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RDT & E Project No. 1M010501A006
BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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A STUDY OF THE VIBRATIONS OF SHALLOW SPHERICAL SHELLS

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B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

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WHHoppmannII/CNBaronet/mec
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ABSTRACT

In this report there is presented a comparison of the theoretically and experimentally determined frequencies and zero displacement lines for the normal mode vibrations of shallow spherical shells. Results are given for the clamped edge and the momentless edge boundary conditions.

Calculations of frequencies and loci of zero displacement components for various shell sizes were made on an IBM 650 computer for the rotationally symmetric vibrations only, as the theory is not available for the case of asymmetric vibrations.

Experiments were performed to determine the vibration characteristics both for the symmetric and the asymmetric vibrations. Several model shells constructed of steel were used for the purpose.

Results of the investigation definitely demonstrate the value of the theory for the determination of the dynamic response of shallow shells.

TABLE OF CONTENTS

INTRODUCTION

In a previous paper [1*], the equations of motion for flexural vibrations of orthotropic shells were developed and simplified so as to obtain solutions for the case of shallow shells. In particular, it was shown in that paper how the equations reduce to cover the case of isotropic shells for which some computed frequencies were given for both the shell with clamped edge and with momentless edge. The equivalent equations for the clamped isotropic shell were given previously in a paper by Eric Reissner [2].

Because of the importance of the problem and because some doubt exists at the present time as to the applicability of the theoretical results, experiments on shell models were considered to be needed. In addition a computational program was also needed to check the previous calculations [3] and to extend them to cover a larger number of shell parameters.

The present report presents experimentally determined frequencies and zero displacement loci for both symmetric and asymmetric vibrations. It also provides theoretically determined frequencies and zero displacement loci computed with an IBM 650 computer for the symmetric vibrations.

THEORY

The thin shell theory for small amplitude vibrations which is under consideration in this report has been amply discussed in texts by A. E. H. Love, S. Timoshenko, and others. It is basically a stress resultant theory in which the resultant of the stress distribution through the thickness of the shell at any point may be represented as a force and a moment per unit length of the line formed by the intersection of the middle surface and a plane normal to that surface. In such a case the defining differential equations of motion are linear. Despite this fact however they are in general quite difficult to solve and consequently very few solutions to the dynamical equations for shells have been published. It turns out that in the important case of shallow shells, the equations can be solved and the frequencies, as well as lines of zero

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* Superscript numbers refer to references at end of report.

displacement, can be found. The frequency and displacement, equations have been given in a previous paper[1]. A summary of equations are provided in the Appendix to the present paper.

In the previous paper, frequencies computed with the aid of electric desk calculators and Tables of Functions [4] were tabulated. There the assumed shells had chord length of 12, thickness of 1/4, and sagittas of 0.5, 1.0, and 1.6. In the present study frequencies and zero displacement lines of the normal modes were obtained with the IBM 650 computer and Tables of Functions [4,5,6] for shells of chord length 12, thicknesses of 1/16, 1/8, 1/4 and sagittas of 0.5 and 1.6. The calculations were carried out for clamped and momentless edge conditions. All of these calculated results are given in the Appendix in Tables II-IV.

