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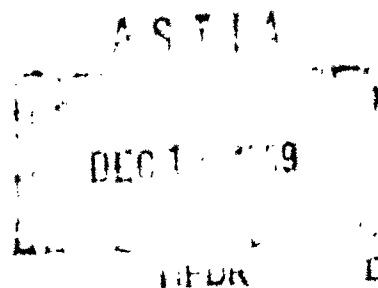
NAVAL PROVING GROUND
DAHLGREN, VIRGINIA

REPORT No. 16-44



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TRAJECTORIES OF BRITISH UP3 ROCKETS FORWARDED
FIRED FROM AIRCRAFT



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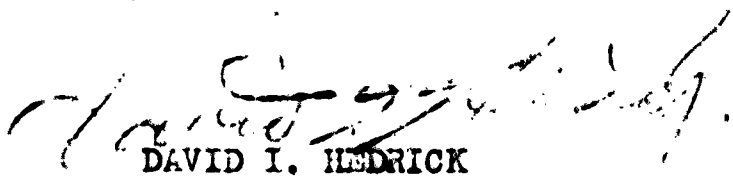
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NAVAL PROVING GROUND
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REPORT No. 16-44

TRAJECTORIES OF BRITISH UP3 ROCKETS
FORWARD FIRED FROM AIRCRAFT

APPROVED:



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Commanding Officer.

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INTRODUCTION

This report consists of two parts. The first covers experimental determinations of rocket trajectories and the second, calculations of the trajectories based on independently determined values of thrust and drag. The observed and calculated trajectories agree within the limits of the inherent dispersion of the rockets and the expected errors in measurement of thrust and drag.

Portions of rocket trajectories have previously been measured photographically, but this report describes for the first time, as far as is known, photographic measurement of the complete trajectories of rockets. This was accomplished by means of five ⁵ Mitchell high-speed motion picture chronographs with lenses of ~~six~~ inches focal length. The method of rectifying the data so obtained is described.

Rocket trajectories were calculated also on the basis of experimentally determined thrust curves for the rocket motors and experimentally determined values for the ballistic coefficients of the rockets. Trajectories were computed by numerical integration using as variables θ , the angle between the horizontal and the tangent to the trajectory, and $v-v_1$, the velocity differential due to air resistance and gravity.

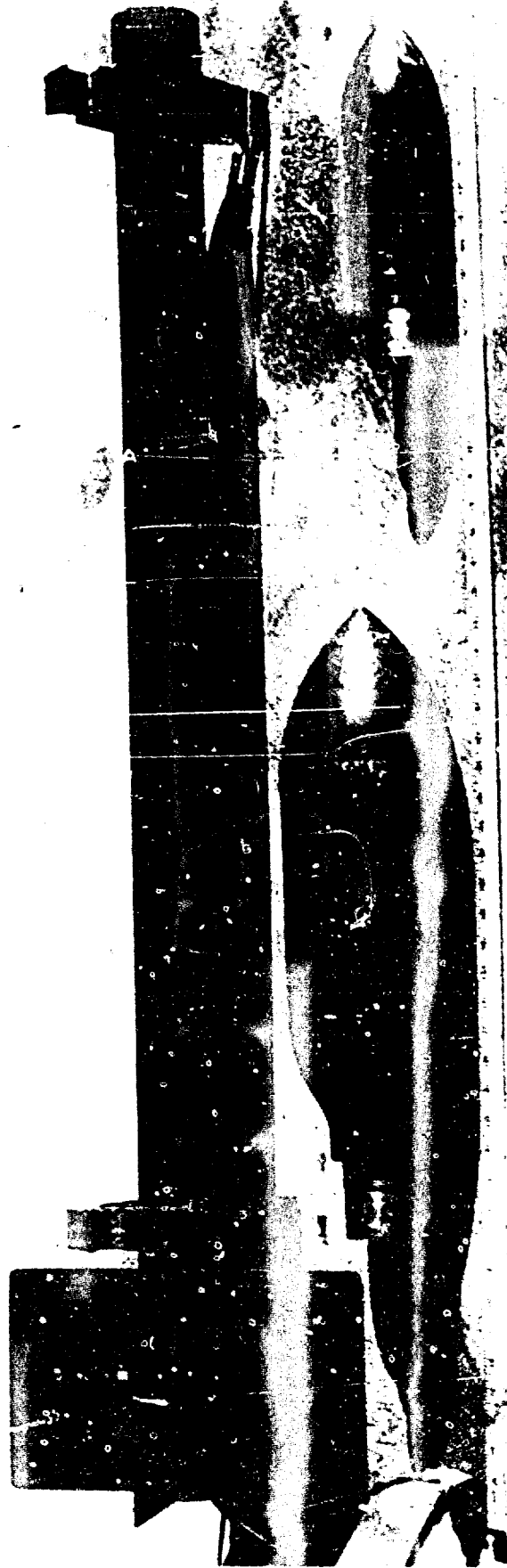
The agreement of the calculations with the observations indicates the validity of the method of calculating trajectories. From these trajectories, aiming data can be obtained.



NFB Photo No. 22477 (E1) Launcher rails for UP3 of SB24 plane, 1 Nov. 1943 OIA 2000111



NFO Photo No. 22478 (EX) UP3, Motor, 60 lb. head, 25 lb. head, 3 Nov. 1943, CONFIDENTIAL



The differential corrections for variations in initial conditions were calculated and their relative importance appraised. An analysis of the dispersion of the rockets and the results of an investigation of the causes of errors are given.

The rockets were British UP3 (shown in NPG Photo No. 22478). The motor, No. 1, Mk 11, has an 11 lb. cruciform powder grain. (The entire motor weighs 35 lb.) The 25 lb. AP and the 60 lb. HE heads were used. The rockets were fired either one at a time or in pairs from under opposite wings of the airplane. An underslung wing installation of a British Mark I projector (7.5 ft. rails) was used on an SB2A plane (NPG Photo Nos. 22476 and 22477).

NPG PHOTO. NO. 24622

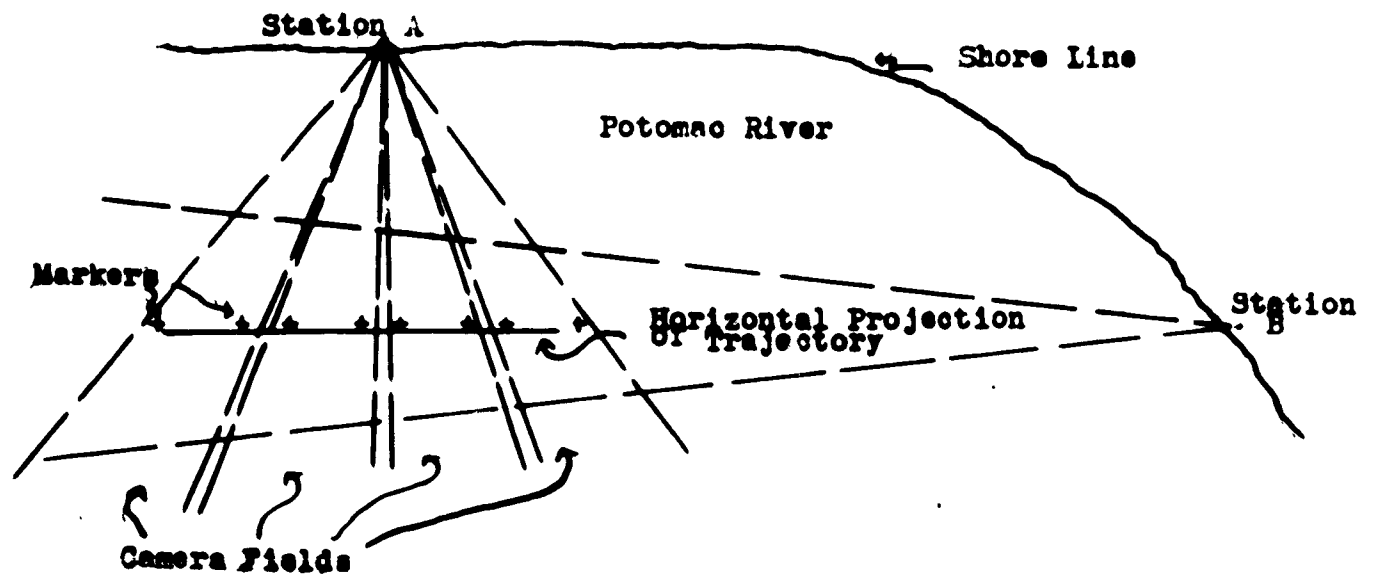
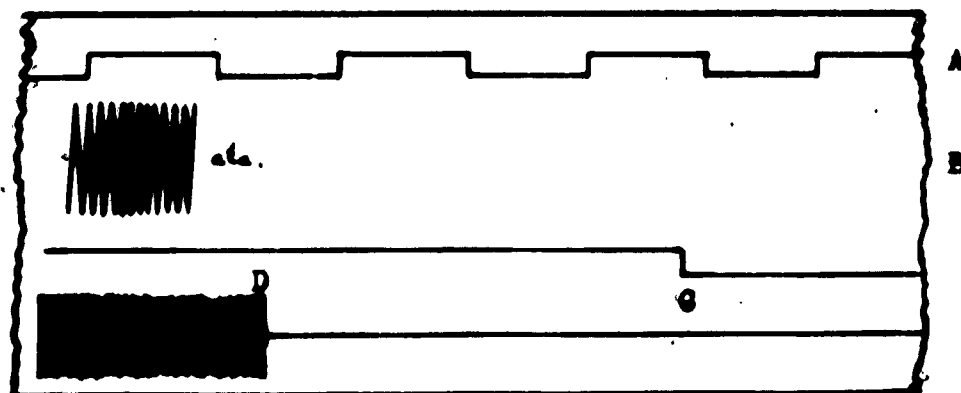


