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NANOSECOND PULSE BREAKDOWN STUDY

Lawrence C. Scholz

ARMOUR RESEARCH FOUNDATION
of
Illinois Institute of Technology
Technology Center
Chicago 16, Illinois

ARF-A217-7
(Final Report)

Contract No. AF 30(602)-2780

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Prepared
for
Rome Air Development Center
Air Research and Development Command
United States Air Force
Griffiss Air Force Base
New York

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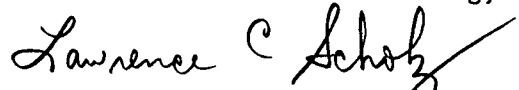
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FOREWORD

This report, ARF-A217-7, was prepared by the Armour Research Foundation of the Illinois Institute of Technology for Rome Air Development Center under Contract No. AF 30(602)-2780. The work was done in the period 15 May to 15 November 1962.

Respectfully submitted,

ARMOUR RESEARCH FOUNDATION
of Illinois Institute of Technology



Lawrence C. Scholz, Research Physicist
Plasma and Electron Physics Research

APPROVED BY:



S. W. Kash, Manager
Plasma and Electron Physics Research

LCS:bw

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ABSTRACT

A summary of microwave gas breakdown theory is presented with special emphasis on nanosecond pulse breakdown. The various approaches previously presented in the literature are compared and shown to be identical as far as practical results. It is suggested that in general, for pressure-time products greater than 1 torr-nanosecond, measurements made under cw conditions will be satisfactory. A program of future investigation is outlined to test this suggestion and to gather the needed data on pulse breakdown.

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TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	iii
I. INTRODUCTION	1
II. BASIC PROCESSES OF BREAKDOWN	3
1. General Considerations	3
2. Diffusion	7
3. Ionization and Attachment	10
III. THE THEORY OF PULSE BREAKDOWN	17
1. Single Pulse Breakdown	17
2. Multiple Pulse Breakdown	23
IV. MICROWAVE TESTING METHODS	26
V. SUGGESTIONS FOR FUTURE WORK	31
1. Pulse Breakdown Measurements	31
2. Measurement of Ionization and Attachment Coefficients	32
3. Microscopic Theory of Breakdown	33
REFERENCES	39
APPENDIX - DETAILS OF THE CALCULATION OF THE DIFFUSION COEFFICIENT	I-1
TABLE	
I BINDING ENERGIES OF NEGATIVE IONS	13
II ATTACHMENT CROSS SECTIONS	14
FIGURE	
1 Reflection Coefficient vs. f_p/f .	35
2 Breakdown Field as Function of p .	36
3 Dimension of Ridged Waveguide.	37
4 Schematic Design of Pulse Generator Using Circulator.	37
5 Pulse Generator Using Resonant Ring Structure.	38

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NANOSECOND PULSE BREAKDOWN STUDY

I. INTRODUCTION

This report summarizes a study of breakdown initiated by intense short-duration microwave pulses. We will present the results of our analysis concerning prior theoretical and experimental approaches and then give a critic of these methods. The importance of individual processes in a gas that effect breakdown will be discussed. A future program is also outlined that incorporates a combined empirical and theoretical attack on the problem.

The problem of breakdown for nanosecond pulses arises out of the need for higher peak pulse power in shorter pulses to increase the range and range resolution of present radar systems. Previous studies of pulsed microwave breakdown have been directed primarily towards the problem of satellite communication. The radar detection problem is somewhat different in that the equipment is all ground based. Thus, because of the well known variation of breakdown with pressure, a breakdown minimum will occur at an altitude of about 100,000 ft. That is, unless some special focusing scheme is used we may expect breakdown to occur in the pressure range of 1 atmosphere to about 1 torr. To allow for pressurized equipment in our analysis we will consider pressures from 1 torr to 6 atm. This is a convenient division since it turns out that in the low pressure region a different mechanism must come into play. We will consider frequencies in the gigacycle range, although as will be shown this is not critical. The analysis will apply to pulse lengths of 1-1000 nanoseconds.

The general plan is to first discuss the basic processes of breakdown including the generation and loss of electrons, criteria for breakdown, and the

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interaction of the e. m. wave with the ionized gas. We shall discuss the processes and their relative importance for short pulse breakdown. After reduction of the loss processes by physical arguments to attachment we shall derive the equations that govern breakdown. Microwave testing methods and future experiments are also discussed.

II. BASIC PROCESSES OF BREAKDOWN

1) General Considerations

We are considering the problem of breakdown in free air, that is, in a medium not bounded by a container. This greatly affects the type of analysis we shall apply to the problem. In addition we are interested in breakdown because it affects the propagation of the microwave energy. The problem we are examining is a general one of the interaction of an e. m. field and an ionized gas. This problem arises in communicating with a re-entering vehicle surrounded by a thermally ionized gas, in plasma diagnostics, and in the propagation of signals strong enough to cause ionization. The general character of the problem should be kept in mind both in defining breakdown and in searching the literature.

In general use the term breakdown indicates failure of a normally insulating substance so that it becomes conducting. In a gas this means the presence of free electrons obtained by ionization of the atoms. In d. c. or Townsend measurements one may use the free electron density as a direct indication of breakdown, that is one may observe the temporal growth of the current density. For microwave frequencies this direct measurement is neither convenient, nor simple. The conductivity, which is directly related to the free electron density will strongly affect the propagation of electromagnetic waves in the gas. We may therefore use the wave causing breakdown as a direct measuring device to determine the electron concentration. We must define an ionization level that we may use as a breakdown criterion. Sometimes an arbitrary electron density or an arbitrary ratio of final to initial densities is used. We will use a density that is directly connected to the propagation properties of the wave. This is the "resonant plasma density",

which may be defined as the electron density n at which the plasma angular frequency ω_p is equal to the applied angular frequency ω . The plasma frequency is (Ref. 1)

$$\omega_p^2 = \frac{4\pi e^2}{m} n, \quad (1)$$

And we may solve for the breakdown density n_b in terms of the signal frequency.

$$n_b = 1.24 \times 10^{-8} f^2, \quad (2)$$

where e and m are the electron charge and mass respectively.

We shall show how this relates to the propagation of microwaves. As a simple approximation consider the case of a wave normally incident on an abrupt air-plasma interface. The reflection coefficient R for this case in terms of a generalized index of refraction η is

$$R = \frac{\eta - 1}{\eta + 1} \quad (3)$$

For a homogeneous isotropic plasma we have (Ref. 2)

$$\eta^2 = 1 - \frac{\omega_p'^2}{1 - i \frac{\nu_c}{\omega}} \quad (4)$$

where ν_c is the collision frequency of the electrons. Figure 1 shows the magnitude of the reflection coefficient R as a function of f_p/f . We see that in the collisionless case there is an abrupt reflection at $f = f_p$. The other

cases are not so sharply affected and appreciable transmission occurs for $f_p > f$. However, we have thus far not included attenuation which also affects the propagation. The breakdown criteria given above has also been used by Margenau and others.

We will now study the processes that govern the growth of the free electron density in a gas. In a high frequency field an electron will oscillate back and forth, and because of inertial effects will be out of phase with the field. The electron will not gain energy during these oscillations, unless it happens to collide with a gas atom. When an elastic collision occurs the electron, because of its very much smaller mass, will keep most of its energy, but will be randomly scattered with a different phase angle. It will continue to oscillate back and forth until it has another collision. If the number of collisions per cycle is high the electron can gain appreciable energy from the field. Since the motion is randomized the electron gas is in effect heated up.

