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TRANSLATION

REFLECTION OF SUDDEN CHANGES IN DENSITY FROM THE AXIS OF SYMMETRY

BY

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BY: D. A. Mel'nikov

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REFLECTION OF SUDDEN CHANGES IN DENSITY FROM THE AXIS OF SYMMETRY

by

D. A. Mel'nikov

The flow of a supersonic axially symmetrical stream from a nozzle, with pressure in the stream greater or less than the pressure in the surrounding medium, is accompanied by the formation of jumps of condensation, which are reflected from the axis of symmetry. The existing methods enable one to compute the flow in such a stream, with the exception of the area of the reflection of the jumps in condensation from the axis. Below there is stated the problem with its solution obtained for the flow of a gas in the region of the reflection of an incident jump in condensation from the axis of symmetry and the formation of a central jump in condensation converting a supersonic flow into a subsonic one. The accepted system of flow has been confirmed by experiment. There is shown the possibility of reflecting an incident jump in condensation from the axis of symmetry without the formation of a subsonic flow.

1. Let us take as one of the possible systems of flow of a gas in the region of reflection from the axis of symmetry of an incident jump in condensation the following one. Close to the nozzle the incident jump is split into two jumps in condensation (Fig. 1). The first reflected jump in condensation KR goes in the direction from the axis of symmetry. The second, the central jump in condensation KP, converts the supersonic flow near the axis into a subsonic one. From the point of splitting (in Fig. 1, point point K) there goes the line of flow KL, which proves to be the surface of a tangential break. The subsonic flow back of the central jump in condensation is rapidly accelerated and becomes supersonic, i. e., the area

of subsonic flow proves to be closed inside the supersonic flow. The flow behind the reflected jump in condensation in the general case can be completely supersonic or partially subsonic.

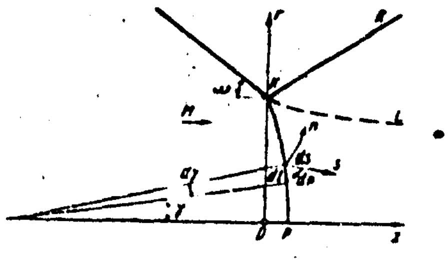

Fig. 1

Let us seek a solution determining the subsonic flow behind the central jump.

Let us introduce the dimensionless parameters u, v, the components of the vector of velocity $V = \sqrt{u^2 + v_2}$ referred to the maximum velocity in the cylindrical system of coordinates z, r with the beginning of the coordinates z = 0, which runs through (Fig. 1) the point K. p and ρ are the pressure and density, referred to the pressure and density of the retarded flow K before the jumps in condensation. k is the index of the adiabatic curve. Let us use the basic equations of an ideal gas:

$$u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r} + \frac{k-1}{2k} \frac{1}{\rho} \frac{\partial p}{\partial z} = 0 \qquad (1.1)$$

$$V^2 + p/\rho = 1 \qquad (1.2)$$

$$\frac{\partial (r\rho u)}{\partial z} + \frac{\partial (r\rho v)}{\partial r} = 0 \qquad (1.3)$$

The pressure p and the angle θ between the vector of velocity and the positive direction of the axis z we will consider as the unknown functions of z and r. Let us introduce into the consideration also the entropy function $\varphi = p / \rho^k$. The equation (1.1) we convert to the form

$$\frac{\partial}{\partial z}\left[r\left(\rho u^2 + \frac{k-1}{2k} p\right)\right] + \frac{\partial (r\rho u v)}{\partial r} = 0 \qquad (1.4)$$

Let us solve the problem by the method integral relationships (see, for example, [1]). In the general case we divide the subsonic flow between the axis of symmetry and the line of the flow running through the point K onto the band N by equidistant lines. The first line coincides with the

(Fig. 1) with the line of flow KL proceeding from the point K. The zero line is the axis of symmetry. Let us derive a formula with N = 1.

