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# TRANSLATION

ON THE PROBLEM OF SYNTHESIZING BLOCK DIAGRAMS FOR SELF-ADJUSTING CONTROL SYSTEMS FOR TURBO-JET ENGINES WITH AFTERBURNERS

By

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# UNEDITED ROUGH DRAFT TRANSLATION

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# ON THE PROBLEM OF SYNTHESIZING BLOCK DIAGRAMS FOR SELF-ADJUSTING CONTROL SYSTEMS FOR TURBOJET ENGINES WITH AFTERBURNERS

V. A. Bodner and Yu. A. Ryazanov

## 1. Statement of the Problem

The choice of optimum regimes of operation in aircraft power plants under various flight conditions for the purpose of obtaining the most advantageous characteristics in the aircraft is made difficult by the fact that these regimes are determined by a great many interrelated parameters and characteristics. It is not feasible to set these regimes directly because of the great complexity of the optimum relationships. But since the engine must operate at the optimal regimes or close to them, it is natural to pose the problem of creating automatic control systems which would ensure the choice of these regimes. Included among such systems are the various optimalizing systems.

As the external flight conditions change the characteristics of turbojet engines with afterburners [ATE] vary over wide ranges and in the end this leads to a change in the dynamic parameters of the engines as objects of control. In this case the dynamic parameters of ATE's may vary over side ranges in rather short periods of time. As a result of this the transmission ratio of the controllers, chosen as optimal for one set of flight conditions, may prove unsatisfactory under another set of conditions.

In this paper we are posing the problem of synthesizing block diagrams of self-adjusting control systems for ATE's which ensure optimalization of the transients in the control circuits for the speed of revolution and gas temperature. Moreover the specific natures of the variation in the dynamic parameters of the engine and parameters of the controllers under different flight conditions is taken into account.

## 2. The Effect of Flight Conditions on the Dynamic Characteristics of ATE's

Let us consider a single-shaft ATE (Fig. 1) in which the basic controlled parameters are: speed of revolution of compressor n, gas temeprature in front of the turbine $T_3$, the gas temperature in the afterburner $T_a$, the extent of the decrease in the pressure in the turbine $\pi_T$, and the parameters determining the operation of the diffuser. For the control actions we are employing: the main fuel flow $G_T$, the jet nozzle cross section F, and the organs of diffuser control.*

In determining the dynamic parameters of an ATE and the parameters of the controllers with respect to the variable flight regimes it was assumed that the diffuser is controlled and guarantees optimal variation with respect to flight velocity for the pressure recovery factor σ.

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* V. A. Bodner. Aircraft Engine Automation, Oborongiz, 1956.

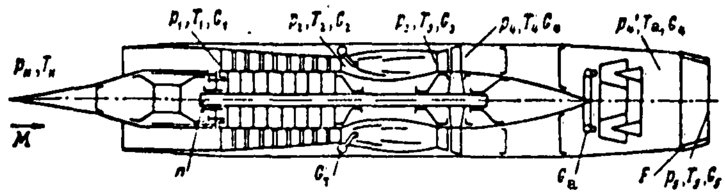

Fig. 1. Diagram of ATE

The equations for the ATE as an object of control shall be taken in the following form:

$$\left.\begin{aligned} &(\tau_1 p+1)\,x_1 = l_{13}x_3 + l_{15}x_5;\\ &x_3 = k_{3O}\mu_O - l_{31}x_1;\\ &x_4 = k_{4a}\mu_a - l_{41}x_1 + l_{43}x_3 - l_{45}x_5;\\ &x_5 = k_{5F}\mu_F + l_{53}x_3 - l_{54}x_4, \end{aligned}\right\} \qquad (1)$$

where $\tau_1$, $l_{13}$, $l_{15}$, $k_{3O}$, $l_{31}$, $k_{4a}$, $l_{41}$, $l_{43}$, $l_{45}$, $k_{5F}$, $l_{53}$, $l_{54}$ are the coefficients characterizing the dynamic characteristics of an ATE as an object of control;

$$x_1=\frac{\Delta n}{n};\quad x_3=\frac{\Delta T_3}{T_3};\quad x_4=\frac{\Delta T_a}{T_a};\quad x_5=\frac{\Delta \pi_T}{\pi_T};$$

$$\mu_O=\frac{\Delta G_T}{G_T};\quad \mu_a=\frac{\Delta G_a}{G_a};\quad \mu_F=\frac{\Delta F}{F}.$$

We may state the following relationships for the determination of the coefficients in Eqs. (1):

$$\left.\begin{aligned} &\tau_1=\frac{\pi}{60}\,\frac{Jn}{M_n};\quad l_{13}=\frac{1}{2};\quad l_{15}=\frac{k-1}{2k}\,\frac{1}{\pi_T^{\frac{k-1}{k}}-1};\\ &l_{31}=4\,\frac{\pi_K^{\frac{k-1}{k}}-1}{\pi_K^{\frac{k-1}{k}}}\,\frac{T_3-T_2}{T_3+T_2}\left(\frac{k}{k-1}-\frac{T_2}{T_3-T_2}\right);\\ &k_{3O}=2\,\frac{T_3-T_2}{T_3+T_2};\quad k_{4a}=\frac{T_a-T_4}{T_4};\quad l_{43}=\frac{1}{2}\,\frac{T_a+T_4}{T_a};\\ &l_{41}=\frac{2k}{k-1}\,\frac{\pi_K^{\frac{k-1}{k}}-1}{\pi_K^{\frac{k-1}{k}}}\,\frac{T_a-T_4}{T_a};\quad l_{45}=\frac{k-1}{k}\,\frac{T_4}{T_a};\\ &k_{5F}=1;\quad l_{53}=l_{54}=\frac{1}{2}, \end{aligned}\right\} \qquad (2)$$

where $\pi_K$ is the pressure ratio in the compressor and $\pi_T$ is the expansion ratio in the turbine.

It is possible to set up the block diagram for an ATE as an object of control depicted in Fig. 2 in accordance with Eq. (1). As is apparent from Fig. 2, the ATE as an object of control is a complex dynamic system having internal communications and actions.

