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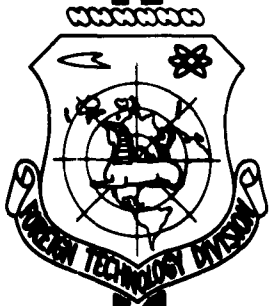
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TRANSLATION
ON THE SIMILARITY OF SYSTEMS OF REGULATING
GAS TURBINES
By
Ye. M. Syuzyumova

**FOREIGN TECHNOLOGY
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ON THE SIMILARITY OF SYSTEMS OF REGULATING
GAS TURBINES

Ye. M. Syuzumova

The use of similarity theory methods for investigating regulatory systems is connected with the transition to relative coordinates and relative times. The method of relative times noted by Stoloda in his time has been further developed in the works of M. S. Kheyfets [1] and A. V. Shcheglyayev [2].

The similarity of automatic regulating systems may be approached in two ways, in both cases using the P theorem of dimensional theory [3, 4] which demonstrates that from the total number of a system it is always possible to exclude k parameters, where k is the number of basic dimensional units. Thus, in mechanical regulating systems with three dimensional units one may always exclude three parameters. This, however, does not introduce substantial simplifications into the calculations of automation systems with a large number of degrees of freedom and accordingly with a large number of parameters. For computing such systems an efficient method proposed by Stoloda may be used: curtailing the number of parameters in the system - excluding all the statistical parameters - and converting all the

reducing parameters into so-called "times", i.e., giving them the dimensionality of time in the first or second degree. Since in this process all the parameters of the system have only one dimensionality, then in accordance with the P theorem one parameter may additionally be excluded, taking one of the times as unity and proceeding to relative times. Although the P theorem thus allows us to exclude a lesser number of parameters than when it is directly used the total number of excluded parameters for any developed regulatory systems will be more in this case.

We will clarify this by an example. Let us examine the simplest linear turbine regulatory system with one stress. The equations of this system in absolute coordinates are

$$\left. \begin{aligned} b_{11}\dot{\omega} + c_{12}m = 0; \\ b_{22}\dot{m} + c_{21}m - c_{21}\omega = 0, \end{aligned} \right\} \quad (1)$$

where b_{11} , b_{22} , c_{12} , c_{21} , and c_{22} are constant coefficients; ω , the angular velocity of the turbine; and m , the travel of the servomotor.

From these equations it follows that the system depends, as it were, on five parameters b_{11} , b_{22} , c_{12} , c_{21} , and c_{22} . Direct application of the P theorem would make it possible to exclude three parameters in this mechanical system with three units.

Using the second method let us transform system (1) reducing it to relative coordinates

$$\left. \begin{aligned} T_a\dot{\varphi} + \mu = 0; \\ T_1\dot{\mu} + \mu - \delta = 0, \end{aligned} \right\} \quad (2)$$

where φ is the relative value of the turbine's angular velocity; μ , the relative travel of the servomotor; δ , the degree of inhomogeneity; T_a , the machine's time for accelerating to a nominal value of angular velocity; T_1 , the period of the servomotor.

We reduce the number of parameters to three: T_a , T_1 , and δ . Then we proceed to the relative coordinate $\psi = \frac{\Delta\omega}{\delta\omega_0} = \frac{q}{\delta}$. In place of the machine's accelerating time T_a to a nominal value of angular velocity ω_0 we introduce time T_M of accelerating the machine to the value $\delta\omega_0$. As a result of this transformation δ does not enter the equations of the system in an obvious form, but of course remains in them in the unobvious form of $T_M = \delta T_a$.

The transition to the relative coordinate ψ excludes δ from Eqs. (2) and diminishes the number of parameters to two, T_M and T_1 :

$$\begin{cases} T_M \dot{\psi} + \psi = 0; \\ T_1 \ddot{\psi} + \psi = 0. \end{cases} \quad (3)$$

It still remained to introduce the relative times. Let us refer all times to one, e.g., to the machine's time and denote them by τ . Then $\tau_M = T_M/T_M = 1$ and $\tau_1 = T_1/T_M$. System (3) after this will have an even simpler form; it will depend on one parameter, τ_1 :

$$\begin{cases} \dot{\psi} + \psi = 0; \\ \tau_1 \ddot{\psi} + \psi = 0. \end{cases} \quad (4)$$