Let $r_1 = r_1(z)$ be the equation of the line of the flow KL. Let us integrate the equations (1.3) and (1.4) by r from r = 0 to $r = r_1(z)$. In the integrals obtained we will approximate the subintegral functions according to r through their values with r = 0 and $r = r_1(z)$. After the conversions to the variables p, θ, and φ, taking into consideration that

$$\frac{d\varphi_0}{dz} = \frac{d\varphi_1}{dz} = \theta_0 = 0 \qquad (1.4a)$$

from the integrated equations (1.3) and (1.4) we will get

$$\frac{dp_1}{dz} = \Big\{2r\rho_1 V_1^2 \sin\theta_1 \Big(2\cos\theta_1 - \frac{V_0}{V_1}\Big)\frac{d\theta_1}{dz} + 2\,\mathrm{tg}\,\theta_1 \Big[\frac{k-1}{2k}(p_1 - p_0) + 2\rho_1 V_1^2 \cos\theta_1 \Big(\frac{V_0}{V_1} - \cos\theta_1\Big)\Big]\Big\}\Big\{2r_1\Big[\frac{\rho_1 V_1^2}{kp_1}\cos\theta_1\Big(\cos\theta_1 - \frac{V_0}{V_1}\Big) + \frac{k-1}{2k}\Big(1 - 2\cos^2\theta_1 + \frac{V_0}{V_1}\cos\theta_1\Big)\Big]\Big\}^{-1} \qquad (1.5)$$

$$\frac{dp_0}{dr} = \Big\{2r_1\cos\theta_1\Big(\frac{k-1}{2k}\frac{V_0}{V_1} - \frac{\rho_1 V_0 V_1}{kp_1}\Big)\frac{dp_1}{dz} + 2r_1\rho_1 V_1 V_0 \sin\theta_1 \frac{d\theta_1}{dz} - 2\,\mathrm{tg}\,\theta_1(\rho_0 V_0^2 + 2\rho_1 V_1 V_0 \cos\theta_1)\Big\}\Big\{r_1\Big(\frac{\rho_0 V_0^2}{kp_0} - \frac{k-1}{2k}\Big)\Big\}^{-1} \qquad (1.6)$$

We will make use also of the equation of the line of flow passing through the point K:

$$dr_1/dz = \mathrm{tg}\,\theta_1 \qquad (1.7)$$

The subscripts 1 and 0 refer to the parameters on the first and zero lines of flow.

Still one more equation can be obtained from the boundary conditions on the line of flow passing through the point K and forming the surface of the tangential break. Let us consider that with $r > r_1(z)$ the flow is completely supersonic. Let us write the equation which connects p and θ along the characteristic II of the family:

$$d\theta_c + \frac{\sin\mu_c\cos\mu_c}{kp_c}dp_c - \frac{dr}{r}\frac{\sin\theta_c\sin\mu_c}{\sin(\theta_c - \mu_c)} + \frac{\sin 2\mu_c}{2k(k-1)}d\ln\varphi_c = 0 \qquad (1.8)$$

Here μ is Mach's angle. The subscript c relates to the supersonic flow with $r > r_1(z)$.

The system of equations (1.5)—(1.8) determines the unknown functions $p_0(z)$, $p_1(z)$, $\theta_1(z)$, and $r_1(z)$. The initial parameters with the subscript 1 with $z = 0$ for integrating the system of equations are determined if one knows the point of splitting K on the incident jump in intensity. From the condition of symmetry the central jump on the axis at the point P will be a straight jump in intensity. Therefore the initial parameters with the subscript 0 are known if one knows the coordinate z_p of the point P. The value $d\theta_1/dz$ which enters into the equations (1.5) and (1.6) remains unknown with $z = 0$. Let us write the equations (1.1) and (1.3) in the system of curved-line coordinates s, n (Fig. 1):

$$\frac{\partial\theta}{\partial s} = -\frac{p}{\rho V_s}\frac{k-1}{2k}\frac{\partial \ln p}{\partial n} \tag{1.9}$$

$$\frac{\partial\theta}{\partial n} = \left(\frac{k-1}{2k}\frac{p}{\rho V_s} - \frac{1}{k}\right)\frac{\partial \ln p}{\partial s} - \frac{\sin\theta}{r} \tag{1.10}$$

