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US ARMY ELECTRONICS RESEARCH & DEVELOPMENT ACTIVITY

**A METHOD FOR USING MEASURED RATE
TO IMPROVE RADAR RANGE DATA**

BY

WILLIAM L. SHEPHERD

USA ERDA-4

FEBRUARY 1963

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**WHITE SANDS MISSILE RANGE
NEW MEXICO**

**A METHOD FOR USING MEASURED RATE
TO IMPROVE RADAR RANGE RATE**

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**INSTRUMENTATION DEPARTMENT
U. S. ARMY ELECTRONICS RESEARCH AND DEVELOPMENT ACTIVITY
WHITE SANDS MISSILE RANGE
NEW MEXICO**

A B S T R A C T

Two least squares polynomial methods for using radar range rates to obtain improvement of radar range are given. One method involves integration of smoothed range rate, the other is a modification of the interpolating polynomial fitting both range and range rate.

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I N T R O D U C T I O N

Pulse type radars are presently being built which incorporate Doppler techniques for measuring range rate. These measurements will be independent of and in addition to measurements of range. It is anticipated that measured range rates will be much more precise than measured ranges.

In this report we develop two least squares methods which will permit the use of range rate information to obtain improved estimates of range.

The first method consists of fitting a least squares approximating polynomial to range rate data without reference to range, integrating the polynomial, and determining the constant of integration so as to obtain the integral curve which best fits the range data in the sense of least squares. It is to be noted that this does not involve numerical integration from a tie-in point.

The second method is an adaptation of the interpolating polynomial which fits the entire collection of range and range rate data. A simple weighted least squares approximating polynomial is fitted to the entire collection of data.

The first method permits one to smooth range rate information without degradation caused by imprecise range. The introduction of the simple weights in the second method permits some flexibility in adapting the method to particular cases where measures of precision for the range and for the range rate are available.

For greater ease of presentation only quadratic fits to range are considered.

THE INTEGRAL METHOD

Suppose we have a set of measured values of range from a radar site of a moving point, and a corresponding set of independently measured values of range rate, according to the following table.

Time	-kh,	(-k + 1) h, ..., 0, ..., (k - 1) h, kh
Range	r_{-k} ,	r_{-k+1} , ..., r_0 , ..., r_{k-1} , r_k
Range Rate	$\dot{r}_{-k}$,	$\dot{r}_{-k+1}$, ..., $\dot{r}_0$, ..., $\dot{r}_{k-1}$, $\dot{r}_k$

Notice we have a table with an odd number of columns. This restriction, though not necessary, permits desirable simplifications of form.

Let the required approximating polynomial be

$$r(t) = a_0 + a_1 t + a_2 t^2, \quad (1)$$

with derivative

$$\dot{r}(t) = a_1 + 2a_2 t. \quad (2)$$

We shall determine a_1 and a_2 so that $\dot{r}(t)$ is the least squares fit to the rate data, then determine a_0 so that $r(t)$ is the least squares fit to the range data.

Set

$$g(a_1, a_2) = \frac{1}{2} \sum_{i=-k}^k (a_1 + 2a_2 h_i - \dot{r}_i)^2.$$

The normal equations for (2) are

$$\frac{\partial g(a_1, a_2)}{\partial a_1} = \frac{\partial g(a_1, a_2)}{\partial a_2} = 0.$$

Thus

$$\sum_{i=-k}^k (a_1 + 2a_2 h i - \dot{r}_i) = 0, \quad (3)$$

$$\sum_{i=-k}^k (a_1 + 2a_2 h i - \dot{r}_i) 2h i = 0. \quad (4)$$

Set

$$\sum_{i=-k}^k i^p = s_p^{(k)}.$$

Using $s_1^{(k)} = 0$ and solving (3) and (4) for a_1 , and a_2 , we get

$$a_1 = \frac{1}{2k+1} \sum_{i=-k}^k \dot{r}_i \quad (5)$$

$$a_2 = \frac{1}{2h s_2^{(k)}} \sum_{i=-k}^k i \dot{r}_i. \quad (6)$$

We now determine a_0 so that $r(t)$ is the least squares fit to the range data under the constraint that a_1 and a_2 are given by (5) and (6).

Set

$$f(a_0) = \frac{1}{2} \sum_{i=-k}^k (a_0 + a_1 i h + a_2 i^2 h^2 - r_i)^2.$$

The normal equation for (1) is

$$\frac{\partial f}{\partial a_0} = \sum_{i=-k}^k (a_0 + a_1 i h + a_2 i^2 h^2 - r_i) = 0. \quad (7)$$

Then

$$a_0 = \frac{1}{2k+1} \sum_{i=-k}^k r_i - \frac{h^2 s_2^{(k)}}{2k+1} a_2. \quad (8)$$

Equation (1), with a_1 and a_2 given by (5) and (6) and a_0 given by (8) gives a formula for smoothed range whose derivative must be the smoothed range rate obtained from the independently measured range rate.

