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RESEARCH IN INFRARED INTERFEROMETRY

AND OPTICAL MASERS

Bruce Billings

Baird-Atomic, Inc.
33 University Road
Cambridge, Massachusetts

Contract No. AF 19(604)-2264

FINAL REPORT

VOLUME I

21 February 1963

Project 7670

Task 76703

Geophysics Research Directorate

Air Force Cambridge Research Laboratories

Office of Aerospace Research

United States Air Force

Bedford, Massachusetts

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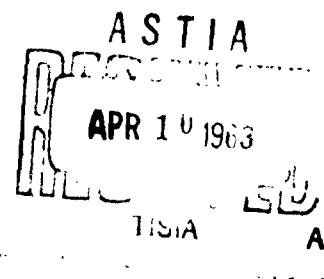Cambridge 38, Massachusetts

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VOLUME I

INTERFEROMETRIC SPECTROSCOPY

ABSTRACT

This final report describes the work performed on Contract AF 19(604)-2264 during the period 1 March 1957 through 31 August 1962.

For purposes of presentation the report is broken into two distinct and independent sections. The first section deals with the interferometric spectroscopy studies performed during the early years of this program. Included are results and conclusions. The second section deals with the maser research performed during the last six months of this program. Included are theoretical considerations of photons in a laser cavity, techniques for isolating a single photon state, macroscopic effects associated with circularly polarized operation of a laser, conclusions.

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VOLUME I

INTERFEROMETRIC SPECTROSCOPY

1. SCOPE

The earlier phases of this contract covered an investigation into the possibility of obtaining near-infrared spectra by recording interferograms from a two-beam interferometer and deriving the spectra by numerical methods. The basic purpose of the program was to discover experimentally the circumstances under which this method is more advantageous than conventional spectroscopy.

The effort naturally divides into two phases:

a. design and construction of an interferometer and recording of interferograms.

b. investigation of computational techniques for making the transformation from interferogram to spectrum.

During most of the program, effort on these two phases was carried on concurrently.

2. INTRODUCTION

Consider a Michelson interferometer as modified by Twyman and Green, that is, with a collimated input beam and the output beam condensed onto a small detector. Assume that one of the end mirrors of the interferometer is moved to change the optical path in one arm of the interferometer and the output of the detector is recorded.

If the source radiated only a single line with a wavenumber, $\nu \, cm^{-1}$, the intensity at the detector would be a 100 percent modulated sine wave of frequency $2 \nu v$ where v is the speed of the moving mirror, and the factor of two is required because the change in optical path is twice the distance the mirror moves. For a wide-band source the modulation of light intensity is the sum (or integral) of the modulations corresponding to the individual frequencies present in the radiation. In this case the recorded output of the detector is called an interferogram and is in fact the Fourier transform of the spectrum of the source, optical system and detector response.

The spectrum of a source can be found by mathematically performing the inverse transformation from interferogram to spectrum. To perform this calculation one must, as a minimum, measure the amplitude of the interferogram at points corresponding to equally spaced mirror positions, of spacing $1/2 \, \Delta \nu$ where $\Delta \nu$ is the total frequency range in the spectrum. The general formulation of the required transformation is well-known; the major mathematical effort has been devoted to selecting and programming a method of performing the transformation which is efficient both in terms of recovering available information and in speed and precision of digital computing. The method selected adds the requirement that a precise mirror position scale must be added to the recorded interferogram.

This method of getting a spectrum is of interest because it has two important advantages over conventional spectroscopic methods. First the Twyman-Green interferometer has no limitation corresponding to the practical limits to slit

heights in a grating spectrometer, and calculation shows that for the same resolution and collimated beam cross section the interferometer can accept 30 to 100 times more energy from a given source. This advantage in luminosity for an interferometer is well known and in principle can always be used to provide superior performance in some way. Also the detector is receiving radiation from each resolved frequency interval throughout the recording time rather than only for the time taken to scan this frequency interval across the slit of a grating spectrometer. Thus the interferometer has an integration time advantage in the ratio of one to the number of intervals resolved, provided the recording time is the same for the interferometer as for the equivalent spectrometer. This fact is of no advantage if the detector is radiation noise limited but in the usual case for the infrared of a detector limited by its own internal noise there is a gain in signal/noise ratio equal to the square root of the number of intervals resolved. In a case of this type for a given resolution, instrument size, recording time and source the interferometer may have an S/N advantage of 1000 or more times.

The following paragraphs describe the several variations of instruments and techniques used for recording interferograms, the method of calculation of spectra from interferograms, some results of the combined process and the practical considerations which indicate the most promising areas of application of this method.

3. INTERFEROMETERS AND INTERFEROGRAMS

In order to give a complete review of the experimental effort, this portion describes in chronological order the several designs of interferometers and associated optics built and used to record interferograms.

3.1   Interferometer No. 1

The first interferometer built was a conventional Michelson with 4-inch optics. Calcium fluoride plates were used for the beam splitter and compensator, with an evaporated germanium film on the beam splitter. The detector was a lead sulfide (PbS) cell. The screw and ways of a standard Hilger 1-inch interferometer were used to translate the moving end mirror. A test interferogram recorded with this instrument is shown in figure 1.

After assembly and limited testing, the instrument was taken to the Jung-fraujoch, Switzerland, to attempt to record OH emission in the night sky. Some interferograms were made but no attempt was made to reduce data because the instrumental operation was patently unsatisfactory. The difficulty seemed to be a combination of the effects of overloading the small Hilger ways, and variations in the optical paths in the interferometer caused by vibrations and by air disturbances.

3.2   Interferometer No. 2

The attempted field operations clearly indicated the need for improvement of instrumental stability and accuracy. A laboratory interferometer and photometric system were built in order to investigate these difficulties and to attempt to discover to what extent and how best they could be avoided.

The instrument was placed on double anti-vibration mountings and the inter-ferometer proper enclosed in a Celotex box which acted as a light shield and reduced thermal gradients in the interferometer structure, optics and paths. As a result of these two precautions, interferometric noise was always less than noise due to other causes. Interferometric noise is defined as that due to spurious
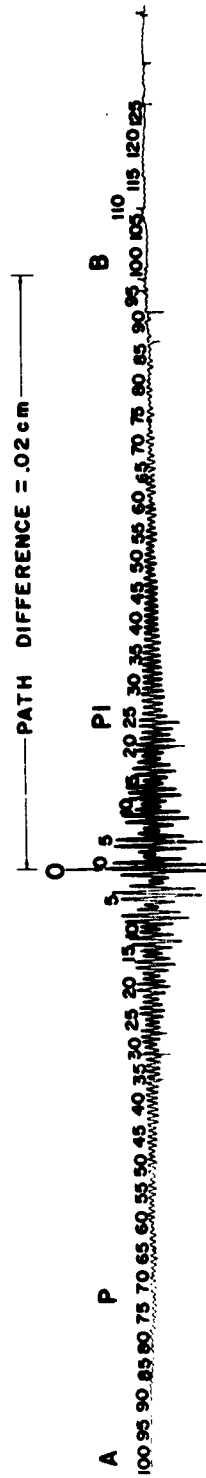
Figure 1. Typical Interferogram Made With Interferometer No. 1

variations of the path difference between the two arms of the interferometer caused either by unwanted relative motion of the optical components or by changing thermal gradients thus causing changes in refractive index in the air filling the paths and these unwanted changes in path difference.

The instrument was a Michelson interferometer used in the manner invented by Twyman and Green and is shown diagramatically in figure 2. C is an iris diaphragm which forms the entrance aperture of the instrument, light from the source, A, being focused onto it by the concave mirror, B. D is a collimating mirror so that an essentially parallel beam falls on the phase splitting mirror, E. This is very lightly aluminized on its rear surface and in this case was like the compensating plate, F, a plate of optical quality fuzed quartz three inches in diameter and about 1/2-inch thick. G is the fixed plane mirror and H the moving plane mirror whose mount can be moved along accurately lapped ways by means of a lead screw driven by a synchronous motor through suitable reduction gearing. Using pieces of Teflon sheet as bearings for the mirror mount on its ways and maintaining correction lubrication of the screw gearing and bearings, no trouble was experienced in obtaining smooth movement of the moving mirror.

