63-3-1

FTD-TT- 62-1768

TRANSLATION

EMPLOYING THE KARMAN METHOD FOR CALCULATING A TURBULENT BOUNDARY LAYER ON A PLATE IN A GAS FLOW

By

L. G. Loytsanskiy and Yu. V. Lapin

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE
OHIO

FTD-TT- 62-1768/1+2

UNEDITED ROUGH DRAFT TRANSLATION

EMPLOYING THE KARMAN METHOD FOR CALCULATING A TURBULENT BOUNDARY LAYER ON A PLATE IN A GAS FLOW

BY: L. G. Loytsanskiy and Yu. V. Lapin

English Pages: 13

SOURCE: Russian Book, "Tekhnicheskaya Gidromekhanika", Politikhnicheskiy Institute, Nr. 217, 1961, pp 7-16

S/563-61-0-217

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.	PREPARED BY: TRANSLATION SERVICES BRANCH FOREIGN TECHNOLOGY DIVISION WP-AFB, OHIO.

Employing the Karman Method for Calculating a Turbulent Boundary Layer on a Plate in a Gas Flow

by

L. G. Loytsanskiy; Yu. V. Lapin

Introduction

In the theory of turbulent boundary layer in a compressible fluid at greater velocities two historical trends have been noticed. The first one of these, discovered by F.I.Frankl and V.V.Voyshel [1] was based on direct transfer of the semiempirical theory by Karman into gas dynamic formulae. Van Drast [2] followed an analogous path, but with the application of Prandtl formulae.

In reports of the second trend were used known Dorodnitsyn [3] variables and it has been assumed, that the corresponding formulas of the semiempirical theory of turbulences should be compiled in these variables [4].

The answer to the question on which trend offers results more closer to the truth, could be given only by experiments at greater M-numbers, but by the time of appearance of both trends experimental data were still lacking. At M-numbers, not much exceeding one, both trends have given, naturally, results slightly differing from each other.

Only relatively recently, after data have been obtained on the friction resistance of flat surfaces at greater M-numbers (reaching up to 8 , it became clear, that only the first trend offers proper results.

We would like to point out, that in the Frankl/Voyshel report the authors stood along the road of directly generalizing the Karman method, having simplified only with the assumption of friction stress constancy across the boundary layer. Following this path, they at first find the form of the profile of velocity in the cross section of the layer, then in an ordinary way they find the so-called law of "Resistivity", i.e. relationship between local coefficient of friction and R-number of the boundary layer (R-number = Reynolds or Re-number). Then, excluding this Re-number from equation of the "law of resistivity" and from equation of pulses, they obtain the sought for relationship between the coefficient of resistance and the Re-number, formulated by the velocity of an oncoming flow and the abscissa of given point on the plate.

The degree of approximation, adopted by Frankl and Voyshel, enabled them to personally carry out calculations but only up to M-numbers, slightly exceeding one. The change over to greater M-numbers requires, apparently, either an even greater complexity and besides that a complex calculating method by these authors, or direct application of numerical methods.

It is shown in this report that when applying the Karman method and assuming friction stress constancy across the boundary layer, there is a simpler way of formulating a solution, requiring no introduction of the concept concerning the Reynolds number of the boundary layer and compilation of the "law of resistivity". This method considerably simplifies the investigation of problems pertaining to a turbulent boundary layer in a gas flow. The use of a simple asymptotic break down allows comparatively easily to generalize the Karman theory of plate resistance in an incompressible liquid for the case of a gas flow with greater M-numbers.

1. Turbulent Friction on a Plate in a Flow of Incompressible Liquid.

The integral equation of pulses for a plate, around which an incompressible liquid flows at zero angle of attack, has the form of :

$$\frac{d\delta^{**}}{dx} = \frac{\tau_\omega}{\varrho_\infty U_\infty^2}. \qquad (1.1)$$

where x -coordinate, directed along the surface of the plate; U_∞ and ϱ_∞ - velocity and density of the oncoming flow; τ_ω - friction stress on the wall; δ^{**} - thickness of pulse loss, expressable by equality:

$$\delta^{**} = \int_0^\infty \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy. \quad (1.2)$$

In expression (1.2) we substitute the universal coordinates:

$$\varphi = \frac{u}{v_*}, \quad \eta = \frac{v_* y}{\nu}; \quad (1.3)$$

here....y -coordinate along the normal to the surface of the plate ; coefficient of kinematic viscosity; $v_* = \sqrt{\frac{\tau_\omega}{\varrho}}$ - dynamic velocity.

