

UNCLASSIFIED

AD 400 723

*Reproduced
by the*

**ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA**



UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

AD

400 723

CA
AS AU 111.



WATERTOWN ARSENAL LABORATORIES

RECTANGULAR TENSILE BAR WEAKENED BY SURFACE CRACKS
PART I, STRESS ANALYSIS

TECHNICAL REPORT WAL TR 811.8/5

BY

OSCAR L. BOWIE

DATE OF ISSUE - MARCH 1963

ASTIA
APR 12 1963

AMC CODE 5011.11.838
BASIC RESEARCH IN PHYSICAL SCIENCES

D/A PROJECT 1-A-0-10501-B-010

WATERTOWN ARSENAL
WATERTOWN 72, MASS.

The findings in this report are not to be construed as an official Department of the Army position.

ASTIA AVAILABILITY NOTICE

Qualified requesters may obtain copies of this report from ASTIA

DISPOSITION INSTRUCTIONS

Destroy; do not return

AD

Elasticity

Crack stresses

RECTANGULAR TENSILE BAR WEAKENED BY SURFACE CRACKS
PART 1, STRESS ANALYSIS

Technical Report WAL TR 811.8/5

By

Oscar L. Bowie

Date of Issue - March 1963

AMC Code 5011.11.838
Basic Research in Physical Sciences

D/A Project 1-A-O-10501-B-010

WATERTOWN ARSENAL
WATERTOWN 72, MASS.

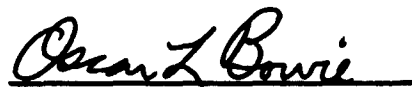
WATERTOWN ARSENAL LABORATORIES

TITLE

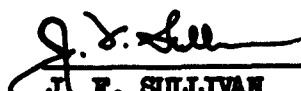
RECTANGULAR TENSILE BAR WEAKENED BY SURFACE CRACKS
PART I, STRESS ANALYSIS

ABSTRACT

The stress analysis of a rectangular tensile bar weakened by surface cracks is formulated in terms of mapping functions of a complex variable. Although an approximate solution of this problem has been previously considered by Tait, the present derivation specifies the proper load-free boundary conditions on the lateral edges of the rectangle as well as allowing for the variation of both crack length and length/width ratios of the plate. Determination of the stress function is reduced to the solution of a linear system of simultaneous equations.


OSCAR L. BOWIE
Mathematician

APPROVED:


J. F. SULLIVAN
Director
Watertown Arsenal Laboratories

INTRODUCTION

In the area of crack studies, a problem of practical importance is the simple geometry of a rectangular tensile specimen weakened by surface cracks. In order to apply such theories as that of Irwin¹, it is necessary to determine the stress distribution in the vicinity of the crack root. Although the behavior of the stress in such a neighborhood is known qualitatively, quantitative results require a formal solution of the stress problem as a whole. It is this matter which is considered in this report for the plane problem shown in Figure 1.

R. J. Tait² has considered the problem of determining the state of stress due to surface cracks in an infinite elastic strip of finite width. Unfortunately, in his analysis, load-free conditions on the strip surfaces are only partially prescribed. In fact, a mixed boundary value problem was assumed in his analysis which, in effect, is intrinsic to an infinite sheet containing a row of periodically spaced cracks.

In the present report, we shall consider surface cracks in rectangular plates of finite length and width with the proper load-free boundary conditions on the lateral edges of the rectangle. In addition to introducing the desirable modification of Tait's analysis, it is possible to study the additional parameter, length/width ratio and its effect on the stress concentration. The polynomial approximation method used by the author³ in the analysis of an infinite plate containing radial cracks does not appear well suited to the present problem due to slowness of convergence. Thus, an alternate approximation scheme is devised.

CONVENTIONAL COMPLEX VARIABLE FORMULATION

Muskhelishvili's⁴ original complex variable statement of the plane problem in rectangular coordinates depends on the representation of Airy's stress function, $U(x,y)$, in terms of the two analytic functions, $\varphi(Z)$ and $\psi(Z)$, of the complex variable $Z=x + iy$ as

$$U(x,y) = \operatorname{Re} \left[\bar{Z} \varphi(Z) + \int^Z \psi(Z) dZ \right]. \quad (1)$$

With this representation, the stress and displacement components can be written as

$$\sigma_y + \sigma_x = 2 \left[\varphi'(Z) + \overline{\varphi'(Z)} \right] = 4 \operatorname{Re} \left[\varphi'(Z) \right], \quad (2)$$

$$\sigma_y - \sigma_x + 2i T_{xy} = 2 \left[\bar{Z} \varphi''(Z) + \psi'(Z) \right], \quad (3)$$

$$2\mu (u+iv) = K\varphi(Z) - Z \overline{\varphi'(Z)} - \overline{\psi(Z)}, \quad (4)$$

where the prime notation denotes differentiation with respect to Z , the bars denote complex conjugates, and μ and K are material constants. A useful relation for handling stress boundary conditions is provided by

$$\varphi(Z) + Z \overline{\varphi'(Z)} + \psi(Z) = 1 \int_0^s (X_v + iY_v) ds = f_1(s) + if_2(s), \quad (5)$$

where $X_v ds$ and $Y_v ds$ represent the x - and y - components of the force acting on the element of arc ds from the positive side of the normal v .