Originally the parallel beam of radiation leaving the instrument was focused onto a PbS detector, a conventional mechanical chopper being used to interrupt the radiation at 450 cps. The output of the detector was amplified in a narrow band 450 cps amplifier, the amplified signal rectified and displayed on a pen recorder. To record the washed-out fringes whose amplitude was often only about one percent of the total signal, arrangements were made in the electronics to back off the d-c part of the rectified signal. In this way it was possible to make the a-c part of the fringes fill an appreciable part of the recorder paper. This, however, proved unsatisfactory as drift throughout the system could not be controlled adequately and the very large 450 cps signal corresponding to the d-c level of the washed-out fringes overloaded the amplifier before the much smaller part of the signal corresponding to the a-c component of the fringes was amplified sufficiently to give adequate deflections of the recorder pen.
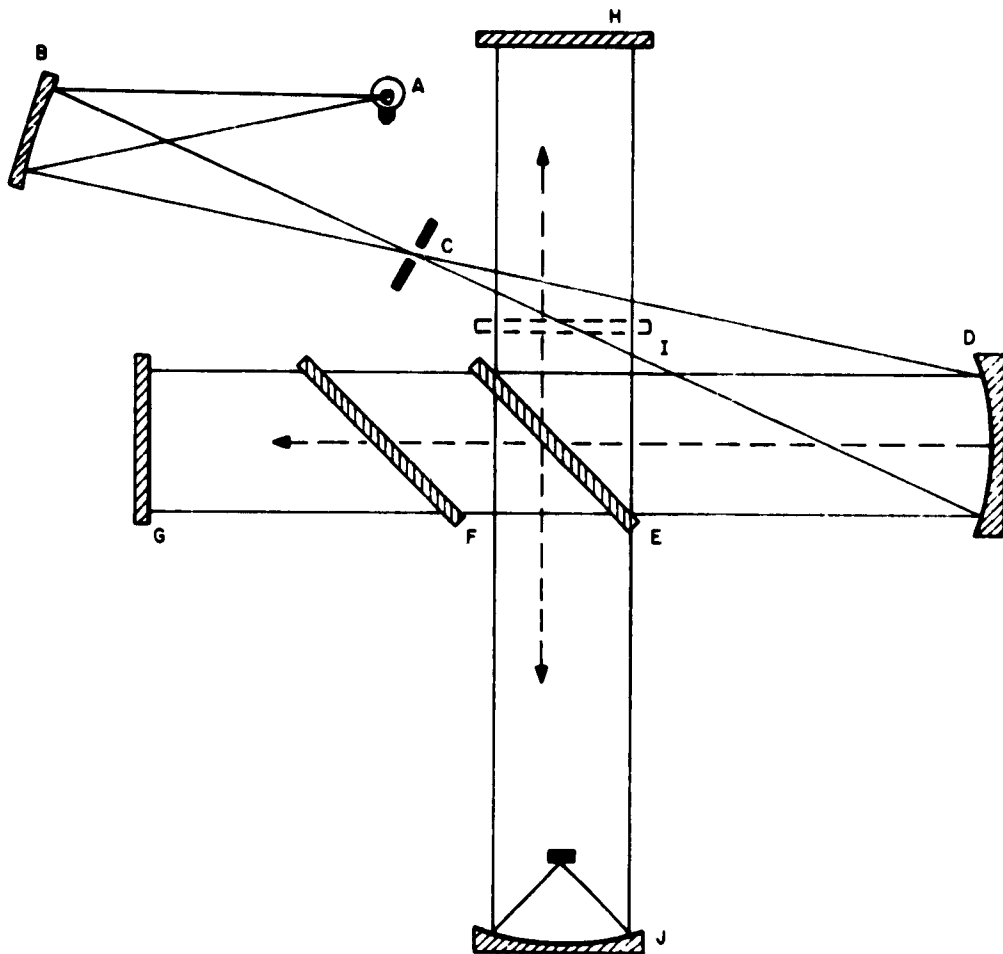
Figure 2. Diagram of Interferometer No. 2

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To avoid these difficulties, a two beam method of working was introduced which has proved much more satisfactory. A plane mirror, I, in figure 1, of half the height of the interferometer mirrors G and H was placed in front of the moving mirror, H, so as to obscure the lower half of H. As the path difference between light reflected from this mirror and from the lower half of G is large and unchanging, and also because I is neither of high quality nor interferometrically aligned, it suppresses interference effects in the lower half of the exit beam which thus forms a reference beam whose intensity is equal to the mean intensity of the upper half of the beam. Two equal apertures are placed one above the other in the parallel exit beams so that the upper aperture passes radiation in which interference is taking place and the lower aperture passes only the unchanging reference radiation. These two beams are interrupted in antiphase by means of a sectored chopper driven by a synchronous motor. Both beams are focused onto a single PbS detector whose output is amplified as before. Now, however, the 450 cps signal entering the amplifier is proportional to the difference in level in the two beams and when the fringe amplitude is small the gain of the amplifier can be raised so that the fringes are recorded at a reasonable level.

This is an inefficient use of the interferometer as only about five percent of the area of the interfering wavefronts is used. However, it was still necessary to underrun the tungsten lamp source to avoid overloading the PbS detector. This demonstrates the energy advantage of this method referred to above.

With this apparatus it was possible to record interference fringes from broad or narrow sources over path differences of several centimeters corresponding to a spectral resolution of about 1 cm^{-1}. Figure 3 shows interferograms taken with a PbS detector and various sources. These interferograms showed a great deal better resolution and repeatability than those made with Interferometer No. 1.
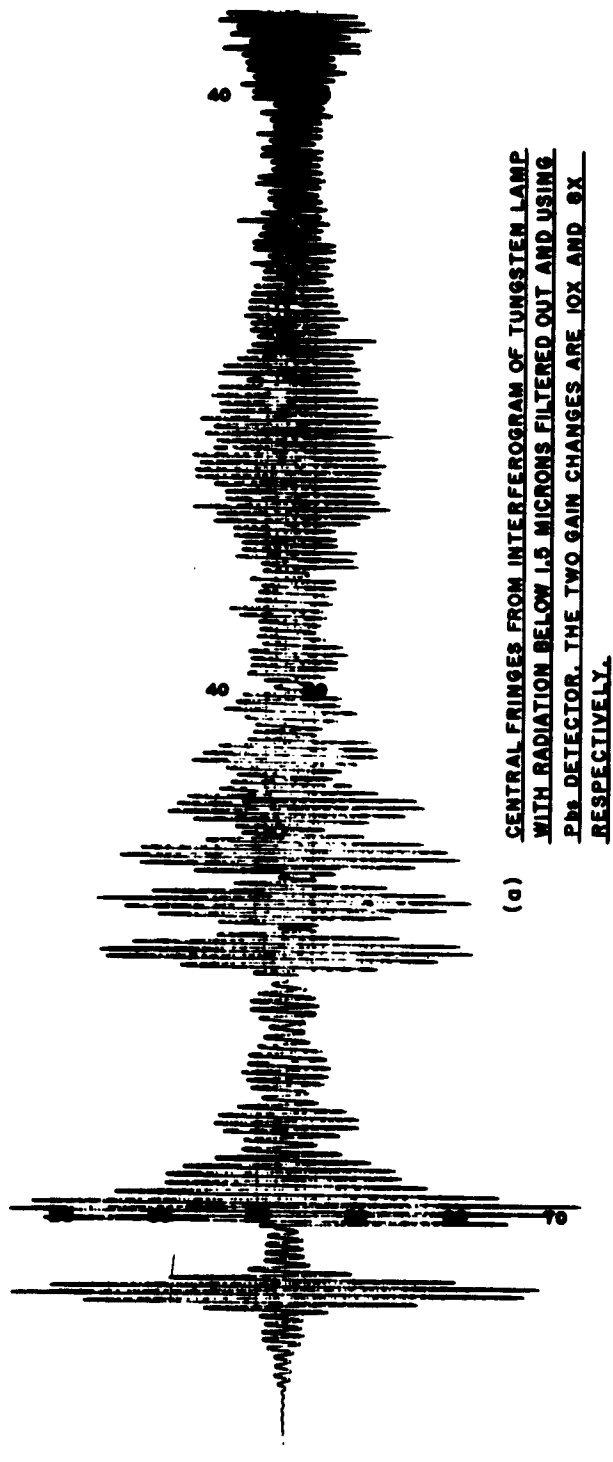
(a) CENTRAL FRINGES FROM INTERFEROGRAM OF TUNGSTEN LAMP WITH RADIATION BELOW 1.5 MICRONS FILTERED OUT AND USING PbS DETECTOR. THE TWO GAIN CHANGES ARE 10X AND 9X RESPECTIVELY.