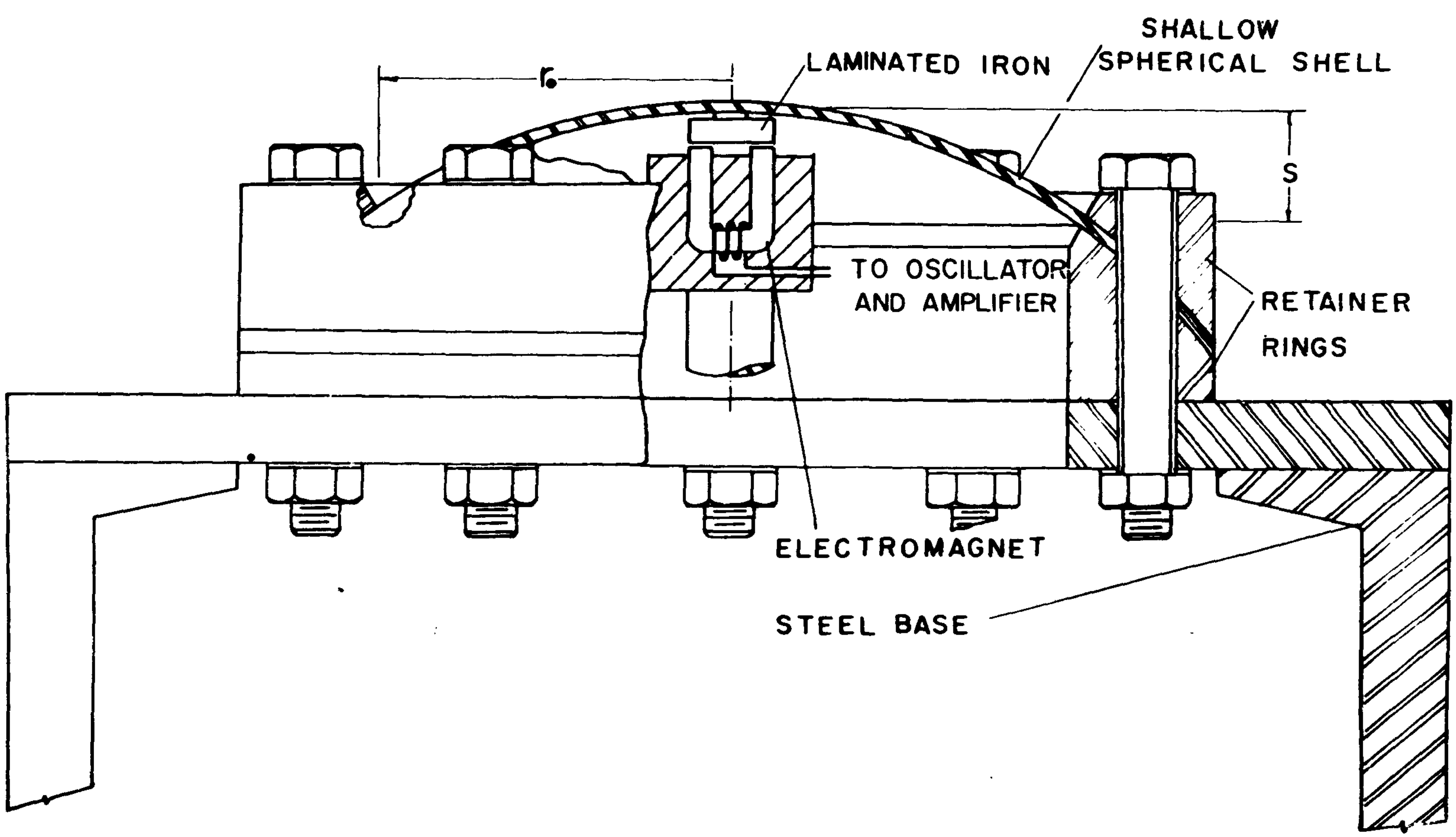
VIBRATION EXPERIMENTS WITH THIN SHALLOW SPHERICAL SHELLS

For the purpose of making experiments two model shells were designed and constructed. An arrangement of the shell with the device used to provide a clamped edge boundary is shown in Fig. 1. The momentless edge boundary condition was obtained by using the clamping ring the same as for the clamped case but machining a deep peripheral groove 0.84 times the thickness of the shell and very close to the clamping ring, in the same manner described elsewhere for use with vibrating plates [7].

The shells were of steel and they were vibrated by means of an electromagnetic oscillator shown schematically in Fig. 1. The resonant tuned magnetic circuit and power pack to obtain sufficient driving power had been previously developed [8]. The lines of zero displacement were found with the use of a crystal pickup whose output was fed to two of the plates of a cathode ray oscilloscope. The driving frequency from an audio-oscillator were fed into the other two plates of the C.R.O. The technique is quite satisfactory. Its use with a capacitive type pickup instead of the crystal is fully described in another paper [8].

