THE UNIVERSITY OF TENNESSEE

DEPARTMENT OF ELECTRICAL ENGINEERING

FIRST ORDER IMPEDANCES OF A
CIRCULAR ANTENNA ARRAY

by
C. E. Hickman

Scientific Report No. 5
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Prepared For
ELECTRONICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
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ABSTRACT

This report reviews the integral equation solution for a circular antenna array and presents a limited set of data for first order impedances. The array consists of identical, parallel dipoles which are equally spaced around the circumference of a circle. The more accurate first order impedances calculated in this report are an addition to the numerical data available for this type of antenna array. Extensive tables of the less accurate modified zeroth order impedances are given in Scientific Report No. 4, AFCRL 542.

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INTRODUCTION

This report considers the calculation of first-order impedances for a circular antenna array. The array consists of identical, parallel, cylindrical dipoles which are equally spaced around the circumference of a circle. The dipoles are of half-length h, radius a, and are center driven by a voltage V_i where the index i runs from 1 to m and identifies each element of the array. Figure 1 shows the geometry of the system.

The integral equation solution for the current distribution and impedances of the system were derived in detail in Scientific Report No. 4, AFCRL 542. This method of solution, as well as all other known methods, is only approximate and a process of iteration is used to improve the accuracy of the results. This process may be extended to theoretically derive any order solution. The integrals obtained in these higher order solutions are quite complicated and their evaluation requires numerical analysis techniques.

Extensive tables and graphs of the modified zeroth order impedances are presented in Scientific Report No. 5. The purpose of this report is to briefly review the integral equation solution and to present a limited set of data for the more accurate first order impedances.

REVIEW OF INTEGRAL EQUATION SOLUTION

A brief review of the technique for determining the current distribution along the antenna will be given first.

Each of the m elements of the array is excited by a voltage applied at the base gap. There are m terminal currents in response to these voltages, and since the array is a linear system these voltages and currents must be related by a set of m equations of the form

$$
\begin{aligned}
V_1 &= I_1(o)\, Z_{11} + I_2(o)\, Z_{12} + \ldots + I_m(o)\, Z_{1m}, \\
V_2 &= I_1(o)\, Z_{12} + I_2(o)\, Z_{11} + \ldots + I_m(o)\, Z_{2m}, \\
&- \\
V_m &= I_1(o)\, Z_{1m} + I_2(o)\, Z_{2m} + \ldots\, I_m(o)\, Z_{11},
\end{aligned}
\qquad (1)
$$

where

$$
\begin{aligned}
I_1(z) &= I_1(o)\, f_1(z), \\
I_2(z) &= I_2(o)\, f_2(z), \\
&- - - - - - - - - \\
I_m(z) &= I_m(o)\, f_m(z).
\end{aligned}
\qquad (2)
$$

In order to obtain a solution of the simultaneous equations of (1), the procedure of Reference 1 will be used. If it is assumed that $\beta a \ll 1$, there will be only a z component of current in each antenna. If the elements are assumed to be perfect conductors, the boundary condition that must be satisfied is that the electric field tangential to each element on its cylindrical surface must be zero. It is further assumed that the current at the ends of the antenna is zero, and the base gap

$2\delta = 0$. Using these boundary conditions and electromagnetic theory, it can be shown that the z component of the electric field at any point in space is given by

$$E_z = -\frac{j\omega}{\beta^2}\left(\frac{\partial^2 A_z}{\partial z^2} + \beta^2 A_z\right) , \tag{3}$$

where A_z is the vector potential and $\beta^2 = \omega^2/c^2$. The point at which E_z is evaluated lies on the surface of the antenna, and from the boundary conditions E_z must equal zero. The solution of the resulting differential equation for the vector potential yields

$$\left[A_z(z)\right]_{r=a} = -\frac{j}{c}\left(C_m \cos \beta z + \frac{V_m}{2} \sin \beta |z|\right) . \tag{4}$$

$$(-h \leq z \leq h)$$

The vector potential at any point in space may be expressed by the integral

$$\bar{A} = \frac{\mu}{4\pi}\int \frac{\bar{J}\, e^{-j\beta R}}{R}\, dv, \tag{5}$$

where $\bar{J}$ is the current density, R is the distance from the current element to the point at which $\bar{A}$ is found, and the integration covers all the volume of space in which there are currents. If this integral is applied to the circular array, the vector potential at a point on antenna m becomes

$$A_z = \frac{\mu}{4\pi} \sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} \int_{-h}^{h} I_{zi}(\bar{z})\, \frac{e^{-j\beta R_{mi}}}{R_{mi}}\, d\bar{z} , \tag{6}$$

where $I_{zi}(\bar{z})$ is the current in the i th element, and

$$R_{mi} = \sqrt{(z-\bar{z})^2 + d_{mi}^{\;2}} \;, \qquad (7a)$$

where d_{mi} is the distance from the i th element to element number m. If i = m

$$R_{mn} = \sqrt{(z-\bar{z})^2 + a^2} \;. \qquad (7b)$$

The vector potential found in (4) is now equated to that found in (6) to give

$$\int_{-h}^{h} \sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} I_{zi}(\bar{z}) \frac{e^{-j\beta R_{mi}}}{R_{mi}} d\bar{z}$$

$$= -j \frac{4\pi}{\eta} \left(C_m \cos \beta z + \frac{V_m}{2} \sin \beta |z| \right), \qquad (8)$$

where the intrinsic impedance $\eta = \sqrt{\frac{\mu}{\epsilon}}$.

Similar integral equations can be written for each of the m elements of the array. The solution of the resulting set of m simultaneous equations has not been accomplished, even for the case of m = 2 . However, if all of the m equations are identical, the method used by King[2] for the isolated case can be used to obtain an approximate solution. For the m simultaneous integral equations to be identical requires that each element be in the same total environment as every other element. The geometrical situation of each element is already the same. If the electrical environment of each element is to be the same, the scalar potential V_i at the i th element must be the same in magnitude for all values of i, and the phase difference of voltages V_i and V_{i+1} must be independent of i. These conditions are satisfied if V_i has the form

$$V_i^{(n)} = V^{(n)} e^{j\frac{2\pi n i}{m}} \;, \qquad (9)$$

in which n is any integer and $V^{(n)}$ is a constant. Any arbitrary set of m voltages V_i can be expressed in terms of $V_i^{(n)}$ by

$$V_i = \sum_{n=1}^{m} V_i^{(n)} = \sum_{n=1}^{m} V^{(n)} e^{j \frac{2\pi n i}{m}}, \qquad (10)$$

provided $V^{(n)}$ is chosen properly. To determine $V^{(n)}$ so that (10) is true, multiply both sides of (10) by

$$e^{-j \frac{2\pi q i}{m}},$$

where q is any integer from 1 to m, inclusive. This expression is then summed with respect to i, from 1 to m, giving

$$\sum_{i=1}^{m} V_i \; e^{-j \frac{2\pi q i}{m}} = \sum_{n=1}^{m} V^{(n)} \left[\sum_{i=1}^{m} e^{j \frac{2\pi}{m} (n-q) i} \right]. \qquad (11)$$

The sum in the bracket on the right is a geometric progression of the form

$$\left| \sum_{i=1}^{m} e^{j \frac{2\pi}{m} (n-q) i} \right| = \left| \frac{\sin \pi (n-2)}{\sin \frac{\pi}{m} (n-q)} \right|, \qquad (12)$$

which is equal to zero if $n \neq q$ and is equal to m if $n = q$. Then

$$V^{(q)} = \frac{1}{m} \sum_{i=1}^{m} V_i \; e^{-j \frac{2\pi q i}{m}} . \qquad (13)$$

It follows that any set of m voltages V_i can be written as the sum of m sets of voltages $V_i^{(n)}$. The sets $V_i^{(n)}$ are called the sequence components of V_i, and $V^{(n)}$ is called the n th sequence voltage. As a result, each of the m simultaneous equations of (1) gives an integral equation of the form of (8), so that instead of m simultaneous equations, there are m uncoupled integral equations. These uncoupled equations can be solved approximately, and the superposition of these solutions gives the actual currents on the elements of the array.

An iteration process is used to solve for the true current given by the integral equation. The integral equation to be solved for the unknown current distribution is

$$\int_{-h}^{h} I_z^{(n)}(\bar{z})\, K^{(n)}(z,\bar{z})\, d\bar{z}$$

$$= -j\,\frac{4\pi}{\eta}\left[C_n^{(n)}\cos\beta z + \frac{V^{(n)}}{2}\sin\beta|z|\right] , \qquad (14)$$

where

$$K^{(n)}(z,\bar{z}) = \sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} e^{j\frac{2\pi mi}{m}}\;\frac{e^{-j\beta R_{mi}}}{R_{mi}} . \qquad (15)$$

The method of solution of the integral equation will be briefly summarized here. Let

$$I_z^{(n)}(z) = I_o^{(n)} f^{(n)}(z) , \qquad (16a)$$

and

$$I_z^{(n)}(\bar{z}) = I_o^{(n)} f^{(n)}(\bar{z}) , \qquad (16b)$$

so that

$$I_z^{(n)}(\bar{z}) = I_z^{(n)}(z)\,\frac{f^{(n)}(\bar{z})}{f^{(n)}(z)} . \qquad (16c)$$

Define the relative distribution function $g^{(n)}(z,\bar{z})$ by

$$g^{(n)}(z,\bar{z}) = \frac{f^{(n)}(\bar{z})}{f^{(n)}(z)} \quad . \tag{17}$$

A good approximation to this relative distribution function is

$$g^{(n)}(z,\bar{z}) \doteq \frac{\sin\ \beta(h-|\bar{z}|)}{\sin\ \beta(h-|z|)} \quad . \tag{18}$$

Next, the function $\psi^{(n)}(z)$ is defined by

$$\psi^{(n)}(z) = \int_{-h}^{h} g^{(n)}(z,\bar{z})\ K^{(n)}(z,\bar{z})\ d\bar{z}$$

$$= \frac{4\pi}{\mu}\ \frac{\left[A_z^{(n)}\right]_{r=a}}{I_z^{(n)}(z)} . \tag{19}$$

Because of the factor $1/R$ in the integral for vector potential, nearby currents contribute more to A_z than more distant currents. Therefore, it can be argued that $\psi^{(n)}(z)$ must be nearly constant over the range $-h < z < h$. As $z \rightarrow h$, $\psi^{(n)}(z) \rightarrow \infty$, since $f^{(n)}(h) = 0$ and A_z is not zero. It should be possible, however, to select a constant $\psi^{(n)}$ such that

$$\psi^{(n)}(z) = \psi^{(n)} + \gamma^{(n)}(z), \tag{20}$$

where $\gamma^{(n)}(z)$ is small except near $z = \pm h$. After considerable manipulation, it can be shown that the function $\psi^{(n)}(z)$ can be written in terms of the integrals $C_d(h,z)$ and $S_d(h,z)$ which are tabulated by Mack[3]. Three integral functions are important in solving for $\psi^{(n)}(z)$ and other functions to be considered later. They are

$$E_d(h,z) = \int_{-h}^{h} \frac{e^{-j\beta R}}{R}\ d\bar{z}, \tag{21a}$$

$$C_d(h,z) = \int_{-h}^{h} \cos \beta\bar{z} \; \frac{e^{-j\beta R}}{R} \, d\bar{z} \, , \tag{21b}$$

$$S_d(h,z) = \int_{-h}^{h} \sin \beta \left|\bar{z}\right| \; \frac{e^{-j\beta R}}{R} \, d\bar{z} \, , \tag{21c}$$

where

$$R = \sqrt{(z-\bar{z})^2 + d^2} \quad , \tag{21d}$$

and d = center-to-center distance between elements in an array of antennas

d = a = the antenna radius for single antennas.

The expression for $\psi^{(n)}(z)$ becomes

$$\psi^{(n)}(z) = \frac{\displaystyle\sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} e^{j\frac{2\pi mi}{m}} \left[\sin\beta h \, C_{d_{mi}}(h,z) - \cos\beta h \, S_{d_{mi}}(h,z) \right]}{\sin\beta(h - |z|)} \, . \tag{22}$$

The function $\psi^{(n)}$ is real and equal to the magnitude of $\psi^{(n)}(z)$ at a suitably chosen reference. Since the current is largest at $z = 0$ if $h \leq \lambda/4$ and at $z = h - \lambda/4$ if $h > \lambda/4$, the function $\psi^{(n)}$ is defined as

$$\psi^{(n)} = \left| \psi^{(n)}(o) \right| \, , \qquad h \leq \lambda/4 \, , \tag{23a}$$

$$\psi^{(n)} = \left| \psi^{(n)}(h - \lambda/4) \right| \, , \qquad h > \lambda/4 \, . \tag{23b}$$

Numerical results for $\psi^{(n)}(z)$, $\psi^{(n)}$, and $\gamma^{(n)}(z)$ are graphically presented in Figures 2-13. These results were calculated for a six element circular array of dipoles with half-length $h = \lambda/4$ and a height-to-radius ratio $h/a = 75$. Three circumference values are used. They are $\beta\rho = 0.5$, $\beta\rho = 1.0$ and $\beta\rho = 3.0$ wavelengths. These graphs illustrate that $\gamma^{(n)}(z)$ is small except near the ends and that $\psi^{(n)}(z)$ is fairly constant except near the ends. These were the desired results.

The true current is found by assuming that the trigonometric terms in the integral equation are a first approximation to the current. This is called the zeroth order current. The true current is then

$$I_z^{(n)}(z) = \left[I_z^{(n)}(z) \right]_0 + \left[I_z^{(n)}(z) \right]_c , \tag{24}$$

where the second term on the right is a correction to the zeroth order current. This correction term involves the function $\gamma^{(n)}(z)$ which is assumed to be small. It also contains the relative distribution function $g^{(n)}(z,\bar{z})$. If this function is chosen carefully, this term is also small.

Since the correction term involves the true current, it still cannot be found exactly. However, an approximate correction can be obtained by substituting the zeroth order current in place of the true current in the correction term. This gives a first order correction term given by $\left[I_z^{(n)}(z) \right]_{c_1}$. The first order current then is

$$\left[I^{(n)}(z) \right]_1 = \left[I_z^{(n)}(z) \right]_0 + \left[I_z^{(n)}(z) \right]_{c_1} . \tag{25}$$

This process may be carried out again to obtain a second order solution, or indeed, a solution of any order. The integrations become quite difficult, and only first order solutions are considered here.

Again, after considerable manipulation and the evaluation of the constant $C_n^{(n)}$, the first order current may be written as

$$\left[I_z^{(n)}(z) \right]_1 = \frac{j2\pi V^{(n)}}{\eta\psi^{(n)}} \frac{\sin\beta(h-|z|) + \dfrac{M_1^{(n)}(z)}{\psi^{(n)}}}{\cos\beta h + \dfrac{F_1^{(n)}(h)}{\psi^{(n)}}} , \tag{26}$$

where

$$M_1^{(n)}(z) = F_1^{(n)}(z)\sin\beta h - G_1^{(n)}(z)\cos\beta h$$
$$+ G_1^{(n)}(h)\cos\beta z - F_1^{(n)}(h)\sin\beta z . \tag{27}$$

All of the above functions are completely defined in Reference 1.

FIRST ORDER IMPEDANCES

The first order impedance may be obtained by setting z = 0 in (26), which gives the terminal current, and then solving for the ratio $V^{(n)} / \left[I_z^{(n)}(o)\right]_1$, which is the terminal impedance.

The first order impedance is then

$$Z_1^{(n)} = - \frac{j\eta\psi^{(n)}}{2\pi} \frac{\cos\beta h + \dfrac{F_1^{(n)}(h)}{\psi^{(n)}}}{\sin\beta h + \dfrac{M_1^{(n)}(o)}{\psi^{(n)}}} . \tag{28}$$

The first order impedance requires the evaluation of $F_1^{(n)}(h)$ and $M_1^{(n)}(o)$. These functions can be written in terms of the integrals $C_d(h,z)$, $S_d(h,z)$, and $E_d(h,z)$. The function $F_1^{(n)}(h)$ is defined as

$$F_1^{(n)}(h) = - \int_{-h}^{h} (\cos\beta\bar{z} - \cos\beta h)\, K^{(n)}(h,\bar{z})\, d\bar{z} . \tag{29}$$

This can be integrated in the same manner as the integrals for $\psi^{(n)}(z)$ giving

$$F_1^{(n)}(h) = - \sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} e^{\frac{j2\pi mi}{m}} C_d(h,h)$$

$$+ \cos\beta h \sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} e^{\frac{j2\pi mi}{m}} E_d(h,h) . \tag{30}$$

If z = 0 is substituted into (27), the function $M_1^{(n)}(o)$ becomes

$$M_1^{(n)}(o) = F_1^{(n)}(o) \sin\beta h - G_1^{(n)}(o) \cos\beta h + G_1^{(n)}(h) . \tag{31}$$

The integrals associated with this function are

$$F_1^{(n)}(o) = -(1 - \cos \beta h)\, \gamma^{(n)}(o) - \int_{-h}^{h} \cos \beta \bar{z}\, K^{(n)}(o,\bar{z})\, d\bar{z}$$

$$+ \cos \beta h \int_{-h}^{h} K^{(n)}(o,\bar{z})\, d\bar{z} + \psi^{(n)}(o) - \cos \beta h\, \psi^{(n)}(o), \quad (32a)$$

$$G_1^{(n)}(o) = \sin \beta h\, \gamma^{(n)}(o) - \int_{-h}^{h} \sin \beta|\bar{z}|\, K^{(n)}(o,\bar{z})\, d\bar{z}$$

$$+ \sin \beta h \int_{-h}^{h} K^{(n)}(o,\bar{z})\, d\bar{z} - \sin \beta h\, \psi^{(n)}(o), \quad (32b)$$

and

$$G_1^{(n)}(h) = - \int_{-h}^{h} (\sin \beta|\bar{z}| - \sin \beta h)\, K^{(n)}(o,\bar{z})\, d\bar{z}\, . \quad (32c)$$

Using the fact that

$$\psi^{(n)} = \psi^{(n)}(z) - \gamma^{(n)}(z)\, , \quad (20)$$

and the relationships of (21), $M_1^{(n)}(o)$ becomes

$$M_1^{(n)}(o) = \sin\beta h\ \psi^{(n)} + \sin\beta h \sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} e^{\frac{j2\pi mi}{m}} E_d(h,h)$$

$$- \sin\beta h \sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} e^{\frac{j2\pi mi}{m}} C_d(h,0)$$

$$+ \cos\beta h \sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} e^{\frac{j2\pi mi}{m}} S_d(h,0)$$

$$- \sum_{i=-\frac{m-1}{2}}^{\frac{m-1}{2}} e^{\frac{j2\pi mi}{m}} S_d(h,h)\ . \qquad (33)$$

Define

$$F_1^{(n)}(h) \equiv \alpha_1^{I} + j\,\alpha_1^{II} \qquad (34a)$$

$$M_1^{(n)}(o) \equiv \beta_1^{I} + j\,\beta_1^{II}\ . \qquad (34b)$$

The first order impedance given by (28) then becomes

$$Z_1^{(n)} = \frac{-j\eta\psi^{(n)}}{2\pi} \quad \frac{\cos\beta h + \dfrac{\alpha_1^{I}}{\psi^{(n)}} + j\,\dfrac{\alpha_1^{II}}{\psi^{(n)}}}{\sin\beta h + \dfrac{\beta_1^{I}}{\psi^{(n)}} + j\,\dfrac{\beta_1^{II}}{\psi^{(n)}}}\ . \qquad (35)$$

Rationalizing (35) gives

$$R_1^{(n)} = \frac{\eta}{2\pi} \; \frac{\alpha_1^{II}\left[\sin\beta h + \dfrac{\beta_1^{I}}{\psi^{(n)}}\right] - \beta_1^{II}\left[\cos\beta h + \dfrac{\alpha_1^{I}}{\psi^{(n)}}\right]}{\left[\sin\beta h + \dfrac{\beta_1^{I}}{\psi^{(n)}}\right]^2 + \left[\dfrac{\beta_1^{II}}{\psi^{(n)}}\right]^2} \qquad (36)$$

and

$$X_1^{(n)} = -\frac{\eta\psi^{(n)}}{2\pi} \; \frac{\left[\cos\beta h + \dfrac{\alpha_1^{I}}{\psi^{(n)}}\right]\left[\sin\beta h + \dfrac{\beta_1^{I}}{\psi^{(n)}}\right] - \dfrac{\alpha_1^{II}\,\beta_1^{II}}{\left[\psi^{(n)}\right]^2}}{\left[\sin\beta h + \dfrac{\beta_1^{I}}{\psi^{(n)}}\right]^2 + \left[\dfrac{\beta_1^{II}}{\psi^{(n)}}\right]^2} \,. \qquad (37)$$

For the case of an isolated antenna, it is known that first order data are inaccurate. The first order impedance spiral almost coincides with the more accurate second order spiral, except corresponding values are for slightly different lengths. Table I shows the variation of the first order impedance near $h = \lambda/4$ for $h/a = 75$. The second order impedance for an isolated antenna having $h/a = 75$ and $h = \lambda/4$ is $Z_2 = 86.5 + j41.7$ ohms. From Table I it may be noted that the first and second order data are closest when $h = 0.254\lambda$. Since for large separation, the sequence impedances must approach the impedance of the isolated antenna, first order data were obtained for $h = 0.254\lambda$. These data are called modified first order impedances for $h = \lambda/4$. The usefulness of these values is based on the assumption that the first order sequence impedance spiral and the second order sequence impedance spiral would almost coincide. Since second order data are practically non-existent, this assumption has not been proved. An extensive amount of numerical work would be required to validate the assumption.

A limited number of first order impedances were calculated using Equation (35). The cases considered are for arrays with a circum-varying from 0.6 to 12 wavelengths and which are composed of $m = 4$, 6 and 8 elements. Results of these computations are presented in three ways. Sequence resistances and reactances are tabulated in Tables II, III and IV. Results are presented graphically in Figures 14-37 by plotting (1) resistance and reactance as a function of the circumference for all sequences, and (2) reactance versus resistance for all sequences. The modified zeroth-order impedances for arrays of dipoles of half-length $h = \lambda/4$ are plötted on these same graphs for comparison.