Here, s is taken to be along and in the direction of the lines of flow, and n along the normal to them. The direction of the coordinate n is chosen in such a way that the mutually perpendicular directions n and s coincide with the direction r and z, respectively, with a turn at the angle θ. The differentials dn and ds can be connected with the element dl of the central jump (Fig. 1):

$$\begin{aligned} dn &= dl\cos(\gamma-\theta) = R\cos(\gamma-\theta)\,d\gamma \\ ds &= -dl\sin(\gamma-\theta) = -R\sin(\gamma-\theta)\,d\gamma \end{aligned} \tag{1.11}$$

Here R is the radius of the curvature of the central jump in density and γ is the angle between the normal to the central jump and the positive direction of the axis z.

Let us introduce into the consideration the obvious relationships

$$\frac{d\theta}{d\gamma} = \frac{\partial\theta}{\partial s}\frac{ds}{d\gamma} + \frac{\partial\theta}{\partial n}\frac{dn}{d\gamma}, \qquad \frac{d\ln p}{d\gamma} = \frac{\partial \ln p}{\partial s}\frac{ds}{d\gamma} + \frac{\partial \ln p}{\partial n}\frac{dn}{d\gamma} \tag{1.12}$$

From the system of equations (1.9)—(1.12) let us establish the dependence between $\partial \ln p / \partial s$ and $\partial\theta / \partial s$ and we will definitely get the supplementary equation for determining $d\theta_1/dz$ with $z = 0$:

$$\frac{d\theta_1}{dz} = \left\{ -r_1 \frac{d \ln p_1}{dz} \left[\frac{d\theta_1}{d\gamma} \sin(\gamma_1 - \theta_1) + \frac{d \ln p_1}{d\gamma} \left(\frac{k-1}{2k} \frac{p_1}{\rho_1 V_1^2} - \frac{1}{k} \right) \cos(\gamma_1 - \theta_1) \right] + \right.$$
$$+ \frac{d \ln p_1}{d\gamma} \sin\theta_1 \cos(\gamma_1 - \theta_1) \cos\theta_1 \Big\} \Big\{ r_1 \Big[\frac{2k}{k-1} \frac{\rho_1 V_1^2}{p_1} \cos(\gamma_1 - \theta_1) \frac{d\theta_1}{d\gamma} - \qquad (1.13)$$
$$\left. - \frac{d \ln p_1}{d\gamma} \sin(\gamma_1 - \theta_1) \Big] \right\}^{-1}$$

The products $d\theta/d\gamma$ and $d \ln p/d\gamma$ are determined from the relationships for the oblique jump in density taking into consideration the variability of the gas-dynamic parameters before the central jump in density. The equations (1.5) and (1.13) determine the values $\partial p_1/\partial z$ and $d\theta_1/dz$ with $z = 0$.

From the system of equations (1.9)—(1.12) let us express also the radii of the curvature of the central jump:

at the point K with $z = 0$

$$R_1 = \left(-p_1 \cos\theta_1 \frac{d \ln p_1}{d\gamma} \right) \left[\frac{dp_1}{dz} \sin(\gamma_1 - \theta_1) + \frac{2k}{k-1} \rho_1 V_1^2 \cos(\gamma_1 - \theta_1) \frac{d\theta_1}{dz} \right]^{-1} \qquad (1.14)$$

at the point P with $z = z_p$

$$R_0 \quad 2p_0 \frac{d\theta_0}{d\gamma} \left[\frac{dp_0}{dz} \left(\frac{k-1}{2k} \frac{p_0}{\rho_0 V_0^2} - \frac{1}{k} \right) \right]^{-1} \qquad (1.15)$$

