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ROCKET PROPULSION ESTABLISHMENT

WESTCOTT, BUCKINGHAMSHIRE

R.P.E. TECHNICAL NOTE No: 184

SOME CONSIDERATIONS ON FACTORS AFFECTING THE DESIGN AND PERFORMANCE OF SOLID PROPELLENT ROCKET MOTORS:

(A). THE RELATIVE IMPORTANCE OF SPECIFIC IMPULSE AND PROPELLENT DENSITY

(B). THE SHAPE OF THE MOTOR BODY AS AN ADDITIONAL VARIABLE WHICH MAY BE USED TO ACHIEVE A GIVEN THRUST-TIME PROGRAMME

by

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ROCKET PROPULSION ESTABLISHMENT

WESTCOTT

SOME CONSIDERATIONS ON FACTORS AFFECTING THE DESIGN AND PERFORMANCE OF SOLID PROPELLENT ROCKET MOTORS:

A THE RELATIVE IMPORTANCE OF SPECIFIC IMPULSE AND PROPELLENT DENSITY

B THE SHAPE OF THE MOTOR BODY AS AN ADDITIONAL VARIABLE WHICH MAY BE USED TO ACHIEVE A GIVEN THRUST-TIME PROGRAMME

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W.R. Maxwell

SUMMARY

A The relative importance of an increase in propellent density as compared with an increase in propellent specific impulse for various applications of solid propellent motors is examined, and it is shown that for ballistic missile applications an increase in propellent specifio impulse is always much more valuable than the same percentage increase in propellent density, although an increase in propellent density is always beneficial. In cases where the payload is high, e.g. for the boost motors of a medium range surface to air weapon such as Seaslug, then for constant missile volume an increase in propellent density is almost as valuable as a similar increase in specific impulse.

B The shape of the body of a solid propellent motor is discussed as a design variable which may be used to obtain a given thrust-time programme. It is concluded that a cone and cylinder shape with a simple cylindrioal conduit has certain attractions which make it worth considering for some applications.

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A THE RELATIVE IMPORTANCE OF SPECIFIC IMPULSE AND PROPELLENT DENSITY

1 INTRODUCTION

In comparing the relative merits of liquid and solid propellent motors for ballistic missile applications it is accepted that for the longer range missiles the solid propellent version will consist of more stages than its liquid counterpart. However, for the solid motors to be an attractive proposition it is necessary that the motors of the separate stages should be of such a size that they can be filled and transported without undue difficulty. Although by clustering it should be possible to keep down the size of individual motors the size of the complete missile is not without importance - especially if the missile is to be carried in a submarine (e.g. Polaris). Once the payload and the number of stages have been fixed the factors which have the greatest effect on missile weight are the specifio impulse of the propellent and the ratio, propellent weight/inert weight for each motor. The general optimism in the U.S.A. in 1957-1958 which led to the forecasting of practical specifio impulses in the 250-260 lb sec/lb range within the very near future has not been justified, especially when coupled with the use of light-weight components, and some attention has therefore been turned to the possibility of inoreasing the propellent weight/inert weight ratio by inoreasing the propellent density, e.g. by the incorporation of heavy metals such as zirconium. Sometimes the term "density specific impulse" - specific impulse x propellent density - is used as a measure of propellent performance, the implication being that specific impulse and propellent density are of equal importance.

Studies of the relative importance of specific impulse and propellent density in relation to the performance of certain types of missile using liquid propellents have already been made[1] but little has been written on the relative importance of these factors in missiles using solid propellents. Rumbel, Friedman and Henderson[2] have compared the performance of missiles of constant motor volume and various mass ratios when filled with a series of propellents of different specific impulse and density but no analytical relationships were derived. The essential simplicity of solid propellent motors lends itself well to a simple general treatment and this is given below for two different restrictions on design: viz (1) constant missile volume (2) constant missile weight. The treatment is approximate yet is sufficiently accurate to reveal the essential points. It refers to motors used to boost missiles to the required velocity under conditions where the effects of drag and gravity on the velocity at all burnt, while not negligible, are nearly constant and therefore need not be taken into account. Only single stage missiles are dealt with, but the arguments can easily be applied to multi-stage missiles.

2 CASE 1: CONSTANT MISSILE VOLUME

For a single stage missile launched from rest we can write:

$$V = I.g\,\ell n\,\frac{M_p + M_B}{M_B} - (\text{gravity} + \text{drag})\text{corrections} \qquad (1)$$

where

V = the missile velocity at the end of powered flight when all the propellent has been consumed (taken to be a measure of missile performance)

I = propellent specific impulse

g = the appropriate acceleration due to gravity

M_p = the weight of propellent consumed

M_B = weight of missile without propellent = constant.