THEORETICAL AND EXPERIMENTAL RESULTS

The experimentally determined frequencies for the case of symmetric vibrations, both for the clamped edge and momentless edge boundary conditions, together with the corresponding theoretical results are given in Table I. In that



r₀
LAMINATED IRON
SHALLOW
SPHERICAL SHELL
S
TO OSCILLATOR
AND AMPLIFIER
RETAINER
RINGS
ELECTROMAGNET
STEEL BASE

Fig. 1

TABLE I

SYMMETRICAL MODES OF FLEXURAL VIBRATIONS OF SHALLOW ISOTROPIC SPHERICAL SHELLS

FREQUENCIES $\times(10^{-3}$ rad./sec.)

	h (in.)	n	S = 0 Theor.	S = 0.5 Theor.	S = 0.5 Exper.	S = 1.0 Theor.	S = 1.6 Theor.	S = 1.6 Exper.
CLAMPED EDGE	0.0625	1	1.1	6.6		11.8		
		2	4.3	8.8		14.2	20.0	
		3	9.6	11.5		17.8	24.0	
		4	17.0	18.0		21.3		
	0.125	1	2.2	7.9	6.5	13.2	19.1	15.4
		2	8.6	11.0	10.0	17.6	24.6	22.3
		3	19.1	20.0	17.1	23.0	29.9	29.2
		4	33.6	34.4	29.3	36.0	39.0	42.2
	0.250	1	4.4	9.1	7.2	15.7	22.4	18.5
		2	17.1	18.4	15.7	22.0	29.4	22.3
		3	38.2	36.6	33.2	40.0	42.8	34.2
		4	68.0	68.1	56.5	69.0	70.4	55.0
MOMENTLESS EDGE	0.0625	1	0.5	6.4		11.5		
		2	3.2	8.8		13.6	19.4	
		3	7.9	10.3		17.7	22.4	
		4	14.9	15.9		19.8		
	0.125	1	1.1	7.7	6.2	12.6	18.7	15.2
		2	6.4	9.8	10.1	17.6	23.4	19.6
		3	15.9	17.0	16.5	20.5	29.7	29.2
		4	29.7	26.7	28.0	31.8	35.4	40.6
	0.250	1	2.1	8.8	7.2	15.5	21.0	17.3
		2	12.8	14.3	14.5	19.6	29.2	21.0
		3	31.8	32.4	30.2	34.0	37.4	33.7
		4	59.2	59.6	51.6	60.6	61.6	52.5

Table h is the thickness, S is the sagitta length, and n is the mode number.

The experimentally determined loci of zero radial displacements for two thicknesses of shell are given in Tables II and III along with corresponding theoretical results. Definitions of symbols are in Fig. 2.

The experimentally determined frequencies and loci of zero displacement components for asymmetric vibrations are given in Table IV. Because the loci for these vibrations are not simply circles, sketches of the loci for the several modes which were investigated are shown.

DISCUSSION AND CONCLUSIONS

It is concluded that the results of the investigation demonstrate that the theory describes very well the nature of the vibrations of shallow spherical shells. It is also considered that the same theoretical results can be used for describing the vibrations of any very shallow shells of revolution which have positive curvature everywhere and which, further, have the same base diameter and sagitta. In these cases the shell is then really assumed to be equivalent to the osculating sphere at the apex. The modal shapes for the shallow shells are similar to those for flat circular plates but the frequencies are much higher, indicating greater stiffness.

It is interesting to note that for the fundamental vibration of the symmetric type there are two nodal circles, one at the boundary and another closer to the center. As the sagitta or rise of the shell approaches zero the latter circular node approaches closer to the boundary and in the limit, which is a flat plate, it becomes coincidental with the boundary. It is rather surprising that the frequencies for the clamped edge condition and the momentless edge condition are so nearly equal in the various modes. Further study should be made of this point in order to assess the influence of membrane stresses as opposed to that of flexural stresses. It would be pertinent in such a study to compare the potential energy corresponding to stretch and the potential energy corresponding to flexure as was done by Arnold and Warburton for cylindrical shells in order to assay the anomaly of complexity of nodal pattern as related to frequency of vibration [9].