The excursion of the electrons in a high frequency field is rather small so that the effects are confined to the volume of the gas, and electrode effects will be small. On the other hand, in a d. c. discharge the electrons are swept out of the gas and breakdown is controlled by secondary effects at the electrodes and walls. (Ref. 3) This may also happen for lower frequency r. f. discharges. (Ref. 4) In high pressure discharges there may also be a transport of photons to the electrodes causing photoelectric emission. However, we shall concern ourselves here only with volume effects. We now write an equation for the creation and destruction of free electrons. Electrons will be created only by ionization in the volume. We may break this into two terms: a constant due to cosmic rays or other natural sources plus a part due to collisional ionization. Electrons may be lost by diffusion, by attachment to form stable negative ions,

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and by recombination with the positive ions. The general equation governing the electron density is then

$$\frac{dn}{dt} = \nu_i n - \nu_a n - \alpha n^2 - \nabla \cdot \Gamma + S \quad (5)$$

where ν_i is the number of ionizing collisions per electron per second, ν_a is the attachment coefficient, α is the recombination coefficient, which depends on the product of the electron and positive ion densities, which we assume are the same, although this may not be correct if the negative ion density is appreciable. $\nabla \cdot \Gamma$ is the divergence of the diffusion current density and S is a general source term. For example, S may represent the source term due to x-rays introduced to eliminate statistical variation in the measurement during an experiment.

It can easily be seen that recombination is negligible for small ion densities, especially if attachment is important. Loeb (Ref. 5) gives $\alpha_{\text{air}} \approx 10^{-6} \text{ cm}^3/\text{ion sec}$. This leads to a loss rate of 2 ions per cm^3 per microsecond if n is $10^6/\text{cm}^3$. Attachment under the same conditions will be about $10^3 p$ per cm^3 per microsecond, where p is the pressure in torr. For these low densities the diffusion rate will also always be much greater than the recombination rate. In the afterglow when the electronic field is removed recombination may be important; however, for electron attaching gases, attachment may always be dominant because attachment cross-sections are usually high for low energy electrons, and in the afterglow we expect thermal electrons. In C. W. breakdown Herlin and Brown (Ref. 6) have shown that diffusion is the dominant loss mechanism, but in our case especially at higher pressures we shall show that diffusion is negligible.

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2) Diffusion

In this section we shall discuss the processes of diffusion and the importance of diffusion as a loss mechanism. It will be shown that in the pressure time regime under consideration diffusion is a negligible loss mechanism.

As discussed in Appendix I the particle current density

$$\Gamma = -\nabla(Dn) \quad , \quad (6)$$

where D is the diffusion coefficient and is a measure of the mean-squared-distance a particle diffuses per unit time. This may be substituted into equation (5), Where α is considered negligible, and $\mathcal{V} = \nu_i - \nu_a$. This reduces to

$$\frac{\partial n}{\partial t} = \mathcal{V}n + D\nabla^2 n + S \quad (7)$$

if we assume that D is not a function of r . General solutions to this equation can be found, but they are not too useful because they depend on the geometry, and except in the most elementary cases they are extremely difficult to obtain. We can obtain all the information we require by realizing that the solution will involve a characteristic diffusion length (Λ) which will be of the order of the actual physical dimensions of the system (i. e. $\frac{L}{\pi}$ for infinite parallel plates with spacing L). Therefore we may calculate a characteristic diffusion time (Λ^2/D), and assume that diffusion is negligible if the pulse length is much smaller than the diffusion time. We have shown in the appendix

that

$$D = \frac{2}{3m} \left\langle \frac{u}{\nu_c} \right\rangle^* \quad (8)$$

where u is the energy of the electrons, and ν_c is the collision frequency. In general ν_c is a function of the velocity. The variation of ν_c with velocity will depend on the interaction potentials. The exact solution of this problem is extremely difficult, however there are two approximations often made that lead to simple results. These are, hard sphere collisions and an inverse fifth power repulsion law, which lead to constant collision frequency (ν_c) and constant mean free path (λ) respectively. While it is not expected that either of these assumptions accurately describes the interaction in a gas they have provided reasonable solutions for many of the common phenomena. Either of these assumptions will yield a similar result for D .

Assume that ν_c is independent of velocity then we find for air that

$$D = 2.2 \times 10^5 \frac{\langle u \rangle}{p} \quad (9)$$

where $\langle u \rangle$ is in electron volts and we have used $\nu_c = 5 \times 10^9 p$ as given by Brown. (Ref. 8) We may also write

$$D = \frac{n^{-1}}{3} \int \frac{v^2}{\nu_c} f(v) dv \quad (10)$$

* The brackets $\langle \rangle$ denote an average over the velocity distribution function in this case $\langle u/\nu_c \rangle = \int d\nu (u/\nu_c) f(\nu)$.

which after substitution of $\lambda = v/\nu_c$ and assuming λ is constant gives

$$D = \frac{1}{3} \lambda \langle v \rangle \quad (11)$$

The mean free path is just $\frac{\langle v \rangle}{\nu_c}$. If we use our value for ν_c , we may compare these two diffusion coefficients

$$\frac{D}{\bar{D}} = \frac{\langle v^2 \rangle}{\langle v \rangle \langle v \rangle} = \frac{\langle v^2 \rangle}{\langle v \rangle^2} \quad (12)$$

for a Maxwellian distribution this ratio is 1.2. We seek a value for $\langle u \rangle$. This depends on the gas and the fields but it certainly is not greater than the ionization potential of say 10 ev for air. So that $D = \frac{2.2 \text{ cm}^2}{p \text{ sec}}$ and for $p\lambda^2 > 1$ we have diffusion times in the order of a microsecond. We may therefore neglect diffusion in our theory of nanosecond pulse breakdown. If one wants to consider extremely low pressure or perhaps an experiment where spacings are made very small, then diffusion must be taken into account. For repetitive pulses diffusion as well as recombination is important in the afterglow between pulses.

These approximate diffusion coefficients depend on the average electron energy, which is a function of E/p . In a gas discharge the electron distribution is altered from the equilibrium case (Maxwellian), and although the assumption of isotropy is reasonable we do not know the actual distribution very well. However, it can be seen that diffusion is a function of the average energy, which will not be greatly affected by the shape of the distribution function. We may expect the greatest change in the distribution in the high energy tail and in the low energy region. This of course will greatly affect the ionization

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and attachment rates, which depend on the high and low energy electrons and not on the average. In addition we have made our calculations for free electron diffusion. In an ionized gas we also have positive ions and except at the lowest densities we do not expect the electrons to diffuse freely, but rather we expect space charge forces to cause the ions to be dragged along with the electrons. This is known as ambipolar diffusion and its rate is much slower than free diffusion, and it is discussed in Allis (Ref. 29).

3) Ionization and Attachment

We are left with only two mechanisms that directly affect the electron density, these are ionization and attachment. Excitation is important as a secondary mechanism, which will affect the electron velocity distribution. Since ionization and attachment are a function of the velocity distribution excitation will indirectly affect these coefficients. However, we are using a semi-empirical approach that does not include the excitation mechanism explicitly. Ionization can occur by varied mechanisms, such as photo-ionization, high field ionization, and collisions with excited atoms or ions. Photo-ionization may be important in high pressure streamers, but we need not consider it here. Collisions with excited atoms (Penning Effect) are important in some rare gas mixtures and may be important in a gas discharge if impurities are present. It is not important unless excited states for the impurity exist with energy levels greater than the ionization potential of the main constituent, therefore we consider only electron-atom collision processes.

