SP-1181/000/00

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17 April 1963

(SP Series)

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A-1162

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ABSTRACT

This paper develops a method for obtaining confidence bands in polynomial regression when the observations are independently distributed with constant but unknown variance. The bands may be obtained, in principle, over arbitrary sets of the independent variable with exact preassigned confidence coefficients. In general, difficult distribution problems result when specific applications are attempted. The major portion of this paper is concerned with first degree polynomials since some progress has been made here. A table is provided to obtain a constant width confidence band which contains the true but unknown straight regression line for values of the independent variable in some arbitrarily selected interval with an exact preassigned confidence coefficient. The present method is compared with the classical hyperbolic band for the whole regression line.

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[1]The author wishes to express his indebtedness to Mr. Vance A. Griffitts who did all the programming for the table.

1. INTRODUCTION AND SUMMARY

The basic problem considered in this paper is the following. Suppose for every $t \epsilon (-\infty, \infty)$, Y_t is a normal random variable with unknown variance σ^2 and mean value m_t given by a polynomial of known degree $r \geq 1$ and unknown coefficients. Let I be a subset of interest in $(-\infty, \infty)$. Based on mutually independent observations it is desired to construct simultaneous confidence intervals for m_t, $t \epsilon I$, with preassigned probability $1-\alpha$. It should be pointed out that the material discussed here is close to methods called "multiple comparisons" in other contexts.

A well known result occurs when the set I contains only one point, Graybill [1, pp. 121-122]. It must be emphasized that if intervals are computed by that technique for every t, no confidence statement may be made about the resulting band (a hyperbola for $r=1$) containing the unknown regression line, i.e., that method does not provide simultaneous coverage of the ordinates of the regression line. Less known is the work of Working and Hotelling [2] in which a hyperbolic confidence band is obtained for the whole regression line when it is assumed the variance is known. The method is easily extended to the unknown variance case and provides a hyperbolic band valid for the whole regression line, Scheffé [3, pp. 52,53]. Hoel [4] extends the method of Working and Hotelling for the straight line regression in such a way as to make it possible to find an optimum confidence band. The optimum band is defined to be that band of an admissible class of bands such that its expected total area is a minimum. Also, in [4] the case of polynomial regression of degree two or higher is considered and a procedure similar to the first degree case is outlined. However, in these cases the confidence bands possess confidence coefficients $\geq 1 - \alpha$.

The present study was undertaken to extend some of the results described above. Ordinarily an experimenter is not interested in coverage of the whole regression curve. On the contrary, interest lies in only a bounded interval or even a finite set of points. The restriction of the above described bands to bounded sets of interest yield confidence coefficients $\geq 1 - \alpha$ (even in the first degree case). A method for providing a band that is valid only for the set of interest may yield a more efficient band. Secondly, it would be desirable to maintain a uniform degree of accuracy over the set of interest, i.e., the width of the band is the same for all values of the independent variable t in the set of interest.

This paper develops a general method for obtaining confidence bands of arbitrary shape and over any arbitrary subset of the line when the observations are independently normally distributed. The shape is arbitrary in the sense that if w is any positive function defined over the subset I of interest in $(-\infty,\infty)$, then the width of the band for $t \in I$ is proportional to $w(t)$. Thus, by selecting $w(t) = 1$, $t \in I$, the resulting band has the same width for every $t \in I$.

In general, difficult distribution problems result when specific applications are attempted. The major portion of this paper is concerned with first degree polynomials since some progress has been made here. A table is provided to obtain a confidence band which contains the true regression line for values of the independent variable in an arbitrarily selected interval of interest $[a,b]$ with an exact confidence coefficient. The band has the same width for all values $t \in [a,b]$. The table is constructed for use in the following situation: (1) The sample size n is even; (2) If observations are made at the values

$t_1, t_2, \ldots, t_n$ of the independent variable then $\bar{t} = \frac{1}{n} \sum_{i=1}^{n} t_i = \frac{a+b}{2}$. Defining $[A,B]$ as the interval in which observations are permissible a best solution obtains if in addition (3) $\frac{A+B}{2} = \frac{a+b}{2}$, i.e., the observation interval $[A,B]$ is symmetrically located with respect to the interval of interest $[a,b]$; (4) (2) is realized by making half the observations at A and half at B. The solution is best in the sense that for a given n, (B-A)/(b-a), and probability of coverage this particular experimental configuration achieves the smallest bandwidth.

The important feature of the band provided by the present method is that it is uniformly wide over $[a,b]$. In order to get some idea of its efficiency it was compared to the band that arises by merely considering the restriction of the hyperbolic one to the interval $[a,b]$, though in this case the probability of coverage is no longer $1-\alpha$ but $> 1 - \alpha$. The comparison was made in terms of the areas of the bands. To be more specific for a given n, (B-A)/(b-a), and probability of coverage, the best band (i.e., minimum area) was computed by the present method. The experimental configuration to achieve this also provides the minimum area over $[a,b]$ for the hyperbolic band. The ratio of the two areas was then considered as a measure of the efficiency. Roughly, the result is that for $(B-A)/(b-a) > 3/2$ the present method is more efficient and for $(B-A)/(b-a) < 3/2$ the restriction of the hyperbolic band to $[a,b]$ yields smaller areas. More specific calculations will be presented in a later section of the paper.

2. GENERAL TECHNIQUE

Suppose that for every $t \in (-\infty, \infty)$, Y_t is a normal random variable with unknown variance σ^2 and expectation given by a polynomial $\beta_o + \beta_1 t + \dots + \beta_r t^r$ of unknown coefficients and known degree r. Let $I \subset (-\infty, \infty)$ be the set of interest. For preassigned confidence coefficient $1-\alpha$ and positive function w defined on I it is desired to obtain simultaneous confidence intervals for $E[Y_t] = m_t$, $t \in I$, such that the length of the interval for each $t \in I$ is proportional to w(t).

Suppose independent observations are made at the time points $t_1, t_2, \dots, t_n$ where the number of distinct observation points is $\geq r + 1$ and the number of observations is $> r + 1$ (this ensures that σ^2 may be estimated since only $r + 1$ distinct points are needed for the estimability of the linear parameters). Let $\hat{\beta}' = (\hat{\beta}_o \; \hat{\beta}_1 \dots \hat{\beta}_r)$ denote the vector of least squares estimates for $\beta' = (\beta_o \; \beta_1 \dots \beta_r)$ given by

$$\hat{\beta} = (T'T)^{-1}T'Y$$

where

$$T = \begin{pmatrix} 1 & t_1 & t_1^2 \dots t_1^r \\ 1 & t_2 & t_2^2 \dots t_2^r \\ \cdot & \cdot & \cdot \; \dots \; \cdot \\ \cdot & \cdot & \cdot \; \dots \; \cdot \\ \cdot & \cdot & \cdot \; \dots \; \cdot \\ 1 & t_n & t_n^2 \dots t_n^r \end{pmatrix}$$

and $Y' = (y_1 \; y_2 \dots y_n)$ is the vector of observations at the points $t_1, t_2, \dots, t_n$.

Denote by

$$\hat{\sigma}^2 = \frac{1}{n-r-1} (Y-T\hat{\beta})'(Y-T\hat{\beta})$$

the independent unbiased estimator of σ^2 based on n-r-1 degrees of freedom. Let $\hat{m}_t$ be the best linear estimate of m_t given by $\sum_{j=o}^{r} \hat{\beta}_j t^j$. From the function

$$\frac{\hat{m}_t - m_t}{w(t)\hat{\sigma}}$$

and for any pair of numbers (δ_1, δ_2) with $\delta_1 < \delta_2$ let

$$V(\delta_1, \delta_2) = \left\{ \frac{\hat{\beta}-\beta}{\hat{\sigma}} : \delta_1 < \frac{\hat{m}_t - m_t}{w(t)\hat{\sigma}} < \delta_2, \ t\epsilon I \right\}$$

in the space of the random vector $\frac{\hat{\beta}-\beta}{\hat{\sigma}}$, whose distribution is parameter free and calculable [5]. These are sufficient conditions to obtain

$$[\hat{m}_t - \delta_2 w(t)\hat{\sigma}, \ \hat{m}_t - \delta_1 w(t)\hat{\sigma}], \ t\epsilon I ,$$

as simultaneous confidence intervals of confidence coefficient $P[V(\delta_1,\delta_2)]$ [6]. The width of the band for any $t\epsilon I$ is $(\delta_2-\delta_1)\ w(t)\hat{\sigma}$.

To insure the existence of at least one pair (δ_1,δ_2) to acquire the probability $1-\alpha$, an additional restriction must be imposed on the function w. The set $V(\delta_1,\delta_2)$ may be written as

$$\bigcap_{t\epsilon I} \left\{ \frac{\beta-\beta}{\hat{\sigma}} : \delta_1 < \frac{\hat{m}_t - m_t}{w(t)\hat{\sigma}} < \delta_2 \right\} .$$

Since

$$\frac{\hat{m}_t - m_t}{w(t)\hat{\sigma}} = \frac{1}{w(t)} (1 \ t \ t^2 \ \dots \ t^r) \left(\frac{\hat{\beta}-\beta}{\hat{\sigma}} \right)'$$

it follows that each set in the above intersection consists of the points between two parallel hyperplanes which are perpendicular to $(1 \ t \ \dots \ t^r)'$ and are at distances $(w(t)|\delta_2|)/(\sum_{j=0}^{r}|t|^j)^{1/2}$ and $(w(t)|\delta_1|)/(\sum_{j=0}^{r}|t|^j)^{1/2}$ from the origin. Hence, if there exist constants $m > 0$ and $M > 0$ such that $m \leq w(t)/(\sum_{j=0}^{r}|t|^j)^{1/2} \leq M$ for $t \epsilon I$ then and only then does there exist a pair (α_1, α_2) (actually many pairs) such that the required probability is attained.

It is conjectured that optimum confidence intervals are obtained whenever δ_2 is taken > 0 and $\delta_1 = -\delta_2$. The optimum is in the sense that for a given confidence coefficient $1-\alpha$ the difference $\delta_2-\delta_1$, and hence the length of the confidence intervals, will be minimized. This conjecture is based on: (1) The fact that the density function for the random vector $\frac{\hat{\beta}-\beta}{\hat{\sigma}}$ is constant on concentric $(r+1)$ - dimensional ellipsoids with center at origin and decreases monotonely with distance from the origin, and (2) The set $V(\alpha_1, \alpha_2)$ in this situation is symmetrical with respect to the origin and probably has a maximum volume for any fixed difference $\delta_2-\delta_1$.

It should be emphasized again that the real difficulty here is the calculation of δ_1 and δ_2 to achieve probability $1-\alpha$ when any specific applications are attempted. Progress has been made for the case $r=1$, I an interval, and $w(t) = 1$ for $t \epsilon I$, i.e., a band which has the same width over the interval of interest. The major portion of the remainder of the paper is devoted to this problem.

However, for some special examples the general case specializes properly to well known results. E.g., if I is a single point only, say t_o, $w(t_o) = 1$, $\delta_1 = -\delta_2$, and $\delta_2 > 0$, it can be shown that

$$\delta_2 = t_{\frac{\alpha}{2};n-r-1}[(1\ t_o\ \dots\ t_o^r)(T'T)^{-1}(1\ t_o\ \dots\ t_o^r)']^{1/2},$$

where $t_{\frac{\alpha}{2};n-r-1}$ is the upper $\alpha/2$ point of a t-variable with n-r-1 degrees of freedom, so that

$$P[\sum_{j=0}^{r}\hat{\beta}_j t_o^j - \delta_2\hat{\sigma} \leq \sum_{j=0}^{r}\beta_j t_o^j \leq \sum_{j=0}^{r}\hat{\beta}_j t_o^j + \delta_2\hat{\sigma}] = 1-\alpha ,$$

[1, p. 122]. Similarly, consider the set of all linear combinations $\{\beta_o u_o + \dots + \beta_r u_r : (u_o\ u_1\ \dots\ u_r) \in E_{r+1}\}$. Setting $\delta_1 = -\delta_2$ and $\delta_2 > 0$ and defining w for any $(u_o\ u_1\ \dots\ u_r)$ to equal

$$[(u_o\ u_1\ \dots\ u_r)(T'T)^{-1}(u_o\ u_1\ \dots\ u_r)']^{1/2}$$

gives that

$$\delta_2 = (r+1)\ F_{\alpha;r+1,\ n-r-1} ,$$

where $F_{\alpha;r+1,\ n-r-1}$ is the upper α point of a F-variable with r+1 and n-r-1 degrees of freedom. This then gives

$$P[|\sum_{j=0}^{r}\hat{\beta}_j u_j - \sum_{j=0}^{r}\beta_j u_j| \leq (r+1)\ F_{\alpha;r+1,\ n-r-1}$$
$$\times((u_o\ u_1\ \dots\ u_r)(T'T)^{-1}(u_o\ u_1\ \dots\ u_r)')^{1/2}\ \hat{\sigma} : (u_o\ u_1\ \dots\ u_r) \in E_{r+1}] = 1-\alpha$$

[6]. An infinite subset of the above intervals is then a confidence band of confidence coefficient $\geq 1 - \alpha$ for the mean curve. For r=1 this gives a band for the whole line with exact confidence coefficient $1-\alpha$. A little calculation shows this to be the hyperbolic band referred to in Section 1.

3. STRAIGHT LINE REGRESSION

This section contains the analysis in detail of the straight line regression case. For convenience the regression line is written in the form

$$m_t = \beta_o + \beta_1(t-\bar{t})$$

where $\bar{t} = \frac{1}{n}\sum_{i=1}^{n} t_i$, $n > 2$. The t_i's are observation points such that at least two are distinct. The observation at t_i is denoted by y_i. It is supposed that observations may be made only in an interval [A,B] and that a uniformly wide confidence band is required for the interval [a,b], i.e., $w(t) = 1$ for $t \epsilon [a,b]$.

Proceeding as outlined in Section 2, form the function

$$\frac{\hat{\beta}_o - \beta_o}{\hat{\sigma}} + \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}}(t-\bar{t}) \ ,$$

where

$$\hat{\beta}_o = \frac{1}{n}\sum_{i=1}^{n} y_i,$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(t_i-\bar{t})y_i}{\sum_{i=1}^{n}(t_i-\bar{t})^2} \ ,$$

$$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}[y_i-\hat{\beta}_o-\hat{\beta}_1(t_i-\bar{t})]^2$$

are stochastically independent. Determine for $\delta > 0$

$$V(-\delta,\delta) = \left\{\left(\frac{\hat{\beta}_o - \beta_o}{\hat{\sigma}}, \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}}\right): -\delta < \frac{\hat{\beta}_o - \beta_o}{\hat{\sigma}} + \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}}(t-\bar{t}) < \delta, t\epsilon[a,b]\right\},$$

or equivalently the image $V'(-\delta,\delta)$ of $V(-\delta,\delta)$ in the plane of t-variables of n-2 degrees of freedom

$$u = \sqrt{n}\,\frac{\hat{\beta}_o - \beta_o}{\hat{\sigma}}, \quad v = \sqrt{ns}\,\frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}}$$

where

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(t_1-\bar{t})^2 .$$

The resulting set is a parallelogram and is shown in Fig. 1. The density function g of (u,v) is given

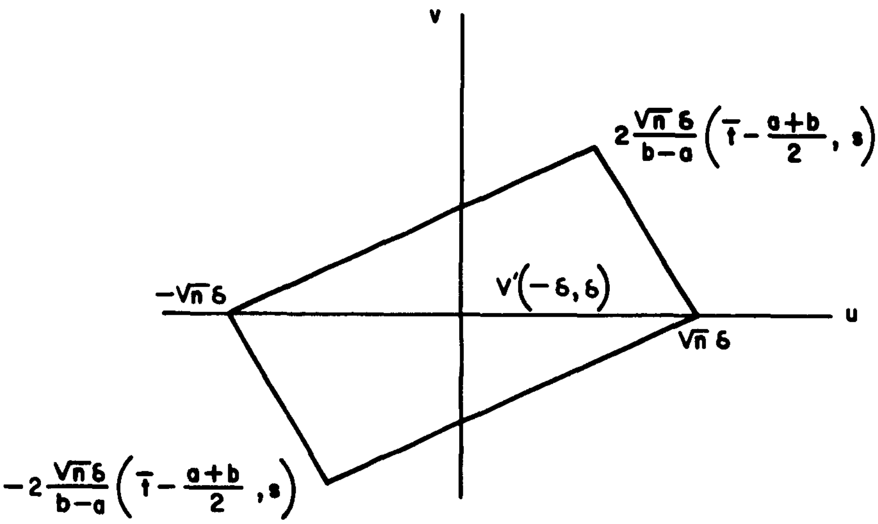

Figure 1

by

$$g(u,v) = \frac{1}{2\pi}\left[1 + \frac{u^2 + v^2}{n-2}\right]^{-\frac{1}{2}n} .$$

From symmetry of the density function we need to consider only the probability of the triangle $T(-\delta,\delta)$ in the upper half plane.

An examination of Fig. 1 illustrates the fact that for a fixed $[a,b]$, $[A,B]$, δ, and n, different values of $\bar{t}$ and s result in different confidence coefficients. The problem of maximizing the confidence coefficient is now investigated.

For a given $\bar{t}$ the claim is that the confidence coefficient is maximized when the variance of the observation points is maximized. For $\bar{t}$ such that the apex of the triangle is in $[-\sqrt{n}\delta, \sqrt{n}\delta]$, this is clear from the fact that if $s_2^2 > s_1^2$ are the variances of two configurations with corresponding triangles $T_2(-\delta,\delta)$ and $T_1(-\delta,\delta)$ respectively, then $T_2(-\delta,\delta) \supset T_1(-\delta,\delta)$. If $\bar{t}$ is such that the apex of $T(-\delta,\delta)$ lies in the complement of $[-\sqrt{n}\delta, \sqrt{n}\delta]$, it is not patently clear that the probability increases with s. That this, however, is the case is shown as follows. Let $h(\xi,\eta) = P[T(-\delta,\delta)]$, where ξ and η are the u and v coordinates of the apex. Then

$$h(\xi,\eta) = \int_0^{\eta} dv \int_{\alpha_1(\xi,\eta)}^{\alpha_2(\xi,\eta)} du \, \frac{1}{2\pi}\left[1 + \frac{u^2 + v^2}{n-2}\right]^{-\frac{1}{2}n} ,$$

where

$$\alpha_1(\xi,\eta) = \frac{\xi + \sqrt{n}\delta}{\eta} v - \delta\sqrt{n} ,$$

$$\alpha_2(\xi,\eta) = \frac{\xi - \sqrt{n}\delta}{\eta} v + \delta\sqrt{n} .$$

Hence

$$2\pi\eta^2 \frac{\partial}{\partial\eta} h(\xi,\eta) = \xi(I_1 - I_2) + 2\sqrt{n}\delta\,(I_1 + I_2),$$

where

$$I_1 = \int_0^\eta dv\, v \left[1 + \frac{\alpha_1^2(\xi,\eta) + v^2}{n-2}\right]^{-\frac{n}{2}} ,$$

$$I_2 = \int_0^\eta dv\, v \left[1 + \frac{\alpha_2^2(\xi,\eta) + v^2}{n-2}\right]^{-\frac{n}{2}} .$$

But $I_1 \geq I_2$ for $\xi \geq 0$ since

$$\alpha_2^2(\xi,\eta) - \alpha_1^2(\xi,\eta) = \frac{4\xi\sqrt{n}\delta v}{\eta}\left[1 - \frac{v}{\eta}\right] \geq 0 , \quad 0 \leq v \leq \eta .$$

Thus for $\xi \geq 0$ (and by symmetry for $\xi \leq 0$)

$$2\pi \frac{\partial}{\partial\eta} h(\xi,\eta) \geq 0 .$$

This proves that for a given $\bar{t}$, $A \leq \bar{t} \leq B$, the variance of the n observation points must be maximized. Intuitively, this is what one would expect.

