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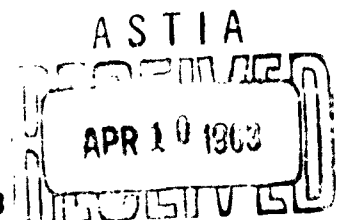
CRACK STRESSES: AN APPLICATION OF MUSKHELISHVILI'S EXTENSION
PRINCIPLE FOR MULTIVALUED MAPPING FUNCTIONS

TECHNICAL REPORT WAL TR 811.8/4

BY

OSCAR L. BOWIE

DATE OF ISSUE - MARCH 1963



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BASIC RESEARCH IN ENGINEERING SCIENCES

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CRACK STRESSES: AN APPLICATION OF MUSKHELISHVILI'S EXTENSION
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ABSTRACT

Muskhelishvili's extension principle for the case of geometries involving corners and cracks and hence multivalued mapping functions is considered. It is shown that a useful application of the procedure can be found in the study of the local stress distribution near the base of a crack. The structures of the stress functions on the Riemann surface defining the mapping function in the neighborhood of σ_0 corresponding to the crack root can be found and the local stress distribution shown to depend on $\phi'(\sigma_0)$.



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I. INTRODUCTION

In the plane theory of elasticity, Muskhelishvili's¹ formulation of the problem in terms of a single complex function defined in both the occupied region and its complement in the plane is well-known. Applications of the concept to regions defined by polynomial and rational mappings of several basic regions have been carried out, e.g., mappings of the unit circle by Kartsvadze². The author³ recently investigated the extension of the concept to multivalued mapping functions of the type encountered in the description of boundary corners.

This report is intended to show a practical application of the extension principle when applied to multivalued mapping functions. In the study of the local stress distribution near the base of a crack, it will be shown that the extension argument provides necessary information as to the character of the stress function. A simple conversion of the stress function defined in the auxiliary complex plane to the stress concentration in the physical plane is thus provided.

II. THE EXTENSION PRINCIPLE AND MULTIVALUED MAPPINGS

For concreteness, we consider regions in the physical Z-plane corresponding to a closed contour C and its interior S_2^+ . It will be assumed that a mapping relation

$$Z = \omega(\zeta) \quad (1)$$

exists which maps conformally the interior of the unit circle in the ζ -plane, S_1^+ , into S_2^+ and carries the unit circle τ into C. The existence of corner points in C implies branch point type of singularities in equation 1 lying on the unit circle, e.g., consider the Schwartz-Christoffel transformation.

Muskhelishvili's original formulation depends on representing the stresses and displacements in terms of two complex functions $\phi(\zeta)$ and $\psi(\zeta)$ holomorphic in S_1^+ as

$$\sigma_y + \sigma_x = 2\operatorname{Re}[\phi'(\zeta)/\omega'(\zeta)], \quad (2)$$

$$\sigma_y - \sigma_x + 2i \tau_{xy} = 2(\overline{\omega(\zeta)}[\phi'(\zeta)/\omega'(\zeta)]' + \psi'(\zeta))/\omega'(\zeta), \quad (3)$$

$$2\mu(u + iv) = \kappa\phi(\zeta) - \omega(\zeta)\overline{\phi'(\zeta)}/\overline{\omega'(\zeta)} - \psi(\zeta), \quad (4)$$

where primes denote differentiation with respect to ζ , bars denote complex conjugates, and μ and κ are material constants. For boundary conditions in terms of the applied load on C, the corresponding conditions on the unit circle $\zeta = \sigma = e^{i\alpha}$ can be written as

$$\phi(\sigma) + \omega(\sigma)\overline{\phi'(\sigma)}/\overline{\omega'(\sigma)} + \overline{\psi(\sigma)} = g(\sigma), \quad (5)$$

where $g(\sigma)$ is prescribed by the mapping and the given applied load.