(b) FROM THE SAME INTERFEROGRAM AS FIGURE (a) BUT FOR PATH DIFFERENCE OF ABOUT 5MM.

Figure 3. Interferogram of Tungsten Lamp

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3.3   Modifications of Instruments

At this time the instrumental revisions considered desirable were:

a. Add a means of recording the position of the moving mirror precisely
in terms of path difference because the computational theory indicated that the
amplitude of the interferogram had to be measured at evenly spaced intervals
for optimum use of data.

b. For further work with a PbS detector, return to the use of the original
Hilger 1-inch glass optics to obtain better long time stability of alignment of
interferometer mirrors.

c. Further identification of the source of noise in the interferograms which
appeared to be associated with the balancing system and was an order of magnitude
above detector noise.

The only reasonable way to determine mirror position to a small part of an
interference fringe period is of course to calibrate the instrument using fringes
from a narrow isolated emission line. The Hg 5461 line was used for the path
difference calibration. It shared the full linear aperture of the interferometer,
being introduced and separated by dichroic mirrors. The transmission of one
of the dichroic mirrors is shown in figure 4. Recording of the mercury fringes
was done with the recorder's auxiliary "pipper" pen. This pen draws shallow
rectangular waves on the edge of the recording paper when actuated by a solenoid.
The output of the photomultiplier detecting the mercury fringes was amplified and
clipped to produce a square wave to operate a relay controlling the solenoid.

During test of the mercury recording system, the light from the mercury
lamp, which is modulated at 120 cps by its power supply, was chopped at 450 cps
and the output of the photomultiplier displayed on an oscilloscope. Examination
of the signal showed a bounce which made it clear that the synchronous chopper
motor was hunting rather than holding synchronous speed. This effect in combina-
tion with the narrow bandpass amplifier would cause an amplitude modulation of
the interferogram and was the cause of the noise noted with Interferometer No. 2.
Replacement of the chopper motor eliminated this difficulty.
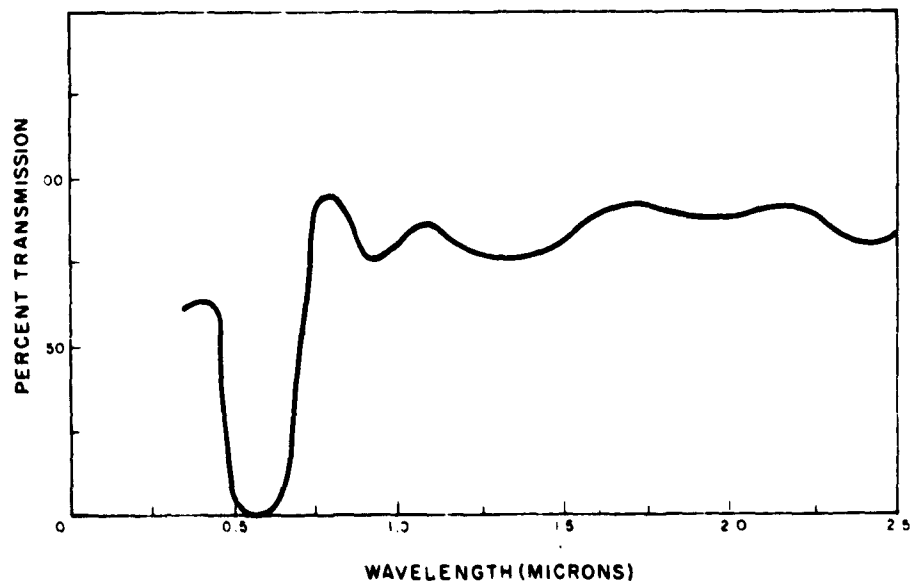
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Figure 4. Transmission of Dichroic Plate at 45-Degree
Incidence

By using a very small area PbS detector it was possible to simplify the primary optical system somewhat as shown in figure 5. There is a virtual image of this detector at the source. Since this detector image acts as an input aperture, so it is not necessary to use an input aperture and focus the source onto it with an extra optical element. Also since the source is much larger than the detector image, lenses can be used rather than mirrors as chromatic and other aberrations are unimportant. Also with this arrangement the position of the large source is not critical.

The balancing system was simplified by omitting the two apertures used with Interferometer No. 2. A stationary mirror is still used in front of and covering the lower half of the moving mirror. The sectored chopper is placed in the parallel beam immediately in front of the condensing lens so that each blade alternately covers the upper and lower half of the beam. By adjusting the size of the beam with an iris diaphragm and moving the center of the chopper blade relative to the beam while the interferometer is out of adjustment a good balance between the two halves of the beam can be obtained fairly readily. Fine adjustment of balance was obtained by a trial and error process of inserting a small object, such as a pointed scribe in the correct position in the beam.

With this interferometer, employing an underrun tungsten lamp and a PbS detector, several interferograms with mercury calibration fringes were recorded. These interferograms contained between several hundred and a few thousand data points and were used for the transformation calculations described below.

3.4   New Interferometer to be used with Lead Selenide and Indium Antimonide

The PbS region in which the work described above was done is not especially interesting, both because very few convenient test samples have absorbtion bands in this region and because there is little need for the energy gains of the interferometric method since powerful sources and sensitive detectors are available. Therefore, a new interferometer was built for use with lead selenide and indium
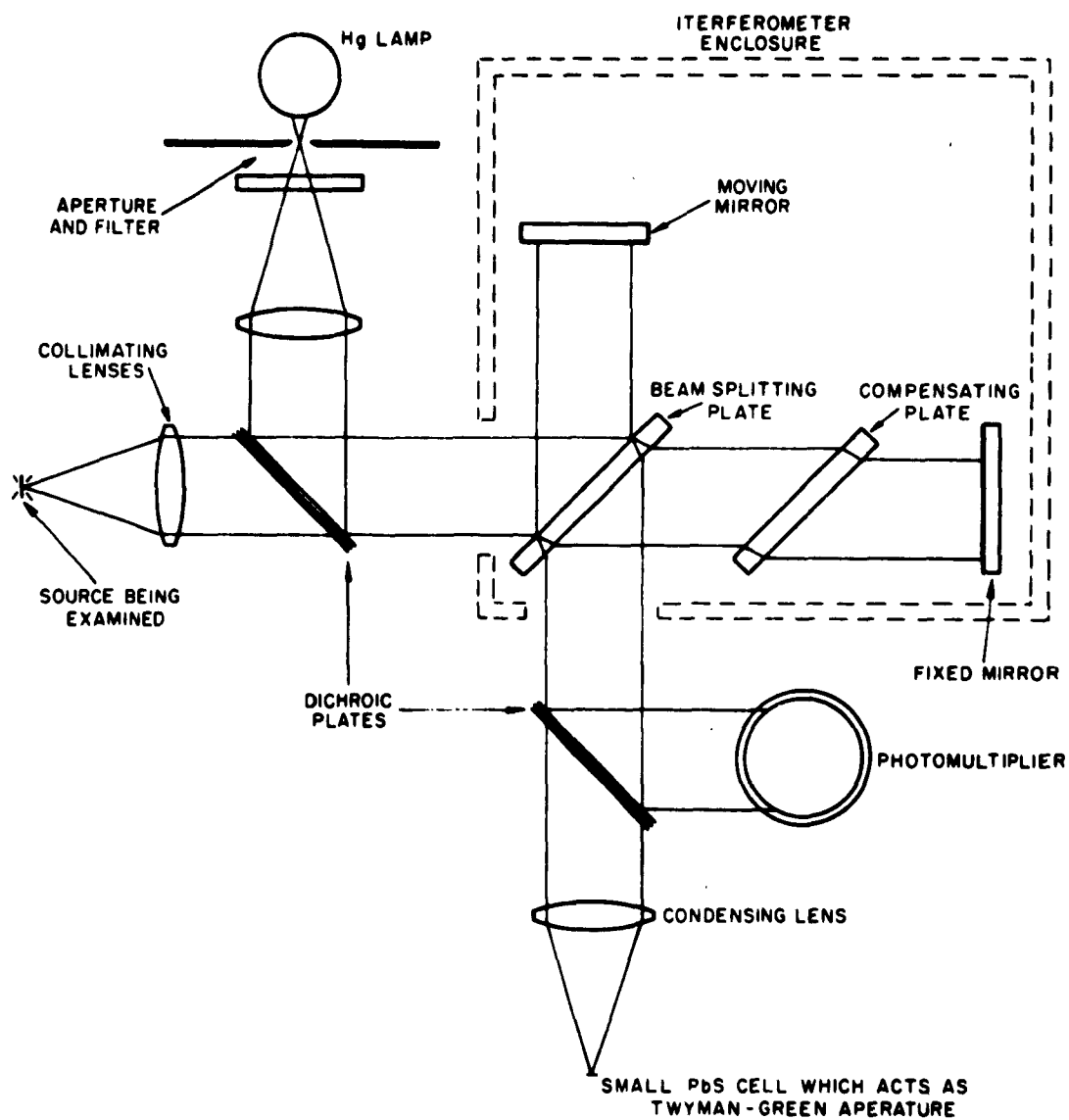
Figure 5. Optical System (Balancing System not shown)