Ionization by collision is essentially a simple process, by which all atoms and molecules may be ionized. There is no difficulty satisfying energy and momentum considerations, because there are two free electrons after collision. We can easily see that although the electron collision frequency is high the

probability that an atom will collide with an electron is much lower, due to the low electron density. We may therefore consider ionization to occur in a single step, except for those atoms that have a very long lived metastable state. Even then it is unlikely the ionization will occur by multiple collision since atom-atom collisions will always be more probable than atom-electron collisions. The probability of ionization upon impact is zero for all electron energies below the ionization energy. It rises with increasing energy until a maximum is reached at 3-4 times the ionization energy. Multiple ionization is possible but not probable with the low ion densities that exist before breakdown. A great deal of data and computation is available on ionization potentials, cross-sections and probability, and Townsend first coefficients. These may be found in the literature and in books such as those by Brown, (Ref. 8) Loeb, (Ref. 5) Handbuch der Physik V XXI and XXII, Franklin and Field (Ref. 9) and in the recent work edited by Bates. (Ref. 10)

Electron attachment is a more special process and only certain elements can form stable negative ions. To form a stable ion the energy after attachment must be less than that of the neutral state. Therefore those elements with nearly filled shells should have the highest electron affinities (energies of attachment). Adding an electron to an outer shell is similar to forming an excited state of the atom, but the similarity is not exact because there is an excess charge which will interact with the nucleus. Quantum mechanical calculations have been attempted for some of the simpler atoms for which wave functions are available. (Ref. 11)

Because attachment is a capture process energy considerations are important. The direct capture process



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is essentially a resonance process. Direct capture by molecules is also possible and important. Another process is dissociative attachment:

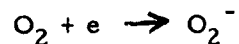


The process provides a ready mechanism for energy and momentum conservation and is likely to be very efficient. There is also the possibility of a non-capture process.

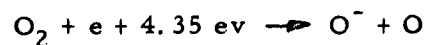


which is of interest in the formation of negative ions, but not as a loss mechanism for electrons in breakdown. A list of atomic negative ions of interest in gas breakdown is given in Table I. (Taken from Branscombe Ref. 12).

There are many molecular attaching gases that are very important and efficient. Buchel'nikova (Ref. 21) gives the following information, see Table II. The cross sections in Table II do not agree exactly with certain other measurements especially for O_2 for which Craggs (Ref. 22) gets 2.3×10^{-16} . Some of these molecular species form more than one negative ion depending on energy. For example in oxygen we see the following



and



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Table I
BINDING ENERGIES OF NEGATIVE IONS

Element	Binding Energy (ev)	Reference and Measurement Method
H ⁻	0.8 $\pm$ 0.1	Surface ionization ⁽¹³⁾
He ⁻	0	Charge exchange ⁽¹⁴⁾
O ⁻	1.465 $\pm$ 0.005	Photodetachment ⁽¹⁵⁾
F ⁻	3.62 $\pm$ 0.09	Surface ionization ⁽¹⁶⁾
	3.47	Surface ionization ⁽¹⁷⁾
S ⁻	2.07 $\pm$ 0.07	Photodetachment ⁽¹⁸⁾
	2.27	Surface ionization ⁽¹⁹⁾
Cl ⁻	3.76 $\pm$ 0.09	Surface ionization ⁽¹⁶⁾
	3.71	Surface ionization ⁽¹⁷⁾
Br ⁻	3.51 $\pm$ 0.06	Surface ionization ⁽¹⁶⁾
	3.53 $\pm$ 0.12	Photoionization ⁽²⁰⁾
I ⁻	3.17 $\pm$ 0.05	Surface ionization ⁽¹⁶⁾
	3.23	Surface ionization ⁽¹⁷⁾
	3.13 $\pm$ 0.12	Photoionization ⁽²⁰⁾

Table II
ATTACHMENT CROSS SECTIONS

Molecule	Cross Section cm ²	Energy (ev)
SF ₆	5.7×10^{-16}	0.00
CCL ₄	1.3×10^{-16}	0.02
CF ₃ I	7.8×10^{-17}	0.05
CCL ₂ F ₂	5.4×10^{-17}	0.15
BCL ₃	2.8×10^{-17}	0.4
HBr	5.8×10^{-17}	0.5
HCL	3.9×10^{-18}	0.6
H ₂ O	4.8×10^{-18}	6.4
O ₂	1.3×10^{-18}	6.2

Each of these has a different cross section and energy dependence. In a complex molecule such as SF_6 all sorts of products have been identified: (Ref. 23) SF_6^- , SF_5^- , SF_4^- etc. and F^- , F_2^- . The most probable reaction involves SF_6^- production, which has a sharp peak at 0.03 ev. (Ref. 24) SF_5^- formation has a much broader peak at 0.2 ev and a much lower cross-section.

It has been recently determined (Ref. 47) that SF_6^- is not completely stable but has a short lifetime ($1/\mu\text{sec}$) after which it dissociates into various products such as $\text{SF}_6 + e$, $\text{SF}_5^- + \text{F}$ etc. Some of the dissociation products yield free electrons and some do not. We know that SF_6 has a large cross section and is an effective inhibitor of breakdown. Because of this metastable character of SF_6^- we should expect different behavior for short pulse breakdown (i. e. pulse length less than the lifetime of the state). This phenomena should be investigated not only in sulphur hexafluoride, but in other polyatomic gases which might also display this metastable character. In fact one should evaluate many gases with the consideration of metastable negative ion formation since most previous experiments have been done under continuous or long pulse conditions. CO_2 forms negative ions by the following reaction



with a peak cross section of $5 \times 10^{-19} \text{ cm}^2$ at 8 ev.

We see that only certain gases will attach electrons, and that they do this in a fairly complex manner (i. e. the functions giving the energy dependence of the cross section are generally peaked and not representable by a simple function as are ionization potentials to a good degree of approximation).

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This is due to the quantum mechanical nature of the process. Further study is required on attachment processes and their coefficients. It may be possible to formulate a mixture of gases that has a high attachment cross section over a wide energy range. This gas would then be an ideal dielectric. In considering breakdown in free air we must consider the presence of water vapor which attaches electron readily.

III. THE THEORY OF PULSE BREAKDOWN

1) Single Pulse Breakdown

We shall first discuss breakdown that occurs during a single pulse. We assume that an equilibrium, quiescent condition exists before the application of the pulse. We shall set up the equations governing the electron buildup and then find solutions for the electric field that will cause the electron density to reach a critical value in a time equal to the pulse length.

After consideration of the physical processes involved, which are discussed above, we can reduce equation (5) to

$$\frac{\partial n}{\partial t} = (\nu_i - \nu_a) n \quad (12)$$

The source term S has been dropped because for short pulse breakdown where the net ionization ν is very large it will be important only as an initial source of electrons. It may be replaced by assuming an initial value for n . The spatial dependence has been removed by the elimination of the diffusion term and by assuming that ν_i and ν_a are independent of position. This is equivalent to assuming that the electron distribution function is isotropic and homogeneous in space. The ionization and attachment rates are average values over the electron distribution. The solution to equation (12) is

$$n = n_0 e^{\nu t},$$

where

$$\nu = \nu_i - \nu_a.$$

These concepts and general approach have worked for c. w. and long pulse breakdown (Refs. 35, 37, 38) where diffusion losses were included. If the neglect of diffusion is justified, and this approximation is better for shorter pulses, this approach should be valid for short pulse breakdown. Only if there were processes which are important for very short times would we expect a discrepancy. Such effects as metastable states of negative ions are not expected to affect the theory, they just require re-evaluation of the constants.

Let us examine the theory for possible limitations in the very short pulse limit. By assuming constant collision frequency we see that there is a lower limit on the time between collisions, and that if the pulse length is less than this time there can be no ionizations. This may be expressed as $\gamma \nu_c \leq 1$. Using our value of collision frequency for air reduces this to $p\tau < 0.2$ nanosecond torr for collisions, and hence ionizations to occur. This puts a lower limit on p of 0.01 torr for a 20 nsec pulse. This theory is not exact because the collision frequency is a function of energy and there will be a distribution of collision times. By considering somewhat more realistic potentials for interaction of electrons and atoms (Ref. 26) one may see that ν_c may decrease with increasing energy. This can lead to run-away electrons because as the collision frequency decreases we must increase the energy to insure a sufficient number of ionizations. This, however, reduces the number of collisions, and we see that breakdown will not occur. This effect will be very sensitive to the interaction potential.

