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TRANSLATION

CALCULATING THERMAL BOUNDARY LAYER IN
A FLOW OF COMPRESSIBLE GAS

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Calculating Thermal Boundary Layer in a Flow of Compressible Gas

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L. M. Zysina-Molozhen

Discussed is an approximate semiempirical method, which allows with sufficient accuracy to calculate a laminary, transient and turbulent zones of thermal boundary layer when a flow of compressible gas is directed around a surface.

The solution of many technical problems is connected with the necessity of calculating heat exchange of a surface with a stream of compressible gas flowing around it. The boundary layer originating thereat in dependence upon the flow condition and the nature of velocity distribution along the surface can be over the entire span of the surface either laminary, transient or turbulent, or on sections of surface -laminary, then transient and finally turbulent.

In this report is introduced and approximate semiempirical method, allowing with satisfactory accurecy to calculate all three zones of thermal boundary layer, originating when a compressible gas is flown around a surface. The nature of the method lies in the following.

We will investigate a plane flow of compressible gas. It is known, that for this case the integral ratio of energy in Dorodnitsayn variables acquires the form

$$\frac{d\delta_T^{**}}{d\xi} + \frac{U'_0\xi}{U_0} = \frac{T_0^*}{T_w}\frac{\mathrm{Nu}_x}{\mathrm{Pr}\mathrm{Re}_x}. \qquad (1)$$

Here

$$\delta_T^{**} = \int_0^{\delta} \frac{\rho}{\rho_0^*}\frac{u}{U_0}\left(1 - \frac{t^*}{t_0^*}\right) dy; \qquad (2)$$

$$U'_{0\xi} = \frac{dU_0}{d\xi};\ U'_0 = \frac{dU_0}{dx};\ \xi = \int_0^x \frac{\rho}{\rho_0}\, dx;$$

$$\mathrm{Nu}_x = \frac{\alpha x}{\lambda^*};\ \mathrm{Re}_x = \frac{U_0 x}{\nu^*};\ \mathrm{Pr} = \frac{\nu^*}{c^*}; \qquad (3)$$

$$t^* = T^* - T_w;\ t_0^* = T_0^* - T_w \qquad (4)$$

(sign* corresponds to retardation paramaters).

It can be written

$$\frac{\partial}{\partial x}(1-\lambda_0^2)^{\frac{k}{k-1}}\frac{\partial}{\partial \xi}+\frac{\partial \eta}{\partial \xi}\frac{\partial}{\partial \eta}, \tag{5}$$

where

$$\lambda_0=\frac{U_0}{\sqrt{2Ic_pT_0^*}};\quad \eta=\int_0^y\frac{\rho}{\rho_0^*}\,dy. \qquad (b)$$

Using formula (5) it is possible to change equation (1) into form of

$$\frac{d\delta_T^{**}}{dx}+\frac{U_0'}{U_0}\delta_T^{**}=\frac{T_0^*}{T_w}(1-\lambda_0^2)^{\frac{k}{k-1}}\frac{\mathrm{Nu}_x}{\mathrm{PrRe}_x}. \tag{7}$$

We will introduce parameters:

$$f_T=\frac{U_0'}{U_0}\delta_T^{**}G_T, \tag{8}$$

$$\chi=\frac{T_0^*}{T_w}(1-\lambda_0^2)^{\frac{k}{k-1}}\frac{\mathrm{Nu}_x}{\mathrm{PrRe}_x}G_T \tag{9}$$

and will assume, that they, changing along the surface around which flow is directed, do synonymously determine all characteristics of thermal boundary layer. We will assume,

that in expression (8) the influence of the longitudinal temperature gradient is characterized by the complex $\frac{U'_0\delta_T}{U_0}$, and G_T does not depend upon $\frac{du}{dx}$ and determines only the effect of the Re-number. In this case the value G_T will be identical for the flow around the profile and for the flow around a plate as well. We will determine the function G_T in accordance with data concerning heat exchange of a plate.