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antimonide detectors at longer wavelengths. The arrangement of this interferometer is shown in figure 6. Several new features were incorporated in this instrument.

The beam splitter and compensator were calcium fluoride plates to permit longer wavelength operation. The interferometer was rearranged to have an angle of incidence of 22-1/2 degrees instead of 45 degrees. Also the beam splitter and compensator were placed in the same holder, almost in contact. These variations of the conventional arrangement were tried merely to assure that no difficulties were encountered. It should be noted that the smaller angle of incidence permits the use of smaller plates for the same linear aperture. The beam splitting coating employed dielectric layers which were 5/4 waves thick at 5461A and therefore 1/4 wave thick at 2.7 microns. This provides a beam splitter which is equally efficient at both wavelength of interest.

The collimation and condensing of the measured radiation is conventional, but a somewhat more elaborate and more convenient beam balancing technique was used. One half of the fixed mirror was coated with a dielectric layer of 1/4-wave mechanical thickness at 2.7 microns before aluminizing. Thus, there is always a path difference of 1/2 wave between the two halves of the beam so that the fringes from opposite halves of the beam are always out of phase. This doubles the energy difference between the halves of the beam and is a more efficient use of the interferometer. To obtain the anti-phase chopping of the two halves of the beam, the chopper wheel is located so that the collimating mirror of the interferometer also images the wheel on the end mirrors of the interferometer. The magnification (in this case 1:1) is chosen so that one blade and one space of the chopper wheel just cover an end mirror.

This interferometer was completely assembled and aligned, but experimental effort terminated before any useful interferograms were recorded.
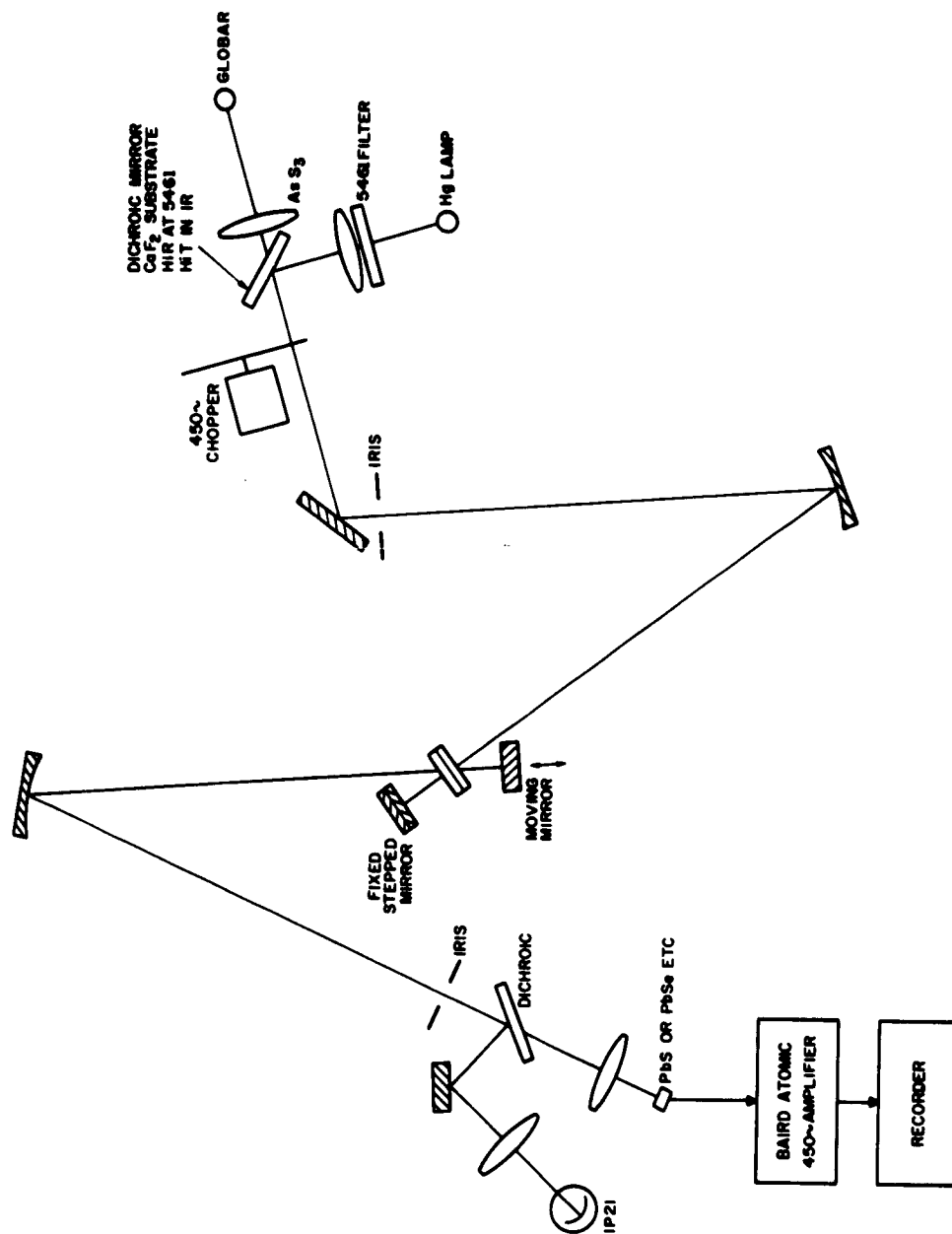
Figure 6. Interferometer No. 4

4. COMPUTING METHODS

Consider a Michelson Interferometer. For a narrow band of wave numbers "$d\nu$", the amplitude and phase in one beam are effectively constant and can be represented by the expression:

$$\rho_1 \, E^{1/2} \cos \left[2\pi\nu(z-vt) + \phi_1 \right] \qquad (1)$$

where:

    $E d\nu$ is the intensity of the beam in the wave number interval $d\nu$ entering the instrument

    ρ_1 is the factor by which the amplitude of the first interfering beam is less than E as a result of the division of amplitude at the beam splitting mirror and any reflection or absorption losses throughout its path

    z is distance along the beam

    v is the wave velocity and t is time

    ϕ_1 gives the phase of the wave leaving the instrument

Note that ρ_1, E and ϕ_1 may all vary with ν.

Similarly the second beam can be represented by:

$$\rho_2 \, E^{1/2} \cos \left[2\pi\nu(z-vt) + \phi_2 + 2\pi\nu\chi \right] \qquad (2)$$

Here "χ" is the path difference between the two beams brought about by altering the position of the moving mirror. The value of "ϕ_2" will depend on the choice of the origin of χ. Putting $\delta = O$ when the moving mirror is in such a position that the optical paths in the two beams are equal seems an obvious choice but, in fact, this position will vary with wave number unless $\phi_1 - \phi_2$ is independent of wave number.

(It is often assumed that, provided a compensator plate is correctly used the fringes formed by a Michelson interferometer will satisfy this requirement. This is equivalent to saying that the fringe pattern is symmetrical about a point corresponding to zero path difference. This is rarely true in practice, and cannot be true for the usual form of the Michelson interferometer because the air gap between the beam splitter and the compensator introduced asymmetry.)

