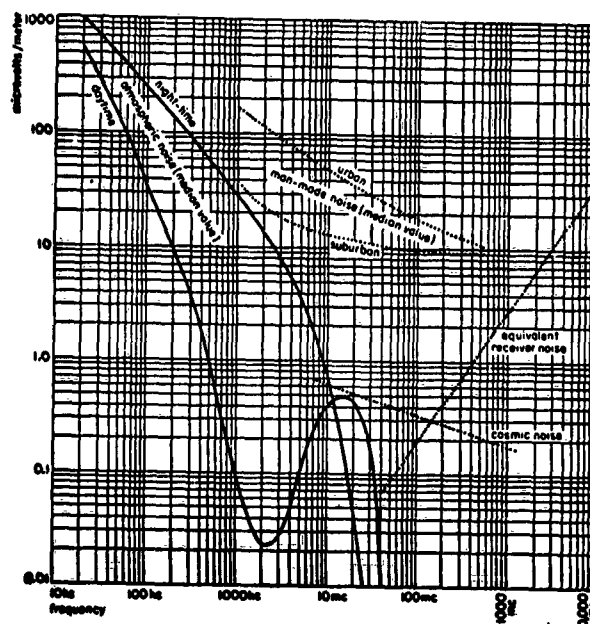


Fig. 3-16 Major Sources of Radio Noise (Ref. 3.4)

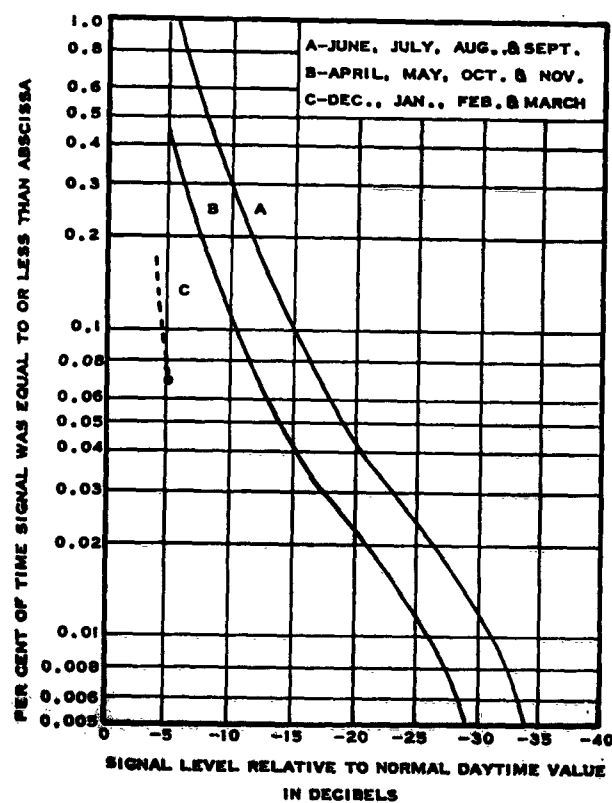


Fig. 3-17 Time Distribution Curves of the Signal Levels Observed on the Southard Hill-Crawford Hill Path. Data of 1947, 1948 and 1950, (Ref. 3.6)

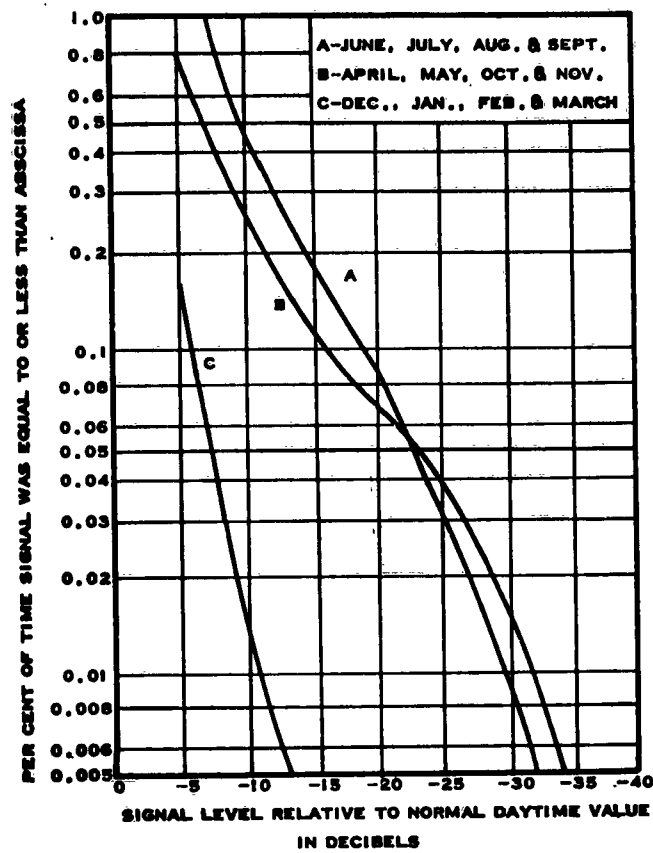


Fig. 3-18 Time Distribution Curves of the Signal Levels Observed on the Southard Hill-Crawford Hill Path. Data of 1947, 1948, 1949 and 1950, (Ref. 3.6)

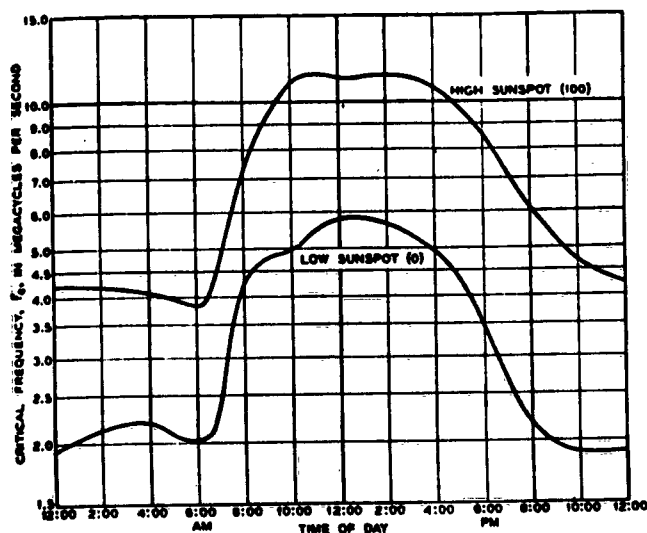


Fig. 3-19 Typical Diurnal Variations of Critical Frequency for January at Latitude 40 Degrees (Ref. 3.7)

3.3.2 Ionospheric Reflection

Radio communication below 100 mc and above 2 mc can be accomplished through reflection from the ionosphere. The limitations on the upper frequency depend on the angle at which the wave enters the particular layer being used and the critical frequency of that layer; i.e., $f_c / \sin \alpha$ where α is that angle and f_c is the critical frequency. The critical frequency is that frequency where no energy is reflected from the ionosphere above. Figure 3-19 gives typical values of critical frequency (Washington, D.C.) showing diurnal fluctuation.

Transmissions through this medium may have ranges on the order of 1000 miles or more. Very narrow bandwidths (one speech channel or less) are attainable. The fast fading follows the Rayleigh distribution, since fading in these circuits is essentially of multi-path origin. Echo delays of up to several milliseconds have been observed. These produce considerable distortion in the communication channel.

In general, the path loss may be attributed to absorption and free path loss. Due to the uncertainty of the path, additional losses may occur. Figure 3-20 gives the absorption loss as a function of frequency.

Communications through this means are at best undependable, due to the continuous variation of the several layers.

3.3.3 Ionospheric Scatter

The reflection of radio waves from many areas of ionized E layer is the next medium of interest. Ionospheric scatter requires high transmitter powers because the signal is attenuated from 80 to 100 db more than line-of-sight transmission through free space. Figure 3-21 shows the median signal levels for ionospheric-scatter transmission.

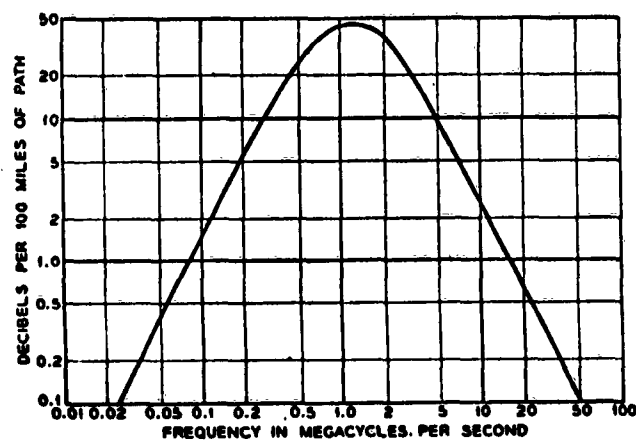


Fig. 3-20 Typical Values of Midday Ionospheric Absorption (Ref. 3.7)

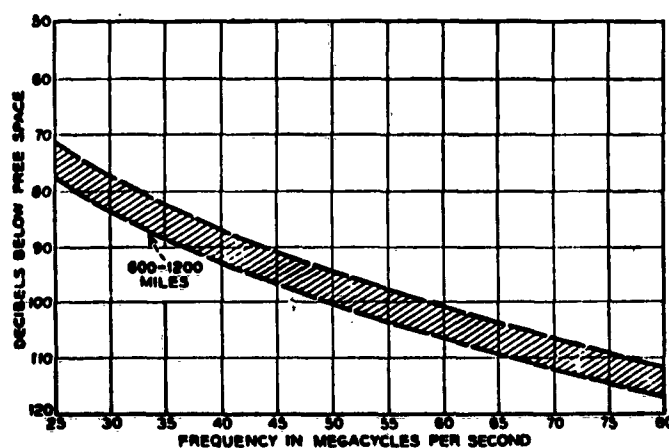


Fig. 3-21 Median Signal Levels for Ionospheric Scatter Transmission (Ref. 3.7)

The ranges of transmission through this medium are typically less than 1000 miles. The bandwidths available are on the order of 30 kc. Fast fading is again of the Rayleigh type since multipath is the major contribution. Slower fading of the hourly median varies approximately normally, with a standard deviation of 6 to 8 db.

The ionospheric-scatter propagation, although it has a much greater attenuation and more selective receivers are necessary, can be considered to be much more reliable than the reflection mode previously mentioned.

3.3.4 Meteor-Scatter Propagation

The ionization tail of meteors as they sweep through the upper atmosphere reaches sufficient size and electron density to deflect an appreciable amount of radio energy. The density of the electrons has a definite effect on the output signal strength enhancement. This is shown in Fig. 3-22. The frequency of operation of this type of propagation is again in the 50 mc range. A nominal channel characteristic bandwidth may be 1 mc. The range of a typical system may be on the order of 500 miles, depending on the scatter angle. Figure 3-23 gives signal energy as a function of scatter angle. Transmission loss as a function of range is given in Fig. 3-24.

Fast fading is again of the Rayleigh type, although, as previously mentioned, blackout due to solar storms account for long term fades. The operation of meteor-scatter propagation is usually accomplished in bursts whenever the received signal intensity is sufficient to ensure signal reception. If it is operated in this manner, very reliable communications can be obtained. This technique, of course, reduces the usable data rate of the system.

3.3.5 Tropospheric-Scatter Propagation

Tropospheric scatter is not dependent on the ionosphere and, therefore, is not completely subject to the blackouts due to solar

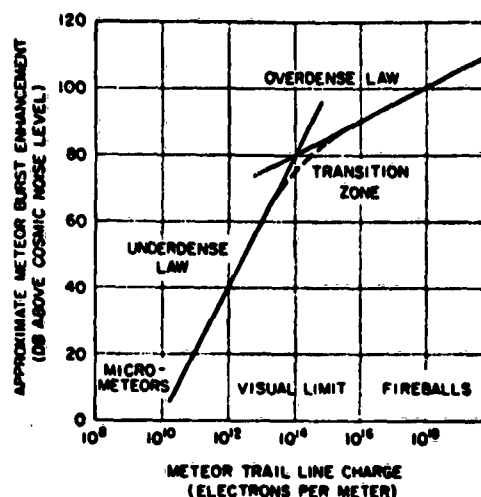


Fig. 3-22 Effect of Overdense and Underdense Trails on Received Signal Strength Enhancement (Ref. 3.3)

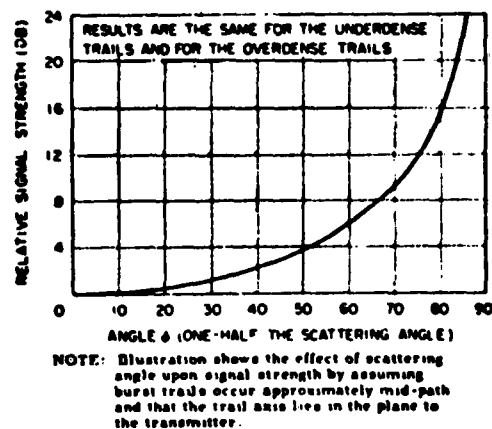


Fig. 3-23 Effect of ϕ on Received Power (Ref. 3.3)

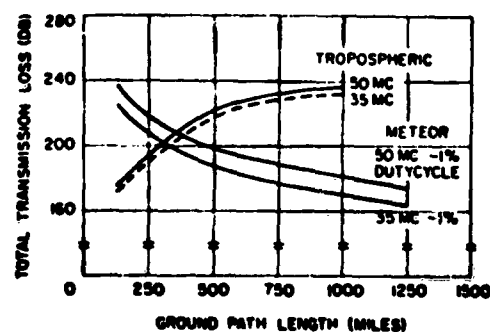


Fig. 3-24 Transmission Loss as a Function of Range and Duty Cycle (Ref. 3.3)

origin. The scatter from the tropospheric region is generally attributed to variations in mean tropospheric refractive index. This change may be due to water vapor variation caused by turbulent convection currents. The range of tropospheric scatter is usually less than 800 miles. Figure 3-25 gives path loss as a function of range for two different operating frequencies. The operating frequency generally varies from 40 mc to 5 kmc.

Fast fading again follows a Rayleigh distribution while slow fading is thought to have a log normal distribution. Figure 3-26 gives average fading rates as a function of percentage of time the rate is exceeded for two operating frequencies. Propagation through this medium becomes relatively variable with seasons. Path loss appears to reach a maximum during the summer months (See Fig. 3-27.) Figure 3-28 gives an idea of the capabilities for communication in this medium.

3.3.6 Satellite Communications

Satellite communications must pass through the various levels of the Earth's atmosphere. The troposphere, stratosphere, and ionosphere will have an effect on the communication via an earth satellite. Depending on the frequency of operation, various problems associated with these different levels will arise. Figure 3-29 shows that noise density reaches a minimum in the range of 500 mc to 10 mc. Satellite communication seems to be inherently limited to this range by many factors. Radio signals below the lower range are reflected back to earth and, therefore, limited from use in satellite communications. Radio signal above the higher range are attenuated by water vapor absorption as shown in Fig. 3-30. Figures 3-31 through 3-33 gives curves of optimum frequency of operation for gain limited, area limited, and mixed antennas. The points of optimum frequency again seem to be in this range.

Operating in this range of frequency, factors such as fading have little effect due to lack of multipath. The distance of communication

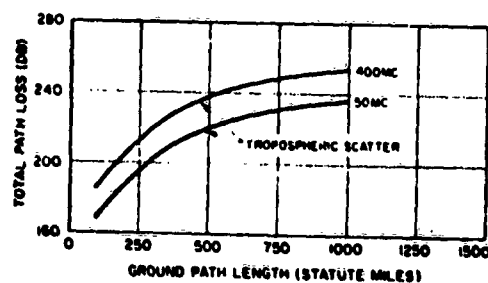


Fig. 3-25 Tropospheric Path Loss (Ref. 3.3)

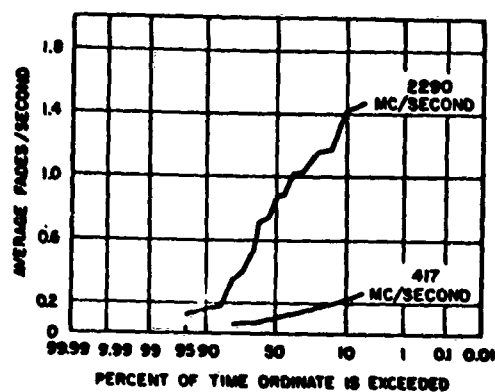


Fig. 3-26 Observed Fading Rate at 417 MC and 2290 MC Using Equal 5° Beam Antennas (Ref. 3.3)

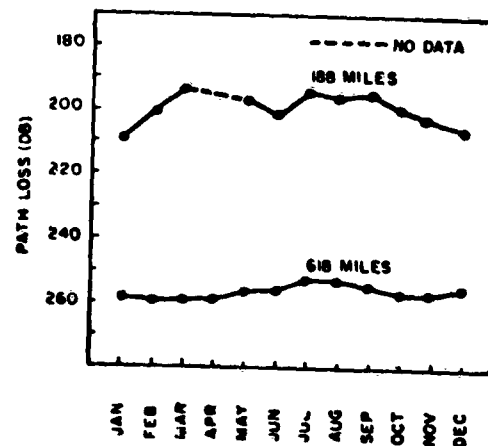


Fig. 3-27 Seasonal Variation of 400-MC Data (Ref. 3.3)

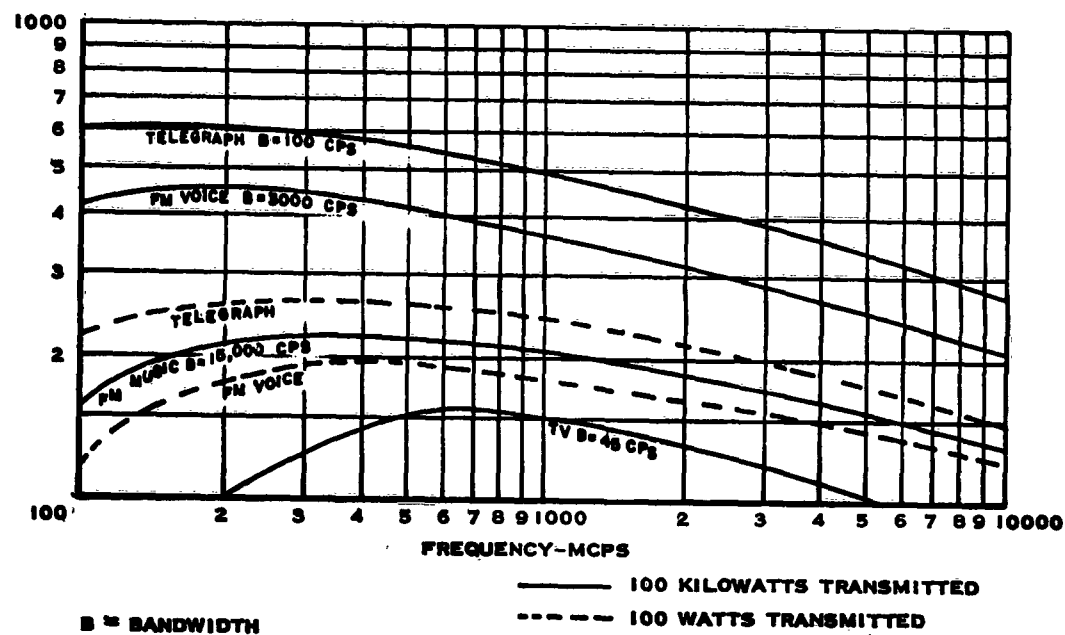


Fig. 3-28 Distance at Which Satisfactory Service of the Types Indicated May be Provided for 99% of the Hours (Ref. 3.8)