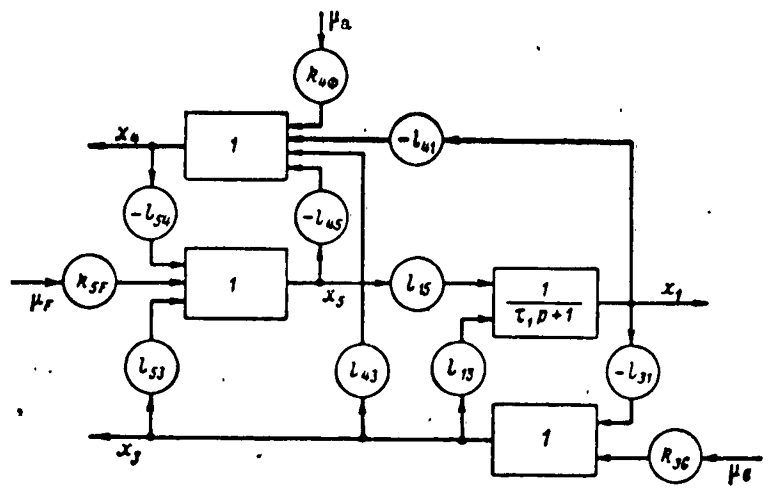

Fig. 2. Block diagram of an ATE as an object of control.

In order to control the operation of the ATE under variable flight conditions a suitable automatic control system is needed. The thrust of the ATE may be varied as a result of a change in the temperature of the gases in front of the turbine as well as of those in the afterburner, in which case it is possible to vary the thrust in the way required without changing the speed of revolution. In this way a more economical operation is achieved in the ATE since efficient operation of the compressor is ensured.

A possible program for the take-off and climb of an aircraft with an ATE from start to attainment of the rated flight altitude and velocity is presented in Fig. 3. Such a take-off and climb program ensures a minimum over-all fuel consumption in a number of cases; for the purpose of decreasing the over-all fuel consumption the initial portion of the take-off may be accomplished without the afterburner with a subsequent increase in afterburning up to the maximum value corresponding to a temperature $T_{a\ max}$. Let us take as an example an ATE with the following arbitrary data: n = 7500 rpm; $T_3 = 1250^\circ$; $T_a = 2250^\circ$; $\pi_{k0} = 5.0$; J = 4 kg·m·sec$^2$. The dynamic parameters listed in Table 1 were determined according to these data using Eq. (2).

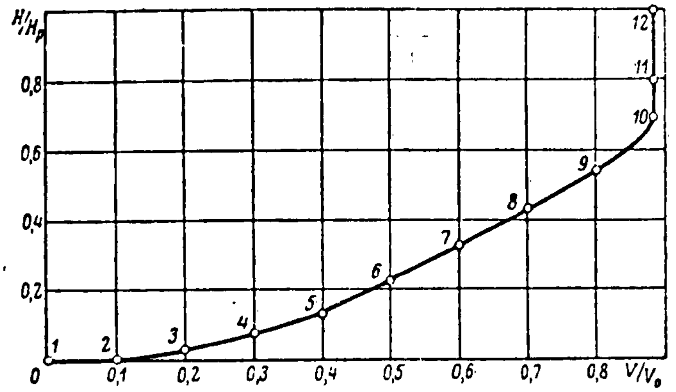


Fig. 3. A possible takeoff and climb program for an aircraft with an ATE.

The nature and limits of the variation in the dynamic parameters of the ATE as an object of control are shown in Fig. 4. Here the corresponding values of the dynamic engine parameters are plotted on the ordinate while the relative variation in the energetic flight altitude $H_e = H + V^2/2g$ are plotted on the abscissa.

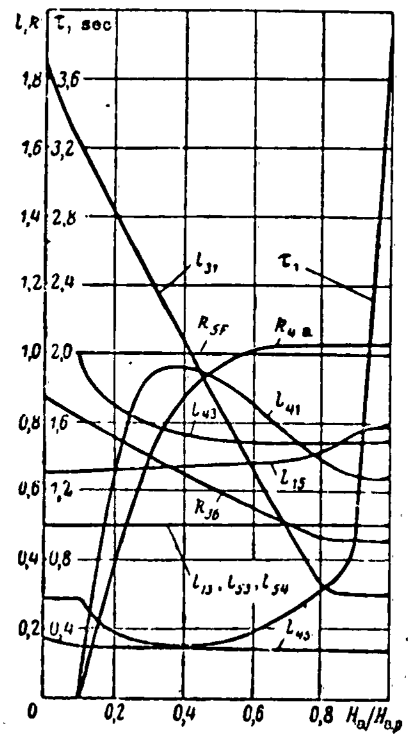


Fig. 4. Variation of dynamic parameters of an ATE according to flight regimes

With simultaneous variation in flight velocity and altitude as prescribed by the take-off and climb program (cf. Fig. 3) the energetic flight altitude $H_e$ is a sufficiently characteristic parameter for determining the flight program of the aircraft.

It follows from a consideration of the curves in Fig. 4 that the time constant $\tau_1$ changes by a factor greater than 10, with the most pronounced increase in $\tau_1$ occurring at high altitudes. The other dynamic parameters of the ATE as an object of control also vary in accordance with flight regimes.

TABLE 1

| No. | $\frac{V}{V_0}$ | $\frac{H}{H_p}$ | $\tau_1$ sec | $l_{13}$ | $l_{15}$ | $l_{31}$ | $k_{3G}$ | $k_{4a}$ | $l_{41}$ | $l_{43}$ | $l_{45}$ | $k_{5F}$ | $l_{53}$ | $l_{54}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0,34 | 0,5 | 0,65 | 1,83 | 0,86 | 0 | 0 | 1,0 | 0,29 | 1,0 | 0,5 | 0,5 |
| 2 | 0.1 | 0 | 0,34 | 0,5 | 0,65 | 1,81 | 0,86 | 0 | 0 | 1,0 | 0,29 | 1,0 | 0,5 | 0,5 |
| 3 | 0.2 | 0,025 | 0,33 | 0,5 | 0,66 | 1,74 | 0,85 | 0 | 0 | 1,0 | 0,29 | 1,0 | 0.5 | 0.5 |
| 4 | 0.3 | 0,075 | 0.3 | 0,5 | 0,66 | 1,61 | 0,82 | 0 | 0 | 1,0 | 0,29 | 1,0 | 0.5 | 0,5 |
| 5 | 0,4 | 0,135 | 0,3 | 0,5 | 0,66 | 1,43 | 0,79 | 0,28 | 0,51 | 0,89 | 0,22 | 1,0 | 0,5 | 0,5 |
| 6 | 0,5 | 0,225 | 0,29 | 0,5 | 0,67 | 1,27 | 0,75 | 0,68 | 0,91 | 0,80 | 0,17 | 1,0 | 0,5 | 0,5 |
| 7 | 0.6 | 0,325 | 0,30 | 0,5 | 0,67 | 1,02 | 0,68 | 0,85 | 0,95 | 0,77 | 0,16 | 1,0 | 0,5 | 0,5 |
| 8 | 0,7 | 0,43 | 0.35 | 0,5 | 0,68 | 0,74 | 0,62 | 1,01 | 0,89 | 0,75 | 0,15 | 1,0 | 0.5 | 0,5 |
| 9 | 0.8 | 0,535 | 0.5 | 0,5 | 0,71 | 0,5 | 0,53 | 1.07 | 0,77 | 0,74 | 0,14 | 1,0 | 0,5 | 0,5 |
| 10 | 0,885 | 0,695 | 0,74 | 0,5 | 0,75 | 0,3 | 0,46 | 1,06 | 0,64 | 0,74 | 0,14 | 1,0 | 0,5 | 0,5 |
| 11 | 0,885 | 0,8 | 1,37 | 0,5 | 0,78 | 0,3 | 0,46 | 1,05 | 0,64 | 0,75 | 0,14 | 1,0 | 0,5 | 0,5 |
| 12 | 0,885 | 1,0 | 3,77 | 0,5 | 0,78 | 0,3 | 0,46 | 1,04 | 0,64 | 0,75 | 0,14 | 1,0 | 0,5 | 0,5 |