TABLE II

LOCI OF RADIAL ZERO DISPLACEMENT (SYMMETRIC CASE)

CLAMPED EDGE (h = 0.125 in.)					
S (in.)	n	η_{w_n} theor. / η_{w_n} exper.			
		i = 1	i = 2	i = 3	i = 4
0.5	1	0.75 / 0.74			
	2	0.30 / 0.32	0.97 /		
	3	0.21 / 0.26	0.57 / 0.58		
	4	0.14 / 0.18	0.43 / 0.44	0.67 / 0.68	
1.6	1	0.47 / 0.49			
	2	0.19 / 0.22	0.49 / 0.69		
	3	0.15 / 0.30	0.41 / 0.42	0.72 /	
	4	0.11 / 0.17	0.33 / 0.44	0.56 / 0.66	0.75 /

MOMENTLESS EDGE (h = 0.125 in.)					
S (in.)	n	η_{w_n} theor. / η_{w_n} exper.			
		i = 1	i = 2	i = 3	i = 4
0.5	1	0.70 / 0.63			
	2	0.29 / 0.27	0.99 /		
	3	0.23 / 0.28	0.57 / 0.63		
	4	0.15 / 0.20	0.42 / 0.47	0.70 / 0.73	
1.6	1	0.50 / 0.52			
	2	0.17 / 0.26	0.72 / 0.70		
	3	0.14 / 0.37	0.45 / 0.40	0.72 /	
	4	0.12 / 0.19	0.37 / 0.49	0.59 / 0.70	0.79 /

TABLE III

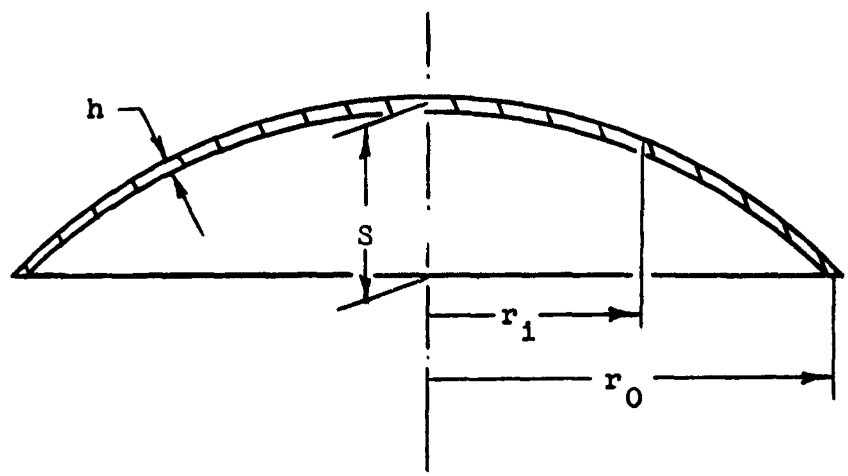
LOCI OF RADIAL ZERO DISPLACEMENT (SYMMETRIC CASE)

CLAMPED EDGE (h = 0.250 in.)

S (in.)	n	η_{w_n} theor. i = 1	η_{w_n} exper. i = 1	η_{w_n} theor. i = 2	η_{w_n} exper. i = 2	η_{w_n} theor. i = 3	η_{w_n} exper. i = 3
0.5	1		0.88				
	2	0.41	0.37				
	3	0.27	0.25	0.61	0.42		
	4	0.17	0.19	0.45	0.44	0.70	0.68
1.6	1	0.70	0.54				
	2	0.42	0.22				
	3	0.23	0.27	0.56	0.56		
	4	0.17	0.19	0.46	0.43	0.70	0.68

MOMENTLESS EDGE (h = 0.250 in.)