It can be shown (Appendix) that for any $\bar{t}$, $A \leq \bar{t} \leq B$, the corresponding maximum s^2 which may be attained by the observation points $\{t_1, t_2, \ldots, t_n\}$ is $(B-A)^2\, f^2(\tau)$, where

$$f^2(\tau) = \frac{k + (n\tau - k)^2}{n} - \tau^2 , \quad \frac{k}{n} \leq \tau \leq \frac{k+1}{n},\ k=0,1,\ldots,n-1,$$

and

$$\tau = \frac{\bar{t}-A}{B-A} .$$

The configuration of observation points to obtain this maximum occurs with k t_i's at B, 1 t_i at $n(\bar{t}-A) - k(B-A) + A$, and $n - (k+1)$ t_i's at A. Thus for a fixed $\bar{t}$, the maximum confidence coefficient for the band of width $2\delta\hat{\sigma}$ is achieved when the coordinates of the apex are

$$u = 2\sqrt{n\delta\ell}(\tau-e) ,$$

$$v = \ell\sqrt{n\delta\ell}\ f(\tau) ,$$

where

$$\ell = \frac{B-A}{b-a} ,$$

and

$$e = \frac{\frac{a+b}{2} - A}{B-A} .$$

Plots of the loci of the apex are shown in Figures 2 and 3 for an even and an odd sample size respectively. Each section of the curve corresponds to the range

$$\frac{k}{n} \leq \tau = \frac{\bar{t}-A}{B-A} \leq \frac{k+1}{n} , \quad k=0,1,\ldots,n-1 .$$

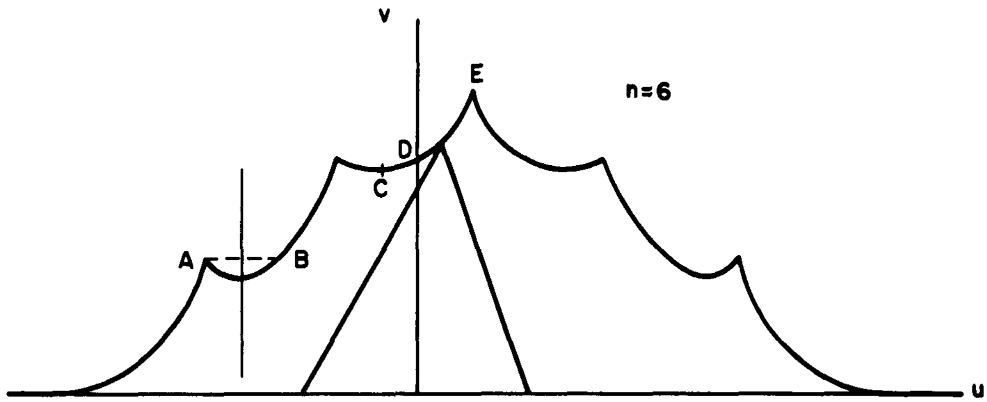

Figure 2

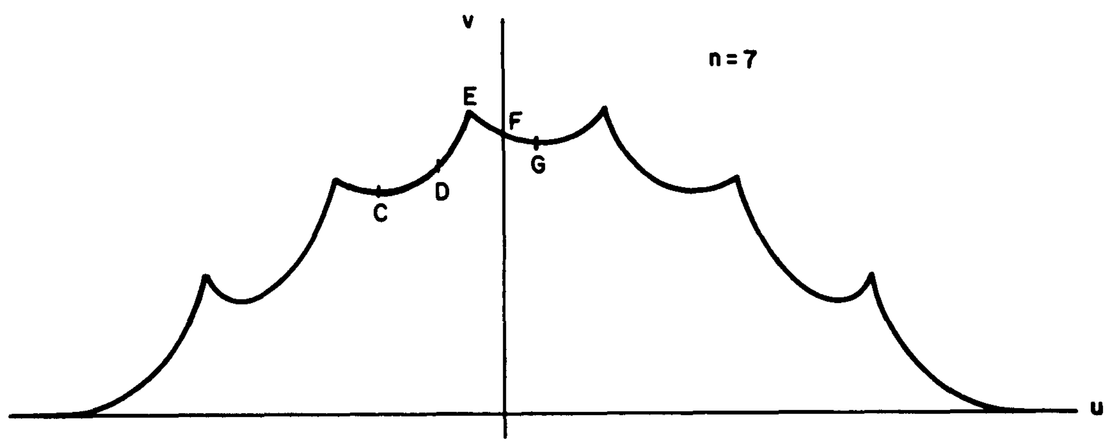

Figure 3

Due to the symmetry of the problem, it may always be assumed that $-\infty < e \leq \frac{1}{2}$, i.e., the midpoint of the interval [a,b] is always to the left of the midpoint of [A,B]. Whenever $e = \frac{1}{2}$, i.e., [A,B] and [a,b] have the same midpoint. The contours are symmetrical with respect to the v-axis. For $e < \frac{1}{2}$, the contours are shifted to the right by the amount $2\sqrt{n}\delta\ \ell(\frac{1}{2} - e)$.

The problem of choosing the best $\bar{t}$ for a fixed [A,B], [a,b] n, and δ is now considered. The best $\bar{t}$ is defined as the one that yields the maximum confidence coefficient when its corresponding maximum variance configuration is used (or equivalently minimizes δ for a given confidence coefficient $1-\alpha$, [A,B], [a,b], and n). Intuitively one would expect the best $\bar{t}$ to be the one whose corresponding maximum variance configuration possesses the highest possible variance of the observation points. This has been proved for the following situations:

1. n even and ≥ 6, $\frac{1}{2}(\frac{n-2}{n-1}) \leq e \leq \frac{1}{2}$. First it is shown that the maximum confidence coefficient must be attained for some point on the apex-contour-curve between the first peak to the left of the v-axis and the highest peak to the right of the v-axis. This follows from the fact that $\frac{\partial}{\partial\eta} h(\xi,\eta) \geq 0$, and that

$$2\pi\eta \frac{\partial}{\partial\xi} h(\xi,\eta) = I_2 - I_1 \leq 0, \quad \xi \geq 0 .$$

This last equation merely states that the probability in the triangle decreases as its apex moves away from the v-axis along a horizontal line. Next observe that each section has an axis of symmetry for a distance (which may be 0, such as for the first and last sections) on either side of a vertical which passes through the point having a horizontal tangent (see the arc AB, k=1, in Fig. 2). Hence if the v-axis intersects any section to the right of the axis of symmetry, the maximum probability of a triangle whose apex lies anywhere on the section occurs when the apex is at or to the right of the v-axis. This is the situation (see Fig. 2) when $\frac{1}{2}(\frac{n-2}{n-1}) \leq e \leq \frac{1}{2}$, which means that the v-axis intersects the first section somewhere on the arc CE.

Now as the apex moves from the v-axis toward the peak, the probability in the triangle, which is one half the confidence coefficient, increases. This follows by writing the confidence coefficient as a function of $\tau = \frac{\bar{t}-A}{B-A}$ in the iterated integral

$$P[V'(-\delta,\delta)] = 2\int_0^{\varphi(\tau)} dv \int_{\nu_1(\tau)}^{\nu_2(\tau)} du \, \frac{1}{2\pi}\left[1 + \frac{u^2+v^2}{n-2}\right]^{-\frac{n}{2}},$$

where

$$\nu_1(\tau) = \frac{2\ell(\tau-e)+1}{2\ell f(\tau)} v - \sqrt{n}\delta,$$

$$\nu_2(\tau) = \frac{2\ell(\tau-e)-1}{2\ell f(\tau)} v + \sqrt{n}\delta,$$

and

$$\varphi(\tau) = 2\sqrt{n}\delta\ell f(\tau).$$

Differentiating with respect to τ gives

$$2\pi\ell f^2(\tau)\frac{\partial}{\partial\tau}P[V'(-\delta,\delta)] = \frac{2\ell}{f(\tau)}[\tau((n-1)e-k) + \frac{k}{n}(1+\frac{k}{n})-ke](J_2-J_1)+f'(\tau)(J_1+J_2)$$

where

$$J_1 = \int_0^{\varphi(\tau)} dv \; v\left[1 + \frac{\nu_1^2(\tau)+v^2}{n-2}\right]^{-\frac{n}{2}},$$

$$J_2 = \int_o^{\varphi(\tau)} dv\ v\left[1 + \frac{v_2^2(\tau) + v^2}{n-2}\right]^{-\frac{n}{2}} .$$

As the apex moves from the v-axis toward the peak (along arc DE on Fig. 2), τ varies from e to $\frac{1}{2}$, $k = \frac{n}{2} - 1$, $f'(\tau) > 0$, and $J_2 - J_1 \leq 0$. But for n=6,8,10,... the coefficient of $J_2 - J_1$ is < 0 along the arc DE and hence $\frac{\partial}{\partial\tau} P[V'(-\delta,\delta)] > 0$. This means that the maximum confidence coefficient is attained when the apex of the triangle is at the point E.

2. n odd and ≥ 3, $\frac{n-1}{2n} \leq e \leq \frac{1}{2}$. In this case the v-axis lies somewhere on arc EG, say F, Fig. 3. The maximum probability is then on arc EF. As the apex moves from E to F, τ varies from $\frac{n-1}{2n}$ to e, $k = \frac{n-1}{2}$, $f'(\tau) < 0$, and $J_2 - J_1 \geq 0$. But the coefficient of $J_2 - J_1$ is < 0 along EG and hence $\frac{\partial}{\partial\tau} P[V'(-\delta,\delta)] < 0$. Thus the maximum confidence coefficient occurs when the apex of the triangle is at E.

3. n odd and ≥ 7, $\frac{n-3}{2(n-1)} \leq e \leq \frac{n-1}{2n}$. Now the v-axis would lie on CE, say D, in Fig. 3, and the maximum probability would lie somewhere on arc DE. As the apex moves from D to E, τ varies from e to $\frac{n-1}{2n}$, $k = \frac{n-3}{2}$, $f'(\tau) > 0$, and $J_2 - J_1 \leq 0$. But the coefficient of $J_2 - J_1$ is < 0 along DE and $\frac{\partial}{\partial\tau} P[V\ (-\delta,\delta)] > 0$, i.e., the maximum confidence coefficient occurs at E.

4. <u>TABLE</u>

From the above it is seen that in general, the maximum confidence coefficient for a band of width $2\delta\sigma$ depends on the parameters $\ell = \frac{B-A}{b-a}$, $e = \frac{\frac{a+b}{2} - A}{B-A}$, and n. Hence, a table which could handle all possible experimental situations would have to contain the value of the confidence coefficient for a range of values of the parameters δ, ℓ, e, and n. This seemed too extensive an undertaking at this time.

The table presented in this paper is constructed for use in the following situation:

(1) n even, specifically, n = 4(2)20(10)30(20)50, ∞.

(2) $\bar{t} = \frac{a+b}{2}$, so that an optimum solution is possible only if $\frac{a+b}{2} = \frac{A+B}{2}$.

Hence the problem is essentially to compute the integral of the function

$$g(u,v) = \frac{1}{2\pi}\left[1 + \frac{u^2 + v^2}{n-2}\right]^{-\frac{1}{2}n}$$

over the triangle shown in Fig. 4.

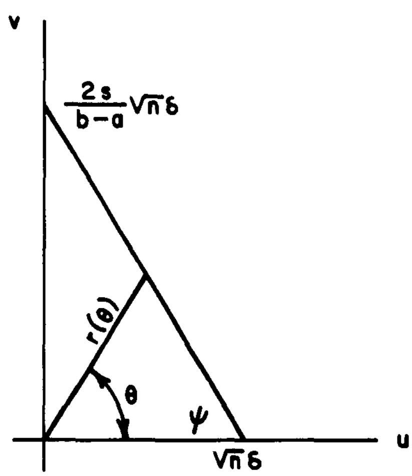

Figure 4

The table consists of 13 pages. At the top of each page are listed two values of a number c = 1(.1)2(.2)3(.4)5(1)6(2)10(10)20, ∞.* When the maximum variance configuration is used, i.e., $\frac{n}{2}$ observations at A and $\frac{n}{2}$ observations at B, $c = \frac{B-A}{b-a} = \ell$. If any other configuration of observation points is used, still maintaining $\bar{t} = \frac{a+b}{2}$, then $c = \frac{2s}{b-a}$ where s is the variance of the observation points. For each value of c, the confidence coefficient is computed for all combinations of n = 4(2)20(10)30(20)50, ∞* and $d = \sqrt{n}\delta$ = 1(.05)2.5(.1)4(.2)5(.5)7(1)10(5)20(10)50.* The confidence coefficient is entered into the body of the table without a decimal point. Each entry is correct to 3 significant figures and a blank space corresponds to a rounding off to 1.

It should be noted that the table is not restricted to those values of $c \geq 1$. Because of the symmetry of the density function g, it follows that for any $c < 1$, the table with heading 1/c may be used. In this case the values in the column $d = \sqrt{n}\delta$ must be multiplied by 1/c.

This table was computed using an expression derived by a technique similar to that of Dunnett and Sobel [5]. The confidence coefficient $1-\alpha$ may be written as

$$\frac{1}{4}(1-\alpha) = \frac{n-2}{2\pi} \int_0^{\pi/2} d\theta \int_0^{r(\theta)} d\rho\,(1+\rho^2)^{-\frac{n}{2}}$$

$$= \frac{1}{4} - \frac{1}{2\pi} \int_{\tan^{-1}c}^{\frac{\pi}{2}+\tan^{-1}c} d\varphi\,[1 + k^2 \csc^2\varphi]^{-\frac{n}{2}+1}$$

*The table is composed of computer print-out and thus the letters c,n, and d appear as capitals.

where

$$\rho^2 = \frac{u^2}{n-2} + \frac{v^2}{n-2} = \delta \sqrt{\frac{n}{n-2}} \frac{\sin\psi}{\sin(\theta+\psi)} ,$$

$$\theta = \tan^{-1}\frac{v}{u} ,$$

$$\psi = \tan^{-1}c ,$$

$$k^2 = \frac{\delta^2 c^2 n}{(n-2)(1+c^2)} ,$$

$$c = \frac{2s}{b-a} ,$$

$$\varphi = \theta + \psi .$$

Define

$$Q_{\frac{n}{2}} = \frac{1}{2\pi} \int_{\tan^{-1}c}^{\frac{\pi}{2}+\tan^{-1}c} d\varphi [1 + k^2 \csc^2\varphi]^{-\frac{n}{2}+1}$$

and consider

$$Q_{\frac{n}{2}} - Q_{\frac{n}{2}-1} = -\frac{1}{2\pi} \int_{\tan^{-1}c}^{\frac{\pi}{2}+\tan^{-1}c} d\varphi [1 + k^2 \csc^2\varphi]^{-\frac{n}{2}+1} k^2 \csc^2\varphi .$$

Making use of the change of variable

$$y = \frac{1}{1 + \left(1 + \frac{1}{k^2}\right)\tan^2\varphi}$$

it is seen after some calculation that

$$Q_{\frac{n}{2}} - Q_{\frac{n}{2}-1} = -\frac{k(1+k^2)^{-\frac{1}{2}(n-3)}}{4\pi}\left\{B_{f_1(c,k)}\left[\frac{1}{2}, \frac{1}{2}(n-3)\right] + B_{f_2(c,k)}\left[\frac{1}{2},\frac{1}{2}(n-3)\right]\right\}$$

where

$$f_1(c,k) = \frac{1}{1+\left(1+\frac{1}{k^2}\right)c^2},$$

$$f_2(c,k) = \frac{1}{1+\left(1+\frac{1}{k^2}\right)\frac{1}{c^2}},$$

and $B_z[p,q] = \int_o^z t^{p-1}(1-t)^{q-1}dt$ is the incomplete beta function.

Now for the case that n is odd and ≥ 3

$$1-\alpha = 1 - 4Q_{\frac{n}{2}}$$

$$= 1-4[(Q_{\frac{n}{2}} - Q_{\frac{n}{2}-1}) + (Q_{\frac{n}{2}-1} - Q_{\frac{n}{2}-2}) + \ldots + (Q_{\frac{5}{2}} - Q_{\frac{3}{2}}) + Q_{\frac{3}{2}}] .$$

But

$$Q_{\frac{3}{2}} = \frac{1}{2\pi}\left[\sin^{-1}\frac{1}{\sqrt{1+(1+n\delta^2)c^2}} + \sin^{-1}\frac{c}{\sqrt{1+(1+n\delta^2)c^2}}\right]$$

so that finally, in terms of the incomplete beta function ratio $I_z[p,q] = B_z[p,q]/B_1[p,q]$,

$$1-\dot{\alpha} = 1 - \frac{2}{\pi}\left[\sin^{-1}\frac{1}{\sqrt{1+(1+n\delta^2)c^2}} + \sin^{-1}\frac{c}{\sqrt{1+(1+n\delta^2)c^2}}\right]$$

$$+ \frac{2k}{\pi}\left[\sum_{j=1}^{\frac{1}{2}(n-3)} \frac{4^{j-1}[(j-1)!]^2}{(1+k^2)^j(2j-1)!}\left(I_{f_1(c,k)}\left[\frac{1}{2},j\right] + I_{f_2(c,k)}\left[\frac{1}{2},j\right]\right)\right], \tag{1}$$

$$n=5,7,9,\ldots$$

$$= 1 - \frac{2}{\pi}\left[\sin^{-1}\frac{1}{\sqrt{1+(1+n\delta^2)c^2}} + \sin^{-1}\frac{c}{\sqrt{1+(1+n\delta^2)c^2}}\right], \quad n=3 .$$

The formula

$$I_z\left(\frac{1}{2}, j\right) = \sqrt{z}\sum_{i=0}^{j-1}\frac{(2i)!}{4^i(i!)^2}(1-z^i)$$

is used for calculating the incomplete beta function ratios in (1).

