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SHOCK, VIBRATION AND ASSOCIATED ENVIRONMENTS

PART III

APRIL 1963

OFFICE OF
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THE SECRETARY OF DEFENSE
Research and Engineering

The 31st Symposium on Shock, Vibration and Associated Environments was held at the Hotel Westward Ho, Phoenix, Arizona on October 1-4, 1962. The U.S. Air Force was host.

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FOREWORD

This section of the Bulletin contains papers on the prediction of environments, the measurement and analysis of environmental data, and current environmental programs. Panel discussions relating to these topics are also included.

Suggestions to improve the Symposia and the Bulletin are always welcome. They should be addressed to Code 4021, U.S. Naval Research Laboratory, Washington, 25, D. C.

April 15, 1963

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Section 1

PREDICTION OF ENVIRONMENTS AND THEIR EFFECTS

UTILIZATION OF DYNAMICALLY SIMILAR STRUCTURAL MODELS IN PREDICTING VIBRATION RESPONSES OF FLIGHT VEHICLES

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This paper describes the results to date of two programs which are directed toward the development of the dynamically similar structural model for the prediction of vibration and strain responses of the full scale vehicle. Results are given from a completed preliminary program which determined the response of simple scaled panels to scaled jet noise. A brief progress report is given of the current program which compares the acoustic vibration transfer functions of a 1/4 scale model of a 377 inch segment of the Snark fuselage with full scale data. Full scale data leading toward an improved understanding of the transmission and acceptance of vibratory energy in a complex structure, are discussed.

INTRODUCTION

The reliability requirements for aerospace vehicle structure and equipment are several orders of magnitude greater than those previously experienced in manned aircraft. A much greater percentage of the vehicles' components can cause catastrophic failures, or missions of limited success. At the same time, the environmental factors resulting in failure, such as noise and vibration, combined with temperature and other dynamic and static loads, are assuming extreme values.

New phenomena in structural dynamics are constantly being disclosed; this accounts, in part, for the manifold reliability problems present in our current vehicles. Obvious solutions or means to avoid these problems, however, are not apparent. Rather, the work has shown the depth and fundamental nature of these roadblocks to good structural design. No technique, no quantitative set of measures, no design procedures or design analyses exist for attaining a stated reliability. Thus, in the field of structural dynamics, the design staff cannot guarantee the accomplishment of effective long life design for new vehicles. The structural dynamicist has a key role to play since inadequacies in his area appear to be a major factor in lack of reliability.

Major structural failures have occurred in primary wing structures. Further, costly and time consuming failures have occurred in secondary structures of all types, including missile and space booster upper stages. Many failures of equipment are merely failures in miniature structures. Significant missile failures have occurred during the brief period spent at transonic or maximum q flight during exit from the atmosphere. These failures have been traced to turbulence, oscillating shocks, or to structural instabilities such as buckling and panel flutter. Often significant structural damage results from combined loads in a short 2- to 3-second period, although this is not generally recognized.

These structural failures concern fatigue to a greater degree than is generally appreciated. Measurements on our production vehicles show acoustic stresses as low as 1000-2000 psi rms near fatigue failure points. The acoustic environments have been galloping upward for many years. The threshold of structural damage was crossed at 140 db and the capability to design around the environment has been slow to develop.

These diverse factors present the design engineer with a serious dilemma. On the one hand, he is asked to guarantee vehicle reliability for operation in increasingly adverse environments, and on the other hand, he is directed to develop optimum, minimum-weight structures and components to maximize vehicle performance capability. These twin objectives are the basis of present research in flight vehicle noise and vibration. They present the designer and research engineer alike, with a serious challenge. The time required for achievement of these goals, together with the degree of this achievement, has a direct effect on the timely and successful accomplishments of national space goals. Therefore, the dynamicist must be considered a prominent member of the design team.

In the early stage of a new vehicle design, the dynamicist must estimate the probable environment, and the response of the vehicle to this enivronment. These preliminary estimates are necessarily based on available empirical data which have considerable scatter and often lead to significant uncertainty. These predictions, however, generally become the basis for design criteria and test specifications. The degree of conservatism of much of the resulting design stems directly from the confidence of the estimator in his data and its applicability to a new situation.

During the design stage, these preliminary estimates are supplemented by some analytical efforts, ranging from sophisticated analyses of basic vehicle modes for consideration of flutter and guidance, to approximate assessments of possible fatigue from combined loads. Most of the estimates, however, are unaltered until the vehicle and its components are sufficiently complete to undergo full scale testing. At this point, components which are unreliable clearly assert themselves, and the dynamicist attempts to direct their salvage. Components which have achieved reliability through overdesign are unnoticed.

It is clear that the achievement of both reliable and optimum structure cannot be accomplished efficiently primarily by tests of the full scale vehicle. This after-the-fact engineering is not reliability-by-design, but is reliability-by-test, a natural consequence of an inadequate state of the art. However, until better analytical tools, empirical or experimental, or both are available to the dynamicist at appropriate stages of the design process, the present procedures are inevitable and reliability is achieved only at great cost in time and money, and at the sacrifice of optimum design.

This paper will review the progress on two Air Force sponsored Norair projects in this area by discussing a simple model, a complex fuselage model, and the measured vibratory characteristics of the complex fuselage.

DYNAMICALLY SIMILAR STRUCTURAL MODELS

One of the most promising alternatives to present procedures is the use of dynamic models which enable the designer to anticipate and prevent problems. This concept, currently under development by Norair under Air Force sponsorship (Refs. 1 and 2) offers several unique advantages to the designer in the prediction of responses of a vehicle to various environments and in the development of reliable optimized design before the design is frozen and the vehicle is constructed.

The concept of a dynamically similar structural model for flight vehicles is an outgrowth of the flutter and force models of the aircraft engineer, and the static load bridge and building models used by the civil engineer. The model offers a complete duplication of the dynamic characteristics and responses of the full scale vehicle at a time which precedes final construction of the full scale vehicle. Thus, the model gives the designer a powerful experimental tool to test out design predictions and methods to optimize the design for actual environments and

[1]Eldred, K., Roberts, W., and White, R., "Structural Vibrations in Space Vehicles," WADD Technical Report 61-62 (March, 1961).
[2]Starr, J. W., et al, "Vibration Attenuation Characteristics of N-69D Fuselage," Northrop Aircraft Report Number NAI-56-114 (February 24, 1956).

to discover unexpected phenomena sufficiently early to take optimum corrective action.

Table 1 gives a brief listing of important model scale laws. These show that when all dimensions are scaled by the factor n, the time domain of the model is reduced by that same factor. Hence, the frequency domain of the model is higher than the full scale vehicle by the factor of 1/n. If the model is excited by the noise of model scale rockets or jets, the excitation frequencies are similarly increased by the factor 1/n. Furthermore the sound pressure spectra at the scaled frequencies and scaled locations in the sound field are identical to full scale pressure spectra. Similarly, the spatial correlations of these pressures over the model vehicle's skin, the diffraction phenomena, ground reflections, and so on, all occur at model scale frequencies exactly as on the full scale vehicle at full scale frequencies. Therefore, the concept of a dynamically similar model excited by scaled engine noise is inherently a complete laboratory duplication of the full scale process.

TABLE 1
Scaling Laws

Characteristics	Scale Factor[a]
Dimensions	n
Thrust	n^2
Acoustic Power	n^2
Total Mass	n^3
Time Scale	1/n
Frequency	1/n
Material	Same
Stress	Same
Sound Pressure Level	Same
Young's Modulus	Same
Damping Ratio C/C_c	Same

[a]Scale factor is ratio of model to full scale component

This analogy can be carried much further, determining responses to many fluctuating aerodynamic phenomena such as separated flow and heating from the scaled model rockets, together with evaluation of the effect of cold liquid propellants and combined static and dynamic loadings, including stiffness nonlinearities. These results have a broad potential use including development of guidance system characteristics, determination and solution of structural fatigue problems, and evaluation of optimum locations and attachments for major equipment items.

The first Norair application of these principles to the sonic fatigue problem was undertaken in a program sponsored by the Flight Dynamics Laboratory of the Aeronautical Systems Division (ASD), "Feasibility of Using Structural Models for Acoustic Fatigue."

Several simple panel designs were investigated in three sizes, full, one-third, and one-sixth scale. The 18-inch square full-scale panels were excited by a J-85 jet engine and the models were excited by scaled hot jets which had identical flow parameters. Figure 1 shows a typical model test configuration.

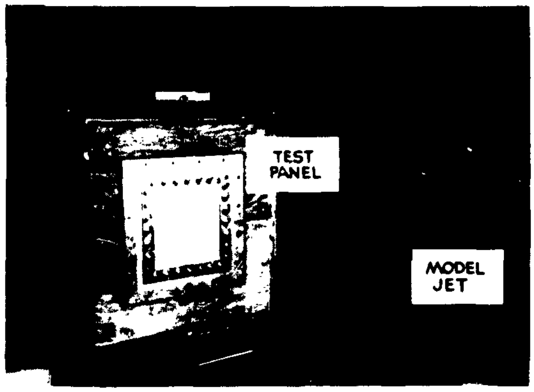


Fig. 1 - Acoustic fatigue model test

The noise spectra over the panel surface scaled with high precision, as illustrated in Fig. 2. The strain response of one design which exhibited a sharp resonant peak in the fundamental mode, is shown in Fig. 3. The response of another design which had considerable nonlinear stiffness, is given in Fig. 4.

These results demonstrate the validity of the scaling laws and their applicability to simple structural models. Further, they show the power of the total model concept where scaled strain response results from scaled jet flow, an end-to-end process which eliminates all intermediate estimates and predictions, together with their inevitable errors.

SNARK FUSELAGE MODEL (1/4 SCALE)

With these results available, the question arose "can complex structure be modeled for

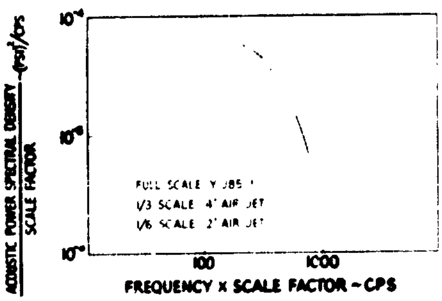


Fig. 2 - Acoustic power spectral density at the center of the panel

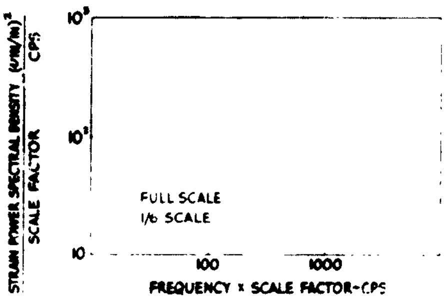


Fig. 3 - Strain power spectral density near one edge of the panel (panel riveted along all four edges)

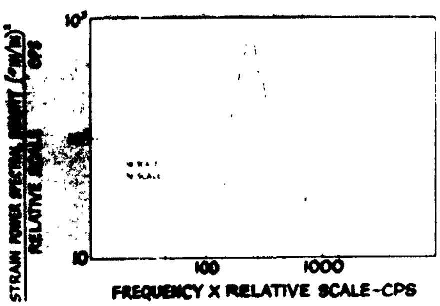


Fig. 4 - Strain power spectral density near one edge of the panel (panel restrained vertically but free to move in plane)

the purpose of predicting vibration response?" The new problems that would arise would presumably be joint transmission, damping, fabrication, and degree of simulation. Other than these points, there were no essential differences or changes to the scaling. A step-by-step state of the art buildup through structures of successively greater complexity was a possible approach, but this is time consuming and costly. Therefore, with ASD support, a bold step was taken from the modeling of a simple panel to the modeling of a 377-inch segment of a Snark fuselage. The design and construction of this model was supported by allotting 15 percent of the total funds in experimental studies to the problems mentioned. Results showed a wide difference in joint characteristics between riveted and bonded structure; the conclusion was reached that it is impossible to simulate riveted structure with a bonded model. Simulation, however, was not reduced by dropping out a substantial proportion of the rivets.

The complexity of the fuselage structure is illustrated in Fig. 5. The forward section has thick skins, longerons, and stiff honeycomb bulkheads and floors. The aft portion of the structure has lightweight skins supported by typical aircraft substructure.

Figure 6 illustrates the model and full-scale structure test configurations. Random acoustic excitation was horn-coupled to an area of approximately 14 square feet on the full scale vehicle over the frequency range of 40-2000 cps. Similar excitation was applied to the model over a scaled surface area throughout a frequency range extending to 8000 cps. The acoustic and vibration measurements were used to compute a transfer function, defined as 10 $\log_{10}$ of the ratio of the mean square accelerations and the mean square acoustic force input (the higher the transfer function, the higher the response).

A comparison of the model and full scale transfer functions, which were computed from the measurements, showed that the model gave correct simulation in several areas as typified by Fig. 7, and contained gross errors at other locations, as shown in Fig. 8. Careful examination of the data showed that the errors were generally associated with honeycomb sandwich structures. The modeling of the honeycomb sandwich was accomplished using correct dimensions and face sheets, but the core was made of foam plastic. Although this technique has been successful in simulating full depth honeycomb structures-wings and empennage for flutter models, study showed the velocity of sound in the material for this particular high frequency application was too low. Subsequent experiments have been completed on the honeycomb sandwich structures to show that direct simulation using honeycomb core of the same density yields a dynamically similar structure. These results will be incorporated together with a few other minor changes into the model to upgrade its simulation.

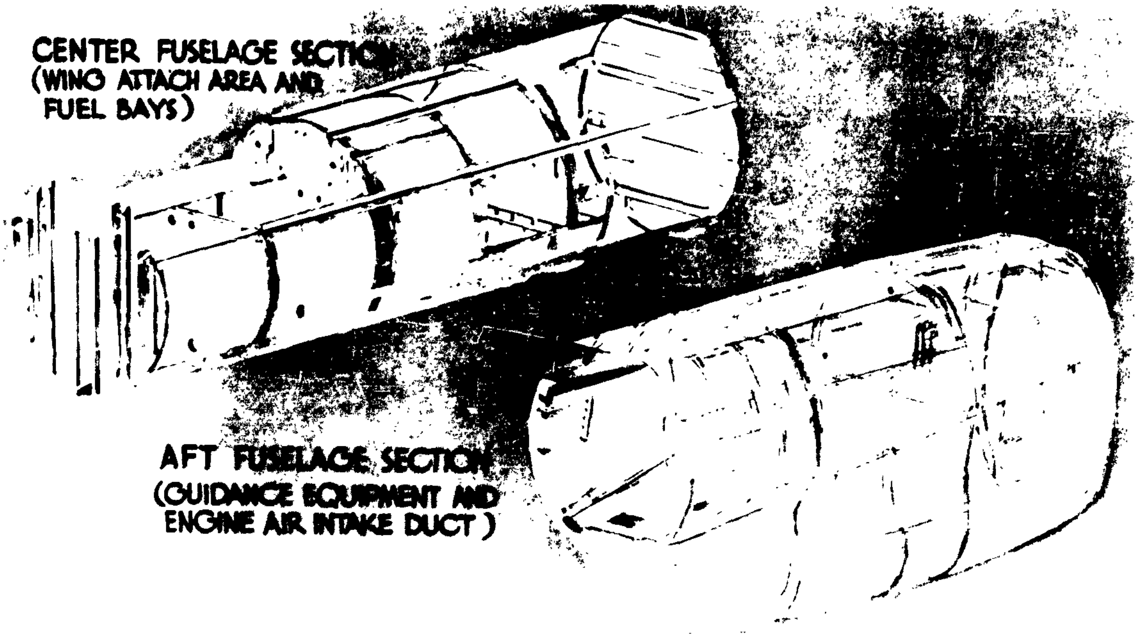
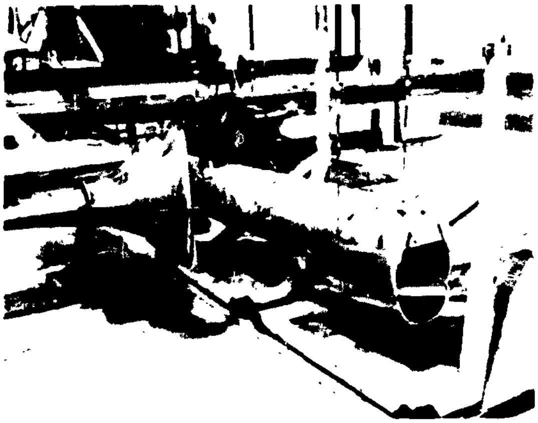
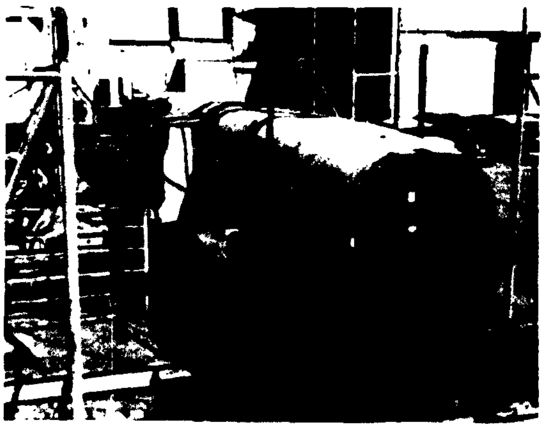


Fig. 5 - Perspective showing complexity of structure chosen



(a) Model



(b) Full scale

Fig. 6 - Model and full-scale structure undergoing excitation from random air modulator

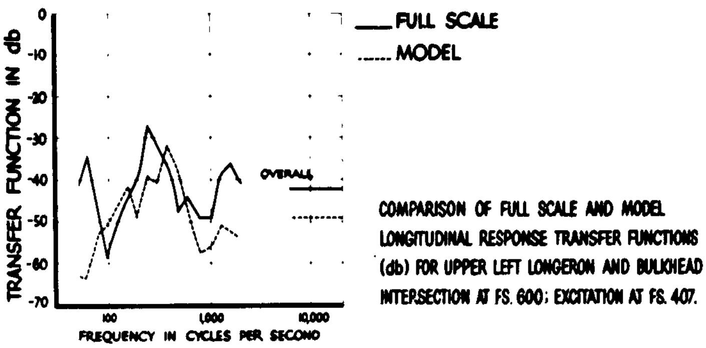


Fig. 7 - Comparison of model and full-scale longitudinal response transfer functions

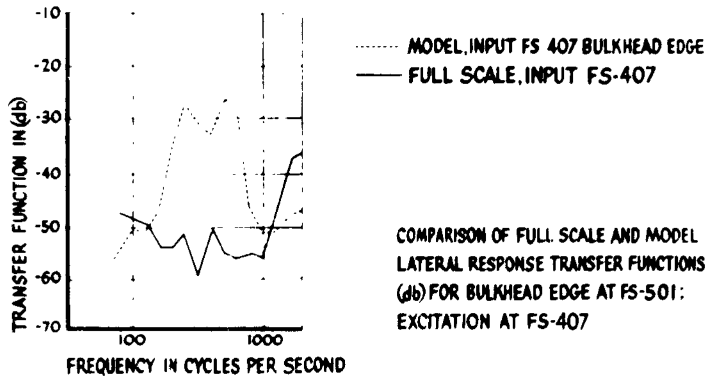


Fig. 8 - Comparison of model and full-scale lateral response transfer functions

The model has served one initial major purpose, validating many of the design simplifications used in its construction, while pointing directly to compromises which required substudy development. This has reduced the cost and time required for advancing the model state of the art to a practical tool.

FULL SCALE RESULTS

An extensive series of full scale measurements has been completed on the full scale vehicle (Ref. 3). These measurements include data taken at 50 points in the structure for each of 4 acoustic input locations. Further, an additional 50 measurements were taken in the compartment between Sta. 536 and 600.

Figure 9 shows the transfer functions obtained from 14 locations for both forward and aft excitation. These transfer functions clearly illustrate the attenuation through the vehicle with distance from the noise source. It is interesting to note that the transfer functions for both forward and aft excitation are similar in magnitude over a considerable portion of the middle of the vehicle. This is remarkable considering that the structural transmission paths from the two ends to the middle are entirely different.

The attenuation for lateral response is summarized for 250 and 2000 cps in Fig. 10. Although some local effects are evident, the data are sufficiently consistent to determine an attenuation constant (α) for these two frequencies. At 2000 cps, α is estimated to be 1.1 db/ft for forward excitation and 1.2 db/ft for aft

[3] "Investigation of a Method for the Prediction of Vibratory Response and Stress in Typical Flight Vehicle Structure," Norair Division of Northrop Corporation (August, 1962).

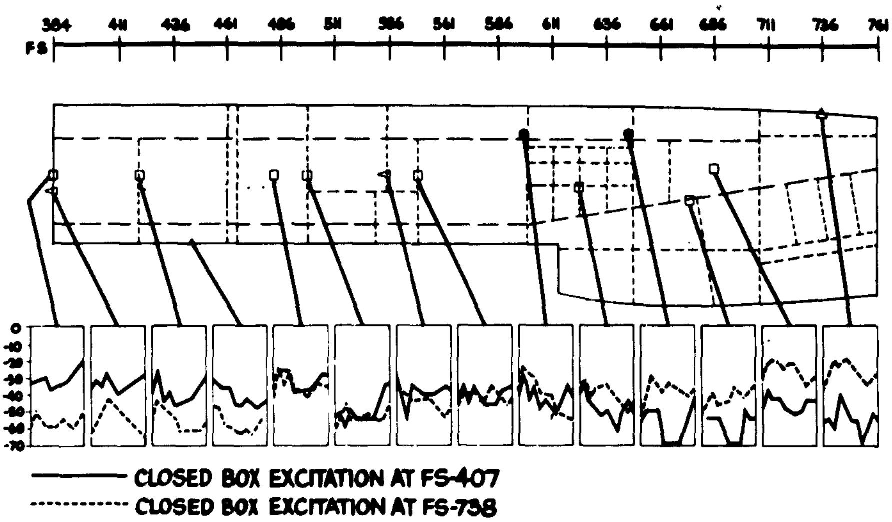


Fig. 9 - Comparison of various full-scale structural responses for lateral excitation at FS-407 and 738 showing the general axial attenuation of response

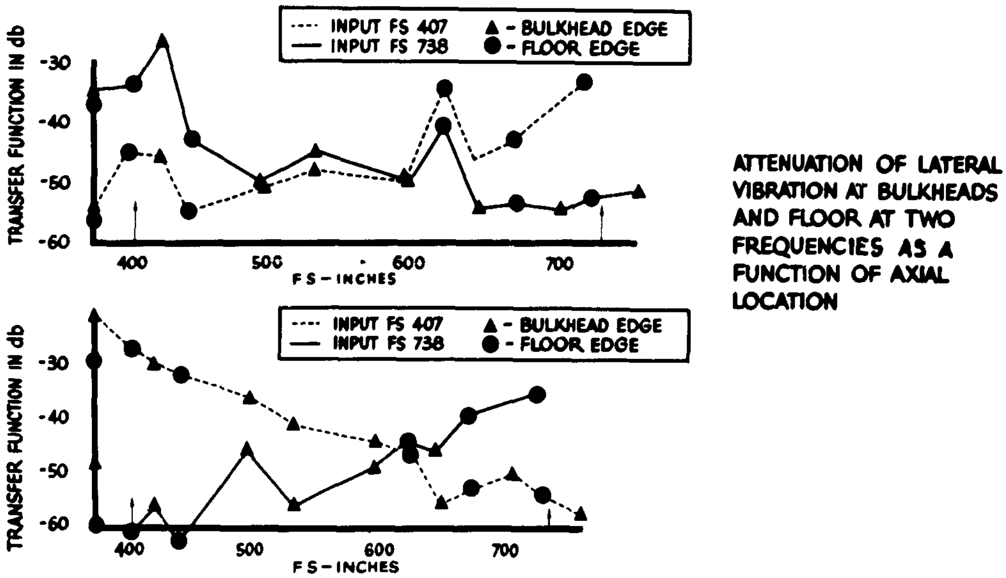


Fig. 10 - Comparison of various full-scale structural responses to excitation at either end of the vehicle showing axial attenuation

excitation. For 250 cps, α is grossly estimated to be 0.84 and 0.48 db/ft for excitation at the forward and aft ends, respectively. These values compare with 0.6 db/ft at 250 cps and 1.2 db/ft at 1000 cps, obtained several years ago on a section of fuselage extending from sta. 300 to 600 with vertical shaker excitation at station 600, Refs. 1 and 2. The variation of these attenuations with frequency is essentially consistent with a constant attenuation per cycle for bending waves whose velocity is proportional to the square root of frequency.

The significance of local structural parameters in controlling the response transfer functions in the mid frequency range was illustrated at the majority of locations throughout the structure. Figure 11 shows the attenuation along a forward longeron and the similarity of response over an extended distance on a contiguous piece of structure. Figure 12 compares the upper and lower longerons at a middle station for excitation at both the forward and aft ends of the vehicle. It is striking that the response in this area is so similar for excitation from the two ends, since the structural transmission paths are markedly different. Figure 13 shows the transfer function to a panel for excitation at three locations. Again, the basic panel resonance characteristic dominates the result, independent of the differing dynamic characteristics of the three transmission paths from the excitation to the panel.

Figure 14 illustrates a typical comparison of the transfer functions for in-plane vibration across a stiff honeycomb structure. Here the transfer functions are very similar, showing that these stiff structures act as rigid links for in-plane transmission for frequencies below longitudinal wave resonances.

A detailed study was made of the vibration response of structure located in the compartment between sta. 536 and 600 with direct external excitation. A brief illustration of the results in Figs. 15 and 16 leads to several significant conclusions. The comparison of the vibration on the directly excited side panel and its mirror image panel on the opposite side of the vehicle again shows a dominance of local effects. The resonant characteristics of both panels are very similar and do not exhibit any significant distortions resulting from the intervening structural path. Since the two panels are directly connected by stiff honeycomb bulkheads and a light honeycomb floor, this result would follow from their tendency to act as rigid links for in-plane vibration transmission. Figure 16 illustrates the vibration perpendicular to the plane for the forward honeycomb bulkhead, which has two obvious resonances, and the

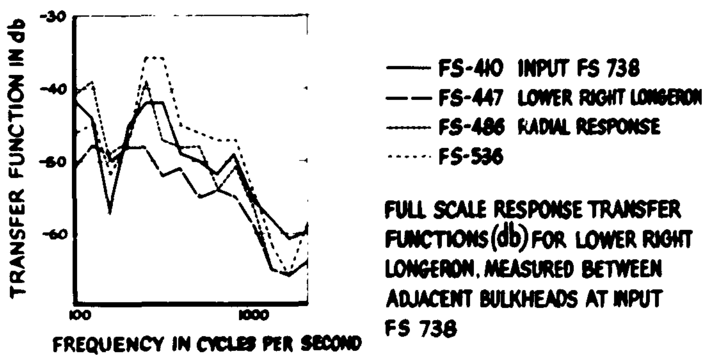


Fig. 11 - Full-scale response transfer functions for the lower right longeron at different stations along the vehicle showing similarity for one type of structural element

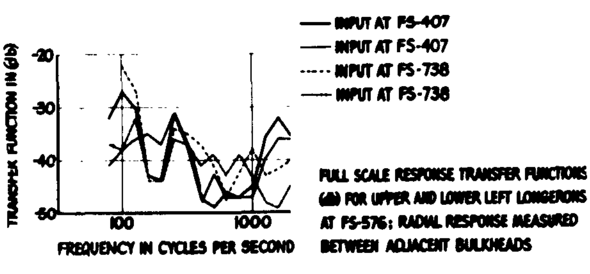


Fig. 12 - Full-scale response transfer functions for the longerons for inputs at different locations

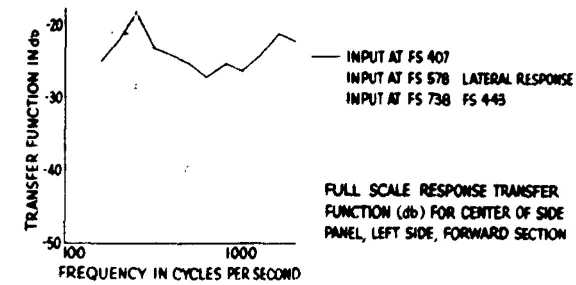


Fig. 13 - Full-scale response transfer functions for a stiff cylindrical panel ,for inputs at different locations

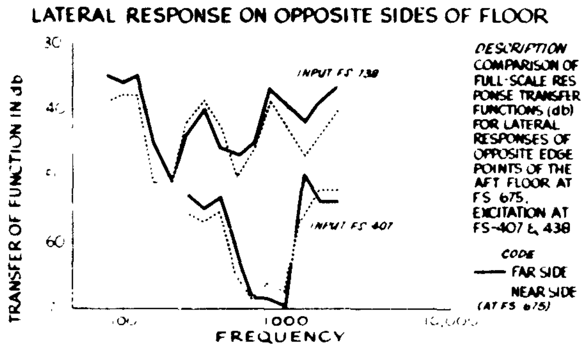


Fig. 14 - Comparison of full-scale response transfer functions showing a stiff structural panel acting as a rigid link for in-plane motion

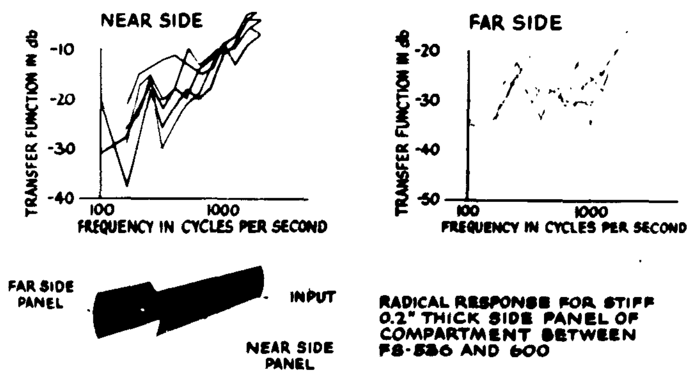


Fig. 15 - Radial response of stiff side panels in the compartment study

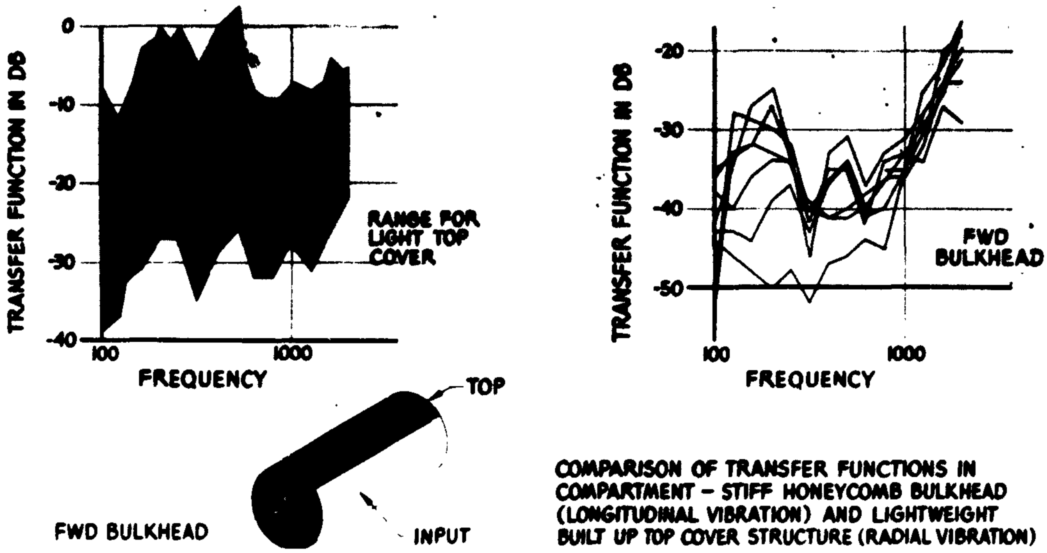


Fig. 16 - Top and forward bulkhead response in the compartment study

lightweight complex top cover structure. This latter structure exhibits very high valued transfer functions which have individual characteristics, depending on location on the complex built-up structure.

These brief results, together with the complete results in Ref. 3, suggest the basic form of a model for transmission and acceptance of vibratory energy through a complex vehicle. There appear to be three general frequency ranges of primary consideration, which are summarized in Figs. 17, 18 and 19.

The low frequency region (Fig. 17) is generally below 100 cps and was not investigated in detail on the present vehicle. However, inspection of many of the transfer functions illustrates that the vibration amplitudes are similar on both ends of the vehicle, as expected for a free body bending.

The medium frequency region (Fig. 18) appears, from the data obtained here, to be generally between 100 and 600 cps. In this region the responses of major panels, bulkheads, and floors are determined primarily by their own individual resonance characteristics, and are primarily in their fundamental and second harmonic modes. In many cases, clear evidence exists of resonance associated with the coupling of two or more connected parts. Although these individual resonances dominate any local response functions, they do not appear to play a dominant role in the transmission of vibration power through the vehicle beyond the region of local influence of the part. Rather, the preponderant portion of the power in this frequency region appears transmitted by the stiff structure, for example, in-plane transmission through floors and bulkheads, and transmission through longerons and heavy framing. Since the responses in this medium frequency region appear to be controlled by individual mass-stiffness

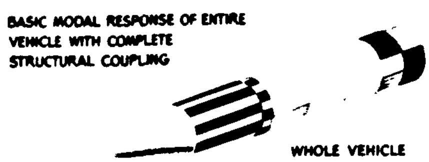


Fig. 17 - Low-frequency response characterized by response in fundamental modes of the vehicle

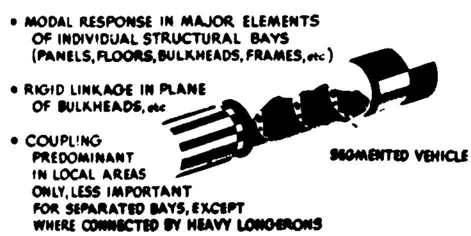


Fig. 18 - Medium-frequency response characterized by resonant response of structural elements

and damping parameters, the response of lightweight flexible structure is considerably greater than that of heavy structure, for example, the bay cover and light main floor of the compartment as compared to the stiffer forward upper floor and bulkhead.

The high frequency region (Fig. 19) in these data, variously above 600-1000 cps, is above the basic modes of panels, bulkheads, floor main frames, and longerons. The motion of these items near to the source of vibratory power tends to have constant velocity. Thus, these elements near the source act as though

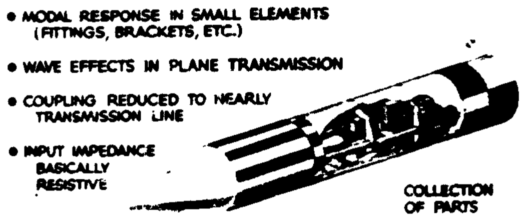


Fig. 19 - High-frequency response characterized by transmission line action with high axial attenuation

their motion is resistance controlled. This resistance control implies that these elements are acting as links in a transmission line which transmits vibratory power to adjacent portions of the vehicle. Furthermore, the evidence suggests that local compartment areas of the structure can be considered reverberant, as seen from the uniform energy density throughout the stiffer structure in the compartment study.

For the present vehicle, the primary damping occurs at the joints. Therefore, within each reverberant area, power is absorbed at each joint in the bay. The major power absorption mechanism, however, is believed to be transmission of power longitudinally through the vehicle, rather than local absorption. This result is apparent when it is recognized that the axial attenuations of the order of 1 db/ft found for the vehicle, reduce the level of energy reflected from distant portions of the vehicle so that it does not contribute significantly to the local reverberant energy density. Thus, definition of the local area of the vehicle in which reverberation is significant depends upon the structural configuration which, in turn, defines major reflecting boundaries, the number of joints, extent of continuous stiff structure, dissimilarity of adjacent structure, and attenuation factor.

* * *

ESTIMATION OF SOUND-INDUCED VIBRATIONS BY ENERGY METHODS, WITH APPLICATIONS TO THE TITAN MISSILE*

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This paper presents a first step in a new approach to the estimation of acoustically induced vibration of complicated vehicles. Although the results of this study are restricted to the estimation of vehicle skin panel motion, the general concepts involved can be extended to the study of vibration transmission to internal structures.

INTRODUCTION

The excessive random vibration of aerospace vehicle structures caused by the aerodynamic and acoustic environment may create critical problems in vehicle design and mission programming. These problems may be associated with the vibration itself or with the sound reradiated by the vibration into the vehicle. Any attempt at estimating or controlling the vehicle vibration must be preceded by a theoretical or empirical understanding, or both, of the processes that control coupling of energy from the aerodynamic and acoustic field to the structure, and the removal of energy from the structure by dissipation or transmission to other areas.

This paper presents a first step in a new approach to the estimation of acoustically-induced space vehicle vibration. This step consists of the extension and application of recent results obtained by Bolt Beranek and Newman, Inc., in research studies of vibration in complex structures. The research studies have been performed under ASD and NASA sponsorship and are reported in detail elsewhere [1-4].

The work described in this paper is restricted to the estimation of vehicle skin panel motion. The general concepts used here are being extended to the study of vibration transmission to internal structures, and the results of this research will be reported in the future. Also, this paper considers only the average values of sound-induced response. Additional work is required to develop estimation procedures for the variances in response that are superposed on these average values.

The noise and vibration data considered in this paper were taken during the static firing of a TITAN I vehicle, because of the quantity of available data. The section entitled "Experimental Results and Observations" presents a brief description of the data, converts the data into a form suitable for comparison with analysis, and proposes the simplified model of the vehicle structure to be considered in the analysis.

The research results to be used in the vibration estimation arise from a statistical treatment of the complex (i.e., multi-modal) vehicle structure. The section entitled "Statistical Treatment of Complex Vibration" presents a descriptive account of some of the reasoning in this statistical treatment and points out how various physical factors enter into the analytic expressions of the estimation procedure. By making certain reasonable assumptions concerning the vehicle motion, it is possible to obtain encouraging agreement between the experimental data and the vibration estimates.

The present study of vibration estimation points up the usefulness of certain additional experimental data in order to refine subsequent estimates. In the final section — "Suggested Measurements," five different areas are

*This work was supported by Space Technology Laboratories, Inc.

proposed in which helpful experimental investigations may be performed.

Appendix A to this paper contains a brief description of another set of experimental data that provides further illustration of some of the concepts mentioned in this report.

GLOSSARY OF SYMBOLS

The equation or figure number in which the symbol first appears is indicated in parentheses following the definition.

$\mathcal{AL}$ acceleration level (Eq. (3))

A_N normalized acceptance function (Eq. (1))

c speed of sound in air (Eq. (4))

c_ℓ longitudinal wave speed in panel material (Eq. (9))

E total sound energy in a reverberant space (Eq. (4))

f frequency (Eq. (5))

f_c coincidence frequency (Eq. (10))

f_m frequencies of panel cross resonance (Eq. (18))

h average panel thickness (Eq. (2))

k wave number (Eq. (17))

K dimensional factor of proportionality (Eq. (1))

ℓ_1, ℓ_2 panel dimensions (Fig. 6)

n density of flexural modes (Eq. (8))

n' modal density corrected for "clumping" (Eq. (19))

P panel perimeter (Eq. (10))

$\overline{p^2}$ mean-square sound pressure in a reverberant space (Eq. (4))

R_{rad} panel radiation resistance (Eq. (8))

R_{tot} total panel structural resistance (Eq. (8))

s panel surface area (Eq. (8))

S_a, S_p acceleration and pressure spectral densities (Eq. (1))

SPL sound pressure level (Eq. (3))

v rms structural velocity (Eq. (21))

V volume of reverberant space (Eq. (4))

x_1, x_2 panel coordinates (Fig. 6)

Y Young's modulus (Eq. (21))

β dimensionless factor of proportionality (Eq. (21))

γ boundary absorption coefficient (Eq. (12))

λ_c wavelength at coincidence (Eq. (10))

ρ density of air (Eq. (4))

ρ_p density of panel material (Eq. (2))

σ rms stress (Eq. (21))

Δf frequency interval (Eq. (5))

ΔN number of resonant acoustic modes (Eq. (5))

EXPERIMENTAL RESULTS AND OBSERVATIONS

The acoustic and vibration measurements of concern in this paper were made by the Martin Company during static firings of TITAN I Missiles J-2 and J-3 at Martin-Denver in the summer of 1960. Figure 1 shows the locations of the transducers that are of interest. The accelerometers were mounted to detect radial motion of the vehicle skin.

The measurements used herein were made during a Sequence Compatibility Firing (SCF). The firing procedure was such that the stage I engines were fired first and the stage II engine was then fired with stage I tanks empty. For convenience we will concentrate on measurements taken between tanks on stage I (acoustic measurement No. 3225 and vibration measurement No. 3214), during the SCF of missile J-3. (However, similar results have been obtained from other skin locations on the TITAN I vehicle, as well as from similar locations on the JUPITER and SATURN vehicles.) The pertinent SCF stage I data are shown in Figs. 2 and 3.

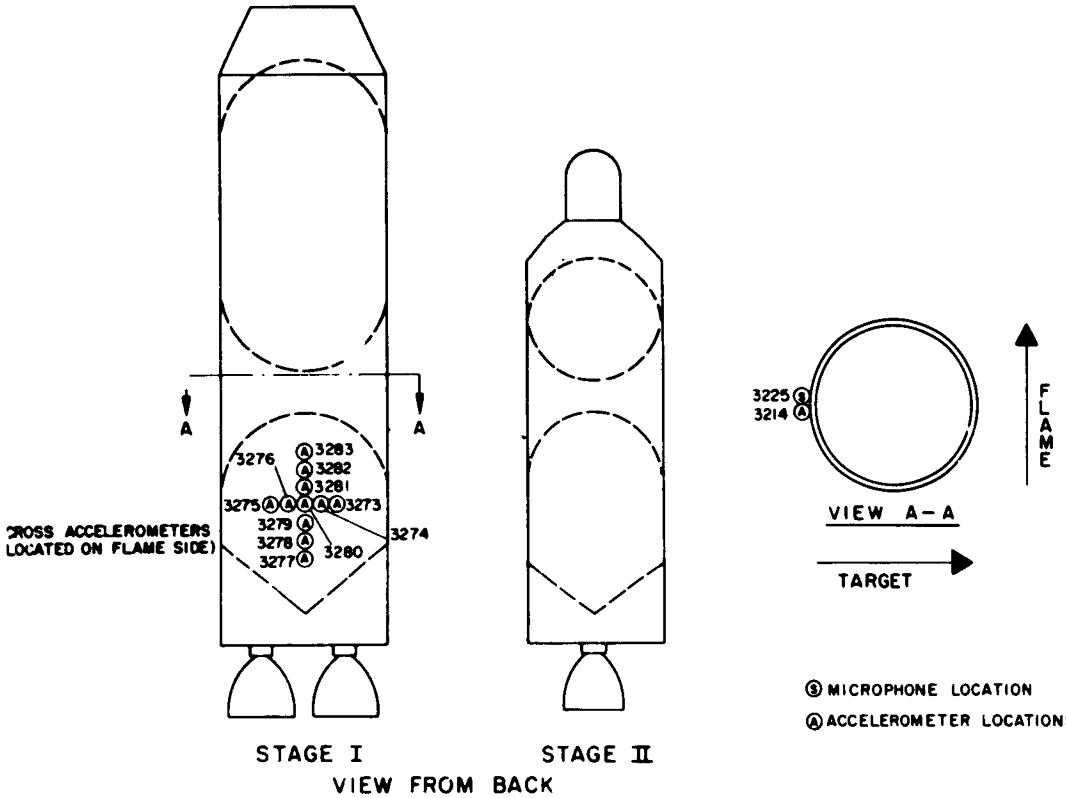

Fig. 1. Approximate locations of acoustic and vibration transducers of interest, TITAN I Missiles J-2 and J-3

Definition of an Acoustic Acceptance Function

In relating the measured data to the results of a generalized analysis, it will be convenient to define a quantity that gives a measure of the structural response in some normalized form. We define the normalized acoustic acceptance function A_N as the ratio of the acceleration spectral density S_a to the pressure spectral density S_p, or

$$A_N = K \frac{S_a}{4S_p} . \quad (1)$$

(S_p is the pressure spectral density that would be measured in the absence of the vehicle structure. Therefore, because of pressure doubling effects, the pressure spectral density at the vehicle surface is $4S_p$.) We assign a value $(\rho_p h)^2$ for the factor of proportionality K, where ρ_p is the density of the vehicle skin panel and h is the average panel thickness. Our expression for A_N is then

$$A_N = \frac{(\rho_p h)^2 S_a}{4S_p} . \quad (2)$$

A value of $A_N = 1$ implies that the structure is responding according to "normal incidence mass law," because the acceleration spectral density times the square of the mass per unit area $[S_a(\rho_p h)^2]$ is equal to the spectrum density of the exciting pressure at the surface $[4S_p]$.

The skin between tanks on stage I of TITAN I is made of aluminum. The average thickness of the tank structure is approximately 0.1 inch thick. By using the proper density and thickness in Eq. (2), we obtain the following expression relating the acceptance function A_N to the measured values of sound pressure level (SPL) and acceleration level (AL):

$$10 \log A_N = AL - SPL + 131 \text{ db}. \quad (3)$$

Equation (3) has been used to obtain values of A_N from the data given in Figs. 2 and 3, and the results are plotted in Fig. 4.

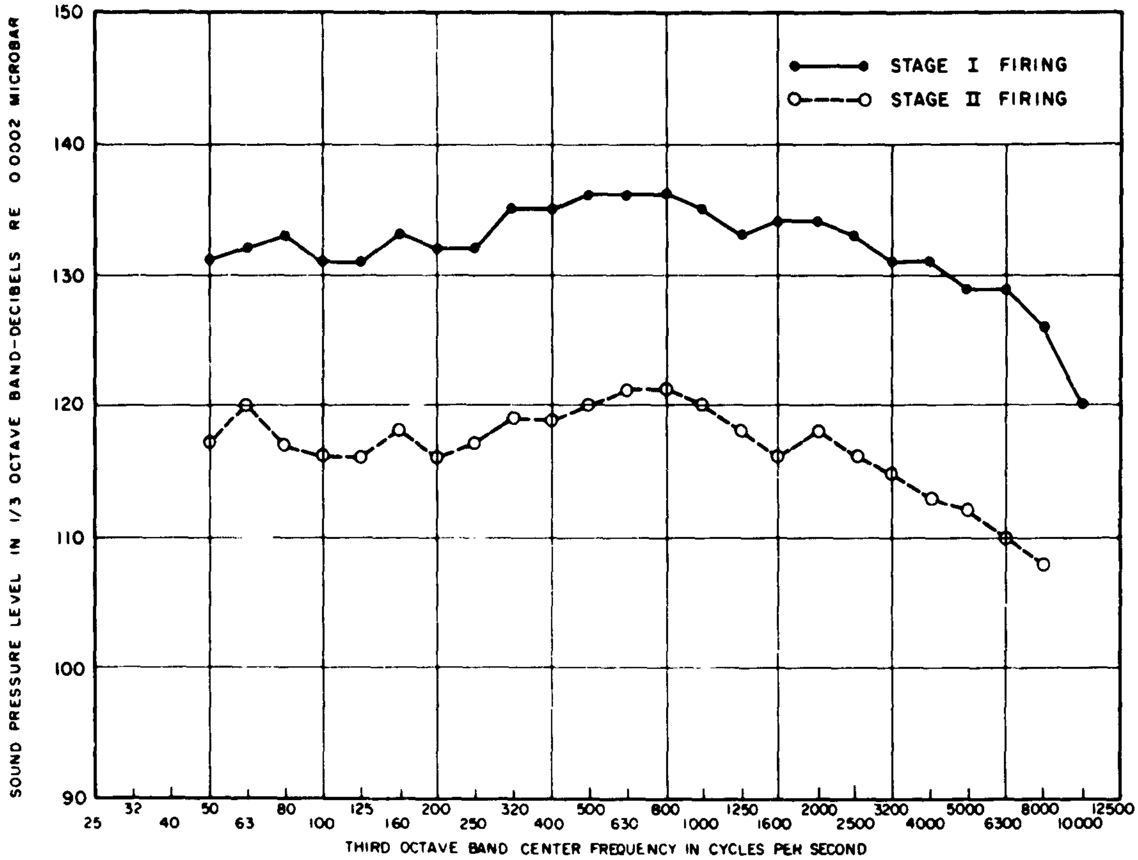

Fig. 2. Acoustic measurements between tanks on stage I, TITAN J-3 SCF, measurement No. 3225

Figures 2 and 3 show peaks in the structural response occurring in the 1/3-octave bands centered at 100, 250, and 500 cps. Narrow-band (3 cps wide) studies of the vibration data in these bands failed to reveal any single mode which is responsible for these peaks. The vibration therefore appears to be strongly multi-modal. In addition, the response values in these frequency regions are considerably in excess of the mass law value. From this we conclude that resonant response is dominant in determining the motion, at least in these frequency regions.

Vibration Correlation and the Choice of a Physical Model

In the study of a complicated physical problem, it is generally necessary to choose an appropriately simplified model for analysis. In the present case of sound-induced vibration in the TITAN skin structure, the simplified model to be chosen consists of a panel or a series of panels. Some insight into the proper choice of size and shape for this panel can be obtained from the cross correlations of vibration data obtained during SCF of the J-2 missile. We will utilize correlations obtained from the cross array of accelerometers on the stage I fuel tank (see Fig. 1). The accelerometers were spaced 12 inches on center. The lowest accelerometer in the array was located on the aft skirt, and all others were located on the fuel tank itself. The tank skin is chemically milled, with thicknesses ranging from approximately 0.050 to 0.090 inch. For simplicity, only data obtained with the stage I engines firing are considered.

Figure 5 shows a sequence of three acceleration cross correlations as functions of delay time. The solid line is the autocorrelation of one of the stage I fuel tank accelerometers, the broken line is the cross correlation of accelerometers with a 1-foot axial separation, and

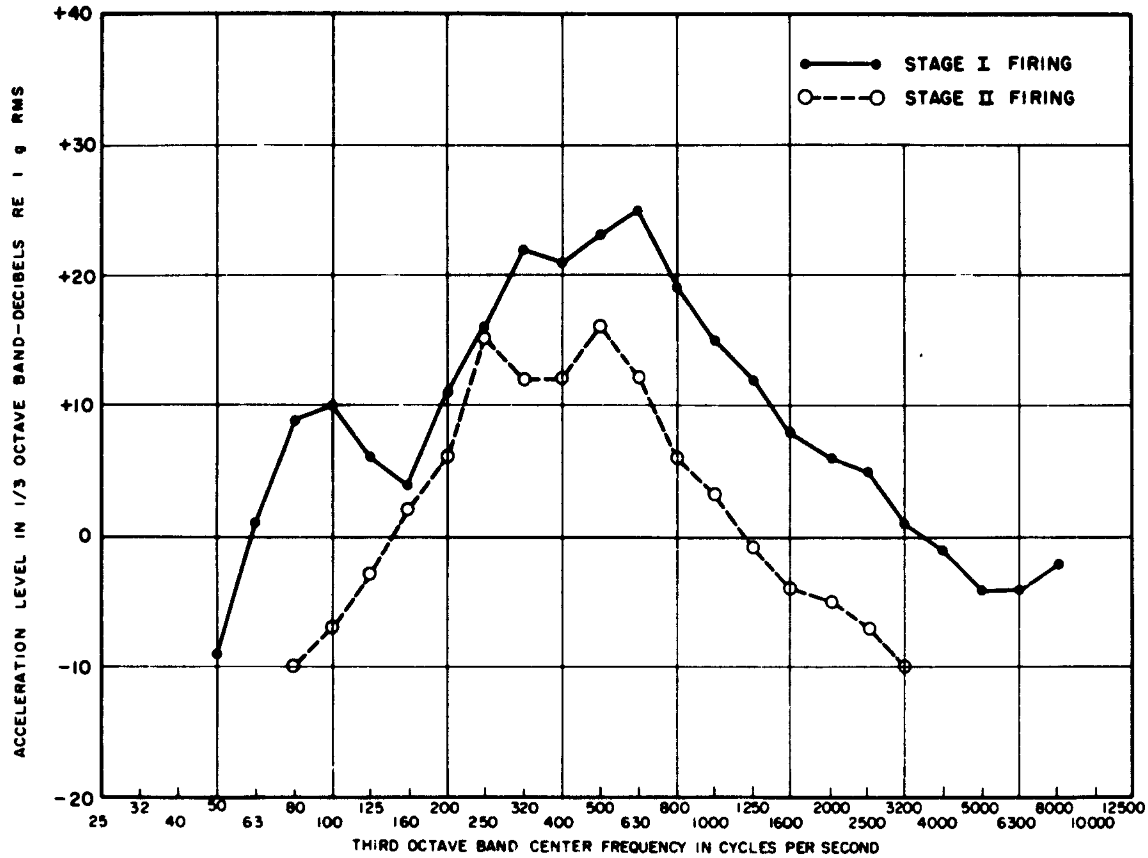

Fig. 3. Vibration measurements between tanks on stage I, TITAN J-3 SCF, measurement No. 3214

the dot-dashed line is the cross correlation of accelerometers with a 2-foot axial separation. The correlations show considerable symmetry about zero time delay. We interpret this symmetry to mean that the vibration field moving axially up the missile is very similar to that moving axially down the missile. There does not seem to be any indication of a preferred axial direction of the vibration field, such as one might expect if all the skin vibration were a forced wave matching an axially traveling sound field. This is in line with our earlier conclusion that resonant response rather than forced motion is dominant in determining the vehicle vibration.

In general, all the correlations from the cross array show behavior similar to that in Fig. 5, both for axial and circumferential separations. However, significantly less correlation is observed in two important instances:

- Little correlation is observed between accelerometer positions on opposite sides of the extruded longitudinal tank stringers.

- Little correlation is observed between positions on the tank structure and the position on the aft skirt.

It appears that the presence of a heavy, continuous, internal member (such as the longitudinal stringers and the ring frame on the bulkhead between the fuel ank and the aft skirt) creates a "correlation boundary." The multiple reflections from these members apparently create a reverberant vibrational field within the panel defined by the members.

The dimensions of an intertank skin panel determined by the stringers and bulkheads are typically 16 × 72 inches. On the basis of the correlation results obtained from the fuel tank

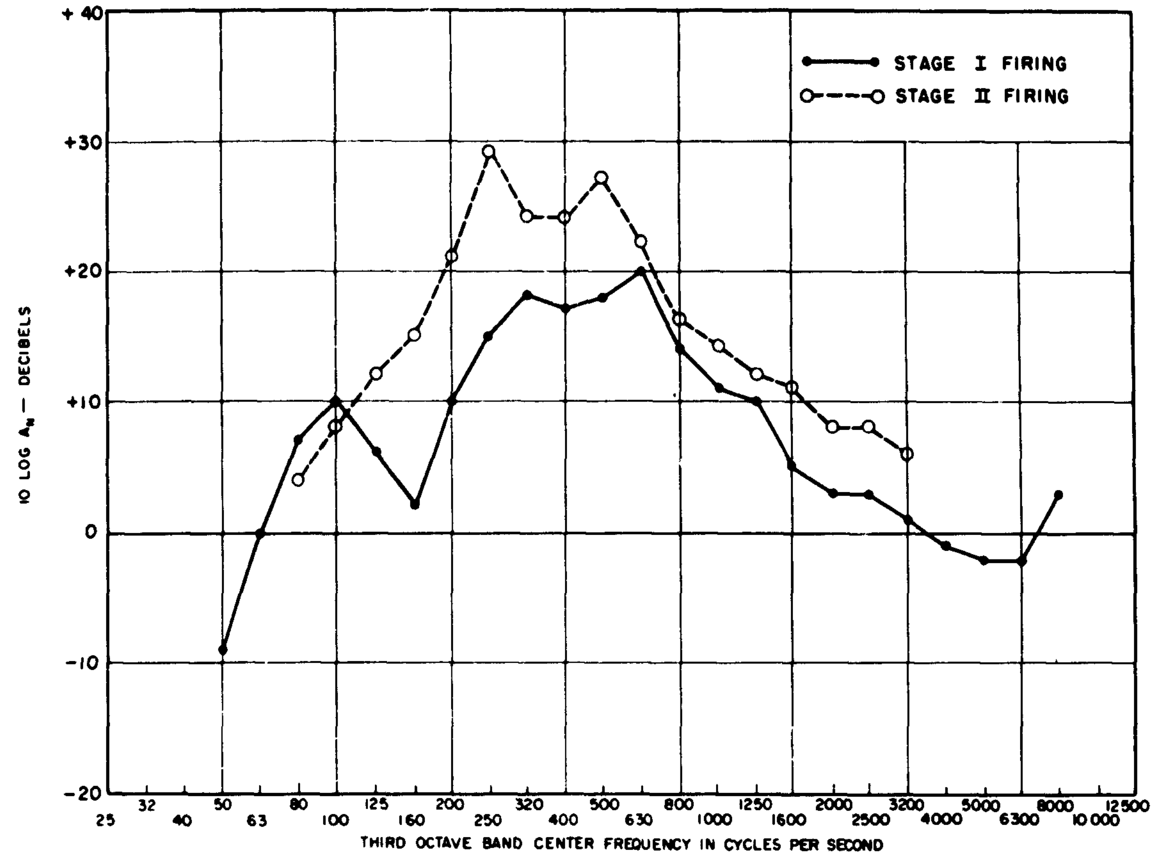

Fig. 4. Acoustic acceptance function for skin panel between tanks on stage I, TITAN J-3 SCF

accelerometer cross array, we assume that similar correlation behavior would be observed on the intertank skin. Thus, the vibration of an intertank panel is assumed to be well correlated over the panel extent of approximately 16 × 72 inches, but essentially no correlation is assumed to be observed between a position in such a panel and a position in an adjacent panel. This choice of panel size and shape suggested by the correlation results permits us to proceed with the analysis of the vibration field.

STATISTICAL TREATMENT OF COMPLEX VIBRATION

In the study of structural vibrations, a traditional approach that has proven fruitful for many years is the direct application of the differential equations governing the structure and the surrounding fluid. For certain systems with deterministic behavior, such an approach is reasonable. When the time behavior is random, workers have found it necessary to adopt a statistical description of the signals, and a large literature exists reflecting the many problems that have been studied in this manner.

When the vibration structure becomes very complicated, with irregular shapes and intricate connections between panels, frames, trusses, and so on, its description in terms of differential equations is still possible. However, as a practical matter, and because of the large number of degrees of freedom, complicated boundary conditions, and inconvenient geometries, the traditional methods of description become very difficult to apply.

When only a few modes are important in the behavior one is trying to explain, simplification of the dynamical description with a few-mode model is a powerful technique which has been widely applied. When the vibration involves many structural modes at once, a few-mode model will not be adequate. The vibration

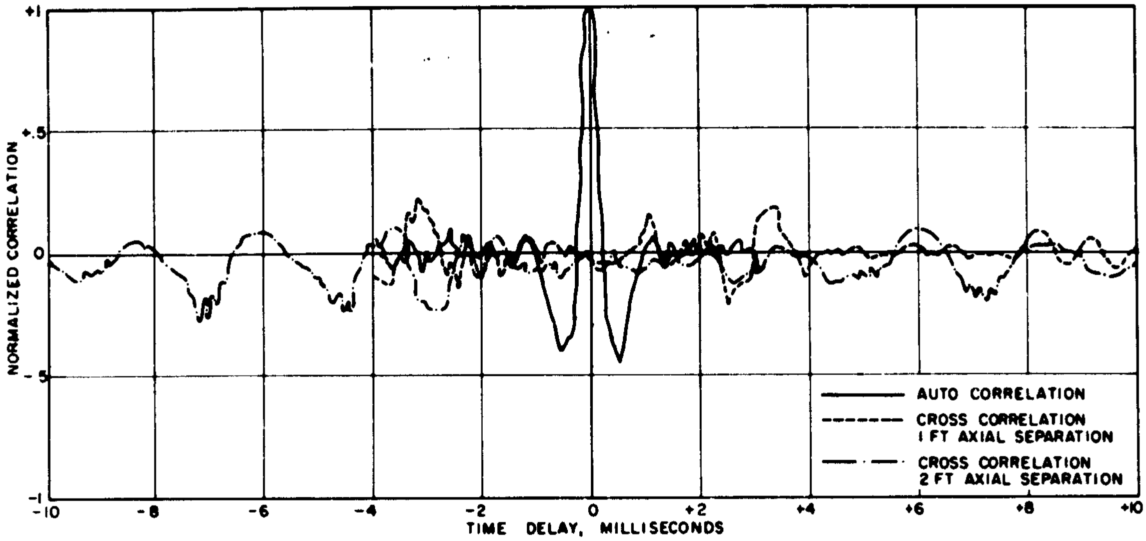

Fig. 5. Typical cross correlations of vibration on stage I fuel tank, TITAN J-2 SCF, stage I engine firing

data discussed in the previous section indicates that the TITAN structure is an instance of such a multi-modal system. We can, therefore, expect that the explanation of the vibration data will require an analysis that involves many modes.

In acoustics, there has been a long tradition of work on multi-modal systems. For example, the response of a room to noise involves very many modes in an irregular space with complicated boundary conditions, and yet there has been considerable success in the prediction of the sound level in rooms. The basis of this success has been a statistical description of the dynamics of the room, using energy as a primary variable. This strongly suggests that the description of the energy distribution in complex structures can be fruitfully carried out in statistical terms in a manner parallel to the analysis of sound in rooms.

Elements of the Statistical Model

The central assumption of our work is that the complex mechanical structure may be described by a set of modes whose dynamics are the same as those of a set of simple harmonic oscillators. This assumption is motivated by the similarity between the sinusoidal temporal dependence of an (idealized) structural mode and the sinusoidal temporal dependence of a harmonic oscillator. For most structures the vibrational modes are observed experimentally to be below critical damping, and resonant response dominates the motion. Our goal is the knowledge of a distribution of energy spatially over the structure and a determination of the energy frequency spectrum. To meet this goal, we need to understand how various parts of the structure contribute to the modal density* and how the energy is shared among the modes. This understanding is provided quantitatively by the statistical description of modal densities and coupling parameters.

A statistical description of modal densities is expressed in terms of areas, lengths, and thicknesses of structures. Thus, two structures of similar overall dimensions will have the same statistical modal density, even though their configurations may be quite different.

The coupling of power into a structure or from one section of structure to another depends on the real part of the impedance (resistance) at the attachment points. Thus, the radiation resistance of a structure governs how much power is fed into it from a sound field, and the point resistance determines how much power is transferred to a connecting truss. (A striking example of the role played by the radiation resistance in a sound-structure coupling is given in Appendix A.)