The equations (1.5) and (1.6) determine the values dp_1/dz and dp_0/dz by expressions which represent a fraction. The denominator of the expression (1.6) becomes a zero at the special point where the velocity V_0 is equal to the velocity of sound. The denominator of the expression (1.5) also becomes zero at a special point close to the velocity V_1, equal to the velocity of sound. Consequently in order that there be a continuous section close to the sonic line it is necessary that the numerators of the equations (1.5) and (1.6) become zero where the denominators are equal to zero. From these two conditions at the special points of the equations (1.5) and (1.6) it is necessary to determine the positions of the points K and P, i. e., r_1 with $z = 0$ and z_p with $r = 0$.

2. The experiments were performed on four axially symmetrical nozzles. Two nozzles with a radius of the exhaust $r_a = 60$ mm had a uniform field of velocities at the exhaust with the figures M = 2.8 and 3.2. Two other noz-

zles had a nonuniform field of velocities at the exhaust. The contours of the latter nozzles were obtained by trimming the equalizing area of the nozzles which have a uniform field at the exhaust with the figures M = 2.67 and 3.37, in such a way that the field of velocities was uniform cutoff. not on the whole exhaust radius, but on the area 0.51 r_a with the value M = 2.67 and on the area 0.46 r_a with M = 3.37. r_a = 30.57 and 41.55 mm with the values M = 2.67 and 3.37, respectively.

Fig. 2, a and b

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There were obtained shadow photographs of the flow into the atmosphere of the supersonic axially symmetrical air streams. Investigation was made of the reflection from the axis of symmetry of the incident jump passing from the exhaust of the nozzle with a pressure in the stream of less than the atmospheric. The region of the reflection from the axis of this incident jump turned out to be in all the investigated nozzles in the rhombus of uniform velocity. Therefore from here on we will characterize the investigated nozzles by the figures of M in the uniform velocity. The pressure of the retardation p_c before the nozzle changed within a broad range.

As an example there are presented in Fig. 2 shadow photographs of the reflection of the incident jump in density from the axis of symmetry, (a) with M = 2.67 and $p_c^o = 10.9 < p^{o*}$ and (b) with M = 3.2 and $p_c^o = 35.1 > p^{o*}$.

On the photographs of the reflection of the incident jump in the nozzles with uniform and nonuniform fields of velocities at the exhaust one does not see the central jump in density in some range of pressure $p_c < p_p$ (Fig. 2 b). The computed pressure p_p was determined from the condition that the pressure on the surface of the nozzle in the exhaust section was equal to the atmospheric. Let us designate the minimum pressure p_c at which there occurs the reflection of the jump from the axis without the formation of a central jump through p^*. The experimental values of the pressures p_p^o and p^{o*} referred to the atmospheric for the tested nozzles are given in Table 1.

Table 1

M number	p_p^o	p^{o*}	p_m^*	Remarks
				Nozzles with uniform field of velocities
				Nozzles with nonuniform field of velocities

On the shadow photographs with $p^* < p_c < p_p$ it is seen that the incident and reflected jumps close to the axis have a form near to the rectilinear. With a decrease in the pressure $p^* < p_c < p_p$ there occurs a reflection of the incident jump from the axis of symmetry without the formation of the visible central jump and the zone of subsonic flow behind it (see diagram in Fig. 3, where AO is the incident and OB the reflected jump in density).

Let us investigate the flow in the case of such a reflection of the incident jump AO in the region around the point of reflection O on the axis. Let us make use of the polar system of coordinates R, γ with the pole in the point of reflection O.