For n even and ≥ 4

$$1-\alpha = 1-4\, Q_{\frac{n}{2}}$$

$$= 1-4[(Q_{\frac{n}{2}} - Q_{\frac{n}{2}-1}) + (Q_{\frac{n}{2}-1} - Q_{\frac{n}{2}-2}) + \ldots + (Q_2-Q_1) + Q_1] .$$

But $Q_1 = \frac{1}{4}$. Hence after some calculation

$$1-\alpha = k\sum_{j=1}^{\frac{n}{2}-1}\frac{(2j-2)!}{(1+k^2)^{j-\frac{1}{2}}[(j-1)!]^2 4^{j-1}}\left(I_{f_1(c,k)}\left[\frac{1}{2},j-\frac{1}{2}\right] + I_{f_2(c,k)}\left[\frac{1}{2},j-\frac{1}{2}\right]\right) \tag{2}$$

$$n=4,6,8,\ldots .$$

The formula

$$I_z\left(\frac{1}{2}, j-\frac{1}{2}\right) = \frac{2}{\pi}\tan^{-1}\sqrt{\frac{z}{1-z}} + \frac{2}{\pi}\sqrt{z(1-z)}\sum_{i=0}^{j-2}\frac{4^i(i!)^2}{(2i+1)!}(1-z)^i$$

is used for evaluating the incomplete beta function ratios appearing in (2).

The actual computations were performed on a Philco 2000 digital computer using equations (1) and (2). For $n \leq 50$, which is the range of finite n in the table, an error analysis showed that the resulting probabilities could be off at most by seven digits in the 7th place. To reduce the size of the table, however, these were rounded off to three figures. This should be sufficient for most applications.

Now

$$\lim_{n\to\infty} g(u,v) = \frac{1}{2\pi}e^{-\frac{1}{2}(u^2+v^2)}, \tag{3}$$

which is the uncorrelated bivariate normal distribution with zero means and unit variances. To make the calculation for $n=\infty$, which amounts to the integral of (3) over the triangle of Fig. 4, a method outlined by Owen [7] was used. For $1 \leq c < \infty$ this gives

$$1-\alpha = 1-4(E+F)$$

where

$$E = T\left(x, \frac{1}{c}\right),$$

$$F = \frac{1}{2}\left[G(x)+G(y)\right] - \left[G(x)G(y) + T\left(y, \frac{1}{c}\right)\right],$$

$$x = \frac{c(\delta\sqrt{n})}{\sqrt{1+c^2}},$$

$$y = \frac{c^2(\delta\sqrt{n})}{\sqrt{1+c^2}},$$

$$T(h,z) = \frac{1}{2\pi}\left(\tan^{-1}z - \sum_{j=0}^{\infty} c_j z^{2j+1}\right),$$

$$c_j = (-1)^j \frac{1}{2j+1}\left[1-e^{-\frac{h^2}{2}} \sum_{i=0}^{j} \frac{(\frac{h^2}{2})^i}{i!}\right],$$

$$G(x) = \frac{1}{2\pi}\int_{-\infty}^{x} e^{-\frac{1}{2}\xi^2}\, d\xi .$$

For $c=\infty$, $E=0$ and

$$F = \frac{1}{2}[1-G(x)]$$

where

$$x = (\delta\sqrt{n}) .$$

These again were performed on the Philco 2000 and the computations were such that the resulting confidence coefficients are correct to three significant figures.

One additional observation is that, as $c\to\infty$ for a fixed $\delta\sqrt{n}$, the confidence coefficient is the area of the function g over an infinite strip parallel to the v-axis. Hence, the values in the table with $c=\infty$ could have been obtained from a t-table. Each column corresponds to a t-variable whose degrees of freedom is two less than the sample size heading.

5. EFFICIENCY

In Scheffé [3, pp. 52, 53], it is seen that a $1-\alpha$ confidence band for the true line consists of all points (t,y) satisfying

$$[y-\hat{\alpha}-\hat{\beta}(t-\bar{t})]^2 \leq F_{\alpha;2,n-2}\ \hat{\sigma}\left[\frac{1}{n}+\frac{(t-\bar{t})^2}{ns^2}\right].$$

This gives a band about the fitted line, bounded by the two branches of a hyperbola. In order to use this for comparison purposes with the method of this paper, it is restricted to just the interval $[a,b]$. The confidence coefficient of this band is, of course, no longer $1-\alpha$ but $\geq 1-\alpha$.

The area A_1 of the hyperbolic band over the interval $[a,b]$ is given by

$$A_1 = 2\hat{\sigma}\sqrt{2F_{\alpha;2,n-2}}\int_a^b dt\left[\frac{1}{n}+\frac{(t-\bar{t})^2}{ns^2}\right]^{\frac{1}{2}}.$$

It is clear that this area is minimized when $\bar{t}=\frac{a+b}{2}$ and s^2 is maximized. Thus if $\frac{a+b}{2}=\frac{A+B}{2}$, s^2 is maximized for n even when $\frac{n}{2}$ observations are at A and $\frac{n}{2}$ observations are at B. In this case $s^2=\frac{1}{4}(B-A)^2$. Thus

$$A_1 = \hat{\sigma}(b-a)c \sqrt{\frac{2F_{\alpha;2,n-2}}{n}} \int_o^{\frac{1}{c}} [1+\xi^2]^{\frac{1}{2}} d\xi$$

$$= \hat{\sigma}(b-a) \sqrt{\frac{2F_{\alpha;2,n-2}}{n}} \left[\left(1 + \frac{1}{c^2}\right)^{\frac{1}{2}} + c \log \frac{\sqrt{1+c^2}+1}{c}\right],$$

where $c = \frac{B-A}{b-a}$. The area A_2 of our band is $2\delta\hat{\sigma}(b-a)$. Hence, the ratio A_1/A_2, which will be referred to as the efficiency of our method, is given by

$$\frac{A_1}{A_2} = \frac{\sqrt{2F_{\alpha;2,n-2}}}{(\delta\sqrt{n})} \quad \frac{1}{2}\left[\left(1 + \frac{1}{c^2}\right)^{\frac{1}{2}} + c \log \frac{\sqrt{1+c^2}+1}{c}\right]. \tag{4}$$

Eq. (4) is valid for any $0 < c < \infty$. It was noted in Section 4 that $\lim_{c\to\infty}(\delta\sqrt{n}) = t_{\frac{\alpha}{2};n-2}$. But

$$\lim_{c\to\infty} c \log \frac{\sqrt{1+c^2}+1}{c} = 1 .$$

Hence

$$\lim_{c\to\infty} \frac{A_1}{A_2} = \frac{\sqrt{2F_{\alpha;n-2}}}{t_{\frac{\alpha}{2};n-2}} . \tag{5}$$

The symmetry of the function g means that $\lim_{c\to o} c\delta\sqrt{n} = t_{\frac{\alpha}{2};n-2}$. Also

$$\lim_{c\to o} c^2 \log \frac{\sqrt{c^2+1}+1}{c} = 0$$

Hence

$$\lim_{c\to\infty} \frac{A_1}{A_2} = \frac{1}{2} \; \frac{\sqrt{2F_{\alpha;n-2}}}{t_{\frac{\alpha}{2};n-2}} . \tag{6}$$

Eqs. (4), (5), and (6) summarize the results of this section. Fig. 5 is a graph of the efficiency, for each of three values of n, as a function of c. The confidence coefficient selected is .95.

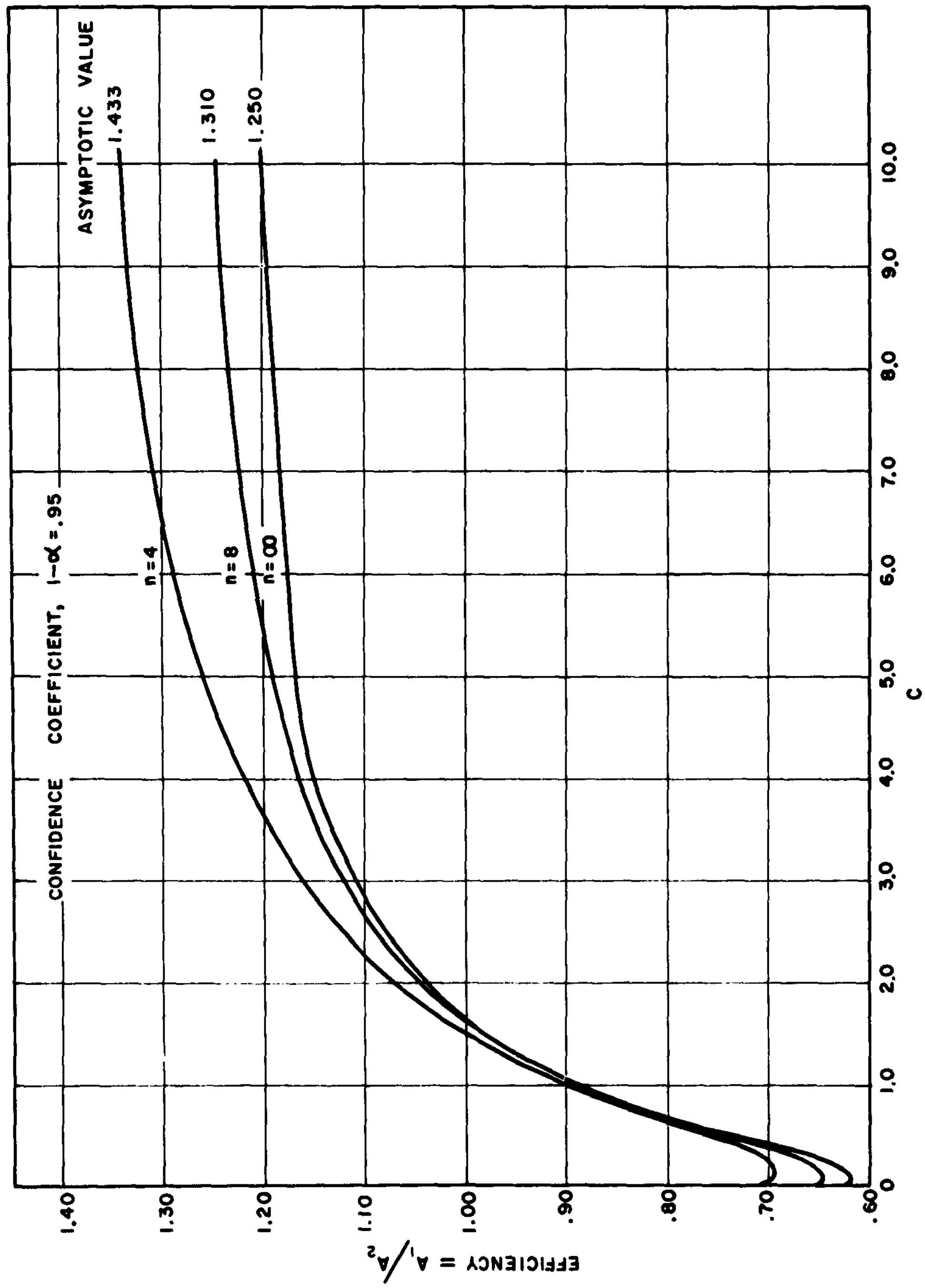
CONFIDENCE COEFFICIENT, 1−α = .95
ASYMPTOTIC VALUE
1.433
1.310
1.250
n=4
n=8
n=∞
EFFICIENCY = A1/A2
C
1.40
1.30
1.20
1.10
1.00
.90
.80
.70
.60
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0

Figure 5.

APPENDIX

Let $\{z_1, z_2, \ldots, z_n\}$ be n points in the unit interval [0,1]. For a fixed $\bar{z} = \frac{1}{n} \sum_{i=1}^{n} z_i$ in [0,1] the problem is to maximize

$$ns^2 = \sum_{i=1}^{n}(z_i - \bar{z})^2 = \sum_{i=1}^{n} z_i^2 - n\bar{z}^2 .$$

The claim is that, to maximize set each z_i equal to 0 or to 1, except for one, keeping $\sum_{i=1}^{n} z_i = n\bar{z}$. For suppose, without loss of generality, $0 < z_1 \leq z_2 < 1$. Then there exists $\delta > 0$ such that

$$0 \leq z_1 - \delta \leq 1 ,$$
$$0 \leq z_2 + \delta \leq 1 ,$$

and

$$(z_1-\delta)^2 + (z_2+\delta)^2 = z_1^2+z_2^2+2\delta^2 + 2\delta(z_2-z_1) > z_1^2+z_2^2 ,$$

i.e., ns^2 may be increased. The actual configuration for any $\bar{z}$ such that $\frac{k}{n} \leq \bar{z} \leq \frac{k+1}{n}$, $k=0,1,\ldots,n-1$ is k z_i's at 1, 1 z_i at $n\bar{z}-k$, and $n-(k+1)$ z_i's at 0. The resulting maximum variance is

$$\frac{k+(n\bar{z}-k)^2}{n} - \bar{z}^2 .$$

Thus for $\{t_1, t_2, \ldots, t_n\} \subset [A,B]$, the maximum variance configuration for a fixed $\bar{t}$, such that

$$\frac{k}{n} \leq \tau = \frac{\bar{t}-A}{B-A} \leq \frac{k+1}{n}, \qquad k=0,1,\ldots,n-1 ,$$

is given by k t_i's at B, 1 t_i at $n(\bar{t}-A) - k(B-A) + A$, and $n-(k+1)$ t_i's at A. The maximum variance is

$$(B-A)^2\left[\frac{k+(n\tau-k)^2}{n} - \tau^2\right].$$

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C = 1.0

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	239	254	259	262	264	265	266	266	267	268	269	271
1.05	258	274	280	284	286	287	288	289	289	291	292	294
1.10	276	294	302	305	308	309	310	311	312	314	315	317
1.15	293	315	323	327	330	332	333	334	335	337	338	341
1.20	311	335	344	349	352	354	355	357	357	360	362	365
1.25	329	355	365	371	374	376	378	379	380	383	385	388
1.30	346	375	386	392	396	399	401	402	403	406	409	412
1.35	363	395	407	414	418	421	423	425	426	429	432	436
1.40	380	415	428	435	440	443	445	447	448	452	455	459
1.45	397	434	449	456	461	465	467	469	471	475	478	483
1.50	413	453	469	477	483	486	489	491	492	497	501	506
1.55	429	471	488	498	503	507	510	512	514	519	523	528
1.60	444	490	508	518	524	528	531	533	535	541	545	551
1.65	459	507	527	537	544	548	552	554	556	562	566	573
1.70	474	525	545	556	563	568	572	574	576	582	587	594
1.75	488	542	563	575	582	587	591	594	596	603	608	615
1.80	502	558	581	593	601	606	610	613	615	622	627	635
1.85	516	574	598	611	619	625	629	632	634	641	647	655
1.90	529	590	614	628	636	642	646	650	652	660	666	674
1.95	542	605	630	644	653	659	664	667	670	678	684	692
2.00	554	619	646	661	670	676	681	684	687	695	701	710
2.05	566	633	661	676	685	692	697	700	703	712	716	727
2.10	578	647	676	691	701	707	712	716	719	728	734	744
2.15	589	660	689	705	715	722	727	731	734	743	750	760
2.20	600	673	703	719	730	737	742	746	749	758	765	775
2.25	611	685	716	733	743	750	756	760	763	772	779	789
2.30	621	697	728	745	756	763	769	773	776	785	793	803
2.35	631	709	741	758	769	776	782	786	789	798	806	816
2.40	641	720	752	770	781	788	794	798	801	811	818	829
2.45	650	730	763	781	792	800	805	809	813	822	830	841
2.50	659	741	774	792	803	811	816	821	824	834	841	852
2.60	676	760	794	812	823	831	837	841	844	854	862	872
2.70	693	778	812	830	842	850	855	860	863	873	880	891
2.80	708	794	829	847	859	866	872	876	880	889	897	907
2.90	722	810	844	862	874	882	887	891	895	904	911	921
3.00	735	824	858	876	887	895	900	905	908	917	924	933
3.10	748	836	871	889	900	907	912	916	920	928	935	944
3.20	759	848	882	900	911	918	923	927	930	938	945	953
3.30	770	859	893	910	921	928	933	936	939	947	953	961
3.40	781	869	903	919	929	936	941	944	947	955	960	968
3.50	790	879	911	928	937	944	948	952	954	961	967	973
3.60	800	887	919	935	944	950	955	958	960	967	972	978
3.70	808	895	926	942	951	956	961	963	966	972	977	982
3.80	816	902	933	948	956	962	966	968	970	976	981	986
3.90	824	909	939	953	961	966	970	973	975	980	984	988
4.00	831	915	944	958	965	970	974	976	978	983	986	991
4.20	844	926	953	966	973	977	980	982	984	988	991	994
4.40	856	936	961	972	979	982	985	987	988	992	994	996
4.60	867	944	967	978	983	986	989	990	991	994	996	998
4.80	876	951	972	982	987	990	991	993	994	996	997	999
5.00	865	956	977	985	989	992	993	995	995	997	998	999
5.50	903	968	985	991	994	996	997	997	998	999	999	
6.00	917	976	990	995	997	998	998	999	999			
6.50	928	982	993	997	998	999	999	999				
7.00	938	986	995	998	999	999						
8.00	951	991	997	999								
9.00	961	994	999									
10.00	968	996	999									
15.00	986	999										
20.00	992											
30.00	996											
40.00	998											
50.00	999											