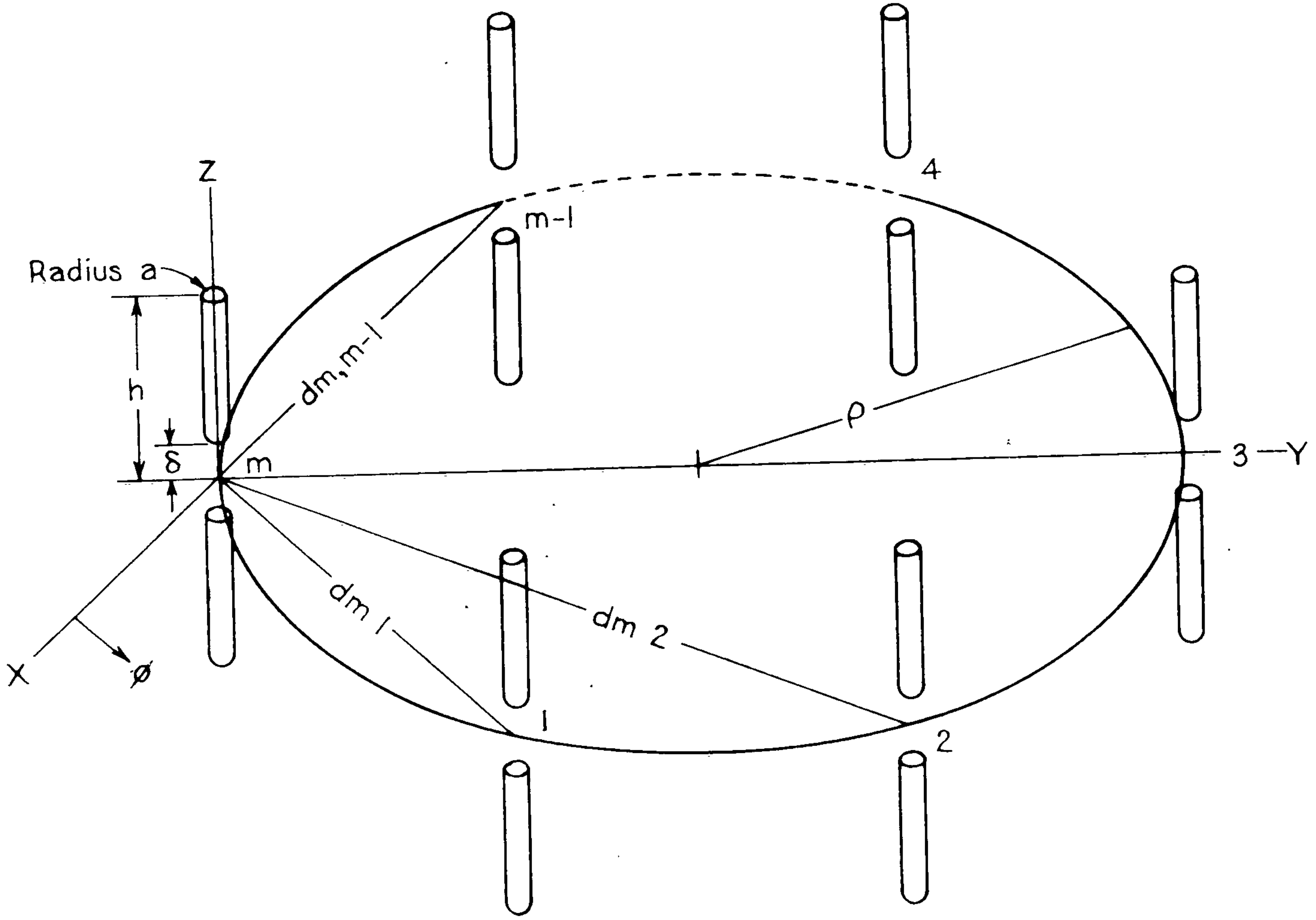

Fig. 1. Perspective view of circular antenna array, showing a cylindrical coordinate system

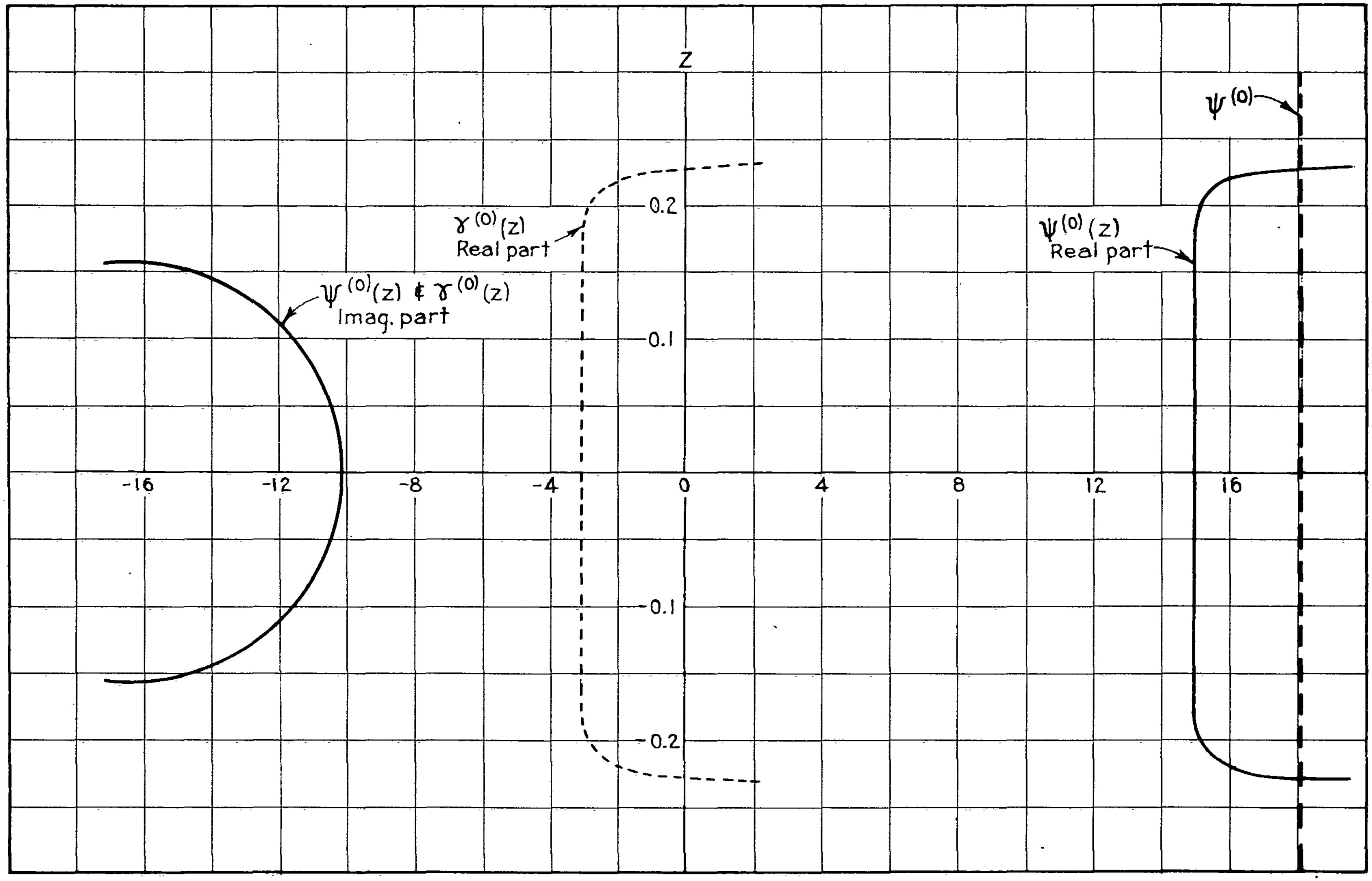

Fig. 2 . The functions $\psi^{(0)}$, $\psi^{(0)}(z)$ and $\gamma^{(0)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 0.5 wavelengths

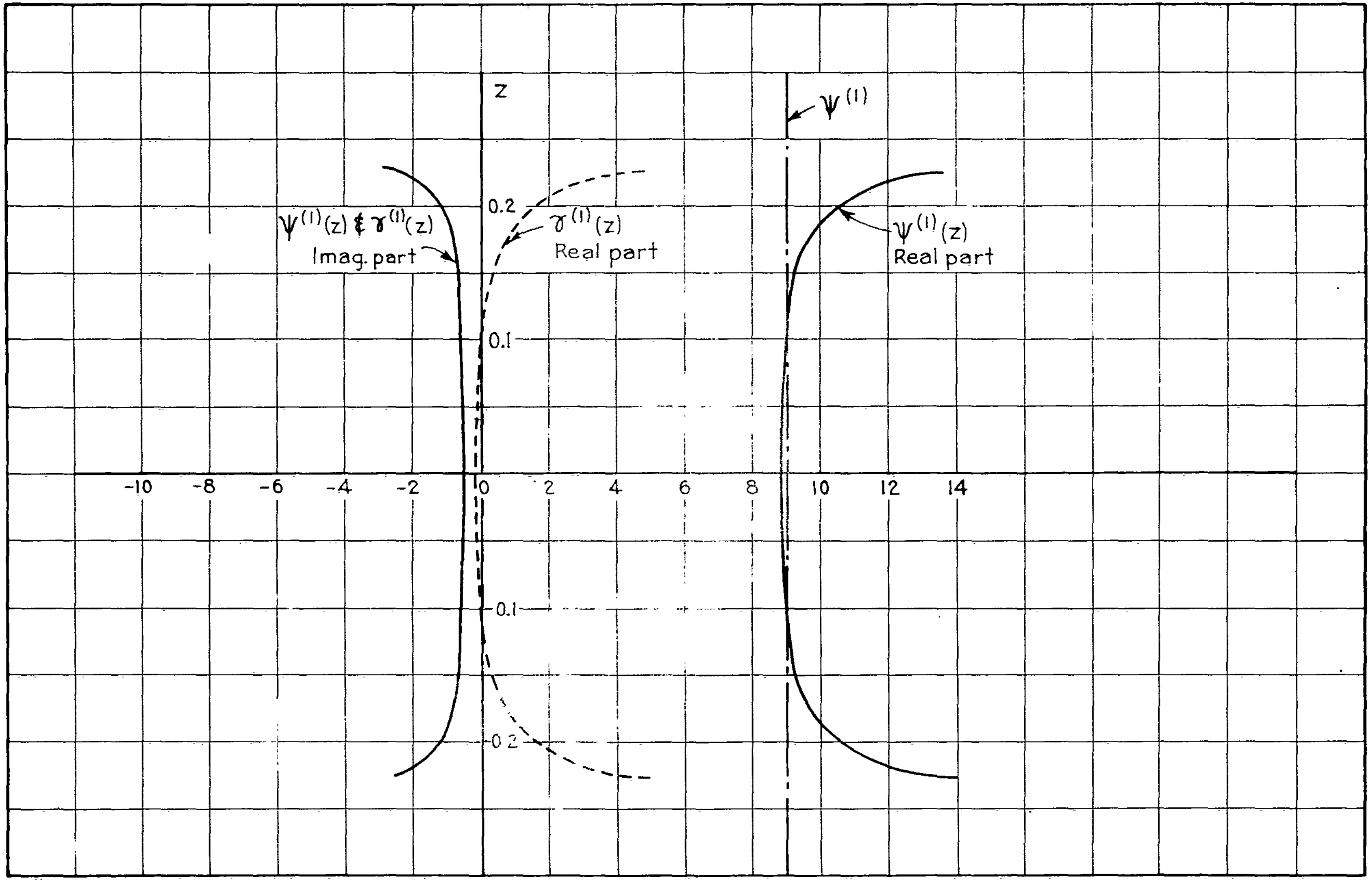

Fig. 3. The functions $\psi^{(1)}$, $\psi^{(1)}(z)$ and $\gamma^{(1)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 0.5 wavelengths

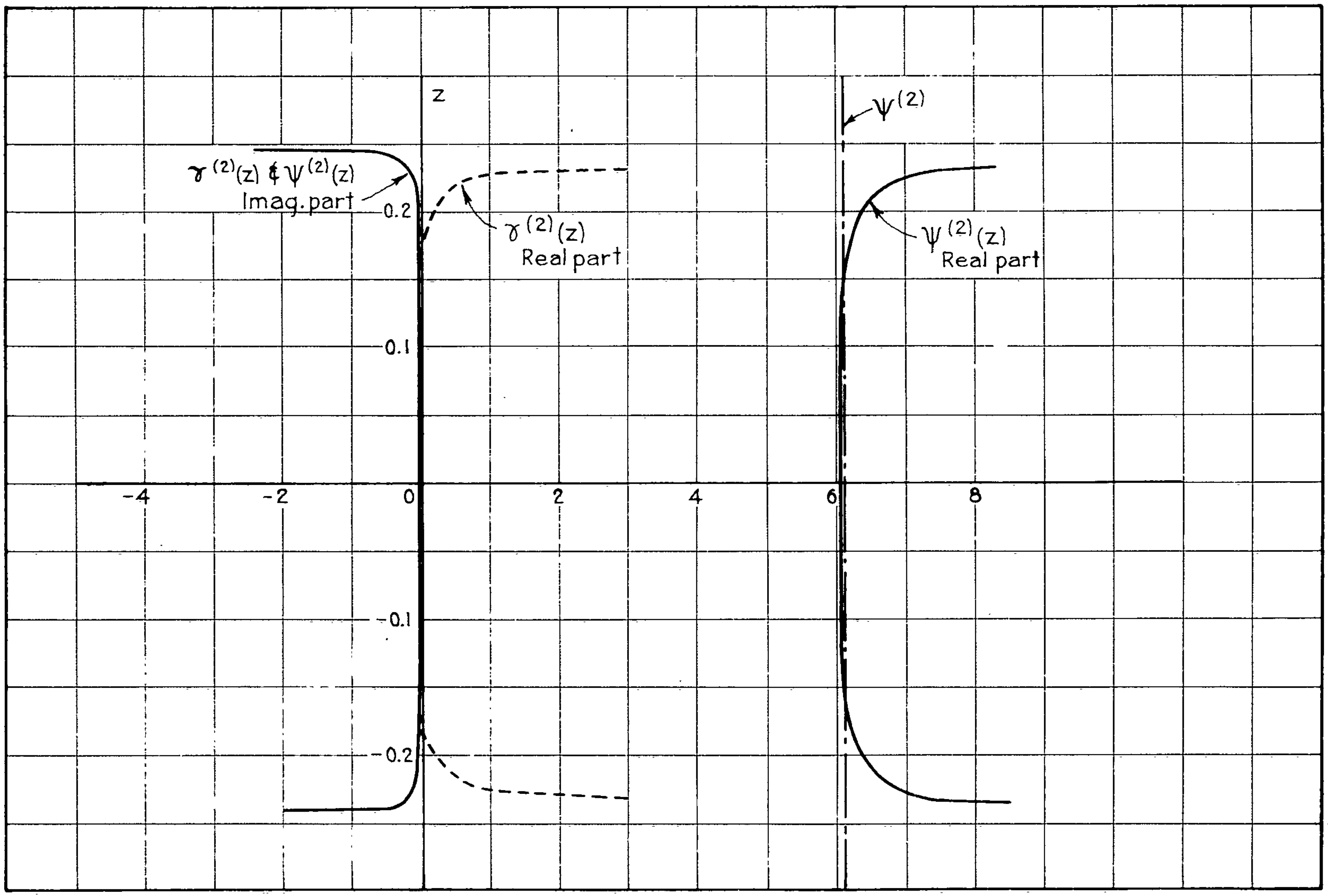

Fig. 4. The functions $\psi^{(2)}$, $\psi^{(2)}(z)$ and $\gamma^{(2)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 0.5 wavelengths

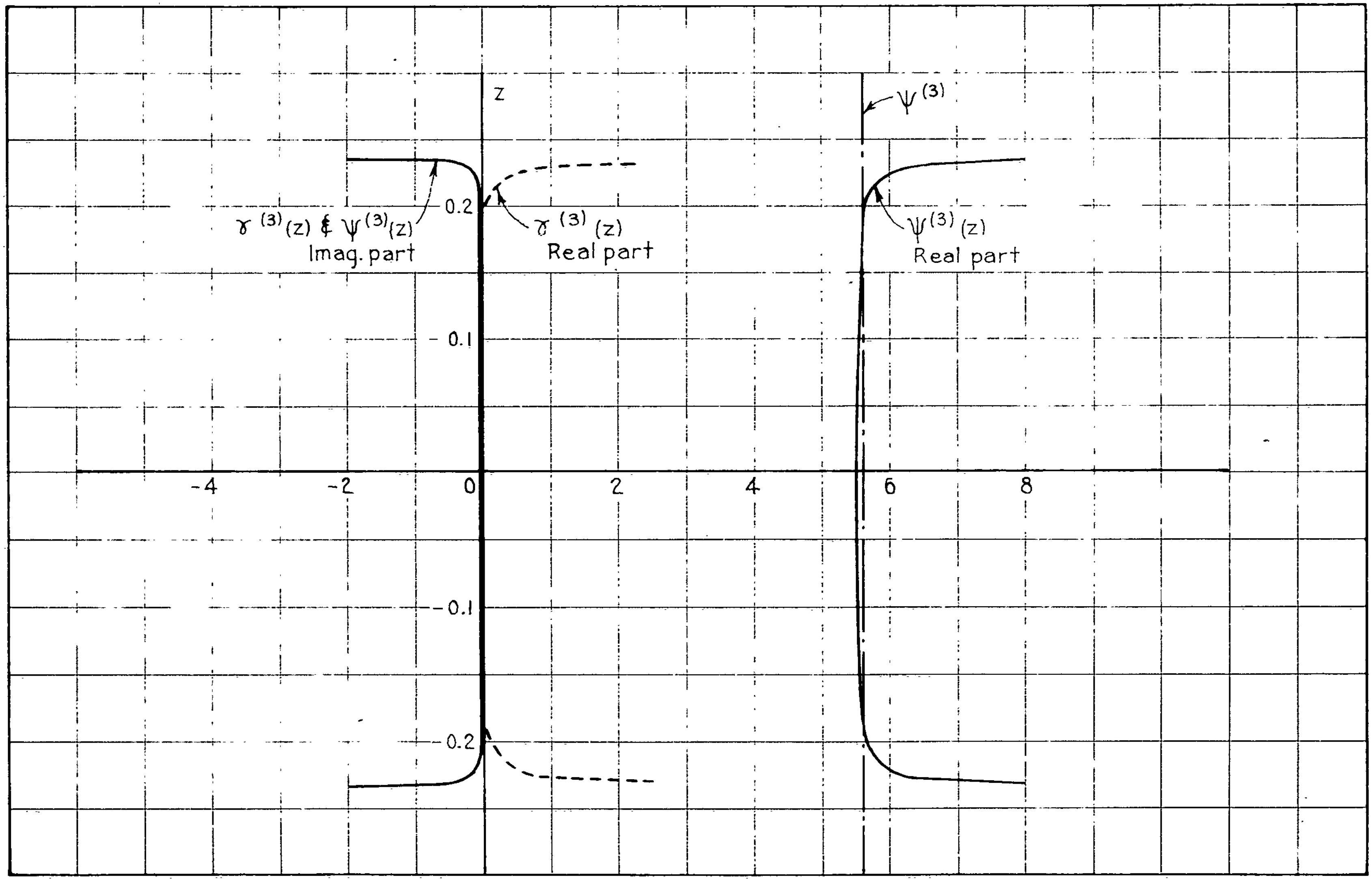

Fig. 5. The functions $\psi^{(3)}$, $\psi^{(3)}(z)$ and $\gamma^{(3)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 0.5 wavelengths

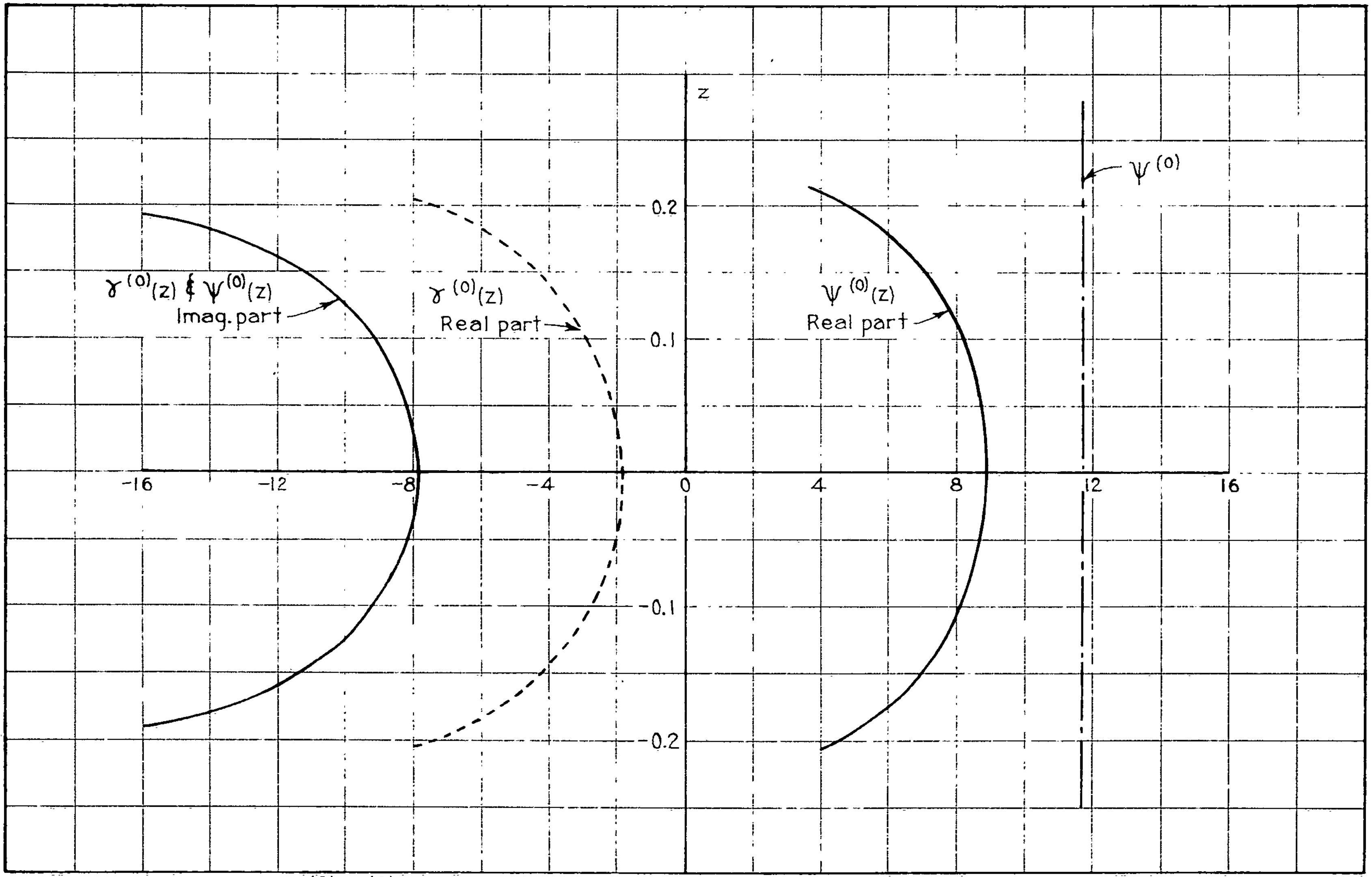

Fig. 6. The functions $\psi^{(0)}$, $\psi^{(0)}(z)$ and $\gamma^{(0)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 1.0 wavelengths