Curvilinear coordinates can be conveniently introduced by considering an auxiliary complex ζ -plane along with a conformal mapping function

$$Z = \omega(\zeta). \quad (6)$$

The analyticity of (6) ensures the analyticity of the stress functions when considered as functions of the variable ζ . New notation is simplified by designating $\varphi(Z) \equiv \varphi[\omega(\zeta)]$ as $\varphi(\zeta)$, etc., leading to such relationships as $\varphi'(Z) = \varphi'(\zeta)/\omega'(\zeta)$, etc.

The stresses and displacements can then be written as

$$\sigma_y + \sigma_x = 2 \left[\varphi'(\zeta)/\omega'(\zeta) + \overline{\varphi'(\zeta)}/\overline{\omega'(\zeta)} \right], \quad (7)$$

$$\sigma_y - \sigma_x + 2i T_{xy} = 2 \left[\frac{\overline{\omega(\zeta)}}{\overline{\omega'(\zeta)}} \left(\frac{\varphi'(\zeta)}{\omega'(\zeta)} \right)' + \frac{\psi'(\zeta)}{\omega'(\zeta)} \right], \quad (8)$$

$$2\mu (u + iv) = K \varphi(\zeta) - \omega(\zeta) \overline{\varphi'(\zeta)}/\overline{\omega'(\zeta)} - \overline{\psi(\zeta)}. \quad (9)$$

When C corresponds to a closed contour in the Z -plane into which the unit circle $\zeta = \sigma = e^{i\theta}$ is mapped by (6), the arc parameter s can be considered as a known function of σ , hence

$$f_1(s) + if_2(s) = g(\sigma). \quad (10)$$

The boundary condition on C (assuming the applied load is given) can thus be written as

$$\varphi(\sigma) + \omega(\sigma) \overline{\varphi'(\sigma)}/\overline{\omega'(\sigma)} + \overline{\psi(\sigma)} = g(\sigma). \quad (11)$$

THE MAPPING FUNCTION

The region occupied by the rectangle with surface cracks will be denoted by S_z with boundary T in the the Z -plane (Figure 1).

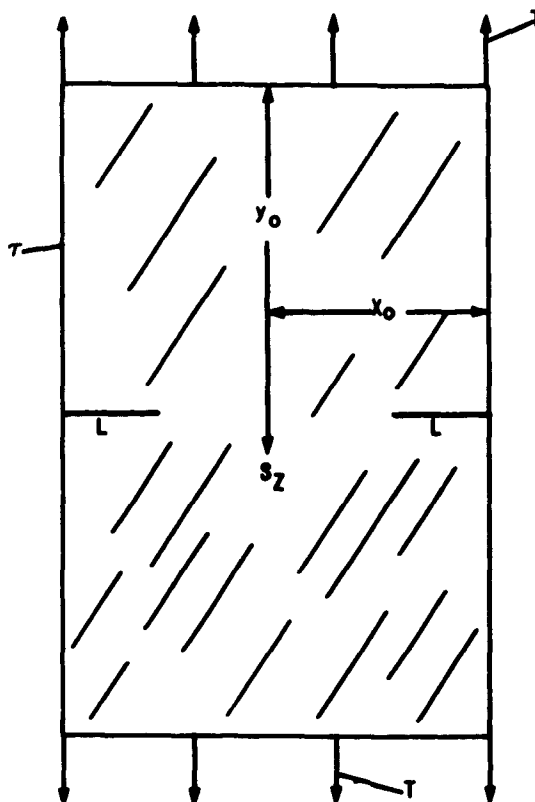


Figure 1

The exact mapping function $w(\zeta)$ is defined as the mapping of the unit circle and its interior into the boundary τ and S_γ , respectively.

By a straightforward consideration of the Schwartz-Christoffel transformation, it can be seen that the exact mapping function corresponds to an appropriate branch of the function

$$Z = -1 \int_0^1 (\zeta^2 + 1) \left[(1 - e^{2i\alpha} \zeta^2)(1 - e^{-2i\alpha} \zeta^2)(1 - e^{2i\beta} \zeta^2)(1 - e^{-2i\beta} \zeta^2) \right]^{-1/2} d\zeta. \quad (12)$$

The exact mapping function has eight branch points located on the unit circle necessary to the description of the eight right angle corners. We define the real parameters in (12) such that $0 < \alpha < \beta \leq \pi/2$, thus, when $\beta = \pi/2$ the circle maps into the limiting case of a rectangle. The crack roots are described by the zeroes of $\omega'(\zeta)$ on the unit circle, i.e., $\zeta = \pm 1$. The ratios L/x_0 and y_0/x_0 can be varied by appropriately adjusting the parameters α and β .