*Modal density is defined as the number of resonances contained in a frequency band that is one unit wide.

Detailed descriptions of the statistical treatment of complex vibration are given in Refs. 1-4. In this paper, we will be concerned only with the results of these analyses as they apply to the experimental data given in the previous section. The reader interested in following a careful derivation of the results is referred to the References.

Formal Evaluation of the Acceptance Function

The formal solution to the vibration estimation problem consists of an evaluation of the pressure and acceleration spectral densities S_p and S_a. In order to evaluate the pressure spectral density, it is convenient to consider first a reverberant sound field contained in a large room of volume V. This assumption of a reverberant sound field introduces the concept of acoustic modes and permits us to use the existing research results, which are based on considerations of energy balance between such acoustic modes and the structural modes. The TITAN SCF configuration involved traveling sound waves near grazing incidence rather than a reverberant sound field. However, results mentioned later in this section show that the coupling between the sound field and the structural motion is relatively independent of the nature of the sound field. This situation encourages us to pursue the reverberant sound field assumption, anticipating that some modification may eventually be required in the analytic interpretation of S_p for a traveling wave.

In a frequency interval Δf, the total sound energy E in the room is [5]

$$E = \frac{\overline{p^2} V}{\rho c^2}, \tag{4}$$

where $\overline{p^2}$ is the mean square sound pressure in the frequency interval Δf, ρ is the air density, and c is the sound speed. This energy is shared by ΔN modes, where [5]

$$\Delta N = \frac{4\pi f^2 V \Delta f}{c^3}. \tag{5}$$

The pressure spectral density S_p is defined as

$$S_p = \frac{\overline{p^2}}{\Delta f}. \tag{6}$$

Combining Eqs. (4), (5), and (6), we obtain

$$S_p = \frac{4\pi f^2 \rho}{c} \times \frac{E}{\Delta N}. \tag{7}$$

The acceleration spectral density is evaluated by multiplying the mean-square acceleration per structural mode by the number of modes in a band 1 cycle wide. It is expressible in terms of the average energy per acoustic mode $E/(\Delta N)$. By omitting the details of the calculation, we obtain

$$A_N = \left[\frac{\pi c h \rho_p}{4 s \rho}\right] [n] \left[\frac{R_{rad}}{R_{tot}}\right], \tag{8}$$

where n is the modal density -- the number of structural modes per cycle, ρ_p is the density of the structural material, s is the surface area of the structure, h is the average thickness of the structure, and R_{rad} and R_{tot} are the radiation resistance and total structural resistance of the panel. The first factor in the right-hand side of Eq. (8) gives the dependence of the acceptance function on the panel geometry and material, the second factor is the modal density, and the third factor determines the degree of coupling between the sound field and the structure [5]. In obtaining this form for the coupling factor, it has been assumed that the radiation resistance R_{rad} is much smaller than the total resistance R_{tot}, that is, the major contribution to energy loss in the panel is from internal losses and reflections of flexural waves at panel boundaries, rather than by reradiation of sound from the panel. The following paragraphs discuss the exact form of this coupling factor further.

Numerical Evaluation of the Acceptance Function

Three unfamiliar quantities appear in Eq. (8), the expression for the acoustic acceptance function A_N. These quantities are the modal density n, radiation resistance R_{rad}, and total structural resistance R_{tot}. We may now evaluate each of these quantities on the basis of relatively idealized assumptions, and thereby obtain a numerical value for A_N. We may then refine our value of A_N by reconsidering our assumptions and making them somewhat more realistic.

Previously, a room acoustics result has been used for the number of acoustic modes per unit bandwidth in a reverberant sound field (Eq. (5)). An analogous result for the density of flexural modes on a flat panel is [6]

$$n = \frac{\sqrt{3}\, s}{c_\ell h}, \tag{9}$$

where c_ℓ is the longitudinal wave speed in the panel material.

For a supported plate in the frequency range where the bending wave speed is less than the speed of sound in air (often called the "below coincidence" or "acoustically slow" range), Maidanik finds for the radiation resistance [3]

$$R_{rad} = \frac{1}{\pi^2} \rho c P \lambda_c \left(\frac{f}{f_c}\right)^{1/2}, \quad f \leq 0.2 f_c \,. \tag{10}$$

where P is the panel perimeter, the length of boundaries along which flexural waves are reflected, λ_c is the wavelength (c/f_c) at the coincidence frequency f_c, and

$$f_c = \frac{\sqrt{3}\, c^2}{\pi c_\ell h} \,. \tag{11}$$

The result given in Eq. (10) for R_{rad} assumes the existence of a diffuse reverberant vibrational field on the plate, that is, the incidence of flexural waves is equally probable for all directions.

For flexural waves propagating in riveted structures, there is evidence that most of the energy dissipation occurs at reflection from the boundaries, rather than by internal losses in propagation. When this is the case, it is convenient to express the energy losses by an absorption coefficient γ, defined as

$$\gamma = \frac{\text{power absorbed by boundary}}{\text{power incident on boundary}}, \tag{12}$$

in a manner completely analogous to the statistical absorption coefficient of room acoustics. By assuming a reverberant vibrational field on a plate, Heckl has shown that the relationship between γ and R_{tot} at frequencies below coincidence is [7]

$$R_{tot} = \frac{2\gamma}{\pi} \rho_P c P h \left(\frac{f}{f_c}\right)^{1/2} . \tag{13}$$

Equations (8), (9), (10), and (13) may now be combined to obtain the simple expression for the acceptance function

$$A_N = \frac{\pi}{8\gamma} , \tag{14}$$

or

$$10 \log A_N = -10 \log \gamma - 4 \text{ db}. \tag{15}$$

There is some limited experimental evidence to indicate that, for aluminum panels and riveted stringers, γ varies slowly with frequency, and values in the range from 10^{-1} to 10^{-3} are typical [6]. In the basence of more detailed information, we will assume a frequency-independent value of 10^{-2} for γ to obtain

$$10 \log A_N = 16 \text{ db}. \tag{16}$$

Comparing Eq. (16) with the transfer functions given in Fig. 4, we see that the analytic result is reasonable over the frequency range from 125 to 800 cps, but is significantly higher than the experimental values at lower and higher frequencies. We will now consider three refinements in our analysis, in the hope of improving the comparison in these lower and higher frequency regions.

The first analytical refinement that we consider concerns the differences between the average modal density for flexural waves on plates given in Eq. (9) and the actual modal density associated with our particular panel. The panel coordinate convention is shown in Fig. 6. We assume that the panel edges are clamped, in order that the lowest structural mode will occur around 100 cps, in agreement with the observed structure of the acceptance function. The wave numbers in the x_1 and x_2 directions are related by the expression

$$k_1^2 + k_2^2 = \frac{\pi\sqrt{3}\, f}{c_\ell h} \,. \tag{17}$$

Because of the long, narrow character of the panel we are studying, there is a tendency for modal "clumping" to occur at the frequencies f_m of cross resonance of the panel

$$f_m = \frac{\pi}{4\sqrt{3}} \frac{c_\ell h}{\ell_2^{\,2}} \left(m + \frac{1}{2}\right)^2, \quad m = 1, 2, 3 \ldots \,. \tag{18}$$

The pattern of resonances marked by "x's" in wave number space is shown in Fig. 7. From the form of Eq. (17), we see that a curve of constant frequency is a circle in this diagram. For example, all points on the circular arc in Fig. 7 correspond to the second cross-resonance frequency f_2.

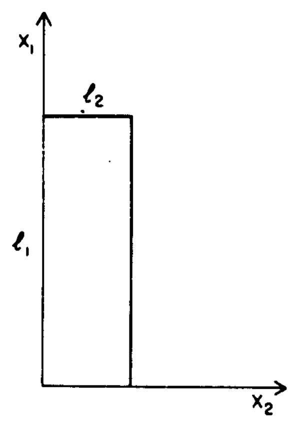

Fig. 6. Panel geomet-

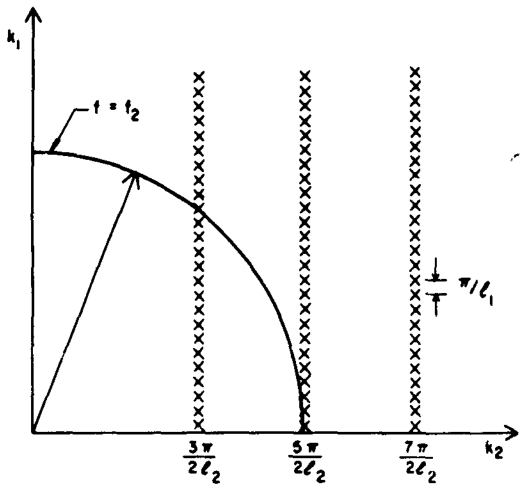

Fig. 7. Modal pattern in wave number space

The shape of Fig. 7 has two important features. First, no modes can occur below f_1, and therefore $A_N = 0$ for frequencies below approximately 100 cps. (Actually there will be some small response below 100 cps, but it will be well below the resonant response at higher frequencies and, therefore, will be neglected here.) Second, the modal "clumping" should lead to greater response than given by Eq. (15), in the frequency bands containing the first few modes. We have derived a modal density which contains this "clumping" effect and approaches the asymptotic expression of Eq. (9) at high frequencies. It is

$$n' = \frac{2\sqrt{3}\,\ell_1}{hc_\ell} \sum \frac{1}{k_1}, \tag{19}$$

where the indicated sum is over the possible values of k_1 consistent with f. The correction factor to be applied to Eq. (15) is therefore

$$\frac{n'}{n} = \frac{2\ell_1}{s} \sum \frac{1}{k_1} = \frac{2}{\ell_2} \sum \frac{1}{k_1}. \tag{20}$$

Equation (20) has been integrated over 1/3-octave bands and is plotted in Fig. 8.

The second refinement considered concerns the loading of the accelerometer on the skin panel. The mass of the accelerometer and its mounting block was approximately 20 gm. A point load of this mass on a vibrating 0.1-inch thick aluminum panel reduces the panel response by 3 db around 1500 cps, and the loading effect increases approximately 6 db per octave above this frequency. This correction is shown in Fig. 8. The modal density and accelerometer loading corrections have been combined with Eq. (16), and the result is plotted as the reverberant estimate (broken line) in Fig. 9. Although far from perfect, the agreement between experiment and theory shown in this figure is considered to be encouraging.

A third refinement that may be considered concerns the effect of the actual nonuniform angular distribution of the incident sound field on the panel. In using Maidanik's expression for R_{rad} (Eq. (10)), we have assumed that the incident sound field is perfectly diffuse. From the TITAN SCF configuration, we expect that the angular distribution of the incident sound field will be heavily weighted toward waves moving in the axial direction near grazing incidence. Calculation of R_{rad} for a diffuse field weighted in this way gives results for the acceptance within 1 db of those calculated for the completely diffuse field. Considering the extreme case, that is, all sound at axially grazing incidence on the structure, we find that the expression for R_{rad} contains a singularity in the region near the first few cross resonances. Maidanik has made a careful study of the behavior of R_{rad} in this range [3]. His results may be used to show that the acceptance function for this grazing incidence sound field exceeds the values calculated from Eq. (16) by approximately 5 db, at the first few cross resonances. This result can be interpreted as an indication of the excellent coupling that can occur between the structure and the unidirectional sound field. This analytical result for the traveling wave sound field is also shown in Fig. 9, and is not very different from the reverberant field estimate.

From these considerations of sound fields with varied angular distributions, it is to be concluded that the response estimate shows a weak dependence on the precise angular distribution of incident sound.

Discussion of Assumptions

In the course of making the response estimates given in Fig. 9, several assumptions have been made in order to utilize the research results available. Consideration of these assumptions may help to indicate the nature of future work required and, in particular, the nature of new experiments.

The following assumptions have been mentioned:

- The incident sound field has been assumed to be reverberant. The present results indicate that the particular angular distribution

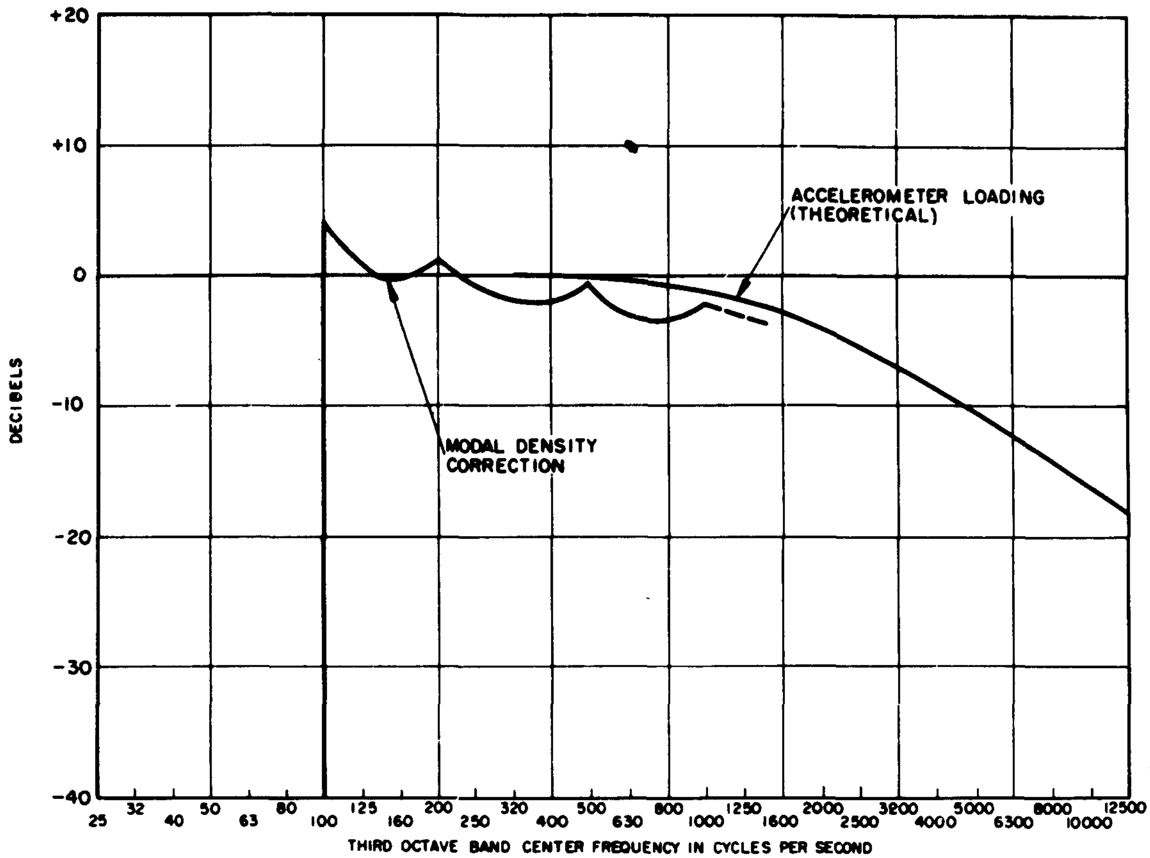

Fig. 8. Modal density and accelerometer loading correction terms

of the sound field does not affect the calculation of R_{rad} significantly. However, some modification in the calculation of S_p may be required for the results to be completely valid for a traveling wave field.

• The vibrational field has been assumed to be reverberant. The correlation results indicate that this is a reasonable assumption.

• The panel dimensions have been assumed to be determined by the longitudinal stringers and circular frames. The correlation data show the reasonableness of this assumption in the frequency region containing most of the vibrational energy. However, the correlation results cannot be used to validate this assumption in other frequency regions. An experimental investigation into this question is suggested in the next section.

• The radiation resistance calculations have all involved the assumption that the frequency is less than 0.2 f_c. For a 0.1-inch aluminum plate, 0.2 f_c is approximately 800 cps. Additional analytical work is needed to extend the frequency region of validity.

• The calculation of the total structural resistance has involved the assumption that the predominant source of energy loss is at the panel boundaries. This assumption appears reasonable, but further studies are required.

• The value of the boundary absorption coefficient γ has been chosen rather arbitrarily, and permits as much as a ± 10 db variation in the number obtained by Eq. (14). Further experimental information here is also desirable.

• The calculation of structural modal densities and resistances assumes that the curvature of the missile panel is not significant in determining these quantities. Considerably more analytical work is required to investigate this matter. However, some additional experimental information concerning modal densities can be readily obtained, as suggested in the next section.

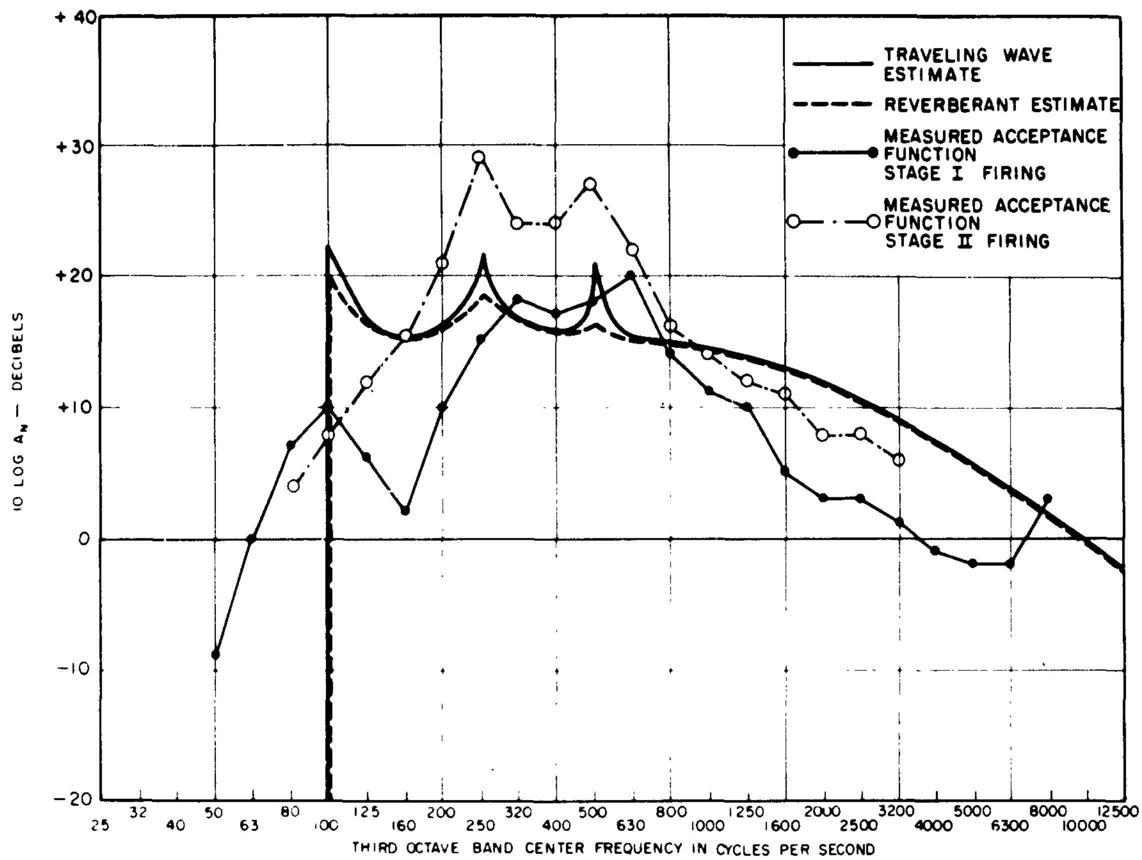

Fig. 9. Comparison of estimated and measured acceptance functions between tanks on stage I, TITAN J-3 SCF, assumed damping $\gamma = 10^{-2}$

- The effects of accelerometer loading have been calculated on the basis of a point mass model. These effects can also be investigated experimentally.

SUGGESTED MEASUREMENTS

The results of the previous section have shown that the statistical analysis of complex structures can provide reasonable estimates of sound-induced vibration, if certain important assumptions concerning the missile motion and concerning values of physical constants are made. In order to improve the theoretical and empirical understanding of the structural response, it is therefore necessary to obtain improved experimental information on the validity of these assumptions. We suggest experimental investigations in five different areas. The first four of these areas are suggested directly by the present study. The last area concerns a related subject, the estimation of sound-induced stress.

Structural Damping

It is clear that the energy loss or structural damping (represented by the boundary absorption coefficient γ) controls the vehicle skin vibration, but there is very little information presently available on the damping of such structures. A careful and proper measurement of structural damping is thus essential in order to obtain any improvement in our understanding of sound-induced vibration.

Structural damping can be measured by providing a uniform random excitation of the structure, removing the excitation "instantaneously," and observing the resulting decay rate of the structural motion. The decay rate should be observed in relatively narrow-frequency bands, certainly no broader than octaves. It is important to note that the excitation must be uniform over the structure. Localized excitation, such as a single shaker, will not provide this uniform excitation. (The decay rate with such localized excitation will present not only dissipated energy loss, but also energy transfer away from the point of excitation. For this reason, localized excitation is not suitable for a damping measurement.) Uniform excitation can be approximated by the use of several shakers distributed around the structure or by acoustic excitation of the whole structure. Care must be taken that the excitation ceases "instantaneously," that is, much faster than the decay rate of the structural motion.

Stringer Vibration

Figure 9 shows that the measured structural response is significantly below the predicted response in the frequency range of approximately 100 to 200 cps. A possible explanation of this difference is the decoupling between sound and structure, which may occur when the vehicle stringers cease to form a stationary support. In other words, the structure will be excited to a lesser degree if the stringers share the panel skin motion at lower frequencies. This behavior can be checked by monitoring the stringer vibration amplitude and the skin vibration amplitude at the same time, and observing whether the relative motion of the stringers is indeed greater below 200 cps than it is above 200 cps.

Accelerometer Loading

The observed decrease of structural response above 1000 cps is greater than has been calculated on the basis of the accelerometer loading. It would, therefore, be valuable to make direct measurements of the loading effects. Such measurements could be made on a structure attached to a shaker, with accelerometers of various masses loading the structure. Alternatively, the structure may be shaken by acoustic excitation. The excitation, either by shaker or by an acoustic source, should be random, and the response should be measured in octave or narrower bands.

Density of Vibrational Resonances

The exact nature of the modal density function n can be determined by pure tone excitation of the structure. A slow sweep through the frequency range below 500 cps and a counting of resonances encountered would determine the extent of modal "clustering," such as has been postulated in Eq. (19).

Stress-Velocity Relations

It has been shown that, for a wide range of wave motions and structures, a relatively simple relation exists between the vibration velocity of a structure and the stress in that structure. This equation can be put in the form

$$\sigma = \beta Y \frac{v}{c_{\ell}}, \qquad (21)$$

where σ is the rms stress, Y is Young's modulus, v is the rms velocity, c_{ℓ} is the speed of longitudinal waves in the structure, and β is a parameter on the order of unity. It would be valuable to obtain detailed information on the behavior of the parameter β as a function of frequency for missile structures. Equation (21) permits this behavior to be determined directly from measurements of the structural vibration and stress.

Notes

It should be noted that all of these tests may be performed with relatively low amplitude excitation. Also, only the test measuring the modal density n uses pure tone excitation. The remaining four tests use random excitation.

The measurements suggested here should provide valuable information directly applicable to the problems of estimating sound-induced vibration and stress in a missile structure.

Appendix A

THE SOUND-INDUCED VIBRATION OF A RIBBED PLATE

The importance of radiation resistance in determining structural response, and a physical picture of how this importance arises, can be obtained by describing briefly a research experiment performed at BBN [3]. In this experiment, a flat aluminum panel 24 × 44 × 0.032 inches was hung in a reverberant sound field, and its velocity spectrum was measured.

Another panel of the same dimensions with a riveted frame-stringer network was hung in the same sound field, and its velocity spectrum was measured. The weight of the second panel was 50 percent greater than the first, the structural damping was greater by a factor of 3, and the static stiffness was much greater. Figure A1 shows the relative vibration of the two panels. One may be surprised to see that the ribbed panel vibrated with an amplitude as much as 14 db greater than the plain panel. Since mean-square resonant response is inversely proportional to damping, the increase in structural damping by a factor of 3 would correspond to a decrease in response of 5 db. Therefore, the addition of ribs increased the sound-vibration coupling by as much as 19 db.

This large increase in sound-vibration coupling is related to the increase in reflecting boundaries provided by the application of the ribs to the panel. A flexural wave will radiate effectively only if the flexural wavelength is longer than the wavelength in the surrounding fluid ("above coincidence"). The presence of discontinuities, such as the ribs, permits incident waves to be scattered upon reflection from these boundaries. Even if the incident waves themselves do not radiate well to the surrounding fluid, the scattered waves will usually contain some components that will be above coincidence and will radiate well. Thus the ribbed panel will radiate sound more effectively than the plain panel because there are more scattered waves on the ribbed panel and therefore more waves that radiate well. By the principle of reciprocity, a panel that radiates sound well will also be readily excited into vibration by incident sound. Thus, because of the possibility of more scattered waves, the ribbed panel responds more readily than does the plain panel. Formally, we say that the radiation resistance of the ribbed panel is greater than that of the plain panel.

The calculation of the radiation resistance of ribbed structures is treated in quantitative detail in Ref. 3.

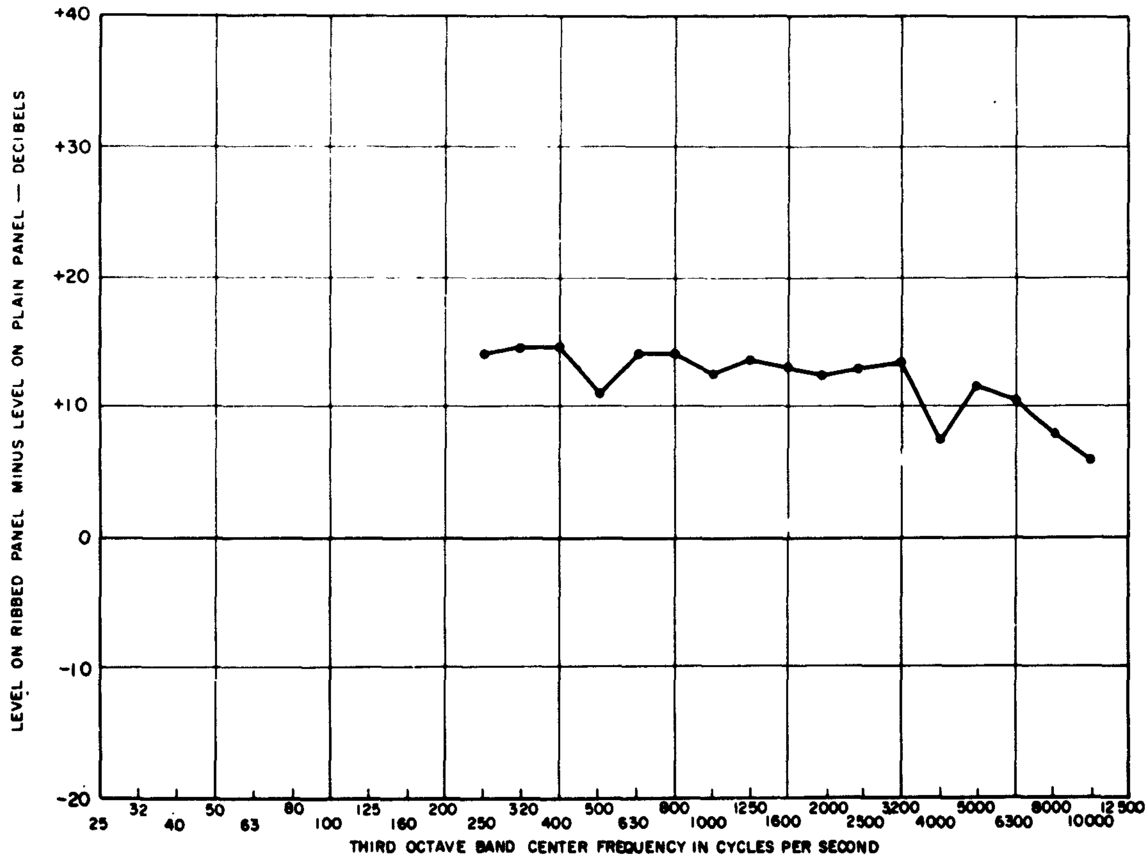

Fig. A1. Relative vibration of plain and ribbed panels in reverberant sound field

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ESTIMATION OF NOISE LEVELS AT THE SURFACE OF A ROCKET-POWERED VEHICLE*

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The general properties of rocket noise fields are discussed, and a procedure is presented for estimating the octave band sound pressure spectrum at a vehicle surface.

INTRODUCTION

The noise generated by a large rocket propulsion system forms an important part of the environment experienced by a vehicle propelled by such a system. This noise may produce vehicle equipment malfunction, vehicle structure fatigue, or communication problems or discomfort for vehicle occupants. In order to assess these problems properly, it is necessary to obtain estimates of the rocket noise field at the vehicle surface, often in the planning stages of the vehicle. Noise measurements have been made on several large rocket-powered systems, including JUPITER, ATLAS, TITAN, and SATURN. This paper presents estimation procedures based on these measurements and presented in a generalized form that permits extrapolation to larger systems.

The firing configuration considered here involves the rocket firing vertically downward, with an exhaust deflector turning the stream into one or more horizontal paths. This geometry is used in many static test firings and surface launches of large contemporary rocket-powered vehicles.

GENERAL PROPERTIES OF ROCKET NOISE FIELDS

For engineering purposes, the acoustic field of a rocket engine may be conveniently divided into the "far field," where the distributed nature of the sound sources is not of prime importance, and the "geometric field" (often referred to as the "near field"), where the spatial distribution of the sound sources must be taken into account. The surface of the space vehicle is typically located in the latter region. (In the immediate vicinity of the source distribution, there is also the "induction field," where sound pressure and particle velocity are generally not in phase.) Because of the distributed nature of the rocket exhaust noise source, a detailed analytical understanding of the near field behavior is extremely complicated, and noise level estimates are generally based upon empirical procedures.

Most of the noise produced by rocket engines is associated with the turbulence occurring in the exhaust stream. The principal noise sources appear to be located in the turbulent interface between the exhaust and the surrounding atmosphere, particularly in the region where the exhaust flow velocity first becomes subsonic. This is a tentative conclusion based on experimental evidence. Use of an exhaust deflector will affect the flow pattern and thus the manner in which the flow becomes subsonic. It may be expected that the rocket noise field will depend somewhat on the type and location of the exhaust deflector.

Although present-day liquid fuel engines vary markedly from solid fuel engines in their design and operation, both types of engines produce exhaust streams of high-velocity hot gases. A typical exhaust velocity is of the order of 7500 fps for both kinds of engines. Because the sound pressures are associated with the exhaust flow,

*This paper was not presented at the Symposium.

we expect that the noise field of the two types of engines will not be markedly different, and experimental evidence to date indicates that this is the case. Of course, exception must be made for nonturbulent noise sources that are particular to any engine design, such as resonant burning in a solid fuel engine or fuel line oscillations in a liquid fuel engine. In well-designed engines these nonturbulent noise sources have been found to be insignificant compared with the usual turbulence noise and consequently will not be considered in this material.

For dynamically similar systems, it may be shown [1,2] that the sound pressure spectra measured at similar positions are the same when given in constant-percentage-frequency bands and when frequency is scaled inversely proportional to a characteristic length. This result has been verified in experiments [3] and gives rise to the use of a nondimensional frequency parameter. This parameter is the so-called Strouhal number, defined by frequency times a characteristic dimension of the system (such as engine nozzle diameter) and divided by a characteristic velocity (such as expanded exhaust velocity). Under the assumption that the characteristic velocity of all rocket systems under consideration is approximately constant, the frequency parameter may be reduced to "frequency times nozzle diameter."

Up to this point, it has been tacitly assumed that only single-nozzle systems are being considered. The inclusion of multiple-nozzle systems requires the definition of an effective nozzle diameter D_{eff} to be used in obtaining the frequency parameter. For a system consisting of n equal nozzles spaced one nozzle diameter D or less apart, the effective nozzle diameter is found from experiments to be

$$D_{eff} = \sqrt{n}\ D. \qquad (1)$$

Limited data indicate that D_{eff} is the same as D if the nozzle separation distance greatly exceeds D.

The fuel-oxidizer mixtures and nozzle geometries used currently in most liquid rockets and some solid rockets are such that the density of the exhaust gases is approximately constant. Under this condition the total rocket thrust is proportional to the total nozzle exit area because, as has been stated, the exhaust velocity does not vary significantly. When this simple relationship between thrust and area exists, the overall sound pressure levels measured at geometrically similar points on two systems are the same. However, where this thrust-area relationship does not hold, a level adjustment must be included in extrapolating levels from one system to another.

In addition to the magnitude ot the sound pressure spectrum, the spatial correlation of sound pressure over the vehicle surface may be important in determining the structural response. For example, if the sound pressures are correlated only over a small area of the structure in question, the pressure field is inefficient in exciting large-scale motions of the structure. Both the longitudinal and angular sound pressure correlations near large space vehicles have been reported by Dyer et al. [4], and will not be discussed further here.

A recent comprehensive review of all aspects of rocket noise has been made by Wiener [5]. The reader is referred to Ref. 5 for a summary of current information on such subjects as long range sound propagation, appropriate acoustic criteria, and rocket noise reduction methods.

ESTIMATION PROCEDURES

On the basis of the previous observations, it is possible to combine available rocket noise data in a generalized form. The scaling results suggest that we utilize constant-percentage-frequency bands. Sound pressure levels given in octave bands are generally sufficiently precise for preliminary estimates of rocket noise. Figures 1-3 show the octave band sound pressure levels to be expected in the open at three positions along a space vehicle on the test stand or prior to lift-off. These positions are near the vehicle surface and are described by a coordinate x measured along the vehicle axis and above the nozzle exhaust plane. Since the shape of large rocket vehicles does not vary greatly, these three positions generally correspond to points near the vehicle tail, halfway along the vehicle, and near the vehicle nose.

The estimation curves are presented in terms of the generalized frequency parameter $f \times D_{eff}$ and the total thrust F. The center (geometric mean) frequency of the octave band in question is f, and D_{eff} is the effective nozzle diameter of the rocket engine system, as given by Eq. (1). The "shaded band" mode of presentation emphasizes the uncertainties involved in extrapolating the measured data to larger systems.

The steps involved in the estimation procedure are as follows:

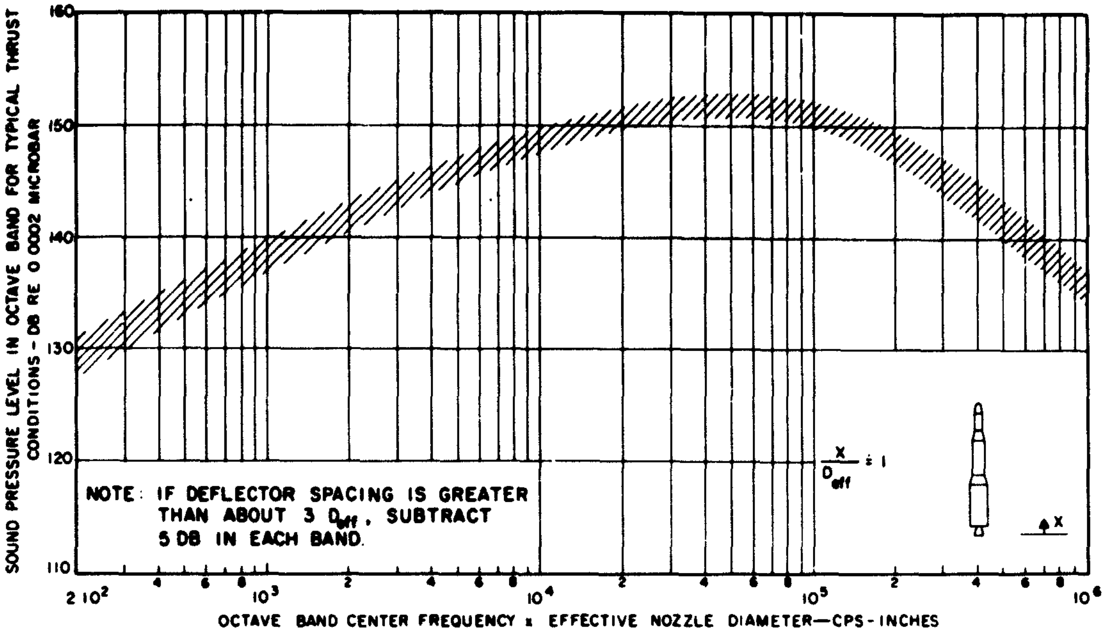

Fig. 1 - Sound pressure levels near tail section of space vehicle-static test or before lift-off

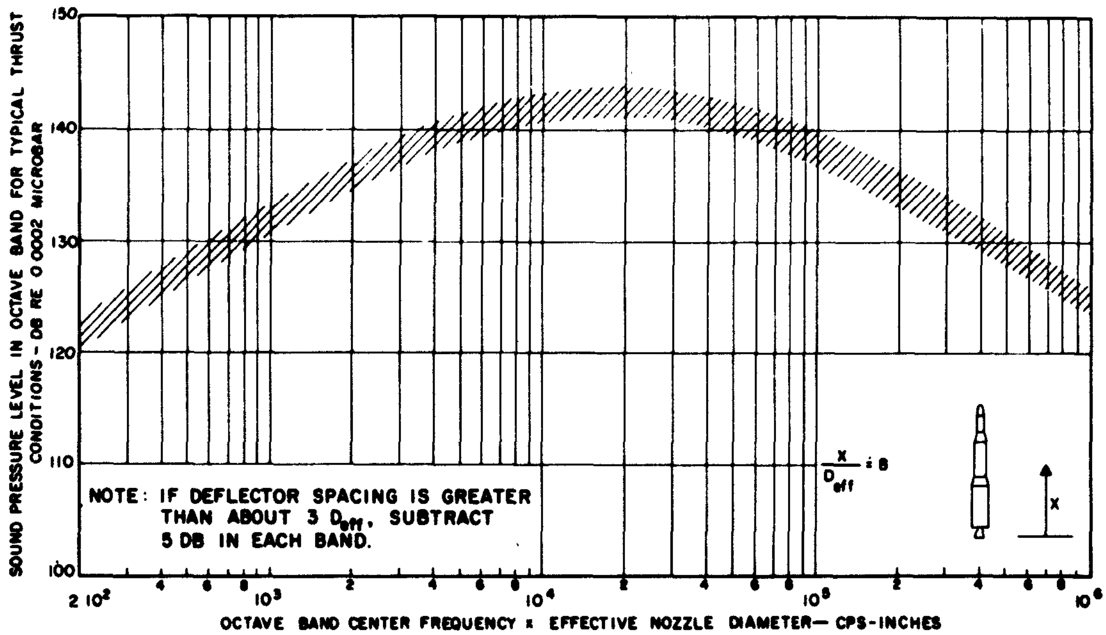

Fig. 2 - Sound pressure levels halfway along length of space vehicle-static test or before lift-off

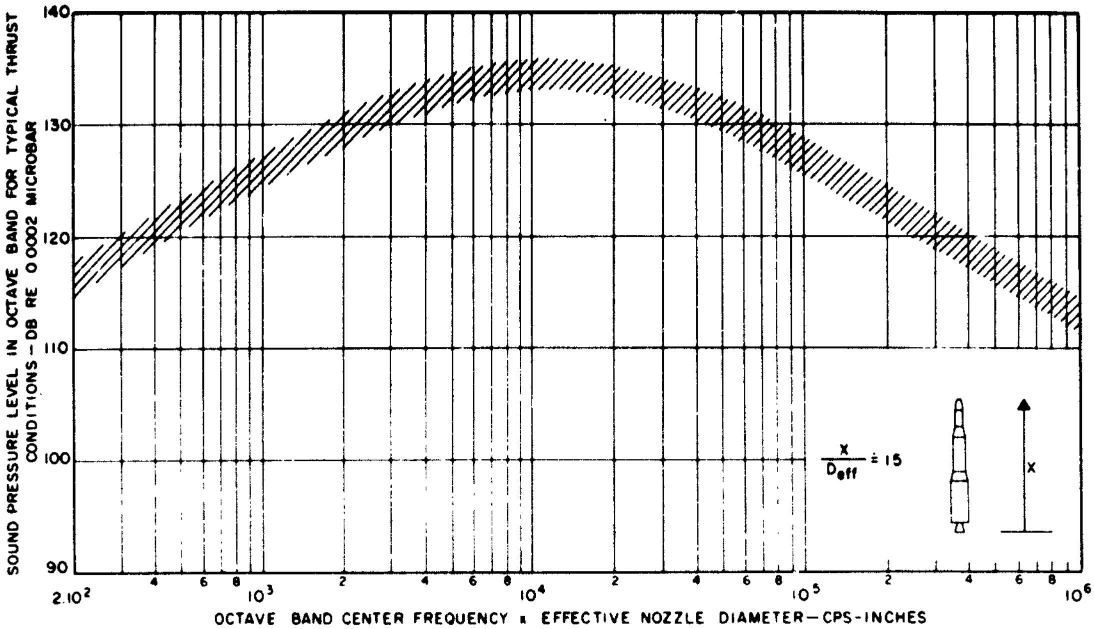

Fig. 3 - Sound pressure levels near nose of space vehicle-static test or before lift off

1. Determine the total system thrust F in pounds and the effective nozzle diameter D_{eff} in inches from Eq. 1.

2. Obtain the octave band sound pressure level estimates from Figs. 1-3 for typical thrust conditions. Replot the abscissas of these curves in terms of frequency, using the appropriate value of the effective nozzle diameter D_{eff}.

3. Determine the typical thrust from Fig. 4.

4. Calculate the quantity

$$10 \log_{10} \frac{\text{total thrust}}{\text{typical thrust}}$$

and add this quantity to the levels obtained in Step 2.

5. For deflector configurations in which the spacing between the rocket exhaust nozzle and the impingement point on the deflector exceeds about three times D_{eff}, make the appropriate level correction in each band as indicated in Figs. 1-3.

The level estimates obtained by this procedure represent the maximum octave band sound pressure levels to be expected, since the levels in the vicinity of the vehicle generally decrease after lift-off. Shielding effects, if present, will reduce the maximum levels below the estimates obtained by this procedure.

As additional data become available on larger rocket-powered systems, it is expected that this procedure will be revised to provide improved estimates for planning purposes.

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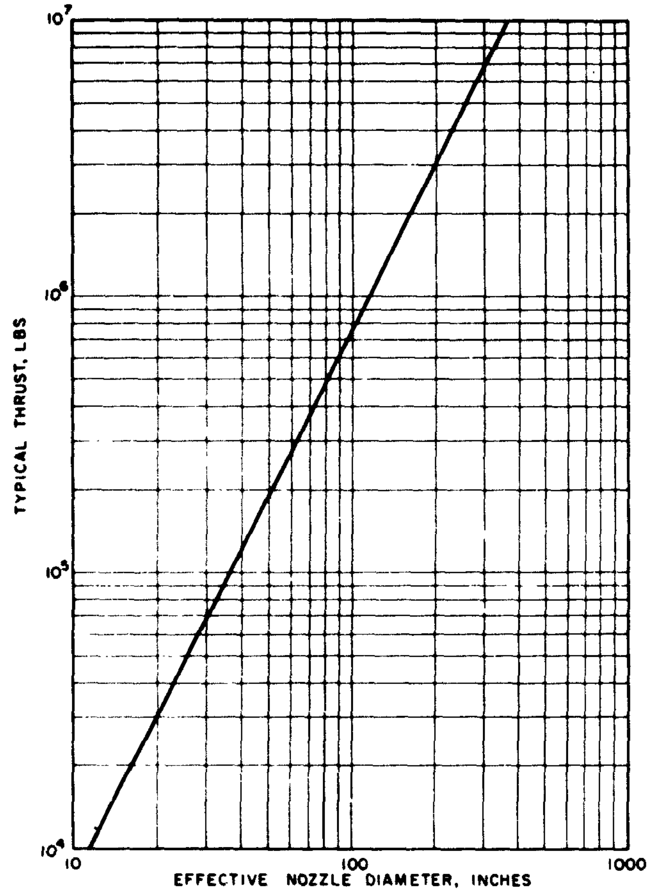

Fig. 4 - Relationship between typical thrust and effective nozzle diameter

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RANDOM FATIGUE DATA

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A test program is described for obtaining fatigue data applicable to the design of structures subjected to random vibration loading. The motivation and test philosophy is discussed. Small cantilever beams subjected to "Band-Limited White Noise" base excitation were tested. The instrumentation used to measure the vibration response consisted of a commercially-available optical displacement tracking device. The estimated bending strain response computed from the measured displacement of the free end of the cantilever is compared with a direct strain measurement. The agreement is found to be good. It is concluded that the optical instrumentation combined with analog-to-digital data processing provides a workable means of accurately and economically obtaining the large amounts of response data required in a random vibration fatigue test.

INTRODUCTION

At the present time, the prediction of structural damage due to random vibration loading is an extremely difficult, if not impossible, task. Cumulative damage predictions from experimental data obtained in constant amplitude sinusoidal fatigue tests, have in general been unsuccessful. Predicted lifetimes differing from the measured by 50 or more are not too uncommon in the literature. Of course this is not entirely the fault of the cumulative damage concept, and the degree of usefulness of the sinusoidal data in predicting "random" fatigue is still an open question. It appears to the authors that if one is to be able to predict (with any confidence) the damage due to random vibration and, thereby, guard against in-service failures, one must build up a backlog of knowledge on the failure characteristics of basic structures when subjected not to sinusoidal stresses, but to random excitation. This paper describes an engineering test program which was designed for such a purpose.

TEST PURPOSE AND PHILOSOPHY

The purpose of the test program was to provide basic data for establishing rational design criteria for structural components subjected to random vibration loading. Internal equipment support structures commonly used in missile and spacecraft design was of primary interest and structural frequencies in the range of 50 to 500 cps were of concern. Such structures must be designed to withstand the vibration environment incurred in both ground and flight test. The tests were conducted at near room temperature since this is the condition that exists in flight during the most severe vibration environment.

The test philosophy employed was basically as follows: (1) Full-scale structural components are difficult and expensive to test. Consequently, one must make use of failure information obtained from small-scale specimens. A certain degree of simulation can and should be included even in the latter case by using the material of interest at the appropriate temperature, humidity, and so on. The obvious objections to coupon testing are understandable in that accurate predictions concerning the fatigue life of an actual structure are difficult to achieve. This, however, is a matter of developing engineering methods and criteria for applying the basic data. For instance, consider the problem faced by the stress analyst in using allowable yield stress, shear stress, bearing stress, and so on. The methods developed for applying these "allowables" (which are usually obtained from simple tests using unrealistic structures) have been very successful in the design of aircraft and missile structure. (2) The specimens tested were simple cantilever

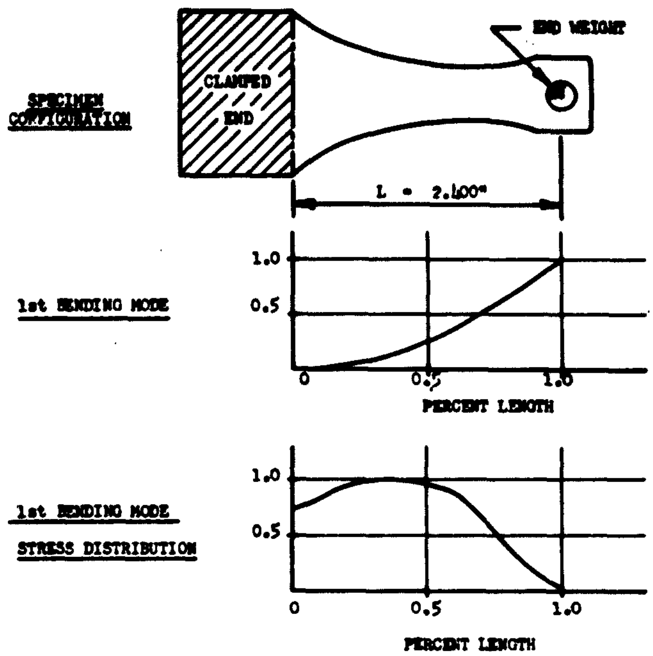

Fig. 1 - Fatigue specimen configuration

beams (Fig. 1), made of 0.063-inch gage, AZ31B - H24 Mag. Alloy. The reason for the peculiar shape is explained in the following paragraph. The cantilever beams were excited in bending by the use of "Band-Limited White Noise Base Excitation" so that only the first bending mode was excited. The cantilever beam was selected over others because of the simplicity of the boundary conditions. The base acceleration excitation was selected since most shaker systems are designed to monitor accelerations. (3) In spite of all the models and theories yet proposed to explain fatigue in engineering materials, the simple fact remains that high, local, time-varying stresses (stress concentrations), if allowed to exist long enough, will initiate a fatigue crack. It is usually not possible to obtain direct measurements of the local stresses at the stress concentration; therefore, predictions based on the measured nominal stress or strain must be relied on. The high dependence of fatigue on local stresses was responsible for the shape of the specimens shown in Fig. 1. A rectangular cantilever beam, when vibrating in the first mode, produces a maximum bending stress at the cantilever edge. The tapered beam used in the present test produces a maximum bending stress at about mid-span; this precludes dealing with stress concentrations at the cantilever edge. Stress concentrations were introduced by means of machined holes at the critical cross section (not shown in Fig. 1), since it was felt that better predictions of the local stresses could be obtained in this way. (4) Since both the forcing functions and the material properties are random processes, the statistics of the problem are important. Essentially, this means that "large" amounts of data must be collected and analyzed. Because of the possibility of extensive and unorthodox computations, it was decided to collect the response data in digital form rather than analog form in order to utilize the greater flexibility afforded by digital computer methods.

INSTRUMENTATION

The instrumentation for such a fatigue test presented a number of problems. Originally, a direct measurement of the bending response due to the random vibration was attempted by means of strain gages. Foil gages were tried first. These, however, proved to be unsatisfactory since the gages failed in fatigue before the response data could be collected. Microscopic examination showed a multitude of fatigue cracks in each gage after a relatively short time (5 to 60 seconds) and moderate rms bending stress (5000 psi). Bakelite and Japanese paper gages were tried, but these also failed in fatigue. Next, an indirect measurement of the bending stress was attempted by mounting a subminiature accelerometer in place of the end weight at the free end of the cantilever. This also proved unsatisfactory because the accelerometers were susceptible to mechanical failure and because of the difficulty involved in physically calibrating them for the high accelerations expected.

The instrumentation finally adopted consisted of a commercially available optical tracking device capable of accurately measuring the end displacement of the cantilever. A good approximation to the corresponding stress response for lightly damped systems can then be obtained by computation. The particular instrument used was the Model 701 Displacement Follower, manufactured by the Optron Corporation. This instrument was designed to produce a voltage output directly proportional to the displacement of a point on a moving object. A small spot of light is focused on a reflective surface. The reflected light activates a photocell servo system which causes the spot of light to follow the motion of the target. This measurement system is particularly attractive for the type of dynamic testing discussed here, since it required no physical contact with the test specimen.

The Displacement Follower has a frequency response essentially flat from 0 to 3 kc, with a 3-db drop at 7 kc. Various full-scale displacements are achieved by use of interchangeable lenses. An interesting feature of this device is a static calibration procedure which displaces the instrument a prescribed amount with respect to the object. This is accomplished by means of a micrometer lead screw.

A schematic of the instrumentation and peripheral equipment used in the test laboratory is shown in Fig. 2. A random vibration console generates a random signal which drives the 150-pound-force electromagnetic shaker. This signal is filtered so as to produce a nominal 20- to 500-cps "Band-Limited White Noise" acceleration at the base of the cantilever beams. No great attempt was made to define this spectrum accurately, but only to insure that it remained constant during a particular sequence of tests. The light spot from the "Optron" was focused on the side of the specimen opposite the center of the end weight. The output signal was then processed through a divider network and dc Amplifier to supply the proper input voltage to the voltage controlled oscillator (VCO). The output of the "Optron" was also monitored on an oscilloscope. The FM signal from the VCO (70-kc carrier frequency, ± 15-percent bandwidth) was transmitted to the Dynamic Ground

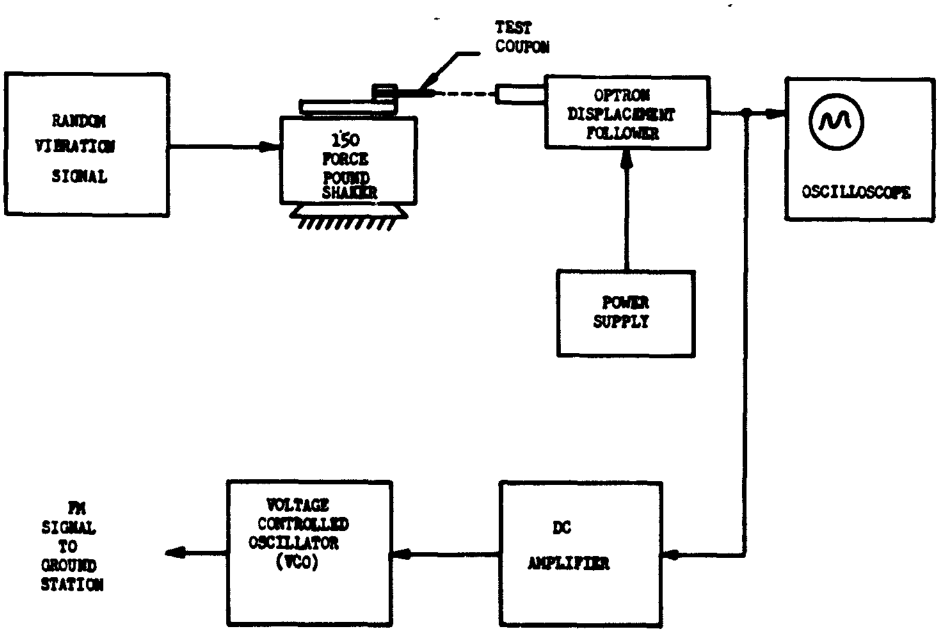

Fig. 2 - Fatigue test instrumentation

Station for recording and processing as described in the next section.

DATA RECORDING AND PROCESSING

In order to capitalize on the advantages afforded by present day, high-speed, digital-computer methods, the data must first be put in digital form. Fortunately the LMSC Dynamic Ground Station, which is normally used to process flight telemetry data, was available for this purpose. Although such a complete facility would not normally be available to the investigator, there are simpler, less expensive, commercially-available systems for putting dynamic data into digital form. A schematic of the portion of the Lockheed System used in recording and converting the data is shown in Fig. 3.

The "Optron" signal is transmitted by coaxial cable from the Test Laboratory area to the Dynamic Ground Station area using an FM carrier frequency of 70 kc and a bandwidth of ±15 percent. This channel was selected on the basis of a requirement for recording vibration response in the frequency range of 50 to 500 cps. The transmitted FM signal is recorded on magnetic tape along with a voice identification and a 50-kc reference signal. A real time oscillograph record is also made in order to monitor the data recorded on the tape. The actual conversion of the analog data to digital form is a very complex process and the details will not be attempted here. The 50-kc reference signal is used in determining the coordinates of the digital signal and thus provides automatic tape speed compensation. Sampling rates up to 8000 samples per second can be selected. For the fatigue test data under discussion, a sampling rate of 1250 samples per second was used. Either a "high density" or "low density" digital format can be chosen. By use of the "high density" mode approximately 1200 seconds of real time data, sampled at 1250 samples per second, can be stored on one 2400-foot binary tape. This tape is in the proper format for reading into the IBM 7090 Digital Computer.

COMPARISON OF ESTIMATED AND MEASURED BENDING RESPONSE

The estimated bending strain computed from the measured displacement of the free end of the vibrating beam is compared with a direct strain gage measurement. A special semiconductor (silicon filament) gage, mounted so that it measured the strain transverse to its principal axis, was found to be "fatigue resistant" for purposes of the direct measurement. The strain gage was mounted at the station where the maximum first mode bending stress occurs (Fig. 1).

The estimated bending strain or bending stress was computed as follows. By use of the Bernoulli-Euler Beam Theory (plane sections remain plane) the undamped equation of motion for a cantilever beam excited by "Base acceleration" was found to be ...

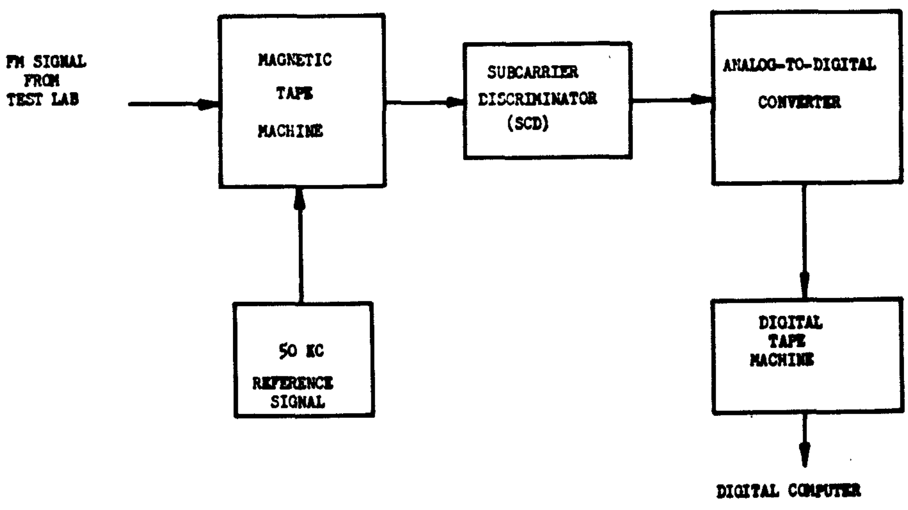

Fig. 3 - Data recording and processing

$$\frac{\partial^2}{\partial x^2}(EIy'') + m(x)\ddot{y} = -m(x)\,\ddot{y}_o\,, \qquad (1)$$

where

$\ddot{y}_o$ is the acceleration of the base,

y is the beam deflection relative to the base,

y'' is the small deflection theory curvature, and

$m(x)$ is the mass distribution.

By expanding y in a series of normal modes, we obtain

$$y(x,t) = \sum \phi_n(x)\; q_n(t)\,, \qquad (2)$$

where the ϕ_n's are orthogonal with respect to the mass distribution $m(x)$ and over the interval 0 to L. The maximum bending strain at any particular cross section of the beam can then be written,

$$\epsilon_b(x,t) = \frac{h}{2}\sum \phi_n''(x)\; q_n(t)\,, \qquad (3)$$

where h is the beam thickness.

Likewise, the bending stress is

$$f_b(x,t) = E\epsilon_b(x,t) = \frac{Eh}{2}\sum \phi_n''(x)\; q_n(t)\,. \qquad (4)$$

Two additional assumptions concerning the known or expected dynamic response of lightly-damped systems will now be made in arriving at the final result. (1) The deflected shape of the beam at any time is essentially that of the first bending mode (Fig. 1). (2) The relative displacement of the free end of the beam is nearly equal to the absolute displacement. Then, denoting the measured displacement at $x = L$ by δ,

$$\delta = y + y_o \cong q_1\,. \qquad (5)$$

These two assumptions, when combined with Figs. (3) and (4), yield

$$\epsilon_b(x,t) \cong \frac{h}{2}\,\phi_1''(x)\;\delta(t)\,, \qquad (6)$$

$$f_b(x,t) \cong \frac{Eh}{2}\,\phi_1''(x)\;\delta(t)\,. \qquad (7)$$

In Fig. 4, a short sample of the time history of the bending strain obtained from Eq. (6) is compared with the strain gage measurement previously described. Both signals are shown as they appear after "digitizing." The first natural frequency of this particular beam is approximately 100 cps. The sampling rate is 1250 cps. The rms bending stress associated with the two signals was found to be the same and equal to 8199 psi or approximately 28-percent of the yield stress for this particular material.

As would be expected, both time histories have the general character of the response of a lightly damped system to a "White Noise" input. The two signals exhibit reasonably good agreement. Figure 4 also serves to illustrate an important point. In general, with measurements of this type it is a good idea to perform some type of low-frequency filtering of the data. In the case of the optical measurement, undesirable rigid body motion of the shaker, which is interpreted as bending response in Eqs. (5) and (6), can be eliminated by filtering. This filtering

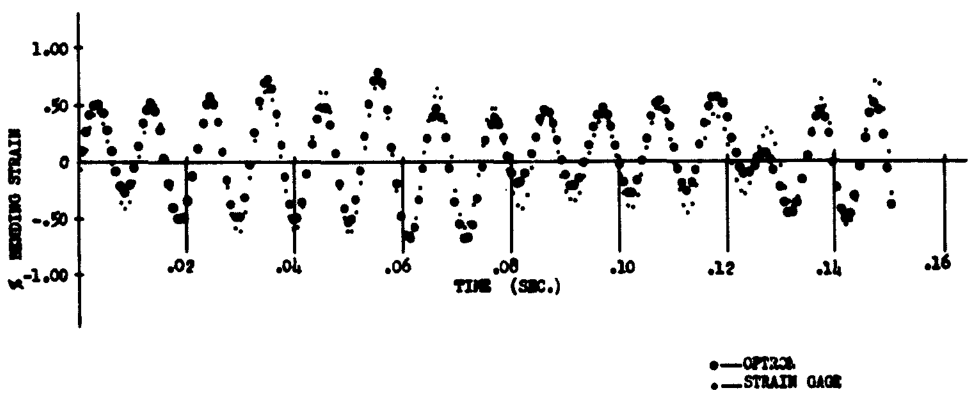

Fig. 4 - Comparison of direct and indirect bending strain measurement

can be performed either by analog means before the data is recorded, or by digital filtering after the digital tape is made. In the case of the test program described here, it was decided to use digital filtering. Thus the estimated bending response shown in Fig. 4 is further improved in later digital operations.

The Cumulative Probability Distribution Function (CDF) is plotted in Fig. 5 for both the estimated bending response and the strain gage measurement. About 20 seconds of data at 1250 samples per second or approximately 25,000 data points were used in computing each CDF. The results are plotted on probability paper such that the normal distribution plots as a straight line. The solid straight line shown is the normal distribution that would be obtained using the variance of the strain gage measurement. The two CDF's are found to be close to the normal and in good agreement with each other. The greatest differences occur at the extreme values. Again the agreement can be improved by low-frequency filtering.

