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AN EQUIVALENT CIRCUIT FOR THE "WOBBLE-BOX"
TRANSDUCERS USED IN THE A.R. L.
TYPE 10 ARRAY

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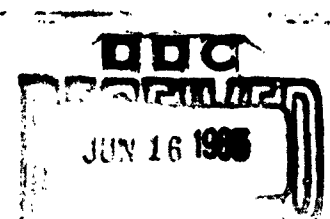
BY
A. S. MERRIWEATHER

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AN EQUIVALENT CIRCUIT FOR THE "WOBBLE-BOX"
TRANSDUCERS USED IN THE A.R.L. TYPE 10 ARRAY

by

A. S. Merriweather

ABSTRACT

The derivation of an equivalent circuit for the "wobble-box" transducers used in the A.R.L. Type 10 array is described, and from it are obtained expressions for the coefficients of the matrix relating input voltage and current to output force and velocity. These expressions will be used in the near future to derive the basic transducer input data required for a Pegasus Computer analysis of the mutual impedance effects in this array.

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20 pages
3 figures

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Glossary of Mathematical Symbols

E	=	electrical input voltage to transducer. (Volt)
I	=	electrical input current to transducer. (Ampere)
I_0	=	D.C. poling current to transducer. (Ampere)
r	=	electrical series resistive loss of coils (ohm)
C	=	tuning and D.C. blocking capacitance (Farad)
L_e	=	"clamped" inductance of coils (Henry)
B_0	=	D.C. Flux density (Weber/metre ²) = $N_{DC} I_0 \mu_0 / 2g$
N_{AC}	=	number of turns on A.C. coil windings
N_{DC}	=	number of turns on D.C. coil windings
μ_0	=	rational permeability of free space ($4\pi \times 10^{-7}$ Henry/metre)
g	=	gap width (metre)
ω	=	$2\pi f$ = pulsatace or angular frequency
ω_0	=	resonant pulsatace = $1/\sqrt{L_1 C_1}$
f	=	frequency (c/s)
V	=	electrical input voltage to transducer primary
G	=	electromechanical force factor (Newton/Ampere) = $2B_0 L_e / \mu_0 N_{AC}$
F_1	=	mechanical input force to transducer (Newton)
F	=	mechanical output force from transducer (Newton)
v_1	=	mechanical input velocity to transducer (metre/s)
v	=	mechanical output velocity from transducer (metre/s)
r_m	=	mechanical damping of transducer springs (kg/s)
S	=	transducer spring stiffness. (Newton/metre)
M_1	=	light or non-radiating mass of transducer (kg)
M_2	=	heavy radiating mass of transducer (kg)
M_s	=	total mass of springs (kg)
$\bar{M}$	=	transducer equivalent mass in air (kg)
ρ	=	density of sea water (kg/metre ³)

- c = velocity of sound in sea water (metre/s)
 A = area of transducer radiating face (metre²)
 a = radiation resistance ratio to full $\rho c A$ loading
 b = radiation reactance ratio to full $\rho c A$ loading
 Z_{11} = series form of acoustic radiation impedance (kg/s) = $\rho c A(a + jb)$
 Z_c = $r + 1/j\omega C + j\omega L_e$
 $1/Z$ = $r_m + S/j\omega + j\omega M_1$
 Q_e = electrical quality factor of transducer = $\omega_0 L_e / R_1$
 Q_m = mechanical quality factor of transducer = $R_1 / \omega_0 L_1 = \omega_0 R_1 C_1$
 R_1 = shunt resistance in electrical equivalent circuit (ohm)
 L_1 = shunt inductance in electrical equivalent circuit (Henry)
 C_1 = shunt capacitance in electrical equivalent circuit (Farad)
 α = transmission matrix coefficient relating input Volts to output force (Volt/Newton)
 β = transmission matrix coefficient relating input Volts to output velocity (Volt/(metre/s))
 γ = transmission matrix coefficient relating input Current to output force (Ampere/Newton)
 δ = transmission matrix coefficient relating input Current to output velocity (Ampere/(metre/s))