In series form,

$$Z = u(\zeta) = -1 \sum_{n=1}^{\infty} A_n \zeta^{2n-1} \quad (13)$$

where the A_n 's are real. The A_n 's may be obtained numerically from a straightforward recursive formulae (Appendix I) determined by comparing (12) and (13). The series (13) converges interior to the unit circle and on any interval of the unit circle not including a branch point.

Unfortunately, the rate of convergence of (13) is slow on the unit circle, thus leading to extremely tedious numerical calculation in the stress analysis. On the other hand, the procedure of polynomial approximation of the mapping function utilized in Reference 3 is plagued by the same slowness of convergence.

We adopt a different approach and define for $K \geq 1$,

$$\omega(\zeta, K) = -1 \int_0^{\zeta} (\zeta^2 + 1) \left[(K^2 - e^{2i\alpha}\zeta^2)(K^2 - e^{-2i\alpha}\zeta^2)(K^2 - e^{2i\beta}\zeta^2)(K^2 - e^{-2i\beta}\zeta^2) \right]^{1/2} d\zeta. \quad (14)$$

Clearly $\omega(\zeta, 1)$ coincides with (12). On the other hand, for $K > 1$ the branch points are moved out to lie on a circle $|\zeta| = K > 1$. The cusps representing the crack roots are still preserved on the unit circle at $\zeta = \pm 1$. Thus, by evaluating (14) for values of $K \approx 1$ on the unit circle, increasingly accurate approximations τ are obtained as $K \rightarrow 1$. On the other hand, the convergence of the series expansions for $K > 1$ will be substantially improved compared with that of (12). The form of the power series (13) is preserved in the expansion of $\omega(\zeta, K)$, thus,

$$\omega(\zeta, K) = -1 \sum_{n=1}^{\infty} A_n \zeta^{2n-1}, \quad (15)$$

where the A_n 's are real and are now considered as functions of the parameter K as well as α and β .

DETERMINATION OF THE STRESS FUNCTIONS

It is necessary to determine the two stress functions $\varphi(\zeta)$ and $\psi(\zeta)$ such that both functions are analytic for $|\zeta| < 1$ and the condition (11) is satisfied on the unit circle. Apart from mathematical elegance, little is gained by any consideration other than a direct power series argument in the present formulation. The difficulty due to the zeroes of $\omega'(\zeta)$ on the unit circle can be eliminated by writing (11) in the form

$$\overline{\omega'(\sigma)} \varphi(\sigma) + \omega(\sigma) \overline{\varphi'(\sigma)} + \overline{\omega'(\sigma)} \psi(\sigma) = \overline{\omega'(\sigma)} g(\sigma). \quad (16)$$

From symmetry considerations,

$$\varphi(\zeta) = 1/T \sum_{n=1}^{\infty} a_n \zeta^{2n-1}, \quad (17)$$

$$\psi(\zeta) = 1/T \sum_{n=1}^{\infty} b_n \zeta^{2n-1}, \quad (18)$$

where the coefficients a_n and b_n are real. If the stress state were that of uniform tension, i.e., no surface cracks were present, then $\varphi(\zeta) = T \times \omega(\zeta)/4$ and $\psi(\zeta) = T \omega(\zeta)/2$. Thus, we assume

$$\varphi(\zeta) = T \omega(\zeta)/4 + \varphi_1(\zeta), \quad (19)$$

$$\psi(\zeta) = T \omega(\zeta)/2 + \psi_1(\zeta), \quad (20)$$

where

$$\varphi_1(\zeta) = 1/T \sum_{n=1}^{\infty} \alpha_n \zeta^{2n-1}, \quad (21)$$

$$\psi_1(\zeta) = 1/T \sum_{n=1}^{\infty} \beta_n \zeta^{2n-1}. \quad (22)$$

Clearly,

$$\begin{aligned} \overline{\omega'(\sigma)} \varphi_1(\sigma) + \omega(\sigma) \overline{\varphi_1'(\sigma)} + \overline{\omega'(\sigma)} \psi_1(\sigma) = \\ \overline{\omega'(\sigma)} \left\{ g(\sigma) - \frac{T}{2} [\omega(\sigma) + \overline{\omega(\sigma)}] \right\} = T \sum_{n=-\infty}^{\infty} d_n \sigma^{2n-1}. \end{aligned} \quad (23)$$

Substitution of (21) and (22) into (23) and comparing coefficients of equal powers of σ leads to the two sets of conditions

$$\sum_{n=1}^{\infty} A_n (2n-1) \alpha_{p+n-1} + \sum_{n=1}^{\infty} \alpha_n (2n-1) A_{p+n-1} = -d_p, \quad p = 1, 2, 3, \dots \quad (24)$$

$$\sum_{n=p+2}^{\infty} A_n (2n-1) \alpha_{n-p-1} + \sum_{n=p+2}^{\infty} \alpha_n (2n-1) A_{n-p-1} \\ - \sum_{n=1}^{p+1} A_n (2n-1) \beta_{p+2-n} = -d_{-p}, \quad p = 0, 1, 2, \dots \quad (25)$$

The set of equations 25 provides for a direct determination of the β_n 's once the α_n 's have been found from the linear system of simultaneous equations 24.