Let us form a basic system of equations of an ideal gas (1.1)—(1.13) and the condition of adiabaticity to the coordinates R, γ. After the conversion from this system we obtain the following equations:

$$\frac{\partial \ln V}{\partial \gamma} = -\left\{\sin\theta \sin(\gamma-\theta) + \sin\gamma\left[R\frac{\partial\theta}{\partial R} + \operatorname{ctg}(\gamma-\theta)\frac{p}{\rho V^2}\frac{k-1}{2k}R\frac{\partial \ln p}{\partial R}\right]\right\} \times$$
$$\times\left\{\sin\gamma\left[\frac{2}{k-1}\frac{\rho V^2}{p}\sin^2(\gamma-\theta)-1\right]\right\}^{-1} + \operatorname{ctg}(\gamma-\theta) R\frac{\partial \ln V}{\partial R} \quad (2.1)$$

$$R\frac{\partial \ln V}{\partial R} = -R\frac{\partial\theta}{\partial R}\operatorname{tg}(\gamma-\theta) - \frac{k-1}{2k}\frac{1}{\cos^2(\gamma-\theta)}\frac{p}{\rho V^2}R\frac{\partial \ln p}{\partial R} +$$
$$+\frac{\partial \ln V}{\partial \gamma}\operatorname{tg}(\gamma-\theta) + \frac{\partial\theta}{\partial\gamma}\operatorname{tg}^2(\gamma-\theta) \quad (2.2)$$

Outside of the basic point with R = 0 in the field between the incident and reflected jumps in density the change in the gas dynamic parameters according to the coordinates R and γ proves to be continuous. The brackets in the denominator of the first member of the equation (2.1) enclose a quantity equal to zero, while the projection of the vector of velocity in the direction of γ is equal to the velocity of sound.

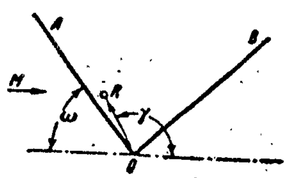

Fig. 3

On the incident and reflected jumps in density if one considers them as rectilinear, the projection of the vector of velocity in the direction of γ coincides with the velocity vector normal to the surface of the jump.

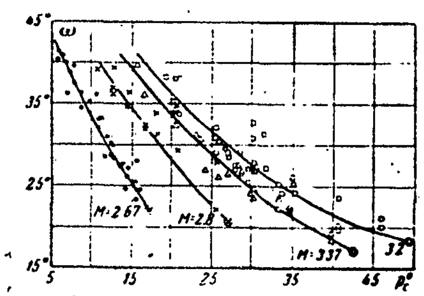

Fig. 4

The normal component of the vector of velocity behind the incident jump is less than the velocity of sound, and in front of the reflected jump it is greater than the velocity of sound. Consequently the square-bracket quantity of the equation (2.1) is negative near the incident jump, positive near the reflected jump, and equal to zero somewhere between them. When the bracket quantity in the denominator of the first member of the equation (2.1) is equal to zero, from the condition of the continuity of flow, also the numerator should be equal to zero. From the latter condition it follows that either each of the values $\partial\theta/\partial R$ and $\partial \ln p/\partial R$ is the order of A/R, where the value A is of an order equal or greater than 1, or one of the indicated products is of the order of $1/R$.

In all these cases the equation (2.2) shows that in the area where the bracket quantity in the denominator of the first member of the equation (2.1) is equal to zero, the derivative $\partial \ln V/\partial R$ is of the order of $1/R$. Consequently in this area with the decrease in R there is an increase in the derivatives $\partial \ln V/\partial R$ and $\partial \theta/\partial R$. With small values of R, when the derivatives $\partial V/\partial R$ and $\partial \theta/\partial R$ are very great, in the equations of motion it is necessary to take into consideration the forces of viscous flow. Consequently, in the case of the reflection of the jump in density from the axis without the formation of a central jump in density, the equations of an ideal gas in the region of $R = 0$ are not valid.

