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ELECTRIC BOAT DIVISION
GROTON, CONNECTICUT

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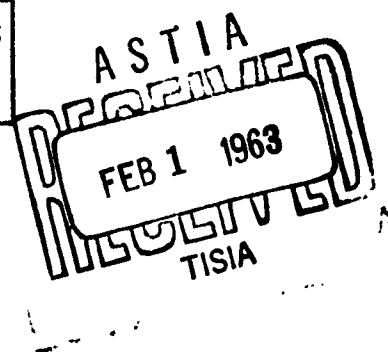
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**Optimal Firing Angles and Their
Acquisition Probability for a
Salvo of Straight-Running Torpedoes (U)**

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**Optimal Firing Angles and Their
Acquisition Probability for a
Salvo of Straight-Running Torpedoes (U)**

by

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Foreword

This report was prepared by the Applied Sciences Department of General Dynamics/Electric Boat as part of the Submarine Integrated Control Program (SUBIC) of the Office of Naval Research. Electric Boat is co-ordinator, under Contract NONr 2512(00), of this program; Cdr. F.R. Haselton, Jr., USN, is Project Officer for ONR; Dr. A.J. van Woerkom is Project Co-ordinator for Electric Boat, and Chief Scientist of the Applied Science Department.

The program is divided into several parts: ship control, weapon and tactical control, engineering control, communications, environmental control, and command control. This report is one of a series dealing with tactical control.

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Abstract

An own-ship with constant course and speed fires a straight-running torpedo at a distant target. Fire control information about the target is based solely on passive sonar and can include bearing acceleration and/or target speed based on screw count. Since screw count speed is absolute (water) speed, the method provides for relating target motion to own-ship, relative to which bearings are measured.

Target is assumed to be straight-running until the torpedo approaches within a certain distance. Then target attempts to evade, and can be located anywhere within an ever-widening circle. Torpedo's probability of acquiring target, that is getting target within its acoustic cone, depends upon the accuracy of the fire control solution and target's evasion capacity. Salvos of more than one torpedo are considered.

If target gets within torpedo's acoustic cone, 100% acquisition is assumed; if it doesn't, zero. Target and torpedo motions after acquisition and motions in depth are not considered.

Sample simulations indicate that the most sensitive parameters are the distance at which target detects an approaching torpedo, solution accuracy which decreases with increasing target range, and number and type of torpedoes in a salvo.

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1

SUMMARY OF CALCULATION PROCESS

The purpose of this report is to describe a method which will determine the best angles at which to fire torpedoes at a target and the probability of acquiring the target for those angles. Information about the target depends on timed sonar bearings and possibly a screw count for speed.

Constant course and speed are assumed for own-ship and the torpedo. A straight-running target is also assumed until the torpedo gets within a certain distance of the target at which time it attempts to evade. Acquisition probability depends on errors in the bearing data and on the target's ability to evade.

Input for the calculation comes from an Automatic Statistical Processing (ASP) solution of a target's bearing-time curve. This solution gives target bearing at some initial time, bearing rate, possibly bearing acceleration and the errors (standard deviations) in these data. The target's water speed, based on screw count, and its error may also be at hand.

From this input is found a probability distribution of the target's location at the time a straight-running torpedo fired from own-ship would acquire it. This distribution is specified for a network of points (i,j) , where i and j both go from 1 to n , so that there are n^2 separate points. For each point is calculated the probability that the target will have a trajectory associated with it, and some parameters to specify that trajectory. Slightly different calculation processes are used for the 4 cases of input data:

- Case I. Bearing, Rate, Acceleration, and Screw-Count speed known.
- Case II. Bearing, Rate, and Acceleration known
- Case III. Bearing, Rate, and Screw-Count speed known.
- Case IV. Bearing and Rate known.

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After the calculations have been made for the n^2 points, the target's motion associated with each of those points is studied. A certain firing angle is assumed for the torpedo. The target is assumed to be straight-running until the torpedo fired at it gets within a certain distance. Target evasion is next considered. The target continues its straight run for a calculable time interval, after which it can be located anywhere within an ever-widening circle. The time of straight-run and rate of increase of circle radius depend on target initial speed and target top speed.

For each of the n^2 points, the target's probability of acquisition by the torpedo depends on the intersection of its path with the torpedo's acoustic head. Torpedo is assumed to be straight running and to run for a certain length of time before its fuel is exhausted. For convenience in calculation, the target's motion is plotted relative to the torpedo. If target passes through torpedo's acoustic head before its circle develops, the acquisition is 100%. Otherwise, the acquisition equals the fraction of target's circle which passes through the acoustic head before the torpedo runs out of fuel.

The acquisition probability for each of the n^2 points is found, multiplied by the probability that that point characterizes the target's motion, and summed over all n^2 points. This equals the total acquisition probability for the torpedo.

Naturally the total acquisition probability depends on the angle at which the torpedo is fired. Calculation is made for five different angles covering a spread outside of which acquisition probability is very small. A firing angle is chosen within the spread which will give a high acquisition probability. If more than one torpedo is to be fired, additional firing angles are chosen which do not seriously duplicate the acquisition effected by previously fired torpedoes. Up to 4 torpedoes may be fired.

The whole calculation can be summarized by a schematic flow chart, Figure 1.

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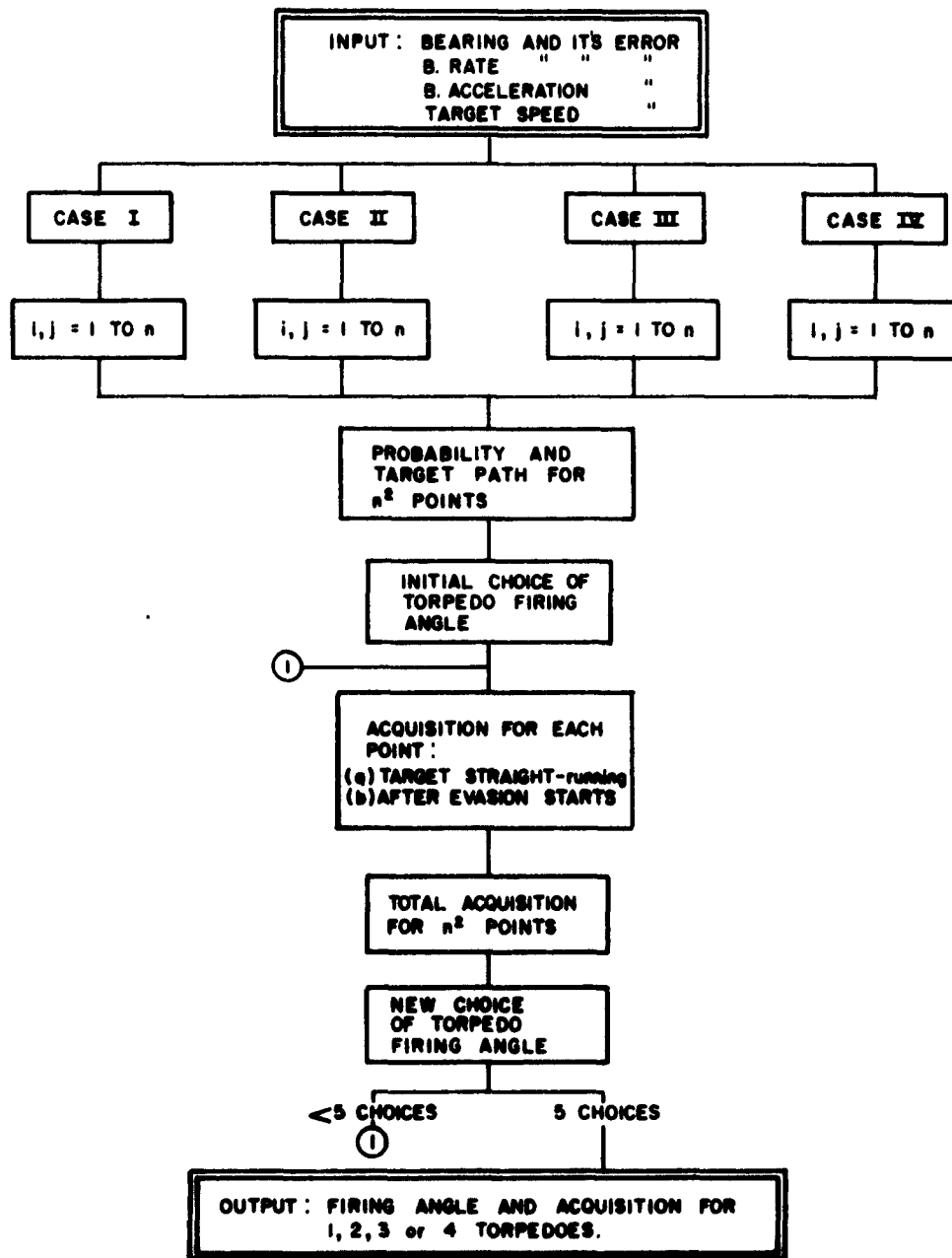


FIGURE 1. SUMMARY OF THE CALCULATION PROCESS

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II

EXPLANATION OF CO-ORDINATE SYSTEMS

Several co-ordinate systems are used in this report. At first, systems are chosen in which the observations (input quantities) are made. Later, transformations are made to a system that appears to be most convenient for calculation.

Ocean Bottom System

This system, being fixed with respect to the earth, is physically the most fundamental. However, the input quantities, speeds and bearings, are not measured relative to the ocean bottom. The ocean bottom reference, therefore, is not computationally convenient since no component in the fire control problem is fixed relative to it. Accordingly, it is not used in this report.

