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Subject: Contract AF 29(601)-4136

To: Headquarters
Air Force Special Weapons Center (SWRCM)
Kirtland Air Force Base, New Mexico

Marked for: Contract No. AF 29(601)-4136
Project No. 4988

Enclosure: (a) The Fourth Semi-Annual Technical Report for the period ending 31 December 1962, ten (10) copies (UNCLASSIFIED)

1. In accordance with the requirements of the subject contract, enclosure copies of the Fourth Semi-Annual Technical Report are submitted herewith.

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cc: Air Force Special Weapons Center (SWRRA)
Kirtland Air Force Base, New Mexico
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Lt. J. L. Dowdell, ACO, AFPRO, SV - w/o encl.

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Contract AF 29(601)-4136

Semi-Annual Technical Summary

for the period ending 31 Dec. 1962

The work discussed in the last semi-annual summary is progressing toward completion and results are being prepared for inclusion in the final report.

In addition to the work described there, a machine program for solution of the spherically symmetric coupled radiative transport and hydrodynamic equations has been formulated.

Radiative Cooling at Very Late Times

To provide a means of calculating radiative cooling at very late times, such that hydrodynamic motions are relatively unimportant, and the hot gas is everywhere optically thin, a simplified code is being developed to calculate the rate of radiative cooling for gases at temperatures below 10,000 subjected to a given variation of pressure versus time. For this calculation the emissivities of Kivel and Bailey are employed, together with newly developed analytic fits to the Gilmore tables[1]. This code is expected to be most useful for low altitude detonations, where the late-time motion can be reasonably approximated by simple models.

General Description

The code developed for late time fireball computations is a generalization of the Richtmyer-von Neumann artificial viscosity technique for a spherical Lagrangian system in which the divergence of the radiative flux appears in the hyperbolic partial differential equation which expresses the conservation of energy for a given mass zone. The basic differential equations are those given by Richtmyer,[2]

$$\frac{\partial U}{\partial t} = -\frac{1}{\rho_0}\left(\frac{R(r,t)}{r}\right)^2 \frac{\partial p}{\partial r} \tag{1}$$

$$\frac{\partial R}{\partial t} = U \tag{2}$$

$$\frac{\partial \mathcal{E}}{\partial t} = \frac{p}{\rho^2}\frac{\partial \rho}{\partial t} + \frac{1}{\rho}\nabla F \tag{3}$$

modified to include this additional radiative term. Without this term, the solution of the equations is a standard initial value problem which can be integrated explicitly. Such solutions are well known; only those modifications required to include the radiative transport will be discussed here.

Formulation of the Radiative Energy Transport

To understand the basic problem of radiative transport consider the standard equation of radiative transfer for a media in local thermodynamic equilibrium and for a plane parallel geometry:

$$\cos\Theta \frac{d I_\nu(x,\cos\theta)}{\rho\, dx} = K_\nu\left(B_\nu(T(x)) - I_\nu(x\cos\theta)\right) \tag{4}$$

where the quantities are defined in the standard terminology as given by Chandrasekhar[3].

This equation represents the change in the specific intensity of the radiation along a ray having a given direction in space. Since an infinite number of directions for rays can be assigned to any unit volume under consideration, it is clear the eq. 4 represents an infinite set of differential equations which are functions of position, direction in space, and radiation frequency.

The solution for this set of equations has been obtained in closed form by Chandrasekhar (ibid pages 55 ff) based on a method in which the radiation field is represented by 2N streams where, in the limit, N approaches infinity. The convergence of this solution is shown to be critically dependent on the proper choice of radiation streams used in the first approximation, (i.e. N = 1), which corresponds to the two stream method of A. Schuster and K. Schwarzschild (1905). Suitable choice of the directions of the two rays used to represent the radiation field results in a formulation of the gross energy transport whose accuracy is considerably greater than the accuracy with which we know the appropriate absorption coefficients. The accuracy of this approach improves when it is applied to thin isothermal slabs as contrasted with the usual application to an atmosphere of essentially infinite extent.

The basic physics of the Schuster-Schwarzschild method has been carried over to spherical geometry. In this application, the inclination of the chosen radiation streams changes with the relative radius of curvature of the spherical zones. When the radius of curvature becomes large compared to the zone thickness the appropriate inclination is that which is applicable for plane parallel geometry. Indeed, in this limit, the exact solution for the flux in terms of exponential integrals may be employed and no approximation is required. However, in the present application, other conditions of the problem made it desirable to maintain the two stream approximation, the choice of inclination being determined by the condition that the difference in these two solutions be held to a minimum. The proper choice of inclination near the center of the sphere has been temporarily postponed by choosing the central zones such that they are optically thick and by requiring that the two stream solution transform into the diffusion approximation which is applicable under these conditions.