To obtain $r(0)$ and $\dot{r}(0)$, the values for smoothed range and range rate at the mid-point of the smoothing interval, we set $t = 0$ in (1) and (2) and get

$$\dot{r}(0) = a_1 \quad (9)$$

$$r(0) = a_0 \quad (10)$$

Inspection of equations (10), (9), (8), (6), and (5) shows that $r(0)$ and $\dot{r}(0)$ may each be expressed as a linear combination of $x_{-k}, \dots, x_k, \dot{x}_{-k}, \dots, \dot{x}_k$.

APPROXIMATING QUADRATIC FOR THE INTERPOLATING POLYNOMIAL

We again refer to the table and equations (1) and (2). Introduce the common weighting factor, c , for the residuals for range and the common factor, d , for the residuals for range rate, and set

$$f(a_0, a_1, a_2) = c^2 \sum_{i=-k}^k \left[r(1h) - r_i \right]^2 + d^2 \sum_{i=-k}^k \left[\dot{r}(1h) - \dot{r}_i \right]^2. \quad (11)$$

The normal equations

$$\frac{\partial f}{\partial a_j} = 0 \quad j = 1, 2, 3$$

may be written after slight simplification as

$$\sum_{i=-k}^k (a_0 + a_1 h i + a_2 h^2 i^2 - r_1) = 0, \quad (12)$$

$$h c^2 \sum_{i=-k}^k (a_0 + a_1 h i + a_2 h^2 i^2 - r_1) i + d^2 \sum_{i=-k}^k (a_1 + 2 a_2 h i - f_1) = 0, \quad (13)$$

$$h^2 c^2 \sum_{i=-k}^k (a_0 + a_1 h i + a_2 h^2 i^2 - r_1) i^2 + 2 h d^2 \sum_{i=-k}^k (a_1 + 2 a_2 h i - f_1) i = 0. \quad (14)$$

Using

$$s_p^{(k)} = 0, \quad p = 1, 3$$

and collecting coefficients of a_0 , a_1 , a_2 , in equations (12), (13) and (14), we get

$$s_0^{(k)} a_0 + h^2 s_2^{(k)} a_2 = \sum_{i=-k}^k r_1, \quad (15)$$

$$(h^2 c^2 s_2^{(k)} + d^2 s_0^{(k)}) a_1 = h c^2 \sum_{i=-k}^k i r_1 + d^2 \sum_{i=-k}^k f_1, \quad (16)$$

$$h^2 c^2 s_2^{(k)} a_0 + (c^2 h^4 s_4^{(k)} + 4 d^2 h^2 s_2^{(k)}) a_2 = c^2 h^2 \sum_{i=-k}^k i^2 r_1 + 2 d^2 h \sum_{i=-k}^k i f_1. \quad (17)$$

$$\begin{aligned}
D &= \begin{vmatrix} s_0^{(k)} & h^2 s_2^{(k)} \\ h^2 c^2 s_2^{(k)} & h^4 c^2 s_4^{(k)} + rh^2 d^2 s_2^{(k)} \end{vmatrix} = h^2 \begin{vmatrix} s_0^{(k)} & s_2^{(k)} \\ h^2 c^2 s_2^{(k)} & h^2 c^2 s_4^{(k)} + 4d^2 s_2^{(k)} \end{vmatrix} \\
&= h^2 \left\{ h^2 c^2 \left[s_0^{(k)} s_4^{(k)} - (s_2^{(k)})^2 \right] + 4d^2 s_0^{(k)} s_2^{(k)} \right\}. \quad (18)
\end{aligned}$$

It can be shown that $D > 0$ for $k \geq 1$. Hence, the normal equations have a unique solution,

$$a_0 = \frac{1}{D} \begin{vmatrix} \sum_{i=-k}^k r_i & h^2 s_2^{(k)} \\ c^2 h^2 \sum_{i=-k}^k i^2 r_i + 2d^2 h \sum_{i=-k}^k i f_i, & h^4 c^2 s_4^{(k)} + 4h^2 d^2 s_2^{(k)} \end{vmatrix}, \quad (19)$$

$$a_1 = \left(hc^2 \sum_{i=-k}^k i r_i + d^2 \sum_{i=-k}^k i f_i \right) / \left(h^2 c^2 s_2^{(k)} + d^2 s_0^{(k)} \right), \quad (20)$$

$$a_2 = \frac{1}{D} \begin{vmatrix} s_0^{(k)} & \sum_{i=-k}^k r_i \\ h^2 c^2 s_2^{(k)}, & c^2 h^2 \sum_{i=-k}^k i^2 r_i + 2d^2 h \sum_{i=-k}^k i f_i \end{vmatrix}. \quad (21)$$

Clearly a_0 , a_1 , and a_2 can be expressed as explicit linear combinations of $\{r_i, f_i\}$. Also, a_0 , a_1 , and a_2 are homogeneous and of degree 0 in c , d . Hence, only the ratio of c and d need be specified.

The equations

$$r(0) = a_0 \quad (22)$$

$$\dot{r}(0) = a_1 \quad (23)$$

with a_0 and a_1 given by equations (19) and (20) give the mid-point smoothed range and range rate. For mid-point range and range rate only, it is not necessary to obtain a_2 .

S U M M A R Y

In this report two methods of treating a combination of measured radar range and range rate are discussed. Each uses least squares theory, but the first permits independent smoothing of range rate. Each method anticipates a situation likely to occur in radar instrumentation.

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