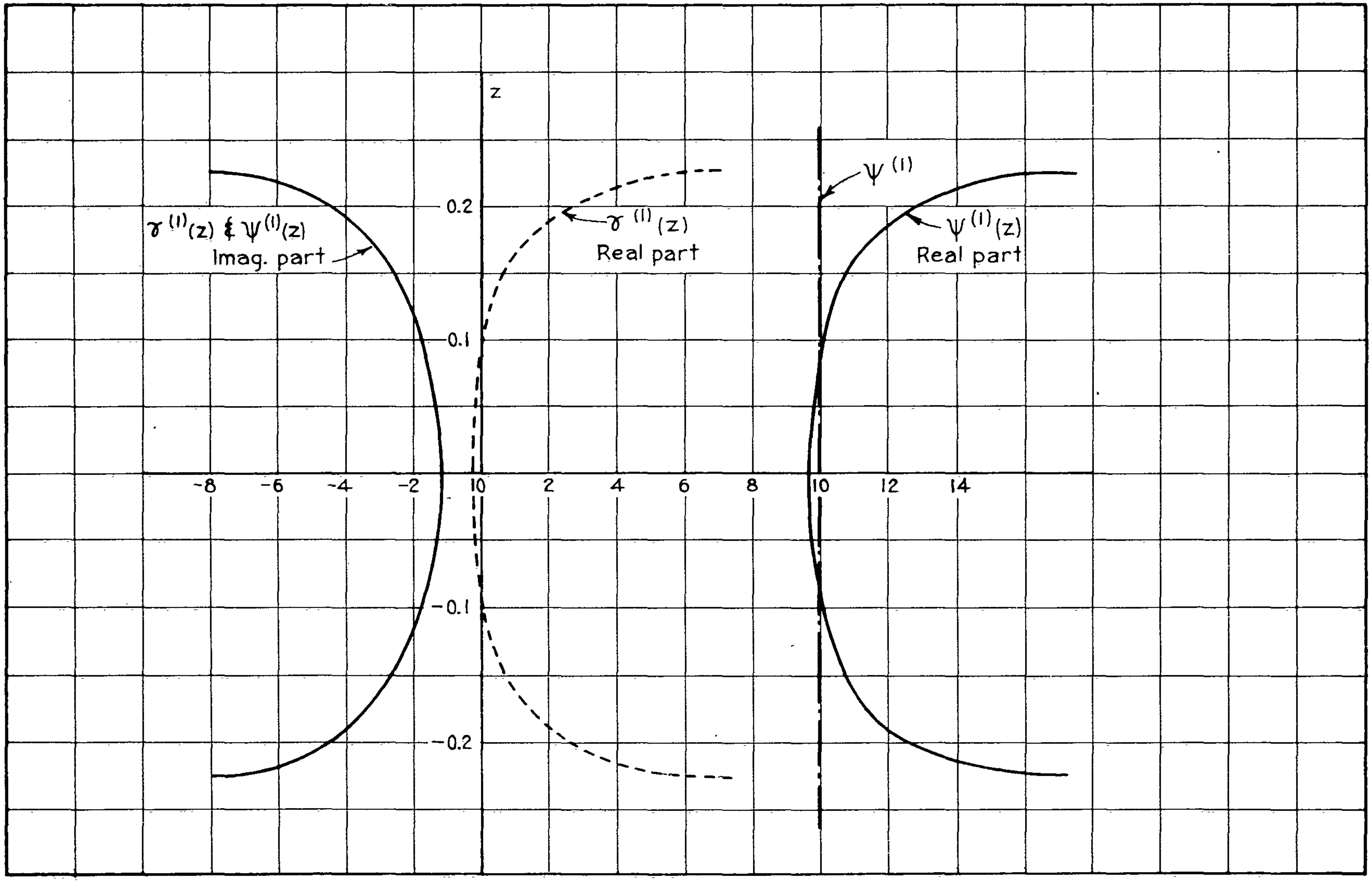

Fig. 7. The functions $\psi^{(1)}$, $\psi^{(1)}(z)$ and $\gamma^{(1)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 1.0 wavelengths

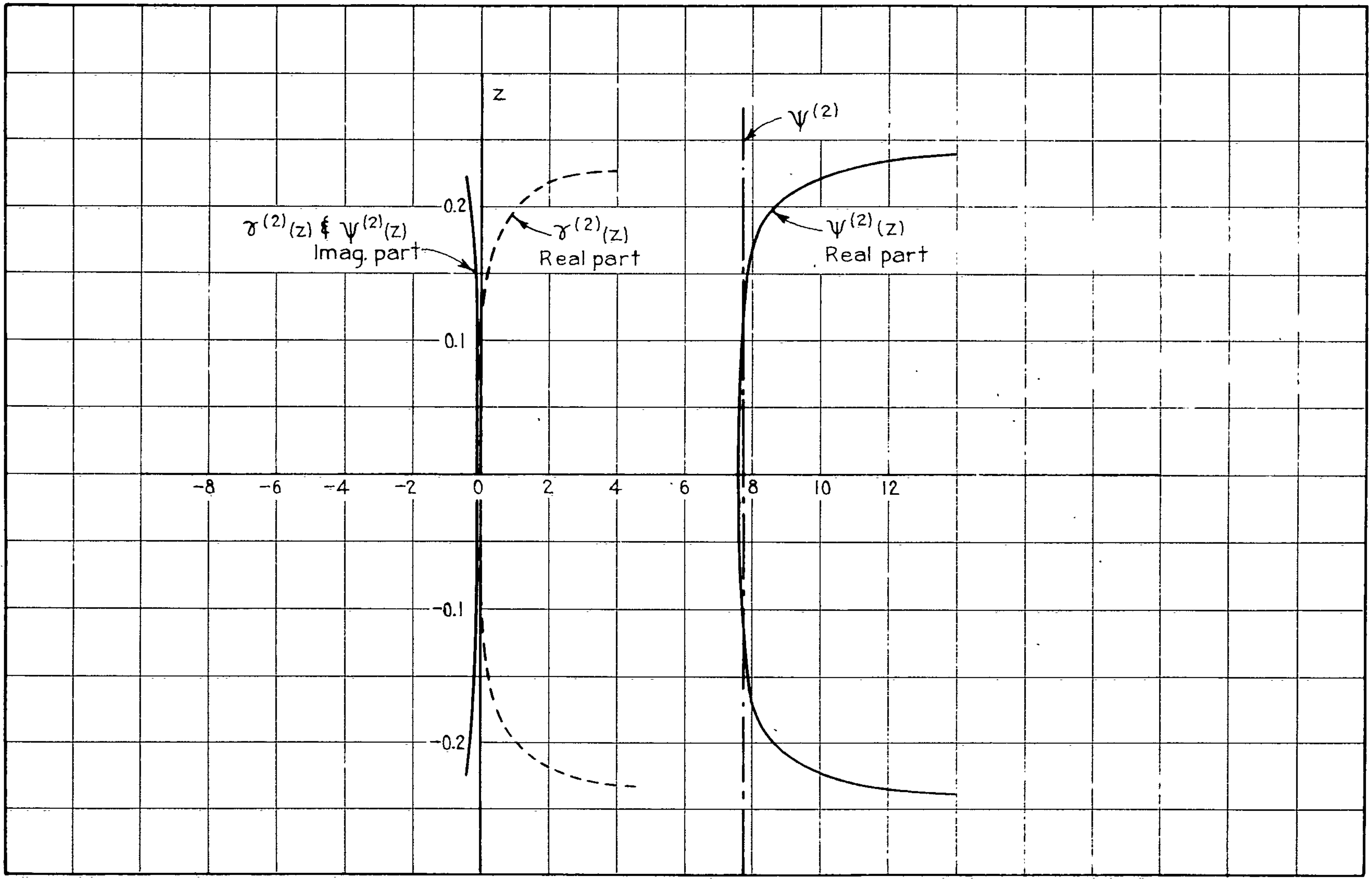

Fig. 8. The functions $\psi^{(2)}$, $\psi^{(2)}(z)$ and $\gamma^{(2)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 1.0 wavelengths

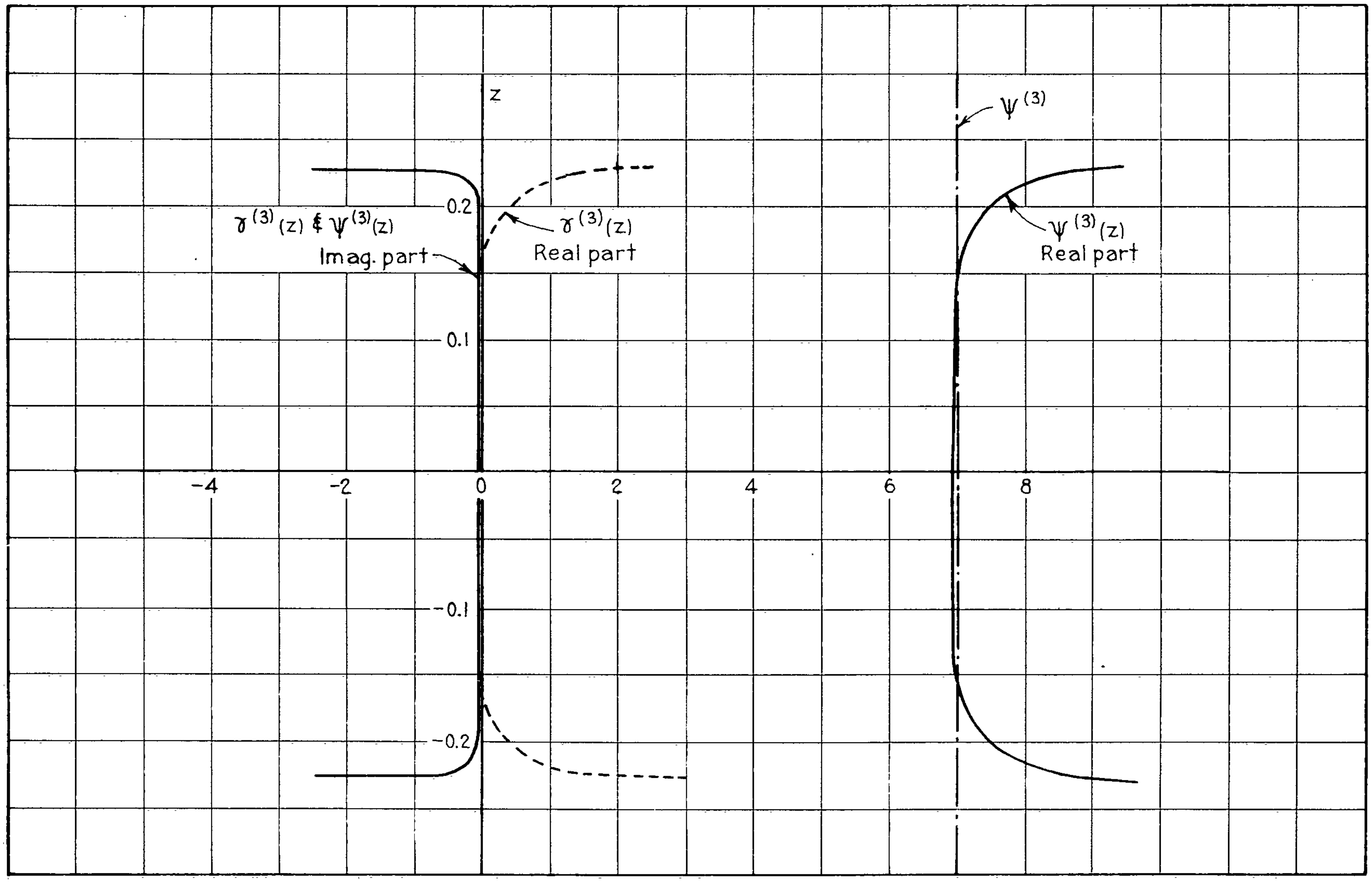

Fig. 9. The functions $\psi^{(3)}$, $\psi^{(3)}(z)$ and $\gamma^{(3)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 1.0 wavelengths

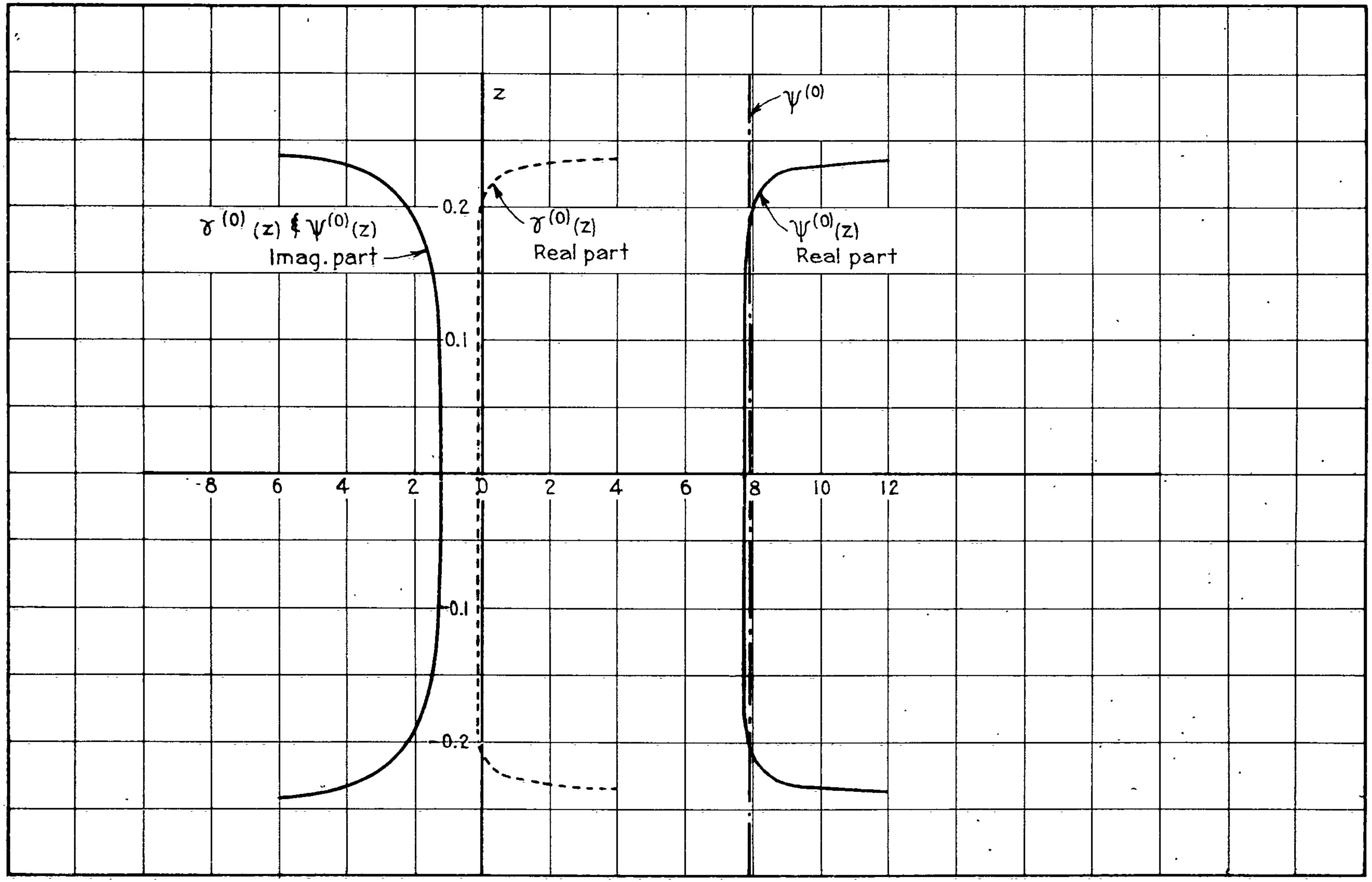

Fig. 10. The functions $\psi^{(0)}$, $\psi^{(0)}(z)$ and $\gamma^{(0)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 3.0 wavelengths

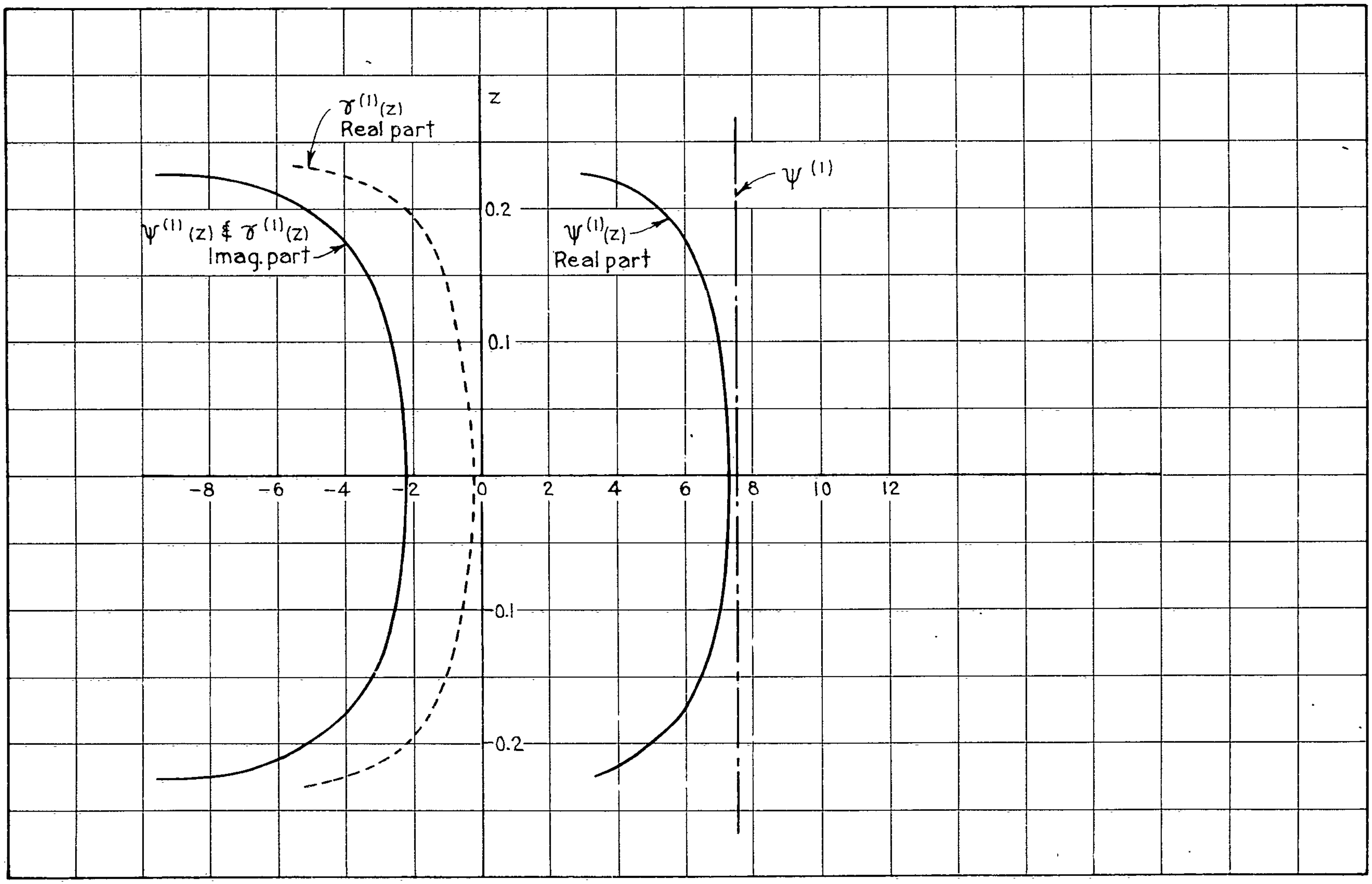

Fig. 11. The functions $\psi^{(1)}$, $\psi^{(1)}(z)$ and $\gamma^{(1)}(z)$ for $\beta h = \pi/2$, h/a = 75, and a circumference of 3.0 wavelengths

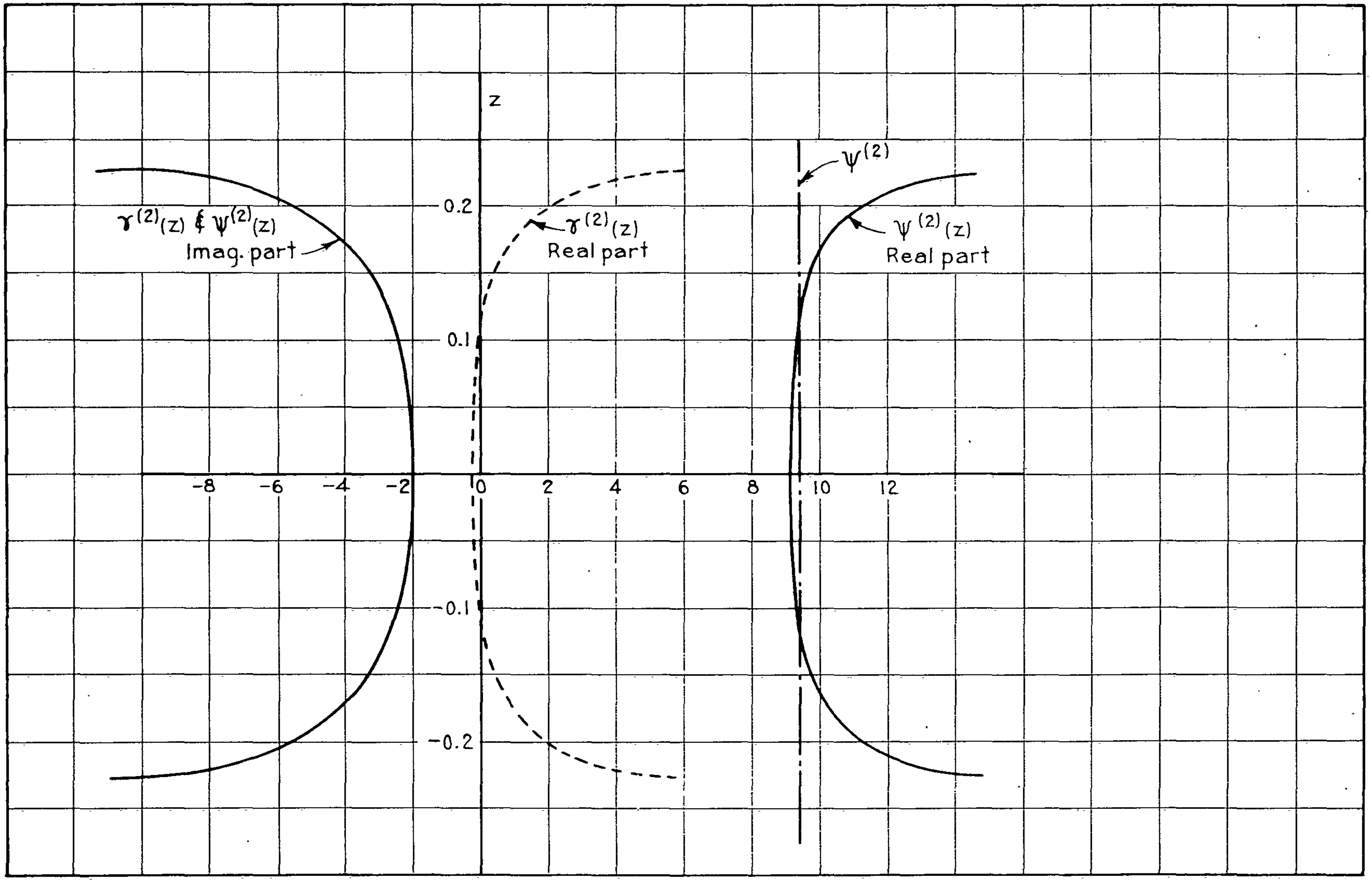

Fig. 12. The functions $\psi^{(2)}$, $\psi^{(2)}(z)$ and $\gamma^{(2)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 3.0 wavelengths

