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Computation of Ships'
Magnetic Fields

by

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Technical Memorandum No. 43
7 August 1958

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ABSTRACT

It is shown that the magnetic moment of a steel ship, whether permanent or induced, can conveniently be represented by an array, in three dimensions, of magnetic dipoles at the centers of rectangular cells which the ship wholly or partly occupies, the magnetic moment of each dipole being equal to that of the portion of the ship's volume which it represents. By this artifice a single computation of magnetic field components of a magnetic dipole at points of a grid centered there, at can be used to find all three magnetic components of the field at any grid point due to the whole ship on any magnetic heading anywhere. The agreement with experimental data should be good at points much closer to the ship than when its magnetic moment is represented by a linear distribution of magnetic poles. An example is computed in some detail.

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COMPUTATION OF SHIPS' MAGNETIC FIELDS

It has been known since the development of degaussing, during World War II, that the magnetic field of a ship, at points where the effects of its local inhomogeneities are not great, is nearly that of a homogeneously magnetized ellipsoid, the principal dimensions of which are nearly the principal dimensions of the ship (length, beam, depth), and which is centered at the ship's centroid. The most complete statement of this generalization is found in a publication (1) of the Bureau of Ordnance hereinafter referred to as OD 8050.

In OD 8050, (pp. 432-463, in Part IV) a method using spheroidal coordinates, applicable where the more general ellipsoid degenerates into a spheroid, is explained, and is applied to the USS OVERTON DD 239, to show how magnetic measurements made on this ship in 1949 can be reproduced by computation. It is correctly stated that such computations are inconvenient, and are too time-consuming for general use.

In another place in OD 8050 (p. 69, in Part I) an empirical formula is given for finding a vector $\underline{M}$ proportional to the magnetic moment of a ship heading in any direction in the local magnetic field of the earth. This magnetic moment, or $\underline{M}$, can safely be used to compute the magnetic field of the ship at distances great with respect to its dimensions.

It is the purpose of this note to point out that it is possible, and not too time-consuming if high speed digital computers are available,

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to combine the two ideas just presented, and to compute relatively near fields of a ship, considered as a uniformly magnetized ellipsoid, by adding up the separately computed field components of suitable transverse slices of the ellipsoid. A trial computation for the OVERTON, done by "hand", using a desk type electrically driven digital machine, has been carried through and will be presented below. The use of the new method for predicting the expected interaction of steel ships of known dimensions with magnetic mines of known, or assumed, characteristics, may turn out to be justified in solving complicated mine warfare problems.

The formula for $\underline{M}$, given in OD 8050, is ,

$$\underline{M} = 0.013X(1+p)\underline{i} + 0.003Y\underline{j} + 0.005Z\underline{k} \quad (1)$$

Here X, Y, Z are components of the earth's local magnetic field along the unit vectors $\underline{i}, \underline{j}, \underline{k}$ fixed in the ship, $\underline{i}$ pointing forward, $\underline{j}$ pointing to starboard, $\underline{k}$ pointing downward. Thus X and Y depend upon the ship's heading, but Z does not. The number p is stated to be "the ratio of permanent to induced longitudinal magnetization", but study of the formula shows that this is not a possible definition for p since the formula would then say that when X becomes zero the "permanent" magnetization has no effect on $\underline{M}$, which is absurd. The mistake must have been committed in simplifying a general formula for use with conventional north and south headings in degaussing work. The general formula, which should have been given, is

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$$\underline{M} = 0.013(X + P)\underline{i} + 0.003Y\underline{j} + 0.005Z\underline{k} \quad (2)$$

in which P stands for a magnetic field, constant in magnitude and in direction along $\underline{i}$, which would account for the permanent longitudinal magnetization as X accounts for the induced longitudinal magnetization.

A more general expression of $\underline{M}_S$, the magnetic moment of the ship, is

$$\underline{M}_S = K_X (x + P) \underline{i} + K_Y Y \underline{j} + K_Z Z \underline{k} \quad (3)$$

where K_X , K_Y , K_Z are factors, all of the same physical dimensions (magnetic moment divided by magnetic field intensity). Since remote field changes, radial and circumferential, can be computed from

$$\Delta H_r = 2M_S \cos \theta / r^3 \quad (4)$$

$$\Delta H_\theta = M_S \sin \theta / r^3 \quad (5)$$

it is clear that, in Gaussian units, here used, the physical dimensions of magnetic moment divided by magnetic field intensity are those of volume, or $[L]^3$. This means that here, as in OD 8050, lengths can be expressed in reduced form, that is as numerical multiples of any convenient length, in both parts of the computation leading to magnetic field values. It also states the well-known fact that scale models can be used in planning the degaussing of ships. It is also clear that all magnetic fields can be expressed in oersteds or in milli-oersteds, or, as is customary in degaussing circles, in gaussses or milli-gaussses. (The use of units of magnetic induction is tolerable because ships and

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mines interact in media which cannot be appreciably magnetized.)

Computation by ship slices can be much shortened by choosing for computation points in the field which are separated, along the direction of the keel, by intervals equal to the thickness of each slice. The numerical factors which appear in computing the field components due to one slice at a longitudinal row of points will reappear for every other slice at other points of the row and so need be computed only once. The expansion of the vectorial formulae for field component of the n^{th} slice, with its resultant magnetic moment $F_n M_s$ at its centroid, yields the following formulae for a well-depermed ship, for which, that is, $P=0$.

$$\Delta H_{xn} = F_n (K_x \alpha X + K_y \alpha' Y + K_z \beta Z) \quad (6)$$

$$\Delta H_{yn} = F_n (K_x \alpha' X + K_y \beta Y + K_z \alpha' Z) \quad (7)$$

$$\Delta H_{zn} = F_n (K_x \beta X + K_y \alpha' Z + K_z \alpha Z) \quad (8)$$

Here F_n is the fraction of the volume of the ellipsoid in the slice and the factors $\alpha, \beta, \gamma, \alpha', \beta', \gamma'$ are numerical factors expressed in the reduced coordinates ξ, η, ζ of the point chosen with respect to the centroid of the slice, and of the reduced radius from that centroid. Explicitly

$$r^2 = x^2 + y^2 + z^2 \quad (9)$$

$$\alpha = \frac{3x^2}{r^5} - \frac{1}{r^3} \quad (10) \quad \beta = \frac{3y^2}{r^5} - \frac{1}{r^3} \quad (11) \quad \gamma = \frac{3z^2}{r^5} - \frac{1}{r^3} \quad (12)$$

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$$\alpha' = \frac{345}{25} \quad (13) \quad \beta' = \frac{355}{25} \quad (14) \quad \gamma' = \frac{354}{25} \quad (15)$$

The relative magnitudes of K_x , K_y , K_z here assumed were consistent with OD 8050, that is

$$K_x : K_y : K_z = 0.013 : 0.003 : 0.005 \quad (16)$$

The common multiplier of the numbers on the right hand side of this proportion, to give K_x , K_y , K_z is $(\ell/2)^3$ where ℓ is the length of the ship (not the length of the substituted ellipsoid). For the OVERTON of length 314 ft and for the chosen unit of length, 40 ft, it follows that the reduced length is $\lambda = 7.35$. The numerical factor used in computations is then $(\lambda/2)^3 = 60.467$.

The unit length, 40 ft, was chosen so that computations for points on a plane 40 ft below the ship's centroid (as in OD 8050) would be especially easy. It was thought better, however, to use slices less than 40 ft thick, and compute fields for points less than 40 ft apart on lines parallel to the ship's course. It can now be concluded that 20 ft would have been about right as the slice thickness and grid interval, but, in fact, 10 ft was chosen. This made the central slices of the OVERTON considerably too thin to be well represented by point magnetic dipoles at their centroids, and this, in turn, makes all computed fields at $\xi = 1$ too small by possibly 3 per cent at peak values, positive or negative. Less than peak values are proportionately, as well as absolutely, closer to the best values obtainable by the slice treatment.

The three numbers given in Eq. (1) and copied in Eq. (2), are

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different for three reasons. Most fundamentally, the steel in a ship is not isotropically arranged, there being different average areas of steel in random plane sections perpendicular to the three vectors $\underline{i}$, $\underline{j}$, $\underline{k}$. Secondly, the resultant magnetic moments induced by equal axial fields in a magnetically isotropic ellipsoid would still differ along different axes, being greater in the directions of longer principal axes because the self-demagnetizing fields are less for these directions. Thirdly, the effect of permanent magnetization is included only in the third term. The numbers given in OD 8050 are only appropriate for ships of conventional shape and structure. Any radical departure from the type of hull on which these numbers were based would change their relative magnitudes. All this means is that more recent experience must be utilized before any extensive computations can be based upon Eq. (2), and that different multipliers K_x , K_y , K_z can be expected for ships of peculiar characteristics.

The spheroid found in OD 8050 to be best suited to calculations of OVERTON field values by spheroidal harmonic analysis was also assumed here. It has foci 352 ft apart and a major axis 353.2 ft long. It was divided into 35 slices, the central slice centered at $x = 0$, $y = 0$, $z = 0$, and the extreme slices centered at $x = \pm 170$ ft, $y = 0$, $z = 0$. (This ignores a very small volume at each end.) Neglecting the curvature of the slice edges the relative volumes were taken as proportional to the areas of their transverse mid-sections. The resulting values of $\mathcal{F}_n$ are given in Table I, and have been adjusted to make $\sum_{n=1}^{35} \mathcal{F}_n = 1$.

