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FLUID AMPLIFICATION

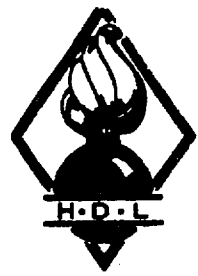
5. Jet Attachment Distance as a Function of Adjacent Wall Offset and Angle

Sheldon G. Levin

Francis M. Manion

31 December 1962

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FOR THE COMMANDER:
Approved by

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CONTENTS

	Page
ABSTRACT	5
1. INTRODUCTION	5
2. THEORETICAL DEVELOPMENT	5
2.1 Analytic Base	5
2.2 Notation	6
2.3 Derivation of Equations	8
2.3.1 Parameter t	8
2.3.2 Attachment Angle	11
2.3.3 Geometry of Attachment	15
2.3.4 Entrainment	17
2.4 Computation	18
3. EXPERIMENTAL PROGRAM	20
3.1 Air-Jet Test Model	20
3.2 Water-Jet Test Model	24
3.3 Comparison of Results	25
4. SUMMARY	25
5. CONCLUSIONS	29
6. REFERENCES	30
Appendix A. Derivation of $R = J/\Delta p$	31

LIST OF ILLUSTRATIONS

Figure

- 1 Mathematical model of the attachment of a two-dimensional jet to an offset, inclined adjacent wall
- 2 Jet stream velocity profiles
- 3 Comparison of Goertler's equation prediction to assumed flow at nozzle exit
- 4 Mathematical model for control volume and attachment point theories
- 5 Schematic of experimental air-jet arrangement
- 6 Isometric view of experimental air-jet arrangement
- 7 Errors in measuring attachment distance
- 8 University of Maryland water table
- 9 University of Maryland water table with mathematical model superimposed
- 10 Schematic of experimental water table arrangement
- 11 Comparison of observed attachment distances with theoretical computations based on control volume model, $\sigma = 1$
- 12 Comparison of observed attachment distances with theoretical computations based on control volume model, $\sigma = 3$
- 13 Comparison of observed attachment distances with theoretical computations based on control volume model, $\sigma = 5$
- 14 Comparison of observed attachment distances with theoretical computations based on control volume model, $\sigma = 7$
- 15 Comparison of observed attachment distances with theoretical computations based on control volume model, $\sigma = 8$
- 16 Comparison of observed attachment distances with theoretical computations based on control volume model, $\sigma = 10$
- 17 Comparison of observed attachment distances with theoretical computations based on control volume model, $\sigma = 15$
- 18 Comparison of observed attachment distances with theoretical computations based on attachment point model, $\sigma = 15$
- 19 Comparison of observed attachment distances with theoretical computations based on attachment point model, $\sigma = 20$
- 20 Comparison of observed attachment distances with theoretical computations based on attachment point model, $\sigma = 25$
- 21 Comparison of observed attachment distances with theoretical computations based on attachment point model, $\sigma = 30$

ABSTRACT

Attachment of a submerged, incompressible, two-dimensional, turbulent jet to an adjacent straight wall (Coanda effect) is analyzed. Parametric equations are developed that predict the point at which the jet attaches as a function of wall angle and offset distance. Computer solutions were obtained for several sets of conditions. Experiments were conducted with both air and water jets at Mach 0.5 equivalent, and results agree well with corresponding computer solutions when the jet spread parameter is also treated as a function of offset distance and wall angle. The equations provide an analytic method, independent of the particular fluid, for predicting the attachment distance, and should be helpful in designing elements based on the Coanda effect; e.g., the fluid flip-flops or bistable elements.

1. INTRODUCTION

The steady-state deflection and attachment of a jet to a nearby surface has been referred to as the Coanda effect. The effect with respect to two-dimensional flow is of interest since two-dimensional flow is approximated in many fluid devices where the flow is constrained between two opposing parallel flat plates, and the proper design of many elements using the Coanda effect requires a means for calculating the attachment distance as wall angle and offset distance are varied.

Borque and Newman (ref 1) as well as Sawyer (ref 2) discuss the deflection and reattachment of a two-dimensional, submerged, incompressible, turbulent jet to an adjacent flat plate, and develop equations to predict the attachment distance as a function of the distance that the wall is offset from the nozzle exit, or, alternatively, as a function of the angle between the wall with zero offset and the axis of the nozzle. A study was initiated to develop a more general expression of attachment distance as a function of both parameters. The study included a theoretical analysis and derivation, computer solutions, and tests with air and water jets.

2. THEORETICAL DEVELOPMENT

2.1 Analytic Base

The Coanda effect, as discussed here, arises as follows. A submerged jet (fig. 1) entrains fluid by means of viscous interaction between the moving stream and the quiescent surrounding fluid. If a wall is located such that the replacement flow is impeded, the static pressure in the fluid between the jet and wall decreases, and the pressure differential across the jet deflects the jet towards the wall. The pressure differential is self-reinforcing until the jet is deflected far enough to attach to the wall. A bubble of reduced static pressure is inclosed between the jet stream and wall maintaining the attachment.

A steady-state flow pattern arises where the mass entrained by the jet from the bubble is returned to the bubble near the attachment point.

The analytic development of a general expression describing the steady-state flow pattern requires the following assumptions:

- (a) The jet flow is incompressible and two-dimensional.
- (b) Jet velocity is uniform at the nozzle exit.
- (c) The jet velocity is independent of the reduced pressure in the bubble.
- (d) The pressure within the separation bubble is uniform.
- (e) Jet momentum flux is conserved, i.e., drag losses due to constraining plates are neglected.
- (f) The centerline of the jet is a circular arc of radius R .
- (g) The nozzle width is small compared with R ; and the attachment-wall length is long compared with the nozzle width.
- (h) The jet exhibits turbulent flow after emerging from the nozzle; i.e., the Reynolds number is high.
- (i) Changes in the jet structure due to the centrifugal force of curvature are negligible.

2.2 Notation

Figure 1 illustrates the jet model. The notation used in figure 1 and in the derivation is as follows:

- D = distance from attachment wall to the side of the nozzle (ft)
- J = jet momentum flux per unit depth (lbf/ft)
- p_o = stagnation pressure of the fluid supplying the jet (lbf/ft²)
- p_B = static pressure of fluid in bubble (lbf/ft²)
- p_{∞} = static pressure of fluid surrounding the jet (lbf/ft²)
- Q = volume flow (ft³/sec)
- Q_e = volume flow at nozzle exit (ft³/sec)
- Q_T = total volume flow at a distance s from the nozzle exit (ft³/sec)
- R = radius of a theoretical circular arc described by the jet centerline (ft)
- s = arbitrary distance along the jet centerline from the nozzle exit (ft)
- s_o = distance from the hypothetical (apparent) origin of the jet to the nozzle exit.
- t = parameter from Goertler's equation for jet velocity profile (dimensionless)

u = jet stream velocity (fps)
 u_e = uniform jet stream velocity at the exit (fps)
 u_0 = jet stream velocity at the centerline (fps)
 w = nozzle width (ft)
 y = distance from the jet centerline to an arbitrary point measured on a line normal to the centerline (ft)
 y' = distance from the jet streamline that passes through the attachment point to the jet centerline, measured on a line normal to the centerline
 x = distance from the attachment point to the offset wall measured along the attachment wall (ft)
 α = angle between the attachment wall and a line parallel to the nozzle axis (rad or deg)
 θ = angle between the jet centerline and the attachment wall (rad or deg)
 ρ = density of the fluid (lbm/ft³)
 σ = spread parameter for a free turbulent jet (dimensionless)
 $g_c = 32.2 \text{ lbm-ft/sec-lbf}$

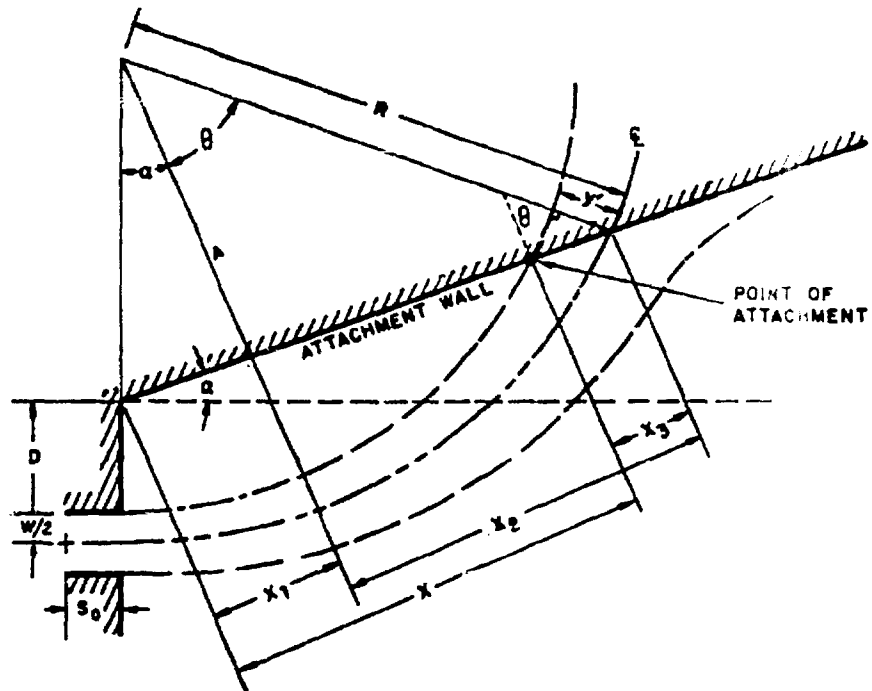


Figure 1. Mathematical model of the attachment of a two-dimensional jet to an offset, inclined adjacent wall.