For example, the coefficient $l_{31}$ decreases by a factor of more than six for an increase in the flight altitude and velocity. The coefficient $k_{3G}$ under the same conditions decreases by a factor of about two, while the coefficients $l_{13}$, $l_{15}$, $k_{5F}$, $l_{53}$ and $l_{54}$ remain practically constant. The variation of the coefficients determining the effect of the engine parameters for a variation in the temperature of the gases $T_a$ in the afterburner ($k_{4F}$, $l_{41}$, $l_{43}$ and $l_{45}$) is determined principally by the law governing the supply of fuel to the afterburner.

## 3. Synthesis of Optimal Controller Parameters

In designing a control system for an ATE there arises the problem of selecting the parameter of the controllers and needed correction systems taking into account certain specifications regarding the quality of the transients in the control process. A fundamental requirement for the control system of an ATE is the need to maintain a given engine thrust with an accuracy to the order of $\pm 2\%$, for which it is necessary to maintain the speed of revolution to an accuracy of $\pm 0.3\%$, while the temperature of the gases in the afterburner and in front of the turbine must be maintained with an accuracy of $\pm 1.5\%$. The transients with respect to the temperature of the gases in front of the turbine should be as free from overshoots as possible. In the limiting case a short (no more than 1-2 sec) overshoot not exceeding $50^\circ$ - - $80^\circ$C is acceptable. The maximum deviation in the speed of revolution under transient conditions should not exceed 200-300 rpm.

As follows from Eqs. (1), an ATE considered as an object of control has three control actions $\mu_G$, $\mu_a$, and $\mu_F$. Since the fuel consumption in the after-burner $\mu_a$ is controlled by the flight velocity controller, we shall limit ourselves to a consideration of a control system consisting of two controllers: one for the speed of revolution and one for the temperature of the gases in front of the turbine. Moreover, it is assumed that the controller of the speed of revolution acts upon the main fuel supply, while the controller for the temperature of the gases in front of the turbine acts upon the cross section of the jet nozzle. The fuel consumption in the afterburner is controlled by the flight velocity controller, which is included in the control system of the aircraft.

We shall carry out the synthesis of the circuits for control of the speed of revolution and the temperature of the gases in front of the turbine under the assumption that under conditions of overspeed the system incurs a disturbance resulting from a readjustment of the gas temperature control $x_{30}$, and without overspeed a disturbance is introduced by a variation in the afterburner fuel supply $\mu_{a0}$.

In deriving the equations of the controls we shall take into account the thermal inertia of the sensitive element of the gas temperature control evaluated as the time constant $\tau_T$.

Let us assume that a correction system with a time constant $T_T$ which serves to compensate for the dynamic aberrations of the thermocouples is used in the gas temperature control. Let us assume the speed of revolution control has a correcting system in the form of two servomotors wired in parallel. The systems for the main and afterburner fuel supply include constant flow controls which are assumed to be ideal.

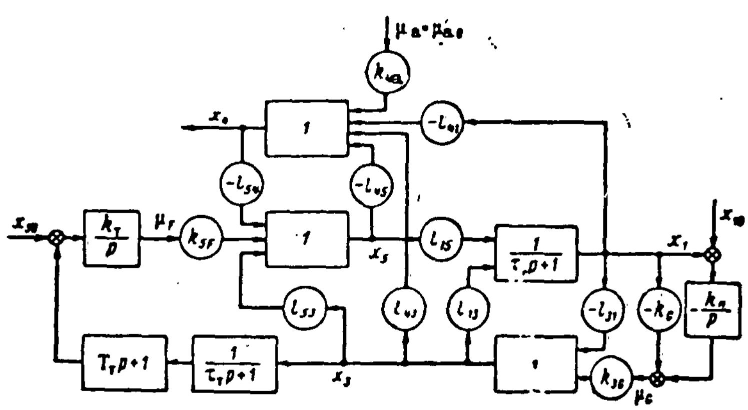

Fig. 5. Block diagram of ATE control system.

With these assumptions we may write:

the equation of the control of speed of revolution

$$\mu_G = -\frac{k_n}{p}(x_1 - x_{10}) - k_G x_1, \quad (3)$$

where $k_n$ is the amplification factor of the astatic circuit of the speed of revolution controller; and

$k_G$ is the amplification factor of the static circuit of the speed of revolution controller;

the equation of the gas temperature controller

$$p\mu_F = k_\tau \left( \frac{T_\tau p + 1}{\tau_\tau p + 1} x_3 - x_{30} \right). \qquad (4)$$

where $k_T$ is the amplification factor of the gas temperature controller;

and the equation of the afterburner fuel consumption regulator

$$\mu_F = \mu_{F0}. \qquad (5)$$

A block diagram for an ATE control system is presented in Fig. 5. It follows from the diagram that the controls for speed of revolution and gas temperature are linked by intra-engine connections and therefore form a single circuit system.

The distinctive feature of such a system is the unambiguous relationship between the transients with respect to gas temperature and speed of revolution, which allows the synthesis to be carried out with respect to one of these parameters.