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Table I

Fractions, f_n , of total magnetic moment of USS OVERTON DD 239 ellipsoid model, contained in transverse slice, n , 10 ft thick.

n	f	f_n
1 35	± 4.25	0.003114
2 34	± 4.00	0.007607
3 33	± 3.75	0.011827
4 32	± 3.50	0.015775
5 31	± 3.25	0.019450
6 30	± 3.00	0.022854
7 29	± 2.75	0.025985
8 28	± 2.50	0.028844
9 27	± 2.25	0.031430
10 26	± 2.00	0.033745
11 25	± 1.75	0.035787
12 24	± 1.50	0.037556
13 23	± 1.25	0.039054
14 22	± 1.00	0.040279
15 21	$\pm .75$	0.041232
16 20	$\pm .50$	0.041912
17 19	$\pm .25$	0.042321
18	0	0.042457

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Computation is straight-forward for any point (ξ, η, ρ) in the field, being the summation for each of the nine terms in Eqs. (6), (7), (8) of 35 products of ξ by appropriate α , ----- α' factors. The values of $\alpha, \beta, \gamma, \alpha', \beta', \gamma'$ are tabulated for a few fixed values of η ($0, \pm 0.5, \pm 1, \pm 2, \pm 3$), for one value of $\int(1)$, and for a range in values of $\int$ ($0, \pm 0.25, \pm 0.50, \text{-----} \pm 9.75, \pm 10.00$) sufficiently great to include points well ahead of the ship and well behind it. The chosen values of $\int$ and η permitted mapping the field of the OVERTON in a rectangle 460 ft long and 240 ft wide, or to points 73 ft ahead and behind, to points about 105 ft off the beam. The assumed values of the earth's magnetic field were $H(=\sqrt{x^2 + y^2}) \pm 0.200$ gauss, $H(=Z) 0.495$ gauss, as at the point where the OVERTON was studied. All H terms directly computed (before summing the 35 items) were recorded to 10^{-8} gauss.

If a grid of points at the corners of square meshes had been completely filled in, by selecting the range and intervals in η to match those in ξ , the computation would not have been as much greater as a mere count of added grid points would suggest, since symmetry reduces the task considerably.

Figs. 1-3, 4-6, 7-9 show $\Delta H_x, \Delta H_y, \Delta H_z$ at the selected grid points for the OVERTON headed $0^\circ, 45^\circ$ and 90° (magnetic) where the earth's field has a horizontal component of 0.200 gauss, a down-pointing vertical component of 0.495 gauss.

Since there are mines which attend only to changes in the total

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$$H_T = (x^2 + y^2 + z^2)^{\frac{1}{2}} = (H_z^2 + H^2)^{\frac{1}{2}} \quad (17)$$

magnetic field it seemed desirable to compute the magnitude of the

$$\left\{ (x + \Delta H_x)^2 + (y + \Delta H_y)^2 + (z + \Delta H_z)^2 \right\}^{\frac{1}{2}} - \left\{ x^2 + y^2 + z^2 \right\}^{\frac{1}{2}} = \Delta H_T \quad (18)$$

resultant of $\Delta H_x, \Delta H_y, \Delta H_z$ (at each point for which these components had been computed). This was done, and Figs. 10-12 present the results graphically for the same three headings.

Results for other cardinal and intercardinal headings can easily be derived by symmetry from those given in the Figures. Interpolation for other headings and for other values of η , may also be feasible.

The behavior of a mine attending to the total magnetic field, or to changes in its vertical component, when a ship passes, can be predicted from curves like those already given. Plotting field level lines on a horizontal projection of the ship and its neighborhood to get actuation widths for mines of known characteristics is a well-known method used in such prediction. Mines attending to changes in the horizontal component need a further step in computation, explained below.

A horizontal-component induction type mine can only attend to changes in $H \phi$ the horizontal component which lies along the axis of the core of its induction coil. The value of this change $\Delta H \phi$, at any grid point, is $\Delta H \phi = \Delta H_x \cos \phi + \Delta H_y \sin \phi$ when ϕ is angle between ship heading and mine axis. It can be assumed, unless the contrary can be

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proved, that all orientations of this axis in the horizontal plane of the sea bottom are equally probable. If this be admitted, the chance that such a mine will be fired when passed by a ship on a given heading depends upon the angle, ϕ , between the core axis and the magnetic meridian, as well as upon the ship's magnetic heading, Θ .

The computation of the changes in H_ϕ at grid points for which ΔH_x and ΔH_y have already been computed is again straightforward. It has been carried out for the same Θ values (0° , 45° , 90°) for a series of ϕ values 0° , 30° , 60° , 90° . What is more interesting than the results for any specific ϕ , for a given Θ , is the mean value $\overline{\Delta H_\phi}$ for random orientations of the mine axes, here approximated by

$$\overline{\Delta H_\phi} = (\Delta H_{0^\circ} + 2\Delta H_{30^\circ} + 2\Delta H_{60^\circ} + \Delta H_{90^\circ})/6$$

Values of $\overline{\Delta H_\phi}$ for $\Theta = 0^\circ$, 45° , 90° are presented in Figs. 13-15. The contour map technique may again be used to get most probable results and to estimate their variances in different parts of real channels.

If it turns out that single-component mines do not lie close to horizontal, but are predominantly at an appreciable angle to the horizontal plane when waiting for ships, the analysis here used will have to be replaced by a similar, but less simple, analysis.

Another method of handling the dependence of actuation probability upon the orientation of horizontal component mines was developed (2) (3) (4) in connection with magnetic mine sweeping. It uses vector diagrams of horizontal field increments at equal time intervals at

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fixed points on the bottom as the field-producing agent passes, and a transparent rotatable overlay showing the assumed orientation of the mine axis and required actuating field minima along that axis. This procedure is time consuming as first proposed, and is not well adapted to machine computing even in its latest form. Since no assumptions were made in the present paper regarding sensitivities or other operating characteristics of mines this method was here inapplicable.

Values of ΔH_z , for $\theta = 0^\circ$, for various values of x and for $y = 0$, $y = 20$ ft, are replotted in Fig. 16 in order to allow comparison with OVERTON data given in OD 8050 (on pp. 453 and 454) from measurements at the Wolf Trap degaussing station near Norfolk, Va. These data being presented in the form of contour maps for $\theta = 0^\circ$ and $\theta = 180^\circ$, it was necessary to interpolate between contours to get curves of ΔH_z vs x for Fig. 16. The depth to magnetometers was not exactly 40 ft, measured from the ship centroid (assumed to be 2 ft above the waterline) but z was nearly enough 40 ft to make comparison interesting.

It is seen that the course of ΔH_z values is very similar in both cases, but that computed values are smaller at the maximum, and show more dissymmetry about the midship section ($x = 0$), so as to have weak negative values near the stern. These differences mean that in the OVERTON the vertical magnetization is greater, and the longitudinal magnetization is less, than is predicted by the factors given for ships in general early in OD 8050. It is suggested that this is due to the fact that a destroyer has more vertical steel in proportion than vessels of greater beam and larger compartments in which decks, bottom, and side

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plating are relatively heavier. This emphasizes the need for establishing relative K -values Eq. (16) for ships of different types if calculated magnetic fields of ships are to be relied upon.

Since the computations here reported were completed I have found a note by R. K. Reber (5) giving a detailed analysis of the magnetic field of a barge 110 feet long, 30 feet wide, and 24 feet deep. The width and depth were therefore almost the same as for the OVERTON, the length only one third as great. Reber did not attempt to compute the field of the barge at nearby points, except for the vertical field component at a point on the bottom under the center of the vessel. He says, in fact: "Accurate theoretical calculations of the horizontal component fields due to longitudinal and transverse magnetization are hardly possible." He does estimate the total longitudinal magnetization for a north or south heading, and the total transverse magnetization for an east or west heading. From these it is easy to find relative values of K_x , K_y , K_z in the notation of the present paper. In order to make comparison easier I have arbitrarily assumed $K_z = 0.005$ in the following equation derived from his findings.

$$K_x : K_y : K_z = 0.0054 : 0.0027 : 0.005 \quad (20)$$

It is seen that the relative values of K_y and K_z agree excellently with corresponding numbers in Eq. (16), but that K_x is only about two fifths of the earlier given value. This is clearly due to the relative shortness of the barge as compared with typical cargo ships.

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The place where the magnetic measurements used by Reber were made was very near where the earlier measurements on the OVERTON were made. Reber assumed for the earth's magnetic field components the values $H_v = 0.53$ gauss, $H_h = 0.18$ gauss, whereas the values used in the present paper were $H_v = 0.495$ gauss, $H_h = 0.200$ gauss. If the present calculations were to be corrected so as to agree with Reber's assumptions M_2 would be about 6 per cent greater, and M_A , for a north heading, would be 10 per cent less. Both of these changes would improve the fit between observed and calculated curves for ΔH_2 in Fig 16.

My thanks are due to Mrs. C. K. Bockelman and Mrs. F. H. Stillinger, Jr. for computation of all field component values used in the Figures, except those presented as "observed" values of ΔH_2 in Fig 16, which I derived from small scale vertical field contour maps in Figures 15.45 and 15.46 in OD 3050, and which are much less precise.

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REFERENCES

- (1) NAVORD OD 8050. Technical Reference Book on Degaussing. 30 November 1951. CONFIDENTIAL (Note: Though dated in 1951 the parts here of interest appeared without essential differences in a much earlier publication, Naval Ordnance Laboratory Report No. 888. 17 November 1943.)
- (2) Bureau of Ships. Minesweeping Section. Report No. 94. Magnetic minesweeping. Theoretical methods for horizontal component mine and information on the J-sweep. By Louis R. Maxfield and Mary C. McKnight. 11 February 1941. CONFIDENTIAL
- (3) Bureau of Ships. Minesweeping Branch. Technical Report No. 107. Theoretical methods of determining the swept path of magnetic sweeps against horizontal component induction mines. By R. K. Reber and Mary C. McKnight. 1 September 1953. CONFIDENTIAL
- (4) Bureau of Ships. Code 620. Scientific Memorandum No. 119. The horizontal component magnetic field of the barge DTMB 3760. By R. K. Reber. 6 March 1945. CONFIDENTIAL

