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OPERATIONS EVALUATION GROUP
Study 684

Ballistic Dispersion of Mk 80 Series Bombs
Delivered in Sticks by the A-4 Aircraft (U)

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1. Enclosure (1), prepared by the Operations Evaluation Group
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the time of issue.

2. From test data, the study calculates the ballistic dispersion
of low-drag bombs dropped in sticks from A-4 aircraft, so that
ballistic dispersion as a function of delivery conditions can be
better predicted.

3. The Chief of Naval Operations approves this study for use by
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OPERATIONS EVALUATION GROUP

STUDY NO. 684

BALLISTIC DISPERSION OF MK 80 SERIES BOMBS
DELIVERED IN STICKS BY THE A-4 AIRCRAFT (U)

By: S.K. Dietz. Operations Evaluation Group

Approved by: *[signature]* For The Director W.G. Leight, Operations Evaluation Group

28 December 1964

This study represents the view of the Operations Evaluation Group at the time of issue. It does not necessarily reflect the official opinion of the Chief of Naval Operations except to the extent indicated in the forwarding letter. It includes information of an operational rather than a technical nature, and should be made available only to those authorized to receive such information.

Enclosure (1) to
CNO ltr Ser 08P03
Dated 28 December 1964

Prepared by the
OPERATIONS EVALUATION GROUP
Center for Naval Analyses

ABSTRACT

The ballistic dispersion of low drag bombs dropped in sticks from the A-4 aircraft is calculated from test data. The data is inadequate to permit determination of whether dispersion depends on slant range or time of fall, but an estimate can be made for delivery parameters of interest.

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I. INTRODUCTION

Kill probability in stick bombing is a function of aiming accuracy, target vulnerability, bomb reliability, the nominal distance between bombs in the stick, and the ballistic dispersion of the bombs. The kill probability resulting from a set of these variables can be determined in a number of ways; the computer program of reference (a) is an example.

The importance of ballistic dispersion is illustrated by figure 1, computed using reference (a). The aiming accuracy used in figure 1 is typical of that achieved in $20°$ glide attacks using contact-fuzed low drag bombs (reference (b)); the target is a revetted jet fighter with a vulnerable area of 23,600 square feet (reference (c)).

As seen in figure 1, the effectiveness of 6 Mk 81 low drag bombs varies greatly as a function of ballistic dispersion and the spacing between bombs. The kill probability for very low ballistic dispersion (5 feet in the plane normal to the bomb trajectory) can be as much as 40 percent greater than that achievable if dispersion is as great as, say, 60 feet. Moreover, for any given dispersion, the kill probability depends on the choice of bomb spacing. In the example, changing the spacing and dispersion can change kill probability by as much as a factor of three. If the ballistic dispersion is definitely known to be less than or equal to 45 feet, optimum spacing is on the order of 100 feet (variation between 80 and 120 feet will not markedly affect kill probability for the particular target/ delivery illustrated). However, for this optimum spacing kill probability is sensitive to the value of ballistic dispersion. For ballistic dispersions known to be between 45 and 90 feet, any spacing up to 120 feet yields close to optimum results. For greater ballistic dispersions, a salvo (i.e., zero spacing between bombs) produces the best results. A knowledge of bomb dispersion is therefore a prerequisite to recommending intervalometer settings to produce the desired bomb spacings and to providing inputs for estimating force requirements.