Thus, using the second method we have succeeded in excluding four parameters in this simplest system, i.e., one more parameter than in direct application of the P theorem. In any developed systems substantially more parameters are excluded with the second method. For example, in the same system of the fifth order the total number of parameters before treating the problem by similarity methods equals 14. When the P theorem is directly applied it proves possible to exclude three parameters and leave 11. With the method of relative coordinates and relative times four parameters remain - the relative times of the four servomotors representing the criteria of similarity of the system. Thus the method of relative coordinates

and relative times is essentially a similarity method using equations of statics (when proceeding to relative coordinates) for preliminary curtailment of the parameters.

Successive similarity transformations with the use of the relative coordinate method [1-3] led to the system actually depending on one parameter instead of the ostensible five. The regions of stability, aperiodicity, and all the other properties of the system must thus be arranged and investigated not in a space of five dimensions but only in one-dimensional space - for straightforward τ_1 . The importance of this result is obvious. It would in general be laborious to investigate more complex systems without systematic application of the relative coordinate method. It is necessary, however, to substantiate the validity of the transformation of Eq. (3) into Eq. (4) and clarify in what relationship the stability and transitional processes of the system characterized by Eqs. (4) will stand to the stability and transitional processes of the initial system of Eq. (3).

For this purpose let us examine a more complicated case - a monocyclic system with two stresses:

$$\left. \begin{aligned} T_M \dot{\psi} + \mu_2 = 0; \\ T_1 \dot{\mu}_1 + \mu_1 - \dot{\psi} = 0; \\ T_2 \dot{\mu}_2 + \mu_2 - \mu_1 = 0. \end{aligned} \right\} \quad (5)$$

A characteristic equation of this system will have the form:

$$T_M T_1 T_2 \omega^3 + (T_M T_1 + T_M T_2) \omega^2 + T_M \omega + 1 = 0. \quad (6)$$

Let us introduce the new variable $w = \frac{1}{T_M} v$:

$$\frac{T_M}{T_M} \cdot \frac{T_1}{T_M} \cdot \frac{T_2}{T_M} v^3 + \left(\frac{T_M}{T_M} \cdot \frac{T_1}{T_M} + \frac{T_M}{T_M} \cdot \frac{T_2}{T_M} \right) v^2 + \frac{T_M}{T_M} v + 1 = 0. \quad (7)$$

Or else designating as before $T_1/T_M = \tau_1$ and $\frac{T_2}{T_M} = \tau_2$ we get

$$\tau_1 \tau_2 v^3 + (\tau_1 + \tau_2) v^2 + v + 1 = 0. \quad (8)$$

The system of Eqs. (5) after transition to relative times will assume the form:

$$\left. \begin{aligned} \dot{\psi} + \mu_2 \psi &= 0; \\ \tau_1 \mu_1 + \mu_1 - \dot{\psi} &= 0; \\ \tau_2 \mu_2 + \mu_2 - \mu_1 &= 0. \end{aligned} \right\} \quad (9)$$

The roots of Eq. (8) differ from those of Eq. (6) by the constant factor T_M , that is, $v = T_M w$. Consequently if Eq. (8) answers to the conditions of stability, aperiodicity, and any other qualitative conditions depending on the properties of the roots, then Eq. (6) also answers to these conditions. This transformation thus does not introduce any new conditions into the investigation of stability and simplifies the investigation by curtailing the number of parameters.

Now let us examine the connection between the transitional processes in both systems, the initial and the transformed. We will designate the roots of the transformed characteristic equation by v_1 , v_2 , and v_3 . The common integral of the system will equal

$$\psi = A_1 e^{-v_1 t} + A_2 e^{-v_2 t} + A_3 e^{-v_3 t}, \quad (10)$$

or else

$$\psi = A_1 e^{-\tau_1 \mu_1 t} + A_2 e^{-\tau_2 \mu_2 t} + A_3 e^{-\mu_2 t}. \quad (11)$$

This integral is the solution for the whole family of similar systems accordingly having the same τ_1 , τ_2 and differing only in the value of T_M . Introducing the new variable $t' = T_M t$ we may write this integral as follows:

$$\psi = A_1 e^{-\omega_1 t'} + A_2 e^{-\omega_2 t'} + A_3 e^{-\omega_3 t'}. \quad (12)$$

Thus, the transition from the characteristic Eq. (6) to the characteristic Eq. (8) is equivalent to linear conversion of coordinate \underline{t} . The transients of the systems connected by a similarity relationship differ only by the scale of the \underline{t} axis.