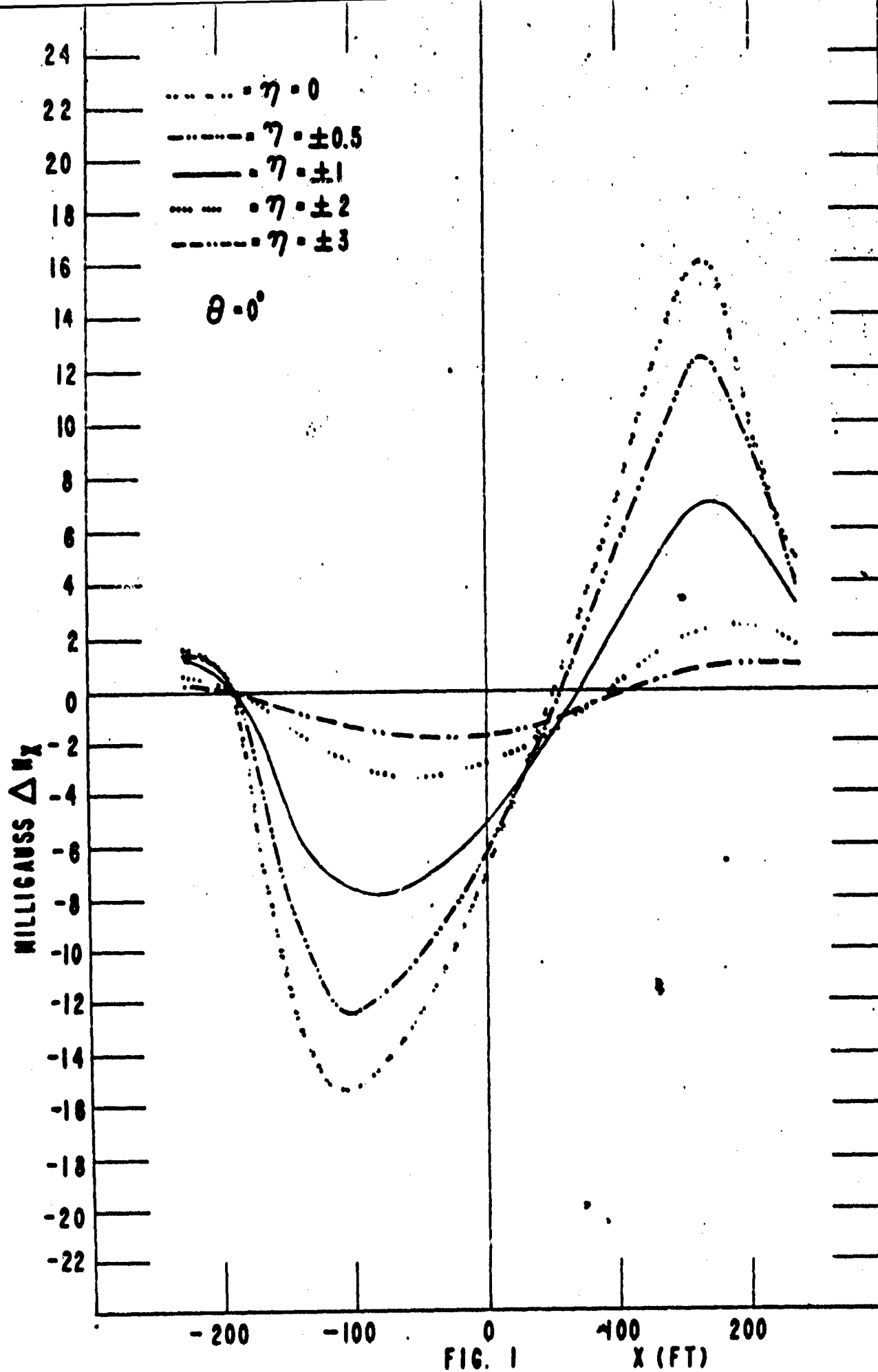
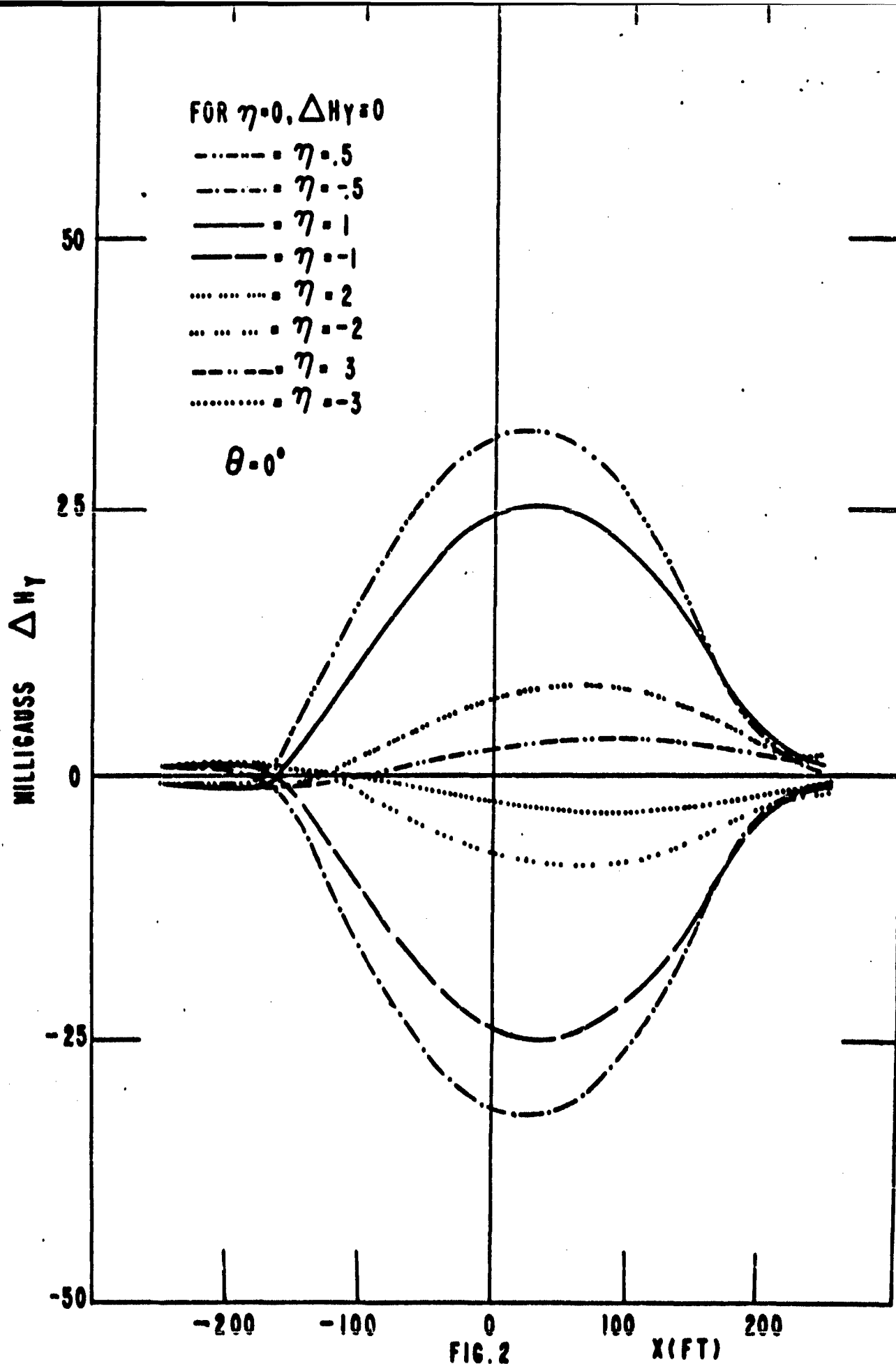