The purpose of this study is to analyze data obtained in recent A-4 bomb drops by Air Development Squadron Five to determine a relationship by which ballistic dispersion can be predicted as a function of delivery conditions. However, measuring ballistic dispersion in range requires an accurate knowledge of the nominal (dispersion-free) distance between bombs in a stick. The inadequacy of current theoretical methods for determining bomb spacing is shown in reference (d). The analysis in this study, therefore, deals primarily with ballistic dispersion in deflection.
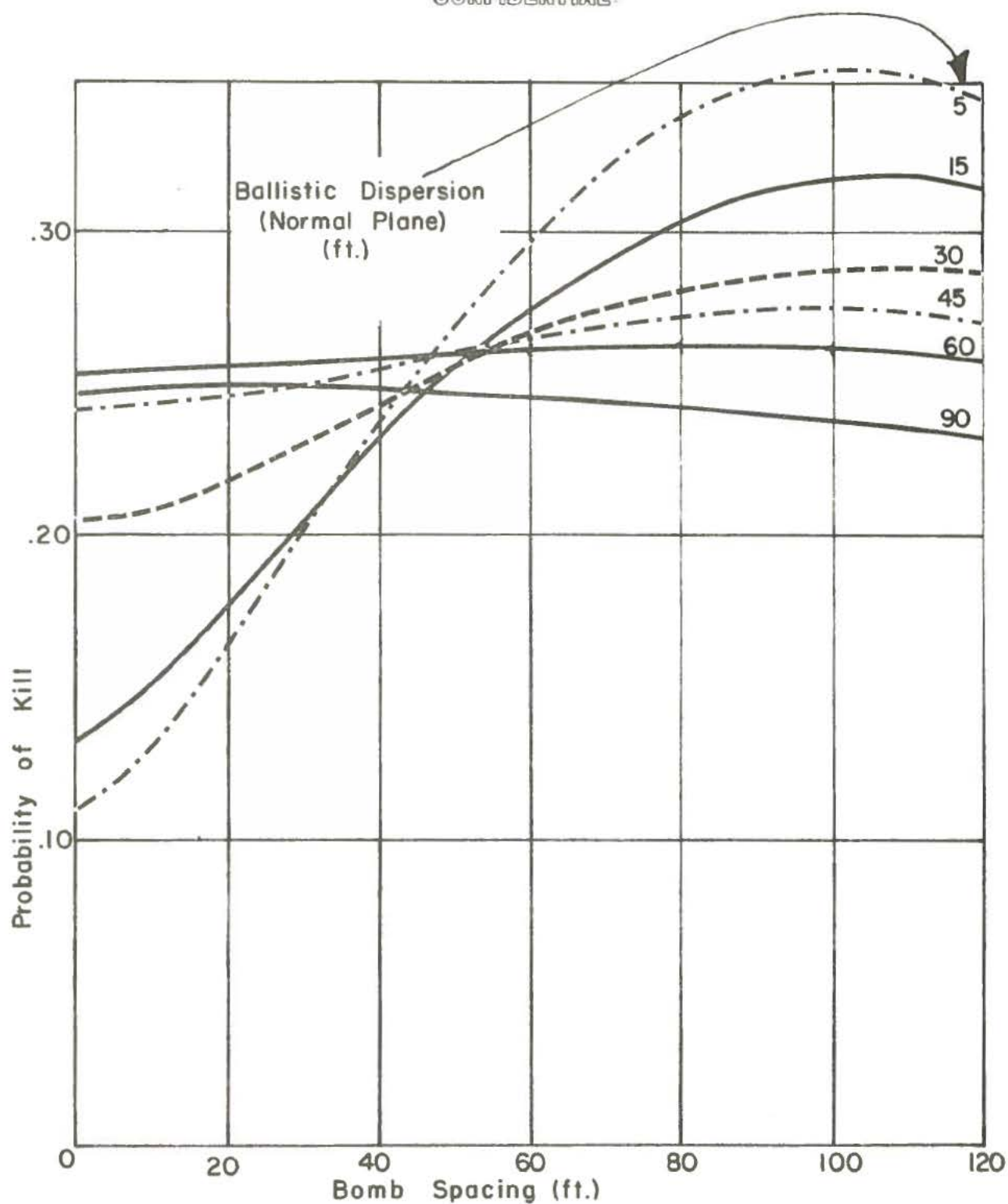
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FIG. 1: EFFECTIVENESS OF 6 MK 81 LOW DRAG BOMBS AS A FUNCTION
OF BALLISTIC DISPERSION AND BOMB SPACING

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If it is true, as usually assumed, that ballistic dispersion is circularly distributed around the bomb trajectory, then knowledge of deflection dispersion permits prediction of dispersion in range. Reference (c) assumes that ballistic dispersion is circularly-normally distributed about the bomb trajectory and that it is a linear function of slant range, and that low drag bombs of the Mark 80 series display a dispersion of 3 mils* normal to the trajectory. Using the empirically-adjusted bomb spacing formula developed in reference (d), range ballistic dispersion is calculated by a method derived in this study, for the narrow range of delivery conditions into which over half of the data falls. This calculation confirms the assumption of equal range and deflection ballistic dispersion in the normal plane.

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*With a 3 mil ballistic error, standard deviation of ballistic error is 3×10^{-3} x slant range.

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II. CONCLUSIONS

- For release at slant ranges between 3100 and 3800 feet, or for a time of fall of 4 to 5 seconds, ballistic dispersion is 15 to 20 feet for Mk 81 bombs dropped from an A-4. These slant ranges and times of fall correspond to those resulting from the standard tactics used in 20° and 30° contact-fuzed, high speed glide deliveries and 45° delay-fuzed delivery.

- An effective technique is developed for calculating ballistic dispersion in range from stick bombing impact data.

- Analysis of test data confirms the customary assumption that range and deflection ballistic dispersion are equal in a plane normal to the trajectory.

- The available data is inadequate to validate the theory that ballistic dispersion is a linear function of slant range or, alternatively, that it is a function of time of fall.

- Additional data is required to formulate a theory for predicting ballistic dispersion on the basis of radically different release conditions. However, in view of the prospective replacement of low-drag bombs by Snakeye, the required large number of drops of low drag bombs does not appear warranted.

III. THE TEST DATA

The test data analyzed in this study was obtained from 13 sticks of bombs dropped by Air Development Squadron Five (VX-5) in 1962 and 1963 using A-4 aircraft equipped with the Douglas Multiple Carriage Bomb Rack (MBR). The data has been divided into 2 sources which have the following distinguishing characteristics:

Data Source 1 .

- Of 12 runs which were made, 4 were rejected because of inadequate camera coverage.

- Mk 86 water/sand-filled (WSF) bombs were dropped.

- Two bombs, which were unstable due to leakage, were excluded.

- Delivery conditions were recorded by Askania camera coverage.

- Airspeed at release varied from 300 to 400 knots.

Data Source 2

- Five runs were made.

- Mk 81 and 82 inert (plaster-filled) bombs were dropped.

- Five bombs which oscillated persistently and appeared unstable were included.

- Delivery conditions were recorded by radar tracking.

- Airspeed at release was 450 knots for all drops.

The 2 sources of data have the following common characteristics:

- One stick of bombs was dropped on each run; most sticks consisted of 6 bombs.

- A constant-speed, constant-dive angle delivery was employed.

- The MBR intervalometer controlling bomb release was set at 0.06 seconds.

- All passes were made over a bull-dozed path leading to a clearly-marked range target.