The actual formulation of the flux is then carried out using the formal integral solution [3] of the equation of transfer for the specific intensity along the representative rays. The source function is assumed to be that which is appropriate under conditions of local thermodynamic equilibrium and is expanded in terms of the temperatures, and derivatives of the temperatures, at the zone boundaries. While the code has been written to allow the spectrum to be divided into any desired number of frequency bands, the initial computations have been carried out using the Rosseland mean absorption coefficients for the purposes to be enumerated later.

The numerical solution is carried out by explicit integration of the basic equations, except for the energy equation, using the well known Richtmyer-von Neumann method [4]. The integration of the energy equation cannot be carried out explicitly since the radiative flux term for any particular zone is dependent upon the temperatures of the neighboring zones. Since the density is known at this stage of the solution, it is convenient to express the energy equation in terms of zone temperatures alone. The solution is then carried out by assuming an approximate temperature for each zone and using a Newton-Raphson iteration scheme to adjust the zone temperatures until changes in temperatures on subsequent iterations are less than a pre-determined amount.

This method of solution involves the solution of N linear equations in N unknowns, where N is the number of zones which is usually of the order of a hundred. Such a system is in principle easily soluble by computer methods, but the procedure is time consuming. In practice only a few zones in each direction measurably influence the temperature of a particular zone which results in a simplification. In a first solution of the problem a five zone centered system has been used which results in a matrix having five terms across the diagonal. After suitable scaling, such a system is rapidly soluble by direct elimination and back substitution.

As part of the check out procedure, the equation of state and formulation of the mean absorption coefficient given by Brode (ibid) were used so as to reproduce his results over an appropriate period of fireball history. Using 100 zones, and 3 iterations per hydrodynamical cycle, the code proceeds successfully about 200 time steps per hour. With minor modifications, the code has also been applied to a problem in thermal ablation in fireballs and to problems in stellar stability.

Interpolation formulae are currently being written for mean absorption coefficients for chosen spectral bands appropriate to late fireball development.

Initial Energy Deposition in Variable Density Atmosphere

When a nuclear device is detonated in the atmosphere, a large percentage of the energy released is radiated away and is deposited in the surrounding atmosphere. Since the air density exhibits a large variation with altitude, it follows that the scale of distances over which this energy is absorbed by the air will be dependent upon the burst altitude. Consequently the source density may also exhibit a dependence on altitude.

The sequence of radiative and hydrodynamic events which follow this initial phase are dependent upon the radial energy distribution. Thus, one would expect the scaling laws, for various weapons effects as a function of altitude, to also exhibit a dependence on the initial energy deposit. It is therefore of interest to perform computations of this distribution as a function of weapon yield and altitude.

Because of the large numer of cases of interest, a machine code was written to perform the appropriate computations. The basic equations involved are

$$\frac{dE}{dR} = \frac{1}{4\pi R^2} \int W_\nu K(\rho\nu)\, \rho(A)\, e^{-\int_0^R K(\rho\nu)\, \rho(A)\, dR}\, d\nu \qquad (5)$$

$$\rho = \rho(A) \qquad (6)$$

$$K_\nu = K(\rho\nu) \qquad (7)$$

$$T = T(E, \rho) \qquad (8)$$

where $\frac{dE}{dR}$ = energy per unit volume deposited at a distance R from the point of detonation.

ρ = air density as a function of altitude A

K_ν = absorption coefficient of air

W_ν = total energy of frequency ν radiated by the source. Equation 6 was evaluated using an empirical fit to the ARDC Standard Atmosphere (5).

The absorption coefficients used were those of Gilmore (6) for cold air. Stripping was neglected since it appears to have only a small effect upon the later development.

In coding the problem, the radiative output of the source was divided into 11 separate spectral regions and a corresponding average opacity was chosen for each of these regions. A minor modification of the code allows a larger number of spectral regions to be used when desired. The code carries out radial computations for the downward direction and at directions in space differing by 10 degrees, or any multiple of 10 degrees, as may be desired. The printed output for each direction in space lists the radius from the point of detonation for 35 different points, and the air density, altitude, and energy density for each point.

Temperatures were determined using the tabulated equation of state data given by Gilmore[7], and Hilsenrath and Beckett[8]. Initial runs have shown that a better representation of the equation of state for air is desirable.

The initial runs included a variety of yields and altitudes including cases applicable to previous field test situations and cases of interest for future planning. Considerable success was obtained in predicting a number of the effects observed in earlier field tests, but a better handling of the equation of state is indicated before the results are presented in scaling law form.

Various aspects of the role of debris in determining ionization levels are being studied. The work on ionization of debris atoms has been mentioned in previous reports. A second interesting question is how to reconcile the large dimensions of observed debris clouds with the small values found in hydrodynamic calculations. Finally, we note that in low altitude detonations, dust-like impurity particles may be formed and play a significant role in electron recombination, as recently suggested by Rosen[10].

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