FIG. 1



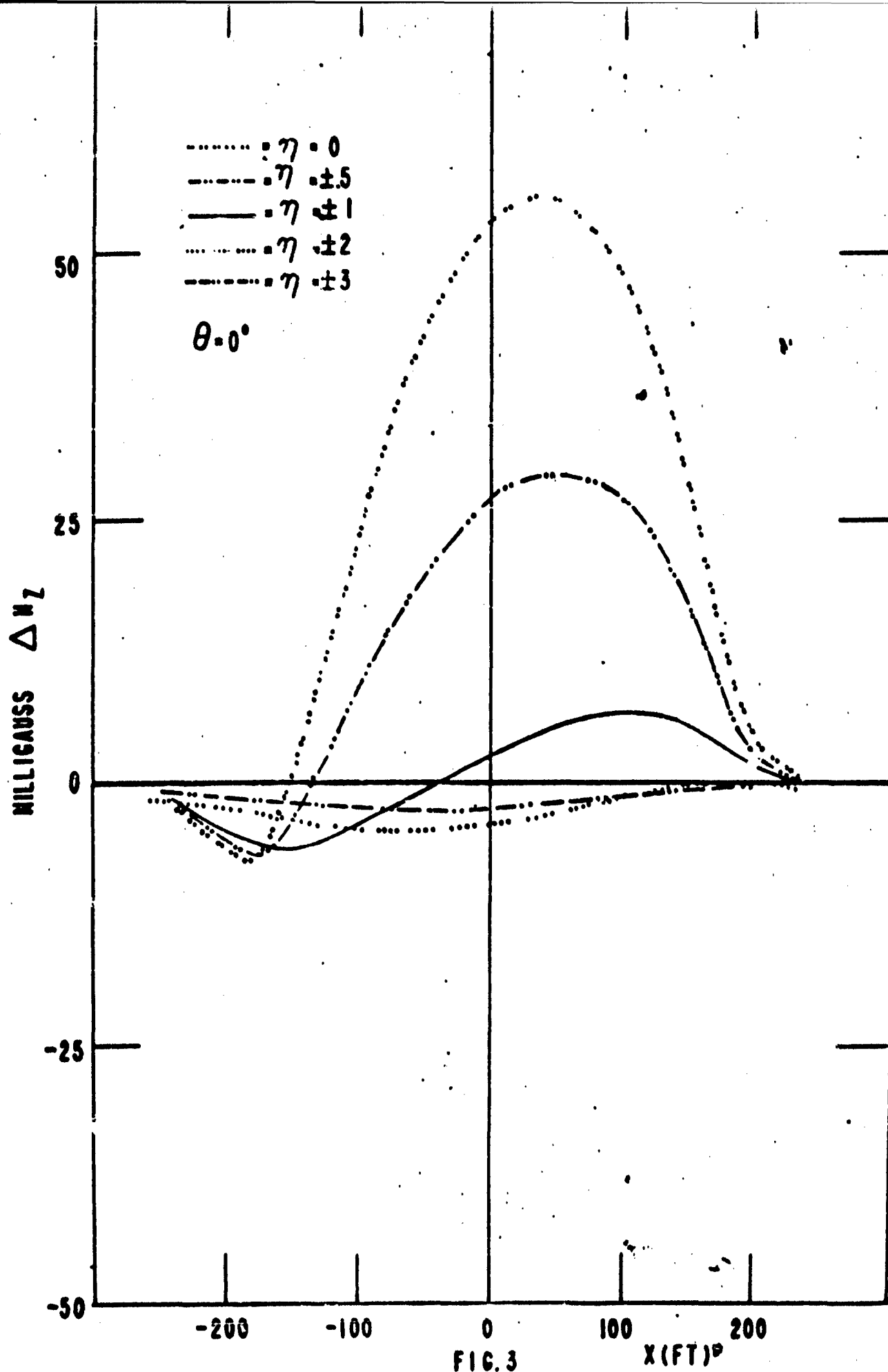


FIG. 3

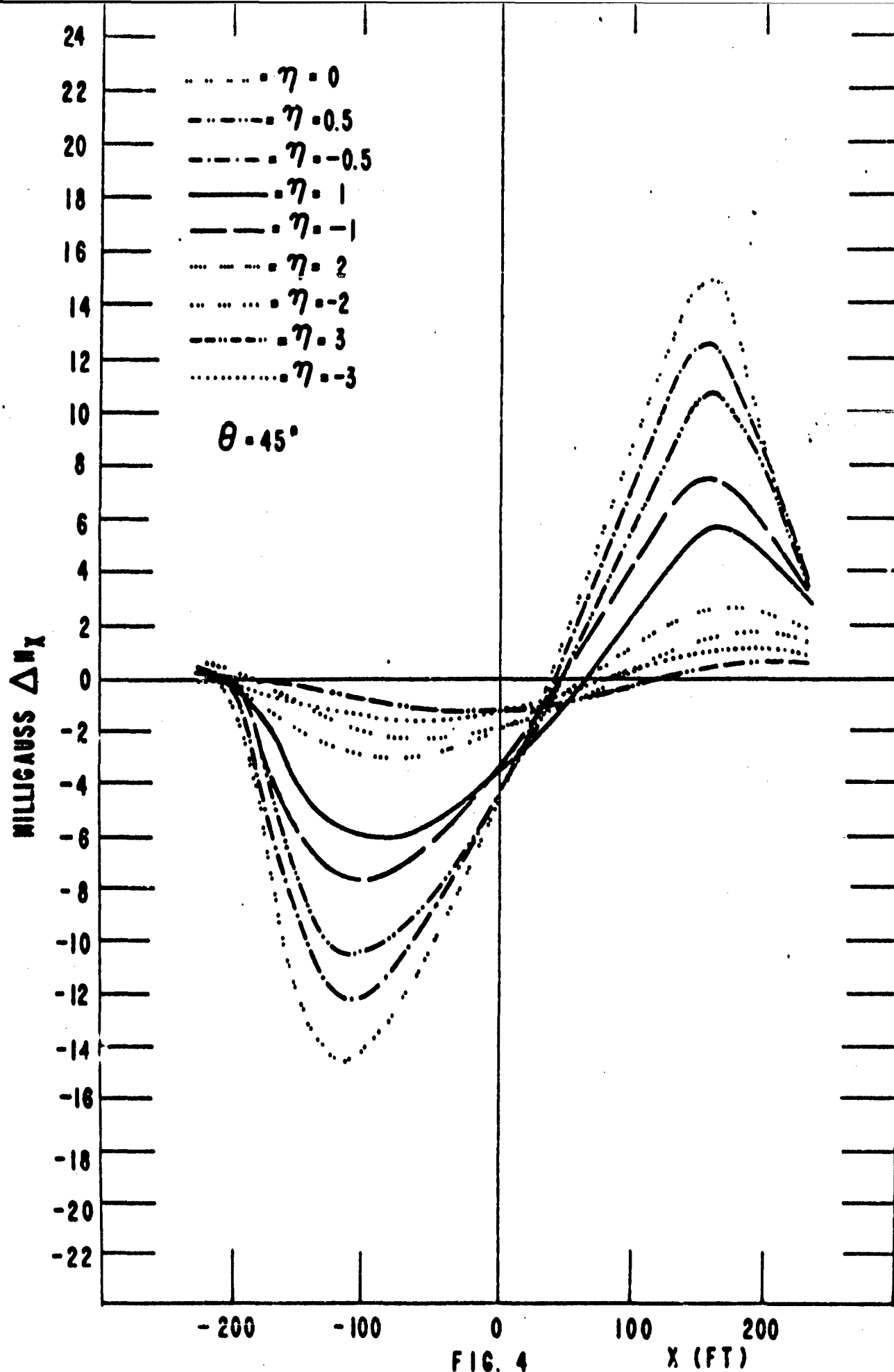
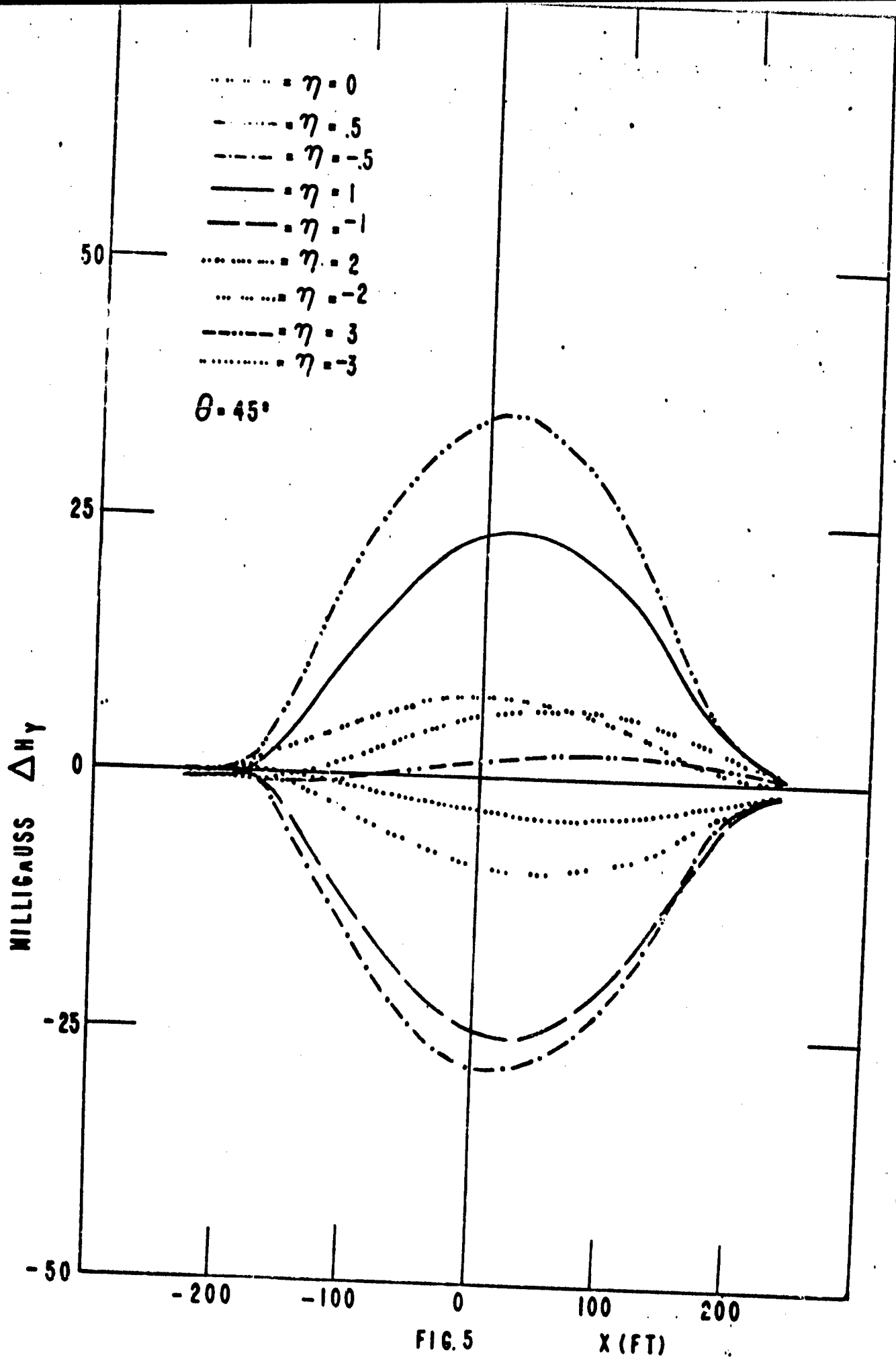