Then

$$\delta^{**} = \frac{\nu h}{U_\infty}\int_0^\infty \frac{\varphi}{h}\left(1 - \frac{\varphi}{h}\right) d\eta, \quad (1.4)$$

where

$$h = \frac{U_\infty}{v_*}. \quad (1.5)$$

Equality (1.4) can also be written in form of :

$$\delta^{**} = \frac{\nu h^2}{U_\infty}\int_0^h \frac{\varphi}{h}\left(1 - \frac{\varphi}{h}\right) \dot{\eta}\, d\left(\frac{\varphi}{h}\right). \quad (1.6)$$

The value $\dot{\eta} = \frac{d\eta}{d\varphi}$ in the laminary sublayer is equal to one ($\dot{\eta} = 1$). In the turbulent nucleus to determine $\dot{\eta}$ we use the Karman ratio:

$$\tau = \varrho \varkappa^2 \frac{u'^4}{u''^2}, \quad (1.7)$$

where x - turbulence constant, equalling 0.4 (the hatchure sign in the equality (1.7) designates a derivative by y). Adopting a simplified assumption about the friction stress constancy across layer $\tau = \tau_\omega$ and transition in (1.7) toward universal coordinates (1.3) we will obtain:

$$\frac{\varphi'^2}{\varphi''} = -\varkappa. \quad (1.8)$$

The minus sign appears after ext action of the root in connection with the fact, that $\varphi'' < 0$. In equation (1.8) we change the argument and function places, then we obtain an equation:

$$\frac{\ddot{\eta}}{\dot{\eta}} = \varkappa, \quad (1.9)$$

from which it is evident that

$$\dot{\eta} = Ce^{\varkappa\varphi}. \quad (1.10)$$

If we should designate by the letter f the magnitude of the derivative $\frac{d\varphi}{d\eta}$ on the boundary of the laminary sublayer $\eta = \eta_a = a$ from the side of the turbulent nucleus, and writing

$$f = \left(\frac{d\varphi}{d\eta}\right)_{\eta=\eta_a-0} = \varphi'(a+0) = \frac{1}{\varkappa a}. \qquad (1.11)$$

then equation (1.10) will acquire the form of

$$\eta = \frac{e^{-\varkappa a}}{f} e^{\varkappa\varphi}, \qquad (1.12)$$

where a - turbulence constant, equalling 11.5.

Returning to equation (1.6) we want to point out, that, strictly speaking, the integral in the right part of this equation should have been broken down into two sections: $0 \leq \varphi \leq a$ and $a \leq \varphi \leq h$ and place in each one of these a value η, namely $\eta = 1$ in the first and according to equation (1.12) - in the second. But in view of the relatively thin (relative thinness) of the laminary sublayer it is possible to bypass the first section, extending the second one to the wall. As is shown by calculations in first approximation, at greater h no noticeable difference is obtained.

In this way, eliminating η from equation (1.6) through (1.12) we will obtain the following expression for pulse loss thickness:

$$\delta^{**} = \frac{\nu h e^{-\varkappa a}}{f U_\infty} \int_0^1 \bar{u}(1-\bar{u}) e^{\varkappa h \bar{u}} d\bar{u}, \qquad (1.13)$$

where $\bar{u} = \frac{\varphi}{h}$.

The value $\varkappa h$, as is evident from its determination, considerably exceeds one. In these cases it is convenient to use the following general idea of the integral (containing the exponential function), which is obtained as result of integrating by parts:

$$\int e^{\varkappa h\psi} f(\psi) d\psi = \frac{e^{\varkappa h\psi}}{\varkappa h}\left[f(\psi) - \frac{f'(\psi)}{\varkappa h} + \frac{f''(\psi)}{\varkappa^2 h^2} - \cdots\right]. \qquad (1.14)$$

In this case, when the series in square parentheses remains undisrupted, formula (1.14) can be considered as an asymptotic series, expressing the value of the integral at greater h.