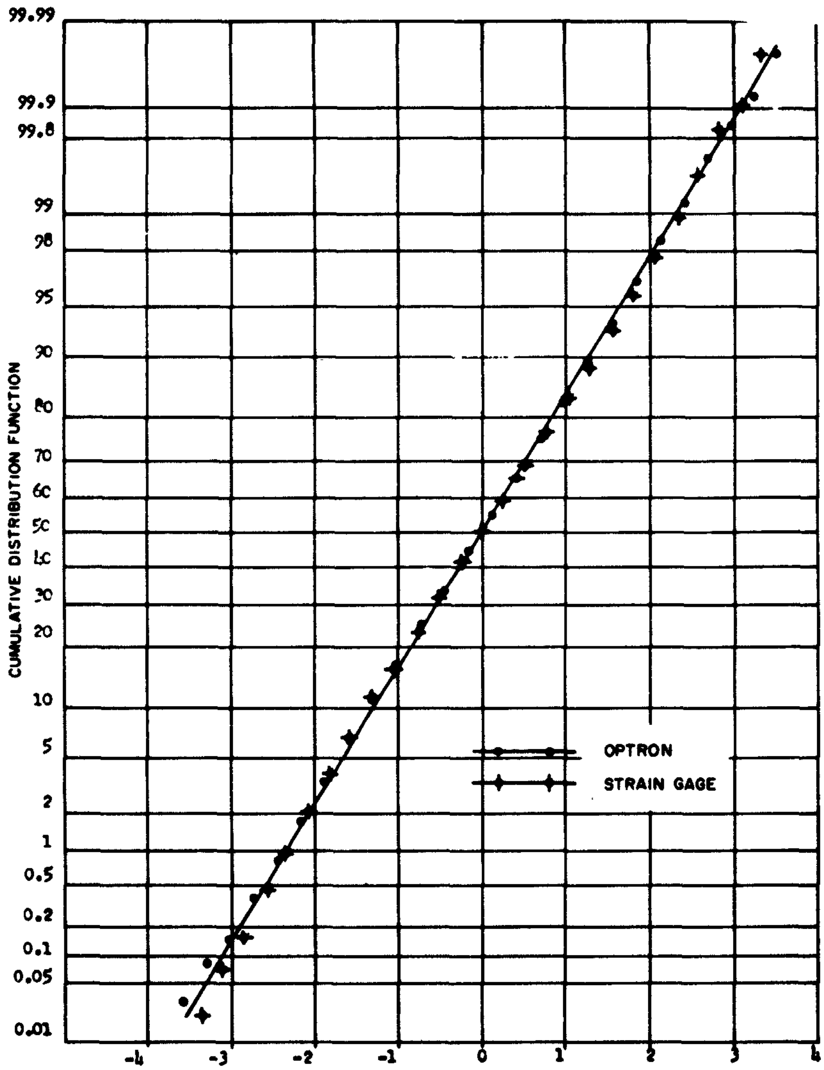

Fig. 5 - Cumulative distribution function of direct and indirect bending strain measurement

CONCLUSIONS

Although the analysis of the random fatigue data and its application to structural design problems is still under consideration, it appears that a workable and economical test program for collecting basic data has been achieved.

* * *

A GENERALIZED RESPONSE EVALUATION PROCEDURE FOR MULTIDEGREE SPRING MASS SYSTEMS*

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This paper outlines, in generalized language, a procedure for evaluating the response of linear multidegree of freedom spring mass systems to shock and to other forms of disturbances as well. The novel feature is a concise summary of the relations which permit the responses of multidegree systems to be defined in terms of the responses of corresponding single degree systems. Such responses are typically reported by shock spectra. The concept of effective modal mass also is treated. Material is included which can be useful to persons working in the field, but this presentation is designed expressly for the nonspecialist.

INTRODUCTION

Shock environments are typically reported in terms of shock response spectra. The use of data in this form for predicting the shock response of single degree of freedom systems is elementary and widespread, but the ease with which such data can be used to predict the shock response of multidegree of freedom systems is not so widely appreciated.

Table 1 offers a concise, generalized summary of the relations which permit the shock response spectra to be used in predicting the behavior of linear, multidegree of freedom, spring mass systems. These equations can, in fact, be applied in evaluating the response to all types of disturbances in all forms of mathematically linear, second-order, undamped, physical systems. With the explanation offered herein, application of these equations is within the grasp of any graduate engineer.

The shock response spectrum owes its value to the fact that two simple spring mass systems mounted on a single base and having identical natural frequencies will experience identical responses to a given disturbance of the base, regardless of the specific mass values of the two systems. It can be shown that the natural motion of a multidegree of freedom system is the sum of wholly independent, natural modes of motion, each of which is analogous to the motion of a single degree of freedom system. It can also be shown that the response of each mode of a multidegree system to a given disturbance is completely analogous to the response to the same disturbance of the single degree system having the same natural frequency. This leads to the development of a response factor which is the ratio of these two response magnitudes. The factor is, in effect, a measure of the degree to which a given mode of a multidegree system "participates" in the response associated with the single degree system. The factor is commonly known as a "participation factor," and it is the burden of this paper to show how this factor can be determined and applied to the response evaluation problem.

The response evaluation procedure outlined here has three basic phases; these are discussed in successive sections of this paper. It is first illustrated how the equations of motion of linear, elastic, multidegree of freedom systems can be represented by the first equation of Table 1. By way of illustrating this routine but crucial facet of the response evaluation problem, the equations of motion are developed in considerable detail for several specific configurations. The second phase of the response evaluation procedure consists of determining the natural frequencies and amplitude ratios of the pertinent system. This facet of the problem is discussed

*This paper was not presented at the Symposium.

TABLE 1
Generalized Equations for
Response Evaluation of Spring Mass Systems

if:	$m_n \ddot{u}_n + \sum_{j=1}^{N} K_{nj} u_j = m_n \ddot{s}_n$ $K_{nj} = K_{jn}$: $j = 1, 2 \ldots N$ ω_i^2 = mode i eigenvalue (square of circular natural frequency) α_{ni} = n^{th} element of mode i eigenvector (amplitude ratio) $\ddot{\delta}_i + \omega_i^2 \delta_i = -\ddot{s}$ (reference system equation of motion) $r_n = \ddot{s}_n / \ddot{s}$ $n = 1, 2 \ldots N$ $i = 1, 2 \ldots N$
then:	$\gamma_i = \sum_{n=1}^{N} [m_n \alpha_{ni} r_n] \Big/ \sum_{n=1}^{N} \left[m_n (\alpha_{ni})^2\right]$ (participation factor) $u_{ni} = \alpha_{ni} \gamma_i \delta_i$ (modal displacement) $u_n = \sum_{i=1}^{N} u_{ni}$ (total relative displacement) $\ddot{u}_n + \ddot{s}_n = \sum_{i=1}^{N} \left[(\omega_i)^2 u_{ni}\right]$ (total absolute acceleration)

briefly and specific algebraic formulations are offered for two and three degree of freedom systems. Use of the equations of Table 1 to determine actual response magnitudes constitutes the third phase of the response evaluation procedure. The manner in which these equations can be used is stated and illustrated with specific examples.

Additional material is included which treats the concept of effective modal mass and the application of that concept in response evaluation problems.

GENERAL EQUATIONS OF MOTION

D'Alembert's Principle

The essential first step of the response evaluation procedure is the application of Newton's second law of motion to each of the degrees of freedom of the system. As formulated by D'Alembert, the law provides that the summation of all the forces acting on a mass in a given direction is equal to zero, where the reaction to an acceleration of the mass in that direction is regarded as a force acting to oppose the acceleration, equal in magnitude to the product of the mass and its absolute value of acceleration. A corresponding relation exists between moments applied to a mass, its mass moment of inertia, and the consequent rotary accelerations of the mass.

A Single Degree of Freedom System

Consider first the simple single degree of freedom system of Fig. 1(a), where both the mass and the system base are constrained to move in a single direction. The dimension x is the displacement of the mass c.g. from its relaxed position relative to the system base, and the dimension s is the "absolute" displacement of the system base relative to the inertial frame of reference (indicated in Fig. 1(a) by the star).

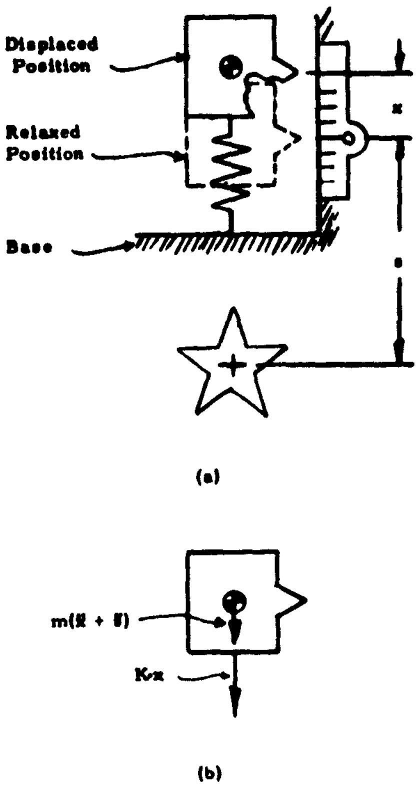


Figure 1

If the value of the mass is m and the spring rate is K, the application of D'Alembert's principle provides the following equation of motion (see Fig. 1(b) and note that the reaction $m(\ddot{x}+\ddot{s})$ is positive in the sense opposite to that of increasing x and s values):

$$m(\ddot{x}+\ddot{s}) + Kx = 0. \tag{1}$$

It can be shown that when the foundation acceleration, $\ddot{s}$, is equal to zero, the mass can experience a "natural" or "characteristic" motion of the form indicated by the equations:

$$x = B \sin(\omega t + \phi), \tag{2}$$

$$\omega = \sqrt{K/m}. \tag{3}$$

In Eq. (2), t is time, ω is the (circular) natural frequency of oscillation, and B and ϕ are constants which can take on any real values (including zero) as a result of an "excitation" of the system.

A Single Mass, Two Degree of Freedom System

Consider next the system of Fig. 2(a) which is equal to the system of Fig. 1(a) except that an additional degree of freedom exists — specifically, rotation within the plane of the figure, "I" being the moment of inertia of the mass about its c.g. Application of D'Alembert's principle to both the translation and rotation coordinates provides the following two equations of motion (see Fig. 2(b)):

$$m(\ddot{x}+\ddot{s}) + K_1(x - L_1\psi) + K_2(x + L_2\psi) = 0, \tag{4}$$

$$I(\ddot{\psi}+\ddot{\theta}) - K_1(x - L_1\psi)L_1 + K_2(x + L_2\psi)L_2 = 0. \tag{5}$$

The elastic coefficients of Eqs. (4) and (5) can be isolated as follows:

$$K_{xx} = K_1 + K_2, \tag{6}$$

$$K_{x\psi} = K_{\psi x} = K_2 L_2 - K_1 L_1, \tag{7}$$

$$K_{\psi\psi} = K_2 L_2^2 + K_1 L_1^2, \tag{8}$$

thus permitting the two equations of motion to be restated as:

$$m\ddot{x} + K_{xx}x + K_{x\psi}\psi = -m\ddot{s}, \tag{9}$$

$$I\ddot{\psi} + K_{\psi x}x + K_{\psi\psi}\psi = -I\ddot{\theta}. \tag{10}$$

It can be shown that when the two base acceleration parameters $\ddot{s}$ and $\ddot{\theta}$, are equal to zero, the mass can experience "natural" or "characteristic" motion in either or both of two sinusoidal forms, as indicated by the following equations:

$$x = x_1 + x_2, \tag{11}$$

$$\psi = \psi_1 + \psi_2, \tag{12}$$

$$\left.\begin{aligned} x_2 &= B_1 \sin(\omega_1 t + \phi_1), \\ x_2 &= B_2 \sin(\omega_2 t + \phi_2), \end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} \psi_1 &= b_1 \sin(\omega_1 t + \phi_1), \\ \psi_2 &= b_2 \sin(\omega_2 t + \phi_2), \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} \beta_1 &= b_1/B_1, \\ \beta_2 &= b_2/B_2, \end{aligned}\right\} \tag{15}$$

$$\omega_1^2 = \frac{1}{2}\left[R_1 - \sqrt{R_1^2 - 4R_o}\right],$$
$$\omega_2^2 = \frac{1}{2}\left[R_1 + \sqrt{R_1^2 - 4R_o}\right], \qquad (16)$$

$$\beta_1 = \left[m\omega_1^2 - K_{xx}\right]/K_{x\psi},$$
$$\beta_2 = \left[m\omega_2^2 - K_{xx}\right]/K_{x\psi}. \qquad (17)$$

$$R_o = \left[K_{xx}\,K_{\psi\psi} - K_{x\psi}^2\right]/mI. \qquad (18)$$

$$R_1 = K_{xx}/m + K_{\psi\psi}/I. \qquad (19)$$

In the preceding equations, t is time, ω_1 and ω_2 are the (circular) natural frequencies of two "natural modes of motion," and B_1 and φ_1, and B_2 and φ_2 are constants which can take on any real values (including zero) as a result of an excitation of the system. This indicates that the mass is capable of experiencing natural sinusoidal translation at frequency ω_1 and also at frequency ω_2. Each of Eqs. (13) is completely analogous to Eq. (2) which describes the natural behavior of a single degree of freedom system.

The preceding equations show that as the mass experiences a translatory oscillation at either of the two natural frequencies, it simultaneously experiences a rotary oscillation of proportionate magnitude at the same frequency, the β_1 and β_2 values being natural "amplitude ratios" which relate the translatory and rotary portions of the motion and which are (like the natural frequencies) determined exclusively by the physical constants of the system.

Inspection of Fig. 2(a) demonstrates that the translation, x' of a point located a distance, L, from the c.g. is given by the following equation:

$$x' = x - L\psi. \qquad (20)$$

If the case is considered where the quantities x_2 and ψ_2 are equal to zero, the values of ψ and x' will be as indicated by Eqs. (21) and (22):

$$\psi = \beta_1 x. \qquad (21)$$

$$x' = x(1 - L\beta_1). \qquad (22)$$

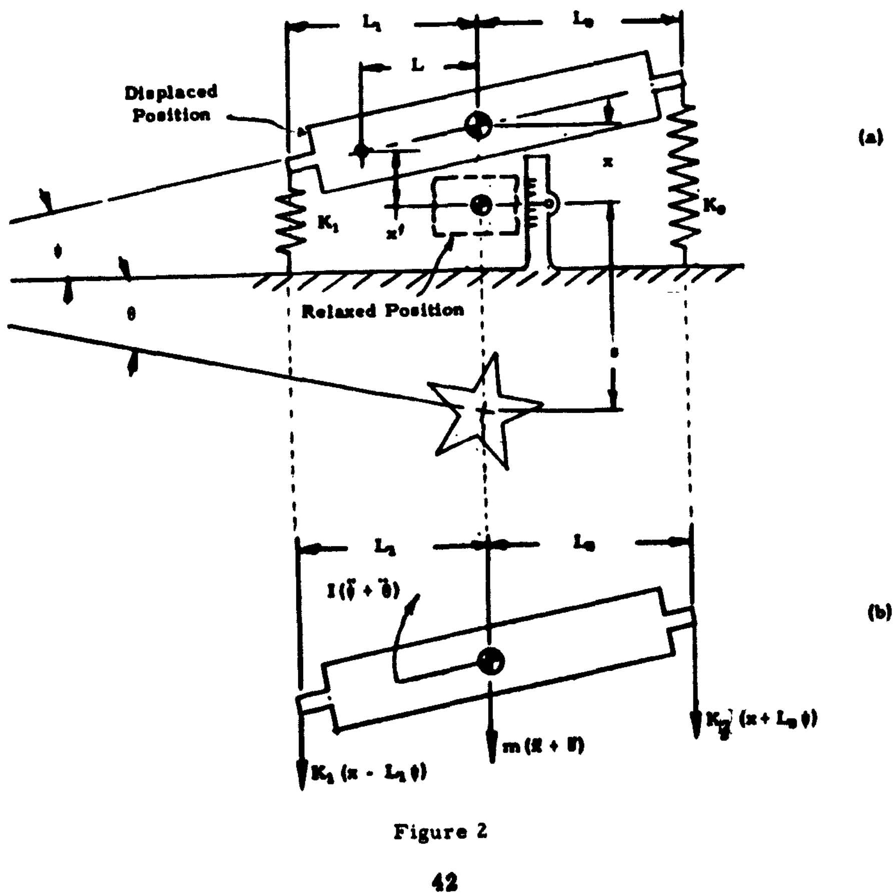


Figure 2

It can be seen that if the length, L, were equal to the reciprocal of the amplitude ratio, β_1, the value x' would be continuously zero regardless of the amplitude of motion and it can be said that there is a natural mode of motion which consists of a rotary oscillation at frequency ω_1, with a center of oscillation located a distance $(1/\beta_1)$ from the c.g. It can be shown in a similar manner that there is a second natural mode of motion having a natural frequency ω_2 with a center of oscillation located a distance $(1/\beta_2)$ from the c.g. (Note: For the configuration of Fig. 2, the two amplitude ratios are always of opposite sign and the centers of rotation of the two modes are on opposite sides of the c.g.)

If the coefficient $K_{x\psi}$ is equal to zero (see Eq. 7), one of the two amplitude ratios is equal to zero and the other is equal to infinity, and, in turn, one of the natural centers of oscillation lies at the c.g. and the other lies an infinite distance from the c.g. This indicates, in effect, that one of the two natural modes is pure translation and the other is pure rotation. In such a system, the x and ψ coordinates are said to be uncoupled.

It may be noted from inspection of Eqs. (16), (18), and (19) that the ω_1^2 and ω_2^2 values are independent of the excitation of the system, being determined exclusively by the stiffness and inertia coefficients of the system. It might be said that the ω_1^2 and ω_2^2 terms are the system's "own" natural values. The German word for "own" is "eigen," and from this we have one of the most popular "status" words in our technical vocabulary — "eigenvalue." For the undamped, spring-mass systems of interest here, an eigenvalue is simply the square of a (circular) natural frequency. Perhaps even more effective for impressing lay listeners is the word "eigenvector" which, for the systems of interest here, simply denotes the one or more amplitude ratios necessary to describe the displacement characteristics of a natural mode of motion.

A Systematic Notation for Two Degree of Freedom Systems

For the sake of establishing a more general form of notation, new symbols can be defined as in Eqs. (23) through (31):

$$u_1 = x\,, \tag{23}$$

$$u_2 = \psi\,, \tag{24}$$

$$s_1 = s\,, \tag{25}$$

$$s_2 = \theta\,, \tag{26}$$

$$m_1 = m\,, \tag{27}$$

$$m_2 = I\,, \tag{28}$$

$$K_{11} = K_{xx}\,, \tag{29}$$

$$K_{12} = K_{21} = K_{x\psi}\,, \tag{30}$$

$$K_{22} = K_{\psi\psi}\,. \tag{31}$$

Equations (10) and (11) can therefore be restated as:

$$m_1\ddot{u}_1 + K_{11}u_1 + K_{12}u_2 = -m_1\ddot{s}_1\,, \tag{32}$$

$$m_2\ddot{u}_2 + K_{21}u_1 + K_{22}u_2 = -m_2\ddot{s}_2\,. \tag{33}$$

The establishment of a more general form of notation may be pursued with the definition of additional symbols as in Eqs. (34), (35), and (36):

$$A_{11} = B_1\,, \qquad A_{12} = B_2\,, \tag{34}$$

$$A_{21} = b_1\,, \qquad A_{22} = b_2\,, \tag{35}$$

$$\alpha_{21} = \beta_1\,, \qquad \alpha_{22} = \beta_2\,. \tag{36}$$

Adding a second subscript to the terms u_1 and u_2 to denote the particular parts of the motions associated with a specific natural mode, permits Eqs. (11) through (15) to be restated as Eqs. (37) through (41):

$$u_1 = u_{11} + u_{12}\,, \tag{37}$$

$$u_2 = u_{21} + u_{22}\,, \tag{38}$$

$$\begin{aligned} u_{11} &= A_{11}\sin(\omega_1 t + \phi_1)\,, \\ u_{12} &= A_{12}\sin(\omega_2 t + \phi_2)\,, \end{aligned} \tag{39}$$

$$\begin{aligned} u_{21} &= A_{21}\sin(\omega_1 t + \phi_1)\,, \\ u_{22} &= A_{22}\sin(\omega_2 t + \phi_2)\,, \end{aligned} \tag{40}$$

$$\begin{aligned} \alpha_{21} &= A_{21}/A_{11}\,, \\ \alpha_{22} &= A_{22}/A_{12}\,. \end{aligned} \tag{41}$$

In these equations, the first subscript of a double subscript denotes a specific one of the two motion coordinates while a second subscript or a single subscript denotes the natural mode of motion with which the quantity is associated.

A Two Mass, Two Degree of Freedom System

The approach used to establish the equations of motion for the foregoing cases can be applied to any multidegree, spring mass system. By way of illustration, consider the configuration of Fig. 3(a) where s denotes the absolute displacement of the base (with respect to the inertial frame of reference) and u_1 and u_2, respectively, denote the displacement, relative to the base, of masses m_1 and m_2 from their relaxed positions. Application of D'Alembert's principle provides the following two equations (see Fig. 3(b)):

$$m_1(\ddot{u}_1 + \ddot{s}) + K_1 u_1 - K_c(u_2 - u_1) = 0 . \qquad (42)$$

$$m_2(\ddot{u}_2 + \ddot{s}) + K_c(u_2 - u_1) + K_2 u_2 = 0 . \qquad (43)$$

If the following definitions are established,

$$K_{11} = K_1 + K_c . \qquad (44)$$

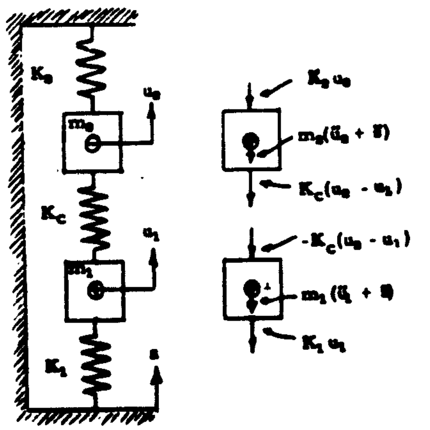
$$K_{12} = K_{21} = K_c . \qquad (45)$$

$$K_{21} = K_2 + K_c . \qquad (46)$$

$$s_1 = s_2 = s . \qquad (47)$$

then Eqs. (32), (33), and (37) through (41), established for the configuration of Fig. 2(a), are fully applicable to the configuration of Fig. 3(a).

The resulting natural motion of the two mass systems is likewise the sum of two sinusoidal motions, although in this case, the physical concept of center of rotation is not directly applicable. The independence of the two natural modes of motion, however, is every bit as real, the system having two distinct natural frequencies, and corresponding to each frequency, an amplitude ratio which indicates the relative motion of the two masses associated with that mode. For the particular case described, the two masses will oscillate in the same direction in one of the system's natural modes of motion, and in opposite directions for the second natural mode of motion.



(a) (b)

Figure 3

Three Degree of Freedom Systems

If D'Alembert's principle is applied to a three degree of freedom, spring mass system, the resulting equations of motion can be stated as follows:

$$m_1\ddot{u}_1 + K_{11}u_1 + K_{12}u_2 + K_{13}u_3 = -m_1\ddot{s}_1 . \qquad (48)$$

$$m_2\ddot{u}_2 + K_{21}u_1 + K_{22}u_2 + K_{23}u_3 = -m_2\ddot{s}_2 . \qquad (49)$$

$$m_3\ddot{u}_3 + K_{31}u_1 + K_{32}u_2 + K_{33}u_3 = -m_3\ddot{s}_3 . \qquad (50)$$

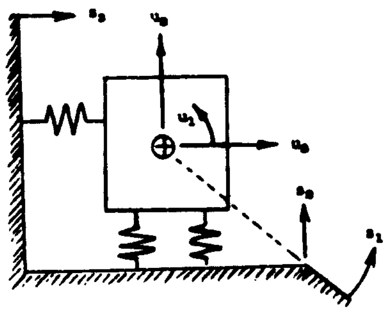


Figure 4

As applied to the single mass system of Fig. 4, for example, the terms s_1, s_2, and s_3 denote one absolute rotation and two absolute translations of the base, respectively. The terms u_1, u_2, and u_3 denote the corresponding displacements of the mass from its relaxed position relative to the base. The term m_1 is equal to the moment of inertia of the mass about its c.g. and the terms m_2 and m_3 are each equal to the magnitude of the mass. The K terms denote appropriate combinations of spring stiffness and location dimensions in the manner of the K terms of Eqs. (6), (7), and (8). (Note: It is presumed in the discussion of this specific example, that motions both of the mass and of the base are restricted to motions within the plane of the figure.)

It can be shown that when the three base acceleration parameters ($\ddot{s}_1$, $\ddot{s}_2$, and $\ddot{s}_3$) are all equal to zero, a system described by Eqs. (48), (49), and (50) can experience "natural" motion in any one or combination of three sinusoidal forms (or modes) as indicated by the following equations which correspond to the "two degree" equations — Eqs. (37) through (41).

$$u_1 = u_{11} + u_{12} + u_{13}, \tag{51}$$

$$u_2 = u_{21} + u_{22} + u_{23}, \tag{52}$$

$$u_3 = u_{31} + u_{32} + u_{33}, \tag{53}$$

$$u_{11} = A_{11} \sin (\omega_1 t + \phi_1),$$

$$u_{12} = A_{12} \sin (\omega_2 t + \phi_2),$$

$$u_{13} = A_{13} \sin (\omega_3 t + \phi_3), \tag{54}$$

$$u_{21} = A_{21} \sin (\omega_1 t + \phi_1),$$

$$u_{22} = A_{22} \sin (\omega_2 t + \phi_2),$$

$$u_{23} = A_{23} \sin (\omega_3 t + \phi_3), \tag{55}$$

$$u_{31} = A_{31} \sin (\omega_1 t + \phi_1),$$

$$u_{32} = A_{32} \sin (\omega_2 t + \phi_2),$$

$$u_{33} = A_{33} \sin (\omega_3 t + \phi_3), \tag{56}$$

$$\alpha_{21} = A_{21}/A_{11},$$

$$\alpha_{22} = A_{22}/A_{12},$$

$$\alpha_{23} = A_{23}/A_{13}, \tag{57}$$

$$\alpha_{31} = A_{31}/A_{11},$$

$$\alpha_{32} = A_{32}/A_{22},$$

$$\alpha_{33} = A_{33}/A_{13}. \tag{58}$$

The pattern of subscripts in these equations is similar to that of Eqs. (37) through (41) with the qualification that two amplitude ratios are necessary to define a natural mode of motion. In the illustrative example of Fig. 4, the center of rotation of each natural motion can be established by the amplitude ratios in the same manner as was done for the two degree of freedom system of Fig. 2(a), two amplitude ratios being required to locate the center which in this case can lie anywhere in the plane.

Generalizing the Equations of Motion

The fundamental approach to establishing the equations of motion and describing the natural modes of motion for the two- and three-degree systems treated in the preceding paragraphs can be validly applied to linear elastic, undamped, spring mass systems having any number of degrees of freedom. The total number of degrees of freedom to be considered in a given case is exactly equal to the number of motion coordinates which are necessary to describe the motion of interest, and the total natural (unforced) motion is the sum of an equal number of sinusoidal motions. Since the approach outlined here consists of applying D'Alembert's principle (or Newton's second law of motion) to each of the degrees of freedom, an "N" degree of freedom system will have "N" equations of motion, each of these equations including consideration of up to "N" coordinates. Since these equations are similar in form for all the systems considered here, it is possible to express them in a generalized form which can be applied to systems having any number of degrees of freedom. Note, for example, that Eq. (59) can be made to represent either Eq. (32) or (33) by substituting for the letter n in Eq. (59) the numeral 1 or 2, respectively:

$$m_n \ddot{u}_n + K_{n1} u_1 + K_{n2} u_2 = -m_n \ddot{s}_n. \tag{59}$$

Similarly, all three equations, (48), (49), and (50) can be represented by the single expression of (60) if the right hand portion of that expression is understood to indicate all the forms of that expression which can be obtained by successively substituting for the letter n, the numerals 1, 2, and 3:

$$m_n \ddot{u}_n + K_{n1} u_1 + K_{n2} u_2 + K_{n3} u_3 = -m_n \ddot{s}_n; \quad n = 1, 2, 3. \tag{60}$$

In addition, the expressions of (59) and (60) can themselves be shortened by introducing the summation "operator"

$$\left[\sum_{j=A}^{B}\right]$$

which denotes the summation of all the forms of the indicated expression which can be established by successively substituting for j, each of the integers from A to B. This permits Eqs. (59) and (60) to be restated in the following manner:

$$m_n \ddot{u}_n + \sum_{j=1}^{2} K_{nj} u_j = -m_n \ddot{s}_n ; \quad n = 1, 2, \tag{61}$$

$$m_n \ddot{u}_n + \sum_{j=1}^{3} K_{nj} u_j = -m_n \ddot{s}_n ; \quad n = 1, 2, 3. \tag{62}$$

The pattern of Eqs. (61) and (62) suggests the ultimate generalized form of Eq. (63) below wherein the term N denotes the total number of coordinates (and hence degrees of freedom) involved. This is the basic, generalized equation of motion of linear elastic, undamped spring mass systems, the first equation of Table 1,

$$m_n \ddot{u}_n + \sum_{j=1}^{N} K_{nj} u_j = -m_n \ddot{s}_n ; \quad n = 1, 2, \ldots N. \tag{63}$$

While the several equations of motion represented by this general form may be validly written in any order, some computational benefits may follow from a particular order. Where the coefficients K_{11}, K_{22}, K_{nn}, and so on, are considered to represent the "main diagonal" of the array of "K" coefficients, it generally contributes to arithmetic simplicity to arrange the equations in such an order that the non-zero coefficients are concentrated near the main diagonal.

Equations (64), (65), and (66) are generalized expressions which describe the natural motion of the system of Eq. (63) and correspond to Eqs. (51) through (58) for the three degree of freedom system. The subscript n in these equations denotes the particular coordinate with which the term is associated, and the subscript i denotes the particular natural mode of motion with which the term is associated:

$$\left.\begin{aligned} u_n &= \sum_{i=1}^{N} u_{ni} , && (64) \\ u_{ni} &= A_{ni} \sin(\omega_i t + \phi_i) , \quad i = 1, 2 \ldots N && (65) \\ \alpha_{ni} &= A_{ni}/A_{1i} , \quad i = 2, 3 \ldots N && (66) \end{aligned}\right\} n = 1, 2 \ldots N.$$

In the foregoing discussions, the amplitude ratio sets (all the terms associated with a particular natural mode) were normalized on the #1 coordinate, with one less term being considered in a set than there were significant coordinates in the problem. Actually, the amplitude ratio set may be normalized validly on any one of the significant coordinates. It is convenient to consider that there is an amplitude ratio term for every significant coordinate, the one which is unity denoting on which coordinate the set is normalized. Such an amplitude ratio set is known as the eigenvector, each individual ratio of the set being an "element" of the eigenvector. The eigenvector is sometimes referred to as a mode shape since in certain configurations the elements of the eigenvector actually constitute a plot of the shape assumed by the system when the particular mode is excited.

Limiting Conditions

It has been pointed out that the foregoing algebra is applicable to "linear" systems. The strict mathematical definition of the term linear is somewhat involved but for our purposes, a spring mass system can be described as linear if each of the force reactions within the system is proportional to the first power of the motion parameter which causes it (e.g., spring force reaction proportional to displacement).

Equations (30) and (45) indicate an "elastic reciprocity" for the two degree of freedom systems. This reciprocity is an inherent feature of ordinary linear spring mass systems having any number of degrees of freedom. This quality, which is stated in generalized form by Eq. (67), is essential for the valid application of the response evaluation procedure treated here:

$$K_{nj} = K_{jn} . \tag{67}$$

For a system which includes a mass not restrained from rotation (either by physical constraint or by symmetry), the validity of these response evaluation procedures is subject also to the applicability of the following conditions:

1. Rotation of system masses is sufficiently small to justify the approximation that the angle of rotation is equal to its sine. This condition precludes the introduction of nonlinearity associated with variable moment arms for springs.

2. Translation coordinate axes for any mass intersect its c.g. and are parallel to its principle axes of inertia. This qualification eliminates "inertia" coupling which complicates the evaluation of natural characteristics and renders Table 1 inapplicable.

3. High-speed rotating masses such as electric motor rotors are not a part of the system. The "gyroscopic" effect of such rotating masses introduces nonlinear, inertia coupling.

4. Translations of a system mass are small compared to its radii of gyration about axes perpendicular to the direction of translation. The moment of inertia values associated with conventional coordinates are reckoned from a point fixed within the inertial frame of reference, and displacements of the mass accordingly cause variation of the inertia value, rendering the equations of motion nonlinear.

Reference 1 contains information dealing with principle axes of inertia. Reference 2 includes additional information relating specifically to the determination of principle axes. Appendix IV of Ref. 2 deals in particular with conditions 2, 3, and 4, presented in the previous paragraph, and is based on a similar treatment which may be found in Ref. 3.

THE NATURAL MODE SOLUTION

An essential step in the response evaluation procedure is the evaluation of the natural frequency (square root of the eigenvalue) and the amplitude ratios (eigenvector) for each natural mode of motion. For systems having four or more coupled degrees of freedom, this is the most complicated arithmetic routine in the response evaluation procedure. Fortunately, there are widely available digital computer routines which, on the basis of inertia (m_n) and stiffness (K_{nj}) parameters, turn out eigenvalue and eigenvector solutions with great speed, reliability, precision, and economy. Matrix iteration methods for evaluating complex systems are beyond the scope of this paper, but are covered in a number of texts, including Refs. 4, 5, and 6. An appreciation of the solution can be gained by consideration of Eq. (68) which is applicable to the system described by Eq. (63):

$$\alpha_{ni}\,\omega_i^2 = \sum_{j=1}^{N}\left(\frac{K_{nj}}{m_n}\right)\times\alpha_{ji}\,; \quad n = 1,2\ldots N. \tag{68}$$

Since, for any value of i, one of the amplitude ratios (eigenvector elements) is equal to unity, Eq. (68) will represent N specific simultaneous

TABLE 2
Natural Mode Solutions for Two and Three Degree of Freedom Systems

	ω_i^2 = Eigenvalue $\quad$ α_{ni} = n^{th} element of Eigenvector
Two Degrees of Freedom	$m_n\ddot{u}_n + K_{n1}u_1 + K_{n2}u_2 = 0; \quad n = 1, 2$
	$R_o = \left[K_{11}\times K_{22} - K_{12}^2\right]/m_1 m_2$
	$R_1 = (K_{11}/m_1) + (K_{22}/m_2)$
	$(\omega_i^2)^2 + R_1(\omega_i)^2 + R_o = 0$
	$\left.\begin{array}{l}\alpha_{1i} = 1\\ \alpha_{2i} = K_{12}/(m_2\omega_i^2 - K_{22})\end{array}\right\}\; i = 1, 2$
Three Degrees of Freedom	$m_n\ddot{u}_n + K_{n1}u_1 + K_{n2}u_2 + K_{n3}u_3 = 0; \quad n = 1, 2, 3$
	$R_o = (K_{11}K_{22}K_{33} + 2K_{12}K_{13}K_{23} - K_{11}K_{23}^2 - K_{22}K_{13}^2 - K_{33}K_{12}^2)/m_1 m_2 m_3$
	$R_1 = \left[m_1(K_{22}K_{33} - K_{23}^2) + m_2(K_{11}K_{33} - K_{13}^2) + m_3(K_{11}K_{22} - K_{12}^2)\right]/m_1 m_2 m_3$
	$R_2 = (K_{11}/m_1) + (K_{22}/m_2) + (K_{33}/m_3)$
	$(\omega_i^2)^3 + R_2(\omega_i^2)^2 + R_1(\omega_i^2) + R_o = 0$
	$\left.\begin{array}{l}\alpha_{1i} = 1\\ \alpha_{2i} = \dfrac{K_{12}K_{13} - K_{23}(m_1\omega_i^2 - K_{11})}{K_{12}K_{23} - K_{13}(m_2\omega_i^2 - K_{22})}\\ \alpha_{3i} = \left[(m_1\omega_i^2 - K_{11}) - \alpha_{2i}K_{12}\right]/K_{13}\end{array}\right\}\; i = 1, 2, 3$

equations containing N unknowns, one ω_i^2 and $(N-1)\,\alpha$ values. When these equations are manipulated to eliminate the α terms, a single N^{th} degree polynomial results providing N different solutions for ω_i^2 (presuming all coordinates are coupled). For each value of ω_i^2, one set of α_{ni} terms can be determined from the same N equations. Specific coefficients for the eigenvalue polynomial and eigenvector relationships for two and three degree of freedom systems are listed in Table 2 for convenience.

In evaluating natural frequencies as indicated herein, it is important to consider that the natural dimension of the eigenvalue is time to the power of -2. The square root of the eigenvalue, therefore, is the so-called "circular" natural frequency, whose dimension is radians per unit time. The numerical value of the circular natural frequency is 2π times the numerical value of the "cyclic" natural frequency whose natural dimension is cycles per unit time.

It should also be remembered that an eigenvector element can be either positive or negative and that this sense is an essential part of the element's mathematical quality.

APPLICATION OF THE RESPONSE EQUATIONS

The General Response Evaluation Equations

The total relative displacement response to a multidegree of freedom system to a given base disturbance may be visualized as the sum of the responses of each of the natural nodes of the system to the same disturbance. The response of any one mode is proportional to a "modal" participation factor and to the response, to the same disturbance, of a reference single degree of freedom system having the same natural frequency. Table 1, which is a summary of the generalized equations for response evaluation of spring mass systems, indicates the formulation of these modal participation factors (γ_i) and the relation for establishing modal displacement responses (u_{ni}). It also indicates relationships for determining total relative displacement responses (u_n) and total absolute acceleration responses $(\ddot{u}_n + \ddot{s}_n)$ in terms of the individual modal response parameters. The subscript i in the table denotes any particular one of the several natural modes.

Table 1 is applicable where the equations of motion have been formulated and the corresponding eigenvalues (ω_i^2) and eigenvectors (α_{ni}) resolved, and where the displacement responses (δ_i) of reference single degree of freedom systems to the pertinent base disturbance are known for each of the system natural frequencies (ω_i). The ratio (r_n) which each of the system base acceleration functions $(\ddot{s}_n)$ bears to the reference base acceleration function $(\ddot{s})$ is also required.

Derivation of various forms of the participation factor can be found in a number of sources, including Ref. 7 (which does not identify the participation factor by name but uses the symbol γ_n). Justification of the specific equations of Table 1, in essentially equal language, is contained in Appendix II of Ref. 2.

Illustrative Example — Simple Two Mass System

By way of illustrating the response evaluation procedure, consider the two degree of freedom system and the two reference single degree systems illustrated in Fig. 5. It was shown previously that application of D'Alembert's principle results in Eqs. (42) and (43) which can be restated as Eqs. (32) and (33), with the qualification that $\ddot{s}_1 = \ddot{s}_2 = \ddot{s}$. It can be concluded that the ratios, r_1 and r_2 are each equal to unity and the participation factor for mode i can be stated as follows:

$$\delta_i = \left[m_1\alpha_{1i} + m_2\alpha_{2i}\right] \Big/ \left[m_1(\alpha_{1i})^2 + m_2(\alpha_{2i})^2\right]; \quad i = 1, 2. \qquad (69)$$

The relative displacements (u_n) and the absolute accelerations $(\ddot{u}_n + \ddot{s})$ of masses 1 and 2 are given by Eqs. (70) through (73).

$$u_1 = \alpha_{11}\gamma_1\delta_1 + \alpha_{12}\gamma_2\delta_2, \qquad (70)$$

$$u_2 = \alpha_{21}\gamma_1\delta_1 + \alpha_{22}\gamma_2\delta_2. \qquad (71)$$

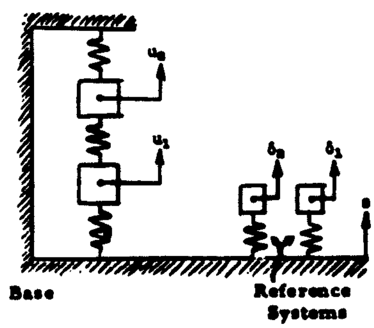


Figure 5

$$\ddot{u}_1 + \ddot{s} = \omega_1^2 \alpha_{11} \gamma_1 \delta_1 + \omega_2^2 \alpha_{12} \gamma_2 \delta_2 . \quad (72)$$

$$\ddot{u}_2 + \ddot{s} = \omega_1^2 \alpha_{21} \gamma_1 \delta_1 + \omega_2^2 \alpha_{22} \gamma_2 \delta_2 . \quad (73)$$

Illustrative Example — "Split" Base Motion

The manner in which the response evaluation procedure may be applied to another type problem is illustrated in the following example.

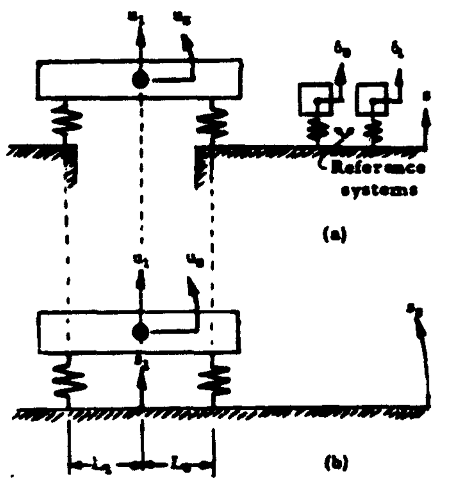


Figure 6

Consider the system of Fig. 6(a) which consists of a single mass mounted on two springs as indicated and subject to translation in the u_1 direction and rotation in the u_2 direction. Consider also that the system experiences a shock disturbance which consists exclusively of a translation, in the s direction, of the right-hand portion of the base. A vehicle striking a bump could experience such a disturbance. This system may be considered equivalent to the system of Fig. 6(b) mounted on a solid base, which as it experiences a translation s_1, simultaneously experiences a rotation s_2, which effectively cancels the translation of the base at the point where it supports the left-hand spring. The appropriate relations between base acceleration parameters $\ddot{s}_1$, $\ddot{s}_2$ and $\ddot{s}$ can be determined geometrically as follows:

$$\frac{s_1}{L_1} = \frac{s}{L_1 + L_2} . \quad (74)$$

$$s_2 = \frac{s}{L_1 + L_2} . \quad (75)$$

As was demonstrated in earlier paragraphs, application of D'Alembert's principle to the system of Fig. 6(b) leads to the equations of motion, Eqs. (32) and (33), and it can be concluded from Eqs. (74) and (75) that the ratios r_1 and r_2 are as given by Eqs. (76) and (77) and the participation factor for mode i is given by Eq. (78):

$$r_1 = L_1/[L_1 + L_2] . \quad (76)$$

$$r_2 = 1/[L_1 + L_2] . \quad (77)$$

$$\gamma_i = [(m_1 L_1 \alpha_{1i} + m_2 \alpha_{2i})/(L_1 + L_2)] / [m_1(\alpha_{1i})^2 + m_2(\alpha_{2i})^2] ; \qquad i = 1, 2 . \quad (78)$$

The response equations, Eqs. (70) through (73) are applicable to this system.

Illustrative Example — "Misaligned" Base Motion

As a further illustration, consider the system of Fig. 7, which consists of a single mass m, having a moment of inertia I, mounted elastically on a base which experiences a disturbance which is exclusively a translation in the direction indicated in the figure by the alignment of the reference single degree of freedom systems. As was stated previously, application of D'Alembert's principle to this configuration leads to the equations of motion, Eqs. (48), (49), and (50), where m_1 is equal to I, and m_2 and

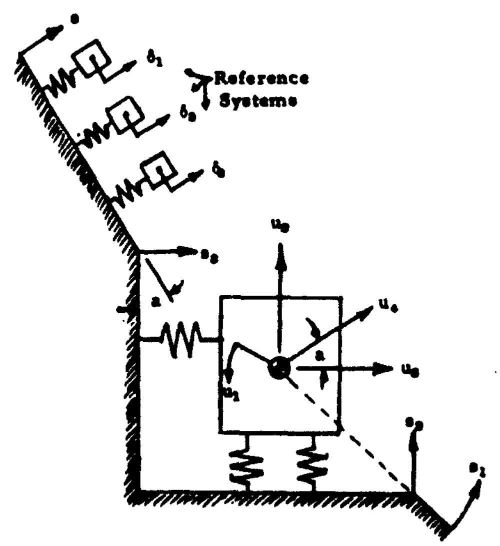


Figure 7

m_3 are each equal to m. Inspection of Fig. 7 shows that the base acceleration functions, $\ddot{s}_1$, $\ddot{s}_2$, and $\ddot{s}_3$ are related to the fundamental acceleration function $\ddot{s}$ as indicated by the following equations:

$$\ddot{s}_1 = 0 , \quad (79)$$

$$\ddot{s}_2 = \ddot{s} \times \sin(a) , \quad (80)$$

$$\ddot{s}_3 = \ddot{s} \times \cos(a) . \quad (81)$$

It can be concluded from these relations that the ratios r_1, r_2, and r_3 are equal to zero, $\sin(a)$, and $\cos(a)$, respectively, and the participation factor for mode i can be stated as follows:

$$\gamma_i = \left[m\alpha_{2i} \sin(a) + m\alpha_{3i} \cos(a)\right] / \left[I(\alpha_{1i})^2 + m(\alpha_{2i})^2 + m(\alpha_{3i})^2\right]; \quad i = 1,2,3. \quad (82)$$

Equation (82) can be reduced to:

$$\gamma_i = \left[\alpha_{2i} \sin(a) + \alpha_{3i} \cos(a)\right] / \left[\frac{I}{m}(\alpha_{1i})^2 + (\alpha_{2i})^2 + (\alpha_{3i})^2\right]; \quad i = 1,2,3. \quad (83)$$

The relative displacement responses are given by Eqs. (84), (85), and (86). The corresponding absolute acceleration responses can be evaluated in a similar fashion according to the following equation of Table 1:

$$u_1 = \alpha_{11}\gamma_1\delta_1 + \alpha_{12}\gamma_2\delta_2 + \alpha_{13}\gamma_3\delta_3 . \quad (84)$$

$$u_2 = \alpha_{21}\gamma_1\delta_1 + \alpha_{22}\gamma_2\delta_2 + \alpha_{23}\gamma_3\delta_3 . \quad (85)$$

$$u_3 = \alpha_{31}\gamma_1\delta_1 + \alpha_{32}\gamma_2\delta_2 + \alpha_{33}\gamma_3\delta_3 . \quad (86)$$

Base Disturbance Qualities

In presenting the foregoing response equations (e.g., Eq. (86)), no special qualifications have been made as to time, and there is no need for such a qualification since the equations presented are valid for all values of time. Considering Eq. (84) for example, if δ_1, δ_2, and δ_3 are each known as functions of time, then the displacement response u_1 can be determined as a function of time, accordingly.

If the acceleration of a given base is known as a function of time, the relative displacement response of a reference single degree of freedom spring mass system mounted on the base can be determined according to Duhamel's integral of Eq. (87). In the language of Table 1, this integral indicates, for any time t', the relative displacement response δ_i, due to all the base acceleration $\ddot{s}$, occurring between the times t_o and t':

$$\delta_i = \frac{1}{\omega_i} \int_{t_o}^{t'} \ddot{s} \times \sin\left[\omega_i(t - t')\right] dt . \quad (87)$$

For certain classical acceleration functions, algebraic solutions are available for the Duhamel integral. Automatic computer routines which can evaluate this integral for any arbitrary acceleration function, are widely available.

Application of the Shock Spectrum

In a typical instance, the variation of the reference displacements with time are not known, the disturbance being reported as a shock response spectrum which plots, as a function of the natural frequency of simple spring mass "reference" systems, the peak response magnitude which such systems experience when subjected to the base disturbance. The response parameters most commonly reported are relative displacement, velocity (pseudo), and absolute acceleration. In the absence of specific qualification to the contrary, response spectra may be presumed to report peak magnitudes without regard to sign. In the language of Table 1, the foregoing displacement, velocity, and acceleration spectra are equal to

$$|\delta_i|\text{max}, \quad \omega_i|\delta_i|\text{max}, \quad \text{and} \quad \omega_i^2|\delta_i|\text{max} ,$$

respectively, where the parallel lines enclosing an algebraic expression denote its magnitude, exclusive of sign. More detailed discussion of shock response spectra may be found in a number of sources including chapter 2 of Ref. 2.

Where only the peak magnitude of the δ_i functions are known, the response equations must be restricted to consideration of peak magnitudes. Equation (86) for example, would take the form of Eq. (88),

$$|u_1|\text{max} = |\alpha_{11}\gamma_1| \times |\delta_1|\text{max} + |\alpha_{12}\gamma_2| \times |\delta_2|\text{max} + |\alpha_{13}\gamma_3| \times |\delta_3|\text{max}. \quad (88)$$

The peak responses of several spring mass systems of differing natural frequencies to the same disturbance do not necessarily occur at the same time and hence the summation of the individual mode response magnitudes (as in Eq. (88)) represents a limiting value of the total response rather than a specific maximum value. There are certain considerations, however, which suggest that this value is not generally a grossly conservative value. If a given amplitude of motion exists in a given natural mode of motion of an undamped system at the cessation of an external disturbance, hypothetically the system will oscillate at that amplitude and at the mode's natural frequency indefinitely. If more than one mode is so excited, then at some times their peak magnitudes of oscillation will add. As the separation of mode frequencies increases, the probability increases that such addition of mode amplitudes will occur before actual inherent damping appreciably reduces the mode amplitudes. Further, experience has shown that the greatest portion of a limiting response value is typically accounted for by a single mode and that, in general, not more than two or three modes contribute significantly. Where two or more mode frequencies are very close or where some unusual quality of the base disturbance is known, special treatment may be in order.

Combination of Response Values of Several Coordinates

When "maximum magnitude" type data is used for the reference displacements, special caution is appropriate regarding the evaluation of response dimensions which must be established by combining the response values of two or more of the motion coordinates used in the problem analysis. In general, the total response values of two or more coordinates should not be combined to evaluate the response value of an alternate coordinate, but rather, the peak response magnitude of the desired coordinate should be individually established for each natural mode of motion and the several modal values summed to establish the total value. By way of illustration, consider the system of Fig. 7 and the question — What is the peak relative displacement response (u_4) of the mass c.g. in the direction of the base motion? Inspection of Fig. 7 shows the relation between the response coordinate u_4 and the three problem coordinates given by the following equation:

$$u_4 = u_2 \sin(a) + u_3 \cos(a) . \qquad (89)$$

Direct substitution of the limiting values established for the coordinates u_2 and u_3 in Eq. (89) would introduce unnecessary conservatism since, in at least one of the natural modes of motion, the sense of the u_2 portion of the motion will oppose the sense of the u_3 portion. However, Eqs. (85), (86), and (89) can be combined to justify Eq. (90):

$$\begin{aligned} u_4 = & \left[\alpha_{21} \sin(a) + \alpha_{31} \cos(a)\right] \gamma_1 \delta_1 \\ & + \left[\alpha_{22} \sin(a) + \alpha_{32} \cos(a)\right] \gamma_2 \delta_2 \\ & + \left[\alpha_{23} \sin(a) + \alpha_{33} \cos(a)\right] \gamma_3 \delta_3 . \end{aligned} \qquad (90)$$

Consideration of Eq. (90) may be restricted to actual and limiting magnitudes in the manner done for Eq. (88) without losing sight of any algebraic cancellation which might occur in the bracketed coefficients of Eq. (90) due to the opposite signs of the α terms.

When a system base is simultaneously subjected to two or more disturbances (e.g., vertical and horizontal motions) and the relationships between the disturbances is not known, the response to each disturbance must be evaluated independently and the total system response regarded as the sum of the individual values.

Force Disturbances

In the foregoing treatment, the only disturbances considered were accelerations of the base. A system may be subject to disturbing forces applied directly to the mass (e.g., a wind force). In such a case, application of D'Alembert's principle will result in an equation of motion including the force. This may be accommodated by the response evaluation procedure described here by dividing the force F_n, applied to mass m_n, by the value m_n to establish an equivalent base acceleration function $\ddot{s}_n$.

EFFECTIVE MODAL MASS

The Effective Mass Concept

A valuable concept pertinent to the response evaluation of linear, multidegree of freedom systems concerns the effective mass of a natural mode of motion. This quantity is significant in the evaluation of the reaction applied to a base by a spring mass system as a result of base motion, and is also useful in establishing simplified configurations to approximate the characteristics of distributed mass systems and other complex multidegree of freedom systems.

Consider the systems of Fig. 5 where each of the natural frequencies of the two mass system is duplicated by the natural frequency of one of the two reference systems and where base motion is restricted to the s direction, as indicated. Suppose that mass values could be selected for the two reference systems such that the total force applied by them to the base, as a result of base motion, is equal to the corresponding total force applied by the two mass system. If such were the case, the total dynamic reaction of the two reference systems on the base would be indistinguishable from the dynamic reaction of the two mass system on the base, and the reference system mass values could, therefore, be regarded as the "effective" mass values of the corresponding natural modes of the two mass system. Such effective masses can be established. Specific relationships are developed in the following paragraphs.

Development of the Effective Mass Equation

The energy introduced into an elastic system by a given base displacement is equal to the product of the force applied by the base and the displacement of the base. Therefore, the force reactions of two elastic systems on a supporting base are completely equal if the energy introduced into one of the systems by any given base motion is equal to the energy introduced into the second system by the same base motion. Consider, for example, a system having N degrees of freedom and a group of N single degree of freedom reference systems mounted on a common base in the manner of Fig. 5, where each of the reference systems has a particular mass value, m_i, which experiences a relative displacement response, δ_i, due to a foundation motion, s. The total energy introduced into the N degree system is equal to the sum of the energies introduced into each natural mode of motion, and the total energy introduced into the group of reference systems is equal to the sum of the energies introduced into each individual reference system. If the energy introduced into each natural mode of the multidegree system is equal to the energy introduced into the corresponding reference system, the reaction on the base of the group of reference systems will be identical to that of the multidegree system. Each reference system mass value would be the effective mass of the corresponding mode of the multidegree system.

Let E denote the energy introduced into one mode of a multidegree system by a given base disturbance; E' the energy introduced into the corresponding reference system of mass m_i, by the same base disturbance; v_i denote the velocity response amplitude of mass m_i; and v_{ni} the corresponding velocity amplitude of coordinate n in mode i of the multidegree system. The energy contained in mode i of the multidegree system is equal to the sum of the peak kinetic energies attained by the N masses, m_n. Accordingly, the two energy values E and E' can be established by Eq. (91) below, and where these energy values are equal, Eq. (92) follows:

$$E = \frac{1}{2}\sum_{n=1}^{N} m_n (v_{ni})^2 ; \quad E' = \frac{1}{2} m_i v_i^2 . \tag{91}$$

$$m_i = \sum_{n=1}^{N} m_n \left(\frac{v_{ni}}{v_i}\right)^2 . \tag{92}$$

The velocity response amplitude ratio (v_{ni}/v_i) is equal to the displacement response amplitude (u_{ni}/δ_i) which can be deduced from the equation of Table 1 to justify Eq. (93) and in turn, the "effective mass" Eq. (94):

$$m_i = \sum_{n=1}^{N} m_n (\gamma_i \alpha_{ni})^2 , \tag{93}$$

$$m_i = \gamma_i^2 \sum_{n=1}^{N} m_n (\alpha_{ni})^2 . \tag{94}$$

The summation of Eq. (94) is the term known as the "generalized mass" of mode i and incidentally is the denominator of the participation factor (γ_i) equation of Table 1.

Applications of the Effective Mass Concept

The dynamic reaction of an elastic system on its base resulting from a disturbance of the base is often a matter of considerable interest since the "base" of one system is, perhaps more often than not, one element of a larger elastic system. The chief value of the effective mass concept is that it permits those characteristics of a complex elastic system which bear significantly on its base reaction to be described effectively by the relatively few physical parameters of a simple spring mass system. Where it can be established that all of a complex system's natural modes do not participate significantly in the response to a given disturbance, the nonparticipating natural modes may be discounted.

In its simplest application, an N degree of freedom spring mass system, mounted on a base constrained to move only in a single straight line, is replaced by N single degree of freedom spring mass systems aligned in the same direction (for example, see Fig. 5). The natural frequency and effective mass of each mode of the multidegree system is reflected in one of the single degree systems. It should be noted that the participation factors and, hence, the values of effective modal mass, are affected by the direction of base freedom.

It is often the case that the base of an elastic system is sensitive not only to the force reactions but to the moment reactions as well. Consider, for example, the system of Fig. 8(a), supported on a base subject to a displacement in the s direction for which a suitable dynamic model is the N mass model of Fig. 8(b). If one natural mode of this system were to be represented by a single degree of freedom system mounted on a mass-less bar as in Fig. 8(e), it would obviously make a great deal of difference to the base reaction whether the effective mass was at the right or left end of the system. The proper location of that mass (in the z direction of Fig. 8(c)) is that point at which the base could be balanced while the mass was oscillating freely in the pertinent mode. This point would coincide with the center of the peak acceleration reactions of the several masses of the dynamic model. The peak acceleration reaction of each mass is equal to the product of the mass and its peak acceleration magnitude. For any given mode, this latter quantity is proportional to the peak displacement of the mass which is, in turn, proportional to the corresponding amplitude ratio (eigenvector element). The balance point location is hence at the center of action of the $(m_n \alpha_{ni})$ products. The z_i dimension of Fig. 8(e), which gives the "effective" location of the effective mass of mode i, is given by the following equation:

$$Z_i = \frac{\sum_{n=1}^{N} Z_n m_n \alpha_{ni}}{\sum_{n=1}^{N} m_n \alpha_{ni}} . \tag{95}$$

The r_n values of Table 1 would all be equal to unity for the configuration described, and it may be noted that the denominator of Eq. (95) is the resulting numerator of the participation factor equation of Table 1.

Several modes of a system such as that of Fig. 8 may be reflected in a model such as that of Fig. 9, which includes several effective masses, each located at appropriate positions on the mass-less bar. It should be remembered that the sum of all the effective modal masses of a given system should equal the total mass of the system. If, for a system such as that of Fig. 8(a), the available data indicate that the sum of the effective masses of the presumed significant modes is appreciably less than the total

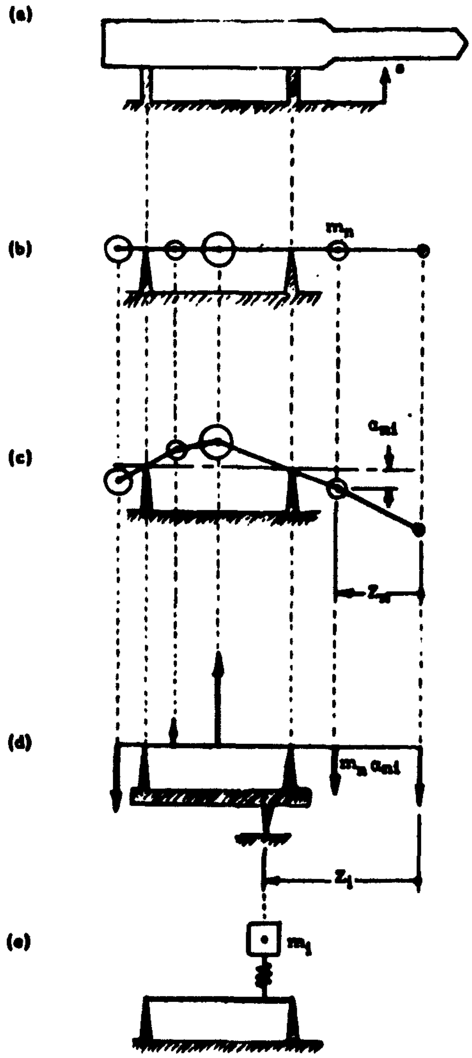


Figure 8

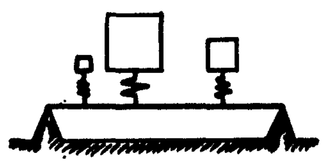


Figure 9

mass, it may be that the difference is concentrated near the support points and is, in effect, not elastically supported.

The proper location for an effective modal mass for a rigid body system such as that of Fig. 7 may be developed by regarding the body as a combination of two masses equally spaced about the c.g., establishing amplitude ratios for the two masses based on the effective center of rotation of the pertinent mode, and then proceeding as in the foregoing examples.

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RESPONSE OF LIGHTLY DAMPED STRUCTURES TO RANDOM PRESSURE FIELD*

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A completely rational design of structures subjected to random loads is possible only when adequate information is available on the forcing input and on the behavior of materials to random stresses. When this information is not available, one must depend on semi-empirical methods of analyzing structural response. To facilitate this procedure, existing theory on the response of a multidegree of freedom system is extended to the case where all the generalized forces are random. The result is simplified when the forcing function is due to small scale turbulence. The theory is illustrated in the case of a simply supported beam.

INTRODUCTION

A class of chronic problems in mechanical design is the failure of parts under random loads. In the case of airplanes and missiles, the design of these parts under steady and quasi-steady loads has been refined to such an extent that random loads, heretofore disregarded, are sometimes more critical for design.

In the past ten years, a number of writers [1-7] have dealt with the subject. All these writers have regarded the mechanical system as a linear device which converts the forcing function (input) into a response (output); the mathematical technique for describing the relationship was transferred from electrical engineering. In this way, Liepmann [1] has discussed the response of an airplane wing passing through turbulent air. His analysis was predicated on a certain structure of turbulence derived from measurements in wind tunnels. Fung [2] extended this work to cover the case of airplane response to gusts. Miles [3] has presented some general considerations relating to a panel subjected to the random pressure fluctuations of a jet. He used the "cumulative damage" criterion for failure prediction, but his work was incomplete due to lack of detailed information on the forcing input. Press and Houbolt [4] and Diederich [5] extended the work cited above on the response of airplanes to gusts. A somewhat different aspect of the problem was treated by Thomson and Barton [6]. They considered a lightly damped system with a number of degrees of freedom and found an expression for the power density of the response.† A related work was published by Eringen [7] who restricted his discussion to beams and plates, but suggested simple representations of the forcing function.

It is clear from the work accomplished that rational design for random loads depends on the understanding of two fundamental areas. The first is the knowledge of the input. When the input is due to the action of a turbulent stream on the structure, the information required is beyond the present level of understanding of fluid dynamics. The second area where more knowledge is required is the mechanism of material failure. The theory of cumulative damage as used by Miles [3] does not take into account the time sequence of loadings; this criterion of damage is therefore questionable.

Notwithstanding the problems involved in rational design, it is of considerable practical

*This paper was not presented at the Symposium.