This conclusion can be made clearer and simultaneously extended to a linear systems with constant servomotor velocity.

The method of transition to relative times set forth above - transformation of the variable and of the characteristic equation - is not suitable for linear systems containing elements with constant velocities; since in such systems there is not characteristic equation, this transition is nevertheless possible for them, too.

We will note that in system (5) all the values are dimensionless except the T parameters which possess the dimensionality of time. The transition from one unit of time to another, that is, linear transformation of coordinate \underline{t} in the initial equations or their integrals should not entail changes in the other coordinates. We may select a unit of time so that one of the parameters of system (5) will be excluded. Let us select for this the value of T_M as the unit of time instead of 1 sec. It is obvious that all the parameters T_M , T_1 , and T_2 having time dimensionality must also be measured by this unit. Their new values are

$$\tau_1 = \frac{T_1}{T_M}; \quad \tau_2 = \frac{T_2}{T_M}; \quad \tau_3 = \frac{T_3}{T_M}.$$

In this the parameter of T_M is excluded. This method of transition to relative times is general for linear and ailinear systems. An ailinear system with two extensions is characterized by the equations

$$\left. \begin{aligned} T_M \dot{x}_1 + x_1 &= 0; \\ \dot{x}_2 &= \frac{1}{T_1} x_1; \\ \dot{x}_3 &= \frac{1}{T_2} x_2. \end{aligned} \right\} \quad (13)$$

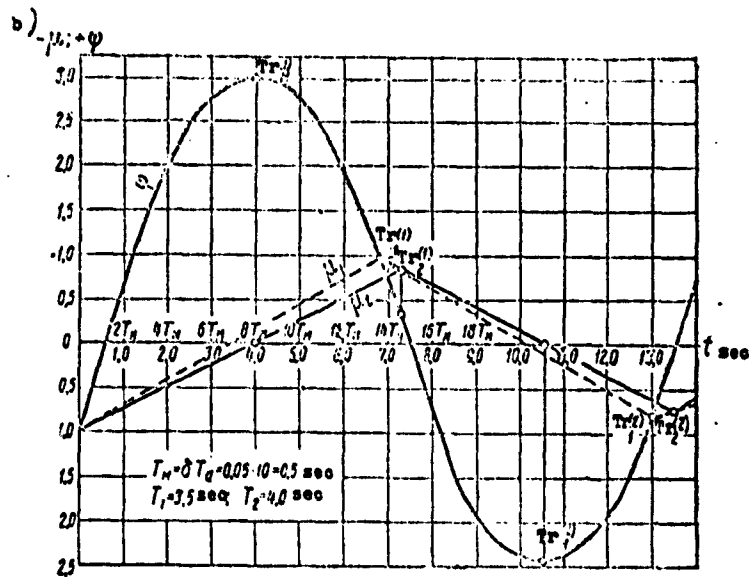
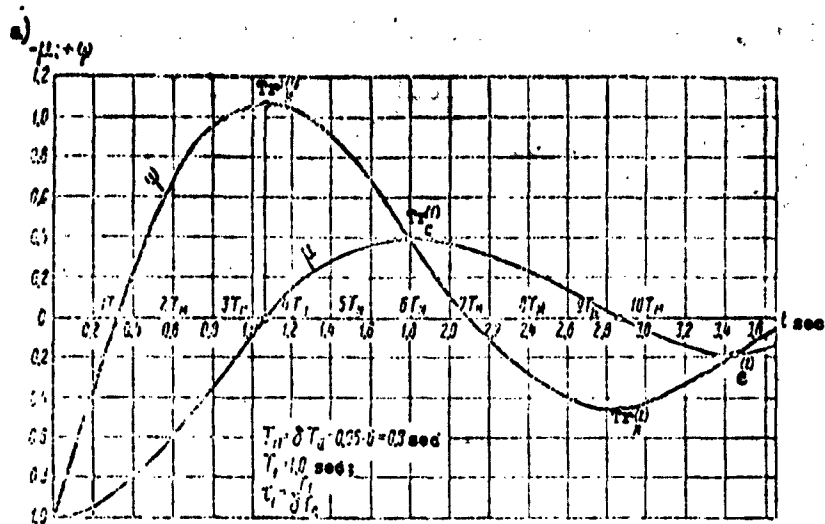


Fig. 1. Transients in linear (a) and ailinear (b) regulating systems with one extension.