The sum of equations (1) and (2) gives the instantaneous amplitude of the resultant beam emerging from the instrument:

$$E^{1/2} \left[\rho_1 \cos \overline{2\pi \nu(z-vt) + \phi_1} + \rho_2 \cos \overline{2\pi \nu(z-vt) + \phi_2 + 2\pi\nu\chi} \right].$$

By suitable manipulation this can be shown to be:

$$E^{1/2} A \cos \overline{2\pi \nu(z-vt) + \alpha} \tag{3}$$

where

$$A = \sqrt{\rho_1^2 + \rho_2^2 + 2\rho_1\rho_2 \cos \overline{\phi_1 - \phi_2 - 2\pi\nu\chi}}$$

and

$$\tan \alpha = \frac{\rho_1 \sin \phi_1 + \rho_2 \sin \overline{\phi_2 + 2\pi\nu\chi}}{\rho_1 \cos \phi_1 + \rho_2 \cos \overline{\phi_2 + 2\pi\nu\chi}} \ .$$

The intensity of the combined beams is the time average of the square of the instantaneous combined amplitude and is thus:

$$E_o(\nu) = \overline{E A^2 \cos^2 \overline{2\pi\nu(z-vt) + \alpha}}$$

$$= \frac{1}{2} E A^2$$

$$= \left[\frac{\rho_1^2 + \rho_2^2}{2} E + \rho_1\rho_2 E \cos \overline{\phi_1 - \phi_2 - 2\pi\nu\chi} \right]$$

If we examine this with a detector (or filter-detector combination) whose wave number response is given by $S(\nu)$ we obtain for the output of the detector:

$$O(\chi) = \int_0^\infty S(\nu)\, E_o(\nu)\, d\nu$$

and we see that

$$O(\chi) = \frac{1}{2} \int_0^\infty (\rho_1^2 + \rho_2^2)\, E\, S\, d\nu + \int_0^\infty \rho_1\rho_2\, E\, S\, \cos(\phi_1 - \phi_2 - 2\pi\nu\chi)\, d\nu.$$

The first term is independent of "χ" while the second contains "χ" only as a cosine term so that the first term is the mean value of O. This term is removed from our interferograms by the beam balancing technique so only the second term has to be considered here.

We can write for the varying part of O:

$$O'(\chi) = O(\chi) - \overline{O(\chi)}$$

$$= \int_0^\infty \rho_1\rho_2\, E\, S\, \cos(\phi_1 - \phi_2 - 2\pi\nu\chi)\, d\nu$$

$$= \int_0^\infty \left[\rho_1\rho_2\, E\, S\cos(\phi_1 - \phi_2)\cos 2\pi\nu\chi + \rho_1\rho_2\, E\, S\sin(\phi_1 - \phi_2)\sin 2\pi\nu\chi\right] d\nu \qquad (4)$$

Whence by the rules of Fourier transformation:

$$\rho_1\rho_2\, E\, S\cos(\phi_1 - \phi_2) = 2\int_{-\infty}^\infty O'(\chi)\cos \overline{2\pi\nu\chi}\, d\chi = A(\nu) \qquad (5)$$

$$\rho_1\rho_2\, E\, S\sin(\phi_1 - \phi_2) = 2\int_{-\infty}^\infty O'(\chi)\sin \overline{2\pi\nu\chi}\, d\chi = B(\nu) \qquad (6)$$

By squaring and adding these equations we obtain:

$$\rho_1\rho_2\, E\, S = \sqrt{A^2 + B^2}$$

We note that this expression holds for any value of v so that it can be obtained without knowledge of ϕ.

Now $S(v)$ is limited to a finite range of wavenumber, say v_1 to v_2, outside of which it is sufficiently close to zero to be ignored. Thus since the harmonic content of S is known to be limited, we can apply the sampling theorem and sample $O'(\chi)$ at discrete points, replacing the integral signs in equations (5) and (6) with summations over the "n" sampling points from $-\infty$ to $+\infty$.

The sampling procedure is simplified, so that we can sample O' at equal intervals of χ, if we chose v_1 and v_2 so that $v_2 - v_1 = \Delta v$ and $v_1 = L \Delta v$ where L is either zero or an even positive integer. The sampling interval is the reciprocal of twice the wavenumber interval, which means, for example, that to sample an interferogram of a spectrum lying entirely between one and two microns, the sampling interval is one micron, (that is, the amplitude of the interferogram must be enumerated for every micron of path difference or one-half-micron mirror movement.

In practice it is not possible to have n or χ vary from minus to plus infinity. Limiting n, in fact, sets the resolution with which S can be found.

We limit χ to an experimentally reasonable value and use the simplifications of the sampling method to rewrite the transformation equations as:

$$A_m = \frac{1}{\Delta v} \sum_{n=-N}^{M} O' \left(\frac{n}{2\Delta v}\right) \cos \left(\frac{m n \pi}{N}\right) \tag{7}$$

$$B_m = \frac{1}{\Delta v} \sum_{n=-N}^{N} O' \left(\frac{n}{2\Delta v}\right) \sin \left(\frac{m n \pi}{N}\right) \tag{8}$$

where

$$m = 1, 2, \ldots \ldots N$$

Here the interferograms have been sampled at 2N equally spaced points and we get N independent values of A and B.

In this form the equations are quite simple and the calculation would present no difficulty if it were not for the large number of values of O' obtained from the interferograms. The difficulties involved will be discussed in terms of the IBM 704 machine used on this project.

The first difficulty is that, while the machine can compute the cosine of a given angle in one millisecond, a direct application of equations would require the computation of $2N^2$ cosines for A. This would take 33 minutes if N is 1000 and 56 hours if N is 10,000.

Two methods have been used by other workers to avoid this difficulty. In one method a table of cosine values was read into storage and sines and cosines of the required angles found by polynomial interpolation. This method requires that a large portion of the machine's memory be devoted to storing of the cosines and has been found to be the slowest of the three methods described here.

The second method involves the use of recurrence formula. The cosine of θ can be developed to give in turn the cosines of integral multiples of θ by the standard relation:

$$e^{in\theta} = \cos n\,\theta + i \sin n\,\theta = (\cos \theta + i \sin \theta)^n \tag{9}$$

Let $t_n = 0$, $t_{n-1} = y_{n-1}$ and use the recurrence formula based on (9) in the form:

$$t_\alpha = (2 \cos \theta)\, t_{\alpha+1} - t_{\alpha+2} + y_\alpha \tag{10}$$

where

$$\alpha = n-2,\ n-3 \ldots \ldots 0$$

To obtain:

$$A(\theta) = t_0 - t_1 \cos \theta$$

$$B(\theta) = t_1 \sin \theta$$

The power of this method lies in the achievement of both sine and cosine transforms for a particular frequency with the need to compute only one cosine.

However, terms involving cos M θ, cos (N-1)θ...etc. have been developed as a power series of cos θ. Thus, when the number of terms gets large, errors are introduced even if the original cosine is known to seven or eight places. In practice it seems that with single precision appreciable errors occur with N less than 1000.

Our own method was developed to avoid these errors that accumulate as higher and higher powers of cos θ are evaluated. It was recognized that, in the whole scheme represented by equations (7) and (8), the $2N^2$ cosines and the $2N^2$ sines involve only N+1 different numerical values. These are the values of the cosines in the first N+1 terms in the expansion of A_1, that is, the cosines of π/N multiplied by 0, 1, 2 up to N. All the other cosines and sines needed can be found by use of the formulae:

$$\cos \theta = \cos (2M\pi \pm \theta)$$

and:

$$\sin (\frac{\pi}{2} - \theta) = \cos \theta$$

It was also noted that for any m, O'_n and O'_{2M-n+2} are multiplied by cosines having the same value and by sines having values different only in sign. We therefore form the series $O'_n \pm O'_{N-n+2}$ with O'_1 and O'_{N+1} by themselves as the terminal values and store them. The machine computes and stores the necessary cosines and then draws the paired O' values and the required cosines from storage in the correct sequence to be multiplied together and summed.

All three of the methods discussed have been programmed and tested on the IBM 704. A description of the program is given in Appendix I.