By examining the experimental data (fig.1) it becomes evident that when the physical constants are referred to retardation parameters * the formulae for calculating heat exchange retain the very same form as for the case of a noncompressible flow. For the transient zone of the boundary layer the effect of compressibility is reflected on the coordinates of the beginning and ending of the transition (kh_{beg}, kh_{end}), and the development of the transition process after its origination takes place in such a way, that lines $Nu_x = N(Re_x)$ remain parallel to each other over the entire investigated range of change in M number.

It is evident, that the curves can be approximated by a family

*Footnote —— During the calculating all physical constant are referred to temperature of retardation.

$$\mathrm{Nu}_x = B\,\mathrm{Re}_x^{n} \qquad (10)$$

in which the values B and n are different, but unchanged for each flow condition in the boundary layer. For the transient zone the coefficient B appears, as shown by experiments, to be a variable value, changing with the change in value of Re number, corresponding to the point of beginning of transition Re_{xhbeg}. This value is determined by the initial turbulence of the flow [4], magnitude of the temperature factor and M-number, i.e. the value B retains constancy within limits of each concrete experiment, but can change with the change in initial and practical conditions of the process.

Substituting expression (10) in the integral ratio of energy for plate

$$\frac{d\delta_T^{**}}{dx} = \frac{T_0}{T_w}\left(1-\lambda_0^2\right)^{\frac{k}{k-1}} \frac{\mathrm{Nu}_x}{\mathrm{Pr}\,\mathrm{Re}_x} \qquad (11)$$

and using formula (9) it is possible to obtain for function G_T the following expression:

$$G_T = (m-1)\left[\frac{\mathrm{Pr}}{(m-1)\,B}\right]^{m-1} \frac{Z}{\left[\frac{T_0}{T_w}\left(1-\lambda_0^2\right)^{\frac{k}{k-1}}\right]^{m-1}}\,\mathrm{Re}_T^{**m}\,. \qquad (12)$$

where $m = \frac{1-n}{n}$.

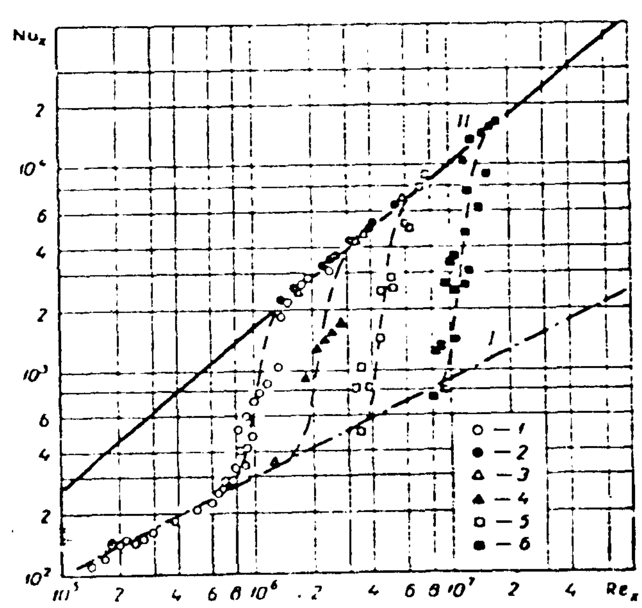

Fig.1.Dependence $\mathrm{Nu}_x = N(\mathrm{Re}_x)$ at various values of M-number.
1-0.24; 2- 0.44; 3-0.52; 4-0.73; 5- 1.07; 6- 1.43; I-Nu_x=0.297 Re_x-laminary bounda-

ry layer; II- $Nu_x = 0.0255\ Re_x^{4/5}$ - turbulent; $T_w/T_0^* \approx 1$.

We will determine function X in such a way that for a plate it will be possible to accept

$$\chi \left[\frac{T_0^*}{T_w} (1 - \alpha_0^2)^{\frac{k}{k-1}} \right]^{-(m+1)} = 1. \qquad (13)$$

Then for G_T is obtained a formula

$$G_T = A Re_T^{**m}, \qquad (14)$$

which is perfectly analogous to formula for an incompressible flow [1].