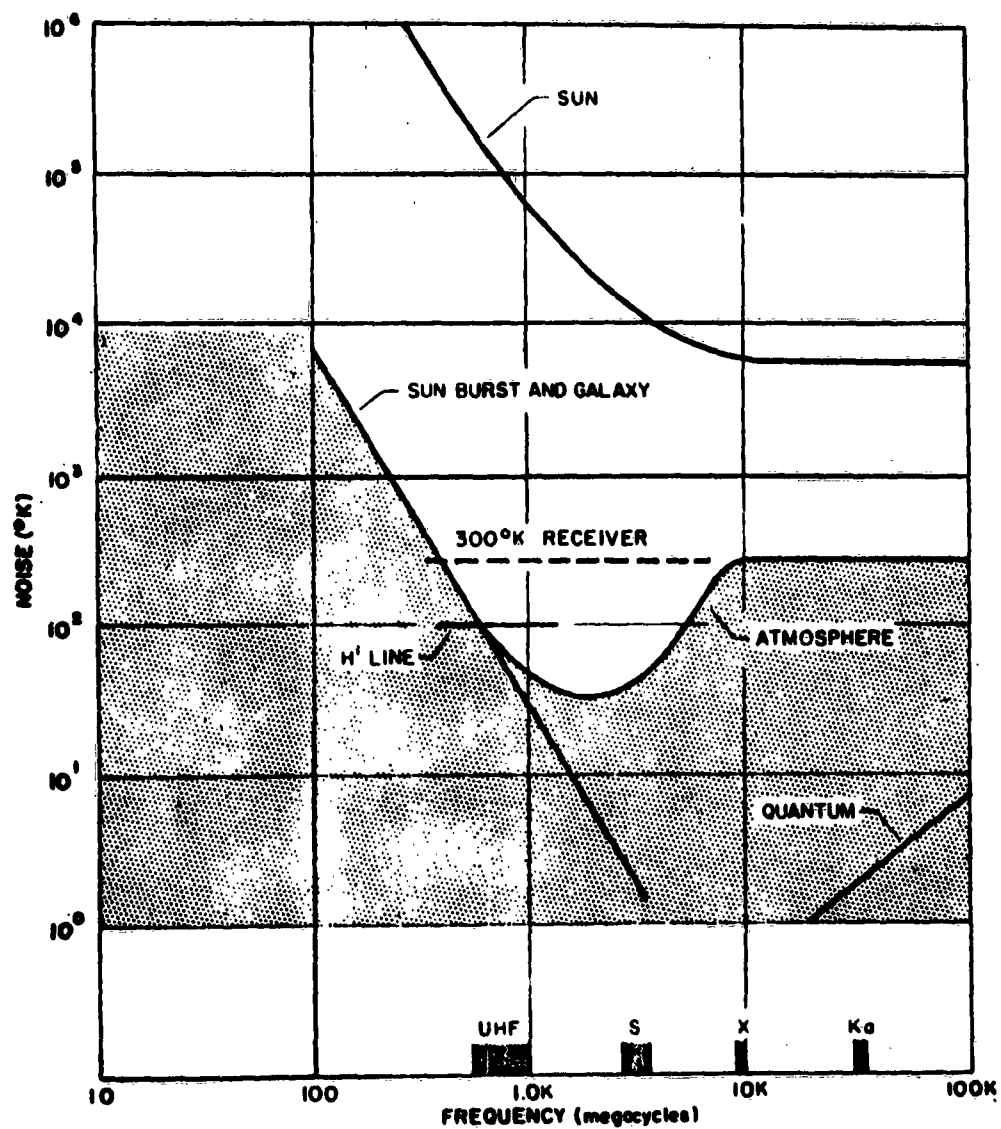


Fig. 3-29 Antenna Background Noise (Ref. 3.9)

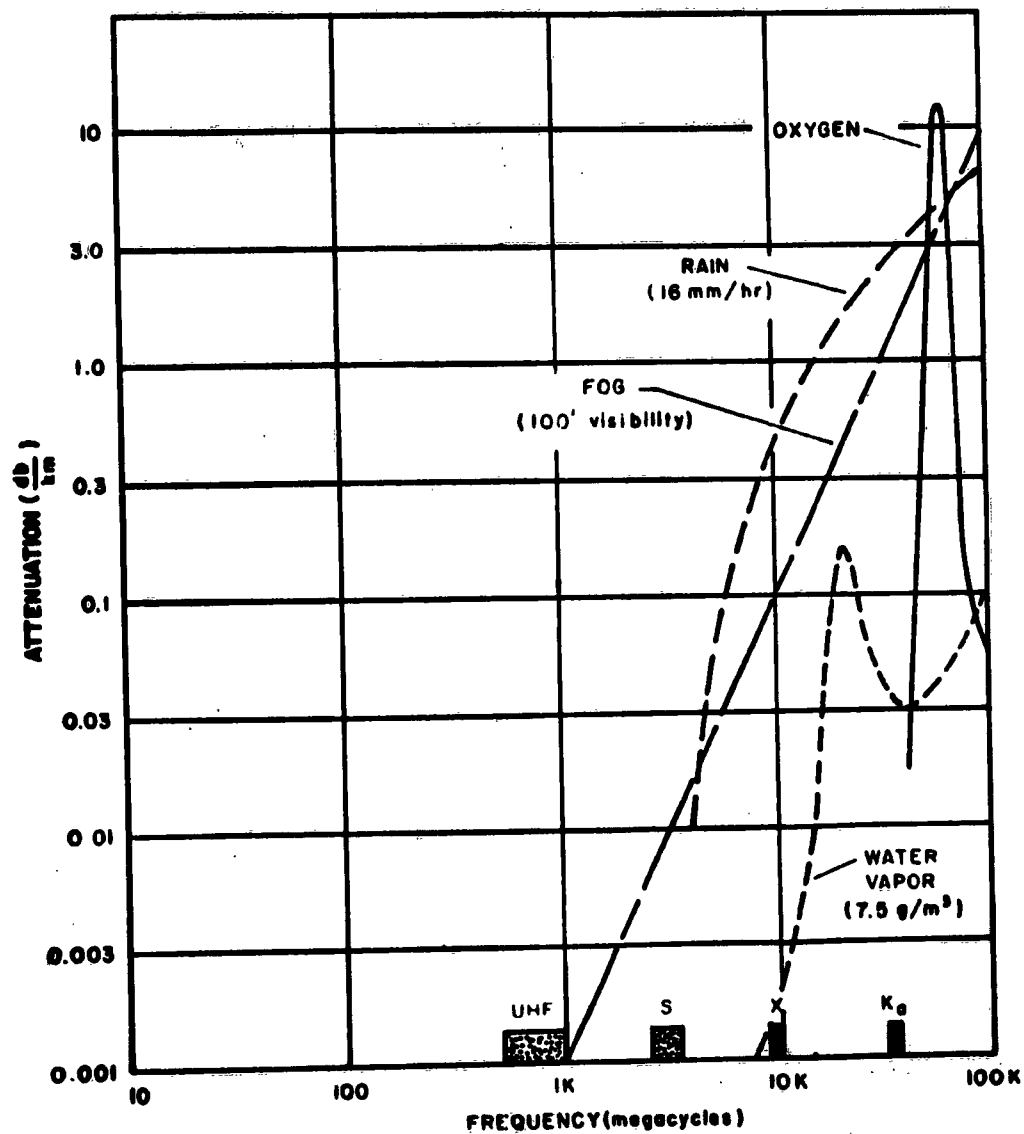


Fig. 3-30 Atmosphere Attenuation (Ref. 3-9)

Δf : 1.0 mc
 P : 10^3
 R : 4×10^6 km
 G_t : 4.5×10^4 (30° BEAM)
 G_r : 4.5×10^4 (1° BEAM)
 — : ATMOSPHERE (80 km, 100' FOG)
 ---- : FREE SPACE

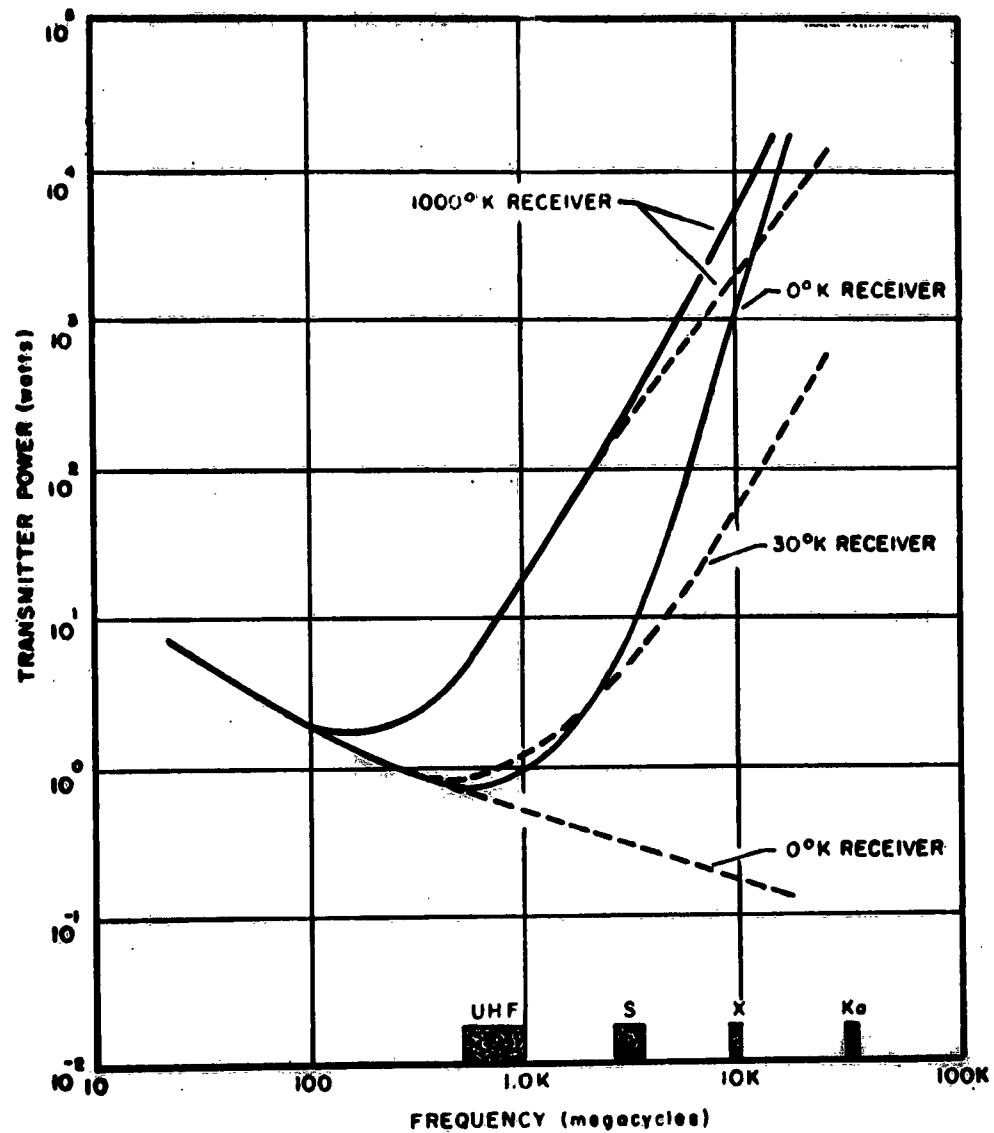


Fig. 3-31 Optimum Frequency for Minimum Transmitter Power-Gain Limited Antennas (Ref. 3.9)

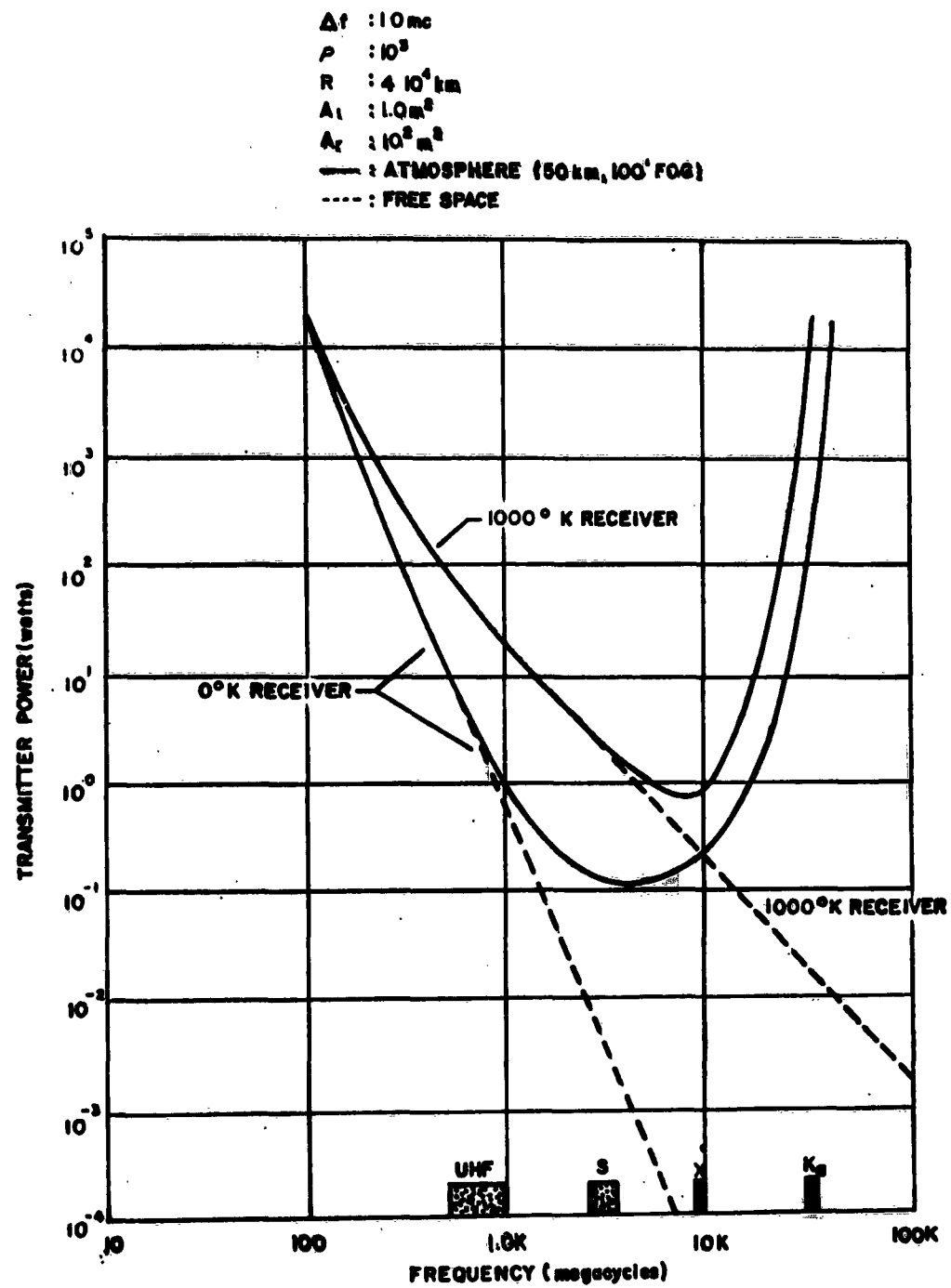


Fig. 3-32 Optimum Frequency for Minimum Transmitter Power-Area Limited Antennas (Ref. 3.9)

Δf : 1.0 mc
 P : 10^3
 R : $4 \cdot 10^4$ km
 G_1 : 1
 A_r : 10^2 m²
 — : ATMOSPHERE (50 km, 100' FOG)
 --- : FREE SPACE

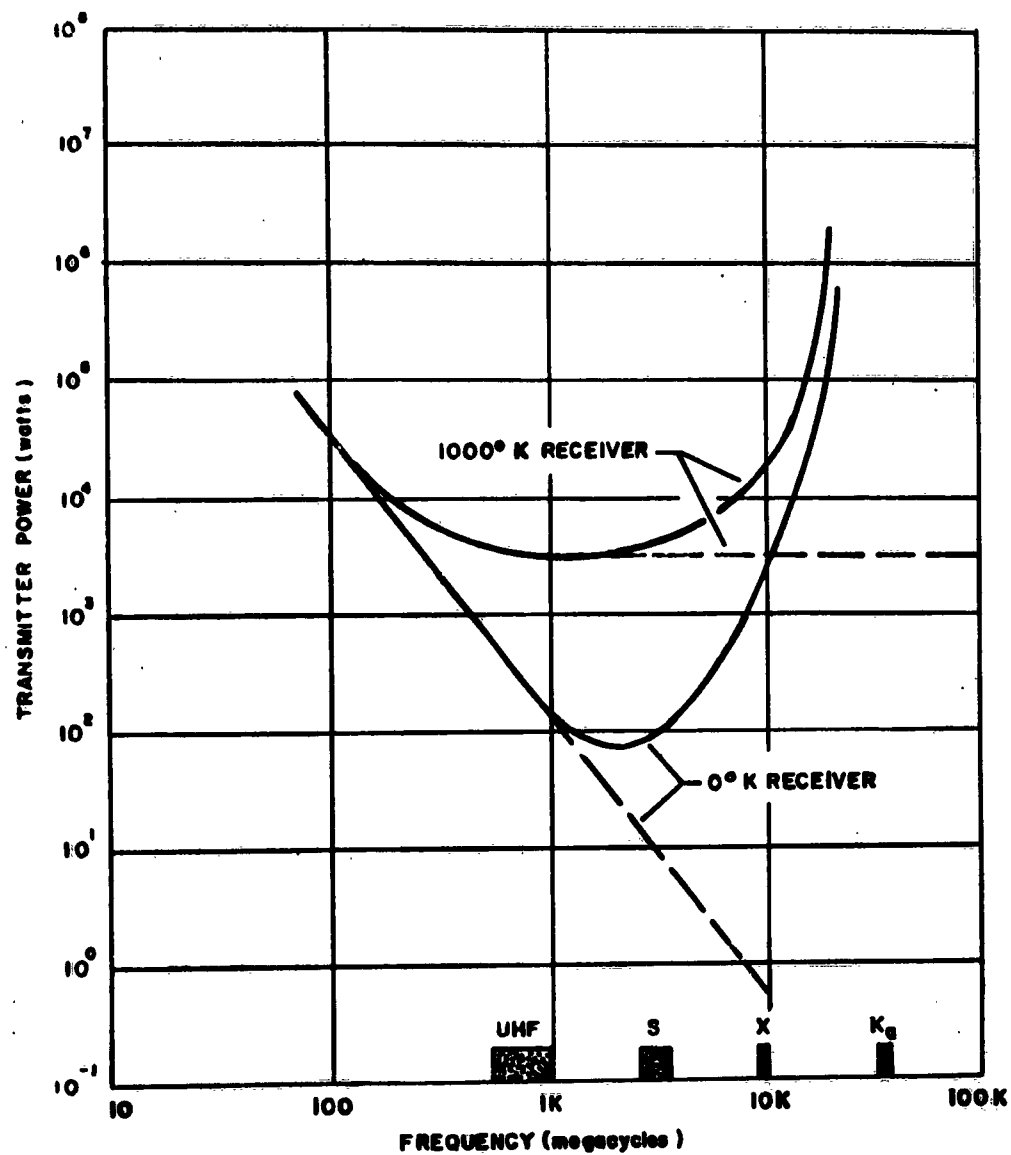


Fig. 3-33 Optimum Frequency for Minimum Transmitter Power-Mixed Limited Antennas (Ref. 3.9)

will depend greatly on the type and complexity of the satellite system involved. The attenuation will drop off as free-space radiation and, therefore, the sensitivity of receivers need not be exceptionally great.