As the mean free path increases with decreasing pressure more and more electrons will be swept to the walls at each cycle. This causes secondary effects which are discussed by Brown. (Ref. 4) For the higher

pressures $\nu_c > \omega$ and this is not important. Therefore as long as $p\tau > 0.2$ and p is not too low we should expect the theory to hold.

We must now investigate the calculation of ν . To do this in detail from elementary principles requires a knowledge of the distribution of electron energies, the attachment and ionization cross sections as a function of energy, and the collision frequency as a function of energy. Even if we had all this information, we would still have an extremely difficult problem. To find the distribution function one must solve the Boltzmann equation

$$\frac{\partial f}{\partial t} + \nabla \cdot \bar{\nabla}_r f + \frac{\bar{F}}{m} \cdot \nabla_v f = \left(\frac{\partial f}{\partial t} \right)_{\text{coll}} \quad (13)$$

The collision term on the right involves the effects of all collisions (elastic, exciting, attaching and ionizing) which in general not known exactly and usually varies in a complex manner. Brown and Allis, (Ref. 27) and Desloge and Matthysse (Ref. 28) have given approximations for this term. A general solution to this is virtually impossible and in electrical discharge problems one usually expands f in spherical harmonics (see Allis (Ref. 29) or Delcroix (Ref. 26)). Even then, however, a solution is not easily obtained except by approximate methods and usually only with great difficulty. The Boltzmann equation approach has been used to give a theory of breakdown for hydrogen, (Ref. 30) helium, (Ref. 31) helium without excited states, (Ref. 32) and neon. (Ref. 33) To apply this to other gases, especially to the more complex molecular species, would be very difficult. In any case, there is an extreme lack of cross-section information for many of the gases in which we have interest. The Boltzmann equation may also be used in a more general way to establish the importance of relaxation times for the various collision

processes and to gain insight into the effects of certain perturbations.

The usual study of breakdown or ionization processes avoids calculation from first principles although some work of this kind has been done. (Ref. 34) The theory usually used has been semi-empirical utilizing empirically determined ionization coefficients and theoretically supplying the other details of the calculation. This is a very useful method and further work along these lines is suggested. Empirical determination of the constants has the advantage that one automatically obtains averages over the distribution and that the difficult problem of accounting for excitation is automatically included in the measurement. The only problem then is to determine whether the long time c. w. measurements of ionization, and attachment differ from very short pulse measurements because of the finite time it takes to establish the steady state distribution. This also depends on the electron-atom, and electron-electron interactions and varies with the form of the potentials. Delcroix (Ref. 26) has a discussion of the problem, and calculates relaxation times for two cases. Except for very small values of $p \tau$ we expect that this effect will not be important.

The way to evaluate breakdown, then is to use experimental data. For air Gould and Roberts (Ref. 35) used d. c. coefficients and an equivalent field method to determine microwave coefficients. This approach may lead to errors, because there is an a priori assumption made that the distribution functions are the same for the equivalent values of E/p . This is not known to be true and the agreement may be fortuitous. Holstein (Ref. 36) has, however, shown that electron distributions in microwave discharges approach those in the d. c. case for a limited range. The method of Gould and Roberts may be used to calculate breakdown for gases in which microwave data is not

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available, but for which Townsend measurements have been made. Two other papers on breakdown air by MacDonald, (Ref. 37) and by Kelly and Margenau (Ref. 38) both utilize Brown's (Ref. 39) experimental microwave ionization data obtained from breakdown measurements. This coefficient $\beta = \nu/DE^2$ is a microwave coefficient for the diffusion limited case in analogy to the mobility limited Townsend ionization. The approach in these papers is somewhat different, but the essential results are the same. As before, we have the equation

$$\frac{\partial n}{\partial t} = (\nu_i - \nu_a) n \quad , \quad (14)$$

and

$$n(t) = n_0 e^{\nu t}$$

We define breakdown such that the electron density at the end of a pulse is to be

$$n_b(\tau) = 1.24 \times 10^{-8} f^2 \quad (15)$$

so that breakdown will occur if

$$\nu \tau = 4.6 \log f - 26.0 \quad (16)$$

where n_0 was chosen as $3 \times 10^3/\text{cm}^3$.

Kelly and Margenau utilize the modified Boltzmann equation and introduce a distribution function

$$f(v, t) = e^{-\mathcal{V}t} \phi(v) \quad (17)$$

The only result of this is to formally show that $\mathcal{V} = \mathcal{V}_1 - \mathcal{V}_2$, which was somewhat implicit in their choice of the distribution function.

We have a combination of theory and empirical data which will allow us to calculate the breakdown fields. We can calculate the value of $\mathcal{V}^*$ required to cause breakdown, as a function of frequency, with equation (16). Herlin and Brown (Ref. 39) have tabulated ϕ as a function of $(E/\rho)^2$. Therefore if one knows D as a function of E/ρ it is possible to relate the net ionization $\mathcal{V}$ and the field strength E . Unfortunately ϕ is also a function of $p\lambda$ (λ is the wavelength which corresponds to the frequency f), however when $p\lambda$ is greater than 200 all the curves merge. For $p\lambda$ 200 it is convenient to write

$$\mathcal{V}^* = D_p \cdot \left(\frac{E}{\rho}\right)^2 \cdot p\tau \cdot \phi, \quad (18)$$

which is obtained from Herlin's coefficient by judiciously multiplying by the correct factors. For smaller values of $p\lambda$ we must multiply equation (18) by λ , but for the higher pressures we can obtain a universal breakdown curve from equation (18). All the factors are either known or may be calculated. The only difficulty is with D_p . We have previously shown that

$$D_p = 2 \times 10^5 \langle u \rangle.$$

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The difficulty is in evaluating $\langle u \rangle$. Margenau uses a distribution function that he has previously (Ref. 40) shown to be approximately correct, whereas MacDonald uses the $\langle u \rangle$ data of Healy and Read, (Ref. 41) which was taken in dc discharges. Margenau's distribution is practically independent of E/p , and gives $Dp = 1.6 \times 10^6$, whereas MacDonald obtains $Dp \approx 1.2 \times 10^4 E/p$ (for higher pressures). These will be equal for $E/p = 130$, which means that their calculations of Dp are about equal in the range where breakdown occurs. This is of course to be expected, because the data for calculation of these values is supposedly based on actual measurements in gas breakdown or at least on a theory that is known to yield proper values. The agreement is, however, far from perfect and we expect that this discrepancy may be overcome by attempting to measure diffusion losses directly. Although the diffusion coefficient is not sensitive to the shape of the distribution it does depend directly on the value of the energy, which in turn must be known absolutely. Unfortunately we do not generally know the average energy either directly or indirectly.

Using equation (18) and a value of $Dp = 1.36 \times 10^6$ we have plotted a universal breakdown curve (Figure 2). This curve agrees with previous calculations of Gould and Roberts (Ref. 35) and at the longer pulse lengths agrees with the published measurements of MacDonald. This curve can be used to predict breakdown in planning experiments, so that one may predict the requirements of the equipment needed to do pulse measurements.

2) Multiple Pulse Breakdown

It is possible over a series of pulses to build up the electron density so that breakdown occurs after N pulses. We can write the following two equations for this situation.

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The buildup equation during each pulse is

$$\frac{\partial n}{\partial t} = \nu n \quad , \quad (12)$$

and the decay equation between pulses is

$$\frac{\partial n}{\partial t} = - \nu_a n - \alpha n^2 + D \nabla^2 n \quad (18)$$

We include the recombination and diffusion in the decay because of the longer time between pulses. In a strongly attaching gas we may simplify equation (18) to

$$\frac{\partial n}{\partial t} = - \nu_a n \quad (18a)$$

The electron density after N pulses is

$$n(t) = n_0 e^{N \nu \tau_1 - (N-1) \nu_a \tau_2} \quad (19)$$

or

$$\ln \frac{n}{n_0} = N \nu \tau_1 - (N-1) \nu_a \tau_2 \quad (19a)$$

There is a $-\nu_a$ term in ν but it will have a different value than ν_a during the decay period because the electrons in the long period between pulses will come into equilibrium with the gas and hence will have a Maxwellian

distribution at a temperature of about 300°K . The attachment cross section is high for low energy electrons so that the attachment during decay will be an efficient process. Unfortunately in the microwave measurements of ϕ there is no way to separate the ionization and attachment terms. Either d. c. data will have to be used, or a direct measurement may have to be made in the afterglow. This should be a prime consideration in the determination of new coefficients.