M.K.S. System of Units

- Voltage = V = [Volt] = [Newton.metre]/[Coulomb] = [F.L]/[I.T]
 Current = I = [Ampere]
 Charge = Q = [Coulomb] = [Ampere.second] = [I.T]
 Force = F = [Newton] = [kilogramme.metre]/[second²] = [M.L]/[T²]
 Length = L = [metre]
 Mass = M = [kilogramme]
 Time = T = [second]

INTRODUCTION

Type 10 is a bottom-laid, tripod-mounted, low-frequency, high power underwater acoustic projector array constructed for the A.R.L. Veronica Research Programme into low frequency, high power Active Sonar Systems. The original array consisted of 66 electromagnetic (variable-reluctance) "wobble-box" transducers arranged on a rotating framework 14 feet x 7 feet, giving a horizontal beam angle of 20° and a vertical beam angle of 40° , at the nominal operating frequency of 1 kc/s. The "wobble-box" transducers forming the Type 10 array are similar to the U.S. Naval Research Laboratory XEM-3B units but modified to A.R.L. designs by Stamp and Porteous [1 & 2].

Except for a break for emergency repairs from October 1961 to June 1962, the Type 10 array has been continuously in use at Perranporth, Cornwall for the Veronica Research Programme. However, trials carried out in Plymouth Sound in September 1961 and February 1962, prior to laying operations at Perranporth and reported by Field in [3], indicated certain anomalies in the operating conditions of the array, the most important of which were excessive amplitudes of motion and large phase differences in the region of resonance. These anomalies were assumed to be caused by mutual impedance effects and it was therefore decided to fix an input voltage limit to the whole array of 1.0 Kv R.M.S., which would result in amplitudes not exceeding the 2 mils peak to peak limit of the transducers. However, on the 14th October 1963 an accidental voltage overload of 1800 volts R.M.S. occurred which ultimately put the array completely out of action. It was therefore lifted so that the necessary repairs could be made, and on inspection it was found that one cable leading to a transducer unit had caught fire and damaged a number of adjacent cables and five transducers had been seriously damaged.

Although the five transducers which were damaged, appeared to be randomly distributed over the array, it was strongly suspected that their failure was caused by the mutual impedance effects which had been indicated by the earlier measurements. It was therefore decided to carry out a theoretical "post-mortem" on the Type 10 array using the Pegasus Computer Programme for acoustic array interactions developed by Group M, A.R.L. The computations will be started in the near future and it is hoped that they may indicate a significant correlation between the positions in the array of the units suffering the largest mutual impedance effects and the positions of the five transducers which failed due to the voltage overload.

In order to carry out the theoretical analysis, it is first necessary to use some form of electromechanical representation of the individual "wobble-box" transducers which form the Type 10 array. The most accurate representation, which is valid over a wide frequency band, is by means of the experimentally determined transmission matrix coefficients relating input voltage and current to output force and velocity. However, these coefficients are difficult to measure in practice and therefore for heavily mass-loaded, low-frequency transducers, the coefficients derived from a simple "lumped" equivalent circuit are usually taken as a first approximation. The purpose of this note is to describe the derivation of a simple "lumped" equivalent circuit for the "wobble-box" transducers and from the circuit to obtain expressions for the transmission matrix coefficients. The coefficients derived from the equivalent circuit will be used in the near future to obtain the basic transducer input data required for the Pegasus Computer analysis of the mutual impedance effects in the Type 10 array.

2. THE "WOBBLE-BOX" VARIABLE-RELUCTANCE TRANSDUCER

(a) The "lumped" equivalent circuit

The original 1 kc/s "wobble-box" transducer which was used as the basis for the design of the transducers used in the A.R.L. Type 10 array was developed by J. Chervenak of the U.S. Naval Research Laboratory.