2.3 Derivation of Equations

2.3.1 Parameter (t)

The parameter t , which is taken from the velocity-profile equation given by Goertler (ref 3), is derived as follows (fig. 1):

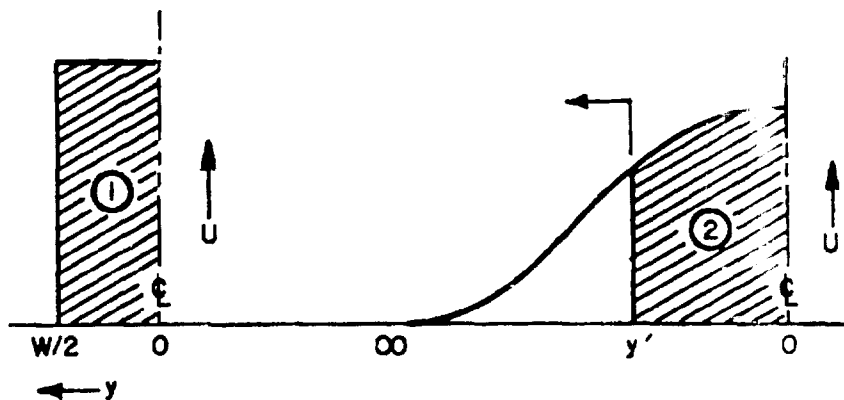
Goertler gives the jet stream velocity u as a function of the distance s the jet has traveled, the distance y from the centerline, and the jet centerline velocity u_0 at a distance s from the nozzle exit, thus,

$$u = u_0 \operatorname{sech}^2 \left(\frac{\sigma y}{s + s_0} \right) \quad (1)$$

$$u_0 = \left[\frac{3J\sigma g_c}{4\rho(s + s_0)} \right]^{1/2}$$

where s_0 is the distance from the nozzle exit to the apparent jet origin (the point from which the jet appears to emanate) y is an arbitrary distance from the centerline, σ is the jet spread parameter, and J is the jet momentum flux.

The velocity profile at the nozzle exit is uniform (fig. 2a) by assumption. A representative jet velocity profile at s per Goertler's expression is illustrated in figure 2b.



(a) Jet stream velocity profile assumed at $s = 0$

(b) Jet stream velocity profile at a distance $s > 0$ per Goertler's expression

Figure 2. Jet stream velocity profiles.

In general, volume flow is not conserved, but by assumption, there exists a line of constant volume flow which is called the attachment streamline. The attachment streamline is a distance $w/2$ from the jet centerline at the nozzle exit and a distance y' from the centerline at a distance s from the nozzle. The fluid is incompressible and two dimensional by assumption; hence, one half the jet volume flow is

$$\frac{Q}{2} = \int_0^{y'} u dy$$

To solve for s , one half the volume flow $Q/2$ at the nozzle exit is first assumed to be equal to one half the volume flow $Q/2$ at $s > 0$ using Goertler's expression, i.e.,

$$\frac{Q_e}{2} = \frac{Q}{2} \text{ or } u_e \frac{w}{2} = \int_0^{y'} u dy \quad (2)$$

where w is the nozzle width and u_e is the velocity at the nozzle exit. Substituting (1) into (2)

$$\int_0^{y'} \left[\frac{3J\sigma g_c}{4\rho(s + s_o)} \right]^{1/2} \text{sech}^2 \left(\frac{\sigma y}{s + s_o} \right) dy = u_e \frac{w}{2}$$

By integrating and noting that $\tanh 0 = 0$,

$$\left[\frac{3}{4} \frac{J\sigma g_c}{\rho(s + s_o)} \right]^{1/2} \left(\frac{s + s_o}{\sigma} \right) \tanh \left(\frac{\sigma y'}{s + s_o} \right) = u_e \frac{w}{2}$$

which simplifies to

$$\left[\frac{3}{4} \frac{J(s + s_o)g_c}{\rho\sigma} \right]^{1/2} \tanh \left(\frac{\sigma y'}{s + s_o} \right) = u_e \frac{w}{2} \quad (3)$$

which is an expression for the volume flow for half the stream at the nozzle exit. Since the jet momentum flux at the nozzle exit is

$J = \frac{\rho u_e^2 w}{g_c}$, the normalized volume flow for half the stream becomes

$$\left[\frac{3(s + s_o)}{w\sigma} \right]^{1/2} \tanh \left(\frac{\sigma y'}{s + s_o} \right) = 1$$

Then the dimensionless parameter t is defined as

$$t = \tanh \frac{\sigma y}{s + s_0} \quad (4)$$

and

$$t'^2 = \tanh^2 \left(\frac{\sigma y'}{s + s_0} \right) = \frac{w\sigma}{3(s + s_0)} \quad (5)$$

which is the equation of the streamline.

Figure 3 shows that Goertler's equation does not adequately represent the volume flow at the nozzle exit. However, the approximation is made: the volume flow at the nozzle exit $u_e(w/2)$ is forced to be equal to the volume flow at the nozzle exit given by Goertler's equation $\int_0^\infty u dy$. This permits an evaluation of s_0 .

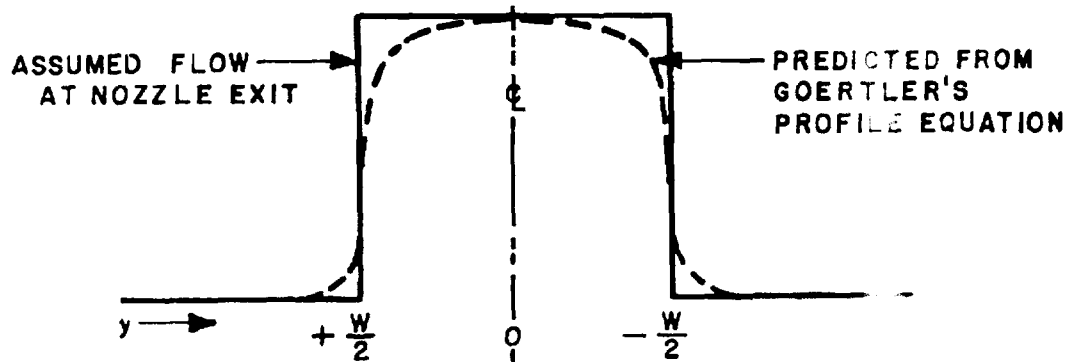


Figure 3. Comparison of Goertler's equation prediction with assumed flow at nozzle exit.

$$\int_0^\infty u dy = u_e \frac{w}{2}$$

and proceeding as with equation (2)

$$\left[\frac{3}{4} \frac{J(s + s_0) g_0}{\rho \sigma} \right]^{1/2} \tanh \left(\frac{\sigma y}{s + s_0} \right) \Big|_0^{\infty} = u_e \frac{w}{2}$$

$$\left[\frac{3}{4} \frac{J(s + s_0) g_c}{\rho \sigma} \right]^{1/2} = u_e \frac{w}{2} \quad (6)$$

At the nozzle exit $s = 0$ and $J = \frac{\rho u_e^2 w}{g_c}$

then

$$\left[\frac{3}{4} \frac{\rho u_e^2 w s_0 g_c}{4 \rho \sigma} \right]^{1/2} = u_e \frac{w}{2}$$

which becomes

$$s_0 = \frac{\sigma w}{3} \quad (7)$$

Substituting (7) into (5) yields

$$\frac{3s}{\sigma w} = \frac{1}{t'^2} - 1 \quad (8)$$

for the attachment streamline.

2.3.2 Attachment Angle

Two approaches that may be used to express the attachment angle θ as a function of t are illustrated in figure 4. The first

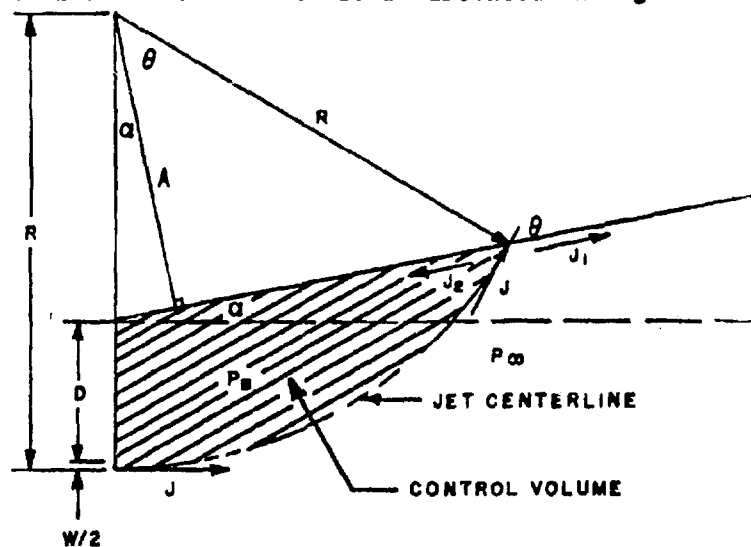


Figure 4. Mathematical model for control-volume and attachment-point theories.

approach evaluates the forces (J , J_1 , and J_2) acting at the attachment point and assumes that the momentum flux component parallel to the wall is conserved. This representation of the attachment mechanism will be called the attachment-point model. The second approach considers the forces due to $(p_\infty - p_B)$ acting on the control volume inclosed by the centerline of the jet, the attachment wall and offset wall, shown as the shaded region in figure 4. The two models considered are attempts to represent the mechanism of attachment. There seemed no reason to prefer either model, and both were developed.