IV. MICROWAVE TESTING METHODS

Experimentally we require a fast rising pulse that can supply the required field strength. The field strength is related to power density so it is a matter of getting fast pulses and power. We do not, for purposes of testing, need actual short pulses although to supply the power we need in a long pulse will be difficult. We may divide this problem into two: the problem of rise time, and the problem of field strength.

We may relate the required field strength to power density by the Poynting theorem

$$\vec{S} = \vec{E} \times \vec{H} \quad (20)$$

By specifying the geometry we may relate E and H through an impedance for simple modes. The following cases are particularly useful:

The Plane Wave in Free Space

$$P = \frac{1}{Z} \frac{E^2}{377}$$

E in Peak Volts/cm
P in W/cm²

Rectangular Wave Guide

$$TE_{10} \text{ mode} \quad P = \frac{E^2 ab}{4 Z}$$

where a and b are the dimensions of the guide and Z is the impedance (about 500 ohms).

We cannot use a normal section of waveguide as a test section for two reasons. First, we must handle the same power everywhere so that we can

have a breakdown anywhere in the system, and second, because the guide is parallel we cannot predict where the breakdown would occur. Furthermore, to reduce our peak power requirement we wish to intensify the fields. The fields in a resonant cavity are of course very high, and this is used in c. w. breakdown for the evaluation of the ionization coefficients. However high Q cavities have a long time constant, $t = 2 Q/\omega$, which is of the order of a microsecond for X band cavities. This makes the cavity unsuited for pulsed measurement.

A simple method for obtaining higher fields is to taper the waveguide. In the TE_{10} mode the propagation is not affected by the height, so that we can reduce the height and increase the power density and hence the field strength. The field strength at constant power is inversely proportional to b^2 . A further gain may be realized by using ridged waveguides. The complete equations for power and field distribution (Ref. 42) are somewhat involved, and one may examine the original papers for a complete description. However, we may calculate the field strength in the following approximate but simpler manner. Consider a ridge waveguide with the following dimensions (Figure 3). From Hopfer (Ref. 43) we find that $\lambda_c = 3.5 A$ compared to $\lambda_c = 2 a$ for rectangular guide, therefore we may reduce A to $0.8a$, and S will be $0.4a$, B will be $0.8b$ and D will be $0.24b$. The fields are concentrated in the region of the ridge so that we see that much greater power densities are experienced. Hopfer gives information that also allows one to calculate the field. For this case we find that the guide would have $E/p = 40$ with 28 KW, whereas a tapered rectangular guide with the same actual height would require 108 KW. This is only an approximate calculation but shows the advantage in the ridged guide. The ridge guide may be tapered. To provide a transition from rectangular

guide it must be tapered in both dimensions (that is the width and the ridge gap). There is a minor problem with fringing fields at the corners, but this can be eliminated by contouring the ridge. The tapered ridge guide provides a convenient mechanism for providing high fields with modest power requirements; it also provides a localized breakdown because the rectangular guide that feeds the ridged section will have much lower fields. Loss of electrons by diffusion may have to be included for longer pulses because the diffusion length is small.

There is also the possibility of using mismatched sections to produce higher fields. If the test section were placed a quarter wavelength in front of a short we expect the following sequence. A traveling wave propagates down the guide toward the test point. The amplitude builds up in a time equal to the rise time, but there will be a backward wave reflected from the short which will set up a standing wave giving a E field maximum at the test point. Since the propagation time is very short (over a half wavelength path) in comparison to the rise time, we expect the field to rise to $2 E_0$, with a time characteristic of the pulse rise time. The time between the arrival of the traveling wave and the establishment of a standing wave is extremely short ($\approx 10^{-10}$ sec). We must remember that in a wave guide the pulse shape will be degraded due to dispersion if long propagation paths are used so that we must keep our wave guide length at a minimum.

We also considered the production of fast rising pulses. It appears that only an experimental program can provide new solutions to this problem. Presently available magnetrons have pulse rise times in the order of 20-50 nsec; limited by mode changing when faster drive pulses are used. It has been reported (Ref. 48) that special designs to eliminate mode changing can lower

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the rise time to say 1.0 nsec. This has not been fully exploited because present rise times are probably satisfactory with microsecond pulses. We can of course use a fast rise long pulse for testing if a source were available. It is unlikely that we can achieve the high peak powers required for breakdown in nanosecond times in a long pulse.

In order to get fast rise time one can consider either pulse shaping or a new generating scheme. If the wave guide could be suddenly shorted while a wave were propagating in it we would expect a sharp reflection. A useful fast switch is a gas discharge (e. g. , the TR Tube). We have seen that for $\omega > \omega_c$ the reflection as the electron density approaches the plasma resonant density is rather abrupt, so that we expect a sharp reflection from a rapidly ignited low pressure discharge. This could provide the means for pulse shaping. A circulator could be used to distribute the pulse to the test section and keep the reflected and incident waves separated. This system appears feasible and depends mainly on the fast switching which is available. We have been informed by W. Quinn of RADC, however, that this has been tried and has not been very satisfactory. This kind of pulse shaper would depend on the fast switch, the magnetron pulse and peak power available and on the properties of the circulator. It might be possible to develop this system, but there has appeared in the literature (Ref. 45) recently a more promising device, the variable-coupling ring resonator which can provide fast rise, short, high peak-power pulses. This device consists of a resonant re-entrant wave guide coupled to a straight section of guide. (Figure 5). This device has been previously used as a field intensity magnifier, and as an elimination filter. An electric field intensity gain in the ring of about 10 can be obtained, but because it is a high Q device the rise time is too long to make it useful

to us as a method of obtaining high fields. However, if after the ring is charged the coupling coefficient of the ring is suddenly changed to unity, it is possible to take out all the energy in a single pulse. The rise time is limited by the switch in the coupler and the pulse length depends on the ring length. Rise times of 3 nsec with pulse length of 20 nsec and a peak power of 750 KW have been achieved. (Ref. 45) The switching again depends on a rapidly ignited gas discharge. This appears to be the best method presently available for obtaining short, fast high energy pulses, and would certainly make a useful device for breakdown experiments.

V. SUGGESTIONS FOR FUTURE WORK

One of the main aims of this project was the investigation of the desirability and the requirements for doing pulse breakdown experiments. We, have shown in preceeding sections that a theory exists for predicting breakdown fields. This theory is essentially phenomenological and requires the use of empirically determined coefficients. This is entirely suitable for the guidance of system designers, but it does indicate a lack of fundamental knowledge of the processes involved in microwave breakdown. It is extremely desirable to have a detailed microscopic understanding of the processes involved in breakdown, and ultimately this approach is best. However, there are inherent difficulties in the microscopic approach, and we, therefore suggest the following three part program, which is designed to give us the data on breakdown that is required for system design, and to give a more basic understanding of the processes.

1) Pulse Breakdown Measurements

A primary requirement is for actual measurements under pulsed conditions. We suggest a series of pulse breakdown measurements in at least the following gases: air, humid air, CO_2 , SF_6 , and a none attaching rare gas. These measurements would be made using presently available techniques such as the variable-coupling ring resonator, along with tapered wave guide. The suggested gases were chosen because they represent the important cases encountered in present systems and because they represent a range of attachment cross sections (see Table II). The rare gas or other non-attaching gas is suggested as a check on the approximations made in the theory. If diffusion and recombination are negligible and attachment is absent then there are no loss mechanisms and breakdown should be controlled

entirely by the ionization coefficient. This direct measurement of the breakdown fields is not enough since it gives only data, and does not provide an explanation of the phenomena. To provide the understanding we must perform supplementary experiments.