The applied load will be expanded on the basis of the exact geometry and loading. Thus, if

$$g(\sigma) - \frac{T}{2} \left[w(\sigma) + \overline{w(\sigma)} \right] = \sum_{K=-\infty}^{\infty} C_{2K-1} \sigma^{2K-1}, \quad (26)$$

then it can easily be shown (Appendix 2), that

$$C_{2K-1} = -\frac{2iT}{\pi} \frac{\cos(2K-1)\beta}{2K-1} \sum_{n=1}^{\infty} A_n \sin(2n-1)\beta \\ + \frac{2iT}{\pi} \int_{\beta}^{\pi/2} \sum_{n=1}^{\infty} A_n \sin(2n-1)\theta \sin(2K-1)\theta d\theta, \quad K = 0, \pm 1, \pm 2, \dots \quad (27)$$

For computational purposes an alternate form of C_{2K-1} is now presented.

$$C_{2K-1} = -\frac{2iT}{\pi} \int_{\beta}^{\pi/2} \left[X(\beta) - X(\theta) \right] \sin(2K-1)\theta d\theta, \quad (28)$$

where $X(\theta)$ is the real part of $w(e^{i\theta})$. Integration by parts yields

$$C_{2K-1} = \frac{2iT}{(2K-1)\pi} \int_{\beta}^{\pi/2} \frac{dX}{d\theta} \cos(2K-1)\theta d\theta. \quad (29)$$

Using (12) to compute $dX/d\theta$, it follows that

$$C_{2K-1} = \frac{2iT}{(2K-1)\pi} \int_{\beta}^{\pi/2} \frac{\cos \theta \cos (2K-1)\theta d\theta}{(\cos 2\alpha - \cos 2\theta)^{1/2} (\cos 2\beta - \cos 2\theta)^{1/2}}. \quad (30)$$

From the definition of the d_n 's in (23), it follows that

$$T d_p = 1 \sum_{n=1}^{\infty} A_n (2n-1) C_{2(p+n)-3}, \quad p = 0, \pm 1, \pm 2, \dots \quad (31)$$

When approximate mappings 15 are used, the loading conditions must be considered as approximated. On the other hand, as $K \rightarrow 1$ both the geometry and the applied load converge to the exact problem.

DETERMINATION OF THE STRESS CONCENTRATION IN THE VICINITY OF THE CRACK ROOTS

The stress concentration in the vicinity of either of the crack roots is the prime aim of the analysis. If polar coordinates (r, γ) are introduced with the origin chosen as the crack root, the stresses vary to the first order* as

$$\sigma \approx L(\gamma)/\sqrt{r}. \quad (32)$$

The determination of $L(\gamma)$ in terms of the present analysis will now be considered.

Assume that ζ_0 is the point on the unit circle which maps into the crack root Z_0 . Then $w'(\zeta)$ has a simple zero at $\zeta = \zeta_0$. Furthermore, $w(\zeta)$ is analytic in the neighborhood of ζ_0 on the Riemann surface defining the mapping function, thus,

$$Z - Z_0 = \sum_{n=2}^{\infty} s_n (\zeta - \zeta_0)^n. \quad (33)$$

By reversion of series,

$$\zeta - \zeta_0 = \sum_{n=1}^{\infty} t_n (Z - Z_0)^{n/2}, \quad (34)$$

* $L(\gamma)$ is essentially proportional to Irwin's factor K .

where

$$s_2 t_1^2 = 1, \text{ etc.}$$

thus,

$$\zeta - \zeta_0 \approx \frac{1}{\sqrt{s_2}} (z - z_0)^{1/2}. \quad (35)$$

If we introduce polar coordinates,

$$z - z_0 = r e^{i\gamma}, \quad (36)$$

then

$$\zeta - \zeta_0 \approx \frac{\sqrt{r}}{\sqrt{s_2}} e^{i\gamma/2} = \frac{\sqrt{2r}}{\sqrt{w''(\zeta_0)}} e^{i\gamma/2}. \quad (37)$$

In order to utilize these approximations to examine the stress field local to the notch root, it is clear from (7) and (8) that the character of $\varphi(\zeta)$ and $\psi(\zeta)$ in the neighborhood of $\zeta = \zeta_0$ is required. If we define

$$\psi(\zeta) = -\bar{\varphi}(1/\zeta) - \bar{w}(1/\zeta) \varphi'(\zeta)/w'(\zeta) \quad (38)$$

where

$$\bar{f}(1/\zeta) \equiv \overline{f(1/\zeta)}, \quad (39)$$

and substitute into (11), it can be shown that $\varphi(\zeta)$ is analytic in the neighborhood of $\zeta = \zeta_0$ whereas $\psi(\zeta)$ has a simple pole in this neighborhood.