Since the missile volume and configuration are kept constant, as also are the launching conditions, and since it is proposed to investigate the effects of small changes of specific impulse and density, the gravity and drag correction may be regarded as constant to a high degree of approximation. Differentiating equation (1) we obtain

$$dV = I.g \frac{M_B}{M_p + M_B} \frac{dM_p}{M_B} + g.\ell n \frac{M_p + M_B}{M_B} dI$$

or

$$\frac{dV}{V+(\text{gravity}+\text{drag})\text{corrections}} = \frac{M_p/(M_p + M_B)}{\ell n\left[(M_p + M_B)/M_B\right]} \frac{dM_p}{M_p} + \frac{dI}{I} \quad . \quad (2)$$

If V is constant then

$$\frac{dM_p}{M_p} = -\frac{dI}{I} \frac{M_p + M_B}{M_p} \ell n \frac{M_p + M_B}{M_B} \quad . \quad (3)$$

If the volume of the missile is constant the volume of the motor will also be constant and $M_p \propto \rho$ where ρ is the propellent density.

Thus

$$\frac{d\rho}{\rho} = -\frac{dI}{I} \frac{M_p + M_B}{M_p} \ell n \frac{M_p + M_B}{M_B} \quad . \quad (4)$$

If we put

$$a = M_p/(M_p + M_B)$$

then

$$\frac{d\rho}{\rho} = \frac{dI}{I} 2.3026 \frac{1}{a} \log_{10}(1 - a) \quad . \quad (5)$$

The above method of analysis is similar to that used by Forbister in determining the relative importance of specific impulse and motor weight for a missile using liquid propellents[3].

The function, $2.3026 \frac{1}{a} \log_{10}(1-a)$, is plotted against a in Fig.1. It will be seen that when the mass ratio a is high (0.90 or above) as in a long range ballistic missile an increase of about 3 per cent in propellent density is required to compensate for a loss of 1 per cent in propellent specific impulse. For the boosts of a ground to air anti-aircraft missile such as Seaslug, however, where the ratio, boost propellent weight/total missile weight is of the order of 0.3, a small percentage increase in propellent density is nearly as valuable as a similar percentage increase in specific impulse.

The above analysis does not apply to sustainer motors since it neglects the effects of drag and sustainer motors are of comparatively low thrust and long duration, whose sole purpose is to overcome drag. It is however easy to see that here also an improvement in specific impulse is more beneficial than a proportionate improvement in propellent density. For suppose the specific impulse of the propellent in a sustainer motor is lowered by 1 per cent but the total impulse is kept constant by increasing the propellent density by 1 per cent. The missile will now be heavier and the same boost motors will therefore accelerate it to a lower velocity. At the same time because the missile is heavier it will need larger wings to manoeuvre under the same conditions and therefore at equivalent speeds the drag will be higher and the weapon less efficient.

3 CASE 2: CONSTANT MISSILE WEIGHT

Here again we start from the equation

$$V = I.g \ \ell n \frac{M_T}{M_B} - (\text{gravity} + \text{drag})\text{corrections} \tag{6}$$

where M_T, the total missile weight $(= M_p + M_B)$, is supposed constant. We also write

$$M_B = M_W + M_S$$

where M_W = payload = constant

and M_S = mass of empty missile body, which will be assumed to be the same as the mass of the motor body. The conditions of constant total missile weight and constant payload weight require that any change in propellent density shall be accompanied by a change in charge volume and therefore in motor body volume, since we shall assume that the motor loading density is held constant. The motor operating pressure will also be kept unchanged. If we keep the shape of the motor body constant, then the linear dimensions $\propto v^{1/3}$, where v = volume of motor body, the surface area $\propto v^{2/3}$ and the wall thickness $\propto v^{1/3}$, so that $M_S \propto v$.

We can therefore write

$$M_S = q \, V_\rho$$

where V_ρ is the propellent volume (= v × density of loading) and q is the constant mass of hardware required per unit volume of propellent for any particular motor design (i.e. mass of motor body/internal volume of motor × density of loading). If the propellent density is ρ, then $M_S = q \, M_p/\rho$.

We write

$$M_W = x \, M_T$$

where M_W, M_T and therefore also x, are constant.

Since

$$M_T = M_p + M_S + M_W \ ,$$

we have

$$M_T = \frac{\rho}{q} M_S + M_S + x \, M_T$$

or

$$M_S = \frac{q(1-x)}{\rho + q} M_T$$

and

$$M_B = M_S + M_W = \left\{\frac{q(1-x)}{\rho + q} + x\right\} M_T$$

$$= \left\{\frac{q + \rho x}{\rho+q}\right\} M_T \quad . \tag{7}$$

Equation (6) may therefore be written as

$$V = I.g \, \ell n \frac{\rho+q}{\rho x+q} - (\text{gravity} + \text{drag})\text{corrections} \quad . \tag{8}$$

Therefore

$$dV = dI.g \, \ell n \frac{\rho+q}{\rho x+q} + I.g \frac{1}{\rho+q} \frac{q(1-x)}{\rho x+q} d\rho$$

or

$$\frac{dV}{V+(\text{gravity}+\text{drag})\text{corrections}} = \frac{dI}{I} + \frac{1}{\rho+q} \ell n \frac{1/(\rho+q)}{\rho x+q} \frac{\rho q(1-x)}{(\rho x+q)} \frac{d\rho}{\rho} \quad .$$

