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TITLE: Integral equations for the harmonics on the surface of a body of revolution

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TEXT: When any wave $u_o(M)$ hits the closed surface S of a body of revolution on which $r = f(z)$, $a \leqslant z \leqslant b$, the total field will be $u(M) = u_o(M) + v(M)$. $v(M)$ is determined by the boundary value problem $\Delta v + k^2 v = 0$, $v|_S = u_o|_S$, $\frac{\partial v}{\partial R} + ikv = O(1/R)$ for $R \rightarrow \infty$. Other boundary conditions can be treated in a similar way. The function $v(M)$ is determined by its values on S and by its derivatives in the directions of the normal to the surface by means of Green's formula. The author generalizes a method established by N. N. Govorun (DAN SSSR, 126, no. 1, 49, 1959; 132, no. 1, 91, 1960) to obtain a first-kind Fredholm integral equation for the determination of

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$v(M)$. This equation has a kernel without any singularities:

$$\int_a^b \left\{ r^\nu A^{-\frac{2\nu+1}{4}} H^{(2)}_{\nu+1/2}(kA^{1/2}) \frac{\partial v_\nu}{\partial n} - v_\nu(z) \frac{\partial}{\partial n}\left(r^\nu A^{-\frac{2\nu+1}{4}} H^{(2)}_{\nu+1/2}(kA^{1/2})\right)\right\} \times$$
$$\times f(z)\sqrt{1+f'^2(z)}\,dz = 0, \qquad (1)$$

where $A = (z-\eta)^2 + r^2$, $a < \eta < b$. The H are Hankel functions, $\vec{n}$ is the unit vector directed along the surface normal to the outside, $v_\nu(z)$ are the harmonics of $v(M)$ upon S. Thus,

$$v(M)|_S = \sum_{\nu=-\infty}^{\infty} v_\nu(z) e^{i\nu\varphi}, \qquad \frac{\partial v}{\partial n}\Big|_S = \sum_{\nu=-\infty}^{\infty} \frac{\partial v_\nu}{\partial n}(z) e^{i\nu\varphi}$$

When this steady case is to depend on time it is formally subjected to a Fourier transformation with the aid of the Kirchhoff-Sobolev formula, resulting in an integro-functional equation. An electromagnetic wave is dealt with also. This procedure is a translation of the results found for the scalar case into a vectorial analog. The field outside S is given in

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terms of the field strengths $\vec{E}$ and $\vec{H}$ upon S according to the Stratton-Ch'u formulas. It is pointed out that the equations always have solutions when the boundary conditions of the differential equations can be fulfilled. The uniqueness of the solutions obtained is demonstrated.

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