Figure 1.1



- A Second Signals
- B 50 cycle wave
- C Motion of Indicator which appears in all camera records
- D Firing of Rocket

Figure 1.2

Part I

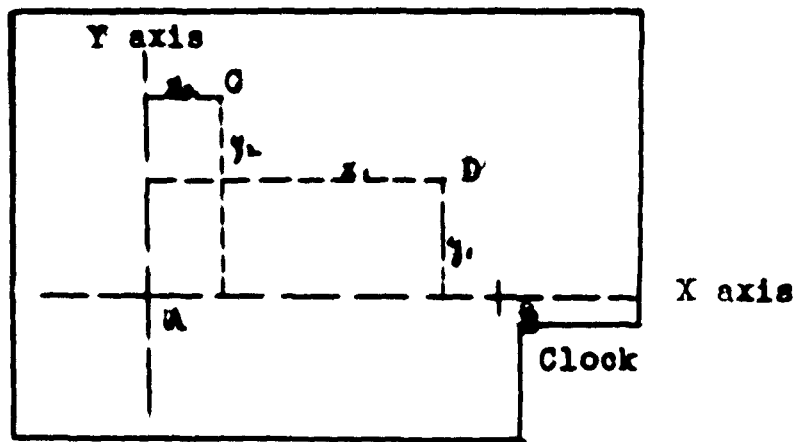
PHOTOGRAPHIC DETERMINATION OF ROCKET TRAJECTORIES

The trajectory of the rocket was photographed by five 35 mm Mitchell Cameras equipped with six-inch focal length lenses and located as shown in Figure 1.1. The four cameras at Station A covered a range of about 3,000 ft. with small angles of separation between the camera fields. These cameras were run at 128 frames per second. The single camera at Station B furnished angular data which were combined with the data from Station A to triangulate various points on the trajectory. Two markers were placed in the field of each camera at Station A in order to calibrate the lenses and relate the fields of the various cameras. The markers were in the form of one-foot-square targets placed on poles in the river along the proposed path of the rocket. These markers, which were all at the same height above the water, formed a horizontal reference plane. As all rockets were released from an altitude of 250 feet or less, the height of the field of the six-inch lense was in all cases large enough to photograph simultaneously the point of rocket release and the markers.

Each camera recorded in the lower right corner of the picture frame a solenoid indicator and a chronometer driven from a tuning-fork-controlled 50 cycle generator. The chronometer provided timing for its particular camera and the solenoid indicators, controlled by a single key, provided a common time between cameras. An oscillograph was used to record:

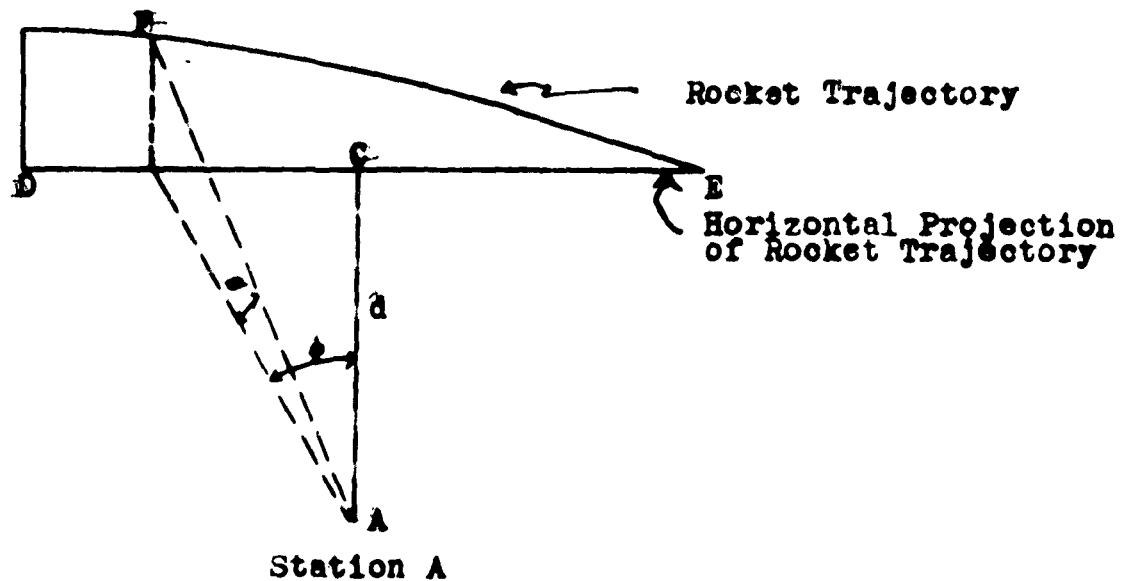
- (1) The current in the solenoid indicators.
- (2) A radio signal (received from the firing aircraft) which was cut off at the instant of firing.
- (3) A 50 cycle signal as a time reference.
- (4) One second interval signals from the chronometers.

NPG PHOTO. NO. 24777



A and B	Markers
C	Position of airplane
D	Position of rocket

Figure 1.3



AC is perpendicular to DE and of length d
F position of rocket at any time

Figure 1.4

A sample of the oscillograph record is shown in Figure 1.2. The timing system permitted events in each camera field to be referred to the instant of firing as a common time origin.

The angle of elevation of the launching rails at the instant of firing was determined by a torpedo-point-of-aim camera mounted under the wing of the plane. This camera took a picture when the firing circuit was closed. The camera on the plane was aligned with the rails by aiming both camera and rails at a distant point (4,500 ft. away). This point was determined by the machine gun sight which was aligned with the thrust line of the plane (a line parallel to the propeller shaft). The fact that the rails and the camera were located several feet below the machine gun sight resulted in their being tilted up 2.5 minutes above the thrust line.

The point of release was in the center of the field of the first camera at Station A. This allowed the plane to be photographed for about one second before the rocket was fired.

Reduction of the Data

The measurements of the Mitchell Camera film were made by means of a 20 power single frame projector. Every fourth or fifth frame was projected and the positions of the rocket and airplane, with respect to the markers, were plotted. The clock was read for each frame thus measured. A set of rectangular coordinates was erected on the screen (Figure 1.3), the origin being at one of the markers and the x-axis being horizontal. The x and y coordinates of each recorded position of the airplane and rocket were measured in arbitrary units. The x and y coordinates were converted to angles. This was done by calibrating the field of the camera, e.g., so many units of distance were equivalent to one degree of angle. This was accomplished by erecting a sur-

veyor's transit at the position of the camera and measuring the angle between the two markers which appeared in the camera record. The markers were projected on the screen along with the plane and rocket. The distance between the markers was measured. This distance, with the angle between the markers, determines the factor for converting distances to angles. This factor was taken as a constant over the entire field of the six inch lens, an assumption which leads to a final maximum error of about one foot in the position of the plane or rocket.

The angles measured in all the cameras were converted to angles with respect to one centrally located marker. The relative positions of the centrally located marker, station A, and station B were accurately determined by triangulation. These three points were laid off on a plotting table to the scale of 400 feet to the inch. The start of the trajectory and the impact of the rocket were plotted using the horizontal angles as measured from the camera records from stations A and B. A perpendicular, AC, (Figure 1.4), was drawn from the position of the camera station A to the horizontal projection of the trajectory. All horizontal angles were converted to angles with respect to line AC by an addition or subtraction and were denoted by ϕ . Vertical angles were denoted by θ . The distance AC was denoted by d and measured on the plotting table (in feet). X and Y coordinates of the rocket are computed by the following equations:

$$\begin{aligned} X &= d \tan \phi \\ Y &= d \tan \theta \sec \phi. \end{aligned}$$

X coordinates were changed to coordinates with respect to an origin directly beneath the point of firing of the rocket. The times of the clock, as read from the camera records, were converted to times measured from the instant of firing.