Solving Eqs. (1) and (3)-(5) simultaneously we obtain the equations of a closed ATE control system relative to $x_1$ and $x_3$:

$$\left. \begin{aligned} A(p) x_1 &= B_{11}(p) x_{10} - B_{13}(p) x_{30} - B_{1a}(p) \mu_{a0}; \\ A(p) x_3 &= B_{31}(p) x_{10} + B_{33}(p) x_{30} + B_{3a}(p) \mu_{a0}, \end{aligned} \right\} \qquad (6)$$

where

$$A(p) = a_0 p^4 + a_1 p^3 + a_2 p^2 + a_3 p + a_4;$$

$$a_0 = \tau_1 \tau_\tau,$$

$$a_1 = \tau_1 + \tau_\tau - \tau_\tau L_{15} l_{54} l_{41} + \tau_\tau L_{15} (l_{53} - l_{54} l_{43}) k_{3G} k_G + \tau_\tau k_{3G} l_{13} k_G + \\ + \tau_\tau L_{15} (l_{53} - l'_{54} l_{43}) l_{31} + \tau_\tau l_{13} l_{31};$$

$$a_2 = 1 - L_{15} l_{54} l_{41} + l_{13} l_{31} + l_{13} k_{3G} k_G + L_{15} (l_{53} - l_{54} l_{43}) l_{31} + \\ + L_{15} k_{5F} k_{3G} k_G T_\tau k_\tau + L_{15} (l_{53} - l_{54} l_{43}) k_{3G} k_G + L_{15} k_{5F} l_{31} T_\tau k_\tau + \\ + \tau_\tau l_{13} k_{3G} k_n + \tau_\tau L_{15} (l_{53} - l_{54} l_{43}) k_{3G} k_n;$$

$$a_3 = L_{15} k_{5F} k_{3G} k_G k_\tau + L_{15} (l_{53} - l_{54} l_{43}) k_{3G} k_n + L_{15} k_{5F} l_{31} k_\tau + \\ + l_{13} k_{3G} k_n + L_{15} k_{5F} k_{3G} T_\tau k_n k_\tau;$$

$$a_4 = L_{15} k_{5F} k_{3G} k_n k_\tau;$$

$$B_{11}(p) = b_{011} p^2 + b_{111} p + a_4;$$

$$B_{13}(p) = b_{013} p^2 + b_{113} p;$$

$$B_{1a}(p) = b_{01a} p^3 + b_{11a} p^2;$$

$$B_{31}(p) = b_{031} p^3 + b_{131} p^2 + b_{231} p;$$

$$B_{33}(p) = b_{033}p^2 + b_{133}p + a_4;$$
$$B_{3a}(p) = b_{03a}p^3 + b_{13a}p^2 + b_{23a}p;$$
$$b_{011} = \tau_T l_{13} k_{3G} k_n + L_{15}(l_{53} - l_{54} l_{43}) \tau_T k_{3G} k_n;$$
$$b_{111} = l_{13} k_{3G} k_n + L_{15}(l_{53} - l_{54} l_{43}) k_{3G} k_n + L_{15} k_{5F} k_{3G} T_T k_n k_T;$$
$$b_{013} = \tau_T L_{15} k_{5F} k_T;$$
$$b_{113} = L_{15} k_{5F} k_T;$$
$$b_{01a} = \tau_T L_{15} l_{54} k_{4a};$$
$$b_{11a} = L_{15} l_{54} k_{4a};$$
$$b_{031} = \tau_T \tau_1 k_{3G} k_n;$$
$$b_{131} = \tau_T k_{3G} k_n + \tau_1 k_{3G} k_n - \tau_T L_{15} l_{54} l_{41} k_{3G} k_n;$$
$$b_{231} = k_{3G} k_n - L_{15} l_{54} l_{41} k_{3G} k_n;$$
$$b_{033} = \tau_T L_{15} k_{5F} k_{3G} k_G k_T + \tau_T L_{15} k_{5F} l_{31} k_T;$$
$$b_{133} = \tau_T L_{15} k_{5F} k_{3G} k_n k_T + L_{15} k_{5F} k_{3G} k_G k_T + L_{15} k_{5F} l_{31} k_T;$$
$$b_{03a} = \tau_T k_{15} l_{54} k_{4a} k_{3G} k_O + \tau_T L_{15} l_{54} k_{4a} l_{31};$$
$$b_{13a} = \tau_T L_{15} l_{54} k_{4a} k_{3G} k_n + L_{15} l_{54} k_{4a} k_{3G} k_G + L_{15} l_{54} k_{4a} l_{31};$$
$$b_{23a} = L_{15} l_{54} k_{4a} k_{3G} k_n,$$

where $L_{15} = \frac{l_{15}}{1 - l_{54} l_{45}}$.

Let us carry out a synthesis of the ATE control system with respect to the basic controlled parameters $x_1$ and $x_3$. We shall use the method of expanding the sought transfer functions $\pi_1(p)$ and $\pi_3(p)$ and the specified transfer functions $\pi_{10}(p)$ and $\pi_{30}(p)$ into a McLaurin series with respect to p in the neighborhood $p \to 0$, after equating the corresponding derivatives of these transfer functions.

The synthesis of the system will be carried out for disturbances resulting from readjustment of the gas temperature control $x_{30}$ with a subsequent analysis for disturbances resulting from variation in the afterburner fuel supply $\mu_{a0}$. The disturbance to the system resulting from readjustment of the speed of revolution controller $x_{10}$ is not being considered since it is absent in the important regimes of operation. As is apparent from Eqs. (6), a deviation in the speed of revolution $x_1$ in the ATE control system under consideration does not coincide in sign with the disturbing actions $x_{30}$ and $\mu_{a0}$. This means that for an increase in the gas temperature due to a readjustment of the temperature control and for an increase in the afterburner fuel supply a decrease in the speed of revolution will occur under the transient conditions. This is a favorable circumstance for the ATE from the point of view of durability, since the temporary increase in the temperature of the gases in front of the turbine above the maximum value occurs at a reduced speed of revolution. Taking this into account, let us carry out the synthesis of the system with respect to the gas temperature $x_3$.

The transfer function with respect to $x_3$ for the distrubance $x_{30}$ will have the form

$$\Pi_{33}(p)=\frac{b_{033}p^2+b_{133}p+a_4}{a_0p^4+a_1p^3+a_2p^2+a_3p+a_4}. \tag{7}$$

It is advantageous to use the equation

$$\Pi_{330}(p)=\frac{1}{\tau_x^2p^2+2\xi\tau_xp+1}. \tag{8}$$

for the specified transfer function.