If V is held constant then

$$\frac{d\rho}{\rho} = \frac{dI}{I} 2.3026 \frac{(\rho+q)(\rho x+q)}{(1-x)\rho q} \log_{10} \frac{\rho x+q}{\rho+q} \quad . \tag{9}$$

Values of $\quad 2.3026 \frac{(\rho+q)(\rho x+q)}{(1-x)\rho q} \log_{10} \frac{\rho x+q}{\rho+q}$

are plotted against ρ for various values of x and q in Fig.2 and Fig.3. For a long range ballistic missile (ρ = 1.7 gm/cc, x = 0.05 gm/cc, q = 0.16 gm/cc) it takes about 3.5 per cent improvement in propellent density to compensate for 1 per cent loss in propellent specific impulse, a somewhat similar result to case 1. For the boost motors of a ground-to-air anti-aircraft missile such as Seaslug (ρ = 1.6 gm/cc, x = 0.5 gm/cc, q = 0.60 gm/cc approximately) we find it takes nearly 3 per cent improvement in propellent density to compensate for 1 per cent loss in specific impulse. This at first sight seems a little surprising when compared with the corresponding result under case 1. The explanation is however not hard to find; any loss in propellent specific impulse <u>at constant missile weight</u> can only be compensated for by an improvement in the ratio $(M_p+M_B)/M_B$ which means a reduction in M_B since (M_p+M_B) is constant. But the only way to reduce M_B is to reduce the weight of the motor body which in the case under consideration is a fairly small fraction of M_B. An appreciable reduction in motor volume is necessary to effect a reduction in motor body weight which will affect M_B significantly and this means that in order to load sufficient propellent a rather considerable increase in propellent density will be required.

It is interesting to note that as the propellent density approaches zero (a hypothetical case) the change in propellent density required to compensate for a 1 per cent change in propellent specific impulse becomes smaller and approaches 1 per cent in all cases.

4 CONCLUSIONS

It is concluded that in ballistic missile and similar applications where the payload is a small fraction and the propellent a high fraction of the total missile weight, an improvement in propellent specific impulse is much more valuable than a similar improvement in density. Improvements in propellent density are however of significance and are worth having. In cases where the payload forms a large fraction of the total missile weight, e.g. for the boost motors of a medium range ground-to-air anti-aircraft weapon, such as Seaslug, then at constant missile volume an improvement in propellent density is almost as valuable as a similar improvement in propellent specific impulse.

B THE SHAPE OF THE MOTOR BODY AS AN ADDITIONAL VARIABLE WHICH MAY BE USED TO ACHIEVE A GIVEN THRUST-TIME PROGRAMME

1 INTRODUCTION AND GENERAL PRINCIPLES

Nearly all the world's solid propellent rocket motors of which we have knowledge are cylindrical in shape with end closures of approximately semi-ellipsoidal or hemi-spherical form. The propellent charge is normally of the central burning type and remains sensibly constant in length. The pressure inside the motor is kept constant or varied in a prescribed manner by designing the perimeter of the central conduit to remain constant in length (and therefore the burning surface of the charge remains constant) or to vary with time in a prescribed manner. Sometimes the charge is in two parts, the perimeter of the central conduit in one part increasing in size while that in the other decreases in size, as in the slotted tube charge; by suitably adjusting the lengths of the two parts of the charge the burning surface may be maintained constant or caused to increase or decrease as required. Sometimes charges have more than one conduit and here again the surface area of the propellent and hence the motor pressure may be kept constant or varied by suitable choice of the cross-sectional forms of the different conduits.

There is however another way of varying the propellent surface which has been rarely exploited but which may have advantages in certain applications. This method involves choosing a motor tube of such a shape that the length of the charge conduit varies during burning so that together with the variation in conduit perimeter the required area of propellent burning surface is obtained during the burning period. A simple example of this is shown in Fig.4 where the perimeter of the simple cylindrical conduit increases at the same rate as the length of the conduit decreases. The body consists of a plain cylindrical section and a tapering section, which although generally conical in form is not an exact cone in the ideal theoretical case. The equation for the generating curve for this tapering section can be found simply as follows:

Let ℓ be the length of the parallel part of the tube and R its radius. Also let r be the radius of the central conduit at any time during burning, and x the length of conduit in the tapering section (total length of conduit = ℓ + x).