The stress analysis requires the determination of the two functions $\phi(\zeta)$ and $\psi(\zeta)$ which must be analytic in S_ζ^+ and obey the condition, equation 5, on the unit circle. Muskhelishvili's extension principle consists essentially in replacing $\psi(\zeta)$ by an extended definition of $\phi(\zeta)$. Using the same form of extension as Kartzivadze, we define $\phi(\zeta)$ in the region exterior to the unit circle, S_ζ^- , as

$$\phi(\zeta) = -\omega(\zeta)\bar{\phi}'(1/\zeta)/\bar{\omega}'(1/\zeta) - \bar{\psi}(1/\zeta), \quad \zeta \in S_\zeta^-, \quad (6)$$

where

$$\bar{f}(1/\zeta) = \overline{f(1/\bar{\zeta})}. \quad (7)$$

Clearly $\phi(\zeta)$ as defined by equation 6 is analytic in S_ζ^- with the possible exception of the point at infinity. It follows from equation 6 that

$$\psi(\zeta) = -\bar{\phi}(1/\zeta) - \bar{\omega}(1/\zeta)\phi'(\zeta)/\omega'(\zeta), \quad \zeta \in S_\zeta^+. \quad (8)$$

It is evident for the mapping class considered that $\psi(\zeta)$ as defined by equation 8 is analytic in S_ζ^+ with the possible exception of $\zeta = 0$.

One of the conditions on the extended definition of $\phi(\zeta)$ is therefore the insurance of the analyticity of $\psi(\zeta)$ at $\zeta = 0$ in equation 8. The remaining condition arises from the satisfaction of the boundary condition equation 5. Replacing $\bar{\psi}(\sigma)$ in equation 5 by the definition equation 8, we find

$$\phi^+(\sigma) - \phi^-(\sigma) + [\omega^+(\sigma) - \omega^-(\sigma)]\bar{\phi}^{+'}(\sigma)/\bar{\omega}^{+'}(\sigma) = g(\sigma), \quad (9)$$

where the notation $f^-(\sigma)$, $[f^+(\sigma)]$, is defined as the value of $f(\zeta)$ as $\zeta \rightarrow \sigma$ through S_ζ^- , $[S_\zeta^+]$, respectively.

For polynomial mapping functions, equation 9 reduces to the so-called problem of linear relationship, i.e.

$$\phi^+(\sigma) - \phi^-(\sigma) = g(\sigma) \quad (10)$$

and the solution can be found in terms of a polynomial and a Cauchy integral.

When branch points in the mapping function lie on the unit circle the simplification equation 10 is no longer possible. The mapping function equation 1 must be considered as defined on an appropriate Riemann surface. With little loss in generality, we assume the appropriate branch of the mapping function lies on the top sheet and the branch cuts are intervals of the unit circle joining the branch points. It is evident that across the branch cuts, $\omega^+(\sigma) - \omega^-(\sigma) \neq 0$ and hence the simplification equation 10 is no longer possible.

Although the effectiveness of the extension principle in the case of multivalued mapping functions as a numerical procedure has little advantage over other techniques, one can argue that considerable insight into the

character of the stress functions is gained. A practical use of such insight will now be demonstrated in the study of the local stress distribution near the base of cracks occurring in geometries involving corner points.

III. AN APPLICATION OF THE EXTENSION PRINCIPLE

Let us assume that part of C corresponds to a crack; thus, there corresponds an interval τ_c on the unit circle which maps into the crack. Let σ_0 denote the point on τ_c which maps into the tip of the crack Z_0 . Then

$$\omega'(\zeta) = (\zeta - \sigma_0)W(\zeta), \quad (11)$$

where $W(\zeta)$ is analytic in S_ζ^+ and nonvanishing in S_ζ^+ and τ_c . A finite number of branch points are assumed on the unit circle necessary to the description of the corner points of C .

Since C is a continuous closed curve, the branch cuts can be taken as intervals of the unit circle. Furthermore, it is easy to show that no cut is necessary on τ_c . By a rotation, the crack can be considered as falling on the real axis. Then $\omega(1/\zeta)$ is the analytic continuation of $\omega(\zeta)$ across τ_c by Schwartz's reflection theorem. Since $\omega(\zeta)$ can be continued analytically across τ_c , certainly $\omega(\zeta)$ is analytic at $\zeta = \sigma_0$. Noting that $\omega'(\sigma_0) = 0$, it follows that in the neighborhood of σ_0 ,

$$Z - Z_0 = \sum_{n=2}^{\infty} S_n (\sigma - \sigma_0)^n. \quad (12)$$

If polar coordinates (r, θ) are introduced by

$$Z - Z_0 = re^{i\theta} \quad (13)$$

then by reversion of series,

$$\zeta - \sigma_0 \approx \sqrt{(Z - Z_0)/S_2} = \sqrt{r/S_2} e^{i\theta/2}. \quad (14)$$