The data has been reduced on an individual stick basis with standard deviation in deflection (i.e., deflection ballistic dispersion) computed for each stick, rather than for all bombs dropped. This was done for 2 reasons: first, it is essential

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to treat separately those runs where delivery conditions differ markedly; second, not all aircraft were directly over the bull-dozed path and cross-range wind undoubtedly differed from stick to stick. This has the effect of introducing a constant offset for each stick; however, the offset differs from stick to stick. Measurement of the standard deviation of deflection ballistic dispersion for each stick is not affected by this offset, whereas the over-all standard deviation computed for all sticks combined would be influenced by the offset.

The release conditions, vacuum trajectory parameters (computed by the method of reference (e)), and standard deviation of ballistic dispersion for each stick are given in table I; the raw impact data is summarized in appendix A.

Ballistic dispersion in deflection, S_{BD}, was determined from the formula given below and illustrated in figure 2.

$$S_{BD} = \left[\frac{\Sigma(Z_i - \overline{Z})^2}{n-1} \right]^{1/2}$$

where Z_i = the impact distance of the ith bomb measured from a reference line parallel to the flight path

$\overline{Z}$ = the mean distance of bomb impact from the reference line

n = the number of bombs in the stick.

Table I shows that the inclusion or exclusion of unstable bombs makes very little difference in the ballistic dispersion of the sticks of data source 2. Therefore, and because the inert bombs in this data source presumably approximate the behavior of bombs used in combat, this analysis will include the unstable bombs of data source 2. The 2 unstable bombs of data source 1 will still be excluded, however, since their behavior is attributed to water or sand leaks and thus is not typical of live bombs.
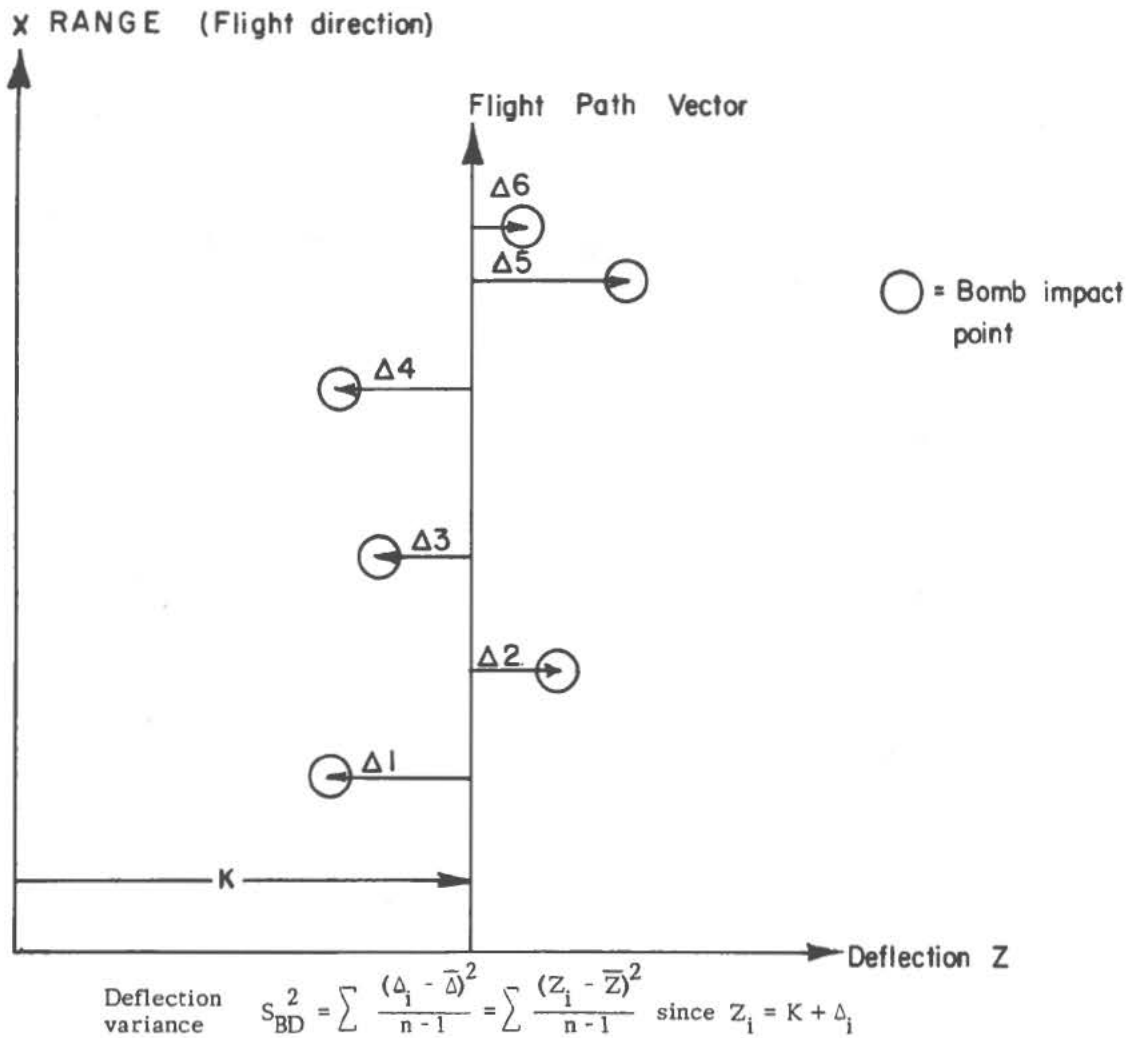
TABLE I