Then, for the burning surface to remain constant in area,

$$2\pi r(\ell+x) = 2\pi R\ell$$

$$r x = R\ell - r\ell$$

or

$$x = \frac{(R-r)\ell}{r} .$$

The surface of revolution generated by this shape is not very attractive as a pressure vessel and would be somewhat difficult to manufacture. It is possible to get the burning surface at the beginning and end of burning the same by replacing this surface by the enveloping frustum of a cone. This would give rise to a hump-backed pressure time curve, the height of the hump above the level curve produced by the correct shape depending on the pressure exponent of the propellent being used. With many propellents however the height of this hump would be unacceptable in a high performance motor. A better solution would be to use a shorter conical section (the dotted line in Fig.4a) and compensate in some way for the low initial burning surface. It is tempting to suggest that this might be done by the increase in rate of burning of the propellent early in the burning time due to erosion. Experience has shown however that though erosion must be allowed for it can rarely be arranged to be a useful factor in design. If it were decided to make use of erosion to boost the initial pressure, it would be best from a purely geometrical point of view to have the conical part of the motor body at the nozzle end rather than at the head end, but then more thermal insulation would be required.

A more attractive proposition would be to increase the initial burning surface by a series of small castellations in the surface of the charge (Fig.3). It is not difficult to design these so that the perimeter of the charge will degenerate into a circle when the necessary fraction of the web has been burned through.

An alternative design of motor body which is based on the same principles as that in Fig.4a is shown in Fig.4b, which has a 'conical' section at each end.

Several years ago two motors were fired at R.P.E. to test the design principle shown in Fig.4a. They had an internal diameter of $5\frac{1}{2}$ inches, a parallel length of 23.5 inches and a conduit diameter of 2.80 inches. The conical portion was a simple frustum of a cone designed to give the same surface area at the beginning of burning as at the end and the propellent used was plastio propellent R.D.2307. Both motors functioned correctly and gave pressure-time curves which were a close approximation to what would be expected.

The Aerojet JATO 15-KS-1000 which has been made in large numbers for the assisted take-off of aircraft uses the same principle as that shown in Fig.4b except that it has a loose charge inhibited on the outside and therefore one of the conical ends has to be removable. No attempt to 'optimise' the length of cone appears to have been made in this design and the pressure-time curve has a rather large hump.

2 CIGARETTE BURNING MOTORS

A tube in the form of a frustum of a cone tapering away fron the nozzle end may find use in sustainer motor applications or in ballistic missile applications where a thrust which decreases with time may be required.

3 DESIGN AND MANUFACTURE OF COMPONENTS

The manufacture of tubes with a tapering bore does not appear to present any serious problems and Bristol Aerojet Ltd. has manufactured a wrapped and welded tube with a maximum bore of 10 inches without difficulty. However if the weight of these conical tubes is to be kept close to the theoretical optimum, then the wall thickness must decrease in the required manner towards the narrow end. In the opinion of Bristol Aerojet Ltd. this could best be done by chemical etching, a sheet of steel being suspended vertically in the etching solution and gradually raised to produce the required variation in thickness. There is some fair experience of this method of reducing metal thickness and no serious difficulties are anticipated.

The weight of the short cone (DB in Fig.4a) having the required variation in thickness is about 15 per cent greater than the equivalent cylinder of the same diameter as the parallel part of the motor body holding the same amount of propellent at the same volumetric loading density. The weight of the whole motor body in Fig.4a is however less than 5 per cent greater than the equivalent cylindrical body of the same diameter holding the same amount of propellent at the same loading density, the ends being excluded in both cases. The ends on the motor in Fig.4a (cone and cylindrical motor) will however be lighter than the ends on the equivalent cylindrical motor because the front closure is of smaller diameter and the rear end can be bottled over to a greater extent because the diameter of the cylindrical former used in the pressing operation will be less than of its star shaped counterpart for the cylindrical motor. On the other hand, the cone and forward end closure of the cone and cylinder motor will probably require more thermal insulation than the forward end closure of the equivalent cylindrical motor and this will add a little weight to the motor body and displace a little propellent. It is not possible to state which motor body is likely to be the lighter for a given application without doing a design study for a specific application and gaining more practical design experience with cone and cylinder motors. It is clear however that any difference between them is not likely to be large. However, for motors of equal internal volume filled with plastic propellent the cone and cylinder type is likely to give a higher total impulse than the corresponding cylindrical motor with star-centred conduit in all cases where a medium or long burning time is required. This is because the web thickness is greater and a faster burning propellent can be used, and generally speaking with plastic propellent a higher burning rate is accompanied by a higher specific impulse. In addition, the cone and cylinder motor is theoretically sliverless whereas a star centred charge always gives slivers which either burn away at reduced pressure or are left unburned or only partially burned. A further point in favour of the cone and cylinder motor is that because of the axially symmetrical flow in the cylindrical conduit nozzle erosion problems should be less severe.

The same kind of arguments that have been given for and against the motor in Fig.4a apply in general to the motor in Fig.4b, but here the cone at the nozzle end of the motor would need more thermal insulation than the cone at the forward end. In any real motor design the discontinuity where a cone joins the parallel part of the motor body would be replaced by a gentle curve so that one shape could pass smoothly into the other and stress concentrations would be reduced to a minimum.

The above discussion has been concentrated on motors required to give a constant thrust but motor body shape can clearly be used as a design variable where other thrust-time programmes are required.