REFERENCES

- 3.1 Alexander, Gryb & Nast, "Capabilities of the Telephone Network for Data Transmission," The Bell System Tech. Journal, pp. 431-476; May 1960.
- 3.2 Howard L. Yudkin, "Some Results in the Measurement of Impulse Noise on Several Telephone Circuits," Lincoln Lab. MIT, ASTIA AD247887.
- 3.3 Hughes Aircraft Co., Communications Division, "Survey of Digital Communications", TR-1; 15 April 1961. ASTIA AD277176.
- 3.4 IT & T Corp., Reference Data for Radio Engineers, Fourth Ed.; 1957.
- 3.5 C. L. Mack, "Diversity Reception in UHF Long Range Communications," Proc. IRE, p. 1281; October 1955.
- 3.6 A. B. Crawford and W. C. Jakes, "Selective Fading of Microwave," Bell System Tech Journal; January 1952.
- 3.7 Kenneth Bullington, "Radio Propagation Fundamentals, "The Bell System Tech Journal; p. 593; May 1957.
- 3.8 Westinghouse Electric Corp., "Tropospheric Scatter at Elevation," Tech. Note 1; May 20, 1962.
- 3.9 C. T. McCoy, Philco Corporation Research Division, Report 279 Space Communications; 10 January 1959.

SECTION 4

COMPARISON OF DIGITAL DATA TRANSMISSION EQUIPMENT

4.1 CRITERIA

This section compares the digital data transmission equipment described in Section 2 and a method is suggested for selecting a system to meet a user's particular requirements.

The criteria used for comparison of the systems are:

1. Maximum Data Transmission Rate. This is the actual maximum rate of transmission of the input data over a "standard" telephone voice channel, and is not always as great as the given bit transmission rate (due to coding or redundancy).
2. Reliability. An indication of reliability can be obtained by examining the complexity of the system, reliability calculations, experimental results, and actual field use. Only limited information from a few of these sources has been available.
3. Maintainability. An indication of maintainability can be obtained by examining the complexity of the system, the construction and layout (e.g., use of modular construction, adequate test points, built-in test provisions), and test and maintenance procedures developed for the equipment.
4. Availability and Cost. Some of the equipment considered in Section 2 is still experimental and/or developmental in nature and is not available "off-the-shelf." The AT&T Dataphone and A-1 systems are provided on a rental basis directly by the telephone facility. Since these factors vary so much with time, and depend on the optional features included, no merit weighting is given. A user should contact the manufacturer

or his representatives to obtain current information when it is needed. In general, the cost of a given system will increase with increasing equipment complexity and amount of optional equipment required by the user.

5. Error Performance. This is a complex criterion since it is a function of many variables. These functional relationships will be discussed. Experimental data gives the best indication of error performance, but such information is not available for all equipment or under all conditions. Theoretical results can give some comparisons and will indicate bounds on best obtainable performance. The factors that are important in determining error performance include:
 - a. Transmission Medium. Transmission media are discussed in Section 3. Contributing factors include amplitude distortion, envelope delay, fading, frequency displacement, multipath jitter, white and gaussian noise, and impulse (burst) noise.
 - b. Modulation and Detection Techniques. Each modulation and demodulation technique has its advantages and disadvantages with respect to error performance, simplicity, reliability, and cost. A technique good for one set of conditions may be poor for another set.
 - c. Signal-to-Noise Ratio. Signal-to-noise ratio is a function of the transmitted signal power, and of factors a and b above. However, error rate is a function of signal-to-noise ratio; therefore, to have a meaningful error rate stated for a system, the corresponding signal-to-noise ratio and transmission medium characteristics must also be stated.
 - d. Transmission Rate. Error rate is also a function of the transmission rate, generally an increasing function. Thus, it is necessary that the transmission rate (bits per second) also be given when an error rate is stated.
 - e. Coding. Each of the systems considered will accept coded binary data. This coding can be used to decrease the data error rate by providing redundancy or through error detection and correction (see Section 5). Some of the systems code the data into a particular format before transmission.

This may be done to increase the allowable data transmission rate within a given bandwidth (as with Lenkurt "Duobinary" coding) or to facilitate retrieval of synchronization from the data (as with COBI and CTDS). Limited error detection is also provided by "Duobinary" coding with the addition of available receiving terminal equipment.

Error rates can be minimized by matching a system to its operating conditions (especially the transmission medium it must work into) and/or by using coding. Thus, one would use the modulation and demodulation techniques least susceptible to the existing media effects (such as fading, or impulse noise), and use techniques such as line equalization, diversity reception, and/or coding when the improvement is worth the cost.

The above listed factors are not in a useful form for purposes of system comparison. A set of criteria is needed which can be used to compare each system according to the relative degree to which each criterion adversely affects the error performance. The criteria to be used are:

1. Amplitude distortion
2. Envelope delay
3. Fading
4. Frequency displacement
5. Multipath jitter (phase fluctuations)
6. Gaussian and white noise
7. Impulse (burst) noise
8. Other considerations (these will be comments only, and will not be weighted).

Both experimental and theoretical evaluations of the equipment are used if available; unfortunately, much of the information was available only from manufacturers' literature. When little information was available, the rating spread was made narrow to compensate for the incompleteness.

The comparisons are summarized in Table 4-1. In addition, each type of equipment is assigned a weighting (merit) number within each comparison category. These numbers, 1 through 9, indicate a judgment of the particular system relative to the other systems considered. The higher the number the greater the relative merit of the system within the particular category. Data rate will not be assigned a weighting, since it either meets or does not meet user requirements.

These weighting numbers can be used to guide the selection of a system for a particular use. A reasonable method would be to weight each of the comparison categories according to the importance of that category in the potential user's particular situation. This weighting number is then multiplied times each merit number in the table. The sum of these new numbers for a particular system can then be taken as a "figure-of-merit" for the system.

4.2 COMPARISON

The evaluation and comparison of systems according to the criteria presented in the preceding paragraph is recognized to be partly subjective. The information summarized in Table 4-1 will help, however, to indicate the reasons for the weightings given. The numbers in the boxes are the merit weightings, given as discussed in Paragraph 4.1. More complete and/or up-to-date information may alter these weightings somewhat.

A major difference between systems, and a factor in determining error performance, is the modulation technique. A brief discussion and comparison of the effects of the modulation technique on error performance is given in the following discussion (Refs. 4.1 and 4.2). See Section 3 of this report for further comments about the media effects.

CRITERIA	User's weighting of criteria	RIXON	LENKURT	ROBERTSHAW-FULTON	STROMBERG CARLSON	AT
		"SEBIT" 24B	"DUOBINARY" 26B	PHASE "MULTILOK"	MODEL S-C-301	"DATA"
Maximum Data Transmission Rate		2400 bits/sec	2400 bits/sec	3000 b/s (synch) / 1500 b/s (non-synch)	2400 bits/sec	1200 bits/sec
Reliability		6 Solid-state construction using Mil Spec or approved equivalent components throughout. Manufacturer's proven mean-time-between failure record in excess of 7500 hours. Binary simplicity.	6 Solid-state construction. High-quality Mil and industrial components. Binary simplicity.	4 Tube and solid-state construction. Built to high quality commercial standards. 3 and 5 phase levels, rather than binary.	5 Solid-state construction. Binary	5 Solid-state construction. Binary
Maintainability		6 Replaceable sub-assemblies, printed wiring, plug-in connectors. Simple check-out procedure, integral scope. 300 internal test jacks many with normal oscilloscope patterns provided. Front panel adjustment of equalizer, with scope indication. Random test data signal generator and complete performance monitor included.	6 Plug-in modular packaging. Simplicity of alignment and servicing due to simple functioning basic modules. Built-in pattern generator for complete system alignment. Oscilloscope for equalizer adjustment.	5 Plug-in printed circuit cards.	5 Plug-in printed circuit modules.	5 Maintained by terminal equip
Availability and Cost ²		Operational. Available off-the-shelf.	In production. Available off-the-shelf.	Experimental.	Experimental.	Available on carrier. Open
Error Performance ³	Amplitude Distortion	5 Equalization required	6 Equalization required.	6/6 Equalization required.	5 Equalization required.	6 Equalization data rates.
	Envelope Delay	5 Equalization required.	5 Equalization required.	5 Equalization required.	5 Equalization required.	5 Equalization data rates.
	Fading ⁵	5 Use "Seppath".	5 Use HF2X	5	4	Data phone wire line trans
	Frequency Displacement	6	5	5	6	5
	Multipath Phase Jitter ⁶	5 Use "Seppath".	5 Use HF2X		5	Data phone wire line trans
	Gaussian and White Noise	4	4	3/4	4	5
	Impulse (Burst) Noise	4	4	3/4	3	5
	Other Considerations ⁴		Very wide dynamic range.			
Comments ⁴		Considerable operating experience	Developed from systems having considerable operating experience.			Considerable
System Figure of Merit ⁵						

1 Determined by user's requirements

2 This information should be obtained or verified at its time of use, so it is not weighted.

3 See the general comparison in Section 4.2.

4 See also the Remarks in Table 2.1.

5 Found for particular user requirements as explained in Sections 4.1 and 4.3.

6 Radio channel effects. Combat by using frequency multiplexed channels and diversity reception.



ROBERTSHAW-FULTON PHASE "MULTILOK"	STROMBERG CARLSON MODEL S-C-301	AT & T "DATA PHONE"	HALLICRAFTERS CTDS-2400/M2C	MIT LINCOLN LABORATORY "COSI"	AT & T (BELL) A-1 (DDT and DDR)	COLLINS "KIMEPLEX" AM/GCS-4
0 b/s (synch) / 1500 b/s (non-synch)	2400 bits/sec	1200 bits/sec	1300 bits/sec	1200 bits/sec	1600 bits/sec	5400 bits/sec
Tube and solid-state construction. It is to high quality commercial standards. 3 and 5 phase levels, better than binary.	5 Solid-state construction. Binary	5 Solid-state construction, simple terminal equipment. Binary system	5 Solid-state construction. Binary simplicity.	5 Solid-state construction. Binary simplicity.	4 Tube construction. Binary simplicity.	3 Solid-state construction. 5 phase levels. Manufacturer specifies: 1000 hours mean-time-between failures, 15 minutes mean-time-to-repair.
Plug-in printed circuit cards.	5 Plug-in printed circuit modules.	5 Maintained by commercial carrier	5 Modular construction. Printed wiring cards.	5 Modular constructions. Printed wiring cards.	5 Maintained by commercial carrier	5 Drawer-mounted plug-in modular circuit cards. Self-contained integral test facility. Manufacturer states less than 5 hours per year preventive maintenance for continuous operation.
Experimental.	Experimental.	Available on lease from common carrier. Operational.	Operational.	Experimental.	Available on lease from common carrier. Operational.	Operational.
6 Equalization required.	5 Equalization required.	6 Equalization required for high data rates.	6 Equalization may be required.	6	5 Equalization may be required.	6 Equalization may be required.
Equalization required.	5 Equalization required.	5 Equalization required for high data rates.	5 Equalization may be required.	5	5 Equalization may be required.	5 Equalization required.
	4	Data phone available only for wire line transmission.	6	6	A-1 available only for wire line transmission.	5
	6	5	5	5	6	4
	5	Data phone available only for wire line transmission.	5	5	A-1 available only for wire line transmission.	5
4	4	5	6	6	4	3
4	3	5	5	5	3	3
		Considerable operating experience				Narrow dynamic range.

Comments as explained in Sections
using frequency multiplexed

2

TABLE 4-1 COMPARISON OF DIGITAL DATA TR

<u>PARSONS</u> S-C-301	<u>AT & T</u> "DATA PHONE"	<u>HALLICRAFTERS</u> CTDS-2400/M2C	<u>MIT LINCOLN LABORATORY</u> "COB"	<u>AT & T (WELL)</u> A-1 (DDT and DDR)	<u>COLLINS</u> "KINEPLEX" AM/GCS-4	<u>COLLINS</u> "KINEPLEX" TR-2100-2
	1200 bits/sec	1300 bits/sec	1200 bits/sec	1600 bits/sec	5400 bits/sec	2400 bits/sec
struction.	5 Solid-state construction, simple terminal equipment. Binary system	5 Solid-state construction. Binary simplicity.	5 Solid-state construction. Binary simplicity.	4 Tube construction. Binary simplicity.	3 Solid-state construction. 8 phase levels. Manufacturer specifies: 1000 hours mean-time-between failures, 15 minutes mean-time-to-repair.	4 Solid-state construction. 4 phase levels. Manufacturer specifies: greater than 3000 hours mean-time-between failures, less than 15 minutes mean-time-to-repair.
d circuit	5 Maintained by commercial carrier	5 Modular construction. Printed wiring cards.	5 Modular constructions. Printed wiring cards.	5 Maintained by commercial carrier	5 Drawer-mounted plug-in modular circuit cards. Self-contained integral test facility. Manufacturer states less than 5 hours per month preventive maintenance for continuous operation.	6 Drawer-mounted plug-in etched wire circuit cards. Self-contained integral test facility. Manufacturer states less than 5 hours per month preventive maintenance for continuous operation.
	Available on lease from common carrier. Operational.	Operational.	Experimental.	Available on lease from common carrier. Operational.	Operational.	Operational.
quired.	6 Equalization required for high data rates.	6 Equalization may be required.	6	5 Equalization may be required.	6 Equalization may be required	6 Equalization may be required.
quired.	5 Equalization required for high data rates.	5 Equalization may be required.	5	5 Equalization may be required.	5 Equalization required.	5 Equalization required.
	Data phone available only for wire line transmission.	6	6	A-1 available only for wire line transmission.	5	5
	5	5	5	6	4	5
	Data phone available only for wire line transmission.	5	5	A-1 available only for wire line transmission.	5	5
	5	6	6	4	3	4
	5	5	5	3	3	4
					Narrow dynamic range.	Narrow dynamic range.
	Considerable operating experience					Considerable operating experience on similar TR-206.

TABLE 4-1 COMPARISON OF DIGITAL DATA TRANSMISSION EQUIPMENT

An obvious comparison is that phase and frequency modulation techniques are less susceptible to sudden and short term amplitude variation effects than amplitude modulation. Phase modulation techniques usually use coherent or differentially coherent detection and its corresponding improvement in error performance, at the expense of increased complexity. The complexity increases with the number of phase levels. Curves of error probability as a function of signal-to-noise ratio are plotted in Fig. 4-1 for Gaussian noise, and Fig. 4-2 for impulse noise, for various transmission systems. These curves indicate superiority of phase modulation for error performance.

Of the various devices used within the telephone system, only companders present a serious problem to digital data transmission (Ref. 4.1). FSK and PSK systems are relatively immune to the effects of these devices. But AM and multi-tone systems evidencing amplitude variations in the envelope of the composite signal activate the companders, resulting in distortion of the data signal. Keyed carrier (AM) systems, on the other hand, are relatively insensitive to frequency translation in the channel, while FSK and PSK systems are adversely affected by this characteristic, common to carrier facilities in the telephone plant.

Many sources conclude that phase modulation is technically the optimum solution for telephone circuit data transmission problems. The main reasons for this are that (Ref. 4.1 and 4.2) in phase modulation, all available power is utilized for intelligence conveyance, the signal is not as adversely effected by companders in the circuit, and it trades data speed for signal-to-noise ratio. Following phase modulation (PSK and differentially coherent PSK), are FSK and keyed carrier (AM), in that order. Other factors, as this report shows, must be considered in choosing a system and making comparisons. In general, the higher performance system requires a more complex equipment installation and involves greater maintenance costs, while the lower performance systems are simpler to implement, smaller in size, and probably less costly to maintain.