FIG. 4



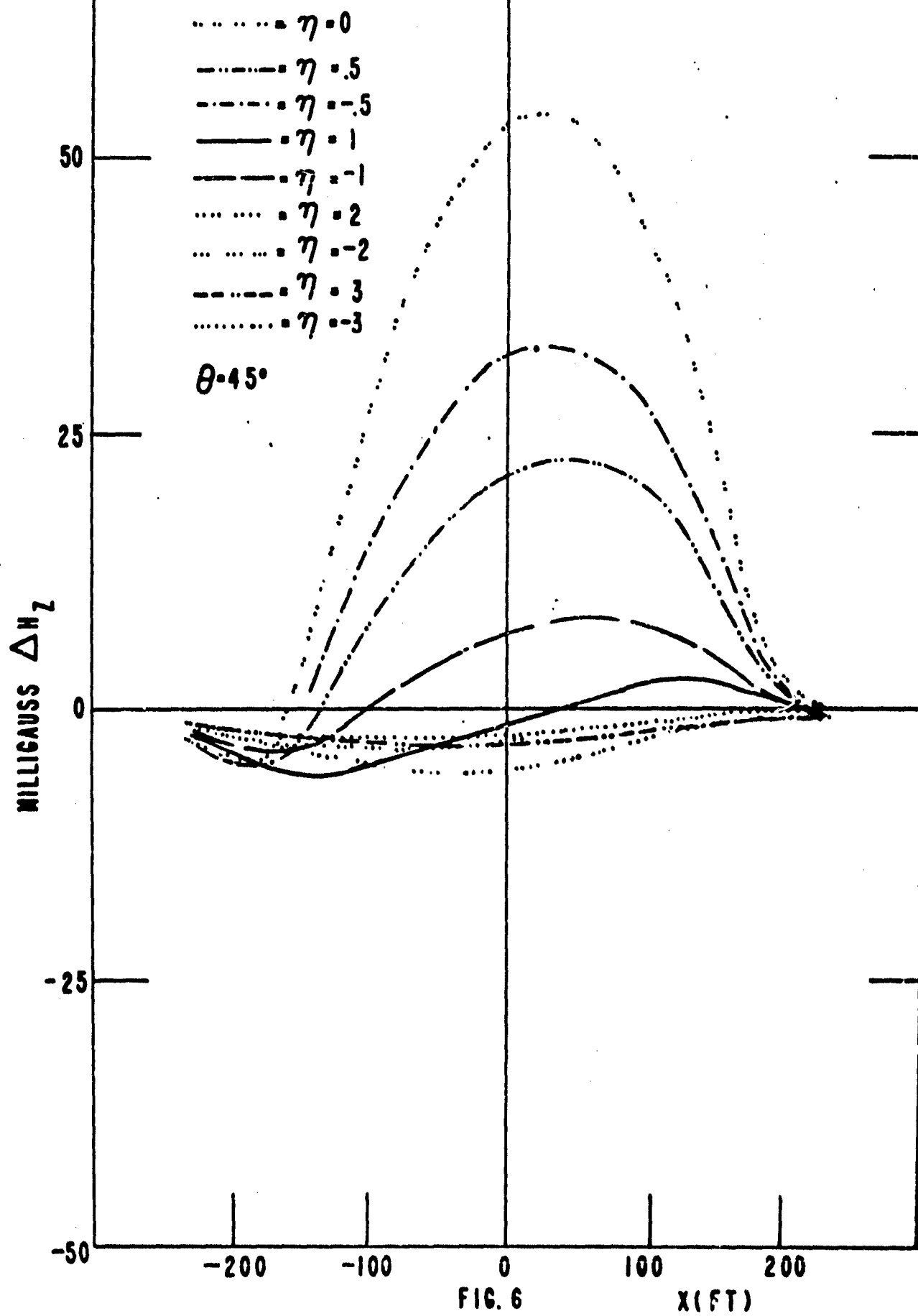


FIG. 6

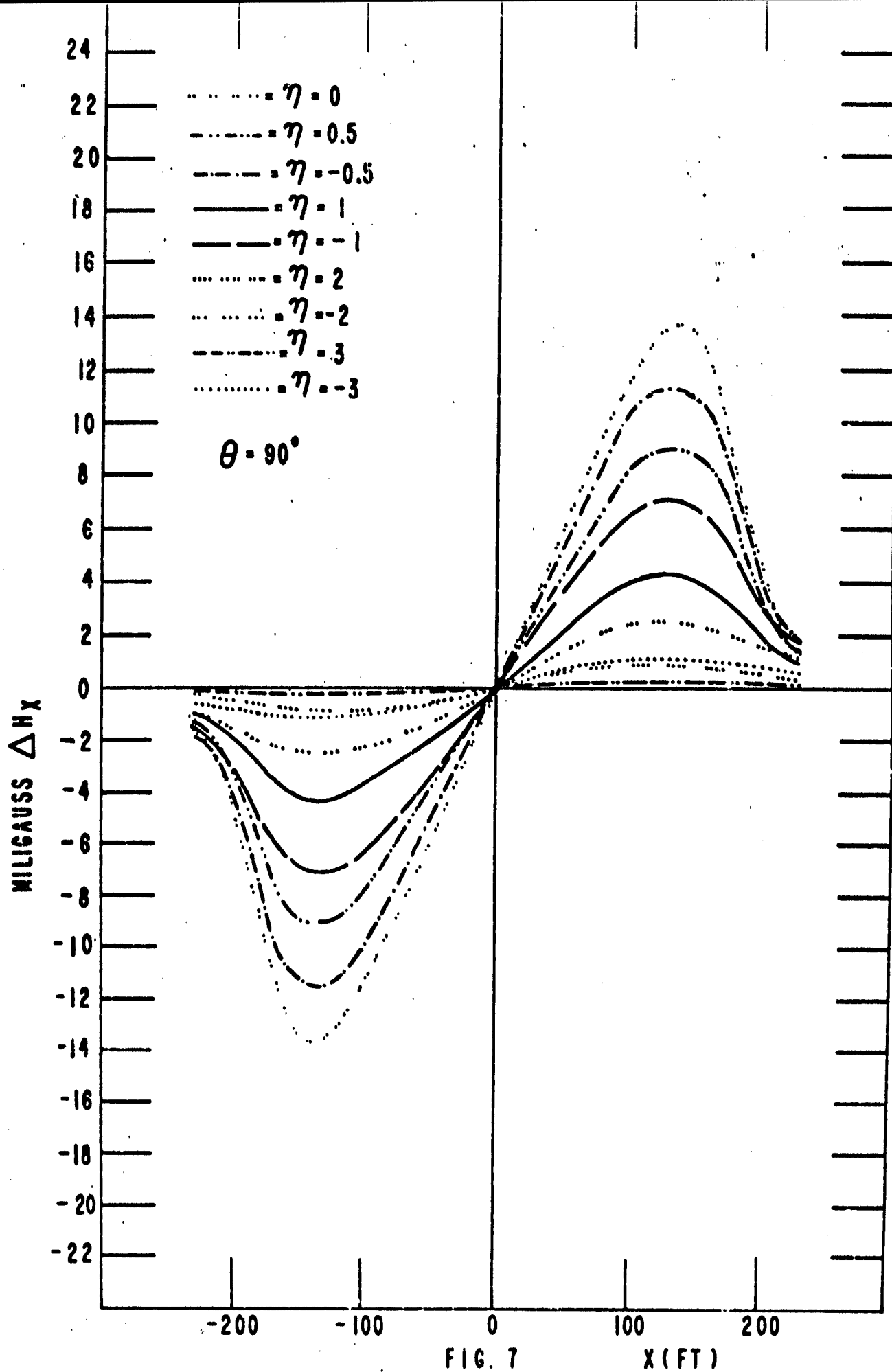
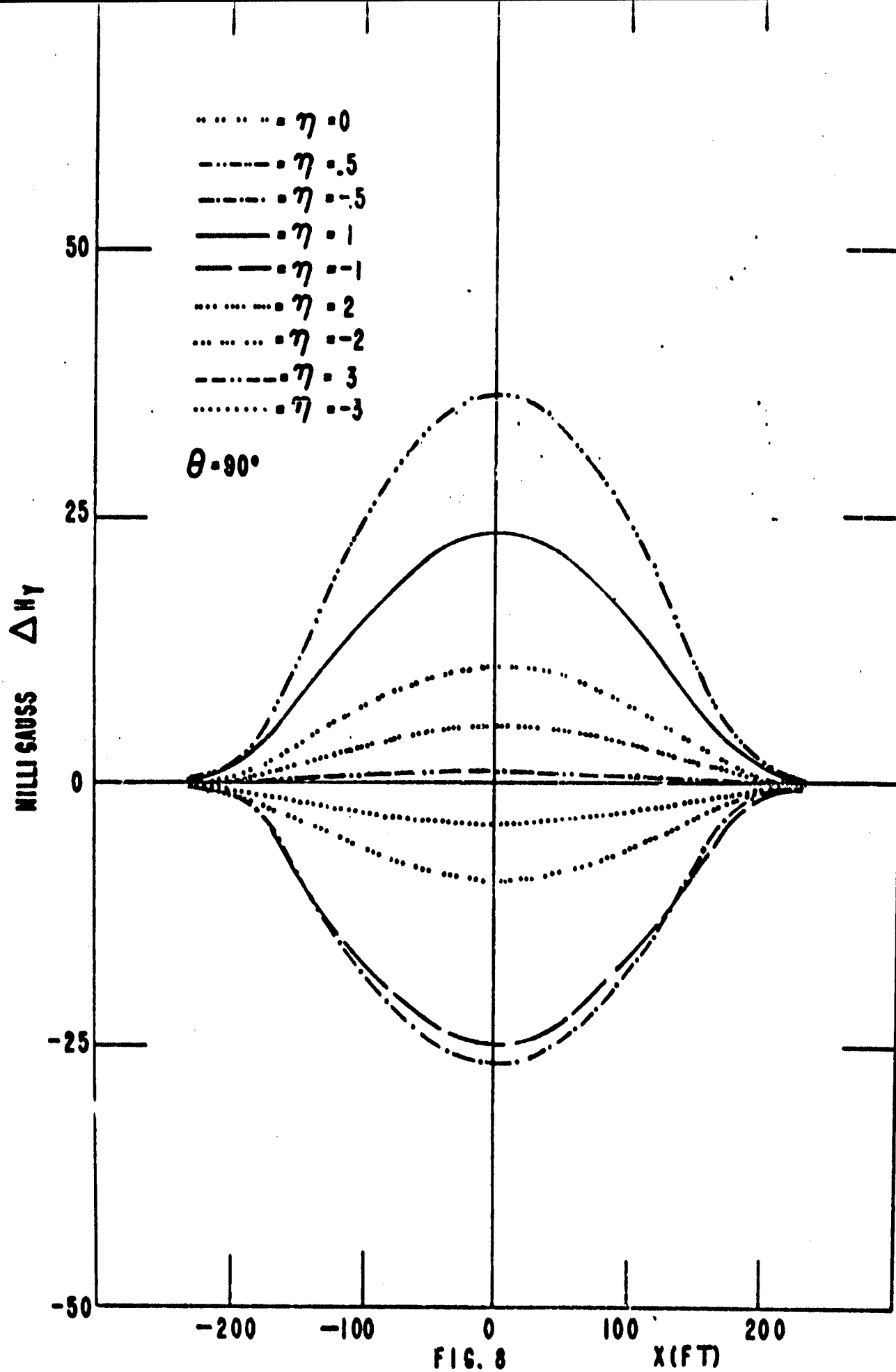