After the transition to relative times the system of equations take the form

$$\left. \begin{aligned} \dot{\psi} + \mu_2 &= 0; \\ \dot{\mu}_1 &= \frac{1}{\tau_1}; \\ \dot{\mu}_2 &= \frac{1}{\tau_2}. \end{aligned} \right\} \quad (14)$$

The integral of this system in absolute times is:

$$\left. \begin{aligned} \psi &= -\frac{1}{2T_1 T_2} t^2 + \frac{\mu_{20}}{T_2} t + \psi_0; \\ \mu_1 &= \frac{1}{T_1} t + \mu_{10}; \\ \mu_2 &= \frac{1}{T_2} t + \mu_{20}; \end{aligned} \right\} \quad (15)$$

and in relative times:

$$\left. \begin{aligned} \dot{\psi} &= -\frac{1}{2\tau_2} t^2 + \mu_{20} t + \psi_0; \\ \dot{\mu}_1 &= \frac{1}{\tau_1} t + \mu_{10}; \\ \dot{\mu}_2 &= \frac{1}{\tau_2} t + \mu_{20}. \end{aligned} \right\} \quad (16)$$

Equations (9) and (14) characterize the families of similar systems. The criteria of similarity for both families are the relative times τ_1 and τ_2 .

The similarity method permits us to proceed from the investigation of individual systems to that of families of similar systems. Let us clarify this by examples. The transients in linear and ailinear systems with one stresses which depend on one parameter are shown in Fig. 1.

Figure 2 gives the linear system transients described by Eqs. (9) and the ailinear system transients with two extensions (Eq.14).

If the curves of the transients (see Fig. 2) are found by solving equation systems (9) and (14), then the time unit

is T_M . Thus the time axis must be provided with a scale of $1T_M$, $2T_M$, $3T_M$, etc. In this the curves describe the transients of the whole family of similar systems with the same τ_1 and τ_2 and different T_M . For curves characterizing the transients of a family of similar linear (Fig. 2a) and a linear (Fig. 2b) systems accordingly

$$\tau_1 = \frac{T_1}{\delta T_a} = \frac{0,1}{0,3} = 0,33; \quad \tau_2 = \frac{T_2}{\delta T_a} = \frac{0,25}{0,3} = 0,833;$$

$$\tau_1 = \frac{T_1}{\delta T_a} = \frac{3,5}{0,5} = 7; \quad \tau_2 = \frac{T_2}{\delta T_a} = \frac{4,0}{0,5} = 8,0.$$

In order to characterize the transient process for a concrete system we should substitute the value of T_M in the scale of $1T_M$, $2T_M$, $3T_M$, etc., i.e., give this scale a concrete value.

Let there be two systems with the respective parameters

$$T_a = 6 \text{ sec}; \quad \delta = 0,05, \quad T_1 = 0,1 \text{ sec}, \quad T_2 = 0,25 \text{ sec}$$

and

$$T_a = 15 \text{ sec}, \quad \delta = 0,01, \quad T_1 = 0,2 \text{ sec}, \quad T_2 = 0,5 \text{ sec}$$

For the first system:

$$T_M = \delta T_a = 0,05 \cdot 6 = 0,3 \text{ sec.}$$

$$\tau_1 = \frac{T_1}{T_M} = \frac{0,1}{0,3} = 0,33, \quad \tau_2 = \frac{T_2}{T_M} = \frac{0,25}{0,3} = 0,833;$$

for the second system:

$$T_M = 0,01 \cdot 15 = 0,15 \text{ sec.}$$

$$\tau_1 = \frac{0,2}{0,15} = 1,33, \quad \tau_2 = \frac{0,5}{0,15} = 3,33.$$

Since in both systems τ_1 and τ_2 are accordingly equal, these systems, for all the difference in their parameters, are similar and are described by the general curves of the transients (Fig. 2). In order to obtain the curves characterizing the transient for the first system we should substitute $T_M = 0,3$ in the

common scale of $1T_M$, $2T_M$, $3T_M$, etc., i.e., transfer to the scale
 $1 \cdot 0.3$, $2 \cdot 0.3$, $3 \cdot 0.3$, etc.