The transducer is a two-mass, resonant, electromechanical device in which the non-radiating mass is spring-suspended at the centre of a box-like housing which is the radiating mass and moves in a piston-like fashion. Vibration is forced by periodic variation of the magnetic attraction across parallel air gaps in the magnetic circuits of two electromagnets which are electrically phased for push-pull operation. Radiation of acoustic energy takes place from one end of the box only; the other end is acoustically shielded with a layer of pressure-release rubber. A D.C. polarizing current flows through the two sets of coils and causes a strong magnetic force of attraction across both air gaps. A superposed alternating current flows through the coils in such a direction that the magnetic force is decreased across one air gap and increased across the other. This "push-pull" action causes the centre mass to move backwards while the fixed pole-pieces move forwards. During the next half cycle of A.C. the displacements are reversed. The rigidity of the masses and the disposition of the spring sections permit the box to vibrate as a "lumped-mass" (i.e. in a piston-like manner) but in opposite phase to the centre mass. Maximum vibration occurs when the alternating current is adjusted in frequency to the mechanical resonance. The distribution of vibrational energy between the heavy and light masses is inversely proportional to the mass ratio, which in this case is 2.3:1. Inasmuch as the D.C. source is connected across two points which are at the same A.C. potential, no chokes are required in the D.C. line. Blocking condensers, which are also used for tuning the transducer, prevent the D.C. from bypassing the coils. A full description of the design, construction and operation of the original "wobble-box" transducers is given by Chervenak [4].

The electromechanical equivalent circuit for the U.S. Naval Research Laboratory XEM-3B "wobble-box" transducers was originally derived by Dr. F. Firestone by using the Mobility Analogy. This analogy is frequently used for the representation of electromagnetic vibrating systems because it produces an equivalent circuit which is antireciprocal. All electromagnetic vibrating systems are antireciprocal and therefore their equivalent circuits must also have this property if they are to be completely accurate. The basic "lumped" equivalent circuit for the USNRL "wobble-box" transducers is given in Fig.1 and this will be used as the circuit which represents the transducers used in the Type 10 array.

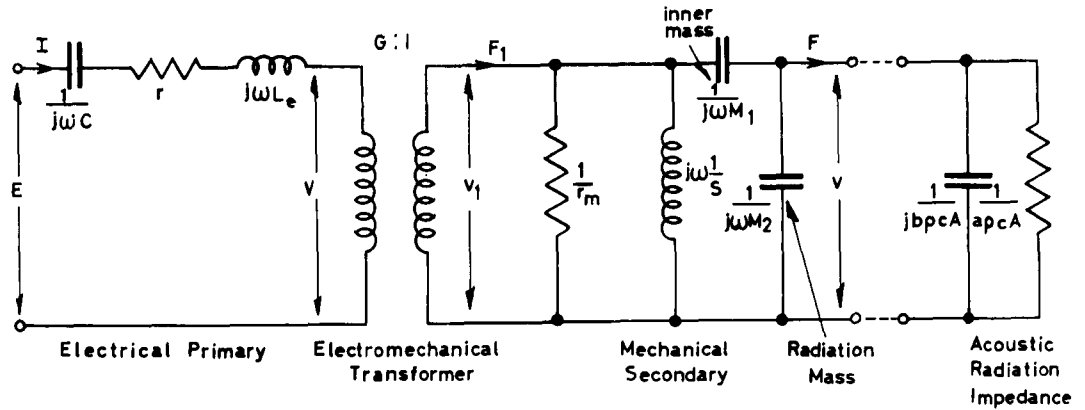


FIG.1 ELECTROMECHANICAL EQUIVALENT CIRCUIT FOR
A "WOBBLE-BOX" TRANSDUCER

The symbols used in the circuit shown in Fig.1 are defined in the Glossary of Mathematical Symbols which appears at the front of this note. It should be carefully noted that when the Mobility Analogy is used for the representation of electromagnetically coupled vibrating systems, the currents flowing in the meshes of the mechanical side of the system are taken as proportional to forces and the voltages as proportional to velocities. This is the inverse of the convention used for electrostatic and piezoelectric systems and automatically produces an equivalent circuit which is antireciprocal.