4 CONCLUSIONS

It is concluded that motor bodies of the 'cone and cylinder' type may offer significant advantages in certain applications and that solid propellent motor and missile designers should not confine their attention exclusively to bodies of a cylindrical shape. Variation in motor body shape is an additional design factor which it may sometimes be possible to use either to improve motor performance or to achieve a better missile design.

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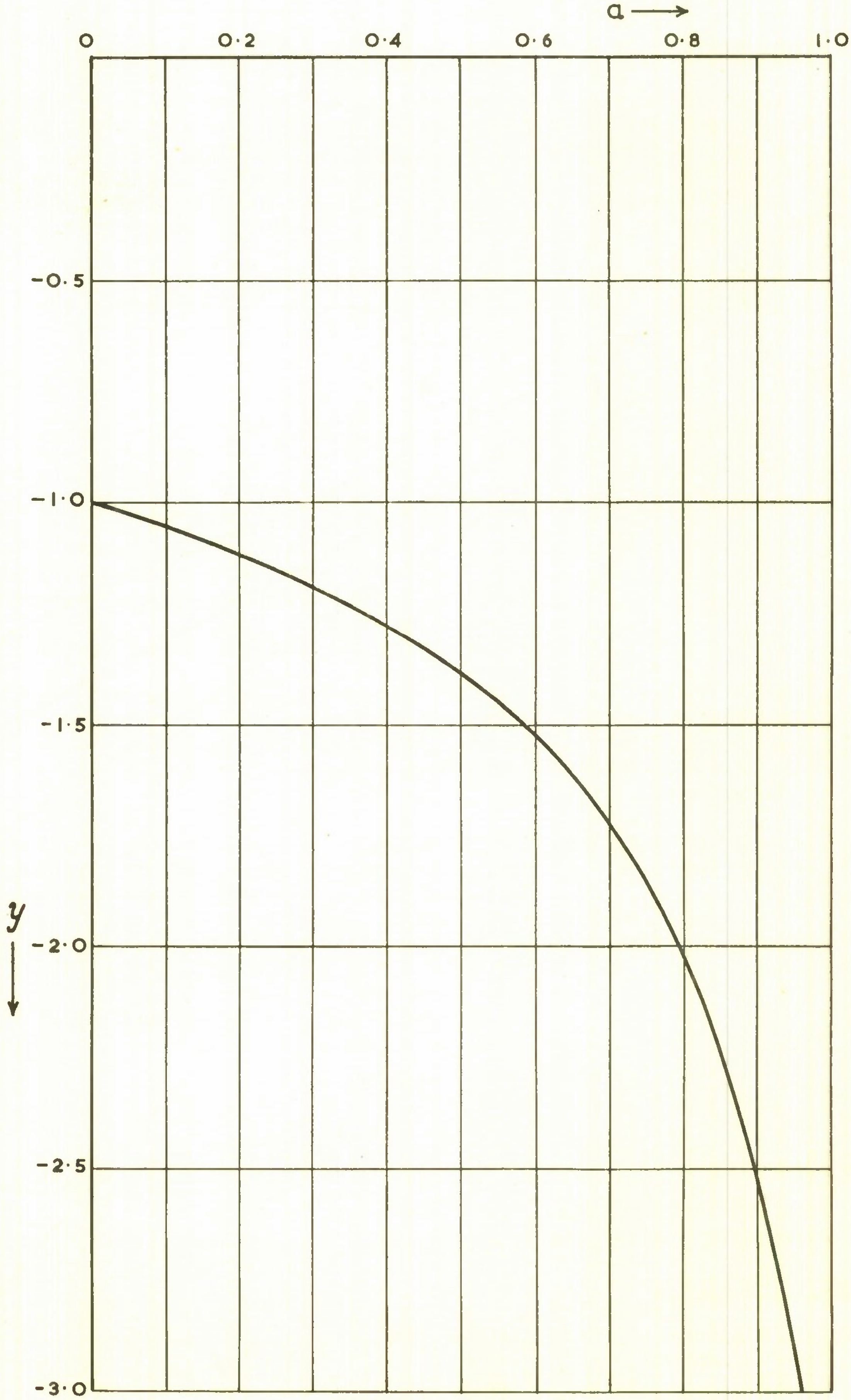