It turns out that our method involved almost the same number of operations as the recurrence formula method and is therefore equally fast while eliminating the inaccuracy of that method. Our method is similar to the direct summation method in that cosine values are stored, but is faster because for each multiplication in our method, the older method requires several multiplications.

5. RESULTS AND CONCLUSIONS

Several of the interferograms recorded on Interferometer No. 3 were transformed by the methods described above. The resulting spectra showed the shape and features expected; that is, the general shape was that for the combination of tungsten lamp, high pass filter and PbS detector spectral characteristics; also the 1.8-micron atmospheric water vapor band could be distinguished. This transformation output data was delivered to AFCRC with the interferometer some time ago.

On the experimental side the most important development is undoubtedly the ingenious beam balancing technique to remove the d-c level of the interferogram. This technique was suggested by Dr. T.K. McCubbin of Pennsylvania State University during summer employment at Baird-Atomic.

We believe our method of performing the transformation computations is near optimum. However, the time involved is still excessive if a large number of input points is involved. Our experience is that 1000 points can be transformed in about 12 minutes. Since the machine time goes as the square of the number of input points, 100,000 points would take approximately three months to transform. This is prohibitive from the financial viewpoint.

It should be noted that it is possible to obtain high resolving power by this method if the overall bandwidth of the energy in the interferogram is limited by an auxiliary filter.

Because of the additional effort and expense involved in performing the transformation, we believe that the use of the interferometric technique is preferable to conventional methods only if a small number of data points (in the order of 100) are used or if the energy gain of the technique makes it the only method for obtaining a desired result.

APPENDIX I

PROGRAM DESCRIPTION

The following is a summary of the formulae and description of the IBM #704 program which was used to obtain the Fourier Transform of a discrete set of data points.

PROBLEM: It is required to obtain the Fourier Transform of a set of N points which are evenly spaced throughout an interval of 2π radians where y_k is the kth point of a set

$$y_1, \ y_2, \ \ldots, \ y_n \ .$$

The Fourier Transforms of the set of N points for a given θ are given by

$$A(\theta) = \sum_{k=1}^{N} y_k \cos (k-1) \theta$$

$$B(\theta) = \sum_{k=1}^{N} y_k \sin (k-1) \theta$$

$A(\theta)$ is the Fourier Cosine transform and $B(\theta)$ its corresponding Sine transform.* The methods used to obtain the transform for a given value of θ are described below.

Method 1 - Recursion Formulae

Define:

$$t_{N+1} = 0$$

$$t_N = y_N \ .$$

Solve for t_k in the following relationship:

$$t_k = (2 \cos \theta) \ t_{k+1} - t_{k+2} + y_k \ ,$$

for $k = N-1, \ldots, 1 \ .$

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*It should be noted that as opposed to standard harmonic analysis $A(\theta)$ and $B(\theta)$ as defined are lacking the factor $2/N$ for $\theta \neq 0$, $1/N$ for $\theta = 0$.

Then $A(\theta) = t_1 - t_2 \cos \theta$,

$B(\theta) = t_2 \sin \theta$.

Method 2 - Direction Summation

Method 3 - Construction of Even and Odd Functions

An even function F and a corresponding odd function G were constructed as follows:

$$F_1 = y_1$$

$$F_k = y_k + y_{N-(k-2)}, \quad k = 2, \ldots, \frac{N}{2}$$

$$F_{\frac{N}{2}+1} = y_{\frac{N}{2}+1}$$

and

$$G_1 = y_1$$

$$G_k = y_k - y_{N-(k-2)}, \quad k = 2, \ldots, \frac{N}{2}$$

$$G_{\frac{N}{2}+1} = y_{\frac{N}{2}+1} \quad .*$$

Note that this necessitates that the number of points N be even.

Then $$A(\theta) = \sum_{k=1}^{N/2+1} F_k \cos (k-1) \theta$$

$$B(\theta) = \sum_{k=1}^{N/2+1} G_k \sin (k-1) \theta \quad .$$

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* See, for example, W. E. Milne, Numerical Calculus, Princeton University Press, 1949, pp. 294-305.

LIMITATIONS OF METHODS:

There are limitations to each of the methods previously described - the limitations being those imposed by either storage considerations, timing, or those of an arithmetic nature imposed by the numerical process. These latter type limitations are evident in "round-off" type errors.

Method 1: No storage limitations; handles data in blocks. In processing data (N = 1000) it was noticed that this method in several instances differed from the results of Methods 2 and 3. The difference was small, amounting to 2 or 3 units in the 6th or 7th place. It is possible that the differences were attributable to the manner in which the data were processed and to the variations in the magnitudes of the numbers involved.

Method 2: Limitations imposed are of two types: storage and numerical. The machine used in this program was an 8K IBM #704; it was necessary to reserve approximately 3000 storages for the program leaving 5000 for data. A maximum of 4001 cosines is permitted to be stored internally in the machine; the y values are processed in blocks of 500. A maximum of 16,000 data points is allowed since only the cosines for angles in Quadrant I are stored. It follows that N must be divisible by 4.

Numerical limitations could influence the accuracy of the results inasmuch as the sizes of the numbers may fluctuate widely. It would be better if numbers of the same order of magnitude could be processed together. This, however, would present some difficult problems in data handling, and hence it would not be expedient to do so. If it is known that the magnitudes of the numbers would fluctuate widely, it would be better to reprogram this section and make use of double precision techniques.

Method 3: The limitations of this method are similar to those mentioned for Method 2. In this case, however, the even and odd functions constructed (F, G) are stored internally. A maximum of 4000 data points is permissible. If more than 4000 points were to be handled, then use would have to be made of magnetic tape storage and, again, several problems of data manipulation arise, in which case Method 3 would have to be reprogrammed.

Several additional points concerning all the methods should be brought out, viz.:

(1) The machine used was an 8K IBM# 704. If a "larger" machine were used, more data could be handled more efficiently and more rapidly, inasmuch as tape use could be held to a minimum.

(2) When the problem was programmed, the range of N was to have been quite extensive, i.e., $0 < N \leq 10^5$, and as such the programming was meant to be as flexible as possible. The storage restrictions on each of the three methods have been stated. Until production was actually accomplished, no estimate of timing was available. On the basis of several production cases, timing for the different methods is given below.

Method 1	12 min., 35 sec.	
Method 2	15 min.$^+$	1000 input, 500 output points
Method 3	12 min., 53 sec.	

Method 2 is the slowest, undoubtedly due to greater use of magnetic tapes; Methods 1 and 3 are comparable. Assuming it takes 1.5×10^{-3} sec/input-output point (Methods 1, 3), it is easily estimated that 10^5 points input with 0.5×10^5 output points would take

$$(0.5)(10^{10})\, 1.5 \times 10^{-3} = 0.75 \times 10^7 \text{ sec} = 90 \text{ days}.$$

It appears that the upper bound on the number of points is rather optimistic. In general it appears that Method 3 is most practical from a consideration of timing and accuracy. It is for this reason that Method 3 was incorporated into "Version II."

PROGRAM

Output from the program consists of $A(\theta)$, $B(\theta)$, θ, ϕ, ν and λ where

$$\theta = \beta\, (\overline{\Delta \theta}), \quad \theta, \Delta \text{ input}, \quad \beta \text{ is sequence count (radians)}$$

A, B are previously defined

$$C = \sqrt{A^2 + B^2}$$

$$\phi = \tan^{-1} (B/A) \text{ (expressed in degrees)}$$

$$\nu = \frac{\theta}{2\pi\,(\delta x)} \text{ cm}^{-1}$$

$$\lambda = \frac{2\pi\,(\overline{\delta x})}{\theta}\, 10^4 = \frac{10^4}{\nu} \text{ cm}$$

There are two distinct versions of the program which will be numbered I, II. Version I contains the three (3) different methods of computing, whereas Version II contains only Method 3 (SHOCUT).

It is possible to accomplish several different options in either Version I or II, as follows. (How to do this will be indicated in a later section.)

Versions I and II

(1) Five floating point variables from the list A, B, C, ϕ, ν, λ, θ may be selected to be punched out on cards (off line, from tape 6).

(2) Weighting (aphodizing) can be accomplished in several ways:

    a. No weighting

    b. Weighting by prescribed functions

    c. Weighting by values read in on y value card, e.g., for each y_k there is a W_k.

