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TRANSLATION

GYROSCOPE DRIFT CAUSED BY FRICTION OF THE GIMBAL BEARINGS
AND REDUCTION OF THIS DRIFT BY THE USE OF ROTATING
BEARINGS

By

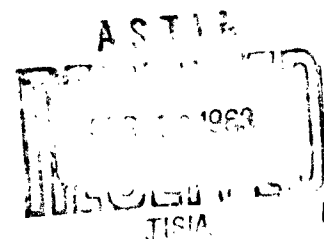
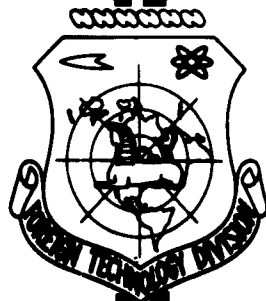
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FOREIGN TECHNOLOGY DIVISION

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GYROSCOPE DRIFT CAUSED BY FRICTION OF THE GIMBAL BEARINGS
AND REDUCTION OF THIS DRIFT BY THE USE OF ROTATING BEARINGS

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GYROSCOPE DRIFT CAUSED BY FRICTION OF THE GIMBAL BEARINGS
AND REDUCTION OF THIS DRIFT BY THE USE OF ROTATING
BEARINGS

Ye. M. Rodionov

A study of the causes of unsteady drift is of practical interest in connection with increasing the stability of gyroscopic instruments and systems, since instability of the drift with respect to magnitude and sign precludes the possibility of effective suppression of the drift by ordinary methods, i.e., by balancing or by means of correction devices.

This article is devoted to a consideration of one of the causes of unsteady drift, namely, the rolling-resistance moments of the gimbal bearings, and an evaluation is given of the effectiveness of methods of reducing the drift by oscillatory rotations of the outer rings of the gimbal bearings. When studying the behavior of gyroscopes, the rolling-resistance moment of the bearings of the gimbal suspension is usually introduced in the form of a friction moment with a constant angle of rotation; at best only the asymmetry of the friction during rotation in various directions is taken into account.

However, the rolling-resistance of a real ball bearing is in

essence not only a moment of friction forces (sliding, rolling, fluid friction), but is highly unstable with respect to the angle of rotation of the movable ring. The presence of radial gaps and the deviations in shape (ovalization of the rings and balls) which inevitably occur during manufacture, as well as individual roughnesses and microroughnesses of the rolling surfaces, lead to the appearance of additional moments differing in essence from friction moments.

These moments, in contrast to friction moments, do not change sign during a reversal of motion and in our opinion are one of the main causes of both the asymmetry of the rolling-resistance moment of a bearing during rotation in various directions and the instability of the drift of real gyroscopes.

The rolling-resistance moment M of a bearing can be represented in the form of a sum:

$$M = \pm M_{fr} + M_a; \quad (1)$$

$$M_a = \frac{dx}{d\alpha} \cdot N, \quad (2)$$

where

M_{fr} is the moment of all the friction forces of the bearing (rolling; sliding, fluid friction) acting on the shaft;

α is the angle of rotation of the movable ring relative to the stationary ring of the bearing;

x is the translational motion of the movable ring of the bearing in the direction of the load N acting on it and is caused by the presence of gaps, ovalization of the rings and balls, microroughnesses and individual roughnesses of the rolling surfaces;

M_a is the active moment arising during translational displacements of the movable ring.

Our studies showed that in the instrument ball bearings operating under purely radial loads without mutual axial displacement of the rings the radial load is distributed over no more than two balls, regardless of the total number of balls in the bearing. This law of distribution of the load in a bearing is maintained up to fairly large radial loads on the bearing. On bearing No. 23, for example, this law is maintained for loads not exceeding 2.15 kg.

On the basis of the law of distribution of the load, we obtained the dependence of the rolling-friction moment $M_{r.fr}$ on the angle of rotation α of the movable ring. This dependence is shown in a graph (Fig. 1b) for bearing No. 23 and a load $N = 300$ g (the other components of the friction moment M_{fr} were not studied).