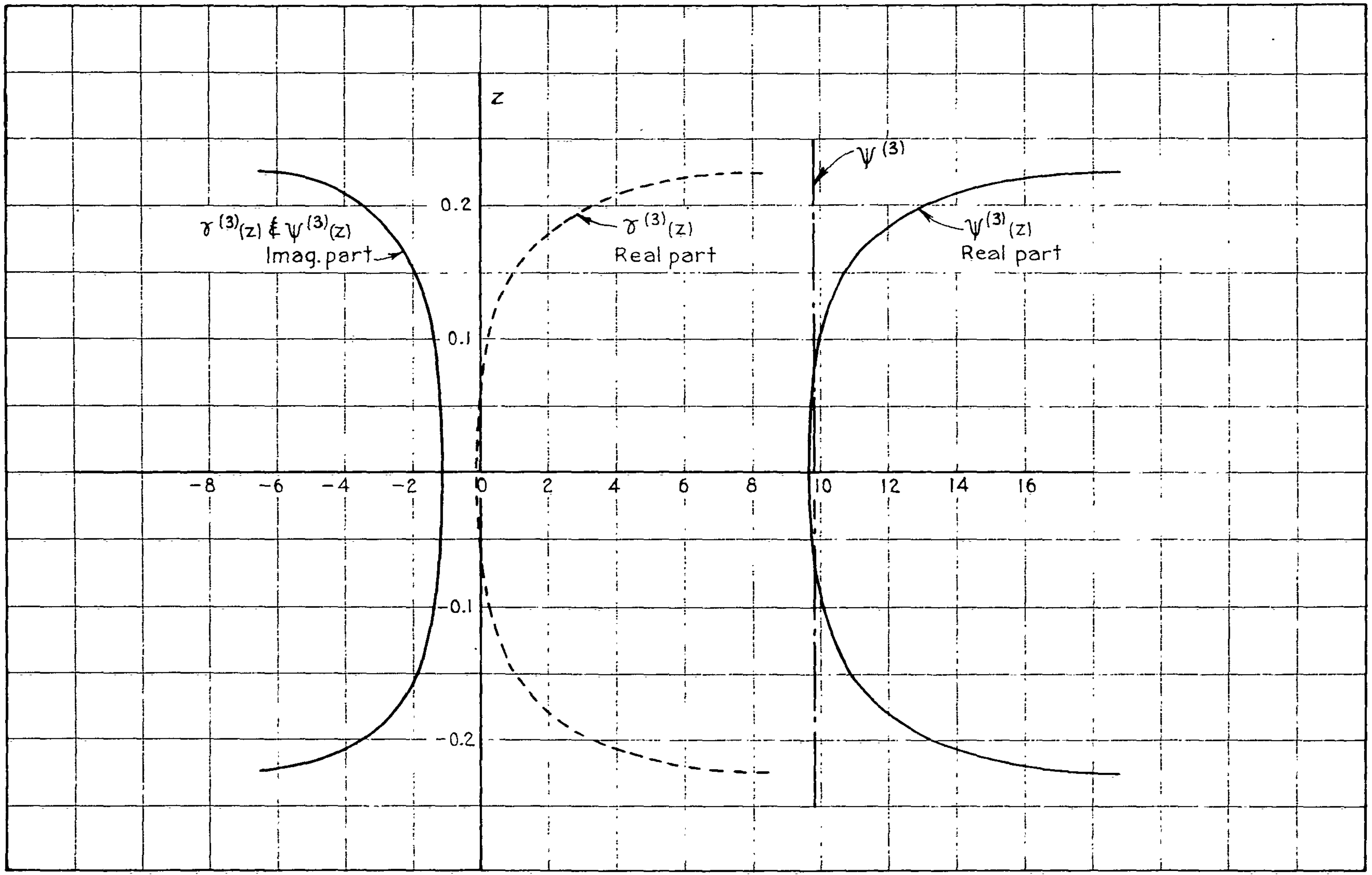

Fig. 13. The functions $\psi^{(3)}$, $\psi^{(3)}(z)$ and $\gamma^{(3)}(z)$ for $\beta h = \pi/2$, $h/a = 75$, and a circumference of 3.0 wavelengths

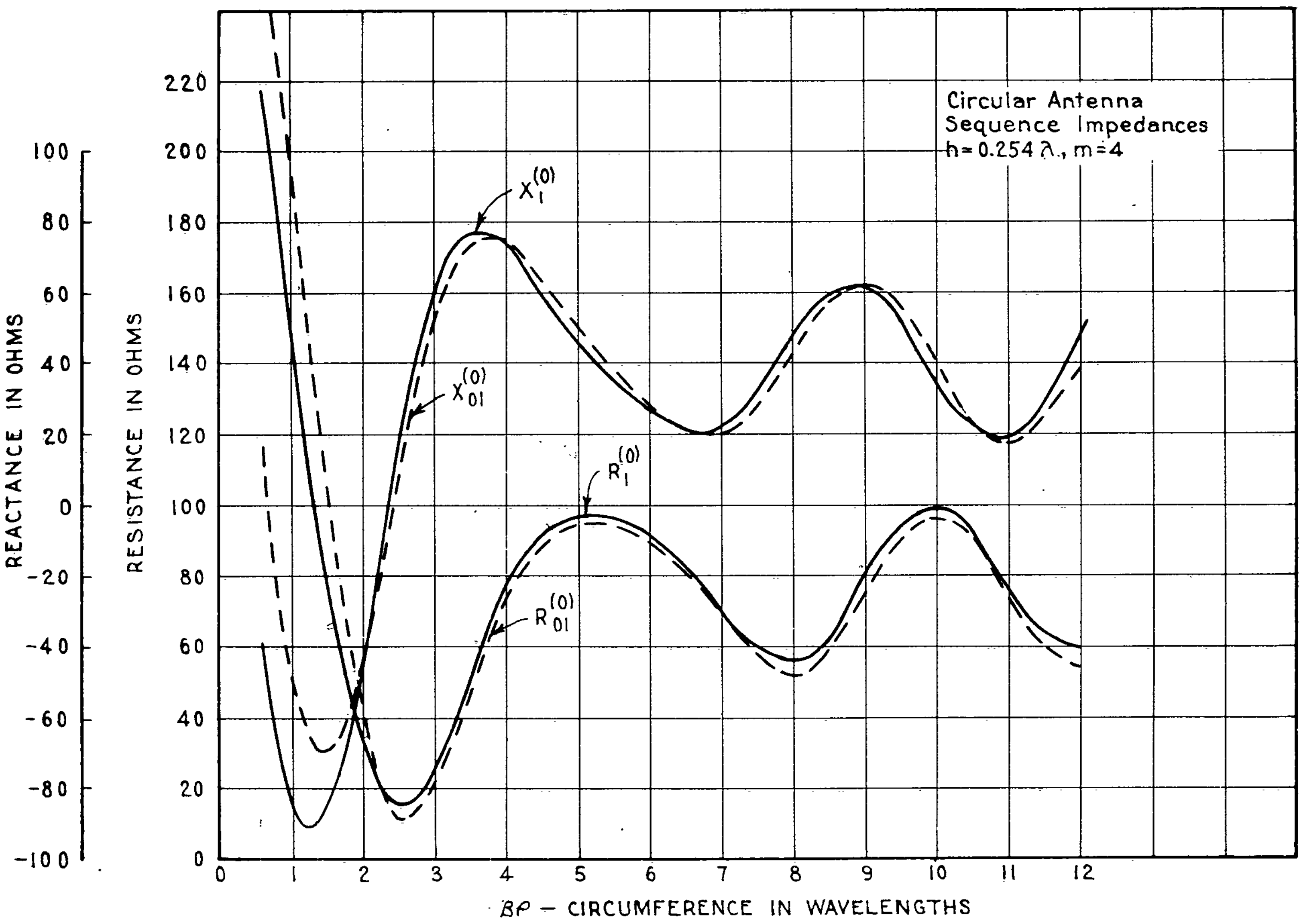

Fig. 14. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 4, n = 0

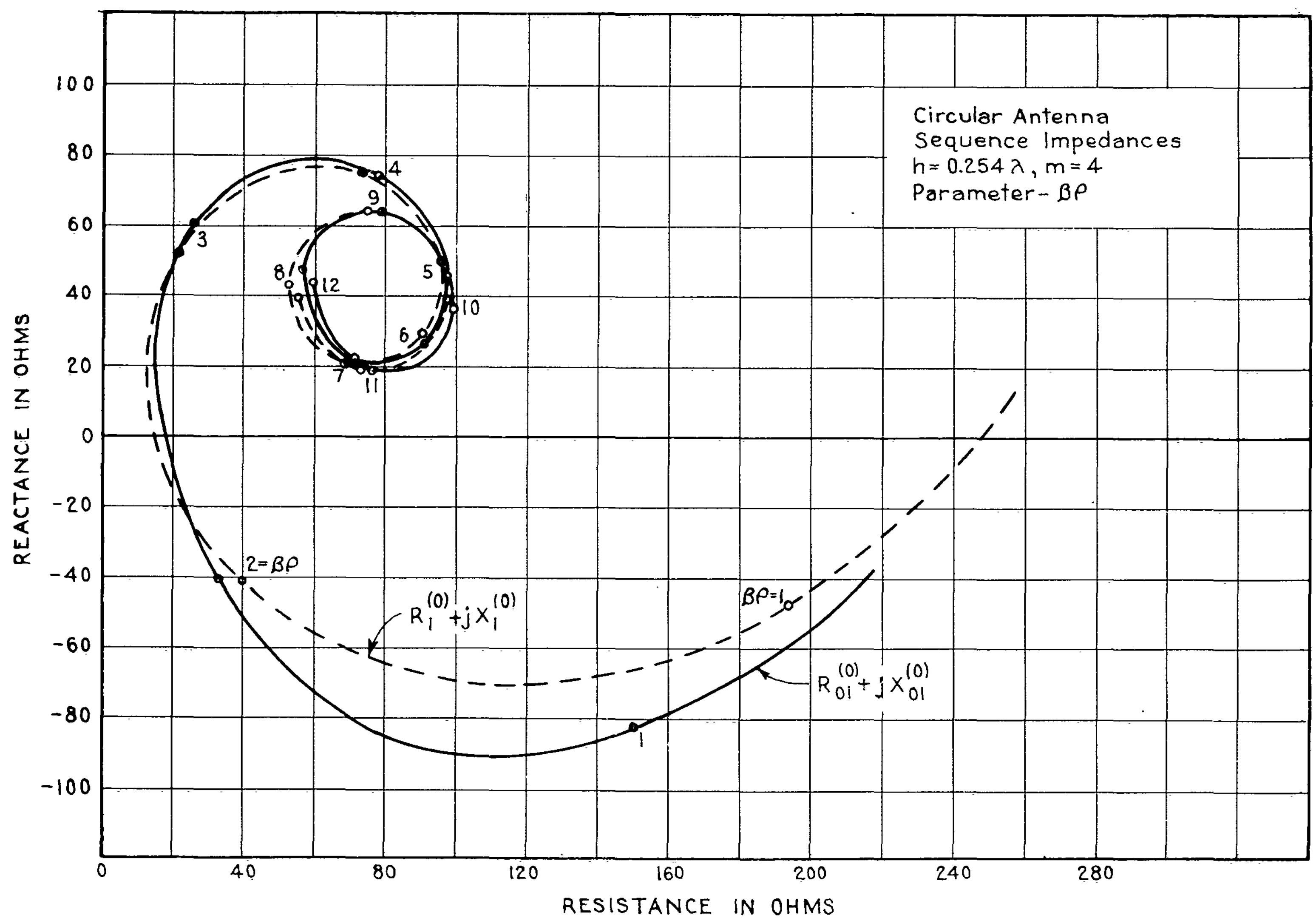

Fig. 15. Sequence reactance versus sequence resistance; parameter is βρ, m = 4, n = 0

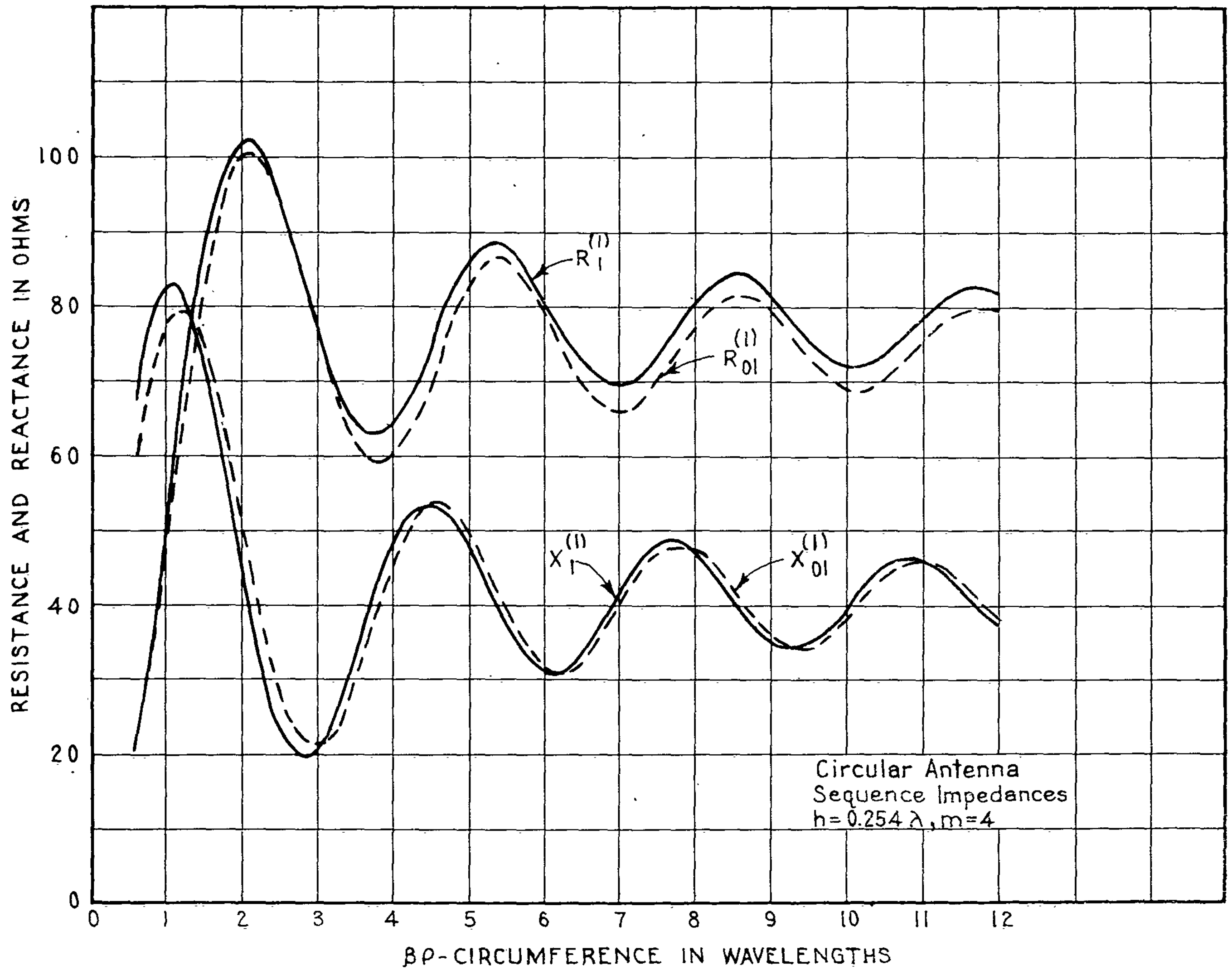

Fig. 16. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 4, n = 1

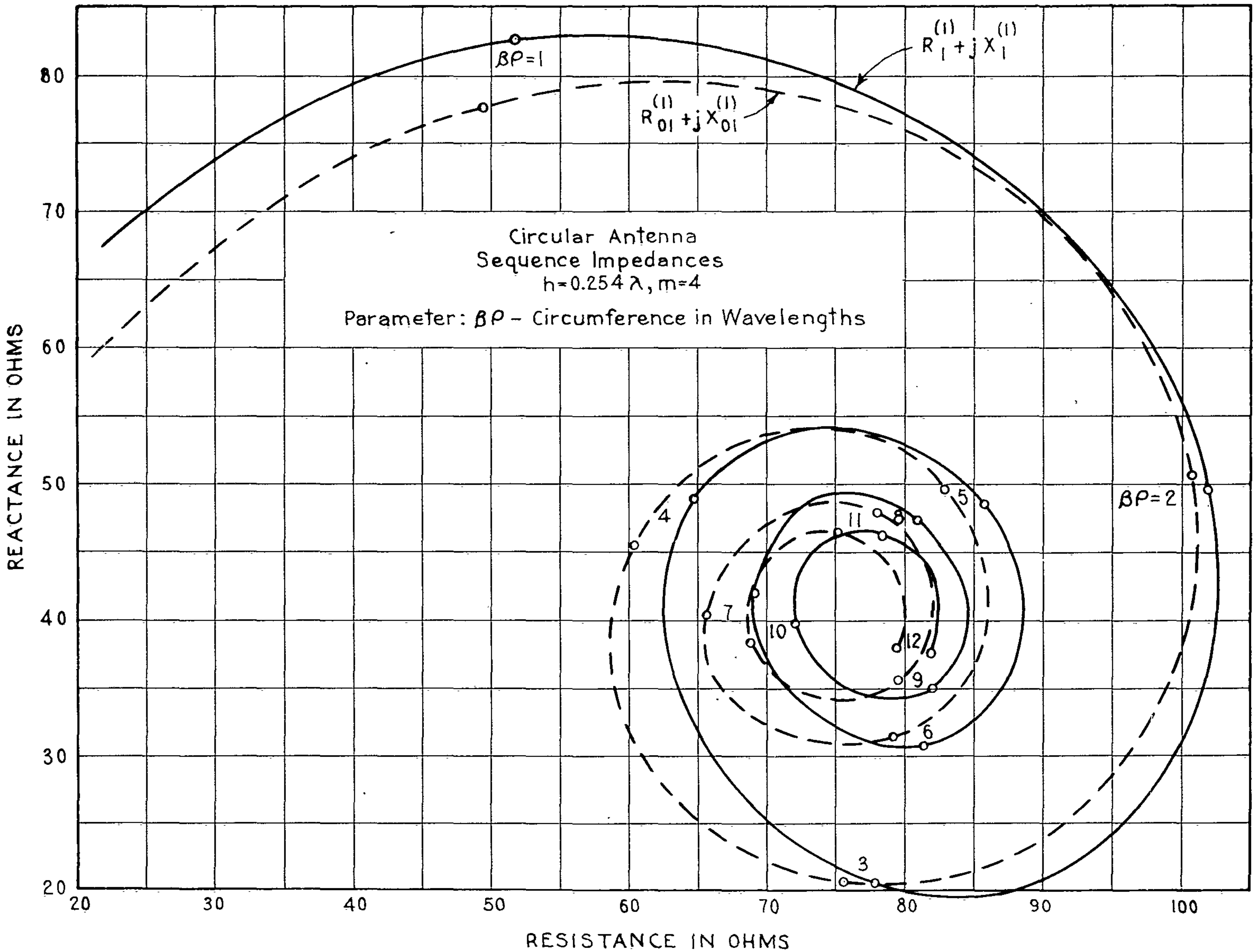

Fig. 17. Sequence reactance versus sequence resistance; parameter is $\beta\rho$, m = 4, n = 1