Introducing formulas (8), (9) and (14) into equation (7) it is possible with the aid of noncomplex calculations to bring same into the following form:

$$\frac{df_T}{dx} = [(m+1)\chi - 2f_T] \frac{U_0'}{U_0} - \frac{U_0''}{U_0'} f_T. \qquad (15)$$

For an incompressible flow was shown experimentally [2], that the function, set in square parentheses

$$F_T = (m+1)\chi - 2f_T \qquad (16)$$

can be well expressed by formula

$$F_T = a - 2f_T. \qquad (17)$$

The value a was found to be independent from the temperature gradient and is determined only by the condition of flow in boundary layer.

We will assume, that for a compressible flow is also possible to write an analogous formula. If a does not depend upon the longitudinal pressure gradient, then it, apparently, should have an identical value as during a flow around a profile, and during the flow around a plate. Then, by using formula (13) it is possible to determine a by formula

$$a = (m+1) \left[\frac{T_0^*}{T_w} (1 - \alpha_\infty^2)^{\frac{k}{k-1}} \right]^{m+1}, \qquad (18)$$

where α_∞ - value α_0 for an perturbed flow.

If we would consider flows, characterized by condition $T_w/T_0^* = \Theta = \text{const}$, then it is apparent that for each flow condition a will be a constant value depending only upon the flow condition.

In this case equation (15) can be easily integrated and allows to determine δ_T^{**} for the thermal boundary layer:

$$\delta_T^{**} = \left(\frac{a}{A}\right)^{\frac{1}{m+1}} \frac{(\nu_0^*)^{\frac{m}{m+1}}}{U_0} \left\{ \int_{x_н}^{x} U_0 dx + \nu_0^* \, (\mathrm{Re}_{T_н}^{**})^{m+1} \, \frac{A_л}{a} \right\}. \qquad (19)$$

Here A_1 corresponds to the value of the coefficient A in formula (14) for laminary boundary layer, if the calculation is made for the transient zone, and corresponds to the value A for transient zone if the calculation is made for the turbulent part of the boundary layer. Consequently $\mathrm{Re}_{Tn}^{**} = \frac{U_0 \delta_{Tn}^{**}}{\nu_0}$ is determined for coordinates of the beginning of change over from calculation of laminary section in first case and coordinates of the end of transition from calculation of transient zone in second. For convenience of and to speed calculation value a for each condition of flow in the boundary layer can be calculated by formula (18) in the necessary range of parameter changes and presented in form of curves (fig.2).

Using formulas (19), (9), (16)-(18) it is possible to obtain an expression for the calculation of local values of heat exchange coefficient

$$\mathrm{Nu}_x = \left(\frac{m+1}{A}\right)^{\frac{1}{m+1}} \frac{\mathrm{Pr}}{1-m} \left[\frac{1-\alpha_x^2}{1-\alpha_0^2}\right]^{\frac{k}{k-1}} \mathrm{Re}_x \left\{ \int_{x_н}^{x} \frac{U_0 dx}{\nu_0^*} + (\mathrm{Re}_{T_н}^{**})^{m-1} \frac{A_л}{a} \right\}^{-\frac{m}{m+1}}. \qquad (20)$$

Comparing (20) with the corresponding formula for incompressible flow [4], it is possible to detect, that when all physical constants are referred to flow temperature retardation T_0^* there is a formal analogy between the type of formula for calculating intensity of heat exchange in a compressible gas flow Nu_x and in an incompressible flow Nu'_x. By comparing these formulas it is easy to obtain a relation

$$\mathrm{Nu}_x = \mathrm{Nu}'_x \left[\frac{1-\alpha_x^2}{1-\alpha_0^2}\right]^{\frac{k}{k-1}} = \mathrm{Nu}'_x \left[\frac{1-\frac{k-1}{2}\lambda_x^2}{1-\frac{k-1}{2}\lambda_0^2}\right]^{\frac{k}{k-1}} \qquad (21)$$

It is evident from this formula, that for the case of a flow directed around a plate, when the rate on the outer limit of the boundary layer is equal to the speed of the oncoming flow and $\alpha_0 = \alpha_\infty$ or $\lambda_0 = \lambda_\infty$ respectively, at above described representation of formulas should come to a ratio

$$Nu_x = Nu'_x. \qquad (22)$$

As already mentioned, this formal ratio corresponds well with experimental data.