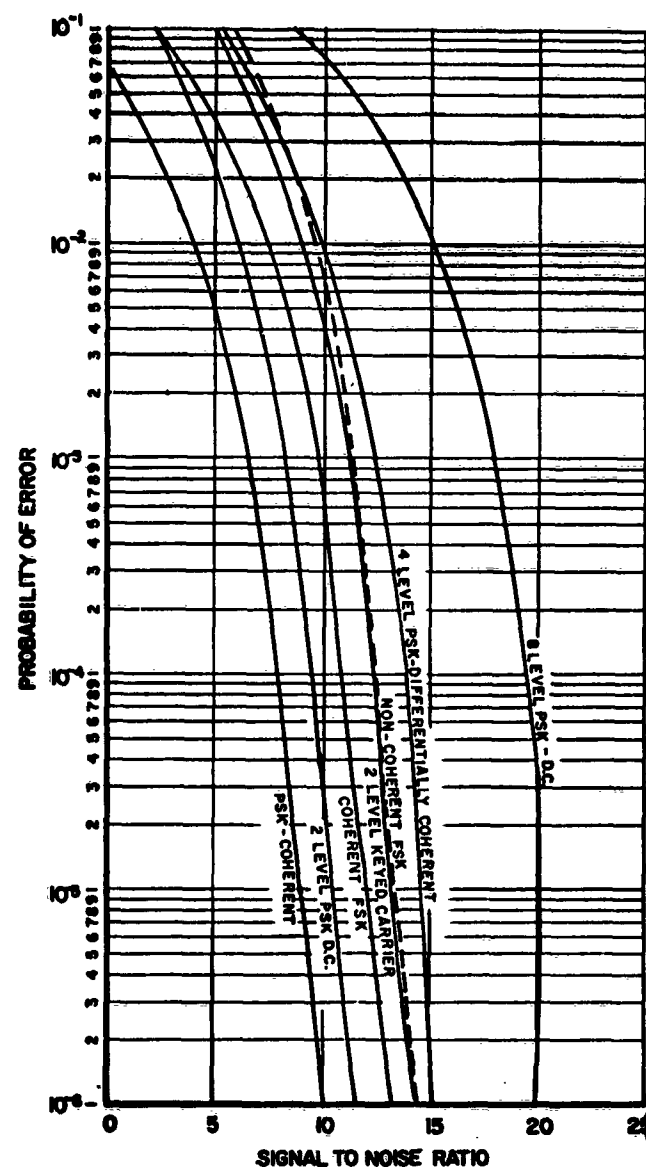


FIG. 4-1 Comparison of Probability of Error Due to Gaussian Noise for Various Transmission Systems

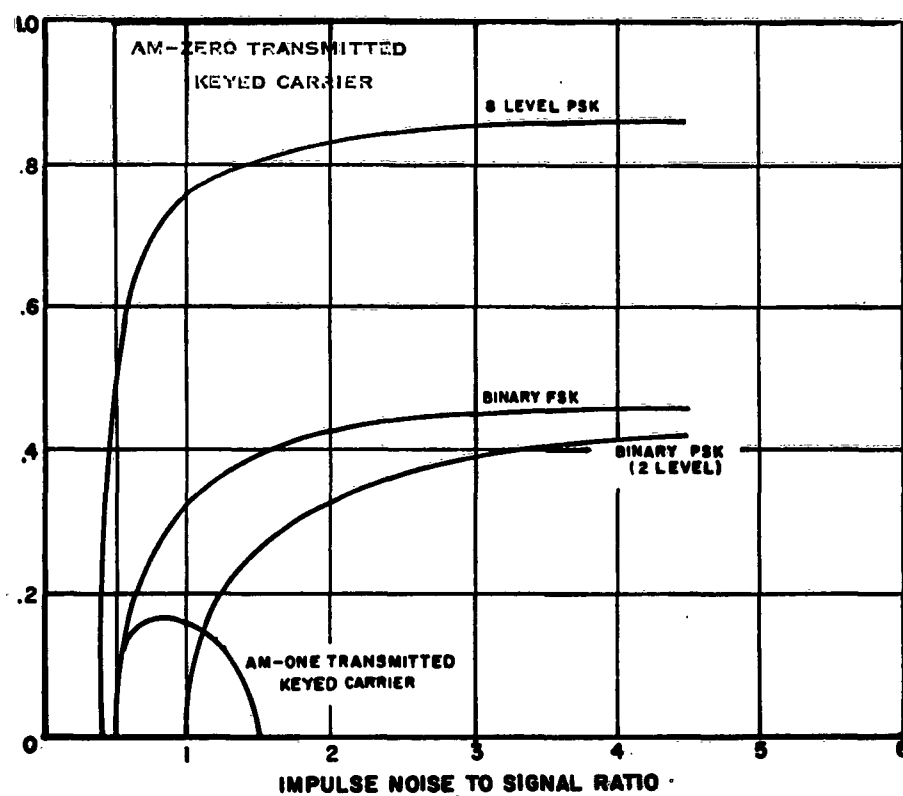


Fig. 4-2 Comparison of Probability of Error Due to Impulse Noise for Various Transmission Systems

Propagation factors in radio transmission, such as HF radio transmission, make it impossible to use a high order of time-division multiplexing, ruling out shorter pulses and wide bandwidths. A receiver with large dynamic range is needed due to frequency selective fading of as much as 30 to 40 db encountered on HF links. Diversity receiving techniques are used to provide improved (more reliable) reception in the presence of fading.

4.2.1 Experimental Error Statistics

A few papers presenting experimental error statistics and comparisons are available at the present time. Some of the conclusions made and results given are summarized below. Hofmann concludes, from tests on the Bell A-1 facility over various media (Ref. 4.4), that: (1) digital errors are of a burst nature, both within a word and over many words; consequently, averaging error performance over long periods is a poor means of describing the performance of a data system; (2) background noise on the telephone circuits is of little consequence in affecting error rate since a degradation in signal-to-noise ratio does not materially change the error rate; (3) the error rate is not fixed--it may vary over many magnitudes on a circuit over different periods of time; (4) the longer telephone circuit used shows a higher preponderance of high error rate days than does the short circuit used, though the error rate is not a direct function of the length of the circuit; (5) the undersea cable, because it is free of the influence of switching stations, people, and nature, has the lowest error rates of the circuits used. Note that the Bell A-1 is a vestigial sideband, 3-level AM system.

Morey (Ref. 4.5) states that early tests indicated that COBI, as compared with CTDS, had greater tolerance of gaussian and impulse noise, less internally generated noise, and increased tolerance of nonlinear phase characteristics of the communication channel.

Lincoln Laboratory of MIT has been conducting an extensive program of measuring and recording errors which occur during medium-speed digital data transmission over various leased private-wire transmission facilities. Some of the results of these tests on four different data systems (CTDS, Milgo, Bell A-1, and Collins TE-206), over two types of telephone circuits (K carrier and microwave), have been presented and/or discussed in Refs. 4.6, 4.7, and 4.8. Curves presented (Refs. 4.7 and 4.8) include distribution of error in erroneous code words, distribution of burst lengths in erroneous code words, runs of error-free code words, and runs of erroneous code words. No clear-cut superiority could be indicated by the data.

Data taken to compare the error performance of 3 modems at 2400 bits/second over specially treated, leased voice channels are summarized in Appendix 4-A of Ref. 4.9. The modems used were the Collins AN/GSC-4 (operating well below its maximum rate), the Rixon Sebit 24 (an earlier version of the sebit 24B considered in Table 2-1), and the Stromberg Carlson S-C-301. The data are presented only as "weighted average error rates per kilomele." Unfortunately, it was assumed that the error rates varied directly with distance, and each test was weighted in proportion to its time duration to arrive at the weighted average error rates per kilomele. Consequently, the results lose some significance. The Collins modem generally gave the best bit and message (2000 bits per message) error rate. It was also indicated that the Collins modem did not require as critical an adjustment of its built-in equalizers as the others.

Tests of the COBI modem over private wire services in the United Kingdom (Ref. 4.10) gave an average bit error rate, over 673.5 hours of operation with 16 bits per word, of 3 bit errors in 10^6 bits received and an average word error rate of 0.057 per minute. As in tests in the U. S., the circuit was found to be error-free for periods up to 24 hours and inoperable during some finite short periods. The error rate varied over more than 2 magnitudes on some days, and the average daily bit error rate varied greatly from day to day.

In summary, it may be said that errors are due much more to impulse noise, short duration line interruptions, and other forms of interference than to white noise. The performance of a data system depends upon its freedom from, or immunity to, such interference. No one system is yet able to claim maximum immunity to all such forms of interference. None of the experimental results available so far give a valid conclusion of a "best" system, or even a "best" modulation method.

4.3 SYSTEM SELECTION -- AN ILLUSTRATION

To illustrate the use of the evaluation method presented in this section, assume a hypothetical situation, as follows:

A user requires a data transmission system of high reliability for operation in a remote area over leased telephone lines. A moderate error rate (1 in 10^4 bits at worst) is acceptable. Only three systems meeting the required data rate are determined to be available within the required short delivery schedule. Only a minimum of down time can be tolerated. The link is short, and of moderate but consistent quality.

A reasonable Table of Comparison for this user can now be made (following Table 4-1), such as shown in Table 4-2. Note that a small variation in user's weighting will not change the relative results by very much.

According to this procedure, System A would be the best choice although not by a very impressive margin. The addition of comments on such things as performance proven in field operation, or past company performance, could help to make the choice more conclusive.

4.4 GENERAL DISCUSSION

The digital data transmission systems discussed in this report are not the only ones that could be considered. The Hughes Aircraft Company HC-270 Digital Data Transceiver (Refs. 4.11 and 4.12) uses differentially coherent, four-level phase shift keying to obtain data rates up to 4800

TABLE 4-2
Illustration of System Comparison

CRITERIA	USER'S WEIGHTING	SYSTEM A	SYSTEM B	SYSTEM C
Reliability	8	6 (48)	4 (32)	5 (40)
Maintainability	8	6 (48)	5 (40)	5 (40)
Availability and Cost	3	4 (12)	4 (12)	6 (18)
Amplitude Distortion	2	5 (10)	6 (12)	5 (10)
Envelope Delay	2	5 (10)	5 (10)	5 (10)
Frequency Displacement	1	6 (6)	5 (5)	4 (4)
Gaussian Noise	3	5 (15)	4 (12)	4 (12)
Impulse Noise	5	5 (25)	5 (25)	4 (20)
Comments				
Figure of Merit		174	148	154

Note: Figures in parentheses are products of user's weight and system weight

bits/second. The Milgo Electronic Corporation Model 1652 Data Transmission System (Refs. 4.13 and 4.14) is essentially an FSK system. Stelma, Incorporated, developed the AN/TYC-1 for the Signal Corps (Ref. 4.15). This is a rugged, highly reliable, field-transportable FSK digital data terminal which will transmit up to 1200 bits/second. The common carriers (AT&T and Western Union) have a variety of full-time (leased) and part-time (dial-up) services (Ref. 4.16) available, including a Western Union 1200/2400 baud FM carrier transceiver (Ref. 4.8). IBM developed an experimental low-cost data transmission system (Ref. 4.17) to determine speed and reliability limitations on transmitting binary data over private telephone lines. More recently, a group at IBM working with Dr. Emil Hopner have developed systems for using TV channels at 20 Mbps and voice phone lines at 8000 bps using some new techniques (Ref. 4.18).

It should be noted that, at the present time, FCC tariff regulations prohibit the use of any modem equipment other than AT&T supplied units on the dial system (Ref. 4.8). Commercial modems are allowed only on leased circuits.

Enticknap and Lerner (Ref. 4.19) present a summary of considerations important to a designer of data transmission systems. Their paper discusses how data systems may be designed now in such a way that specialists can exercise their skills most effectively, thus allowing the system to take advantage of new coding or modulation equipment without unnecessary replacement of entire data systems and/or modifications to data sources or sinks.

The Armour Research Foundation report previously referenced (Ref. 4.1) gives the results of an investigation and study of the techniques and methods necessary for the transmission of digital data over standard telephone circuits in order to establish the basis for an optimum signalling system. It is a fairly complete and useful reference which includes information on channel capacity, transmission media, encoding

and decoding, modulation techniques, and a discussion of optimum line utilization. A good summary of wireline digital data transmission problems and techniques is given on Pages 2 to 17 of Ref. 4.1.

Finally, it should be emphasized that the data presented in this section is only a starting point in the comparison of digital data transmission systems. This data should be checked to be sure it is current, and further data should be obtained directly from cognizant engineers and operational records where possible. Manufacturers' literature and theoretical evaluations are not entirely adequate at the present time. Conclusive comparisons of equipment can only be made by including results of carefully planned and controlled experimental evaluations and operational data, with conditions and equipment and circuit configurations completely specified for each case. Experimental comparisons will not be adequate unless equipment responses are compared on the basis of identical inputs that accurately represent the conditions to which the equipment will be subjected in actual operation.

REFERENCES

- 4.1 Data Line Techniques, Armour Research Foundation Technical Report; 1 April 1961. AD 263 144.
- 4.2 Survey of Digital Communications, Hughes Aircraft Company, Communication Division Report No. TR-1; 13 April 1961. AD 277 176.
- 4.3 A. B. Glenn, "Comparison of PSK vs FSK and PSK-AM vs FSK-AM Binary-Coded Transmission Systems," IRE Trans. on Communication Systems; June 1960.
- 4.4 E. J. Hofmann, "Measured Error Distributions on the Bell A-1 Facility Over Various Media," presented at the National Electronics Conference, October 1960.
- 4.5 K. H. Morey, The COBI Data Transmission Modem, MIT Lincoln Laboratory Technical Report No. 263; 23 May 1962.
- 4.6 E. J. Hofmann, Collected Error Statistics Utilizing Four Modems Over Various Telephone Circuits, MIT Lincoln Laboratory Report No. 25G-0028; 23 January 1961. AD 251 571.
- 4.7 A. B. Fontaine and R. G. Gallager, Error Statistics and Coding for Binary Transmission Over Telephone Circuits, MIT Lincoln Laboratory Report No. 25G-0023; 30 October 1960.
- 4.8 R. A. De Kimpey, Relation of Some Data Transmission Error Statistics to the Data Line Translator, Beckman/Systems Division Technical Memorandum No. TM-176; 5 April 1961.
- 4.9 UNICOM, Progress Report for the Ninth Quarter, 1 July through 30 September 1961, Vol. II; 30 September 1961. AD 269 144.
- 4.10 P. L. Grant, COBI Data Transmission Testing in the United Kingdom, MIT Lincoln Laboratory Report No. 25G-8,; 27 March 1962. AD 275 152.
- 4.11 G. L. Evans, E. Enriquez, and Q. C. Wilson, "A High Speed, Serial, Four-Phase Data Modem for Regular Telephone Circuits," GLOBECOM V Convention Record, pp. 100-104.
- 4.12 J. E. Toffler and J. N. Buterbaugh, "The HC-270, a Four-Phase Digital Data Transceiver," presented at the 1961 Western Electronic Show and Convention, San Francisco; Part 36/1.

REFERENCES (CONT'D)

- 4.13 C. R. Scott and W. H. Butler, "Real-Time Data Transmission System," IRE Trans. on Communication Systems, Vol. CS-7, No. 3, pp. 201-205; September 1959.
- 4.14 E. J. Hofmann and P. L. Grant, Error Statistics on a Long-Haul Looped Telephone Circuit Utilizing the Milgo Data System, MIT Lincoln Laboratory Report. No. 25G-0010; 17 June 1960.
- 4.15 Development of Digital Data Terminal AN/TYC-1, Final Report from Stelma, Incorporated, Stamford, Connecticut; 30 November 1961. AD 271 985.
- 4.16 A Survey of High Speed Data Communications, Weather Systems Center, United Aircraft Corporation; 22 June 1962. AD 283 924.
- 4.17 E. Hopner, "An Experimental Modulation-Demodulation Scheme for High-Speed Data Transmission," IBM Journal, pp. 74-84; January 1959.
- 4.18 G. Comstock, "Low-Cost Terminals Triple Phone, TV Link Data Rates," Electronic Design, pp. 8-11; January 18, 1963.
- 4.19 R. G. Enticknap and R. M. Lerner, The Design of Data Transmission Systems, MIT Lincoln Laboratory Report No. 25G-0022; 4 October 1960. AD 243 926.

SECTION 5

ERROR DETECTING AND ERROR CORRECTING CODES

5.1 GENERAL

It is evident from the preceding discussion that error rates on the order of 1 error in 10^5 can be achieved with typical equipment configurations. If slightly better performance is required, a judicious choice of equipment and modulation scheme plus greater output power would probably suffice. However, if the performance must be increased by several orders of magnitude, coding appears to be the only solution.

The possible improvements that can be achieved with coding will be the subject of this section. Both error correcting and error detecting codes will be considered; and it will be shown that error detecting codes are far more promising, but are only useful over two-way links.

Error detecting and error correcting codes are extremely sensitive to the statistics of the errors. Only two types of errors will be considered here:

1. Random errors caused by white noise interference (which might occur over microwave links)

2. Burst errors caused by the impulse noise on telephone circuits.

However, a few general comments will be made concerning paths with combined statistics, such as might occur over tandem of radio and telephone circuits.

5.2 RANDOM ERRORS DUE TO WHITE NOISE

It has been well established that error correcting codes can greatly reduce the number of random errors that occur. However, in this section, the validity of this conclusion when the following two constraints are placed upon the system will be examined:

1. Constant data rate.
2. Constant power.

These constraints affect the analysis in the following manner. In order to maintain the same data rate when additional (checking) bits are added to the word, each bit must be reduced in time so that the entire word with checking bits occupies the same time interval as the original word. This, of course, reduces the signal energy per bit. The constant power constraint prevents the use of more power in order to return the signal energy per bit to the original value. The significance of the two constraints is that the coded word is transmitted with the same power and in the same length of time as the uncoded word by reducing the energy per bit. The justification for these constraints is that systems should be compared by equating the significant parameters; in this case, the transmission time per word and the energy per word. These are the significant parameters because the words of the uncoded and coded messages contain the same amount of information rather than the individual bits.

An analysis of error correcting codes under these constraints is carried out in Appendix A. Although a completely general solution is not obtained, the results are very significant. An example is worked out using the bit error probability curves for FSK envelope detection. For this case, it is shown that there is little, if any, advantage to be gained by the use of error correcting codes under these particular constraints.* These results are of general interest for more than just FSK envelope detection because of the similarity in shape of bit error rate curves for many different modulation schemes as derived in Appendix B. Therefore, the usefulness of error correcting codes for any modulation scheme under the assumptions and constraints of this section can be questioned.

* In the example, the number of bits prior to encoding was chosen to be 5. A larger number would result in more efficient codes and, hence, better performance but would carry the penalty of additional complexity.

The reason for the lack of effectiveness of error correcting codes is quite clearly the strong dependence of the probability of error on the signal energy per bit (E). It is seen from Fig. B-1 that, for typical situations, a reduction of E to half its original value will increase the error rate more than a hundred-fold. Since many error correcting codes require almost as many checking bits as information bits, the value of E is often halved, and the poor results are understandable.