C = 1.1

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	257	273	280	283	285	286	287	288	288	290	291	293
1.05	276	295	302	306	308	309	311	311	312	314	316	318
1.10	295	316	324	328	331	333	334	335	336	338	340	342
1.15	313	337	346	351	354	356	358	359	360	363	364	367
1.20	332	358	369	374	378	380	382	383	384	387	389	392
1.25	350	379	391	397	401	403	405	407	408	411	414	417
1.30	368	400	413	419	424	427	429	430	431	435	438	442
1.35	385	420	434	442	446	450	452	454	455	459	462	466
1.40	402	440	456	464	469	472	475	477	478	483	486	491
1.45	419	460	476	485	491	495	497	499	501	506	509	515
1.50	436	479	497	506	512	516	519	522	523	529	533	538
1.55	452	498	517	527	534	538	541	544	545	551	555	561
1.60	467	517	537	547	554	559	562	565	567	573	577	584
1.65	482	535	556	567	574	579	583	586	588	594	599	606
1.70	497	552	574	586	594	599	603	606	608	615	620	628
1.75	511	569	592	605	613	619	623	626	628	635	641	648
1.80	525	585	610	623	632	637	642	645	647	655	660	669
1.85	539	601	627	641	649	655	660	663	666	674	679	688
1.90	552	617	643	658	667	673	677	681	684	692	698	707
1.95	565	632	659	674	683	690	695	698	701	709	716	725
2.00	577	646	674	690	699	706	711	715	718	726	733	742
2.05	589	660	689	705	715	722	727	730	733	742	749	759
2.10	600	673	703	719	730	737	742	746	749	758	765	775
2.15	611	686	717	733	744	751	756	760	763	773	780	790
2.20	622	698	730	747	757	765	770	774	777	787	794	804
2.25	633	710	742	759	770	778	783	787	791	800	807	818
2.30	643	722	754	772	783	790	796	800	803	813	820	831
2.35	652	733	766	783	795	802	808	812	815	825	832	843
2.40	662	743	777	795	806	814	819	823	827	837	844	855
2.45	671	754	787	805	817	824	830	834	838	847	855	866
2.50	680	763	797	815	827	835	840	845	848	858	865	876
2.60	696	782	816	834	846	854	859	864	867	877	884	894
2.70	712	799	833	852	863	871	876	881	884	893	901	911
2.80	726	814	849	857	878	886	892	896	899	908	915	925
2.90	740	829	863	881	892	900	905	909	912	921	928	937
3.00	753	842	876	894	904	912	917	921	924	933	939	948
3.10	765	854	887	905	915	923	928	931	934	943	949	957
3.20	775	865	898	915	925	932	937	940	943	951	957	965
3.30	786	875	907	924	934	940	945	948	951	959	964	971
3.40	796	884	916	932	942	948	952	955	958	965	970	976
3.50	805	893	924	939	948	954	959	962	964	970	975	981
3.60	814	900	931	946	955	960	964	967	969	975	979	985
3.70	822	908	937	952	960	965	969	971	974	979	983	988
3.80	830	914	943	957	965	970	973	975	977	982	986	990
3.90	837	920	948	961	969	973	977	979	981	985	988	992
4.00	844	926	953	965	972	977	980	982	984	988	991	994
4.20	856	936	961	972	979	982	985	987	988	991	994	996
4.40	867	944	967	978	983	987	989	990	991	994	996	998
4.60	877	951	973	982	987	990	992	993	994	996	997	999
4.80	886	957	977	986	990	992	994	995	995	997	998	999
5.00	894	962	981	989	992	994	995	996	997	998	999	
5.50	910	972	988	993	996	997	998	998	999	999		
6.00	924	979	992	996	998	998	999	999	999			
6.50	934	984	994	997	999	999	999					
7.00	943	988	996	998	999							
8.00	956	993	998	999								
9.00	965	995	999									
10.00	971	997	999									
15.00	987	999										
20.00	993											
30.00	997											
40.00	998											
50.00	999											

C = 1.2

D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	273	292	299	302	305	306	307	308	309	311	312	314
1.05	293	314	322	326	329	331	332	333	334	336	338	340
1.10	312	336	345	350	353	355	357	358	359	361	363	366
1.15	331	358	368	374	377	380	381	383	384	386	389	392
1.20	350	380	391	398	401	404	406	407	408	412	414	416
1.25	369	401	414	421	425	428	430	432	433	437	440	444
1.30	387	423	437	444	449	452	454	456	457	461	465	469
1.35	405	443	459	467	472	475	478	480	482	486	489	494
1.40	422	464	480	489	495	499	501	504	505	510	514	519
1.45	439	484	502	511	517	521	524	527	528	534	538	544
1.50	456	503	522	533	539	544	547	549	551	557	561	567
1.55	472	522	542	553	560	565	569	571	573	579	584	591
1.60	487	541	562	574	581	586	590	593	595	601	606	613
1.65	503	559	581	594	601	607	611	614	616	623	628	635
1.70	517	576	600	613	621	626	631	634	636	643	649	657
1.75	532	593	618	631	640	646	650	653	656	663	669	677
1.80	545	609	635	649	658	664	669	672	675	683	689	697
1.85	559	625	652	666	676	682	687	690	693	701	707	716
1.90	572	640	668	683	693	699	704	708	710	719	725	735
1.95	584	655	683	699	709	716	721	724	727	736	743	752
2.00	596	669	698	714	724	731	736	740	743	752	759	769
2.05	608	682	712	729	739	746	752	756	759	768	775	785
2.10	619	695	726	743	754	761	766	770	773	783	790	800
2.15	630	708	739	756	767	775	780	784	787	797	804	814
2.20	641	720	752	769	780	788	793	797	801	810	817	828
2.25	651	731	764	782	793	800	806	810	813	823	830	841
2.30	661	742	775	793	804	812	818	822	825	835	842	853
2.35	670	753	786	804	816	823	829	833	837	846	854	864
2.40	679	763	797	815	826	834	840	844	847	857	864	875
2.45	688	773	807	825	836	844	850	854	857	867	875	885
2.50	697	782	816	835	846	854	859	864	867	877	884	894
2.60	713	800	834	852	864	871	877	881	885	894	901	911
2.70	728	816	850	868	880	887	893	897	900	909	916	926
2.80	742	830	865	883	894	901	907	911	914	923	929	938
2.90	755	844	879	896	908	914	919	923	926	934	940	949
3.00	767	856	890	907	917	924	929	933	936	944	950	958
3.10	779	867	900	917	927	934	939	942	945	953	958	966
3.20	789	878	910	926	936	943	947	950	953	960	965	972
3.30	799	887	919	934	944	950	954	957	960	967	971	978
3.40	809	896	926	942	951	956	960	963	966	972	976	982
3.50	818	903	933	948	957	962	966	969	971	977	981	986
3.60	826	911	940	954	962	967	971	973	975	980	984	989
3.70	833	917	945	959	967	971	975	977	979	984	987	991
3.80	841	923	951	963	971	975	978	980	982	986	989	993
3.90	847	929	955	967	974	979	981	983	985	989	991	995
4.00	854	934	959	971	977	981	984	986	987	991	993	996
4.20	866	943	966	977	982	986	988	990	991	994	996	997
4.40	876	950	972	982	986	989	991	992	993	996	997	999
4.60	885	957	977	985	990	992	994	995	995	997	998	999
4.80	894	962	981	988	992	994	995	996	997	998	999	
5.00	901	967	984	991	994	995	996	997	998	999	999	
5.50	917	976	990	994	997	998	998	999	999			
6.00	929	982	993	997	998	999	999	999				
6.50	939	986	995	998	999	999						
7.00	947	990	997	999	999							
8.00	959	994	996	999								
9.00	967	996	999									
10.00	973	997										
15.00	988	999										
20.00	993											
30.00	997											
40.00	998											
50.00	999											

C = 1.3

D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	288	309	317	321	324	325	327	327	328	330	332	334
1.05	308	332	341	346	349	351	352	353	354	357	358	361
1.10	328	355	365	370	374	376	378	379	380	383	385	388
1.15	348	377	389	395	399	401	403	404	406	409	411	415
1.20	367	399	412	419	423	426	428	430	431	435	437	441
1.25	386	421	435	443	448	451	453	455	456	460	463	468
1.30	404	443	458	466	472	475	478	480	481	486	489	494
1.35	422	464	481	490	495	499	502	504	506	510	514	520
1.40	440	485	503	512	518	522	525	528	530	535	539	545
1.45	457	505	524	534	541	545	549	551	553	559	563	569
1.50	474	524	545	556	563	568	571	574	576	582	587	593
1.55	490	543	565	577	584	589	593	596	598	605	609	617
1.60	505	562	585	597	605	610	614	617	620	626	632	639
1.65	521	580	604	617	625	631	635	638	640	648	653	661
1.70	535	597	622	636	644	650	655	658	660	668	674	682
1.75	549	614	640	654	663	669	674	677	680	688	694	702
1.80	563	630	657	672	681	687	692	696	698	707	713	722
1.85	576	645	673	688	698	705	710	713	716	725	731	740
1.90	589	660	689	705	715	721	726	730	733	742	749	758
1.95	602	674	704	720	730	737	743	746	749	758	765	775
2.00	613	688	719	735	746	753	758	762	765	774	781	791
2.05	625	701	732	749	760	767	773	777	780	789	796	806
2.10	636	714	746	763	774	781	787	791	794	803	810	821
2.15	647	726	758	776	787	794	800	804	807	817	824	834
2.20	657	738	771	788	799	807	812	817	820	829	837	847
2.25	667	749	782	800	811	819	824	829	832	841	849	859
2.30	677	760	793	811	822	830	836	840	843	853	860	871
2.35	686	770	804	822	833	841	846	851	854	863	871	881
2.40	695	780	814	832	843	851	856	861	864	873	881	891
2.45	703	789	823	841	853	860	866	870	873	883	890	900
2.50	711	798	832	850	862	869	875	879	882	892	899	909
2.60	727	815	849	867	878	886	891	895	898	908	915	924
2.70	742	830	864	882	893	900	906	909	913	921	928	937
2.80	755	844	878	895	906	913	918	922	925	934	940	949
2.90	768	856	890	907	917	924	929	933	936	944	950	958
3.00	780	868	901	918	928	934	939	942	945	953	958	966
3.10	791	878	911	927	937	943	947	951	953	960	966	972
3.20	801	888	920	935	944	950	955	958	960	967	972	978
3.30	810	897	927	943	951	957	961	964	966	972	977	982
3.40	819	905	935	949	957	963	967	969	971	977	981	986
3.50	828	912	941	955	963	968	971	974	976	981	985	989
3.60	836	919	947	960	967	972	975	978	980	984	987	991
3.70	843	925	952	964	972	976	979	981	983	987	990	993
3.80	850	930	956	968	975	979	982	984	985	989	992	995
3.90	856	935	961	972	978	982	985	986	988	991	993	996
4.00	862	940	964	975	981	984	987	988	990	993	995	997
4.20	873	948	971	980	985	988	990	992	993	995	997	998
4.40	883	955	976	984	989	991	993	994	995	997	998	999
4.60	892	961	980	988	991	993	995	996	996	998	999	999
4.80	900	966	983	990	993	995	996	997	997	999	999	
5.00	907	970	986	992	995	996	997	998	998	999		
5.50	922	978	991	995	997	998	999	999	999			
6.00	934	984	994	997	998	999	999					
6.50	943	988	996	998	999							
7.00	950	991	997	999								
8.00	961	994	999									
9.00	969	996	999									
10.00	975	998										
15.00	989	999										
20.00	994											
30.00	997											
40.00	998											
50.00	999											

C = 1.4

D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	302	325	334	338	341	343	344	346	346	349	351	353
1.05	323	349	358	364	367	369	371	372	373	376	378	381
1.10	343	372	383	389	393	395	397	398	399	403	405	409
1.15	363	395	407	414	418	421	423	425	426	429	432	436
1.20	383	418	431	439	443	446	449	450	452	456	459	463
1.25	402	440	455	463	468	472	474	476	478	482	485	490
1.30	420	462	478	487	492	496	499	501	503	508	511	517
1.35	438	483	501	510	516	520	523	526	527	533	537	542
1.40	456	504	523	533	539	544	547	550	552	557	562	568
1.45	473	524	544	555	562	567	570	573	575	581	586	592
1.50	490	543	565	577	584	589	593	596	598	604	609	616
1.55	506	562	585	598	605	611	615	618	620	627	632	640
1.60	521	581	605	618	626	632	636	639	641	648	654	662
1.65	537	598	623	637	646	652	656	659	662	669	675	683
1.70	551	615	642	656	665	671	675	679	682	689	695	704
1.75	565	632	659	674	683	690	694	698	701	709	715	724
1.80	579	648	676	691	701	707	712	716	719	727	734	743
1.85	592	663	692	708	718	724	729	733	736	745	751	761
1.90	604	677	707	723	734	741	746	750	753	761	766	778
1.95	617	691	722	739	749	756	761	765	768	777	784	794
2.00	628	705	736	753	764	771	776	780	783	793	800	810
2.05	640	718	750	767	777	785	790	794	798	807	814	824
2.10	650	730	762	780	791	798	804	808	811	820	828	838
2.15	661	742	775	792	803	811	816	820	824	833	840	851
2.20	671	753	786	804	815	823	828	832	836	845	853	863
2.25	681	764	797	815	826	834	840	844	847	857	864	874
2.30	690	774	808	826	837	845	850	855	858	867	875	885
2.35	699	784	818	836	847	855	860	865	868	877	884	895
2.40	708	793	827	846	857	864	870	874	877	887	894	904
2.45	716	802	837	855	866	873	879	883	886	895	902	912
2.50	724	811	845	863	874	882	887	891	894	904	910	920
2.60	739	827	861	879	890	897	902	906	909	918	925	934
2.70	753	842	875	893	903	911	916	920	923	931	937	946
2.80	767	855	888	905	916	922	927	931	934	942	948	956
2.90	779	867	899	916	926	933	938	941	944	951	957	964
3.00	790	878	910	926	936	942	946	950	952	959	965	971
3.10	801	888	919	935	944	950	954	957	960	966	971	977
3.20	811	897	927	942	951	957	961	964	966	972	976	982
3.30	820	905	934	949	957	963	966	969	971	977	981	986
3.40	828	912	941	955	963	968	971	974	976	981	984	989
3.50	836	919	947	960	967	972	975	978	979	984	987	991
3.60	844	925	952	965	972	976	979	981	983	987	990	993
3.70	851	931	957	969	975	979	982	984	985	989	992	995
3.80	857	936	961	972	978	982	985	987	988	991	993	996
3.90	864	941	965	975	981	985	987	989	990	993	995	997
4.00	869	945	968	978	984	987	989	990	991	994	996	998
4.20	880	953	974	983	987	990	992	993	994	996	997	999
4.40	890	959	979	986	990	993	994	995	996	997	998	999
4.60	898	965	982	989	993	995	996	997	997	998	999	
4.80	905	969	985	991	994	996	997	998	998	999	999	
5.00	912	973	988	993	996	997	998	998	999	999		
5.50	926	980	992	996	998	999	999	999	999			
6.00	937	986	995	998	999	999						
6.50	946	989	997	999	999							
7.00	953	992	998	999								
8.00	964	995	999									
9.00	971	997	999									
10.00	976	998										
15.00	989											
20.00	994											
30.00	997											
40.00	998											
50.00	999											

C = 1.5

D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	315	340	349	354	358	360	361	362	363	366	368	371
1.05	336	364	375	380	384	386	388	390	391	394	396	399
1.10	357	388	400	406	410	413	415	417	418	421	424	428
1.15	377	411	425	432	436	439	442	443	445	448	451	456
1.20	397	434	449	457	462	465	468	469	471	475	479	483
1.25	416	457	473	481	487	491	493	495	497	502	505	510
1.30	435	478	496	505	511	515	518	521	522	527	531	537
1.35	453	500	519	529	535	540	543	545	547	553	557	563
1.40	471	521	541	552	558	563	567	569	571	577	582	588
1.45	488	541	562	574	581	586	590	593	595	601	606	613
1.50	504	560	583	595	603	608	612	615	617	624	629	637
1.55	520	579	603	616	624	630	634	637	639	646	652	660
1.60	536	597	622	636	644	650	654	658	660	668	674	682
1.65	551	615	641	655	664	670	674	678	681	688	694	703
1.70	565	632	659	673	683	689	694	697	700	708	714	723
1.75	579	648	676	691	701	707	712	716	718	727	733	742
1.80	593	663	692	708	718	725	730	733	736	745	751	761
1.85	605	678	708	724	734	741	746	750	753	762	769	778
1.90	618	693	723	740	750	757	762	766	769	778	785	795
1.95	630	706	737	754	765	772	777	781	784	793	800	810
2.00	641	719	751	768	779	786	792	796	799	808	815	825
2.05	652	732	764	782	792	800	805	809	812	822	829	839
2.10	663	744	777	794	805	813	818	822	825	835	842	852
2.15	673	755	788	806	817	825	830	834	838	847	854	864
2.20	683	766	800	817	829	836	842	846	849	858	866	876
2.25	693	777	810	828	839	847	852	857	860	869	876	886
2.30	702	787	820	838	849	857	863	867	870	879	886	896
2.35	711	796	830	848	859	867	872	876	879	889	896	905
2.40	719	805	839	857	869	876	881	885	888	897	904	914
2.45	727	814	848	866	877	884	889	893	897	906	912	922
2.50	735	822	856	874	885	892	897	901	904	913	920	929
2.60	750	837	871	889	899	906	911	915	918	927	933	942
2.70	764	851	885	902	912	919	924	928	930	939	945	953
2.80	776	864	897	913	923	930	935	938	941	949	954	962
2.90	788	875	907	924	933	940	944	947	950	957	962	969
3.00	799	886	917	933	942	948	952	955	958	964	969	976
3.10	809	895	926	941	949	955	959	962	964	971	975	981
3.20	819	904	933	948	956	961	965	968	970	976	980	985
3.30	828	911	940	954	962	967	970	973	975	980	984	988
3.40	836	919	946	959	967	971	975	977	979	984	987	991
3.50	844	925	952	964	971	976	978	981	982	987	989	993
3.60	851	931	957	968	975	979	982	984	985	989	992	995
3.70	858	936	961	972	978	982	984	986	988	991	993	996
3.80	864	941	965	975	981	984	987	988	990	993	995	997
3.90	870	945	968	978	983	987	989	990	991	994	996	998
4.00	876	949	971	981	986	989	990	992	993	995	997	998
4.20	886	956	977	985	989	992	993	994	995	997	998	999
4.40	895	962	981	988	992	994	995	996	997	998	999	999
4.60	903	967	984	991	994	995	996	997	998	999	999	
4.80	910	972	987	993	995	997	997	998	998	999		
5.00	918	975	989	994	996	997	998	999	999	999		
5.50	930	982	993	997	998	999	999	999				
6.00	940	987	995	998	999	999						
6.50	949	990	997	999	999							
7.00	955	992	998	999								
8.00	965	995	999									
9.00	973	997	999									
10.00	978	998										
15.00	990											
20.00	994											
30.00	997											
40.00	999											
50.00	999											