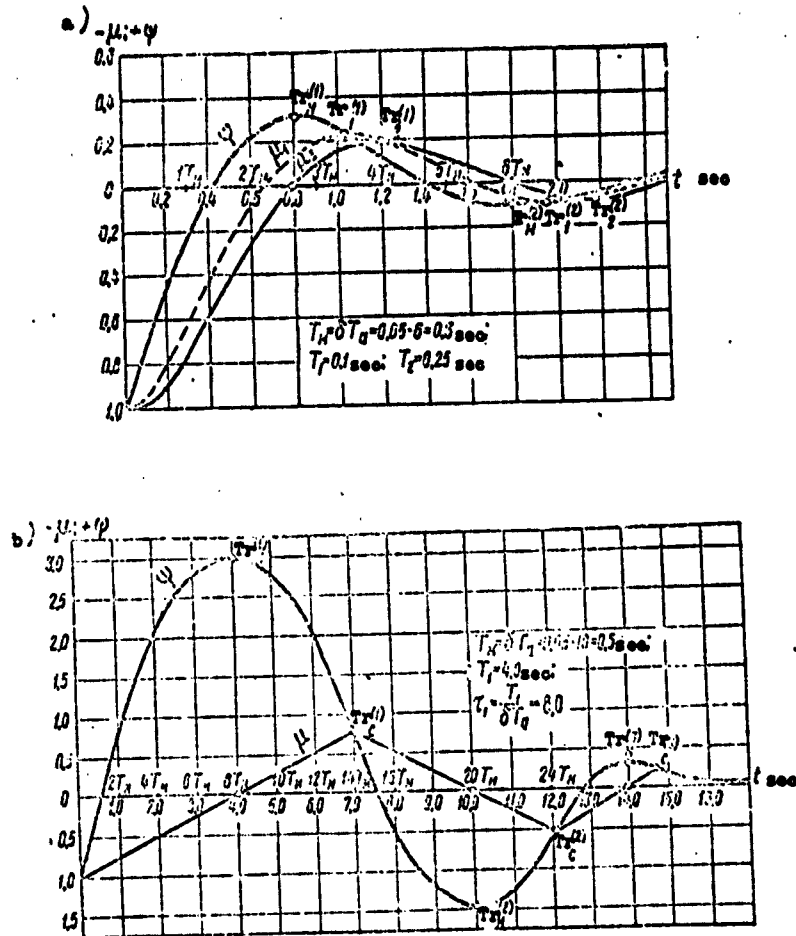


Fig. 2. Transients in linear (a) and linear (b) regulating systems with two extensions.

The curves characterizing the transients described by Eqs. (14) of a family of linear similar systems (Fig. 2) with parameters $\tau_1 = 7$ and $\tau_2 = 8$ refer in equal degree, for example, to systems with the following parameters

- 1) $T_1 = 5 \text{ sec}$, $\delta = 0,05$, $T_1 = 2,1 \text{ sec}$, $T_2 = 2,4 \text{ sec}$;
- 2) $T_1 = 4 \text{ sec}$, $\delta = 0,1$, $T_1 = 2,8 \text{ sec}$, $T_2 = 3,2 \text{ sec}$;
- 3) $T_1 = 10 \text{ sec}$, $\delta = 0,05$, $T_1 = 3,5 \text{ sec}$, $T_2 = 4 \text{ sec}$.

These systems consequently have the same τ_1 and τ_2 . The characteristics of the transients for each of these may be obtained by using Fig. 2b and substituting in the time scale the appropriate value of T_M .

In order to obtain the characteristics of the transient of a family of similar systems there is no need to convert the equations of the transient to relative times. We may solve these equations in absolute times for any system and then enter the scale of relative times on the abscissa axis.