An error detection scheme might prove useful in this situation because fewer checking bits are required and, therefore, E is not reduced as much as in error correcting schemes. However, knowledge that an error has occurred somewhere in a word is of little help unless the word can be retransmitted. In general, this implies that a return communication link to the transmitter is required.

5.3 BURST ERRORS DUE TO IMPULSE NOISE

It has just been shown that error correcting codes perform poorly against random noise (under given constraints) due to a strong dependency of error rate on E . However, the dependence of error rate on E is much smaller for errors occurring over telephone circuits which are characterized by impulse-type noise. For example, Alexander, Gryb, and Nast (Ref. 5.1) have shown that a reduction in transmitted power by 5 db will approximately double the error rate. By contrast, for white noise interference, a reduction of 5 db would increase the error rate by approximately 3 orders of magnitude. It would be expected, therefore, that the constant power, constant data rate constraints will not appreciably affect the telephone circuits. Unfortunately, as will be shown in the next paragraph, the burst nature of the errors considerably reduces the effectiveness of error correction for telephone lines.

A considerable amount of data concerning the statistics of the error bursts have been obtained by Alexander, Gryb, and Nast (Ref. 5.1). Using this data, they have shown that typical burst error correcting

codes can correct only a moderate number of errors. For example, they show that a typical code capable of correcting bursts up to length 20 will correct only 79 percent of the error bursts. Codes capable of correcting larger bursts would be more effective but also more costly.

On the other hand, error detecting codes can be quite effective. For example, Fontaine and Gallager (Ref. 5.2) have shown, using typical telephone error patterns, that the mean time between undetected error can be reduced to several years through the use of moderate block sizes (100-500 bits/block) and only a small amount (3%-11%) of checking bits. Statistics for European links (with error rates of approximately 10^{-4}) published by Jones (Ref. 5.3) indicate that only 1%-3% of the blocks need to be retransmitted due to detected errors.

The necessity for a return path for effective use of error detecting codes presents a number of problems. Some of these problems, and their suggested solutions, are considered by Reiffen, Schmidt and Yudkin (Ref. 5.4), and by Metzger and Morgan (Ref. 5.5).

5.4 COMBINED LINKS

If a combination of telephone lines and radio links were used, the errors would be a combination of burst and random errors. Error detecting codes would probably be most effective but, due to the random errors, many more repeats of blocks might be required. In this eventuality, a little more redundancy in the form of extra-check bits would provide single error correcting capability and might considerably reduce the number of requests for repetition.

REFERENCES

- 5.1 A. A. Alexander, R. M. Gryb, and D. W. Nast, "Capabilities of the Telephone Network for Data Transmission," Bell System Tech. J., Vol. 39, pp. 431-476; May 1960.
- 5.2 A. B. Fontaine and R. G. Gallager, "Error Statistics and Coding for Binary Transmission Over Telephone Circuits," Proc. IRE, Vol. 49, pp. 1059-1065; June 1961.
- 5.3 W. T. Jones, "Transmission of Data Over Telephone-Type Circuits," Electrical Communication, Vol. 37, No. 2, pp. 87-102; 1961.
- 5.4 B. Reiffen, W. G. Schmidt, and H. L. Yudkin, "The Design of an Error-Free Data Transmission System for Telephone Circuits," Communication and Electronics, pp. 224-231; July 1961.
- 5.5 J. J. Metzger and K. C. Morgan, "Coded Feedback Communication Systems," Proc. National Electronics Conf., Vol. 16, pp. 250-257; October 1960.

APPENDIX A
WORD ERROR PROBABILITIES

APPENDIX A

WORD ERROR PROBABILITIES

A.1 GENERAL

In this appendix we shall investigate word error probabilities in a binary symmetric channel when all errors in the channel are mutually independent. It differs from previous work in this area in that two constraints are placed upon the system.

The first of these is that a constant data rate must be maintained. That is, when parity check bits are added, each bit must occupy less time so that the entire word occupies the same time as the original. The second constraint is that the total energy needed to transmit the word remains constant. That is, we are not allowed to increase the transmitted power in order to compensate for the shorter time interval of each bit.

A.2 CODING PARAMETERS

For this study, the Slepian group alphabets (Ref. A.1 and A.2) were used. These codes consist of words with n bits of which k are information bits and $n-k$ or r are parity check bits. It has been shown that a code with r parity check bits can correct no more than $2^r - 1$ error patterns. The Slepian codes are optimum in that they can correct all of these errors.* If B_s is the number of s -tuple error patterns that a code can correct and

$$\sum_{s=1}^n B_s = 2^r - 1 \quad (A-1)$$

then the probability that a word is received in error and is corrected

* In the last analysis, all error correcting codes may be reduced to a Slepian equivalent. For example, the Bose-Chaudhuri (9,5), (13,5), and (15,5) codes are the first single, double, and triple error correcting codes as are the Slepian (9,5), (13,5), and (15,5) codes but the Slepian codes correct additional error patterns as well.

is

$$Q_{n,k} = \sum_{s=1}^n B_s p^s q^{n-s} \quad (A-2)$$

where

p = probability that a single bit is in error.

$q = 1-p$

Since the probability that a word is received correctly is q^n , the probability that a word is still in error after reception and error correction is given by

$$P_{n,k} = 1 - q^n - Q_{n,k} \quad (A-3)$$

A.3 SIGNIFICANCE OF CONSTANT DATA RATE

It is well known that a word error probability may be reduced by several orders of magnitude by using an error correcting code* and reducing the data rate. In general, $P_{n,k}$ is a monotonically decreasing function of r because of the increasing number of error patterns that can be corrected.

It is evident, however, that for a particular value of k and $P_{n,k}$, the data rate may be reduced to an undesirable level.

If a constant data rate must be maintained, each bit must be reduced in time by an amount such that the coded word, $k+r$, has the same duration as the original uncoded word. However, if the energy per bit per one sided noise density, denoted by α , is fixed initially at α_0 , the new value is $k\alpha_0/n$ and, consequently, the shortening of the bit time increases the bit error probability to p_n , and the word error probability prior to correction, P_n :

* An example is provided by Slepian (Ref. A.1) If a 5-bit word with no error correction is transmitted through a binary symmetric channel with bit error probability $p = 0.001$, the word error probability is $P = 0.00449$, whereas the addition of 5 parity check bits, with a concomitant 50% reduction in the signalling rate, yields a symbol error probability $p = 0.000024$.

$$P_n = 1 - q^n \quad (A-4)$$

We are aware, on the other hand, that the coding imposes constraints on the bits, via linear dependence, so that errors show up as inconsistencies and may be corrected, reducing the word error rate to $P_{n,k}$.

$$P_{n,k} = P_n - \sum_{s=1}^n B_s p^s q^{n-s} = P_n - Q_{n,k} \quad (A-5)$$

Efficient coding, at least under the constraints of this appendix, takes place when the second effect outweighs the first.

A.4 METHOD

Normally, one would start by taking the equation $P_{n,k} = P_n - Q_{n,k}$, make the proper substitutions for the variables and find those values of r which minimize $P_{n,k}$. Since r is a discrete variable, one may not use the derivative of $P_{n,k}$ to find the best values of r .

Given the values of α_0 and k and the modulation scheme T , the bit error probability p_n that results from using r check bits may be determined from

$$p_n = f_T \left\{ \frac{k\alpha_0}{k+r} \right\} \quad (A-6)$$

Some typical functions, (for white noise interference) are derived in Appendix B and are summarized in Table A-1.

TABLE A-1

BIT ERROR PROBABILITY EQUATIONS AND APPROXIMATIONS

Modulation	Defining Equation	Approximation $\alpha \geq$
FSK	$p_k = 0.5 e^{-0.5\alpha}$	$p_k = 0.5 e^{-0.5\alpha}$
PM and BI-ORTHO	$p_k = \int_{\frac{1}{(2\alpha)^2}}^{\infty} (2\pi)^{-\frac{1}{2}} e^{-\frac{x^2}{2}} dx$	$p_k = 0.142 e^{-0.955\alpha}$
AM SYNC	$p_k = \int_{-\infty}^{\infty} \left\{ (2\pi)^{-\frac{1}{2}} e^{-\frac{x^2}{2}} \right\} \left\{ \int_{x+(2\alpha)^2}^{\infty} \frac{1}{(2\pi)^{\frac{1}{2}}} e^{-\frac{z^2}{2}} dz \right\} dx$	$p_k = 0.2 e^{-0.553\alpha}$
ORTHO	$p_k = \int_{\frac{1}{(\alpha)^2}}^{\infty} (2\pi)^{-\frac{1}{2}} e^{-\frac{x^2}{2}} dx$	$p_k = 0.2 e^{-0.553\alpha} \quad \alpha \leq 12.5$ $p_k = 0.231 e^{-0.558\alpha} \quad \alpha > 12.5$

To fix ideas, if we are using FSK signal with envelope detection, the uncoded bit error rate is

$$p_k = \frac{1}{2} e^{-\alpha/2} \quad (\text{A-7})$$

while the coded bit error rate is

$$p_n = \frac{1}{2} e^{\frac{-k\alpha_o}{2n}} \quad (\text{A-8})$$

and consequently

$$p_n = \frac{1}{2} (2p_k)^{\frac{k}{k+r}} \quad (\text{A-9})$$

Since $q_n = 1 - p_n$, the first term of Eq. (A-5) is

$$P_n = 1 - \left[1 - \frac{1}{2} (2p_k)^{\frac{k}{k+r}} \right]^{k+r}; \quad (\text{A-10})$$

$Q_{n,k}$ is somewhat more complicated. The B_s in each term are of the form $(C_s) \binom{k+r}{s}$ where C_s is 0, 1, or a fraction:

$$\begin{aligned} Q_{n,k} = & C_1 \binom{k+r}{1} p_n q_n^{k+r-1} + C_2 \binom{k+r}{2} p_n^2 q_n^{k+r-2} \\ & + C_3 \binom{k+r}{3} p_n^3 q_n^{k+r-3} \dots C_{k+r} \binom{k+r}{k+r} p_n^{k+r} q_n^0 \end{aligned} \quad (\text{A-11})$$

To date, no function relating the B_s , n , and k has been found. For convenience, the B_s have been tabulated in Table A-2 for $k = 4, 5, 6$ and $6 \leq n \leq 16$.

TABLE A-2

S-FOLD ERROR PATTERNS CORRECTED BY A SLEPIAN n, k CODE

$k=4$	n	$s=1$	2	3	4	5	6	7	$A_{n,k}$
	6	3							$3p$
	7	7							$21p^2$
	8	8	7						$21p^2$
	9	9	22						$14p^2$
	10	10	39	14					$6p^2$
	11	11	55	61					$104p^3$
	12*	12	66	144	33				$74p^3$
	13*	13	78	246	173	1			$40p^3$
	14*	14	91	364	511	42	1		$490p^4$
	15*	15	105	455	993	475	4		$372p^4$
	16*	16	120	560	1582	1680	134	3	$236p^4$
$k=5$	n	$s=1$	2	3	4	5			
	7	3							$4p$
	8	7							$21p^2$
	9	9	6						$30p^2$
	10	10	21						$24p^2$
	11*	11	46	6					$9p^2$
	12*	12	63	52					$3p^2$
	13*	13	78	152	12				$134p^3$
	14*	14	91	294	112				$70p^3$
	15*	15	105	455	420	28			$945p^4$
	16**	16	120	560	1351				$469p^4$

* See Ref. A.2

** See Ref. A.3

TABLE A-2 (CONT'D.)

k=6	n	s=1	2	3	$A_{n,k}$
	8	3			$5p$
	9	7			$2p$
	10	10	5		$40p^2$
	11	11	20		$35p^2$
	12*	12	50	1	$16p^2$
	13*	13	72	42	$6p^2$
	14*	14	91	150	$214p^3$

* See Ref. A.2

For example, using Eq. (A-11) and Table A-2, we have

$$Q_{10,4} = 10q^9p + 39q^8p^2 + 14q^7p^3$$

because the code corrects 10 of the 10 single, 39 of the 45 double, and 14 of the 120 triple errors. Since the probability that the symbol contains no error is q^{10} , Equation (A-5) becomes

$$\begin{aligned}
 P_{10,4} &= P_n - Q_{10,4} \\
 &= 1 - q^{10} - 10q^9p - 39q^8p^2 - 14q^7p^3 \\
 &= 6p^2(1-p)^8 + 106p^3(1-p)^7 + \dots + p^{10} \\
 &= 6p^2 + 64p^3 + \dots + p^{10}
 \end{aligned}$$

Furthermore, $6p^2$ is defined to be $A_{n,k}$, which is an approximation of $P_{n,k}$ when the bit error probability, p , is very small. This will be discussed in more detail later.

A.5 NUMERICAL INVESTIGATION OF THE $n,5$ CODES USING FSK

Since it was apparent that no general formula could be set down that would include all the variables, it was decided to choose the parameters for a system and work it through as an example.

Binary frequency-shift keying was chosen as a modulation scheme so that the channel transition probability is given by

$$p_n = \frac{1}{2} e^{\frac{-k\alpha_0}{2n}} \quad (A-8)$$

The values of α_0 chosen were 10, 15, and 25.

The word with 5 information bits was chosen because it is a good choice for the transmission of English text and is a good vehicle for PCM.

The above parameters were inserted in Equation (A-5) and the resultant values of $P_{n,k}$ are presented in Table A-3 and plotted in Fig. A-1.

- It is quite unexpected that the shape of the curves and the relative position of the codes should change so drastically as we vary α_0 . We see from the figure that no form of encoding (up to 11 parity check bits) yields a $P_{n,k}$ less than the uncoded word for $\alpha_0 = 10$ or 15.

Computation shows that there is a breakeven point at $\alpha_0 = 17.8$; i.e., below this value, no coding is best and above this value the (9,5) code has the advantage.

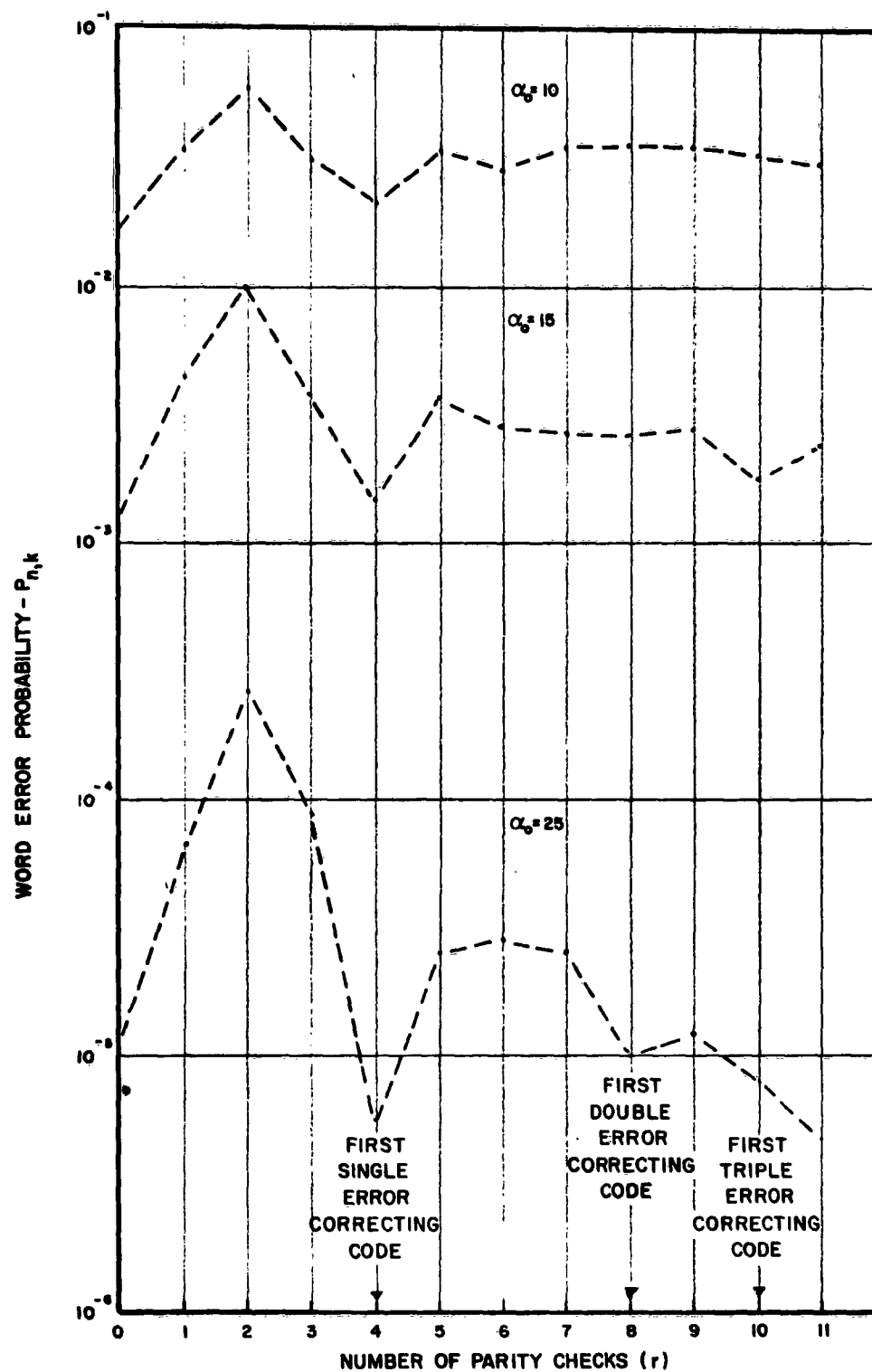


Fig. A-1 Error Probability for a Word Containing 5 Information Bits and r Parity Checks

A-9

TABLE A-3

$P_{n,5}$ as a Function of n and α_0 for FSK Modulation				
n,k	r	$\alpha_0 = 10^*$	$\alpha_0 = 15^{**}$	$\alpha_0 = 25^{***}$
5,5	0	1.7×10^{-2}	1.3×10^{-3}	9.0×10^{-6}
6,5	1	3.4×10^{-2}	4.5×10^{-3}	6.6×10^{-5}
7,5	2	5.8×10^{-2}	9.9×10^{-3}	2.6×10^{-4}
8,5	3	3.2×10^{-2}	3.8×10^{-3}	8.6×10^{-5}
9,5	4	2.1×10^{-2}	1.5×10^{-3}	5.5×10^{-6}
10,5	5	3.4×10^{-2}	3.6×10^{-3}	2.5×10^{-5}
11,5	6	2.9×10^{-2}	2.9×10^{-3}	2.8×10^{-5}
12,5	7	3.5×10^{-2}	2.7×10^{-3}	2.5×10^{-5}
13,5	8	3.7×10^{-2}	2.6×10^{-3}	9.8×10^{-6}
14,5	9	3.6×10^{-2}	2.8×10^{-3}	1.2×10^{-5}
15,5	10	3.4×10^{-2}	1.8×10^{-3}	8.0×10^{-6}
16,5	11	2.9×10^{-2}	2.4×10^{-3}	4.7×10^{-6}

$$* p_5 = 3.35 \times 10^{-3}$$

$$** p_5 = 2.7 \times 10^{-4}$$

$$*** p_5 = 1.8 \times 10^{-6}$$

Surprisingly enough, at $\alpha_0 = 25$, the (16,5) code is better than the (9,5) code. That is, 11 parity checks are slightly better than 4.