From equations 2 and 3 it is clear that an estimate of the stresses requires a knowledge of the local behavior of the stress functions. For simplicity, we assume the crack surfaces are free from applied load. Then it is possible to define $g(\sigma)$ so that $g(\sigma) = 0$ on τ_c . From equation 9 it is clear that $\phi^+(\sigma) = \phi^-(\sigma)$ on τ_c for this case. Thus, $\phi(\zeta)$ as defined by equation 6 is the analytic continuation of $\phi(\zeta)$, $\zeta \in S_\zeta^+$, across the interval τ_c (in the top sheet of the Riemann surface for the mapping function). In particular, $\phi(\zeta)$ is analytic at $\zeta = \sigma_0$, thus,

$$\phi(\zeta) = \phi'(\sigma_0) + \phi''(\sigma_0)(\zeta - \sigma_0) + \dots \quad (15)$$

is valid in a neighborhood of σ_0 overlapping points in S_L^+ and S_L^- . Thus, it follows easily that

$$\sigma_x + \sigma_y \approx \text{Re} \left\{ \frac{2\sqrt{2} [\phi'(\sigma_0)] e^{-i\theta/2}}{\sqrt{\omega''(\sigma_0)} \sqrt{r}} \right\}. \quad (16)$$

To approximate equation 3, the structure of $\psi(\zeta)$ is necessary. From the preceding arguments, $\phi'(\zeta)/\omega'(\zeta)$ is analytic at σ_0 except for a simple pole, thus,

$$\phi'(\zeta)/\omega'(\zeta) = a_{-1}(\zeta - \sigma_0)^{-1} + a_0 + a_1(\zeta - \sigma_0) + \dots \quad (17)$$

Since $\phi(\zeta)$ and $\omega(\zeta)$ are analytic at σ_0 then so are $\bar{\phi}(1/\zeta)$ and $\bar{\omega}(1/\zeta)$; thus, from equation 8, $\psi(\zeta)$ has a simple pole at σ_0 . We can write

$$\psi(\zeta) = A_{-1}(\zeta - \sigma_0)^{-1} + A_0 + A_1(\zeta - \sigma_0) + \dots \quad (18)$$

Comparison of equations 8 and 18 and the series expansions above yields

$$A_{-1} = -a_{-1}\bar{Z}_0 = -\phi'(\sigma_0)\bar{Z}_0/2S_2,$$

$$A_0 = -\overline{\phi(\sigma_0)} - a_0 \bar{Z}_0, \quad (19)$$

$$A_1 = \overline{\phi'(\sigma_0)}/\sigma_0^2 - \bar{S}_2 a_{-1}/\sigma_0^4 - \bar{Z}_0 a_1.$$

It is then a straightforward matter from equation 3 to show

$$\sigma_y - \sigma_x + 2i \tau_{xy} \approx \frac{\sqrt{2} e^{-i\theta/2}}{\sqrt{r\omega''(\sigma_0)}} \left[\frac{\overline{\phi'(\sigma_0)}}{\sigma_0^2} - \frac{\phi'(\sigma_0)}{2\sigma_0^4} - \frac{\phi'(\sigma_0)e^{-2i\theta}}{2} \right]. \quad (20)$$

OBSERVATIONS

It was shown in the preceding section that the stresses in the vicinity of an unloaded crack can be approximated by equations 16 and 20. Since $\omega''(\sigma_0)$ can be calculated directly from the mapping function, the key computation is the evaluation of $\phi'(\sigma_0)$. Thus, for geometries involving corner points and cracks, the extension argument can be used to show that the local stresses in the crack vicinity depend on the evaluation of $\phi'(\sigma_0)$, independent of the procedure adopted, e.g., polynomial approximation, power series, integral equations, etc.

Generalizations of the arguments used in Section III to a larger class of geometries are self-evident. Furthermore, the restriction of the loading conditions on the crack surface can be removed by a modification of the argument. In particular, if $g(\sigma) \neq 0$ on the interval corresponding to the crack surface,

$$\phi(\zeta) = \phi_1(\zeta) + \frac{1}{2\pi i} \int_{\gamma_c} \frac{g(t)dt}{(t - \zeta)}, \quad (21)$$

where $\phi_1(\zeta)$ is analytic in the neighborhood of σ_0 . Thus, the character of $\phi(\zeta)$ and hence $\psi(\zeta)$ in the vicinity of σ_0 is known and an argument similar to that of Section III can again be carried out.

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