Attachment-Point Model—An expression for θ in terms of t is obtained as follows: From figure 4 one can write

$$J_1 - J_2 = J \cos \theta \quad (9)$$

The J 's can be written as integrals of the form

$$\int \rho u^2 dy$$

Using the Goertler equation (1), this becomes

$$\int \rho u^2 dy = \rho \left(\frac{3J\sigma}{4\rho(s + s_0)} \right) \left(\frac{s + s_0}{\sigma} \right) \int \operatorname{sech}^4 \left(\frac{\sigma y}{s + s_0} \right) d \left(\frac{\sigma y}{s + s_0} \right)$$

Integrating and substituting the value of t from (4)

$$J_2 = \int_{y'}^{\infty} \rho u^2 dy = \frac{J}{4} (3t - t^3) \Big|_{y'}^{\infty}$$

Since $t = \tanh \frac{\sigma y}{s + s_0}$, $\tanh 0 = 0$, and $\tanh \infty = 1$,

$$J_2 = \frac{J}{4} (3-1) - \frac{J}{4} (3t' - t'^3) = \frac{J}{2} - \frac{J}{4} (3t' - t'^3) \quad (10)$$

Now

$$J_1 = \int_{-\infty}^{y'} \rho u^2 dy = \int_{-\infty}^0 \rho u^2 dy + \int_0^{y'} \rho u^2 dy$$

Since

$$J = \int_{-\infty}^{\infty} \rho u^2 dy$$

and since u is symmetric

$$\frac{J}{2} = \int_{-\infty}^0 \rho u^2 dy = \int_0^{\infty} \rho u^2 dy$$

Thus

$$J_1 = \frac{J}{2} + \int_0^{y'} \rho u^2 dy$$

Proceeding as with J_2 ,

$$J_1 = \frac{J}{2} + \frac{J}{4} (3t' - t'^3) \quad (11)$$

Inserting the values of J_1 and J_2 from (10) and (11) into (9)

$$J \cos \theta = J \left(\frac{1}{2} + \frac{3}{4} t' - \frac{1}{4} t'^3 \right) - J \left(\frac{1}{2} - \frac{3}{4} t' + \frac{1}{4} t'^3 \right)$$

and finally

$$\cos \theta = \frac{3}{2} t' - \frac{t'^3}{2} \quad (12)$$

Control-Volume Model—The force equation states that the momentum flux returned to the low-pressure region p_B balances the pressure difference times the area normal to the wall. This can be expressed as

$$J \cos \alpha - J_1 = (p_{\infty} - p_B) \left(D + \frac{w}{2} \right) \cos \alpha \quad (13)$$

From figure 4

$$\cos \alpha = \frac{A}{R - D - w/2} \text{ and } \cos \theta = \frac{A}{R}$$

thus

$$R - \frac{A}{\cos \alpha} = D + \frac{w}{2}$$

Substituting $\cos \theta = \frac{A}{R}$ gives

$$R \left(1 - \frac{\cos \theta}{\cos \alpha} \right) = D + \frac{w}{2} \quad (14)$$

Using (14) and the approximation $\Delta P = \frac{J}{R}$ (justified in appendix A) in (13)

$$J \cos \alpha - J_1 = \left(\frac{J}{R}\right) R \left(1 - \frac{\cos \theta}{\cos \alpha}\right) \cos \alpha$$

which simplifies to

$$\cos \alpha - \frac{J_1}{J} = \cos \alpha - \cos \theta$$

Hence

$$\cos \theta = \frac{J_1}{J} \quad (15)$$

which is equivalent to (9) with $J_2 = 0$.

Substituting the value of J_1/J from equation (11) gives

$$\cos \theta = \frac{\frac{J}{4} (3t' - t'^3) + \frac{J}{2}}{J} = \frac{1}{2} + \frac{3}{4} t' - \frac{t'^3}{4} \quad (16)$$

As in (12), $\cos \theta$ again involves only y , s , and σ , and not α . Equations (12) and (16) are the same as those obtained by Borque and Newman even though the wall angle α has been considered in this derivation.

Comparison of Models—Two different expressions for $\cos \theta$ as a function of t' have been derived. In the process, two expressions relating (J, J_1, J_2) and $\cos \theta$ have occurred.

$$\text{attachment-point model: } J_1 - J_2 = J \cos \theta \quad (9)$$

$$\text{control-volume model: } J_1 = J \cos \theta \quad (15)$$

The primary reason for this difference in (9) and (15) is that the difference of pressure $p_\infty - p_B = \Delta p$ was neglected in the attachment-point model. The reaction of the stream to J_2 tends to move the attachment point downstream, hence to increase x . The effect of Δp on the jet is to decrease x by shrinking the trapped bubble. Thus the consideration of Δp tends to reduce the magnitude of J_2 . If the control volume model were more nearly correct—as measured by the discrepancy between the theory and experimental data—then one might conclude that the effect of Δp tends to cancel J_2 . However, if the attachment-point model agreed more closely with experimental data, then this would not be so.

2.3.3 Geometry of Attachment

Substituting (7) into (4)

$$\tanh^{-1} t' = \frac{\sigma y'}{s + \frac{\sigma w}{3}}$$

Then solving for y' and substituting the value of $3s/\sigma w$ from (8)

$$y' = \frac{3s + \sigma w}{3\sigma} \tanh^{-1} t' = \frac{w}{3} \left[\frac{1}{t'^{1/2}} - 1 + 1 \right] \tanh^{-1} t'$$

Finally

$$y' = \frac{w}{3t'^{1/2}} \tanh^{-1} t' \quad (17)$$

For the case where s is the distance from the nozzle exit to the attachment point, (fig. 4)

$$s = R (\theta + \alpha) \quad (18)$$

and combining (8) and (18) yields

$$\frac{1}{t'^{1/2}} - 1 = \frac{3R(\theta + \alpha)}{\sigma w}$$

Hence

$$\frac{R}{w} = \frac{\sigma}{3(\theta + \alpha)} \left(\frac{1}{t'^{1/2}} - 1 \right) \quad (19)$$

Again using figure 4

$$A = (R - D - \frac{w}{2}) \cos \alpha = R \cos \theta$$

or

$$R - D - \frac{w}{2} = \frac{R \cos \theta}{\cos \alpha}$$

solving for $\frac{D}{w}$

$$\frac{D}{w} = \frac{R}{w} \left(1 - \frac{\cos \theta}{\cos \alpha} \right) - \frac{1}{2} \quad (20)$$

Now substituting (19) into (20)

$$\frac{D}{w} = \frac{\sigma}{3(\theta + \alpha)} \left(\frac{1}{t'^2} - 1 \right) \left(1 - \frac{\cos \theta}{\cos \alpha} \right) - \frac{1}{2} \quad (21)$$

This is the first of the required parametric equation. It reduces to the form found by Borque and Newman as shown below.

At $\alpha = 0$, (21) becomes

$$\frac{D}{w} = \frac{\sigma}{3\theta} \left(\frac{1}{t'^2} - 1 \right) (1 - \cos \theta) - \frac{1}{2}$$

which is Borque's equation (15); and at $\alpha = \theta$, it reduces to $-\frac{1}{2}$. (The negative sign arises from the convention that positive D is directed away from the centerline of the jet while positive R is directed away from the common vertex of α and θ .)

Referring to figure 1,

$$x_1 = (R - D - \frac{w}{2}) \sin \alpha$$

$$x_2 = R \sin \theta$$

$$x_3 = y' / \sin \theta$$

$$\text{and } x = x_1 + x_2 - x_3$$

Combining these yields

$$\frac{x}{w} = \frac{(R - D - \frac{w}{2}) \sin \alpha}{w} + \frac{R \sin \theta}{w} - \frac{y'}{w \sin \theta} \quad (22)$$

Substituting (17) and (19) into (22)

$$\frac{x}{w} = \frac{\sigma}{3(\theta + \alpha)} \left(\frac{1}{t'^2} - 1 \right) (\sin \alpha + \sin \theta) - \frac{\tanh^{-1} t'}{3t'^2 \sin \theta} - \left(\frac{D}{4} + \frac{1}{2} \right) \sin \alpha \quad (23)$$

which is the second of the required parametric equations. If the value $\alpha = 0$ is used (23) reduces to

$$\frac{x}{w} = \frac{\sigma}{3\theta} \left(\frac{1}{t'^2} - 1 \right) \sin \theta - \frac{\tanh^{-1} t'}{3t'^2 \sin \theta}$$

which is Borque's equation (17) for the case, $\alpha = 0$.