It is interesting to look at Slepian's example (see footnote, p.A-2) under this new regime. A bit error probability of 10^{-3} can be achieved with an $\alpha_0 = 12.5$. Encoding into a 10,5 code increases p_n to 0.022 and $P_{n,k} = 0.009$. This is approximately 375 times the value of 0.000024 one obtains by halving the data rate.

It is natural to enquire how $P_{n,5}$ behaves as r increases for some α_0 . It has been shown (Ref. A.4) that the fraction of the possible errors that a code can correct depends only on the number of information bits and is 2^{-k} . However, as the number of parity checks increases, so does the absolute number of error patterns that can occur.

In the expression for $P_{n,k}$ these error patterns must be weighted by their probability of occurrence. However, the probability of occurrence depends on the bit error probability and, as we see from the curves of Appendix B, p_k is driven to unity for fixed α_0 and increasing redundancy. Consequently, all the curves of Fig. A-1 eventually turn up and $P_{n,k}$ approaches unity as r increases (albeit in a badly behaved fashion).

For the sake of example, let us take the Bose-Chaudhuri (36,5) code (Ref. A.5) which corrects all errors up to and including 6-fold errors. Let us assume that the analogous Slepian code corrects no 7-fold or higher error patterns.

$$P_{36,5} = 1 - \sum_{s=0}^6 \binom{36}{s} p_n^s q_n^{36-s}. \quad (A-12)$$

For

$$\alpha_0 = 25, \quad p_n = \frac{1}{2} e^{-\frac{1}{2} \left(\frac{5 \times 25}{36} \right)} = 0.088.$$

We may approximate $P_{36,5}$ by a normal distribution with parameters

$$\mu = np = (36) (0.088) = 3.168$$

$$\sigma^2 = npq = [(36) (0.088) (0.912)] = 2.89$$

$$\begin{aligned}
 P_{36,5} &= \frac{1}{\sqrt{2\pi}\sigma} \int_{x_1}^{\infty} e^{-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2}} \\
 &= \frac{1}{\sqrt{2\pi}(2.89)} \int_{x_1=6\frac{1}{2}}^{\infty} \exp \frac{(x-3.168)^2}{5.78} dx \quad (A-13)
 \end{aligned}$$

or letting $t = \frac{x-\mu}{\sigma}$, $t_1 = \frac{6\frac{1}{2}-\mu}{\sigma} = 1.96$

$$P_{36,5} = \frac{1}{\sqrt{2\pi}} \int_{t=1.96}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt \approx 0.025 \quad (A-14)$$

This $P_{n,k} \approx 0.025$ must be contrasted with a word error rate of approximately 9×10^{-6} which we could have obtained by sending an uncoded word of 6 bits at $\alpha_0 = 25$.

A.6 APPROXIMATIONS

Although we cannot solve our initial problem analytically with any degree of sophistication, we find that we are in a position to set down a reasonably easy method of approximation. For small values of p ($p < 10^{-3}$) we may approximate $P_{n,k}$ fairly closely by functions of the form $R(p)^s$ which are tabulated in Table A-2 as $A_{n,k}$. These approximations are derived from the binomial coefficients as follows.

Let C_x be the first coefficient that is not unity. Then $P_{n,k}$ may be written

$$P_{n,k} = \sum_{s=x}^n (1 - C_s) \binom{k+r}{s} p^s q^{k+r-s} \quad (A-15)$$

If the substitution $q = 1 - p$ is made, the expression expanded, and terms collected, $P_{n,k}$ may be written

$$P_{n,k} = R_1 p^s + R_2 p^{s+1} + R_3 p^{s+2} + \dots + R_n p^{k+r} \quad (A-16)$$

where the R_i are constants. The $R_1(p)^s$ term in equation n is the largest term and is defined to be $A_{n,k}$. The rest are neglected because of their rapidly decreasing magnitude.

An upper bound to the error introduced by neglecting higher terms in p may be found (Ref. A-6) by letting $x + 1 = \lambda(k+r)$. Then

$$\sum_{s=\lambda(k+r)}^{k+r} p^s q^{k+r-s} \leq \left(\frac{p}{\lambda}\right)^{\lambda(k+r)} \left(\frac{1-p}{1-\lambda}\right)^{(1-\lambda)(k+r)}$$

providing $\lambda > p$.

If we are given α_0 and k, one may evaluate $A_{n,k}$ for as many r as desired and choose among them in order to achieve or better any desired symbol error probability. For example, suppose we are given $\alpha_0 = 30$, $k = 6$. We find the $A_{n,6}$ are

(6,6)	9.0×10^{-7}	(11,6)	5.9×10^{-7}
(8,6)	3.2×10^{-5}	(12,6)	1.1×10^{-6}
(9,6)	5.2×10^{-5}	(13,6)	2.6×10^{-8}
(10,6)	1.4×10^{-7}	(14,6)	1.1×10^{-7}

If our design value had been $P_{n,k} = 10^{-6}$, we would most likely choose to do no encoding. It had been 10^{-7} , we find that although the (13,6) is the only code that provides the desired performance, the $P_{n,k}$ for the (10,6) code is close enough that there might be merit in reconsideration of the design value.

A.7 CONCLUSIONS

The following observations are made:

1. At moderate values of α_0 , the reduction in the symbol error probability gained by encoding is unimpressive if it exists at all.
2. If a large value of α_0 is available initially and the ratio $P_{k,k}/P_{n,k}$ is impressive, it must be remembered that there will be a set of r parity equations that must be solved to correct the errors. If the code corrects any s -tuple errors, this set of equations must be iterated s times. Consequently, the price of improvement in the word error probability is related to the price of the equipment to perform the error correction.
3. A glance at Fig. A-1 shows that it is never profitable to add less than 3 parity check bits to a symbol containing 5 information bits. Comparable figures for other codes are never less than 3 for 4 information bits and never less than 5 for 6 information bits. One might be tempted to speculate on the existence of a rule "never less than $k-1$ parity checks," but this reasoning is fallacious, as a glance at Fontaine and Peterson (Ref. A.2) will show.

REFERENCES

- A.1 D. Slepian, "A Class of Binary Signaling Alphabets," Bell System Tech. J., vol. 35, p. 203; January 1956.
- A.2 A. B. Fontaine and W. W. Peterson, "Group Code Equivalence and Optimum Codes," IRE Trans. on Circuit Theory, vol. CT-6, p. 60; May 1959.
- A.3 S. Reiger, "Error Rates in Data Transmission," Proc. IRE, vol. 46, p. 919; May 1958.
- A.4 C. M. Melas, "Reliable Data Transmission Through Noisy Media," Trans. AIEE, Communications & Electronics, p. 501; November 1961.
- A.5 R. C. Bose and D. K. Ray-Chaudhuri, "Further Results on Error Correcting Binary Group Codes," Case Institute of Technology, Publication 1020, ASTIA AD 251 137.
- A.6 W. W. Peterson, "Error Correcting Codes," MIT Press and J. Wiley and Sons, Appendix A; 1961.

APPENDIX B
ERROR PROBABILITIES IN A BINARY CHANNEL

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ERROR PROBABILITIES IN A BINARY CHANNEL

B.1 ASSUMPTIONS, DEFINITIONS AND RESULTS

We are concerned with sending a random sequence of 1's and 0's, each bit of which is allotted T seconds. The probabilities of 1's and 0's are equal to $1/2$ and it is assumed that there is no inter-symbol influence. The autocorrelation of such a sequence, now considered as 1's and -1's, is

$$\begin{aligned}\phi_{aa}(\tau) &= 1 - \frac{|\tau|}{T} & |\tau| \leq T \\ &= 0 & |\tau| > T\end{aligned}\tag{B-1}$$

The corresponding spectrum is the Fourier transform of the autocorrelation

$$\Phi_{aa}(f) = T \frac{\sin^2 \pi f T}{(\pi f T)^2}\tag{B-2}$$

which has its first zeros at $\pi|f|T = \pi$ or $|f| = 1/T$. We will assume that the video bandwidth is $B = \frac{1}{2T}$. Note that this is the Nyquist bandwidth for the given bit rate.

The assumption of $B = \frac{1}{2T}$ is the minimum possible value for B ; if a larger value of B is used, the corresponding value of $\alpha = E/N_0$ will be increased for a specified bit error probability since

$$\alpha = \frac{E}{N_0} = \frac{ST}{N/2B} = (2BT) \frac{S}{N} = (2BT) \rho\tag{B-3}$$

and for all systems, except the matched-filter systems, the bit error probability is a function of ρ .

B-1

For example, Fig. B-1 shows, for phase modulation, $\alpha = 6.93$ for $p = 10^{-4}$ when $B = \frac{1}{2T}$. If $B' = 2B = 1/T$ and p is held constant, $\alpha = 13.86$ for $p = 10^{-4}$.

On the other hand, the curves for the matched-filter orthogonal and bi-orthogonal systems are independent of bandwidth.

The receiver is assumed to sample the output of the detector at some instant during the bit time. The effect of bandwidth on the signal rise time has been ignored.

For the matched-filter systems, discussed in Paragraphs B.5 and B.6, it has been assumed that the integrator is sampled at the end of the bit interval. The method and power to achieve this timing has been ignored.

The signal power will be assumed constant over each bit interval and its long-time average value will be S .

White Gaussian noise having a spectral density

$$W_N(f) = \frac{N_0}{2} \quad -\infty < f < \infty \quad (B-4)$$

will be assumed.

The noise power is N .

The signal-to-noise power ratio at the receiver input is ρ .

The ratio of signal energy to the noise spectral density will be denoted by

$$2\alpha = \frac{E}{N_0/2} = \frac{2E}{N_0} \quad \text{or} \quad \alpha = \frac{E}{N_0} \quad (B-5)$$

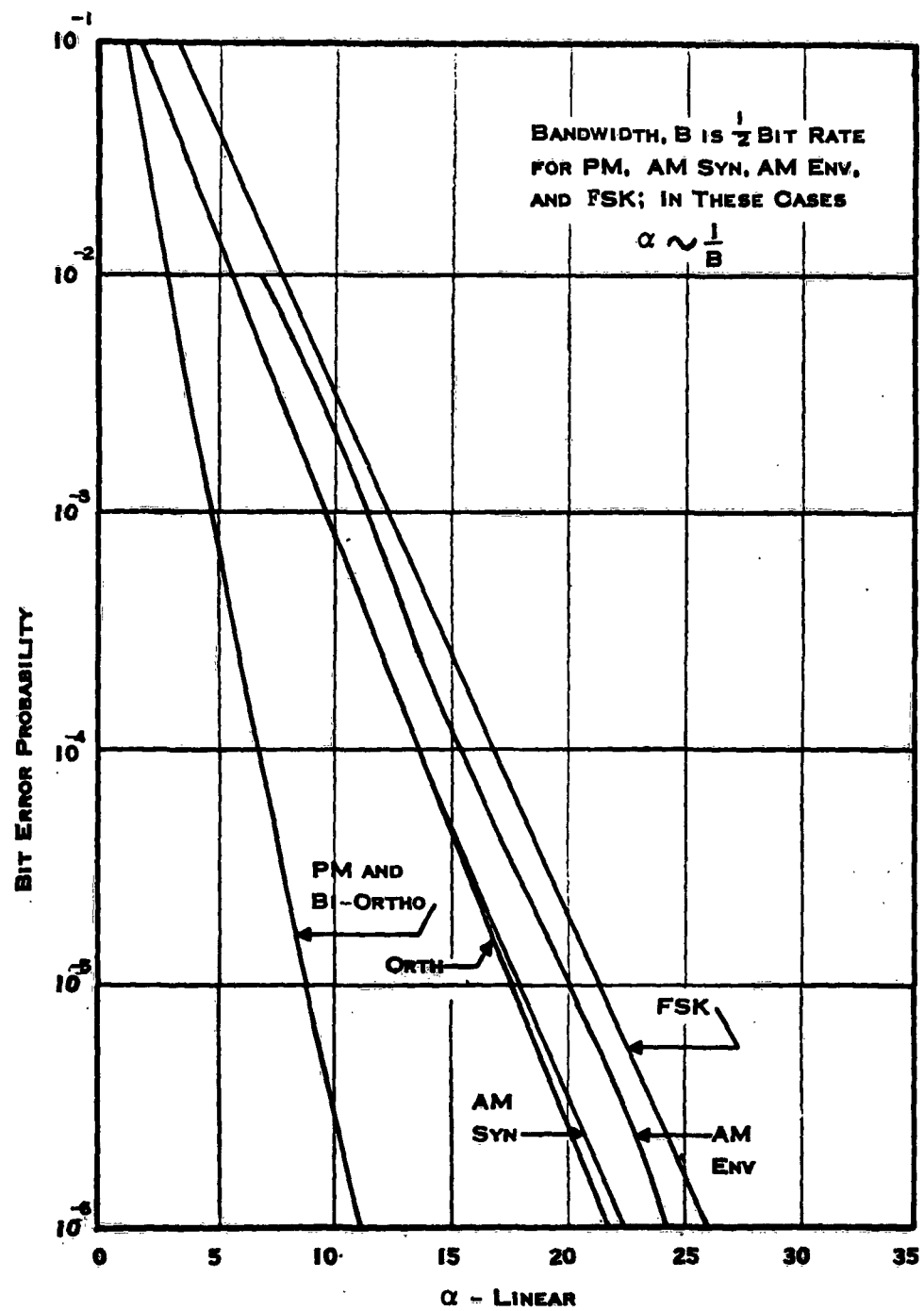


Fig. B-1 Bit Error Probability vs α , Signal Energy/Bit/One-Sided Noise Spectral Density

It is assumed that by the end of each bit interval, the receiver is forced to make the decision whether a 1 or a 0 was transmitted. The decision may be correct or there may occur a Type I error -- deciding that a 1 was sent when in fact a 0 was sent -- or a Type II error deciding that a 0 was sent when in fact a 1 was sent.

The probability of a Type I or a Type II error will be denoted as P_I and P_{II} , respectively. Since 1's and 0's are sent with equal probability, the probability of error is

$$P = \frac{1}{2} P_I + \frac{1}{2} P_{II}. \quad (B-6)$$

If $P_I = P_{II} = P$, the subscripts may be omitted. The probability of a correct decision is $1-P$.

In all cases, the threshold will be selected to give the minimum total probability of error. Except for envelope detection of on-off AM, where the optimum threshold is a function of the parameter α , the optimum threshold is midway between the means of the two signal conditions.

The results of the paper in the form of curves of Bit Error Probability for the various systems vs α are summarized in Fig. B-1 (α linear) and Fig. B-2 (α db).

B.2 DOUBLE-SIDEBAND AMPLITUDE MODULATION

In this section, envelope (noncoherent) detection and synchronous (coherent) detection are considered. The probabilities of error are calculated on the assumption that the audio output of the detector is sampled near the end of the bit interval.

Let the data be $a(t) = \pm 1$ where $a(t)$ is constant in each bit interval. Let ω_c be the carrier frequency. Then the double-side-band signal is

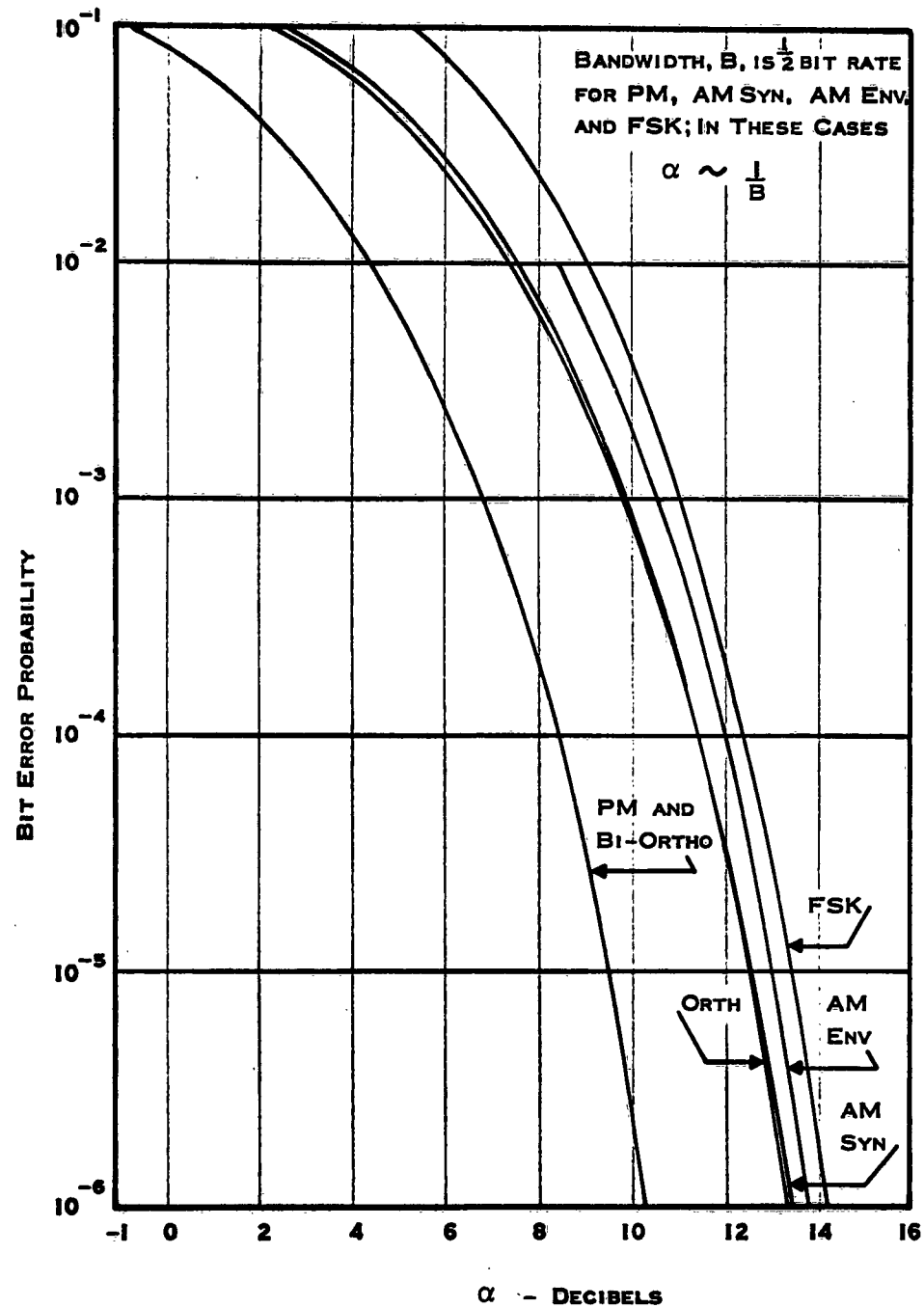


Fig. B-2 Bit Error Probability vs. α , Signal Energy/Bit/One-Sided Noise Spectral Density

B-5

$$s(t) = A [1 + m a(t)] \sin \omega_c t. \quad (B-7)$$

In the case of envelope (noncoherent) detection $m \leq 1$. For $m = 1$, the process may be called on-off keying.