Making in (1.13) an integration with the aid of (1.14) we will obtain an expression for pulse loss thickness:

$$\delta^{**} = \frac{e^{-\varkappa a}\nu}{f\varkappa^2 U_\infty} e^{\varkappa h}\left(1 - \frac{2}{\varkappa h}\right). \qquad (1.15)$$

We will also obtain an expression for the form-parameter H;

$$H = \frac{\delta^*}{\delta^{**}}. \quad (1.16)$$

where δ^* - thickness of squeezing out the boundary layer, expressed by equation

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy. \quad (1.17)$$

Using a method analogous to the one shown above, it is possible to find the following expression for the thickness of squeeze out:

$$\delta^* = \frac{e^{-\varkappa a} \nu}{f \varkappa \cdot U_\infty} e^{\varkappa h}. \quad (1.18)$$

Compiling the ratio δ^*/δ^{**} according to equations (1.18) and (1.15) we will obtain a term for H:

$$H = \frac{1}{1 - \frac{2}{\varkappa h}}. \quad (1.19)$$

which at Re numbers $10^6 - 10^7$ gives a value H = 1.3, well conforming with the experiment.

Turning to expression (1.15), we want to mention, that in view of the exponential dependence of pulse loss thickness upon h, when it is being calculated it is possible to disregard the second component in comparison with the first one. As was shown by calculations, at greater values h the error, obtainable when calculating the friction coefficient, is insignificant at such an approximation.

Having accepted for pulse loss thickness such an approximation, we substitute its expression in the integral formula of pulses (1.1). After simple transform we will obtain the following equation:

$$\frac{e^{-\varkappa a}}{f \varkappa^2} h^2 d e^{\varkappa h} = dR_x, \quad 1.20$$

where

$$R_x = \frac{U_\infty x}{\nu}. \quad (1.20a)$$

Taking the integral from both parts of this equation and utilizing the boundary condition $R_x = 0$ at h = 0, we will have in this approximation

$$\frac{e^{-\varkappa a}}{f \varkappa^2} h^2 e^{\varkappa h} = R_x. \quad (1.21)$$

Logarithming (1.21) and turning away from variable h to c_f according to equation

$$h = \sqrt{\frac{2}{c_f}}. \quad (1.22)$$

we will obtain the known Karman formula

$$\frac{1}{\sqrt{c_f}} = A + B\lg(R_x c_f). \quad (1.23)$$

The just now explained order of deriving the Karman formula is not in itself important, but only as an explanation to a more complex derivation of Karman formula generalization for the case of a gas flow.

2. Turbulent boundary layer in the flow of a compressible gas

We will confine ourselves to the case when the Prandtl number equals one (Pr=1) and will discuss the flow around a plate, as thermally insulated ("lamellar thermometer") and with discharge of heat as well.

We will assume, that in any given section of the boundary layer is valid the following known ratio between the absolute temperature of the gas T and its longitudinal velocity u

$$\frac{T}{T_w} = 1 - \omega\left(\frac{\varphi}{h}\right) - \gamma\left(\frac{\varphi}{h}\right)^2,$$

where

$\omega = 1 - \frac{T_t}{T_w}$ ($\omega > 0$ when the surface is heated and $\omega < 0$ when cooled); gamma $= \frac{k-1}{2} M_\infty^2 \frac{T_\infty}{T_w}$;

$T_t = T_\infty(1 + \frac{k-1}{2} M_\infty^2)$ - temperature of the thermometer, equalling the temperature of plate surface T_w in the absence of heat transfer ($\omega = 0$).