In order to determine the active moment M_a , it is necessary to know the shape of the actual parts of the bearing, so as to represent the effective errors of a part of the bearing in the form of a function of the angle of rotation of this part. When the effective errors of the parts of a bearing are known, by taking into account the law of distribution of the load over the balls, we can represent the translational displacements of x of the movable ring of the bearing in the form of a function of the effective errors of the parts of the bearing and the angle of rotation α of the movable ring. After differentiating with respect to α and multiplying by the magnitude of the load N , we obtain the active component of the resistance moment M_a of the bearing also in the form of a function of the effective errors and the angle of rotation of the bearing.

It was found impossible to determine exactly by analytical means the active moments of the bearing caused by deviations in the shape of the parts of the bearing not only because the magnitude of the errors and the shape of the actual parts of the bearing are

random, but also because the nature of the moment M_a and its maximum values, as was shown by our studies, depend very significantly on the initial respective positions of the rings and the balls of the bearing. Taking into account that during the operation of a bearing under conditions of vibration the separator together with the balls describes undetermined movements, it proves to be impossible to determine M_a exactly even in the case of an actual bearing, even if we know exactly the actual shape of all of its parts and the position in which the rings of the bearing are set on the shaft and in the housing.

Of all the components of the active moment M_a it is the moment M_g , caused by the radial gap in the radial bearings, that possesses the greatest determinateness.

A graph of this moment is shown in Fig. 1b for bearing No. 23 and a load $N = 300$ g. In plotting the graph of this moment it was assumed that the balls roll without slipping.

The active moments arising in the balls of radial bearings as a result of ovalization of the rings were determined for the following preconditions.

1. The radial bearings are loaded with a purely radial load without mutual axial displacement of the rings.
2. The radial load is absorbed by no more than two balls of a bearing.
3. The races of the rings and balls of a bearing are in the form of an ellipse with a maximum difference $\underline{t}$ between the diametral dimensions (ovalization).
4. In the operation of the bearing the balls roll without slipping.

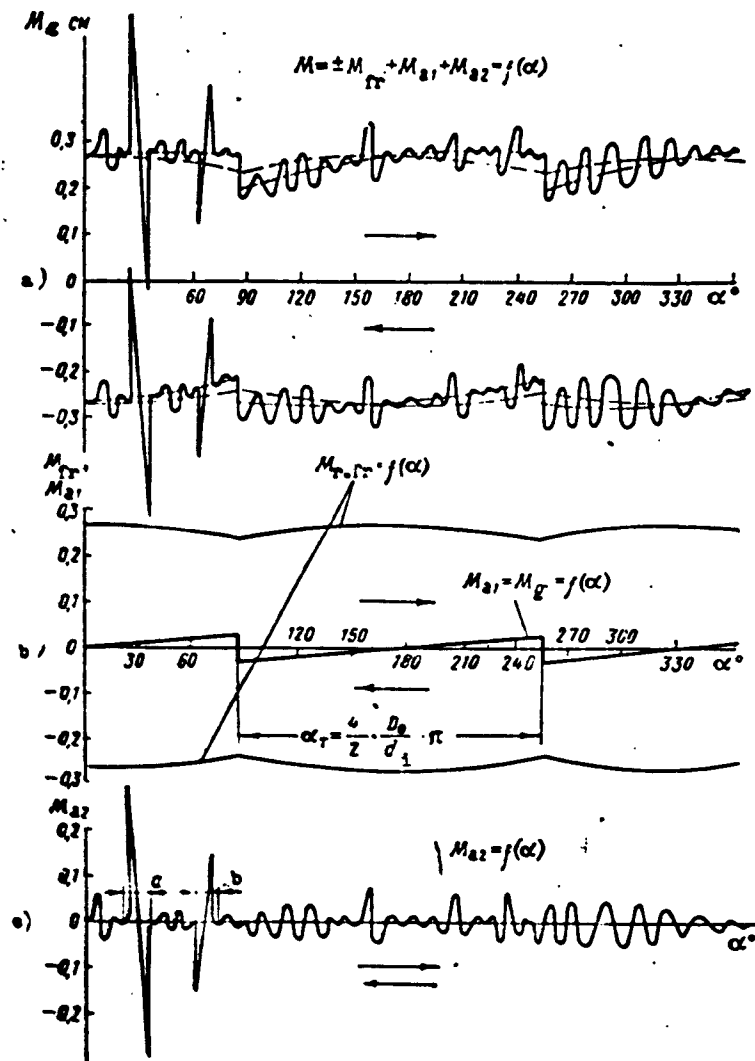


Fig. 1. Illustrative graph of the rolling-resistance moment of bearing No. 23 and the components of this moment.