Taking $\alpha = 0$ and $\frac{D}{w} = -\frac{1}{2}$

$$\frac{x}{w} = \frac{\sigma}{3\alpha} \left(\frac{1}{t'^{1/2}} - 1 \right) \sin \alpha - \frac{\tanh^{-1} t'}{3t'^{1/2} \sin \alpha}$$

which is Borque's equation (23) for the 0 offset case assuming $\alpha = 0$. Since $\frac{D}{w}$ is measured from the centerline, $-\frac{1}{2}$ corresponds to 0 offset.

This more general theory is expected to give more accurate results for small angles when an offset $\frac{D}{w} \geq 1$ is used, since the separation bubble will be larger and the assumption that $\alpha = 0$ is not necessary.

2.3.4 Entrainment

The parameter σ is a floating constant that accounts for the geometrical spread of the jet due to entrainment. Previous experimental work by Reichardt (ref 4) estimated the value of σ as 7.67 for a straight jet; however, due to the curvature in this case, the entrainment, hence the value of σ , would be expected to be different. To obtain an expression for the entrained volume flow in terms of s and σ , (3) is required.

$$\left[\frac{3}{4} \frac{J(s+s_0)}{\rho\sigma} \right]^{1/2} \tanh \frac{\sigma y}{s+s_0} = \frac{Q}{2}$$

If y is taken from 0 to ∞ , then $\frac{Q_T}{2}$ is equal to the total volume flow on one side of the jet centerline, including the entrained flow. Thus

$$\left[\frac{3}{4} \frac{J(s+s_0)}{\rho\sigma} \right]^{1/2} \tanh(\infty) = \frac{Q_T}{2} \quad (24)$$

or

$$\left[\frac{3}{4} \frac{J(s+s_0)}{\rho\sigma} \right]^{1/2} = \frac{Q_T}{2}$$

and since

$$J = \rho u_0^2 w$$

(24) becomes

$$\frac{u_0^2}{2} \left[3 \left(\frac{s+s_0}{w\sigma} \right) \right]^{1/2} = \frac{Q_T}{2} \quad (25)$$

Since $\frac{Q_e}{2} = \frac{u_e w}{2}$ from (2)

and defining $q' = \frac{Q_T}{u_e w} = \frac{Q_T}{Q_e}$

and substituting into (25)

$$q' = \left[3 \frac{(s + s_0)}{w\sigma} \right]^{1/2} \quad (26)$$

The entrained flow, i.e., the excess over the flow at the nozzle exit is $Q_T - Q_e$ and is normalized as

$$\frac{Q_T - Q_e}{Q_e} = q' - 1$$

At the origin $Q_e = Q_T$ so that the entrained flow is zero. Hence rewriting (26) as

$$q' - 1 = \left[\frac{3(s + s_0)}{w\sigma} \right]^{1/2} - 1 \quad (27)$$

yields an expression for the normalized entrained volume flow in terms of s and σ . The value of q' varies directly as s , the distance the stream travels, and inversely as σ , the spread parameter and w , the nozzle width. It also must be true that the entrained flow varies directly as the separation bubble size, since the bubble size also varies with s and σ . The separation bubble size, hence the entrainment, is determined by D/w and α . If the entrained flow is determined by s/σ , then D/w and α must affect the distance s and the spread parameter σ .

2.4 Computation

It was not possible to obtain an explicit expression for x/w as a function of α and D/w in a form that could be used for computation. Instead, parametric equations were derived and the attachment distance was computed indirectly.

First (12) and (16) give two different expressions for $\cos \theta$ in terms of the parameter t' and will be referred to as the point and the volume equations, respectively. Second, (21) allows the computation of D/w , the offset distance, in terms of t' , α , and σ . Finally (23) permits the computation of x/w in terms of the same quantities. For each selected value of σ and α , a range of values for t' were selected, so that θ was less than 90 deg. This required preliminary computations to

determine the range for t' , since the range differed for the point and volume equations and for each value of σ and α used. When the ranges were determined, the values of θ , R/w , y/w , D/w , x/w were calculated at small intervals on each of the selected σ values and for $\alpha = 0, 15, 30, 45, 55$ deg. An example of such a page of computations, which was done on an IBM 7090 computer, is included as table I. For each σ and α condition, the x/w that corresponded to $D/w = 0, 2, 4, 10$ was found by interpolation. The results are shown in Table II. This operation was repeated and the resulting points plotted and connected for each of the D/w values for $\alpha = 0$ to 55 deg. Each page corresponds to a value of σ and α , in addition to the family of four theoretical curves, the experimental data points. Figures 11 through 17 present the families of curves for σ in the range 1 to 15 for the control-volume model; and figures 18 through 21 present the families for values of $\sigma = 15, 20, 25, 30$ for the attachment-point model.

Table I. Sample Computer Print-out Sheet, $\sigma = 2$,

T'	COS θ	SIN θ	θ	R/w	y/w	D/w	x/w
1.4000000E-01	6.0431399E-01	7.9674625E-01	5.2620707E 01	3.6172246E 01	4.7933862E 00	1.3310	1.3310E 01
1.4900000E-01	6.1092300E-01	7.9149002E-01	5.2343926E 01	3.2139847E 01	4.5078318E 00	1.2220	1.2220E 01
1.5800000E-01	6.1751391E-01	7.8655994E-01	5.1865383E 01	2.8744838E 01	4.2550555E 00	1.0500	1.0500E 01
1.6700000E-01	6.2408963E-01	7.8135595E-01	5.1385088E 01	2.5910683E 01	4.0297607E 00	9.2400	9.2400E 00
1.7600000E-01	6.3083704E-01	7.7607790E-01	5.0903054E 01	2.3474611E 01	3.8277331E 00	8.1700	8.1700E 00
1.8500000E-01	6.3716708E-01	7.7072570E-01	5.0419292E 01	2.1378086E 01	3.6455801E 00	7.2500	7.2500E 00
1.9400000E-01	6.4367465E-01	7.6529925E-01	4.9933811E 01	1.9560243E 01	3.4805317E 00	6.4000	6.4000E 00
2.0300000E-01	6.5015883E-01	7.5978405E-01	4.9446624E 01	1.7973519E 01	3.3303329E 00	5.7300	5.7300E 00
2.1199999E-01	6.5661796E-01	7.5422334E-01	4.8957738E 01	1.6537900E 01	3.1930799E 00	5.2300	5.2300E 00
2.2099999E-01	6.6305152E-01	7.4857376E-01	4.8468716E 01	1.5348078E 01	3.0671938E 00	4.8700	4.8700E 00
2.3000000E-01	6.6945824E-01	7.4284946E-01	4.7974916E 01	1.4254730E 01	2.9513481E 00	4.5200	4.5200E 00
2.3900000E-01	6.7583701E-01	7.3705110E-01	4.7480997E 01	1.3219245E 01	2.8444094E 00	4.2000	4.2000E 00
2.4799999E-01	6.8218674E-01	7.3117799E-01	4.6985418E 01	1.2405065E 01	2.7454106E 00	3.9000	3.9000E 00
2.5699999E-01	6.8850634E-01	7.2523031E-01	4.6488107E 01	1.1618453E 01	2.6535206E 00	3.6100	3.6100E 00
2.6599999E-01	6.9479471E-01	7.1920812E-01	4.5989314E 01	1.0907937E 01	2.5680204E 00	3.3200	3.3200E 00
2.7499999E-01	7.0105078E-01	7.1311136E-01	4.5488806E 01	1.0263875E 01	2.4882855E 00	3.0300	3.0300E 00
2.8399999E-01	7.0727342E-01	7.0694011E-01	4.4986688E 01	9.6781111E 00	2.4137708E 00	2.7300	2.7300E 00
2.9299999E-01	7.1346155E-01	7.0069438E-01	4.4482911E 01	9.1437125E 00	2.3439982E 00	2.4300	2.4300E 00
3.0199999E-01	7.1961408E-01	6.9437423E-01	4.3977541E 01	8.6547523E 00	2.2785488E 00	2.1300	2.1300E 00
3.1099999E-01	7.2572994E-01	6.8747970E-01	4.3470584E 01	8.2061402E 00	2.2170443E 00	1.7900	1.7900E 00
3.1999999E-01	7.3180798E-01	6.8151087E-01	4.2961987E 01	7.7934431E 00	2.1591809E 00	1.4500	1.4500E 00
3.2899999E-01	7.3784717E-01	6.7494713E-01	4.2451816E 01	7.4129745E 00	2.1046028E 00	1.1400	1.1400E 00
3.3799999E-01	7.4384636E-01	6.6835044E-01	4.1940056E 01	7.0613016E 00	2.0531076E 00	0.8300	0.8300E 00
3.4699999E-01	7.4980450E-01	6.6165944E-01	4.1426714E 01	6.7355708E 00	2.0044403E 00	0.5200	0.5200E 00
3.5599999E-01	7.5572048E-01	6.5489631E-01	4.0911794E 01	6.4332451E 00	1.9583905E 00	0.2100	0.2100E 00
3.6499999E-01	7.6159320E-01	6.4805540E-01	4.0395303E 01	6.1520923E 00	1.9147675E 00	0.0000	0.0000E 00
3.7399999E-01	7.6742158E-01	6.4114282E-01	3.9877244E 01	5.8901424E 00	1.8734001E 00	-0.3000	-0.3000E 00
3.8299999E-01	7.7320651E-01	6.3415675E-01	3.9357622E 01	5.6456509E 00	1.8341329E 00	-0.6000	-0.6000E 00
3.9199999E-01	7.7894091E-01	6.2709713E-01	3.8836442E 01	5.4170685E 00	1.7968251E 00	-0.9000	-0.9000E 00
4.0099999E-01	7.8462969E-01	6.1994612E-01	3.8313706E 01	5.2030156E 00	1.7613489E 00	-1.2000	-1.2000E 00
4.0999999E-01	7.9026973E-01	6.1275913E-01	3.7789420E 01	5.0022599E 00	1.7275877E 00	-1.5000	-1.5000E 00
4.1899999E-01	7.9585997E-01	6.0548012E-01	3.7263587E 01	4.8136988E 00	1.6954353E 00	-1.8000	-1.8000E 00
4.2799999E-01	8.0139303E-01	5.9812972E-01	3.6738207E 01	4.6363619E 00	1.6647945E 00	-2.1000	-2.1000E 00
4.3699999E-01	8.0688683E-01	5.9070614E-01	3.6207287E 01	4.4692987E 00	1.6355765E 00	-2.4000	-2.4000E 00
4.4599999E-01	8.1232085E-01	5.8321010E-01	3.5676828E 01	4.3117659E 00	1.6076999E 00	-2.7000	-2.7000E 00
4.5499999E-01	8.1770090E-01	5.7584334E-01	3.5144832E 01	4.1630181E 00	1.5810898E 00	-3.0000	-3.0000E 00
4.6399999E-01	8.2302564E-01	5.6800422E-01	3.4611303E 01	4.0223979E 00	1.5556776E 00	-3.3000	-3.3000E 00
4.7299999E-01	8.2829403E-01	5.6029367E-01	3.4076239E 01	3.8893095E 00	1.5314001E 00	-3.6000	-3.6000E 00
4.8199999E-01	8.3350494E-01	5.5251201E-01	3.3539446E 01	3.7632103E 00	1.5081992E 00	-3.9000	-3.9000E 00
4.9099999E-01	8.3865730E-01	5.4465948E-01	3.3001522E 01	3.6436072E 00	1.4860211E 00	-4.2000	-4.2000E 00
4.9999999E-01	8.4374999E-01	5.3673440E-01	3.2461859E 01	3.5300697E 00	1.4648164E 00	-4.5000	-4.5000E 00
5.0899999E-01	8.4878192E-01	5.2874308E-01	3.1920649E 01	3.4221258E 00	1.4445397E 00	-4.8000	-4.8000E 00
5.1799999E-01	8.5375203E-01	5.2067983E-01	3.1377982E 01	3.3194595E 00	1.4251488E 00	-5.1000	-5.1000E 00
5.2699999E-01	8.5865918E-01	5.1234698E-01	3.0833748E 01	3.2217047E 00	1.4068049E 00	-5.4000	-5.4000E 00
5.3599999E-01	8.6350231E-01	5.0434487E-01	3.0287987E 01	3.1285443E 00	1.3888723E 00	-5.7000	-5.7000E 00
5.4499999E-01	8.6828032E-01	4.9607389E-01	2.9746700E 01	3.0396866E 00	1.3719174E 00	-6.0000	-6.0000E 00