S (in.)	n	η_{w_n} theor. i = 1	η_{w_n} exper. i = 1	η_{w_n} theor. i = 2	η_{w_n} exper. i = 2	η_{w_n} theor. i = 3	η_{w_n} exper. i = 3
0.5	1	0.93					
	2	0.42	0.42				
	3	0.24	0.28	0.59	0.64		
	4	0.16	0.20	0.47	0.47	0.71	0.74
1.6	1	0.68	0.55				
	2	0.39	0.20	0.92			
	3	0.21	0.30	0.54	0.60		
	4	0.16	0.20	0.47	0.47	0.74	0.73



h = thickness, S = sagitta

$\eta = \frac{r_i}{r_0}$ Location of radial zero displacement circle

i = 1,2,3 ...

w_n = radial displacement

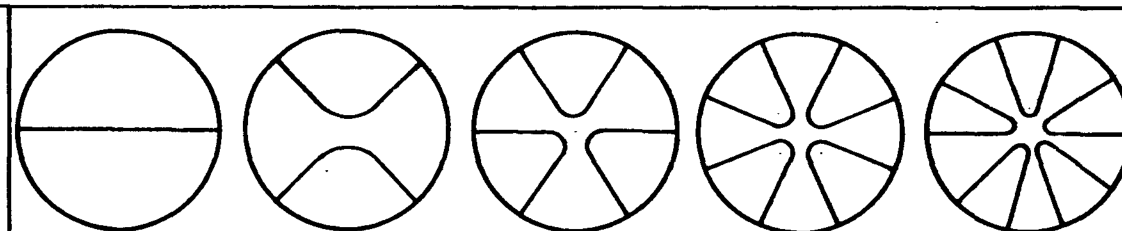
v_n = tangential displacement

Fig. 2

TABLE IV

ASYMMETRICAL MODES OF FLEXURAL VIBRATIONS
OF SHALLOW ISOTROPIC SPHERICAL SHELLS

EXPERIMENTAL DATA (LOCI OF RADIAL ZERO DISPLACEMENT)



r_0 = 6.00 in.			$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
h in.	S in.	BOUNDARY CONDITIONS	FREQUENCIES $\times(10^{-3}$ rad./sec.)				
0.125	0.50	MOMENTLESS	7.2	9.5	11.8	15.0	18.6
		CLAMPED	7.3	9.3	11.9	15.2	19.1
0.125	1.60	MOMENTLESS	16.0	17.9	18.8	20.7	22.4
		CLAMPED	16.3	18.0	19.4	21.4	23.6
0.250	0.50	MOMENTLESS	8.9	13.2	18.0	24.2	31.4
		CLAMPED	9.7	14.0	19.5	26.4	34.2
0.250	1.60	MOMENTLESS	18.1	19.2	23.1	27.6	
		CLAMPED	17.9	20.2	24.9	29.2	

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APPENDIX

COMPUTATIONS FOR DETERMINATION OF FREQUENCIES AND NODAL CIRCLES OF SYMMETRIC VIBRATIONS

To carry out computations the following dimensionless parameters were introduced

$$\begin{aligned} \xi_n &= p_n r_0 \sqrt{\rho/E} \\ \epsilon &= 2\sqrt{3}\, r_0/h \\ \delta &= 4\sqrt{3}\, s/h \\ \eta &= r/r_0 \end{aligned} \tag{1}$$

Applying these definitions the resulting eigenvalue problem is defined by the following equations:

$$\left\{x^2 - (1-\nu^2)\xi^2\right\}\left\{x^4 - (1-\nu^2)\xi^2\epsilon^2 + 2\delta^2(1+\nu)\right\} - (1+\nu)^2\delta^2x^2 = 0 \tag{2}$$

CASE A. Clamped Edge.