($N = 300$ g, $\mu = 0.0002$, $g = 15\mu$)

Each component of the active moment M_a , caused by the radial gap and by the ovalization of the rings and balls, is a complex transcendental function of the angle of rotation of the bearing. These functions undergo a discontinuity at the moment when a transfer of the load from one pair of balls to another occurs. The maximum values of the active moments attainable in a bearing under the most

unfavorable conditions can be determined from the following formulas, which are valid if the preconditions mentioned above are observed.

The maximum value of the moment resulting from the presence of a radial gap.

$$M_z = \pm \frac{1}{2} \frac{d_1}{D_0} N \left(\frac{\pi}{2} \sin \frac{\pi}{z} - \frac{5}{3} \delta \cos \frac{2\pi}{z} \right), \quad (3)$$

where δ is the rapprochement of the rings due to elastic deformations

In the case of standard bearings

$$\delta = 96 \cdot 10^{-6} \sqrt{\frac{N}{d_b}} \text{ cm}; \quad (4)$$

N is the radial load on the bearing in kg;

d_1 is the diameter of the inner race of the bearing in mm;

d_b is the diameter of a ball in mm;

D_0 is the diameter of the circumference of the centers of the balls in mm;

z is the number of balls in the bearing.

The maximum value of the moment resulting from ovalization t_0 of the outer stationary ring of the bearing

$$M_0 = \pm \frac{1}{2} \frac{d_1}{D_0} \frac{1}{\cos \frac{\pi}{z}} \left[3 \sin \frac{\pi}{z} + \sin 3 \frac{\pi}{z} \right] t_0 N. \quad (5)$$

The maximum value of the moment resulting from ovalization t_1 of the inner movable ring of the bearing

$$M_1 = \pm \frac{1}{4 \sin \frac{2\pi}{z}} \left[\left(2 - \frac{1}{2} \frac{d_1}{D_0} \right) \sin \frac{3\pi}{z} - \left(2 - \frac{3}{2} \frac{d_1}{D_0} \right) \sin \frac{\pi}{z} \cos \frac{2\pi}{z} \right] t_1 N. \quad (6)$$

The maximum value of the moment resulting from ovalization t_b of the balls of the bearing

$$M_b = \pm \frac{1}{2} \frac{d_1}{d_b} \frac{1}{\cos \frac{\pi}{z}} t_b N. \quad (7)$$

The total active moment caused by the radial gap and ovalization

of the rings and balls is a complex and at the same time random periodic function, an analytical determination of the nature of which is hardly possible. However, the order of the maximum value of this total moment can be determined by summation according to the formula

$$M_{a1} = \pm \sqrt{M_s^2 + M_b^2 + M_{b1}^2 + M_{a1}^2} \quad (8)$$

The periodicity of this moment corresponds approximately to that angle of rotation of the bearing at which transference of the load from one pair of loaded balls to another occurs. This angle is determined from the expression:

$$\alpha_T = \frac{4}{z} \frac{D_0}{d_1} \pi.$$

In bearing No. 23 for $N = 300$ g, $t_0 = 0.003$ mm, $t_b = 0.00025$ mm, and $z = 7$ we obtain

$$M_{a1} \approx 0.1 \text{ g-cm} \quad \alpha_T \approx 168^\circ.$$

The moment M_{a1} can be represented in the form of a trigonometric series:

$$M_{a1} = \sum_{i=1}^m M_i \sin k_i (\alpha + \varphi_i). \quad (9)$$

The parameters of this series can be determined more accurately only by experiment. However, as a first approximation it can be assumed that the initial harmonic has an amplitude M_{a1} and a period α_T and that the amplitude decreases with increasing frequency. In addition to the above-mentioned active moments, caused by a disruption in the macrogeometry of the parts of the bearing (ovalization of the rings and balls) and by the presence of gaps in the bearing, active moments also arise as a result of individual minute impurities, scratches, dents, and blisters and as a result of roughnesses and

instability of the microhardness on the rolling surfaces. These moments, caused by the microgeometry of the surfaces, are also of periodic nature, but vary with a much greater frequency with respect to the angle of rotation of the bearing.