†Reference [6] is marred by a number of defects due probably to motation inconsistencies. It appears that the intention was to obtain the response of the system under the action of a single random force.

importance to redesign structures which fail under random loads in service. The most direct approach to the problem is: (1) to attach strain gages to critical areas of the structure, (2) to measure the strains in service, (3) to analyze the output, and (4) to try to duplicate as closely as possible in the laboratory the motion of the structure. If failures occur under these laboratory conditions, then a redesign can be started. However, the question of redesign hinges upon answering the questions: What will be the response of the redesigned structure to the same environment? What is the response at points other than where the strain gages were located? These are the practical questions with which the following study is concerned.

We imagine that the structure itself is a test instrument and seek to infer the nature of the forcing function. It is a question here not so much of an absolute answer as it is a relative answer, namely, to predict the effects of changes in design. Our procedure is somewhat to improve the general formulation of Thomson and Barton [6] for structures of very little damping. A simplification is then made when the forcing pressures are due to turbulence, along the lines of Eringen [7], for the case where the length and time scales are assumed very small. The output of one strain gage can then be used to estimate the structural characteristics of the forcing pressure, and the effects of redesign can be evaluated.

NOTATION

a_n, b_n	Fourier coefficients
c	ordinate of beam element
$f(t)$	function of t
$g(\omega)$	transform of $f(t)$
g	damping coefficient
m	mean per unit length
t	time
w	transform of W
x	space coordinates
$A(t), B(t)$	dynamic variables
E	Young's modulus
F_a	generalized force
I	section moment
$P(x,t)$	pressure distribution
Q_a	generalized coordinate
R	reaction of beam
T	time interval
$\mathfrak{T}$	kinetic energy
U	potential energy
$W(x,t)$	deflection function
δ	Dirac delta function
ϵ	strain
$\rho_{AB}(\omega)$	power density corresponding to $A(t)$, $B(t)$
τ	time displacement
$\phi_a(x)$	normal mode
ω	(angular) frequency

Superscripts

$*$	complex conjugate

Subscripts

T	function section

DEFINITION OF POWER DENSITY

The power density $\rho(\omega)$ corresponding to a function of time, $f(t)$, is defined slightly differently by various writers. It is desirable therefore to present here the background of our definition:

Let $f(t)$ be a real-valued function defined on the interval $-\infty < t < \infty$. Define for large T

$$f_T(t) = f(t), \qquad |t| < T, \qquad\qquad (1)$$
$$= 0, \qquad |t| > T.$$

Let $g_T(\omega)$ be the Fourier transform of f_T, i.e.,

$$f_T(t) = \int_{-\infty}^{\infty} e^{i\omega t}\, g_T(\omega)\, d\omega. \qquad (2)$$

Since $f_T(t)$ is real, it is readily shown that

$$g_T(-\omega) = g_T(\omega)^* , \qquad (3)$$

the asterisk represents "complex conjugate." The function $f_T(t)$ may be expanded in the interval $-T < t < T$ into a Fourier series

$$f_T(t) = \sum_{-\infty}^{\infty} a_n \, e^{in\pi t/T} . \qquad (4)$$

The difference between two successive angular frequencies is therefore

$$\Delta\omega = \frac{\pi}{T} . \qquad (5)$$

The representations of Eqs. (2) and (4) agree if one writes

$$a_n = g_T(\omega) \, \Delta\omega , \qquad (6)$$

$$\omega = \frac{n\pi}{T} . \qquad (7)$$

The power (in the electrical sense) furnished by the function $f_T(t)$ in Eq. (4) for the frequency ω is

$$|a_n|^2 + |a_{-n}|^2 .$$

Using Eqs. (6) and (3), this takes the value

$$|g_T(\omega)|^2 \, \Delta\omega^2 + |g_T(-\omega)|^2 \, \Delta\omega^2 = 2|g_T(\omega)|^2 \, \Delta\omega^2 .$$

The density relative to the ω axis is obtained by dividing by $\Delta\omega$. Then, substituting from Eq. (5), one finds for the density

$$\frac{2\pi}{T} \, |g_T(\omega)|^2 .$$

The power density is then defined in the limit $T \to \infty$,

$$\rho(\omega) = \lim \frac{2\pi}{T} \, |g_T(\omega)|^2 . \qquad (8)$$

The function $\rho(\omega)$ has the property

$$\rho(-\omega) = \rho(\omega) . \qquad (9)$$

With this definition, one can show that the autocorrelation function derived from $f(t)$ is a Fourier transform of $\rho(\omega)$ in the following sense:

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(t+\tau) \, f(t)^* \, dt = \int_0^{\infty} \cos \omega\tau \, \rho(\omega) \, d\omega . \qquad (10)$$

This statement may be proved by substituting into the left side of Eq. (10) from Eq. (2) and interchanging the order of integrations,

$$\int_{-T}^{T} f(t+\tau) \, f(t)^* \, dt = \int_{-T}^{T} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i\omega(t+\tau) - i\omega' t} \times g_T(\omega) \, g_T(\omega')^* \, d\omega \, d\omega' \, dt . \qquad (11)$$

But

$$\int_{-T}^{T} e^{i\omega t - i\omega' t} \, dt \cong 2\pi \, \delta(\omega - \omega') ,$$

where δ is Dirac's delta function. Substituting into Eq. (11) and carrying out the integration on ω', one finds

$$\begin{aligned}
\int_{-T}^{T} f(t+\tau) \, f(t)^* \, dt &\cong 2\pi \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i\omega\tau} \, \delta(\omega - \omega') \times g_T(\omega) \, g_T(\omega')^* \, d\omega \, d\omega' , \\
&\cong 2\pi \int_{-\infty}^{\infty} e^{i\omega\tau} \, g_T(\omega) \, g_T(\omega)^* \, d\omega \\
&= 4\pi \int_0^{\infty} \cos \omega\tau \, |g_T(\omega)|^2 \, d\omega ,
\end{aligned}$$

where the approximation approaches equality as $T \to \infty$ (see Ref. 1, Appendix 6).† Dividing by $2T$ and passing to the limit, one obtains

$$\begin{aligned}
\lim \frac{1}{2T} \int_{-T}^{T} f(t+\tau) \, f(t)^* \, dt &= \int_0^{\infty} \cos \omega\tau \, \lim \left[\frac{2\pi}{T} \, |g_T(\omega)|^2 \right] d\omega , \\
&= \int_0^{\infty} \cos \omega\tau \, \rho(\omega) \, d\omega ,
\end{aligned}$$

which is the required relation. This relation may be generalized as follows. Let $f'(t)$ be a second function of t and let g_T' be the corresponding Fourier transform —

†These representations make use of the delta function. They are unrealistic as introduced by Eringen because the delta function becomes infinite when the independent variable is zero.

$$f'_T(t) = \int_{-\infty}^{\infty} e^{i\omega t} g'_T(\omega)\, d\omega .$$

Then the joint power density of the pair of functions f, f' is defined by the limit

$$\rho_{ff'}(\omega) = \lim_{T\to\infty} \frac{2\pi}{T} g_T(\omega)\, g'_T(\omega)^* . \qquad (12)$$

Proceeding just as before, one finds

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(t+\tau)\, f'(t)^*\, dt = \int_0^{\infty} \cos \omega\tau\, \rho_{ff'}(\omega)\, d\omega . \qquad (13)$$

POWER DENSITY OF COMPOSITE STRUCTURES

Let x represent a point on the structure (x is represented by one coordinate for a one-dimensional structure, two coordinates for a two-dimensional structure, and so on). Let $\phi_a(x)$, $a = 1, 2, \ldots$, represent a complete system of normal modes, normal and orthogonalized with respect to the kinetic energy of the system. Let the deflection of the point x at time t be represented by the function $W(x, t)$. Write

$$W(x,t) = \sum_a Q_a(t)\, \phi_a(x) , \qquad (14)$$

where Q_a are the generalized coordinates. Then the kinetic energy $\mathfrak{T}$ and potential energy U have the respective forms

$$\mathfrak{T} = \frac{1}{2} \sum_a \dot{Q}_a^2 , \qquad (15)$$

$$U = \frac{1}{2} \sum \omega_a^2\, Q_a^2 , \qquad (16)$$

where ω_a is the natural frequency of the mode $\phi_a(x)$.

Let $P(x,t)$ be the external pressure applied to point x at time t. The generalized force in mode $\phi_a(x)$ is

$$F_a(t) = \int P(x,t)\, \phi_a(x)\, dx . \qquad (17)$$

The equations of motion of the system under the action of the pressure $P(x,t)$ are

$$\ddot{Q}_a + \omega_a^2 Q_a = F_a(t) .$$

Introduce the Fourier transforms of Q_a, F_a, and W, according to the equations

$$Q_a(t) = \int_{-\infty}^{\infty} e^{i\omega t}\, q_a(\omega)\, d\omega ,$$

$$F_a(t) = \int_{-\infty}^{\infty} e^{i\omega t}\, f_a(\omega)\, d\omega , \qquad (19)$$

$$W(x,t) = \int_{-\infty}^{\infty} e^{i\omega t}\, w(x,\omega)\, d\omega .$$

Then Eq. (18) may be solved in the transform as follows

$$q_a = \frac{1}{-\omega^2 + \omega_a^2} f_a(\omega) .$$

Hence the Fourier transform of Eq. (14) is given by

$$w(x,\omega) = \sum_a \frac{f_a(\omega)}{-\omega^2 + \omega_a^2} \phi_a(x) . \qquad (20)$$

If $F_a(t)$ does not have a Fourier transform, then the preceding equation must be modified. Assume that the system is stable so that when the external forces $F_a(t)$ are removed, the coordinates Q_a approach zero. Then Eq. (20) may be replaced by the approximations

$$w_T(x,\omega)^2 \cong \sum_a \frac{f_{aT}(\omega)}{-\omega^2 + \omega_a^2} \phi_a(x) . \qquad (21)$$

In general, one is interested in certain quantities $A(t)$, $B(t)$, and so on, which are linear functions of the deflection $W(x, t)$. These functions can be written in the form of a definite integral on x.

$$A(t) = \int p(x)\, W(x,t)\, dx ,$$
$$B(t) = \int q(x)\, W(x,t)\, dx . \qquad (22)$$

For instance, in the case of a beam, x can be taken as the one dimensional coordinate x along the beam. The deflection $A(x_0,t)$ at a point x_0 can be written

$$A(x_0,t) = \int \delta(x-x_0)\, W(x,t)\, dx, \quad p(x) = \delta(x-x_0). \qquad (23)$$

The strain ϵ at a fibre distant c from the neutral axis is given by

$$\epsilon = c \frac{\partial^2 A(x_o, t)}{\partial x_o^{\,2}} = c \int \delta''(x - x_o)\, W(x,t)\, dx, \tag{24}$$

so that $p(x)$ for ϵ is

$$p(x) = c\, \delta''(x - x_o). \tag{25}$$

Multiply throughout Eq. (21) by $p(x)$ and integrate on x, and repeat with the multiplier $q(x)$. Let

$$a_T(\omega) = \int w_T(x, \omega)\, p(x)\, dx ,$$

$$b_T(\omega) = \int w_T(x, \omega)\, q(x)\, dx .$$

Then

$$a_T(\omega)^2 \cong \sum_a \frac{a_a\, f_{aT}(\omega)}{-\omega^2 + \omega_a^2},\quad a_a = \int p(x)\, \phi_a(x)\, dx,$$

$$b_T(\omega) \cong \sum_a \frac{b_a\, f_{aT}(\omega)}{-\omega^2 + \omega_a^2},\quad b_a = \int q(x)\, \phi_a(x)\, dx. \tag{26}$$

These formulae are based on the assumption that there is no damping in the system. Suppose now that the natural frequencies ω_a are well separated. Suppose further that the damping in the vicinity of each natural frequency is small and can be represented by a phase displacement in the restoring force, independent of the frequency. Then the term ω_a^2 must be replaced by a term of the form $\omega_a^2(1 + i\, g_a)$ where g_a is independent of the frequency ω. The formulae for $a_T(\omega)$ and $b_T(\omega)$ must then be replaced by the following:

$$a_T(\omega) \cong \sum_a \frac{a_a\, f_{aT}(\omega)}{-\omega^2 + \omega_a^2(1 + i\, g_a)} ,$$

$$b_T(\omega) \cong \sum_a \frac{b_a\, f_{aT}(\omega)}{-\omega^2 + \omega_a^2(1 + i\, g_a)} .$$

Then the joint power density of the pair of functions $A(t)$, $B(t)$ is

$$\rho_{AB}(\omega) = \lim \frac{2\pi}{T}\, a_T(\omega)\, b_T(\omega)^* , \tag{27}$$

(cont.)

$$\rho_{AB}(\omega) = \lim \frac{2\pi}{T} \sum_{a,\beta} \times \frac{a_a\, b_\beta^*\, f_{aT}(\omega)\, f_{\beta T}(\omega)^*}{\left[-\omega^2 + \omega_a^2(1 + i g_a)\right]\left[-\omega^2 + \omega_\beta^2(1 - i g_\beta)\right]} . \tag{27}$$

We have changed from the approximation to the equality because of the division by T and the subsequent passage to the limit.

It is now proposed to multiply both sides of Eq. (27) by $\cos \omega\tau\, d\omega$ and to integrate on ω. Inasmuch as the damping is supposed to be small, our interest is in the asymptotic form of the integral developed in orders of g_a where $g_a \to 0$. When $a \neq \beta$, the contribution to the integral is finite in this limit. However, when $a = \beta$, the integral is not finite. In fact, one can show by contour integration the following relation

$$\int_0^\infty \frac{dz}{(z^2 - a^2)^2 + b^4} = \frac{\pi}{2a\, b^2} .$$

Therefore, for small g_a,

$$\int_0^\infty \frac{\cos \omega\tau\; f_{aT}(\omega)\; f_{aT}(\omega)^*\, d\omega}{(-\omega^2 + \omega_a^2)^2 + \omega_a^4 g_a^2}$$

$$\cong \cos \omega_a\tau\; f_{aT}(\omega_a)\; f_{aT}(\omega_a)^* \int_0^\infty \frac{d\omega}{(-\omega^2 + \omega_a^2)^2 + \omega_a^4 g_a^2}$$

$$= \frac{\pi}{2}\, \frac{\cos \omega_a\tau\, |f_{aT}|^2}{\omega_a^3\, g_a} ,$$

which shows that the terms in Eq. (27) for which $\beta = a$ contribute to the final integral terms of order g_a^{-1}. Disregarding terms of lower order and using Eq. (13), one finds

$$\lim \frac{1}{2T} \int_{-T}^{T} A(t + \tau)\, B(t)^*\, dt$$

$$= \int_0^\infty \cos \omega\tau\; \rho_{AB}(\omega)\, d\omega , \tag{28}$$

$$\cong \frac{\pi}{2} \sum_a \frac{\rho_{aa}(\omega_a)}{\omega_a^3\, g_a}\, a_a\, b_a^*\, \cos \omega_a\tau ,$$

where

$$\rho_{aa}(\omega) = \lim \frac{2\pi}{T} f_{aT}(\omega) f_{aT}(\omega)^* . \qquad (29)$$

In particular, the mean square of $A(t)$ is found by putting $B = A$ and $\tau = 0$.

$$\left\langle A^2 \right\rangle = \lim \frac{1}{2T} \int_{-T}^{T} A(t) A(t)^* dt ,$$

$$\cong \frac{\pi}{2} \sum_a \frac{\rho_{aa}(\omega_a)}{\omega_a^3 g_a} |a_a|^2 . \qquad (30)$$

It is possible to take an inverse Fourier transform of the two right sides in Eq. (28) and thereby to derive an approximate formula for ρ_{AB} in terms of ρ_{aa}. The result is

$$\rho_{AB}(\omega) \cong \sum_a A_a \delta(\omega - \omega_a) , \qquad (31)$$

where

$$A_a = \frac{\pi}{2} \frac{\rho_{aa}(\omega_a)}{\omega_a^3 g_a} a_a b_a^* . \qquad (32)$$

Therefore, to a first approximation, $\rho_{AB}(\omega)$ consists of a series of "bar-functions." A more accurate analysis, taking into account previously disregarded terms, would show the presence in Eq. (31) of summands which are continuous functions. However, we can accept Eq. (31) as correct within the approximation indicated. Then the A_a can be expressed approximately in terms of an empirically determined ρ_{AB} by integrating with respect to ω across each ω_a,

$$A_a = \int \rho_{AB}(\omega) d\omega , \qquad (33)$$

where the integration is on the range $1/2(\omega_a + \omega_{a-1}) < \omega < 1/2(\omega_a + \omega_{a+1})$.

A comparison of Eqs. (32) and (33) shows that the quantities $\rho_{aa}(\omega_a)$ can be calculated from the empirically determined power spectrum $\rho_{AB}(\omega)$ provided that certain dynamic data is known. This includes the spectrum of natural frequencies and the damping coefficients g_a associated with these frequencies, both of which can be obtained from vibration tests. Quantities a_a and b_a are known from the geometry of the system (see Eq. (26)).

The quantities $\rho_{aa}(\omega_a)$ are the values at the natural frequencies of the power density of the corresponding generalized forces, Eq. (17). When the system is changed in some minor way, it is convenient to assume that the ω_a are changed only slightly and that $\rho_{aa}(\omega_a)$ is practically unchanged. On the other hand, the coefficients a_a and b_a would change, and the corresponding change in the power density could be calculated.

LIMITING FORMS OF THE POWER DENSITY

The quantities $\rho_{aa}(\omega_a)$ described in the preceding section depend both on the environment and on the structure itself. It would be desirable in some way to extract information from an empirical power density which would throw light on the environment independent of the structure. This would be a means of assembling empirical information which could be correlated between different tests and configurations.

If one considers the random pressure environments within turbulent flows, it seems out of the question, at the present time, to develop a rational method of inference of the structure of the turbulent field from the power density of the structure. Without this, the only alternative is to assume a model of turbulence and to derive as much information as possible about this model with the hope of correlating this data.

If one considers a specific point x at the surface of the structure, then the mean square pressure P^2 is given by

$$\langle P^2 \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} P_T(x,t) P_T(x,t) dt .$$

The limiting correlation function

$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} P_T(x,t) P_T(x+\xi, t+\tau) dt ,$$

therefore reduces to $\langle P^2 \rangle$ for $\xi, \tau \to 0$. It is likely that the length ℓ_0 and time t_0 scales of turbulence are so small in comparison with the dimensions and periods of the structure that the correlation function, regarded as a function of ξ and τ, is essentially like a product of Dirac delta-function. It is therefore convenient to assume that the following form exists:

$$\lim \frac{1}{2T} \int_{-T}^{T} P_T(x,t) P_T(x+\xi, t+\tau) dt$$

$$= \left\langle P^2(x) \right\rangle \Delta\left(\frac{\xi}{\ell_0}\right) \Delta\left(\frac{\tau}{t_0}\right) . \qquad (34)$$

Here $\Delta(s)$ represents a function of s such that

$$\int_{-\infty}^{\infty} \Delta(s)\, ds = 1,$$

$$\Delta(0) = 1.$$

and $\Delta(x/\ell_o)$ is the product of $\Delta(x_i/\ell_o)$, one for each dimension. Then restricting x to points on the surface of the structure,

$$\int_{-\infty}^{\infty} \Delta(t/t_o)\, dt = t_o ,$$

$$\int_{-\infty}^{\infty} \Delta(x/\ell_o)\, dx = \ell_o^2 . \qquad (35)$$

In the limits $t_o \to 0$, $\ell_o \to 0$, the functions $t_o^{-1}\,\Delta(t/t_o)$, $\ell_o^{-2}\,\Delta(x/\ell_o)$ behave like Dirac delta functions in one and two dimensions respectively. Using the properties of the delta functions, one can find simple expressions for the power densities $\rho_{aa}(\omega)$. The inverse of Eq. (19) gives

$$f_{aT}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}\, F_{aT}(t)\, dt ,$$

where, from Eq. (17),

$$F_{aT}(t) = \int P_T(x,t)\, \phi_a(x)\, dx .$$

Substituting into Eq. (29) one finds

$$\rho_{aa}(\omega) = \lim \frac{2\pi}{T}\, f_{aT}\, f_{aT}^{*},$$

$$= \lim \frac{1}{2\pi T}\iiiint e^{i\omega(t'-t)}\, P_T(x,t) \times P_T(x',t')\, \phi_a(x)\, \phi_a(x')\, dt\, dt'\, dx\, dx' .$$

Here the integration on x and x' is over the surface of the structure, and t and t' range from $-\infty$ to $+\infty$. Replace t' by $t+\tau$ and x' by $x+\xi$. Then

$$\rho_{aa}(\omega) = \lim \frac{1}{2\pi T}\iiiint e^{i\omega\tau}\, P_T(x,t) \times P_T(x+\xi,\ t+\tau)\, \phi_a(x+\xi)\, dt\, d\tau\, dx\, d\xi .$$

Integrate first with respect to t, applying Eq. (33), then integrate on ξ and τ using the properties of the delta function in the limit ℓ_o, $t_o \to 0$,

$$\rho_{aa}(\omega) = \frac{1}{\pi}\iiint e^{i\omega\tau}\, \langle P^2(x)\rangle \times \Delta\left(\frac{\xi}{\ell_o}\right) \Delta\left(\frac{\tau}{t_o}\right) \phi_a(x)\, \phi_a(x+\xi)\, d\tau\, d\xi\, dx ,$$

$$= \frac{1}{\pi}\int \langle P^2(x)\rangle\, \ell_o^2\, t_o\, \phi_a(x)^2\, dx . \qquad (36)$$

This is as far as one can go without additional assumptions. If, however, $\langle P^2(x)\rangle$ is independent of x, $\langle P^2(x)\rangle = \langle P^2\rangle$, and if ℓ_o and t_o are also independent of x, then these quantities can be brought outside the integral sign. Since the $\phi_a(x)$ are normalized, the resulting integral is unity. Therefore,

$$\rho_{aa}(\omega) = \frac{1}{\pi}\, \langle P^2\rangle\, \ell_o^2\, t_o . \qquad (37)$$

Equations (28) and (30) may be then simplified using Eq. (36) to the following respective forms:

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A(t+\tau)\, B(t)^{*}\, dt \cong \frac{1}{2}\, \langle P^2\rangle\, \ell_o^2\, t_o \sum \frac{a_a\, b_a^{*} \cos \omega_a \tau}{\omega_a^3\, g_a} ; \qquad (38)$$

$$\langle A^2\rangle \cong \frac{1}{2}\, \langle P^2\rangle\, \ell_o^2\, t_o \sum{}' \frac{|a_a|^2}{\omega_a^3\, g_a} . \qquad (39)$$

REACTION OF A SIMPLY SUPPORTED BEAM

Another kind of application of the power density is illustrated by the following problem. A beam extending from $x = 0$ to $x = L$ is simply supported at the two ends. A strain gage is mounted at the top of the beam at the mid-point $x = L/2$, and the response to a random loading is recorded. It is required to find the mean square reaction $\langle R^2\rangle$ at the end $x = L$.

Let m represent the mass per unit length, let EI represent the bending stiffness, and let c represent the ordinate of the strain gage relative to the neutral axis. Suppose the beam vibrates principally in a single normal mode $\phi(x)$ which is supposed normalized according to

$$\int_0^L m\, \phi^2 dx = 1 . \qquad (40)$$

Let the deflection of the beam at any instant be $Q(t)\ \phi(x)$. Then under the pressure $P(x,t)$, the equation of motion is

$$-\int m\phi^2 dx\ \ddot{Q} - \int EI\phi_{xx}^2 dx\ Q + \int P(x,t)\,\phi dx = 0. \quad (41)$$

Now make a virtual deflection consisting of a rigid body motion $w = x$. One obtains the equation

$$-\int m\phi x\,dx\ \ddot{Q} + \int P(x,t)\,x\,dx + RL = 0. \quad (42)$$

Truncate the time dependent functions in Eqs. (41) and (42) to the interval $-T<t<T$ and take Fourier transforms. Use Eq. (40), and let ω_o represent the natural frequency corresponding to $\phi(x)$. In accordance with earlier notation, let

$$F(t) = \int P(x,t)\,\phi(x)\,dx\,, \int F_1(x) = \quad P(x,t)x\,dx\,,$$

let $f_T(\omega)$ and $f_{1T}(\omega)$ represent Fourier transforms of the truncated functions F_T and F_{1T}, respectively, and let ρ_{FF}, ρ_{QQ}, and ρ_{RR} represent the power densities of the function F, Q, and R, respectively. From Eq. (41)

$$q_T(\omega) \doteq -\frac{f_T(\omega)}{\omega^2 - \omega_o^2}\,. \quad (43)$$

and from Equation (42),

$$r_T(\omega) \doteq -\frac{\int m\phi x\,dx}{L}\,\omega^2\,q_T(\omega) - \frac{f_{1T}(\omega)}{L}\,.$$

Eliminating $q_T(\omega)$, one obtains

$$r_T(\omega) \doteq \frac{\int m\phi x\,dx}{L}\cdot\frac{\omega^2\,f_T(\omega)}{\omega^2 - \omega_o^2} - \frac{f_{1T}(\omega)}{L}\,,$$

$$\doteq \frac{\int m\phi x\,dx}{L}\,\frac{\omega_o^2\,f_T(\omega_o)}{\omega^2 - \omega_o^2} + \cdots\cdots, \quad (44)$$

$$\doteq \frac{\int m\phi x\,dx}{L}\,\frac{\omega_o^2\,f_T(\omega)}{\omega^2 - \omega_o^2} + \cdots\cdots,$$

where the dots indicate terms which are finite at $\omega \to \omega_o$. These finite terms are not shown explicitly because they will be of lower order in the damping coefficient. Squaring both sides, multiplying by $2\pi/T$, and passing to the limit $T\to\infty$, one has

$$\rho_{RR}(\omega) = \left(\frac{\int m\phi x\,dx}{L}\right)^2 \omega_o^4\,\rho_{QQ}(\omega)\,. \quad (45)$$

Integrating on ω and applying Eq. (10) for $\tau = 0$, one obtains

$$\left\langle R^2\right\rangle = \left(\frac{\int m\phi x\,dx}{L}\right)^2 \omega_o^4 \left\langle Q^2\right\rangle.$$

The strain ϵ at the strain gage is given by

$$\epsilon = c\,\phi_{xxo}\,Q\,,$$

where the subscript o means that ϕ_{xx} is evaluated at $x = L/2$. Eliminating Q, one finally obtains

$$R^2 = \left[\frac{\int m\phi x\,dx}{c\,L\,\phi_{xxo}}\right]^2 \omega_o^4 \left\langle \epsilon^2\right\rangle.$$

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SPATIAL CORRELATION IN ACOUSTIC-STRUCTURAL COUPLING AND ITS EFFECT ON STRUCTURAL RESPONSE PREDICTION TECHNIQUES*

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The responses of flat rectangular panels with simply-supported or clamped boundary conditions to specified acoustic fields, are examined analytically and numerically to improve response prediction techniques for practical problems. Analytical development for beam-like structures, found in the literature, was modified and extended to rectangular panels. The prediction of panel response can be accomplished with knowledge of the power spectrum and spatial correlation of the acoustic field and the panel mechanical impedance.

Numerical results of the acoustic-structural coupling coefficient frequency spectrum indicate a marked sensitivity to changes in the spatial correlation, a measure of the coherence of the force distribution over the panel. Consequently, as the spatial correlation decreases, the panel mode shapes lose their wavelength selectivity and, hence, respond at all frequencies.

The analytical work is of value in its ability to define numerical limits on the response of panels with specified boundary conditions to acoustic plane wave fields with various degrees of spatial correlation. Extrapolation to practical problems can result in a better estimation of the order of magnitude of the expected response.

A standard size panel is chosen to define curves of the acoustic-structural coupling coefficients. These curves can be applied to any size rectangular panel with similar boundary conditions by multiplying the ordinates and abscissae by the proper scale factor. These scale factors can represent aspect ratio, thickness, weight, stiffness, or almost any other physical panel parameter.

INTRODUCTION

Analysis of structural vibrations excited by random loads can be accomplished for a few specific practical combinations of load and structure. The method used is that of generalized harmonic analysis. This method, as applied to the acoustic structural problem, develops a set of coefficients [1, 2, and 3] which couple the acoustic field characteristics to those of the structure.

The prediction of structural response can be accomplished with knowledge of the power spectrum and spatial correlation of the acoustic field and the structural mechanical impedance. Application to rectangular panels with specified boundary conditions excited by particular traveling plane wave pressure fields will be developed analytically and numerically.

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SYMBOLS

a = point on panel surface area

a_o = reference point

A = panel area = ℓw, sq in.

$\bar{A}$ = amplitude of pressure wave

$\bar{A}_o$ = reference amplitude

c = speed of sound = 13,200 in./sec

C_o = exponential decay constant

f = frequency, cps

f_m = mode shape in y-direction, normalized

f_n = mode shape in x-direction, normalized

F_m = mode shape in y-direction

F_n = mode shape in x-direction

F = arbitrary mode shape

k = wave number = $2\pi f/c$, cycles/in.

ℓ = panel length, in.

m = transverse (y) modal number = 1,2,3, ...

n = longitudinal (x) modal number = 1,2,3,...

p = time dependent pressure wave

p_d = damping pressure

p_o = reference pressure

t = time, sec

T = period = $1/f$, sec/cycle

w = panel width, in.

Y = mechanical impedance

ω = radial frequency = $2\pi f$

ω_s = natural frequency of sth mode

μ = mass density/unit area

THEORY

The general theory is well developed in Refs. 1 through 4. The main features of the theory are summarized. (It will be assumed that the cross coupling between modes is negligible in the following derivation.)

The displacement of any point on a structure during any arbitrary vibration can be represented by

$$g(a,t) = \sum_{s=1}^{\infty} \zeta_s(t)\, f_s(a), \tag{1}$$

where $\zeta_s(t)$ is the generalized coordinate of the sth mode and $f_s(a)$ is the normalized mode shape.

Consider the response of the sth mode to a time dependent load. The Lagrange equation of motion is of the form

$$M_s \ddot{\zeta}_s + C_s \dot{\zeta}_s + K_s \zeta_s = L_s(t), \tag{2}$$

where

$$M_s = \int_A \mu f_s^2(a)\, da, \qquad \text{generalized mass,}$$

$$C_s = \int_A p_{d_s}(a)\, f_s^2(a)\, da, \quad \text{generalized damping,}$$

$$K_s = \omega_s^2 M_s, \qquad \text{generalized stiffness,}$$

$$L_s(t) = \int_A p(a,t)\, f_s(a)\, da, \quad \text{generalized force.}$$

Solution of Eq. (2) yields the power spectral density of the displacement for the sth mode,

$$w_{\zeta_s}(\omega) = \frac{w_L(\omega)}{|Y(\omega)|^2}, \tag{3}$$

where, $w_L(\omega)$ = power spectral density of the generalized force,

$$|Y(\omega)|^2 = M_s^2\left[(\omega_s^2 - \omega^2)^2 + C_s^2\omega^2/M_s^2\right],$$

mechanical impedance squared. Thus, the mean square displacement is

$$\bar{\zeta}^2 = \int_0^\infty w_{\zeta_s}(\omega)\, d\omega.$$

The power spectral density of the generalized force at frequency ω is

$$w_L(\omega) = \int_A \int_A \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} p(t,s)\, p(t,s')\, f(s)\, f(s')\, dt\, ds\, ds', \tag{4}$$

where we define

$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} p(t,s)\, p(t,s')\, dt$ = spatial correlation of the pressures in a narrow- (1 cycle) frequency band centered at ω.

The power spectral density of the noise pressures in a narrow band at ω and some reference point s_o, chosen such that $p(t,s)/p(t,s_o) \leq 1.0$, becomes

$$w_{p_o}(\omega) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left[p(t,s_o)\right]^2 dt. \tag{5}$$

The spatial correlation function, $R_\omega(s,s')$ is then defined as

$$R_\omega(s,s') = \frac{\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} p(t,s)\, p(t,s')\, dt}{w_{p_o}(\omega)}. \tag{6}$$

Thus,

$$w_L(\omega) = w_{p_o}(\omega) \times A^2 \times \left[\frac{1}{A^2}\int_A \int_A R_\omega(s,s')\, f_s(s)\, f_s(s')\, ds\, ds'\right], \tag{7}$$

where the term in brackets is in nondimensional form, and is usually defined as the acoustic-structural coupling coefficient or by other authors as joint acceptance squared, $j_{ss}^2(\omega)$. Hence, the power spectral density of the generalized force is simply

$$w_L(\omega) = w_{p_o}(\omega) \times A^2 \times \left[j_{ss}^2(\omega)\right]. \tag{7}$$

And, finally, the mean square response becomes

$$\bar{\zeta}_s^2 = \int_0^\infty \frac{w_{p_o}(\omega) \times A^2 \times j_{ss}^2(\omega)}{|Y_s(\omega)|^2}\, d\omega. \tag{8}$$

APPLICATIONS

Plane Wave Spatial Correlation Function

The spatial correlation R_ω, and the power spectral density of the reference mean squared pressure $w_{p_o}(\omega)$, are characteristics of the exciting acoustic field. The reference point a_o, is usually chosen such that $w_{p_o}(\omega)$ is larger or as large as that anywhere in the area of interest. Thus, R_ω is normalized with a maximum value of one.

Consider a system of sinusoidal plane pressure waves traveling in the x-direction

$$p(t,a) = \bar{A} \sin(\omega t - kx),$$

where, $\bar{A}$ is the pressure amplitude at x. The spatial correlation function R_ω, becomes

$$R_\omega(a,a') = \frac{\lim\limits_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} p(t,a)\, p(t,a')\, dt}{\lim\limits_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left[p(t,a_o)\right]^2 dt}$$

$$= R_\omega(x,x') = \frac{\bar{A}\bar{A}' \int_0^{2\pi} \sin(\omega t - kx) \sin(\omega t - kx')\, d(\omega t)}{\bar{A}_o^2 \int_0^{2\pi} \sin^2(\omega t - kx_o)\, d(\omega t)}.$$

Therefore, integrating, we obtain

$$R_\omega(x,x') = \frac{\bar{A}\bar{A}'}{\bar{A}_o^2} \cos k(x-x'),$$

which is independent of y-coordinate and of time t. Now, to obtain a more realistic correlation function, assume that the relationship between the pressure amplitudes $\bar{A}/\bar{A}_o$ and $\bar{A}'/\bar{A}_o$ approximates a decaying exponential with increasing separation magnitude, $|x - x'|$, i.e.,

$$\frac{\bar{A}\bar{A}'}{\bar{A}_o^2} = \exp\left[-\frac{k}{C_o}|x-x'|\right],$$

where, $1/C_o$ is the decay constant. Finally,

$$R_\omega(x,x') = e^{-\frac{k}{C_o}|x-x'|} \cos k(x-x') \tag{9}$$

represents a spatial correlation function for a field of sinusoidal plane pressure waves traveling along the x-axis, whose magnitude is damped exponentially with increasing separation, $|x - x'|$, between measurement points.

Simply-Supported Rectangular Panel

The acoustic-structural coupling coefficient $j^2(\omega)$, is defined in Eq. (7) as

$$j^2(\omega) = \frac{1}{A^2} \int_A \int_A R_\omega(a,a')\, F(a)\, F(a')\, da\, da'. \tag{10}$$

Consider a homogeneous rectangular panel of length ℓ, and width w, simply-supported on all four sides and lying in the x-y plane (Fig. 1). The normal mode shapes $F(a)$, of the panel are

$$F(a) = f_n(x)\, f_m(y) = \sin\left(\frac{n\pi x}{\ell}\right) \sin\left(\frac{m\pi y}{w}\right) . \tag{11}$$

where,

$n = 1, 2, 3, \ldots$

$m = 1, 2, 3, \ldots$

$A = \ell \times w.$

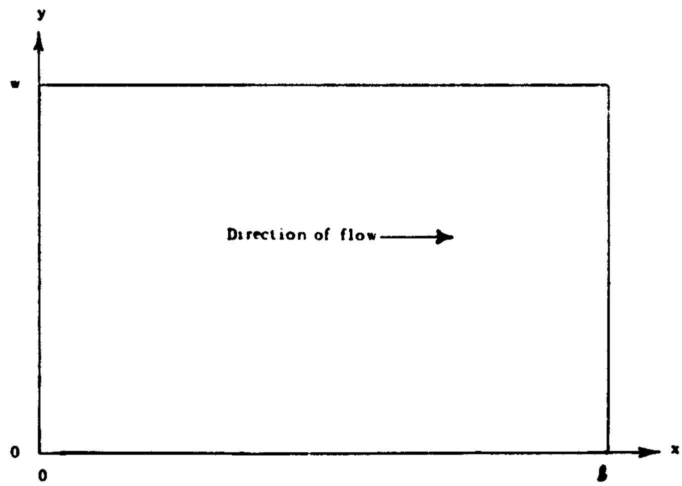

Fig. 1 - Rectangular panel of length (ℓ) and width (w) coordinate system

Now, exciting the panel with a plane pressure wave traveling parallel to the x-y plane and along the -axis, the acoustic-structural coupling coefficients become

$$j^2_{mmnn}(\omega) = \frac{1}{\ell^2 w^2} \int_0^w \int_0^w \int_0^\ell \int_0^\ell e^{-\frac{k}{C_o}|x-x'|} \cos k(x-x') \times \sin\left(\frac{n\pi x}{\ell}\right) \sin\left(\frac{n\pi x'}{\ell}\right) \times$$

$$\times \sin\left(\frac{m\pi y}{w}\right) \sin\left(\frac{m\pi y}{w}\right) dx\, dx'\, dy\, dy' . \tag{12}$$

The x-integrations and y-integrations are separable, hence,

$$j^2_{mmnn}(\omega) = \left[\frac{1}{w^2} \int_0^w \sin\left(\frac{m\pi y'}{w}\right) dy' \int_0^w \sin\left(\frac{m\pi y}{w}\right) dy\right] \times$$

$$\times \left[\frac{1}{\ell^2} \int_0^\ell \int_0^\ell e^{-\frac{k}{C_o}|x-x'|} \cos k(x-x') \sin\left(\frac{n\pi x}{\ell}\right) \sin\left(\frac{n\pi x'}{\ell}\right) dx\, dx'\right]$$

$$= \left[j^2_{mm}(\omega)\right] \times \left[j^2_{nn}(\omega)\right] = \Gamma_y(\omega) \times \Delta_x(\omega) . \tag{13}$$

Integrating Eq. (13) [5], yields,

$$\Gamma_y(\omega) = \frac{1}{w^2}\left[\frac{w}{m\pi} - \frac{w}{m\pi}\cos m\pi\right]^2 = \begin{cases} 0, & m \text{ even} \\ \dfrac{4}{m^2\pi^2}, & m \text{ odd}, \end{cases} \tag{14}$$

and

$$\Delta_x(\omega) = \frac{1}{\ell^2}\left[\frac{e^{-T\ell}}{2}\left\langle\left[\frac{P}{T^2+P^2}+\frac{Q}{T^2+Q^2}\right]\times\left[\frac{\langle -T\sin P\ell - P\cos P\ell\rangle}{T^2+P^2}+\frac{\langle -T\sin Q\ell - Q\cos Q\ell\rangle}{T^2+Q^2}\right]\right.\right.$$
$$\left. - \left[\frac{T}{T^2+Q^2}-\frac{T}{T^2+P^2}\right]\left[\frac{\langle Q\sin Q\ell - T\cos Q\ell\rangle}{T^2+Q^2} - \frac{\langle P\sin P\ell - T\cos P\ell\rangle}{T^2+P^2}\right]\right\rangle$$
$$\left. + \frac{\ell}{2}\left[\frac{T}{T^2+Q^2}+\frac{T}{T^2+P^2}\right] + \frac{1}{2}\left[\frac{Q}{T^2+Q^2}+\frac{P}{T^2+P^2}\right]^2 - \frac{1}{2}\left[\frac{T}{T^2+Q^2}-\frac{T}{T^2+P^2}\right]^2\right], \tag{15}$$

where,

$$T = \frac{k}{C_o},$$

$$P = \left(\frac{n\pi}{\ell} + k\right),$$

and

$$Q = \left(\frac{n\pi}{\ell} - k\right).$$

Finally,

$$j^2_{mmnn}(\omega) = \begin{cases} 0, & m \text{ even}; \; n = 1,2,\ldots \\ \Gamma_y(\omega)\times\Delta_x(\omega), & m \text{ odd}; \; n = 1,2,\ldots, \end{cases} \tag{16}$$

for a rectangular panel, simply-supported on all sides.

Clamped Rectangular Panel

The acoustic-structural coupling coefficients for the panel of Fig. 1, which is clamped on all four sides, are

$$j^2_{mmnn}(\omega) = \frac{1}{\ell^2 w^2}\int_0^w\int_0^w\int_0^\ell\int_0^\ell e^{-\frac{k}{C_o}|x-x'|}\cos k(x-x')\,F_n(x)\,F_n(x')\,F_m(y)\,F_m(y')\,dx\,dx'\,dy\,dy', \tag{17}$$

where

$$F_n(x) = A_n f_n(x) = A_n\left[\cosh\left(\frac{\gamma_n x}{\ell}\right) - \cos\left(\frac{\gamma_n x}{\ell}\right) - \bar{C}_n\left\langle\sinh\left(\frac{\gamma_n x}{\ell}\right) - \sin\left(\frac{\gamma_n x}{\ell}\right)\right\rangle\right],$$

$$F_m(y) = B_m f_m(y) = B_m\left[\cosh\left(\frac{\epsilon_m y}{w}\right) - \cos\left(\frac{\epsilon_m y}{w}\right) - \bar{K}_m\left\langle\sinh\left(\frac{\epsilon_m y}{w}\right) - \sin\left(\frac{\epsilon_m y}{w}\right)\right\rangle\right],$$

A_n and B_m are normalization constants,

$$
\gamma_n = \begin{cases} 1.506\pi , & n = 1 \\ (2n+1)\,\dfrac{\pi}{2} , & n = 2, 3, \ldots \end{cases} ,
$$

$$
\epsilon_m = \begin{cases} 1.506\pi , & m = 1 \\ (2m+1)\,\dfrac{\pi}{2} , & m = 2, 3, \ldots \end{cases} ,
$$

$$
\bar{C}_n = \frac{\sin \gamma_n + \sinh \gamma_n}{\cosh \gamma_n - \cos \gamma_n} ,
$$

and

$$
\bar{K}_m = \frac{\sin \epsilon \ + \sinh \epsilon_m}{\cosh \epsilon_m - \cos \epsilon_m} .
$$

The x-integrals and y-integrals are again separable, hence,

$$
\begin{aligned}
j^2_{mmnn}(\omega) &= \left[\frac{1}{w^2} \int_0^w F_m(y')\, dy' \int_0^w F_m(y)\, dy \right] \times \\
&\quad \times \left[\frac{1}{\ell^2} \int_0^\ell \int_0^\ell e^{-\frac{k}{C_0}|x-x'|} \cos k(x-x')\, F_n(x)\, F_n(x')\, dx\, dx' \right] \\
&= \left[j^2_{mm}(\omega) \right] \times \left[j^2_{nn}(\omega) \right] = \nabla_y(\omega) \times \Omega_x(\omega) .
\end{aligned} \tag{18}
$$

Integrating Eq. (18) [5] yields

$$
\nabla_y(\omega) = \begin{cases} 0 , & m \text{ even,} \\ \dfrac{B_m^2}{\epsilon_m^2} \left[\sinh \epsilon_m - \sin \epsilon_m - \bar{K}_m \cosh \epsilon_m - \bar{K}_m \cos \epsilon_m + 2\bar{K}_m \right]^2 , & m \text{ odd,} \end{cases} \tag{19}
$$

and

$$
\begin{aligned}
\Omega_x(\omega) = \frac{A_n^2}{2\ell^2} \cdot \Bigg[& \frac{\ell}{\gamma_n} \Bigg\langle \frac{(1-\bar{C}_n)V}{V^2+k^2} \Bigg[\frac{(1-\bar{C}_n)(e^{2\gamma_n}-1)}{2} + \Big\langle 1 - e^{\gamma_n}(\sin\gamma_n + \cos\gamma_n) \Big\rangle \\
& + \bar{C}_n \Big\langle 1 + e^{\gamma_n}(\sin\gamma_n - \cos\gamma_n) \Big\rangle + (1+\bar{C}_n)\gamma_n \Bigg] - \frac{(1+\bar{C}_n)U}{U^2+k^2} \Bigg[\frac{(1+\bar{C}_n)(e^{-2\gamma_n}-1)}{2} \\
& + \Big\langle 1 + e^{-\gamma_n}(\sin\gamma_n - \cos\gamma_n) \Big\rangle - \bar{C}_n \Big\langle 1 - e^{-\gamma_n}(\sin\gamma_n + \cos\gamma_n) \Big\rangle - (1-\bar{C}_n)\gamma_n \Bigg] + \frac{1}{2} \Bigg[\frac{S+\bar{C}_nT}{T^2+S^2} - \frac{R-\bar{C}_nT}{T^2+R^2} \Bigg] \\
& \times \Bigg[(1-\bar{C}_n) \Big\langle 1 + e^{\gamma_n}(\sin\gamma_n - \cos\gamma_n) \Big\rangle + (1+\bar{C}_n) \Big\langle 1 - e^{-\gamma_n}(\sin\gamma_n + \cos\gamma_n) \Big\rangle - (1 - \cos 2\gamma_n) \\
& - \bar{C}_n \sin 2\gamma_n + 2\bar{C}_n\gamma_n \Bigg] + \frac{1}{2} \Bigg[\frac{T-\bar{C}_nS}{T^2+S^2} + \frac{T+\bar{C}_nR}{T^2+R^2} \Bigg] \times \Bigg[(1-\bar{C}_n) \Big\langle 1 - e^{\gamma_n}(\sin\gamma_n + \cos\gamma_n) \Big\rangle
\end{aligned} \tag{20}
$$

(Continued)

$$
- (1+\bar{C}_n) \left\langle 1 + e^{-\gamma_n}(\sin\gamma_n - \cos\gamma_n) \right\rangle + \sin 2\gamma_n - \bar{C}_n(1-\cos 2\gamma_n) + 2\gamma_n \Bigg] \Bigg\rangle
$$

$$
- \left[\frac{(1-\bar{C}_n)V}{V^2+k^2} + \frac{(1+\bar{C}_n)U}{U^2+k^2} - \frac{T-\bar{C}_nS}{T^2+S^2} - \frac{T+\bar{C}_nR}{T^2+R^2} \right] \times \left[(1-\bar{C}_n) \left\langle \frac{e^{-U\ell}(-U\cos k\ell + k\sin k\ell) + U}{U^2+k^2} \right\rangle \right.
$$

$$
+ (1+\bar{C}_n) \left\langle \frac{e^{-V\ell}(-V\cos k\ell + k\sin k\ell) + V}{V^2+k^2} \right\rangle - \left\langle \frac{e^{-T\ell}\left[(S-\bar{C}_nT)\sin S\ell - (T+\bar{C}_nS)\cos S\ell\right] + (T+\bar{C}_nS)}{T^2+S^2} \right\rangle
$$

$$
\left. - \left\langle \frac{e^{-T\ell}\left[(R+\bar{C}_nT)\sin R\ell - (T-\bar{C}_nR)\cos R\ell\right] + (T-\bar{C}_nR)}{T^2+R^2} \right\rangle \right] + \left[\frac{(1-\bar{C}_n)k}{V^2+k^2} + \frac{(1+\bar{C}_n)k}{U^2+k^2} - \frac{S+\bar{C}_nT}{T^2+S^2} - \frac{R-\bar{C}_nT}{T^2+R^2} \right]
$$

$$
\times \left[(1-\bar{C}_n) \left\langle \frac{e^{-U\ell}(-U\sin k\ell - k\cos k\ell) + k}{U^2+k^2} \right\rangle + (1+\bar{C}_n) \left\langle \frac{e^{-V\ell}(-V\sin k\ell - k\cos k\ell) + k}{V^2+k^2} \right\rangle \right.
$$

$$
\left. + \left\langle \frac{e^{-T\ell}\left[(T+\bar{C}_nS)\sin S\ell + (S-\bar{C}_nT)\cos S\ell\right] - (S-\bar{C}_nT)}{T^2+S^2} \right\rangle + \left\langle \frac{e^{-T\ell}\left[(T-\bar{C}_nR)\sin R\ell + (R+\bar{C}_nT)\cos R\ell\right] - (R+\bar{C}_nT)}{T^2+R^2} \right\rangle \right] \Bigg] , \qquad (20)
$$

where

$n = 1, 2, 3, \ldots,$

$T = k/C_o,$

$R = (k + \gamma_n/1),$

$S = (k - \gamma_n/1),$

$V = (T + \gamma_n/1),$

$U = (T - \gamma/1)$, and

A_n and B_m are normalization constants.

A few calculated values of A_n and B_m are ($A_n \neq B_m$, $n = m$):

$A_1 = B_1 = 0.62966$

$A_2 = B_2 = 0.62571$

$A_3 = B_3 = 0.66131$

$A_4 = B_4 = 0.66111$

$A_5 = B_5 = 0.66135$

$\vdots \qquad \vdots$

Finally,

$$
j^2_{mmnn}(\omega) = \begin{cases} 0, & m \text{ even};\ n = 1, 2, 3, \ldots \\ \nabla_y(\omega) \times \Omega_x(\omega), & m \text{ odd};\ n = 1, 2, 3, \ldots \end{cases} \qquad (21)
$$

for a rectangular panel clamped on all four sides.

Additional Variations

Equations (14), (15), (19), and (20) can be combined to include the following beams and rectangular panels:

1. A beam of length ℓ, simply-supported at both ends, or a panel simply-supported on edges perpendicular to flow and free on the other two edges,

$$j_{nn}^2(\omega) = \Delta_x(\omega) . \tag{22}$$

2. A beam of length ℓ, clamped at both ends, or a panel clamped on edges perpendicular to flow and free on the other two edges,

$$j_{nn}^2(\omega) = \Omega_x(\omega) . \tag{23}$$

3. A panel with edges parallel to flow clamped, and the other two edges simply-supported,

$$j_{mmnn}^2(\omega) = \nabla_y(\omega) \times \Delta_x(\omega) . \tag{24}$$

4. A panel with edges parallel to flow simply-supported and the other two edges clamped,

$$j_{mmnn}^2(\omega) = \Gamma_y(\omega) \times \Omega_x(\omega) . \tag{25}$$

Summary

Equations (16) and (21) through (25) represent the acoustic-structural coupling coefficients for beams and rectangular panels with specific combinations of clamped or simply-supported boundary conditions excited by a plane pressure wave field traveling in the x-y plane parallel to the x-axis. The complexity of Eqs. (14), (15), (19), and (20) requires a computer program to perform the numerical calculations.

NUMERICAL RESULTS

An IBM 7090 FORTRAN computer program was written to compute numeric results for Eqs. (16) and (21) through (25). The results of a few parametric calculations are illustrated in succeeding figures.

The acoustic-structural coupling coefficients, $j^2(\omega)$, for a rectangular panel with appropriate boundary conditions 100 inches in length (ℓ = 100 inches) and arbitrary width w, were calculated for the first five mode shapes ($n = 1, 2, \ldots, 5$) in the direction of flow, the first mode shape ($m = 1$) in the transverse direction and three values of the correlation function decay constant ($C_o = \infty$, 20, and 1). Figure 2 illustrates the general shape of the correlation function obtained for the three values of the decay constant, C_o ($C_o = \infty$, 20, and 1). Figures 3, 4, and 5 show $j^2(\omega)$ for the panel simply-supported on all four sides (Eq. (16)). Figures 6, 7, and 8, show $j^2(\omega)$ for the panel clamped on all four sides (Eq. (21)). Figures 9, 10, and 11 show $j^2(\omega)$ for the panel with the edges parallel to flow clamped and the other two edges simply-supported (Eq. (24)). Figures 12, 13, and 14 show $j^2(\omega)$ for the panel with the edges parallel to flow simply-supported and the other two edges clamped (Eq. (25)).

Numerical values for Eqs. (22) and (23) can be obtained from Figs. 3, 4, and 5 and 12, 13 and 14, respectively. The value of $\Gamma_y(\omega)$ for $m = 1$ is $4/\pi^2$. Hence, if each value of $j^2(\omega)$, obtained in the preceding paragraph, is multiplied by $\pi^2/4$ the $j^2(\omega)$'s for a simple end-supported beam (panel) and for a clamped end beam (panel) result.

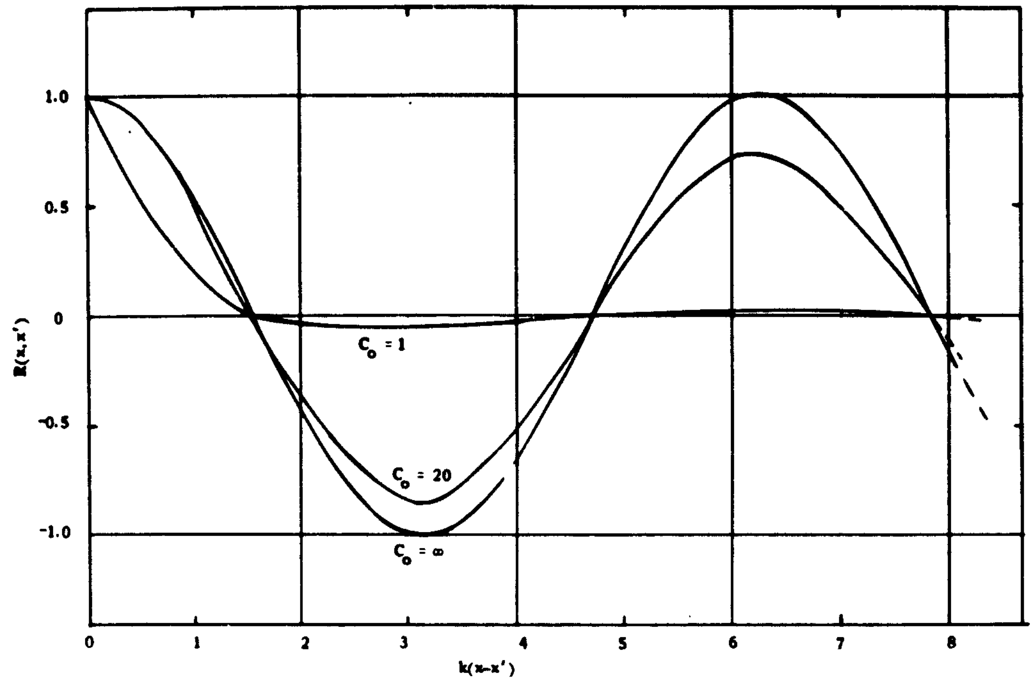

Fig. 2 - Spatial correlation functions for $C_o = \infty$, $C_o = 20$, $C_o = 1$, $[R_\omega = e^{-k/C_o|x-x'|} \cos k(x - x')]$

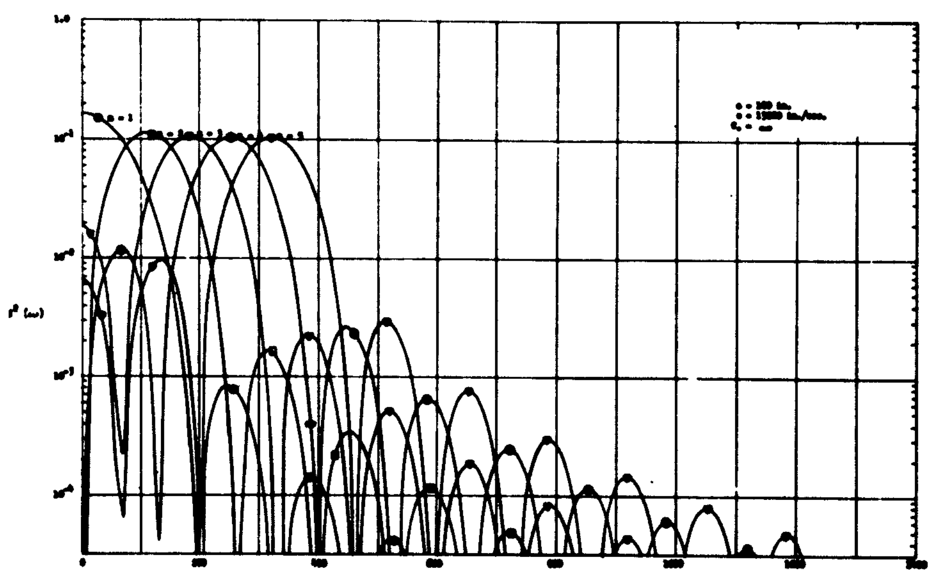

Fig. 3 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with all four sides simply-supported: $(C_o = \infty)$

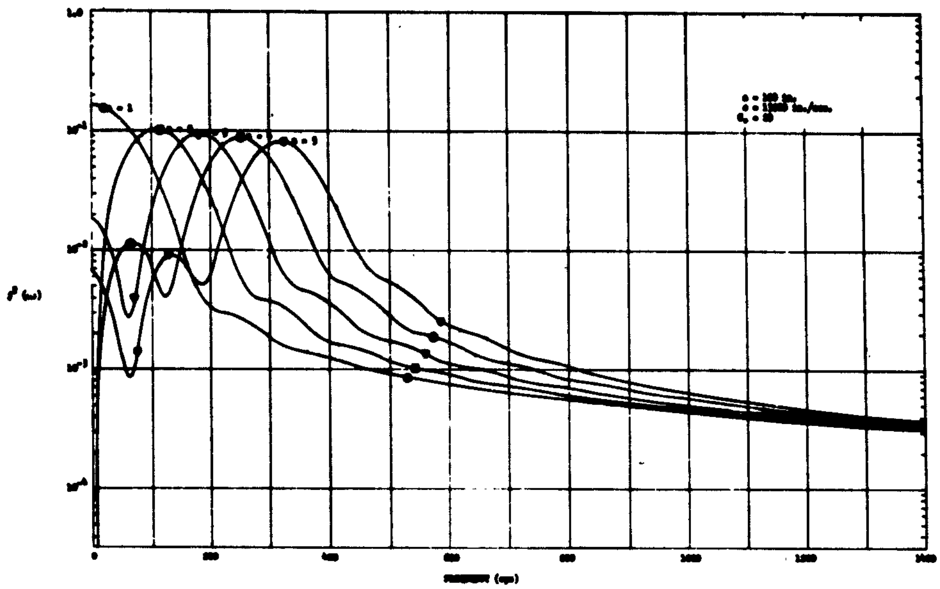

Fig. 4 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with all four sides simply-supported: (C_o = 20)

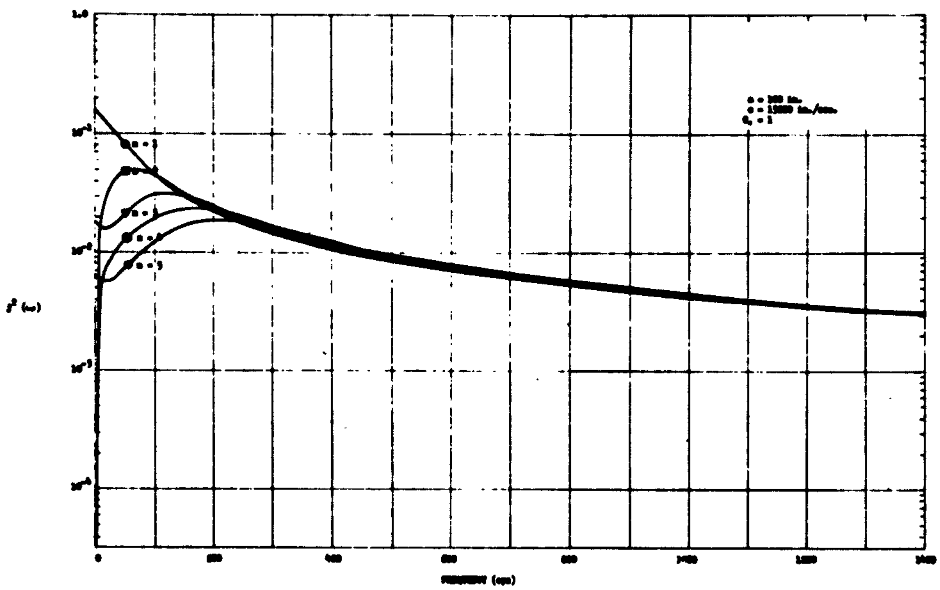

Fig. 5 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with all four sides simply-supported: (C_o = 1)

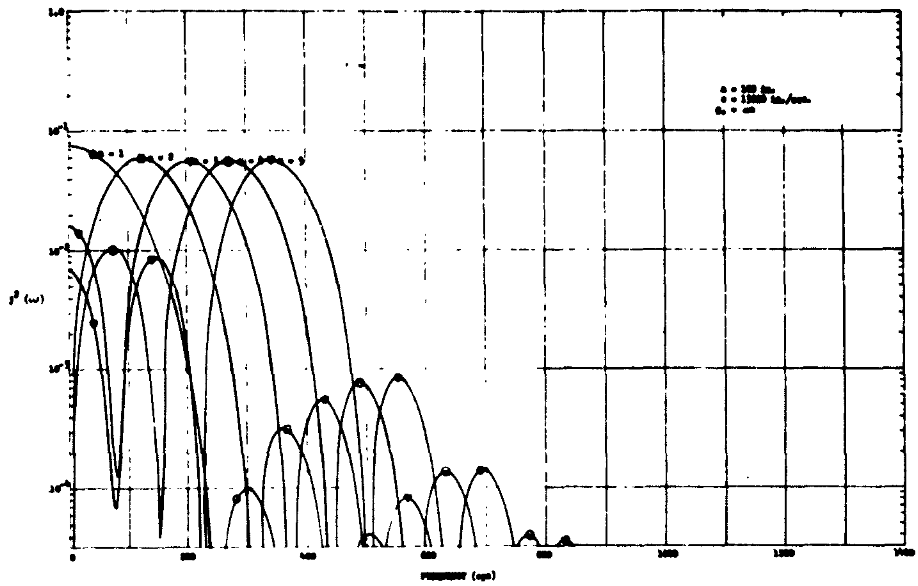

Fig. 6 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with all four sides clamped: ($C_o = \infty$)