The numbers on this page are to be interpreted as whole
plied by a power of ten; thus 4.0999998E-01 is read .4.

Table II. Interpolated values of $\frac{x}{w}$ and θ for selected $\frac{D}{w}$

D/w	x/w	θ
0.	-9.6919863E-01	3.1628
1.0000000E 00	1.0929889E 00	4.0564
2.0000000E 00	2.8548701E 00	4.4175
4.0000000E 00	6.2584630E 00	4.7725
6.0000000E 00	9.6303045E 00	4.9605
8.0000000E 00	1.2998519E 01	5.0818
1.0000000E 01	1.6368838E 01	5.1684

3. EXPERIMENTAL PROGRAM

Experiments were conducted with both air and water to determine the attachment distance as a function of offset distance and wall angle. The test models were designed to obtain essentially two-dimensional incompressible turbulent flow.

The test with a Mach 0.5 air jet was conducted with nozzle widths of 0, 2, 4, and 10 nozzle widths; the wall angle was in increments from 0 to 55 deg for each offset. Tests with simulated Mach 0.5) were conducted with offset distances of 0, 2, 4, and 10 nozzle widths; and the wall angle was moved in 2-deg increments or less for each offset. The attachment distances for computer solutions of the parametric equations for various values of the spread parameter σ are plotted in figures 11 through 21.

3.1 Air-Jet Test Model

The test model used in air-jet tests is shown in figure 5. The nozzle aspect ratio (nozzle height to width) is 8, which is a satisfactory approximation of two-dimensional flow, particularly where measurements are made halfway between the constraining

The nozzle is 1/32-in. wide; the attachment width is 10 nozzle widths long, and can be rotated through 60 deg and offset by 10 nozzle widths.

The attachment point is the location on the wall where the stream divides, so that all fluid on one side of the bubble and all fluid on the opposite side continues along the surface of the wall, the dynamic pressure vector is zero at the surface of the stream beyond the attachment point, but back toward the inclosed bubble. Therefore, the point on the surface at which the total dynamic pressure goes through a null is

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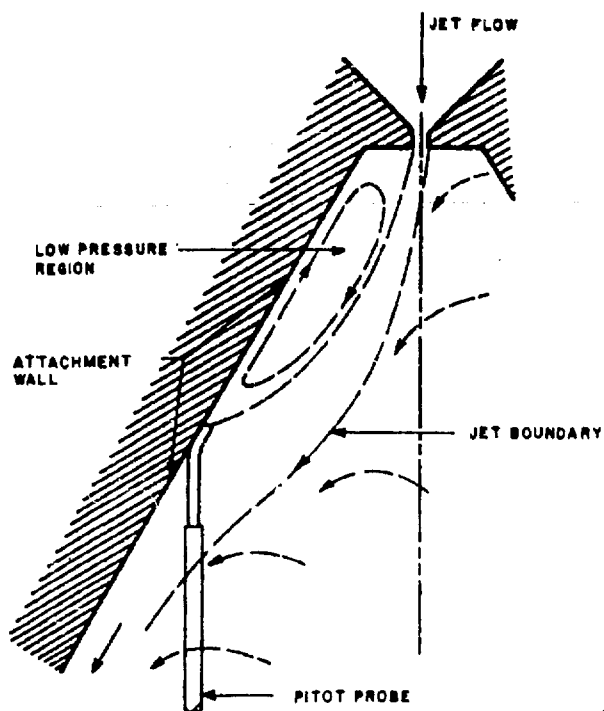


Figure 5. Schematic of experimental air-jet arrangement.

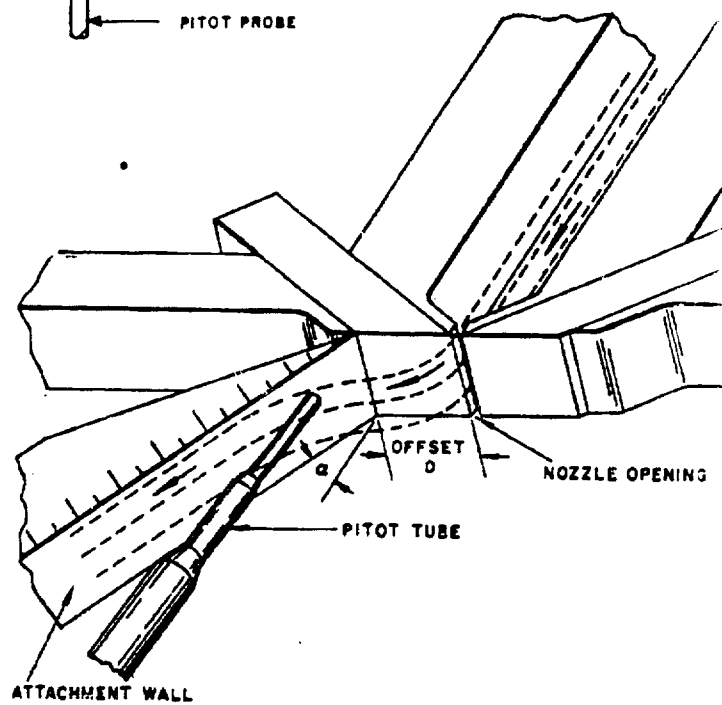


Figure 6. Isometric view of experimental air-jet arrangement.

estimate of the attachment point. To determine the location of the null, a single, 0.022-in. o.d., 0.012-in. i.d. pitot tube was placed parallel to the nozzle axis and moved along the attachment wall with the pitot probe bent slightly to follow the wall. The pitot was mounted on a calibrated jeweler's three-slide rest which measured the distance along the wall, with height adjustment maintained halfway between the constraining plates as shown in figure 6. The point at which the dynamic pressure went through zero was recorded as the attachment point. The data obtained are plotted as open symbols on figures 11 and 21. This method was used in preference to static probes, which would have required moving for each test.

The output of the pitot was fed to a pressure-voltage transducer, amplified, and readout on an x-y recorder. This was to be an extremely sensitive arrangement and a few thousandths of an inch of pitot movement was readily detectable on the recorder. When the attachment bubble was very small, as in the case of zero offset and small α , instability due to the probe size was encountered. When α was less than 25 deg at zero offset, the width of the bubble was less than the probe diameter, and the attachment point could not be located.