Experimental graphs of the rolling-resistance moments of the bearings corroborate the existence of these moments*. In order to estimate the order of the moments resulting from the microroughnesses of the rolling surfaces, we can determine the maximum peak of the moment arising when a ball rolls across an individual absolutely hard projection of height h from the formula

$$M_h = \pm \frac{1}{2} \sqrt{\frac{h}{d_b}} D_0 \cdot N. \quad (10)$$

The nature of the change in this moment with respect to the angle of rotation can be seen from the graph in Fig. 1c, part a.

When a ball rolls across a dent (groove, blister) of width l , the maximum peak of the moment

$$M_l = \pm \frac{l}{4} \frac{D_0}{d_b} N. \quad (11)$$

A graph of this moment with respect to the angle of rotation is shown in Fig. 1c, part b. In deriving formulas (10) and (11), the deformations of the parts of the bearing and the impurities were not taken into account (when the deformations are taken into account, M_h and M_l are decreased). In bearing No. 23, for example, when $N = 300$ g, $h = 1\mu$ and $M_h = \pm 0.77$ g-cm, while the period of the total variation of this moment $\alpha_h \approx 3^\circ$; in the same bearing, when $l = 1\mu$

$$M_l = \pm 0.06 \text{ g-cm}$$

*See, for example V. A. Pavlov's book "Gyroscopic Instruments", Oborongiz, 1954, p. 300.

while the period of the total variation of this moment $\alpha_1 = 0.03^\circ$.

An estimate of the order of magnitude of these moments caused by the microroughnesses of the surfaces enables us to conclude that antifriction bearings are very sensitive to contamination even by the most minute particles. Taking into account also that the elastic deformations of the parts of the bearing considerably exceed the roughness height of the rolling surfaces, we can assume that instability of the microhardnesses of a rolling surface may predominate over the effect of the surface finish during the formation of active moments in the bearing.

The active moment M_{a2} , caused by the microroughnesses and the instability of the microhardness of the rolling surfaces, is a periodic and at the same time random function of the angle of rotation of the bearing and can also be represented in the form of a trigonometric series

$$M_{a2} = \sum_j M_j \sin k_j (\alpha + \varphi_j). \quad (12)$$

The amplitude M_j of this series decreases with increasing frequency. The parameters of this series can be determined more accurately only by experiment.

Taking the above-said into account, the total rolling-resistance moment of a bearing is determined by the expression:

$$M = M_{\Sigma} + \sum_i M_i \sin k_i (\alpha + \varphi_i) + \sum_j M_j \sin k_j (\alpha + \varphi_j). \quad (13)$$

An illustrative analytical graph of this moment $M = f(\alpha)$ for a bearing rolling in both directions is shown in Fig. 1a. This graph is not the result of actual tests of the bearing, but contains typical elements inherent even in experimental graphs. This graph of the total moment M can be broken down into three basic components

(cf. Fig. 1):

1) the friction moment $M_{fr} = f_1(\alpha)$, which changes direction when the bearing reverses (only the rolling-friction moment is shown on the graph);

2) the active moment $M_{a1} = f_2(\alpha)$, caused by the presence of a gap and by a disruption of the macrogeometry of the parts of the bearing (only the moment resulting from the radial gap is shown on the graph);

3) the active moment $M_{a2} = f_3(\alpha)$, caused by the microroughnesses and by the instability of the microhardness of the rolling surfaces.

The active moments M_{a1} and M_{a2} do not change direction when the bearing reverses. The asymmetry of the rolling-resistance moment of the bearing, when the latter rolls in different directions, is caused by the active moments M_{a1} and M_{a2} .

As an illustration of the effect of the rolling-resistance moment of the ball bearings of the gimbal suspension on the behavior of a gyroscope, let us consider the motion of a very simple astatic three-degrees-of-freedom gyroscope mounted on a rotating base (Fig. 2).