(3) The average $\dfrac{\sum\limits_{k=1}^{N} y_k}{N}$ may or may not be subtracted from each point y_k (effectively subtracting d.c. component).

(4) Correction for drift is permitted as desired.

(5) Constants as described in first footnote (page 1) may or may not be used.

(6) The number of transforms desired is a specified variable.

Version I

(1) Can select any of the three methods.

(2) This method always starts with $\beta = 1$ or A_o, B_o.

Version II

(1) Can force a manual dump and similarly can start, bypassing certain preliminary calculation portions.

(2) Can require calculation and print-out of A; B; or A, B, C, ϕ.

(3) Can start with calculation of any prescribed mth transform where

       $M = \beta - 1$

and proceed to calculate as many transforms as desired.

PROGRAM DESCRIPTION
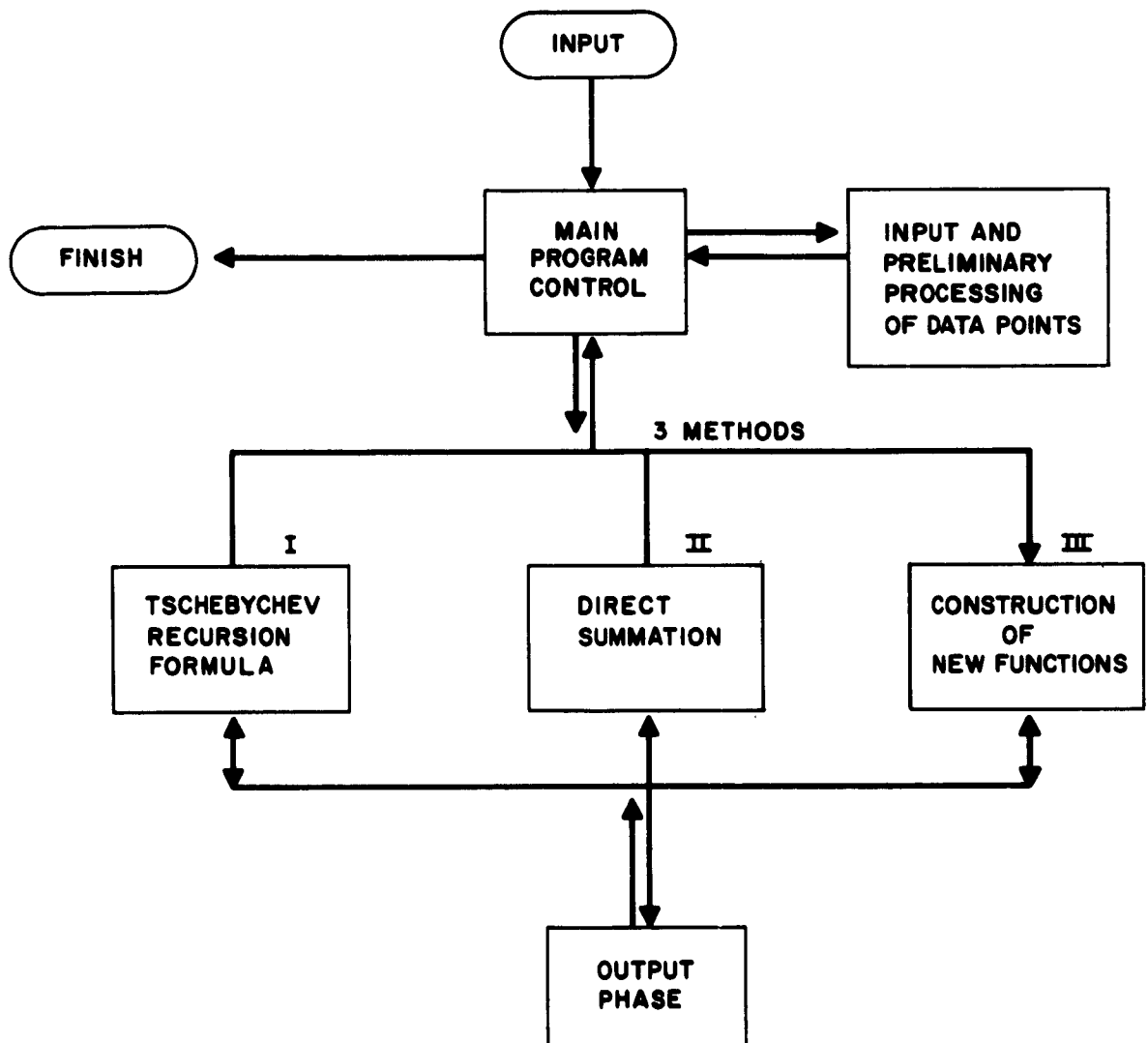
Input to program should be in format described below, noting remarks.

CARD 1 — HOLLERITH CARD TO IDENTIFY DECK (TO BE USED AS DESIRED)

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CARD 2 — CONTROL DATA CARD AND BASIC INFORMATION

θ_0	$\Delta\theta$	Δx	α	NT	NW	NCON	NPCH	NSTAT	NCSB
10 11	20 21	30 31	40	41 43 44 46 47	49 50	52 53	55 56	58 59	61 62 64 65 67 68 70

CARD 3 — (WHEN NPCH IS <u>ZERO</u> THIS CARD IS <u>NOT</u> NEEDED)

NO.1	NO.2	NO.3	NO.4	NO.5
1 5 6	10 11	15 16	20 21	25

CARD 4 — ALL DATA CARDS TO BE IN THIS FORMAT. THERE WILL BE SET OF THESE CARDS.

n	Y	W
10 11	25 26	40

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Card 1

Card 1 is an input card used to identify this particular case being run. Seventy-two alphabetic or numeric characters may be inserted.

Card 2

Card 2 is a "control" card which also contains information basic to this case necessary in the program.

θ_o = Floating point number used in calculation of the mth transform, e.g.,
$\theta_m^{rad} = (\theta_o + m\overline{\Delta\theta})2\pi$. Note that θ_o is not in radians but is given as a fraction or multiple of 2π. The program will accomplish the conversion to radians.

θ_o should be zero for Methods 2 and 3, but need not be zero for Method 1.

$\Delta\theta$ = Floating point number used as described in preceding paragraph. Note that it is given as a (decimal) fraction of 2π radians. $(1/\Delta\theta$ must be evenly divisible by 4.)

Δx = Floating point number used in calculation of λ and μ. Given in cm.

α = Floating point number used in drift correction.

$y_k = y_k + \alpha(k-1)$. Corrected for drift input.

NT = Integer. This gives the subscript of the last set of transforms desired; for example,

$$(A_1, B_1), \dots (A_k, B_k), \dots (A_{NT}, B_{NT})$$

NW = Integer. Describes the method of weighting (aphodizing):

NW < 0. Indicated weighting value is on card with y value (e.g., -1, -2, ...).

NW = 0. Indicates no weighting.

NW > 0. Indicates y values are to be weighted by "functions" given in a subroutine called weight. Which function to use will be determined by the value of NW, e.g., if NW = +1, +2, +3, ... etc.

NM = Integer. This indicates the method to be used:

    NM < 0 Tschebychev   (Method 1) (use -1) (No limit to number of input points.)

    NM = 0 Shocut   (Method 3) (Limit 4000 input points).

    NM > 0 Dirsum   (Method 2) (use +1) (Limit 16,000 input points).

NCON = Integer

    NCON = 0. Do not multiply by constant factor.

    NCON $\neq$ 0. Multiply by constant factor (See first footnote, page 1.)

NPCH = Integer. This integer controls input of card 3 and writing of data on tape 6 for use of tape-to-card conversion. The cards would then be used for automatic plotting purposes.

    NPCH $\neq$ 0. Expect to read in card 3. Write out specified data on tape 6. Have tape 6 punch cards off-line.

    NPCH = 0. Do not expect card 3. Do not write data on tape 6.

NSTAT = Integer. Used only with Version II. This indicates the subscript of the first transform to be calculated; e.g., sequence will now be

$$(A_{NSTAT}, B_{NSTAT}), \ldots (A_k, B_k), \ldots (A_{NT}, B_{NT})$$

(NSTAT > 0; negative and 0 subscripts not to be used.)

NCSB = Integer. Used only with Version II. Code to tell which transforms to compute; e.g.,

    NCSB = 1. Calculate cosine transform only

    NCSB = 2. Calculate sine transform only

    NCSB = 3. Calculate both cosine and sine transforms.