Two graphs were made: X against Y and X against t. From the X vs. t graph, values of X for each one-tenth of a second were taken. Values of Y corresponding to these values of X were taken from the X vs. Y graph. This gave a table of X and Y for equal time intervals from which were calculated velocities and accelerations.

The X and Y coordinates of the airplane were determined in the same way. These coordinates gave the ground speed of the plane and the angle of dive.

The record from the torpedo camera on the plane gave the sum of the angle of dive and the attitude of the plane. Pictures previously taken on the ground with this camera determined the point of aim with respect to fiducial marks on the edge of the film. This camera was calibrated in the same manner as the Mitchell Cameras. The distance between the horizon (corrected for curvature of the earth) and the point of aim was converted to an angle, which is the sum of the dive angle, the attitude, and the inclination of the camera above the thrust line. As the dive angle was determined with the Mitchell Cameras, the attitude angle can be determined. In cases where the horizon was not distinct, the target was used as a reference point and its angle below a horizontal plane through the torpedo camera was calculated from the Mitchell Camera records.

Reduction of Trajectories to Standard Conditions

The trajectories were reduced to: (1) horizontal launching, (2) ground speed of 200 kts., and (3) no range wind. The angle of launching was composed of three parts: the angle of climb or dive, the attitude of the plane, and the inclination of the rails. The last two angles were added. The effective fraction of this combined angle was determined by adding vectorially the launching velocity and the velocity of the plane. The resultant angle was used, the assumption

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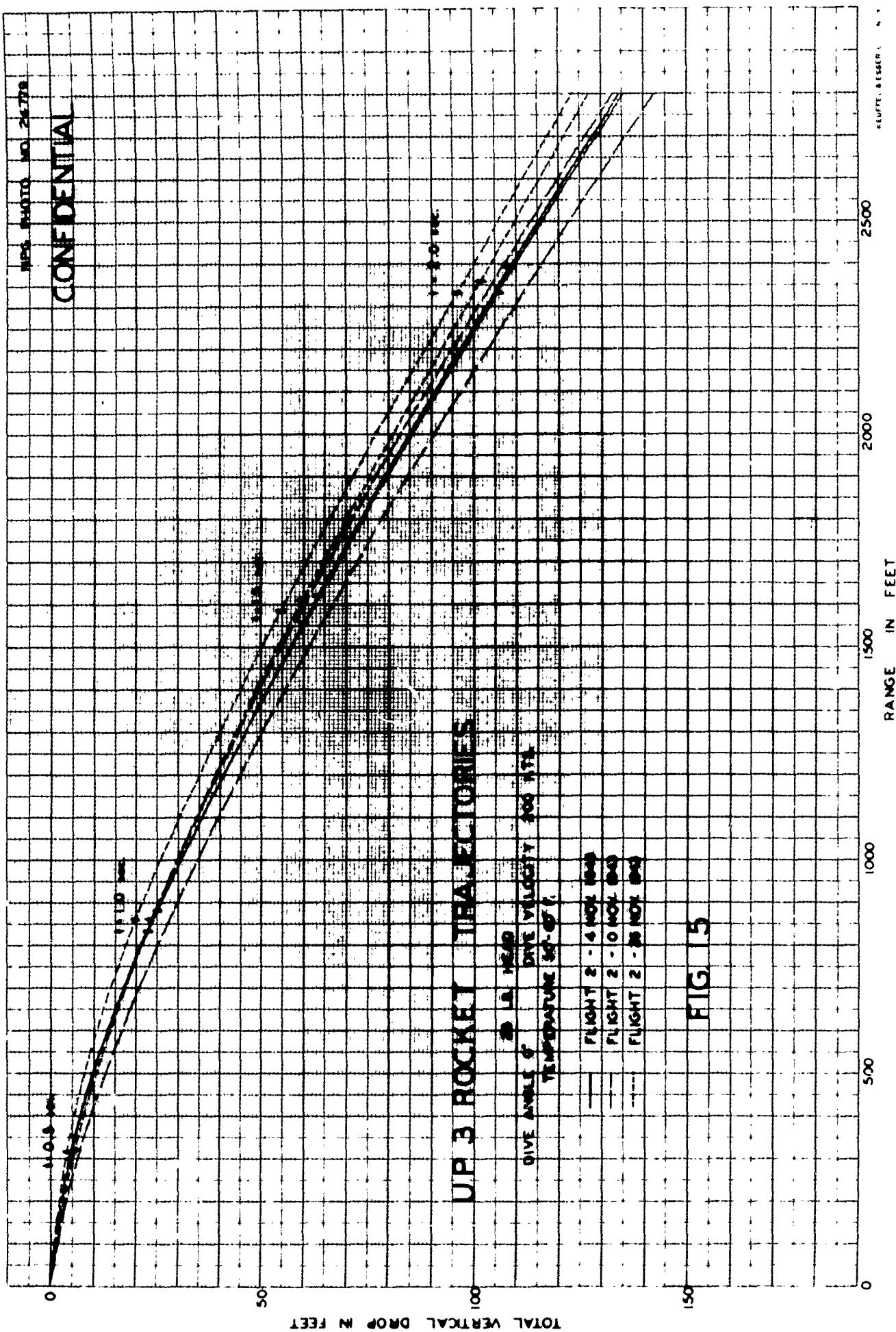
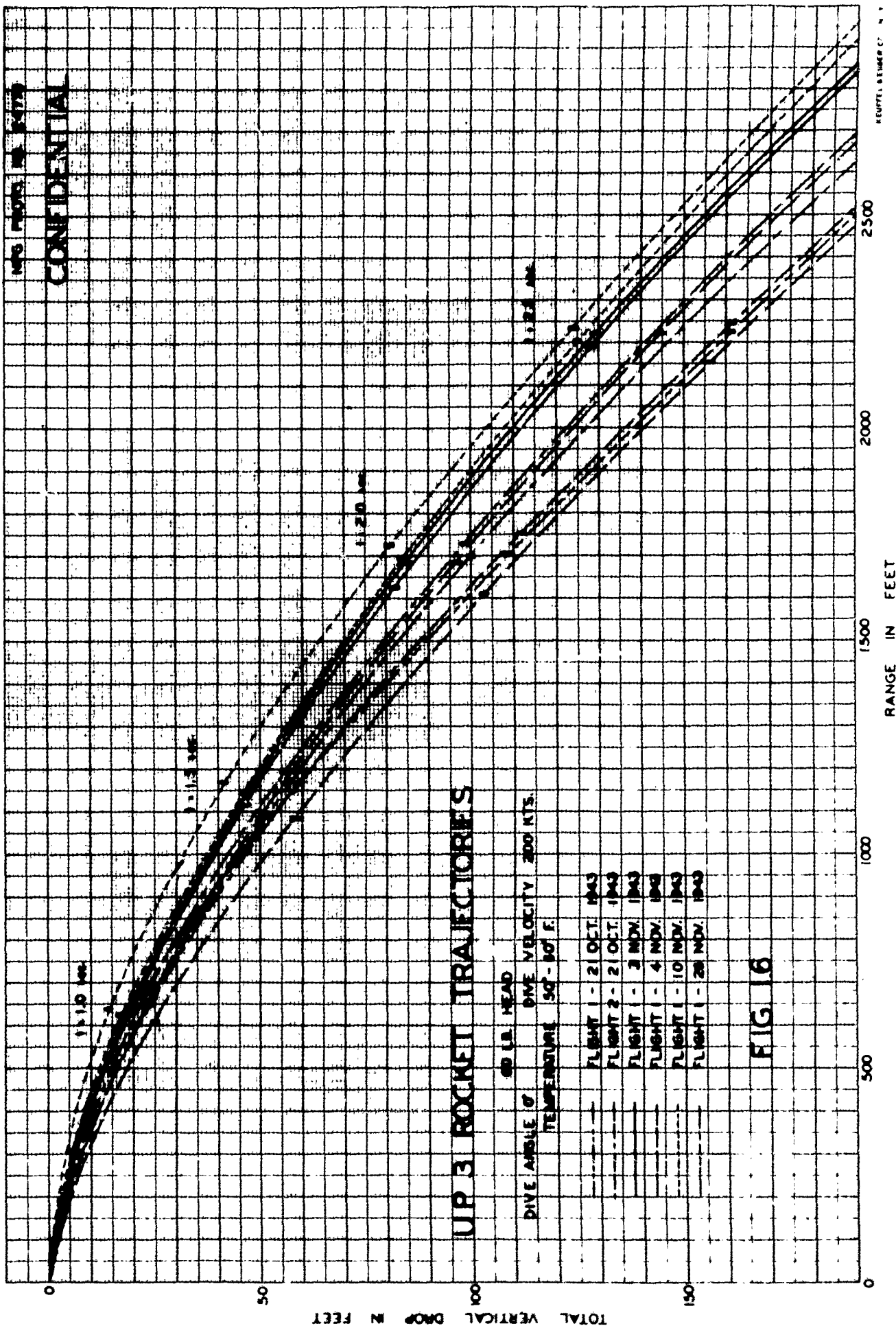


FIG 15

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being made that the rocket immediately aligned itself with the relative air stream. This resultant angle was added to the angle of dive and a simple rotation of the trajectory was performed. Due to the small angle of rotation, less than one degree in all cases, no account was taken of the change of direction of the force of gravity.