DELIVERY CONDITIONS AND BALLISTIC DISPERSION

Data Source	Run	Release Conditions			Vacuum Trajectory Parameters		Deflection Ballistic Dispersion	
		Dive Angle (Degrees)	Speed (Knots)	Altitude (Feet)	Time of Fall (Sec)	Slant Range (Feet)	Number	$S_{Ballistic\ Dispersion}$ (Feet)
1	1	0	304	2532	12.5	6925	6	31.0
	2	24.7	318	2382	7.1	4187	4	12.7
	3	34.8	303	2483	6.3	3632	5	16.4
	4	23.1	366	2531	7.1	4759	5	50.7
	5	42.4	396	2439	4.6	3347	6	16.2
	6	26.9	337	2486	6.8	4246	6	24.6
	7	21.8	409	2356	6.5	4795	6	29.6
	8	42.7	400	2241	4.3	3080	6	17.6
2	1	32.5	450	2200	4.6	3661	4 (6)*	9.1 (7.9)*
	2	27.5	450	1830	4.3	3455	5 (6)	20.5 (20.6)
	3	41.9	450	2720	4.7	3794	5 (6)	12.0 (21.7)
	4	44.5	450	2620	4.3	3526	4 (6)	11.9 (23.4)
	5	44.5	450	2850	4.7	3818	5 (6)	17.8 (16.4)

*Figure in parentheses include additional data from unstable bombs.

X RANGE (Flight direction)

Flight Path Vector

$\Delta 6$

$\Delta 5$

$\bigcirc$ = Bomb impact point

$\Delta 4$

$\Delta 3$

$\Delta 2$

$\Delta 1$

K

Deflection Z

Deflection variance $\quad S_{BD}^2 = \sum \dfrac{(\Delta_i - \overline{\Delta})^2}{n-1} = \sum \dfrac{(Z_i - \overline{Z})^2}{n-1}$ since $Z_i = K + \Delta_i$

where: n = number of bombs in stick

Z_i = deflection coordinate of ith impact point

K = a constant separating the x axis and the flight path vector

FIGURE 2

IV. ANALYSIS

Deflection ballistic dispersion is displayed in figure 3 as a function of release slant range for the sticks of both data sources. The substantial scatter is at least partly due to the small sample sizes. This is illustrated by the 90 percent confidence intervals around the average dispersions for individual drops. These confidence intervals were calculated using the fact that the ratio of sample standard deviation to true standard deviation has a distribution of $\left[x^2 / \text{degrees of freedom} \right]^{1/2}$.

Alternatively, ballistic dispersion is sometimes assumed to be proportional to time of fall, rather than slant range. Figure 4 shows sample ballistic dispersion against time of fall. As with slant range, there is substantial scatter and no clearout correlation, which can also be attributed to the small sample size (as shown by the 90 percent confidence intervals).

The problem of small samples can to some extent be overcome by pooling the data from those sticks released under similar conditions. Table II summarizes groups of similar slant ranges; similarly, runs are grouped by time of fall in table III.

In table IV, the average ballistic dispersion is given for each slant range group separated by data source and with the 2 sources combined. This table shows that the deflection ballistic dispersions of the 2 data sources are comparable for the 2 common slant range groups. However, an over-all difference between the 2 data sources stems from the erratically varying dispersion at range groups for data source 1 not covered in data source 2. Similarly, it can be seen that the time of fall groupings give comparable results for the 2 data sources in the small zone of overlap; however, wide variation in dispersion is evident for the longer times of fall represented only in data source 1.