C = 1.6

D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	327	354	364	370	373	375	377	378	379	382	384	388
1.05	348	378	390	396	400	403	405	406	407	410	413	417
1.10	369	402	415	422	427	430	432	433	435	438	441	445
1.15	390	426	440	448	453	456	459	460	462	466	469	474
1.20	410	449	465	473	479	482	485	487	488	493	497	502
1.25	429	472	489	498	504	508	511	513	515	520	523	529
1.30	448	494	512	522	528	533	536	538	540	546	550	556
1.35	466	515	535	546	552	557	560	563	565	571	575	582
1.40	484	536	557	568	576	581	584	587	589	595	600	607
1.45	501	556	578	590	598	603	607	610	612	619	624	631
1.50	517	575	599	612	620	625	629	632	635	642	647	655
1.55	533	594	619	632	641	646	651	654	656	664	669	677
1.60	549	612	638	652	661	667	671	674	677	685	691	699
1.65	563	629	656	671	680	686	691	694	697	705	711	720
1.70	578	646	674	689	698	705	710	713	716	724	731	740
1.75	592	662	691	706	716	723	727	731	734	743	749	758
1.80	605	677	707	723	733	740	745	748	751	760	767	776
1.85	617	692	722	738	749	756	761	765	768	777	783	793
1.90	630	706	737	753	764	771	776	780	783	792	799	809
1.95	642	719	751	768	778	786	791	795	798	807	814	824
2.00	653	732	764	781	792	799	805	809	812	821	828	838
2.05	664	744	777	794	805	812	818	822	825	834	841	851
2.10	674	756	789	806	817	825	830	834	837	847	854	864
2.15	684	767	800	818	829	836	842	846	849	858	865	875
2.20	694	778	811	829	840	847	853	857	860	869	876	886
2.25	703	788	821	839	850	858	863	867	870	879	886	896
2.30	712	798	831	849	860	867	873	877	880	889	898	906
2.35	721	807	840	858	869	876	882	886	889	898	905	914
2.40	729	815	849	867	878	885	890	894	897	906	913	922
2.45	737	824	857	875	886	893	898	902	905	914	920	930
2.50	745	832	865	883	893	900	906	909	912	921	927	936
2.60	759	846	879	897	907	914	919	923	926	934	940	948
2.70	772	860	892	909	919	926	931	934	937	945	950	958
2.80	785	872	904	920	930	936	941	944	947	954	959	967
2.90	796	883	914	930	939	945	949	953	955	962	967	973
3.00	807	893	923	938	947	953	957	960	962	968	973	979
3.10	817	901	931	946	954	959	963	966	968	974	978	983
3.20	826	910	938	952	960	965	969	971	973	979	982	987
3.30	834	917	945	958	965	970	973	976	978	983	986	990
3.40	842	924	950	963	970	974	978	980	981	986	989	992
3.50	850	930	956	967	974	978	981	983	984	988	991	994
3.60	857	935	960	971	977	981	984	986	987	991	993	996
3.70	863	940	964	975	980	984	986	988	989	992	994	997
3.80	870	945	968	978	983	986	988	990	991	994	996	997
3.90	875	949	971	980	985	988	990	992	993	995	997	998
4.00	881	953	974	983	987	990	992	993	994	996	997	999
4.20	890	959	979	986	990	993	994	995	996	997	998	999
4.40	899	965	982	989	993	995	996	997	997	998	999	
4.60	907	970	986	992	994	996	997	998	998	999	999	
4.80	914	974	988	993	996	997	998	998	999	999		
5.00	920	977	990	995	997	998	998	999	999			
5.50	933	983	994	997	998	999	999	999				
6.00	943	988	996	998	999	999						
6.50	951	991	997	999								
7.00	957	993	998	999								
8.00	967	996	999									
9.00	974	997										
10.00	979	998										
15.00	990											
20.00	995											
30.00	998											
40.00	999											
50.00	999											

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D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	338	367	378	384	387	390	392	393	394	397	400	403
1.05	360	392	404	411	415	418	420	421	422	426	429	433
1.10	381	416	430	437	442	445	447	449	450	454	457	462
1.15	401	440	455	463	468	472	474	476	478	482	485	490
1.20	421	463	480	488	494	498	501	503	504	509	513	518
1.25	441	486	504	513	519	524	527	529	531	536	540	546
1.30	460	508	527	537	544	548	552	554	556	562	566	572
1.35	478	529	550	561	568	573	576	579	581	587	592	598
1.40	496	550	572	584	591	596	600	603	605	611	616	623
1.45	513	570	593	605	613	619	623	626	628	635	640	648
1.50	529	589	613	626	635	640	644	648	650	657	663	671
1.55	545	608	633	647	655	661	666	669	671	679	685	693
1.60	560	625	652	666	675	681	686	689	692	700	706	714
1.65	575	642	670	685	694	700	705	709	711	719	726	735
1.70	589	659	687	702	712	719	723	727	730	738	745	754
1.75	603	675	704	719	729	736	741	745	748	756	763	772
1.80	616	690	719	736	746	753	758	761	764	773	780	789
1.85	628	704	734	751	761	768	773	777	780	789	796	806
1.90	640	718	749	766	776	783	788	792	795	804	811	821
1.95	652	731	762	779	790	797	803	807	810	819	826	836
2.00	663	743	775	793	803	811	816	820	823	832	839	849
2.05	674	755	788	805	816	823	829	833	836	845	852	862
2.10	684	767	799	817	828	835	840	845	848	857	864	874
2.15	694	777	810	828	839	846	852	856	859	868	875	885
2.20	703	788	821	839	849	857	862	866	869	878	885	895
2.25	713	797	831	848	859	867	872	876	879	888	895	905
2.30	721	807	840	858	869	876	881	885	888	897	904	913
2.35	730	816	849	867	877	885	890	894	897	906	912	921
2.40	738	824	857	875	886	893	898	902	905	913	920	929
2.45	745	832	865	883	893	900	905	909	912	921	927	936
2.50	753	840	873	890	900	907	912	916	919	927	934	942
2.60	767	854	887	903	914	920	925	929	932	939	945	953
2.70	780	867	899	915	925	931	936	940	942	950	955	963
2.80	792	878	910	926	935	941	946	949	951	958	963	970
2.90	803	889	919	935	944	950	954	957	959	966	970	976
3.00	814	898	928	943	951	957	961	963	966	972	976	981
3.10	823	907	936	950	958	963	967	969	971	977	981	986
3.20	832	915	943	956	963	968	972	974	976	981	984	989
3.30	840	922	949	961	968	973	976	978	980	984	988	991
3.40	848	928	954	966	973	977	980	982	983	987	990	993
3.50	855	934	959	970	976	980	983	985	986	990	992	995
3.60	862	939	963	974	980	983	986	987	988	992	994	996
3.70	869	944	967	977	982	986	988	989	990	993	995	997
3.80	874	948	970	980	985	988	990	991	992	995	996	998
3.90	880	952	973	982	987	989	991	993	993	996	997	998
4.00	885	956	976	984	988	991	993	994	995	997	998	999
4.20	895	962	980	988	991	993	995	996	996	998	999	999
4.40	903	967	984	990	993	995	996	997	997	999	999	
4.60	911	972	987	992	995	996	997	998	998	999		
4.80	917	975	989	994	996	997	998	999	999	999		
5.00	923	979	991	995	997	998	999	999	999			
5.50	936	984	994	997	999	999	999					
6.00	945	989	996	998	999							
6.50	953	991	998	999								
7.00	959	993	998	999								
8.00	968	996	999									
9.00	975	997										
10.00	980	998										
15.00	991											
20.00	995											
30.00	998											
40.00	999											
50.00	999											

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c \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	349	379	391	397	401	403	405	407	408	411	414	418
1.05	370	404	417	424	428	431	434	435	437	440	443	447
1.10	392	428	443	451	456	459	461	463	465	469	472	477
1.15	412	452	468	477	482	486	489	491	492	497	500	506
1.20	432	476	493	502	508	512	515	517	519	524	528	534
1.25	452	498	517	527	534	538	541	543	545	551	555	561
1.30	471	520	540	551	558	563	566	569	571	577	581	588
1.35	489	542	563	575	582	587	590	593	595	602	606	613
1.40	506	562	585	597	605	610	614	617	619	626	631	638
1.45	523	582	606	619	627	632	636	639	642	649	654	662
1.50	540	601	626	640	648	654	658	661	664	671	677	685
1.55	555	620	646	660	668	674	679	682	685	692	698	707
1.60	571	637	664	679	688	694	699	702	705	713	719	728
1.65	585	654	682	697	706	713	718	721	724	732	738	747
1.70	599	670	699	714	724	731	736	739	742	751	757	766
1.75	613	686	715	731	741	748	753	757	760	768	775	784
1.80	626	700	731	747	757	764	769	773	776	785	791	801
1.85	638	715	745	762	772	779	784	788	791	800	807	817
1.90	650	728	759	776	787	794	799	803	805	815	822	832
1.95	661	741	773	790	800	807	813	817	820	829	836	846
2.00	672	753	785	802	813	820	826	830	833	842	849	859
2.05	683	765	797	815	825	833	838	842	845	854	861	871
2.10	693	776	808	826	837	844	849	853	856	866	872	882
2.15	703	786	819	837	847	855	860	864	867	876	883	893
2.20	712	796	829	847	858	865	870	874	877	886	893	902
2.25	721	806	839	856	867	874	880	884	887	895	902	911
2.30	729	815	848	865	876	883	888	892	895	904	911	920
2.35	738	824	857	874	884	892	897	901	904	912	919	927
2.40	745	832	865	882	892	899	904	906	911	920	926	935
2.45	753	839	872	889	900	907	912	915	918	926	933	941
2.50	760	847	880	896	907	913	918	922	925	933	939	947
2.60	774	861	893	909	919	926	930	934	936	944	950	957
2.70	787	873	904	920	930	936	941	944	947	954	959	966
2.80	799	884	915	930	939	945	950	953	955	962	967	973
2.90	809	894	924	939	948	953	957	960	962	969	973	979
3.00	820	903	932	947	955	960	964	966	969	974	978	983
3.10	829	911	940	953	961	966	969	972	974	979	983	987
3.20	838	919	946	959	966	971	974	976	978	983	986	990
3.30	846	926	952	964	971	975	978	980	982	986	989	992
3.40	853	932	957	968	975	979	982	983	985	989	991	994
3.50	860	937	961	972	978	982	984	986	987	991	993	996
3.60	867	942	965	976	981	985	987	988	990	993	995	997
3.70	873	947	969	979	984	987	989	990	991	994	996	998
3.80	879	951	972	981	986	989	991	992	993	995	997	998
3.90	884	955	975	983	988	990	992	993	994	996	997	999
4.00	889	958	977	985	989	992	993	994	995	997	998	999
4.20	898	964	982	989	992	994	995	996	997	998	999	
4.40	906	969	985	991	994	996	997	997	998	999	999	
4.60	914	973	988	993	996	997	998	998	998	999		
4.80	920	977	990	995	997	998	998	999	999			
5.00	926	980	992	996	997	998	999	999	999			
5.50	938	985	995	998	999	999	999					
6.00	947	989	997	999	999							
6.50	955	992	998	999								
7.00	961	994	998	999								
8.00	970	996	999									
9.00	976	998										
10.00	980	998										
15.00	991											
20.00	995											
30.00	998											
40.00	999											
50.00	999											

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c \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	358	390	402	409	413	416	418	420	421	424	427	431
1.05	380	415	429	436	441	444	447	448	450	454	457	461
1.10	402	440	455	463	468	472	474	476	478	482	486	491
1.15	422	464	481	490	495	499	502	504	506	510	514	519
1.20	442	487	505	515	521	525	528	531	532	538	542	548
1.25	462	510	529	540	546	551	554	557	559	564	569	575
1.30	481	532	553	564	571	576	579	582	584	590	595	601
1.35	499	553	575	587	594	600	603	606	608	615	620	627
1.40	516	574	597	609	617	623	627	629	632	639	644	651
1.45	533	593	618	631	639	645	649	652	654	662	667	675
1.50	549	612	638	651	660	666	670	673	676	683	689	697
1.55	565	630	657	671	680	686	691	694	697	704	710	719
1.60	580	648	675	690	699	706	710	714	716	724	731	739
1.65	594	664	693	708	717	724	729	732	735	743	750	759
1.70	608	680	709	725	735	742	746	750	753	761	768	777
1.75	622	696	725	741	751	758	763	767	770	778	785	794
1.80	634	710	740	757	767	774	779	783	786	795	801	811
1.85	647	724	755	772	782	789	794	798	801	810	817	826
1.90	658	737	769	785	796	803	808	812	815	824	831	840
1.95	670	750	782	799	809	816	822	825	829	838	844	854
2.00	680	762	794	811	822	829	834	838	841	850	857	867
2.05	691	773	805	823	833	841	846	850	853	862	869	878
2.10	701	784	817	834	845	852	857	861	864	873	880	889
2.15	710	794	827	844	855	862	867	871	874	883	890	899
2.20	719	804	837	854	865	872	877	881	884	893	899	909
2.25	728	813	846	863	874	881	886	890	893	902	908	917
2.30	737	822	855	872	883	890	895	898	901	910	916	925
2.35	745	830	863	880	891	898	903	906	909	918	924	933
2.40	752	838	871	888	898	905	910	914	917	925	931	939
2.45	760	846	878	895	905	912	917	920	923	931	937	945
2.50	767	853	885	902	912	918	923	927	930	937	943	951
2.60	780	866	898	914	924	930	935	938	941	948	953	961
2.70	793	878	909	925	934	940	945	948	950	957	962	969
2.80	804	889	919	934	943	949	953	956	958	965	969	975
2.90	815	899	928	943	951	956	960	963	965	971	975	981
3.00	825	908	936	950	958	963	966	969	971	976	980	985
3.10	834	915	943	956	964	968	972	974	976	981	984	989
3.20	842	923	949	962	969	973	976	978	980	984	987	991
3.30	850	929	955	966	973	977	980	982	983	987	990	993
3.40	858	935	959	970	977	980	983	985	986	990	992	995
3.50	864	940	964	974	980	983	986	987	989	992	994	996
3.60	871	945	967	977	983	986	988	989	991	993	995	997
3.70	877	949	971	980	985	988	990	991	992	995	996	998
3.80	882	953	974	983	987	990	991	993	994	996	997	998
3.90	888	957	976	985	989	991	993	994	995	997	998	999
4.00	893	960	979	986	990	993	994	995	996	997	998	999
4.20	901	966	983	989	993	995	996	997	997	998	999	
4.40	909	971	986	992	995	996	997	998	998	999	999	
4.60	916	975	989	994	996	997	998	998	999	999		
4.80	923	979	991	995	997	998	998	999	999			
5.00	928	981	992	996	998	998	999	999	999			
5.50	940	986	995	998	999	999						
6.00	949	990	997	999	999							
6.50	956	992	998	999								
7.00	962	994	999									
8.00	971	996	999									
9.00	977	998										
10.00	981	998										
15.00	991											
20.00	995											
30.00	998											
40.00	999											
50.00	999											

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D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	367	401	414	421	425	426	430	432	433	437	440	444
1.05	389	426	440	448	453	456	459	460	462	466	469	474
1.10	411	451	466	475	480	484	487	489	490	495	498	503
1.15	431	475	492	501	507	511	514	516	518	523	527	532
1.20	452	498	517	527	533	537	540	543	545	550	554	560
1.25	471	521	541	551	558	563	566	569	571	577	581	587
1.30	490	543	564	575	582	587	591	594	596	602	607	614
1.35	508	564	586	598	606	611	615	618	620	627	632	639
1.40	525	584	608	620	628	634	638	641	643	650	656	663
1.45	542	604	628	642	650	656	660	663	666	673	678	686
1.50	558	622	648	662	671	677	681	684	687	694	700	709
1.55	574	640	667	681	691	697	701	705	707	715	721	730
1.60	589	657	685	700	709	716	720	724	727	735	741	750
1.65	603	674	702	718	727	734	739	742	745	753	760	769
1.70	617	690	719	735	744	751	756	760	763	771	777	787
1.75	630	704	734	751	761	767	772	776	779	788	794	803
1.80	642	719	749	766	776	783	788	792	795	803	810	819
1.85	654	732	763	780	790	797	802	806	809	818	825	834
1.90	666	745	777	794	804	811	816	820	823	832	839	848
1.95	677	758	789	806	817	824	829	833	836	845	852	861
2.00	688	769	801	819	829	836	841	845	848	857	864	873
2.05	698	780	813	830	841	848	853	857	860	869	875	885
2.10	708	791	824	841	851	858	864	868	871	879	886	895
2.15	717	801	834	851	861	869	874	878	881	889	896	905
2.20	726	811	843	860	871	878	883	887	890	898	905	914
2.25	735	820	852	869	880	887	892	896	899	907	913	922
2.30	743	828	861	878	888	895	900	904	907	915	921	930
2.35	751	836	869	886	896	903	908	911	914	922	928	937
2.40	759	844	876	893	903	910	915	918	921	929	935	943
2.45	766	852	884	900	910	917	921	925	928	935	941	949
2.50	773	858	890	907	916	923	927	931	934	941	947	954
2.60	786	871	903	918	929	934	938	942	944	951	957	964
2.70	798	883	913	929	938	944	948	951	953	960	965	971
2.80	809	893	923	938	946	952	956	959	961	967	972	977
2.90	820	903	932	946	954	959	963	965	967	973	977	982
3.00	829	911	939	952	960	965	969	971	973	978	982	986
3.10	838	919	946	958	966	970	973	976	978	982	985	990
3.20	847	926	952	964	970	975	978	980	981	986	989	992
3.30	854	932	957	968	975	979	981	983	985	988	991	994
3.40	862	938	961	972	978	982	984	986	987	991	993	996
3.50	868	943	966	976	981	985	987	988	989	992	994	997
3.60	874	947	969	979	984	987	989	990	991	994	996	998
3.70	880	952	972	981	986	989	991	992	993	995	997	998
3.80	886	955	975	984	988	991	992	993	994	996	997	999
3.90	891	959	978	986	990	992	993	994	995	997	998	999
4.00	896	962	980	987	991	993	994	995	996	998	998	999
4.20	904	967	984	990	993	995	996	997	997	998	999	
4.40	912	972	987	992	995	996	997	998	998	999	999	
4.60	919	976	989	994	996	997	998	998	999	999		
4.80	925	979	991	995	997	998	999	999	999			
5.00	930	982	993	996	998	999	999	999	999			
5.50	942	987	995	998	999	999						
6.00	950	990	997	999	999							
6.50	957	993	998	999								
7.00	963	994	999									
8.00	971	997	999									
9.00	977	998										
10.00	982	999										
15.00	992											
20.00	995											
30.00	998											
40.00	999											
50.00	999											