See attached page for Figure 2 (6a)

Fig.2. Dependence $a=a(\lambda)$ for laminary section of boundary layer (a); for transient section (b) and for turbulent (c) at various T_w/T_0^*: 1-0.5; 2-0.6; 3-0.7; 4-0.8; 5-0.9; 6-1.0; 7-1.2; 8-1.4; 9-1.6; 10-1.8; 11-2.0

When calculating the coordinate of the point of the beginning of change over kh_n can be determined by the Dorodnitsyn-Loytsyanskiy method [3]. For an approximate determination of the length of the transient zone we could recommend the following simple considerations. The processing of experiments (results) listed in fig.1, and analogous results of other authors showed, that the parameter r_x, characterizing the relationship between coordinates of end Kh_{end} and beginning of transition kh_{beg}, does not depend upon M number. This property of the parameter r_x allows to determine its value by empirical curves, obtained for an incompressible flow and mentioned in report [4].

Designations

ρ_0^* - density, corresponding to retardation parameters; U_0, T_0- rate and temperature outside of the boundary layer; T_w -temperature of wall.

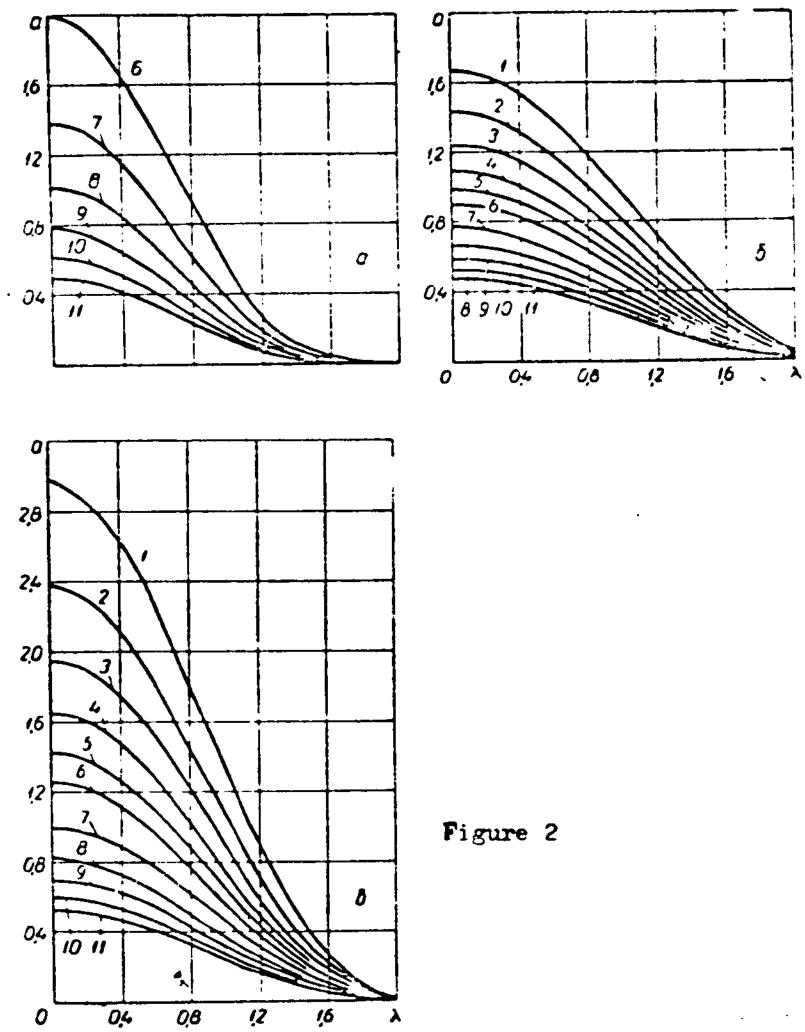

Figure 2

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