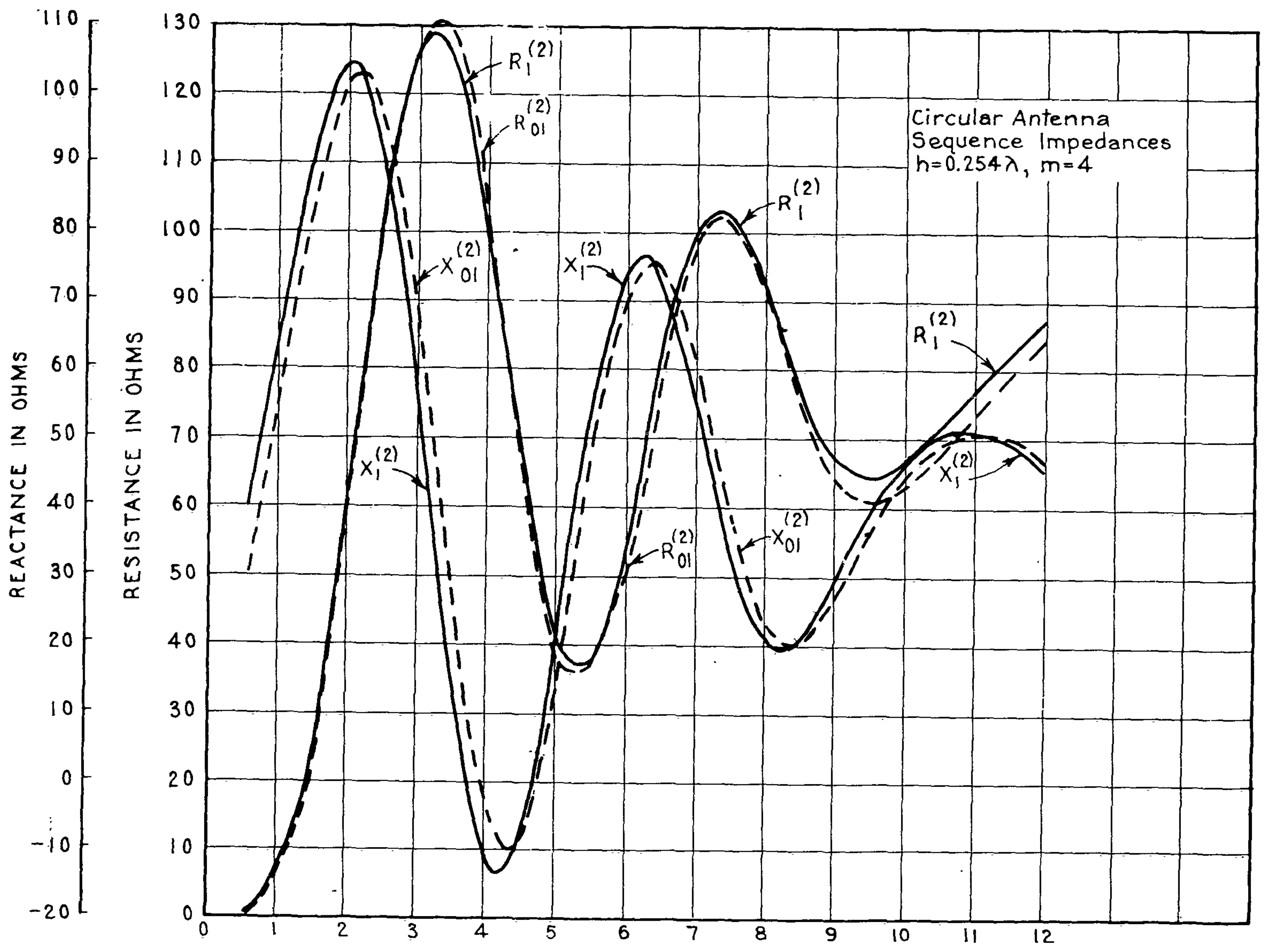

Fig. 18. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 4, n = 2

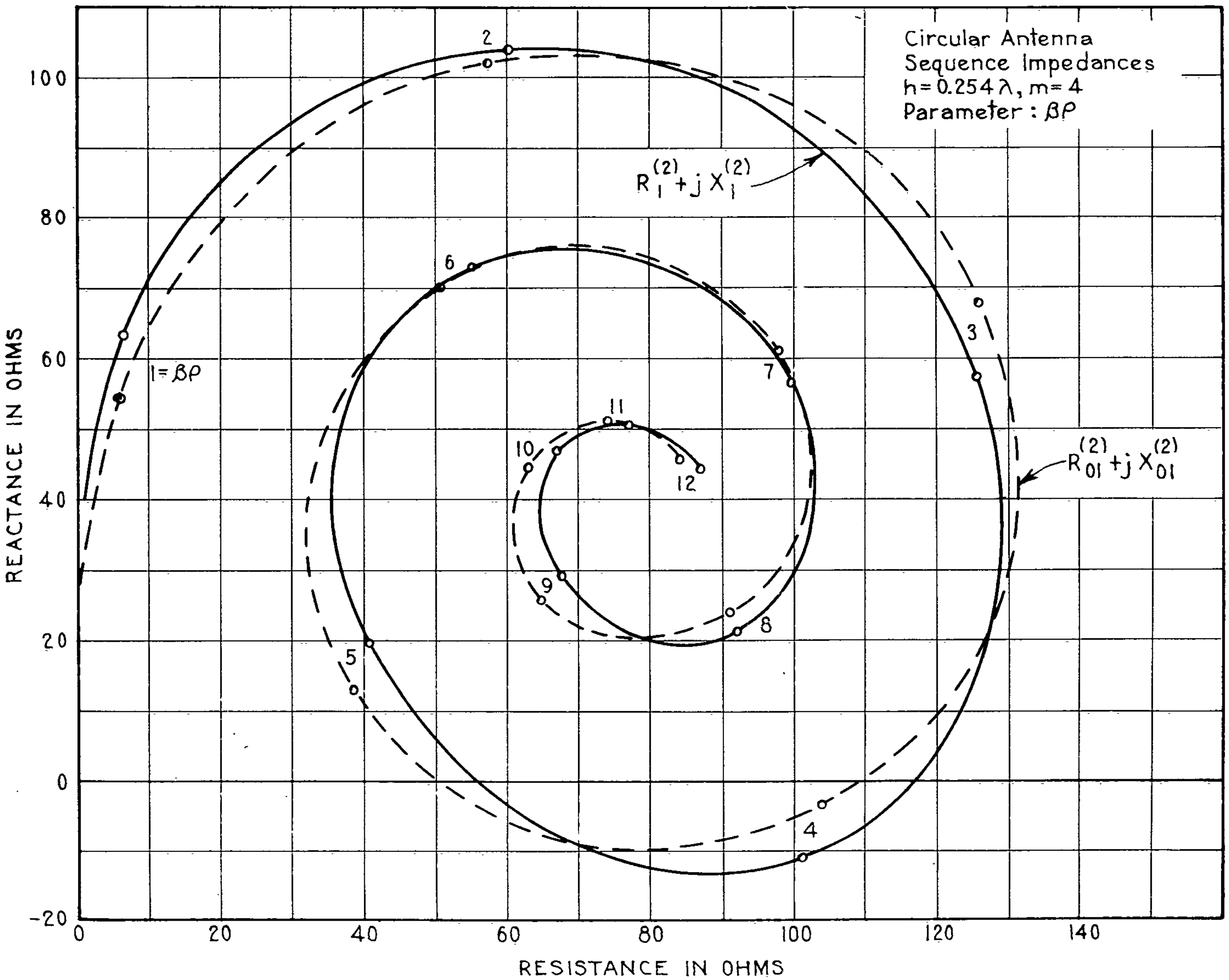

Fig. 19. Sequence reactance versus sequence resistance; parameter is βρ, m = 4, n = 2

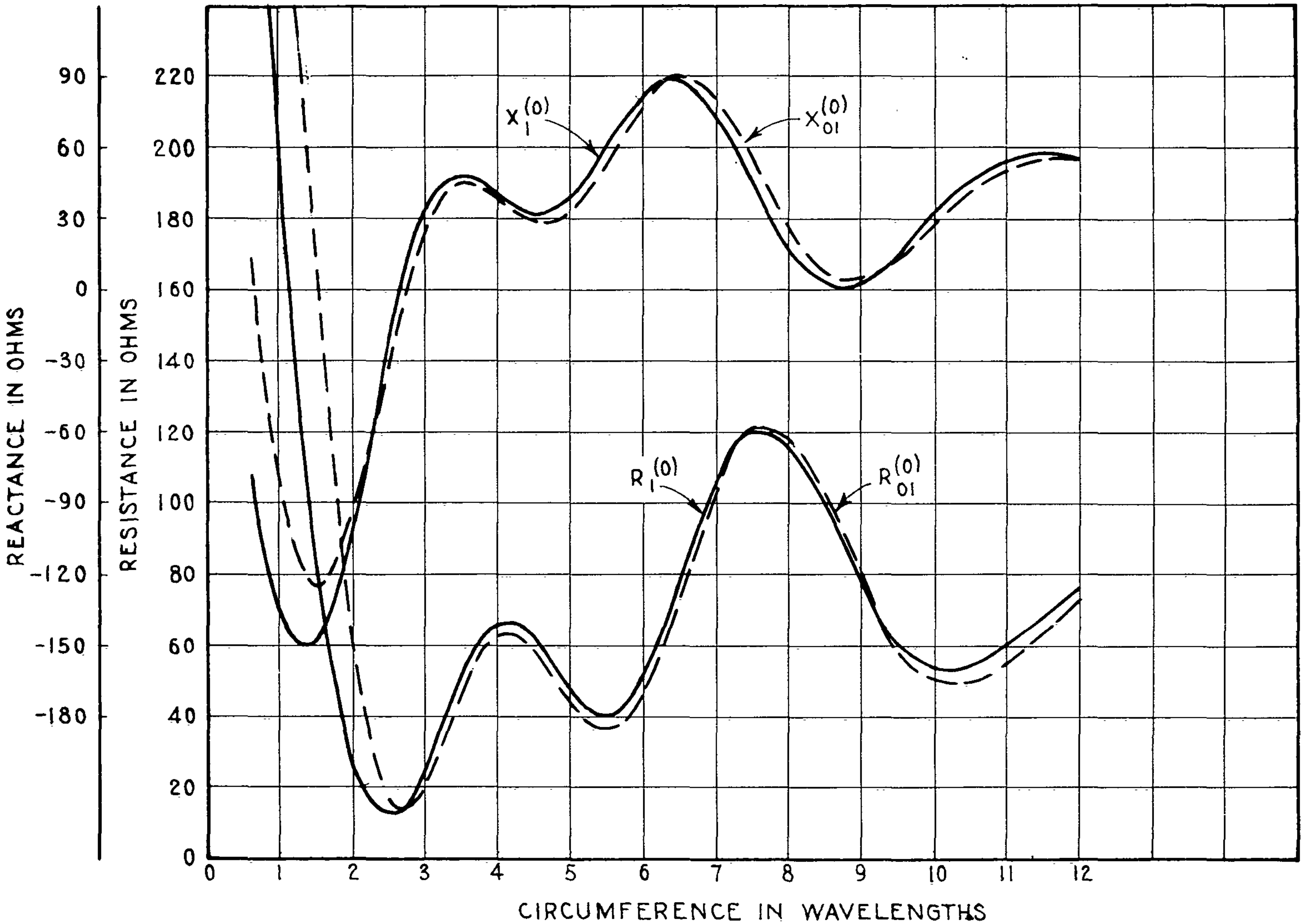

Fig. 20. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 6, n = 0

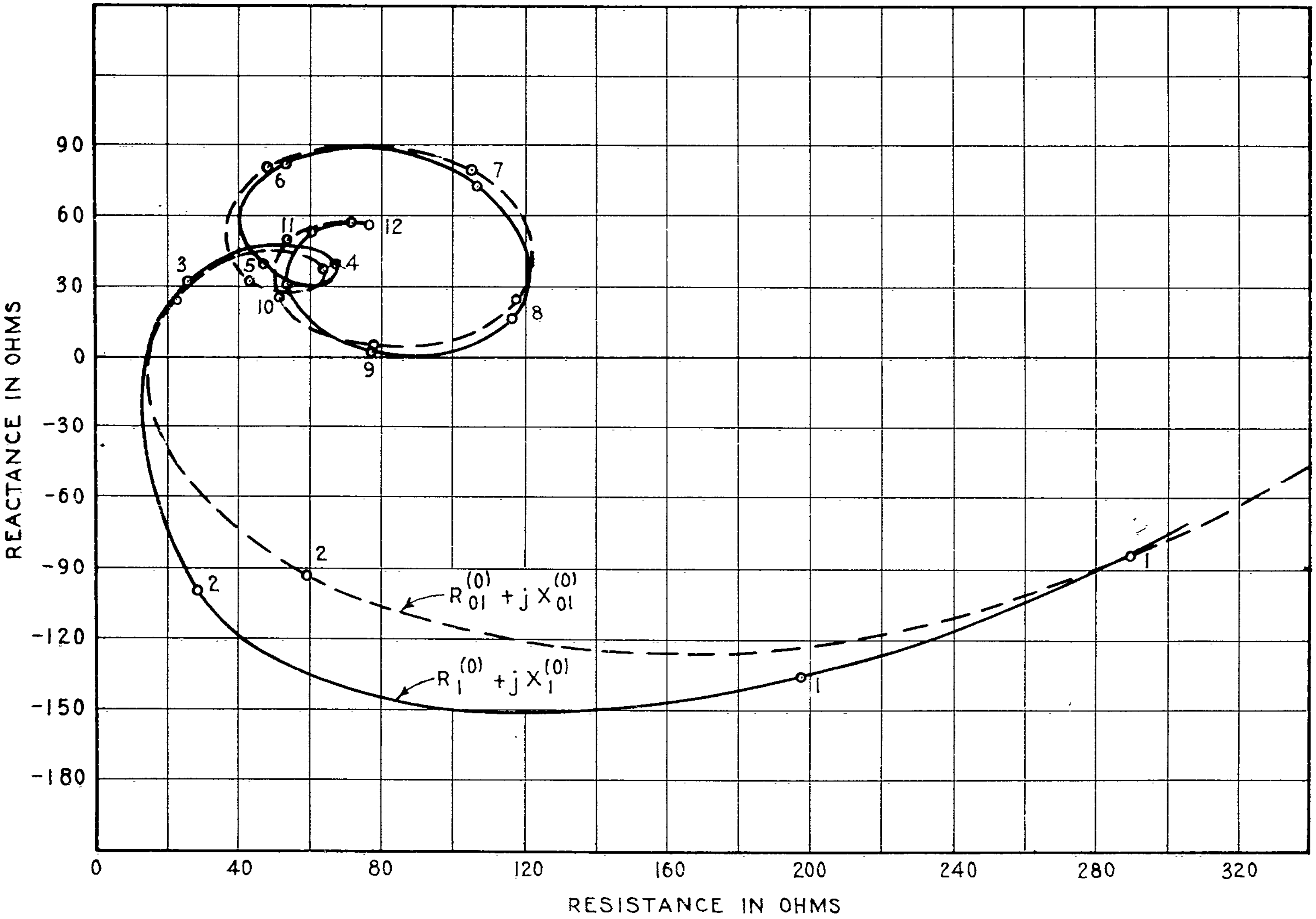

Fig. 21. Sequence reactance versus sequence resistance; parameter is $\beta\rho$; m = 6, n = 0

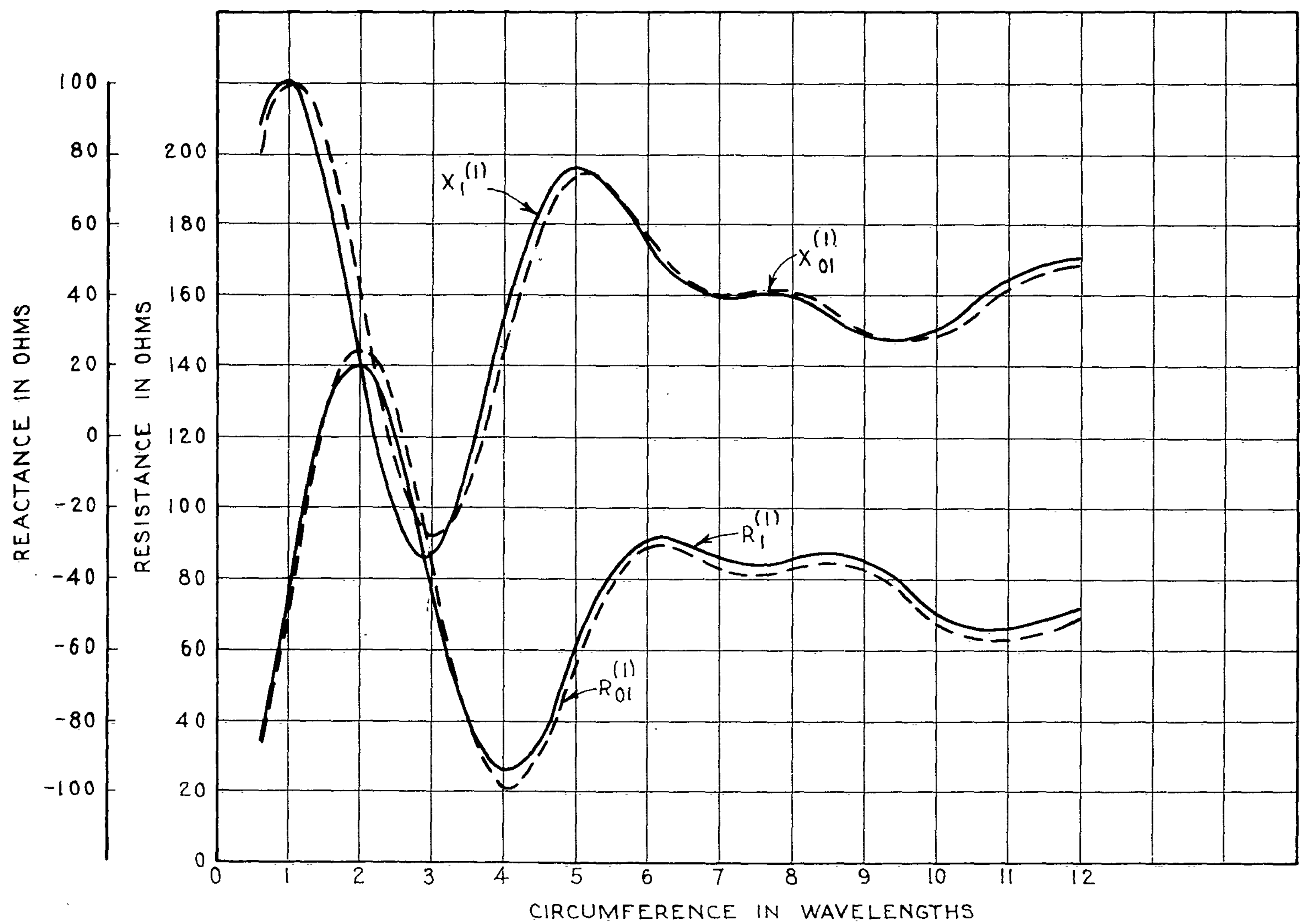

Fig. 22. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 6, n = 1

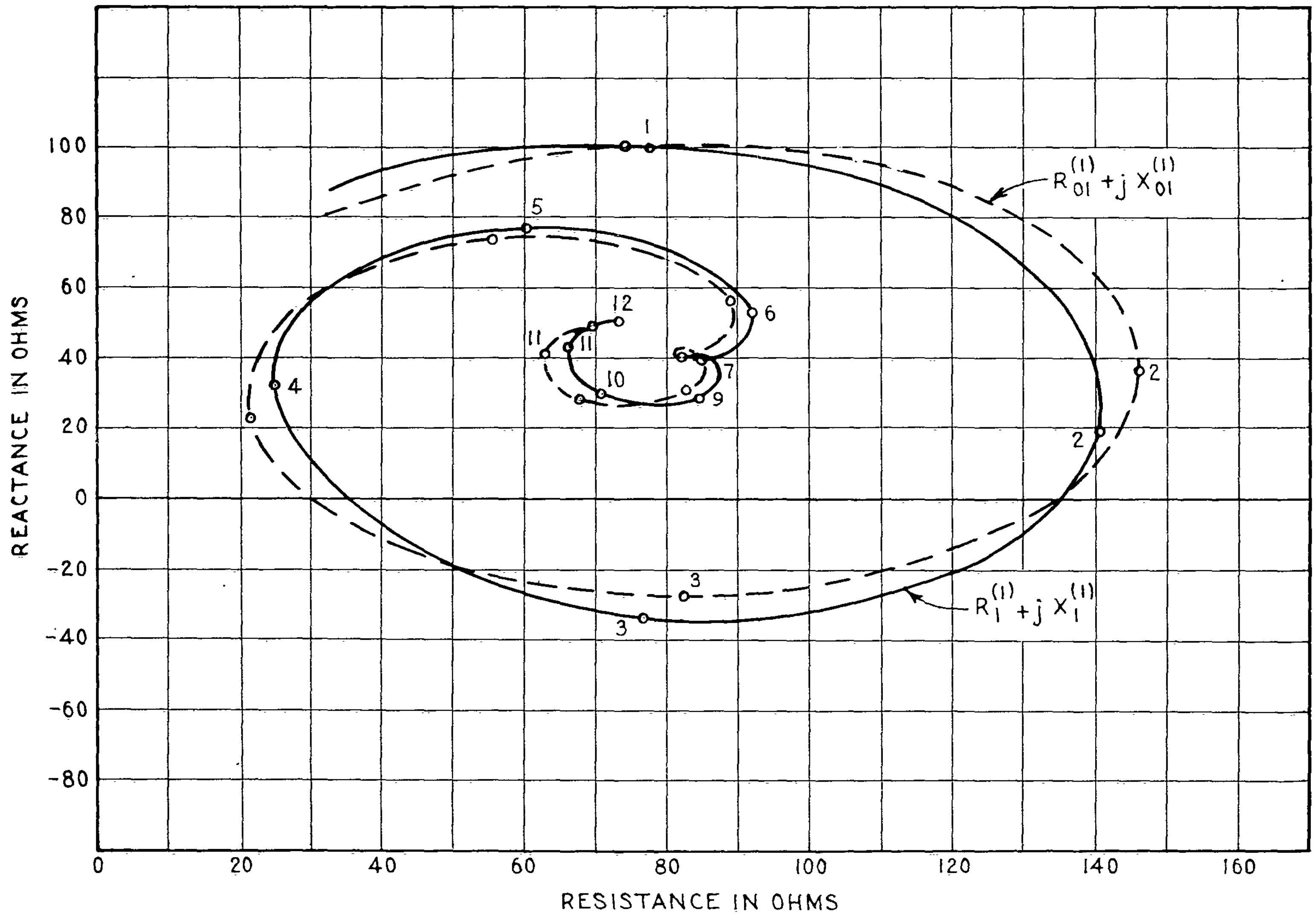

Fig. 23. Sequence reactance versus sequence resistance; parameter is $\beta\rho$, m = 6, n = 1