It follows fairly directly from (7) that

$$\sigma_x + \sigma_y \approx \text{Re} \left\{ \frac{2\sqrt{2} [\varphi'(\zeta_0)]}{\sqrt{w''(\zeta_0)} \sqrt{r}} e^{-i\gamma/2} \right\}. \quad (40)$$

From (8), it follows from a somewhat more difficult calculation, that

$$\sigma_y - \sigma_x + 2i\tau_{xy} \approx \frac{\sqrt{2} e^{-i\gamma/2}}{\sqrt{w''(\zeta_0)} r} \left[\frac{\overline{\varphi'(\zeta_0)}}{\zeta_0^2} - \frac{\varphi'(\zeta_0)}{2\zeta_0^4} - \frac{\varphi'(\zeta_0)}{2} e^{-2i\gamma} \right]. \quad (41)$$

In the calculation of (40) and (41), the key quantity is $\varphi'(\zeta_0)$. Whereas the calculation of $\omega''(\zeta)$ can be made directly from the closed form of the mapping function, the determination of $\varphi'(\zeta_0)$ requires the solution of the system of equations 24.

SUMMARY

The stress analysis for a rectangular tensile bar weakened by surface cracks has been presented. A mapping function which permits variation of the crack depth and length/width ratio of the plate is utilized. Determination of the stress field in the neighborhood of the crack root requires the calculation of $\varphi'(\zeta_0)$ where ζ_0 is the point on the unit circle corresponding to the crack root. A procedure for systematically approximating $\varphi'(\zeta_0)$ is suggested by the limiting process as $K \rightarrow 1$ in the mapping function.

Calculations are underway for pertinent values of the parameters and the results will be presented in a subsequent report.

APPENDIX I

RECURSIVE FORMULAE FOR THE MAPPING COEFFICIENTS

From (14),

$$w'(\zeta, K) = -1 (\zeta^2 + 1) / (\zeta^4 - 2K^2 \zeta^2 \cos 2\alpha + K^4)^{1/2} (\zeta^4 - 2K^2 \zeta^2 \cos 2\beta + K^4)^{1/2}.$$

Let

$$w'(\zeta, K) = -1 \sum_{n=1}^{\infty} M_n \zeta^{2n-2}.$$

Then,

$$\begin{aligned} -(\zeta^2 + 1)^2 = & - \left\{ \zeta^8 + \zeta^6 K^2 (-2 \cos 2\beta - 2 \cos 2\alpha) \right. \\ & + \zeta^4 (2K^4 + 4K^4 \cos 2\alpha \cos 2\beta) + \zeta^2 (-2K^6 \cos 2\alpha - 2K^6 \cos 2\beta) \\ & \left. + K^8 \right\} \left\{ \sum_{n=1}^{\infty} M_n \zeta^{2n-2} \right\}^2. \end{aligned}$$

Let

$$2\epsilon_1 = -2 \cos 2\alpha - 2 \cos 2\beta$$

$$2\epsilon_2 = 2 + 4 \cos 2\alpha \cos 2\beta$$

$$C_2 = M_1^2$$

$$C_3 = 2M_1 M_2$$

$$C_4 = 2M_1 M_3 + M_2^2$$

$$C_5 = 2M_1 M_4 + 2M_2 M_3 \quad \text{etc.}$$

Then,

$$\begin{aligned} (\zeta^4 + 2 \zeta^2 + 1) = & \left[\zeta^8 + 2 K^2 \epsilon_1 \zeta^6 + 2 K^4 \epsilon_2 \zeta^4 + 2 K^6 \epsilon_1 \zeta^2 + K^8 \right] \\ & \left[C_2 + C_3 \zeta^2 + C_4 \zeta^4 + \dots \right]. \end{aligned}$$

Equating coefficients of equal powers of ζ yields

$$c_2 k^8 = 1$$

$$k^8 c_3 + 2 k^6 \epsilon_1 c_2 = 2$$

$$k^8 c_4 + 2 k^6 \epsilon_1 c_3 + 2 k^4 \epsilon_2 c_2 = 1$$

$$k^8 c_5 + 2 k^6 \epsilon_1 c_4 + 2 k^4 \epsilon_2 c_3 + 2 k^2 \epsilon_1 c_2 = 0$$

$$k^8 c_6 + 2 k^6 \epsilon_1 c_5 + 2 k^4 \epsilon_2 c_4 + 2 k^2 \epsilon_1 c_3 + c_2 = 0$$

etc.