Card 3

    Numbers No. 1 through No. 5 refer to the "columns" of information which it is desired to punch.

Value	Number to Use
θ	(2)
ν	(3)
λ	(4)
A	(5)
B	(6)
C	(7)
ϕ	(8)

Note that in Version II, A, B, C or ϕ <u>may or may not</u> be available for punching depending on NCSB. If they are not calculated, they are not available for punching, hence they should <u>not be called for.</u>

Card 3 should be omitted when NPCH = 0.

Card 4

Actually there will be a card of this form for each input point. When NW is (-), a value of W associated with its corresponding y will be on a card. The number N will <u>be left blank</u> except on the <u>last</u> card for that case.

A <u>non-zero</u> number N must be placed on the last card. This serves a twofold purpose insofar as the program is concerned:

(1) It identified the last card of the case and, hence, the program "knows" to begin computations.

(2) When

$$n = +1 \quad \text{The average } \frac{\sum_{i=1}^{N} y_i}{N} \text{ is subtracted from each value } y_i.$$

$$n = -1 \quad \text{The average is } \underline{\text{not}} \text{ subtracted from each value } y_i.$$

In all cases,

(1) Floating point numbers can be placed anywhere in the prescribed field - <u>but</u> there <u>must</u> be a decimal point in the proper place in the number in that field.

(2) Integers are to be placed as far to the right as possible in their respective field. <u>No decimal</u> point is to be included.

(3) Input cases may be stacked with an EOF between cases on logical tape 2. Operator will have to know the number of cases being processed.

Output:

Output will consist of:

(1) Line identifying program

(2) Card 1 of input

(3) Print-out of input data on input card 2 properly identified

(4) Column readings

(5) Series of lines with data as follows:

β, θ, ν, λ, A, B, C, ϕ

(6) Line that indicates end of case, number of points processed and average value of y (d. c. components).

Output is on tape 3.

Output for tape to cards will be on tape 6 in format (5F10.5) and will be those variables selected. If some other format is desired, a binary correction or recompilation will have to be made.

Restart Ability:

Version I cannot be stopped in the middle of a case and taken off the machine to be completed at a later date. If the operator has knowledge of how many files (cases) there are on Input tape 2, he could stop machine on an end of file (kth EOF), take program off and run again at a later date, provided that he positions tape 2 to the kth EOF. None of the other tapes need be saved, as each case is completely independent of the previous cases.

Version II can be stopped in the middle of a case and can be reset in the following manner.

Stop

(1) Put SSW 5 down.

(2) Program will dump necessary data on tape 5. (Tape 5 will previously have been rewound, data will be written on it, and program will rewind again.) Save tape 5.

(3) A single data card will be punched out on line. This card contains the correct value of subscript for next set of transforms.

(4) Save tape 2, noting the number of files completed.

(5) Tape 3 will contain output data which can be listed. Then tape 3 need not be saved. If output for tape-to-card conversion is on tape 6, this tape should be punched; tape 6 need not then be saved.

(6) Tape 4 was used as an intermediate tape and need not be saved.

(7) Program will stop with HPR 1414_8.

Start

(1) Reposition tape 2, spacing up to the correct file.

(2) Reposition tape 5, making certain tape is rewound.

(3) Replace all tapes previously necessary (3, 4, 6) by "new" tapes (or by old ones if output data have been listed and/or punched). Make certain these tapes are rewound.

(4) SSW 4 should be in down position until output starts going out on tape 3, SSW 4 should then be placed in its normal (up) position.

(5) Feed in program deck followed by program transfer, then by single data card punched by step (3) of Stop.

(6) Proceed as usual, completing remainder of cases. (All sense switches will now be in UP position.)

Running Procedure

Set-up of Deck: Binary deck
                Transfer

Input is read in under program control.

Tapes : Input: Tape 2 (Decimal output)

        Output: (Normal) Tape 3
                          Tape 6 will go from tape to cards off-line
                          Both 3 and 6 are decimal tapes.
        Auxiliary: Tapes 4, 5 used in computing. These are binary tapes.

Sense Switches: Normal position - UP for all sense switches.

PROGRAMMED STOPS

Stop	Subroutine	Reason	Procedure
HPR1000$_8$	Shocut	Number of data points (y_i) greater than 4000.	Take program off machine. Return to sender to check.
HPR 1414$_8$	Shocut	Manual dump by machine operator	Take program off machine. Take card out of punch unit. Insert behind transfer and binary deck. Count number of files completed on tape 2. List tape 3 off line and punch tape 6 is used. Save tape 5. Save tape 2.
HPR 202$_8$	Dirsum	NCOS > 4000 There are more than 4000 angles in the first quadrant to be completed.	Take program off machine. Return to sender to check input data.
HPR 0, 1	Main program End of file	EOF between cases on BCD tape 2	If there are more cases on tape, press start to let machine read in more data. If all cases are finished, take off machine.

Version II <u>only</u>

A manual dump of data can be accomplished in Version II only, by placing SSW 5 in DOWN position. A binary write-out of information will occur on tape 5, and a single card will be punched on-line. This program will then stop with an HPR 1414_8 - take off machine. Place card just punched behind transfer of binary deck. Save tapes 2,5 for completion of these cases. Note the number of files passed on tape 2 (k files) so that it can be spaced to the proper position when restarting. Tape 3 contains output which is to be listed off-line; tape 6 <u>may</u> contain data to be punched (off-line tape-to-card conversion) and used for automatic plotting purposes.

To restart:

(1) Set SSW 4 DOWN.

(2) Replace tapes 5 and 2. Tape 5 should be at beginning of tape. Space tape 2 to the kth end of file.

(3) Feed in binary program deck.

(4) When tape 3 begins moving, set SSW 4 in UP position so program will proceed normally with rest of cases.

The contents of the Binary decks for the two versions of the interferometer program should be as follows:

Version I

  1. Main Deck (Version I)
  2. Dirsum
  3. Tscheb
  4. Shocut (Version I)
  5. Output (Version I)
  6. IInpt
  7. Quad
  8. Preset
  9. Weight (some version)
10. Library Subroutines including ATANQRF

Version II

  1. Main Deck (Version II)
  2. Shocut (Version II)
  3. Output (Version II)
  4. IInpt

5. Quad
6. Preset
7. Weight (some version)
8. Library Subroutines including ATANQRF

Note that IInpt, Quad, Preset, Library Routines, and some version of Weight, are necessary to both versions.

Those subroutines and/or main programs followed by parentheses are peculiar to that version only and are not to be used with the other version.

Special Program to Check Input Data

A special program to "check" the input data to the interferometer program (I.P.) has been written. The data for the I.P. is to be punched as designated in the writeup of the I.P. and should then be transferred to Input Tape 2 as required.

This special program will use the data as it appears on Tape 2, and will check to see that any two successive values y_i starting at some subscript k do not exceed some given bound E

$$y_{i+1} - y_i \quad < E \quad , \quad i > k \quad .$$

The values k and E will be supplied (1 for each of the cases on Tape 2) on data cards following the transfer card of the binary deck. The format will be as follows:

1	k	10 11	E	25 26 - - - - - - - 72

(a)
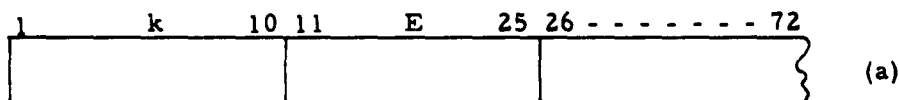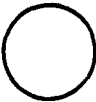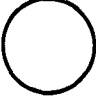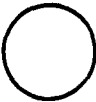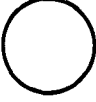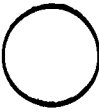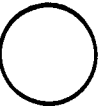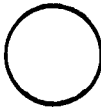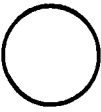
k is an integer.
E is a floating number and hence requires a decimal point.

The deck will be in the order

(1) Binary deck of program and subroutines

(2) Transfer

(3) Set of cards of form (a) one card for each case on Tape 2

The output will list the case checked by sequence count and will indicate each error. If no errors occur in that case, there will only be a printout to indicate that particular case has been completed. No heading is output with this information.