The ground speed was corrected to 200 kts. by assuming no air resistance. The correction to X to be added = -1.689 (ground speed - 200) $\times$ (corresponding time). This was justified by the smallness of the corrections.

The correction for range wind was determined by integrating a trajectory.

The temperatures of the rockets when fired were within the limits of $55^{\circ}\text{ F} \pm 5^{\circ}\text{ F}$. Since the differences in temperature were small, no attempt was made to correct the trajectories to exactly 55° F . An examination of the data showed no correlation of range with these small differences in temperature.

The resultant trajectories, 10 for the 60 lb. head and 6 for the 25 lb. head, are presented in Figures 1.5 and 1.6.

Vertical Dispersion

The variations in temperature, weight of motors, and weight of heads were so small that the trajectories were not corrected for them. As only the vertical dispersion was studied, the trajectories were not corrected for cross-winds.

The true mean dispersion was computed by multiplying the mean deviation by $\sqrt{n/n-1}$ where n is the number of values used to find the mean deviation.

Vertical Dispersion

True mean dispersion

Level flight- no range wind- ground speed of 200 kts.

Range	1200 ft.	1800 ft.	2400 ft.
60 lb. head (ten trajectories)	± 6 ft.	± 11 ft.	± 15 ft.
25 lb. head (six trajectories)	± 2 ft.	± 3 ft.	± 4 ft.

Part 2

CALCULATION OF ROCKET TRAJECTORIES

It is clear that preparation of a range table for rockets would be simplified if rocket trajectories could be computed somewhat as trajectories for projectiles are computed. Theoretically, the problem is straightforward. One obtains a table showing the thrust, $q(t)$, of the rocket motor as a function of time, another table showing the mass, $m(t)$, of the rocket as a function of time and a third giving the acceleration due to air resistance, $r(v,m)$, (hereafter called retardation) of a rocket as a function of its velocity and mass. Then one assumes that there is no yaw and integrates (for example) the equations

$$m \frac{dv}{dt} = q(t) - m r(v,m) - m g \sin \theta$$

2.1

$$\frac{d\theta}{dt} = - \frac{g \cos \theta}{v}$$

where v is the velocity, θ is the angle between the tangent to the trajectory and the horizontal, and g is the gravitational acceleration. Using a variety of initial conditions, one obtains a range table.

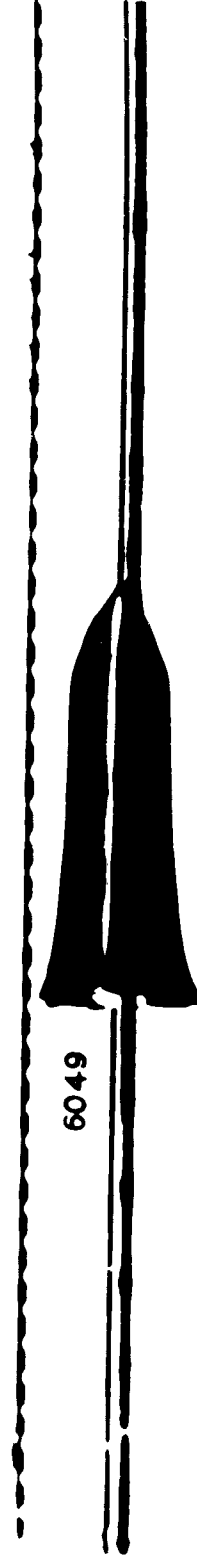
This is exactly what has been done in this section. The work was regarded as an experiment in making this sort of computation rather than as an attempt to obtain an actual range table. It was desired to find out how much must be known about a given type of rocket in order to predict its trajectory with satisfactory accuracy.

6052

WV



6049



ULTRA 8000

WV

6013

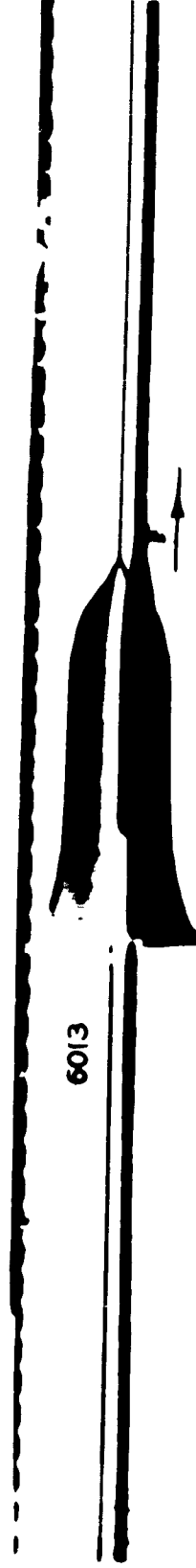


FIG. 2.1

NPG PHOTO. NO. 24586
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It should be noted that calculations are based on the evidence of a single thrust record which was not made under local supervision. Perfect agreement between these calculations and observations was not expected and was not obtained. The agreement was good enough to indicate that the method is sound and is worth using under better controlled conditions. The effects, given later, of various perturbing conditions are believed to be substantially correct. They provide a basis for setting up the conditions under which information about rockets should be obtained in the future.

Approximate tables of $q(t)$, $m(t)$ and $r(v,m)$ were computed. The table for $q(t)$ was obtained from a record of a static thrust test of the rocket. From this $m(t)$ is obtained by assuming that the thrust is proportional to the rate the mass is changing, and by integrating the equation

$$2.2 \quad q(t) = k \frac{dm}{dt}.$$

The constant k is evaluated by equating the total area inside the thrust curve to k times the total weight of powder burned. Finally, it was assumed that the retardation is given by

$$2.3 \quad r(v,m) = \frac{J_6(v) v}{C_1 m}$$

where $J_6(v)$ is the standard Army drag function and $C_1 m$ is the ballistic coefficient of the rocket when its weight is m . The assumption 2.3 is not justifiable except as a first approximation. The drag function of a rocket is undoubtedly different from that of a projectile. It seems unnecessary to consider carefully the exact character of the drag function in view of ignorance of other, more important factors. For example, the average thrusts of two rocket motors, judging by the static thrust tests, see Figure 2.1 below, may differ by 220 pounds, and this under laboratory conditions,

The greatest air retardation (based on Equation 2.3) entering these computations was about 170 poundals. Since this latter is probably an overestimation of the actual retardation*, it can be seen that the true retardation need not be very accurately known.

The Equations 2.1 are more easily handled in the following form, which defines a new variable v_1 , the velocity the rocket would have in the absence of air and gravity. $m_1(t)$ is given the subscript 1 to distinguish it from the case where a different head is used on the same motor and the complete rocket weighs $m_2(t)$. In this case the vacuum velocity is denoted by v_2 .

$$\frac{dv_1(t)}{dt} = \frac{q(t)}{m_1(t)}$$

$$2.4 \quad \frac{d(v-v_1)}{dt} = - \frac{v J_6(v)}{C_1 m} - g \sin \theta$$

$$\frac{d\theta}{dt} = - \frac{g \cos \theta}{v}$$

Substituting $q(t) = k \frac{dm}{dt}$, the solution of the first equation is

$$2.5 \quad v_1(t) = k \log_e \frac{m_1(t)}{m_1(0)}.$$

*Cf The Calculation of Rocket Trajectories, February, 1942 PDE Boncath, communicated by C.S.P.D.E. In the sample calculations found in this paper an elaborate resistance law is used. Although the rocket considered in this paper is somewhat lighter than the UP3, the retardation used is about half that used here at corresponding velocities.

The remaining two equations are integrated by numerical integration. J_0 changes so rapidly during the burning of the propellant that a very small interval (1/10 sec.) must be used if the trajectories are to be comparable to each other. The integration is straightforward. Listed below are the various quantities used in the present calculations of the trajectories of the British UP3 rockets fitted with 25-lb and 60-lb heads. These trajectories are compared with the photographed trajectories of Section 1. In making this comparison it is important to note that none of the constants or functions used is arbitrary or was determined so as to make the theoretic trajectories fit the photographed ones. It is possible to compute the equation of a curve which fits a given set of observations to any desired degree of accuracy. The attempt here was to see whether the trajectories could be predicted before any rockets were fired.