Despite the fact that the data does not provide a method of predicting ballistic dispersion, the common slant range groups shown in table IV are representative of a number of widely used delivery tactics, as shown by table V. The analysis suggests that deflection ballistic dispersion lies between 15 and 20 feet for these tactics and slant ranges. Additional data might permit the development of a definitive method of predicting low-drag bomb dispersion for other delivery conditions, but preliminary calculations indicate that many runs would be required. In view of the imminent introduction of Snakeye, an extensive effort does not appear to be warranted.
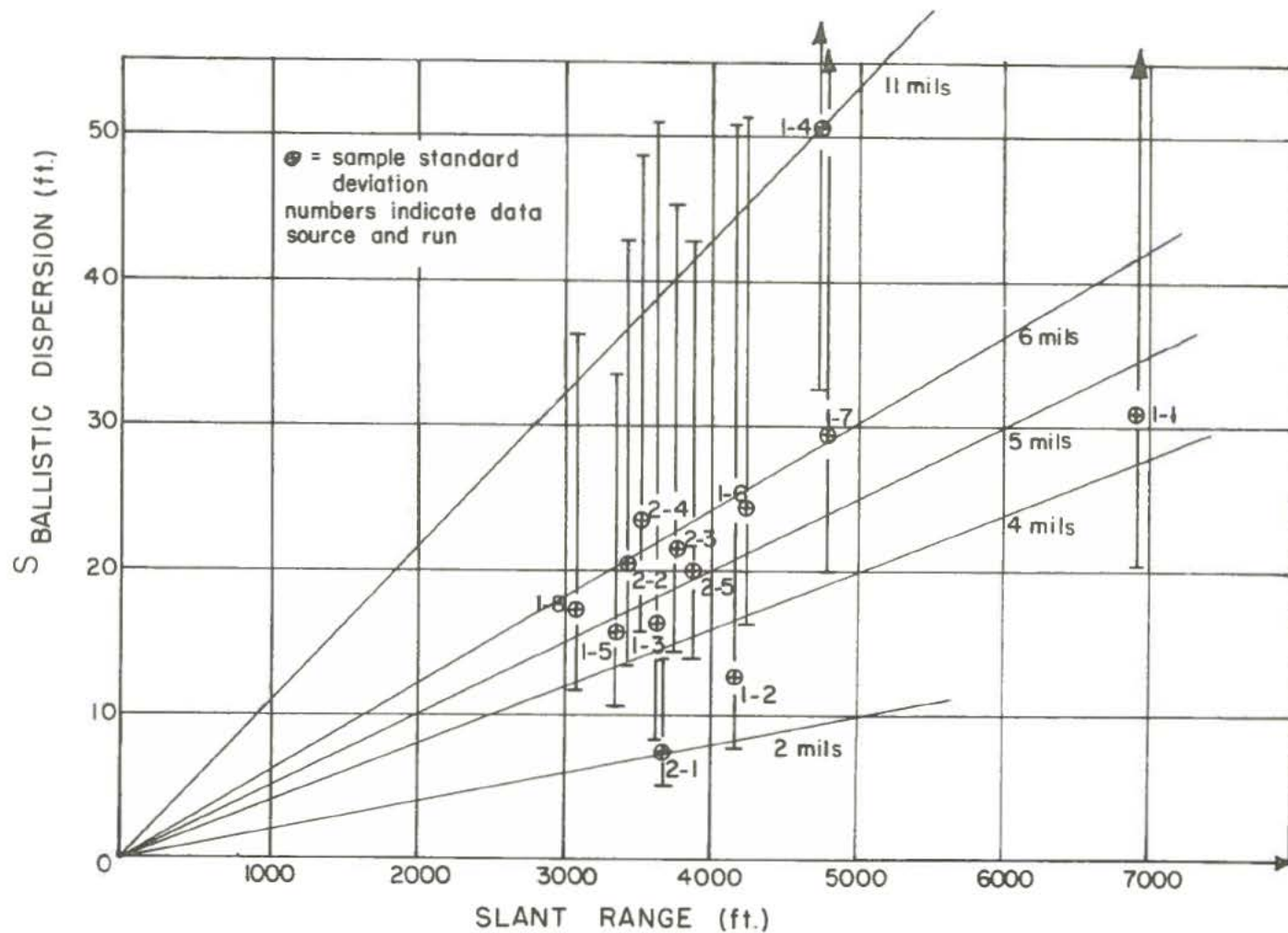
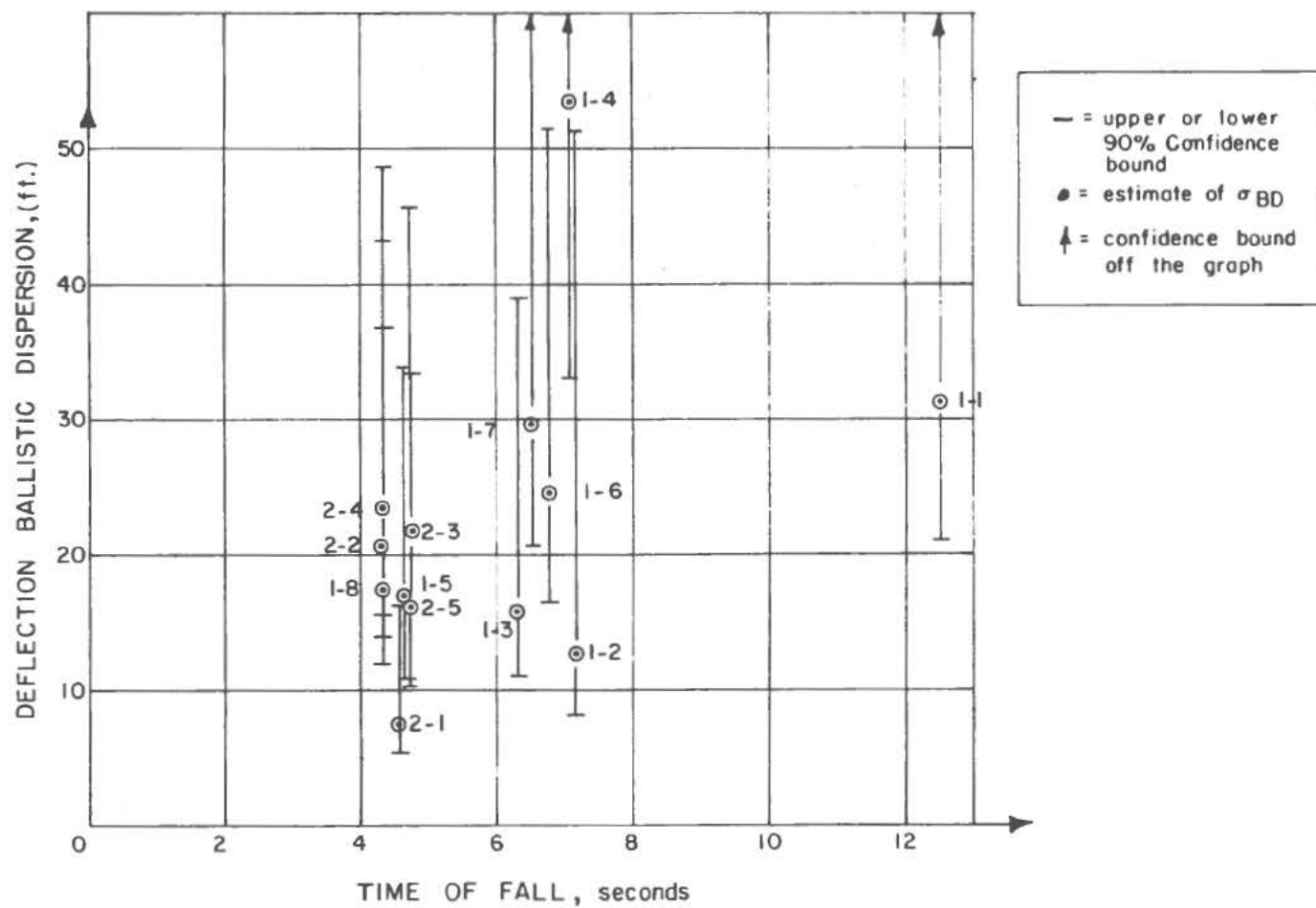FIG. 3: BALLISTIC DISPERSION AS A FUNCTION OF SLANT RANGE