(b) Units and dimensions of the components of the equivalent circuit

(i) The force factor

It can be seen from Fig.1 that the "force factor" is simply equal to the step-down ratio $G:1$ of the ideal electromechanical transformer. A theoretical value for this factor can be derived by substituting values for the basic electrical parameters of the transducer in the following two equations:-

$$G = \frac{2B_o L_e}{\mu_o N_{AC}} \quad (1)$$

$$B_o = \frac{N_{DC} I_o \mu_o}{2g} \quad (2)$$

However, in the Type 10 "wobble-box" transducers, there is a common A.C. and D.C. winding and therefore $N_{AC} = N_{DC} = N$.

Consequently, the force factor G given by equation (1) reduces to:-

$$G = L_e I_o / g \quad (3)$$

On substituting the units and dimensions for L_e , I_o and g in the M.K.S. system in equation (3), it follows that the units and dimensions of the force factor G are given by:-

$$G = \frac{[V.T]}{[L]} = \frac{[F]}{[I]} = \frac{[\text{Newton}]}{[\text{Ampere}]} \quad (4)$$

Equation (4) indicates that the force factor, has the units Newton/Ampere and is therefore a measure of the mechanical force applied to the transducer per unit input current.

(ii) The damping component

The mechanical damping component r_m , of the transducer spring has the units M.K.S. mechanical ohms or kilogramme/second. Therefore, the damping component $1/r_m$ in the equivalent circuit shown in Fig.1 has the units second/kilogramme, and dimensions given by:-

$$\frac{1}{r_m} = \frac{[T]}{[M]}$$

The electrical shunt resistance R_1 , which is the equivalent of $1/r_m$, when the latter is reflected through the ideal transformer into the electrical side of the system is given by:-

$$R_1 = \frac{G^2}{r_m} = \frac{[F][L]}{[I^2][T]} = \frac{[V]}{[I]} \quad (5)$$

It can be seen from equation (5) that the units of G^2/r_m are Volt/Ampere and therefore this component is correctly interpreted as equivalent to an electrical resistance.

(iii) The spring stiffness component

The shunt inductive reactance $j\omega/S$ represents the mechanical reactance of the transducer spring stiffness. The stiffness S , has dimensions of force/length and therefore:-

$$\frac{j\omega}{S} = \frac{[L]}{[T][F]} = \frac{[L][T^2]}{[T][M][L]} = \frac{[T]}{[M]} \quad (6)$$

Equation (6) shows that the mechanical inductive reactance $j\omega/S$ has the dimensions of time/mass and is therefore measured in second/kilogramme.

The electrical shunt inductance L_1 , which has a reactance equivalent to $j\omega/S$ when the latter component is reflected into

the electrical side of the system is given by:-

$$L_1 = \frac{G^2}{S} = \frac{[F^2][L]}{[I^2][F]} = \frac{[F][L]}{[I^2]} = \frac{[V][T]}{[I]} \quad (7)$$

From equation (7) it follows that the units of G^2/S are volt.second/Ampere, which also has the dimensions of the Henry, and therefore this component is correctly interpreted as equivalent to an electrical shunt inductance.

(iv) The mass components

The capacitive reactances $1/j\omega M_1$ and $1/j\omega M_2$ represent the mechanical reactances of the transducer light and heavy masses and are therefore measured in second/kilogramme. However, when the transducer is driven in air, these two reactances can be combined into a single shunt capacitive reactance of $1/j\omega \bar{M}$. $\bar{M}$ is known as the "equivalent mass" of the transducer in air and is given by $\bar{M} = M_1 M_2 / (M_1 + M_2)$. It should be noted that $\bar{M}$ does not include a correction for the finite mass of the transducer springs.

The electrical shunt capacitance C_1 , which is the equivalent of $\bar{M}$ when the latter component is reflected through the ideal transformer into the electrical side of the system is given by:-

$$C_1 = \frac{\bar{M}}{G^2} = \frac{[M][I^2]}{[F^2]} = \frac{[Q^2]}{[F][L]} = \frac{[Q]}{[V]} \quad (8)$$

It can be seen from equation (8) that the units of $\bar{M}/G^2$ are Coulomb/volt, which also has the dimensions of the Farad, and therefore this component is correctly interpreted as equivalent to an electrical shunt capacitance.