If the platform rotates at a speed ω , the gyroscope is acted on along the y-axis by a friction moment $M_{fr.y}$, which for the sake of simplicity of our remaining argumentation we shall regard as constant ($M_{fr.y} = \text{const}$).

Under the action of the moment $M_{fr.y}$ there arises the precession

$$\dot{\alpha} \approx \frac{M_{fr.y}}{H \cdot \cos \alpha}. \quad (14)$$

Taking into account that before testing the gyroscopes the gimbals are set up perpendicularly ($\alpha = 0$) and that this angle has small values during the tests, we can assume that $\cos \alpha \approx 1$ and then

$$\dot{\alpha} \approx \frac{M_{fr.y}}{H} = \text{const}. \quad (15)$$

During the motion of the inner gimbal there arises a moment M_x , which is the sum of the friction moments of the sliding contacts and the rolling-resistance moments of the horizontal bearings:

$$M_x = \pm M_{\pi} + M_{a1} + M_{a2}. \quad (16)$$

Here M_{fr} is the friction moment of sliding, rolling, and fluid friction of the horizontal bearings and of friction of the sliding contacts.

The precession caused by the moment M_x , taking the smallness of α into account, is equal to:

$$\dot{\phi} = \frac{M_x}{H} = \frac{M_{\text{гр}}}{H} + \frac{M_{21}}{H} + \frac{M_{22}}{H}. \quad (17)$$

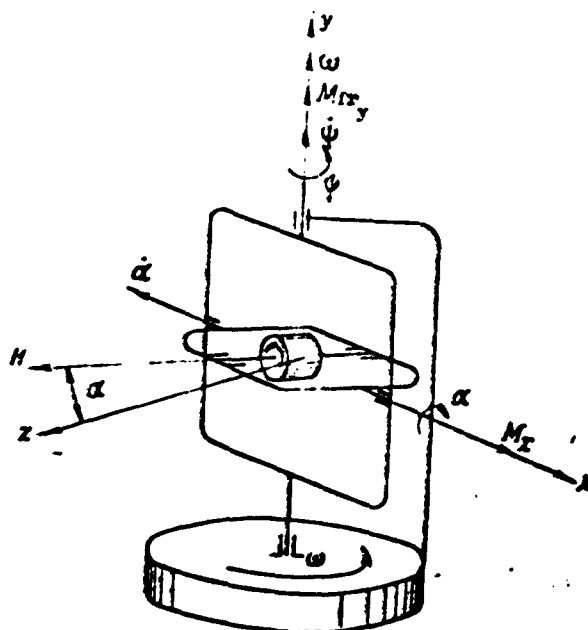


Fig. 2. Diagram of a gyroscope on a rotating base.

During the test time τ the gyroscope will rotate through the following angles:

$$\begin{aligned}
\alpha &= \dot{\alpha} \tau = \int_0^{\tau} \frac{M_y}{H} d\tau = \frac{M_y}{H} \tau; \\
\psi &= \int_0^{\tau} \frac{M_x}{H} d\tau = \pm \frac{1}{H} \int_0^{\tau} M_{\tau} d\tau + \\
&+ \frac{1}{H} \int_0^{\tau} M_{a1} d\tau + \frac{1}{H} \int_0^{\tau} M_{a2} d\tau.
\end{aligned} \tag{18}$$

Assuming for the sake of simplicity that the friction moment on the x-axis $x(M_{fr})$ is constant with respect to the angle of rotation and taking into account the values of M_{a1} and M_{a2} obtained from formulas (9) and (12), we obtain

$$\begin{aligned}
\psi &= \pm \frac{M_{fr}}{H} \tau + \frac{1}{H} \sum M_i \int_0^{\tau} \sin k_i (\dot{\alpha} \tau + \varphi_i) d\tau + \\
&+ \frac{1}{H} \sum M_j \int_0^{\tau} \sin k_j (\dot{\alpha} \tau + \varphi_j) d\tau, \\
\psi &= \pm \frac{M_{fr}}{H} \tau + \frac{1}{H} \sum \frac{M_i}{k_{i2}} [\cos k_i \varphi_i - \cos k_i (\dot{\alpha} \tau + \varphi_i)] + \\
&+ \frac{1}{H} \sum \frac{M_j}{k_{j2}} [\cos k_j \varphi_j - \cos k_j (\dot{\alpha} \tau + \varphi_j)].
\end{aligned} \tag{19}$$

The latter expression shows that the friction moment M_{fr} on the x-axis causes a unidirectional drift of the gyroscope behind the rotating platform (Fig. 3a). When the motion of the platform reverses, the gyroscope drift also changes.