Noting, that at a pressure constancy over the profile of the layer takes place an equality

$$\frac{\varrho}{\varrho_w} = \frac{T_w}{T} = \frac{1}{1 - \omega\left(\frac{\varphi}{h}\right) - \gamma\left(\frac{\varphi}{h}\right)^2}, \quad (2.2)$$

we will compile analogous with (1.8) the basic Karman formula, which in the presence of compressibility of the medium will have the form of:

$$\frac{\varphi''}{\varphi'^2} = -\frac{\varkappa}{\sqrt{1 - \omega\left(\frac{\varphi}{h}\right) - \gamma\left(\frac{\varphi}{h}\right)^2}} \quad (2.3)$$

Substituting, as before in par.1. the argument and function places, we will obtain instead of (2.3) an equation

$$\frac{\ddot{\eta}}{\dot{\eta}} = \frac{\varkappa}{\sqrt{1 - \omega\left(\frac{\varphi}{h}\right) - \gamma\left(\frac{\varphi}{h}\right)^2}}, \quad (2.4)$$

as first integral in which is served by

$$\eta = C \exp\left(\frac{\varkappa h}{\sqrt{\gamma}} \arcsin \frac{\sqrt{\gamma}\frac{\varphi}{h} - \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}}\right). \quad (2.5)$$

To determine the constants C it is necessary to deal with the value of the derivative $\frac{d\varphi}{d\eta}$ on the boundary of the laminary sublryer from the side of the turbulent nucleus. A simpler assumption appears to be the requirement, that this value should be of the very same magnitude $f = \frac{1}{\varkappa a}$, as in the case of low velocities; in other words, it is assumed, that φ (a+0) does not depend upon the gamma and omega coefficients. Such an assumption is equivalent to disregarding a change in density along the length of the laminary sublayer; it leads to the following result:

$$\eta = \frac{1}{f} \exp\left[\frac{\varkappa h}{\sqrt{\gamma}}\left(\arcsin \frac{\sqrt{\gamma}\,\bar{u} - \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}} - \arcsin \frac{\sqrt{\gamma}\,\bar{u} + \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}}\right)\right]. \quad (2.6)$$

One can be convinced easily, that at omega = gamma= 0 the right part of equation (2.6) coincides with the corresponding equation (1.12) for incompressible fluid.

After this we will take up the thickness of pulse loss of the boundary layer. In case of a compressible gas this value is expressed by equation:

$$\delta^{**} = \int_0^{\infty} \frac{\rho u}{\rho_\infty U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy. \quad (2.7)$$

Using equations (2.2) and (2.6) and making transforms, analogous as in previous paragraph, we will obtain the following expression for the thickness of pulse loss:

$$\delta^{**} = \frac{(1-\omega-\gamma)\,h^2 \nu_w}{f U_\infty} \int_0^1 \frac{\bar{u}(1-\bar{u})}{1-\omega\bar{u}-\gamma\bar{u}^2} \times$$

$$\times \exp\left[\frac{\varkappa h}{\sqrt{\gamma}}\left(\arcsin \frac{\sqrt{\gamma}\,\bar{u} - \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}} - \arcsin \frac{\sqrt{\gamma}\,\bar{a} + \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}}\right)\right]. \quad (2.8)$$

Introducing a new variable

$$\psi = \frac{1}{\sqrt{\gamma}} \arcsin \frac{\sqrt{\gamma}\,\bar{u} + \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}}. \quad (2.9)$$

we transform (2.8) into form of

$$\delta^{**} = \frac{(1-\omega-\gamma)\nu_\infty h^2 \sqrt{1-\frac{\omega^2}{4\gamma}}}{U_\infty \gamma} \int_{\psi_a}^{\psi_\infty} \frac{\exp[\varkappa h(\psi-\psi_w)]}{\cos\sqrt{\gamma}\psi} \times (\sin\sqrt{\gamma}\psi - \sin\sqrt{\gamma}\psi_w)(\sin\sqrt{\gamma}\psi_\infty - \sin\sqrt{\gamma}\psi)\,d\psi, \qquad (2.10)$$

where

$$\psi_\infty = \frac{1}{\sqrt{\gamma}} \arcsin \frac{\sqrt{\gamma} + \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1-\frac{\omega^2}{4\gamma}}};$$

$$\psi_w = \frac{1}{\sqrt{\gamma}} \arcsin \frac{\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}}; \qquad (2.11)$$

$$\psi_a = \frac{1}{\sqrt{\gamma}} \arcsin \frac{\sqrt{\gamma}\,\bar{a} + \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}}.$$