C = 2.2

D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	384	419	433	441	446	449	451	453	454	459	462	466
1.05	406	445	460	469	474	477	480	482	484	488	491	496
1.10	427	470	487	496	501	505	508	510	512	517	521	526
1.15	448	494	512	522	528	532	535	538	539	545	549	555
1.20	468	517	537	547	554	558	562	564	566	572	576	582
1.25	487	540	560	572	579	584	587	590	592	598	602	609
1.30	506	561	583	595	603	608	612	614	617	623	628	635
1.35	524	582	605	618	626	631	635	638	640	647	652	660
1.40	541	602	626	640	648	653	658	661	663	670	676	683
1.45	558	621	647	660	669	675	679	682	685	692	698	706
1.50	574	640	666	680	689	695	700	703	706	713	719	727
1.55	589	657	684	699	708	715	719	723	725	733	739	748
1.60	604	674	702	717	727	733	738	741	744	752	758	767
1.65	618	690	719	734	744	751	755	759	762	770	776	785
1.70	631	705	735	751	760	767	772	776	779	787	793	802
1.75	644	720	750	766	776	783	788	791	794	803	809	818
1.80	656	734	764	781	791	798	803	806	809	818	824	834
1.85	668	747	778	794	805	811	816	820	823	832	838	848
1.90	679	759	791	807	818	825	830	833	836	845	852	861
1.95	690	771	803	820	830	837	842	846	849	857	864	873
2.00	700	782	814	831	841	848	854	857	860	869	875	885
2.05	710	793	825	842	852	859	864	868	871	880	886	895
2.10	720	803	835	852	863	869	875	878	881	890	896	905
2.15	729	813	845	862	872	879	884	888	891	899	905	914
2.20	738	822	854	871	881	888	893	897	899	908	914	922
2.25	746	831	863	879	889	896	901	905	908	916	922	930
2.30	754	839	871	887	897	904	909	912	915	923	929	937
2.35	762	847	878	895	904	911	916	919	922	930	936	944
2.40	769	854	885	902	911	918	922	926	929	936	942	950
2.45	776	861	892	908	918	924	929	932	935	942	947	955
2.50	783	868	899	914	924	930	934	937	940	947	953	960
2.60	796	880	910	925	934	940	944	947	950	957	961	968
2.70	807	891	920	935	943	949	953	956	958	964	969	975
2.80	818	901	929	943	951	957	960	963	965	971	975	980
2.90	828	909	937	951	958	963	967	969	971	976	980	985
3.00	837	917	944	957	964	969	972	974	976	981	984	988
3.10	846	925	950	962	969	974	976	979	980	985	988	991
3.20	854	931	956	967	974	978	980	982	984	988	990	993
3.30	861	937	961	971	977	981	983	985	987	990	992	995
3.40	868	942	965	975	981	984	986	988	989	992	994	996
3.50	875	947	969	978	983	986	988	990	991	994	995	997
3.60	881	951	972	981	986	988	990	992	993	995	996	998
3.70	886	955	975	983	988	990	992	993	994	996	997	999
3.80	891	959	978	985	989	992	993	994	995	997	998	999
3.90	896	962	980	987	991	993	994	995	996	997	998	999
4.00	901	965	982	989	992	994	995	996	997	998	999	999
4.20	909	970	985	991	994	996	997	997	998	999	999	
4.40	916	974	988	993	996	997	998	998	998	999		
4.60	923	978	990	995	997	998	998	999	999			
4.80	929	981	992	996	998	998	999	999	999			
5.00	934	983	993	997	998	999	999	999				
5.50	945	988	996	998	999	999						
6.00	953	991	997	999								
6.50	960	993	998	999								
7.00	965	995	999									
8.00	973	997	999									
9.00	978	998										
10.00	982	999										
15.00	992											
20.00	996											
30.00	998											
40.00	999											
50.00	999											

C = 2.4

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	398	436	451	459	464	467	470	472	473	477	481	485
1.05	420	461	478	486	492	496	498	500	502	507	510	516
1.10	441	486	504	513	519	523	526	529	530	535	539	545
1.15	462	510	529	539	546	550	553	556	558	563	567	573
1.20	482	533	554	564	571	576	579	582	584	590	594	601
1.25	501	555	577	589	596	601	604	607	609	616	620	627
1.30	520	577	600	612	620	625	629	631	634	640	645	652
1.35	538	597	621	634	642	648	652	655	657	664	669	677
1.40	555	617	642	655	664	670	674	677	679	686	692	700
1.45	571	636	662	676	684	690	695	698	700	708	714	722
1.50	587	654	681	695	704	710	715	718	721	728	734	742
1.55	602	671	699	714	723	729	734	737	740	748	754	762
1.60	616	688	716	731	741	747	752	755	758	766	772	781
1.65	630	703	732	748	757	764	769	772	775	783	789	798
1.70	643	718	748	764	773	780	785	788	791	800	806	815
1.75	656	732	762	778	788	795	800	804	807	815	821	830
1.80	668	746	776	792	802	809	814	818	821	829	836	845
1.85	679	758	789	806	816	823	828	831	834	843	849	859
1.90	690	770	802	818	828	835	840	844	847	855	862	871
1.95	701	782	813	830	840	847	852	856	859	867	874	883
2.00	711	793	824	841	851	856	863	867	870	878	884	893
2.05	721	803	835	851	862	868	873	877	880	888	895	903
2.10	730	813	845	861	871	878	883	887	890	898	904	913
2.15	739	822	854	870	880	887	892	896	898	907	913	921
2.20	747	831	863	879	889	896	900	904	907	915	921	929
2.25	755	839	871	887	897	903	908	912	914	922	928	936
2.30	763	847	879	895	904	911	915	919	922	929	935	943
2.35	771	855	886	902	911	918	922	925	928	936	941	949
2.40	778	862	893	908	918	924	928	932	934	941	947	954
2.45	785	868	899	914	924	930	934	937	940	947	952	959
2.50	791	875	905	920	929	935	939	943	945	952	957	964
2.60	804	886	918	931	939	945	949	952	954	961	965	971
2.70	815	897	926	940	948	953	957	960	962	968	972	978
2.80	825	908	934	947	955	960	964	966	968	974	978	983
2.90	835	915	942	954	962	966	970	972	974	979	982	987
3.00	844	922	948	960	967	972	975	977	978	983	986	990
3.10	852	929	954	965	972	976	979	981	982	986	989	992
3.20	860	935	959	970	976	980	982	984	985	989	991	994
3.30	867	941	964	974	979	983	985	987	988	991	993	996
3.40	874	946	968	977	982	985	988	989	990	993	995	997
3.50	880	950	971	980	985	988	990	991	992	994	996	998
3.60	886	954	974	983	987	990	991	992	993	996	997	998
3.70	891	958	977	985	989	991	993	994	995	996	998	999
3.80	896	961	979	987	990	993	994	995	996	997	998	999
3.90	901	964	981	988	992	994	995	996	996	998	999	999
4.00	905	967	983	990	993	995	996	997	997	998	999	
4.20	913	972	987	992	995	996	997	998	998	999	999	
4.40	920	976	989	994	996	997	998	998	999	999		
4.60	926	979	991	995	997	998	999	999	999			
4.80	932	982	993	996	998	999	999	999	999			
5.00	937	984	994	997	998	999	999	999				
5.50	947	989	996	998	999							
6.00	955	992	998	999								
6.50	961	994	998	999								
7.00	967	995	999									
8.00	974	997	999									
9.00	979	998										
10.00	983	999										
15.00	992											
20.00	996											
30.00	998											
40.00	999											
50.00	999											

C = 2.6

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	410	450	466	474	479	483	486	488	489	494	497	502
1.05	432	476	493	502	509	511	514	516	518	523	527	532
1.10	454	500	519	529	535	539	542	544	546	551	555	561
1.15	474	524	544	554	561	565	569	571	573	579	583	589
1.20	494	547	568	579	586	591	594	597	599	605	610	616
1.25	513	569	591	603	610	616	619	622	624	630	635	642
1.30	532	590	614	626	634	639	643	646	648	655	660	667
1.35	549	611	635	648	656	661	665	669	671	678	683	691
1.40	566	630	655	669	677	683	687	690	693	700	705	713
1.45	582	648	675	689	697	703	708	711	713	721	726	734
1.50	598	666	693	708	717	723	727	730	733	741	746	755
1.55	613	683	711	726	735	741	746	749	752	760	765	774
1.60	627	699	728	743	752	758	763	767	769	777	783	792
1.65	640	714	743	759	768	775	780	783	786	794	800	809
1.70	653	729	758	774	784	790	795	799	802	810	816	825
1.75	666	743	773	789	798	805	810	814	816	825	831	840
1.80	678	756	786	802	812	819	824	827	830	838	845	854
1.85	689	768	799	815	825	832	837	840	843	851	858	867
1.90	700	780	811	827	837	844	849	852	855	864	870	879
1.95	710	791	822	838	848	855	860	864	867	875	881	890
2.00	720	802	833	849	859	866	871	874	877	885	892	900
2.05	729	812	843	859	869	876	881	884	887	895	901	910
2.10	738	821	852	869	878	885	890	893	896	904	910	919
2.15	747	830	861	877	887	894	898	902	905	913	919	927
2.20	755	838	870	886	895	902	906	910	913	920	926	934
2.25	763	846	877	893	903	909	914	917	920	927	933	941
2.30	771	854	885	900	910	916	921	924	927	934	940	947
2.35	778	861	892	907	917	923	927	930	933	940	945	953
2.40	785	869	898	914	923	929	933	936	939	946	951	958
2.45	792	875	904	919	928	934	938	942	944	951	956	963
2.50	798	881	910	925	934	939	943	947	949	955	960	967
2.60	810	892	921	935	943	949	952	955	957	964	968	974
2.70	821	902	930	943	951	956	960	963	965	970	974	980
2.80	831	911	938	951	958	963	966	969	971	976	980	984
2.90	841	919	945	957	964	969	972	974	976	981	984	988
3.00	849	926	951	963	969	974	977	979	980	984	987	991
3.10	857	933	957	968	974	978	980	982	984	987	990	993
3.20	865	939	962	972	978	981	984	985	987	990	992	995
3.30	872	944	966	976	981	984	986	988	989	992	994	996
3.40	878	949	970	979	984	987	989	990	991	994	995	997
3.50	884	953	973	981	986	989	991	992	993	995	996	998
3.60	890	957	976	984	988	990	992	993	994	996	997	999
3.70	895	960	978	986	990	992	993	994	995	997	998	999
3.80	900	963	981	988	991	993	995	995	996	998	998	999
3.90	904	966	983	989	992	994	995	996	997	998	999	999
4.00	908	969	985	991	993	995	996	997	997	998	999	
4.20	916	974	987	993	995	997	997	998	998	999	999	
4.40	923	977	990	994	996	998	998	999	999	999		
4.60	929	980	992	996	997	998	999	999	999			
4.80	934	983	993	997	998	999	999	999	999			
5.00	939	985	994	997	998	999	999					
5.50	949	989	997	999	999							
6.00	957	992	998	999								
6.50	963	994	999	999								
7.00	968	996	999									
8.00	975	997										
9.00	980	998										
10.00	984	999										
15.00	993											
20.00	996											
30.00	998											
40.00	999											
50.00	999											

C = 2.8

D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	421	463	479	488	493	497	499	502	503	508	511	516
1.05	443	488	506	515	521	525	528	530	532	537	541	546
1.10	465	513	531	542	548	552	555	558	560	565	569	575
1.15	485	536	556	567	574	579	582	584	586	592	596	603
1.20	505	559	580	592	599	604	607	610	612	618	623	629
1.25	524	581	603	615	623	628	632	634	637	643	648	655
1.30	542	602	625	638	646	651	655	658	660	667	672	679
1.35	559	622	646	659	668	673	677	680	683	689	695	702
1.40	576	641	666	680	688	694	698	701	704	711	717	724
1.45	592	659	685	700	708	714	718	722	724	732	737	745
1.50	607	676	704	718	727	733	738	741	743	751	757	765
1.55	622	693	721	736	745	751	756	759	762	769	775	784
1.60	636	709	737	752	762	768	773	776	779	787	793	801
1.65	649	724	753	768	778	784	789	792	795	803	809	818
1.70	662	738	767	783	793	799	804	807	810	818	824	833
1.75	674	751	781	797	807	813	818	822	825	833	839	847
1.80	686	764	794	810	820	827	831	835	838	846	852	861
1.85	697	776	807	823	833	839	844	848	850	859	865	873
1.90	708	788	818	834	844	851	856	859	862	870	876	885
1.95	718	799	829	845	855	862	867	870	873	881	887	896
2.00	727	809	840	856	866	872	877	881	883	891	897	906
2.05	737	819	849	865	875	882	886	890	893	901	907	915
2.10	746	828	859	875	884	891	895	899	902	909	915	923
2.15	754	836	867	883	893	899	904	907	910	917	923	931
2.20	762	845	875	891	900	907	911	915	917	925	930	938
2.25	770	852	883	898	908	914	918	922	924	932	937	945
2.30	777	860	890	905	915	921	925	928	931	938	943	951
2.35	784	867	897	912	921	927	931	934	937	944	949	956
2.40	791	873	903	918	927	933	937	940	942	949	954	961
2.45	798	880	909	923	932	938	942	945	947	954	959	965
2.50	804	885	914	929	937	943	947	950	952	958	963	969
2.60	816	896	924	938	946	952	955	958	960	966	970	976
2.70	826	906	933	946	954	959	963	965	967	972	976	981
2.80	836	915	941	954	961	965	969	971	973	978	981	986
2.90	845	922	948	960	966	971	974	976	978	982	985	989
3.00	854	929	954	965	971	975	978	980	982	986	988	992
3.10	862	936	959	970	976	979	982	984	985	988	991	994
3.20	869	941	964	974	979	982	985	986	988	991	993	995
3.30	876	946	968	977	982	985	987	989	990	993	995	997
3.40	882	951	971	980	985	988	989	991	992	994	996	998
3.50	888	955	974	983	987	990	991	992	993	995	997	998
3.60	893	959	977	985	989	991	993	994	994	996	998	999
3.70	898	962	980	987	990	993	994	995	996	997	998	999
3.80	903	965	982	988	992	994	995	996	996	998	999	999
3.90	907	968	984	990	993	995	996	997	997	998	999	
4.00	911	970	985	991	994	996	996	997	998	999	999	
4.20	919	975	988	993	996	997	998	998	998	999		
4.40	925	978	990	995	997	998	998	999	999	999		
4.60	931	981	992	996	998	998	999	999	999			
4.80	936	984	994	997	998	999	999	999				
5.00	941	986	995	998	999	999	999					
5.50	951	990	997	999	999							
6.00	958	993	998	999								
6.50	964	995	999									
7.00	969	998	999									
8.00	976	997										
9.00	981	998										
10.00	984	999										
15.00	993											
20.00	996											
30.00	998											
40.00	999											
50.00	999											

C = 3.0

D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	431	474	490	499	505	509	512	514	515	520	524	529
1.05	453	499	517	527	533	537	540	542	544	549	553	559
1.10	474	523	543	553	559	564	567	569	571	577	581	587
1.15	495	547	567	578	585	590	593	596	598	604	608	614
1.20	514	569	591	603	610	615	618	621	623	629	634	640
1.25	533	591	614	626	633	639	642	645	647	654	659	665
1.30	551	612	635	648	656	661	665	668	670	677	682	689
1.35	568	631	656	669	678	683	687	690	693	699	705	712
1.40	585	650	676	690	698	704	708	711	713	721	726	734
1.45	601	668	695	709	717	723	728	731	733	741	746	754
1.50	616	685	712	727	736	742	746	750	752	760	765	773
1.55	630	702	729	744	753	759	764	767	770	778	783	792
1.60	644	717	745	760	770	776	781	784	787	795	800	809
1.65	657	732	761	776	785	792	796	800	803	810	816	825
1.70	670	746	775	790	800	806	811	815	817	825	831	840
1.75	682	759	789	804	814	820	825	829	831	839	845	854
1.80	693	771	801	817	827	833	838	842	844	852	858	867
1.85	704	783	813	829	839	845	850	854	857	865	871	879
1.90	714	794	825	841	850	857	862	865	868	876	882	890
1.95	724	805	835	851	861	868	872	876	879	886	892	901
2.00	734	815	845	861	871	877	882	886	888	896	902	910
2.05	743	824	855	871	880	887	891	895	898	905	911	919
2.10	752	833	864	879	889	895	900	903	906	914	919	927
2.15	760	842	872	888	897	903	908	911	914	921	927	935
2.20	768	850	880	895	905	911	915	919	921	929	934	942
2.25	776	857	887	903	912	918	922	925	928	935	940	948
2.30	783	865	894	909	918	924	929	932	934	941	946	953
2.35	790	871	901	916	924	930	934	938	940	947	952	959
2.40	796	878	907	921	930	936	940	943	945	952	957	963
2.45	803	884	913	927	935	941	945	948	950	956	961	967
2.50	809	890	918	932	940	946	949	952	954	961	965	971
2.60	820	900	925	941	949	954	958	960	962	968	972	978
2.70	831	909	936	949	956	961	965	967	969	974	978	983
2.80	841	918	944	956	963	967	970	973	974	979	982	987
2.90	850	925	950	962	968	972	975	977	979	983	986	990
3.00	858	932	956	967	973	977	979	981	983	987	989	992
3.10	865	938	961	971	977	980	983	985	986	989	992	994
3.20	872	944	965	975	980	984	986	987	988	991	993	996
3.30	879	949	969	978	983	986	988	990	991	993	995	997
3.40	885	953	973	981	986	988	990	991	992	995	996	998
3.50	891	957	976	984	988	990	992	993	994	996	997	998
3.60	896	960	978	986	989	992	993	994	995	997	998	999
3.70	901	964	981	988	991	993	994	995	996	997	998	999
3.80	905	967	983	989	992	994	995	996	997	998	999	999
3.90	910	969	985	990	993	995	996	997	997	998	999	
4.00	914	972	986	992	994	996	997	997	998	999	999	
4.20	921	976	989	994	996	997	998	998	999	999		
4.40	927	979	991	995	997	998	998	999	999			
4.60	933	982	993	996	998	998	999	999	999			
4.80	939	985	994	997	998	999	999	999				
5.00	943	987	995	998	999	999	999					
5.50	952	990	997	999	999							
6.00	959	993	998	999								
6.50	965	996	999									
7.00	970	998	999									
8.00	977	998										
9.00	981	998										
10.00	985	999										
15.00	993											
20.00	996											
30.00	998											
40.00	999											
50.00	999											