Using the method of synthesis which involves expanding the transfer functions into a McLaurin series with respect to p in the vicinity $p \to 0$, it is possible to write the following system of equations, obtained by equating the corresponding derivatives of $\pi_{33}(p)$ and $\pi_{330}(p)$:

$$\left.\begin{aligned} a_4&=a_4;\\ 2\xi\tau_xa_4+b_{133}&=a_3;\\ \tau_x^2a_4+2\xi\tau_xb_{133}+b_{033}&=a_2;\\ \tau_x^2b_{133}+2\xi\tau_xb_{033}&=a_1;\\ \tau_x^2b_{033}&=a_0. \end{aligned}\right\} \tag{9}$$

Substituting the expressions for the coefficients from Eqs. (6) into Eqs. (9), we obtain after transformations

$$\left.\begin{aligned} k_T &= \frac{l_{13} + L_{15}(l_{53} - l_{54}l_{43})}{2\xi\tau_x L_{15}k_{5F}}; \\ k_n &= \frac{1 - L_{15}l_{54}l_{41}}{\tau_x^2 k_{3G} L_{15} k_{5F} k_T}; \\ k_G &= \frac{\tau_1}{\tau_x^2 L_{15} k_{5F} k_{3G} k_T} - \frac{l_{31}}{k_{3G}}; \\ T_T &= \tau_T. \end{aligned}\right\} \tag{10}$$

Knowing the expression for the parameters of the controllers of the speed of revolution ($k_n$ and $k_G$) and of the controller for the temperature of the gases ($k_T$ and $T_T$), as well as the values of the parameters of the ATE as an object of control (see Table 1), we shall determine the required values of $k_n$, $k_G$, $k_T$, and $T_T$ from the conditions that the transient in the temperature of the gases in front of the turbine associated with a disturbance in $x_{30}$ is constant under all considered flight conditions. We shall take for the required transient one with a control time no greater than 3 sec and with an overshoot not greater than $10^o$-$20^oC$ over a period of 1.0-2.0 sec. We then obtain $\tau_x$ = 0.5 sec and $\xi$ = 0.8.

The results of a calculation of the parameters of the controllers of the speed of revolution and gas temperature , necessary to ensure the unaltered quality of the transient with respect to $x_3$ in the presence of a disturbance $x_{30}$ under the variable external conditions are given in Table 2.

The time constant $\tau_T$ was chosen equal to 5.0 sec for $p_2$ = 10,000-12,000 kg/m$^2$ in accordance with data on the dynamic characteristics of the thermocouples. In this case the variation of $\tau_T$ was determined by the equation

$$\frac{\tau_{T1}}{\tau_{T2}} = k\left(\frac{G_{a1}}{G_{a2}}\right)^{0.5}, \tag{11}$$

where $G_a$ is the rate of air flow through the engine.

It is necessary to supplement the synthesis just carried out with an analysis of a system for controlling an ATE with respect to all parameters, taking into account the possible disturbances on the system.

TABLE 2

| No. | $\frac{V}{V_0}$ | $\frac{H}{H_p}$ | $k_T$ 1/sec | $k_n$ 1/sec | $k_Q$ | $T_T=\tau_T$ sec |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0,82 | 7,5 | 0,45 | 2,5 |
| 2 | 0,1 | 0 | 0,82 | 7,5 | 0,43 | 2,5 |
| 3 | 0,2 | 0,025 | 0,82 | 7,6 | 0,42 | 2,5 |
| 4 | 0,3 | 0,075 | 0,82 | 8,25 | 0,39 | 2,4 |
| 5 | 0,4 | 0,135 | 0,90 | 6,11 | 0,41 | 2,3 |
| 6 | 0,5 | 0,225 | 0,99 | 5,0 | 0,49 | 2,4 |
| 7 | 0,6 | 0,325 | 1,0 | 5,28 | 0,89 | 2,4 |
| 8 | 0,7 | 0,43 | 1,01 | 5,94 | 1,85 | 2,5 |
| 9 | 0,8 | 0,535 | 0,99 | 7,19 | 4,09 | 2,9 |
| 10 | 0,885 | 0,695 | 0,94 | 8,55 | 7,81 | 3,7 |
| 11 | 0,885 | 0,8 | 0,90 | 8,48 | 14,98 | 5,0 |
| 12 | 0,885 | 1,0 | 0,90 | 8,38 | 42,5 | 8,3 |

## 4. Analysis of Control System for an ATE Operating Under Variable External Conditions

The analysis of the control system of an ATE under variable flight conditions was carried out by the method of structural mathematical simulation using electronic simulators. The results of the analysis of the control system are presented in the graphs of the transients (Fig. 6 and 7) for disturbances resulting from readjustment of the gas temperature controller $x_{30}$ (see Fig. 6a and 7a) and from a change in the afterburner fuel supply $\mu_{a0}$ (see Fig. 6b and 7b).

The transients in the ATE control system for variable flight conditions for the case of a variation in the parameters of the speed of revolution and gas temperature are given in Fig. 6a and 6b using the results of the synthesis in Table 2. The transients in the gas temperature associated with a disturbance resulting from readjustment of the $x_{30}$ controller remain unaltered with respect to all flight conditions (see Fig. 6a), as follows from the conditions of the system synthesis . The quality of the control processes relating to gas temperature in the presence of a disturbance in $\mu_{a0}$ remains practically unchanged, there being a temperature deviation of 80°C which lasts 1.5-2.0 sec. The quality of the transients with respect to the take-of and climb conditions; the greatest deviation in the speed of revolution reaches 180 rpm and is observed at flight velocities $V/V_0 = 0.5$-$0.7$. For a further increase in flight altitude and velocity the deviation in the speed of revolution decreases.

A comparison of the results of an analysis of the effect of various combinations of controller parameters, speed of revolution, and gas temperature under variable flight conditions is shown in Fig. 7. Here are shown the transients for a disturbance resulting from readjustment of $x_{30}$ and a change in the afterburner fuel supply $\mu_{a0}$ for the parameters $k_T$, $k_n$, $k_G$, and $T_T$ obtained in the synthesis of an ATE control system (curve 1) which parameters are variable with respect to flight conditions. For comparison we have presented the transients in a system with constant $k_T$, $k_n$, and $k_G$ for the flight regime $H/H_p =$ $= 0.8$ and $V/V_0 = 0.885$ with a variable time constant in the correction system of the gas temperature control (curve 2); with constant parameters $k_T$, $k_n$, $k_G$, and $T_T$ for the regime $H/H_p = 0.8$ and $V/V_0 = 0.885$ (curve 3) and with constant parameters $k_T$, $k_n$,$k_G$, and $T_T$ for the regime $H/H_p = 0$ (curve 4). As is apparent from Fig. 7, the system with variable parameters $k_T$, $k_n$, $k_G$, and $T_T$ is most advantageous. However, it is entirely possible to limit the variation to the parameters of the correction systems $k_G$ and $T_T$, which simplifies the system without particularly impairing the quality of the control processes. A system with constant parameters chosen for the regime $H/H_p = 0$ and $V/V_0 = 0$ is unable to provide for the operation of an ATE since the system becomes unstable beginning with $H/H_p = 0.6$. When the constant contoller parameters are selected according to the regime $H/H_p = 0.8$ and $V/V_0 = 0.885$ the transients are damped at low altitudes, while at high altitudes they become oscillatory with large overshoots.