Let the transient be calculated for a specific machine with parameters $T_M = T_0 + \delta T_1 = 0,05 \cdot 6 = 0,8 \text{ sec}$, $T_1 = 0,1 \text{ sec}$, $T_2 = 0,25 \text{ sec}$ (Fig. 2a). The absolute values (lower scale) are entered along the abscissa axis. In order that the curves characteristic of the transient of the specific machine indicated above be generalized for all the systems similar to it, it is sufficient that the scale of relative times be entered on the abscissa axis, i.e., that the value of $T_M = 0,3 \text{ sec}$ be adopted as the unit of time. Then we may use these curves for the system in which $T_0 = \delta T_1 = 0,05 \cdot 12 = 0,6 \text{ sec}$, $T_1 = 0,2 \text{ sec}$, $T_2 = 0,5 \text{ sec}$ and τ_1 , τ_2 are the same as in the initial system; consequently it is similar to the original system. For this purpose it is sufficient to reconvert the scale of relative times into a new scale of absolute times by substituting in it the value of $T_M = 0,6 \text{ sec}$. Thus, the characteristics of the transient obtained for any concrete system may be extended to a family of similar systems.

It is obvious that in the transition to relative times we may select as the time unit not the machine's time T_M , but the time of one of the servomotors. This selection, however, cannot be arbitrary. The machine in the monocyclical system represents the only astatic element (in case there is no self-regulation). Since the order of the elements of a monocyclical system is indifferent from the point of view of dynamics, the times of all the servomotors symmetrically enter the characteristic Eq. (6) and only the machine's time T_M disturbs this symmetry. Exclusion of this time leads to the characteristic Eq. (8), perfectly symmetrical relative to servomotor times τ_1 and τ_2 . This in its turn leads to perfectly symmetrical conditions of stability and aperiodicity. For example, according to Hurwitz the stability condition is

$$A_1 = \tau_1 + \tau_2 - \tau_1\tau_2 > 0.$$

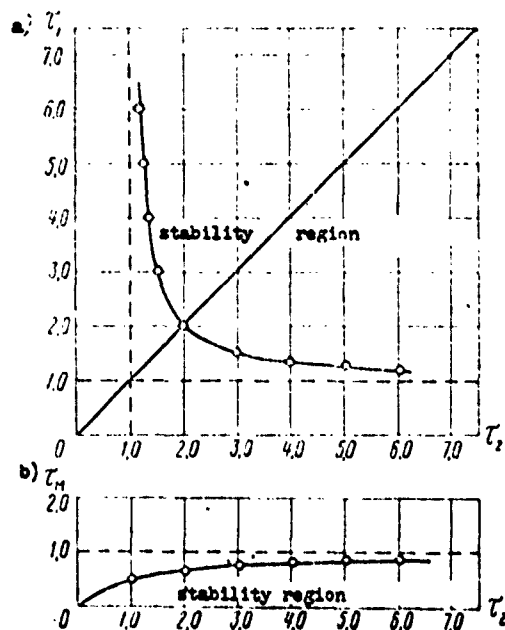


Fig. 3. Boundary lines of stability region of regulating system with two stresses characterized by equations
a) $\tau_1 + \tau_2 - \tau_1\tau_2 = 0$; b) $\tau_M + \tau_M \tau_2 - \tau_2 = 0$.

The boundary curve of the region of stability $\Delta_2 = \tau_1 + \tau_2 - \tau_1 \tau_2 = 0$ represents an equilateral hyperbola with asymptotes $\tau_1 = 1$ and $\tau_2 = 1$ (Fig. 3a). This symmetry is disturbed when all the times are referred not to the machine's time but to that of one of the servomotors, for example, in system (5) when referred to time T_1 . In this process the characteristic equation of the system will assume the form

$$\tau_M \tau_2 v^3 + (\tau_M + \tau_M \tau_2) v^2 + \tau_M v + 1 = 0. \quad (17)$$

This equation is asymmetrical in respect to τ_2 and τ_M ; and the stability conditions are also asymmetrical in respect to these two parameters (Fig. 3b):

$$\Delta_2 = (\tau_M^2 + \tau_M^2 \tau_2) - \tau_M \tau_2 > 0.$$

It is easy to demonstrate that even for complex polycyclical systems the choice of T_M as the unit of time retains its advantages. The selection, then, of the time of one of the servomotors as the time unit lessens the value of the relative coordinate method to a considerable degree.

The similarity method set forth allows us to extend the results of the investigation of regulatory systems to a family of similar systems.

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