FIG. 4: BALLISTIC DISPERSION AS A FUNCTION OF TIME OF FALL

TABLE II

SLANT RANGE GROUPINGS

Group	Runs	Slant Range	Airspeed	Release Dive Angle	Altitude
S - I	1 - 8	3080	400	42.7	2241
S - II	1 - 5	3347	396	42.4	2439
	2 - 2	3455	450	27.5	1830
	2 - 4	3526	450	44.5	2620
S - III	1 - 3	3632	303	34.8	2483
	2 - 1	3661	450	32.5	2200
	2 - 3	3794	450	41.9	2720
	2 - 5	3818	450	44.5	2850
S - IV	1 - 2	4187	318	24.7	2382
	1 - 6	4246	337	26.9	2486
S - V	1 - 4	4759	366	23.11	2531
	1 - 7	4795	409	21.8	2356
S - VI	1 - 1	6925	304	0	2532

TABLE III
TIME OF FALL GROUPING

Group	Runs	Time of Fall	Airspeed	Release Dive Angle	Altitude
T - I	2 - 4	4.3	450	44.5	2620
	2 - 2	4.3	450	27.5	1830
	1 - 8	4.3	400	42.7	2241
T - II	2 - 1	4.6	450	32.5	2200
	1 - 5	4.6	396	42.4	2439
	2 - 3	4.7	450	41.9	2720
	2 - 5	4.7	450	44.5	2850
T - III	1 - 3	6.3	303	34.8	2483
	1 - 7	6.5	409	21.8	2356
	1 - 6	6.8	337	26.9	2486
T - IV	1 - 2	7.1	318	24.7	2382
	1 - 4	7.1	366	23.1	2531
T - V	1 - 1	12.5	304	0	2532

TABLE IV

DEFLECTION BALLISTIC DISPERSION (FEET)   (GROUPED DATA)

Group	Slant Range* or Time of Fall#	S_{BD}(ft)/Number of Impacts		
		Source 1	Source 2	Combined
S - I	3090	17.6/6		17.6/6
S - II	3300 - 3500	16.2/6	22.0/12	20.3/18
S - III	3600 - 3800	16.4/5	16.2/18	16.3/23
S - IV	4100 - 4300	20.9/10		20.9/10
S - V	4700 - 4800	40.4/11		40.4/11
S - VI	6921	31.0/6		31.0/6
T - I	4.3	17.6/6	21.8/12	20.2/18
T - II	4.6 - 4.7	16.2/6	16.3/18	16.3/24
T - III	6.4 - 6.8	24.6/17		24.6/17
T - IV	7.1	40.6/9		40.6/9
T - V	12.6	31.0/6		31.0/6
	Combined	29 /44	19 /30	

*Slant range, in feet, for groups labelled "S" from table II.

#Time of fall, in seconds, for "T" groups listed in table III.

TABLE V

SLANT RANGES FOR COMMON A-4 DELIVERY TACTICS

Dive Angle	Tactic Airspeed	Release Altitude	Slant Range
20°	450	1800	4100
30°	450	2500	4300
45°	450	3000*	3980
20°	325	1500	3190

*Delay-fuzed

Ballistic dispersion is generally assumed to be circularly-distributed around the bomb trajectory, so that range ballistic dispersion, measured normal to the trajectory should equal deflection dispersion. For stick bombing, however, the nominal (dispersion-free) spacing between bombs must be determined as a prerequisite to measuring ballistic dispersion. Reference (d) discloses problems in this area, but provides an empirically-adjusted formula for estimating nominal bomb spacing. Using this formula, as explained in appendix B, produces the results of table VI. The normalized range dispersion agrees closely with the deflection dispersion of table IV.

TABLE VI

RANGE BALLISTIC DISPERSION (NORMAL PLANE)

Group	Slant Range	$S_{Ballistic\ Dispersion,\ Range(N)}$	(Number of Impacts)
II	3300 - 3500	16.4 (16)	
III	3600 - 3800	15.2 (23)	
I & II Pooled		15.7 (39)*	

*Two bombs included in table IV are excluded here because lack of camera coverage precluded distinguishing between bombs. This ambiguity affects the calculation of $S_{Ballistic\ Dispersion,\ Range}$, but not that of

$S_{Ballistic\ Dispersion\ Deflection}$.

References: (a) Naval Warfare Analysis Group Interim Research Memorandum, NIRM 12, Usage Manual for a Computer Program to Compute the Effectiveness of Groups of Weapons Against Rectangular and Line Targets   Unclassified   21 Aug 1962

(b) Operations Evaluation Group Study No. 659, Predicted Combat Accuracy of Conventional Air-to-Surface Weapons (U)   Secret   7 Nov 1963

(c) NWIP 20-1 Naval Weapons Selection - Aircraft (U)   Appendix F, A-4 Interim Weapons Selection Data (U)   Confidential

(d) Operations Evaluation Group Study No. 673, Bomb Spacing in A-4B/C Stick Bombing with Low Drag Bombs (U)   Confidential

(e) OEG Research Contribution No. 51, Computer Program for Calculation of Vacuum Trajectory Parameter   Unclassified   18 Feb 1964

APPENDIX A

APPENDIX A

IMPACT DATA

Tables A-I and A-II list the impact data for the 2 sources described in the text. The entries in the body of each table are the range and deflection coordinates (X_{ij}, Z_{ij}) for the jth bomb of the ith run. Four runs of data source 1 have been rejected due primarily to inadequate camera coverage.