Following a bandpass filter of width $2B$ centered at f_c , the noise power is

$$N = 2B N_o. \quad (B-8)$$

The average signal power is

$$\begin{aligned} S &= \frac{1}{2} \left\{ \frac{1}{2} [A(1+m)]^2 + \frac{1}{2} [A(1-m)]^2 \right\} \\ &= \frac{A^2}{4} (1 + 2m + m^2 + 1 - 2m + m^2) \\ &= \frac{A^2}{2} (1 + m^2). \end{aligned} \quad (B-9)$$

Thus,

$$\rho = \frac{S}{N} = \frac{A^2}{2} \frac{(1+m^2)}{2BN_o} = \frac{A^2 T (1+m^2)}{2N_o} = \frac{ST}{N_o} \quad (B-10)$$

and

$$\alpha = \frac{E}{N_o} = \frac{ST}{N_o} = \rho \quad (B-11)$$

The noise voltage may be written

$$n(t) = x(t) \sin \omega_c t + y(t) \cos \omega_c t$$

where $x(t)$ and $y(t)$ have no frequency components above B cycles/sec and

$$E [x^2(t)] = E [y^2(t)] = E [n^2(t)] = N \quad (B-12)$$

B-6

Suppose that envelope detection is used and let $R(t)$ be the envelope. Then, when a 1 is sent,

$$R(t) = \left\{ \left[A(1+m) + x(t) \right]^2 + y^2(t) \right\}^{\frac{1}{2}} \quad (\text{B-13})$$

Since x and y are independent and normally distributed with means zero and variances N (following Bennett, Ref. B.1)

$$p_1(R) = \frac{R}{N} I_0 \left[\frac{A(1+m)R}{N} \right] \exp \left\{ -\frac{1}{2N} \left[R^2 + A^2(1+m)^2 \right] \right\} \quad (\text{B-14})$$

I_0 is the modified Bessel function of the first kind of zero order:

$$I_0(x) = \sum_{n=0}^{\infty} \frac{x^{2n} (-1)^n}{4^n (n!)^2} \quad (\text{B-15})$$

Following Rice, (Ref. B.2)

$$E(R) = (2N)^{\frac{1}{2}} \Gamma\left(\frac{3}{2}\right) {}_1F_1\left(-\frac{1}{2}; 1; -\frac{A^2(1+m)^2}{2N}\right) \quad (\text{B-16})$$

where

$${}_1F_1\left(-\frac{1}{2}; 1; -x\right) = e^{-\frac{x}{2}} \left[(1+x) I_0\left(\frac{x}{2}\right) + x I_1\left(\frac{x}{2}\right) \right]$$

and

$$E(R^2) = A^2(1+m)^2 + 2N. \quad (\text{B-17})$$

Further, when a 0 is sent,

$$R(t) = \left\{ \left[A(1-m) + x(t) \right]^2 + y^2(t) \right\}^{\frac{1}{2}} \quad (\text{B-18})$$

and

$$p_o(R) = \frac{R}{N} I_o \left[\frac{A(1-m)R}{N} \right] \exp \left\{ -\frac{1}{2N} \left[R^2 + A^2(1-m)^2 \right] \right\} \quad (B-19)$$

Then, for envelope detection followed by sampling with a threshold Q , we have, for error probabilities,

$$P_I = \int_Q^\infty p_o(R) dR \quad (B-20)$$

and

$$P_{II} = \int_0^Q p_1(R) dR \quad (B-21)$$

giving

$$P = \frac{1}{2} \int_Q^\infty p_o(R) dR + \frac{1}{2} \int_0^Q p_1(R) dR = P(m, Q, \rho) \quad (B-22)$$

The formal procedure would require

$$\frac{\partial P}{\partial m} = 0 \quad \text{and} \quad \frac{\partial P}{\partial Q} = 0$$

resulting in a curve of P as a function of ρ alone for optimum choice of m and Q .

While it is not completely clear that $m = 1$ is the best choice, it has been used by several authors including Bennett who solves for Q as a function of ρ and finds that Q varies from 0.52 to 0.57 of the signal envelope $2A$. His independent variable is the ratio of the mean signal power in the "on" state to the mean noise power which is 2ρ or 2α . The results of Bennett's computations are given in Table B-1.

TABLE B-1

BIT ERROR FOR ENVELOPE DETECTION OF ON-OFF CW

P (Probability of error)	$\alpha = \rho$ (db)	α
10^{-2}	8.3	6.78
10^{-3}	10.6	11.55
10^{-4}	11.9	15.50
10^{-5}	13.1	20.40
10^{-6}	13.9	24.50
10^{-7}	14.8	30.10

The values in Table I are in fair agreement with those of Montgomery (Ref. B.3) who uses a threshold of 0.5 and whose R is expressed in terms of peak signal power.

Equation (B-7) gives the double-sideband signal as

$$s(t) = A [1 + m a(t)] \sin \omega_c t \quad (B-7)$$

In a coherent detector, the incoming signal is multiplied by a locally generated wave

$$v(t) = 2C \sin (\omega_c t + \phi). \quad (B-23)$$

The incoming wave is

$$\begin{aligned} s(t) + n(t) &= A [1 + m a(t)] \sin \omega_c t \\ &+ n_1(t) \sin \omega_c t + n_2(t) \cos \omega_c t \end{aligned} \quad (B-24)$$

Then the output of the multiplier is

$$\begin{aligned}
 z(t) = & 2A C [1 + m a(t)] \sin \omega_c t \sin (\omega_c t + \phi) \\
 & + 2Cn_1(t) \sin \omega_c t \sin (\omega_c t + \phi) \\
 & + 2Cn_2(t) \cos \omega_c t \sin (\omega_c t + \phi)
 \end{aligned} \tag{B-25}$$

$$\begin{aligned}
 z(t) = & A C [1 + m a(t)] [\cos \phi - \cos (2\omega_c t + \phi)] \\
 & + Cn_1(t) [\cos \phi - \cos (2\omega_c t + \phi)] \\
 & + Cn_2(t) [\sin (2\omega_c t + \phi) + \sin \phi]
 \end{aligned} \tag{B-26}$$

This signal, $z(t)$, is then passed through a low-pass filter to eliminate the components at $2\omega_c$ giving

$$\begin{aligned}
 W(t) = & AC [1 + m a(t)] \cos \phi + Cn_1(t) \cos \phi \\
 & + Cn_2(t) \sin \phi.
 \end{aligned} \tag{B-27}$$

Now $a(t) = \pm 1$

$n_1(t)$ and $n_2(t)$ are independently normally distributed with means zero and variances N .

Thus

$$E [W(t)] = AC [1 + m a(t)] \cos \phi. \tag{B-28}$$

and the variance of $W(t)$, given that the signal and noise are uncorrelated, is

$$\sigma^2 = C^2 N \cos^2 \phi + C^2 N \sin^2 \phi = C^2 N. \quad (B-29)$$

Suppose that a threshold Q is set at

$$Q = AC \cos \phi. \quad (B-30)$$

The probability of a Type I error, i.e., deciding that $a(t) = +1$ when it is in fact -1 is

$$P_I = \int_{Q=AC \cos \phi}^{\infty} \frac{1}{\sqrt{2\pi C^2 N}} \exp \left\{ -\frac{[W - AC(1-m) \cos \phi]^2}{2 C^2 N} \right\} dW \quad (B-31)$$

Let

$$x = \frac{W - AC(1-m) \cos \phi}{C \sqrt{N}}$$

$$dx = \frac{dW}{\sqrt{C^2 N}}$$

$$\text{When } W = AC \cos \phi \quad x = \frac{AC m \cos \phi}{C \sqrt{N}} = \frac{A m \cos \phi}{\sqrt{N}}$$

Then

$$P_I = \int_{\frac{A m \cos \phi}{\sqrt{N}}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{x^2}{2} \right) dx. \quad (B-32)$$

But the system is symmetrical and, therefore,

$$P = P_I = P_{II}.$$

For this system, Equations (B-10) and (B-11) give

$$\rho = \frac{A^2}{2} \frac{(1+m^2)}{N} \quad \text{and} \quad \alpha = \rho$$

Thus

$$\frac{A^2}{N} = \frac{2\rho}{(1+m^2)} = \frac{2\alpha}{(1+m^2)} \quad (\text{B-33})$$

and

$$P = \int_{\sqrt{\frac{2\alpha}{(1+m^2)}} \cdot m \cos \phi}^{\infty} \left(\frac{1}{\sqrt{2\pi}} \right) \exp \left(-\frac{x^2}{2} \right) dx \quad (\text{B-34})$$

For on-off keying $m=1$ and it is obvious that ϕ should be held as close to zero as possible. If $m = 1$ and $\phi = 0$

$$P = \int_{\sqrt{\alpha}}^{\infty} \left(\frac{1}{\sqrt{2\pi}} \right) \exp \left(-\frac{x^2}{2} \right) dx. \quad (\text{B-35})$$

Equation (B-35) is evaluated in Table B-2.

TABLE B-2

BIT ERROR FOR SYNCHRONOUS DETECTION OF ON-OFF CW

Probability of error	$\sqrt{\rho}$	$\alpha = \rho$ (db)	α
10^{-1}	1.28	2.15	1.64
10^{-2}	2.33	7.34	5.44
10^{-3}	3.10	9.84	9.60
10^{-4}	3.72	11.42	13.85
10^{-5}	4.26	12.61	18.18
10^{-6}	4.76	13.59	22.65
10^{-7}	5.20	14.32	27.05

B.3 PHASE MODULATION

A one is sent as $A \sin \omega t$ and a zero as $A \sin (\omega t + \pi) = -A \sin \omega t$. The signal power is $A^2/2$ and

$$\rho = \frac{A^2}{2N} \quad (B-36)$$

Again $B = \frac{1}{2T}$ and $N = 2BN_0$; therefore,

$$\alpha = \frac{E}{N_0} = \frac{ST}{N_0} = \frac{S}{2BN_0} = \frac{S}{N} = \rho$$

Let the incoming signal when a one is sent be

$$A \sin \omega t + n_1(t) \sin \omega t + n_2(t) \cos \omega t \quad (B-37)$$

and let the receiver multiply by $2 \sin (\omega t + \phi)$. Then the multiplier output is

$$\begin{aligned} x(t) &= 2A \sin \omega t \sin(\omega t + \phi) + 2n_1 \sin \omega t \sin(\omega t + \phi) \\ &\quad + 2n_2 \cos \omega t \sin(\omega t + \phi) \\ &= A [\cos \phi - \cos (2\omega t + \phi)] \\ &\quad + n_1 [\cos \phi - \cos (2\omega t + \phi)] \\ &\quad + n_2 [\sin \phi - \sin (2\omega t + \phi)] \end{aligned} \quad (B-38)$$

If this is passed through a low-pass filter to remove the components at twice the carrier frequency, there results

$$y(t) = A \cos \phi + n_1 \cos \phi + n_2 \sin \phi. \quad (B-39)$$

Then y is normally distributed with mean $A \cos \phi$ and variance

$$\sigma^2 = E(n_1^2) \cos^2 \phi + E(n_2^2) \sin^2 \phi = N \quad (\text{B-40})$$

Since transmission of a zero gives a mean of $-A \cos \phi$, the threshold can be set at zero.

Then a Type I error occurs when $-A \cos \phi + n_1 \cos \phi + n_2 \sin \phi > 0$ or

$$P_I = \int_0^{\infty} \frac{1}{\sqrt{2\pi N}} \exp \left[-\frac{1}{2N} (z + A \cos \phi)^2 \right] dz \quad (\text{B-41})$$

$$\text{Let } z' = \frac{z + A \cos \phi}{\sqrt{N}}$$

$$dz' = \frac{dz}{\sqrt{N}}$$

$$\text{when } z = 0 \quad z' = \frac{A \cos \phi}{\sqrt{N}} ;$$

therefore,

$$P_I = \int_{\frac{A \cos \phi}{\sqrt{N}}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{1}{2} z'^2 \right) dz' \quad (\text{B-42})$$

Due to symmetry of this system

$$P_{II} = P_I \quad \text{and, therefore,}$$

$$P = P_I = P_{II} \quad (\text{B-43})$$

From Equation (B-36) $\frac{A}{\sqrt{N}} = \sqrt{2\rho} = \sqrt{2\alpha}$

$$P = \int_{\sqrt{2\alpha} \cos \phi}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{1}{2} x^2 \right) dx. \quad (B-44)$$

If it is possible to hold $\phi = 0$,

$$P = \int_{\sqrt{2\alpha}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{x^2}{2} \right) dx. \quad (B-45)$$

Equation (B-45) is evaluated in Table B-3.

TABLE B-3

BIT ERROR FOR SYNCHRONOUS DETECTION OF $\pm 90^\circ$ PHASE MODULATION

Probability of error	$\sqrt{2\alpha}$	α (db)	α
10^{-1}	1.28	-0.91	0.82
10^{-2}	2.33	4.34	2.72
10^{-3}	3.10	6.84	4.81
10^{-4}	3.72	8.42	6.93
10^{-5}	4.26	9.61	9.09
10^{-6}	4.76	10.58	11.33
10^{-7}	5.20	11.32	13.52

B.4 FREQUENCY-SHIFT KEYING, ENVELOPE DETECTION

At the transmitter, one of two different frequencies, f_1 and f_2 ($f_2 - f_1 > 2B$), is transmitted. The received signal is passed through two bandpass filters of width $2B$ centered at the frequencies f_1 and f_2 . The filter outputs are envelope detected and subtracted. Their difference is compared to a zero threshold.

Suppose f_1 corresponds to a one and the noise density (one-sided) is N_0 ; hence the noise power at the output of each filter is $N = 2BN_0$.

Suppose a one is transmitted, then the amplitude, R , of the envelope out of Filter 1 (signal plus noise) is distributed as

$$p(R_1) = \frac{R_1}{N} I_0\left(\frac{AR_1}{N}\right) \exp\left[-\frac{1}{2N}(R_1^2 + A^2)\right] \quad (B-46)$$

where the received signal is

$$S_1(t) = A \cos(2\pi f_1 t + \theta). \quad (B-47)$$

Similarly the amplitude of the envelope out of Filter 2 (noise alone) is distributed as

$$p(R_0) = \frac{R_0}{N} \exp\left[-\frac{1}{2N}R_0^2\right]. \quad (B-48)$$

An error will result whenever

$$R_1 - R_0 < 0$$

or

$$R_0 > R_1. \quad (B-49)$$

Hence

$$\begin{aligned}
 P &= \int_0^{\infty} p(R_1) \left[\int_{R_1}^{\infty} p(R_o) dR_o \right] dR_1 \\
 &= \int_0^{\infty} \frac{R_1}{N} I_o \left(\frac{AR_1}{N} \right) \exp \left[-\frac{1}{2N} (R_1^2 + A^2) \right] \\
 &\quad \cdot \left[\int_{R_1}^{\infty} \frac{R_o}{N} \exp \left(-\frac{R_o^2}{2N} \right) dR_o \right] dR_1 \quad (B-50)
 \end{aligned}$$

Consider

$$\begin{aligned}
 \int_{R_1}^{\infty} \frac{R_o}{N} \exp \left[-\frac{R_o^2}{2N} \right] dR_o &= - \exp \left[-\frac{R_o^2}{2N} \right] \Big|_{R_1}^{\infty} \\
 &= \exp \left[-\frac{R_1^2}{2N} \right] \quad (B-51)
 \end{aligned}$$

Substituting Equation (B-51) in Equation (B-50)

$$\begin{aligned}
 P &= \int_0^{\infty} \frac{R_1}{N} I_o \left(\frac{AR_1}{N} \right) \exp \left[-\frac{1}{2N} (2R_1^2 + A^2) \right] dR_1 \\
 &= \exp \left[-\frac{A^2}{4N} \right] \int_0^{\infty} \frac{R_1}{N} I_o \left(\frac{AR_1}{N} \right) \exp \left[-\frac{1}{2N} (2R_1^2 + \frac{A^2}{2}) \right] dR_1 \quad (B-52)
 \end{aligned}$$

Now let $x = \sqrt{2} R_1$ and $a = \frac{A}{\sqrt{2}}$

$$\frac{AR_1}{N} = \frac{\sqrt{2} a \cdot \frac{x}{\sqrt{2}}}{N} = \frac{ax}{N}$$

B-17

$$2R_1^2 + \frac{A^2}{2} = x^2 + a^2$$

$$dR_1 = \frac{1}{\sqrt{2}} dx$$

$$P = \exp \left[-\frac{A^2}{4N} \right] \int_0^\infty \frac{x}{2N} I_0 \left(\frac{ax}{N} \right) \exp \left[-\frac{1}{2N}(x^2 + a^2) \right] \frac{dx}{\sqrt{2}}. \quad (B-53)$$

Therefore

$$P = \frac{1}{2} \exp \left[-\frac{A^2}{4N} \right] \quad (B-54)$$

since

$$\int_0^\infty \frac{x}{N} I_0 \left(\frac{ax}{N} \right) \exp \left[-\frac{1}{2N}(a^2 + x^2) \right] dx = 1 \quad (B-55)$$

is the integral of a probability density over its full range.