The assumption was made that the thrust is given by the thrust curve #6049 of Figure 2.1, using the factors given in Table 2.1, which is a copy of the table accompanying the thrust curves. The other curves included in Table 2.1, curves #6013 and #6052, were not clear enough to read.

$\int_0^t q(t) dt$ was determined graphically from an enlarged photograph of the thrust curve. The figure obtained is about 1% higher than the product (average thrust) $\times$ (time of burning), as obtained from the Table. The results are probably accurate to within $\pm 1\%$. m and k were found from the integral of Equation 2.2:

$$2.6 \quad \int_0^t q(t) dt = k [m(t) - m(0)]$$

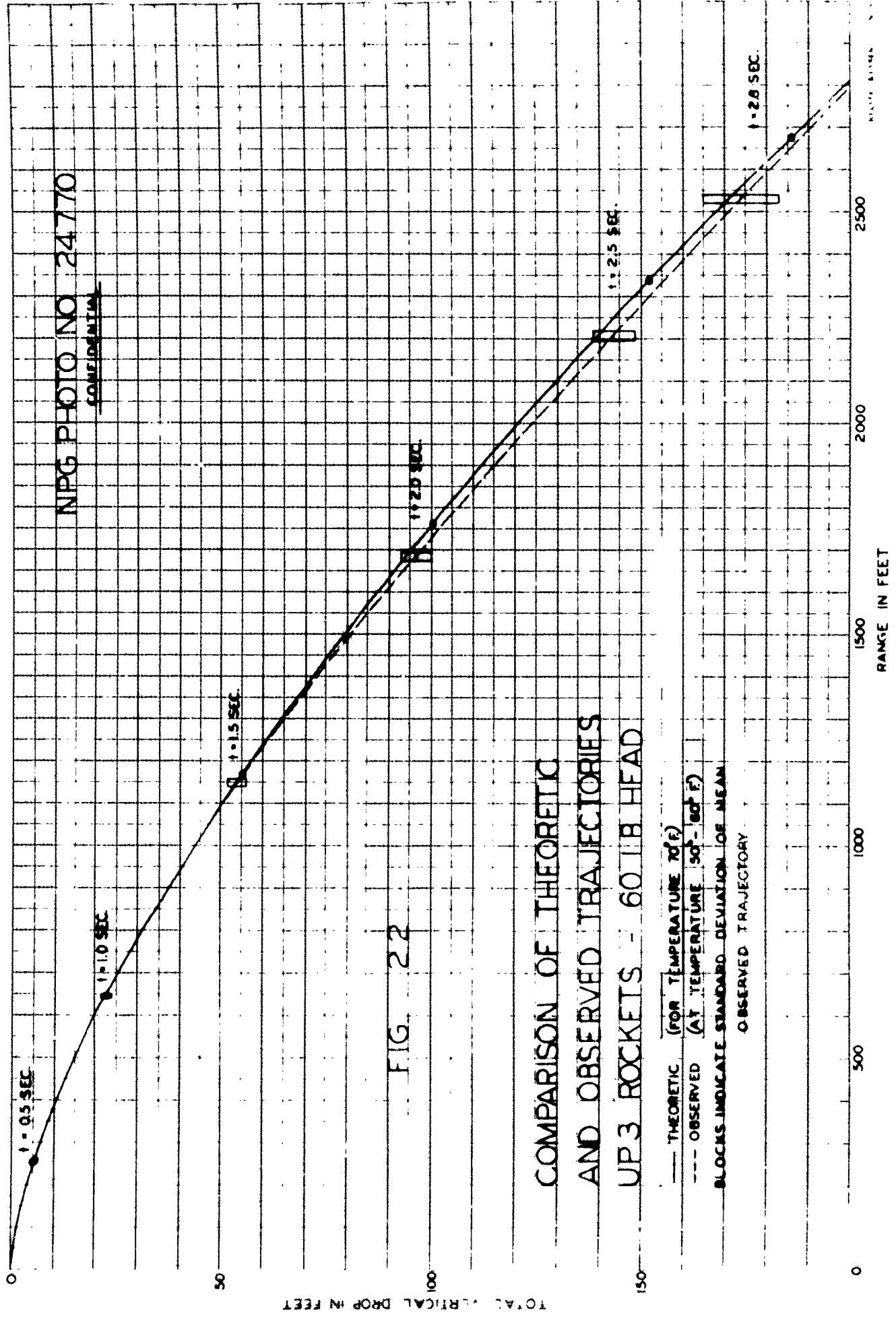
using the assumption that the total weight of pro-

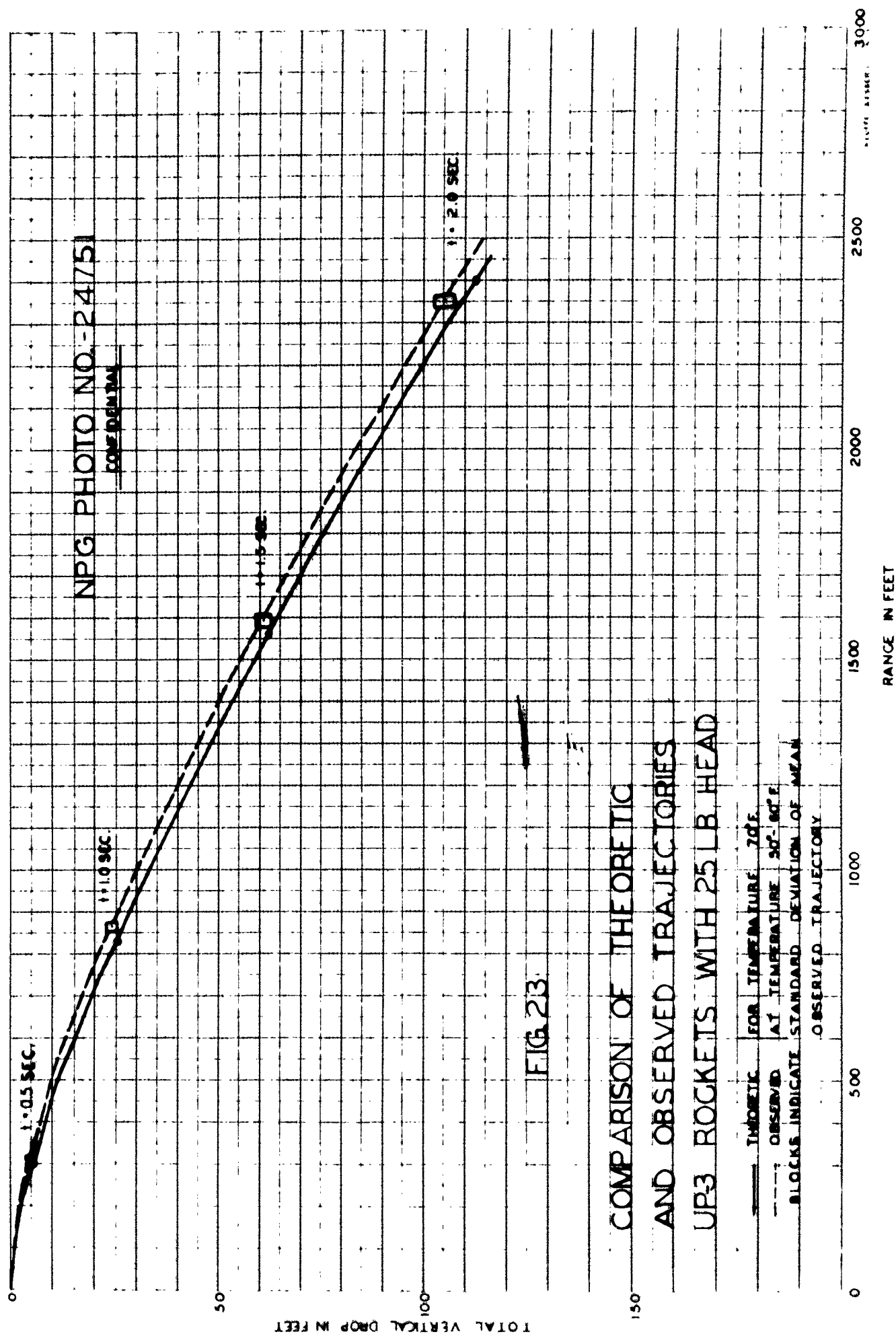
Table 2.1

From records of Naval Powder Factory, Indian Head, Maryland

Round No.	6013*	6049	6052
Date, 1943	2/16	2/17	2/17
Magazine Temperature, °C	20	20	20
Film Time Factor, Sec./in.	0.733	0.733	0.733
Burning Time, Sec.	1.65	1.75	1.68
Aver. Rate of Burning, in./sec.	0.27	0.25	0.26
Pressure Gage No.	4B	4D	4D
Pressure Gage Factor, psi/in.	3550	4100	4100
Max. Film Pressure, psi	----	985	1070
Aver. Film Pressure, psi	----	550	610
Thrust Gage No.	2500B	2500B	2500B
Thrust Gage Factor lb./in.	2700	2700	2700
Max. Thrust, lbs.	2580	2380	2480
Aver. Thrust, lbs.	----	1450	1520

*Pressure record late, due to metal diaphragm in rocket which was not removed but eventually burned through.





pellant expelled from the rocket is 11 lbs. In Table 2.2 are given the resulting values of k ,

$$m(0) - m(t), \quad v_1(t) = k \log_e \frac{m_1(t)}{m_1(0)},$$

(for the rocket with the 60 lb. head) and

$$v_2(t) = k \log_e \frac{m_2(t)}{m_2(0)}$$

(for the rocket with the 25 lb. head).