TABLE A-I

RANGE AND DEFLECTION COORDINATES (X_{ij}, Z_{ij}) FOR THE

IMPACT POINT OF THE ith BOMB OF THE ith RUN

(DATA SOURCE 1)

Flt-Pass	Run$_i$ / Bomb$_j$	1	2	3	4	5	6
2 - 1	1	82, 74	123, 67	172, 19	270, 34	312, 70	307, 106
2 - 2	2	-61, 797	18, 819	3, 827	NR	92, 812	NR
2 - 3	3	-585, -33*	-412, -28	-365, 2	-392, -19	-324, 0	-307, 12
4 - 2	4	-278, 1005	-244, 928	-165, 864	-197, 912	-58, 928	-220, 909*
4 - 3	5	-27, 60	44, 37	38, 14	29, 33	130, 37	43, 19
1 - 2	6	375, 713	443, 692	427, 712	488, 695	503, 728	513, 759
3 - 2	7	-233, 673	-196, 685	-56, 706	-148, 663	-45, 672	-115, 617
3 - 3	8	-13, -118	-2, -137	-18, -145	35, -118	82, -97	62, -110

*Unstable bombs
NR = No Release

TABLE A-II

RANGE AND DEFLECTION COORDINATES (X_{ij}, Z_{ij}) FOR THE

IMPACT POINT OF THE ith BOMB OF THE ith RUN

(DATA SOURCE 2)

Run	Run$_i$ / Bomb$_j$	1	2	3	4	5	6
A	1	-222, 122*	-194, 109	-141, 111*	-134, 111	-117, 129	-103, 114
B	2	-182, 9	-146, 25	-148, 5*	-128, 8	-114, 45	-123, 53
C	3	-258, 21	-225, 22	-275, -23*	-254, 6	-190, 28	-176, 39
D	4	NC	-60, 0	NC	-33.5, -3	25, 18	-22, 20
E	5	-180, -45*	47, 26	-104, -2*	81, 37	71, 23	88, 30
		13, 15*		70, -9			

*Unstable Bombs
NC = No Camera Coverage

APPENDIX B

APPENDIX B
A METHOD FOR DETERMINING BOMB DISPERSION IN RANGE

In stick bombing, impacts are assumed to be normally distributed about idealized (dispersion-free) Mean Points of Impact (MPI), and such MPI's are assumed to be separated from an arbitrary orgin in accordance with specifiable inter-bomb spacings, given the distance from the origin to the MPI of the first bomb. The expression for specifying inter-bomb spacings is given in reference (d) as a function of delivery conditions.

The range coordinate for the jth bomb of a stick may therefore be partitioned into three components:

$$\chi_{ij} = R_{il} + C_{ij} + \epsilon_{ij} ,$$

where the subscript i indicates the run;

R_{il} is the range coordinate of the nominal point of impact of the first bomb of the ith run,

C_{ij} is the separation between MPI's for bomb 1 and j, and

ϵ_{ij} is an independent, normally distributed random variable with standard deviation equal to bomb dispersion in range, σ_{BR}.

A new set of variables, w_{ij}, can be constructed from the χ_{ij} such that each w_{ij} has the form $w_{ij} = \mu_i + \epsilon_{ij}$, where μ_i is the population mean distance from arbitrary orgin to the expected center of the stick. The set of w's, whose variance is an unbiased estimate of range ballistic variance, can be constructed as follows:

$$w_{ij} = x_{ij} + K_{ij}$$

where K_{ij} is given by the following table:

TABLE B-I

j	K_{ij}
1	$+ 3S_{L/2} + S_S + 22.0/\sin \varphi_i$
2	$+ S_{L/2} + S_S + 5.5/\sin \varphi_i$
3	$+ S_{L/2} + 1.8/\sin \varphi_i$
4	$- S_{L/2} + 6.2/\sin \varphi_i$
5	$- (S_{L/2} + S_S) - 17.1/\sin \varphi_i$
6	$- (3S_{L/2} + S_S) - 6.5/\sin \varphi_i$

The spacings, S_L and S_S, are generated by the theoretical expression derived in reference (d) and depend on delivery conditions for a given pass. The remainder of K_{ij} represents a correction factor computed in reference (d) to adjust for discrepancies between theoretical and observed spacings. Figure B-1 illustrates the relationships among these parameters and correction factors.