$$w_n = \frac{dw_n}{d\eta} = v_n = 0 \quad \text{at} \quad \eta = 1 \tag{3}$$

$$D_c = \begin{vmatrix} \dfrac{J_0(x_1)}{x_1} & \dfrac{J_0(x_2)}{x_2} & \dfrac{J_0(x_3)}{x_3} \\ J_1(x_1) & J_1(x_2) & J_1(x_3) \\ \dfrac{J_1(x_1)}{\xi^2(1-\nu^2)-x_1^2} & \dfrac{J_1(x_2)}{\xi^2(1-\nu^2)-x_2^2} & \dfrac{J_1(x_3)}{\xi^2(1-\nu^2)-x_3^2} \end{vmatrix} = 0 \tag{4}$$

CASE B. Momentless Edge.

$$w_n = v_n = M_\phi = 0 \quad \text{at} \quad \eta = 1 \tag{5}$$

$$D_M = \begin{vmatrix} \dfrac{J_0(x_1)}{x_1} , & \dfrac{J_0(x_2)}{x_2} , & \dfrac{J_0(x_3)}{x_3} \\ J_1(x_1)(1-\nu)-x_1J_0(x_1) , & J_1(x_2)(1-\nu)-x_2J_0(x_2) , & J_1(x_3)(1-\nu)-x_3J_0(x_3) \\ \dfrac{J_1(x_1)}{\xi^2(1-\nu^2)-x_1^2} , & \dfrac{J_1(x_2)}{\xi^2(1-\nu^2)-x_2^2} , & \dfrac{J_1(x_3)}{\xi^2(1-\nu^2)-x_3^2} \end{vmatrix} = 0 \tag{6}$$

The eigen functions are given by

$$w_n = - \left\{B_1 \frac{J_0(\eta x_1)}{x_1} + \frac{B_2}{x_2} J_0(\eta x_2) + \frac{B_3}{x_3} J_0(\eta x_3)\right\} \tag{7}$$

$$v_n = -(1+\nu)\left\{B_1 \frac{J_1(\eta x_1)}{(1-\nu^2)\xi^2-x_1^2} + B_2 \frac{J_1(\eta x_2)}{(1-\nu^2)\xi^2-x_2^2} + B_3 \frac{J_1(\eta x_3)}{(1-\nu^2)\xi^2-x_3^2} \right\} \tag{8}$$

where

CASE A. Clamped Edge.

$$\frac{B_2}{B_1} = \frac{x_3^2 - x_1^2}{x_2^2 - x_3^2} \cdot \frac{(1-\nu^2)\xi^2 - x_2^2}{(1-\nu^2)\xi^2 - x_1^2} \frac{J_1(x_1)}{J_1(x_2)} \tag{9}$$

$$\frac{B_3}{B_1} = - \left\{1 + \frac{x_3^2 - x_1^2}{x_2^2 - x_3^2} \cdot \frac{(1-\nu^2)\xi^2 - x_2^2}{(1-\nu^2)\xi^2 - x_1^2}\right\} \frac{J_1(x_1)}{J_1(x_3)} \tag{10}$$

CASE B. Momentless Edge.

$$\frac{B_3}{B_2} = - \frac{\dfrac{(1-\nu^2)\xi^2 - x_1^2}{(1-\nu^2)\xi^2 - x_2^2} \cdot \dfrac{J_1(x_2)}{J_1(x_1)} - \dfrac{x_1 J_0(x_2)}{x_2 J_0(x_1)}}{\dfrac{(1-\nu^2)\xi^2 - x_1^2}{(1-\nu^2)\xi^2 - x_3^2} \cdot \dfrac{J_1(x_3)}{J_1(x_1)} - \dfrac{x_1 J_0(x_3)}{x_3 J_0(x_1)}} = \lambda \qquad (11)$$

$$\frac{B_1}{B_2} = - \left\{ \frac{x_1 J_0(x_2)}{x_2 J_0(x_1)} + \lambda \frac{x_1 J_0(x_3)}{x_3 J_0(x_1)} \right\}$$

All computations were carried out on an IBM 650 computer using floating point arithmetic. The necessary exponential subroutine was included in the floating point subroutine. The Bessel function subroutine used checked to within six significant figures through $x = 13$ with the tables listed in the references.