Ocean Water System

This system is physically equivalent to the bottom system, unless the ocean current vector varies over the trajectory followed by the torpedo in going from own-ship to target. If the current does not vary, the two systems differ only by a constant velocity, which does not affect the dynamical relation between forces and accelerations. The inputs of own-ship speed, target screw count speed, and torpedo speed are measured in this system.

Own-Ship System

Bearings and their derivatives are measured in this system. To combine them with the speed inputs, the latter are converted from the water system to the own-ship system by means of the formulae associated with Figures 2 and 4.

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Torpedo System

In the analysis of target acquisition by the torpedo's acoustic cone, it is convenient to fix one of these two at the origin of a new coordinate system. We choose the torpedo cone. The transformation of target's motion from the own-ship system to the torpedo system is covered by equation (21). Target's motion relative to torpedo is illustrated in Figures 7 and 8.

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III

**OPTIMAL FIRING ANGLES AND THEIR ACQUISITION PROBABILITY
FOR A SALVO OF STRAIGHT-RUNNING TORPEDOES**

3.1 Introduction To The Straight-Running Target

An own-ship traveling at constant course and speed detects a target by means of sonar, and tracks it for a certain time. The sonar information is processed, and own-ship fires a salvo of straight-running torpedoes at different bearing angles chosen to optimize the probability of acquiring the target.

From the sonar is obtained a series of timed bearings and perhaps a screw count speed estimate. A second degree curve is passed through the discrete bearings by means of the theory of least-squares,

$$B(t) = B + b(t-t_0) + c(t-t_0)^2; \quad (1)$$

and initial bearing B , initial bearing rate $\dot{B} = b$, and bearing acceleration $\ddot{B} = 2c$ are found. The target is assumed to be straight-running during the time covered by $B(t)$, and terms higher than second degree are assumed insignificant. The details are given in the ASP (Automatic Statistical Processing) report.¹

The basic geometrical situation is shown in Figure 2. The target speed estimate derived from screw count, S_{TA} , is speed relative to the water. It is used with the speed of own-ship relative to the water, S_{OS} , to find target speed relative to own-ship, u :

$$S_{TA}^2 = S_{OS}^2 + u^2 - 2S_{OS}u \cos(90^\circ - B + \alpha)$$

¹ SUBIC - Mathematical Concepts of the Automatic Statistical Processing Fire Control Computer (U), General Dynamics/Electric Boat report #C417-61-011, October 1961 CONFIDENTIAL

$$\text{or} \quad u = -S_{OS} \sin(\alpha - B) \pm \sqrt{S_{TA}^2 - S_{OS}^2 \cos^2(\alpha - B)} \quad (2)$$

Relative speed, u , must be a real, positive number.

This formula is based on the law of cosines and assumes a co-ordinate system fixed with respect to the water. Bearing B is measured with respect to own-ship's axis: $B = 0^\circ$ if 90° on the port side. Own-ship's axis is assumed to be pointed in the direction of own-ship's course relative to the water. Any difference in water current between own-ship and target is neglected.

In the figure, triangle $O T_0 T_1$ shows the motion of target relative to own-ship. The relative angle-on-the-bow, α , is shown in the ASP report to depend on $\dot{B}$ and $\ddot{B}$:

$$\tan \alpha = \frac{2\dot{B}}{\ddot{B}} \quad -\pi < \alpha < \pi \quad (3)$$

Bearing rate $\dot{B}$ depends on u , α , and range R

$$\dot{B} = \frac{u \sin \alpha}{R} \quad (4)$$

from which range can be found.

The signs of $\dot{B}$ and $\ddot{B}$ and the quadrant of α are controlled as follows:

- a) Bearing increasing, target closing. $\dot{B}+$, $\ddot{B}+$, $0 < \alpha < \pi/2$
- b) Bearing increasing, target opening. $\dot{B}+$, $\ddot{B}-$, $\pi/2 < \alpha < \pi$
- c) Bearing decreasing, target closing. $\dot{B}-$, $\ddot{B}-$, $-\pi/2 < \alpha < 0$
- d) Bearing decreasing, target opening. $\dot{B}-$, $\ddot{B}+$, $-\pi < \alpha < -\pi/2$.

When the torpedo is fired, the target is at position T_0 . The torpedo moves along angle β so that the base of its acoustic cone just acquires the target when the target reaches a certain position T_1 , as shown in Figure 1. If the torpedo cone altitude is z and its speed S_{TO} , then it traverses its cone altitude in time

$$\Delta = \frac{z}{S_{TO}} \quad (5)$$

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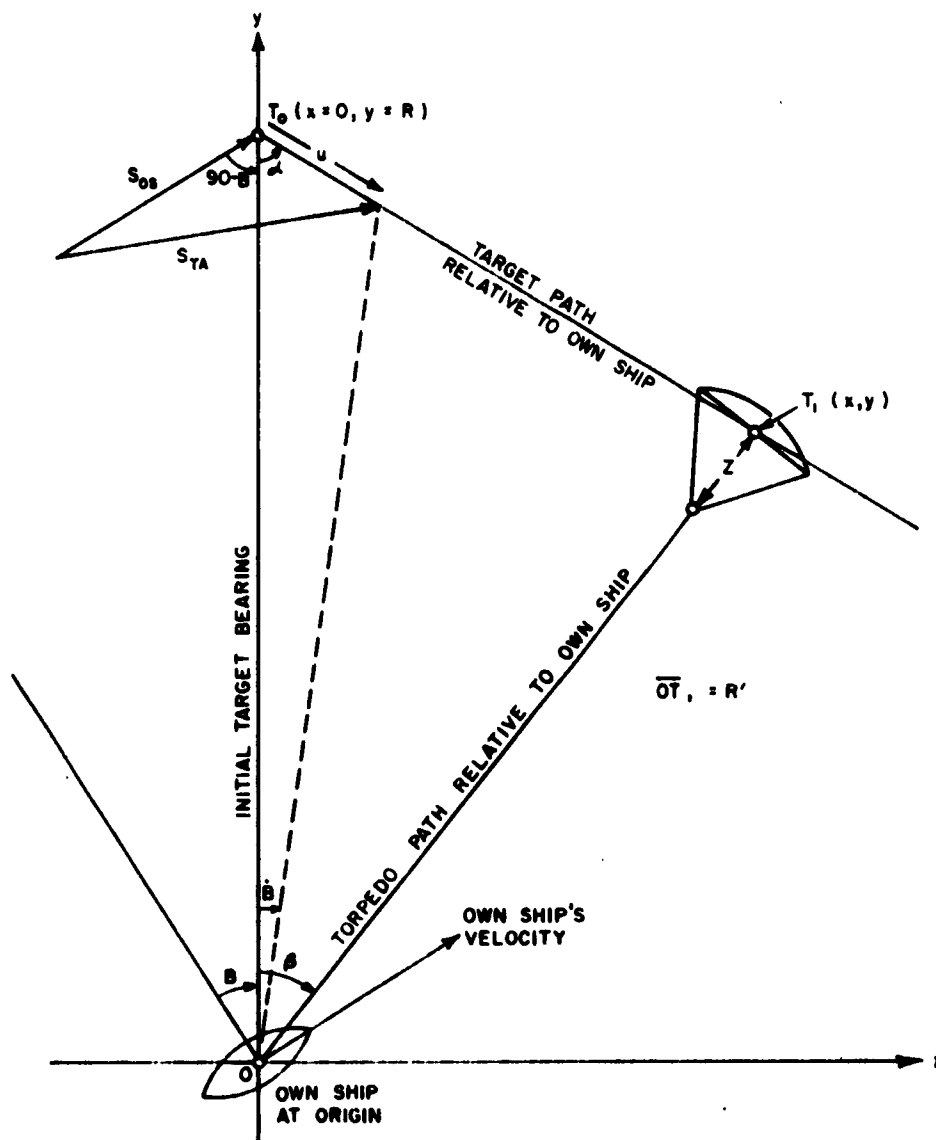


FIGURE 2. MOTIONS RELATIVE TO OWN SHIP

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The co-ordinates of T_1 are given by two equations

$$x = ru \sin \alpha = (r + \Delta) S_{TO} \sin \beta \quad (6a)$$

$$y = R - ru \cos \alpha = (r + \Delta) S_{TO} \cos \beta \quad (6b)$$

which are solved simultaneously to give torpedo travel time r and firing angle β (i.e., the angle of torpedo's motion relative to own-ship's motion).

Purpose of the Backset Δ - If the sonar inputs $B, \dot{B}, \ddot{B}, S_{TA}$ and auxiliary data S_{OS}, z, S_{TO} are exactly known, then equations (2) through (6) can be solved in that order to find r and β . At the instant when target is at T_0 , a straight-running torpedo moves along angle β , and acquisition is assured at point T_1 . However if one of the inputs is uncertain, it is only possible to say that target will be located somewhere along an "acquisition locus" $T_1' T_1''$ where it may be acquired by a torpedo moving at an appropriate angle β . If the backset Δ is used, as provided in equations (5) and (6), one torpedo will acquire not only the target corresponding to β but also nearly all targets along $T_1' T_1''$ intercepted by the torpedo's acoustic head: specifically all targets whose own angles of acquisition lie between

$$\beta - \beta_1 \text{ and } \beta + \beta_1, \text{ where}$$

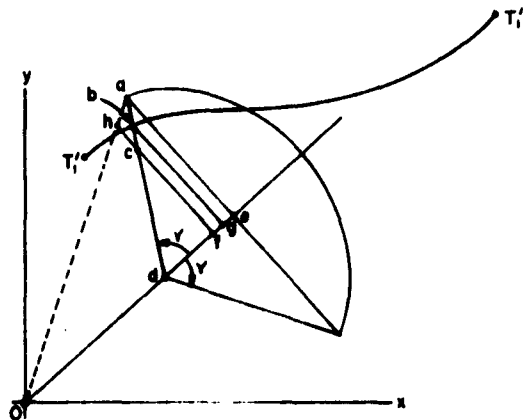
$$\frac{z \tan \gamma}{R} \left(1 - \frac{z \tan^2 \gamma}{2R} \right) < \tan \beta_1 < \frac{z \tan \gamma}{R}$$

See Figure 3.