The analysis of the ATE control system just carried out shows that in order to ensure the stable operation of a system with the quality of the control processes prescribed beforehand it is necessary during the takeoff and climb of the aircraft to vary the parameters of the correction systems of the controllers, especially the time constant $T_T$ of the temperature controller and the coefficient $k_G$ of the speed of revolution controller. The magnification of the speed of revolution controller $k_n$ and of the gas temperature controller $k_T$ may remain constant under all flight conditions.

## 5. Synthesis of Block Diagrams of Circuits for the Self-Adjustment of the Parameters of the Correcting Systems

The variation of the parameters of the correcting systems may be carried out by means of closed self-adjusting systems in which the parameters are varied in such a way as to minimize the errors characterizing the reaction of the system to external disturbances. Minimum reaction of the system is ensured by a system which optimalizes the parameters of the correction systems.

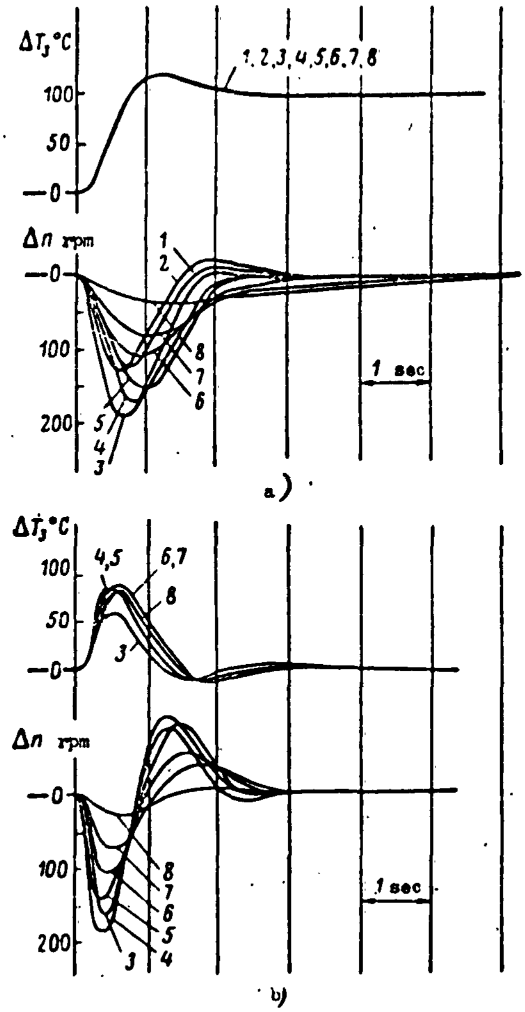


Fig. 6. Transients in an ATE control system.

a) for a disturbance resulting from a readjustment of the gas temperature control; b) for a disturbance resulting from a change in the afterburner fuel supply;

1—$H/H_p$=0; $V/V_0$=0; 2—$H/H_p$=0,075; $V/V_0$=0,3; 3—$H/H_p$=0,225; $V/V_0$=0,5; 4—$H/H_p$=0,43; $V/V_0$=0,7; 5—$H/H_p$=0,535; $V/V_0$=0,8; 6—$H/H_p$=0,695; $V/V_0$=0,885; 7—$H/H_p$=0,8, $V/V_0$=0,885, 8—$H/H_p$=1,0; $V/V_0$=0,886.

When external disturbances are acting upon the system it is possible to evaluate the quality of the control processes with the aid of the following expression:

$$\eta=\int_0^t |\Delta x(\tau)|^n e^{-\frac{t-\tau}{T}}d\tau = e^{-\frac{t}{T}}\int_0^t |\Delta x(\tau)|^n e^{\frac{\tau}{T}}d\tau, \quad (12)$$

where n is an exponent greater than 0.

Usually the values n = 1 and n = 2 are taken for n. Obviously any value n > 0 will ensure the formation of an integral error having an extremum with respect to the system parameters.

The choice of whole values for n is made for the sake of convenience in the computation. In the realizati of the relationship the exponent n may also have fractional values satisfying the condition n > 0.

Since relationship (12) represents the reaction of the inertial member with time constant T to an input signal of the form $|\Delta x|^n$, the realization of this relationship ma be a series connection of a member to realize the operation $|\Delta x|^n$ and an inertial member (Fig. 8). Accordingly,