FIG. 7



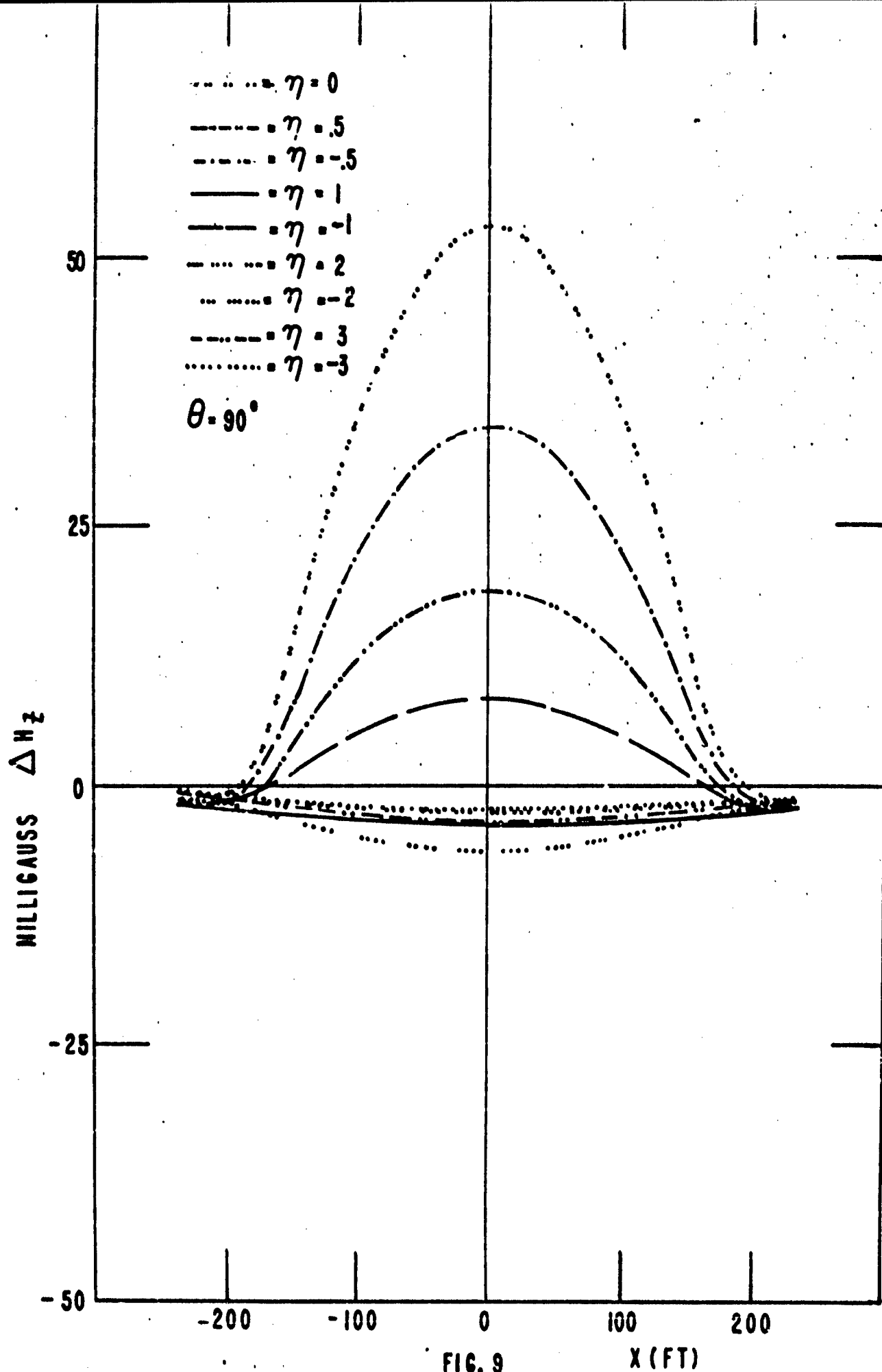
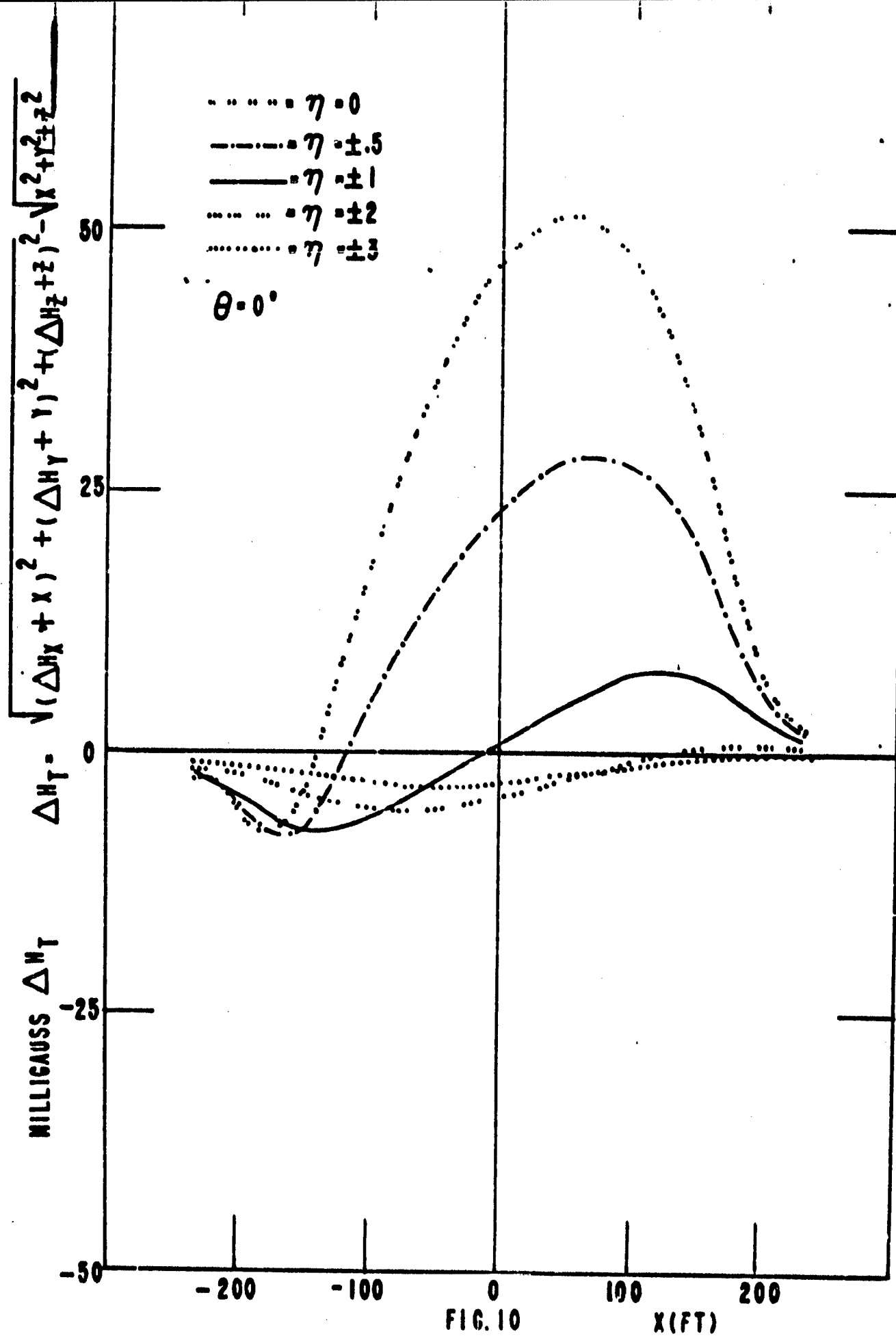


FIG. 9



$$\text{MILLICAUSS } \Delta H_T = \sqrt{\Delta H_x^2 + x^2} + (\Delta H_y + y)^2 - \sqrt{x^2 + y^2 + z^2}$$

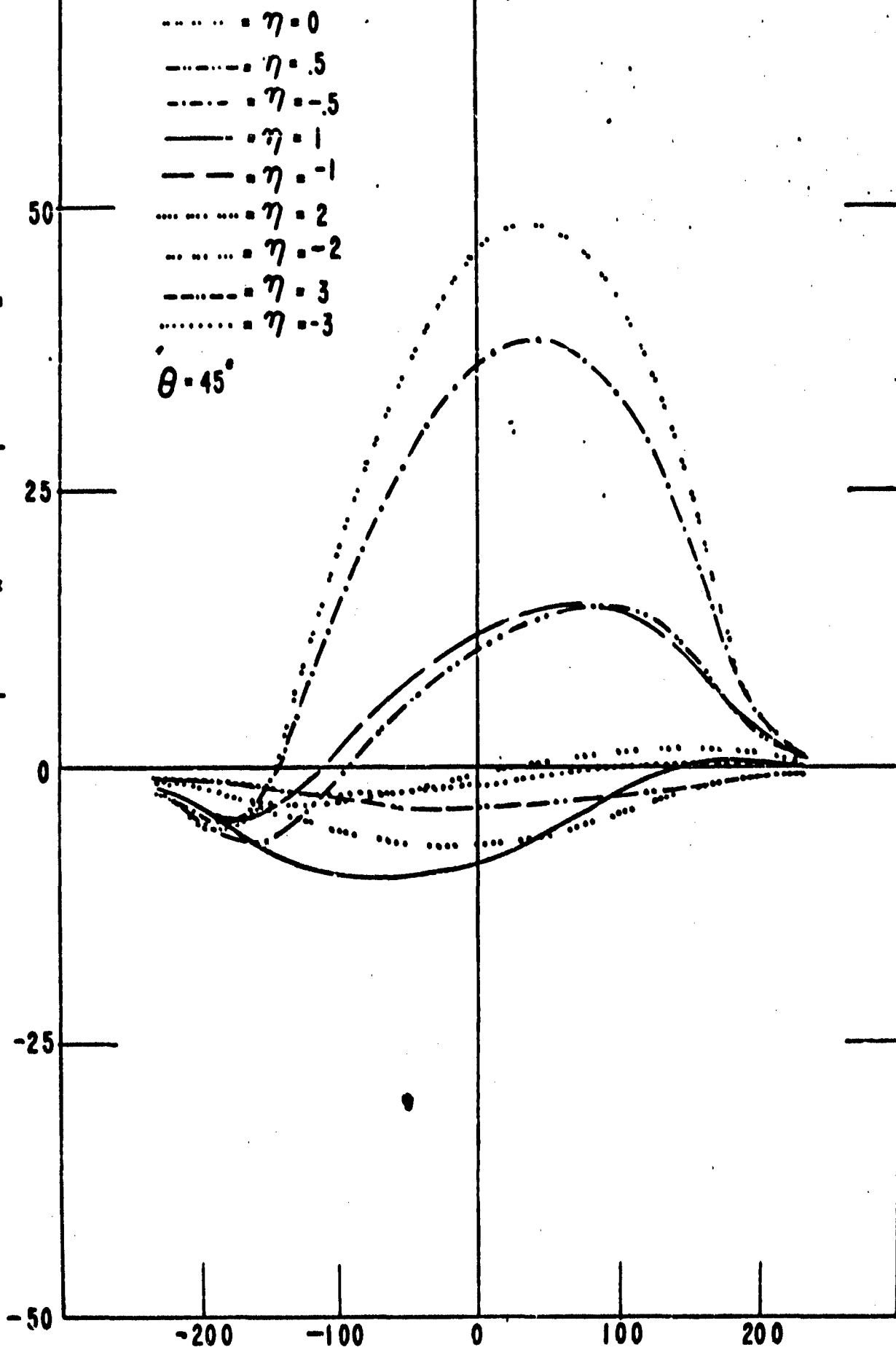


FIG. II

X (FT)

$$\Delta H_T = \sqrt{(\Delta H_x + x)^2 + (\Delta H_y + y)^2 + (\Delta H_z + z)^2} - \sqrt{x^2 + y^2 + z^2}$$