An inherent error is associated with this method of measurement, because the pitot opening is not a true point, and the angle which the attaching jet stream makes with the wall, changes.

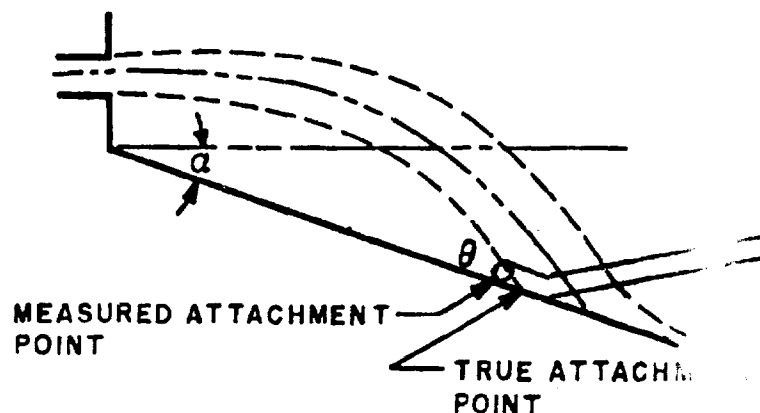


Figure 7. Errors in measuring attachment distance.

The pitot tube has an opening of about 1/3 nozzle width and tends to average the high and low pressures. The smaller the angle, the greater will be the averaging, which will tend to give an apparent attachment point further downstream than the actual value. This effect will decrease to a minimum as the attachment angle approaches 90 deg, as seen from figure 7.

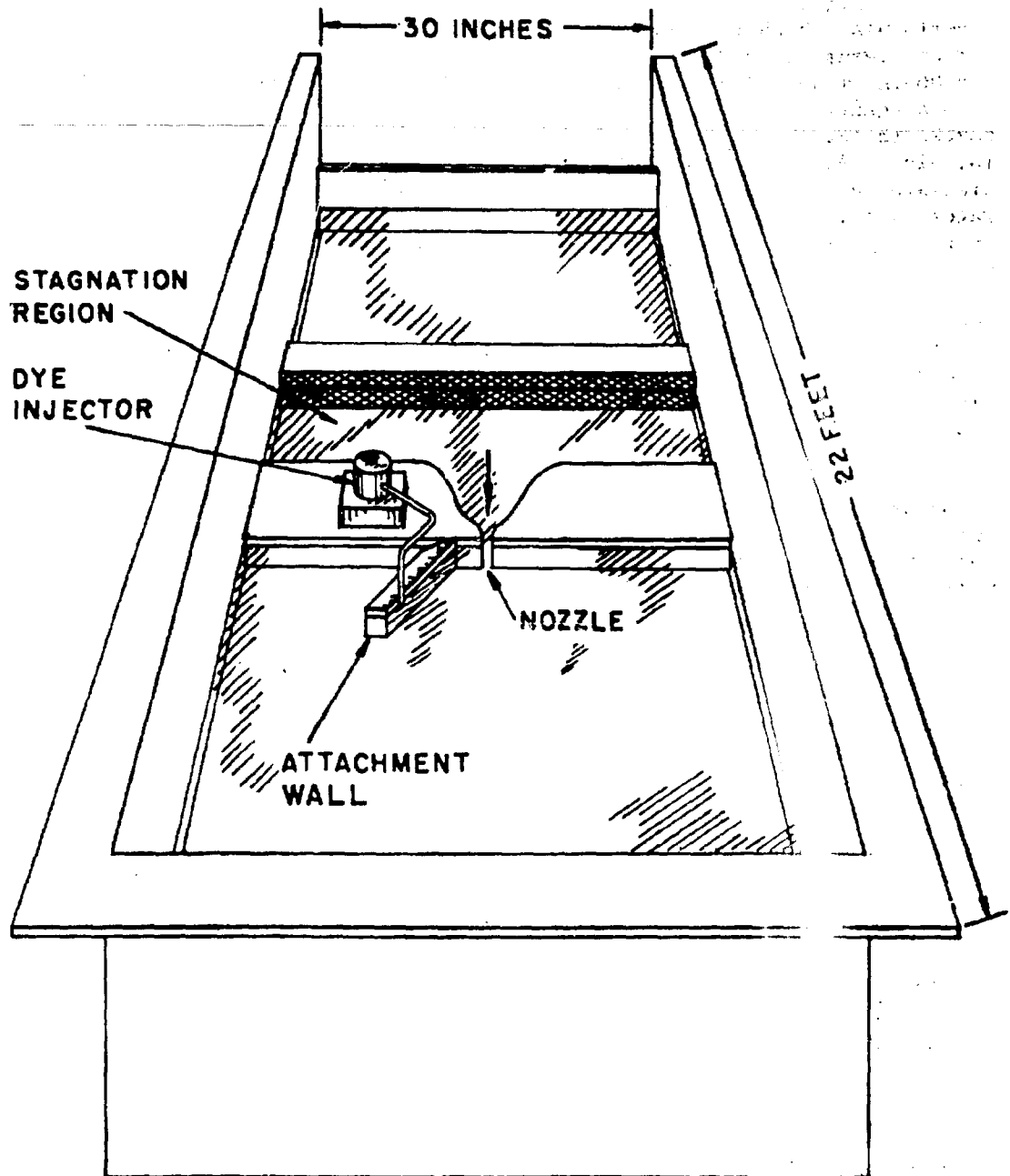


Figure 8. University of Maryland water table.

3.2 Water-Jet Test Model

Figures 8, 9, and 10 illustrate the water table used in these experiments, which were conducted* at the University of Maryland Wind Tunnel Operations Department (ref 5). The water table is 22 ft long and 30 in. wide. The water flow was gradually channeled from the 30-in. wide stagnation region to the nozzle throat, which was 0.5 in. wide. An estimate of the simulated Mach number on the water table, provided by the University (ref 6), is Mach number = $[2(d_0 - d)/d]^{1/2}$ where d_0 = stagnation region depth, d = depth at nozzle exit. Water depth readings were recorded at the nozzle exit and stagnation region; the simulated Mach number used was 0.5

The distance along the attachment wall was measured by introducing a dye and noting the point of stagnation on the wall. The dye was introduced

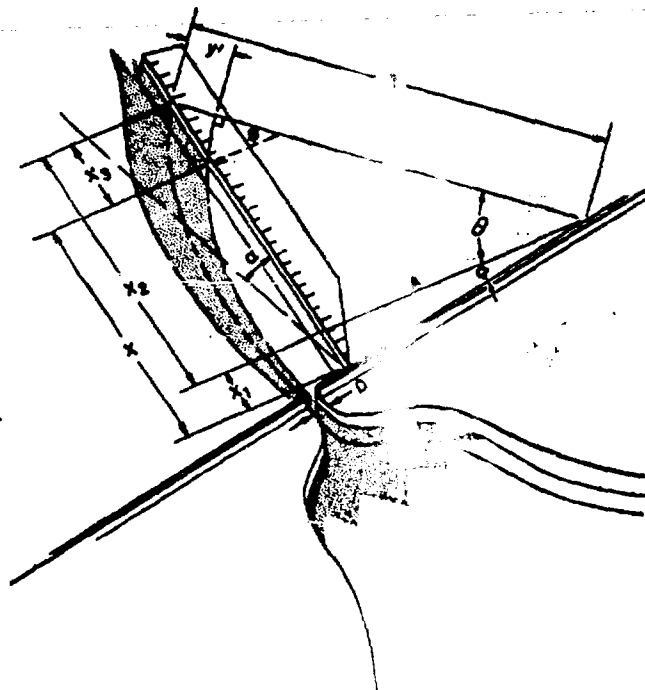


Figure 9. University of Maryland Water table. Mathematical model superimposed.

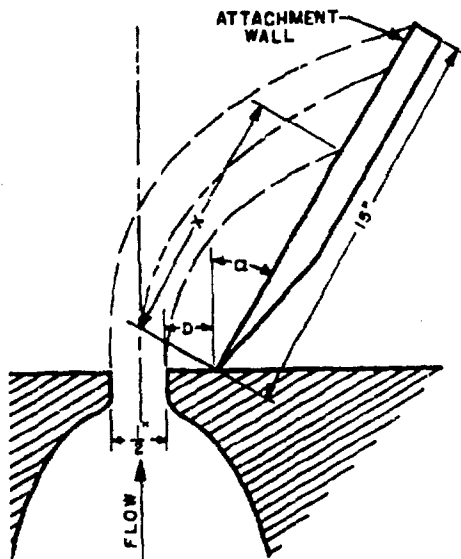


Figure 10. Schematic of experimental water table.

via a small probe placed perpendicular to the water table and adjacent to the attachment wall. The water table data are plotted as solid symbols on Figures 11 through 21. Offsets of 0, 1, and 2 nozzle widths were used, and, for each offset, the attachment wall was moved at 2-deg increments as far as the table width would permit. With the geometry chosen, it was impossible to increase the angle α to greater than 40-deg, and the attachment wall was only 25 nozzle widths long. Reducing it further to obtain greater angles was not considered.