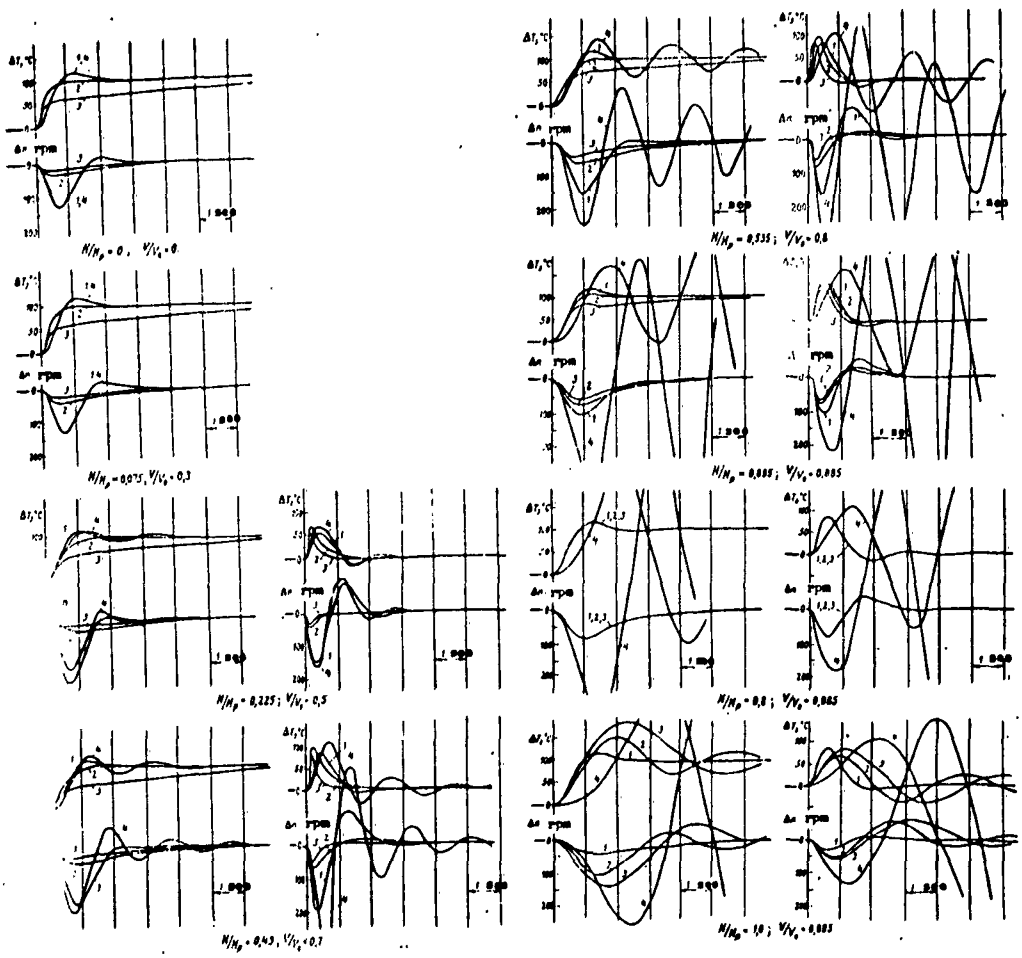

Fig. 7. Transients in an ATE control system.

a) for a disturbance resulting from readjustment of the gas temperature control; b) for a disturbance resulting from a change in the afterburner fuel supply; 1) a system with variable parameters $k_T$, $k_n$, $k_G$, and $T_T$; 2) system with constant parameters $k_T$, $k_n$, and $k_G$ for the regime $H/H_p = 0.8$, $V/V_0 = 0.885$ and variable $T_T$; 3) system with constant parameters $k_T$, $k_n$, $k_G$, and $T_T$ for the regime $H/H_p = 0.8$, $V/V_0 = 0.885$; 4) system with constant parameters $k_T$, $k_n$, $k_G$, and $T_T$ for the regime $H/H_p = 0$, $V/V_0 = 0$.

$$(Tp+1)\eta=|\Delta x|^{n}. \qquad (13)$$

In order to find the extremum of η it is possible to use one of the known methods such as the gradient method wherein the components of the gradient are measured by means of synchronous detectors.

The method for finding the extremum is as follows. Let the functional η depend on the slowly varying parameters $m_1$, $m_2$, ..., $m_k$. We shall vary each of these parameters according to the harmonic law with small amplitudes $\Delta m_1$, $\Delta m_2$, ..., $\Delta m_k$ and frequencies $\omega_1$, $\omega_2$, ..., $\omega_k$, i.e.,

$$m_1=m_{10}+\Delta m_1 \sin \omega_1 t;\quad m_2=m_{20}+\Delta m_2 \sin \omega_2 t;\quad \ldots;\\ m_k+m_{k0}+\Delta m_k \sin \omega_k t. \qquad (14)$$

Then with an accuracy to the squared terms we obtain

$$\eta=\eta_0+\frac{\partial\eta}{\partial m_1}\Delta m_1 \sin \omega_1 t+\frac{\partial\eta}{\partial m_2}\Delta m_2 \sin \omega_2 t+\ldots\\ \ldots+\frac{\partial\eta}{\partial m_k}\Delta m_k \sin \omega_k t, \qquad (15)$$

where $\frac{\partial\eta}{\partial m_1}, \frac{\partial\eta}{\partial m_2}, \ldots, \frac{\partial\eta}{\partial m_k}$ are the components of the gradient of the function η.

If we multiply the function η from Eq. (15) in turn by $\Delta m_1 \sin \omega_1 t$, $\Delta m_2 \sin \omega_2 t$, ..., $\Delta m_k \sin \omega_k t$ and performing an averaging, we will find

$$\bar{u}_1=\frac{1}{2}\Delta m_1^2\frac{\partial\eta}{\partial m_1};\quad \bar{u}_2=\frac{1}{2}\Delta m_2^2\frac{\partial\eta}{\partial m_2};\quad \ldots;\quad \bar{u}_k=\frac{1}{2}\Delta m_k^2\frac{\partial\eta}{\partial m_k}. \qquad (16)$$

Obviously at the extremum the components of the gradient $\partial\eta/\partial m_1$, $\partial\eta/\partial m_2$, ..., $\partial\eta/\partial m_k$ are equal to zero. Accordingly, the circuit for finding the extremum and measuring the components of the gradient should include a setter for the variation of the parameters of the correction system and a synchronous detector (Fig. 9).

The circuit* of the self-adjusting optimalizing system (Fig. 10) should include a member to obtain the power of the absolute value of the error $|\Delta x|^n$, a member containing the synchronous detectors $D_1$, $D_2$, $D_3$ for the determination of the components of the gradient of the function $\eta$, a member of integrating slave mechanisms and a system for obtaining the reference signals. The actuating system for the optimalizing self-adjustment of the parameters of the correction systems should be executed in the form of an integrating member for smoothing the variable components of the signal. The choice of the oscillation frequencies $\omega_1$, $\omega_2$, ..., $\omega_k$ involved in finding the extremum should be made from the conditions of a sufficiently high-speed self-adjustment circuit; moreover the frequency should be less than the frequency of the oscillations in the main control circuit. The amplification factor of the self-adjustment circuit is chosen from the stability conditions of this circuit. Since the processes in the self-adjustment circuit are slower than in the control circuit, the amplification factor of this circuit may be taken fairly small.

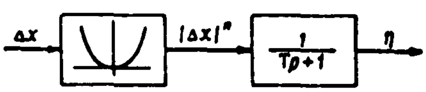


Fig. 8. Arrangement for obtaining the signal $|\Delta x|^n$.

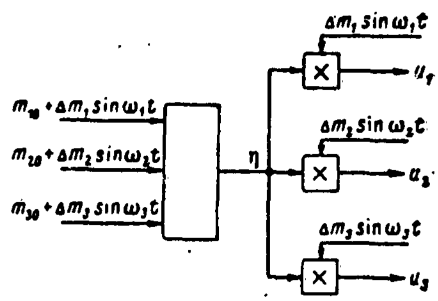


Fig. 9. Arrangement of setter for variation of parameters and synchronous detector.

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* Cf. A. A. Krasovskiy. Izv. Akad. Nauk SSSN, OTN, Energetika i Avtomatika, No. 3, 1960.