To determine the ballistic coefficients of the rockets, two unloaded rockets of each type were dropped as bombs and their Gavre coefficients determined by the usual Navy standard bomb calibration methods. Then a J_0 coefficient was computed which gave the same air resistance at low velocities. The weights (83.7 and 49 lbs, respectively) were divided out to obtain G_1 and G_2 .

The resulting computed trajectories are compared to the photographed trajectories in Tables 2.3 and 2.4 and in Figures 2.2 and 2.3. The release conditions are horizontal flight at a speed of 338 f.s. and no wind. The point of release is taken as the origin.

In Table 2.4, y values corresponding to given x values are compared rather than positions at corresponding times. In both tables, photographed positions are given $\pm$ the standard error of the mean position. The figures in the second table are more reliable (since we know the position better than position-as-a-function-of-time) and are of more interest in determining accuracy of rocket fire.

There is a significant difference between the two trajectories. At $t = 2.5$ sec, the x -coordinate of the theoretic trajectory is 2339 ft., whereas that of the mean of the photographed trajectories is 2207 ft. with a standard error of the mean of 12 ft. The method used has not yielded completely

Table 2.2

<u>t (sec)</u>	<u>m(0)-m(t) (lbs)</u>	<u>v₁(t) (ft./sec.)</u>	<u>v₂(t) (ft./sec.)</u>
.0	.000	.0	.0
.1	.850	67.5	106.8
.2	1.726	137.5	218.2
.3	2.518	201.6	320.7
.4	3.253	261.4	416.9
.5	3.953	318.9	509.7
.6	4.633	375.1	601.0
.7	5.299	430.7	691.3
.8	5.954	485.7	781.7
.9	6.600	540.3	872.0
1.0	7.244	595.2	962.4
1.1	7.888	650.5	1053.8
1.2	8.532	706.1	1147.2
1.3	9.173	762.0	1240.9
1.4	9.787	815.9	1331.8
1.5	10.330	863.8	1413.0
1.6	10.737	900.0	1474.6
1.7	10.967	920.5	1509.6
1.8	11.000	923.4	1514.6

$$k = -7.48 \times 10^3 \text{ ft./sec.}$$

$$m_1(0) = 94.7 \text{ lbs.}$$

$$m_2(0) = 60.0 \text{ lbs.}$$

Table 2.3

t (sec.)	<u>Theoretic Trajectory</u>		<u>Photographed Trajectory</u>		<u>Theoretic Trajectory</u>		<u>Photographed Trajectory</u>	
	(60-lb head)		(60-lb head)		(25-lb head)		(25-lb head)	
	x(ft.)	y(ft.)	x(ft.)	y(ft.)	x(ft.)	y(ft.)	x(ft.)	y(ft.)
.5	251	-5.1	259±4	-5.6±.4	300	-5.6	315±5	-4.9±.3
1.0	644	-22.7	644±7	-22.2±1.2	829	-25.4	865±11	-24.1±1.2
1.5	1168	-55.2	1149±10	-54.0±2.1	1560	-62.1	1593±16	-61.0±1.9
2.0	1762	-100.4	1683±12	-96.6±3.6	2398	-112.4	2352±14	-105.1±2.7
2.5	2339	-152.1	2207±12	-143.6±5.9				
2.8	2675	-186.1	2530±9	-173.9±9.1				

Entries are quoted ± the standard error where

$$\text{standard error} = \sqrt{\sum (\text{deviations})^2 / (n-1)}$$

Table 2.4

x(ft.)	<u>Theoretic Trajectory</u> (60-lb head)		<u>Photographed Trajectory</u> (60-lb head)		<u>Theoretic Trajectory</u> (25-lb head)		<u>Photographed Trajectory</u> (25-lb head)	
	y		y		y		y	
500	-15.4		-15.2±.9		-12.1		-10.9±1.5	
1000	-43.9		-43.7±1.8		-33.5		-31.6±1.2	
1500	-79.6		-81.0±3.1		-59.3		-57.1±1.4	
2000	-120.8		-124.6±4.3		-87.5		-85.5±1.8	
2500	-168.2		-171.3±7.3		-119.6		-116.6±2.4	

Entries are quoted ± the standard error where

$$\text{standard error} = \sqrt{\sum \frac{(\text{deviations})^2}{n(n-1)}}$$

satisfactory trajectories. In the next section, where variations from standard conditions are considered, it will be seen that none of them appears to be large enough to account for the difference. The resistance law is of course subject to question, but the retardation would have to be doubled, approximately, to account for the discrepancy. An assumption that the thrust used is too large, especially toward the end of burning, would account for most of it. For instance, if burning ceased at $t = 1.5$ sec. instead of at $t = 1.7$ sec., the results would be very good.

It is to be noted that these trajectories (both theoretic and photographed) do not agree with trajectories published by the British and reproduced in an OSRD report, Abstract of British Reports in Forward Firing From Aircraft. * No satisfactory explanation for the disagreement has been found.

*OSRD Report CIT UMC 7, 4 Aug. 1943.

DIFFERENTIAL CORRECTIONS

In discussing differential corrections for rocket trajectories, the point of view adopted is that of one who is interested in reducing an observed trajectory to standard conditions. First the method of computing the corrections is indicated, then the effect of applying them to the observed trajectories is examined. The remarks in the text apply to the rocket with 60 lb. head unless a specific statement is made to the contrary.

It will appear later, from observations of pairs of rockets fired simultaneously, that even if all the rockets had been fired under the same conditions, the coordinates at, for example, $t = 2$ sec. would be expected to have standard deviations of 37 feet in x and 3 feet in y , or of 3 mils in angle measured at the point of firing. With these numbers in mind, it is proposed to answer the following question for each correction: How accurately must the magnitude of a given perturbation be known in order to make the correction for it be accurate to ± 1 ft. in y and ± 12 ft. in x , or ± 1 mil in angle? Throughout this discussion the elements of the standard trajectory are written as v, θ, x, y and those of the perturbed trajectory V, Θ, X, Y .

Because of the roughness and rapid variation of the velocity, it is not safe, unless one takes special precautions, to calculate a differential correction by taking the difference between two trajectories integrated from different initial conditions. If this method is followed, one integrates Equations 2.1 with initial velocity, for example, $= 338$ f.s. and obtains v and θ , and then integrates the same equations with initial velocity, for example, $= 348$ f.s. obtaining V and Θ . But it is important that the differences, $V-v$ and $\Theta-\theta$, contain no errors due to numerical integration of rapidly varying thrust data. The best way

to avoid the difficulty seems to be to take the differences $\frac{d\theta}{dt} - \frac{d\phi}{dt}$ and $\frac{dV}{dt} - \frac{dv}{dt}$ and make sure that $\theta - \phi$, and $V - v$ are the integrals of these quantities. Instead of calculating X , and x , and then differencing, one calculates $\frac{dX}{dt} - \frac{dx}{dt} = V \cos \theta - v \cos \phi$ and integrates to find $X - x$. Similarly for $Y - y$.

On difference equations can be integrated directly. For changes in velocity and angle only, these equations are:

$$2.7 \quad \frac{dw}{dt} = - \left(J_6 + \frac{dJ_6}{dv} v \right) \frac{w}{C_1 m} - g \cos \theta \phi$$

$$\frac{d\phi}{dt} = \frac{g \cos \theta}{v^2} w + \frac{g \sin \theta}{v} \phi$$

where w is the velocity differential $V - v$, and ϕ is the angle differential, $\theta - \phi$. These equations are most easily handled in the form

$$\frac{dw}{dt} = - J_6 (n-2 + 2) \frac{w}{C_1 m} - g \cos \theta \phi$$

$$\frac{d\phi}{dt} = - \frac{d\theta}{dt} \left(\frac{w}{v} + \tan \theta \phi \right)$$

where $n - 1 = \frac{d \log J_6(v)}{d \log v}$. Tables of $n - 2$ are already in existence.