*Contract DA-49-186-502-ORD-913

3.3 Comparison of Results

The attachment-point equations give satisfactory agreement with the experimental results for offset distances of 0, 2, and 4 nozzle widths at values of α less than 40 deg using $\sigma = 15$ and $\sigma = 20$ (fig. 18, 19). Agreement is poor for other values of α and for an offset distance of 10 nozzle widths and does not appear to improve for other values of σ .

The control-volume computations agree with the experimental results, but a single value of σ does not fit for all conditions. If it is assumed that the value of σ for which the data and theory agree is the effective σ for the particular D/w and α , then the following observations can be made by examining figures 11 through 17: as α increases, the value of σ decreases, and as D/w increases, the value of σ decreases. The relations can be very crudely estimated as shown in the chart below.

Approximate values of σ for combinations of α and D/w

$D/w \backslash \alpha$	0	10	20	30	40
0	-	12	10	8	6
2	12	10	8	6	4
4	10	8	6	4	2
10	2.5	1	1	1	1

These results are consistent with the development given in Sec. 2.3.4; i.e., σ is a function of D/w and α hence of bubble size.

The width of the water table severely limits the wall length and the angle α through which it may turn. For the data available, it is noted that the theoretical control-volume computations for $\sigma = 7$ and 8 (fig. 14, 15) show excellent agreement with the data for $D/w = 0, 2$, and 4 nozzle widths over the range of possible values of α . The computations of $\sigma = 6$ agree better with the $D/w = 4$. The theoretical attachment-point computations agree closely with the data for $D/w = 0$ using a $\sigma = 15$ and for $D/w = 2$ and 4 using a $\sigma = 20$.

The water and air jet results show excellent agreement over the entire range. Since two different fluids, two different devices, and two different methods of measuring the attachment were used by two independent sets of experimenters, the excellent agreement is remarkable.

4. SUMMARY

Equations were developed that predict the attachment distance x/w in terms of offset D and wall angle α for a two-dimensional, incompressible

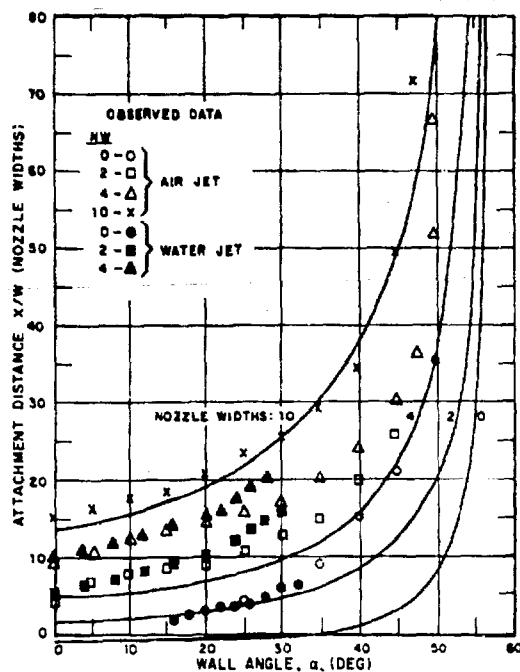


Figure 11. $\sigma = 1$

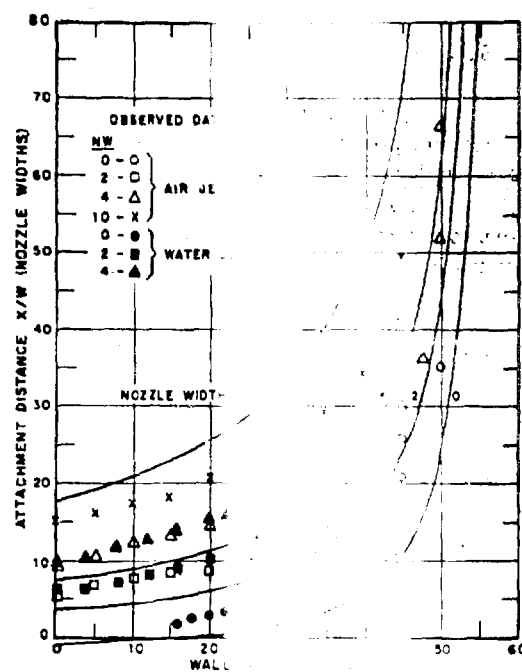


Figure 12. $\sigma = 1$

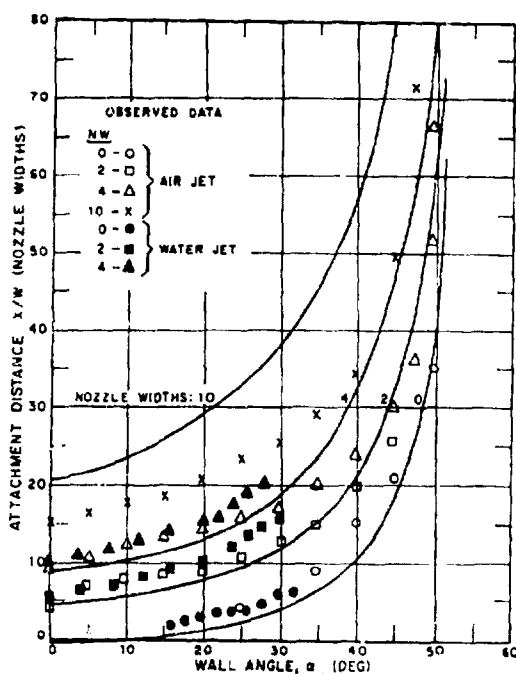


Figure 13. $\sigma = 5$

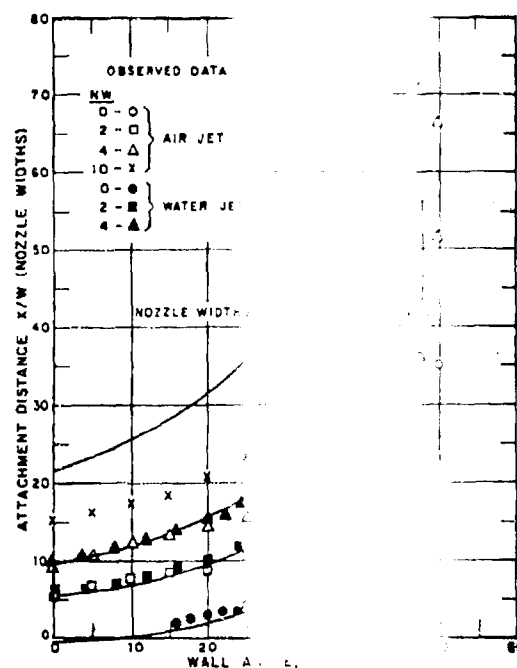


Figure 14. $\sigma = 5$

Figures 11 through 14. Comparison of observed attachment distances with theoretical computations based on control volume analysis.

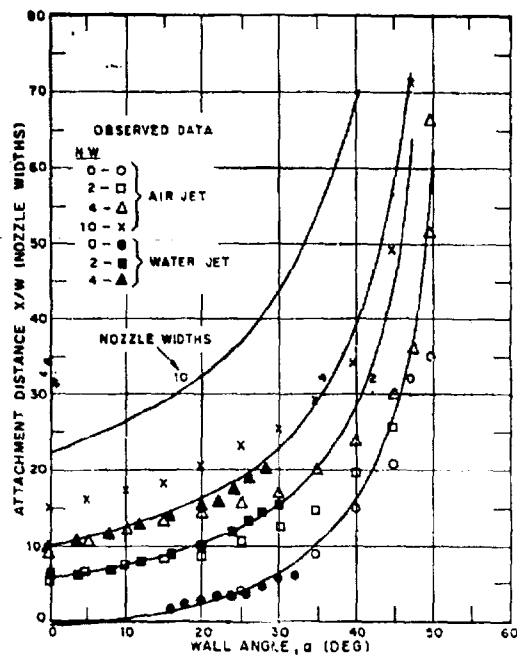


Figure 15. $\sigma = 8$

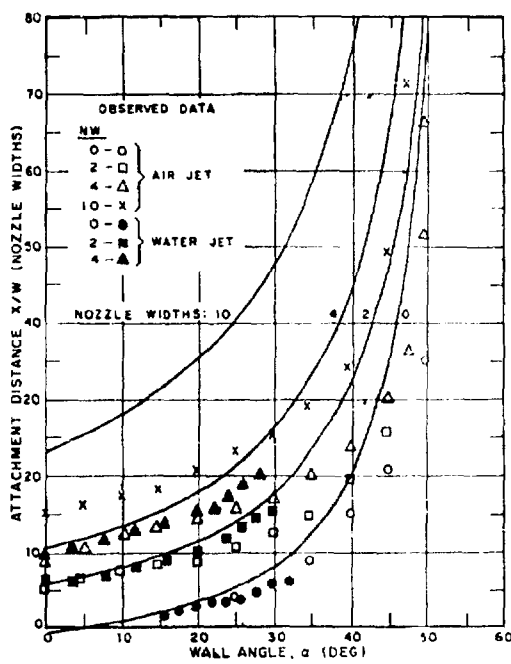


Figure 16. $\sigma = 10$

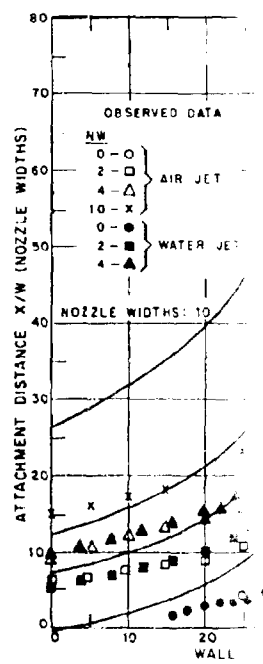


Figure 17.