To calculate the determined integral, situated in the right side of equation (2.10), we will again assume that the method of symtotic analysis, the same as in the previous paragraph. Intergrating with an accuracy to the member, containing $\varkappa^3 h^3$ in the denominator and substituting approximation ψ_a in form of

$$\psi_a = \psi_w + \frac{a}{h}. \qquad 2.12$$

we will obtain the following expression for pulse thickness loss

$$\delta^{**} = \frac{e^{-\varkappa a}(1-\omega-\gamma)\nu_\infty}{\gamma\varkappa^2 U_\infty} e^{\varkappa h(\psi_\infty-\psi_w)}\left(1 - \frac{2-1.5\omega-\gamma}{\varkappa h\sqrt{1-\omega-\gamma}}\right). \qquad (2.13)$$

In an analogous way it is possible to obtain the expression for the thickness of displacement

$$\delta^{*} = \frac{e^{-\varkappa a}(1+\gamma)\nu_\infty}{\gamma\varkappa^2 U_\infty} e^{\varkappa h(\psi_\infty-\psi_w)}\left[1 - \frac{\gamma\left(3-1.5\omega-\gamma-\frac{\omega}{2\gamma}\right)}{\varkappa h(1-\gamma)\sqrt{1-\omega-\gamma}}\right]. \qquad (2.14)$$

Compiling, according to equations (2.14) and (2.13), the ratio $\delta^*/\delta^{**} = H$, we will find:

$$H = \frac{(1+\gamma)\left[1-\frac{\gamma\left(3-1.5\omega-\gamma-\frac{\omega}{2\gamma}\right)}{\varkappa h(1-\gamma)\sqrt{1-\omega-\gamma}}\right]}{(1-\omega-\gamma)\left[1-\frac{2-1.5\omega-\gamma}{\varkappa h\sqrt{1-\omega-\gamma}}\right]}. \qquad (1.15)$$

In a specific case of flow of an incompressible fluid (gamma=0) and in the absence of heat transfer (omega = 0) we obtain a term for H, perfectly coinciding with the corresponding equation of the previous paragraph (1.19).

Having assumed, as in previous paragraph, for pluse loss thickness a more cruder approximation (disregarding the second component in eq. (2.13) we will substitute this equation

$$\delta^{**} = \frac{e^{-\varkappa a}\nu_{\omega}(1-\omega-\gamma)}{j\varkappa^2 U_{\infty}} e^{\varkappa h(\nu_{\infty}-\nu_{\omega})}$$

into an integral equation of pulses, coinciding in form with (1.1). After simple transforms we will obtain an equation:

$$\frac{e^{-\varkappa a}}{j\varkappa^2} h^2 d e^{\varkappa h(\nu_{\infty}-\nu_{\omega})} = dR_{x\omega}. \quad (2.17)$$

where

$$R_{x\omega} = \frac{U_{\infty}x}{\nu_{\omega}}. \quad (2.17a)$$

Having taken the integral from both parts of this equation and using boundary condition $R_{x\omega} = 0$ at $h = 0$, we will obtain in this approximation

$$\frac{e^{-\varkappa a}}{j\varkappa^2} h^2 e^{\varkappa h(\nu_{\infty}-\nu_{\omega})} = R_{x\omega}. \quad (2.18)$$

Equation (2.18) transforms at the limit into a corresponding expression for incompressible fluid

$$\frac{e^{-\varkappa a}}{j\varkappa^2} h_0 e^{\varkappa h_0} = R_{x0}. \quad (2.19)$$

Next we will separate both parts of equation (2.18) into conformal two parts in equation (219), and then we will have

$$\left(\frac{h}{h_0}\right)^2 e^{\varkappa h(\nu_{\infty}-\nu_{\omega})-\varkappa h_0} = \frac{\varrho_{\omega}\mu_{\infty}}{\varrho_{\infty}\mu_{\omega}}. \quad (2.20)$$