3. THE TYPE 10 "WOBBLE-BOX" TRANSDUCER

(a) Experimental data

A full description of the Type 10 "wobble-box" transducer is given by Stamp in [1]. Each coil in the transducer has 160 turns and there are 6 coils in all (3 on each side of the centre mass). The 3 coils in each set are connected electrically in parallel and the A.C. and D.C. windings are common, and therefore have the same number of turns. The resultant inductance of the 3 parallel coils on each side of the centre mass is 38 mH, and the series combination of these two sets of coils has an inductance of 60 mH and an electrical Q of 65 at 1 kc/s.

The remaining data for the "wobble-box" transducer, which will be used in the derivation of the components of the "lumped" equivalent circuit are as follows:-

D.C. Poling Current $I_0 = 9$ Amps
 Average gap width $g = 0.01$ inch
 Mass of inner mass $M_1 = 11.265$ kg
 Mass of outer box $M_2 = 26.040$ kg
 Total mass of springs $M_3 = 2.192$ kg

Total mass of transducer
 $(M_1 + M_2 + M_3) = 39.497$ kg

Average mechanical quality factor Q_m (in air) = 24

Average shunt resistive loss R_1 (in air) = 120 ohm

Average resonant frequency f_0 (in air) = 945 c/s

(b) A.C. and D.C. connections and transducer "clamped" impedance

The electrical connections of the A.C. and D.C. lines, coils, and D.C. blocking condensers are as shown in Fig. 2:-

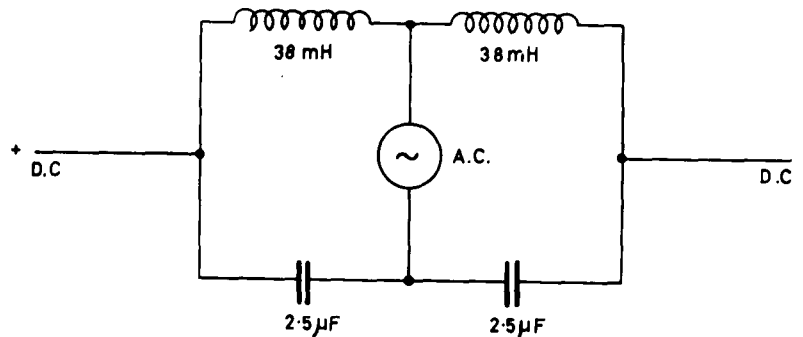


FIG. 2 A.C. AND D.C. CONNECTIONS TO "WOBBLE-BOX" TRANSDUCER

It can be seen from Fig. 2 that the two halves of the circuit are effectively in series with the A.C. source but are electrically in parallel with each other. Therefore, the "clamped" input impedance of the transducer consists of a capacitance of $5 \mu F$ in series with an inductance of 19 mH. This means that the inductance L_e in Fig. 1 is equal to 19 mH and the capacitance C to $5 \mu F$.

When the two transducer coils are connected in series they have a resultant inductance of 60 mH and a Q of 65 at 1 kc/s. Therefore, the series resistive loss of this 60 mH coil is given by:-

$$r = \frac{120\pi}{65} = 5.8 \text{ ohm.}$$

It follows that the series resistive loss of the effective "clamped" inductance of 19 mH is $5.8/4 = 1.45$ ohm and therefore the resistance r in Fig.1 is equal to 1.45 ohm.

(c) The "Equivalent Mass" in air

In order to take account of the mass of the transducer springs, their total mass M_3 is split in the ratio of the inner and outer masses M_1 and M_2 . Therefore, as $M_1/M_2 = 2.31$ and $M_3 = 2.192$ kg it follows that $2.192/3.310 = 0.662$ kg must be added to M_1 and $2.31 \times 0.662 = 1.53$ kg to M_2 . The effective mass of the inner, non-radiating mass of the transducer is therefore 12.795 kg and the heavy external radiating box 26.702 kg, and the "equivalent" mass $\bar{M}$ in air is given by:-

$$\bar{M} = \frac{26.702 \times 12.795}{39.497} = 8.65 \text{ kg.}$$

(d) The "lumped" electrical equivalent circuit in air

The electrical equivalent circuit in air which corresponds to the electromechanical equivalent circuit given in Fig.1, but with the acoustic radiation impedance and tuning condenser removed is shown in Fig.3:-