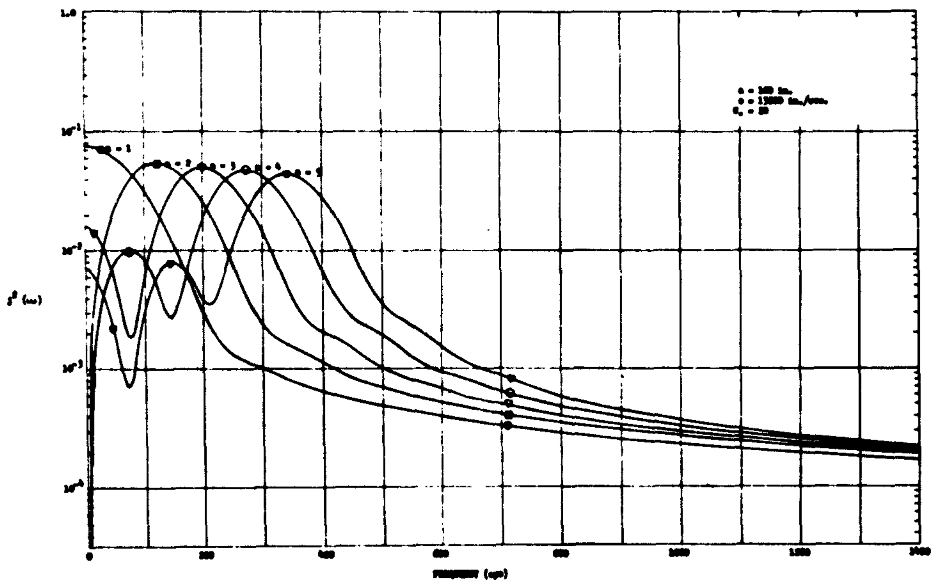

Fig. 7 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with all four sides clamped: ($C_o = 20$)

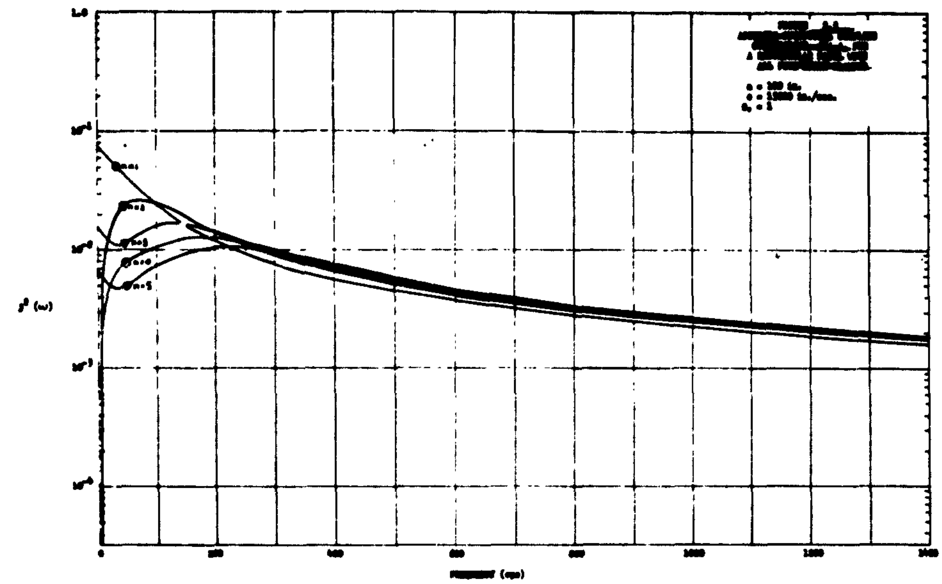

Fig. 8 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with all four sides clamped: (C_o = 1)

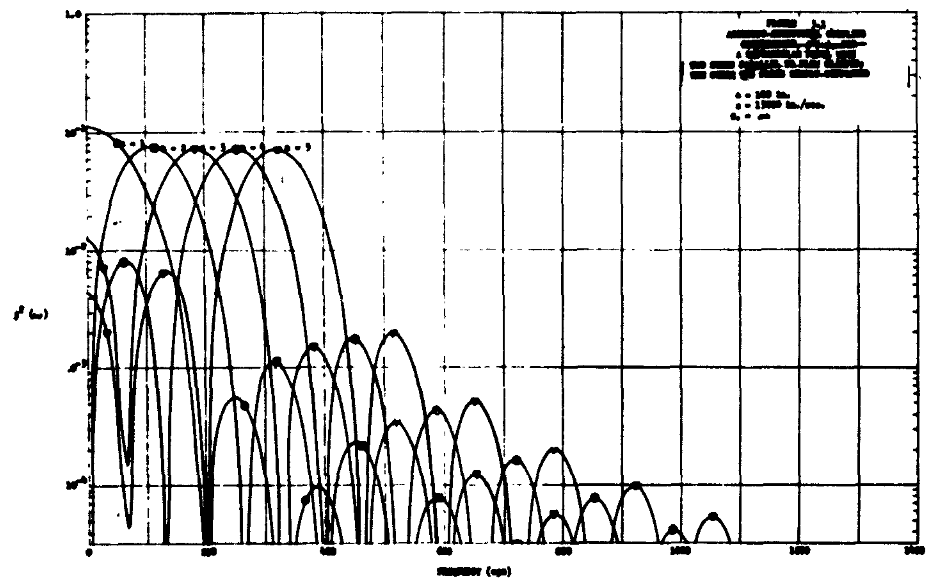

Fig. 9 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with the two sides parallel to flow clamped and the other two sides simply-supported: (C_o = ∞)

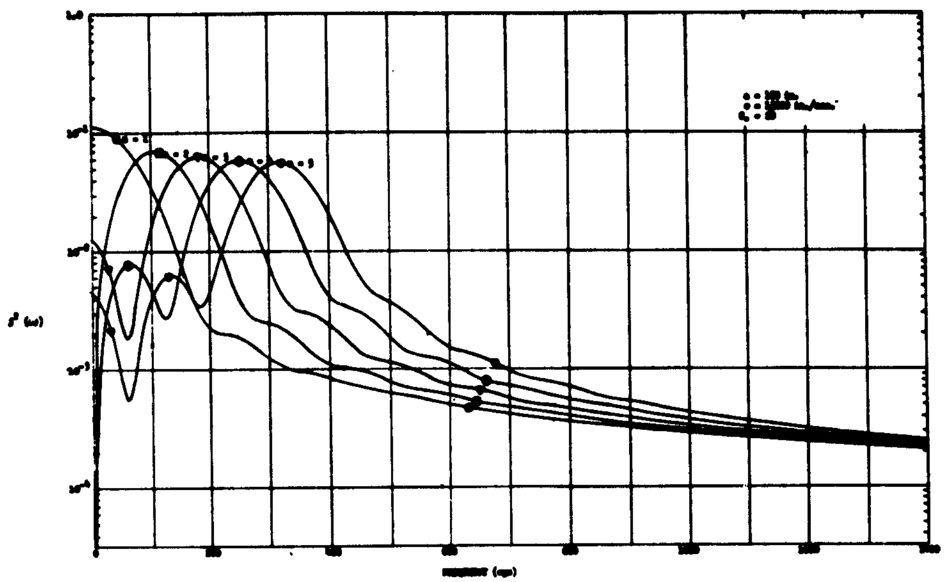

Fig. 10 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with the two sides parallel to flow clamped and the other two sides simply-supported: $(C_o = 20)$

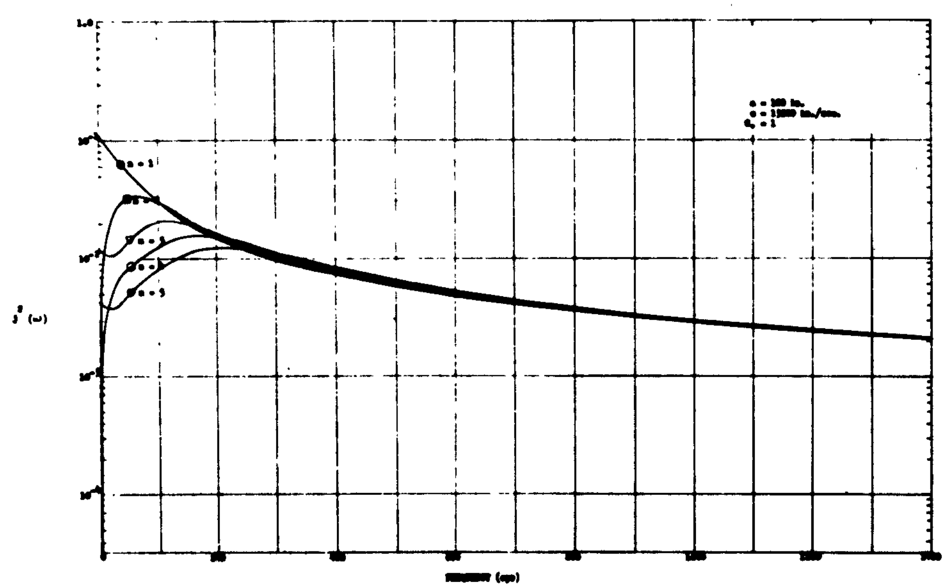

Fig. 11 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with the two sides parallel to flow clamped and the other two sides simply-supported: $(C_o = 1)$

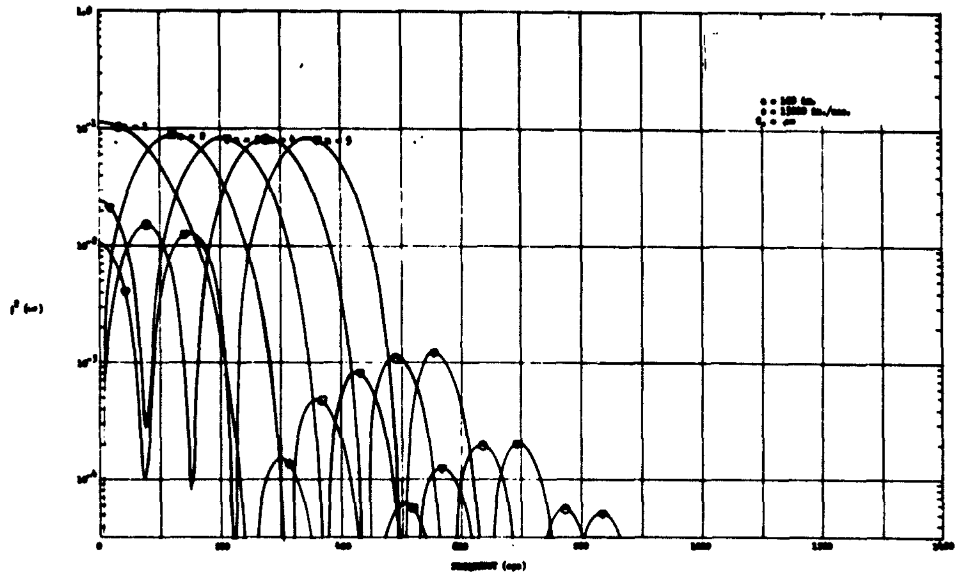

Fig. 12 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with the two sides parallel to flow simply-supported and the other two sides clamped: ($C_o = \infty$)

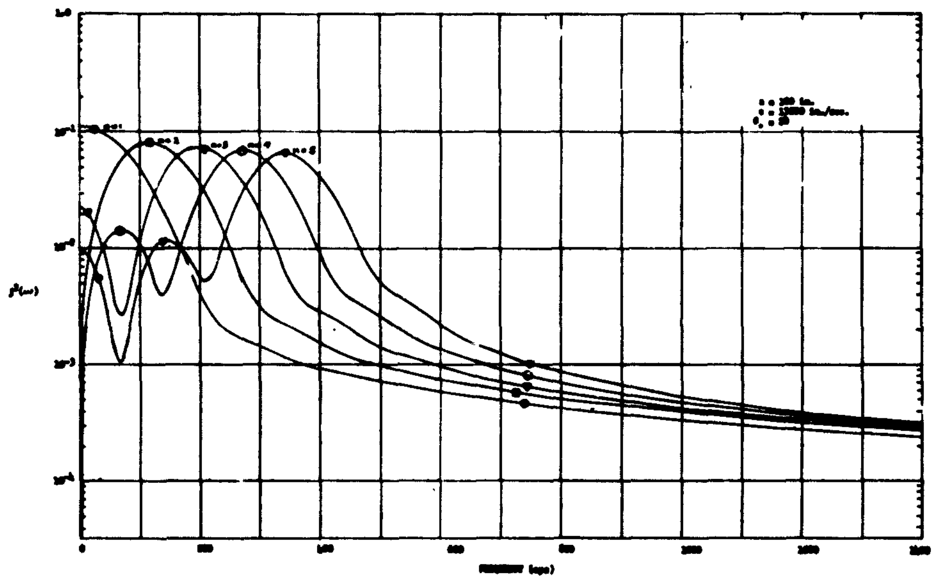

Fig. 13 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with the two sides parallel to flow simply-supported and the other two sides clamped: ($C_o = 20$)

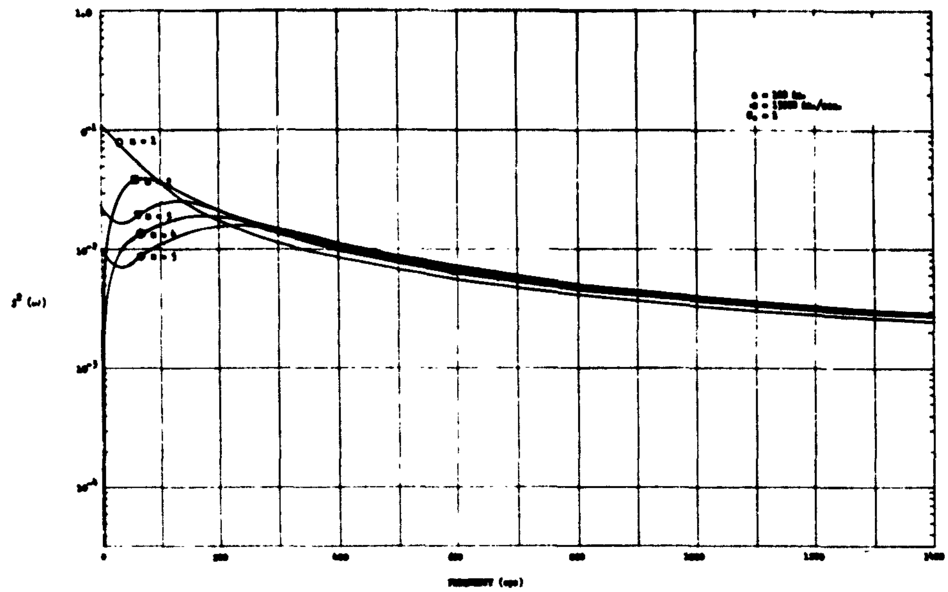

Fig. 14 - Acoustic-structural coupling coefficient, $j^2(\omega)$, frequency spectra for a 100-inch long rectangular panel with the two sides parallel to flow simply-supported and the other two sides clamped: (C_o = 1)

DISCUSSION OF RESULTS

Close inspection of Figs. 3 through 14 reveals a significant change in the $j^2(\omega)$ frequency spectrum with variation of the decay constant C_o. The trend indicates that as the value of C_o varies from $C_o = \infty$ to $C_o = 20$ to $C_o = 1$, the shape of the $j^2(\omega)$ frequency spectrum changes from a definite pattern of peaks and valleys to an almost continuous curve for each mode shape in the direction of flow.

The acoustic wavelength selectivity of the mode shapes, when $C_o = \infty$, is obvious in Figs. 3, 6, 9, and 12. The principal maximum peak occurs at wavelength coincidence, i.e., when the acoustic wavelength and the panel mode shape "wavelength" are in the region where they match. The rest of the spectrum consists of a series of harmonic and subharmonic peaks and valleys.

This definite pattern becomes smoother and more continuous as the spatial correlation decreases. Figures 4, 7, 10, and 13 illustrate this change, when $C_o = 20$. Note that the magnitude increases everywhere except in the coincidence region. Also, observe, that the curves seem to come togethr (coalesce) at higher acoustic frequencies.

The apparent coalescence observed here is realized in Figs. 5, 8, 11, and 14 for $C_o = 1$. The "coincidence" region has all but disappeared and an almost constant curve results for all mode shapes.

The maximum $j^2(\omega)$ frequency spectrum occurs for $C_o = 1$. As C_o decreases, the $j^2(\omega)$ spectra fall off rapidly in magnitude, though, keeping smooth shapes similar to those for $C_o = 1$.

The rectangular panel used for computation of the curves of Figs. 3 through 14 was 100 inches in length, specifically, to facilitate the use of the data for panels with similar boundary conditions, differing in length only. Multiply the frequency scale by the ratio of the reference length (in this case, $\ell_{ref.} = 100$ inches) to the new length, i.e.,

$$f' = \frac{\ell_{ref.}}{\ell'} \times f_{scale} = \frac{100}{\ell'} \times f_{scale} \text{ (cps)},$$

where,

ℓ' = new length (inch),

$\ell_{ref.}$ = 100 inches, and

f_{scale} = abscissa of Figs. 3 through 14.

CONCLUSIONS

Measurements have indicated that the near field of a source can be very uncorrelated. Hence, the need for an analytical approximation of the near field correlation characteristics resulted in Eq. (9). The far field is essentially free of perturbations and a cosine correlation distribution is considered a fairly good approximation. Exponential damping of the cosine spatial correlation function approximates the expected decrease in spatial correlation, with increasing separation between measurement points in the direction of flow in the near field.

The numerical results of the acoustic-structural coupling coefficient frequency spectra indicate a marked sensitivity to changes in the spatial correlation, a measure of the coherence of the force distribution over the panel. Consequently, as the spatial correlation decreases, the panel mode shapes lose their wavelength selectivity. Higher response occurs at all acoustic frequencies except at the coincidence frequency.

Equation (8) shows the dependence of the mean square response on $j^2(\omega)$. Therefore, this analytical work is of value in its ability to define numerical limits on the response of beams and panels with specified boundary conditions to acoustic plane wave pressure fields with varying degrees of spatial correlation.

Extrapolation to practical problems can be made within the limitations of the analysis. A better estimation of the order of magnitude can be obtained for the expected response.

The prediction of panel response can be accomplished with knowledge (either analytically or experimentally) of the power spectrum and spatial correlation distributions of the acoustic field and the panel mechanical impedances.

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* * *

Section 2

TRANSPORTATION ENVIRONMENTS

A COMPARISON OF SHOCK AND VIBRATION DATA FOR AIR, RAIL, SEA, AND HIGHWAY TRANSPORTATION

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An increasing freight loss and damage record, coupled with limited available shock and vibration criteria for cargo transported on existing equipment, has created a strong need for Department of the Army participation in this area. Scientific data needed is in addition to, and separate from, those data required for research and development considerations. A long-range program has been initiated and is now in progress to study the shock and vibration environments occasioned on cargoes, particularly Army sensitive cargoes, by all modes of transportation. Data are being collected concurrently for highway, rail, air, and ocean shipping.

The first efforts in the program have shown both some similarities and many basic differences among the modes of transportation as regards shock and vibration measured data. Perhaps the largest difference occurs with the time history involved in both the shocks and the vibrations. When the vibrations transmitted to the cargo by all modes of transportation are analyzed, it is seen that the basic measured frequency has varied from 1000 to 0.25 cps.

It was decided that some of the preliminary results, covering a sampling of the environment from each mode, should be studied prior to the pursuit of the remainder of the program. The purpose of this paper is to present the original findings in the program; to give a broad idea of the nature of the initial data, and to describe the general plan for the rest of the program. Also included are a general discussion of the instruments and data methods used in the program to date.

INTRODUCTION

All of us, at one time or another, have been in a position in which we have had to make a judicious selection regarding a mode of travel. Perhaps this has been a shipment of a heavy, delicate item, like a piano, or a light, delicate item, like a camera. If the choice of mode of shipment were ours, we, prior to decision, would ponder, in general terms, the shock and vibration encountered in railroad classification yards, the various highway pavement imperfections, and other mode-input shock and vibration factors that might damage our valuable possessions. The final choice for the selection of the mode of shipment would probably be made on the basis of our experience and personal knowledge, with no recourse to facts or scientific data. Even for most major military and commercial shipments made today, choices of shipping means are necessarily made in a similar manner, with little input from scientific data. There normally exist two or more methods of shipment for a given situation, but there does not exist a specific technical procedure for selecting the most economic, safe, and prompt means of shipment when the effect of shock and vibration imparted to the cargo is considered.

DISCUSSION

General

With the introduction of shock and vibration specialists and with the advances made in instrumentation and engineering methods during the last several years, it would appear that great strides would be evident in establishing efficient and effective transportation. Unfortunately, any payoff for our efforts has either been obscured or is nonexistent.

Table 1 illustrates the first point in the state of the art as it pertains to railroads and trucking. Table 1 presents the annual freight loss and damage costs for the years 1956 through 1960 and the damage causes, ratio, and cost. When water and air modes of transport are considered, these figures can be more than doubled. Note that 78 percent of the damage occurs enroute or during movement. The steady increase seems to be more than a reflection of cargo-cost increases. It should be noted that these costs do not include packaging costs, which are many times the damage costs. Packaging costs are not the costs of the cans or containers that contain, say, string beans, but they are the costs of the items that protect the cans. As long as these figures are continually increasing, it is difficult to see any practical payoff for many of our efforts in shock and vibration.

It would further appear that reasonably accurate predictions regarding shock and vibration environment for a given cargo and transportation mode would be routine at this time. It has been our experience that there are as many predicted environments for a given situation as there are experts or organizations requested to give this information. This environmental information is needed, and it is needed now.

For the last 10 years, shock and vibration engineers have complained strenuously about the vast amounts of field data being collected with crude instrumentation and naive scientific input. The effect of these criticisms appears to be that, at the present time, little or no shock and vibration data are being collected in the

TABLE 1
Loss and Damage Costs
For the Years 1956 - 1960

Freight Loss and Damage Costs			
Year	Railroads (Million $)	Trucking (Million $)	Total (Million $)
1960	119.9	18.6	138.5
1959	115.6	12.6	128.2
1958	114.1	16.5	130.6
1957	112.8	14.6	127.4
1956	113.9	10.5	123.4
Average (5 yr period)	115.6	14.6	130.2

Damage Causes - Ratio - Cost		
How Damaged	Percentage of Total Damage	Av. Yearly Cost
Enroute - Not Packaged	16	20.8
Packaged	62	80.7
Improper Handling	4	5.2
Accidents	6	7.8
Fire	2	2.6
Miscellaneous	10	13.1
	100	130.2

field for actual cargo shipments. It is certainly more logical to use more scientific methods, instrumentation, and mathematics on selected transportation problems and to try to correlate these findings with less complex methods that could be economically used to cover a full range of cargoes and transportation modes. We encourage more efforts in this field and, in particular, the collection of shock and vibration data for timely, up-to-date shipments of cargo. Highly technical methods of study would be preferred, but even current data with so called crude methods would be useful.

Presently, the transportation engineering environmental problem is being pursued by individuals, research institutes, manufacturers, and various agencies. Each has its own interpretation of the problem. The costs of pursuit of a problem in this manner are too great. Perhaps the scarcity of available environmental data is due to this shotgun approach.

We in the Office of the Chief of Transportation have recognized this lack-of-coordination problem. Several transportation engineering programs have been set up that will encourage joint studies and coordinated effort by research institutes, industries, and Government agencies to assure maximum use of data and interchange of information. These programs are intended primarily to guarantee logistic support adequate to deliver items to the intended area that will then function with a high degree of reliability. Coordination and joint projects have been established with AEC, Sandia Corporation, Ordnance Corps, Bureau of Explosives, National Academy of Science, AE&SW Board at Fort Bragg, and the Transportation Corps. Although considerable work has been done and additional work is planned and in progress, it is still necessary to broaden the scope of the work to include appropriate research institutes, industry, and industrial associations, as well as all interested Government agencies.

Our ultimate goal is to prepare a manual or a method to be used as a logistic tool for assigning values to cargo environment. Shock and vibration criteria are perhaps the most difficult factors to work into effective logistics. Costs and speeds are much more consistent and predictable than the shock and vibration to which the cargo is subjected. It is realized that considerable background work has been conducted separately for rail, sea, air, and highway methods of travel. When a shipment involves choice of any of these modes, or a combination of modes, it is necessary that the data be across-the-board so that no prejudice by the selector is involved because of his specialized background. Also of importance is the fact that the accuracy of evaluating shock and vibration by all modes must be comparable, or at least relatable, so that no mode is penalized by inaccuracy or naive testing.

Cost is normally the primary consideration for most commercial items. A considerable portion of the work to date in this field has been performed with shipping cost as the controlling factor. For normal shipments by trucks or other standard equipment, cost is perhaps still the major factor determining the way in which the item is to be shipped. For a shipment of defense sensitive cargo, it is easily seen that shock and vibration and their potential damaging results far outweigh any cost considerations. This, of course, does not mean that the items should be handcarried on soft pillows, but it does mean that shock and vibration considerations should have first priority in logistic transportation of sensitive items.

For sensitive cargoes, the need for accuracy and safety in transportation is greater than for any previously known cargo. The need for safety is apparent. The need for accuracy or for small safety factors is inherent in military usage, to assure good mobility and transportability.

Efforts are currently in progress to study shock and vibration of the cargo for all modes of travel. Sample tests and evaluations have been conducted to date by each mode. A quick review of the preliminary test methods, instruments, sample results, and differences will be given.

Rail

A brief background on the Army Warhead Section Transporter System is given to illustrate the degree of importance that the Department of the Army has placed on ensuring the safe functioning movement of cargo for logistic support.

Previously, sensitive items had been shipped in standard boxcars, which have a general mechanical arrangement such as that shown in the lower portion of Fig. 1. The cushioning in these commercial cars was designed for full load (say, 100,000 pounds) or, at least, average load (say, 60,000 pounds). The Army items are relatively light, and the mechanical performance of the car for loads of this type is more nearly that of an empty car. Obviously, the shocks for the sensitive items have to be low, and the nature of the light masses and light cars are

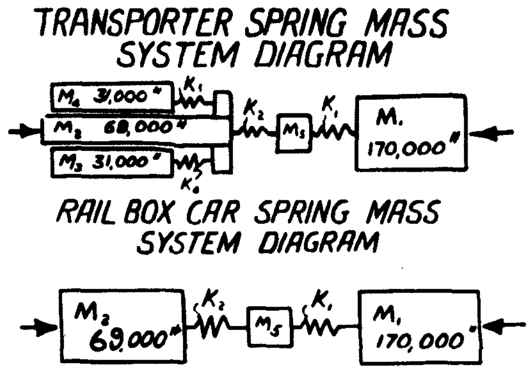

Fig. 1. Cushioning comparison

conducive to high shock loading. The cushions shown as K_1 and K_2 are limited by regulation in size and travel, and are quite stiff, since they are designed for normal railroad usage.

It became necessary to cushion these relatively light items in a heavy mass system, as shown in the upper half of Fig. 1. Cushions K_1 and K_2 are, again, standard railroad draft gears to allow for proper train operation and to fit in with the mass requirements of normally loaded cars. K_3 and K_4 are independent cushions that do not directly affect the train action and that can be designed, as to travel and spring rate, to accommodate light cargoes. The main cushion units can and will be improved to obtain optimum performance, but the system is such that cushion adjustments, or perhaps replacement, can be made without affecting the rest of the system.

The resulting arrangement, the rail car with vans affixed, is shown in Fig. 2. The main cushions are located on the car at the center of each van. Inherent in this transporter system is a logistic requirement to minimize handling of the items during logistic movements. The system is designed so that the vans may be removed and fixed to a bogie to form a trailer unit with a highway vehicle, as shown in Fig. 3. This transfer is made without auxiliary equipment. A transfer of the vans to any mode of transportation is made with crane facilities. The reduction in the amount of handling and reloading of items represents a vast logistic improvement, since the amount of inspecting and functional checking required is consequently reduced.

As soon as the van becomes a highway vehicle, the shock and vibration criteria for the transporter system are extended to the highway mode. Field studies were conducted for this mode of transport, and necessary regulatory approvals were obtained; however, these are not included in the scope of this presentation.

Whereas it became necessary to design the cushioning and mass system specifically for the light mass systems, the tiedown arrangement (as shown in Fig. 4) for securing the items to the vans was designed for universal usage. Five or six Army items with very different geometries have been secured and tested satisfactorily with various arrangements of the tiedown

Fig. 2. Army warhead transporter system in rail arrangement

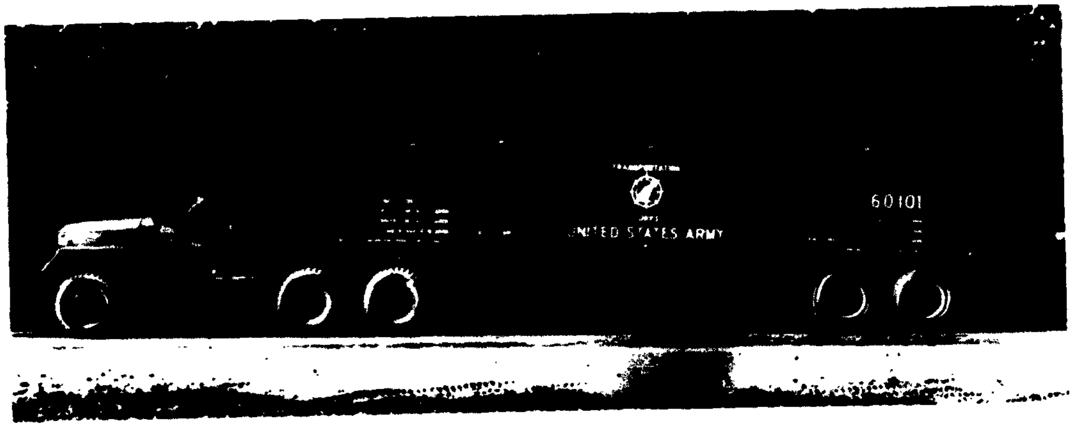

Fig. 3. Army warhead transporter system in highway arrangement

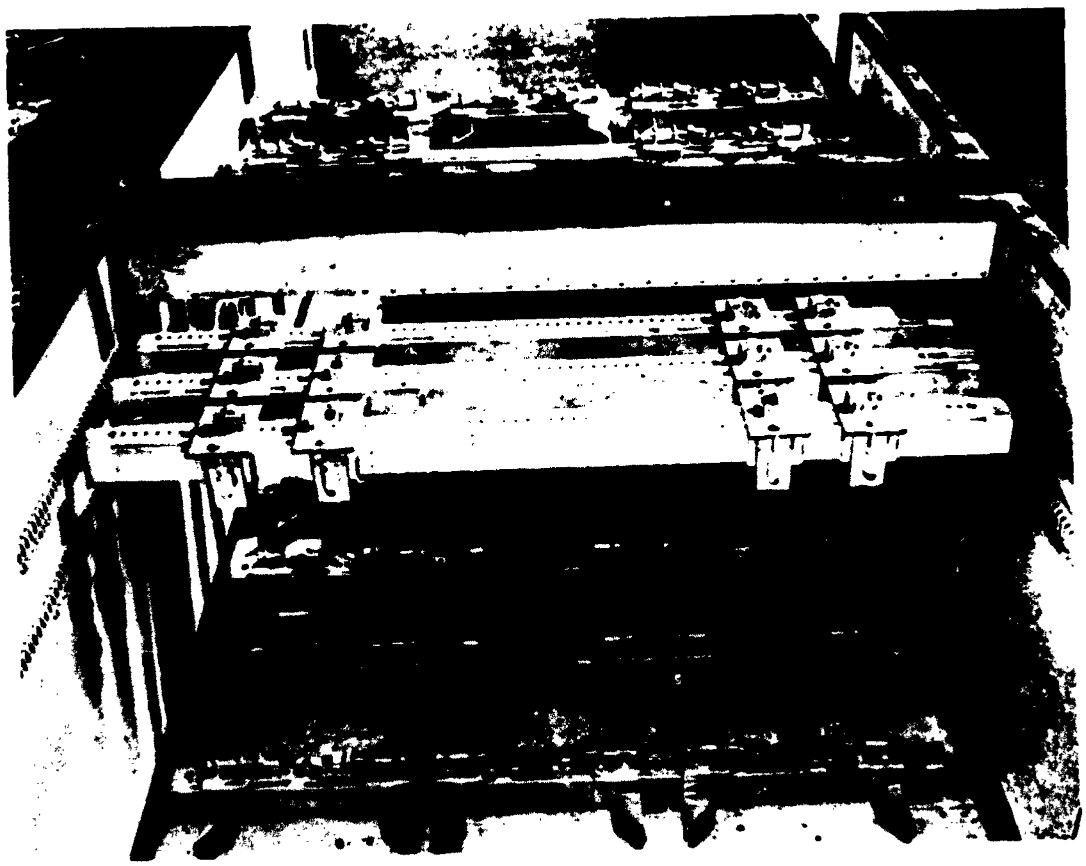

Fig. 4. Transporter van, interior view

components shown. It is ironical that the improvement in the tiedown system came at the same time as the removable-van method, which reduced the required number of times that the tiedown system would be used.

For the rail portion of the test program, initial efforts were devoted to impact tests of the Army Warhead System car and other cars. (Evaluations of shock and vibration for cargoes and rail car during transit are planned for the program but are scheduled for a later date.) The impact tests simulated shock loadings imparted to cargo as a result of flat or automatic switching in rail yards. The test procedure involved impacting, at various speeds, a fully loaded car into a stationary test car loaded with sensitive cargoes. Additional limited testing was conducted with the test car backed up with several loaded cars, as well as with the test car being used as the impacting car. The data reported here cover the test wherein the test car was free to roll. The test procedure very closely simulated other rail impact testing work that had been conducted previously both by industry and by the military. As noted before, the major factor that necessitated the test was the extremely light weight of the cargo and the urgent need for the shock and vibration criteria.

Previous rail impact testing conducted by Government and industry has been conducted primarily with fully loaded test cars for the purpose of applying the most severe loads on the car structure and cushioning units. Because of the hazardous nature of the sensitive cargoes, safety regulations precluded loading the car to its maximum weight capacity or cube capacity. The tests more nearly simulated an empty or lightly loaded car, with emphasis on the small quantity of cargo in the car. The sensitive-cargo tests resulted in different shock and vibration values, higher accelerations, higher frequencies, and, in general, less predictable data. Instrumentation used in the impact tests included SR-4 strain gages, unbonded accelerometers, and potentiometer-type displacement gages. All recording of the transducer outputs was made with Consolidated Engineering Corporation's recording oscillographs. The frequency response of this system was limited during these tests by the oscillograph galvanometers, which were flat from 0 to 160 cps.

The data reduction for the rail test program was performed manually. Trace deflections were measured with a scale, and the data were transferred to graphs or tables by hand. Auxiliary analysis, principally for energy and force studies, was performed on a digital computer. In this instance, the data were typed on the computer cards manually.

The results of the rail tests showed that both the lateral and the vertical accelerations become very important design factors because of their relatively high magnitude. Figure 5 illustrates some of the measured accelerations plotted against impact speed. It should be noted that the input load is transmitted through several cushioning units on its way to the cargo. As shown in the illustration, the acceleration reductions were accompanied by pulse-time increases. In other data, the reduction in g forces, as a result of the cushioning units, is not as immediately apparent as shown in the illustration. The shock forces were often changed to a much higher frequency, since the shock was transmitted through the mechanical system with anywhere from a slightly increased to a noticeably increased acceleration reading, but always with an accompanying higher frequency. In working with relatively heavy masses or cargoes, the g reduction or effectiveness of the cushioning unit is readily apparent (as a direct reduction in force or acceleration level), because all the values possess nearly the same frequency, and usually the predominant values are of low frequency. It is realized that if the response of the recording instruments had been higher, higher frequency shock would have been detected. The instrumentation was well within the frequency range of the main structural members, as verified by strain gage data.

Figure 6 is a plot of the sample oscillograph record and is presented to show the relative shape or waveform. Throughout the test, the bulk of the data traces had approximately the same shape as the one shown, with increasing amplitude accompanying increasing load. The maximum longitudinal acceleration occurred nearly simultaneously with the peak coupler force, with the vertical and lateral maximums occurring many milliseconds later.

Air

The tests to check the shock and vibration environment on cargoes in military aircraft were conducted as part of a larger program. This basic program was designed to check the strength of tiedown equipment for securing various sensitive cargoes in Army aircraft. For these tests, a full range of Army aircraft, both rotary and fixed wing, were used. The test procedure was to fly each aircraft with a particular cargo through a series of standard maneuvers. All items were flown with all aircraft, except in those cases where the combined weight of the items and instrumentation was too heavy for the plane.

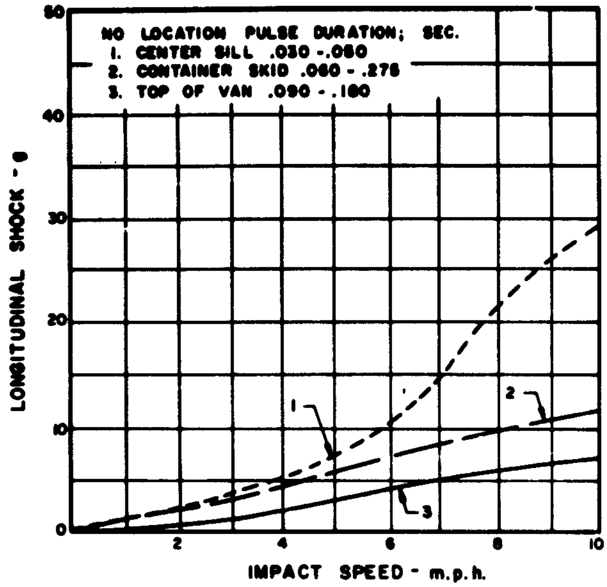

Fig. 5. Rail impact, Army warhead transporter

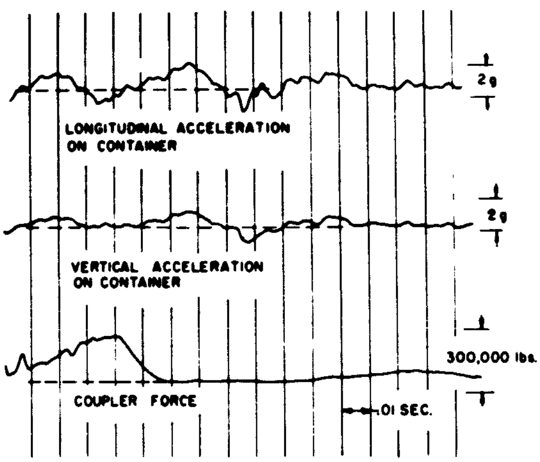

Fig. 6. Rail impact oscillograph, 6.05-mph Army warhead transporter

The shock and vibration instrumentation was activated for several seconds during each maneuver. Instrumentation for the test consisted of unbonded strain-gage-type accelerometers and CEC recording oscillographs. Various other types of instrumentation, including mechanical statistical-type accelerometers, were used as secondary methods, when the weight of the item precluded the use of oscillograph recording. The size and weight of the required instrumentation offered limitations for many of the aircraft. In several instances, many channels had to be deleted for aircraft carrying near-capacity load.

The resulting data were reduced manually. The oscillograph records were visually inspected to select an area that contained a high acceleration level vibration, and, in most cases, this area contained the peak acceleration reading for the period. The amplitude of this vibration was measured and tabulated for approximately 50 cycles. These points were used to form the acceleration versus percent occurrence curve, a sample of which is shown in Fig. 7. If any shocks were superimposed upon the vibration, which was normally the case, the amplitude of the shock and the pulse-time were recorded. In most instances, two separate frequencies of vibration were noted. These vibrations were separated manually and analyzed individually.

For the aircraft tests, the vibration measured from cargo to cargo and aircraft to aircraft did not group in any particular frequency range. Vibrations were found at frequencies ranging from 3 to 300 cps, the limitation on the high range being the response of the recording equipment. It is currently planned to do additional work in the higher frequency range with automatic recording and data analysis equipment. A sample of the oscillograph record is shown in Fig. 8. For most of the records taken,

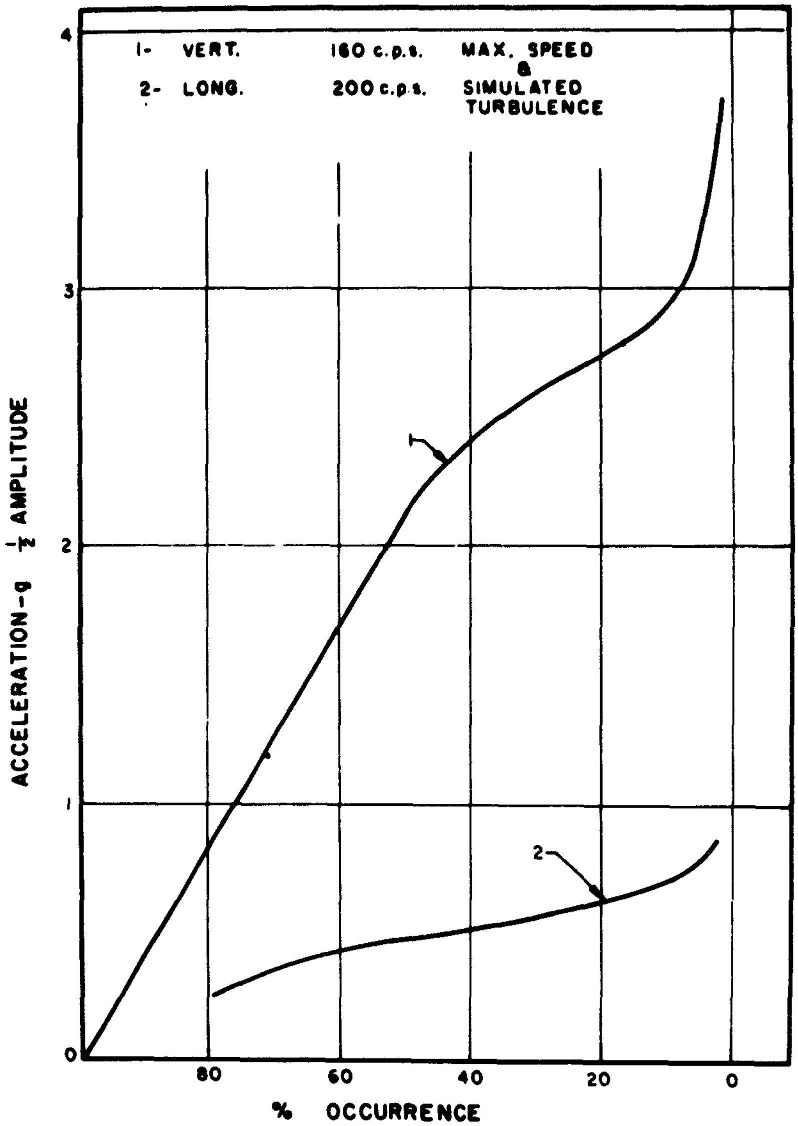

Fig. 7. Caribou aircraft vibration

the vibration was reasonably consistent throughout the recording period. The highest shocks recorded on the cargo to date were experienced on a cargo slung under a helicopter at the time of touchdown and release. These values are not available for publication.

The maximum vibration environments found during the initial portion of the testing included vertical accelerations of 4 g, 160 to 240 cps, on which was superimposed a 0.9 g at 45 to 60 cps. Lateral and longitudinal readings were all under 1 g, within a frequency range of 45 to 250 cps. None of the maximum accelerations continued at a high acceleration level for more than a few seconds and were usually associated with a particular portion of a flight maneuver.

Sea

As with the tests conducted on aircraft, the shock and vibration tests on ocean-going vessels were also conducted as an adjunct to an existing program. The basic test program was designed to measure bending moment stresses in the hulls

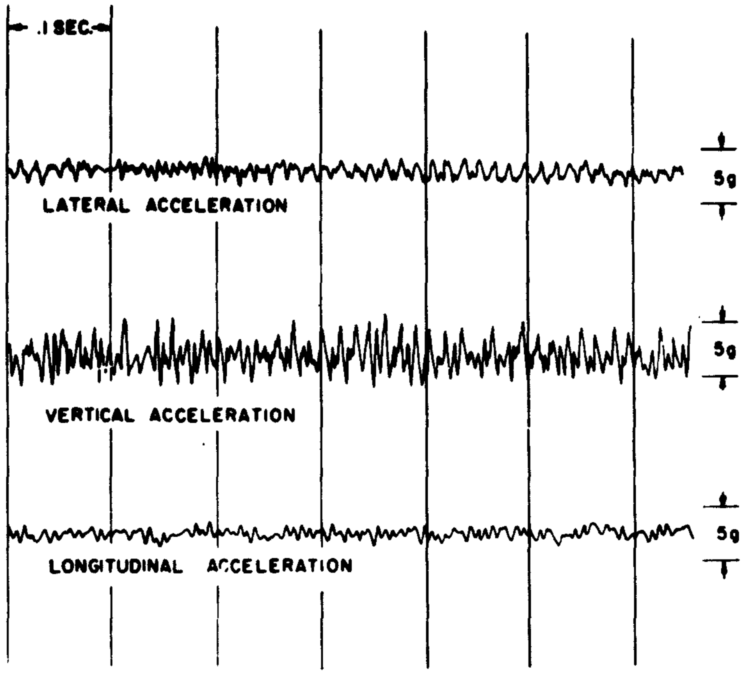

Fig. 8. Oscillograph record, Caribou aircraft

of cargo ships during actual sea operation. Additional accelerometer equipment was added to the existing program by the Army so that the environment of the cargo could be studied in conjunction with the ship's structure. To date, several voyages have been conducted with the instrumented cargo ship in the North Atlantic. The routes selected are the roughest commercial trade routes.

The data presented here are from a voyage from Rotterdam to New York in January 1962. These data were for the most severe portion of the most severe voyage to date.

In this study, the basic instrumentation consisted of accelerometers (unbonded strain-gage type) used in conjunction with a magnetic tape recording system. The recording equipment was designed especially for these tests to allow for the recording of extremely slow vibrations over long time intervals. The recording system was designed to be active for 30 minutes and inactive for 3 hours 30 minutes, this cycle running continuously. Also included in the design was a built-in device to trigger activation of the instrument whenever a preset stress level in the ship hull, as measured by strain-gages, was reached. This made the instruments active continuously throughout the worst periods of ship vibration.

The recording system was frequency modulated and had a flat response from 0 to 60 cps at the tape speed used during testing Initial data were reduced manually. The magnetic tapes were played into a recording oscillograph; frequency and acceleration levels were measured from these records. Additional work in this program will be performed with an analogue-type data reduction machine. This will, for a given data interval, give such values as a complete plot of the histogram, average values, root-mean-squared values, and the peak reading. It is anticipated that the analysis range can be extended to the entire 1/2-hour recording interval to facilitate analysis of all data recorded.

Sample results of the work performed with manual analysis are shown in Figs. 9 and 10, which give the distribution of the vibration amplitudes and a sample time plot. A point of particular interest with these data is the extremely low frequency of the principal vibration. Most of the data investigated in this program to date give the frequency of approximately 0.25 cps, or 4 seconds to complete one cycle. This vibration is occasioned by the general wave action on the ship. The highest acceleration reading encountered for the initial instrumented voyage was 0.7 g. Two important factors to be considered in weighing these relatively low-amplitude

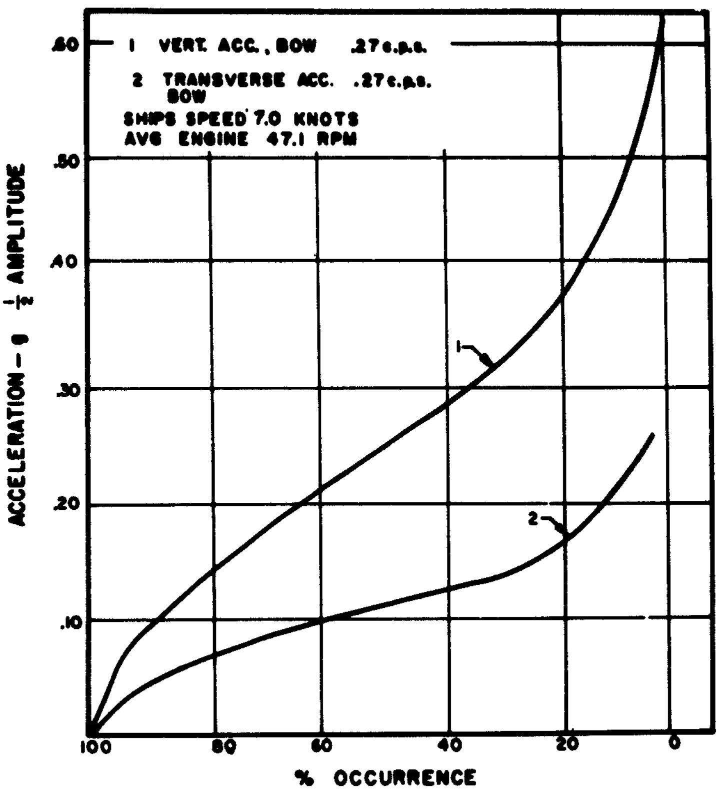

Fig. 9. Rough seas acceleration in bow
(Run 171 West)

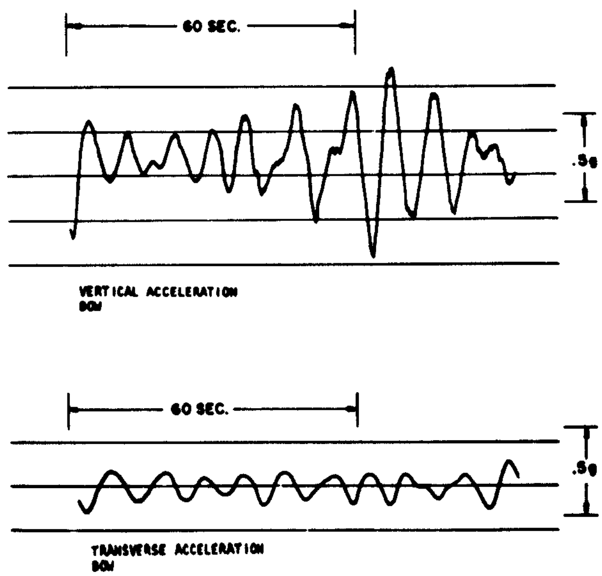

Fig. 10. Oscillograph record, sea voyage
(Run 170 West)

data are that the damage to cargo is substantial by this mode of transportation, and that the static force on the item will be multiplied many times due to stacking of cargoes. The 0.7 g reading mentioned does not seem to be severe unless one considers that the acceleration reading would be the same for a bottom container with 10 more stacked on top of it. This situation would result in an equivalent static acceleration of 7 g on the bottom container for a considerable period of time.

In the sample vibration trace, there exists a higher low-frequency amplitude superimposed on the principal 0.25 cps vibration. This is referred to as the slamming acceleration and is not directly related timewise to the basic ocean wave action. Complete analysis has not been made of this 10-cps vibration, but, in at least one instance, it was found to be over 1 g in severity.

Highway

The sampling used here for the highway vehicle portion of the test program was taken from a test in which various military test vehicles were run over a relatively short test course. The most severe test course, as regards shock and vibration, was a 300-foot long log road at Camp Wallace, Virginia. Tests over many other types of roads were also conducted, including runs on paved surfaces in both very good and very poor condition, over railroad crossings, and on gravel roads. The first tests in this program were conducted with instrumentation consisting of unbonded strain-gage accelerometers and CEC recording oscillographs. The relatively short recording time for these instruments has necessitated tests over carefully designed and short test roads.

More recently, work has been conducted with statistical-type accelerometers, both powered and nonpowered. Some tests have been conducted with a digital tape recording system, which is to be used in conjunction with digital computer data analysis. This system, due to its relatively short recording time, again limits the records to short periods of time. Figure 11 is a plot of some of the accelerometer data recorded with the oscillograph. These data were published last year, but they are included again since they still contain the highest acceleration levels, recorded to date, for low-type roads. A sample oscillograph trace, giving the general wave shape and nature of the vibration, is shown in Fig. 12. It should be noted that the frequency range of the recording equipment was from 0 to 90 cps. Complete coverage of shocks for these tests was not possible, since test records were limited to several seconds. For the data analyzed, the vibration portion contributed the highest acceleration readings.

EVALUATION

It can be seen that a considerable number of instruments, reduction methods, and test procedures have been used for this program. The instruments included recording oscillographs, digital tape, magnetic tape (both direct and frequency modulated), and both mechanical and powered statistical accelerometers. It should be noted that none of the various instrument arrangements had the same sensitivity, frequency response, length of recording, size, capacity, cost, or availability. Almost all of the items tested required different instrument arrangements. Data reduction methods ranging from manual computation to analogue and digital computers were employed. In all cases, the instruments and data reduction methods were selected to do a specific job. With frequency requirements ranging from 0.25 to 1000 cps, recording times ranging from 3 seconds to 4 hours, and instrument weight ranging from 1 to 1000 pounds, it was necessary to employ the wide variation of methods.

Engineers active on the program are getting an understanding of the correlation of shocks and vibrations among all modes of transportation. The necessary data are being collected, but the procedure is complex and requires a great variety of instrumentation and data reduction methods. It is hoped that, after careful study, both the instruments and the methods can be simplified so that the findings can be put to use by operating personnel. An important consideration in this type of work is the cost of testing. We have made every effort in this program to combine our shock and vibration tests with existing programs to reduce costs. It would be highly impractical to sail ocean liners around the world for no other reason than to measure several accelerations.

The program, to date, has been conducted almost exclusively with extremely light cargo masses. In the Army, there are many movements similar to commercial movements wherein weight is a considerable factor and the cargoes are quite massive. The program scope will be broadened to include work with heavy cargo masses.

The major control on shock and vibration in transportation is through the design and adjustment of cushion units. For cargoes

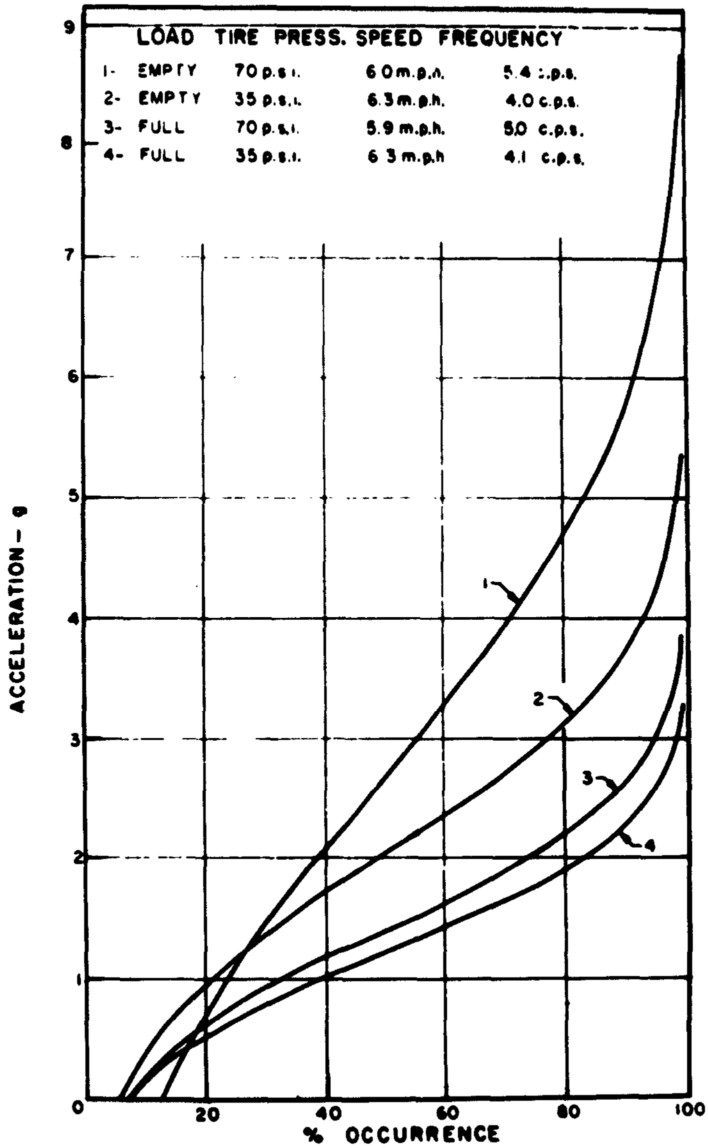

Fig. 11. Highway vehicle vibration, log road

transported by all modes, the cushion units have many requirements. Most present cushioning devices would perform poorly in ocean shipping, since these devices are dependent on velocity for operation and the input of 0.25 cps is normally considered in the static range. Some units will have to be designed for use by all modes of transportation. With the illustrated acceleration-frequency variations, the design may involve more than one cushion unit for multimode use.

It is realized that many will gain, both Government and industry, from this type of program. It is further realized that to obtain maximum usage within a reasonable time frame, many will have to participate in a program of this type involving all modes of transportation. It is planned that a single set of methodology will result. This information will be published in one cover and made available to both the Department of Defense and industry.

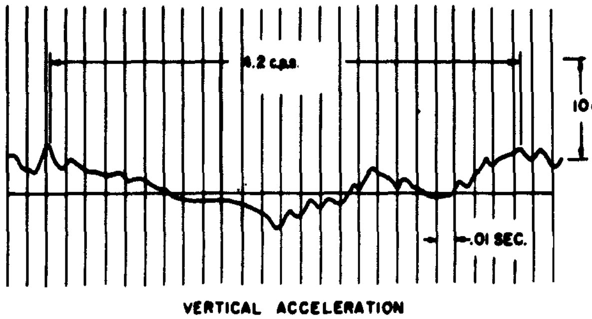

Fig. 12. Highway vehicle oscillograph record (6 mph, log road)

CONCLUSIONS

Time history differences encountered in vibration by various modes of transportation can now be measured and analyzed successfully by employing a wide variety of instrumentation and data reduction equipment. It is hoped that the testing and evaluating procedure can be simplified so that more items can be studied economically.

It appears that a manual or a method could be developed to be used in the scientific selection of the most efficient mode, or combination of modes, of transport for items with consistent and accurate safety assurance. It is important that the end results be a product of many Government agencies and industry and that this work be published under one cover and available to all who need the information.

* * *

SHOCK AND VIBRATION ON RAILROAD MOVEMENT OF FREIGHT

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Shock and vibration data in three planes during rail shipment are presented in parameters suitable for package design for rail shipment and in terms familiar to the packaging engineer rather than the car designer. The analysis is based on three fundamental curves, some empirical data, and thousands of impact measurements.

INTRODUCTION

One of the more pressing problems concerning rail movement of freight is the cost of properly securing lading to the car and protecting it from impact shock. The highway truck has an advantage in this regard because friction between rubber tires and pavement govern the maximum longitudinal forces developed, except for dock impacts. Rail, however, has the advantage in vertical and lateral shock and vibration control, and, with proper design and operation, can meet the truck's longitudinal conditions.

Adverse criticism has been received by the railroads because of their past reluctance to take the necessary steps to reduce lading damage. We now have the hardware to provide this protection; the rail industry, as a whole, is rapidly applying this hardware to its more critical loads.

The purpose of this paper is to acquaint the shipper with the railroad environment and with the characteristics of the equipment available for controlling it.

INSTRUMENTATION

First, a few comments about instrumentation. The instrument in general use on the railroads to measure shock due to longitudinal impact, is the impact register or ride recorder. This device consists of a mass restricted from movement in all directions except longitudinal, where its motion is restrained by a spring. A stylus attached to this mass marks its movement on a proper chart fed continuously by a clock mechanism. The chart is divided into zones which are interpreted in terms of speed of impact for average conditions in accordance with Table 1.

TABLE 1
Ride Recorder Interpretation

Zone (Movement)	Remarks (Handling)	Approximate Speed (mph)
to Zone 2	Normal	4
to end Zone 2	Borderline	5
to Zone 3	Rough	6
to end Zone 3	Rough	7
to Zone 4	Rough	8
to end Zone 4	Severe Rough	9
to Zone 5	Severe Rough	10
to end Zone 5	Severe Rough	11

These registers are dependable, rugged, and have background relationships between their readings and actual car lading damage. They are limited by a resonant frequency of about 10 cps and can indicate only the shock envelope. They do not measure force or acceleration, but for a rough estimate of acceleration, measure the movement of the stylus in 32nds of inches, divide by 4, and use the resulting value as acceleration (g).

In order to correlate impact recorder readings with actual forces, high-frequency accelerometers and both magnetic tape and oscillographic recording equipment have been provided. This equipment, together with power supplies and associated electronics is housed in a railroad car equipped with living facilities so that multichannel shock, strain, speed, and other information may be recorded continuously on road tests or stationary impact tests.

It is, of course, the actual g forces which are of interest to the packaging engineer; it is these forces which will be discussed in the balance of the paper. It is well to point out the basic fact that the coefficient of friction between two bodies is the g force necessary to slide one body on the other. This is important because for most car-to-package surfaces, this coefficient is less than one; a package, unless otherwise restrained, will move at car floor accelerations of less than 1 g.

DISCUSSION ON SHOCK AND VIBRATION

There are three principal forces affecting damage to cars and lading in railroad shipment.

Lateral

Lateral forces are due to side play between wheels and rails, lateral clearances in trucks, rocking of the car or slack in lading. Damage resulting from lateral motion is almost nonexistent on properly packaged, restrained commodities.

Vertical

Vertical forces are due to track or wheel irregularities, improper springing, and excessive road and impact speeds. With properly snubbed, long travel, truck springs in modern trucks on average rails, very little damage arises from vertical vibration. The forces, generally, are less than 1 g at frequencies of 2 or 3 cps in loaded cars, as indicated in Curve 4 (Fig. 1). The average continuous acceleration is about 1/4 g. Shock excited vibration from irregular wheel conditions, worn rail joints, and highway and rail crossings increases the g factor, but, generally speaking, this is not a serious source of damage. The trend toward the use of high-speed trucks and welded rail will improve the present condition.

Vertical shock due to longitudinal car impact is more serious. Here the lading is actually raised from the car floor and on its return sustains a much higher g loading than the car itself. Tests have shown cars receiving 4-g, and lading 9-g, vertical accelerations during severe impact. The load is moved from its as-packed condition and is prone to further damage from otherwise light shocks and vibrations. The time duration of these peak shock forces is generally less than 0.01 second.

Longitudinal

Longitudinal vibration damage is similar to lateral and can be considered of no significance.

Longitudinal shock damage due to slack run in and out is not serious in a properly loaded car. On some railroads where the roadbed is undulating and the train is long, these shocks can disarrange the lading in a poorly loaded car. These shocks can be reduced by use of Type F (Tightlock) couplers,* reducing train speed or length, and by smoothing out the roadbed profile.

Longitudinal shocks, due to car switching in flat or hump yards, are the most serious sources of lading and car damage.