C = 3.4

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	447	492	510	519	525	529	531	534	535	540	544	549
1.05	469	517	536	546	552	556	559	562	563	569	573	578
1.10	490	541	561	572	578	583	586	588	590	596	600	606
1.15	511	564	585	597	603	608	612	614	616	622	626	633
1.20	530	586	609	620	628	633	636	639	641	647	652	658
1.25	548	608	631	643	651	656	660	662	665	671	676	683
1.30	566	628	652	665	673	678	682	685	687	694	699	706
1.35	583	647	672	685	694	699	703	706	708	715	720	728
1.40	599	666	691	705	713	719	723	726	729	736	741	749
1.45	615	683	710	724	732	738	742	745	748	755	761	768
1.50	630	700	727	741	750	756	760	764	766	773	779	787
1.55	644	715	743	758	767	773	777	781	783	791	796	805
1.60	657	730	759	773	783	789	793	797	799	807	813	821
1.65	670	745	773	788	798	804	808	812	815	822	828	836
1.70	682	758	787	802	812	818	823	826	829	836	842	851
1.75	694	771	800	815	825	831	836	839	842	850	856	864
1.80	705	783	812	828	837	844	848	852	854	862	868	876
1.85	716	794	824	839	849	855	860	863	866	874	880	888
1.90	726	805	835	850	860	866	871	874	877	885	890	899
1.95	735	815	845	860	870	876	881	884	887	895	900	908
2.00	745	825	855	870	879	885	890	894	896	904	910	917
2.05	753	834	864	879	888	895	899	902	905	912	918	926
2.10	762	842	872	887	897	903	907	910	913	920	926	933
2.15	770	850	880	895	904	910	915	918	920	928	933	940
2.20	778	858	887	902	911	917	922	925	927	934	939	947
2.25	785	865	894	909	918	924	928	931	934	940	945	952
2.30	792	872	901	916	924	930	934	937	939	946	951	956
2.35	799	879	907	921	930	936	940	942	945	951	956	962
2.40	805	885	913	927	935	941	945	947	950	956	961	967
2.45	811	891	918	932	940	945	949	952	954	960	965	971
2.50	817	896	923	937	945	950	954	956	958	964	968	974
2.60	828	906	932	945	953	958	961	964	966	971	975	980
2.70	838	915	940	953	960	964	968	970	972	977	980	985
2.80	848	923	947	959	966	970	973	975	977	981	984	988
2.90	856	930	954	965	971	975	977	979	981	985	988	991
3.00	864	936	959	969	975	979	981	983	984	988	990	993
3.10	871	942	964	973	979	982	984	986	987	990	993	995
3.20	878	947	968	977	982	985	987	989	990	992	994	996
3.30	884	952	971	980	985	987	989	991	992	994	996	997
3.40	890	956	975	983	987	989	991	992	993	995	997	998
3.50	896	960	977	985	989	991	993	994	994	996	997	999
3.60	901	963	980	987	990	993	994	995	995	997	998	999
3.70	905	966	982	989	992	994	995	996	996	998	999	999
3.80	910	969	984	990	993	995	996	997	997	998	999	
3.90	914	971	986	991	994	996	997	997	998	999	999	
4.00	918	974	987	992	995	996	997	998	998	999	999	
4.20	925	977	990	994	996	997	998	998	999	999		
4.40	931	981	992	996	997	998	999	999	999			
4.60	936	983	993	997	998	999	999	999	999			
4.80	941	986	994	997	998	999	999					
5.00	945	988	995	998	999	999						
5.50	954	991	997	999	999							
6.00	961	993	998	999								
6.50	967	995	999									
7.00	971	996	999									
8.00	978	998										
9.00	982	999										
10.00	986	999										
15.00	994											
20.00	996											
30.00	998											
40.00	999											
50.00	999											

C = 3.8

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	461	507	525	534	540	544	547	549	551	556	560	565
1.05	482	532	551	561	567	571	574	577	579	584	588	593
1.10	503	555	576	586	593	597	601	603	605	611	615	621
1.15	523	578	599	611	618	622	626	628	630	636	641	647
1.20	542	600	622	634	641	646	650	653	655	661	665	672
1.25	561	621	644	656	664	669	673	676	678	684	689	696
1.30	578	641	665	677	685	691	695	697	700	706	711	718
1.35	595	659	684	698	706	711	715	718	721	727	732	740
1.40	611	677	703	717	725	731	735	738	740	747	753	760
1.45	626	694	721	735	743	749	753	757	759	766	772	779
1.50	640	711	738	752	761	767	771	774	777	784	789	797
1.55	654	726	754	768	777	783	787	791	793	801	806	814
1.60	667	741	769	783	792	798	803	806	809	816	822	830
1.65	680	754	783	798	807	813	817	821	823	831	837	845
1.70	692	767	796	811	820	827	831	835	837	845	851	859
1.75	703	780	809	824	833	839	844	847	850	858	863	872
1.80	714	791	821	836	845	851	856	859	862	870	875	883
1.85	725	803	832	847	856	863	867	870	873	881	886	894
1.90	734	813	842	858	867	873	878	881	884	891	897	905
1.95	744	823	852	867	877	883	887	891	893	901	906	914
2.00	753	832	861	877	886	892	896	900	902	909	915	923
2.05	761	841	870	885	894	900	905	908	910	918	923	931
2.10	770	849	878	893	902	908	912	916	918	925	930	938
2.15	777	857	886	901	909	915	920	923	925	932	937	944
2.20	785	864	893	908	916	922	926	929	932	938	943	950
2.25	792	871	900	914	923	928	932	935	938	944	949	956
2.30	799	878	906	920	929	934	938	941	943	950	954	961
2.35	805	884	912	926	934	939	943	946	948	954	959	965
2.40	812	890	917	931	939	944	948	951	953	959	963	969
2.45	818	896	923	936	944	949	952	955	957	963	967	973
2.50	823	901	927	940	948	953	957	959	961	967	971	976
2.60	834	910	936	949	956	960	964	966	968	973	977	982
2.70	844	919	944	956	962	967	970	972	974	978	982	986
2.80	853	927	950	962	968	972	975	977	978	983	986	989
2.90	861	933	956	967	973	977	979	981	982	986	989	992
3.00	869	939	961	971	977	980	983	984	986	989	991	994
3.10	876	945	966	975	980	983	986	987	988	991	993	996
3.20	882	950	970	979	983	986	988	989	990	993	995	997
3.30	889	954	973	981	986	988	990	991	992	995	996	998
3.40	894	958	976	984	988	990	992	993	994	996	997	998
3.50	899	962	979	986	990	992	993	994	995	997	998	999
3.60	904	965	981	988	991	993	994	995	996	997	998	999
3.70	909	968	983	989	992	994	995	996	997	998	999	999
3.80	913	970	985	991	994	995	996	997	997	998	999	
3.90	917	973	987	992	995	996	997	997	998	999	999	
4.00	921	975	988	993	995	997	997	998	998	999	999	
4.20	927	979	990	995	997	998	998	999	999	999		
4.40	933	982	992	996	997	998	999	999	999			
4.60	939	984	994	997	998	999	999	999				
4.80	943	986	995	998	999	999	959					
5.00	947	988	996	998	999	999						
5.50	956	992	997	999	999							
6.00	963	994	998	999								
6.50	968	995	999									
7.00	972	997	999									
8.00	979	998										
9.00	983	999										
10.00	986	999										
15.00	994											
20.00	997											
30.00	998											
40.00	999											
50.00	999											

C = 4.2

D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	472	519	537	547	553	557	560	562	563	568	572	577
1.05	493	543	563	573	579	583	586	589	591	596	600	606
1.10	514	567	587	598	605	609	612	615	617	622	626	632
1.15	534	589	611	622	629	634	637	640	642	647	652	658
1.20	553	611	633	645	652	657	661	663	665	672	676	683
1.25	571	631	654	667	674	679	683	686	688	694	699	706
1.30	588	651	675	688	695	701	704	707	710	716	721	728
1.35	604	669	694	707	715	721	725	728	730	737	742	749
1.40	620	687	712	726	734	740	744	747	749	756	762	769
1.45	635	704	730	744	752	758	762	765	768	775	780	788
1.50	649	719	746	760	769	775	779	782	785	792	797	805
1.55	663	734	762	776	785	791	795	798	801	808	814	822
1.60	676	749	776	791	800	806	810	814	816	824	829	837
1.65	688	762	790	805	814	820	825	828	830	838	843	851
1.70	700	775	803	818	827	833	838	841	844	851	857	865
1.75	711	787	816	831	840	846	850	854	856	864	869	877
1.80	722	798	827	842	851	857	862	865	868	875	881	889
1.85	732	809	838	853	862	868	873	876	879	886	892	899
1.90	741	819	848	863	872	878	883	886	889	896	902	909
1.95	751	829	858	873	882	888	892	895	898	905	911	918
2.00	759	838	867	882	891	897	901	904	907	914	919	927
2.05	768	846	875	890	899	905	909	912	915	922	927	934
2.10	776	855	883	898	906	912	916	920	922	929	934	941
2.15	784	862	890	905	913	919	923	926	929	935	941	947
2.20	791	869	897	912	920	926	930	933	935	942	946	953
2.25	798	876	904	918	926	932	936	939	941	947	952	958
2.30	804	882	910	924	932	937	941	944	946	952	957	963
2.35	811	888	916	929	937	942	946	949	951	957	961	967
2.40	817	894	921	934	942	947	951	953	955	961	965	971
2.45	823	899	926	939	946	951	955	958	960	965	969	975
2.50	828	905	930	943	951	955	959	961	963	969	972	978
2.60	839	914	939	951	958	963	966	968	970	975	978	983
2.70	848	922	946	958	964	969	971	974	975	980	983	987
2.80	857	929	953	963	970	974	976	978	980	984	987	990
2.90	865	936	958	968	974	978	980	982	983	987	990	993
3.00	873	942	963	973	978	981	984	985	987	990	992	995
3.10	880	947	967	976	981	984	986	988	989	992	994	996
3.20	886	952	971	980	984	987	989	990	991	994	995	997
3.30	892	956	974	982	987	989	991	992	993	995	996	998
3.40	897	960	977	985	989	991	992	993	994	996	997	998
3.50	902	963	980	987	990	992	994	995	995	997	998	999
3.60	907	966	982	988	992	994	995	996	996	998	998	999
3.70	912	969	984	990	993	995	996	996	997	998	999	999
3.80	915	972	986	991	994	995	996	997	997	999	999	
3.90	920	974	987	992	995	996	997	998	998	999	999	
4.00	923	976	989	993	996	997	998	998	998	999		
4.20	930	980	991	995	997	998	998	999	999	999		
4.40	935	983	993	996	998	998	999	999	999			
4.60	940	985	994	997	998	999	999	999				
4.80	945	987	995	998	999	999	999					
5.00	949	989	996	998	999	999						
5.50	957	992	998	999								
6.00	964	994	998	999								
6.50	969	996	999									
7.00	973	997	999									
8.00	979	998										
9.00	984	999										
10.00	987	999										
15.00	994											
20.00	997											
30.00	998											
40.00	999											
50.00	999											

C = 4.6

D \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	481	529	547	557	563	567	570	572	574	579	582	588
1.05	502	553	572	583	589	593	596	599	600	606	610	615
1.10	523	576	597	607	614	619	622	624	626	632	636	642
1.15	542	598	620	631	638	643	646	649	651	657	661	667
1.20	561	620	642	654	661	666	669	672	674	680	685	691
1.25	579	640	663	675	683	688	692	694	696	703	707	714
1.30	596	659	683	696	703	709	713	715	718	724	729	736
1.35	612	677	702	715	723	729	732	735	738	744	749	757
1.40	628	694	720	733	742	747	751	754	757	763	769	776
1.45	642	711	737	751	759	765	769	772	775	782	787	794
1.50	656	727	753	767	776	782	786	789	791	798	804	812
1.55	670	741	768	783	791	797	801	805	807	814	820	828
1.60	682	755	783	797	806	812	816	820	822	829	835	843
1.65	695	768	796	811	820	826	830	833	836	843	849	857
1.70	706	781	809	824	833	839	843	846	849	856	862	870
1.75	717	793	821	836	845	851	855	859	861	868	874	882
1.80	728	804	832	847	856	862	867	870	872	880	885	893
1.85	738	814	843	858	867	873	877	880	883	890	896	903
1.90	747	824	853	868	877	883	887	890	893	900	905	913
1.95	756	834	862	877	886	892	896	899	902	909	914	922
2.00	765	843	871	886	894	900	904	908	910	917	922	930
2.05	773	851	879	894	902	908	912	915	918	925	930	937
2.10	781	859	887	901	910	915	920	923	925	932	937	944
2.15	788	866	894	908	917	922	926	929	932	938	943	950
2.20	796	873	901	915	923	929	932	935	938	944	949	955
2.25	802	880	907	921	929	934	938	941	943	949	954	960
2.30	809	886	913	926	934	940	943	946	948	954	959	965
2.35	815	892	918	932	940	945	948	951	953	959	963	969
2.40	821	897	924	937	944	949	953	955	957	963	967	973
2.45	827	903	928	941	949	953	957	959	961	967	971	976
2.50	832	907	933	945	953	957	961	963	965	970	974	979
2.60	843	917	941	953	960	964	967	969	971	976	979	984
2.70	852	925	948	959	966	970	973	975	976	981	984	988
2.80	861	932	954	965	971	975	977	979	981	985	987	991
2.90	868	938	960	970	975	979	981	983	984	988	990	993
3.00	876	944	965	974	979	982	984	986	987	990	992	995
3.10	882	949	969	977	982	985	987	989	990	992	994	996
3.20	889	953	972	981	985	985	989	991	992	994	996	997
3.30	894	958	975	983	987	990	991	992	993	995	997	998
3.40	900	961	978	985	989	991	993	994	994	996	997	999
3.50	905	965	981	987	991	993	994	995	996	997	998	999
3.60	909	968	983	989	992	994	995	996	996	998	999	999
3.70	914	970	985	990	993	995	996	997	997	998	999	
3.80	918	973	986	992	994	996	997	997	998	999	999	
3.90	922	975	988	993	995	996	997	998	998	999	999	
4.00	925	977	989	994	996	997	998	998	998	999		
4.20	931	980	991	995	997	998	998	999	999			
4.40	937	983	993	996	998	998	999	999	999			
4.60	942	986	994	997	998	999	999	999				
4.80	945	987	995	998	999	999	999					
5.00	950	989	996	998	999	999						
5.50	959	992	998	999								
6.00	965	994	998	999								
6.50	970	996	999									
7.00	974	997	999									
8.00	980	998										
9.00	984	999										
10.00	987	999										
15.00	994											
20.00	997											
30.00	999											
40.00	999											
50.00	999											

C = 5.0

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	489	537	555	565	571	575	578	580	582	587	591	596
1.05	510	561	581	591	597	601	604	607	609	614	618	623
1.10	530	584	605	615	622	626	630	632	634	639	644	650
1.15	549	606	627	639	646	650	654	656	658	664	668	675
1.20	568	627	649	661	668	673	677	679	681	687	692	698
1.25	586	647	670	682	690	695	698	701	703	710	714	721
1.30	603	666	690	702	710	715	719	722	724	731	736	742
1.35	619	684	709	721	729	735	739	742	744	751	756	763
1.40	634	701	726	740	748	753	757	760	763	769	775	782
1.45	648	717	743	757	765	771	775	778	780	787	792	800
1.50	662	732	759	773	781	787	791	794	797	804	809	817
1.55	676	747	774	788	797	802	807	810	812	819	825	832
1.60	688	761	788	802	811	817	821	824	827	834	840	847
1.65	700	774	801	816	824	830	835	838	840	848	853	861
1.70	712	786	814	828	837	843	847	851	853	860	866	874
1.75	722	798	826	840	849	855	859	863	865	872	878	886
1.80	733	808	837	851	860	866	870	874	876	883	889	897
1.85	743	819	847	862	870	876	881	884	886	894	899	907
1.90	752	829	857	871	880	886	890	893	896	903	908	916
1.95	761	838	866	880	889	895	899	902	905	912	917	924
2.00	769	846	874	889	897	903	907	911	913	920	925	932
2.05	777	855	882	897	905	911	915	918	920	927	932	939
2.10	785	862	890	904	912	918	922	925	927	934	939	946
2.15	793	870	897	911	919	925	929	932	934	940	945	952
2.20	800	876	904	917	925	931	935	938	940	946	951	957
2.25	806	883	910	923	931	936	940	943	945	951	956	962
2.30	813	889	915	929	937	942	945	948	950	956	961	966
2.35	819	895	921	934	941	947	950	953	955	960	965	970
2.40	825	900	926	939	946	951	954	957	959	964	968	974
2.45	830	905	931	943	950	955	958	961	963	968	972	977
2.50	836	910	935	947	954	959	962	964	966	971	975	980
2.60	846	919	943	954	961	965	968	971	972	977	980	985
2.70	855	927	950	961	967	971	974	976	977	982	984	988
2.80	863	934	956	966	972	976	978	980	981	985	988	991
2.90	871	940	961	971	976	980	982	984	985	988	991	993
3.00	878	945	966	975	980	983	985	987	988	991	993	995
3.10	885	950	970	978	983	986	988	989	990	993	994	996
3.20	891	955	973	981	985	988	990	991	992	994	996	997
3.30	897	959	976	984	988	990	992	993	993	995	997	998
3.40	902	962	979	986	990	992	993	994	995	996	998	999
3.50	907	966	981	988	991	993	994	995	996	997	998	999
3.60	911	968	983	989	992	994	995	996	997	998	999	999
3.70	916	971	985	991	994	995	996	997	997	998	999	
3.80	920	973	987	992	995	996	997	997	998	999	999	
3.90	923	976	988	993	995	997	997	998	998	999	999	
4.00	927	978	990	994	996	997	998	998	999	999		
4.20	933	981	992	995	997	998	998	999	999			
4.40	938	984	993	996	998	999	999	999	999			
4.60	943	986	995	997	998	999	999	999				
4.80	948	988	996	998	999	999						
5.00	951	989	996	998	999	999						
5.50	959	992	998	999								
6.00	966	995	999	999								
6.50	971	996	999									
7.00	974	997	999									
8.00	980	998										
9.00	984	999										
10.00	987	999										
15.00	994											
20.00	997											
30.00	999											
40.00	999											
50.00	999											