FIG. 1 VARIATION OF $y = 2 \cdot 3026 \frac{1}{a} \log_{10}(1-a)$ WITH a

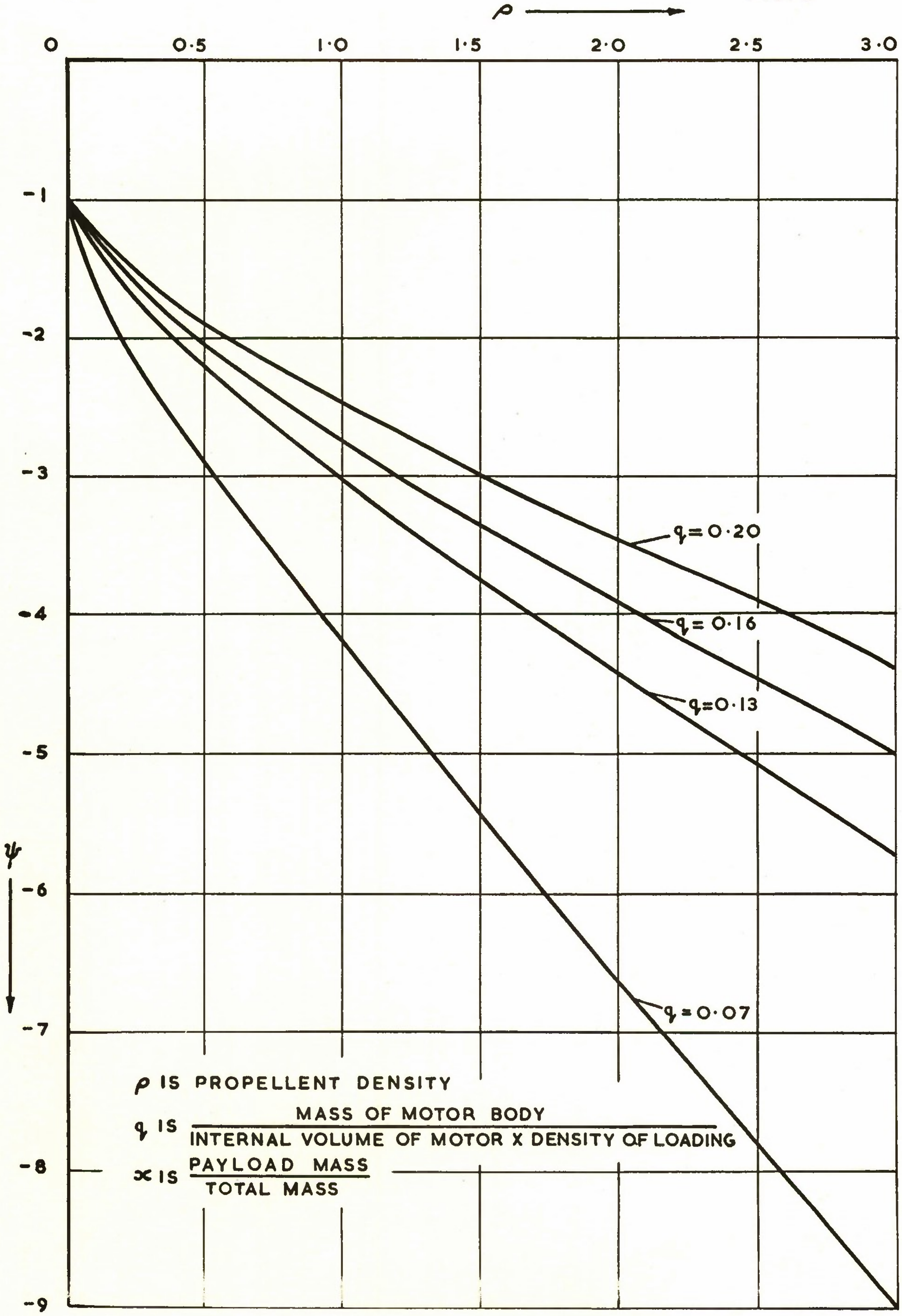

FIG. 2 VARIATION OF $\psi = 2{\cdot}3026 \dfrac{(\rho+q)(\rho x+q)}{(1-x)\,\rho q} \log_{10} \dfrac{(\rho x+q)}{(\rho+q)}$

WITH ρ FOR VARIOUS VALUES OF q FOR $x = 0{\cdot}05$

(BALLISTIC MISSILE CASE)