With the present car and shock mitigating equipment in general use, switching speeds must not exceed 5 mph to prevent lading damage on secured loads, and 3 mph for loads shipped unsecured. The damage impulse increases as the square of the speed, a 7-mph impact is almost twice as damaging as an impact at 5 mph.

The energy contained in moving equipment is proportional to its mass and to the square of its speed, and this energy must all be accounted for when impact occurs. Some energy is absorbed in the draft gears, some in the car structures, some in the lading, and some is left in the movement of the cars after the couplers part. Uncontrolled energy absorption in the car structure means bending or shifting of car parts with respect to one another. Uncontrolled energy absorbed in the lading results in direct damage.

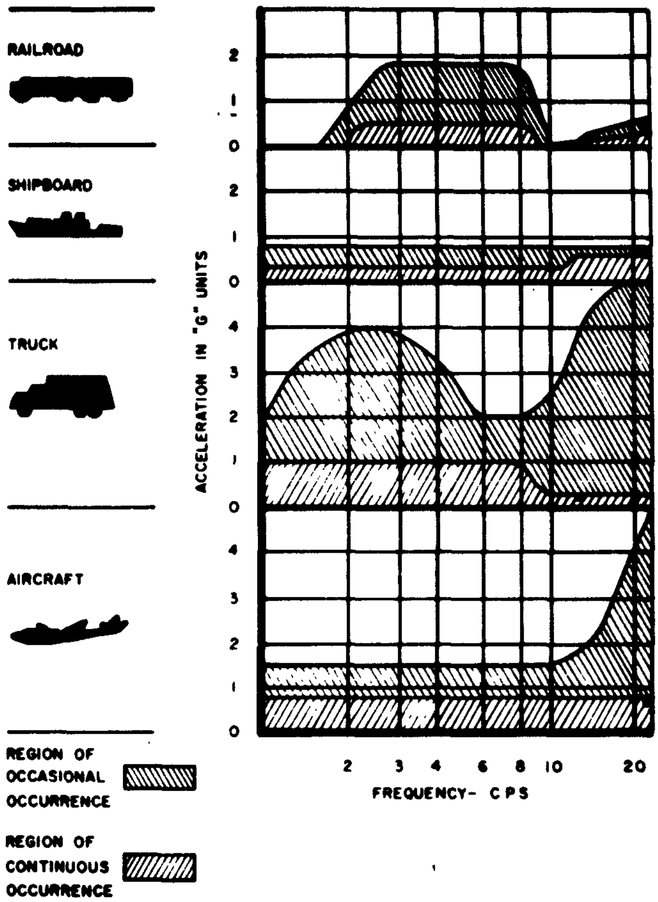

Fig. 1 - Curve 4: Transportation vertical vibration spectra

The railroad's approach to the prevention of impact damage has been to strengthen car bodies and to compartmentize loads. This is a commendable system in some respects, but note the ultimate result. The beef-up car with the compartmentized load, on striking conventional equipment, does not absorb energy in the car or load so the shock energy goes into the struck equipment. Thus, it becomes necessary to make all loads and structures rigid. When this has been done, the end result is a system poorer than the starting system. An expedient solution for a few cars is not the answer; there must be energy absorption in each unit and not an energy shift from one car to another.

Improvements in the capacity and travel of draft gears are constantly being made. The AAR minimum requirement of 18,000-foot-pound capacity for 24-5/8-inch gear pockets is exceeded by two and three times with modern gears, but at the expense of the closure force. In the 36-inch gear pocket, still higher capacities are possible. The cushion tube, which uses both gears in impact, more than doubles the gear capacity by increasing the closure distance and also divides the reaction between both ends of the car. This is a highly effective and economical scheme. Hydraulic draft gears and sliding center sills have considerable promise because they will protect car and lading at higher impact speeds than is possible with friction and rubber gears. High capacities are inherent in the hydraulic system, but to obtain their full potential, closure travel must be much longer than that of conventional gears. Their use will increase car lengths between coupler faces and could cause undesirable actions in the train if any number of cars so equipped are operated together.

Shock mitigation can also be provided within the car. For example, the New York Central Air Flotation System protects the lading from shock and vibration through the use of pneumatic dunnage units. In addition to protecting lading against longitudinal shocks, it dampens vertical forces and reduces vibration frequency. This system will continue to be useful for protecting the most delicate ladings and can be used in most of the present equipment.

Longitudinal shock duration varies with impact speeds and the amount of cushioning, but as an average, the conventional draft gear closure time is 0.05 second; the over solid stop shock lasts about 0.03 second. A 30-inch cushion shock at 8 mph might last 1.0 second.

It is impossible to be exact, but with a shipping industry moving toward 21-inch drop tests for their own package handling, approximately the equivalent to a 9-mph impact with conventional railroad cars, 7-inch hydraulic control cushion movement at the draft gear will be sufficient for practically all commodities shipped in the future in everything except over-the-highway containers. These containers will have to have an additional cushion protection of about 8 to 10 inches to produce shock effects compatible with the competition of highway truck shipment.

Because of some published reports giving average railroad impact speeds as 7.4 mph, a table is included in this report to show that this average speed is in the neighborhood of 5 mph. This analysis is based on System-wide averages of 10,000 measurements per year over a 2-year period.

Data for the 7.4-mph average impact speed is taken from J. M. Roehm's ASME Paper No. 52-5A-41 and is based on 555 impacts measured in the Chicago area in the winter of 1949-50. Pullman Standard published a curve based on 1568 car impacts over a period of 3 months in 1950. A comparison of these two sources is shown in Table 2.

Since a single classification yard handles 2000 to 3000 cars per day, any figures based on 555 impacts over one winter period in one city should not be given too much weight.

A 6-mph impact is an overspeed coupling and constitutes about 50 percent of all overspeed couplings. About 30 percent of all impacts are overspeed couplings. Unfortunately, some yards over some periods have had nearly 100 percent of their equated average impacts at overspeed and it is believed that Roehm's figures contain sizable amounts of these measurements.

DISCUSSION OF CURVES

To obtain a means for comparing different methods of reducing longitudinal shock due to impact, three curves have been constructed based on calculations and empirical data.

Curve 1 (Fig. 2) shows acceleration (g) at the car floor plotted against cushioned movement, at various impact speeds, based on a dead stop with the lading receiving the optimum cushion in the space in which it can move. No shock absorber can reduce the shock level below the values shown on these curves.

During railroad impacts, dead stops rarely occur. In the discussion and on the other curve sheets, the speeds are 2 mph higher than these dead-stop speeds. This 2-mph figure is derived from our experience on a wide variety of bumper cars and car consists, and is about the closest estimate that can be made to the speeds shown on most data observed with impact registers. It should be noted that the duration of shock, with all but the longer-travel hydraulic

TABLE 2
Comparison of Average Impact Speed Data from Two Sources

Impact Speed	Percent of Total Number of Impacts		
	Roehm[a] (%)	Pullman Standard[a] (%)	NYC (%)
Below 6 mph	38	70	70
6-7 mph	11	17	17.4
7-8 mph	15	7	6.0
8-9 mph	11	3	3.1
9-10 mph	11	2	2.3
Over 10 mph	15	1	1.2

[a]Harris and Crede, Shock & Vibration Handbook, Vol. 3, pages 45-31.

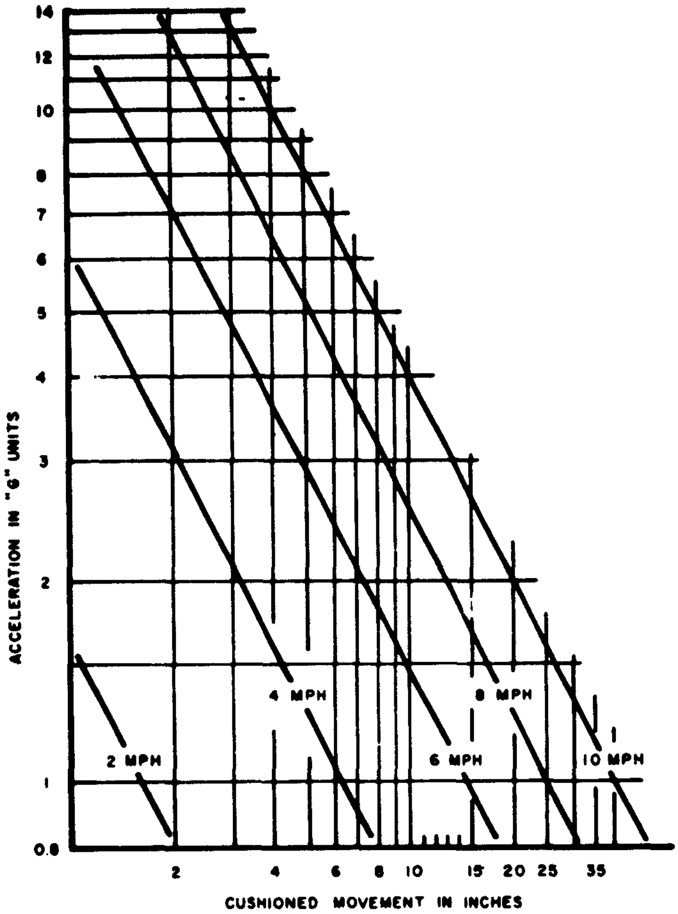

Fig. 2 - Curve 1: Acceleration plotted against movement for dead-stop impacts and 100 percent effective cushions. (Add 2 mph to these speeds when using curves for railroad car impacts.)

gear, is so short that the contribution of more than two or three cars in the bumper consist may be ignored.

This curve is useful for separating truth from fiction. If a manufacturer claims his shock absorber has a 10-inch movement and reduces the car acceleration to 2 g in a 10-mph impact, it is obvious that the test conditions were overly favorable to his device. The curve indicates 2.5 g at 10 inches and 8 (10-2) mph.

Curve 2 (Fig. 3) shows acceleration (g) at the car floor with respect to impact speed for various types of available shock absorbers. The numbers are derived from Curve 1 and from the force/displacement curves for the various devices. Based on considerable data, the friction or rubber devices were rated at 40-percent efficiency and the hydraulic system at 80-percent efficiency, reflecting the differences in their force/displacement curves.

It is important to remember that these curves represent the best conditions. No single device could obtain all the points on its curve with different weights of cars. Good design would aim for these conditions at 8-mph dead-stop with nominal load, and other speeds and loads would be above these curves.

It should also be noted that these numbers apply only if the energy-absorbing capacity of the gear has not been exceeded. For example, line (a) (Fig. 3), representing two conventional gears, will assume a discontinuity at

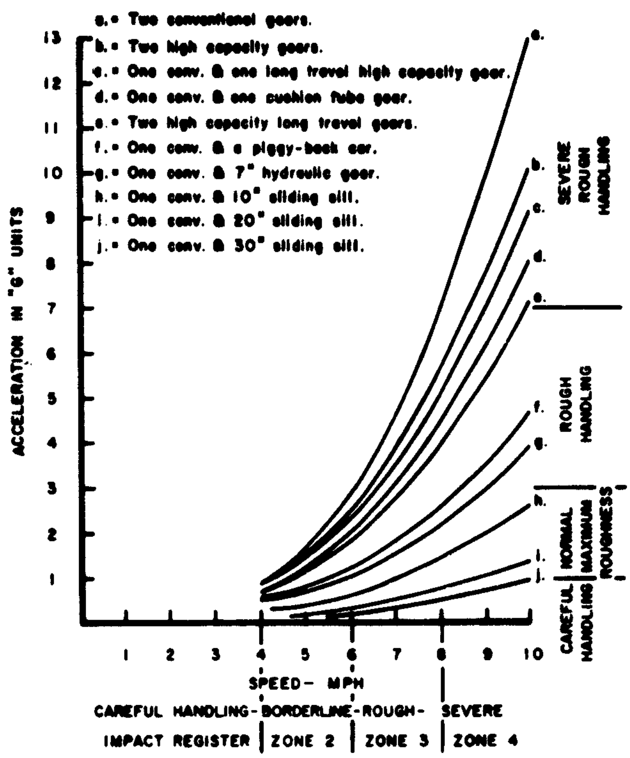

Fig. 3 - Curve 2: Acceleration of car floor to be expected for various impact speeds and types of draft gear or cushion. (Curves cover optimum conditions of cars and lading weight within capacity of shock absorbers. Speeds shown represent railroad car impacts not dead stops.)

about 5 mph and increase to as much as 15 g with nominal loads. This discontinuity results from solid closure due to insufficient energy-absorbing capacity.

It is therefore necessary for complete analysis to refer to Curve 3 (Fig. 4) to determine whether capacity is adequate.

Curve 3 shows energy in foot-pounds with respect to the rail weight of the car, in tons, at various speeds. For shock absorbing devices connected to a container or highway vehicle, use the gross weight of these vans only.

Any moving object has a potential energy equal to one-half its mass times the square of its speed. All shock absorbers have an energy capacity equal to the average force necessary to move the absorber multiplied by the distance the absorber is moved. Using the railroad impact as 2 mph higher in equivalent speed than the dead stop speed, the lines in Fig. 4 show the capacities of the various devices at various speeds and loads. If the capacities indicated are not exceeded, the devices may be expected to perform in accordance with Curve 2 (Fig. 3).

From the curve, it is determined that conventional draft gears are satisfactory, capacity-wise, at 4 mph for any weight lading, but at 6 mph, the cars should not weight over 32 tons. Piggy-back and Flexi-Van equipment have the capacity for 25 tons at 8 mph.

It is inherent in hydraulic devices to secure various capacities by design and the figures shown would be optimum only for certain ladings at one particular weight. They are probably fairly close for most cars.

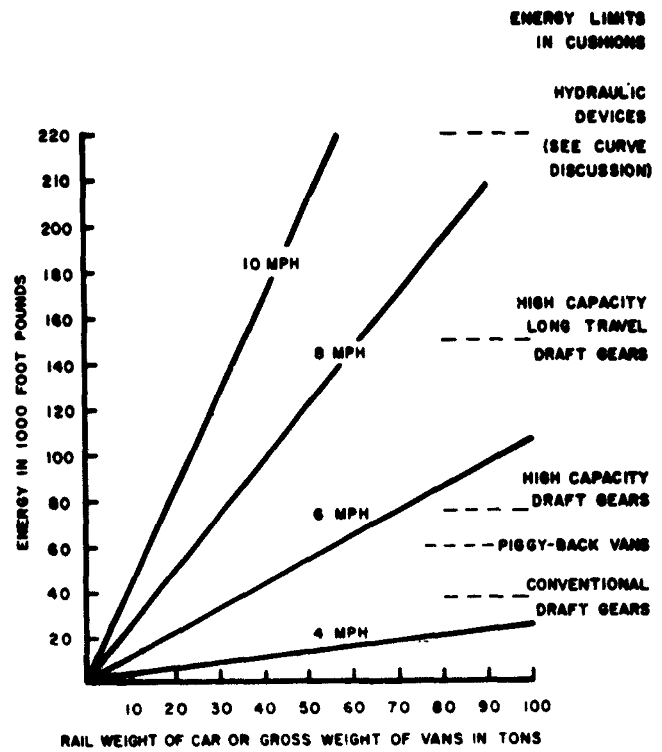

Fig. 4 - Curve 3: Energy at various speeds and weights (speeds shown represent railroad car impacts, not dead stops.)

DISCUSSION

Mr. S. Grabowski (Army Tank Automotive Command): Can you tell us where the transducers were located on the body of these freight cars?

Mr. Shackson: There were a variety of locations. In addition to obtaining general information, we were interested in testing specific tie-down configurations or specific loads. Generally, the observations at the car floor are based on the average of several transducer locations; one generally above each kingpin bolster and one at the midpoint of the car.

Mr. Grabowski: Do you think the readings are affected by the location of the transducers, depending on where they are located?

Mr. Shackson: I'm sure they are. We find considerable scatter between various points on the car floor.

J. Fowler (STL): Do you have enough data to say what is expected maximum of coupling? We're generally not interested in the average.

Mr. Shackson: Yes. I believe I showed 1 or 1.2 percent in excess of 10 mph. Actually, we have measured impacts of 14 mph. I'm certain that they occur at even greater speeds than that, although the ability of a crew driving a switch engine probably hasn't yet produced the ultimate result, the final impact speed, resulting in the destruction of the car. I do believe that the statistical percentage which I have quoted there of 1.2 would apply to impacts in the range of 10 to 12 mph with an undetermined small percentage in excess of 12 mph.

Mr. Floury (Korfund Dynamics): Would the use of a single car to transport a special device, such as a nuclear device, result in a little easier handling than you have described?

Mr. Shackson: This service is of course available and is being used on a number of special weapons transport moves at the present time. The most severe condition which occurs is that which may be produced by slack run-in enroute, and the impacts are not nearly as high as in normal usage.

Mr. Christensen (Aetron): I heard that railroad crews always handle cars with special care if they realize they are bugged. How do you take all these averages without giving any warning? Are they average conditions without special favor?

Mr. Shackson: About half of the observations which are reported here were obtained with bugged cars. Of course, in the fully automatic yard the human element doesn't exist, it is a question of the rolling characteristics of the car. On our system about half of our hump yards are fully automatic. We find that our property protection people are rather clever in concealing the fact that the car is bugged. They seem to be more clever than shippers in this respect. The balance of the observations are based on crews, personnel actually hidden in the yards with stop watches and at measured distances, timing these impacts; they correlate quite well with the measurements from the bugged cars.

Mr. Paladino (BuShips): Mr. Kennedy, in the previous paper, said that the packaged equipment has a greater percent of casualties than the unpackaged equipment. Possibly from your experience you could enlighten us?

Mr. Shackson: I'm afraid I'll have to pass that one on to the previous speaker.

Mr. Kennedy (U.S. Army Transportation Engineering Agency): I think the answer to that would be quite obvious. It's packaged because it's more fragile to start with. You would expect more loss on a shipment of packaged cameras than you would on unpackaged coal, for instance.

Mr. Paladino: That isn't what I'm speaking of. We spend a lot of money in designing packages for the kind of gear you are speaking of. Now if it is the package that is at fault, then we are not getting the package that we are designing for and this is what I am trying to find out. I realize coal is not going to get damaged, but there are a lot of containers which cost a heck of a lot of money and if they are not doing the job then we should know about it.

Mr. Shackson: Certainly the containers can be made stronger but you can always find environments that will be stronger still. But of course, if there is no container at all, then it can't break and it can't run your loss and damage costs up. I believe that in the past you have been forced to package in a rather sophisticated manner because the environment to which your item was subjected was unknown. One of the points that I would like to make is that we have been able, with the data which we have obtained, to move equipment which previously required sophisticated packaging, essentially without packaging. This can be done simply in virtue of the characteristics which are obtainable now with commercially available gear. We have had good results in such cases because there is a psychological advantage in having a naked piece of equipment which has to be handled with kid gloves as opposed to something that looks indestructible.

* * *

ROAD TRANSPORT DYNAMICS*

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An analytical approach for the dynamic design of road transport vehicles is described and illustrated for the case of the MINUTEMAN Missile Transporter-Erector. Details are also given of road-roughness criteria and test methods.

INTRODUCTION

Much study and design effort has been devoted to protecting cargo from the damaging dynamic environment resulting from road transport. This effort, however, has been concentrated at determining the vehicle cargo bed environments, followed by attempts to generalize the data to the extent that all the detailed characteristics of the environment are lost. What remains then for the designer is an envelope of the maximum environment of a large number of different types of vehicles. This approach results in a design that is often extremely conservative, or that, because of the coincidence of cargo and vehicle resonant frequencies, produces a cargo environment that is much higher than necessary. Techniques involving cut-and-try developmental testing, or simulated equipment testing to determine the design environment, have also been used. For large complex systems or for any expensive system, these approaches are too time consuming and costly. Given the capabilities of the digital and analog computers available today, analyses of adequate complexity for the design of cargo-vehicle systems for structural loads can be conducted economically without resorting to expensive developmental testing or to shooting-in-the-dark design. This analytical approach has been used in the design of the MINUTEMAN Missile road vehicles by the Boeing Company.

There are four dynamic environments involved in road transport which produce significant loadings. First, the fore and aft environment associated with accelerating and braking; second, the lateral environment associated with travel around curves; third, and most significant, the vertical environment associated with traverse of rough roads; and fourth, the fore and aft and lateral environment associated with unsymmetrical vertical excitation. The first two environments are relatively low and the resulting structural loads, which are essentially static, can be determined from general braking and accelerating characteristics for the class of vehicle, the speed, and the minimum curve radius. The last two are dependent upon the roughness of the road surface and can be handled by one and two dimensional dynamic analysis of the system. The frequencies of concern in the dynamic environment are, those up to approximately 30 cps, as they can produce significant structural loads or displacements. Frequencies above 30 cps exist in road vehicles, but, although they can excite local structural resonances, they do not cause major structural responses or significant displacements in cargo isolation systems.

DYNAMIC ENVIRONMENT FOR ANALYSIS

The use of this analytical approach requires the determination of a basic dynamic environment which is independent of the vehicle or cargo design, namely the road roughness. This environment has a particular advantage in that, once established and confirmed, it is valid for all vehicles traveling the particular class of road. The determination of this type of environment appears at first to present an insurmountable task. This would appear to be confirmed

*This paper was not presented at the Symposium.

by the definite lack of any such published data. However, a general knowledge of road surface characteristics and a review of vehicle dynamic characteristics provides some simplification of the problem.

Unpaved and flexibly-paved road surfaces are generally characterized by a roughness having a random wavelength and low-level amplitude, occasional washboard areas of relatively high level with nearly constant wavelength, and occasional single bumps such as chuckholes of short length and high amplitude. Washboards can occur with a wide range of wavelength, although any given washboard area has a nearly constant wavelength. The frequency appears to depend upon vehicle axle response frequency and the speed of vehicles traveling the road at the particular area. Rigid pavements generally have a random roughness for sections which are broken, and regular inputs or single sharp steps associated with pavement joints.

The vehicle and cargo comprises a complex dynamic system, generally, with well defined coupled resonant modes. The modes include the low-frequency bounce and pitch of the sprung mass of the vehicle on the suspension; the axle response frequency, usually in the range of 9 to 12 cps; the higher frequency, relatively low-damped, structural modes of the vehicle and cargo which may produce significant loading up to about 30 cps; and the cargo suspension system modes.

This type of system will develop maximum loading under repetitive inputs having frequencies corresponding to the resonant frequencies of the system. The nearly sinusoidal washboards or rigid pavement joints will, therefore, generally, produce the highest responses.

For the design of the system, the highest loading which will occur during the operational life is required. Because of the wide variation in washboard length or joint spacing and vehicle speed, it can reasonably be assumed that the most critical wavelength and vehicle speed would be encountered at some time during the life of the vehicle.

What remains then is the determination of the amplitude of the washboard surface and single bump or chuckholes, which will be encountered for various classes of roads and various operational lives. Because of the limited amount of road surface-roughness measurements, arbitrary preliminary criteria were established for the MINUTEMAN transportation system.

For unpaved secondary roads, the roughness was assumed to be represented by lengths of continuous sinusoidal washboards, each length of constant wavelength, but different lengths having wavelengths ranging from 2 to 25 feet, with 2-inch double amplitude. Single chuckholes with lengths from 2 to 4 feet and 4 inches deep were also assumed. Both the chuckhole and the washboard were considered to be the full width of the road surface, such that the input occurs simultaneously to both wheels on an axle.

For paved primary and secondary roads, the roughness was assumed to be represented by sinusoidal washboards, the same as for the unpaved road, but with 1-inch double amplitude. A single bump of 2 inches in height or a 2-inch deep chuckhole was assumed.

The maximum response of the system was found to occur when the wavelength was most critical for the vehicle axle spacing. In the vehicles we have studied, either a pure bouncing input or a pure pitching input has been found to give the greatest response. This required that the input to all axles at each end of the vehicle be in phase, but that the inputs to the two ends be either in phase or 180 degrees out of phase. For a two axle vehicle the most critical wavelength will be the shortest within the 2- to 25-foot range, which is either an even multiple or an even multiple plus 1/2 wavelength, of the axle spacing as shown in Fig. 1. For a given

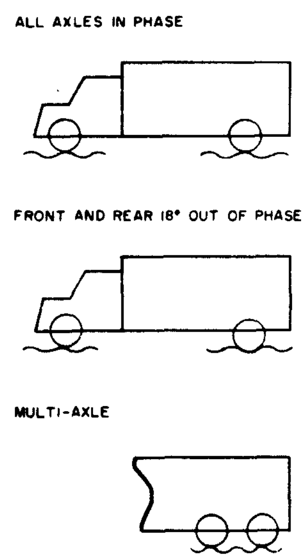

Fig. 1 - Washboard road phasing with axles

speed range, the shortest wavelength results in the widest band of frequency excitation. In a multi-axle vehicle the spacing of the axles can be such that neither type of input will occur. In these cases, the most severe wavelength has been one-half the axle spacing of the multiple axles on either end of the vehicle.

MINUTEMAN TRANSPORTER-ERECTOR DESIGN ANALYSIS

The following description of the MINUTEMAN Transporter-Erector analysis is presented as an example of a large complex system in which this analytical approach was used. Other and much less complex analyses were used in the design of the individual rocket motor transporters, an environmental control unit transported on a flat bed trailer, and several truck and semitrailer configurations used to transport missile components and electronic equipment.

The dynamic analysis model for the Transporter-Erector is shown in Fig. 2. The missile is represented as a flexible beam, supported inside the Transporter-Erector container at six points by torsion bar suspensions and hydraulic dampers. The container, represented by a flexible beam, is supported at the fifth wheel by a four-axle tractor and at the rear by a three-axle rear carriage. The tractor and rear carriage have air suspension with shock absorbers. The analysis model includes the axle mass, tractor and rear carriage beam stiffness, and the tire stiffness. There are five significant coupled modes of this system in the frequency range up to 20 cps, which is the highest excitation frequency possible for the speed and critical wavelength limitations. These modes are: 1.3 cps, primarily bounce of total sprung mass; 1.5 cps, primarily pitch of total sprung mass; 9 cps, primarily missile first mode bending; 10 cps, primarily axle bounce frequency; and 16 cps, primarily container bending.

The coupled mode associated with the damped missile suspension system, which when uncoupled is approximately 3.3 cps, is not excited by inputs to the transporter-erector tires.

Both the vehicle suspension and the missile suspension inside the container have nonlinear spring and damping characteristics in the actual hardware. The nonlinear characteristics are used in the analysis. The force-displacement characteristics of the vehicle air suspension, the force-velocity characteristics of the vehicle shock absorbers, and the nonlinear missile suspension dampers are shown in Fig. 3.

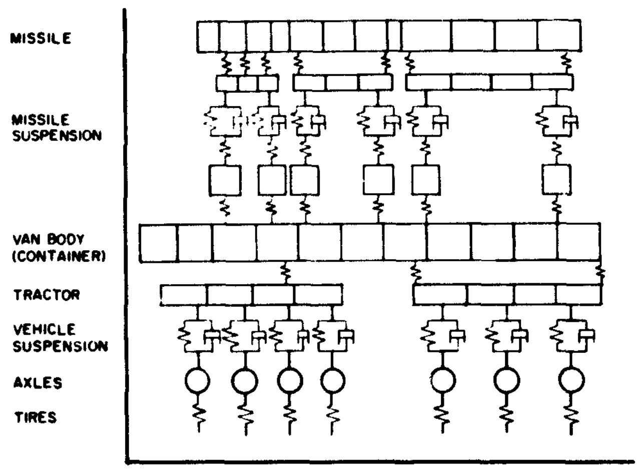

Fig. 2 - Analytical model

The analysis was conducted using the CEA analog computer in which the various portions of the mechanical structural system are replaced by analogous electrical components. Digital computer techniques have also been applied to comparable systems.

Once the road roughness criteria had been established, it remained to determine what

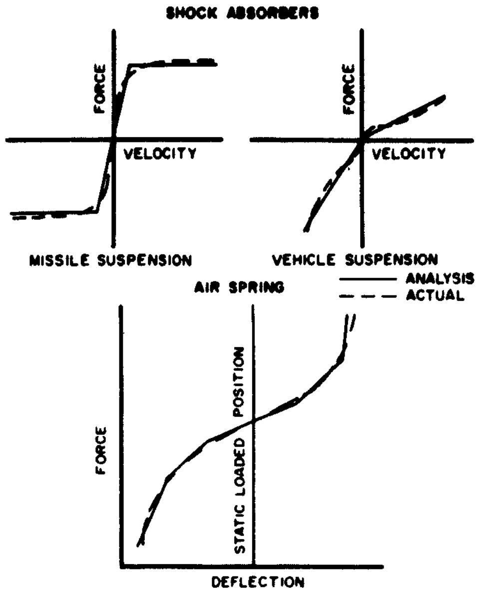

Fig. 3 - Nonlinear component representation on the analog

particular occurrence would be most critical in producing loads and accelerations in the Transporter-Erector. A study simulating the full operating speed range over a wide range of washboard wavelengths was conducted. For this study the washboard amplitude was held constant. It was found that a speed over any wavelength that produced an input frequency corresponding to a resonant mode of response was significant. If, in addition, the wavelength was such that it forced all axles simultaneously the highest response over that amplitude of washboard was obtained. However, for the Transporter-Erector configuration no wavelength satisfied this condition. The most critical wavelength was found to be one-half the axle spacing of the rear carriage axles. Maximum loads in the missile occurred when traversing this washboard spacing at 18 mph.

For the vehicle on its suspension system, maximum responses occur at either the frequency of the bounce or pitch of the vehicle on its suspension, or at the frequency of axle response. The amplitude of response at the low frequency is due mainly to the amplitude of road input and the damping of the vehicle suspension. The axle response frequency for the Transporter-Erector is approximately 10 cps and is the resonant motion of the axle mass between the vehicle suspension spring and the tire as a spring. The axle mass and tire stiffness are the main factors that determine this frequency and since both are usually standard commercial components little can be done to shift this axle resonant frequency. In the Transporter-Erector and most of the vehicles studied, the maximum loads and accelerations in the cargo are excited by this axle resonance. The methods used to reduce the cargo response have been to minimize the axle motion, to use cargo isolation systems with resonant frequencies of 3 to 5 cps, and to design vehicle and cargo structure to be stiff enough that flexible modes occur well above axle resonance. All of these methods lend themselves to analytical solution.

In formulating a mathematical model of a vehicle for study it is mandatory that the axle masses and tire stiffnesses be determined and included to obtain satisfactory results. In the physical system the tires can bounce clear of the road. As an inclusion of tire bounce results in a discontinuous dynamic system, it is desirable not to include this in the analysis. Reasonably good agreement between analysis and test results, both in frequency and amplitude, have been obtained with most vehicles when tire bounce was eliminated.

For the Transporter-Erector analysis special circuits were developed to include tire bounce and the analysis was conducted both with and without tire bounce. Inclusion of tire bounce increased the level of missile response about 10 percent. However, this may have been a somewhat singular occurrence because of the small frequency separation between the missile bending mode (9 cps) and the axle response mode (10 cps) and therefore other systems may exhibit entirely different results.

The results from the analysis of axle motion show that more than one type of bounce exists. At a low amplitude of input, near the axle resonant frequency, or at a higher amplitude for frequencies other than the axle resonance, a form of bounce occurs (Fig. 4(a)) in which the tire comes free from the road as it passes over the crest of the disturbance. This type of bounce will increase as the amplitude of road roughness is increased up to a certain amplitude. Beyond this amplitude, the bounce suddenly changes to the hopping type of action shown in Fig. 4(b).

The analysis has shown tire bounce to be either regular, or to appear almost random, in amplitude. Whether the hop is regular or random is somewhat dependent on the amplitude of input and tire stiffness, but it is influenced primarily by the amount of damping in the truck

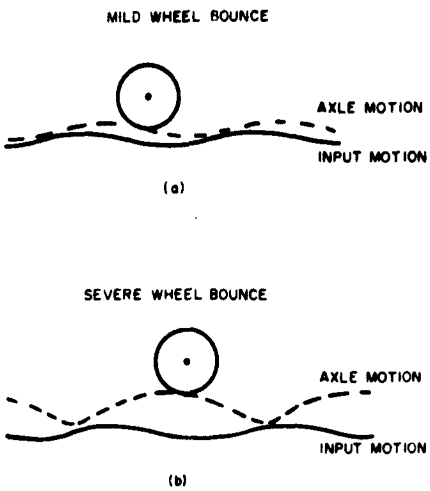

Fig. 4 - Types of wheel bounce

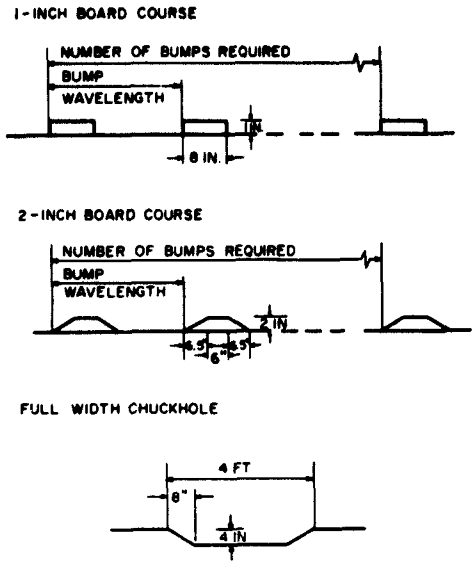

Fig. 5 - Test course shapes

suspension. With the tire free to bounce in the analysis, the axle usually exhibits a resonant peak and diminishes in response to the point where tire bounce ceases as input frequency increases.

DYNAMIC TESTING

Qualification tests were conducted of the MINUTEMAN vehicles to the arbitrary road roughness criteria assumed for design purposes. These tests were instrumented to insure that the allowable missile structural loads and accelerations were not exceeded.

To conduct these tests the dyanmic test courses shown in Fig. 5 were developed. The 1-inch amplitude washboard is simulated by boards 1 inch high, 8 inches wide, and 15 feet long, transverse to the direction of travel. The 2-inch washboard is simulated by tapered boards 2 inches high and 19 inches wide and 15 feet long. The course is designed so that spacing can be adjusted. The most critical spacing is selected from the results of the analyses. For the Transporter-Erector this spacing is 33.5 inches. The length of the course is selected to produce the maximum response of the very low damped structural resonances. Generally this length need not be greater than 1 to 1-1/2 times the wheelbase.

For both test course amplitudes, simple cross-sectional shapes were used for the boards (Fig. 5). During design of the course, there was concern over higher frequency harmonics in the axle response if too prismatic a shape was used. An instrumented vehicle was tested over a variety of shapes ranging from simple rectangular to almost sinusoidal. Higher harmonics existed for every shape tested, but were negligible at the speed corresponding to the critical axle bounce frequency. Although harmonics excite resonant modes at frequencies other than the input frequency, results are readily interpreted and the simple shapes are satisfactory for simulation of sinusoidal washboards.

The chuckhole is simulated by a depression 4 feet long and 15 feet wide with sloping sides (Fig. 5). For the 4-inch chuckhole, the system responses, except in the vicinity of the axles, are less than those obtained over the 2-inch course.

The use of the sinusoidal-type vehicle test course equivalent to the maximum road roughness of various classes of roads, provides a means of determining, by test, the maximum responses which can be expected in the life of the system. In addition, the sinusoidal test course allows a relatively simple analysis to be used for design with only a verification test of the final hardware configuration necessary if qualification of the system is required.

TEST - ANALYSIS COMPARISON

To provide a verification of the analytical method, the following comparison of the

Transporter-Erector test results and the analysis results have been made.

The missile structural loads and accelerations measured during the test have agreed very well with the predicted levels from the analysis.

Figure 6 shows analytical results for the missile bending moments at one critical missile

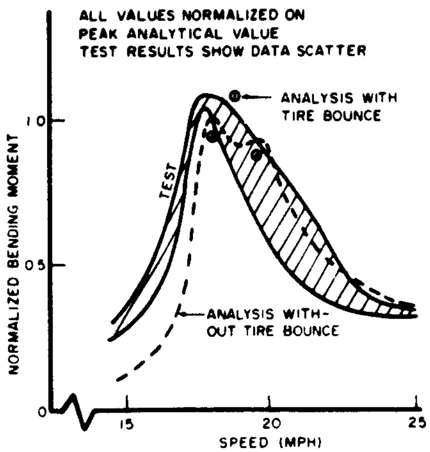

Fig. 6 - Comparison of analytical and test results

station versus vehicle forward speed, for travel over a 1-inch, double-amplitude, washboard with a wavelength of 33-1/2 inches. The results are shown for the speed range which induces maximum response.

The analytical results both with and without tire bounce in the configuration are shown for travel over a continuous sinusoidal washboard. The results with tire bounce are shown as separate points at the specific speeds for which the analysis was conducted.

The measured bending moment data from the test of the Transporter-Erector over the 1-inch washboard course, are also shown in Fig. 6. The band associated with the test results represents the scatter of test data. Because of tire bounce, the actual vehicle is a discontinuous dynamic system resulting in different responses for similar tests at the same speed over the course. Random response and different levels for the same speed have also been found in the analyses, which have included tire bounce.

The moments have all been normalized to the maximum moment for the analysis without tire bounce to allow a percentage comparison.

The maximum test levels at the critical speed are approximately 10 percent higher than the analytical results for a configuration without tire bounce. For an analysis configuration with tire bounce, the maximum test results are equal, but occur at approximately a 1-mph lower speed. This variation in speed is well within both the test data and analysis result accuracies. For the Transporter-Erector, analyses without tire bounce are sufficiently accurate for preliminary studies and for variation of design parameters for optimization studies. However, an analysis with tire bounce is required to establish accurately the maximum levels. For other vehicle configurations, the tire bounce characteristics may be found to be more dominant and will have to be included even for preliminary studies.

The determination of the level and characteristics of the damping of the vehicle-cargo system is one of the most important steps in an analysis, as the response is generally directly related to the level of damping. Commercial suspension systems for road vehicles have nonlinear characteristics due to friction in leaf springs, or to the orifice characteristics of the shock absorbers.

Failure to account for the nonlinear characteristics of the missile suspension system dampers in the initial Transporter-Erector design analyses, resulted in missile loads and accelerations of approximately twice the design levels, during the initial qualification tests.

The solution of this problem demonstrates the advantage of the analysis method. Study of the analytical results showed that the container test responses were consistent with the analysis, whereas the missile test responses were approximately twice as high as the analysis. The problem could then be isolated to the missile suspension system. Laboratory tests showed the dampers to be very nonlinear with much higher damping than that used in the design analyses. Incorporation of the nonlinear damping levels of the actual hardware resulted in the close correlation between analytical and test results shown in Fig. 6.

The analysis was then used to evaluate changes to the dampers without conducting tests of the Transporter-Erector for each change. A change in the dampers, which reduced the load by a factor of 2, was selected based on the results of the analysis. The confirmation test of the Transporter-Erector with revised dampers showed good correlation with the predicted results.

PRELIMINARY ROAD ROUGHNESS CRITERIA

The road roughness criteria selected for the MINUTEMAN system were based on a very limited amount of data and were, therefore, quite arbitrary.

Some measured data have been obtained for MINUTEMAN vehicles under actual operational conditions. However, because of the low probability of occurrence of the critical speed, wavelength, and amplitude required to generate peak loads, only a preliminary indication of the suitability of the criteria has been obtained.

Vehicles required to travel on paved primary and secondary roads were designed for, and tested over, the 1-inch washboard course. Some of these vehicles have been instrumented for road tests ranging up to 10,000 miles. Peak loads occurring during these tests have been between 30 and 50 percent of the maximum levels encountered over the test course.

Not enough data has yet been obtained to verify the 2-inch amplitude washboard for unpaved road operation.

An adequate description is required of road roughness that will produce the same maximum dynamic responses of the vehicle-cargo system as would occur in actual service. The description must take into account the various classes of roads, including construction and maintenance, as well as the expected service life for the system.

A preliminary and, probably, an oversimplified road roughness criteria, which is considered conservative for determining the maximum response of vehicle cargo systems, is shown in Table 1.

CONCLUSION

With the digital and analog computing capability presently available, it is possible to conduct adequate design analyses for all types of road vehicles and cargo. Although this method is extremely well suited for complex systems, such as the MINUTEMAN Missile Transporter-Erector which was presented in this paper as an example, it has been used for design of other simpler road vehicles. This method can be used

TABLE 1

Road Roughness Criteria

Type of Road Encountered	Low Probability of Occurrence[a]		High Probability of Occurrence[b]	
	Sinusoidal Washboard Amplitude (in.)	Single Bump Amplitude (in.)	Sinusoidal Washboard Amplitude (in.)	Single Bump Amplitude (in.)
Well maintained, well constructed, primary and secondary roads with rigid or flexible pavement	3/4	1-1/2	3/8	1
Paved primary and secondary roads with average maintenance or well constructed unpaved roads with good maintenance	1	2	1/2	1-1/2
Poorly maintained, flexibly paved, secondary roads and unpaved roads	2	4 (chuckhole)	1-1/2	3 (chuckhole)

[a]Low probability of occurrence will require a long service life (100,000 miles) if it is to be encountered.
[b]High probability of occurrence indicates levels will be encountered even in a short service life (10,000 miles or less).

NOTE: Washboard to be of critical wavelength and of sufficient length to develop maximum response of lowest mode in the system. Vehicles to be designed for these road roughness criteria at all speeds within the normal vehicle operating range.

quickly and inexpensively to design even the simplest cargo arrangement on any vehicle for which the dynamic characteristics are known.

The only major problem area is the determination of a satisfactory simulation of the road roughness. This problem exists, however, for any design approach short of cut-and-try testing under actual operational conditions. The current effort which is directed toward gathering dynamic environment data, such as cargo bed data, for specific vehicles, should be redirected toward the systematic gathering and categorizing of the basic input environment of road roughness. Once road roughness has been established and verified for various classes of roads, adequate cargo protection under the road transport environment may be quickly and inexpensively obtained through use of the analysis method.

* * *

Section 3
INSTRUMENTATION AND DATA ANALYSIS

PRACTICAL RANDOM VIBRATION MEASUREMENT TECHNIQUES

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A random vibration environment is applied to a test specimen and the resulting data examined. The power spectral density analyzer is described and its use explained. Some pretest checks, which are required to insure reliable data, are described.

INTRODUCTION

The title of this paper is rather all-inclusive. The entire Symposium could be devoted to this one subject and still not cover it completely. So this paper, in order to be practical, will consider only one problem: The problem of applying one specific random vibration environment to one specific test specimen and obtaining some meaningful, repeatable data. The vibration environment shown in Fig. 1 will be applied to the specimen shown in Fig. 2.

THE EDUCATED-GUESS TECHNIQUE

The first attack on the problem will use the educated-guess technique. There are many variations of this technique, but the basic procedure is approximately as follows:

- Bolt the specimen and its holding fixture to a shaker.
- Attach an accelerometer somewhere.
- Feed the accelerometer output signal to an X-Y plotter; run sine-wave sweeps and adjust filters until the X-Y plot is flat over the desired frequency range. Typical successive steps in this process are shown in Fig. 3.
- When the vibration system is equalized (flat) enough to satisfy the test engineer, fire up the random noise generator and run the test. Read the overall g level on one or more meters.

The educated-guess technique may sound facetious, but it is widely used and it does produce data that is truly random. To be successful, this testing technique depends on the fact that the test engineer is an expert in his field. He has tested a large number of items and, thereby, has acquired a background upon which to base expert opinions. His opinions are accepted by his peers; the design program progresses rapidly, unhampered by voluminous test reports. The principal disadvantage of this technique is that it produces no actual random vibration data to compare with Fig. 1.

The educated-guess technique can also be used to obfuscate telemetered flight data. Typical methods are:

- Omit the overall end-to-end data system calibration. Instead, calibrate the components of the data system (accelerometers, amplifiers, subcarrier oscillators, rf link, ground station, and so on) individually and separately, at separate discrete frequencies. Add the separate calibrations together on paper to obtain the overall system calibration. (This calibration technique can result in substantial errors in the final data.)
- Process the accelerometer signal through an analyzer in the missile before

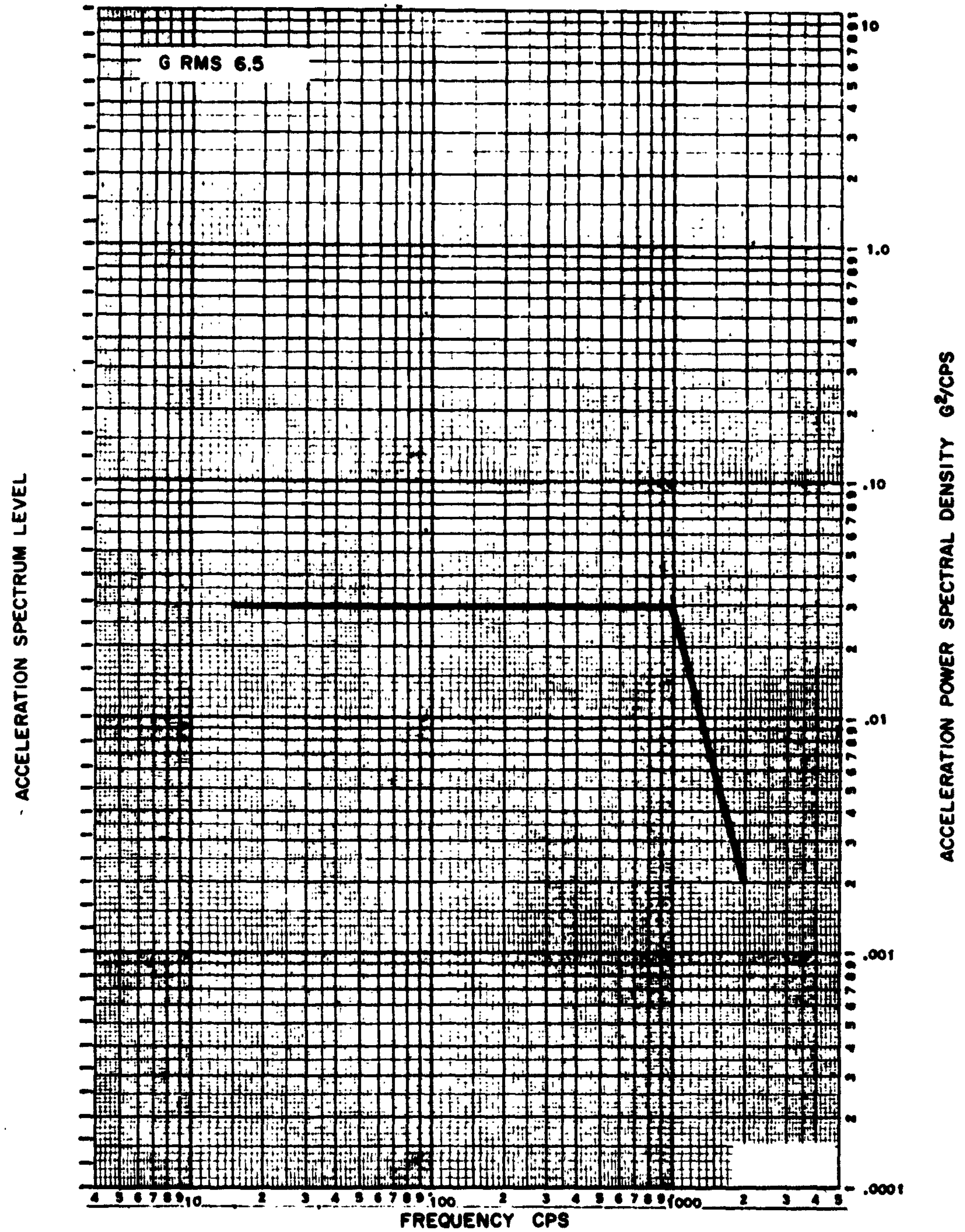

Fig. 1 - Random vibration environment-power spectral density versus frequency

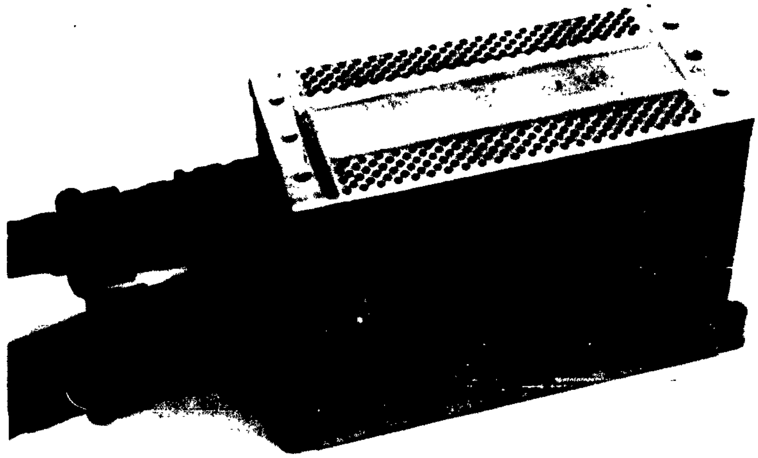

Fig. 2 - Test specimen (The heavy cables will cause excessive cross motion unless restrained by the fixture)

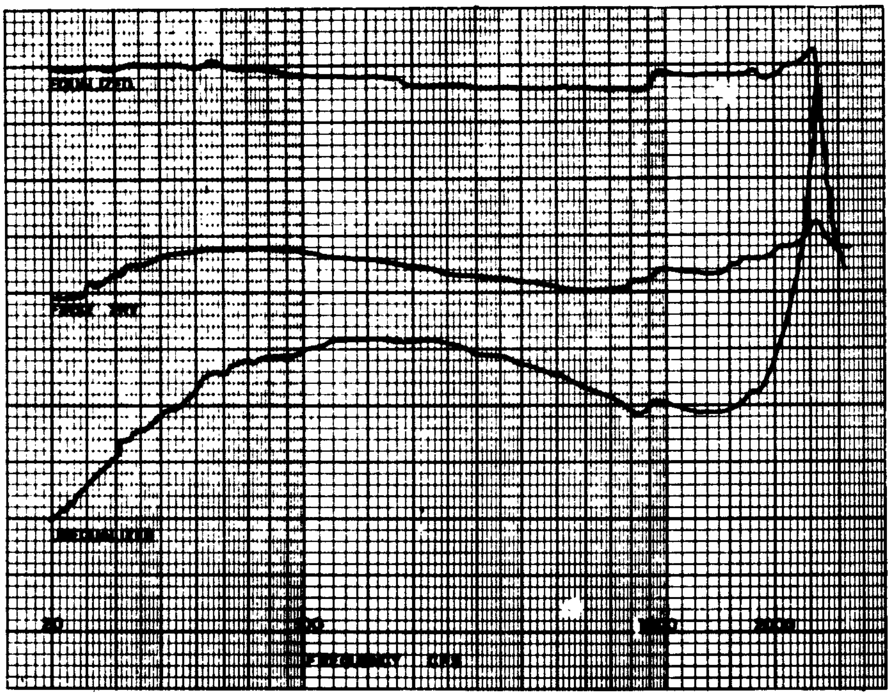

Fig. 3 - Vibration system equalization plot

transmission. The analyzer will effectively remove transient data, nonstationary data, and sine wave spikes. (However, airborne analyzers do offer advantages on large, massive missiles where the data is reasonably stationary for periods of several seconds.)

• Omit the low-pass filter from the accelerometer amplifier. Crystal accelerometer resonances (35 kc) can then beat with a subcarrier oscillator (70 kc) to produce spurious random data.

Other methods for producing truly random flight data are available, but we digress from our laboratory problem.

POWER SPECTRAL DENSITY ANALYSIS

In order to show that a test is performed in compliance with Fig. 1, a graph of g^2/cps versus frequency is required. A power spectral density analyzer will provide such a graph. A block diagram of a power spectral density analyzer is shown in Fig. 4. Much descriptive literature on this instrument is available. The proposed MIL-STD-810 (USAF) will contain some recommendations for its use, so my only comment here is that the data output of a power spectral density analyzer can be greatly influenced by the characteristic curve of its filter. The ideal filter response shape would be flat-topped with vertical sides.

Tolerances and Over-Test Problems

We should now attack the original problem with a power spectral density analyzer, but we cannot, yet, for two practical reasons. First, because of the random nature of random vibrations, precise compliance with Fig. 1 is not

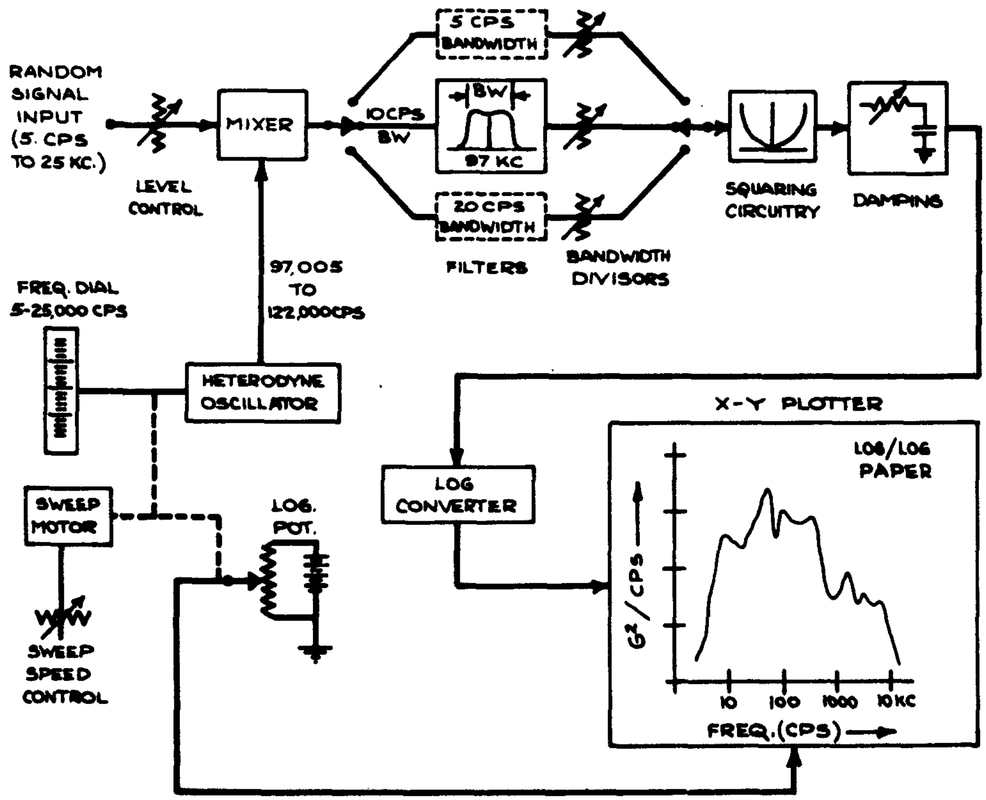

Fig. 4 - Power spectral density analyzer (input signal is heterodyned to 97 kc, passed through a filter of chosen bandwidth, normalized to unity bandwidth, squared, damped, and plotted as g^2/cps [g^2 versus frequency as seen through a window 1 cps wide]. Input level control is always set to utilize the maximum dynamic range of the analyzer, regardless of input signal voltage).

possible. A tolerance must be applied, either in advance by design, or later at the discretion of the test engineer. A reasonable tolerance is shown in Fig. 5. Second, the power spectral density analyzer requires 10 minutes, minimum, to sweep the spectrum. Even slower sweep speeds are recommended by many authorities. Some means must be found to prevent overtesting of the specimen during the 10 or more minutes required to produce the first analyzer record. One preventive measure would be a setup run at reduced g level. This would be satisfactory for linear test specimens, but they are rare. A more satisfactory measure is to interpose a loop tape recorder between the analyzer and the test. A 10-second loop is recorded at full g level, then the loop is played back into the analyzer for the required 10 minutes or more. Two or three 10-second samples of data often suffice for a skilled test engineer to adjust the vibration system within the required tolerance.

Also, there are some pretest checks that must be performed if we are to achieve our goal of meaningful, repeatable data.

Accelerometer Check

A minimum of six calibrated crystal accelerometers are attached to a calibration block, and the calibration block is attached to the shaker (Fig. 6). The shaker is energized sinusoidally and the output signal of each accelerometer is normalized to some standard value, say 10 mv/g, at some standard frequency, by means of a capacitor decade attenuator (Fig. 7). This normalizing procedure promotes interchangeability of accelerometers, reduces meter-reading errors during the test, speeds the data analyzing process, and reduces human error during the data analyzing process. The attenuator settings should be logged. The log provides a check on operator accuracy. Overall data accuracy can be no better than the day-to-day variations shown by this log.

The accelerometer output signals are then fed to an oscillograph. The oscillograph paper speed is set to 0.1 ips. A sine-wave sweep is run to check the linearity and distortion of the accelerometers over the frequency range. Figure 8 shows the oscillograph record of such a sweep.

End-to-End Instrumentation Calibration

The output signal from one of the accelerometers is fed through the tape recorder to the spectral analyzer. The accelerometer is vibrated at 10-g sine wave and this data point is plotted by the spectral analyzer. It must plot as 100 g^2/cps and at the correct frequency. But 100 g^2/cps is an impractical data point, so the signal is attenuated by a factor of 10 at the input of the analyzer. The analyzer is now adjusted to plot the signal as 1 g^2/cps at the correct frequency.

Fixture Survey

The accelerometers are removed from the shaker and firmly attached to the bare fixture, usually with dental cement or Eastman 910. Figure 9 shows four accelerometers located to measure desired axial motion of the fixture, and two accelerometers located to measure undesired cross motion. Figure 9 also shows the vacuum plate used to attach the fixture to the shaker. Attaching fixtures to shakers by the use of vacuum, speeds test setups, especially when using a horizontal oil table, and also eliminates the effects of bolt tightness.

The short posts at the left end of the fixture were designed to give maximum rigidity with minimum weight. In that position posts caused uncontrollable resonances as shown in Fig. 10 and were machined off. The final fixture is shown in Fig. 11 and the final fixture survey record in Fig. 12. Accelerometer location No. 6 (Fig. 12) appears to be a representative location, so accelerometer location No. 6 will be the monitor accelerometer location for all tests conducted on this fixture.

These pretest checks may seem time-consuming, but the redesign of products due to erroneous test data is far more time-consuming, and arguments between laboratories due to non-repeatable test data consume more time yet.

Equalization and Use of Power Spectral Density Analyzer

The next step is to mount the test specimen on the fixture and equalize the vibration system by using low-level sine wave sweeps as shown in Fig. 3 or an automatic equalizer, if available.

We can now apply the power spectral density analyzer to the problem:

- Adjust the vibration system to agree with Fig. 5. This is accomplished by adjusting band-pass filters on a best guess basis, or by the automatic equalizer.

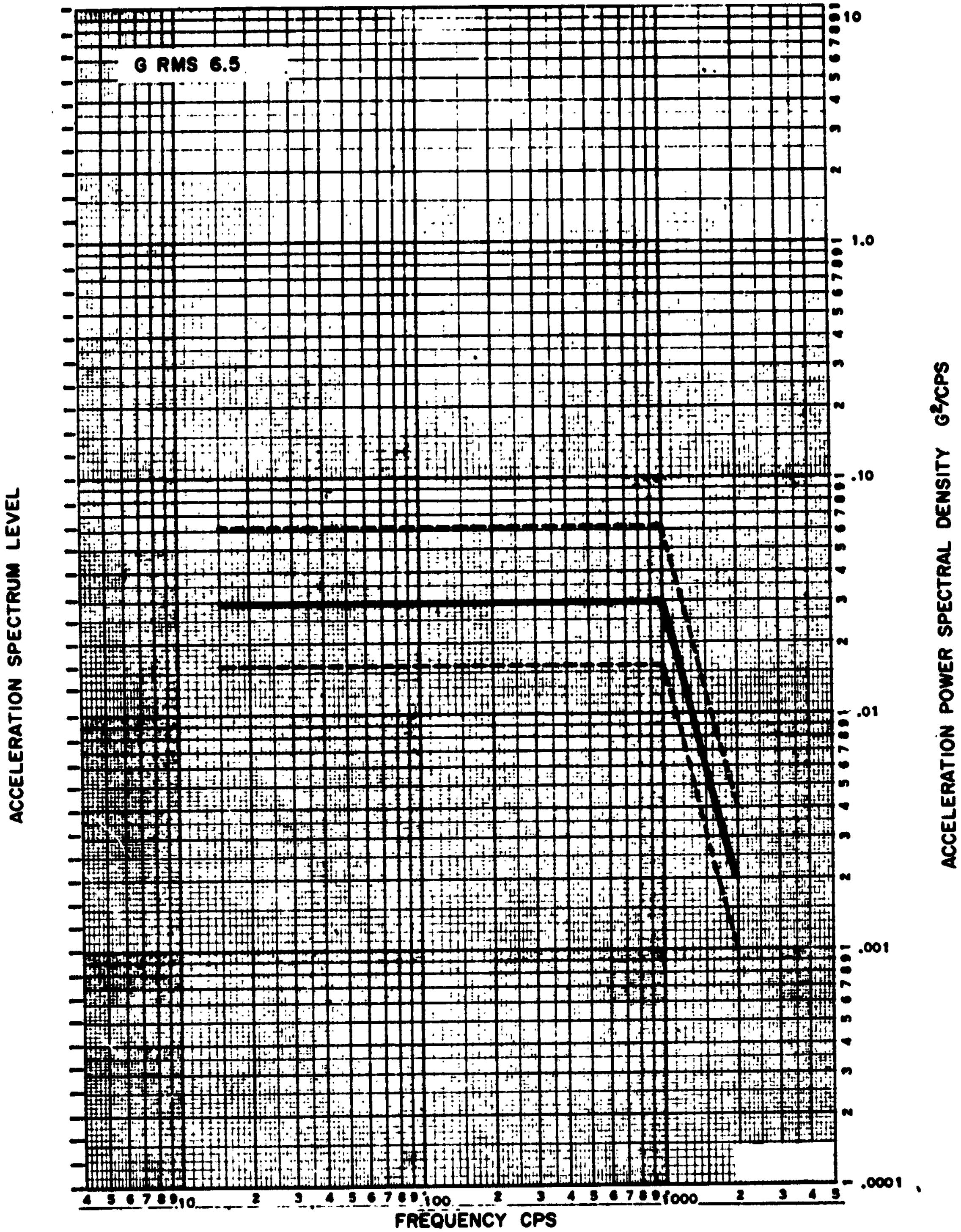

Fig. 5 - Random vibration environment with ±3-db tolerance added

Fig. 6 - Accelerometer calibration block provides uniform g level on top surface to 3000 cps

- Shake the specimen at full random g level for 10 or more seconds to obtain a tape-loop record of the monitor accelerometer signal.