2) Measurement of Ionization and Attachment Coefficients

We have a theory of microwave-pulsed breakdown that requires only that we insert the proper coefficients. The ionization coefficient can be obtained from c. w. breakdown measurements as originally proposed by Herlin and Brown. (Ref. 6) This involves the study of breakdown in a cavity, and yields a net ionization coefficient. It might be possible, by using a second signal as a probe, to determine electron density for fields less than that required for breakdown. This in turn would yield the net ionization coefficient for low values of E/p , and we therefore might be able to separate the attachment and ionization coefficients in a manner similar to that used in the analysis of d. c. discharges.

A similar cavity measurement can also be used to measure the net decay coefficient in the afterglow. In this case it is possible, from the shape of the curves, to separate (at least partially) the attachment and recombination coefficients. These two coefficients, net growth and net decay, would allow us to predict breakdown fields for widely varying conditions under both single pulse and multi pulse conditions. Comparison of such predicted results with the direct pulse breakdown measurements would test the accuracy of the theory. If, as we believe, the theory is sufficiently accurate, then we could substitute c. w. measurements for pulse measurements and still predict breakdown. This would be extremely advantageous since it would eliminate a large number of tedious measurements over a wide range of parameters

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such as pulse width.

This still does not contribute to our understanding of the microscopic processes, in order to do this we must examine a more detailed theory.

3) Microscopic Theory of Breakdown

By a microscopic theory we imply one in which the details of the collision processes are used. This requires a knowledge of the electron velocity distribution and detailed knowledge of the cross sections for the various processes. If one had this detailed information it would be possible to calculate the coefficients discussed in the preceeding section. With a somewhat less detailed knowledge it is possible to compare, or calculate, the microwave ionization from the Townsend first coefficient.

What we propose therefore is to attempt solutions to the Boltzmann equation in order to improve the approximation to the distribution function. This will be used to calculate ionization or attachment coefficients for cases where the cross sections are known. These calculations will be compared to the microwave measurements. The aim being to use both the theoretical calculations.

If one has available a function of v , $g(v)$ and its average $\int g(v) f(v) dv$, it is not possible to establish the distribution function $f(v)$, but it might be possible to use such information to decide between various possible distributions or to adjust constants in a calculated distribution.

The suggested program realizes the usefulness of the phenomenological approach and also realizes the importance of providing immediate data on breakdown. It further realizes the importance of gaining a complete and

detailed understanding of the phenomena. In our suggested program we have tried to stay with presently available techniques and concentrate on the phenomenon of breakdown.

Respectfully submitted,

ARMOUR RESEARCH FOUNDATION
of Illinois Institute of Technology



Lawrence C. Scholz, Research Physicist
Plasma and Electron Physics Research

APPROVED BY:



S. W. Kash, Manager
Plasma and Electron Physics Research



I. B. Fieldhouse,
Assistant Director of Physics Research

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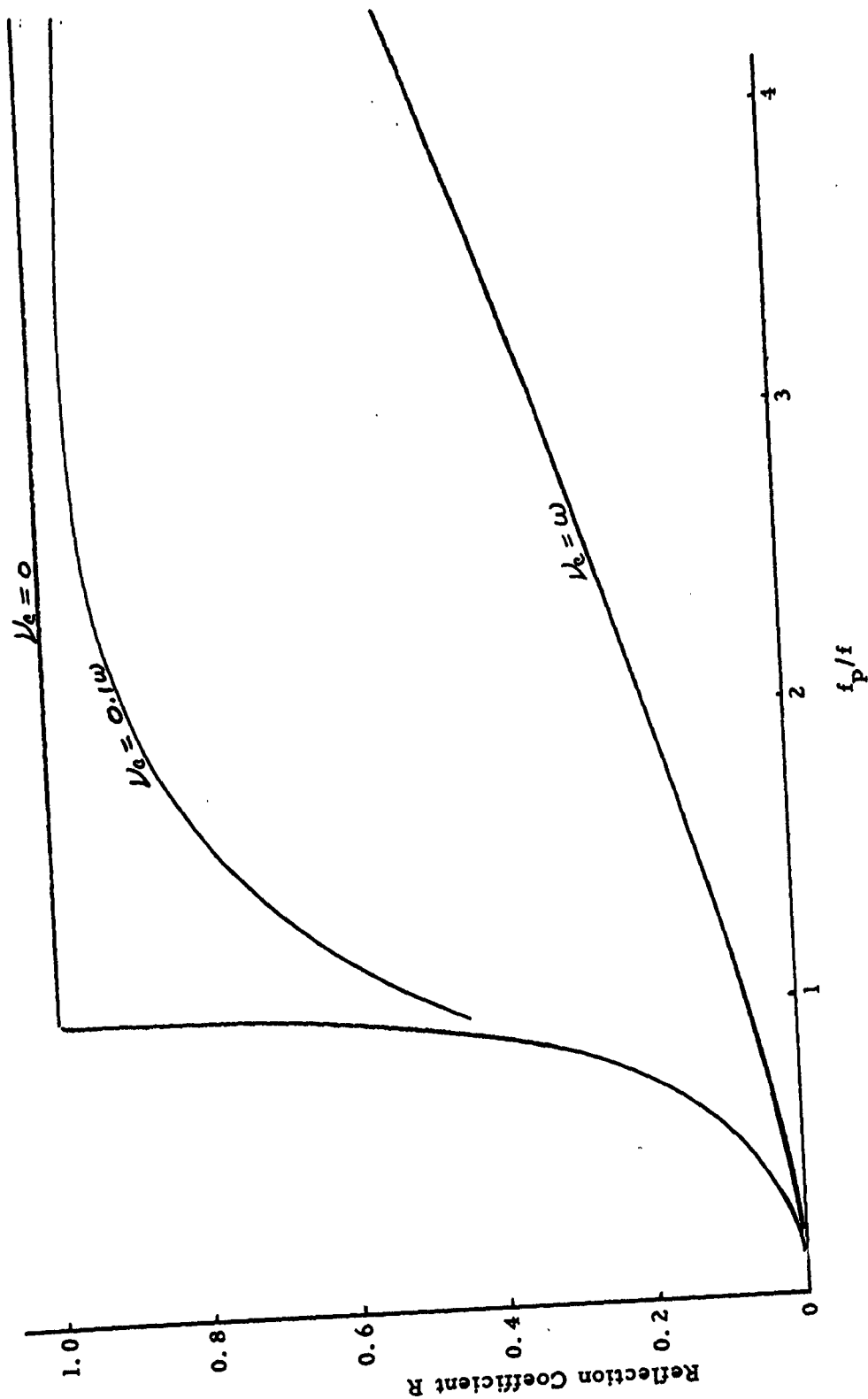


Figure 1. Reflection Coefficient vs. f_p/f .