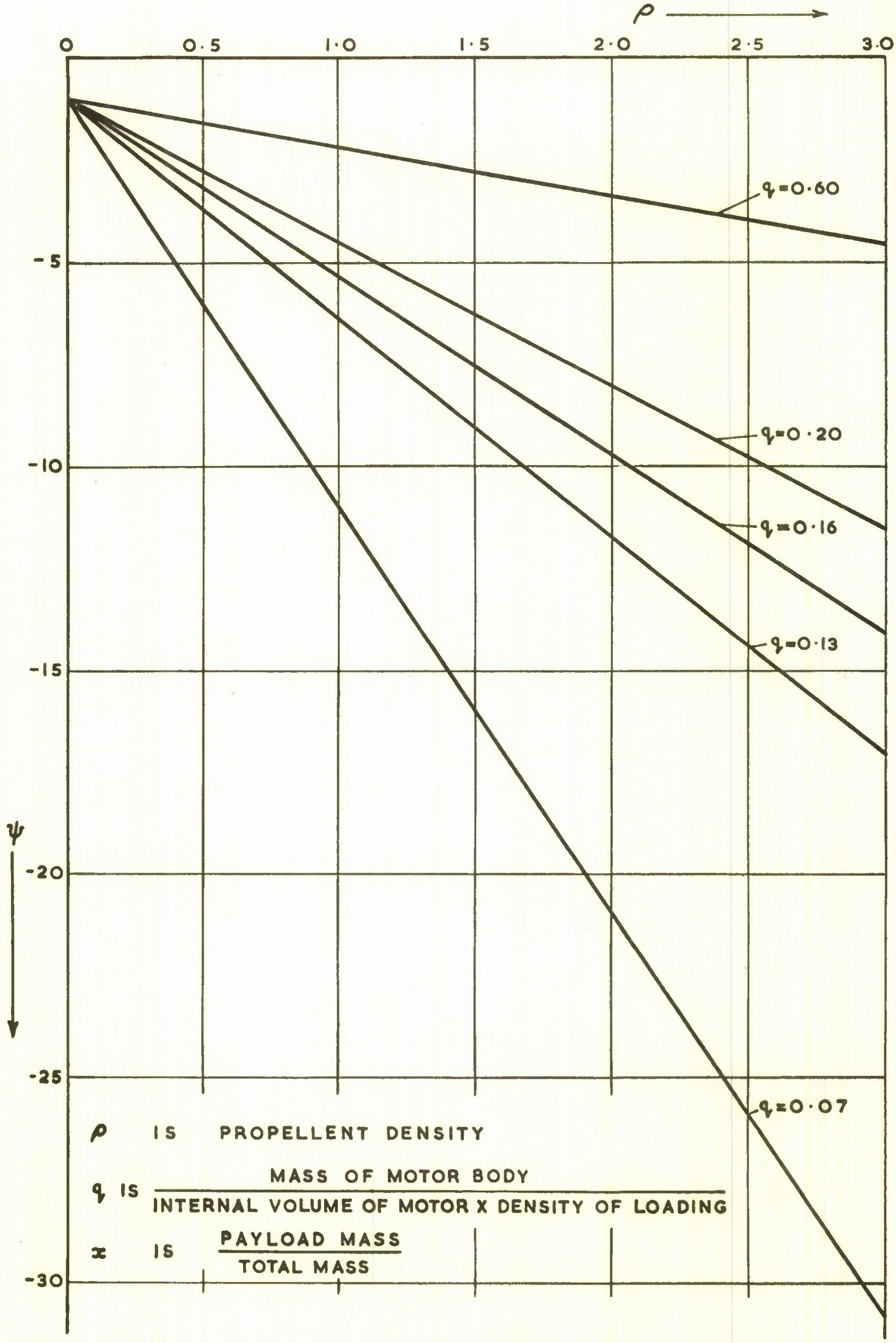

FIG. 3 VARIATION OF $\psi = 2.3026 \dfrac{(\rho+q)(\rho x+q)}{(1-x)\rho q} \log_{10} \dfrac{(\rho x+q)}{(\rho+q)}$

WITH ρ FOR VARIOUS VALUES OF q FOR $x = 0.5$

(SURFACE TO AIR MISSILE CASE)