The significant parameter in Equation (B-54) is

$$\frac{A^2}{4N} = \frac{A^2/2}{2N} = \frac{S}{2 \cdot 2BN_0} = \frac{ST}{2N_0} = \frac{E}{2N_0} = \frac{\rho}{2}.$$

Therefore,

$$P = \frac{1}{2} e^{-\frac{\rho}{2}}.$$

Of course, $\rho = \frac{A^2}{2N}$; therefore $\frac{A^2}{4N} = \frac{\rho}{2}$

Equation (B-54) is evaluated in Table B-4.

TABLE B-4

BIT ERROR FOR FSK DUAL-CHANNEL ENVELOPE DETECTION

Probability of error	$\rho = \alpha$	$\rho = \alpha$ (db)
10^{-1}	3.22	5.06
10^{-2}	7.83	8.95
10^{-3}	12.44	10.96
10^{-4}	17.04	12.31
10^{-5}	21.64	13.37
10^{-6}	26.22	14.20
10^{-7}	30.80	14.88

B.5 ORTHOGONAL MATCHED-FILTER SYSTEMS

When separate signals of duration T and energy E are used to transmit ones and zeros, the system is orthogonal if

$$\int_0^T s_0(t) s_1(t) dt = 0. \quad (B-56)$$

Suppose the receiver multiplies the incoming signal by stored replicas of $s_0(t)$ and $s_1(t)$, integrates, and compares the integrator outputs at time T ; this is matched-filter reception.

Suppose a zero is transmitted; the output of the channel matched to $s_0(t)$ at time T will be

$$y_0(T) = \int_0^T s_0(t) [s_0(t) + n(t)] dt \quad (B-57)$$

while that of the channel matched to $s_1(t)$ will be

$$y_1(T) = \int_0^T s_1(t) [s_0(t) + n(t)] dt. \quad (B-58)$$

Since the signals and the noise are uncorrelated, the means will be

$$E [y_o(T)] = E \int_0^T s_o^2(t) dt = E \quad (B-59)$$

$$E [y_1(T)] = 0 \quad (B-60)$$

For the "zero" channel

$$E [y_o^2(T)] = E \int_0^T dt_1 \int_0^T dt_2 [s_o^2(t_1) + s_o(t_1)n(t_1)]$$

$$[s_o^2(t_2) + s_o(t_2)n(t_2)] = E^2 + E \frac{N_o}{2} \quad (B-61)$$

since

$$E [n(t_1)n(t_2)] = \frac{N_o}{2} \delta(t_1 - t_2). \quad (B-62)$$

Therefore,

$$\sigma^2 = E [y_o^2(T)] - E^2 [y_o(T)] = \frac{EN_o}{2}. \quad (B-63)$$

For the "one" channel

$$\sigma^2 = E [y_1^2(T)] = E \int_0^T dt_1 \int_0^T dt_2 [s_o(t_1)s_1(t_1) + s_1(t_1)n(t_1)]$$

$$[s_o(t_2)s_1(t_2) + s_1(t_2)n(t_2)] = E \frac{N_o}{2}. \quad (B-64)$$

Thus the output of the channel matched to the signal is normally distributed with mean E and variance $E \frac{N_o}{2}$ while the output of the orthogonal channel is normally distributed with mean 0 and variance $E \frac{N_o}{2}$.

An error will occur if the output of the unmatched channel, y_u , exceeds that of the matched channel y_m :

$$P = \int_{-\infty}^{\infty} \left\{ \frac{1}{\sqrt{2\pi \frac{EN_o}{2}}} \exp \left[-\frac{1}{2 \frac{EN_o}{2}} (y_m - E)^2 \right] \right. \\ \left. \cdot \left[\int_{y_m}^{\infty} \frac{1}{\sqrt{2\pi \frac{EN_o}{2}}} \exp \left(-\frac{1}{2 \frac{EN_o}{2}} y_u^2 \right) dy_u \right] \right\} dy_m \quad (B-65)$$

In Equation (B-65), let

$$x = \frac{y_m - E}{\sqrt{\frac{EN_o}{2}}}$$

then

$$P = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{1}{2} x^2 \right)$$

$$\cdot \left[\int_{\sqrt{\frac{EN_o}{2}} x + E}^{\infty} \frac{1}{\sqrt{2\pi \frac{EN_o}{2}}} \exp \left(-\frac{1}{2} y_u^2 \right) dy_u \right] dx$$

and letting $z = \frac{y_u}{\sqrt{\frac{EN_o}{2}}}$,

when $y_u = \sqrt{\frac{EN_o}{2}} x + E, \quad z = x + \sqrt{\frac{2E}{N_o}},$

$$P = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} x^2\right) \left[\int_{x + \sqrt{\frac{2E}{N_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} z^2\right) dz \right] dx. \quad (B-66)$$

Of course,

$$\sqrt{\frac{2E}{N_0}} = \sqrt{2\alpha}. \quad (B-67)$$

This function was computed by Viterbi (Ref. B.4) as tabulated below (Table B-5).

TABLE B-5

BIT ERROR RATE FOR MATCHED-FILTER RECEPTION OF ORTHOGONAL SIGNALS

Probability of error	α	α (db)
10^{-1}	1.61	2.07
10^{-2}	5.40	7.33
10^{-3}	9.55	9.80
10^{-4}	13.75	11.38
10^{-5}	17.95	12.50
10^{-6}	22.15	13.48
10^{-7}	26.35	14.20

B.6 BI-ORTHOGONAL MATCHED FILTER SYSTEMS

When a signal of duration T and energy E and its inverse are used to transmit ones and zeros, the system is bi-orthogonal. Matched-filter reception requires multiplication of the incoming signal by a stored replica of the signal, integration, and sampling at time T .

Suppose $s(t)$ corresponds to a one. If a one is transmitted, the integrator output at time T is

$$y(T) = \int_0^T s(t) [s(t) + n(t)] dt \quad (B-68)$$

while if a zero is transmitted,

$$y(T) = - \int_0^T s(t) [s(t) + n(t)] dt. \quad (B-69)$$

As was shown in Equations (B-59) and (B-63), $y(T)$ is normally distributed with mean $\pm E$ and variance $\frac{EN_0}{2}$.

An error will occur if the output of the integrator is of the wrong sign:

$$P = \int_{-\infty}^0 \frac{1}{\sqrt{2\pi \frac{EN_0}{2}}} \exp \left[- \frac{1}{2 \left(\frac{EN_0}{2} \right)} (y - E)^2 \right] dy. \quad (B-70)$$

Let

$$x = \frac{y - E}{\sqrt{\frac{EN_0}{2}}} \quad (B-71)$$

Then

$$P = \int_{-\infty}^{-\sqrt{\frac{2E}{N_0}}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right) dx$$

$$= \int_{\sqrt{\frac{2E}{N_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right) dx \quad (B-72)$$

Viterbi has also computed this case as tabulated below in Table B-6.

TABLE B-6

BIT ERROR FOR MATCHED-FILTER RECEPTION OF BI-ORTHOGONAL SIGNALS

Probability of error	α	α (db)
10^{-1}	0.82	-0.91
10^{-2}	2.72	4.34
10^{-3}	4.81	6.84
10^{-4}	6.93	8.42
10^{-5}	9.09	9.61
10^{-6}	11.33	10.58
10^{-7}	13.52	11.32

REFERENCES

- B.1 W. R. Bennett, "Methods of Solving Noise Problems," Proc. IRE, vol. 44, pp. 609-638; May 1956.
- B.2 S. O. Rice, "Mathematical Analysis of Random Noise," Bell System Tech. J., vol. 23, pp. 283-332; July, 1944; and vol. 24, pp. 46-156; January, 1945.
- B.3 G. F. Montgomery, "A Comparison of Amplitude and Angle Modulation for Narrow-Band Communications of Binary-Coded Messages in Fluctuation Noise," Proc. IRE, vol. 42, p. 447; February, 1954.
- B.4 A. J. Viterbi, "On Coded Phase-Coherent Communications," IRE Trans. on Space Electronics and Telemetry, vol. SET-7, p. 3; March, 1961.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. A. A. Alexander, R. M. Gryb, and D. W. Nast, "Capabilities of the Telephone Network for Data Transmission," Bell Sys. Tech. J., Vol. 39, pp. 431-476; May 1960
2. Armour Research Foundation, "Data Line Techniques," April 1961. AD 263 114.
3. Beckman Instruments, "Available Modulation Techniques for Digital Data Transmission," Beckman/Systems Division Technical Memorandum TM-145-A; December 1960.
4. W. R. Bennett and F. E. Froehlick, "Some Results on the Effectiveness of Error Control Procedures in Digital Data Transmission," IRE Trans. on Comm. Systems, vol. CS-9, pp. 58-65; March 1961.
5. A. B. Bodonyi, "Effect of Impulse Noise on Digital Data Transmission," IRE Trans. On Comm. Systems, Vol. CS-9, pp. 355-361; December 1961.
6. A. B. Brown and S. T. Meyers, "Evaluation of Some Error Correction Methods Applicable to Digital Data Transmission," IRE Nat. Conv. Record Part 4, pp. 37-55; 1958.
7. K. Bullington, "Radio Propagation Fundamentals," Bell Sys. Tech. J., Vol. 36, pp. 593-625; May 1957.
8. C. R. Cahn, "Performance of Digital Phase-Modulation Communication Systems," IRE Trans. on Comm. Systems, Vol. CS-7, pp. 3-6; May 1959.
9. C. R. Cahn, "Comparison of Coherent and Phase Comparison Detection of a Four-Phase Digital Signal," Proc. IRE, Vol. 47, pp. 1662-1663; September 1959.
10. S. S. L. Chang, "Theory of Information-Feedback Systems," IRE Trans. on Information Theory, Vol. IT-2, pp. 29-40; March 1956.
11. G. Comstock, "Low-Cost Terminals Triple Phone, TV Link Data Rates," Electronic Design, pp. 8-11; January 18, 1963.
12. A. B. Crawford and W. C. Jakes, "Selective Fading of Microwaves," Bell Sys. Tech. J., Vol. 31, pp. 68-90; January 1952.
13. R. A. De Kimpey, "Relation of Some Data Transmission Error Statistics to the Data Line Translator," Beckman/Systems Division Technical Memorandum No. TM-166; 5 April 1961.
14. R. G. Enticknap and R. M. Lerner, "The Design of Data Transmission Systems," MIT Lincoln Laboratory Report 25G-0022; October 1960. AD 243 926.

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15. R. G. Enticknap, "Errors in Data Transmission Systems," IRE Trans. on Comm. Systems, Vol. CS-9, pp. 15-20; March 1961.
16. G. L. Evans, E. Enriquez, and Q. C. Wilson, "A High-Speed, Serial, Four-Phase Data Modem for Regular Telephone Circuits," GLOBECOM V Convention Record, pp. 100-104.
17. A. B. Fontaine and R. G. Gallager, "Error Statistics and Coding for Binary Transmission over Telephone Circuits," Proc. IRE, Vol. 49, pp. 1059-1065; June 1961.
18. E. N. Gilbert, "Capacity of a Burst Noise Channel," Bell Sys. Tech. J., Vol. 39, pp. 1253-1265; September 1960.
19. M. W. Gough, "Microwave Line-of-Sight Propagation," Electronic Engineering, Vol. 30, pp. 237-247; May 1958.
20. P. L. Grant, "COBI Data Transmission Testing in the United Kingdom," MIT Lincoln Laboratory Report No. 25G-8; 27 March 1962; AD 275 152.
21. J. W. Halina, "The Telephone Channel in a Global Communications System," IRE Trans. on Comm. Systems, Vol. CS-9, pp. 247-251; September 1961.
22. E. J. Hofmann, "Measured Error Distributions on the Bell A-1 Facility Over Various Media," Proc. NEC., pp. 37-44; 1960.
23. E. J. Hofmann and P. L. Grant, "Error Statistics on a Long-Haul Looped Telephone Circuit Utilizing the Milgo Data System," MIT Lincoln Laboratory Report 25G-0010; June 1960.
24. E. J. Hofmann, "Collected Error Statistics Utilizing Four Modems Over Various Telephone Circuits," MIT Lincoln Laboratory Report 25G-0028; January 1961. AD 251 571.
25. E. J. Hofmann, "Specifications for a COBI Data Transmission Modem," MIT Lincoln Laboratory Report 25G-0035; July 1961.
26. E. Hopner, "An Experimental Modulation-Demodulation Scheme for High-Speed Data Transmission," IBM Journal, Vol. 3, pp. 74-84; January 1959.
27. A. W. Horton, Jr., H. E. Vaughan, "Transmission of Digital Information Over Telephone Circuits," Bell Sys. Tech. J., Vol. 34, pp. 511-528; May 1955.
28. Hughes Aircraft Co., "Survey of Digital Communications," Comm. Div. Report TR-1; April 1961. AD 277 176.

29. Stelma , Inc., "Development of Digital Data Terminal AN/TYC-1," Final Report from Stelma , Incorporated, Stamford, Connecticut; 30 November 1961. AD 271 985.
30. W. T. Jones, "Transmission of Data Over Telephone-Type Circuits," Electrical Communication, Vol. 37, pp. 85-102; 1961.
31. R. L. Kaylor, "A Statistical Study of Selective Fading of Super-High Frequency Radio Signals," Bell Sys. Tech. J., Vol. 32, pp. 1187-1202; September 1953.
32. J. P. Kelly, J. F. Mercurio, Jr., V. S. Pranzone, "Comparative Performance of Digital Data Modems," The MITRE Corp. Technical Memorandum TM-3037 and TM-3037 S.1; April 1961 and August 1961.
33. C. Laughlin, R. Sullivan, "A Comparison of Error Rates for Coherent and Phase Comparison Detection of Two- and Four-Phase Digital Signals in the Presence of Rayleigh Carrier Fading," Proc. IRE, Vol. 50, pp. 468-469; April 1962.
34. J. G. Lawton, "Theoretical Error Rates of Differentially Coherent Binary and Kineplex Data Transmission Systems," Proc. IRE, Vol. 47, pp. 333-334; February 1959.
35. T. A. Maguire, E. P. G. Wright, "The Examination of Error Distributions for the Evaluation of Error Detection and Error Correction Procedures," IRE Trans. on Comm. Systems, Vol. CS-9, pp. 101-106; July 1961.
36. M. Maniere, R. Benoit-Gonin, "Telegraph Distortion on High-Speed Frequency Shift Data Transmission Systems," IRE Trans. on Comm. Systems, Vol. CS-9, pp. 259-270; September 1961.
37. J. E. Meggitt, "Error Correcting Codes and Their Implementation for Data Transmission Systems," IRE Trans. on Information Theory, Vol. IT-7, pp. 234-244; October 1961.
38. P. Mertz, D. Mitchell, "Transmission Aspects of Data Transmission Service Using Private Line Voice Telephone Channels," Bell Sys. Tech. J., Vol. 36, pp. 1451-1486; November 1957.
39. P. Mertz, "Model of Impulsive Noise for Data Transmission," IRE Nat. Conv. Record, Part 5, pp. 247-259; March 1960.
40. P. Mertz, "Model of Error Burst Structure in Data Transmission," Proc. Nat. Electronics Conf., pp. 232-237; October 1960.
41. J. J. Metzger, K. C. Morgan, "Coded Binary Decision-Feedback Communication System," IRE Trans. on Comm. Systems, Vol. CS-8, pp. 101-112; June 1960.

42. J. J. Metzger, K. C. Morgan, "Coded Feedback Communications Systems," Proc. Nat. Electronics Conf., pp. 250-257; October 1960.
43. K. H. Morey, "The COBI Data Transmission Modem," MIT Lincoln Laboratory Tech. Report 263; May 1962.
44. R. Mosier, R. Clabaugh, "Kineplex, A Binary Transmission System," Trans. AIEE (Comm. and Elect.), pp. 723-727; January 1958.
45. B. Reiffen, W. Schmidt, H. Yudkin, "Design of an Error-Free Data Transmission System for Telephone Circuits," Proc. AIEE (Comm. and Elect.), pp. 224-231; July 1961.
46. L. S. Schwartz, "Some Recent Developments in Digital Feedback Communication Systems," IRE Trans. on Comm. Systems, Vol. CS-9, pp. 51-57; March 1961.
47. C. R. Scott and W. H. Butler, "Real-Time Data Transmission System," IRE Trans. on Communications Systems, Vol. CS-7, No. 3, pp. 201-205; September 1959.
48. K. L. Smith, J. Bowen, L. A. Joyce, "Telephone Circuit Evaluation for Data Transmission," British IRE, Vol. 24, pp. 297-307; October 1962.
49. J. E. Toffler and J. N. Buterbaugh, "The HC-270, a Four-Phase Digital Data Transceiver," presented at the 1961 Western Electronic Show and Convention, San Francisco, Part 36/1.
50. UNICOM, Progress Report for the Ninth Quarter, 1 July through 30 September 1961, Vol. II; 30 September 1961.
51. United Aircraft, "A Survey of High-Speed Data Communications," Weather Systems Center, United Aircraft Corporation; 22 June 1962. AD 283 924.
52. J. H. Vogelmann, "Comparative Evaluation of Communications Transmission Media," IRE Trans. on Comm. Systems, Vol. CS-9, pp. 230-231; December 1959.
53. Westinghouse Electric Corp., "Tropospheric Scatter AJ Evaluation," May 1962. AD 276 695.
54. J. M. Wier, "Digital Data Communication Techniques," Proc. IRE, Vol. 49, pp. 196-210; January 1961.
55. G. A. Wildhagen, "Some Results of Data Transmission Tests Over Leased Telephone Circuits," IRE Trans. on Comm. Systems, Vol. CS-9, pp. 271-274; September 1961.

56. J. M. Wozencraft, M. Horstein, "Coding for Two-Way Channels," Fourth London Symp. on Information Theory; August 1960.
57. E. P. G. Wright, "Error Rates and Error Distributions on Data Transmitted Over Switched Telephone Connections," IRE Trans. on Comm. Systems, Vol. CS-9, pp. 12-15; March 1961.
58. H. L. Yudkin, "Some Results in the Measurement of Impulse Noise on Several Telephone Circuits," Proc. Nat. Electronics Conf., pp. 222-231; October 1960.
59. H. L. Yudkin, "Experiments in the Improvement of the Impulse Response of Telephone Circuits." AD 269 557.

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