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TITLE: ⑥ Operation on chart, improvements

PERIODICAL: ⑤ Shu Hst'eh Hst'eh Pao, v. 10, 3, 1960, 267-275

TEXT: An improved process for operation on charts is proposed by which an optimal flow chart is determined by checking a few circles only. A chart is interpreted to be a continuous linear chart on a plane. A basic flow chart is defined as one which satisfies the following conditions: 1) no opposite flow; 2) no circle having flow on all arcs; 3) number of arcs having flow less than number of points by one. A point P on a continuous linear chart is said to be a movable point, if the chart is still continuous after P and all arcs having P as an end point are deleted from the chart. The following are theorems concerning a basic flow chart: Theorem 1. There exists a basic flow chart. Theorem 2. Optimal flow chart may definitely be found from the basic flow charts. Theorem 3. Suppose that a basic flow chart is given.

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Then, for every arc having no flow (empty arc), there exists the only circle which includes the given empty arc and on which all other arcs have flows. Theorem 4. A basic flow chart is optimal if length of any flow on the outside and inside circles corresponding to all empty arcs in Theorem 3 does not exceed one-half circle length. In proving these theorems, the problems to be solved are expressed by mathematical expressions. Let C_{ij} denote distance between P_i, P_j ; c_{ij} , length of arc connecting P_i, P_j . When there is no arc connecting P_i, P_j , let c_{ij} be a large positive integer M such that direct transportation between P_i, P_j is impossible. Let x_{ij} be the transportation volume from P_i to P_j , then for every point P_i , we have

$$\sum_{\substack{k=1 \\ k \neq i}}^n x_{ik} - \sum_{\substack{k=1 \\ k \neq i}}^n x_{ki} = a_i, \quad i=1, \dots, n \quad (1)$$
$$\left(\sum_{i=1}^n a_i = 0 \right)$$

i. e., total issuing volume of P_i should be equal to the negative number of issuing or

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receiving volume of P_i . But our purpose is to find a set of non-negative x_{ij} ($i, j = 1, \dots, n, i \neq j$) which satisfies (1) and such that the function

$$S = \sum_{\substack{i,j=1 \\ i \neq j}}^n c_{ij} x_{ij} \quad (2)$$

has a minimum value. This is a linear programming problem and that determined by (1) and (2) is called Type A problem. By arranging variables of (1) as $x_{12}, \dots, x_{1n}; x_{21}, \dots, x_{2n}, \dots; x_{n1}, \dots, x_{n, n-1}$, augmented matrix of (1) will have the following form:

$$B = \begin{bmatrix} | & | & \dots & | & -1 & & & & & & a_1 \\ -1 & & & & & & & & & & a_2 \\ & -1 & & & & & & & & & \vdots \\ & & \ddots & & & & & & & & a_{n-1} \\ & & & -1 & & & & & & & a_n \end{bmatrix} \quad (3)$$

Obviously, B has the rank $n - 1$ and thus the basis of (1) should comprise $n - 1$ vari-

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ables. Let x_{ij} be a variable of (1). Transformation of x_{ij} to x_{ji} is called inversion. Let

$$x_{i_1 j_1}, x_{i_2 j_2}, \dots, x_{i_t j_t} \quad (4)$$

be the t ($t \geq 2$) different variables of (1). If (4) itself or the variable set derived after inversion of certain variables in (4) can be arranged in the form

$$x_{k_1 k_2}, x_{k_2 k_3}, \dots, x_{k_t k_1}, \quad (5)$$

(k_1, k_2, \dots, k_n being different)

then, set (4) is called a return route. The total of variables not inverted in arranging set (4) into form (5) is said to be the regular return route; the total of those remaining, the reverse return route. Lemma 4. The necessary and sufficient condition for coefficient row vectors

$$\alpha_{i_1 j_1}, \alpha_{i_2 j_2}, \dots, \alpha_{i_t j_t} \quad (6)$$

of (4) to be linearly dependent is that a portion of variables in (4) forms a return route.

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Theorem 4'. A basic feasible solution set is given. If the length of either the regular or reverse return routes of each pair of nonbasic variables x_{ij} and x_{ji} does not exceed one-half the return route length, the given solution is the optimal solution. Theorem 4 is derived from Theorem 4' and determines the number of circles to be checked in checking a basic flow chart. Suppose that there are n points and s arcs in a given chart. In the basic flow chart, there will be $n - 1$ arcs having flows. Hence

$$\text{Circle number to be checked} = \text{empty arc number} = x - n + 1.$$

There are 8 figures. One English-language reference reads as follows: S. Vajda, Theory of game and linear programming.

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