C = 6.0

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	503	553	571	581	587	591	594	596	598	603	606	612
1.05	524	576	596	606	612	617	620	622	624	629	633	638
1.10	544	599	619	630	637	641	644	647	649	654	658	664
1.15	563	620	642	653	660	664	668	670	672	678	682	688
1.20	582	641	663	675	682	687	690	693	695	701	705	712
1.25	599	660	683	695	703	708	711	714	716	722	727	734
1.30	615	679	702	715	723	728	731	734	736	743	748	754
1.35	631	696	721	733	741	747	750	753	756	762	767	774
1.40	646	713	738	751	759	764	768	771	774	780	785	793
1.45	660	728	754	768	776	781	785	788	791	797	803	810
1.50	674	743	769	783	791	797	801	804	807	814	819	826
1.55	687	757	784	798	806	812	816	819	822	829	834	841
1.60	699	771	798	812	820	826	830	833	836	843	848	856
1.65	711	783	810	825	833	839	843	846	849	856	861	869
1.70	722	795	823	837	845	851	855	859	861	868	873	881
1.75	732	806	834	848	857	863	867	870	872	879	885	892
1.80	742	817	845	859	867	873	877	881	883	890	895	903
1.85	752	827	854	869	877	883	887	890	893	900	905	912
1.90	761	836	864	878	887	892	896	900	902	909	914	921
1.95	770	845	873	887	895	901	905	908	910	917	922	929
2.00	778	853	881	895	903	909	913	916	918	925	930	937
2.05	786	861	888	902	911	916	920	923	925	932	937	943
2.10	793	869	896	909	917	923	927	930	932	938	943	950
2.15	800	876	902	916	924	929	933	936	938	944	949	955
2.20	807	882	909	922	930	935	939	941	944	950	954	960
2.25	814	888	915	928	935	940	944	947	949	955	959	965
2.30	820	894	920	933	940	945	949	951	953	959	963	969
2.35	826	900	925	938	945	950	953	956	958	963	967	973
2.40	831	905	930	942	949	954	957	960	962	967	971	976
2.45	837	910	934	946	953	958	961	963	965	970	974	979
2.50	842	914	939	950	957	961	965	967	969	973	977	981
2.60	852	923	946	957	964	968	971	973	974	979	982	986
2.70	861	930	953	963	969	973	976	978	979	983	986	989
2.80	869	937	958	968	974	977	980	982	983	986	989	992
2.90	876	943	963	973	978	981	983	985	986	989	991	994
3.00	883	948	966	976	981	984	986	988	989	992	993	996
3.10	889	953	972	980	984	987	989	990	991	993	995	997
3.20	895	957	975	982	987	989	991	992	993	995	996	998
3.30	901	961	978	985	989	991	992	993	994	996	997	998
3.40	906	964	980	987	990	992	994	995	995	997	998	999
3.50	911	967	983	989	992	994	995	996	996	998	998	999
3.60	915	970	984	990	993	995	996	996	997	998	999	999
3.70	919	973	986	991	994	996	996	997	997	998	999	
3.80	923	975	988	993	995	996	997	998	998	999	999	
3.90	926	977	989	994	996	997	998	998	998	999	999	
4.00	930	979	990	994	996	997	998	998	999	999		
4.20	936	982	992	996	997	998	999	999	999			
4.40	941	985	994	997	998	999	999	999	999			
4.60	946	987	995	997	999	999	999					
4.80	950	989	996	998	999	999						
5.00	953	990	997	998	999							
5.50	961	993	998	999								
6.00	967	995	999									
6.50	972	996	999									
7.00	976	997	999									
8.00	981	998										
9.00	985	999										
10.00	988	999										
15.00	995											
20.00	997											
30.00	999											
40.00	999											
50.00												

C = 8.0

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	522	572	591	600	606	610	613	615	617	622	625	631
1.05	543	595	614	625	631	635	638	640	642	647	651	657
1.10	562	617	637	648	654	659	662	664	666	672	676	681
1.15	581	638	659	670	677	681	685	687	689	695	699	705
1.20	598	657	680	691	698	703	706	709	711	717	721	727
1.25	615	676	699	711	718	723	727	729	732	738	742	749
1.30	631	694	718	730	737	742	746	749	751	757	762	769
1.35	647	711	735	748	755	761	764	767	769	776	781	788
1.40	661	727	752	765	772	778	782	784	787	793	798	805
1.45	675	742	767	780	788	794	798	801	803	810	815	822
1.50	688	756	782	795	804	809	813	816	818	825	830	837
1.55	700	770	796	809	818	823	827	830	833	839	845	852
1.60	712	783	809	823	831	837	841	844	846	853	858	865
1.65	723	795	821	835	843	849	853	856	859	865	871	878
1.70	734	806	833	847	855	861	865	868	870	877	882	889
1.75	744	817	844	857	866	871	875	879	881	888	893	900
1.80	754	827	854	868	876	882	886	889	891	898	903	910
1.85	763	836	863	877	885	891	895	898	900	907	912	919
1.90	772	845	872	886	894	900	904	907	909	915	920	927
1.95	780	854	880	894	902	908	912	915	917	923	928	935
2.00	788	862	888	902	910	915	919	922	924	930	935	942
2.05	796	869	896	909	917	922	926	929	931	937	942	948
2.10	803	876	902	915	923	928	932	935	937	943	948	954
2.15	810	883	909	922	929	934	938	941	943	949	953	959
2.20	816	889	915	927	935	940	943	946	948	954	958	964
2.25	822	895	920	933	940	945	948	951	953	958	962	968
2.30	828	901	925	938	945	949	953	955	957	963	966	972
2.35	834	906	930	942	949	954	957	959	961	966	970	975
2.40	839	911	935	946	953	958	961	963	965	970	973	978
2.45	845	915	939	950	957	961	964	966	968	973	976	981
2.50	850	920	943	954	960	965	967	970	971	976	979	983
2.60	859	928	950	960	966	970	973	975	977	981	984	987
2.70	867	935	956	966	972	975	978	980	981	985	987	990
2.80	875	941	961	971	976	979	982	983	984	988	990	993
2.90	882	947	966	975	980	983	985	986	987	990	992	995
3.00	889	952	970	978	983	986	987	989	990	992	994	996
3.10	895	956	974	981	985	988	990	991	992	994	996	997
3.20	901	960	977	984	988	990	992	993	993	995	997	998
3.30	906	964	979	986	990	992	993	994	995	996	997	999
3.40	911	967	982	988	991	993	994	995	996	997	998	999
3.50	915	970	984	990	993	994	995	996	997	998	999	999
3.60	919	972	986	991	994	995	996	997	997	998	999	
3.70	923	975	987	992	995	996	997	997	998	999	999	
3.80	927	977	989	993	995	997	997	998	998	999	999	
3.90	930	979	990	994	996	997	998	998	999	999		
4.00	933	980	991	995	997	998	998	999	999	999		
4.20	939	983	993	996	998	998	999	999	999			
4.40	944	986	994	997	998	999	999	999				
4.60	948	988	995	998	999	999	999					
4.80	952	989	996	998	999	999						
5.00	956	991	997	999	999							
5.50	963	993	998	999								
6.00	969	995	999									
6.50	973	996	999									
7.00	977	997	999									
8.00	982	998										
9.00	986	999										
10.00	988	999										
15.00	995											
20.00	997											
30.00	999											
40.00	999											
50.00												

C = 10.0

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	533	583	602	611	617	621	624	626	628	633	636	642
1.05	554	606	625	635	642	646	649	651	653	658	662	667
1.10	573	627	648	658	665	669	672	675	676	682	686	692
1.15	591	648	669	680	687	691	694	697	699	704	709	715
1.20	608	667	689	700	707	712	716	718	720	726	730	737
1.25	625	686	708	720	727	732	736	738	740	746	751	757
1.30	641	703	726	738	746	751	755	757	759	766	770	777
1.35	656	720	743	756	764	769	772	775	777	784	788	795
1.40	670	735	760	772	780	785	789	792	794	801	806	812
1.45	683	750	775	788	796	801	805	808	810	817	822	829
1.50	696	764	789	802	810	816	820	823	825	832	837	844
1.55	708	777	803	816	824	830	834	837	839	846	851	858
1.60	720	790	816	829	837	843	847	850	852	859	864	871
1.65	731	801	827	841	849	855	859	862	864	871	876	883
1.70	741	813	839	852	860	866	870	873	875	882	887	894
1.75	751	823	849	863	871	876	880	883	886	892	897	905
1.80	761	833	859	873	881	886	890	893	895	902	907	914
1.85	770	842	868	882	890	895	899	902	904	911	916	923
1.90	778	851	877	890	898	904	908	910	913	919	924	931
1.95	786	859	885	898	906	912	915	918	920	927	931	938
2.00	794	867	892	906	913	919	922	925	927	934	938	945
2.05	801	874	900	912	920	925	929	932	934	940	945	951
2.10	808	881	906	919	927	932	935	938	940	946	950	956
2.15	815	887	912	925	932	937	941	943	945	951	955	961
2.20	821	893	918	930	938	943	946	948	950	956	960	966
2.25	827	899	923	935	943	947	951	953	955	960	964	970
2.30	833	904	928	940	947	952	955	957	959	964	968	973
2.35	839	909	933	945	951	956	959	961	963	968	972	977
2.40	844	914	937	949	955	960	963	965	967	971	975	979
2.45	849	918	941	952	959	963	966	968	970	974	978	982
2.50	854	923	945	956	962	966	969	971	973	977	980	984
2.60	863	930	952	962	968	972	974	976	978	982	984	988
2.70	871	937	958	968	973	976	979	981	982	985	988	991
2.80	879	943	963	972	977	980	983	984	985	988	991	993
2.90	886	949	968	976	981	984	986	987	988	991	993	995
3.00	892	953	971	979	984	986	988	989	990	993	994	996
3.10	898	958	975	982	986	989	990	991	992	994	996	997
3.20	904	962	978	985	988	991	992	993	994	996	997	998
3.30	909	965	980	987	990	992	993	994	995	997	998	999
3.40	913	968	983	989	992	993	995	995	996	997	998	999
3.50	918	971	985	990	993	995	996	996	997	998	999	999
3.60	922	973	986	991	994	995	996	997	997	998	999	
3.70	925	976	988	993	995	996	997	998	998	999	999	
3.80	929	978	989	994	996	997	998	998	998	999	999	
3.90	932	979	990	994	996	997	998	998	999	999		
4.00	935	981	991	995	997	998	998	999	999	999		
4.20	941	984	993	996	998	998	999	999	999			
4.40	946	986	994	997	998	999	999	999				
4.60	950	988	996	998	999	999	999					
4.80	954	990	996	998	999	999						
5.00	957	991	997	999	999							
5.50	964	994	998	999								
6.00	970	995	999									
6.50	974	997	999									
7.00	978	997	999									
8.00	983	998										
9.00	986	999										
10.00	989	999										
15.00	995											
20.00	997											
30.00	999											
40.00	999											
50.00												

C = 20.0

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	556	605	623	633	639	643	645	648	649	654	658	663
1.05	575	627	646	656	662	666	669	671	673	678	682	687
1.10	594	648	668	678	684	689	692	694	696	701	705	711
1.15	611	667	688	699	705	710	713	716	717	723	727	733
1.20	628	686	707	719	725	730	733	736	738	743	748	754
1.25	644	704	726	737	744	749	752	755	757	763	767	774
1.30	659	720	743	755	762	767	771	773	775	781	786	792
1.35	673	736	759	772	779	784	788	790	792	798	803	810
1.40	687	751	775	787	795	800	803	806	808	815	819	826
1.45	700	765	789	802	810	815	818	821	823	830	835	841
1.50	712	778	803	816	823	829	832	835	838	844	849	856
1.55	724	791	816	829	836	842	846	848	851	857	862	869
1.60	735	803	828	841	849	854	858	861	863	869	874	881
1.65	745	814	839	852	860	865	869	872	874	881	886	892
1.70	755	825	850	863	871	876	880	883	885	891	896	903
1.75	765	834	860	873	881	886	890	892	895	901	906	913
1.80	774	844	869	882	890	895	899	902	904	910	915	921
1.85	782	852	878	890	898	903	907	910	912	918	923	930
1.90	790	861	886	899	906	911	915	918	920	926	931	937
1.95	798	868	893	906	914	919	922	925	927	933	938	944
2.00	806	875	900	913	920	925	929	932	934	940	944	950
2.05	813	882	907	919	927	932	935	938	940	945	950	955
2.10	819	889	913	925	933	937	941	943	945	951	955	961
2.15	826	895	919	931	938	943	946	948	950	956	960	965
2.20	832	901	924	936	943	947	951	953	955	960	964	969
2.25	837	906	929	941	947	952	955	957	959	964	968	973
2.30	843	911	934	945	952	956	959	961	963	968	971	976
2.35	848	916	938	949	956	960	963	965	967	971	975	979
2.40	853	920	942	953	959	963	966	968	970	974	977	982
2.45	858	924	946	956	963	966	969	971	973	977	980	984
2.50	862	928	950	960	966	969	972	974	975	979	982	986
2.60	871	935	956	965	971	974	977	979	980	984	986	989
2.70	879	942	961	970	975	979	981	983	984	987	989	992
2.80	886	947	966	975	979	982	984	986	987	990	992	994
2.90	892	952	970	978	983	985	987	988	989	992	994	996
3.00	899	957	974	981	985	988	989	991	991	994	995	997
3.10	904	961	977	984	988	990	991	992	993	995	996	998
3.20	909	964	980	986	990	992	993	994	994	996	997	998
3.30	914	968	982	988	991	993	994	995	996	997	998	999
3.40	918	971	984	990	992	994	995	996	996	998	998	999
3.50	923	973	986	991	994	995	996	997	997	998	999	999
3.60	926	975	988	992	995	996	997	997	998	999	999	
3.70	930	977	989	993	995	997	997	998	998	999	999	
3.80	933	979	990	994	996	997	998	998	999	999		
3.90	936	981	991	995	997	998	998	999	999	999		
4.00	939	983	992	996	997	998	999	999	999			
4.20	944	985	994	997	998	999	999	999	999			
4.40	949	987	995	997	999	999	999	999				
4.60	953	989	996	998	999	999						
4.80	957	991	997	998	999							
5.00	960	992	997	999	999							
5.50	966	994	998	999								
6.00	972	996	999									
6.50	976	997	999									
7.00	979	998										
8.00	984	999										
9.00	987	999										
10.00	989	999										
15.00	995											
20.00	997											
30.00	999											
40.00	999											
50.00												

C = INF

C \ N	4	6	8	10	12	14	16	18	20	30	50	INF
1.00	577	626	644	653	659	663	666	668	669	674	678	682
1.05	596	647	666	676	682	686	688	691	692	697	701	706
1.10	614	667	687	697	703	707	710	712	714	719	723	728
1.15	631	686	706	717	723	727	731	733	735	740	744	749
1.20	647	704	725	736	742	747	750	752	754	760	764	769
1.25	662	721	742	753	760	765	768	771	773	778	783	788
1.30	677	737	759	770	777	782	785	788	790	796	800	806
1.35	690	752	774	786	793	798	802	804	806	812	817	822
1.40	704	766	789	801	808	813	817	819	821	828	832	836
1.45	716	779	803	815	822	827	831	834	836	842	846	852
1.50	728	792	816	828	835	841	844	847	849	855	860	866
1.55	739	804	828	840	848	853	857	859	861	868	872	878
1.60	749	815	839	852	859	864	868	871	873	879	884	890
1.65	759	826	850	862	870	875	879	882	884	890	895	901
1.70	769	836	860	872	880	885	889	892	894	900	904	910
1.75	778	845	869	882	889	894	898	901	903	909	913	919
1.80	786	854	878	890	898	903	907	909	911	917	922	928
1.85	794	862	886	899	906	911	914	917	919	925	930	935
1.90	802	870	894	906	913	918	922	924	926	932	937	942
1.95	810	877	901	913	920	925	929	931	933	939	943	949
2.00	816	884	908	919	927	931	935	937	939	945	949	954
2.05	823	890	914	925	932	937	940	943	945	950	954	959
2.10	829	896	920	931	938	942	946	948	950	955	959	964
2.15	835	902	925	936	943	947	950	953	955	960	963	968
2.20	841	907	930	941	948	952	955	957	959	964	967	972
2.25	847	912	935	945	952	956	959	961	963	968	971	975
2.30	852	917	939	950	956	960	963	965	966	971	974	978
2.35	857	921	943	953	959	963	966	968	970	974	977	981
2.40	862	926	947	957	963	966	969	971	973	977	980	983
2.45	866	930	950	960	966	969	972	974	975	979	982	986
2.50	870	933	953	963	969	972	975	976	978	981	984	987
2.60	878	940	959	968	974	977	979	981	982	985	988	991
2.70	886	946	964	973	978	981	983	984	985	988	990	993
2.80	893	951	969	977	981	984	986	987	988	991	993	995
2.90	899	956	973	980	984	987	988	990	990	993	994	996
3.00	905	960	976	983	987	989	990	992	992	994	996	997
3.10	910	964	979	985	989	991	992	993	994	996	997	998
3.20	915	967	981	987	991	992	994	994	995	997	998	999
3.30	919	970	984	989	992	994	995	995	996	997	998	999
3.40	923	973	986	991	993	995	996	996	997	998	999	999
3.50	927	975	987	992	994	996	996	997	997	998	999	
3.60	931	977	989	993	995	996	997	998	998	999	999	
3.70	934	979	990	994	996	997	998	998	998	999	999	
3.80	937	981	991	995	997	997	998	998	999	999		
3.90	940	982	992	995	997	998	998	999	999	999		
4.00	943	984	993	996	997	998	999	999	999			
4.20	948	986	994	997	998	999	999	999	999			
4.40	952	988	995	998	999	999	999					
4.60	956	990	996	998	999	999						
4.80	959	991	997	999	999							
5.00	962	993	998	999	999							
5.50	968	995	998	999								
6.00	973	996	999									
6.50	977	997	999									
7.00	980	998										
8.00	985	999										
9.00	988	999										
10.00	990	999										
15.00	996											
20.00	998											
30.00	999											
40.00	999											
50.00												

UNCLASSIFIED

System Development Corporation,
Santa Monica, California
CONFIDENCE BANDS IN STRAIGHT LINE
REGRESSION.
Scientific rept., SP-1181/000/00,
by A. V. Gafarian. 17 April 1963,
45p., 7 refs., 5 figs.
Unclassified report

DESCRIPTORS: Statistical Distribution.
Statistical Functions.

Develops a method for obtaining confidence bands in polynomial regression when the observations

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are independently distributed with constant but unknown variance. Reports that the bands may be obtained over arbitrary sets of the independent variable with exact preassigned confident coefficients. Also reports that difficult distribution problems result when specific applications are attempted. Discusses first degree polynomials since some progress has been made here. Provides a table that obtains a constant width confidence band which contains the true but unknown straight regression line for values of the independent variable in some arbitrarily selected interval with an exact preassigned confident coefficient. Compares the present method with the classical hyperbolic band for the whole regression line.

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