If there were no backset Δ , the torpedo would acquire only the target intercepted by the torpedo itself - the intersection of $T_1' T_1''$ with β - and the advantage of the acoustic head would be lost.

Figure 3

ACOUSTIC HEAD'S INTERCEPTION OF THE ACQUISITION LOCUS



$$O_e \approx O_h \approx R'$$

$$R'^2 = R^2 + (u\tau)^2 - 2Ru\tau \cos \alpha$$

$$de = z$$

$$\angle ade = \gamma$$

$$\angle y_0 e = \beta$$

$$\angle aOc = \left(\arctan \frac{z \tan \gamma}{R} \right) = \frac{z \tan \gamma}{R'}$$

$$\angle bOe = \beta_1$$

$$\angle COe = \left(\arctan \frac{cf}{Of} \right) = \frac{cf}{Of}$$

$$\frac{cf}{ae} = \frac{df}{ac} = \frac{Or' - Od}{z}$$

$$\frac{cf}{Of} = \frac{ae}{z} \left(1 - \frac{Od}{Of} \right) = \tan \gamma \left(1 - \frac{R' - z}{R' \cos \angle aOe} \right)$$

$$R' \cos \angle aOe = R' \left[1 - \frac{1}{R'} \left(\frac{z \tan \gamma}{R'} \right)^2 \right]$$

$$\therefore \frac{df}{dr} = \frac{z \tan \gamma}{R} \left[1 - \frac{z}{2R} \tan^2 \gamma \left(1 - \frac{z}{R} \right) \right]$$

$$= \frac{z \tan \gamma}{R} \left[1 - \frac{z}{R} \tan^2 \gamma \right]$$

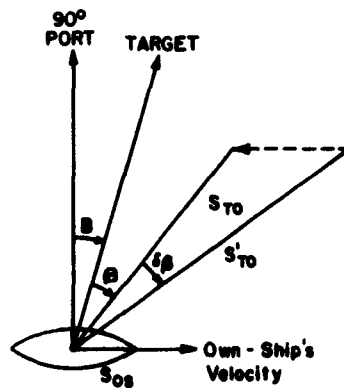
$\angle c0e \sim \angle b0e \sim \angle a0e$

Torpedo Motion - The basic equations (2) to (6) determine torpedo angle β as a function of, among other things, torpedo speed S_{TO} . However, both of these are defined by Figure 2 as being relative to own-ship.

In its path through the water, the torpedo experiences a thrust from the water which is caused by and oppositely directed from own-ship's motion. The torpedo's velocity relative to own-ship equals the vectorial difference of its water velocity minus that of own-ship. See Figure 4.

Figure 4

EFFECT OF FIRING ANGLE ON TORPEDO SPEED RELATIVE TO OWN-SHIP



$$S'_{TO} \sin (B + \beta + \delta\beta) = S_{TO} \sin(B + \beta) + S_{OS} \quad S'_{TO}, S_{TO}, S_{OS} \geq 0$$

$$S'_{TO} \cos (B + \beta + \delta\beta) = S_{TO} \cos (B + \beta)$$

These equations can be rewritten in convenient computational form:

$$S_{TO} = -S_{OS} \sin (B + \beta) + \sqrt{(S_{OS} \sin \{B + \beta\})^2 - S_{OS}^2 + S_{TO}^2} \quad (6c)$$

Torpedo firing angle is given by:

$$(B + \beta + \delta\beta) = \left\{ \arctan \left[\tan (B + \beta) + \frac{S_{OS}}{S_{TO}} \sec (B + \beta) \right] + \frac{5\pi}{2} - \frac{\pi}{2} \operatorname{sign} [\cos (B + \beta)] \right\} \bmod 2\pi \quad (6d)$$

In (6c), the sign of the radical is $\pm$, but must always be taken as $+$ to assure the numerically larger S_{TO} . Whichever sign is taken, the corresponding firing angle $(B + \beta + \delta\beta)$ will be found from (6d). If $+\sqrt{\quad}$ is taken in (6c), S_{TO} is found and then used in (6d) to get the corresponding $(B + \beta + \delta\beta)$.

In the actual calculation, $S_{TO}^{(1)}$ is given but not S_{TO} . An iteration is used. Initially, let $S_{TO}^{(1)} = S_{TO}'$, and apply (2) to (6b) to get an initial estimate for firing angle $\beta^{(1)}$. Put it into (6c) to get a new $S_{TO}^{(2)}$. Use $S_{TO}^{(2)}$ in (2) to (6b), etc. Continue until S_{TO} and β are stabilized; probably 3 or 4 iterations will suffice. β is used in the later calculations of best firing angle and acquisition probability, sections 3.2 to 3.5. When these have been found after equation (48), the firing angle correction, $\delta\beta$, is found from (6d).

3.2 Cases for a Straight-Running Target

The inputs B , $\dot{B}$, S_{OS} , z , and S_{TO} are assumed known, but $\ddot{B}$ and/or S_{TA} may be known only with a considerable error or may be lacking. There are four possible cases. In a calculation of firing angle, the appropriate case must be chosen, its calculation done, and then allowance made for a non straight-running, or maneuvering, target covered in section 3.3.

Case I - $\ddot{B}$ and S_{TA} known with error. Let σ_a be the standard deviation in a , which is found from $\dot{B}$ and $\ddot{B}$ by equation (3). Let σ_S be the standard deviation in S_{TA} .

It is necessary to calculate the target acquisition point $T_1(x,y)$ for different combinations of $(\alpha, \alpha \pm \frac{\sigma_\alpha}{F_1}, \alpha \pm \frac{2\sigma_\alpha}{F_1}, \dots, \alpha \pm F_2\sigma_\alpha)$ and $(S_{TA}, S_{TA} \pm \frac{\sigma_S}{F_1}, S_{TA} \pm \frac{2\sigma_S}{F_1}, \dots, S_{TA} \pm F_2\sigma_S)$ and then assign a probability, P , that the target will occupy each of those positions at the time that the torpedo, if fired at the corresponding angle β , would acquire it. Probability P_{1j} is calculated on the assumption that relative angle-on-the-bow and target speed are normally distributed with means α and S_{TA} and standard deviations σ_α and σ_S , respectively. If these quantities have some other distribution which can be represented by a polynomial, then an alternate formula for P_{1j} is offered. The calculation is as follows:

$$n = 2F_1F_2 + 1 \quad (7)$$

$$\alpha_1 = \alpha + \frac{\sigma_\alpha}{F_1} (i - 1 - F_1F_2) \quad i = 1 \text{ to } n \quad (8)$$

$$S_j = S_{TA} + \frac{\sigma_S}{F_1} (j - 1 - F_1F_2) \quad j = 1 \text{ to } n \quad (9)$$

$$u_{1j} \text{ as in (1)} \quad (2)$$

$$P_{1j} = \exp \left\{ -1/2 \left(\frac{i - 1 - F_1F_2}{F_1} \right)^2 - 1/2 \left(\frac{j - 1 - F_1F_2}{F_1} \right)^2 \right\} \quad (10)$$

$$\text{or } P_{1j} = \begin{bmatrix} g' \\ g = 0 \end{bmatrix} \begin{bmatrix} a_1 g \\ g \end{bmatrix} \begin{bmatrix} h' \\ h = 0 \end{bmatrix} \begin{bmatrix} h' \\ h \end{bmatrix} \quad (10 \text{ alt.})$$

For some i, j there will be two u_j and P_{1j} , for the $+\sqrt{}$ and $-\sqrt{}$ in formula (2). Another index, $k = 1$ and $k = 2$, is used for this. All the α_1, u_{1j}, P_{1j} are stored for use in section 3.3. If B or S_{TA} is lacking, point T_1 is replaced by acquisition locus $T_1 T_1''$. The form of this locus can be judged from a graph of target acquisition points, based on equations (2) through (6), for a typical tactical situation:

$$B = 0^\circ \quad \dot{B} = +2.5 \text{ rad/hr} \quad S_{OS} = 10 \text{ kt} \quad S_{TO} = 40 \text{ kt} \quad z = 700 \text{ yd}$$

$\ddot{B}$ and S_{TA} variable. See Figure 5.

Curves of constant $\ddot{B}$ (dotted lines) are open, with T_1 moving away from the origin as S_{TA} increases. Curves of constant S_{TA} (solid lines) are closed, with T_1 moving first away from the origin and then back again as $\ddot{B}$ goes from $-\infty$ to $+\infty$. These curves exhibit a nesting property: if $S_{TA}^{(1)} < S_{TA}^{(2)}$, the $S_{TA}^{(1)}$ curve lies entirely within the $S_{TA}^{(2)}$ curve. Hence, if $\ddot{B}$ is unknown and S_{TA} is only known to be less than some $S_{TA}^{(2)}$, then the torpedo must be fired within an "acquisition area" bounded by the $S_{TA}^{(2)}$ curve.