Changing over in (2.20) from h to c_f and logarithming, we will obtain:

$$-\lg\frac{c_{f0}}{c_f} + \frac{\varkappa\sqrt{2}\lg e}{c_{f0}}\left[\frac{(\psi_{\infty}-\psi_{\omega})\sqrt{1-\omega-\gamma}}{\sqrt{\frac{c_f}{c_{f0}}}} - 1\right] = \lg\left(\frac{\mu_{\infty}}{\mu_{\omega}}\right). \quad (2.21)$$

Accepting for c_{f_0} a gradual dependence upon the number R_{x0}

$$c_{f0} = 0{,}0263 R_{x0}^{-1/7} \quad (2.22)$$

and introducing designations

$$\left.\begin{aligned} F &= \frac{0.25}{\sqrt{c_{f0}}} = 1{,}55 R_{x0}^{1/14}; \\ G &= \lg\left(\frac{\mu_{\infty}}{\mu_{\omega}}\right); \\ K &= \sqrt{\frac{1-\omega-\gamma}{\gamma}}\left(\arcsin\frac{\sqrt{\gamma}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}} - \arcsin\frac{\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}}\right). \end{aligned}\right\} \quad (2.23)$$

we will obtain a dependence of the friction coefficient upon the parameters of the uncoming flow and conditions on the wall in the following unclear form:

$$-\lg\left(\frac{c_f}{c_{f0}}\right)+FK\sqrt{\frac{c_{f0}}{c_f}}=F+G. \qquad (2.24)$$

Equation (2.24) can be reduced to a transcedental equation with one parameter in form of :

$$\lg N+N=L, \qquad (2.25)$$

where

$$\left.\begin{aligned} L&=\lg\frac{FK}{2}+\frac{F+G}{2};\\ N&=\frac{FK}{2\sqrt{\frac{c_f}{c_{f0}}}}.\end{aligned}\right\} \qquad (2.26)$$

In this way, the expression for the friction coefficient can be written:

$$\frac{c_f}{c_{f0}}=\frac{F^2K^2}{4N^2}. \qquad (2.27)$$

where F an K - known functions, and N is determined from solution of equation (2.25).

As was shown by calculations, equation (2.25) can be replaced by an approximated one, by a simpler equation:

$$N=0{,}123+0{,}820\,L, \qquad (2.28)$$

which appears to be sufficiently accurate in a wide range of changes of all parameters. In this case equation (2.27) acquires the form of:

$$\frac{c_f}{c_{f0}}=\frac{F^2K^2}{4\left[0{,}123+0{,}820\left(\lg\frac{FK}{2}+\frac{F+G}{2}\right)\right]^2}. \qquad (2.29)$$

To determine value G it is necessary to accept a law of change in viscosity due to temperature. In the role of such a law can be accepted either the Sezerlend formula, or the gradual viscosity dependence upon temperature. In the latter case the expression for G has the form of :

$$G=n\lg\left(\frac{T_\infty}{T_w}\right). \qquad (2.30)$$

where n- index of the degree in the $\mu\sim T^n$ law. It would be interesting to compare the described method with the Van-Drayst method, which, as is known, is based on the transfer of the Pranitl formula into gas dynamics. After the Van-Drayst formula is brought to form (2.24) it becomes explained, that the difference lies in the value G, which during the adoption of the gradual law of change in viscosity is expressed by equation:

$$G = \left(n + \frac{1}{2}\right) \lg \left(\frac{T_\infty}{T_w}\right). \quad 2.30a$$

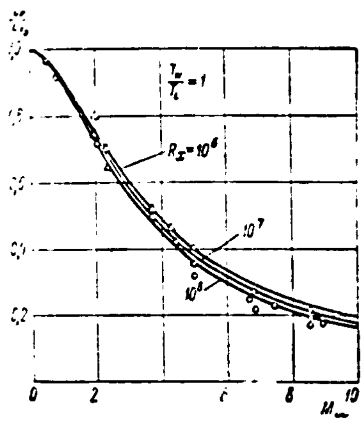

Fig.1.