- Analyze the tape-loop record with the analyzer. A typical first-try plot is shown in Fig. 13.

- Repeat the last three steps until the random vibration environment is within tolerance (Fig. 14).

- Another record should be obtained and analyzed near the end of the test to show that the vibration environment did not change during the test.

We now have two power spectral density graphs, pretest and end-of-test, to compare with the original specification. Both graphs are within the tolerance shown on Fig. 5, so we have apparently met the conditions of the problem. But have we? What about the distribution of energy peaks, for example. A block diagram of a suitable distribution analyzer is shown in Fig. 15, and a plot of peak distribution is superimposed on a standard distribution curve in Fig. 16. The energy distribution appears to be reasonably Gaussian; thus, another unknown in the data has been defined.

Another popular power spectral density analyzer is the comb filter. The comb filter (Fig. 17, left rack) divides the frequency spectrum into 40 discrete bands and reads out the average energy in each band on a separate meter. The meters do not show the amplitude or exact frequency of sharp resonant peaks in the data, so the comb filter is used only where sharp resonances do not exist or are immaterial to the data. However, unknown data, especially flight data, is often played through a comb filter onto a roll of oscillograph paper. The oscillograph roll is then examined visually and portions are selected for more precise analysis.

The power spectral density analyzer can be applied to as many data channels as desired, but a stack of power spectral density graphs might not be too meaningful to a design engineer. Perhaps the data should be digitized, so that a computer can be used to make sense out of it. But perhaps a computer or even a spectral analyzer should not be used, and herein lies the point of this paper.

CONCLUSIONS

This paper has actually presented two opposite philosophies of data gathering.

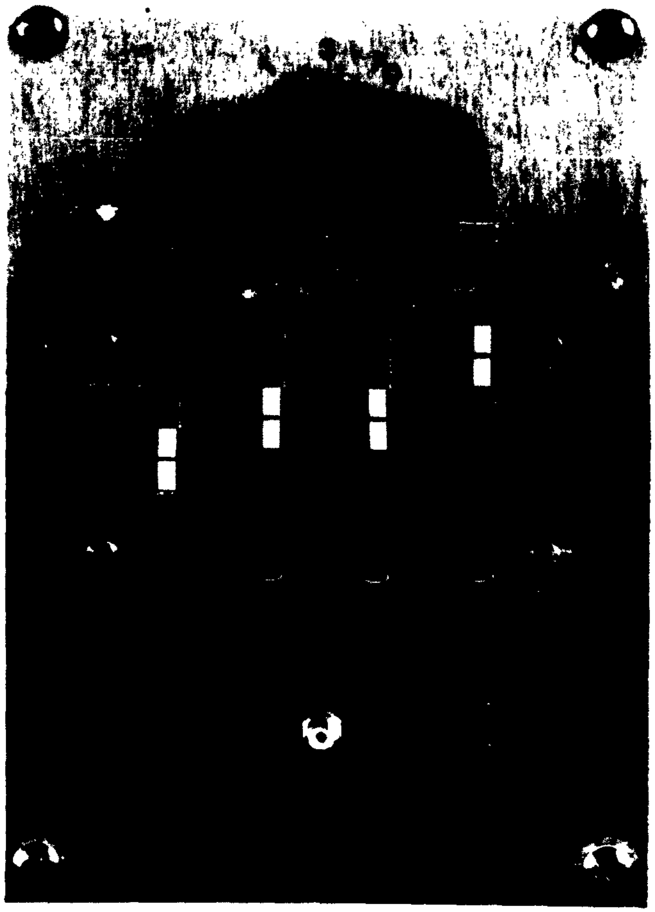

Fig. 7 - Capacitor decade attenuates accelerometer output signal without affecting frequency response

The educated-guess technique leads to test reports that are little more than statements of opinion. I have personally seen data from one test, gathered by two different, independent, instrumentation systems that differed by a factor of 100 to 1 at some frequencies. Computer time or spectral analyzer time is worse than wasted on such data, for the very use of an exotic machine or analysis method imbues the data with a false aura of validity.

Unless the test engineer observes, investigates, measures, and reports all phenomena which affect data validity, his data are probably invalid, no matter what measurement equipment was used, or how accurately it was calibrated. For example, it makes little sense to carefully calibrate an accelerometer system, or to buy a spectral analyzer or a computer, and then omit the fixture survey or ignore vibrator cross motion.

Flight accelerometer data should be listened to, in the raw state, with a loudspeaker, before it is machine analyzed. It should sound like a missile flight; if it does not, there is probably something wrong with it. Invalid data lead to invalid test specifications, the compliance with which can cost thousands of dollars, unnecessarily.

Admittedly it required a lot of thought and work to check each step of the data gathering process for validity and omissions, but it is the only way I know to obtain meaningful, reliable, repeatable vibration data.

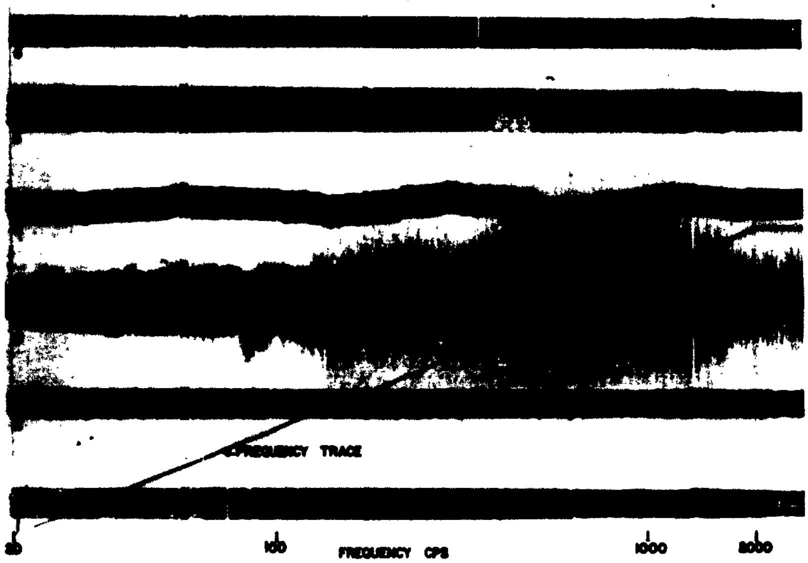

Fig. 8 - Oscillograph record of an accelerometer calibration sweep:
Top trace is monitor accelerometer.
Trace No. 5 shows incorrect attenuator setting.
Traces No. 3 and 4 show defective accelerometers.
Traces No. 1 and 2 show satisfactory accelerometers.

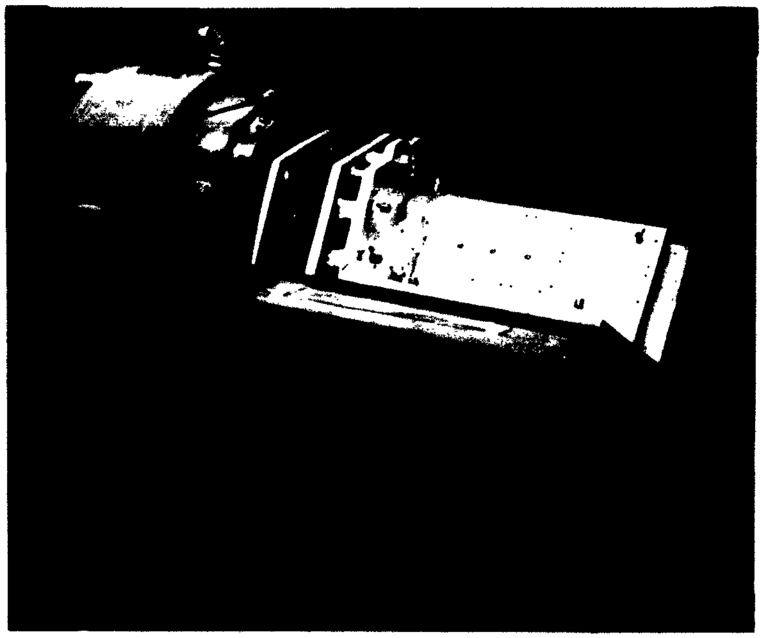

Fig. 9 - Fixture survey (This fixture will test nine specimens simultaneously, three in each axis)

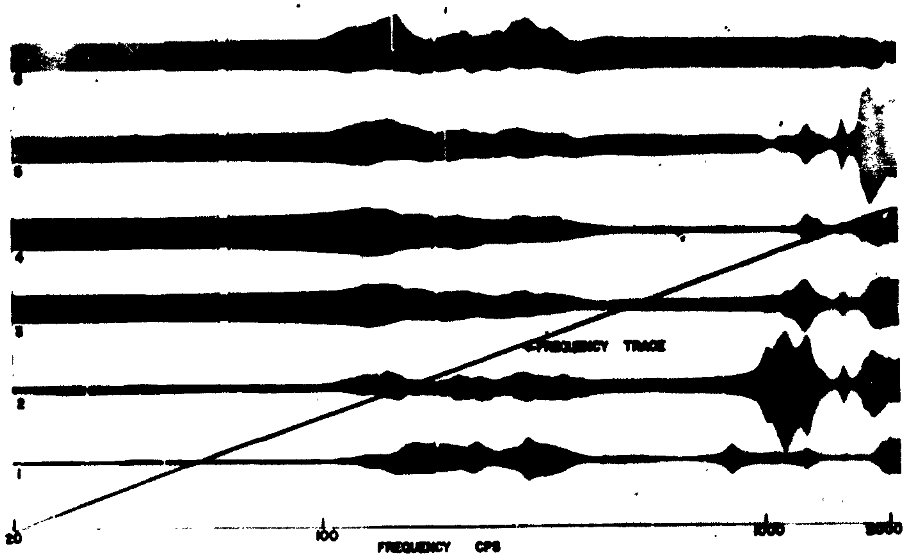

Fig. 10 - Oscillograph record of fixture survey (sine-wave sweep)

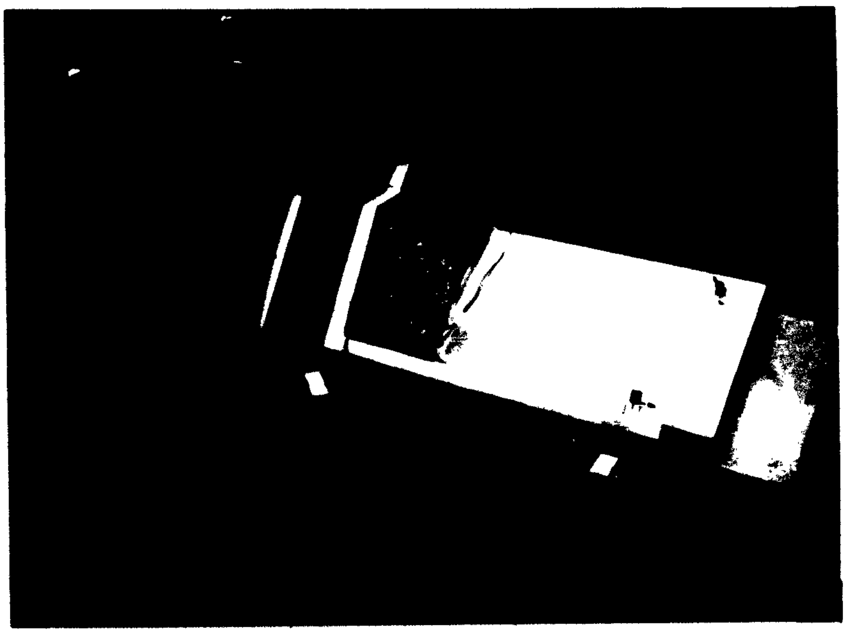

Fig. 11 - Final fixture survey

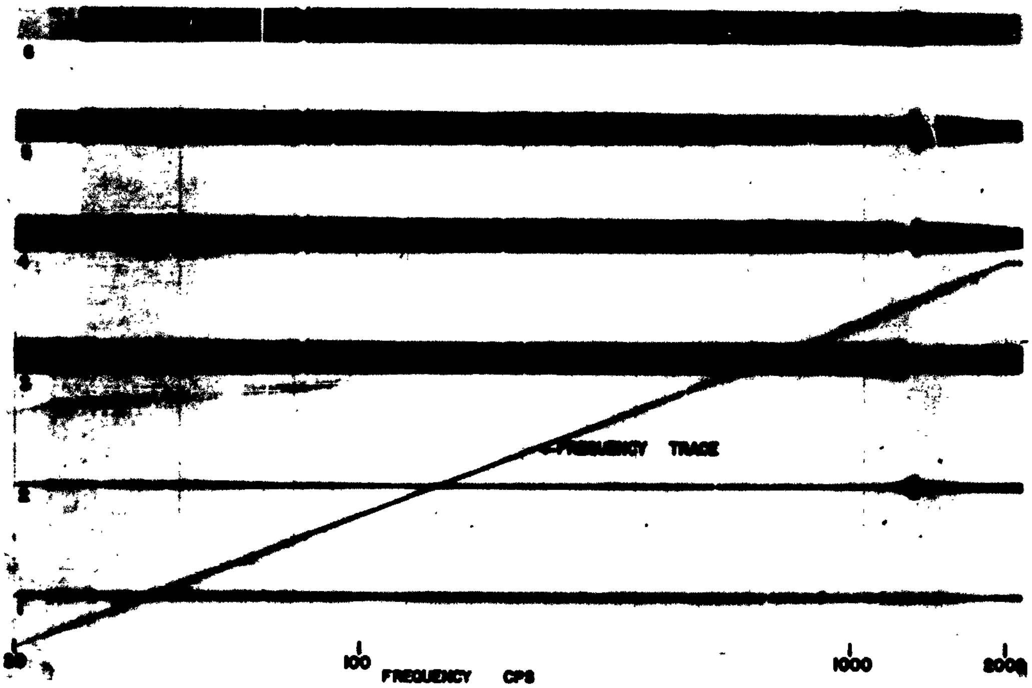

Fig. 12 - Oscillograph record of final fixture survey (sine-wave sweep)

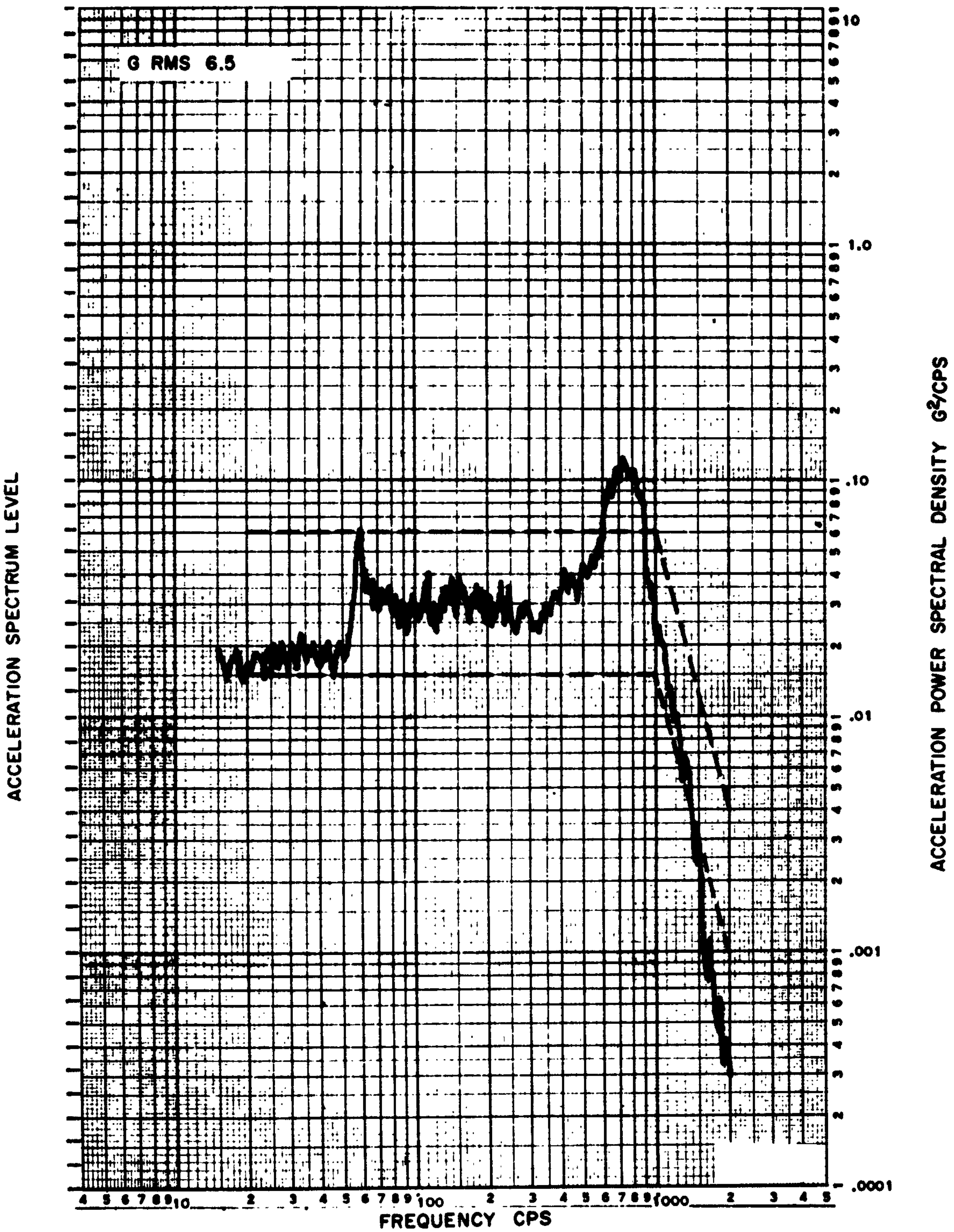

Fig. 13 - Power spectral density versus frequency, first try

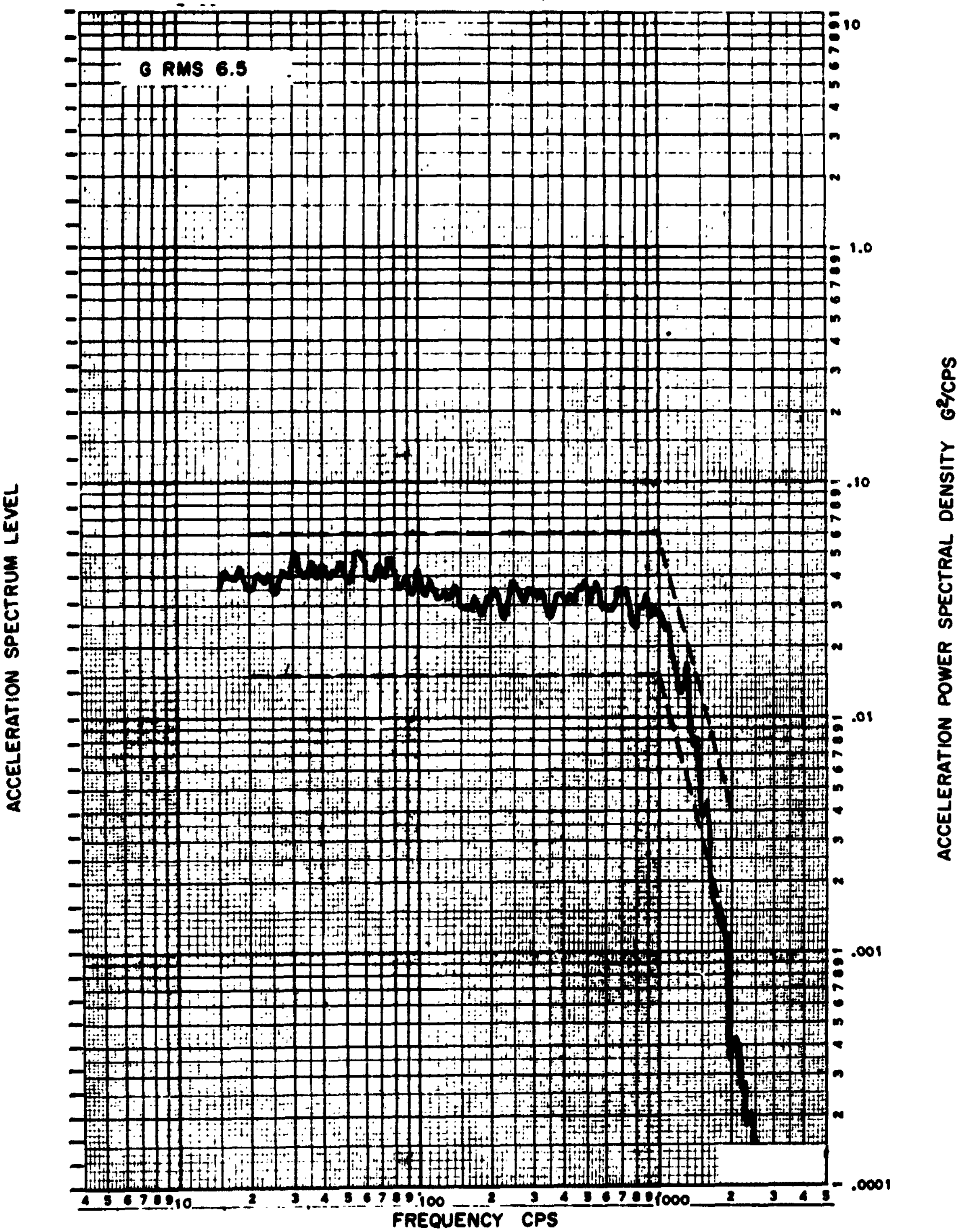

Fig. 14 - Power spectral density versus frequency, final try

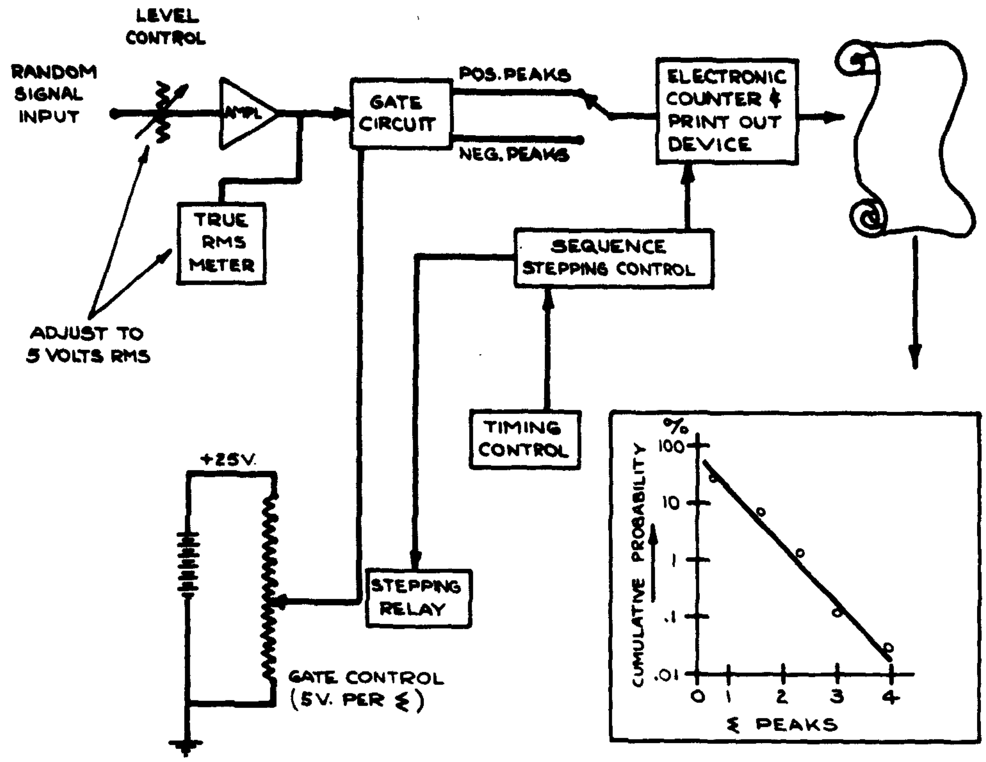

Fig. 15 - Amplitude distribution analyzer (This device counts the number of input signal peaks which exceed each preset level of the gate control)

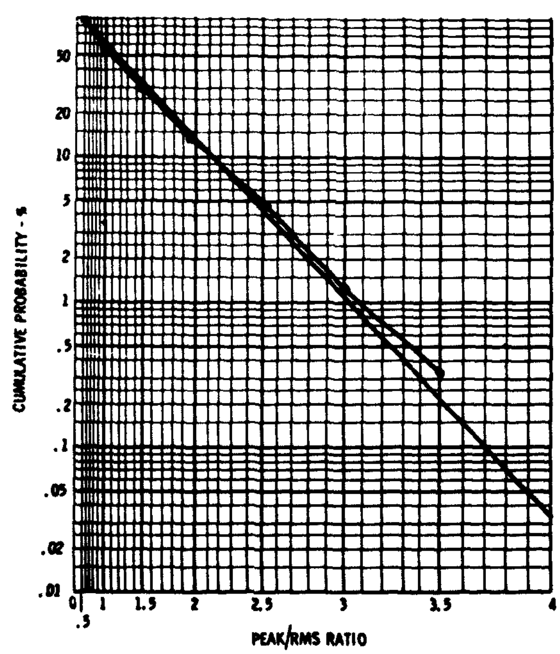

Fig. 16 - Distribution of acceleration peaks

Fig. 17 - Typical comb filter (left rack) and corresponding spectrum shaker (right rack) (automatic spectrum shakers are now available)

DISCUSSION

M. Kaufman (General Applied Science Labs): What is the basis for saying that a square or rectangular filter is the optimum filter for your spectrum analyzer? Is that obvious?

DuBois: It's rather a long story and it involves the humans who look at this data later. The filter shape has an effect on them, and this has an effect on the final data. Should I explain that in more detail? Well, some of the older wave analyzers had the classical filter shape -- the sharp resonant peak -- and when they plotted out data, they tended to plot their own filter shape on the record; a human reviewer looking at this and averaging it, would average it much higher than the data really were.

Colucca (Republic Aviation): Do I understand that when you determine your fixture response you just have a bare fixture with no mass attached to it?

DuBois: Well, actually we continue on with this process. We put the specimens on the fixture and then check it again. I just omitted that detail for the sake of brevity.

D. Schwartz (Kearfott): You mentioned a rude awakening when first using a distribution analyzer on a random noise generator. Will you explain that a little more fully please?

DuBois: Well, the main reason for using a distribution analyzer is to find troubles in your system, a broken spring, a random noise generator, or defects in the electronics; and we find it wise to check our random vibration system with a peak counter from time to time just to be sure that it is working properly.

R. Newman (AC Spark Plug): What was your analysis time constant used in plotting Fig. 14? It looked to me like you had a great deal of smoothing in that figure.

DuBois: I do not remember. It's a five-cycle filter, I recall, but I don't remember the time constant. This could get us into a discussion of tolerances, and tolerances can be quite meaningless. They are anything you want them to be because they depend on the time constant of the analyzer. This new MIL-STD-810 goes into this a little bit.

R. Newman: You mentioned cross spectral analysis. Do you feel it is actually possible to obtain the true transfer function of a mechanical system utilizing steady-state techniques?

DuBois: No I don't. I have been trying to find out, on this trip, just exactly what this cross spectral density analysis technique is good for, and I don't have a real clear answer yet.

R. Newman: Do you feel that random vibration will fully excite resonant components of high q, that is, to a steady-state condition?

DuBois: If I understand your question correctly, I don't see how random vibration can excite high q resonances to a steady-state condition. I would think you would have to use a slow sine-wave sweep to accomplish that.

R. Newman: It would be very difficult to obtain a true transfer function of high q mechanical resonances using random vibration, isn't that true?

DuBois: Excuse me, I don't feel that I am qualified to answer that question. There are, I believe, manufacturers of analysis equipment present at this Symposium that can answer that much better than I can.

J. Ancell (Aerospace): I would like to take a stab at it. I think that you would excite the high q resonance in the same manner that ~~is~~ would be excited by the broadband excitation in the missile flight due to the engine noise or the turbulent buffing pressure fluctuations. So it probably would be a realistic test. Exciting the resonance with a sine-wave sweep, where you really excited it to its maximum would possibly not be the realistic test.

DuBois: Oh I agree it's a realistic test, but I believe the previous gentleman asked if we could excite the sharp resonance to its full peak steady state with random. That is the question I'm not qualified to answer.

R. Kroeger (GE): I would like to take a stab at answering that question. Sometime ago we ran some experiments with printed wiring boards, which is a reasonably simple dynamic system, with various inputs of sinusoidal excitation, random excitation, and combinations of both of these, in various magnitudes. In the simplest case, we were able to apply a uniform spectral density input to this mechanical system on the basis of the square of its transfer function, calculate what the spectrum of the response should look like, and then, by a spectrum analysis of the response, verify that, in fact, that was the response that we would expect. So I would think that the quotient of the spectrum of the response to the spectrum of the input would be the square of the transfer function, and this is exactly what we measured under previous tests.

R. Newman: This doesn't necessarily mean that, if you were to apply a discrete frequency sine-wave to the system and you were to modify it with the transfer function obtained by using this type of analysis technique, that you would truly see the full excitation of a high q resonance. Isn't that correct?

Kroeger: As a matter of fact, in a single degree of freedom system the response is proportional to $\sqrt{q}$. So in the sense of exciting the full resonance you do vibrate at resonance, but you change or filter the response because of the mechanical characteristics of your resonant system. So you do get an rms response that you know something about, it rises to a magnitude which is proportional to $\sqrt{q}$ and in that sense you do excite the full resonance. In the sense that it does not respond to a full q value, where the response is not directly proportional to the q, this is true.

A. Westneat (Gulton Industries): Just in answer to the previous question, we do make transfer function analyzers for random data, and a random input does properly describe the transfer function.

C. Lutz (GE): You spoke a little while ago about digitizing some of this data. Now if you refer to digitizing filtered data, do you have any comments that you would make on the digitizing rate and the response with respect to the analog of a rapidly changing function, perhaps approaching resonance?

DuBois: We, at Boeing, are just getting into this field, and are thinking about the design of a digital data system to digitize random vibration data. We aren't into it deeply enough so that I can answer your question.

* * *

PHASE MEASUREMENT IN VIBRATION TESTING

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Sandia Corporation
Albuquerque, New Mexico

Practical methods are given for using the complex response and complex impedance in solving several problems. Methods of recording complex response and impedance are given, as well as several curves of the sort of transfer functions to be expected. Rapid and practical computer use of complex response is also illustrated.

INTRODUCTION

The response of any point of a linear system is completely determined by the input and by the response transfer function, i.e., the complex ratio of output to input. For very simple systems, it is possible to give the transfer function analytically. For more realistic systems, however, an analytical solution of the transfer function is not practicable, and one must have recourse to a table or graph of the transfer function for various frequencies.

Because the transfer function is a complex ratio; two quantities are required at each frequency. One can record the real and imaginary components (that is, the component in phase with the input and the quadrature component), or the amplitude ratio and phase angle. Vibration tests have traditionally recorded only one of the two required quantities, the amplitude ratio. It will be shown that the utility of a sinusoidal test in which only amplitude is measured, is seriously limited. Whenever phase measurement is mentioned, it is to be understood that the complex transfer function is meant, and that any of the usual means of measurement may be used. All remarks of experimental transfer functions presuppose linearity. Because mechanical systems are invariably nonlinear to some extent, the transfer function should be applied cautiously when environmental levels can be expected to vary widely.

RESPONSE TO ARBITRARY INPUT

The response of a system to sinusoidal input represents a dead end if phase is not measured, since little can be inferred concerning response to nonsinusoidal input. However, use of the complex transfer function permits one to compute the response to any Fourier-transformable input.

A time function, $f(t)$, is Fourier-transformable if the integral

$$\int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

exists and is convergent, if $f(t)$ has only a finite number of discontinuities, a finite number of maxima or minima, and a finite number of infinite points, all in a finite interval $t_1 < t < t_2$. This is, almost invariably, the case with all realizable pulses. The Fourier transform is

$$F(i\omega) = \mathrm{Re}\,[F(i\omega)] + i\,\mathrm{Im}\,[F(i\omega)]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\cos\omega t\, dt - \frac{i}{2\pi}\int_{-\infty}^{\infty} f(t)\sin\omega t\, dt.$$

If we require $f(t) = 0$ for $t < 0$, we can replace the lower limits on the integrals by 0. If, furthermore, $f(t) = 0$ for $t > T_f$, the integrals become

$$F(i\omega) = \frac{1}{2\pi}\int_{0}^{T_f} f(t)\cos\omega t\, dt - \frac{i}{2\pi}\int_{0}^{T_f} f(t)\sin\omega t\, dt.$$

These integrals can be evaluated numerically with a good degree of accuracy, especially when a computer is available. Analog methods exist for direct transformation.

The complex transfer function evaluated by test (Fig. 1) may be represented by:

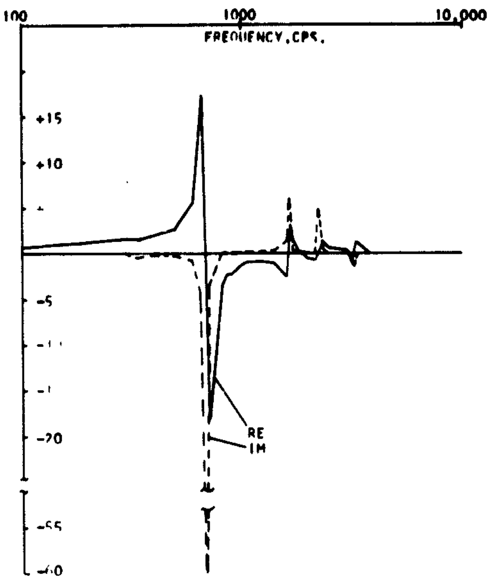

Fig. 1 - A typical response transfer function

$$G(i\omega) = \mathrm{Re}\,[G(i\omega)] + i\,\mathrm{Im}\,[G(i\omega)]\,,$$

and the response in the frequency domain to the input $f(t)$ is

$$H(i\omega) = F(i\omega)\,G(i\omega)$$
$$= \mathrm{Re}\,[H(i\omega)] + i\,\mathrm{Im}\,[H(i\omega)]\,.$$

This response in the time domain is the inverse transform:

$$h(t) = \int_{-\infty}^{\infty} H(i\omega)\,e^{i\omega t}\,d\omega$$

and, because of symmetry,

$$h(t) = 2\int_{0}^{\infty} \mathrm{Re}\,[H(i\omega)]\cos\omega t\,d\omega$$
$$- 2\int_{0}^{\infty} \mathrm{Im}\,[H(i\omega)]\sin\omega t\,d\omega\,.$$

It is to be expected that in practical physical systems, both $H(i\omega)$ and $G(i\omega)$ become very small at high frequency. An upper limit, ω_F, can be chosen such that $H(i\omega)$ is negligibly small for higher frequencies. Then we have approximately,

$$h(t) \approx 2\int_{0}^{\omega_F} \mathrm{Re}\,[H(i\omega)]\cos\omega t\,d\omega$$
$$- 2\int_{0}^{\omega_F} \mathrm{Im}\,[H(i\omega)]\sin\omega t\,d\omega\,.$$

These integrals also can be numerically evaluated with quite high accuracy.

TRIAXIAL VIBRATION

It is well known that in a vibration test conducted primarily in one direction, table motion is generally also present in cross directions. It is also well understood that realistic systems have triaxial response to uniaxial input. Both effects are often lumped under the term cross-talk. Unfortunately, it is impossible to separate these two effects without phase measurement, and any means of recording cross-talk is virtually meaningless without phase information.

Using the subscripts 1, 2, and 3 for the x, y, and z axes, let G_{11} be the response transfer function for output in the x direction due to input in the x direction, let G_{21} be the transfer function for output in the y direction due to input in the x direction, and so on; let I_{11} be input in the x direction in a test presumably in the x direction; let I_{21} be the input in the y direction in a test presumably in the x direction, and so on; let O_{11} be the output in the x direction for a test presumably in the x direction, and let O_{21} be the output in the y direction for a test presumably in the x direction, and so on. One can then formulate the following set of equations:

$$\sum_{j=\ell}^{3} I_{jm}\,G_{\ell j} = O_{\ell m};\quad \ell = 1,2,3;\ m = 1,2,3.$$

We have here nine equations in the nine unknown transfer functions. Equating real and imaginary parts of each side of the equations, we are left with two sets of nine equations each for the real and imaginary parts of the transfer functions, from which the true transfer functions can be computed for each frequency. Modern computer techniques make the solution of such a set of equations extremely rapid, and one can have the true transfer functions for an entire frequency sweep in a very few minutes.

SPRING-MASS MODELS

It is often useful to have a lumped spring-mass model of a system. It is unfortunate, however, that dynamic models from computed spring constants and masses seldom match actual hardware very closely, especially in higher modes. It is possible, however, to use measured transfer functions to compute the characteristics of a spring-mass model to give precisely the same response as the actual hardware. Such a model may then be combined with nonlinear elements.

The equations of motion of a multimass system of n masses can be written in the form

$$m\ddot{x}_\ell + \dot{x}_\ell \sum_{m=1}^{n} D_{\ell m} + x_\ell \sum_{m=1}^{n} (k_{\ell m} - i\gamma_{\ell m}) - \sum_{m=1}^{n} D_{\ell m}\dot{x}_m - \sum_{m=1}^{n} (k_{\ell m} - i\gamma_{\ell m})\, x_m = F_\ell, \quad (\ell = 1, 2, \ldots, n),$$

where

x_ℓ = instantaneous displacement of ℓ-th mass,

m_ℓ = mass of ℓ-th body,

$D_{\ell m}$ = viscous damping coefficient between masses ℓ and m,

$\gamma_{\ell m}$ = solid damping coefficient between masses ℓ and m,

$k_{\ell m}$ = spring constant between masses ℓ and m, and

F_ℓ = external force applied to ℓ-th mass.

The i in the third and fifth terms is a time phase change operator.

Using the transformation $x_\ell(t) = Re_\ell \cos \omega t + Im_\ell \sin \omega t$, where Re_ℓ is the in-phase and Im_ℓ is the quadrature (lagging) component of the amplitude of x_ℓ, and evaluating the equations for $\omega t = 0, \pi/2$; the differential equations become the set of algebraic equations:

$$\left(\sum_{m=1}^{n} k_{\ell m} - \omega^2 m_\ell\right) Re_\ell + \left(\sum_{m=1}^{n} (\omega D_{\ell m} + \gamma_{\ell m})\right) Im_\ell - \sum_{m=1}^{n} k_{\ell m} Re_m - \sum_{m=1}^{n} (\omega D_{\ell m} + \gamma_{\ell m})\, Im_m = Re\,(F_\ell)$$

$$\left(\sum_{m=1}^{n} k_{\ell m} - \omega^2 m_\ell\right) Im_\ell - \left(\sum_{m=1}^{n} (\omega D_{\ell m} + \gamma_{\ell m})\right) Re_\ell + \sum_{m=1}^{n} (\omega D_{\ell m} + \gamma_{\ell m})\, Re_m - \sum_{m=1}^{n} k_{\ell m} Im_m = I_m(F_\ell).$$

This set of $2n$ equations can be readily solved for Re_ℓ and Im_ℓ for all values of ω. Conversely, one can use measured values of Re_ℓ, Im_ℓ, and ω as constants and solve for the values of m_ℓ, $D_{\ell m}$, $\gamma_{\ell m}$, and $k_{\ell m}$. For those who prefer a modal approach, it appears possible to adapt the method to the modal equations.

A verification has been carried out on a small scale. A five-mass system was used. The system was made simple enough that the constants could be computed analytically, but sufficiently complex that results would not be trivial. Masses were measured and spring constants computed. Damping constants were estimated (all damping was assumed to be solid). The response transfer functions were then computed. Sinusoidal vibration and impedance tests were then carried out. Then the constants of the spring-mass system were computed from the experimental transfer functions. Finally, the response transfer functions were computed for the new spring-mass model. The computed and experimental transfer functions for the upper mass are compared in Fig. 2.

RESPONSE TO MULTIPLE INPUTS

When a system has several input points and the service environment is known at each input point, it is not possible to infer the response of any part of the system to simultaneous input at all supports without phase measurement. However, if the system is tested with input at each point in turn, and complex response is measured at all other input points and at the output point, one can readily compute output for simultaneous input. Let a_{ij} be the acceleration at support point j, with input at point i, let $a_{i\,out}$ be the output acceleration for an input at point i, and let G_j be the transfer function from input point j to the output point. One then has the set of n equations:

$$\sum_{j=1}^{n} a_{ij}\, G_j = a_{i\,out}, \quad (i = 1, 2, \ldots, n).$$

Since the quantities are complex, one has actually the set of $2n$ equations:

$$\sum_{j=1}^{n} Re\,[a_{ij}\, G_j] = Re\,[a_{i\,out}]$$

$$\sum_{j=1}^{n} Im\,[a_{ij}\, G_j] = Im\,[a_{i\,out}],$$

which can be rapidly and easily solved for the transfer functions G_j. The phase angle between inputs would seldom be known in service, and the service environment may in fact be random. In the case of sinusoidal or nearly sinusoidal inputs, it is probably best to choose the phase angle between inputs that maximizes output, since the phase angle could probably be expected to assume any value in service. For random input, phasing of inputs has no meaning, but phase measurement is still necessary in computing the transfer functions.

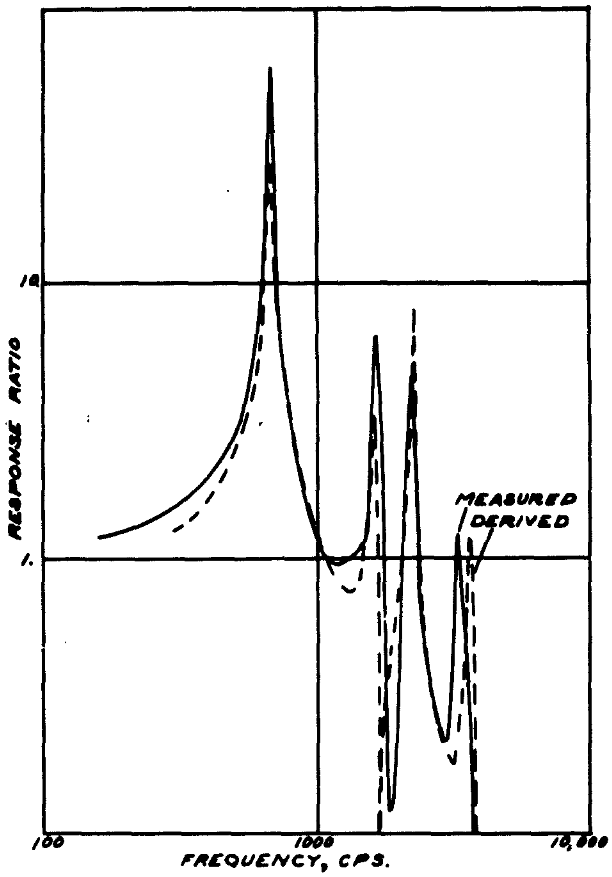

Fig. 2 - Measured and derived responses

IMPEDANCE METHODS

The concept of mechanical impedance appears to lead to great improvements in vibration testing. Unfortunately, the improvements are quite limited if only amplitude of impedance is measured. However, great forward strides are indeed possible if both amplitude and phase are combined.

Mechanical impedance is defined as the complex ratio of a force to the motion associated with the force. Owing to the commonly used electrical analogies, velocity is often chosen as the motion to be used, but mechanical impedance need not be tied to electrical consideration, and it is the author's opinion that the concept will have greater acceptance among vibration engineers with nonelectrical backgrounds if mechanical and electrical impedances are firmly divorced. Since acceleration is the motion most commonly measured, it is most convenient to define impedance as the complex ratio of force to acceleration, and this definition will be used throughout. However, if it is desired to use the force-velocity ratio, one can simply replace acceleration by velocity in the following sections.

CHANGE OF ENVIRONMENT WITH CHANGE OF HARDWARE

If environment is measured with a certain piece of hardware (e.g., a missile flight with dummy payload), it should be clearly understood that different hardware will generally experience a different environment. It can thus be said that the payload helps to determine its own environment. The impedance method allows one to predict the change in environment with change in structure. It is mandatory, however, that phase be measured for impedance measurements to be useful.

If the motion of the attachment point is measured under service conditions without payload, the computation of input with payload is quite simple. One need have no knowledge whatever of the actual forces producing the motion, nor need one know any details of the actual structure involved. In fact, the "black-box" method will be used throughout the impedances. Where equivalent circuits are shown, no attempt is made to indicate the actual structure.

One can measure the impedance of the supporting structure without load. From this, one can infer a fictitious force which would create the measured acceleration if acting through the measured impedance. If the no-load motion in service is a_o, and the no load back impedance is Z_o, the fictitious force is $F_o = a_o Z_o$.

If the measured driving point impedance is Z_{pl}, the motion of the load point with payload installed is

$$a_1 = \frac{a_o Z_o}{Z_o + Z_{pl}} .$$

If the measurement of a_o is made with an actual but known load, say, Z_k, then

$$a_1 = \frac{a_c (Z_o + Z_k)}{Z_o + Z_{pl}} .$$

The assumption is made here that there is no feedback from the payload to the input force. In these calculations, one must use the complex impedance. The magnitude of the impedance cannot be used alone to predict the expected environment.

PREDICTION OF INPUTS AT MULTIPLE SUPPORT POINTS

For the simplest case, assume two points of support. Assume further that measurements of support point motion have been made under service conditions with no load. When a flight without load is impossible (as in missile flights), the method may be modified for loads of known impedance.

An equivalent circuit is shown in Fig. 3(a), where F_i is a fictitious force which causes the same acceleration at points 1 and 2 through the fictitious structure as do the actual forces through the actual structure. The following equations can be written:

$$a_1 Z_1 + (a_1 + a_2) Z_o - F_i = 0$$

$$a_2 Z_2 + (a_1 + a_2) Z_o - F_i = 0.$$

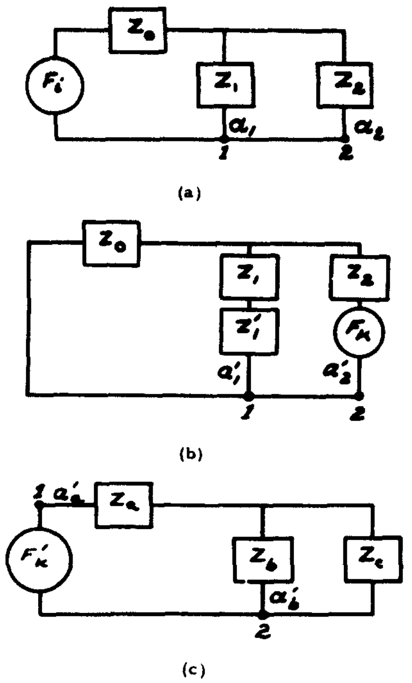

Fig. 3 - Equivalent circuits for prediction of inputs at multiple support points

If a known impedance, Z_1' (such as a fairly heavy mass), is placed on one support point and a known force, F_k, is applied at the unloaded point, the circuit is then as shown in Fig. 3(b). Acceleration at the input point and load point should be measured as well as input force.

The following two equations can now be written:

$$a_2' Z_2 + a_1' Z_1 = F_k - a_1' Z_1'$$

$$a_2' Z_2 + (a_2' - a_1') Z_o = F_k.$$

The quantities Z_1, Z_2, Z_o, and F_i can now be computed by solution of the preceding four equations.

Impedance measurements are now made on the payload, first applying force at point 1 with point 2 free. The equivalent circuit is then as shown in Fig. 3(c). Then,

$$a_b' Z_b + a_a' Z_a = F_k' .$$

We now apply force at point 2 with point 1 free, which yields

$$a_a'' Z_a + a'' Z_b = F_k''.$$

These two equations can now be solved for Z_a and Z_b. Then

$$Z_c = \frac{Z_b a_b'}{a_a' - a_b'} .$$

The problem is now completely solved. The motion of points 1 and 2 with payload installed under service conditions can be found by simultaneous solution of the following equations for A_1 and A_2:

$$A_1 (Z_1 + Z_o + Z_c + Z_a) + A_2 (Z_o + Z_c) = F_i$$

$$A_1 (Z_1 + Z_a) - A_2 (Z_2 + Z_b) = 0.$$

There is a possible check on this method, since the known load can be placed alternately on points 1 or 2. Impedances computed from each point should check closely. Note that field measurements of a_1 and a_2 do not require knowledge of phase. However, all other tests require phase measurements. Again, the method is still perfectly useful if a_1 and a_2 are random.

EFFECT OF CHANGE OF LOAD PATHS OR COMPONENTS

A structure may be tested in one configuration. It may then be desirable to predict the effect of making a considerable change in the structure. For example, suppose a new component is to be added to an existing system. By measuring the complex response transfer function of the existing structure—G_o, the driving point impedance of the old structure—Z_o, the complex driving point impedance of the new component—Z_1, and the complex response transfer function of the new component—G_1, one can compute the effect of the new structure quite easily. An input acceleration a_o requires a force $F_o = a_o Z_o$, for the original structure and gives an acceleration at the mounting point of the new structure of $a_1 = a_o G_o$. Adding the new component requires a force

$$F' = \left[\frac{\left(\frac{Z_o}{1-G_o}\right)\left(\frac{Z_o + Z_1 G_o}{G_o}\right)}{\frac{Z_o}{1-G_o} + \frac{Z_o + Z_1 G_o}{G_o}}\right] \cdot a .$$

The new output at the mounting point is

$$a_1' = \frac{a_o Z_o G_o}{Z_1 G_o (1-G_o) + Z_o}$$

for the same input a_o, and the output at the top of the new component is:

$$a_2 = a_1' G_1 = \frac{a_o Z_o G_o G_1}{Z_o + Z_1 G_o (1-G_o)} .$$

Here again, phase measurements are required.

The examples given are not intended to be exhaustive, but are merely illustrative of the types of problems that can be solved by the response transfer functions. It is hoped that other examples will be suggested.

ACKNOWLEDGMENT

The assistance of Mr. F. J. Perdreauville and Mr. L. T. Wilson in performing experimental work is gratefully acknowledged.

DISCUSSION

M. Kaufman (General Applied Science Labs): Isn't it difficult to measure the phase between two items unless you do extremely fine filtering, when you are dealing with random forcing functions? How do you define phase when the function is not sinusoidal?

Mr. Murfin: Ordinarily the phasing cannot be measured with a random input function. If the sinusoidal input isn't clean, post filtering is required. This is generally a good practice anyway. We measure the phase on a sinusoidal test with close filtering to exclude any possible harmonics, any dirt in the input or the response.

A. Westneat (Gulton Industries): By using cross spectral density techniques you can indeed use random inputs. You get a random output out of the structure you are measuring and you can very easily determine the in-phase- and the quadrature-components of the transfer function. Obviously, the transfer function stays the same regardless of the excitation and if you do use cross spectral density techniques you can extract both components very easily.

Mr. Murfin: That's true. That's at least theoretically possible. I've never tried that.

Mr. Rouault (GE): This is the first such session I have attended on this particular area and I am frankly confused about the mathematics. It has been my understanding that a Fourier transform existed only in certain special cases wherein the function itself was transformable into a Fourier series. In this particular case you have a device, like most electric circuits which does not move in a single plane, but has a minimum of three and possibly five or six motions. This, therefore, would imply that you can get a transfer function and a valid phase only in the fundamental mode. Would you care to comment on this?

Mr. Murfin: I'm not sure that I understand the question. The trick here is that physically realizable pulses are all transformable. I can imagine untransformable pulses.

Mr. Rouault: All physically realizable pulses are not transformable unless the circuit itself is linear.

Mr. Murfin: I mentioned the word linear at the very start of the paper and I should have interjected it in practically every paragraph. If nonlinearities are known, all bets are off.

Mr. Rouault: Well every mechanical system I've ever worked with has been nonlinear.

Mr. Murfin: That is correct to some extent. Now, whether the nonlinearities are so great as to make the use of the Fourier transform impossible is something that you have to decide on each system.

Mr. Rouault: Are you familiar with the tables of phase for the unit slope transformation derived in a Bell Systems Technical Journal paper about 12-14 years ago, in which given the frequency response of a linear circuit you can construct the phase response? But this is only true for a branch of circuits which satisfy some very special requirements. Otherwise, it is not applicable at all. This really leads to the question of the applicability of an equivalent single-stage analog to a very complicated three-dimensional structure. This is the reason why I question its applicability to anything but simple sine-wave measurements taken at a very low rate through a filter, and, therefore, I don't think that any random noise or swept function would give you the answers you're looking for.

Mr. Murfin: I will say that I have tried predicting responses by Fourier-transform methods and they come out at least as well as any other prediction.

D. Keast (BBN): I think it is worth mentioning here, because we've tried to do some of these things too, that these techniques only seem to work when you can treat the system as you have indicated by lumped parameters. You run into difficulties in situations where you are dealing with methods of mechanical energy propagation that are dispersive, such as flectural waves in the skin, in bars, in a truss, and so on, where different frequencies travel at different speeds down the bar. Then you get into some problems that I at least have never seen amenable to mathematical treatment by transfer function analysis.

Mr. Murfin: Yes, that's true. One must have lumped parameters.

J. Barrett (Watervliet Arsenal): In regard to making a transformation from a Fourier analysis to a pulse, we've done this several times with fairly good results. The fellows have predicted the pulses they would get from a given setup using a Fourier-analysis technique and the pulse comes out pretty close. They gave a paper on this over in Springfield just last week. I would like to make the comment on nonlinearities. There is a method for handling nonlinearities, I don't know if it applies here, but the servo people have used the describing function technique which fits right in with Fourier transforms. Essentially, you make a describing function which gives you the response of a nonlinear system, and maybe the first one, two, or three harmonics.

Mr. Murfin: As a matter of fact, when you filter input and output to the frequency of excitation, you are getting a describing function of the fundamental only. This is actually the technique that we used. Now how good it is in the presence of wild nonlinearities, I have no idea. The model that we used had no wild nonlinearities. However, once you have made a lumped spring mass model you can put in whatever nonlinearities you want to if you know they are there.

Mr. Barrett: The nonlinearities I would think of would be sticking and nonlinear springs travel limits, things like this. Are these nonlinearities of the same kind you would be concerned with? Nonlinear spring rates and things like this?

Mr. Murfin: Yes, things like this can be put into the lumped spring mass model.

Mr. Barrett: I think these things are handled with describing functions. I'm not sure though.

Mr. Murfin: It may be so, but I haven't really gone into it.

* * *

AUTOMATED MECHANICAL IMPEDANCE MEASURING INSTRUMENTATION SYSTEM

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This paper describes the automated mechanical impedance and phase angle measuring system developed by, and currently in use at, the Portsmouth Naval Shipyard. The theory of operation of the system is discussed as well as the practical aspects of data reliability and repeatability and system operating characteristics.

INTRODUCTION

It is generally accepted that the concepts and theorems of mechanical impedance provide a powerful tool for the analysis of vibration problems. Many activities, both government and private, are currently engaged in vibration studies involving these concepts and theorems applied to practical problems such as the transmission of sound or the coupled response of mechanical systems. If such studies are to be fruitful, one of the principal requirements is the reliable measurement of the property of mechanical impedance. Since most structures of engineering interest are relatively complex, the task of determining their impedance properties, with any degree of completeness, is one of considerable magnitude and expense.

It is the intent of this paper to describe the automated mechanical impedance measuring system currently in use at the Portsmouth Naval Shipyard and to show its merits with respect to the rapidity with which mechanical impedance data may be determined and recorded and the reliability of the data.

THE PROBLEM

Essentially, the problem of determining the mechanical impedance of a structure is one of determining the complex ratio of a force quantity to a response quantity as a function of frequency either at a single point or between two points on the structure. This ratio is defined as

$$Z = \frac{Fe^{j\omega t}}{Ve^{j(\omega t + \phi)}} = \left|\frac{F}{V}\right| e^{-j\phi} \qquad (1)$$

where

Z = mechanical impedance (lb sec/in.),

F = maximum force amplitude (pounds),

V = maximum velocity amplitude (in./sec),

ϕ = phase angle (degrees),

ω = frequency (rad/sec),

e^j = complex operator, and

t = time (seconds).

The conventional means of establishing this ratio is to excite the structure, at some frequency of interest, with an electrodynamic shaker acting through an impedance head consisting of a force gage and an accelerometer. The amplitude of the output signals from the impedance head force gage and accelerometer (or a transfer accelerometer) are measured, as well as the phase angle between the two signals. With this data the mechanical impedance is calculated for the structure for that particular frequency. If this procedure is repeated for a number of frequencies, the mechanical impedance spectrum for the structure can be determined. An example of this technique is shown in Fig. 1 for the box beam shown in Fig. 2. This particular curve required several hours effort for its construction and is at best only an approximation,

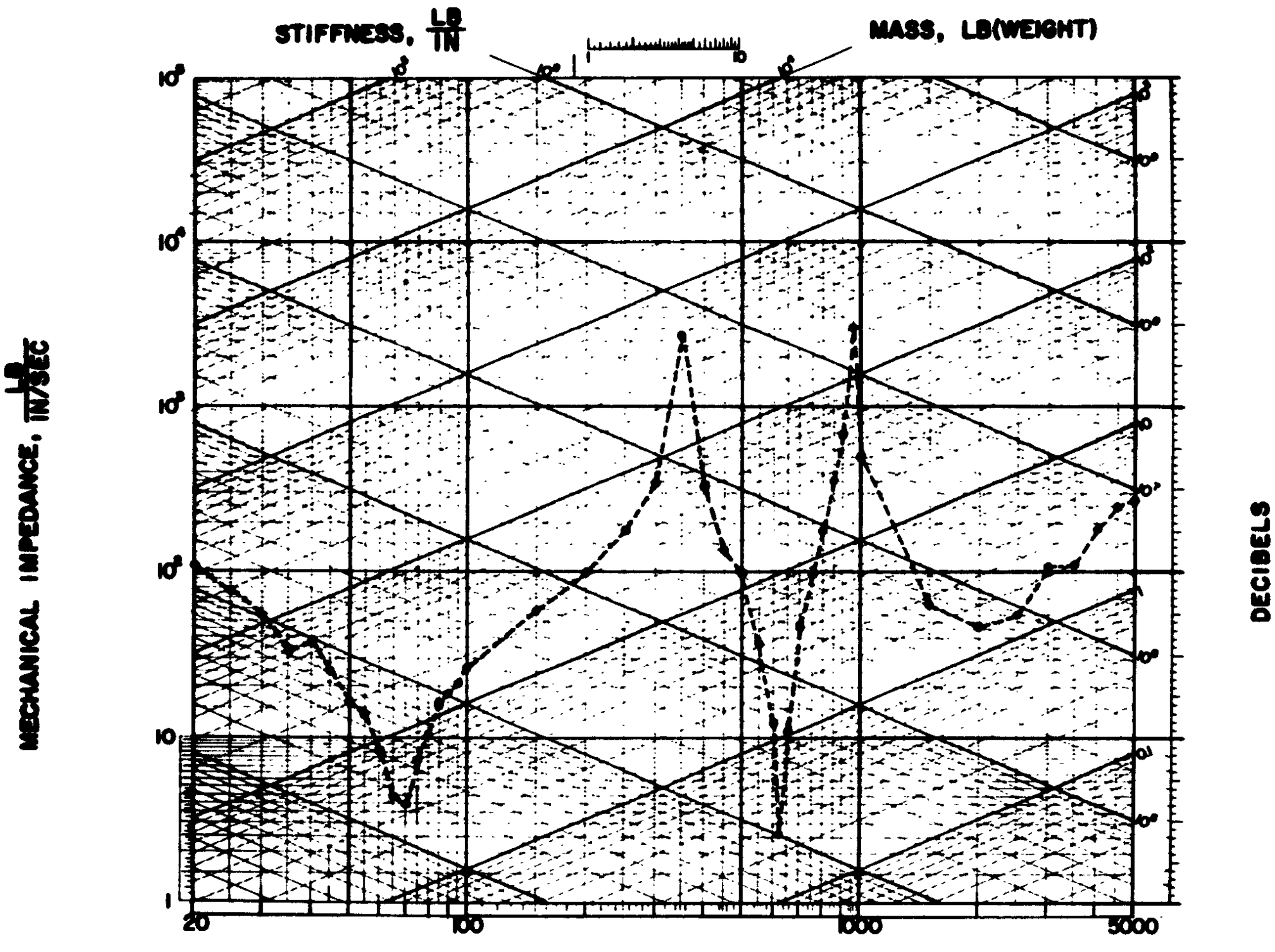


Fig. 1 - Measured impedance of box beam of Fig. 2

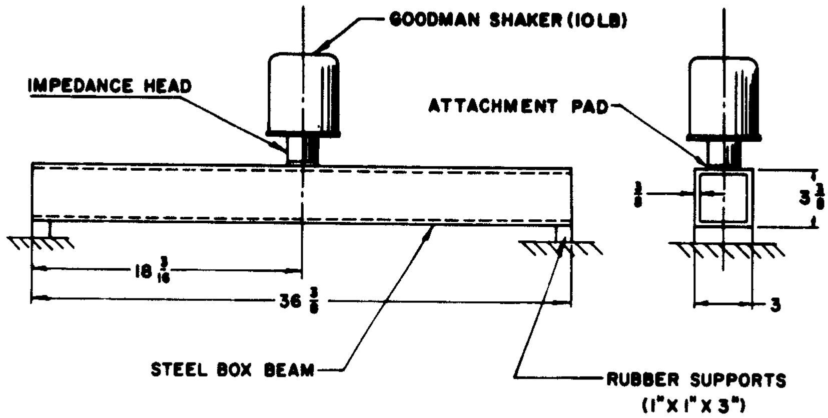


Fig. 2 - Mechanical impedance test beam

since the data are taken at discrete frequencies over the spectrum. It is readily seen, then, that when mechanical impedance information is determined by this method, the measurement process becomes a major portion of any task involving mechanical impedance concepts. There can be little doubt, therefore, that methods for increasing the speed and accuracy of the measurement process are desirable.

THE AUTOMATIC SYSTEM

The instrumentation system employed at the Portsmouth Naval Shipyard has as its designed function the direct recording of mechanical impedance information versus frequency.[1] This system (schematic, Fig. 3) is essentially an automatic data reduction system which continuously processes the signals from the impedance head force gage and accelerometer (or a transfer accelerometer) and records the calculated impedance magnitude, as well as the phase angle magnitude, and polarity on standard impedance and phase angle graph paper.