Equations 2.7 are easy to integrate since w and ϕ are small and change slowly. However, when w and ϕ have been found, one still has to calculate $V = v + w$ and $\Theta = \theta + \phi$, then $\frac{dX}{dt} = \frac{dx}{dt}$ and $\frac{dY}{dt} = \frac{dy}{dt}$

as before. The corrections discussed below are summarized in Tables 2.5 - 2.8.

The Correction for Angle of Release

This was carefully calculated out to $t = 3$ sec. for an angle of glide of 1° . As would be expected for a trajectory with such a short time of flight, the result amounts almost exactly to a rotation of the trajectory. At $t = 3$ sec., $X - x = 4.2$ ft., $Y - y = -50.49$ ft. when calculated by rotating the trajectory whereas $X - x = 2.4$ ft., $Y - y = 50.38$ ft. when calculated by integration of a new trajectory. For small angles, therefore, the corrections are calculated by rotation of the standard trajectory. If one wishes the correction to be good to ± 1 mil the angle of release should be measured to $\pm 3 \frac{1}{2}$ minutes. This is a difficult requirement to meet. It has not been met in the present experiment where the angle was only known to about ± 10 minutes.

As was mentioned in the first section of this report, the angle of release depends not only on the angle between the line of flight and the horizontal, but on the attitude of the plane and the angle at which the rails have been set. At the velocities encountered here, the angle of release is equal to the angle of the flight line plus about $1/4$ the angle

Table 2.5

Corrections (in feet) for Rockets with 60-lb heads.

Time	Angle of Release in-creased 1°		Ground speed * of plane in-creased 10 f.s.		Air speed ** of plane in-creased 10 f.s.		Weight of pro-jectile increased 1 lb.	
(sec.)	δx	δy	δx	δy	δx	δy	δx	δy
.5	0	4.4	5	0	0	0	-4	0
1.0	1	11.2	10	0	0	.2	-3	.1
1.5	1	20.4	15	0	0	.4	-7	.2
2.0	2	30.7	20	0	-1	.8	-11	.2
2.5	3	40.8	25	0	-3	1.3	-14	.3
3.0	4	50.5	30	0	-5	1.7	-17	.3

* Air speed unchanged
 * * Ground speed unchanged

Table 2.6

Corrections (in feet) for Rocket with 60-lb heads.

<u>Range</u> <u>(ft.)</u>	<u>Angle of</u> <u>release in-</u> <u>creased 1°</u>	<u>Ground speed *</u> <u>of plane in-</u> <u>creased 10 f.s.</u>	<u>Air speed **</u> <u>of plane in-</u> <u>creased 10 f.s.</u>	<u>Weight of</u> <u>projectile in-</u> <u>creased 1 lb.</u>
	δy	δy	δy	δy
500 ft.	8.8	.4	.1	-.1
1000 ft.	17.4	.9	.3	-.3
1500 ft.	26.2	1.4	.6	-.5
2000 ft.	35.0	2.0	.9	-.8
2500 ft.	43.9	2.7	1.1	-1.2

* Air speed unchanged

** Ground speed unchanged

Table 2.7

Corrections (in feet) for rockets with 25-lb heads.

<u>Time</u>	<u>Angle of release in- creased 10</u>	<u>Ground speed * of plane in- creased 10 f.s.</u>	<u>Air speed ** or plane in- creased 10 f.s.</u>
sec.	δx δy	δx δy	δx δy
.5	0 5.2	5 0	0 0
1.0	1 14.5	10 0	0 .2
1.5	1 27.2	15 0	-1 .4
2.0	2 41.8	20 0	-2 .7
2.5	4 55.8	25 0	
3.0	5 69.0	30 0	

* Air speed unchanged

** Ground speed unchanged

Table 2.8

Corrections (in feet) for rockets with 25-lb heads.

<u>Range (ft.)</u>	<u>Angle of release in- creased 1</u>	<u>Ground speed * of plane in- creased 10 f.s.</u>	<u>Air speed ** of plane in- creased 10 f.s.</u>
	δy	δy	δy
500	8.7	.3	.1
1000	17.4	.5	.2
1500	25.1	.8	.3
2000	34.9	1.1	.5
2500	43.6	1.4	.6

* Air speed unchanged

** Ground speed unchanged

between the rails and the flight line. The angle between the rails and the horizontal can be accurately determined from the torpedo camera. The angle of the flight line can be determined from photographs of the flight of the plane to within 10 minutes, or less under excellent conditions. Or, if trustworthy curves are available showing the attitude of the plane as a function of its air speed and load, these quantities may be used to determine the angle between the flight line and the rails.

The Correction For Wind and For Speed of The Plane

At The Time of Release

If the air speed remains constant but the ground speed differs from the standard 338 f.s., the correction is merely a horizontal translation of points on the trajectory and

$$x = X - (v_g - 338) t.$$

$v_g - 338$ is the range wind measured along the heading of the plane. The effect of cross wind is to produce a deflection equal to the product of cross wind times the time of fall.

If both air speed and ground speed differ from 338 f.s., the correction is:

$$x - \bar{x} \approx \left(\frac{\partial x}{\partial v_g} \right) \Delta v_g + \left(\frac{\partial x}{\partial v_a} \right) \Delta v_a.$$

The correction for change in ground speed at constant air speed is first evaluated, as above, and to this is added the correction for change in air speed at constant ground speed. The second term is the effect of air resistance and is evaluated by an integration.

In order to get the x correction to within ± 12 ft. at $t = 2$ sec. and the y correction within ± 1 ft., the ground speed of the plane must be known to within ± 6 f.s. or about ± 3.5 knots and the airspeed to within ± 12 f.s. or ± 7 knots. In reducing the trajectories given here, little agreement was found between the figures for ground speed, air speed and surface winds measured at an aerological station two miles away. The last was ignored and the correction based on the first two figures.

From the point of view of the pilot, these corrections are troublesome. He can get his air speed accurately enough, but the wind and therefore his ground speed and the drift are much more doubtful quantities. The problems are the same as those of bombing, though not as great, since the time of flight is short and the trajectory relatively straight.

However, it should be pointed out that the air speed affects the attitude of the plane at the rate of about $.3^\circ$ for 10 knots change in air speed. Thus 8 knots change in air speed changes the angle of release 3.5 minutes and changes the angle between the pilot's sight and his flight line by 13 minutes (or about 4 mils).

Correction For Weight of Rocket

The velocity imparted to a rocket by a given amount of thrust depends on the weight of the rocket. Returning to Equation 2.5

$$v_1(t) = k \log_e \frac{m(t)}{m(0)}$$

let δv be the change in v for a change in $m(0)$ of δm lbs. Then

$$\begin{aligned} \delta v_1(t) &= k \left(\frac{\delta m}{m(t)} - \frac{\delta m}{m(0)} \right) \\ &= \frac{k}{m(0)} \frac{m(0) - m(t)}{m(t)} \delta m. \end{aligned}$$

Using this equation, we calculate

$$V_1 = v_1 + \delta v_1$$

and integrate to get the change in x and y .

In order to get these corrections to within ± 12 ft. and ± 1 ft. at $t = 2$ sec. for x and y respectively, the weight of the rocket must be known to within 1 lb. (The heaviest and lightest rockets of the ten considered here differed in weight by 1.9 lbs. so the correction is worth making).

Ballistic Coefficient

The correction for a 10% change in ballistic coefficient has been calculated in order to see how well this coefficient must be determined. Moreover, the calculation gives one an estimate of the effect of a change in the resistance law. (The effect on the coefficient of change in weight was included in the weight differential.) Integration shows that C , which was taken to be .016, should be known to be within 16% if the x and y of the theoretic trajectory are to be within 12 ft. and 1 ft. respectively of the correct values.

Consideration of Other Perturbing Conditions

An important cause of variation, which was not taken into account here, is temperature. If the methods outlined here are used in preparing trajectories, it is clear that a satisfactory method of computing the temperature corrections is at hand. Thrust curves can be obtained under properly controlled conditions and new trajectories computed to obtain the temperature effect. In the present

instance no attempt was made to obtain these corrections. Such information as was available as to the temperatures of the rockets when they were fired was examined but no significant correlation with range was found. This could be due to a combination of the following reasons:

1. The range of temperature variation was not very great (50° F to 60° F).
2. The temperatures, at firing, were not known to better than ± 5° F since the rockets had been out of doors and unprotected for some time and the ambient temperature was not constant.
3. Other causes of dispersion were present.