These characteristics of the curves will be used in the cases II, III, IV calculations.

Case II - $\ddot{B}$ known with error, S_{TA} less than some upper limit $\bar{S}_{TA}$, R greater than some lower limit $\underline{R}$.

Since $\dot{B}$ and $\ddot{B}$ are known, α is known and has an error σ_α derivable from error in $\ddot{B}$ by differentiating (3). Different speeds S_{TA} are assigned to different ranges R , to which a probability distribution $U(R)$ can be assigned on the basis of search theory.

n and α_1 as in Case I (7), (8)

$$\bar{U} = \text{a function of } \bar{S}_{TA} = -S_{OS} \sin(\alpha - B) + \sqrt{S_{TA}^2 - S_{OS}^2 \cos^2(\alpha - B)} \quad (2)$$

$$\bar{R} = \text{a function of } \bar{U} = \frac{\bar{U} \sin \alpha}{\dot{B}} \quad (4)$$

R_j are distributed evenly between $\underline{R}$ and $\bar{R}$ so that $R_j \pm \frac{1/2}{n} (\bar{R} - \underline{R})$ cover the whole range of $\underline{R}$ to $\bar{R}$. A probability P_{1j} will be calculated which depends partly on α as in Case I and partly on R . The simplest form of the latter would be $U(R) = R_0$, i.e., probability constant with range. This assumes that the probability of a detected

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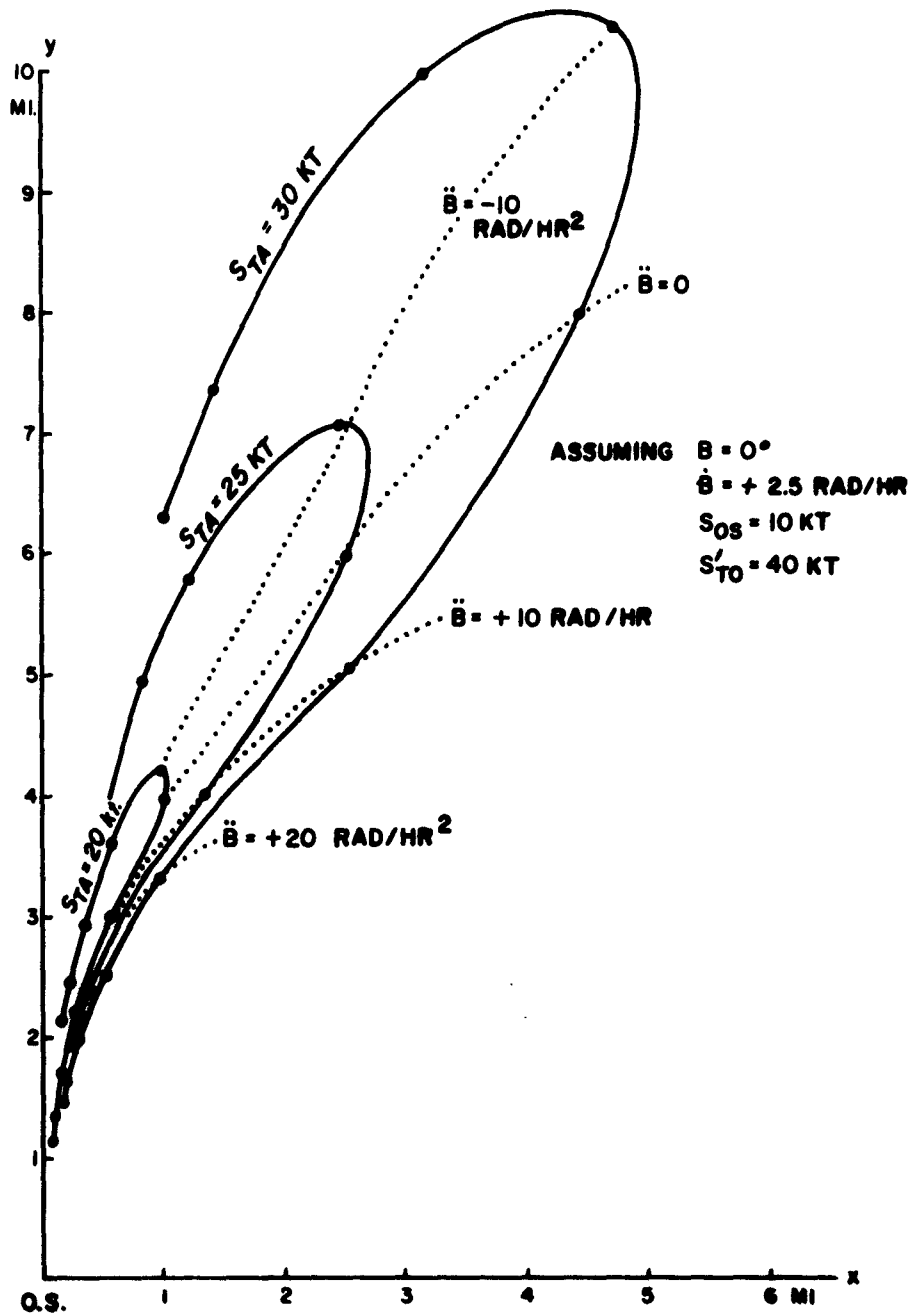


FIGURE 5. ACQUISITION LOCI FOR DIFFERENT $\ddot{B}$, S_{TA}

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target being tracked declines with range just enough to make up for the increase with range of the area of an annulus of fixed thickness within which a target might be found.

$$R_j = (\bar{R} - \underline{R}) \left(\frac{j-1/2}{n} \right) + \underline{R} \quad j = 1 \text{ to } n \quad (11)$$

$$u_{1j} = \frac{R_j \dot{B}}{\sin \alpha_1} \quad (4) \rightarrow (12)$$

$$P_{1j} = \exp \left\{ -1/2 \left(\frac{j-1/2 - F_1 F_2}{F_1} \right)^2 \right\} \left[U_0 + U_1 R_j + U_2 R_j^2 + U_3 R_j^3 \right] \quad (13)$$

$$\text{or } P_{1j} = \left[\sum_{g=1}^3 G_g^{-1} \right] \left[U_0 + U_1 R_j + U_2 R_j^2 + U_3 R_j^3 \right]$$

Store α_1, u_{1j}, P_{1j} for use in section 3.3.

Case III - $\dot{B}$ unknown, S_{TA} known with error, R greater than some lower limit $\underline{R}$.

Since $\dot{B}$ is unknown, α is indeterminate. If $\dot{B}$ is positive, α will lie between 0 and π radians; if negative, between 0 and $-\pi$ radians. For each α , there is an associated range R which has a probability distribution $U(R)$. Unlike Case II, the ranges are not equally distributed from $\underline{R}$ to maximum, so $U(R)$ must be multiplied by $\Delta R \propto |R_1 - R_{1-1}|$.

n and S_j as in Case I (7), (9)

$$\alpha_1 = \frac{\pi}{n} (j-1/2) \cdot \text{sign } \dot{B} \quad (14)$$

u_{1j} as in (2). There can be a positive u_j for $+\sqrt{\quad}$ and for $-\sqrt{\quad}$ (2)

$$R = \frac{u_{1j} \sin \alpha_1}{\dot{B}} \quad (4) \quad (15)$$

$$P_{1j} = \exp \left\{ -1/2 \left(\frac{j-1/2 - F_1 F_2}{F_1} \right)^2 \right\} \cdot \left[U_0 + U_1 R_1 + U_2 R_1^2 + U_3 R_1^3 \right] |R_1 - R_{1-1}| \quad (16)$$

$$\text{or } P_{1j} = \left[\sum_{h=1}^3 h^{-1} U_h \right] \left[U_0 + U_1 R_1 + U_2 R_1^2 + U_3 R_1^3 \right] |R_1 - R_{1-1}|$$

If $R < \underline{R}$, $P_{1j} = 0$.

Store α_1, u_{1j}, P_{1j} for use in section 3.3.

Case IV - $\dot{B}$ and S_{TA} unknown, R greater than $\underline{R}$, S_{TA} less than some upper limit $\bar{S}$.

α_1 is determined as in Case III. A lower limit to target speed, $\underline{S}$, is necessary to maintain the sign of the bearing rate. Then S_{TAj} are distributed evenly between $\underline{S}$ and $\bar{S}$. Ranges R_j are calculated so that a probability can be assigned.

$$n \quad (7)$$

$$\alpha_1 \quad (14)$$

$$\underline{S} = S_{OS} \cos B \operatorname{sign} \dot{B} \quad \text{If } \underline{S} < 0, \text{ let } \underline{S} = 0 \quad (17)$$

$$S_j = \underline{S} + (\bar{S} - \underline{S}) \left(\frac{j-1/2}{n} \right) \quad (18)$$

$$u_{1j} \quad \text{There can be two positive } u_{1j} \text{ as in Case III.} \quad (2)$$

$$R_j \quad (15)$$

$$P_{1j} \quad (16)$$

Store α_1, u_{1j}, P_{1j} for use in section 3.3.

3.3 - Allowance for Target Evasion

The target is assumed to be straight-running during the ASP solution and until the fired torpedo gets within a certain distance, D , of it. At that instant the target attempts evasion by changing course and/or speed. Its ability to evade has been studied by means of simulations run on an IBM 704 calculator and based on the submarine equations of motion.