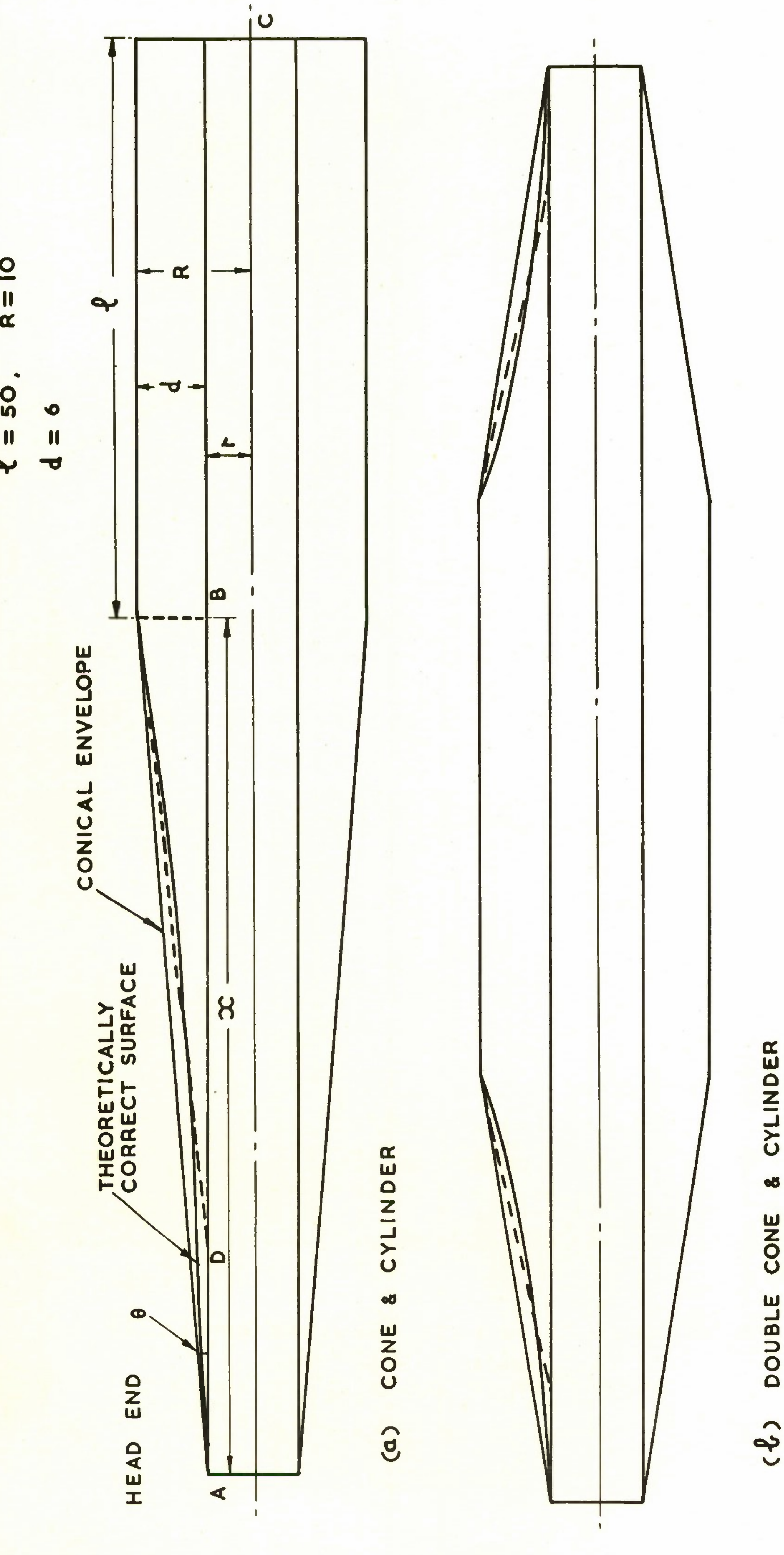

FIG. 4 CONE AND CYLINDER MOTOR BODIES

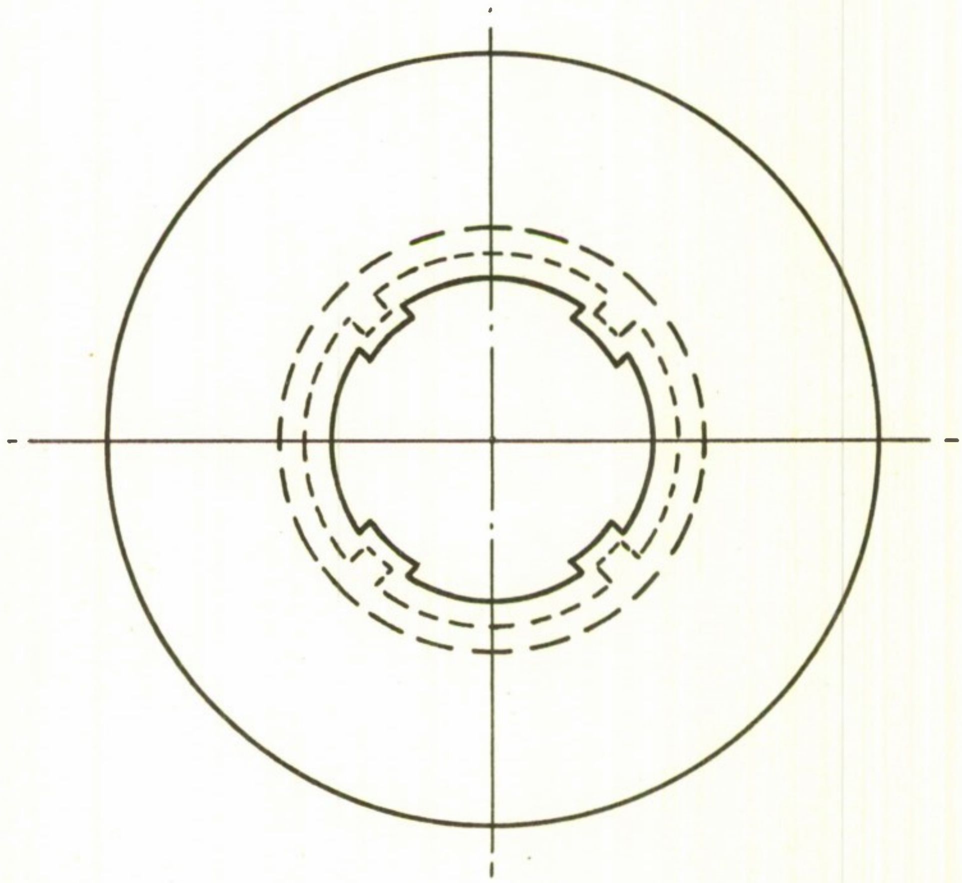

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FIG. 5 INCREASE IN INITIAL PERIMETER OF CYLINDRICAL CONDUIT WHEN USING SHORT CONE

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