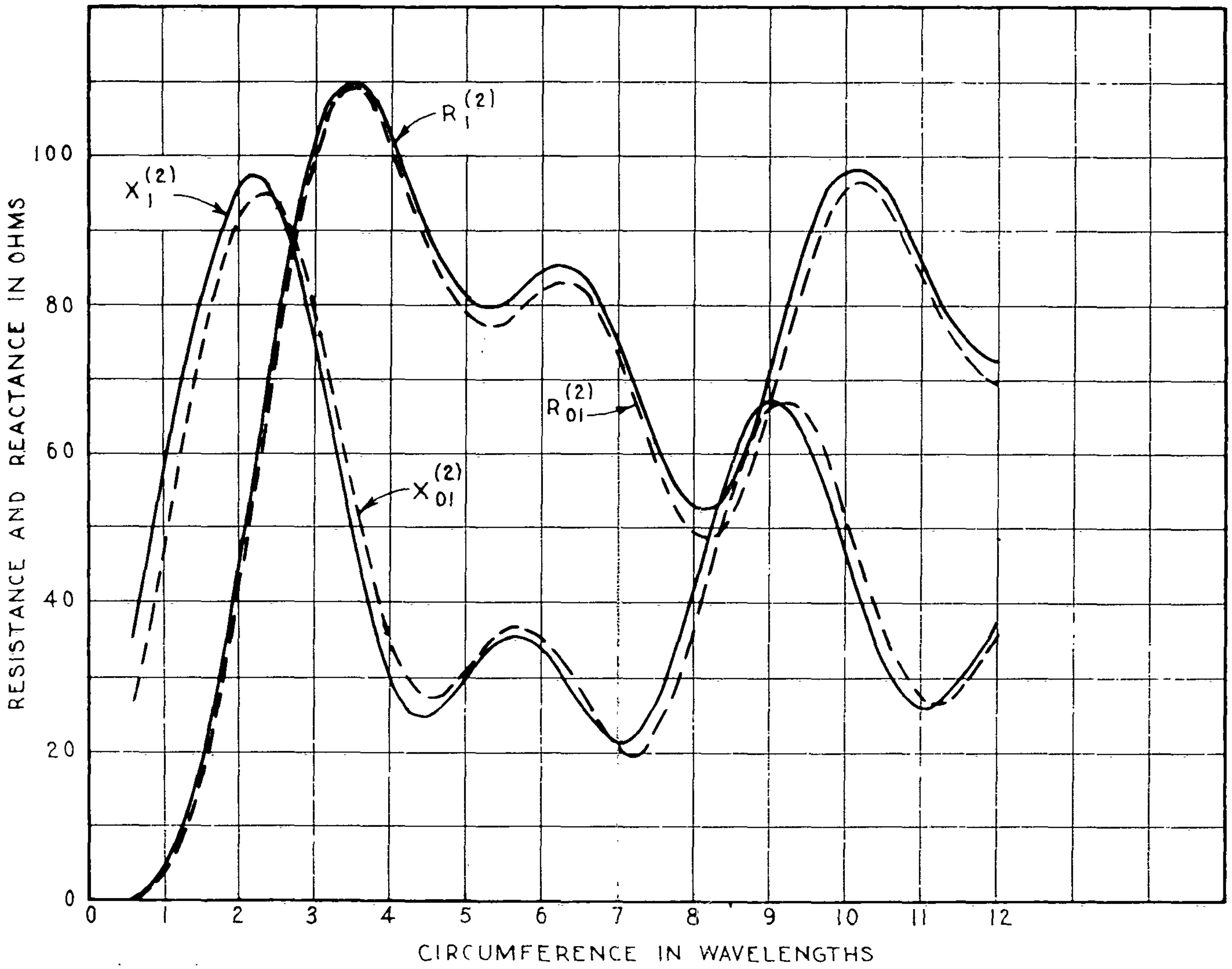

Fig. 24. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 6, n = 2

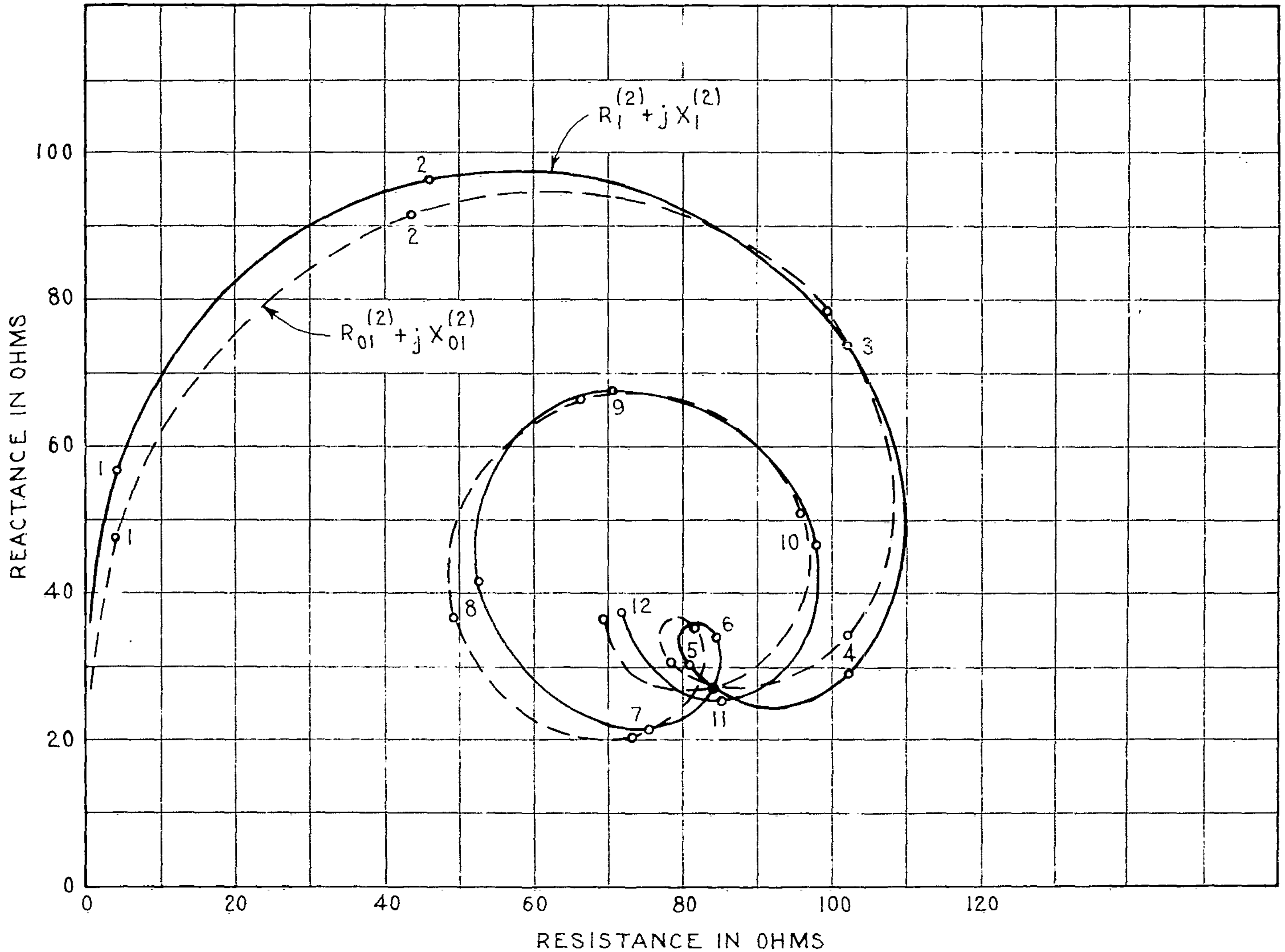

Fig. 25. Sequence reactance versus sequence resistance; parameter is $\beta\rho$, m = 6, n = 2

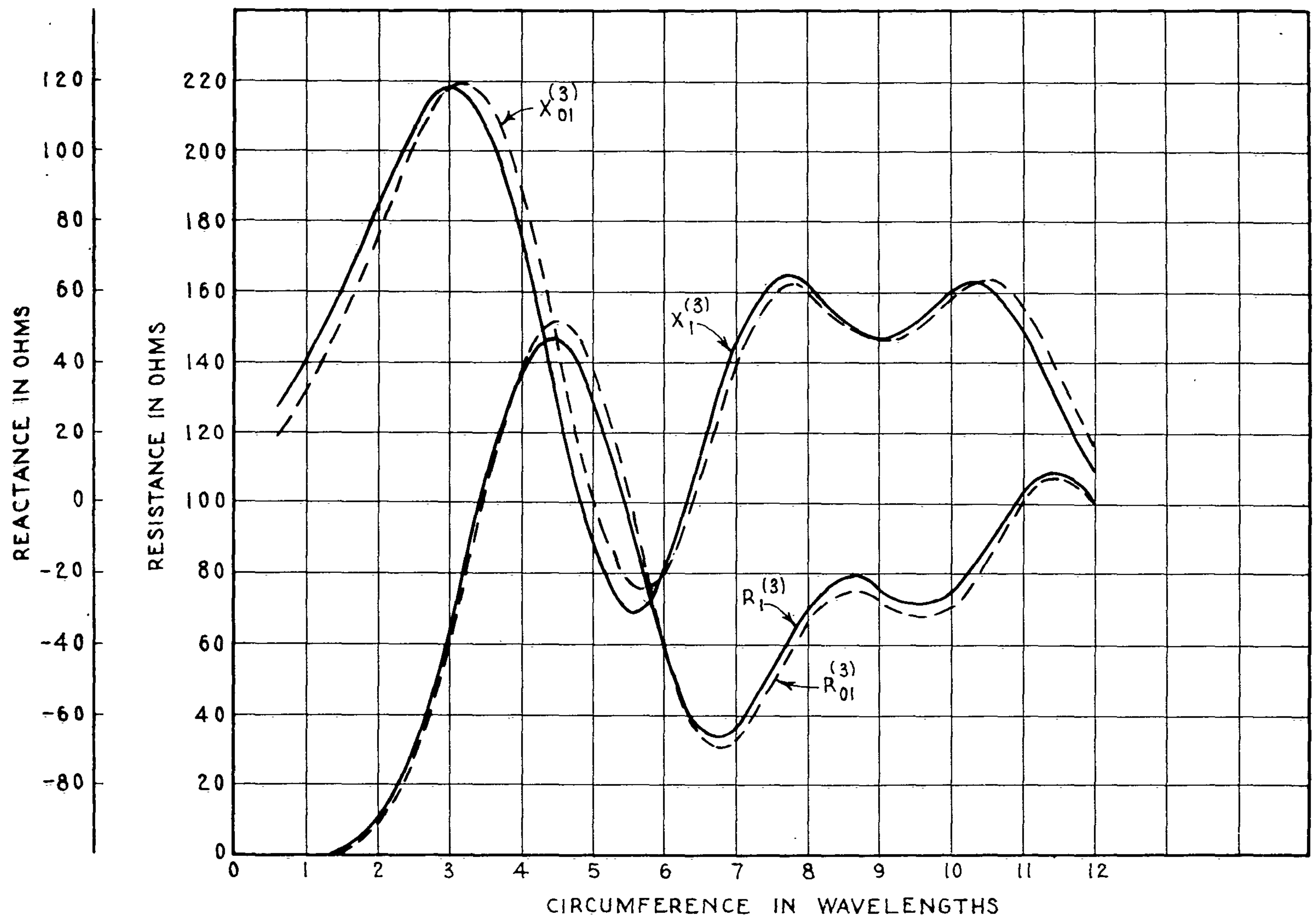

Fig. 26. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 6, n = 3

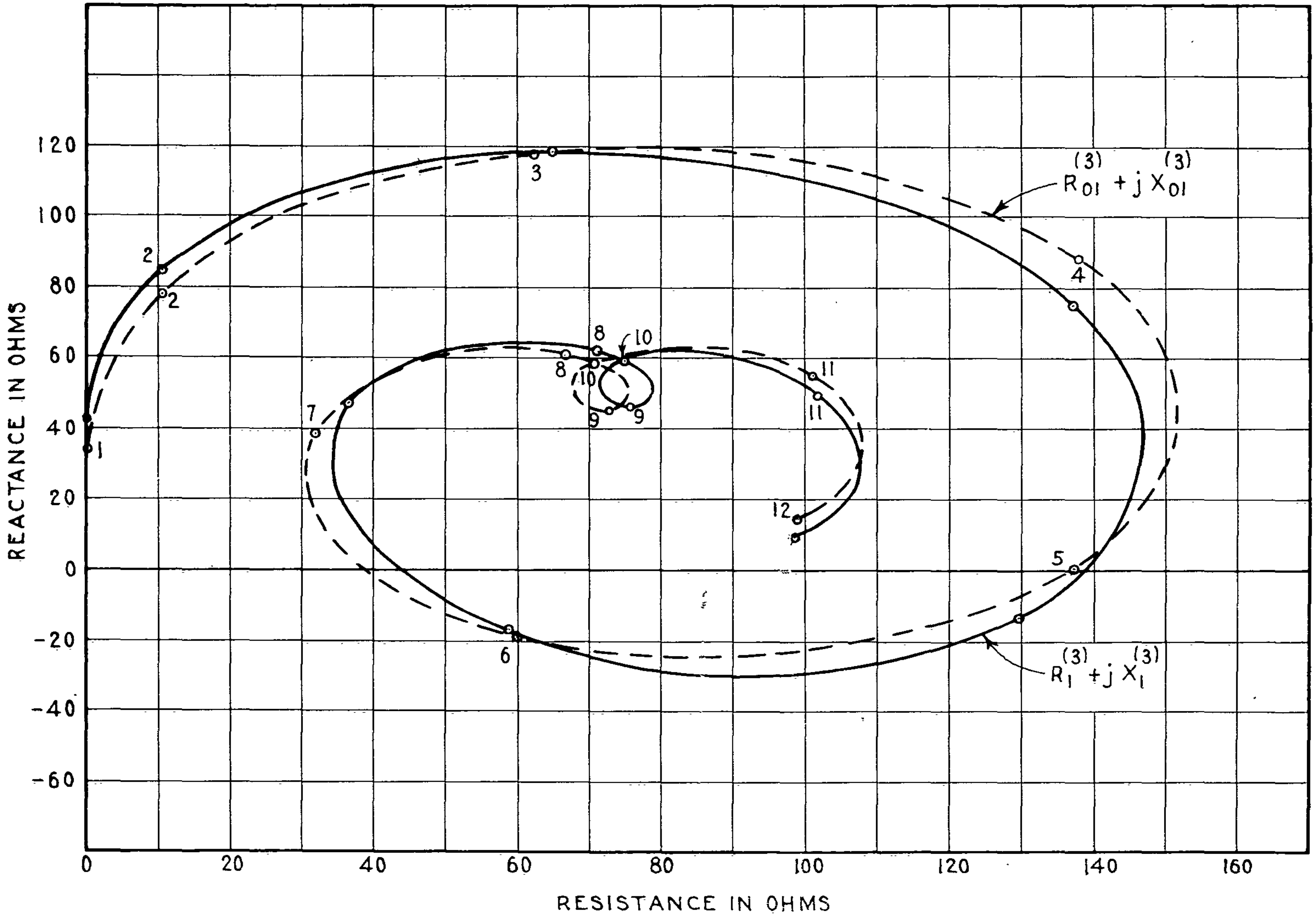

Fig. 27. Sequence reactance versus sequence resistance; parameter is $\beta\rho$, m = 6, n = 3

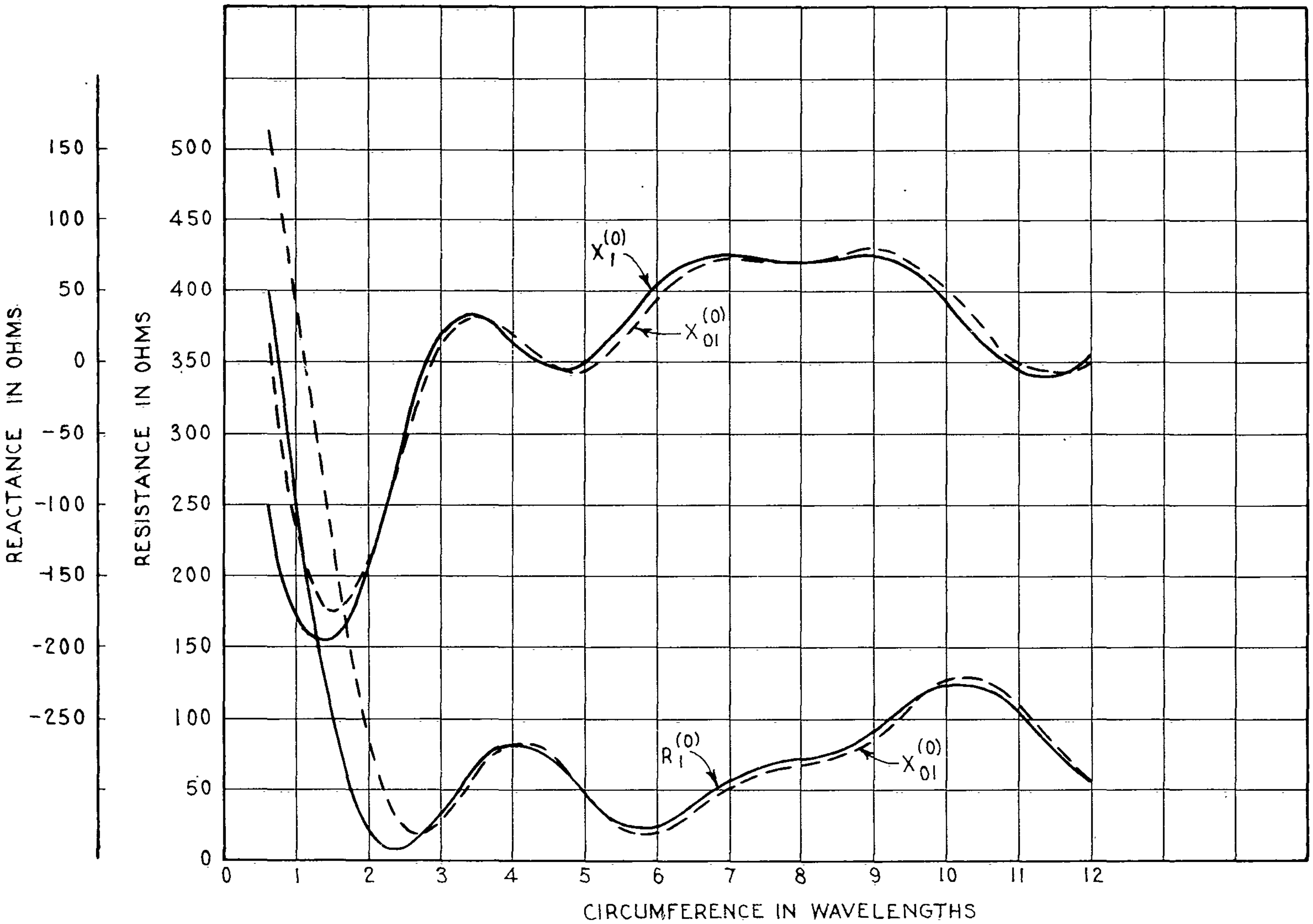

Fig. 28. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; $m = 8$, $n = 0$

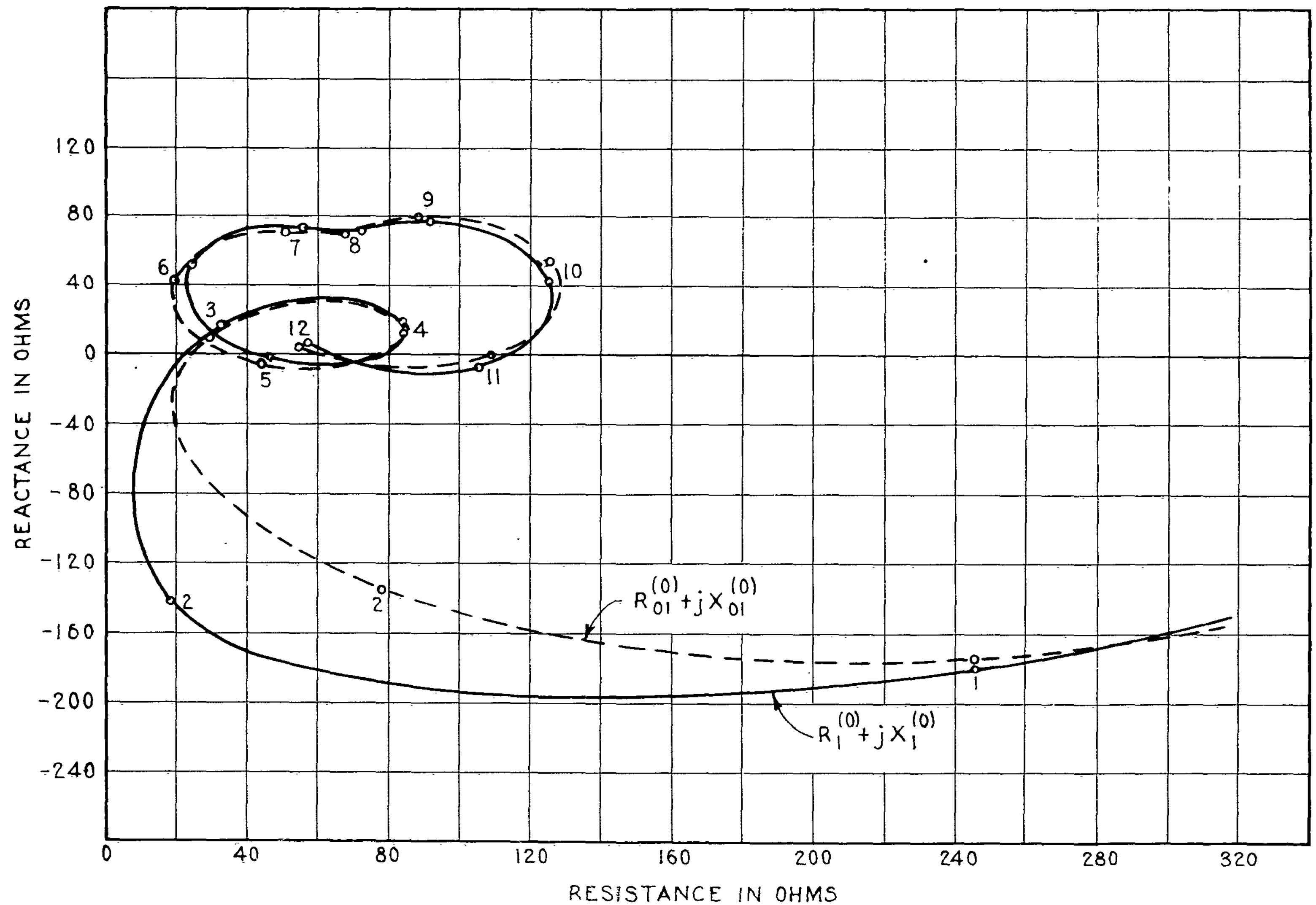

Fig. 29. Sequence reactance versus sequence resistance; parameter is $\beta\rho$, m = 8, n = 0