Thus,

$$M_1 = -1/k^4$$

$$M_2 = -k^4 c_3/2$$

$$M_3 = -k^4 \left[c_4/2 - M_2^2/2 \right]$$

$$M_4 = -k^4 \left[c_5/2 - M_2 M_3 \right]$$

etc.

Thus, in the expansion

$$\omega(\zeta, K) = -1 \sum A_n \zeta^{2n-1},$$

$$A_n = M_n/(2n-1).$$

APPENDIX II

EXPANSION OF THE LOADING FUNCTION

It will be recalled that

$$g(\sigma) = i \int_0^S (X_v + i Y_v) ds.$$

We define

$$\begin{aligned} g(\sigma) &= TX, \quad [-X_0 \leq X \leq 0, y = y_0] \\ &= -TX_0, \quad [X = -X_0, 0 \leq y \leq y_0] \\ &\text{etc.} \end{aligned}$$

Thus,

$$\begin{aligned} g(\sigma) - \frac{T}{2} [w(\sigma) + \overline{w(\sigma)}] &= 0, \quad 0 \leq \theta \leq \beta \\ &= -TX_0 - \frac{T}{2} [w(\sigma) + \overline{w(\sigma)}], \quad \beta \leq \theta \leq \pi - \beta \\ &= 0, \quad \pi - \beta \leq \theta \leq \pi + \beta, \\ &= TX_0 - \frac{T}{2} [w(\sigma) + \overline{w(\sigma)}], \quad [\beta + \pi \leq \theta \leq 2\pi - \beta], \\ &= 0, \quad 2\pi - \beta \leq \theta \leq 2\pi. \end{aligned}$$

If we set

$$g(\sigma) - \frac{T}{2} [w(\sigma) + \overline{w(\sigma)}] = \sum_{k=-\infty}^{\infty} c_k \sigma^k,$$

then

$$C_K = \frac{T}{2\pi} \int_{\beta}^{\pi-\beta} \left\{ -x_0 - \frac{1}{2} \left[w(\sigma) + \overline{w(\sigma)} \right] \right\} e^{-iK\theta} d\theta \\ + \frac{T}{2\pi} \int_{\pi+\beta}^{2\pi-\beta} \left\{ x_0 - \frac{1}{2} \left[w(\sigma) + \overline{w(\sigma)} \right] \right\} e^{-iK\theta} d\theta.$$

It follows from symmetry that

$$C_K = 0, \quad K \text{ even}$$

$$C_K = \frac{T}{\pi} \int_{\beta}^{\pi-\beta} \left\{ -x_0 - \frac{1}{2} \left[w(\sigma) + \overline{w(\sigma)} \right] \right\} e^{-iK\theta} d\theta, \quad K = \pm 1, \pm 3, \dots$$

Again from symmetry,

$$C_K = -\frac{2iT}{\pi} \int_{\beta}^{\pi/2} \left\{ -x_0 - \frac{1}{2} \left[w(\sigma) + \overline{w(\sigma)} \right] \right\} \sin K\theta d\theta, \quad K = \pm 1, \pm 3, \dots$$

Finally,

$$C_{2K} = 0, \quad K = 0, \pm 1, \pm 2, \dots$$

$$C_{2K-1} = -\frac{2iT}{\pi} \frac{\cos(2K-1)\beta}{2K-1} \sum_{n=1}^{\infty} A_n \sin(2n-1)\beta \\ + \frac{2iT}{\pi} \int_{\beta}^{\pi/2} \sum_{n=1}^{\infty} A_n \sin(2n-1)\theta \sin(2K-1)\theta d\theta, \quad K=0, \pm 1, \pm 2, \dots$$

REFERENCES

1. IRWIN, G.R., KIES, J.A., and SMITH, H.L., Fracture Strengths Relative to Onset and Arrest of Crack Propagation, presented at the Sixty-first Annual Meeting of ASTM, Boston, Mass., June 22-27, 1958.
2. TAIT, R.J., Crack in a Strip of Finite Width. See SNEDDON, I.N., Crack Problems in the Mathematical Theory of Elasticity, under Contract Nonr 486(06), North Carolina State College, Raleigh, N.C., May 1961, p. 60-67.
3. BOWIE, O.L., Analysis of an Infinite Plate Containing Radial Cracks Originating at the Boundary of an Internal Circular Hole, Journal of Mathematics and Physics, v. 25, 1956, p. 60-71.
4. MUSKHELISHVILI, N.I., Some Basic Problems of the Mathematical Theory of Elasticity, P. Noordhoff, Ltd., Groningen, Holland, 1953.