The values of parameters used were determined to allow direct comparison with experimental results using shells of thickness 0.25 and 0.125 inches with sagittos of 0.5, 1.0 and 1.6 inches. Table I in the Appendix shows the corresponding values of ϵ and δ. In addition, for comparison with experiment

$r_0 = 6$ inches and $\dfrac{\rho r_0^2}{E} = 8.819 \times 10^{-10}$ sec^2.

The computational procedure used was as follows:

1. Choose ϵ and δ.
2. Choose ξ^2.
3. Compute the six roots of Eq. (2) calling those with positive sign x_1, x_2 and x_3. For the purposes of this work x_1 was always real and x_3 was always pure imaginary.
4. Using the chosen ξ^2 and x_1, x_2 and x_3 compute the values of D_c and D_M.
5. Choose another value of ξ^2 and repeat steps 3 and 4.

TABLE I

h	ϵ
0.0625	332.55
0.125	166.27
0.250	83.13

	h = 0.0625	h = 0.125	h = 0.25
S	δ	δ	δ
0.5	55.42	27.71	13.85
1.0	110.85	55.42	27.71
1.6	177.36	88.68	44.34

The zeros of the resulting graphs of D_c and D_M vs. ξ yield the eigenvalues ξ_n. The values of ξ_n are given in Tables II through IV. The results obtained are accurate to within two in the third digit.

In the eigen function computation the functions

$$v_n' = -\frac{w_n}{B_1}, \quad v_n' = -\frac{v_n}{(1+\nu)B_1} \tag{12}$$

were computed for the clamped case and

$$v_n' = -\frac{w_n}{B_2}, \quad v_n' = -\frac{v_n}{(1+\nu)B_2} \tag{13}$$

were computed for the momentless case. Zeros of the eigen functions are given in Appendix, Tables II-IV.

TABLE II

LOCI OF ZERO DISPLACEMENT COMPONENTS (SYMMETRIC CASE)

CLAMPED EDGE ($\epsilon = 332.55$)					
			η_{v_n} / η_{w_n}		
δ	n	ξ_n	i = 1	i = 2	i = 3
55.42	1	0.197	0.516		
	2	0.261	0.208 / 0.420		
	3	0.341	0.267 / 0.368	0.542 / 0.800	
	4	0.534	0.192 / 0.322	0.417 / 0.516	0.679 / 0.900
110.85	1	0.349	0.325		
	2	0.422	0.235 / 0.375	0.661	
	3	0.529	0.287 / 0.375	0.442 / 0.546	0.891
	4	0.633	0.175 / 0.300	0.435 / 0.517	0.644 / 0.816
177.36	1				
	2	0.593	0.242 / 0.384	0.645	
	3	0.712	0.212 / 0.337	0.424 / 0.522	0.742
	4				

MOMENTLESS EDGE ($\epsilon = 332.55$)					
			η_{v_n} / η_{w_n}		
δ	n	ξ_n	i = 1	i = 2	i = 3
55.42	1	0.189	0.529		
	2	0.261	0.226 / 0.433	0.855	
	3	0.305	0.312 / 0.413	0.558 / 0.846	
	4	0.473	0.200 / 0.342	0.480 / 0.555	0.733
110.85	1	0.342	0.525		
	2	0.403	0.266 / 0.420	0.691	
	3	0.526	0.252 / 0.364	0.433 / 0.571	0.842
	4	0.588	0.160 / 0.316	0.516 / 0.553	0.634 / 0.856
177.36	1				
	2	0.577	0.268 / 0.425	0.690	
	3	0.665	0.218 / 0.350	0.490 / 0.607	0.784
	4				

TABLE III

LOCI OF ZERO DISPLACEMENT COMPONENTS (SYMMETRIC CASE)

CLAMPED EDGE (ϵ = 166.27)