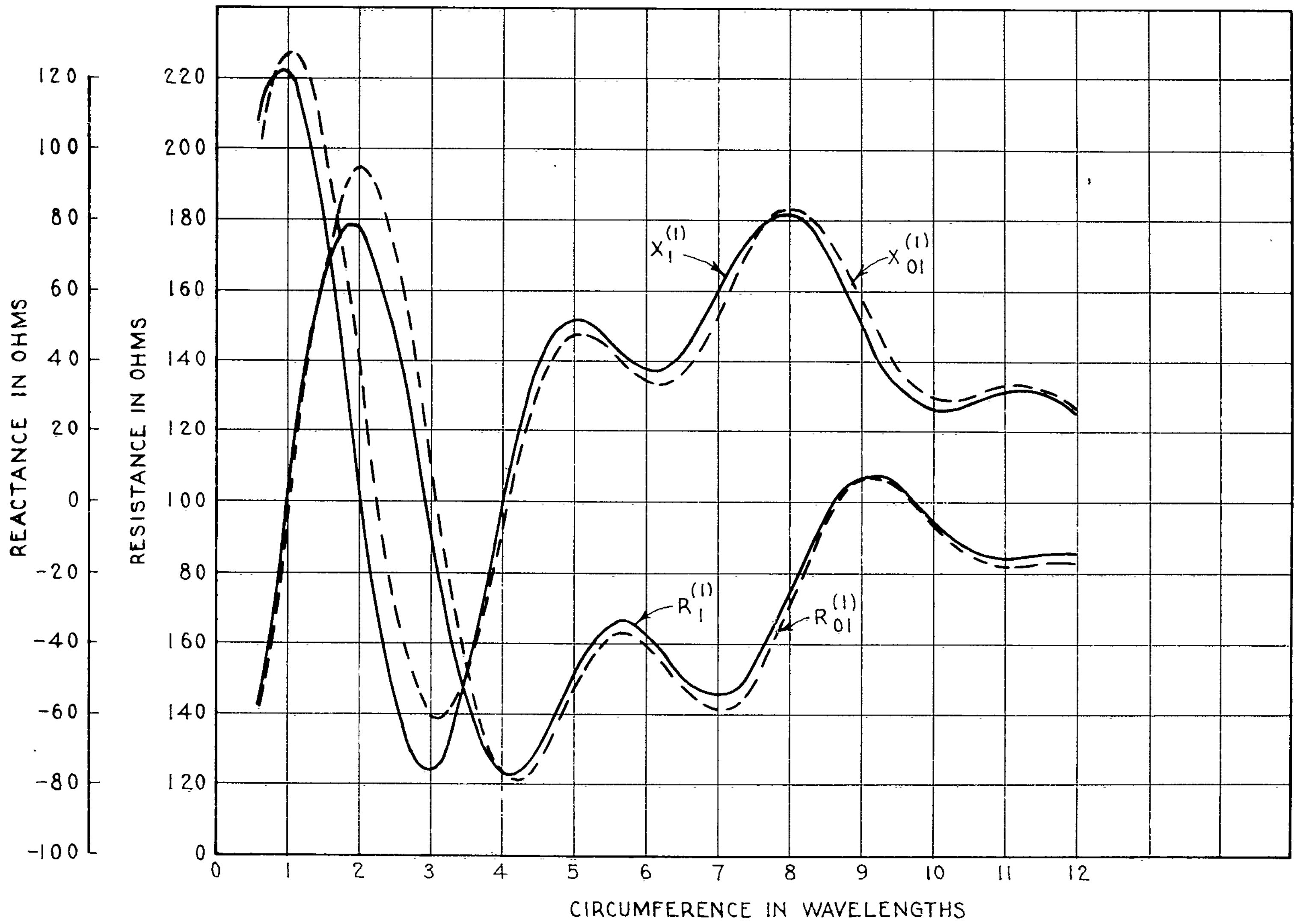

Fig. 30. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 8, n = 1

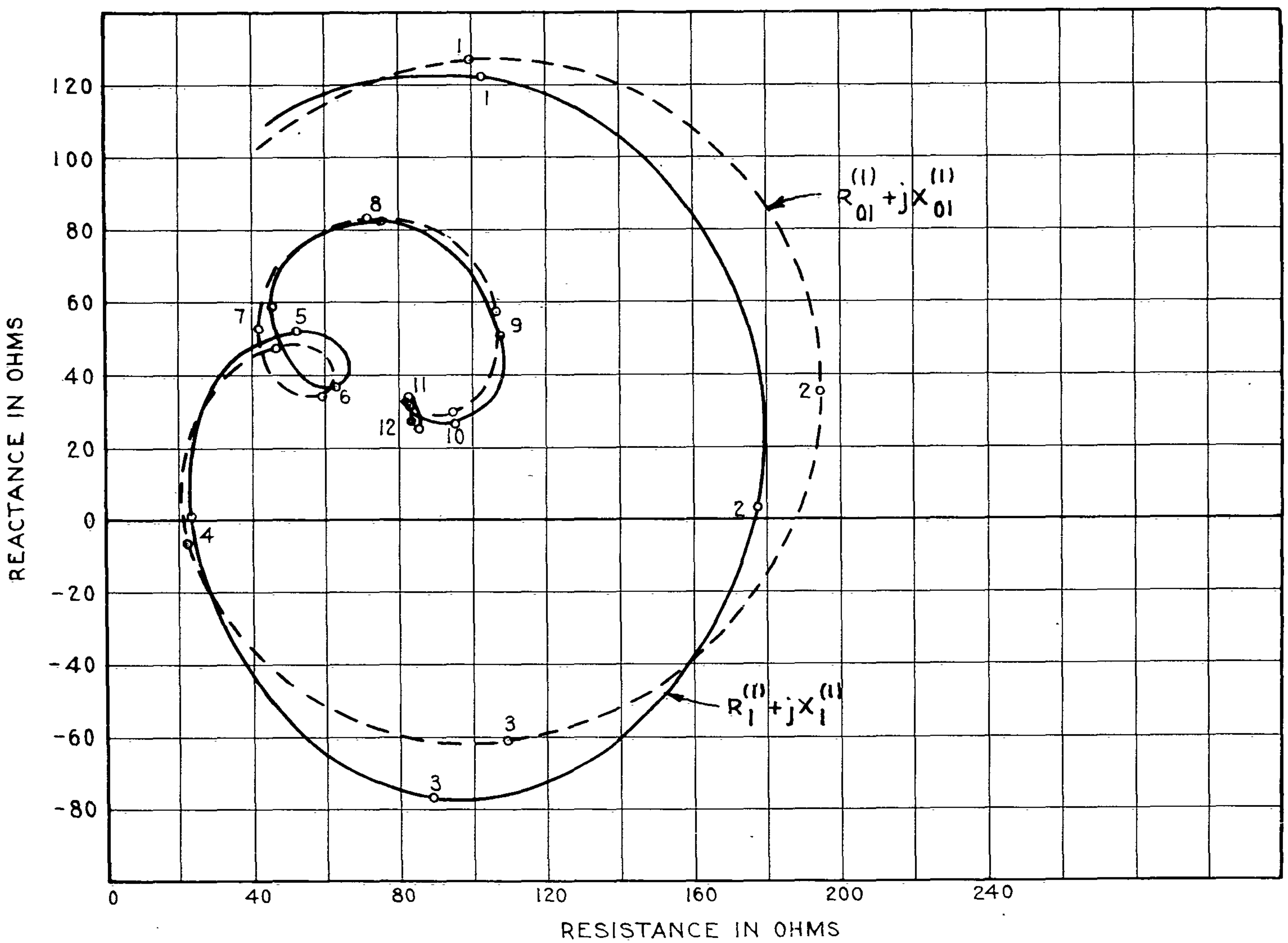

Fig. 31. Sequence reactance versus sequence resistance; parameter is $\beta\rho$, m = 8, n = 1

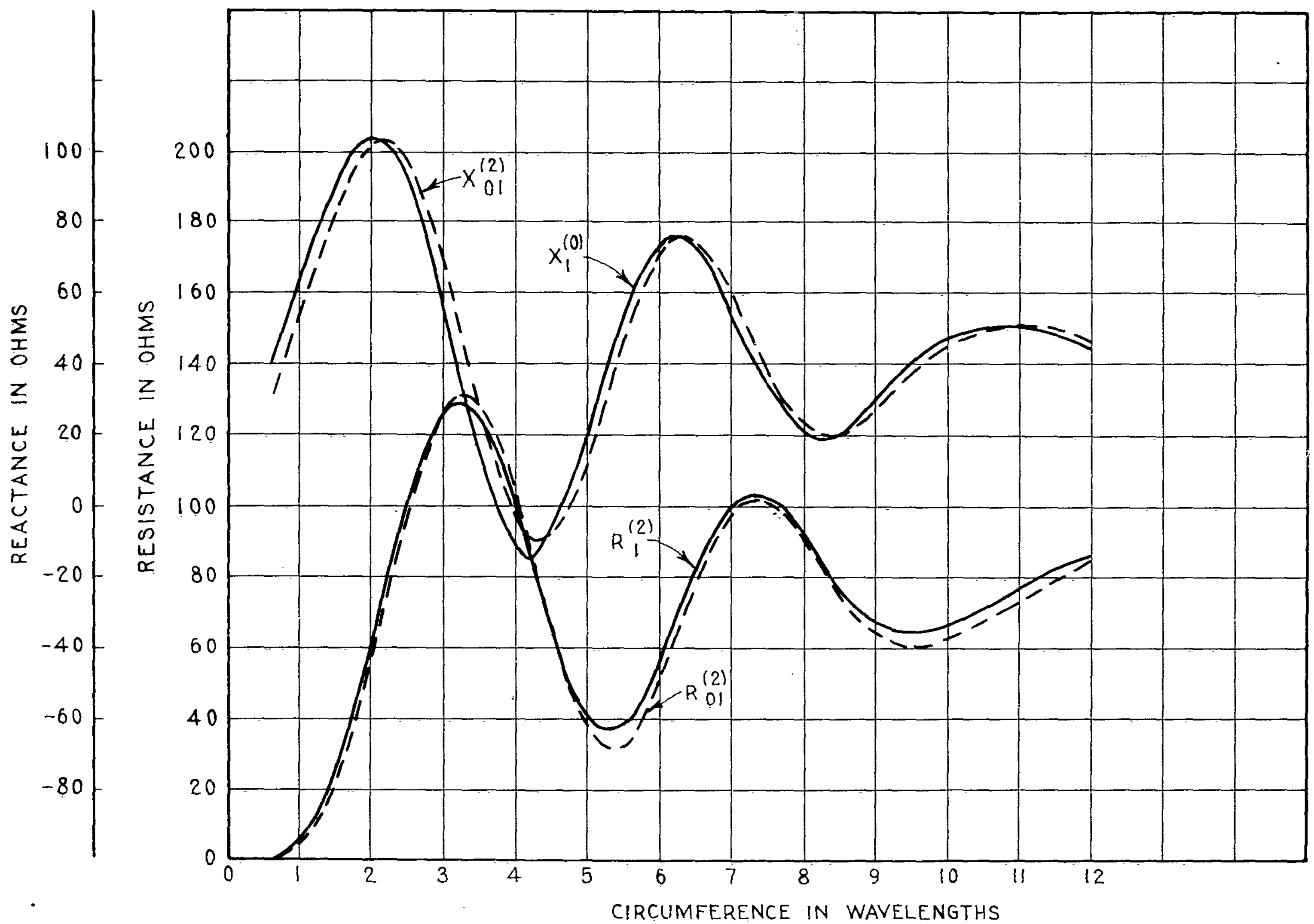

Fig. 32. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 8, n = 2

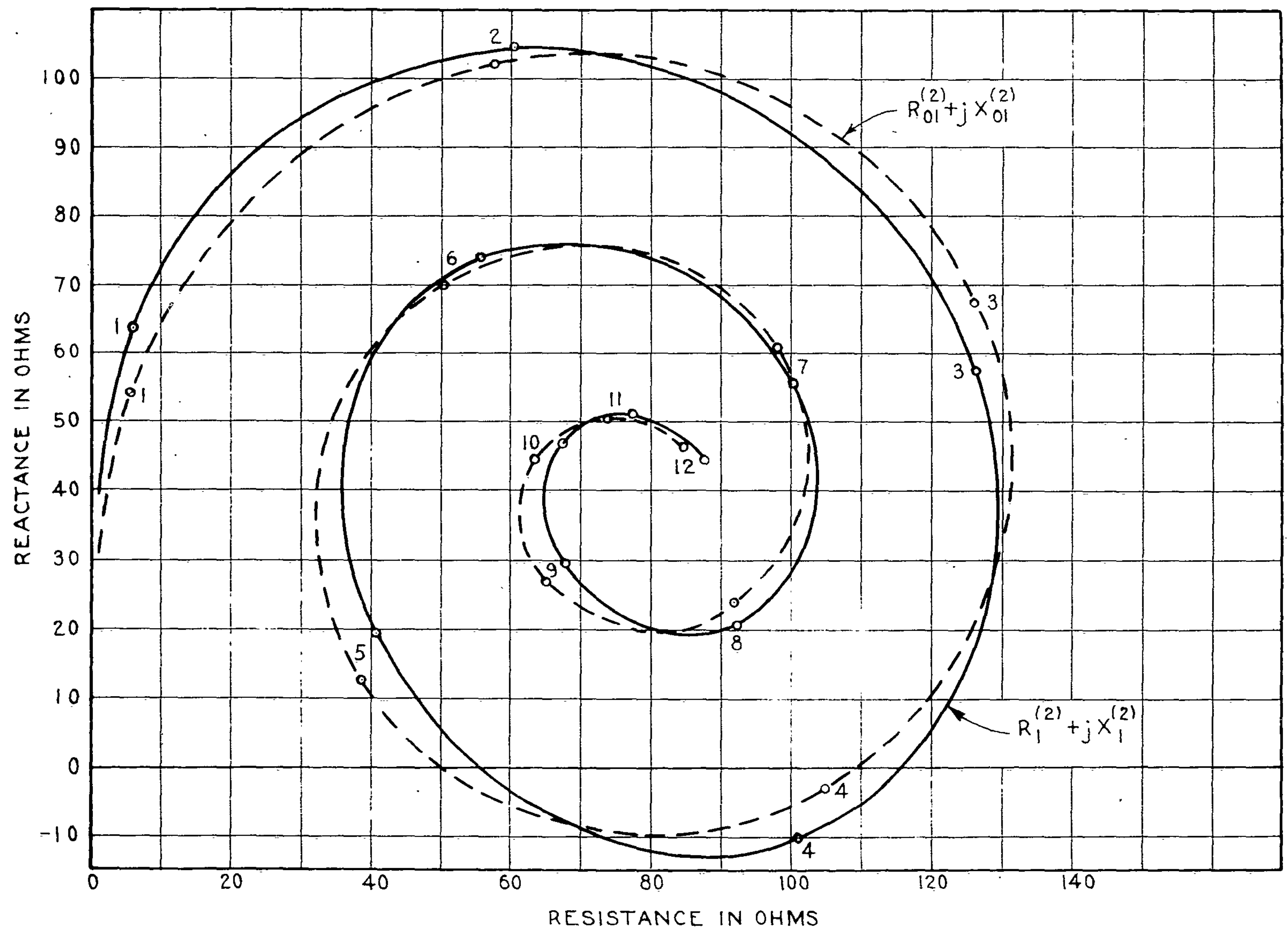

Fig. 33. Sequence reactance versus sequence resistance; parameter is $\beta\rho$, m = 8, n = 2

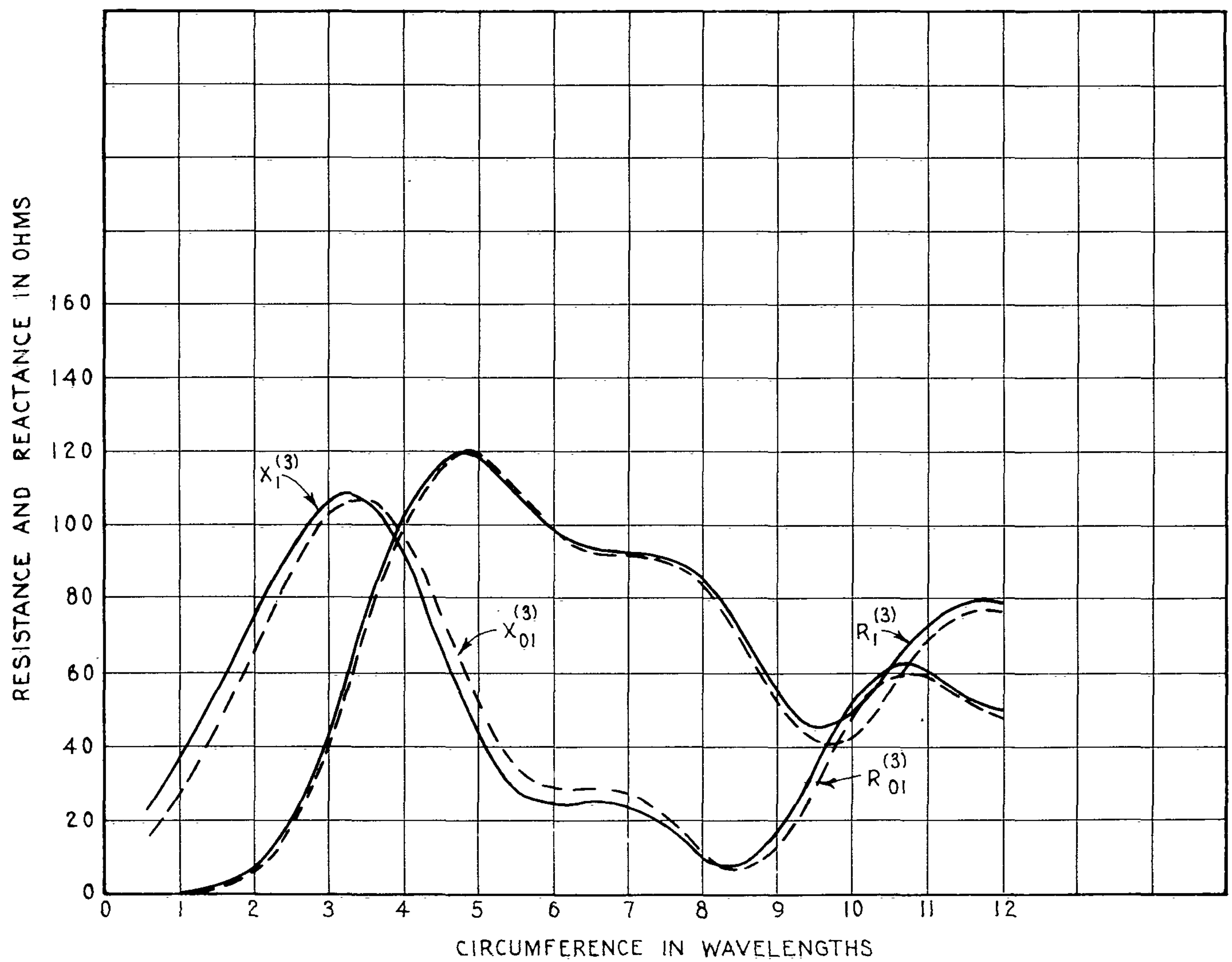

Fig. 34. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 8, n = 3

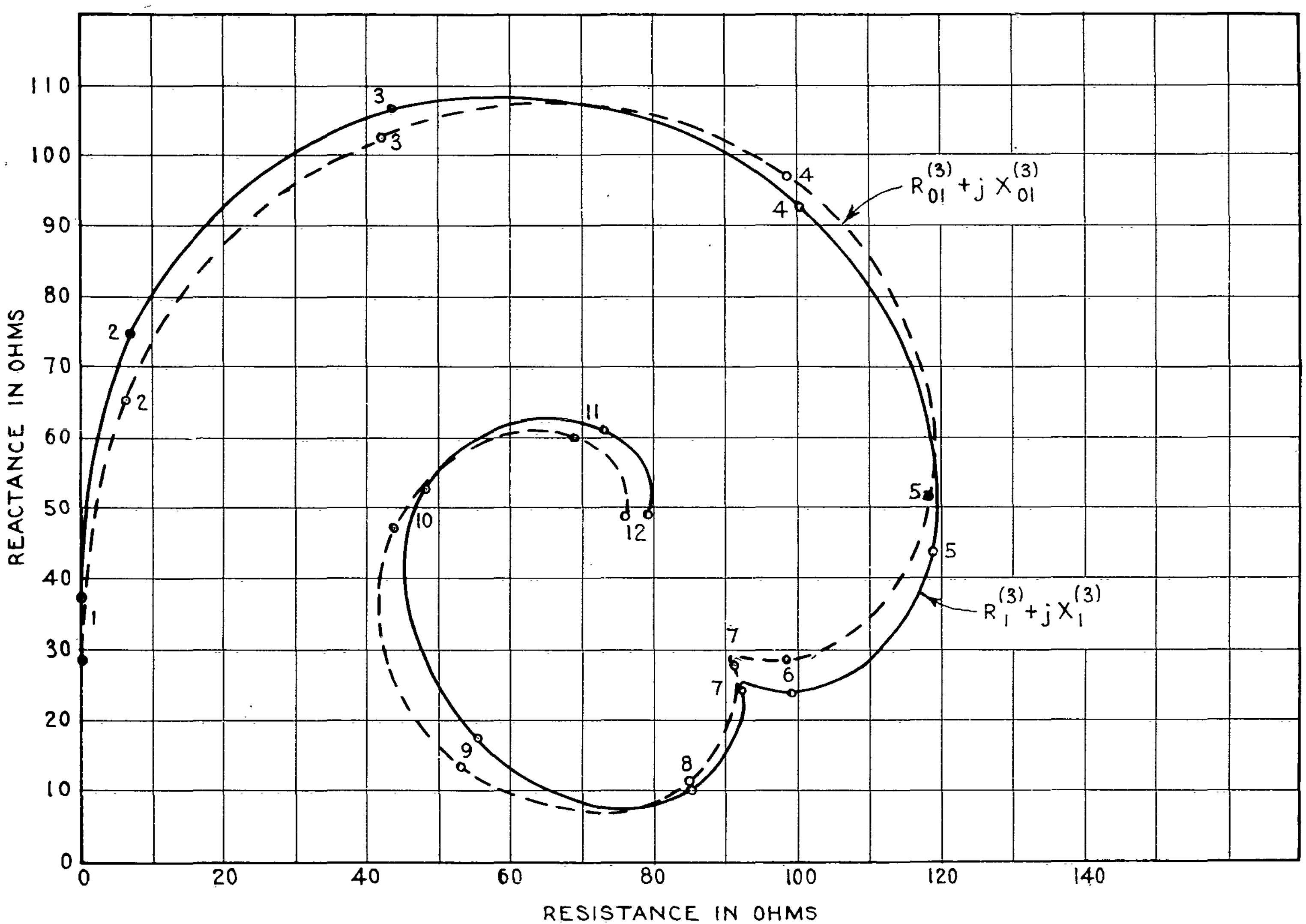

Fig. 35. Sequence reactance versus sequence resistance; parameter is $\beta\rho$, m = 8, n = 3

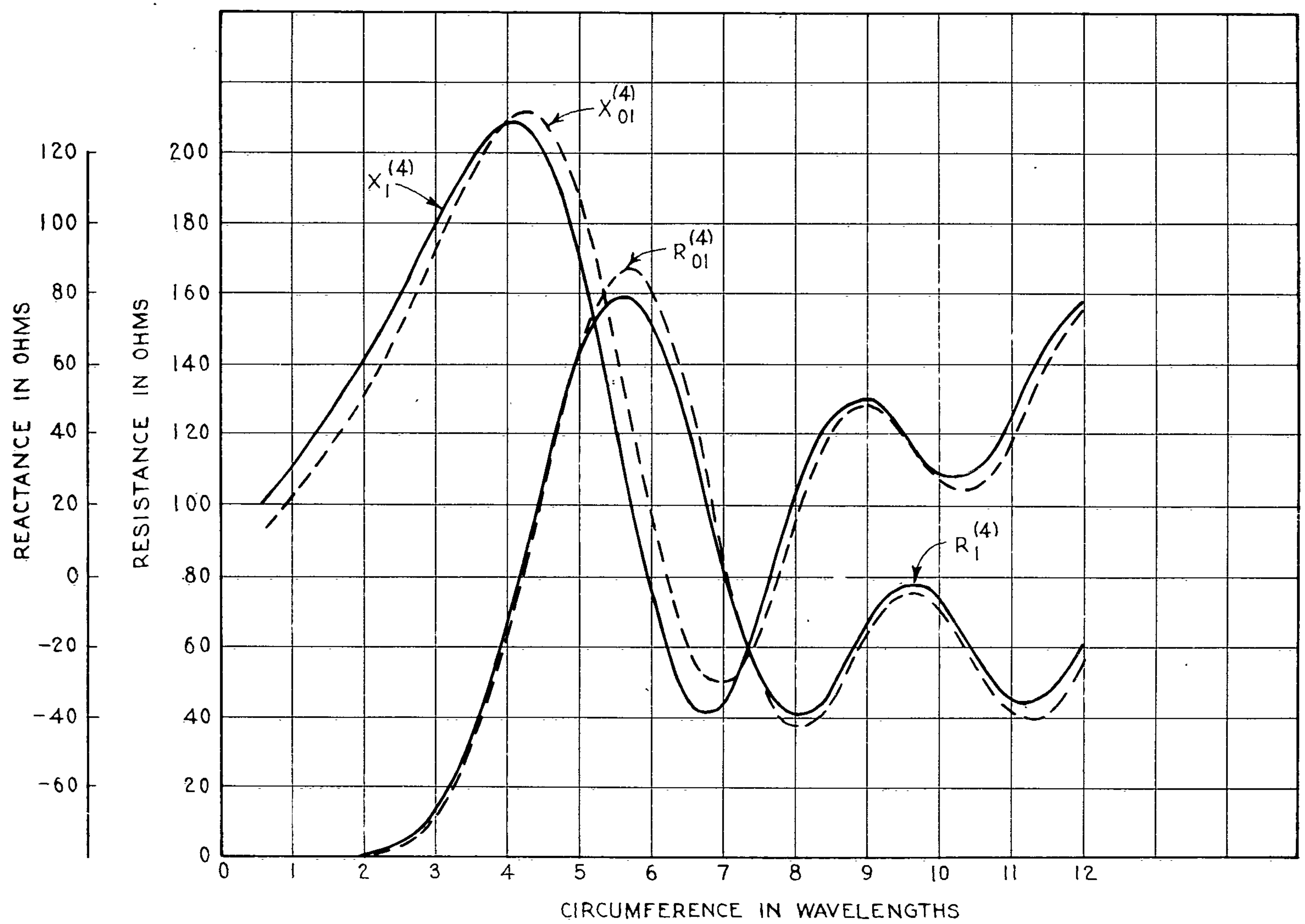

Fig. 36. Comparison of modified zeroth order and first order sequence resistances and sequence reactances as a function of circumference; m = 8, n = 4