In accordance with the shadow photographs there was measured the angle ω between the direction of the incident jump in density in the point K and the axis of symmetry (Figs. 1 and 3). In the case where the central jump is absent the angle ω was measured on the axis of symmetry. The results of the measurements at different pressures of retardation p_c°, referred to the atmospheric, are presented in Fig. 4, where by a little circle there are outlined the computation points with $p_c^{\circ} = p_p^{\circ}$, Mach's angles, corresponding to the number M in the rhombus of uniform velocity.

In the case of the formation of a central jump in density there were computed at the point K the maximum and minimum angles of deviation (of the vector) of velocity behind the incident jump in density θ_{*max} and θ_{*min} and the angles ω_{*max} and ω_{*min} corresponding to them, which characterise the direction of the incident jump, from the condition that behind the reflected jump in density the velocity is equal to the sonic (the values which correspond to the velocity of sound are marked by the subscript $_*$). In accordance with the experiment it was considered that in the reflected jump in density the angle θ increases.

The velocity of the flow behind the reflected jump can be subsonic if the intensity of the incident jump is very great ($\theta < \theta_{*min}$ and $\omega > \omega_{*min}$) or very small ($\theta > \theta_{*max}$ and $\omega < \omega_{*max}$). There were determined also the angle of deviation of the vector of velocity behind the incident jump θ_u and the angle ω_a corresponding to it, from the condition that behind the central jump at the point K the vector of velocity is parallel to the axis of symmetry.

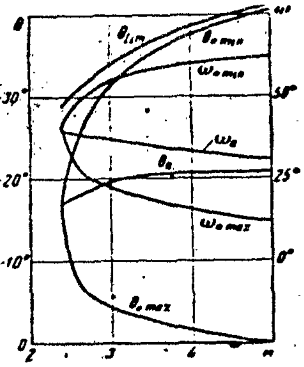

Fig. 5

In Fig. 5 with k = 1.4 as depends on the M number there are presented the values θ_{*max}, θ_{*min}, θ_a, ω_{*max}, ω_{*min}, ω_a, and the limiting angle of the turn of the flow in the oblique jump θ_{lim}.

From the results of the computation one should note that at the point K before the jumps in density, with the values of M less than the critical values of M_* ($M_* = 2.4$ for $k = 1.4$), behind the reflected jump there is always a subsonic flow. Consequently, with the number $M < M_*$ the scheme of the computation of the flow considered in the foregoing section is not accomplished. In Table 2 there are presented the computed values ω_{*max}, ω_{*min}, and ω_a and the experimental values for the central jump $\omega = \omega^*$ with $p_c^o = p^{o*}$ for the numbers M in the rhombus of the uniform velocity of the tested nozzles.

Table 2

M	ω_{*max}	ω_{*min}	ω_a	ω^*
2.80	25°00′	52°30′	38°00′	27°00′
3.20	21 00	56 30	36 30	27 30
2.67	27 30	50 00	38 30	26 00
3.37	20 00	58 00	35 30	21 30

Let us note that with large values of M equaling 2.8, 3.2, and 3.37 the angles $\omega^* > \omega_{*max}$, but with the number M equalling 2.67 where $\omega^* < \omega_{*max}$ in a small range of pressures of retardation ($p_c^o = 13.6$–14.4) the angle ω is less than ω_{*max}. The experimental angles ω presented in Fig. 4 in the presence of a central jump for all the tested nozzles M is considerably less than ω_{*min}.

Consequently, as was assumed in the preceding section, in the tests behind the reflected jump in density there occurred supersonic flow, with the exception of a little range of pressures of retardation at M = 2.67.