Results show that a fast maneuvering target cannot change course or speed markedly until a time Δt after detection of a torpedo.

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Thereafter, the target can be anywhere within a figure which resembles a circle whose radius increases linearly with time. The time after detection Δt and the radius of the circle, r , are functions of time t , initial target speed S_{TA} maximum target speed $\bar{S}$, and detection time t_D .

$$\Delta t = \frac{2500 - S_{TA}(110-2\bar{S})}{100 \bar{S}} \quad (19)$$

$$r = 100 \bar{S} (t - t_D - \Delta t) + S_{TA}(110-2\bar{S}) - 2500 \quad (20)$$

If $r < 0$, let $r = 0$

r is in feet, S_{TA} and $\bar{S}$ in knots, $T = (t - t_D - \Delta t)$ in minutes. Comparison of these simplified expressions with the actual results of a simulation based on $S_{TA} = 17$, $\bar{S} = 25$, $T = 1/2, 1, \dots 3$ minutes are shown in Figure 6. Equations (19) and (20) are fairly adequate for $0 < S_{TA} < 40$, $25 < \bar{S} < 40$. Figure formulae are for a co-ordinate system fixed with respect to the water.

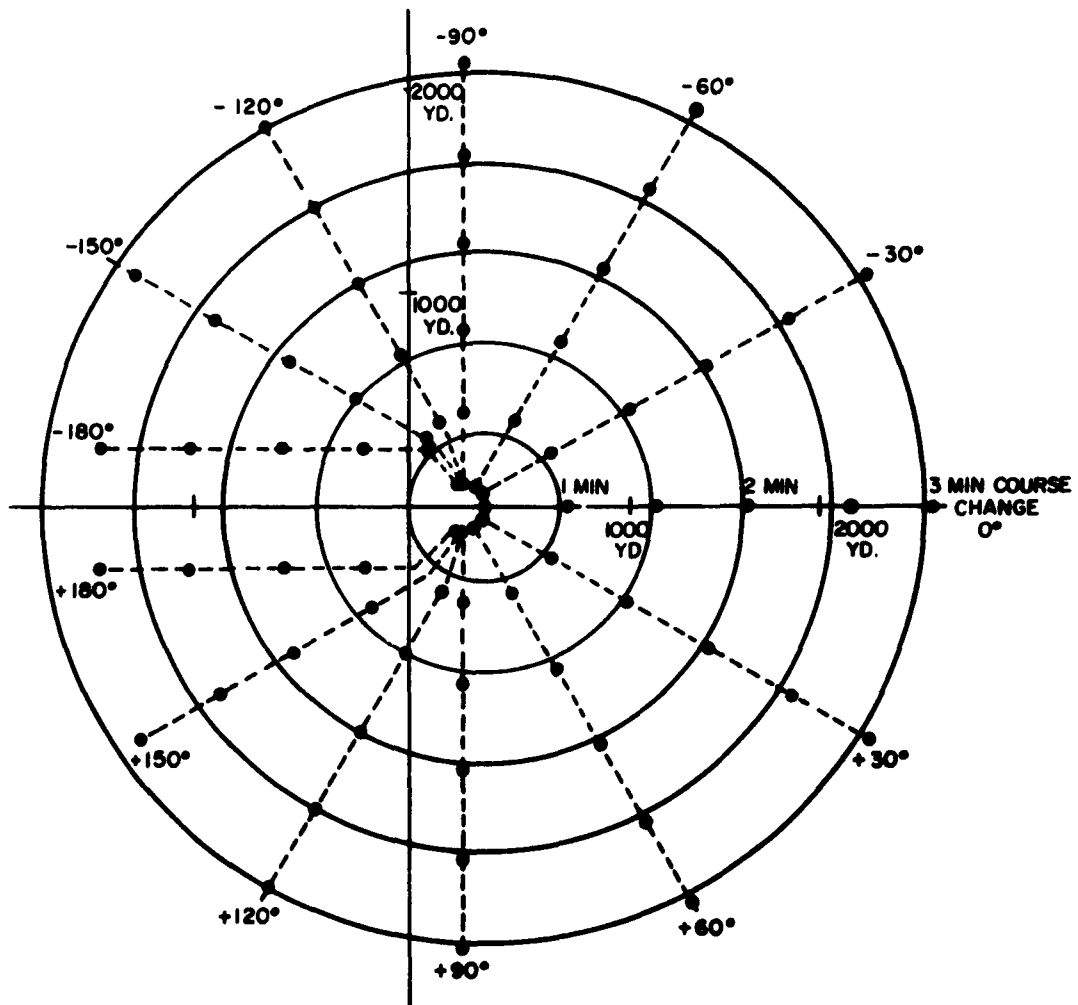
The probability that a torpedo with acoustic head acquires an evading target is found by calculating the target and torpedo paths and determining the likelihood that they simultaneously intersect. Since both time and position are involved, it is useful to imagine a 3-dimensional co-ordinate system with orthogonal axes x and y for position and t for time. In calculating intersections, it is useful for one of the agents - the torpedo - to be imagined stationary in time, so that its position (x, y) and that of its acoustic head are independent of t . The torpedo is placed at the origin. See Figure 7.

Let $t = 0$ when the torpedo is fired. Since both it and target are straight-running until target detects it, the target's trajectory in the (x, y, t) system, relative to the torpedo, is a straight line. When the target passes within some distance D of the torpedo, at time t_D , it still moves along the same straight line until a certain time $t_r = t_D + \Delta t$ when r becomes nonzero. When $t > t_r$, the straight line becomes a cone which flares out with radius $r(t)$, equation (20). The

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INITIAL SPEED 17 KT
FINAL SPEED 25 KT
TIME 1/2, 1, 3 MIN



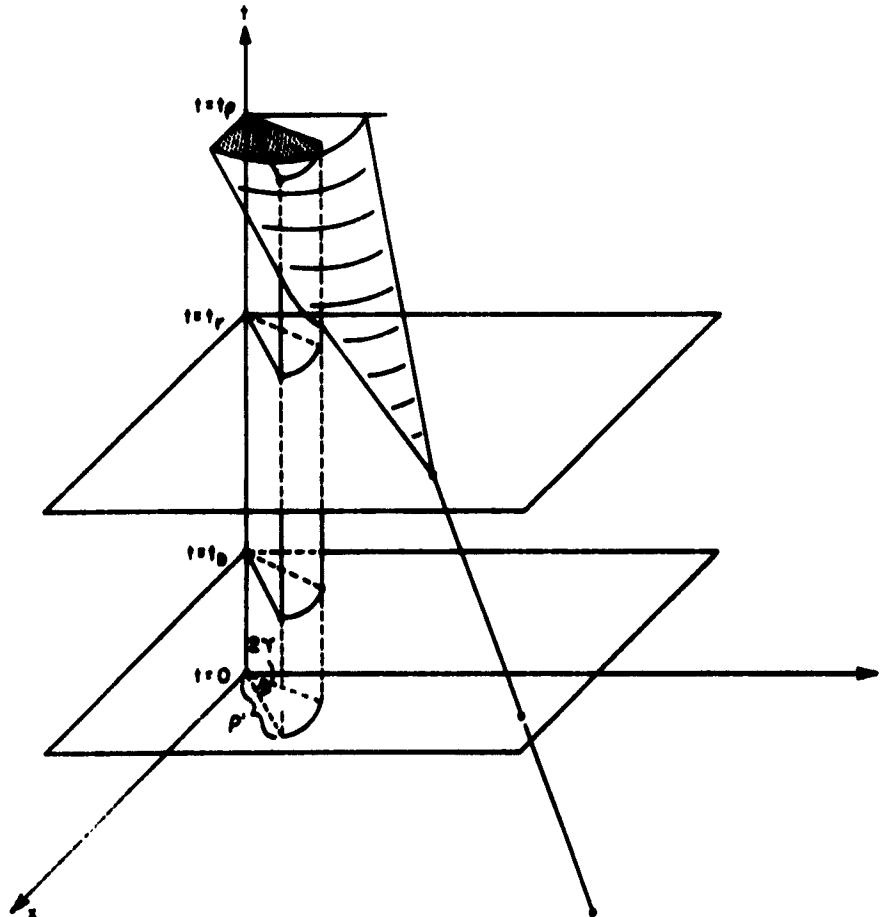
CIRCLES = LOCI OF TARGET POSITIONS AFTER TIMES
1/2, 1, ... 3 MIN. ACCORDING TO EQUATIONS (19), (20).

POINTS = ACTUAL TARGET POSITION ACCORDING TO SIMULATIONS.

FIGURE 6. TARGET EVASION CAPABILITY RELATIVE TO WATER

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Stippled area in cone section at t_p = portion of cone within quadrant $x, y > 0$ which is acquired by torpedo.

FIGURE 1. "TARGET" MOTION RELATIVE TO TORPEDO IN THREE DIMENSIONS

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torpedo's acoustic head is a sector of a circle with search angle 2γ , radius $\rho' = Z \sec \gamma$ center at the origin; and in the (x,y,t) system it rises straight up, parallel to the t -axis. If the target line segments within $0 < t < t_r$ intersect the head, the acquisition probability is 100%. If not, but portions of the cone intersect the head, the probability equals the percentage of points within the cone which intersect the head at any time $t_0 < t < t_p$, where t_p is the time when the torpedo reaches a certain distance ρ from own-ship and runs out of fuel $t_p = \rho \div S_{TO}$. See Figure 7.