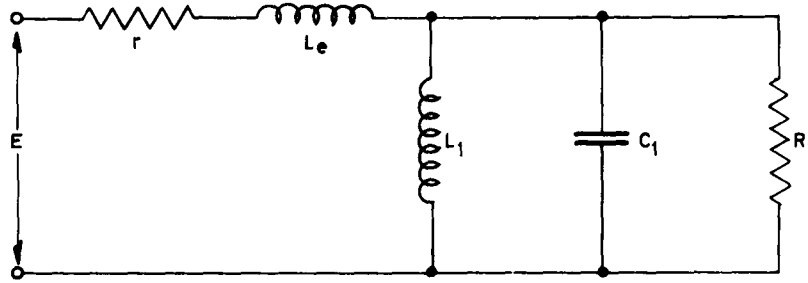


FIG. 3 ELECTRICAL "LUMPED" EQUIVALENT CIRCUIT FOR
"WOBBLE-BOX" TRANSDUCER IN AIR

The electrical Q factor Q_e , mechanical Q factor Q_m , and the resonant frequency f_o of the circuit shown in Fig.3 are given by:-

$$Q_e = \frac{\omega_o L_e}{R_1} \quad (9)$$

$$Q_m = \frac{R_1}{\omega_o L_1} = \omega_o R_1 C_1 \quad (10)$$

$$\omega_o^2 = \frac{1}{L_1 C_1} \quad (11)$$

Using the average experimental values $Q_m = 24$ (in air), $R_1 = 120$ ohm (in air) and $f_o = 945$ c/s (in air), it follows that C_1 and L_1 for the Type 10 "wobble-box" transducers are given by:-

$$C_1 = \frac{Q_m}{R_1 \omega_o} = \frac{24}{120 \times 2\pi \times 945} = 33.7 \mu F$$

$$L_1 = \frac{R_1}{Q_m \omega_o} = \frac{120}{24 \times 2\pi \times 945} = 0.842 \text{ mH}$$

The "equivalent mass" $\bar{M}$ in air, has been computed in paragraph 3(c) and is equal to 8.65 kg. Therefore, using this value and $C_1 = 33.7 \mu F$, the experimental value for the force factor G can be obtained by substituting in equation (8). Thus:-

$$G^2 = \frac{\bar{M}}{C_1} = \frac{8.65 \times 10^6}{33.7} = 0.2565 \times 10^6$$

$$\therefore G = 507 \text{ Newton/Ampere.}$$

Using the above values for G and R_1 , the mechanical damping r_m of the transducer spring in air can be found by substituting in equation (5). Therefore:-

$$r_m = \frac{G^2}{R_1} = \frac{507^2}{120} = 2140 \text{ kg/s.}$$

Similarly, the transducer spring stiffness S , can be found by using equation (7) and the above values for L_1 and G . Thus:-

$$S = \frac{G^2}{L_1} = \frac{507^2 \times 10^3}{0.842} = 0.305 \times 10^9 \text{ Newton/metre.}$$

If it is assumed that the electromechanical constants of the transducer do not change when it is operating in water, it follows that the values of the components of the equivalent circuit shown in Fig.1, which apply to the "wobble-box" transducers used in the Type 10 array are as follows:-

$$r = \text{electrical series resistive loss of coils} = 1.45 \text{ ohm}$$

$$L_o = \text{"clamped" inductance of coils} = 19 \text{ mH}$$

$$C = \text{tuning and D.C. blocking capacitance} = 5 \mu F$$

$$G = \text{electromechanical force factor} = 507 \text{ Newton/Ampere}$$

$$\begin{aligned}
r_m &= \text{mechanical damping of transducer springs} = 2140 \text{ kg/S} \\
S &= \text{transducer spring stiffness} = 0.305 \times 10^9 \text{ Newton/metre} \\
M_1 &= \text{light or non-radiating mass of transducer} = 12.795 \text{ kg} \\
M_2 &= \text{heavy radiating mass of transducer} = 26.702 \text{ kg}
\end{aligned}$$