WATERTOWN ARSENAL
WATERTOWN 72, MASSACHUSETTS

TECHNICAL REPORT DISTRIBUTION

Report No.: WAL TR 811.8/5 Title: Rectangular Tensile Bar Weakened
March 1963 by Cracks, Part I, Stress Analysis

Distribution List approved by 1st Indorsement from Ordnance Weapons
Command, ORDOW-TB, dated 4 January 1962

No. of
Copies

TO

1	Office of the Director of Defense Research and Engineering, Room 3D-1067, The Pentagon, Washington 25, D. C. ATTN: Mr. J. C. Barrett
10	Commander, Armed Services Technical Information Agency, Arlington Hall Station, Arlington 12, Virginia ATTN: TIPDR
1	Advanced Research Project Agency, The Pentagon, Washington 25, D.C. ATTN: Dr. G. Mock
1	Solid Propellant Information Agency, Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland
1	Commanding General, U. S. Army Materiel Command, Washington 25, D. C. ATTN: AMCRD-RS
2	Commanding General, U. S. Army Missile Command, Redstone Arsenal, Alabama ATTN: AMSMI-RB, Redstone Scientific Information Center
1	AMSMI-RRS, Mr. R. E. Ely
1	AMSMI-RKX, Mr. R. Fink
1	AMSMI, Mr. W. K. Thomas
1	AMSMI-RSM, Mr. E. J. Wheelahan
1	Commanding General, U. S. Army Mobility Command, Detroit 9, Michigan
1	Commanding General, U. S. Army Munitions Command, Dover, New Jersey
2	Commanding General, U. S. Army Test and Evaluation Command, Aberdeen Proving Ground, Maryland ATTN: AMSTE, Technical Library

No. of Copies	TO
	Commanding General, U. S. Army Weapons Command, Rock Island, Illinois
1	ATTN: AMSWE-IX, Industrial Division
1	AMSWE-TX, Research Division
1	AMSWE-IM, Industrial Mobilization Branch
1	AMSWE-GU, Security Officer
	Commanding Officer, Diamond Ordnance Fuze Laboratory, Connecticut Avenue and Van Ness Street, N. W., Washington 25, D.C.
1	ATTN: AMXDO-TIB
	Commanding Officer, Frankford Arsenal, Philadelphia 37, Pennsylvania
1	ATTN: SMUFA-1330
1	SMUFA-0270, Library
	Commanding Officer, Picatinny Arsenal, Dover, New Jersey
1	ATTN: AMSMU, Mr. J. J. Scavuzzo, Plastics and Packaging Laboratory
1	Commanding Officer, PLASTEC, Picatinny Arsenal, Dover, New Jersey
	Commanding Officer, Springfield Armory, Springfield 1, Massachusetts
1	ATTN: SWESP-TX, Research and Development Division
	Commanding Officer, Watertown Arsenal, Watertown 72, Massachusetts
1	ATTN: SMIWT-EX, Chief, Engineering Division
1	SMIWT-OE, Industrial Engineering Section
	Commanding Officer, Watervliet Arsenal, Watervliet, New York
1	ATTN: SWEV-RR
	Commanding General, U. S. Army Chemical Warfare Laboratories, Army Chemical Center, Maryland
1	ATTN: Technical Library
	Commanding Officer, Engineering Research and Development Laboratory, Fort Belvoir, Virginia
1	ATTN: Materials Branch
1	Commanding Officer, U. S. Army Environmental Health Laboratory, Army Chemical Center, Maryland
	Commanding General, Quartermaster Research and Development Command, Natick, Massachusetts
1	ATTN: AMXRC, Clothing and Organic Materials Division

No. of
Copies

TO

Headquarters, U. S. Army Signal Research and Development
Laboratory, Fort Monmouth, New Jersey
1 ATTN: Materials Branch

1 Director, Army Research Office, Office, Chief of Research and
Development, The Pentagon, Washington 25, D. C.

1 Commanding Officer, Army Research Office (Durham), Box CM,
Duke Station, Durham, North Carolina

Chief of Research and Development, U. S. Army Research and
Development Liaison Group, APO 757, New York
1 ATTN: Dr. B. Stein

Chief, Bureau of Naval Weapons, Department of the Navy,
Room 2225, Munitions Building, Washington 25, D. C.
1 ATTN: RMMP

Chief, Bureau of Ships, Department of the Navy, Washington 25, D.C.
1 ATTN: Code 344

Chief, Office of Naval Research, Department of the Navy,
Washington 25, D. C.
1 ATTN: Code 423

Chief, Special Projects Office, Bureau of Naval Weapons,
Department of the Navy, Washington 25, D. C.
1 ATTN: SP 271

Commander, U. S. Naval Ordnance Laboratory, White Oak,
Silver Spring, Maryland
1 ATTN: WM

Commander, U. S. Naval Ordnance Test Station, China Lake, California
1 ATTN: Technical Library Branch

Commander, U. S. Naval Research Laboratory, Anacostia Station,
Washington 25, D. C.
1 ATTN: Technical Information Center

U. S. Air Force Directorate of Research and Development,
Room 4D-313, The Pentagon, Washington 25, D. C.
1 ATTN: Lt. Col. J. B. Shipp, Jr.