δ	n	ξ_n	η_{v_n} \ η_{w_n}, $i = 1$	$i = 2$	$i = 3$
27.71	1	0.235	0.737		
	2	0.327	0.317 0.626		
	3	0.596	0.258 0.372	0.578 0.850	
	4	1.02	0.183 0.325	0.436 0.508	0.684 0.916
55.42	1	0.391	0.521		
	2	0.522	0.208 0.425		
	3	0.682	0.266 0.372	0.500 0.799	
	4	1.07	0.185 0.322	0.433 0.518	0.681 0.915
88.68	1	0.569	0.487		
	2	0.730	0.222 0.380	0.695	
	3	0.889	0.300 0.372	0.425 0.625	
	4	1.16	0.175 0.317	0.438 0.512	0.664 0.879

MOMENTLESS EDGE (ϵ = 166.27)

δ	n	ξ_n	η_{v_n} \ η_{w_n}, $i = 1$	$i = 2$	$i = 3$
27.71	1	0.229	0.633		
	2	0.291	0.275 0.650		
	3	0.505	0.283 0.413	0.633 0.950	
	4	0.898	0.205 0.342	0.467 0.564	0.735
55.42	1	0.373	0.530		
	2	0.522	0.222 0.430	0.866	
	3	0.610	0.312 0.408	0.555 0.850	
	4	0.945	0.202 0.344	0.470 0.560	0.730
88.68	1	0.555	0.520		
	2	0.694	0.262 0.421	0.705	
	3	0.883	0.370 0.398	0.400 0.611	
	4	1.05	0.195 0.338	0.486 0.564	0.700 0.950

TABLE IV

LOCI OF ZERO DISPLACEMENT COMPONENTS (SYMMETRIC CASE)

CLAMPED EDGE (ϵ = 83.13)					
			η_{v_n} / η_{w_n}		
δ	n	ξ_n	i = 1	i = 2	i = 3
13.85	1	0.269	0.875		
	2	0.543	0.367 / 0.716		
	3	1.15	0.250 / 0.367	0.416 / 0.866	
	4	2.023	0.192 / 0.338	0.436 / 0.500	0.684 / 0.950
27.71	1	0.468	0.750		
	2	0.654	0.317 / 0.625		
	3	1.19	0.258 / 0.372	0.575 / 0.850	
	4	2.05	0.190 / 0.333	0.433 / 0.500	0.683 / 0.940
44.34	1	0.665	0.545		
	2	0.876	0.224 / 0.480		
	3	1.27	0.266 / 0.405	0.561 / 0.836	
	4	2.09	0.187 / 0.333	0.435 / 0.500	0.680 / 0.916

MOMENTLESS EDGE (ϵ = 83.13)					
			η_{v_n} / η_{w_n}		
δ	n	ξ_n	i = 1	i = 2	i = 3
13.85	1	0.262			
	2	0.425	0.416 / 0.839		
	3	0.962	0.283 / 0.415	0.636 / 0.940	
	4	1.77	0.204 / 0.350	0.471 / 0.558	0.740
27.71	1	0.459	0.646		
	2	0.582	0.275 / 0.650		
	3	1.01	0.286 / 0.408	0.634 / 0.945	
	4	1.80	0.202 / 0.350	0.470 / 0.558	0.734
44.34	1	0.624	0.546		
	2	0.868	0.197 / 0.466		
	3	1.11	0.299 / 0.412	0.604 / 0.880	
	4	1.83	0.200 / 0.350	0.471 / 0.558	0.735

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Vibrations
Structural vibrations -
Shell
Stress analysis -
Theory

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Calculations of frequencies and loci of zero displacement components for various shell sizes were made on an IBM 650 computer for the rotationally symmetric vibrations only, as the theory is not available for the case of asymmetric vibrations.

Experiments were performed to determine the vibration characteristics both for the symmetric and the asymmetric vibrations. Several model shells constructed of steel were used for the purpose.

Results of the investigation definitely demonstrate the value of the theory for the determination of the dynamic response of shallow shells.

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