The three dimensional kinematic model, Figure 7, can be reduced to a two dimensional model which will be a more convenient basis for the actual calculations. The x and y co-ordinates, which denote position relative to the torpedo, are retained. The time co-ordinate is dropped, and the target's motion is shown as a series of points in the xy plane, each point for a particular time. The torpedo's acoustic head is fixed at the origin. For $0 < t < t_r$, the target moves along a line segment. For $t_r < t < t_p$, it can occupy any point within the circle of its maneuver. See Figure 8. If target passes within the acoustic head at any time $0 < t < t_r$, it is acquired. If not, but if in passing from the point $x(t_r)$, $y(t_r)$ to some point in the circle for t_p it goes through the head, then it is acquired. The probability of acquisition during the time $t_r < t < t_p$ equals the fraction of the circle at t_p which has lines going to $x(t_r)$, $y(t_r)$ that pass through the acoustic head.

3.4 - Optimal Firing Angle for One Torpedo

Several angles β are calculated from the u , α found in section 3.2 according to the precepts of section 3.1. Values i, j are chosen to cover the range within which acquisition probability is likely to be non-negligible:

$$(i,j) = (1,1); (1,n); (n,1); (n,n); (1 + F_1 F_2, 1 + F_1 F_2)$$

For each of these, the acquisition probability is calculated according to the method of section 3.1 and by means of the following equations (21-37):

Time Interval $0 < t < t_r$ - Figure 2 shows the motions of a straight-running target and a torpedo relative to own-ship. Target moves at speed u along angle α and torpedo moves at speed S_{TO} along angle β . Target's motion relative to torpedo is

$$\begin{aligned} x_{1j}(t) &= t(u_{1j} \sin \alpha_{1j} - S_{TO} \sin \beta_{ab}) \\ y_{1j}(t) &= R_{1j} - t(u_{1j} \cos \alpha_{1j} + S_{TO} \cos \beta_{ab}) \end{aligned} \quad (21)$$

β_{ab} refers to some one of the angles β_{1j} .

R must be found from u , α , $\dot{B}$ by equation (4). The time t_D occurs when target comes within distance D of torpedo:

$$[x(t_D)]^2 + [y(t_D)]^2 = D^2. \quad (22)$$

The quadratic formula yields two roots, $t_{D1} \leq t_{D2}$. If t_{D1} is positive and less than the time t_p when torpedo runs out of fuel, let $t_D = t_{D1}$. If $t_{D1} < 0$ and $t_{D2} > 0$, then $D < \text{initial range}$ so that target will start to evade as soon as torpedo is fired; i.e., $t_D = \text{zero}$. If $t_{D1} > t_p$ or both roots are negative or imaginary, then the torpedo never can acquire target: $A(\beta_{1j})$ is zero, and we go to the next set of (i, j) .

$$t_r = t_D + \Delta t. \quad (19), (22)$$

Acquisition for $0 < t < t_r$. The acoustic head, Figure 8, is bounded by three segments - $\overline{OT_1}$, $\overline{OT_2}$, and $\overline{T_1T_2}$. If target path $x(0)$, $y(0)$ to $x(t_r)$, $y(t_r)$ intersects any one of these, it is acquired. The intersection tests are as follows:

Let $x(0) = x_1$, $y(0) = y_1$, $x(t_r) = x_2$, $y(t_r) = y_2$. First is found

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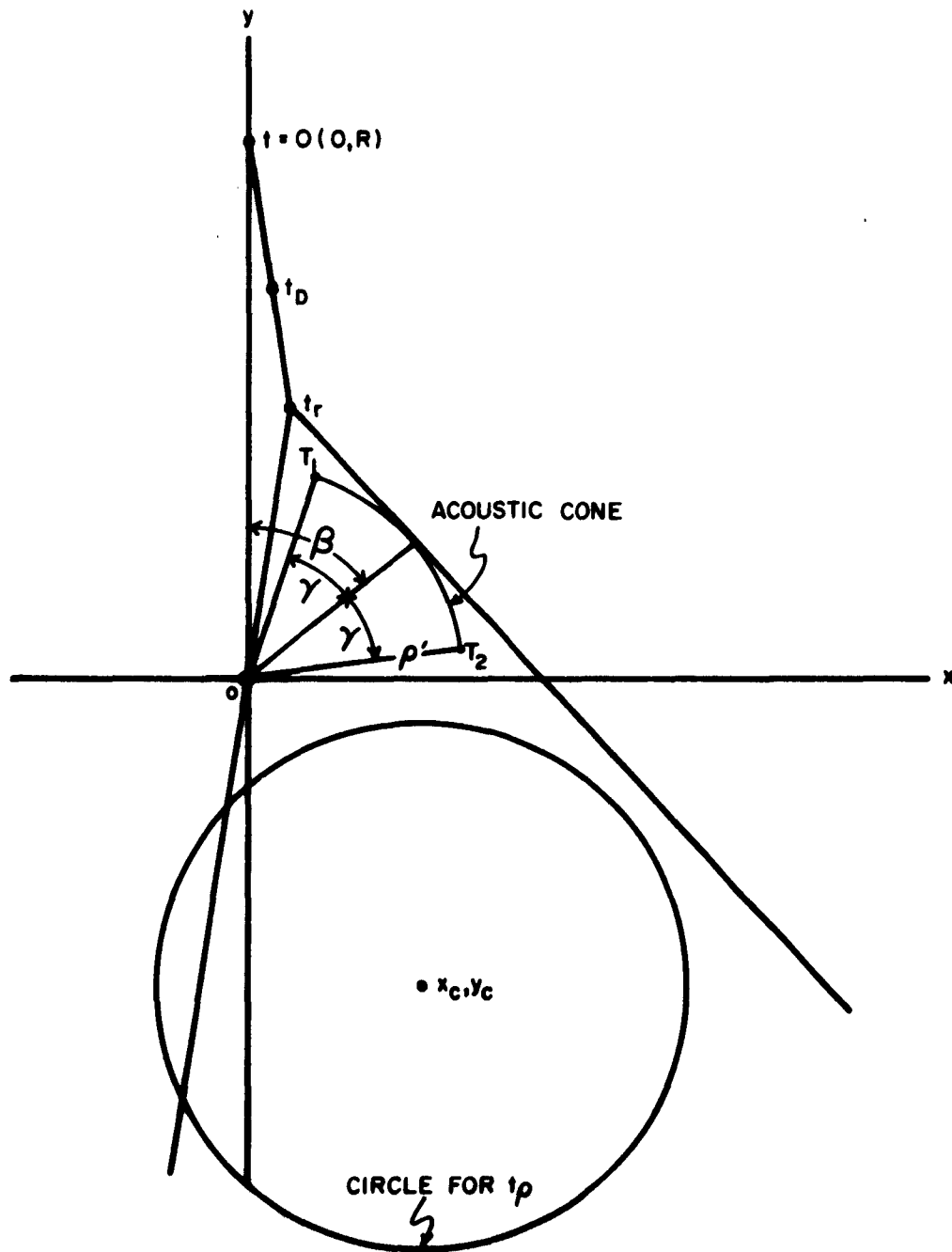


FIGURE 8. TARGET MOTION RELATIVE TO TORPEDO IN TWO DIMENSIONS

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whether or not any point on the line segment (x_1, y_1) to (x_2, y_2) lies within a distance $OT_1 = OT_2 = \rho'$ of the origin.

$$\text{Equation of the line } y = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1) + y_1 \quad (23)$$

$$\text{Equation of the circle } x^2 + y^2 = \rho'^2 \quad (24)$$

(23) is substituted in (24), and roots x', x'' are found from the quadratic formula. If x', x'' are real and one of them lies between x_1 and x_2 , then an intersection exists and the next test, equation (25), is made. If not, then no point of the segment (x_1, y_1) , (x_2, y_2) lies within ρ' of the origin, so omit (25) - (28).

Intersections of (x_1, y_1) to (x_2, y_2) with the line segments $\overline{OT_1}$ and $\overline{OT_2}$ are tested:

$$\text{Equation of the line along } \overline{OT_1} \quad y = x \cot (\beta_{ab} - \gamma) \quad (25)$$

$$\text{Equation of the line along } \overline{OT_2} \quad y = x \cot (\beta_{ab} + \gamma) \quad (26)$$

(25) is substituted into (23). The intersection point (x_r, y_r) is found. If x_r lies between x_1 and x_2 , and if $x_r^2 + y_r^2 < \rho'^2$ an intersection exists and target is acquired. If not, a similar test is made with (26). If it fails, intersection with arc $\widehat{T_1 T_2}$ is tested as follows:

$$y', y'' \text{ found from } x', x'' \quad (23)$$

$P'(x', y')$ lies on the arc $\widehat{T_1 T_2}$ if the following conditions are fulfilled

$$\begin{aligned} \sin \widehat{T_1 T_2} &= \sin (|T_1 P'| + |P' T_2|) \text{ or} \\ \rho'^2 \sin 2\gamma &= |y' \sin(\beta_{ab} - \gamma) - x' \cos(\beta_{ab} - \gamma)| \left[x' \sin(\beta_{ab} + \gamma) + \right. \\ &\quad \left. y' \cos(\beta_{ab} + \gamma) \right] + |x' \cos(\beta_{ab} + \gamma) - y' \sin(\beta_{ab} + \gamma)| \\ &\quad \left[x' \sin(\beta_{ab} - \gamma) + y' \cos(\beta_{ab} - \gamma) \right] \quad (27) \end{aligned}$$

$$\widehat{T_1 P'} \neq \frac{\pi}{2} \text{ or}$$

$$x' \sin(\beta_{ab} - \gamma) + y' \cos(\beta_{ab} - \gamma) = 0 \quad (28)$$

A similar test is applied to $P''(x'', y'')$.