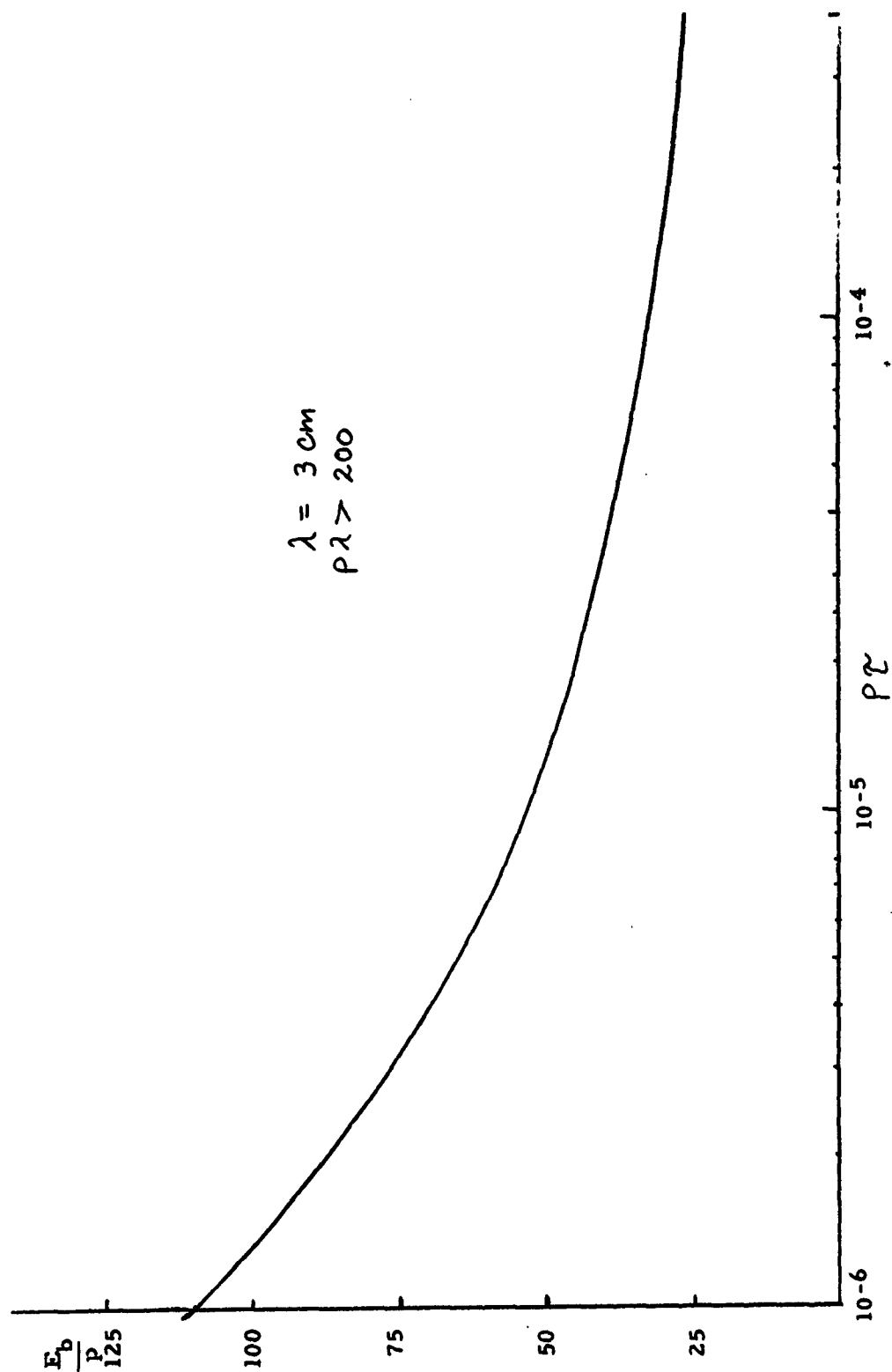


Figure 2. Breakdown Field as Function of pL .

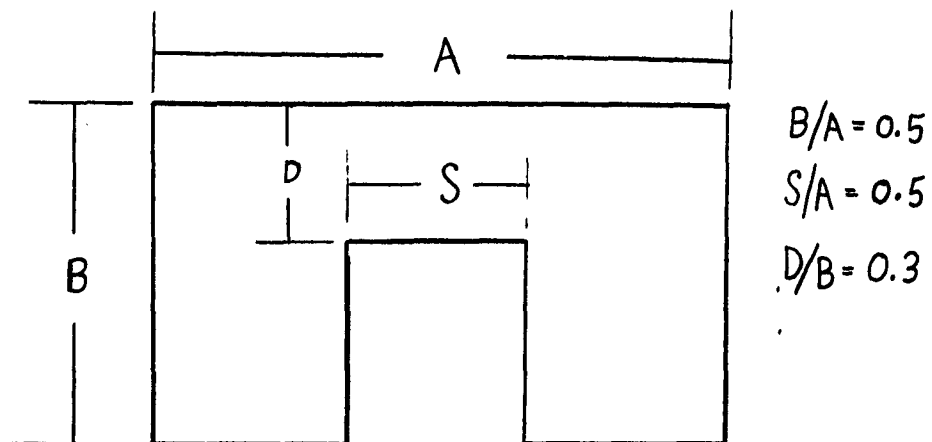


Figure 3. Dimension of Ridged Waveguide.

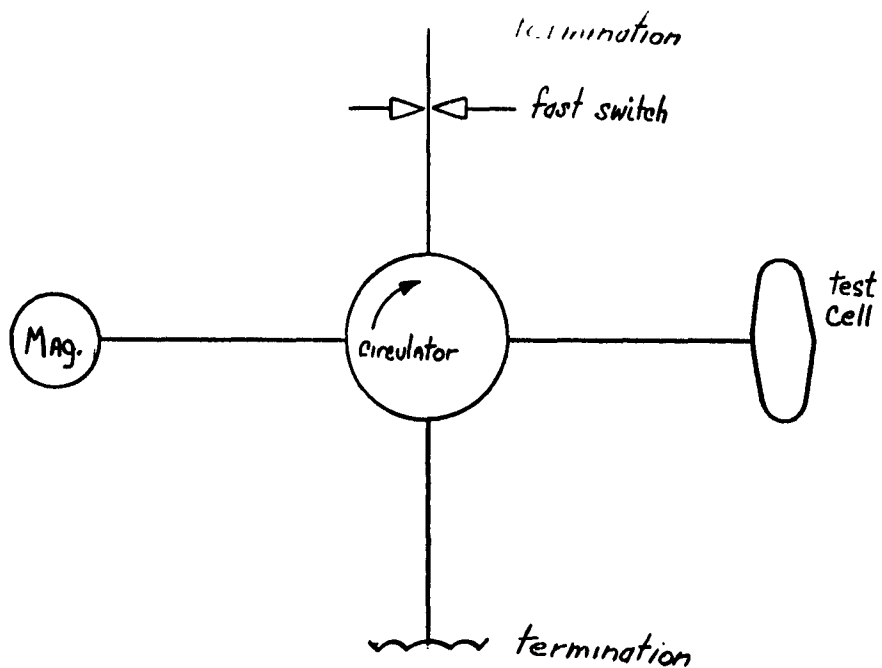


Figure 4. Schematic Design of Pulse Generator Using Circulator.

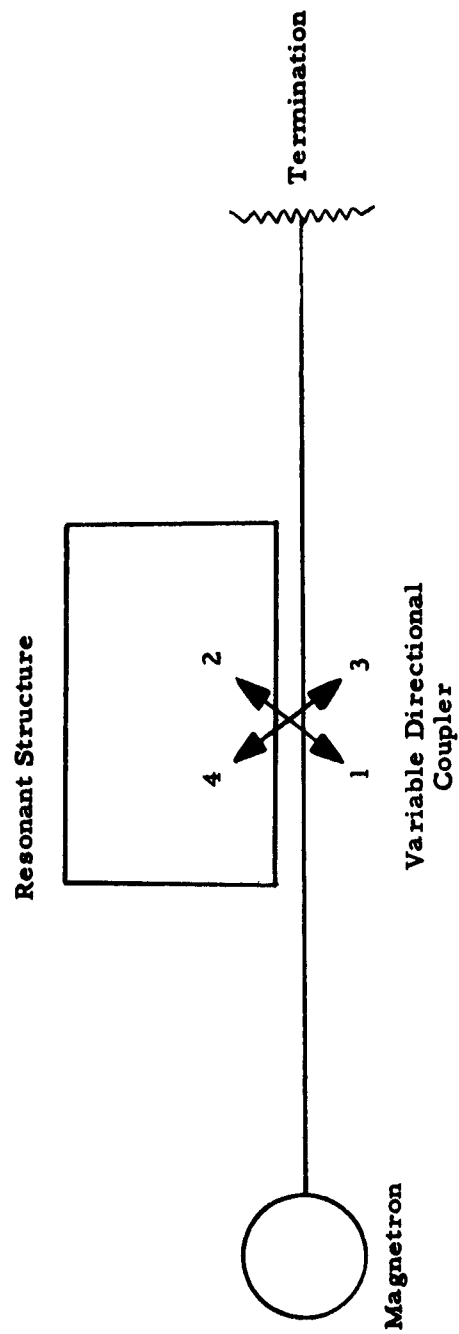


Figure 5. Pulse Generator Using Resonant Ring Structure.