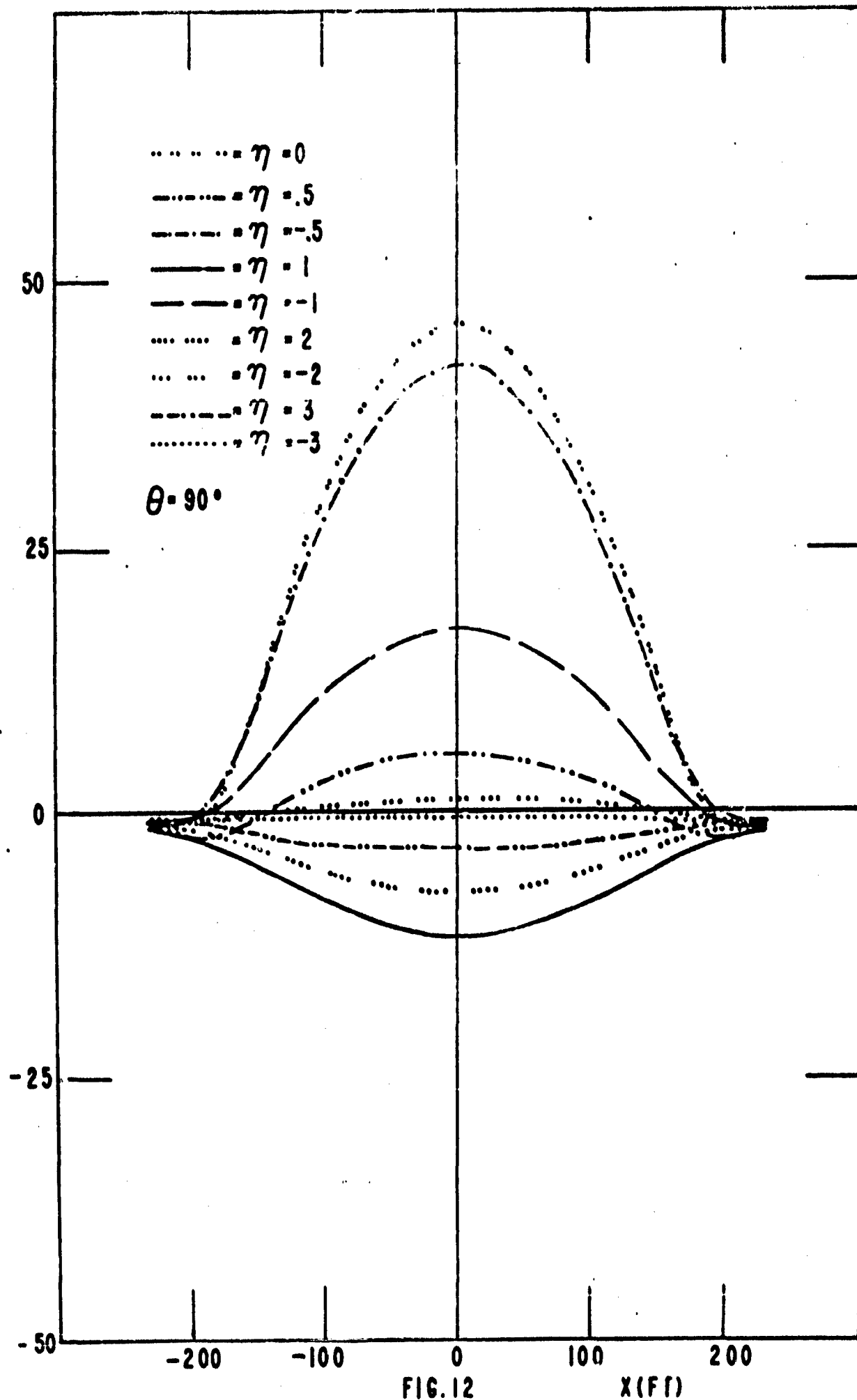
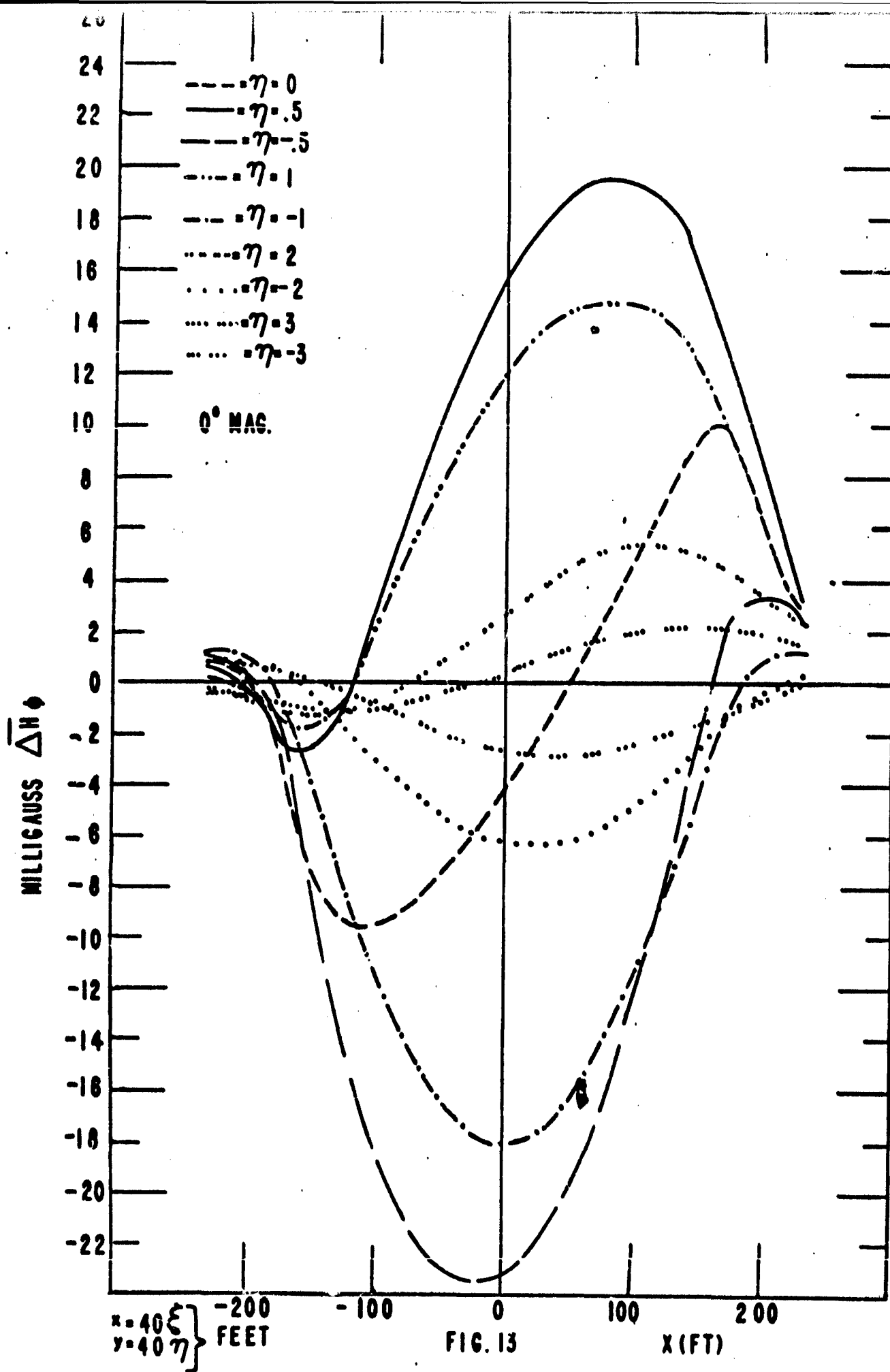


FIG. 12



$\Delta H \Phi$
 MILLIGAUSS

- $\eta = 0$
- $\eta = .5$
- - - $\eta = -.5$
- ... $\eta = 1$
- - - $\eta = -1$
- ... $\eta = 2$
- ... $\eta = -2$
- ... $\eta = 3$
- ... $\eta = -3$

45° MAG.