Time $t_r < t < t_\rho$ - The target can now occupy any point within an ever widening circle of radius $ra(t-t_r)$. The circle's center (x_c, y_c) is the target's position $x(t_r), y(t_r)$ modified by the torpedo's motion after time t_r . The torpedo's position relative to the water is

$$x_{TO} = (S_{TO} \sin \beta_{ab} + S_{OS} \cos B)t, \quad y_{TO} = (S_{TO} \cos \beta_{ab} + S_{OS} \sin B)t \quad (29)$$

At the time t_ρ when the torpedo run ends, the center's position is

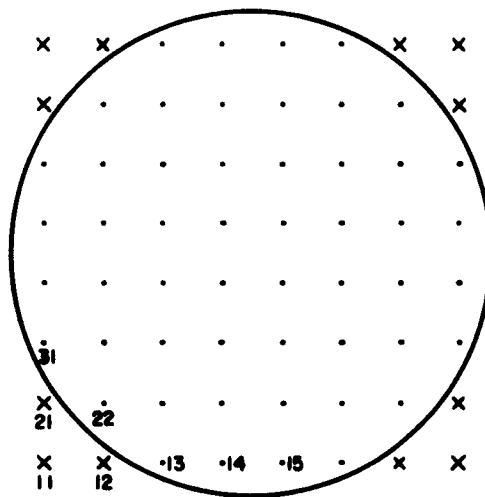
$$x_c(t_\rho) = x(t_r) - [x_{TO}(t_\rho) - x_{TO}(t_r)]; \quad y_c(t_\rho) = y(t_r) - [y_{TO}(t_\rho) - y_{TO}(t_r)] \quad (30)$$

The acquisition probability during $t_r < t < t_\rho$ equals the areal fraction of the circle at time t_ρ which has passed through the acoustic cone since t_r . To find the fraction, a numerical integration is done over a rectangular grid, superposed on the circle as in Figure 9.

$$x_{lm} = x_c + \frac{r(t_\rho)}{K} (2l-1-K) \quad y_{lm} = y_c + \frac{r(t_\rho)}{K} (2m-1-K) \quad l, m = 1 \text{ to } K \quad (31)$$

$$\text{If } (x_{lm} - x_c)^2 + (y_{lm} - y_c)^2 \geq [r(t_\rho)]^2 \text{ omit the point.} \quad (32)$$

FIGURE 9
POSSIBLE TARGET POSITIONS WITHIN THE CONE



Points x do not satisfy (36). Omit them.

Each of the segments $X(t_r)$, $y(t_r)$ to x_{lm} , y_{lm} which satisfy (36) is tested for intersection with the acoustic cone according to (23) through (28) with $x_1 = x(t_r)$, $x_2 = x_{lm}$, $y_1 = y(t_r)$, $y_2 = y_{lm}$. Acquisition probability

$$A_{1j} = \frac{\text{No. of segments } k, l \text{ which have } \alpha < r(t_p) \text{ and intersect cone}}{\text{No. of segments which have } \alpha < r(t_p)} \quad (33)$$

This must be multiplied by the probability P_{1j} that target has α_{1j} , u_{1j} as given for the different cases in section 3.2. The total acquisition probability for firing a torpedo at angle β_{ab} is found by summing over all i, j and dividing by the sum of P_{1j} .

Best Firing Angle For Acquisition During Otkp -

Do (21) - (34) for $i, j = 1 \dots n$ (34)

$$A(\beta_{ab}) = \sum_{i,j=1}^n \left[A_{ij} \cdot P_{ij} \right] \div \sum_{i,j=1}^n P_{ij} \quad (35)$$

Equations (21) through (35) are applied to

$$(a,b) = (1,1); (1,n); (n,1); (n,n); (1 + F_1 F_2, 1 + F_1 F_2) \quad (36)$$

The five angles β_{ab} are arranged in order of size and labeled

$$\beta_1 \leq \beta_2 \leq \beta_3 \leq \beta_4 \leq \beta_5 \quad (37)$$

and the corresponding A_{ij} are labeled $A_1, \dots, A_5$.

If β_1 and β_5 are so chosen that $A(\beta)$ is very small for $\beta < \beta_1$ and $\beta > \beta_5$, it follows that an optimal angle should lie between β_1 and β_5 . This is selected partly to get a large A and partly by random choice, to allow for the target's random evasion maneuver. A line graph of $A(\beta)$ is divided into a number $(H_2 + 1)$ of equal areas by vertical lines and one of these areas is selected by a random number H_1 , between 0 and H_2 . The β which divides that area into two equal parts is the recommended angle. See Figure 10 and equations (39) through (45).

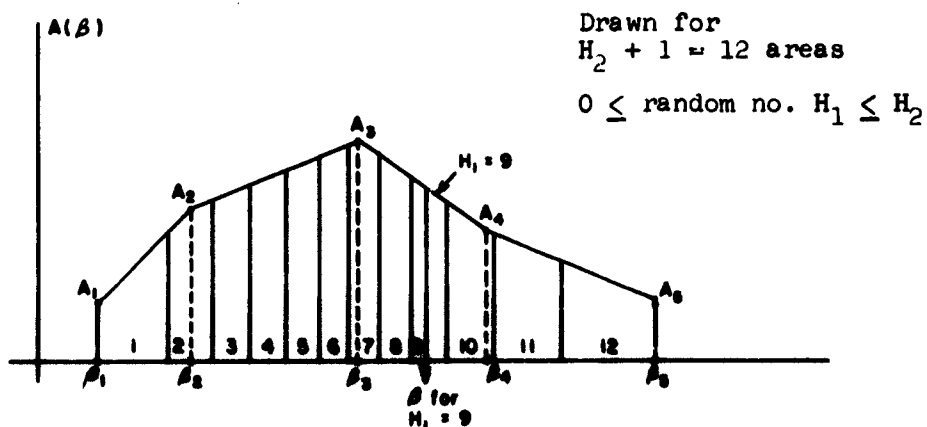


FIGURE 10. ACQUISITION PROBABILITY AS A FUNCTION OF FIRING ANGLE

Total area under curve between β_1 and $\beta_{h'}$,

$$\kappa_1 = 0 \quad \kappa_{h'} = \sum_{h=2}^{h'} 1/2 \left[\beta_h - \beta_{h-1} \right] \left[A(\beta_h) - A(\beta_{h-1}) \right] \text{ if } h' = 2, 3, 4, 5 \quad (39)$$

Fraction of κ bounded by β_1 and $\beta_{h'}$,

$$\phi_{h'} = \kappa_{h'} + \kappa_5 \quad (40)$$

Choose at random an integer H_1 : $0 \leq H_1 \leq H_2$ (41)

Determine ϕ , the areal fraction corresponding to H_1

$$\phi = \frac{H_1 + 1/2}{H_2 + 1} \quad (42)$$

Find h' such that $\phi_{h'} \leq \phi < \phi_{h'+1}$ (43)

the best angle β' must be so fixed that $\phi = \frac{1}{\kappa_5} \int_{\beta_1}^{\beta'} A(\beta) d\beta$.

It is seen from Figure 10 that

$$\begin{aligned} \phi &= \phi_{h'} + \frac{1}{\kappa_5} \int_{\beta_{h'}}^{\beta'} A(\beta) d\beta \\ &= \phi_{h'} + \frac{1}{\kappa_5} \int \left\{ A_{h'} + \left(\frac{\beta - \beta_{h'}}{\beta_{h'+1} - \beta_{h'}} \right) (A_{h'+1} - A_{h'}) \right\} d\beta \end{aligned}$$

$$\text{Let } S_{h'} = \frac{A_{h'+1} - A_{h'}}{\beta_{h'+1} - \beta_{h'}} \quad (44)$$

$$\text{Then } (\phi - \phi_{h'}) \kappa_5 = \left[(A_{h'} - S_{h'} \beta_{h'}) \beta + 1/2 S_{h'} \beta^2 \right]_{\beta_{h'}}^{\beta'}$$

Using algebra and the quadratic formula

$$a_1 = 1/2 S_{h1}, \quad a_2 = A_{h1} - S_{h1}\beta_{h1}, \quad a_3 = \beta_{h1} (1/2\beta_{h1}S_{h1} - A_{h1}) - (\phi - \phi_{h1})A_5$$

$$\beta' = \frac{1}{2a_1} \left[-a_2 \pm \sqrt{a_2^2 - 4a_1a_3} \right] \quad (45)$$

The acquisition probability A^1 associated with β^1 is found by application of equations (21) through (37).

3.5 - Optimal Firing Angles for Several Torpedoes

If a salvo of torpedoes $\beta^{(P)} = \beta^{(1)}, \beta^{(2)}, \dots, \beta^{(N)}$ is to be fired, then the procedure of section 3.4 must be followed for each β^P . However, two modifications are needed: (a) a time delay after firing of the first torpedo; (b) omission of those parts of $P_{1j} \cdot A_{1j}$ that have already been acquired by the preceding torpedoes.