The active moments M_{a1} and M_{a2} , on the other hand, cause oscillatory movements of the gyroscope about the y-axis, and the direction of this motion of the gyroscope does not change when the motion of the platform reverses. Taking into account that $\dot{\alpha} = M_y/H$, we note that the amplitude of the motion of the gyroscope does not depend on the kinetic moment H of the gyroscope and, consequently, cannot be decreased by increasing H . The oscillation amplitude decreases with an increase in the harmonic frequency $k_j \dot{\alpha}$.

The cumulative gyroscope drift resulting from the active moments M_{a1} and M_{a2} during the test time, τ depends substantially on the initial phase of the oscillatory motion (φ_1 and φ_j), the phase at which the test began (Fig. 3b).

In Eq. (19) the expressions in brackets can, within an unlimited period of time τ , vary within different limits, depending on φ : for $k\varphi = 0$, from 0 to +2; for $k\varphi = \pi$, from 0 to -2; for $k\varphi = \pm \frac{\pi}{2}$, from +1 to -1. A change in the phase φ may be caused by undetermined movements of the separator in the working instrument. The latter is one of the causes of the instability of the results of some of the tests.

In order to decrease the drift in precision directional gyroscopes, oscillating and rotating horizontal bearings are used.

Let us compare two methods of decreasing gyroscope drift.

1. Oscillatory rotation of the outer rings of the bearing with an amplitude of several degrees (oscillating bearings).
2. Reversing rotation of the outer rings of the bearings through several turns by means of a motor (rotating bearings).

In the first case oscillatory motion of the outer ring is caused by an electromagnet, and the angle of rotation α_r of the ring is determined from the formula:

$$\alpha_r = \alpha_{rm} \cdot \sin 2\pi f t, \quad (20)$$

where f is the oscillation frequency;

α_{rm} is the amplitude.

The angular oscillation velocity of the outer ring:

$$\dot{\alpha}_r = 2\pi \cdot f \cdot \alpha_{rm} \cdot \cos 2\pi f t. \quad (21)$$