ARDC Flight Test Center, Edwards Air Force Base, California
1 ATTN: Solid Systems Division, FTRSC

No. of Copies	TO
2	AMC Aeronautical Systems Center, Wright-Patterson Air Force Base, Ohio ATTN: Manufacturing and Materials Technology Division, LMBMO
1	National Aeronautics and Space Administration, 1520 H Street, N. W., Washington 25, D. C. ATTN: Mr. G. C. Deutsch
1	Mr. R. V. Rhode
1	Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California ATTN: Dr. L. Jaffe
1	George C. Marshall Space Flight Center, National Aeronautics and Space Administration, Huntsville, Alabama ATTN: M-S&M-M
1	M-F&AE-M
5	Commanding Officer, U. S. Army Materials Research Agency, Watertown 72, Massachusetts ATTN: AMQMR-LXM, Technical Information Section
1	AMQMR-OPT, Office of Programs and Technology
1	AMQMR, Dr. R. Beeuwkes, Jr.
1	Author

69 -- TOTAL COPIES DISTRIBUTED

AD. Accession No. Watertown Arsenal Laboratories, Watertown 72, Mass.
RECTANGULAR TENSILE BAR WEAKENED BY SURFACE CRACKS,
PART I, STRESS ANALYSIS - Oscar L. Bowie
Report No. WAL TR 811.8/5, Mar 1963, 16 pp - illus,
AMC Code 5011.11.838, D/A Proj 1-A-0-10501-B-010,
Unclassified Report

The stress analysis of a rectangular tensile bar weakened by surface cracks is formulated in terms of mapping functions of a complex variable. Although an approximate solution of this problem has been previously considered by Tait, the present derivation specifies the proper load-free boundary conditions on the lateral edges of the rectangle as well as allowing for the variation of both crack length and length/width ratios of the plate. Determination of the stress function is reduced to the solution of a linear system of simultaneous equations.

NO DISTRIBUTION LIMITATIONS

UNCLASSIFIED

1. Elasticity

2. Crack stresses

I. Bowie, Oscar L.

II. AMC Code 5011.11.838

III. D/A Proj 1-A-0-10501-B-010

UNCLASSIFIED

1. Elasticity

2. Crack stresses

I. Bowie, Oscar L.

II. AMC Code 5011.11.838

III. D/A Proj 1-A-0-10501-B-010

AD. Accession No. Watertown Arsenal Laboratories, Watertown 72, Mass.
RECTANGULAR TENSILE BAR WEAKENED BY SURFACE CRACKS,
PART I, STRESS ANALYSIS - Oscar L. Bowie
Report No. WAL TR 811.8/5, Mar 1963, 16 pp - illus,
AMC Code 5011.11.838, D/A Proj 1-A-0-10501-B-010,
Unclassified Report

The stress analysis of a rectangular tensile bar weakened by surface cracks is formulated in terms of mapping functions of a complex variable. Although an approximate solution of this problem has been previously considered by Tait, the present derivation specifies the proper load-free boundary conditions on the lateral edges of the rectangle as well as allowing for the variation of both crack length and length/width ratios of the plate. Determination of the stress function is reduced to the solution of a linear system of simultaneous equations.

NO DISTRIBUTION LIMITATIONS

AD. Accession No. Watertown Arsenal Laboratories, Watertown 72, Mass.
RECTANGULAR TENSILE BAR WEAKENED BY SURFACE CRACKS,
PART I, STRESS ANALYSIS - Oscar L. Bowie
Report No. WAL TR 811.8/5, Mar 1963, 16 pp - illus,
AMC Code 5011.11.838, D/A Proj 1-A-0-10501-B-010,
Unclassified Report

The stress analysis of a rectangular tensile bar weakened by surface cracks is formulated in terms of mapping functions of a complex variable. Although an approximate solution of this problem has been previously considered by Tait, the present derivation specifies the proper load-free boundary conditions on the lateral edges of the rectangle as well as allowing for the variation of both crack length and length/width ratios of the plate. Determination of the stress function is reduced to the solution of a linear system of simultaneous equations.

NO DISTRIBUTION LIMITATIONS

UNCLASSIFIED

1. Elasticity

2. Crack stresses

I. Bowie, Oscar L.

II. AMC Code 5011.11.838

III. D/A Proj 1-A-0-10501-B-010

UNCLASSIFIED

1. Elasticity

2. Crack stresses

I. Bowie, Oscar L.

II. AMC Code 5011.11.838

III. D/A Proj 1-A-0-10501-B-010