4. THE TRANSMISSION MATRIX COEFFICIENT REPRESENTATION OF THE "WOBBLE-BOX" TRANSDUCER

The equivalent circuit shown in Fig.1 is a rather cumbersome form of representation for Digital Computer mutual impedance computations, and it is more useful to represent the transducer as a "black-box" with a transfer function expressed in terms of the four transmission matrix coefficients relating input voltage and current to output force and velocity. These four coefficients are usually designated as α , β , γ and δ and the transducer can therefore be represented by the two equations:-

$$E = \alpha F + \beta v$$

$$I = \gamma F + \delta v$$

These equations are often expressed in matrix form as:-

$$\begin{bmatrix} E \\ I \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} F \\ v \end{bmatrix}$$

If the transducer is reciprocal, the determinant ($\alpha\delta - \beta\gamma$) = +1 but if it is antireciprocal ($\alpha\delta - \beta\gamma$) = -1. The equivalent circuit shown in Fig.1, (excluding the acoustic radiation impedance) can easily be expressed in terms of these four coefficients and it can be shown that for this particular circuit α , β , γ and δ are given by:-

$$\alpha = \frac{\{G + Z_c/ZG\}}{j\omega M_1}$$

$$\beta = \frac{(M_1 + M_2)}{M_1} \left\{ G + \frac{Z_c}{ZG} \right\} - \frac{j\omega M_1 Z_c}{G}$$

$$\gamma = \frac{1}{j\omega M_1 ZG}$$

$$\delta = \frac{1}{G} \left\{ \frac{(M_1 + M_2)}{M_1} \cdot \frac{1}{Z} - j\omega M_1 \right\}$$

where:-

$$Z_c = r + 1/j\omega C + j\omega L_e$$

$$1/Z = r_m + S/j\omega + j\omega M_1$$

From the above expressions for α , β , γ and δ , it can be proved that the determinant ($\alpha\delta - \beta\gamma$) is equal to -1, and therefore the equivalent circuit in Fig.1 represents an anti-reciprocal transducer. This is

correct because it has already been pointed out that electromagnetic transducers are always anti-reciprocal, and this must therefore be true for the above circuit because the Mobility Analogy has been used in the derivation.

A separate Auto-Code Programme has been prepared for the computation of the α , β , γ , δ matrix coefficients. The four coefficients will be determined using the above equations and the basic constants of the "wobble-box" transducer tabulated in para. 3(d). The frequency will be varied in 5 c/s steps from 900 c/s to 1020 c/s using this programme and the results obtained will be used as basic transducer input data for the main Pegasus Computer programme of mutual impedance effects in the Type 10 array. If the interaction effects are large, it may be necessary to increase the bandwidth over which the array computations are made.

5. CONCLUSIONS

An electromechanical equivalent circuit for the "wobble-box" transducers used in the A.R.L. Type 10 array has been derived and from the circuit, expressions for the α , β , γ and δ transmission matrix coefficients have been obtained. From these expressions α , β , γ and δ will be computed in 5 c/s steps from 900 c/s to 1020 c/s, and the coefficients obtained will then be used as the basic transducer input data for the Pegasus Computer programme of array interaction effects in the Type 10 array. It is hoped that the results of this theoretical analysis will indicate a significant correlation between the positions in the array of the transducers suffering the largest mutual impedance effects and the positions of the five "wobble-boxes" which failed due to the accidental voltage overload which occurred on 14th October, 1963.

6. ACKNOWLEDGEMENTS

The author wishes to express his thanks to W. R. Stamp and N. H. Field for the information they supplied on the design and performance of the Type 10 array and to Dr. P. H. G. Crane for useful discussions on transducer equivalent circuits.

A. S. Merriweather (P.S.O.)

ASM/JER

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