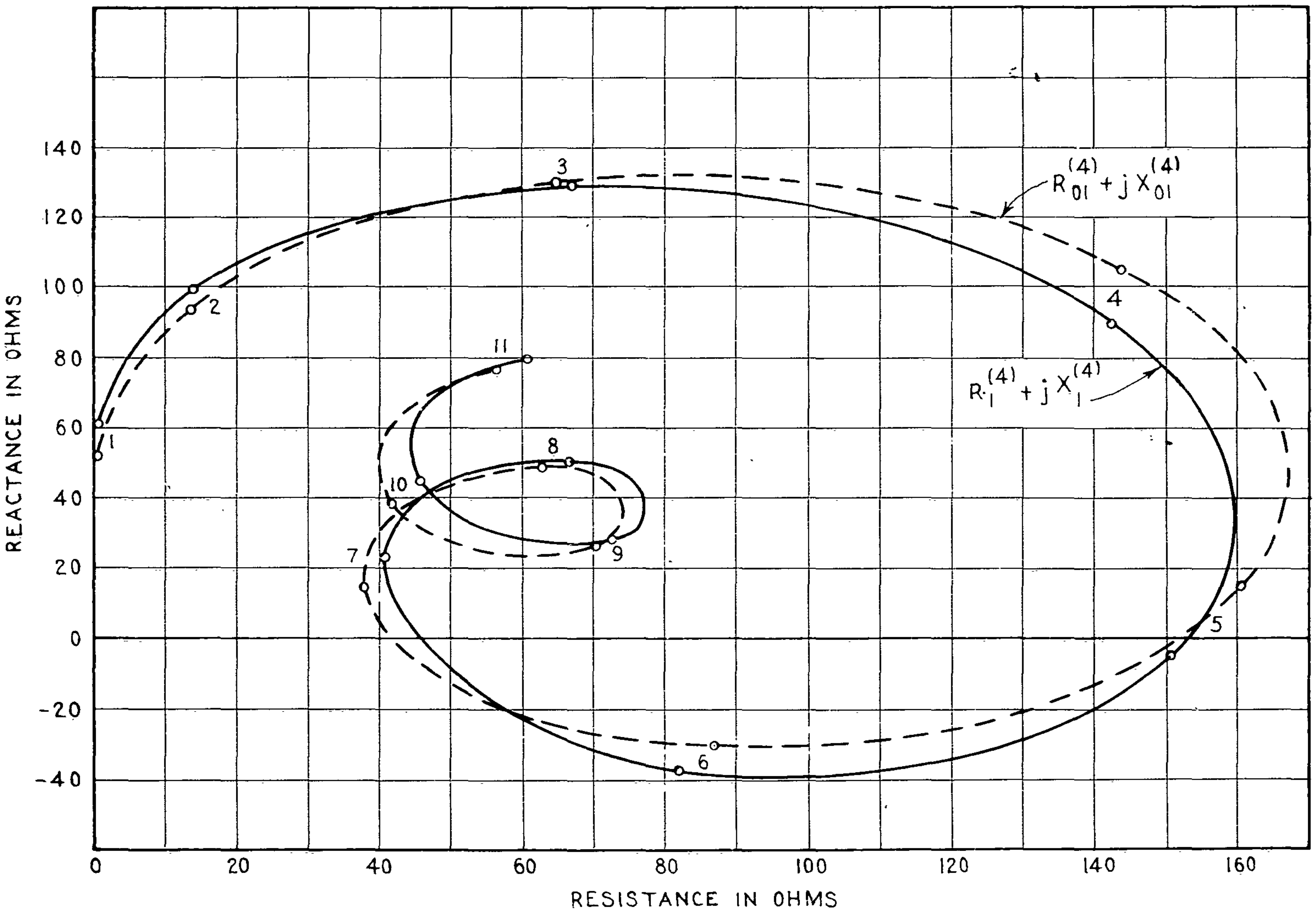

Fig. 37. Sequence reactance versus sequence resistance; parameter is $\beta\rho$, m = 8, n = 4

TABLE I. FIRST ORDER IMPEDANCES OF ISOLATED ANTENNA

h/λ	Z_1
0.252	75.9 + j35.4
0.253	76.8 + j38.0
0.254	77.9 + j41.2
0.255	78.9 + j43.9
0.256	79.9 + j46.5
0.257	80.9 + j49.1

TABLE II. FIRST ORDER SEQUENCE IMPEDANCES FOR CIRCULAR ANTENNA ARRAYS WITH $h = 0.254\lambda$ and $m = 4$

$\frac{2\pi\rho}{\lambda}$	$R_1^{(o)} + jX_1^{(o)}$	$R_1^{(1)} + jX_1^{(1)}$	$R_1^{(2)} + jX_1^{(2)}$
0.6	217.2 - j 37.5	21.8 + j 67.6	0.9 + j 39.5
0.8	183.5 - j 65.2	36.4 + j 78.0	2.6 + j 51.5
1.0	150.6 - j 82.5	52.3 + j 82.7	6.1 + j 63.5
1.2	119.9 - j 90.5	67.8 + j 82.0	11.8 + j 75.3
1.4	92.2 - j 89.5	81.6 + j 76.7	20.2 + j 86.1
1.6	68.4 - j 79.9	92.3 + j 67.9	31.3 + j 95.1
1.8	48.8 - j 62.8	99.2 + j 57.1	44.9 + j101.3
2.0	33.4 - j 40.5	102.2 + j 45.9	60.4 + j104.0
2.2	22.4 - j 16.2	101.6 + j 35.5	76.7 + j102.6
2.4	16.3 + j 7.4	97.8 + j 27.3	92.7 + j 96.8
2.6	15.2 + j 28.6	91.9 + j 21.9	107.0 + j 86.8
2.8	18.7 + j 46.5	84.7 + j 19.7	118.4 + j 73.3
3.0	26.0 + j 60.5	77.4 + j 20.7	126.0 + j 57.3
3.2	35.8 + j 70.4	70.9 + j 24.4	129.2 + j 40.2
3.4	47.0 + j 76.2	66.0 + j 29.9	127.8 + j 23.5
3.6	58.3 + j 78.3	63.2 + j 36.3	122.2 + j 8.7
3.8	68.8 + j 77.2	62.7 + j 42.7	112.9 - j 2.9
4.0	77.8 + j 73.9	64.4 + j 48.2	101.0 - j 10.4
4.2	85.0 + j 68.9	67.9 + j 52.1	87.4 - j 13.0
4.4	90.4 + j 63.1	72.5 + j 54.0	73.4 - j 10.7
4.6	94.0 + j 57.2	77.5 + j 53.8	60.2 - j 3.8
4.8	96.1 + j 51.4	82.2 + j 51.7	49.1 + j 6.7
5.0	97.1 + j 46.0	85.8 + j 48.2	40.9 + j 19.5
5.5	96.0 + j 34.9	88.0 + j 37.1	37.4 + j 52.7
6.0	91.5 + j 26.3	81.4 + j 30.8	55.6 + j 73.6
6.5	83.1 + j 21.0	72.7 + j 33.8	81.8 + j 73.3
7.0	71.3 + j 22.1	69.4 + j 42.2	100.2 + j 55.8
7.5	60.3 + j 31.8	73.6 + j 48.3	102.7 + j 34.2
8.0	56.4 + j 47.1	80.8 + j 47.2	92.2 + j 20.9
8.5	63.7 + j 60.3	84.5 + j 40.7	77.7 + j 20.6
9.0	79.3 + j 63.3	82.0 + j 35.0	67.5 + j 29.2
9.5	94.1 + j 53.5	76.0 + j 34.9	64.5 + j 39.5
10.0	99.2 + j 36.2	72.1 + j 39.9	67.0 + j 46.9
10.5	91.8 + j 22.0	73.5 + j 45.2	71.8 + j 50.3
11.0	76.9 + j 19.3	78.5 + j 46.2	77.0 + j 51.0
11.5	63.7 + j 28.9	82.4 + j 42.3	82.4 + j 49.2
12.0	59.8 + j 43.6	81.9 + j 37.5	87.1 + j 44.4

TABLE III. FIRST ORDER SEQUENCE IMPEDANCES FOR CIRCULAR ANTENNA ARRAYS WITH h = 0.254λ and m = 6

$\frac{2\pi\rho}{\lambda}$	$R_1^{(o)} + jX_1^{(o)}$	$R_1^{(1)} + jX_1^{(1)}$	$R_1^{(2)} + jX_1^{(2)}$	$R_1^{(3)} + jX_1^{(3)}$
0.6	306.2 - j 70.0	32.7 + j 87.1	0.6 + j 35.3	0.0 + j 26.8
0.8	250.5 - j110.4	54.4 + j 98.2	2.0 + j 45.9	0.1 + j 34.5
1.0	197.9 - j135.4	77.7 + j100.5	4.6 + j 56.6	0.2 + j 42.2
1.2	149.9 - j147.9	100.0 + j 94.4	8.8 + j 67.0	0.7 + j 50.1
1.4	107.8 - j150.1	118.8 + j 81.2	15.1 + j 76.8	1.6 + j 58.4
1.6	72.9 - j142.7	132.4 + j 62.8	23.6 + j 85.3	3.3 + j 67.2
1.8	46.5 - j125.7	139.6 + j 41.4	33.9 + j 91.9	6.1 + j 76.3
2.0	28.9 - j 99.8	140.3 + j 19.5	45.8 + j 96.1	10.5 + j 85.7
2.2	18.5 - j 68.2	134.9 - j 0.7	58.6 + j 97.3	16.9 + j 94.8
2.4	13.5 - j 36.1	124.4 - j 17.2	71.4 + j 95.5	25.4 + j103.3
2.6	13.4 - j 7.5	110.3 - j 28.8	83.5 + j 90.6	36.3 + j110.5
2.8	18.0 + j 15.4	93.9 - j 34.3	93.8 + j 83.2	49.4 + j115.6
3.0	26.3 + j 32.1	76.9 - j 33.5	101.9 + j 73.9	64.3 + j118.0
3.2	36.6 + j 42.6	60.5 - j 26.6	107.2 + j 63.5	80.4 + j116.9
3.4	47.2 + j 47.4	46.2 - j 14.8	109.6 + j 53.1	96.9 + j112.0
3.6	56.5 + j 47.6	35.0 + j 0.2	109.3 + j 43.4	112.6 + j103.1
3.8	63.3 + j 44.6	27.8 + j 16.6	106.8 + j 35.4	126.4 + j 90.4
4.0	66.7 + j 40.0	25.0 + j 32.8	102.5 + j 29.4	137.2 + j 74.6
4.2	66.9 + j 35.4	26.6 + j 47.5	97.4 + j 25.8	144.3 + j 56.5
4.4	64.0 + j 32.3	31.9 + j 59.8	91.9 + j 24.6	147.1 + j 37.4
4.6	59.0 + j 31.7	40.0 + j 68.8	87.0 + j 25.2	145.4 + j 18.5
4.8	53.0 + j 33.9	49.9 + j 74.2	83.1 + j 27.3	139.5 + j 1.2
5.0	47.1 + j 39.1	60.3 + j 76.1	80.6 + j 30.0	129.9 - j 13.2
5.2	42.6 + j 46.7	70.1 + j 74.9	79.5 + j 32.8	117.3 - j 23.7
5.4	40.4 + j 55.9	78.6 + j 71.1	79.8 + j 34.9	102.8 - j 29.6
5.6	41.2 + j 65.6	85.1 + j 65.7	81.0 + j 35.9	87.4 - j 30.3
5.8	45.3 + j 74.6	89.4 + j 59.6	82.6 + j 35.6	72.3 - j 26.1
6.0	52.4 + j 82.1	91.5 + j 53.5	84.1 + j 34.0	58.6 - j 17.6
6.2	62.1 + j 86.9	91.8 + j 48.3	84.9 + j 31.4	47.3 - j 5.8
6.4	73.4 + j 88.6	90.8 + j 44.2	84.5 + j 28.1	39.2 + j 7.7
6.6	85.4 + j 86.8	89.0 + j 41.5	82.7 + j 25.0	34.7 + j 21.7
6.8	96.8 + j 81.7	87.0 + j 40.1	79.5 + j 22.6	34.0 + j 34.9
7.0	106.7 + j 73.5	85.3 + j 39.8	75.1 + j 21.4	36.6 + j 46.2
7.5	120.5 + j 45.4	83.7 + j 40.7	61.6 + j 26.5	53.0 + j 62.6
8.0	116.1 + j 17.7	85.7 + j 39.8	52.7 + j 41.8	70.8 + j 61.6
8.5	98.7 + j 2.3	87.3 + j 35.0	56.0 + j 58.9	78.8 + j 52.0
9.0	77.6 + j 2.8	84.8 + j 29.1	70.7 + j 67.3	76.2 + j 46.5
9.5	61.2 + j 14.9	78.1 + j 26.8	88.1 + j 61.9	71.7 + j 50.6
10.0	53.5 + j 31.0	70.7 + j 30.0	97.8 + j 46.6	74.9 + j 59.0
10.5	54.2 + j 44.9	66.2 + j 37.0	95.5 + j 31.7	87.8 + j 61.1
11.0	60.2 + j 53.9	66.0 + j 44.0	85.2 + j 25.8	102.6 + j 49.9
11.5	68.3 + j 57.5	69.0 + j 48.7	75.4 + j 29.8	108.0 + j 28.8
12.0	76.1 + j 56.7	73.2 + j 50.8	72.3 + j 37.7	98.3 + j 9.7

TABLE IV. FIRST ORDER SEQUENCE IMPEDANCES FOR CIRCULAR ANTENNA ARRAYS WITH h = 0.254 λ and m = 8

$\frac{2\pi\rho}{\lambda}$	$R_1^{(o)} + jX_1^{(o)}$	$R_1^{(1)} + jX_1^{(1)}$	$R_1^{(2)} + jX_1^{(2)}$	$R_1^{(3)} + jX_1^{(3)}$	$R_1^{(4)} + jX_1^{(4)}$
0.6	395.2 - j 99.2	43.7 + j109.3	0.9 + j 39.5	0.0 + j 23.9	0.0 + j 20.9
0.8	317.4 - j150.5	72.4 + j121.7	2.6 + j 51.5	0.0 + j 30.7	0.0 + j 26.6
1.0	245.2 - j180.7	103.0 + j122.2	6.1 + j 63.5	0.1 + j 37.4	0.0 + j 32.2
1.2	180.2 - j194.7	131.8 + j111.5	11.8 + j 75.3	0.4 + j 44.3	0.0 + j 37.8
1.4	123.6 - j196.4	155.4 + j 91.4	20.2 + j 86.1	1.0 + j 51.4	0.1 + j 43.5
1.6	76.4 - j188.1	171.6 + j 64.6	31.3 + j 95.1	2.2 + j 58.9	0.2 + j 49.3
1.8	40.5 - j170.7	179.1 + j 34.3	44.9 + j101.3	4.1 + j 66.6	0.4 + j 55.4
2.0	18.2 - j143.6	177.7 + j 3.7	60.4 + j104.0	7.1 + j 74.5	0.9 + j 61.8
2.2	9.0 - j107.9	168.6 - j 24.5	76.7 + j102.6	11.3 + j 82.4	1.7 + j 68.6
2.4	8.1 - j 68.5	153.2 - j 48.1	92.7 + j 96.8	17.1 + j 89.9	3.2 + j 75.9
2.6	12.3 - j 32.1	133.4 - j 65.2	107.0 + j 86.8	24.5 + j 96.7	5.5 + j 83.6
2.8	20.7 - j 3.0	111.5 - j 75.0	118.4 + j 73.3	33.4 + j102.3	8.9 + j 91.6
3.0	32.6 + j 17.4	89.3 - j 76.7	126.0 + j 57.3	43.8 + j106.3	13.8 + j 99.7
3.2	46.6 + j 29.2	68.7 - j 70.4	129.2 + j 40.2	55.3 + j108.3	20.4 + j107.7
3.4	60.4 + j 33.1	50.9 - j 57.1	127.8 + j 23.5	67.3 + j107.9	29.0 + j115.1
3.6	72.0 + j 30.6	37.1 - j 38.9	122.2 + j 8.7	79.3 + j105.1	39.7 + j121.4
3.8	80.0 + j 23.7	27.7 - j 18.6	112.9 - j 2.9	90.6 + j 99.8	52.3 + j126.0
4.0	83.4 + j 14.6	23.3 + j 1.3	101.0 - j 10.4	100.6 + j 92.4	66.7 + j128.3
4.2	82.2 + j 5.4	23.5 + j 19.1	87.4 - j 13.0	108.7 + j 83.3	82.5 + j127.7
4.4	76.8 - j 2.0	27.8 + j 33.5	73.4 - j 10.7	114.6 + j 73.2	98.8 + j123.6
4.6	68.2 - j 6.1	34.9 + j 43.8	60.2 - j 3.8	118.1 + j 62.7	114.9 + j115.9
4.8	57.8 - j 6.1	43.4 + j 49.8	49.1 + j 6.7	119.2 + j 52.7	129.8 + j104.6
5.0	46.9 - j 1.9	51.9 + j 51.9	40.9 + j 19.5	118.3 + j 43.6	142.5 + j 89.9
5.2	36.9 + j 6.1	59.1 + j 50.7	36.5 + j 33.2	115.6 + j 36.0	152.1 + j 72.5
5.4	29.0 + j 16.6	64.0 + j 47.5	36.2 + j 46.5	111.8 + j 30.3	158.0 + j 53.2
5.6	24.1 + j 28.6	66.3 + j 43.3	39.6 + j 58.3	107.4 + j 26.5	159.7 + j 33.2
5.8	22.4 + j 40.7	65.9 + j 39.6	46.4 + j 67.5	102.9 + j 24.4	157.2 + j 13.7
6.0	24.0 + j 51.9	63.2 + j 37.3	55.6 + j 73.6	98.9 + j 23.8	150.6 - j 4.3
6.2	28.3 + j 61.3	58.9 + j 37.2	66.1 + j 76.1	95.8 + j 24.1	140.6 - j 19.4
6.4	34.6 + j 68.4	53.9 + j 39.6	76.8 + j 75.1	93.6 + j 24.8	127.7 - j 30.7
6.6	41.9 + j 72.9	49.4 + j 44.3	86.6 + j 70.9	92.5 + j 25.4	113.0 - j 37.5
6.8	49.3 + j 75.0	46.2 + j 50.9	94.6 + j 64.2	92.1 + j 25.4	97.4 - j 39.3
7.0	55.9 + j 75.3	45.2 + j 58.6	100.2 + j 55.8	92.2 + j 24.4	82.0 - j 36.2
7.5	67.0 + j 72.0	54.1 + j 76.4	102.7 + j 34.2	91.5 + j 18.2	51.1 - j 11.2
8.0	71.6 + j 71.0	75.4 + j 82.2	92.2 + j 20.9	84.8 + j 10.2	40.1 + j 22.2
8.5	77.6 + j 74.7	97.0 + j 71.6	77.7 + j 20.6	70.9 + j 8.2	49.3 + j 45.6
9.0	91.4 + j 76.3	107.6 + j 51.5	67.5 + j 29.2	55.1 + j 17.4	66.7 + j 50.1
9.5	110.6 + j 66.5	104.9 + j 33.7	64.5 + j 39.5	45.9 + j 35.1	77.1 + j 40.0
10.0	124.5 + j 42.8	94.9 + j 26.1	67.0 + j 46.9	48.4 + j 52.7	72.9 + j 28.8
10.5	123.3 + j 13.4	86.3 + j 27.6	71.8 + j 50.3	60.1 + j 62.0	58.3 + j 29.4
11.0	105.6 - j 7.7	83.7 + j 31.2	77.0 + j 51.0	72.9 + j 60.8	45.7 + j 44.4
11.5	79.2 - j 10.0	85.0 + j 30.6	82.4 + j 49.2	79.6 + j 53.9	46.3 + j 65.1
12.0	56.7 + j 6.1	84.5 + j 25.7	87.1 + j 44 4	79.1 + j 49.0	61.7 + j 78.9

CONCLUSIONS

This report reviews the derivation of the integral equation solution for the circular antenna array of parallel dipoles and derives the equations for the sequence impedances. This method, however, yields only an approximate solution for the impedances. It is known that the second order solution agrees more closely with experimental results than the first order solution. The second order equations are quite difficult to evaluate and only a very limited amount of data are available. The first order and second order impedance spirals almost coincide except corresponding impedances are for antennas of different lengths. This report proposes that if the first order impedances are calculated for the height corresponding to the second order impedance that a more accurate set of data may be obtained. The results obtained are listed both graphically and in tables.

It can be seen that there is no great difference between the modified zeroth order solutions and the modified first order solutions obtained for this report. As a result, for many applications the modified zeroth order data would be sufficiently accurate. For more accurate results it is believed that the modified first order impedances could be used. Furthermore, the first order data can be computed much more easily than the second order data.

REFERENCES

1. The Theory of Single-Ring Circular Antenna Arrays, C. E. Hickman, H. P. Neff and J. D. Tillman. Scientific Report No. 4, Contract AF 19(604) - 4967, Electrical Engineering Department, University of Tennessee, July 1961.

2. The Theory of Linear Antennas, R. King, Harvard University Press, Cambridge, Massachusetts, 1956, Chapters 2 and 3.

3. Tables of E(h,z), C(h,z), S(h,z), R. B. Mack and E. W. Mack, Technical Report No. 331, Cruft Laboratory, Harvard University, Cambridge, Massachusetts, November 1960.

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