Although it was recognized that there was some difference in the temperature at which the thrust test was made and that at which the other rockets were fired, this was not corrected for.

Thrust

In order to make a rough estimate of the accuracy to which one should know the thrust-time integral, suppose that the determination of the integral is 5% off. Then k and v_1 are 5% off. The main effect of this change in velocity is to move the rocket about 125 ft. forward or backward along its trajectory at $t = 3$ sec. The curvature of the trajectory is changed slightly as one sees from the equation

$$\frac{d\theta}{dt} = - \frac{g \cos \theta}{v}.$$

It is estimated that the vertical distance through which the trajectory is moved is about 8 ft. at $t = 3$ sec. (This is not the change in the y coordinate at $t = 3$ sec., which may be quite small. See Figure 2.2 to illustrate this point.) It is possible that most of the lack of agreement between photographed and theoretic trajectories is due to error in $q(t)$.

Presence of friction in the thrust apparatus would mean that the thrust is really larger at the beginning and smaller toward the end of burning than the thrust curve indicates. This would cause the theoretic curve to lie to the right during the early part of burning but drop back to its present position toward the end of burning.

A variation in the actual amount of propellant in a rocket motor will have somewhat the same effect. If, for instance, the powder grain is underweight by 1% (.1-lb.) the thrust-time integral for that particular rocket will be 1% short and the effect about $1/5$ of that of the above paragraph.

Another source of error is this: The rocket is still in contact with the rails more than .1 sec. after the firing button is pressed. In integrating, it has been tacitly assumed that Equations 2.1 hold from $t = 0$, on. This seems not to have led to any serious error but is certainly a possible source of discrepancies.

The foregoing analysis indicates that the photographed trajectories have been reduced as carefully as the information warrants. The angle of departure was not known quite as accurately as it should have been, nor was the precise temperature of the propellant or its exact weight.

Even if exact corrections had been made for all these variables, one would not expect to get rid of all of the dispersion, since no two rockets ever burn in exactly the same way or have exactly the same flight. This inevitable variation can be estimated from the distances between rockets fired in pairs.

Tables 2.9 and 2.10 show the extent to which the corrections reduce the variance of the photographed trajectories. If the figures in the lower half of the tables are reduced to standard deviations expressed in mils, it is found that the standard deviation of the ten (corrected) trajectories for the rocket with the 60-lb head varies from 5.4 mils (at $x = 500$) to 6.5 mils (at $x = 2500$ ft.)

Table 2.11 shows the deficiencies of the corrections. Here two independent estimates have been made of the variance to be expected in any sample set of trajectories for UP3 rockets with 60-lb heads. The first is made by finding the variances of samples of rockets which were fired at the same instant from the same airplane, (4 samples of 2 rockets each). The second estimate was made by finding the variances of the means of the samples about their mean (4 samples of 2 rockets each and 2 of one each). Each estimate is based on a different method of finding the variance of the population consisting of UP3 (60-lb head) trajectories. However, if the samples were random, the estimates should be reasonably alike, or if the variations in launching conditions had been completely corrected for, the estimates should be reasonably alike. Obviously the corrections made were not adequate, especially insofar as the variations affect the y coordinates. It is likely that it is the angle of climb or glide which is at fault. This correction

Table 2.9

Rocket with 60-lb head

	Uncorrected photographed tra- jectories		Corrected photographed trajectories	
	Variance in x (ft. ²)	Variance in y (ft. ²)	Variance in x (ft. ²)	Variance in y (ft. ²)
t = .5 sec. (n = 10)	230	2.8	130	1.7
t = 1.0 sec. (n = 10)	658	25.7	413	13.6
t = 1.5 sec. (n = 10)	1161	87.6	889	38.2
t = 2.0 sec. (n = 10)	1305	244.2	1348	104.4
t = 2.5 sec. (n = 8)	1440	491.5	979	244.0
t = 2.8 sec. (n = 5)	971	903.4	321	329.9
x = 500 ft. (n = 10)		12.5		7.3
x = 1000 ft. (n = 10)		60.7		30.0
x = 1500 ft. (n = 10)		176.4		84.4
x = 2000 ft. (n = 10)		346.6		165.5
x = 2500 ft. (n = 6)		801.1		264.0

$$\text{Variance} = \sum \frac{(\text{deviations})^2}{n}$$

Table 2.10

Rockets with 25-lb head

	Uncorrected photographed tra- jectories.		Corrected photographed trajectories.	
	Variance in x (ft. ²)	Variance in y (ft. ²)	Variance in x (ft. ²)	Variance in y (ft. ²)
t = .5 sec. (n = 4)	130	2.8	81	.3
t = 1.0 sec. (n = 4)	511	34.5	378	4.1
t = 1.5 sec. (n = 4)	957	120.5	726	10.4
t = 2.0 sec. (n = 4)	593	291.8	621	21.6
x = 500 ft. (n = 6)		8.5		1.5
x = 1000 ft. (n = 6)		38.4		6.8
x = 1500 ft. (n = 6)		86.9		10.4
x = 2000 ft. (n = 6)		152.5		16.5
x = 2500 ft. (n = 6)		244.6		29.4

$$\text{Variance} = \sum \frac{(\text{deviations})^2}{n}$$

Table 2.11

Rocket with 60-lb head

Corrected Photographed Trajectories

	Estimate of variance of population based on pairs of rockets * (4 pairs)		Estimate of Variance of population based on means of pairs ** (6 means)	
	Variance in x (ft. ²)	Variance in y (ft. ²)	Variance in x (ft. ²)	Variance in y (ft. ²)
t = .5 sec.	32	.2	234	3.3
t = 1.0 sec.	229	1.8	642	25.7
t = 1.5 sec.	853	5.3	1095	72.1
t = 2.0 sec.	1401	9.9	1575	200.6

* Estimate = $\frac{1}{4} \sum (x_j - \bar{x}_j)^2$ where $\bar{x}_j$ is the mean of the pair.

** Estimate = $\frac{1}{3} \sum k_j (\bar{x}_j - \bar{X})^2$ where $\bar{x}_j$ is the mean of a sample and k_j is the number of items in the sample.

According to the Snedecor Tables, the ratio of two corresponding variances would be as high as 15.52 by pure chance only one time in 100.

affects y much more violently than it does x and, as has been mentioned before, one has to know the angle of departure very accurately in order to make the correction properly. If one arbitrarily changes each angle of departure by an angle between $-10'$ and $+10'$ and chosen as advantageously as possible, one can reduce the variance calculated from the means of the samples from 206.1 ft. to 61.6 ft. at $t = 2$ sec. Thus we see that assuming our angles of departure are in error by as much as 10 min. helps to explain the differences in Table 2.11 but does not do so completely.

The variances estimated by finding the difference in position of two rockets fired simultaneously (the numbers in the columns on the left in Table 2.11) should tell one what to expect in the way of dispersion due to random and uncontrollable effects such as irregularities in the burning of the propellant. These are given in terms of standard deviations in Table 2.12.

Conclusion

The method of calculating rocket trajectories and differential corrections for variations in launching conditions given here is satisfactory.

It is shown that the inherent dispersion in range of British UP3 rockets forward fired from a plane in horizontal flight is of the order of ± 3 mils at a range of 500 yards.

A difference of 3.5 min. in angle of release, 25 f.s. in air speed (or 12 f.s. if it is to be used

Table 2.42

Estimate of standard deviation of position
due to random irregularities. Rocket with 60-lb head.

<u>t sec.</u>	<u>x (ft)</u>	<u>y (ft)</u>	<u>Angle (mils) measured from firing point</u>
.5	5.6	.4	3
1.0	15.11	1.3	3
1.5	29.2	2.3	3
2.0	37.4	3.1	3

$$\text{Standard deviation} = \sqrt{\sum \frac{(\text{deviations})^2}{n-1}}$$

in computation of attitude), 10 f.s. in ground speed or 2 lb. in weight of rocket, each cause a one mil difference in aim of the British UP3 rockets forward fired from a plane in horizontal flight at 500 yards range. These figures are recommended as the accuracy required for ranging data on rockets.

If trajectories are to be fitted both in time and space the accuracy of ranging data should be 3.5 minutes in angle of release, 6 f.s. in ground speed, 12 f.s. in air speed and 1 lb. in the weight of the rocket.

No measurements were made here of the temperature coefficient of range, but data already published indicate that the temperature should be known to within $\pm 5^{\circ}\text{F}$ in order to keep the effect of uncertainties in temperature smaller than the inherent space-time dispersion of the rockets.