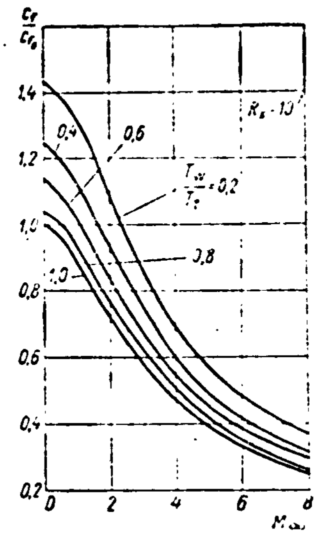

Fig.2

Upon the examination of equation (2.29) it is revealed that at greater values of Re-number the magnitude of the ratio c_f/c_{f_0} depends very weakly upon the Re-number. The truth is, directing the Re-number into infinity ($R_x{}^{o} \to \infty$) or F toward infinity, we obtain the following specific formula for the ratio c_f/c_{f_0}:

$$\frac{c_f}{c_{f_0}} = K^2. \quad (2.30b)$$

This formula may appear to be useful when making approximate calculations of friction resistance at greater Re-numbers.

According to the described method (formula (2.29((the local friction coefficient was calculated. The results of calculation for the case of absence of heat transfer ($T_w/T_t = 1$) and comparison of same with experimental data of different authors [5]-[8] are shown in fig.1. Three curves on this drawing correspond to three different values of Re-number (10^6, 10^7, 10^8).

As is evident from the drawing, the calculated and experimental data are in excellent conformity all the way up to greater M numbers.

The effect of the temperature factor on the friction coefficient is shown in fig.2. We shall examine two available practical values of partial cases.

1) Plate Heat Insulated

In this case the temperature of the wall T_w is equal to the temperature of deceleration:

$$T_w = T_\infty\left(1 + \frac{k-1}{2}M_\infty^2\right). \quad (2.31)$$

and the values omega and gamma res ectively are equal to

$$\omega = 0, \gamma = \frac{\frac{k-1}{2}M_\infty^2}{1+\frac{k-1}{2}M_\infty^2}. \quad (2.32)$$

Function K determining the process will be:

$$K = \sqrt{\frac{1-\gamma}{\gamma}}\arcsin\sqrt{\gamma}. \quad (2.33)$$

2) Plate with heating or cooling at small velocities

In this case, guiding gamma $\to$ 0, we will have:

$$K = \frac{2(\sqrt{1-\omega} + \omega - 1)}{\omega}. \quad (2.34)$$

In the special case of flow of an incompressible fluid in the absence of heat transfer (omega = gamma = 0) the value K = 1.

Literature

1. Frankl', F.I., Voyshel', V.V; Transactions of TSAGI, No. 321, 1937
2. Van-Drayst Ye.K. Mechanics Foreign Literature, collection of translations No.1. 1952
3. Dorodnitsyn, A.A. PMM, vol. IX, 1945, vol. X. 1946
4. Kalikhman, L. E. PMM T. IKH, 1945, T. KH, 1946
5. Chapman D. and Kester R., JAS, N 7, vol. 20, 1953
6. Coles D., JAS, No 10, 1952, vol. 19
7. Hill F. K., JAS, No 1, 1953, vol. 20
8. Lobb R. K., Winkler E. M., Persch J., JAS, No 1, 1955, vol. 22

DISTRIBUTION LIST

DEPARTMENT OF DEFENSE	Nr. Copies
HEADQUARTERS USAF	
AFCIN-3D2	1
ARL (ARB)	1
OTHER AGENCIES	
CIA	1
NSA	6
DIA	9
AID	2
OTS	2
AEC	2
PWS	1
NASA	1
ARMY	3
NAVY	3
RAND	1

MAJOR AIR COMMANDS	Nr. Copies
AFSC	
SCFDD	1
ASTIA	25
TDBTL	5
TDBDP	5
AEDC (AEY)	1
SSD (SSF)	2
AFFTC (FTY)	1
AFSWC (SWF)	1