The modified procedure for $\beta^{(2)}$ et sequentes is the following:

Choose another random $H_1^{(P)}$ (41)

Calculate $\beta^{(P)}$ (42) - (45)

$$x_{1j}(t) = tu_{1j} \sin \alpha_{1j} - t_\delta S_{T0} \sin \beta^{(P)} \quad (46) = (21) \text{ mod.}$$

$$y_{1j}(t) = R_{1j} - tu_{1j} \cos \alpha_{1j} - t_\delta S_{T0} \cos \beta^{(P)}$$

where $\delta_{1,P}$ is the time interval between the launching of the first and Pth torpedoes, and $t_\delta = t - \delta_{1,P}$ if $t > \delta_{1,P}$; $t_\delta = 0$ if $t \leq \delta_{1,P}$.

t_D and t_r are the same as when $P = 1$, since the target maneuver, to which they refer, occurs at the approach of the first torpedo.

However, there are now 4 critical times $0, t_r, t_\rho, \delta_{1,P}$. Along each of the 3 time intervals, target will have a constant speed and direction relative to the torpedo, so that equations (21) through (35) can be used with only slight modification. The 3 nonzero critical times are arranged in order of size

$$0 \leq t^{(1)} \leq t^{(2)} \leq t^{(3)} \quad (47)$$

so that 3 cases are possible:

$$0, t^{(1)} = \delta_{1,P}, \quad t^{(2)} = t_r, \quad t^{(3)} = t_\rho,$$

$$0 \quad t_r \quad \delta_{1,P} \quad t_\rho$$

$$0 \quad t_r \quad t_\rho \quad \delta_{1,P}$$

For whichever of these obtains for torpedo P, then appropriate modification of (21) through (35) will yield $A(\beta^P)$ which is the increment to the acquisition probability for the Pth torpedo. In each case, an acquisition is not counted if it corresponds to an (1.j) or (1.j,l,m) that was already counted during a previous P.

The total acquisition probability for P torpedoes.

$$A^P = A(\beta^1) + \dots + A(\beta^P) \quad (48)$$

For each angle β^k the firing angle with correction $(\delta\beta + \beta)^P$ is calculated from equation (6d). The final output of the program will be

$$(\delta\beta + \beta)^1, A^1; \dots (\delta\beta + \beta)^P, A^P; \dots (\delta\beta + \beta)^N, A^N.$$

Generally, N will not exceed 4 torpedoes in a salvo.

The calculation procedure sketched in the Summary, page 3, and detailed in sections 3.2 through 3.5, is shown in a schematic flow chart, Figure 11.

3.6 - Conclusions

The method detailed in sections 3.2 through 3.5 and outlined in the flow chart is now being programmed on an IBM 704 calculator. The program has been designed to economize on storage and time required for a full solution. Estimates of these quantities for a typical

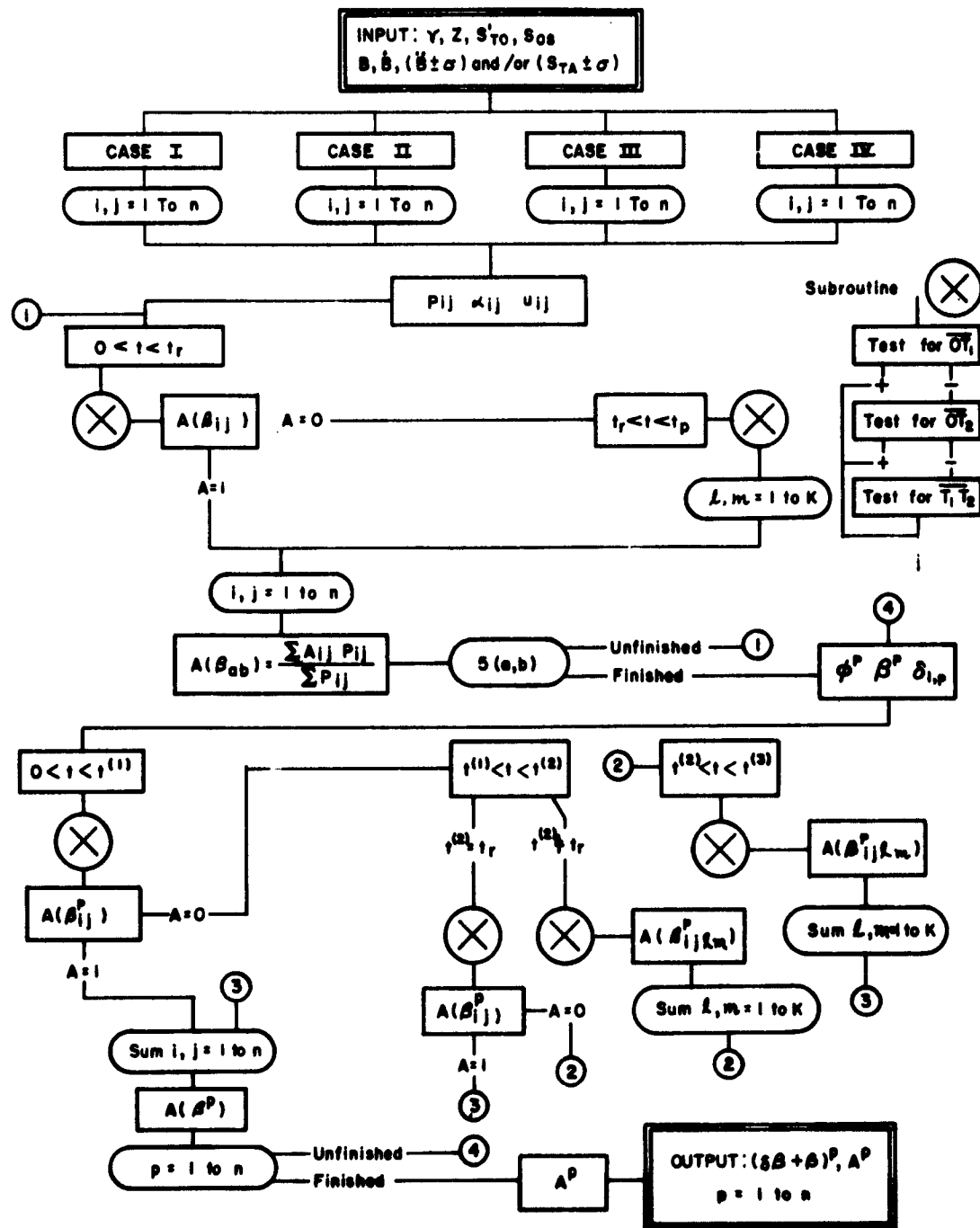


FIGURE 11. FLOW CHART

tactical situation must await completion of the program and simulation of actual tactical cases, within the next 3 months.

The overall efficacy of our method - its ability to produce a high acquisition probability for targets at ranges up to ten miles - and its sensitivity to various input factors cannot be accurately judged until the above simulations are run. However, some graphical analyses suggest that the most critical factors are the distance at which target detects an approaching torpedo, the torpedo speed and size of acoustic cone, and the quality of the ASP fire control solution which degrades rapidly with increasing target range.

A few preliminary graphical analyses were made, assuming that target detects a torpedo 1 mile away, torpedo speed 40 knots, acoustic cone with range 800 yards and half-angle $\gamma = 17^\circ$, a reasonably accurate ASP (automatic statistical processing) solution for a target at range 5 miles, and a known target speed from screw count. Acquisition probability was 35% for 1 torpedo, 50% for 2 torpedoes, and 60% for 4 torpedoes. Acquisition probability was reduced by half if either the target detection = 2 miles or the screw count was lacking.

3.7 - Future Work

In addition to programming the method now in progress and planning tactical simulations, we shall consider improving the method itself by removing present limitations:

a) Straight-Running Own-Ship - If own-ship changes course and/or speed at least once during tracking of target, it can get range information independent of assumed limits on target speed or a screw count. This case is covered in the ASP report (see footnote, page 7).

b) Straight-Running Target - If target sinuates or zigs as a matter of policy before detecting a torpedo, then it naturally becomes more difficult to determine target's position relative to own-

ship. However, this case is being investigated as a sequel to the ASP report.

c) Straight-Running Torpedo - Acquisition probability in Case II or Case III would be greater for a single torpedo if it could travel along the gentle curve of the "acquisition locus", as defined at the end of section 1. Also, if torpedo sinuates around the locus of its forward motion, its effective acoustic cone angle will be increased.

If any of the limitations a), b), or c) be removed, the hypothesis of straight-line motion of target relative to torpedo must be abandoned. This will require considerable complication of equations (23) through (28).

d) Water Resistance on Torpedo - If torpedo is launched from a fast-moving own-ship, it will suffer water resistance which will slow its own speed and perhaps change its course. It could still be straight-running.

e) Torpedo Acoustic Cone - Instead of 100% acquisition within the geometrical cone and zero without, a general formula for % of acquisition as a function of angle and distance from cone could be used. This formula would be based on the sensitivity of a torpedo sonar array.

f) Torpedo Ballistic Dispersion - This factor is neglected, but it could be included on the basis of torpedo mechanical capabilities and inhomogeneities in the water environment.

g) Depth - This factor is assumed fixed, at present.

h) Acquisition Probability - The behavior of the torpedo after it acquires target acoustically and before it hits target is now under study.

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1) Limited Input - Provision will be made for an alternative input of range, bearing, course, and target speed - all relative to own-ship - instead of bearing, rate, acceleration and screw count target speed. Either input can be given by ASP.

It is anticipated that the straight-running limitations, a, b, and c, will be harder to remove than the others. Any generalization of the present program will likely increase storage requirements and calculating time. Whether these changes will sufficiently improve the method's effectiveness to justify their adoption requires further study.

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