The sample variance of the w_{ij} for a fixed run can be expressed as:

$$S_i^2 = \sum_{j=1}^{n} \frac{(w_{ij} - \mu_i)^2}{n_i} \qquad \text{where } n_i \text{ is the number of bombs in } i^{th} \text{ stick.}$$

Although μ_i is unknown, it can be estimated by $\overline{w}_{ij}$ with the loss of only one degree of freedom in the calculation as follows:

$$\overline{S}_i^2 = \sum_{j=1}^{n} \frac{(w_{ij} - \overline{w}_{ij})^2}{n_i - 1} \qquad .$$

The sample bomb dispersion in range, $\overline{S}_i = S_{BR}$, computed for the ith run, is measured in the ground plane. The normal plane range dispersion, $S_{BR(N)}$, can be calculated as $S_{BR(N)} = S_{BR} \sin \varphi$, where φ is the impact angle (measured from the horizontal) of the bomb's velocity vector. Each sample variance is an estimate of the range ballistic variance, σ_{BR}^2. If these variances come from the same population, they can be pooled as follows:
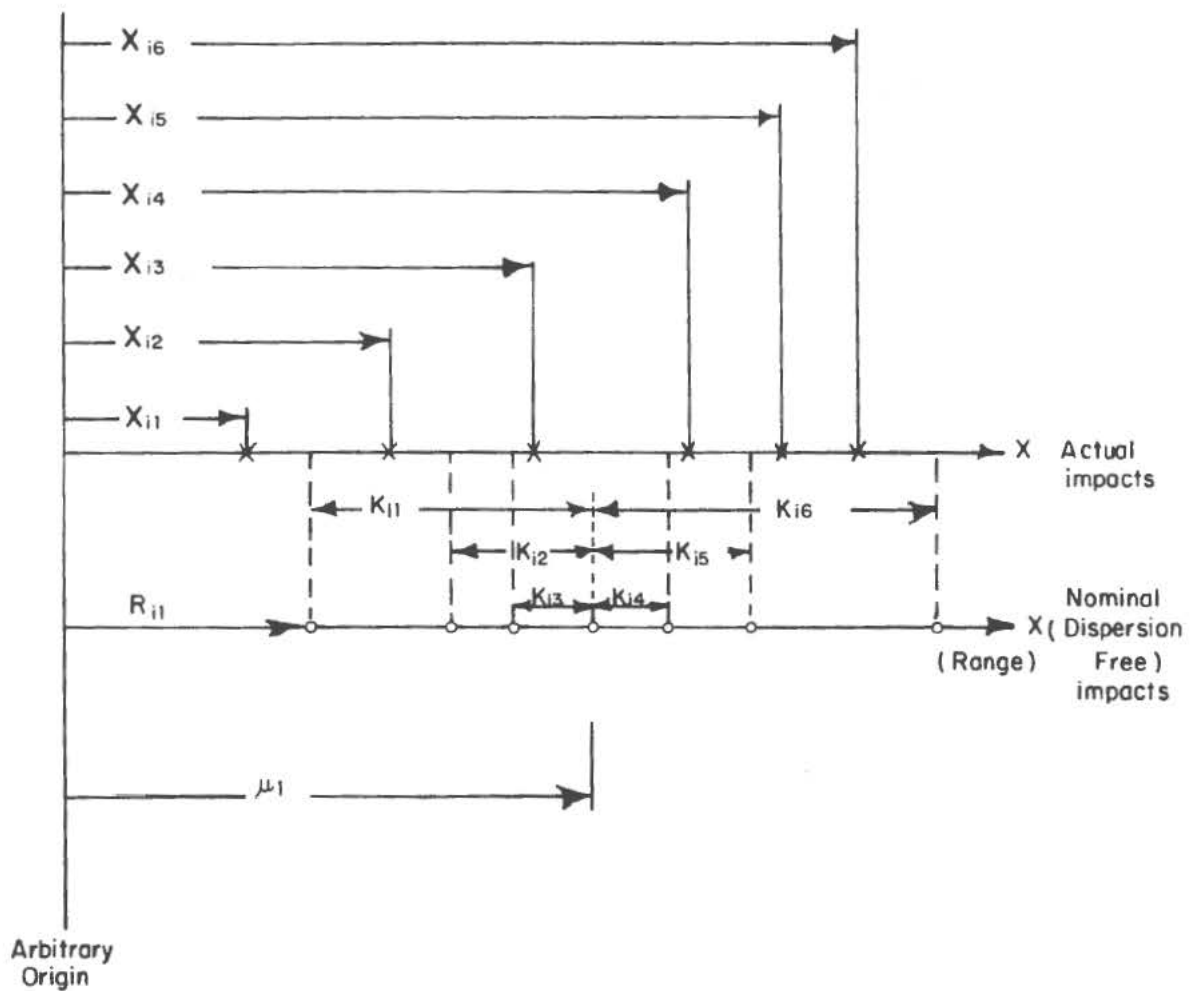
FIG. B-1: SCHEMATIC ILLUSTRATION OF RANGE COORDINATES (X_{ij}) OF
ACTUAL IMPACTS OF A TYPICAL STICK, NOMINAL (DISPERSION FREE)
IMPACTS(O'S) AND POPULATION MEAN AT THE EXPECTED CENTER OF THE
STICK, ALL MEASURED FROM AN ARBITRARY ORIGIN

$$S_{pooled}^2 = \frac{\sum_i (n_i - 1)\overline{S}_i^2}{\sum_i (n_i - 1)}$$

This is an estimate of σ_{BR}^2 with degrees of freedom equal to the total number of bombs dropped minus the number of passes. The assumption that the variances can be pooled is valid only if the variation in delivery conditions from run to run does not significantly change the corresponding population variance for bomb distributions. This requirement is met when the same delivery tactic is used for every run. (It could also be true when the delivery conditions vary from run to run if bomb dispersion is independent of delivery conditions.)

The pooled value for normal plane bomb dispersion in range can be calculated by the above method using $S_{BR(N)}$ instead of S_{BR} for $\overline{S}_i$. This procedure is valid even for different delivery conditions which produce different σ_{BR}, as long as slant range is nearly equal for all runs so that $\sigma_{BR(N)}$ remains constant.