Figures 15 through 17. Comparison of observed attachment distances with theoretical computations based on control volume analysis.

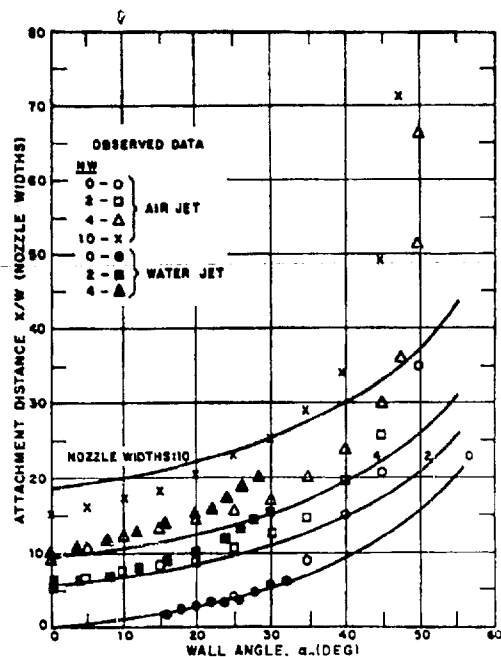


Figure 18. $\sigma = 15$

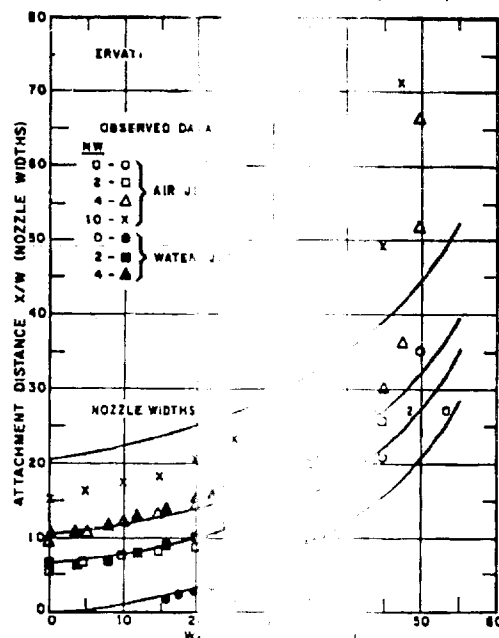


Figure 19.

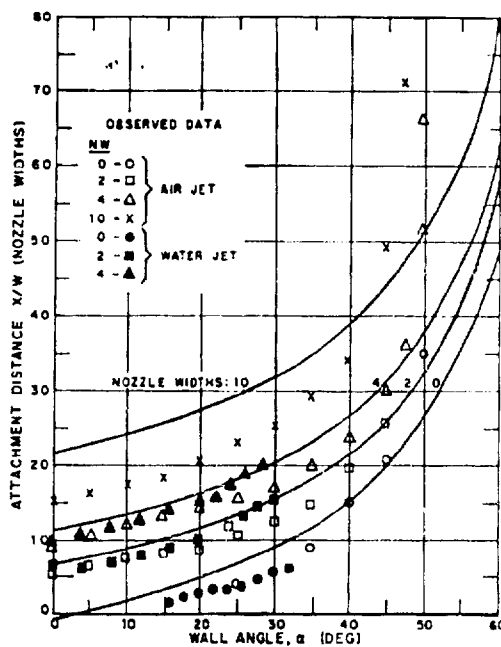


Figure 20. $\sigma = 25$

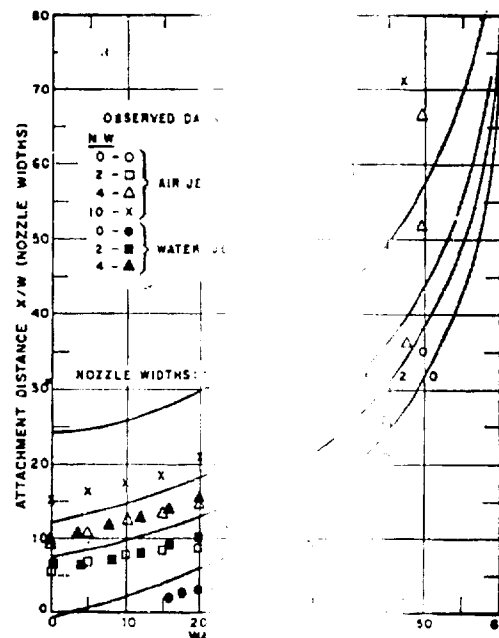


Figure 21.

Figures 18 through 21. Comparison of observed attachment distance with theoretical computations based on attachment distance.

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turbulent jet. The expressions are obtained in terms of t which is defined as

$$t = \tanh [\sigma y / (s + s_0)]$$

where y is the distance from the centerline, s is the distance the jet has traveled, and σ is a spread parameter. Using two expressions for the attachment angle θ were derived.

$$\text{attachment-point model} \quad \cos \theta = \frac{3}{2} - \frac{t^{1/3}}{2}$$

$$\text{control-volume model} \quad \cos \theta = \frac{1}{2} + \frac{3t'}{4} - \frac{t^3}{4}$$

Introducing the geometry, the parametric expressions for D/w developed are

$$\frac{D}{w} = \frac{\sigma}{3(\theta + \alpha)} \left(\frac{1}{t^{1/3}} - 1 \right) \left(1 - \frac{\cos \theta}{\cos \alpha} \right) - \frac{1}{2}$$

and

$$\frac{x}{w} = \frac{\sigma}{3(\theta + \alpha)} \left(\frac{1}{t^{1/3}} - 1 \right) (\sin \alpha + \sin \theta) \frac{\tanh^{-1} t'}{3t'^2 \sin \theta}$$

Computations were performed using both expressions for several values of α .

Experiments were carried out on an airjet model used to determine the attachment point for values of $D/w = 0, 2, 4$ and $\alpha = 0, 5, \dots, 55$ deg. Similar experiments on the water jet for $D/w = 0, 2, 4$, and $\alpha = 0, 2, 4, \dots, 40$ deg. In both cases, curves of x/w versus α formed distinct families that look like hyperbolas with the D/w determining the location of the curves and the α designating the position on the horizontal axis. The water jet results show excellent agreement with the control-volume model computations for $\sigma = 8$ for $D/w = 0, 2$, and 4 over the range of wall angles. The computed value of x/w for airjets using the control-volume model agree with the experimentally determined values of x/w for $D/w = 0, 2$, and 4 over the range of wall angles. This is because entrainment varies inversely as σ (27), hence entrainment varies as D/w and α . Computations based on the attachment-point model for values of $\sigma = 15$, and $\sigma = 20$ and for $0, 2$, and 4 nozzle range of 0 to 35 deg agree with test data.

5. CONCLUSIONS

The equations developed in this report show good agreement with data obtained from water and airjet experimental data.

parameter is allowed to vary. The computations based on the control volume model showed the effective σ was inversely proportional to both D/w and α . The variation in σ is consistent with the fact that the mass flow is inversely related to σ . Using the point model, the effective σ appears to be directly proportional to α . This model neglects the effect of bubble pressure. The control volume model is the preferred analytical representation. The theoretical development requires a number of assumptions. The jet profile equation is used, which assumes that the velocity profile is that of a free jet and is constant. In this report the spread parameter is treated as a floating constant, to be evaluated by comparison with the experimental data. Further work is needed to lead to the development of equations in which σ is a function of D/w and α or of other parameters of known value.

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6. REFERENCES

(1) C. Borque and B. G. Newman, "Reattachment of an Incompressible Jet to an Adjacent Flat Plate," The AIAA Journal, Vol. XI, Aug 1960, p 201

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APPENDIX A. Derivation of $R = J/\Delta p$

This equation can be arrived at using the vel

lations

$$R \omega^2 = a_n, \quad \omega = V/R$$

Writing

$$\text{Normal acceleration} = \frac{\text{pressure drop} \cdot \text{area}}{\text{density} \cdot \text{volume}}$$

$$a_n = \frac{(p_{\infty} - p_B) \cdot A}{\rho A w} = \frac{\Delta p}{\rho w}$$

Substitute into above

$$R \frac{V^2}{R^2} = \frac{\Delta p}{\rho w}$$

Simplify

$$\frac{1}{R} = \frac{\Delta p}{\rho w V^2}$$

Since

$$2 (p_{\infty} - p_{\infty}) w = \rho V^2 w = J$$

$$R = \frac{\rho w V^2}{\Delta p} = \frac{J}{\Delta p}$$

a_n normal acceleration (ft/sec²)

A area at the jet centerline (ft²)

R radius of the jet curvature (ft)

V linear velocity of the jet (ft/sec)

w jet width (ft)

ρ jet density (lbm/ft³)

ω jet angular velocity (rad/sec)

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