In this way the incident jump in density is reflected from the axis of symmetry at little intensity (ω ω^*) without the formation of a central jump and at great intensity ($\omega_{*min} > \omega > \omega^*$) with the formation of a central jump and supersonic flow ($\omega^* > \omega_{*max}$) behind the reflected jump, and with very great intensity ($\omega > \omega_{*min}$) with the formation of a central jump and subsonic flow behind the reflected jum near the point K. In the last case the scheme of computing the flow considered in the foregoing section is not accomplished. With the numbers $M > 3$ this occurs when behind the incident jump in density the angle θ at the point K is close to the limi θ_{lim} (Fig. 5).

The experimental angles ω^* are less than ω_0. Consequently, with small radii of the central jump the angle of deviation of the vector of velocity behind it $\theta_1 > 0$ and the curvature of the central jump is directed to the side opposite the direction of the flow. Proportionally as the intensity of the incident jump increases the radius of the central jump and the angle θ_1 decrease, and with $\theta_1 < 0$ the curvature of the central jump is already directed in the direction of the flow.

Let us determine the maximum ratio of the pressures p_m^* in the incident jump on the axis of symmetry at $p_c^0 = p^{0*}$ in accordance with the measured size of the angle $\omega = \omega$. The values p_m^* are given in Table 1.

The flow in the region of reflection from the axis of the incident jump in density is determined by the whole flow up to the incident jump and the conditions on the boundary of the stream behind it. In this connection it is interesting to note that the values p_m^* for the nozzles with a uniform field prove to be noticeably greater than p_m^* of the nozzels with nonuniform field with identical numbers of M in the rhombus of the uniform velocity.

In accordance with experimental values there were computed at the point K the angles of inclination to the axis of symmetry of the central jump ω_1

and of the vector of velocity behind it θ_1. For all the tested cases ω_1 changes from 87 to 90°, and θ_1 from 0 to ± 6°. Consequently, the central jump as a form close to the rectilinear, which is quite apparent also on the shadow photographs. We approximate the form of the central jump by the polynomial:

$$z = z_p + ar^2 + br^3 + cr^4 \qquad (2.3)$$

The coefficients of the polynomial we will express by the known angles of inclination to the axis of the central jump at the points K and P, the radii of curvature R_0 and R_1, and the condition with $r = r_1$, $z = 0$. By considering the value z_p as small, with the aid of the equations (1.5), (1.6), (1.13), (1.14), (1.15), and (2.3) it is possible to compute the value z_p and all the parameters at $z = 0$ needed for the beginning of the integration of the equation (1.5)—(1.8). In this way, with small values of z_p where the value z_p is unilaterally connected with the value r_1 with $z = 0$ by the equation (2.3) the general solution of the problem is simplified—out of two boundary conditions at special points of the equations (1.5 and (1.6) one can make use only of one for determining the position of the point K on the incident jump in density (r_1 at $z = 0$).

For the tested cases of the reflection of the incident jump in density there were computed the parameters of the subsonic flow with $z = 0$. These computations showed that the gas-dynamic parameters change very little (by 1—2%) in accordance with r. The maximum change in the values ρu which characterize the mass discharge amounted to 2.5%. Taking into consideration this fact, as a first approximation one should consider the subsonic flow as uniform (number of lines N = 0). In this case instead of the equations (1.5) and (1.6) one writes a single equation of discharge

$$r_1^2 p_1^{\frac{1}{k}} \left(1 - \varphi_1^{\frac{1}{k}} p_1^{\frac{k-1}{k}}\right)^{\frac{1}{2}} = \text{const} \qquad (2.4)$$

The subsonic flow is determined in this case by a system of equations

(1.7), (1.8), and (2.4). The position of the point K on the incident jump (r_1 with $z = 0$) is determined by the condition in the critical section of the uniform flow:

$$\theta_1 = 0,\ r = r_*$$

Entered 2-16-1962

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SCFDD	1
ASTIA	25
TDBTL	5
TDBDP	5
AEDC (AEY)	1
SSD (SSF)	2
AFFTC (FTY)	1
AFSWC (SWF)	1
ASD (ASYIM)	1
TDFAD (Mr. Gangel)	4