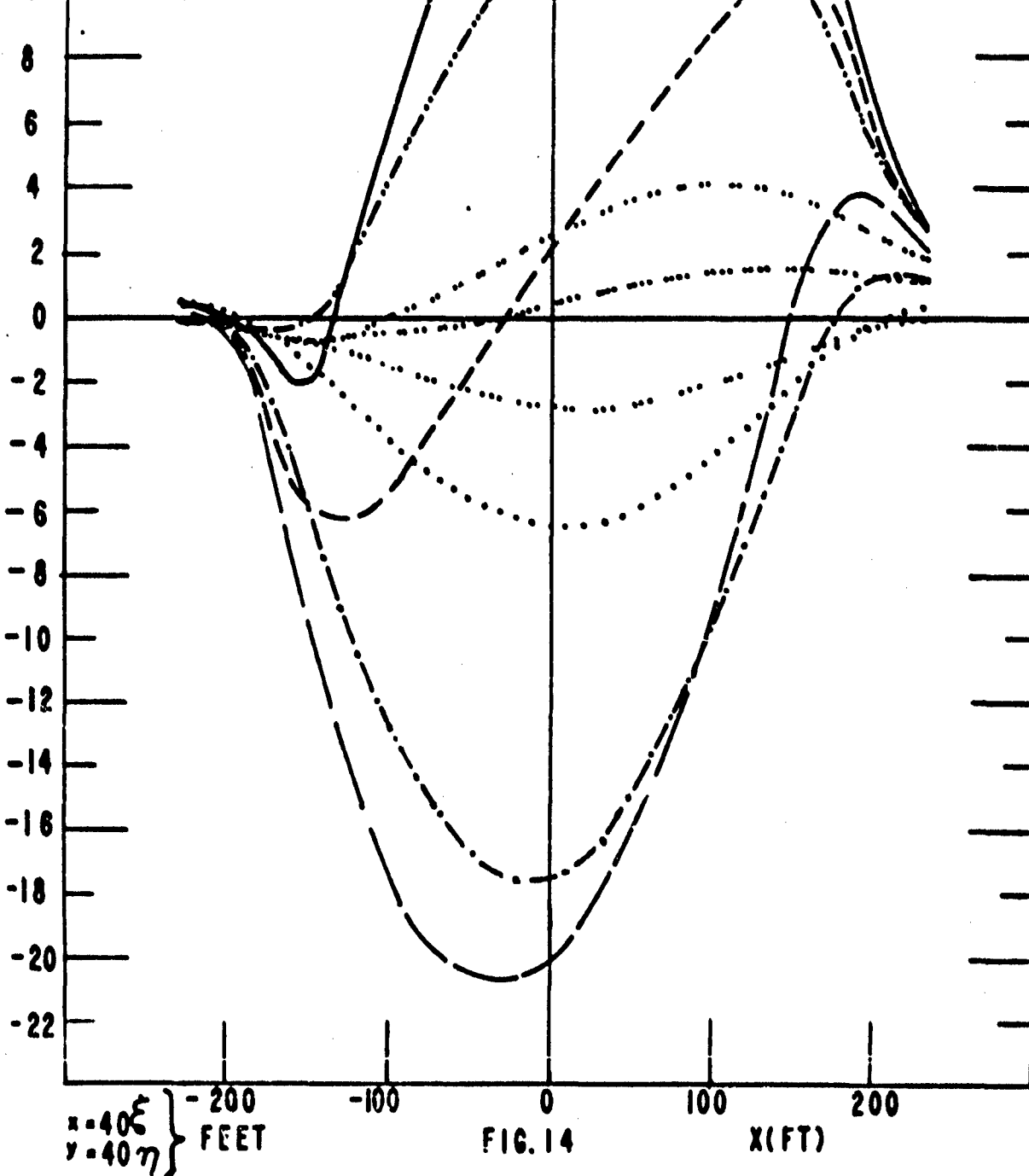
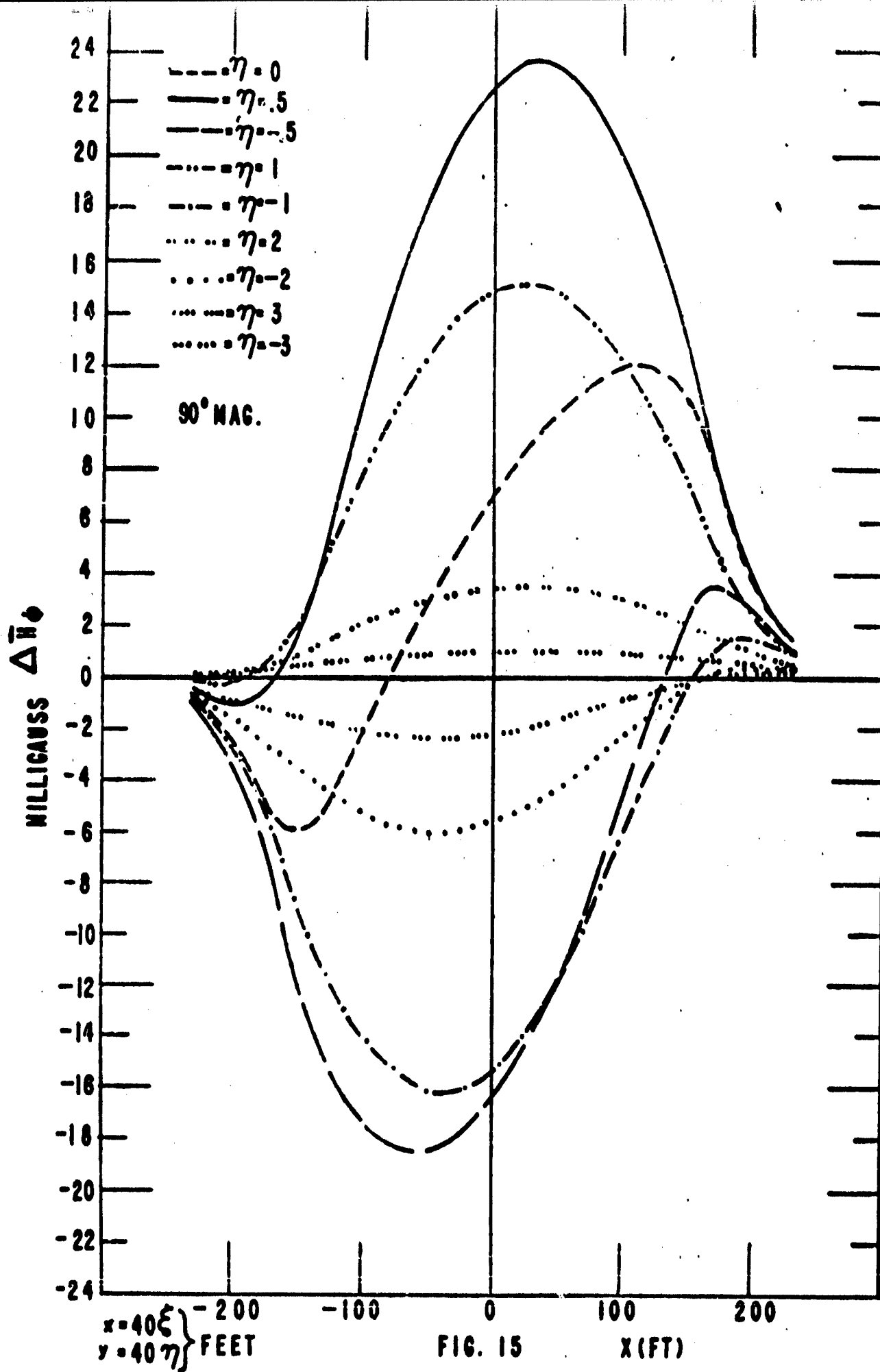


FIG. 14

X (FT)

$x = 40\xi$
 $y = 40\eta$



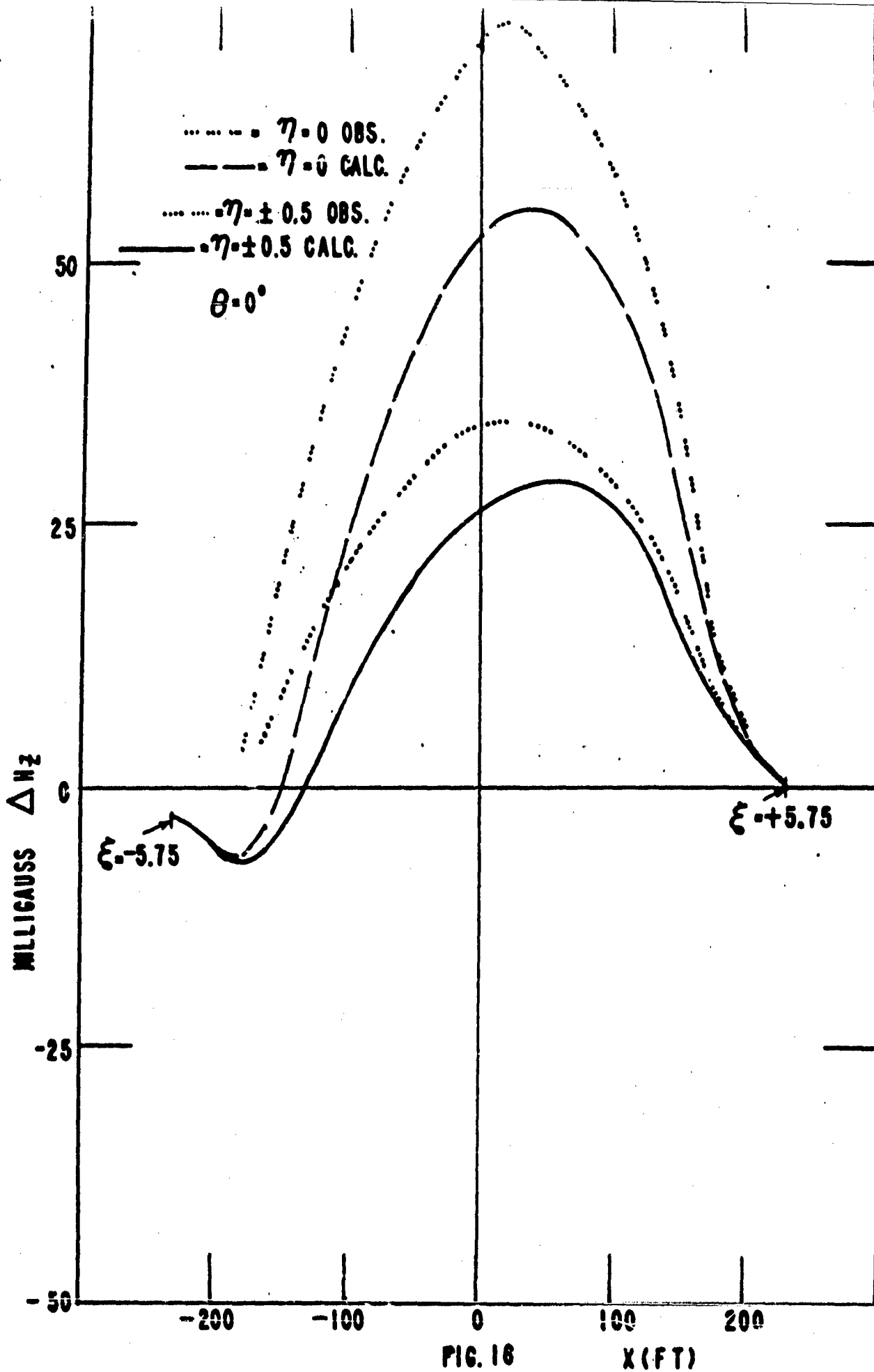


FIG. 16