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LIST OF SYMBOLS USED

D	diffusion coefficient
e	electron charge
E	field strength (volts/meter)
f	frequency (cycles/sec)
f_p	resonant plasma frequency
f	velocity distribution function
ℓ	mean free path (of electrons)
m	mass of electron
n_b	number density of electrons at breakdown
P	pressure (Torr)
R	reflection coefficient for EM waves
S	generalized source term for electron production
S	Poyntings vector (watts/m ²)
u	electron energy
$\langle u \rangle$	electron energy averaged over velocity distribution function
v	electron velocity
$\langle v \rangle$	electron velocity averaged over the distribution function
α	recombination coefficient cm ³ /ion sec
Γ	diffusion particle current density
∇_r	gradient in coordinate space
∇_v	gradient in velocity space
η	generalized index of refraction
β	microwave ionization coefficient
λ	wavelength

Λ	characteristic diffusion length
ν	net ionization rate
ν_a	collision frequency for attachment
ν_c	collision frequency for momentum
ν_i	collision frequency for ionization
τ	pulse length

APPENDIX I
DETAILS OF THE CALCULATION OF THE
DIFFUSION COEFFICIENT

The "particle current" density (excluding charge) may be written as

$$\vec{\Gamma} = -\vec{\nabla}(Dn) \quad (I-1)$$

This equation includes all the diffusion effects due to, density gradients, temperature gradients, and any other effect that might effect D. However we are interested only in density effects and may rewrite this as

$$\vec{\Gamma} = -D\vec{\nabla}n \quad (I-1a)$$

This can be thought of as a defining statement for D. In order to get a mathematical expression and physical interpretation for D we remember that $\vec{\Gamma}$ may also be written as

$$\vec{\Gamma} = n \langle \vec{v} \rangle \quad (I-2)$$

For simplicity consider the one-dimensional case. Let Δx equal the average distance a particle moves between collisions, and τ the time between collisions. Then in a small region of space ($x - \Delta x/2, x + \Delta x/2$) the net current is

$$\Gamma_x = \frac{\Delta x}{\tau} \left[\left(n + \frac{1}{2} \Delta x \frac{\partial n}{\partial x} \right) - \left(n - \frac{\Delta x}{2} \frac{\partial n}{\partial x} \right) \right], \quad (I-3)$$

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which reduces to

$$\Gamma_x = \frac{(\Delta x)^2}{\tau} \frac{\partial n}{\partial x} \quad (I-3a)$$

Therefore we see that

$$D = \frac{(\Delta x)^2}{\tau} \quad , \quad (I-4)$$

which shows that D is the mean-squared-distance a particle moves in unit time. The collision frequency (ν_c) is just equal to the $1/\tau$, so that we may also write

$$D = (\Delta x)^2 \nu_c \quad (I-4a)$$

A more exact calculation may be made by starting from the Maxwell-Boltzmann equation for the electrons.

$$\frac{\partial f}{\partial t} + \vec{\nabla} \cdot \nabla_r f + \frac{\vec{F}}{m} \cdot \vec{\nabla}_v f = \left(\frac{\partial f}{\partial t} \right)_{\text{coll}} \quad (I-5)$$

The implications and approximations are described in many standard works (c. f. Rose (46)). If we assume further that the distribution function $f(v, r)$ is equal to

$$f(v, r) = f_0(\vec{r}, v) + f_1(\vec{r}, \vec{v}) \quad , \quad (I-6)$$

where $f_0(r, v)$ is isotropic, and

$$n(r) = \int d\bar{v} f_0(\bar{r}, v) \quad (I-7)$$

is the number of electrons. We assume f_1 is a small perturbation on the isotropic distribution and that its integral over all phase space is zero. We introduce this distribution function into equation (I-5) and look for an equilibrium solution, whence

$$v_0 \cdot \bar{\nabla}_r (f_0 + f_1) + \frac{\bar{F}}{m} \cdot \bar{\nabla}_v (f_0 + f_1) = \left(\frac{\partial f_0}{\partial t} \right) + \left(\frac{\partial f_1}{\partial t} \right)_{\text{coll}} \quad (I-8)$$

The higher order terms $\bar{\nabla}_r f_1$, $\bar{\nabla}_v f_0$ are neglected and $\left(\frac{\partial f_0}{\partial t} \right)_{\text{coll}}$ equals zero since it is by definition an equilibrium distribution. The (I-8) reduces to

$$\bar{v} \cdot \bar{\nabla}_r f_0 + \frac{\bar{F}}{m} \cdot \bar{\nabla}_v f_0 = \left(\frac{\partial f_1}{\partial t} \right)_{\text{coll}} \quad (I-9)$$

The anisotropy f_1 is generated by the density gradient and the external force $\bar{F}$. We assume that the distribution function would relax to f_0 , in the absence of forcing terms, due to momentum transferring collisions and then

$$\left(\frac{\partial f_1}{\partial t} \right)_{\text{coll}} = -\nu_m f_1, \quad (I-10)$$

where ν_m is the momentum transferring collision frequency (Ref. 29). We

have

$$\begin{aligned}\Gamma &= n \langle \bar{v} \rangle = \int d\bar{v} \bar{v} f = \int d\bar{v} \bar{v} (f_0 + f_1) \\ &= \int d\bar{v} \bar{v} f_1, \end{aligned} \quad (I-11)$$

because $\int d\bar{v} \bar{v} f_0$ is zero due to isotropy of f_0 . We may now substitute for f_1 in equation (I-9) dropping the force term as we are interested only in diffusion due to density gradients. Then

$$\Gamma = - \int d\bar{v} \bar{v} \left(\frac{\bar{v}}{\nu_c} \cdot \bar{\nabla}_r f_0 \right). \quad (I-12)$$

Consider a gradient only in the x direction. Then $\bar{v} \cdot \bar{\nabla}_x f_0 = v_x \partial f_0 / \partial x$ and the flow will also be in the x direction. Therefore

$$\Gamma_x = - \int d\bar{v} v_x \left(\frac{v_x}{\nu_c} \frac{\partial f_0}{\partial x} \right), \quad (I-13)$$

but

$$v_x^2 + v_y^2 + v_z^2 = v^2,$$

so that near equilibrium where the distribution is almost isotropic the component velocities are equal and for any component we have

$$v_x^2 = \frac{v^2}{3} \quad (I-14)$$

The diffusion current is then equal to

$$\Gamma = - \int d\vec{v} \frac{v^2}{3\nu_c} \nabla_r f_0 \quad (I-15)$$

for any arbitrary direction. Because $\vec{v}$ & $\vec{r}$ are independent we have

$$\Gamma = -\nabla \int d\vec{v} \frac{v^2}{3\nu_c} f_0 = -\nabla (Dn) \quad (I-16)$$

Therefore,

$$D = \left\langle \frac{v^2}{3\nu_c} \right\rangle \quad (I-17)$$

This derivation follows closely that given in Rose (Ref. 46) and Delcroix (Ref. 7). We may finally substitute $u = \frac{1}{2} m v^2$ and obtain the following result,

$$D = \frac{2}{3M} \left\langle \frac{u}{\nu_c} \right\rangle \quad (I-18)$$

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