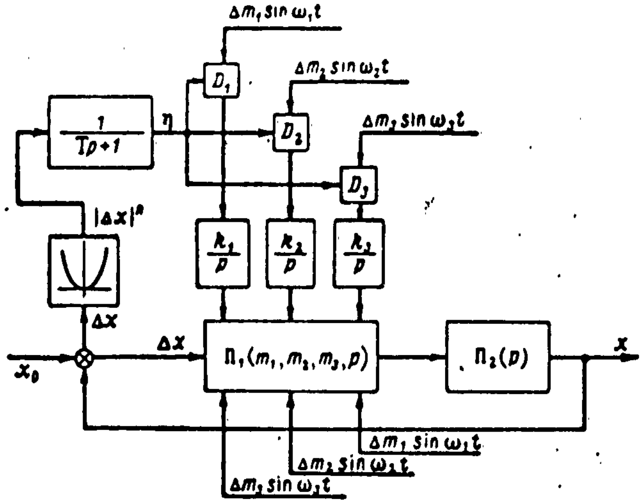


Fig. 10. Block diagram of optimalizing self-adjustment system.

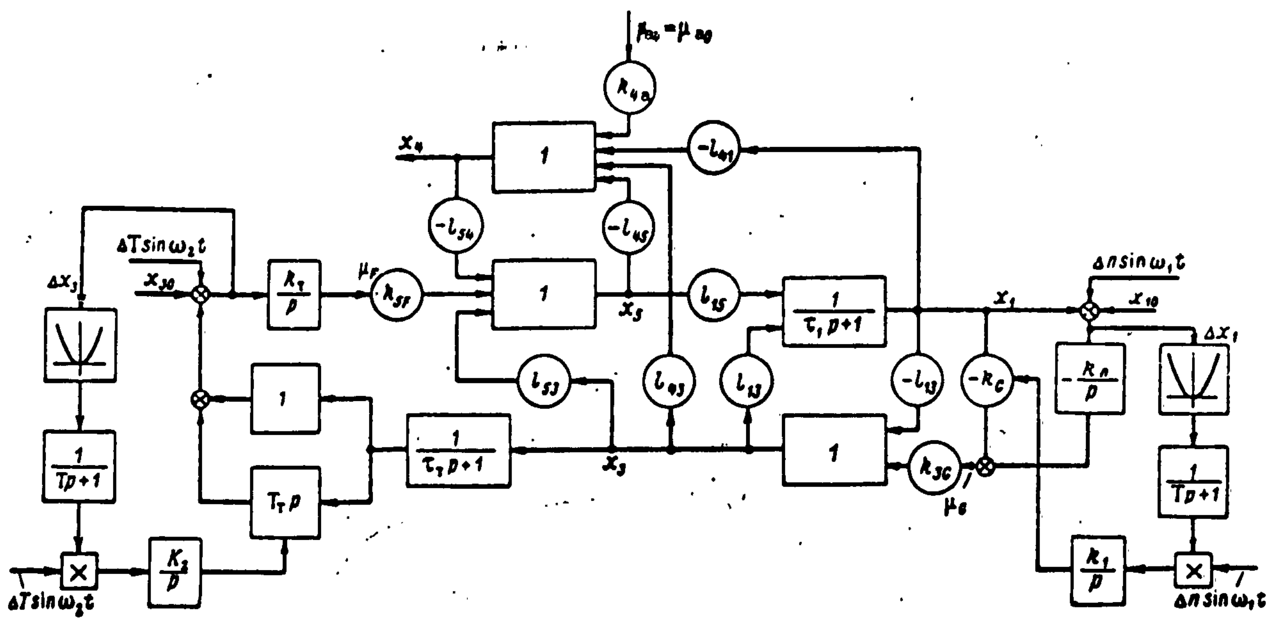


Fig. 11. Block diagram of ATE control system with optimalizing systems for adjustment of parameters of correction systems.

The block diagram of an ATE control system with optimalizing self-adjustment of the parameters of the correction systems $k_G$ and $T_T$ is shown in Fig. 11. The systems for the self adjustment of the parameters $k_G$ and $T_T$ contains members which give $|\Delta x_1|^n$ and $|\Delta x_3|^n$ at the output. Then these signals enter the inertial members, at the outputs of which we obtain

$$\eta_1 = e^{-\frac{t}{T}} \int_0^t |\Delta x_1|^n e^{\frac{\tau}{T}} d\tau; \qquad (17)$$

$$\eta_3 = e^{-\frac{t}{T}} \int_0^t |\Delta x_3|^n e^{\frac{\tau}{T}} d\tau. \qquad (18)$$

The components of the gradients of the functions $\eta_1$ and $\eta_3$ are separated out in synchronous detectors, the signals from which enter the actuating systems which are executed in the form of integrating members. It follows from Fig. 11 that the systems for self-adjustment of the parameters $k_G$ and $T_T$ are practically identical. This circumstance simplifies the realization of these systems to a considerable degree.

Until now it has been assumed that the optimalizing self-adjustment system optimalizes the transient with respect to an integral evaluation criterion. This criterion gives rise to an oscillatory transient in a number of cases. In order to obtain a smoother transient it is possible to take the integral evaluation in the following form

$$\eta = e^{-\frac{t}{T}} \int_0^t [|\Delta x|^n + \nu |\Delta \dot{x}|^n] e^{\frac{\tau}{T}} d\tau, \qquad (19)$$

where $\nu$ is the "weight" coefficient of the derivative of $\Delta x$.

The technical aspects of the realization of expression (19) present no difficulty and are shown in block-diagram form in Fig. 12. The additional elements in this diagram are the differentiating member and the member for obtaining the absolute value of $\Delta x$.

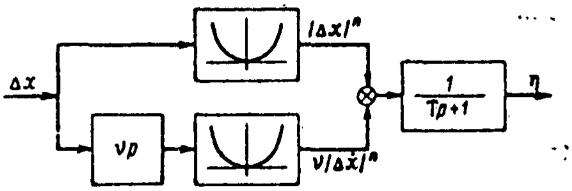


Fig. 12. An arrangement for obtaining the signal $|\Delta x|^n + \nu|\Delta x|^n$.

## Conclusions

1. The dynamic parameters of an ATE vary over wide ranges according to flight regimes and the operational regimes of the engines and this makes it difficult to obtain acceptable quality in the transients when the parameters of the controllers are left unaltered.

2. The investigation that has been made shows that the optimal transient under all flight conditions may be obtained by varying the parameters of the correction circuits of the speed of revolution and gas temperature controls.

3. The choice of the optimal value for the parameters of the correction systems of the controllers may be effected by means of special self-afjustment circuits which include functional formation elements and elements for finding and realizing the extremum of this functional.

4. The self-adjustment circuits used in connection with the circuits for control of speed of revolution and gas temperature in the ATE turn out to be practically identical and this facilitates the realization of these circuits. The use of self-adjustment circuits for obtaining optimal transients permits the creation of ATE control systems which operate over a wide range of flight regimes.

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