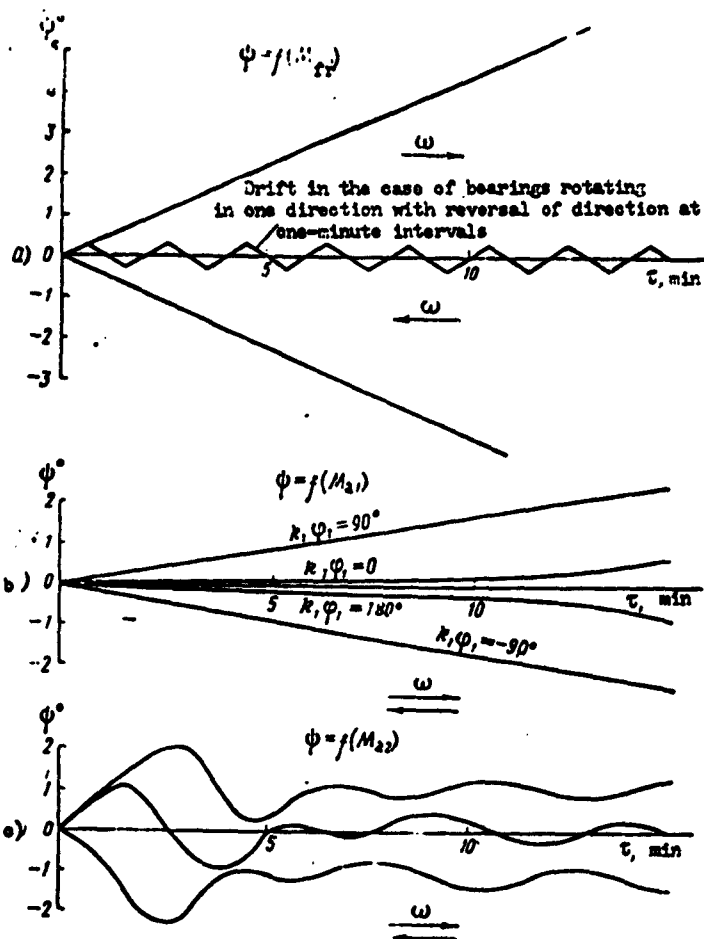


Fig.3. Approximate gyroscope drift ψ , caused by different components of the rolling-resistance moment of the horizontal bearings of the gyroscope.

The inner ring rotates at a constant angular precession velocity $\dot{\alpha}$. The angular velocity of the inner ring relative to the outer ring is the sum of the velocities $(\dot{\alpha} + \dot{\alpha}_r)$. Due to the difference in the time of motion in various directions, there appears a gyroscope drift, which is unidirectional on the average in the direction of rotation of the platform and is caused by the friction moments M_{fr} on the x-axis.

This drift is determined from the expression

$$\psi_{fr} = \frac{M_y \cdot M_{fr}}{2H^2 \cdot a_{fr} \cdot \pi^2 f} \quad (22)$$

The expression obtained determines a unidirectional gyroscope drift, when both horizontal bearings oscillate in one direction (in phase). When the rings of both horizontal bearings oscillate in opposite directions (in antiphase), a unidirectional drift can be determined by the same formula, but instead of M_{fr} we must substitute the difference between the friction moments of the two horizontal bearings.

The use of oscillating bearings greatly decreases gyroscope drift caused by friction on the horizontal axis, but when the oscillation amplitude of the bearing is small the nature of the active moments M_{a1} and the gyroscope drift caused by these moments remains practically unchanged. At low oscillation amplitudes the low harmonics M_{a2} also change very little, while the high harmonics, which produce small drifts as it is, will decrease.

On the other hand, the rotation of the outer rings of the bearings at a constant speed through several turns in first one, then the other direction increases the angular frequency ($k_1 \dot{\alpha}_1, k_j \dot{\alpha}_j$) of all the harmonics and sharply decreases the amplitude of the oscillatory movements of the gyroscope about the y-axis.

When the outer rings of the horizontal bearings rotate at a speed of, let us say, 30 rpm, the gyroscope drift resulting from active moments decreases by a factor of more than 10,000.

The friction moment M_{fr} on the horizontal axis in the case of reversing rotation of the outer rings of the bearings causes an oscillatory motion of the gyroscope about the y-axis; the amplitude $\psi_{fr.r.b}$ of these oscillations is determined from the expression:

$$\psi_{r.r.} = \frac{M_{fr} T}{2H} \quad (23)$$

Here T is the reversing period.

An illustrative graph of these oscillatory movements of the gyroscope is shown in Fig. 3a. In plotting the graph it was assumed that T = 60 seconds.

The amplitude of these oscillations decreases when the rings of the bearing are rotating in opposite directions; the oscillation amplitude in this case can be determined from formula (23), if instead of M_{fr} we substitute the difference between the friction moments in the two horizontal bearings. In the case of a variable angular velocity of rotation, varying harmonically for example, there occurs a unidirectional drift of the gyroscope about the y-axis in the direction of rotation of the platform; the magnitude of this drift can be determined from formula (22).

In the case under consideration a unidirectional gyroscope drift at a maximum angular velocity of 30 rpm does not exceed 0.01 deg/hr.

Thus the use of rotating bearings effectively suppresses the action of both the friction moments and the rolling-resistance moments of the gimbal bearings of gyroscopes, thereby ensuring increased stability of the gyroscope even with bearings of low quality.

The low efficiency of oscillating bearings is due, in our opinion, to the fact that in these bearings the action exerted on the gyroscope by the active components of the rolling-resistance moment of the bearings is, for all practical purposes, not suppressed.

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