An example of the readout of this system for the same impedance curve of Fig. 1 is shown in Fig. 4. A comparison of the two curves shows that much greater detail is resolved by the automatic plot than would be feasible by the conventional method. In addition, it required only about 15 minutes for the automatic plot to plot its curve, as against approximately 4 hours required to plot the curve of Fig. 1. At present, three band sweeps are made to record the impedance magnitude, the phase angle magnitude and the phase angle polarity versus frequency. However, as the proper recorders become available, this information will be recorded during one band sweep and the time reduced to approximately 5 minutes per measurement.

This system consists almost entirely of commercially available components with the exception of the frequency readout potentiometer and the phase angle polarity detector. During the 16 months it has been in operation, it has been used for some 700 impedance measurements made for the Bureau of Ships on various submarine machinery foundations, and has required very little maintenance care.

IMPEDANCE MAGNITUDE

The principal computation to be performed by the instrumentation system is for the impedance magnitude as follows:

$$|Z| = \frac{CE_f E_\omega}{E_a} \qquad (2)$$

or, since all of the variables of Eq. (2) are available in logarithmic form,

$$\log_n |Z| = \log_n C + \log_n E_f + \log_n E_\omega - \log_n E_a, \qquad (3)$$

where

$|Z|$ = Mechanical impedance magnitude (lb sec/in.),

C = Constant of proportionality,

E_ω = Voltage proportional to the frequency of excitation (rad/sec),

[1] J. E. Smith, "Mechanical Impedance Measurements on Fatigue Model F-A1-8, BUSHIPS Project S-F013-11-01, Task 1352," Portsmouth Naval Shipyard Technical Report T100-006 (April 1961).

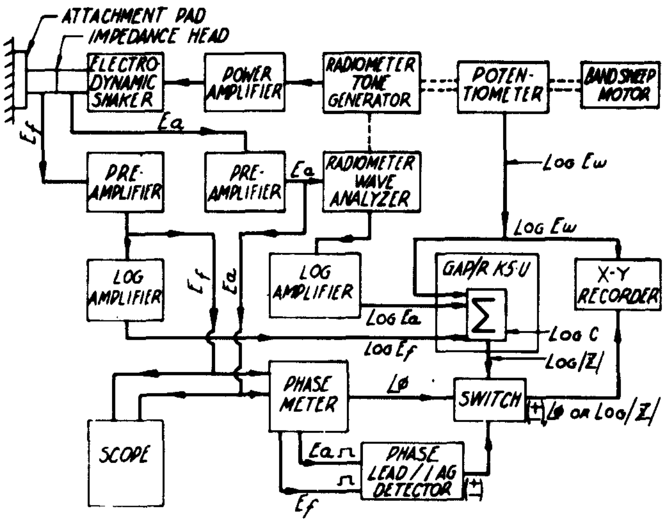


Fig. 3 - Mechanical impedance instrumentation block diagram

E_f = Voltage proportional to the force driving an unknown impedance (pound),

E_a = Voltage proportional to the acceleration response of an unknown driven impedance (in./sec^2), and

n = Common logarithmic base.

The device employed in this system to perform the required computation is the Universal Linear Operator, GAP/R K5-U. This component is an analog computer and performs the following computation:[2]

$$e = e_o + 10^m \sum_{j=1}^{4} (a_j e_j), \tag{4}$$

where

e = Output from K5-U (±50 volts),

e_i = Input to K5-U (±50/a_i volts),

e_o = Constant voltage applied to the output of the K5-U (±60 volts),

a_i = Multipliers for K5-U input (0 ± 11.1), and

10^m = Multiplier for the sum (0 - 10^3).

If the terms of Eqs. (3) and (4) are equated as follows:

$$\log_n |Z| = e$$

$$\log_n C = e_o$$

$$\log_n E_f = 10^m a_1 e_1$$

$$\log_n E_\omega = 10^m a_2 e_2$$

$$\log_n E_a = 10^m a_3 e_3,$$

it is evident that $\log_n |Z|$ will be available at the output of the K5-U. As shown in the instrumentation block diagram, Fig. 3, all input voltages to the K5-U are proportional to the logarithm of the variables E_f, E_a and E_ω to various arbitrary bases (X, Y, and W). Rewriting Eq. (3) as follows:

$$\log_n Z = \log_n C + (\log_n X) \log_x E_f + (\log_n Y) \log_Y E_\omega - (\log_n W) \log_w E_a, \tag{5}$$

it can be seen that

$$10^m a_1 = \log_n X \tag{6}$$

$$10^m a_2 = \log_n Y \tag{7}$$

$$10^m a_3 = \log_n W, \tag{8}$$

and that, therefore, it is the function of the K5-U to convert all input signals to a common

[2]Geo. A. Philbrick Researchers Inc., Boston, Mass., "The Lightning Empiricist," Issue No. 7 (March 1959).

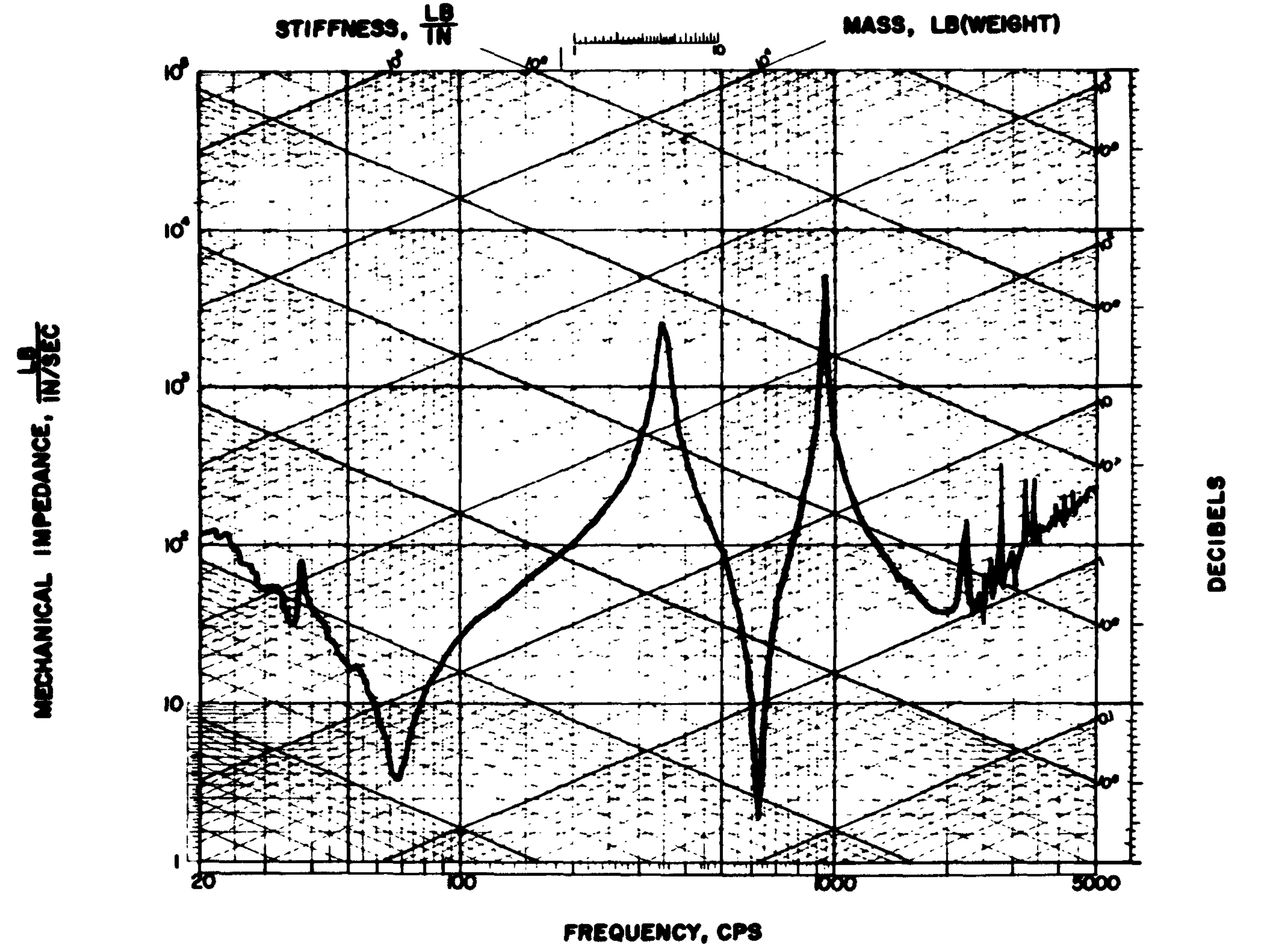


Fig. 4 - Automatic impedance plot for box beam of Fig. 2

logarithmic base and perform the summation operation. The output, then, of the K5-U is $\log_n |Z|$ which may be plotted directly on standard impedance graph paper versus the logarithm of the frequency.

The instrumentation system is calibrated for impedance magnitude by first assuring that the logarithm of each signal is raised to the proper logarithmic base and then measuring a known impedance in order to determine the unknown constant of proportionality. As shown in Eq. (5), each signal must be multiplied by a constant in order to convert it to the proper logarithmic base before summation. This is most easily done by solving Eqs. (6), (7), and (8) for the correct K5-U multiplier (10^{m_a}) as follows:

$$10^{m_{a_1}} = \log_n X = \frac{\log_n(E_{f_1}/E_{f_2})}{\log_x(E_{f_1}/E_{f_2})} \tag{9}$$

$$10^{m_{a_2}} = \log_n Y = \frac{\log_n(E_{\omega_1}/E_{\omega_2})}{\log_y(E_{\omega_1}/E_{\omega_2})} \tag{10}$$

$$10^{m_{a_3}} = \log_n W = \frac{\log_n(E_{a_1}/E_{a_2})}{\log_w(E_{a_1}/E_{a_2})}, \tag{11}$$

where E_1 and E_2 are voltages measured at any convenient point in the system when the variables E_f, E_ω, and E_a are considered individually. Since the value of the denominator in these expressions may be observed as a voltage change at the input to the K5-U and the value of the numerator may be calculated directly, the proper coefficient or multiplier for each input signal to the K5-U may be readily determined. If the impedance head and shaker are then attached to a known impedance, such as a 300-pound weight at low frequencies, the remaining unknown of Eq. (5) ($\log_n C$) is readily determined. This is accomplished by applying a constant voltage (e_0) to the output of the K5-U so that the correct impedance curve for the weight is obtained, as shown in Fig. 5.

The usefulness of this system depends to a great extent on its dynamic range and dynamic response characteristics. It is advantageous to have as large a dynamic range as possible in order to avoid recalibration of the system during a measurement because of excessive dynamic range of the data. The dynamic range of this system is determined primarily by the range of the logarithmic amplifiers employed. These components each have a range of 50 db, indicating that a maximum impedance range of 100 db is available. This range has been found to be adequate for most mechanical systems where the impedance data varies approximately 60-80 db.

The dynamic response characteristics of the system are such that, for most applications, a band sweep rate of 5 minutes from 20 cps to 5 kc is satisfactory. However, since the proper band sweep rate is ultimately determined by the mechanical system being measured, the data is frequently checked by stopping the band sweep motor at frequencies of high Q or resonances and observing the change in the recorded impedance magnitude, if any.

In the foregoing, the discussion was limited to the determination of velocity impedance as given by Eq. (2). By rearranging the quantities in Eq. (2), any one of the six ratios of impedance and mobility could be recorded with equal facility.

PHASE ANGLE MAGNITUDE AND POLARITY

The phase angle magnitude between the force signal and the acceleration response signal is measured in this system with a commercial phase angle meter. This meter accepts the amplified sinusoidal output signals from the force gage and accelerometer and provides a dc output proportional to the phase angle. The phase angle is recorded as an angle between 0 and 180 degrees versus the logarithm of the frequency of excitation. This method of determining phase angles has been found to be satisfactory. When high signal to noise ratios are available phase angle readings accurate to within 1 degree can be made.

The polarity of the phase angle, referenced to the applied force, is determined in the system by means of a special switching circuit employing signals from the phase meter for its operation. This circuit is constructed to produce a voltage output when the acceleration response signal lags the driving force signal and no voltage output when the response is leading. This polarity output is plotted on the phase angle graph paper and is the indication of the phase angle sign. An example of the readout of the system for phase angle magnitude and polarity is shown in Figs. 6 and 7 for the box beam shown in Fig. 2 and the 300-pound weight shown in Fig. 5, respectively.

DATA RELIABILITY

The information recorded by this system agrees in all respects with data obtained in the conventional manner. However, this is no

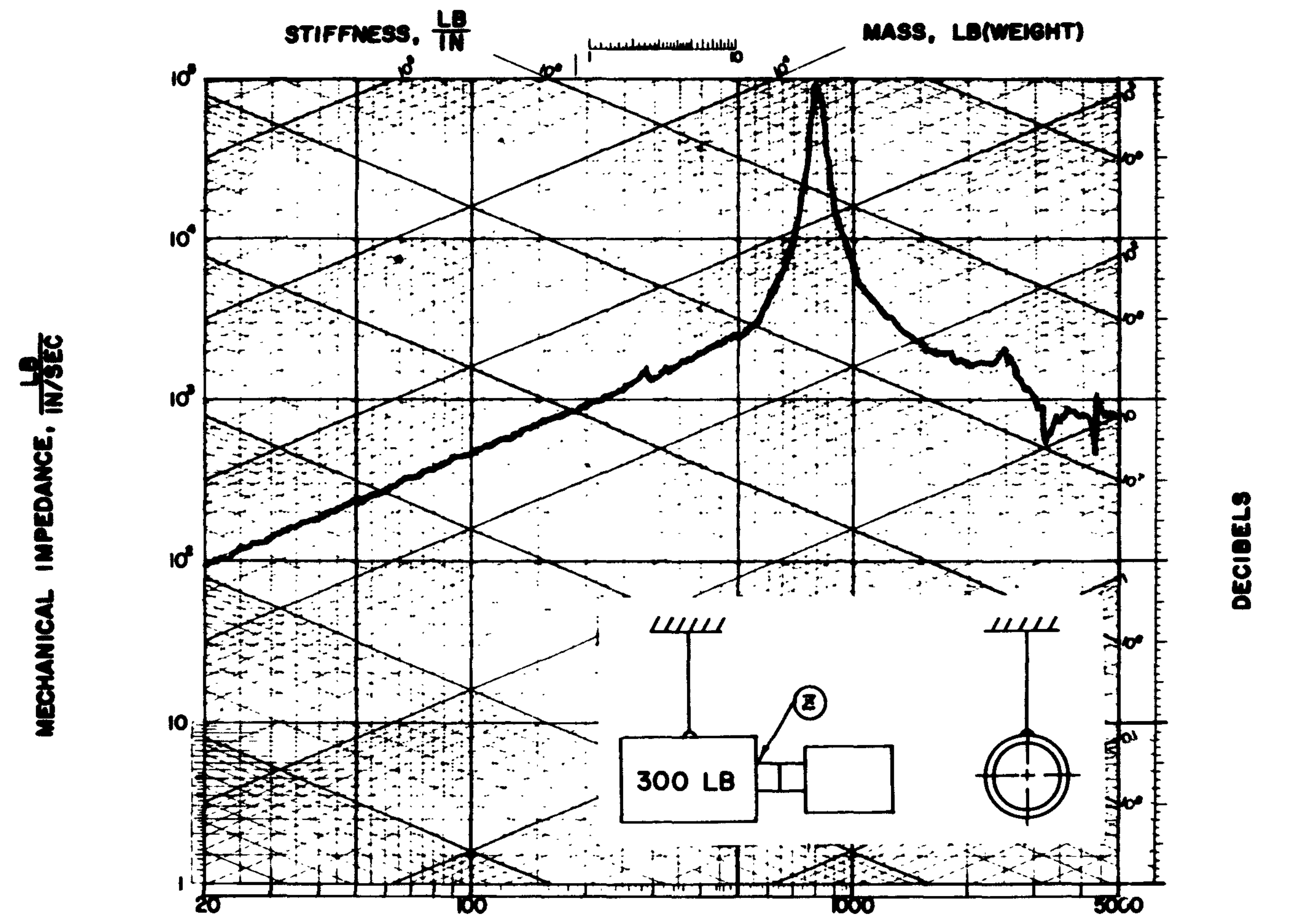


Fig. 5 - Driving point impedance (300-pound weight)

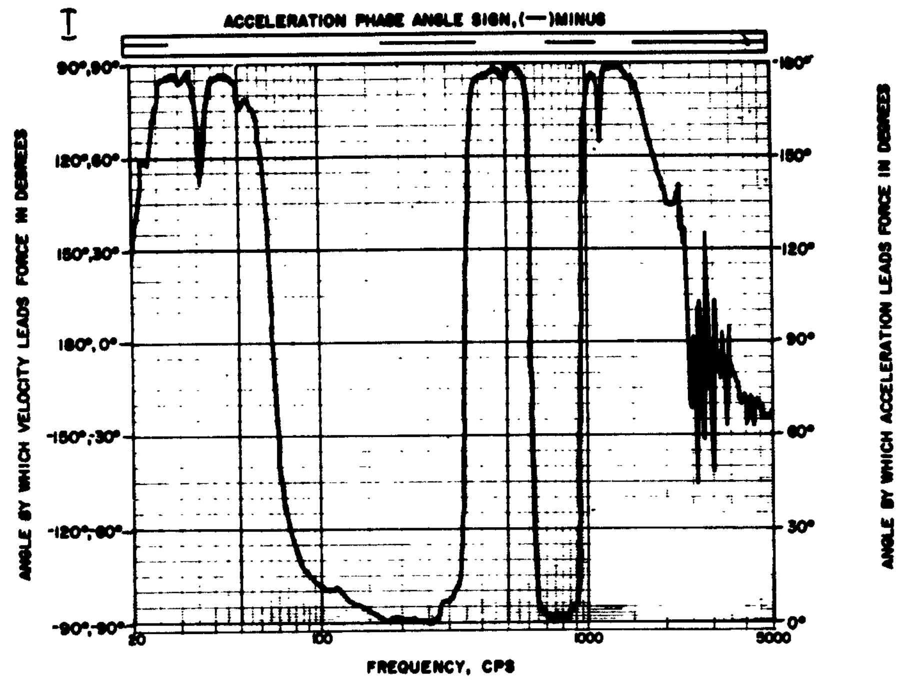


Fig. 6 - Automatic phase angle plot for box beam of Fig. 2

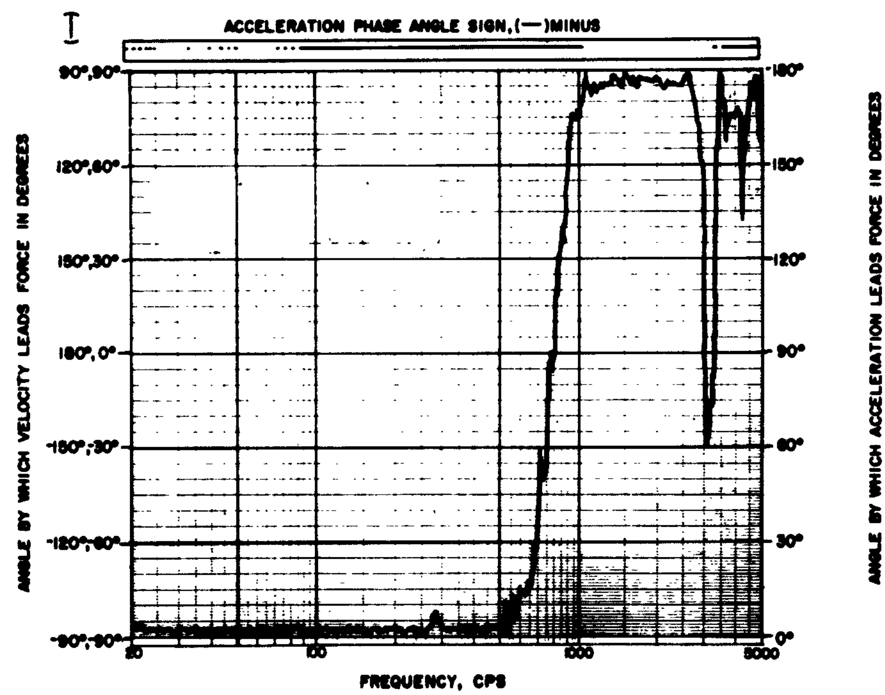


Fig. 7 - Driving point phase angle (300-pound weight)

assurance that mechanical impedance data obtained by either method is correct as defined by Eq. (1). Since no known mechanical impedance information is available for use as a standard for comparison purposes, except for the most simple systems, impedance measurements on a complex structure cannot be checked for accuracy in this way. However, an indication of the validity of the data can be had by using it to predict the behavior of a mechanical system which may then be checked by observation.

The system shown in Fig. 8 is one which has been used for this purpose.[3] This system consists of an exciter and a receiver connected at three points as shown schematically in Fig. 9. The exciter is a flat plate with a permanently attached shaker for a source of internal excitation and the receiver consists of a second flat plate with three permanently attached standard Navy resilient mounts.

Utilizing the theorems of mechanical impedance the following set of vector equations may be written about the terminals of Fig. 9 when the exciter and receiver are connected for a single degree of freedom normal to the plane of the plates.

[3]J. E. Smith, "Preliminary Report on the Development of Analysis Techniques for Submarine Machinery Installations by Mechanical Impedance Methods, BUSHIPS Project S-F013-11-01," Portsmouth Naval Shipyard Technical Report T819-023 (July 1962).

EXCITER TERMINALS

$$
\begin{aligned}
V_1 &= \frac{F_1}{Z_{1-1}} + \frac{F_3}{Z_{1-3}} + \frac{F_5}{Z_{1-5}} + V_{o_1} \\
V_3 &= \frac{F_1}{Z_{1-3}} + \frac{F_3}{Z_{3-3}} + \frac{F_5}{Z_{3-5}} + V_{o_3} \\
V_5 &= \frac{F_1}{Z_{1-5}} + \frac{F_3}{Z_{3-5}} + \frac{F_5}{Z_{5-5}} + V_{o_5}
\end{aligned}
\tag{12}
$$

RECEIVER TERMINALS

$$
\begin{aligned}
V_2 &= \frac{F_2}{Z_{2-2}} + \frac{F_4}{Z_{2-4}} + \frac{F_6}{Z_{2-6}} \\
V_4 &= \frac{F_2}{Z_{2-4}} + \frac{F_4}{Z_{4-4}} + \frac{F_6}{Z_{4-6}} \\
V_6 &= \frac{F_2}{Z_{2-6}} + \frac{F_4}{Z_{4-6}} + \frac{F_6}{Z_{6-6}}
\end{aligned}
\tag{13}
$$

and

$$
V_7 = \frac{F_2}{Z_{2-7}} + \frac{F_4}{Z_{4-7}} + \frac{F_6}{Z_{6-7}} \tag{14}
$$

Since, for any pair of mating exciter and receiver terminals the velocities are equal in magnitude and polarity and the forces existing between them are equal in magniture and opposite in polarity, Eqs. (12) and (13) may be combined accordingly as follows:

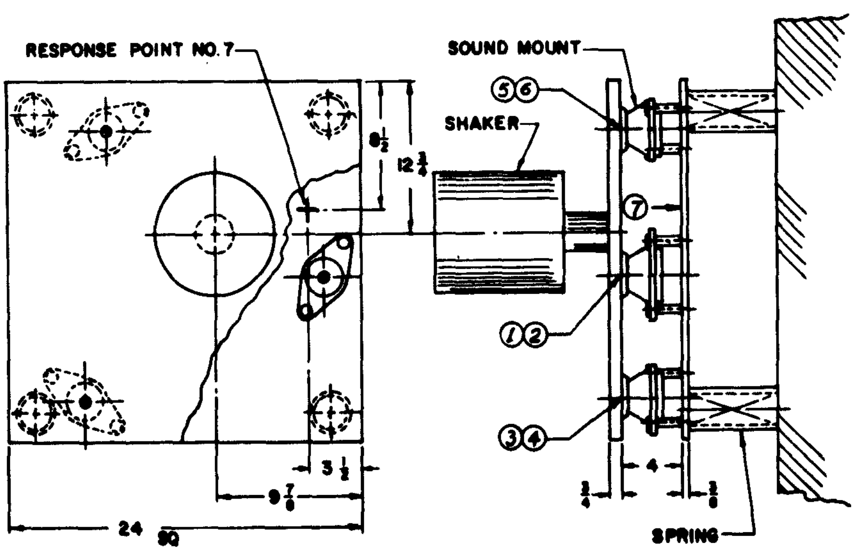


Fig. 8 - Three-connection exciter-receiver system

$$F_2\left[\frac{1}{Z_{1-1}}+\frac{1}{Z_{2-2}}\right]+F_4\left[\frac{1}{Z_{1-3}}+\frac{1}{Z_{2-4}}\right]+F_6\left[\frac{1}{Z_{1-5}}+\frac{1}{Z_{2-6}}\right]=V_{o_1}$$

$$F_2\left[\frac{1}{Z_{1-3}}+\frac{1}{Z_{2-4}}\right]+F_4\left[\frac{1}{Z_{3-3}}+\frac{1}{Z_{4-4}}\right]+F_6\left[\frac{1}{Z_{3-5}}+\frac{1}{Z_{4-6}}\right]=V_{o_3} \qquad (15)$$

$$F_2\left[\frac{1}{Z_{1-5}}+\frac{1}{Z_{2-6}}\right]+F_4\left[\frac{1}{Z_{3-5}}+\frac{1}{Z_{4-6}}\right]+F_6\left[\frac{1}{Z_{5-5}}+\frac{1}{Z_{6-6}}\right]=V_{o_5},$$

NOTE: Subscripts refer to network terminals

where

$F = |F|\, e^{j\alpha}$ = Terminal force (pounds),

$V = |V|\, e^{j\alpha}$ = Terminal velocity (in./sec),

$Z = |Z|\, e^{-j\phi}$ = Mechanical impedance (lb sec/in.),

$V_o = |V_o|\, e^{j\alpha}$ = Terminal free velocity (in./sec),

α = Phase angle referenced to V_{o_1} (degrees), and

ϕ = Phase angle referenced to driving force (degrees).

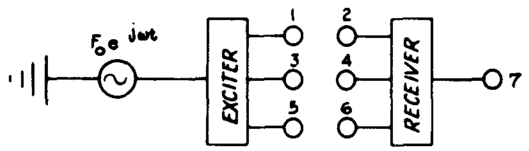


Fig. 9 - Schematic diagram for three-connection, single degree of freedom, exciter-receiver system of Fig. 8

The exciter and receiver mechanical impedance data required by Eqs. (14) and (15) were measured, as well as the required free velocity of the exciter terminals. With this data, Eqs. (15) were solved for the terminal forces which were then used to evaluate Eq. (14) for the response of the receiver terminal (V_7). The results of this calculation are shown in Fig. 10 as a predicted response curve from 20 cps to 5 kc.

After completion of all impedance and free velocity measurements, the two systems were connected and the actual response of the receiver terminal (V_7) was measured. This actual response for purposes of comparison. It was found that the standard error for this particular test was 10 db. Since this was considered to be an excessive spread in the predicted responses and it was unlikely that this effect was due wholly to errors in the impedance measurements, a second system was assembled in order to determine the effect of considering three degrees of freedom of translational motion on the predicted responses.[4] This system, shown in Fig. 11, is again made up of an exciter and a receiver, but is connected at only one point. Since this system is to be considered for three degrees of freedom, its schematic representation (Fig. 12) is equivalent to that used for the previous example and Eqs. (14) and (15), with the proper change in subscripts, are applicable. Solving these equations again for the response of the receiver terminal (V_3) and subsequently measuring its response yielded the curves shown in Fig. 13. The standard error for this set of measurements is approximately 5 db, which shows a significant increase in accuracy over the previous set shown in Fig. 10. Since the effects of rotational motion have not been considered, it is, of course, not possible to assert from these results that the remaining error is due wholly to incorrect impedance information. Nor can it be assumed that the error lies entirely in the neglect of rotational motion, as the following illustration will show.

The impedance curves shown in Fig. 14 were taken on the channel beam or receiver used in the system shown in Fig. 11. These impedance measurements were not used in the calculation of the response of that system, but are included here for illustration purposes only. These curves show the results of measuring transfer impedances between points (1) and (2) on the beam by first driving the beam at point (1) and measuring its response at point (2), and then driving at point (2) and measuring response at point (1). As can be seen from Fig. 14, there is a rather large discrepancy between the two curves at some frequencies which, it might be argued, by the Theorem of Reciprocity ought not to be present. Since it cannot be determined which curve is correct and one is at liberty to

[4] "Supplementary Report on the Development of Analysis Techniques for Submarine Machinery Installations by Mechanical Impedance Methods," Portsmouth Naval Shipyard Technical Report T819-029 (under preparation).

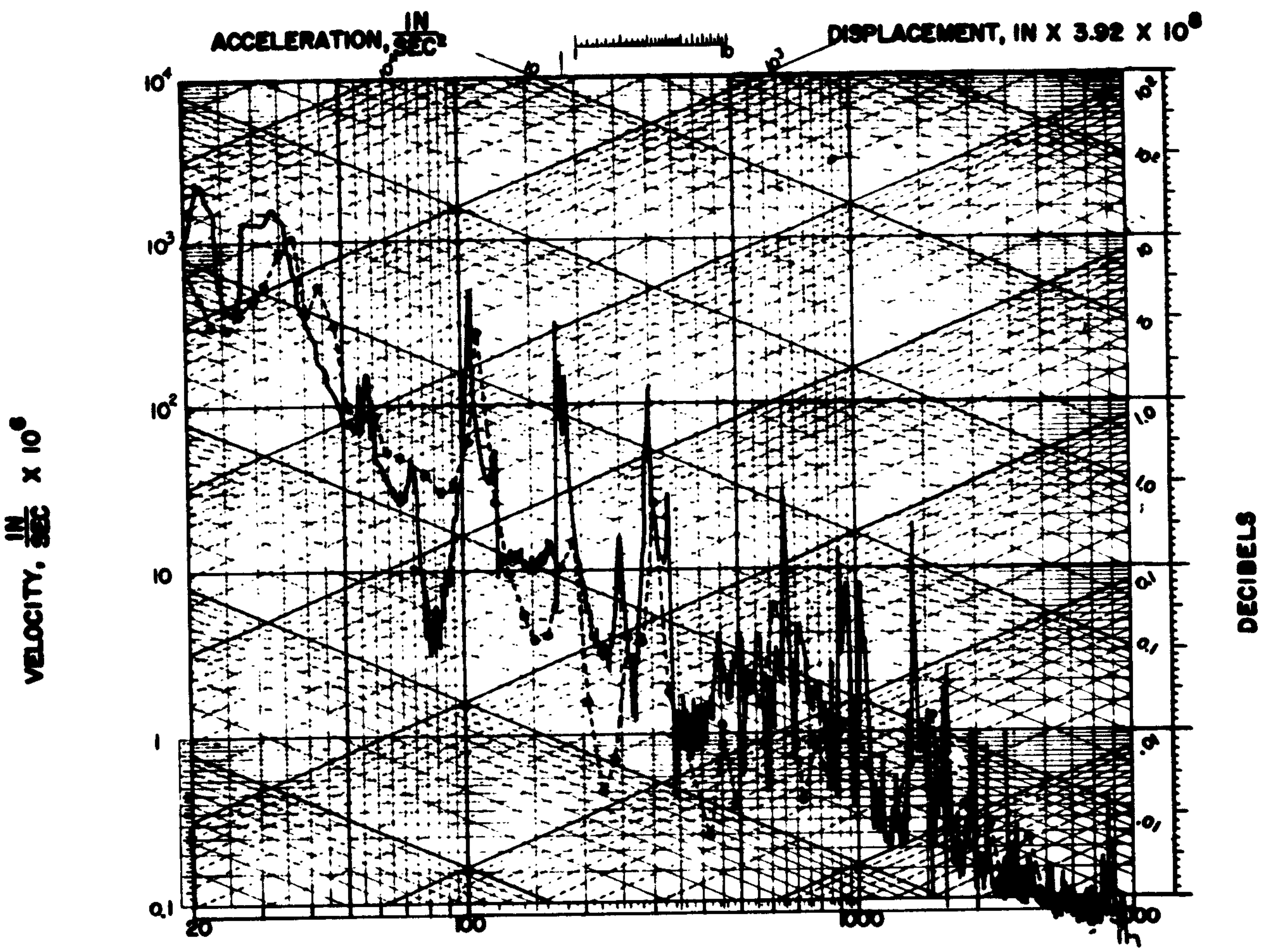


Fig. 10 - Velocity response exciter-receiver system of Fig. 9

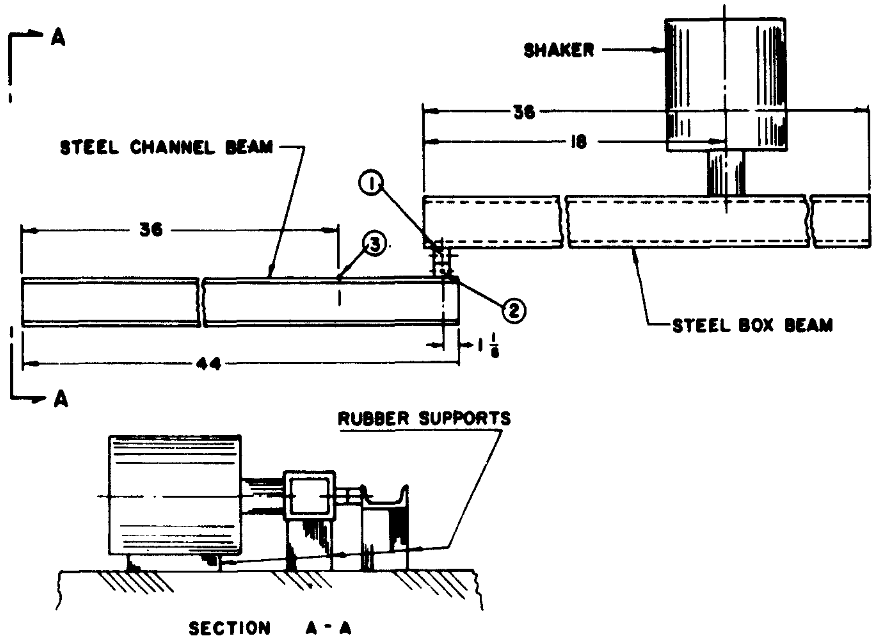


Fig. 11 - Single-connection exciter-receiver system

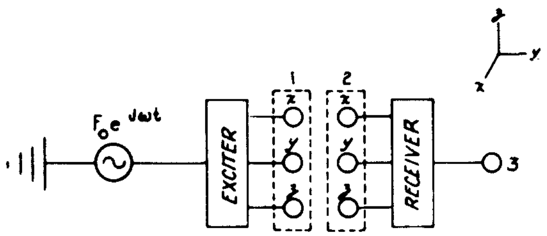


Fig. 12 - Schematic diagram for single-connection, three degree of freedom, exciter-receiver system of Fig. 11

use either curve where transfer impedance information is required, there is a strong possibility that incorrect data could be taken in any set of measurements. As it turns out neither curve is necessarily correct. By applying the same analysis method to the channel beam schematic diagram (Fig. 15) as was used in the previous examples, it can be shown that, for three degrees of translational motion, the measurement of transfer impedances, such as those shown in Fig. 14, is governed by the following relationships:

$$Z_{1-2} = \frac{F_x}{V_a} = Z_{ax} - \frac{Z_{ax}}{V_a}\left[\frac{F_y}{Z_{ay}} + \frac{F_z}{Z_{az}}\right],$$
$$Z_{2-1} = \frac{F_a}{V_x} = Z_{ax} - \frac{Z_{ax}}{V_x}\left[\frac{F_b}{Z_{xb}} + \frac{F_c}{Z_{xc}}\right], \qquad (16)$$

where the subscripts x, y, and z refer to a coordinate axes system about point (1) and the subscripts a, b, and c are a coordinate axes system about point (2). The measurement of forces and velocities at point (1) is made along the coordinate axis (x) and at point (2) along axis (a). These equations show that for strong coupling between the axes of the coordinate systems, one would not expect the two measurements, Z_{1-2} and Z_{2-1}, to agree. Furthermore, neither measurement can be correct unless the sum of the terms within the brackets is close to zero. This condition arises when the coupling is small and a high transfer impedance is present. Needless to say, a similar situation exists for driving point measurements.

It should be noted that an opportunity for increasing the accuracy of impedance measurements is apparent from Eqs. (16). Assuming

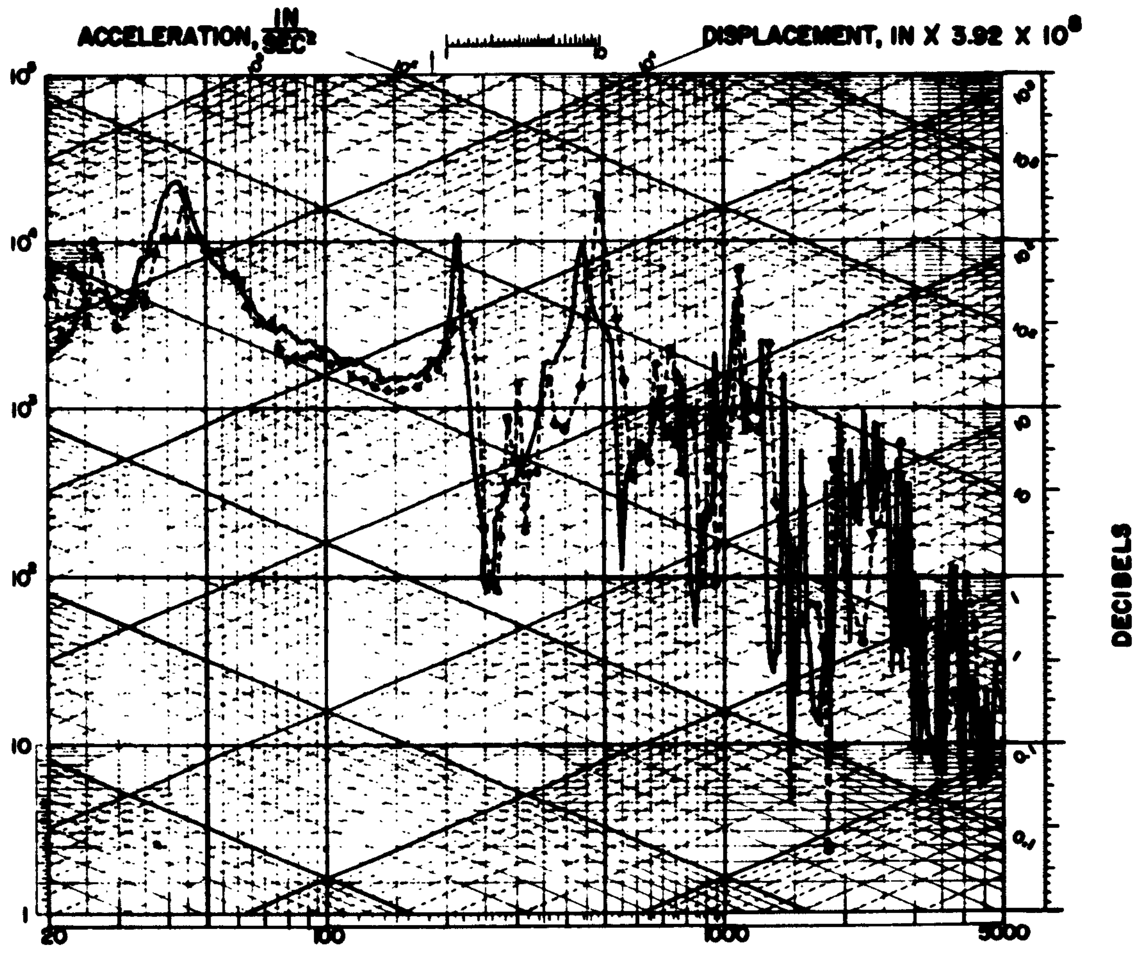
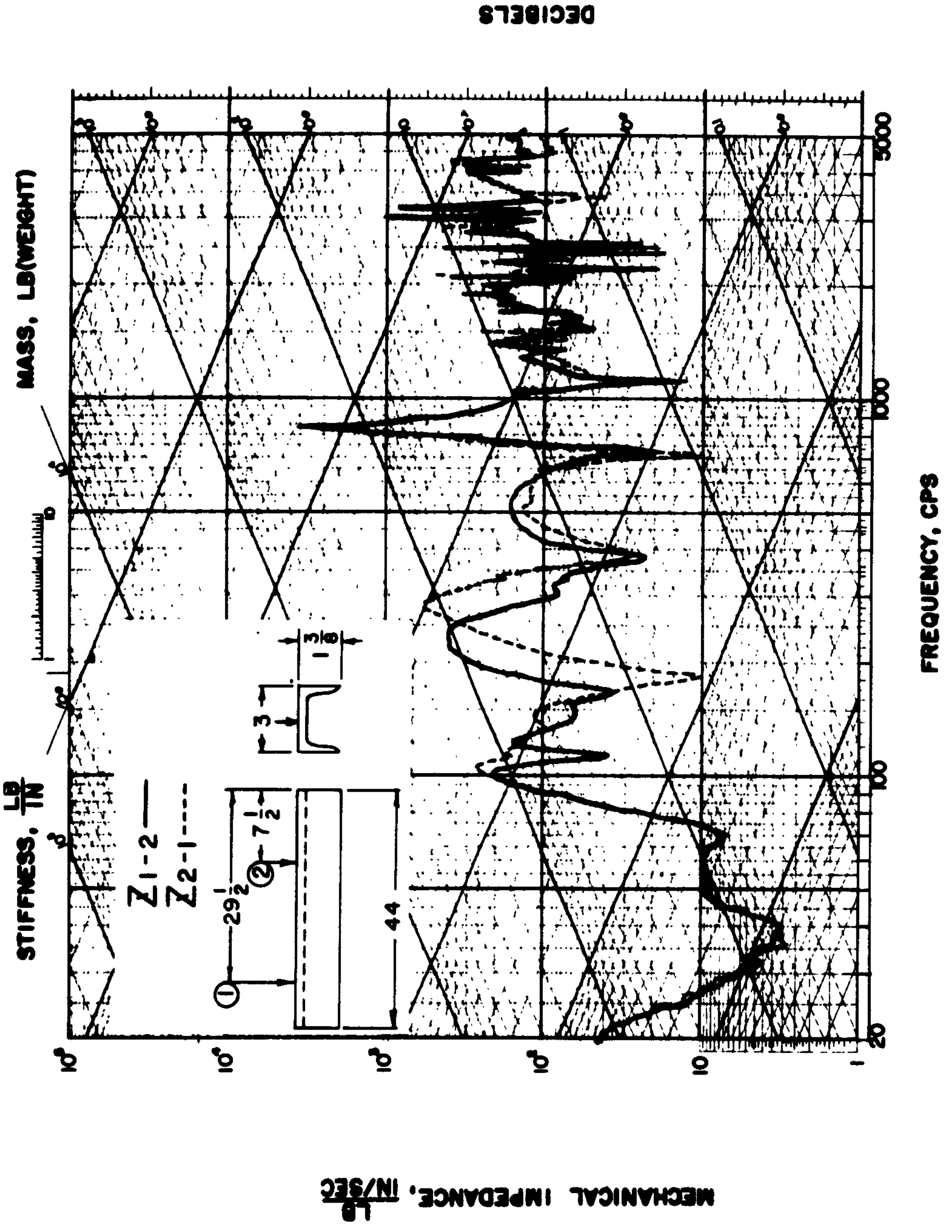


Fig. 13 - Velocity response exciter-receiver system of Fig. 12 (measured______and predicted - - - -)



STIFFNESS, LB/IN
MASS, LB(WEIGHT)
DECIBELS
MECHANICAL IMPEDANCE, LB/IN/SEC
FREQUENCY, CPS
Z_{1-2} ——
Z_{2-1} ----
$29\frac{1}{2}$
$7\frac{1}{2}$
44
3
$1\frac{3}{8}$
20
100
1000
5000

Figure 14

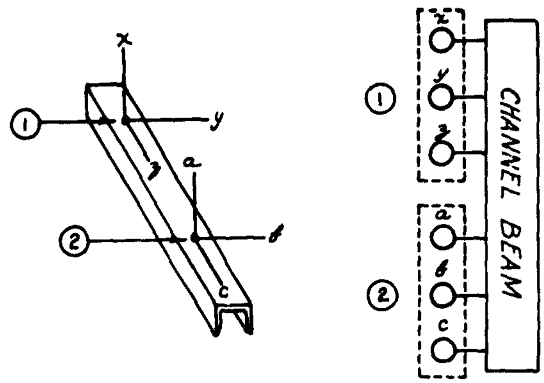


Fig. 15 - Schematic diagram for transfer impedance measurement of Fig. 14

that methods were available for exciting a structure without rigidly coupling the exciter to it, say by means of magnetic coupling, then the force quantities within the brackets of Eqs. (16) would be reduced to zero, and the measurement process would then be more apt to yield the required information.

From the foregoing the absolute precision of mechanical impedance data cannot be established. However, it does provide some insight into the validity of impedance data, as presently measured, and indicates the degree of accuracy to be expected in calculations employing these measurements.

CONCLUSIONS

It can be seen that the use of an automated impedance measuring system, such as described herein, provides a very real advantage in dealing with impedance concepts when applied to complex systems requiring many measurements. In addition, it would seem that there is no loss in accuracy, as compared to the conventional method of measurement, and that the data may be used to advantage where the required precision is of the order experienced in the examples shown. However, the evidence suggests that there is an inherent limitation in the accuracy of mechanical impedance information as it is presently being determined and that additional effort is needed in the areas of impedance heads and shakers in order to assure that the measurement process yields correct data.

ACKNOWLEDGMENTS

Appreciation is expressed to H. Sheffy for his work on the instrumentation system and for performing the tests and to S. E. Woods for her work in preparing the illustrations.

DISCUSSION

D. Stern (GE): You mentioned that you sweep three times, once for impedance, once for phase, and once for polarity. Do you have a two pen plotter?

Mr. Smith: Yes.

Mr. Stern: If you were driving the vibrator with a B&K and a band sweep motor this would give you the frequency signal for the frequency axis for the plotter. Then if you had two pens writing, one could plot the phase and the other would plot the impedance. You could do it in one shot. And if the phase angle meter was for 360 you wouldn't need the polarity. Do you agree with all that?

Mr. Smith: I think you would need a polarity. It has been our experience that the response can lag just as often as it can lead, especially on transfer impedances.

Mr. Stern: You have 0 to 180, is that right?

Mr. Smith: That's right.

Mr. Smith: Suppose it was 0 to 360.

Mr. Smith: Well that's swell, I don't know how you can get it . . .

Mr. Stern: Whose phase meter do you have that gives you a dc signal directly?

Mr. Smith: AD-YU.

R. Bouche (Endevco): On the automatic read-out system, do you have a means for automatically correcting for the impedance between the force transducer and the structure?

Mr. Smith: No, we do not. It has been our experience that this does not produce a tremendous error in the data and I think other investigators have found the same thing.

R. Mustain (Douglas): I would like to know whose impedance head are you using? Is it one you developed yourself?

Mr. Smith: No, we actually have two, we are using one made by Wilcoxon and one provided by the David Taylor Model Basin.

Mr. Mustain: What kind of accuracy are you getting on your phase measurements or have you found this to be a problem?

Mr. Smith: No, I think with lab conditions we can get plus or minus 1 degree with a low background.

Mr. Mustain: This is in agreement with what the vendor has advertised on the phase meter? And you are satisfied with it?

Mr. Smith: Yes we are.

Mr. Mustain: How about repeatability data? Do you think you can repeat your answers well enough with the same instrumentation?

Mr. Smith: I intended to say something about repeatability in the first place. It seems to be a somewhat larger problem now because we are attaching our shaker rigidly to the system and you couldn't expect perfect repeatability, that is, it might repeat the same errors.

Mr. Mustain: This brings up another question then. Your shaker head itself, you don't show how you are holding it, there is bound to be some feedback depending on the structure you are exciting, and so on. How strong a support do you provide for your shaker head and the impedance head itself?

Mr. Smith: We don't provide any. We cement an attachment pad to the structure and we bolt the impedance head and shaker directly to it. The structure supports it entirely.

Mr. C. Sanders (Rocketdyne): Mine is a comment rather than a question. I think it is of interest to note that, for about 2 years, we have been using almost the identical instrument system you described here . . . probably using pretty much the same equipment . . . at least, we have been using an AD-YU phase meter. We have recently put another one, that I cannot recall the name of right now, into service in a parallel system. We have been using it for amplitude ratio plotting much more than mechanical impedance. We have found that it uncovers problems that we were unable to bring to light by the older methods. It is much more repeatable than the older methods we were using, as far as that goes.

* * *

AIRBORNE VIBRATION SPECTRUM ANALYSIS: SOME TECHNIQUES AND LIMITATIONS*

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The large telemetry bandwidth required by flight vibration measurements has led to the use of airborne vibration spectrum analyzers. Criteria for the selection of these instruments are presented. One airborne spectrum analysis system currently in use is described in some detail. Other possible systems are also discussed.

INTRODUCTION

Most flight vehicle development programs include the acquisition of a body of in-flight vibration data. Typically, these data are obtained for two reasons:

- To determine the shock and vibration environment of the vehicle and its components during flight. Knowledge of this environment is required to permit the refinement of ground test levels and to aid in any redesign necessitated by the environment.

- To provide clues to the causes of any malfunctions that may occur during flight.

A difficulty which has been encountered in acquiring flight vibration measurements is the relatively large bandwidth of the data, which requires a correspondingly large telemetry bandwidth for transmission to the ground. A typical vibration measurement has a bandwidth of about 2 kilocycles. On an IRIG Standard FM/FM telemeter (Band E) [1] this requires a base bandwidth of about 27 kc for transmission. This prodigious consumption of bandwidth may displace 25 or more low-frequency measurements. It creates sizable difficulties for those who must allocate a limited telemetry capability to a number of measurement requirements.

Various methods have been employed to overcome this bandwidth problem. Significant among these are single-sideband/frequency modulation (SS/FM) telemetry [2], and airborne data analysis [3]. It is the purpose of this paper to discuss a technique in the latter category - airborne spectrum analysis. This technique involves, in effect, flying data processing equipment on-board the vehicle. The information extracted by this airborne processing equipment can be transmitted with a smaller bandwidth, and thus does not place such extreme requirements on the telemetry system. In fact, the airborne processing equipment presently employed on the TITAN II vehicle analyzes vibration data having a bandwidth of 2 kc, and produces an output having a bandwidth of less than 20 cps, thus reducing the necessary telemetry bandwidth by a factor of 100.

This boon to the telemetry engineer makes the job of the vibration engineer more difficult. Previously, with the broadband data in hand he could sit back and analyze the data at leisure in any of a number of possible ways with a variety of instruments. Now, without a prior opportunity to examine the data, he must make three rather difficult decisions:

1. What information shall be extracted (in flight) from the broadband data; and what shall be discarded?

2. What devices shall be employed to extract the desired information?

*This work was supported by the TITAN Program Office of Space Technology Laboratories, Inc.

3. What are the limitations imposed by the decisions made in 1 and 2?

It is assumed here that the first question has been answered: "information on the spectrum of the data is required." All other information, such as amplitude distribution functions and time waveforms is to be discarded. (This decision is convenient for the purposes of this paper, although certainly not necessary in general. Airborne analyzers are available, or could be designed, to obtain measures of data other than the spectrum). We proceed, then, to provide means for answering the second two questions.

CRITERIA FOR THE SELECTION OF AN AIRBORNE SPECTRUM ANALYSIS TECHNIQUE

Data Requirements

In the introduction it was mentioned that there are two reasons for making flight vibration measurements: to obtain design and test information, and to provide data on in-flight malfunctions. A measure of the value of an airborne spectrum analyzer is, then, how well the data it provides meets these requirements for a specific program.

For typical design and test purposes, the data of greatest interest is usually that with the highest level. During the flight of most vehicles, the highest vibration levels occur at the time of launch, during transonic flight, and at the time of maximum dynamic pressure. Although these time periods may represent a small percentage of the total flight time, they frequently contain all the data of interest. This leads to the criterion:

> CRITERION 1: Errors in the airborne-analyzed spectrum should be a minimum for the highest vibration levels analyzed and for the highest spectral peaks observed.

Unfortunately, some of the most intense vibration excitations are quite brief (i.e., they are almost shocks). Such may be due to the firing of explosive bolts, engine ignition, and so on. Although the application of power spectrum analysis to these transient phenomena is debatable, it is important to realize that some airborne spectrum analysis techniques might not reveal the existence of such excitations, let alone provide information on their spectrum or magnitude. The importance of acquiring information on intense transients becomes more evident when one realizes that such transients are likely to accompany, or even cause, sudden failures of the vehicle or its equipment.

Data Stationarity

In order for a power spectrum analysis to be meaningful, the data must be assumed stationary over the time period analyzed. (A random function is considered to be stationary if all moments of an amplitude probability density estimated from any time segment of the function are independent of the time at which the segment is abstracted. If the probability density function is gaussian, then only the first two moments - the mean and the standard deviation - need be time invariant. Other moments of a gaussian distribution are zero). During spectrum analysis in the laboratory, from the data it is possible to choose a quasi-stationary data sample having a duration, T, and to achieve an accuracy limited only by T and the bandwidth of the analyzer (by reproducing the sample repeatedly). All current airborne analyzers, however, work in real time. The data input changes continuously and is never repeated. The confidence in any particular spectrum from an airborne analyzer is thus inversely related to the flight time required for the analyzer to compute that spectrum because of inherent data nonstationarity. This leads to a second criterion:

> CRITERION 2: Limit the time required to compute a single spectrum to the minimum expected period of stationarity.

In a sense, CRITERIA 1 and 2 are complementary. It is important to maximize accuracy for the highest data levels. If the highest levels are of brief duration, then it is also important to complete an analysis quickly.

Statistical Precision

For the sake of statistical analysis, it is customary to assume that vibration data signals are stationary, gaussian time functions. Of course, actual flight vibration data are neither gaussian nor stationary. However, in the absence of strong pure-tone components, vibration probability density functions are frequently quite similar to gaussian functions between $\pm 3\sigma$. Nonstationarity is circumvented during analysis, with some loss of resolution, by examining only brief portions of the data and assuming these portions to be segments from different infinite, stationary time functions.

In performing a spectrum analysis, one attempts to approximate the exact power spectrum of an infinitely long, stationary, random, time function (ILSRTF) by examining only a finite portion of the function. For random functions, the accuracy of this approximation depends upon the time duration of the finite portion examined, and upon the detail with which the portion is examined (i.e., the bandwidth of the analyzing filter).

If a given vibration measurement is repeated (identically) on a number of (identical) flights, and comparable time portions are analyzed from each flight, there will be differences among the resulting power spectra. The range of these differences is related to the statistical accuracy of each individual spectrum (i.e., how well each individual spectrum represents that of its ILSRTF). The key point here is that the range of these differences determines the accuracy with which the present measurements represent what would be encountered on future (identical) flights. (Ergodicity is assumed. That is, it is assumed that properties approximated from samples of ILSRTF on one flight are indistinguishable from properties determined from samples at the same point in time on a large number of identical flights. If this were not at least roughly true, there would be little point to any flight test program).

A convenient measure of the statistical accuracy of a power spectrum analysis is the number of degrees of freedom, k, of the analysis [4,5,6,7]. Assuming a stationary, gaussian time series having zero mean and a power spectrum which is essentially flat, then:

$$k = 2T\Delta f,$$

where

T = The effective analysis time in seconds, and
Δf = The effective bandwidth of the analyzing filter.

If the spectrum is not flat, k will generally be less than this value, approaching a limit of two for very narrow peaks.

The degrees of freedom, k, is the number of independent observations required to completely specify the analyzing filter output. From the Sampling Theorem, it can be shown that if the spectrum of a time function is flat, continuous, and limited to a rectangular bandwidth Δf, it is completely determined by $2\Delta f$ equally spaced amplitude samples per second [8]. Completely determined means that the time function can be exactly reproduced from the samples by suitable filtering. If $2\Delta f$ samples per second are required, and the time of interest is T seconds, then the necessary number of independent observations is $2T\Delta f$.

In practice, k is determined from the effective analyzer bandwidth and the least of three time periods: the data sample length, the effective detector integration time, or (for scanning analyzers) the time the filter observes a particular point on the spectrum.

When the filter is followed by a mean-square detector (which extracts estimates of the variance for gaussian data), then the detector output has a chi-square distribution for the particular number of degrees of freedom, k [7]. For small values of k, the chi-square distribution is quite unsymmetrical. The median output is always less than the desired average value for the ILSRTF. As k increases, however, the output becomes less variable, approaching a narrow, symmetrical distribution about the desired average. Once k is shown for a given spectrum analysis, consultation of a set of chi-square distribution [6] enables one to determine how close the given spectrum is to the true spectrum, within specified confidence limits.

As an example, a typical power spectral density analysis in a laboratory might have $k = 40$. Then there is a 90-percent chance (90-percent confidence limits) that any spectral value obtained is within the range from 0.65 to 1.5 times the true spectrum. Similarly, if an analysis has 10 degrees of freedom, there is a 90-percent chance that a measured spectral value is within 0.35 to 1.8 times the true one.

On this basis, one may establish a third criterion for the selection of an airborne spectrum analysis technique:

CRITERION 3: Maximize the number of degrees of freedom, k.

Obviously, CRITERION 3 is not compatible with CRITERION 2, and some compromise must be made between the two. This compromise should be based upon a prior assumption about the expected duration of quasi-stationarity during portions of the flight of greatest interest.

Instrument Design

In addition to the fairly general considerations, already mentioned, there are other criteria to be considered in the actual design of the airborne spectrum analyzer.

Certainly in any spectrum analyzer it is necessary to minimize the analysis filter bandwidth if power spectral density is the desired quantity. Therefore:

CRITERION 4: Minimize Δf.

Here again, one is faced with a measure which must be compromised with others: the need to maximize k and to minimize the time to compute a spectrum. For real-time airborne analysis, the key still lies with an estimate of the time period over which a maximum vibration level may be considered quasi-stationary.

For a heterodyne-type airborne analyzer, the effective integration time of the detector low-pass output filter, T_i, should be approximately equal to the time the analyzer filter examines a point on the data spectrum:

CRITERION 5: $T_i \approx \frac{\Delta f}{\dot{f}}$

Here Δf is the effective bandwidth of the analyzing filter, and $\dot{f}$ is the scanning rate in cps/sec. Once the scanning rate and filter bandwidth have been chosen in the compromise between achieving maximum k and minimum analysis time, then T_i is also determined. If it is less than $\Delta f/\dot{f}$, it reduces k without the benefit of a corresponding decrease in analysis time. If T_i is greater than $\Delta f/\dot{f}$, it introduces unnecessary lag errors (i.e., a widening of the apparent analyzer bandwidth, a shift of the spectrum along the frequency scale, and a decrease in the magnitude of spectral peaks).

A second consideration in the selection of heterodyne-type analyzers is to assure that the sweep rate is not so fast that spectral peaks produce only a transient response of the filter. The maximum permissible sweep rate is a function of the detailed filter design and spectrum shape, but several general criteria have been published, based upon different assumptions and filter characteristics.

Some examples are:

$\dot{f} \le \frac{\Delta f^2}{2}$	Moody [5]
$\dot{f} \le 2(\Delta f)^2$	Williams [9]
$\dot{f} << \frac{\pi}{4}(\Delta f)^2$	Chang [10]
$\dot{f} << \pi \Delta f \Delta f_s$	Ratz [11]

where Δf_s is the bandwidth of the narrowest peak in the random data spectrum (not a pure tone).

It appears that a convenient rule-of-thumb would be:

CRITERION 6: $\frac{\dot{f}}{\Delta f^2} << 1$.

There are many other design factors which warrant consideration for particular applications. Such items as filter shape, detector design, calibration procedures, and so on, would, if discussed, tend to turn this paper into a book. A detailed presentation of some of these factors has been published by Ratz [11].

Finally, one should not neglect the consideration of such operational factors as compatibility with telemetry, reliability, stability, ease of maintenance and calibration, weight, size, power consumption, and cost. These might be summarized as:

CRITERION 7: Maximize operational utility.

Before proceeding with the description of a current airborne spectrum analysis system, the various criteria are summarized as follows:

Criteria for the Selection of Airborne Spectrum Analysis Techniques

1. Minimize errors for the highest vibration levels.

2. Limit the time required to compute a single spectrum to the minimum expected period of stationarity.

3. Maximize the number of degrees of freedom, .

4. Minimize the analyzer bandwidth, Δf.

5. $T_i \approx \Delta f/\dot{f}$ (for heterodyne analyzers).

6. $\dot{f}/\Delta f^2 << 1$ (for heterodyne analyzers).

7. Maximize operational utility.

The selection and rank ordering of these criteria are the author's judgments of the significant factors involved in evaluating an airborne analysis technique for general use. Undoubtedly, cases will exist where the relative importance would be different, or where other criteria would be significant.

AN EXAMPLE: THE TITAN II SYSTEM FOR AIRBORNE VIBRATION SPECTRUM ANALYSIS

As an example of how the various conflicting criteria, mentioned previously, have been resolved in practice, the sweep-spectrum analyzer employed on the TITAN II vehicle will be described. This system was selected by the Aerospace Division of the Martin-Marietta Corporation after consideration of their data and telemetry requirements. It is used in conjunction with a smaller number of vibration measurements on FM/FM telemetry.*

Airborne Equipment

Figure 1 is a block diagram of the airborne instrumentation used for those vibration measurements on TITAN II which are processed in flight. The output from each axis of three triaxial accelerometers is amplified to a maximum peak-to-peak voltage of ±2.5 volts. Each data signal is applied to one channel of the nine-channel airborne analyzer. The output from each airborne analyzer channel is then sampled 40 times per second by a PCM encoder. These samples are converted to an eight-bit straight binary code and combined with other data codes and synchronization information into a serial PCM signal. This is used to modulate an FM transmitter.

A simplified block diagram of the analyzer itself is shown in Fig. 2.† A data signal is amplified, passed through a 2-kc, low-pass filter and applied as one input of a balanced modulator. The other input to the modulator is supplied by an oscillator that is swept linearly over the frequency range from about 2900 to 5600 cps. Sweep periods of 2, 4, and 6 seconds are available on this particular unit.‡ However, all flight data have been taken with the 2-second sweep period. The modulator output includes sum and difference frequencies of the sweep oscillator and the data. A three-stage, i-f amplifier following the modulator is tuned to 3 kc,

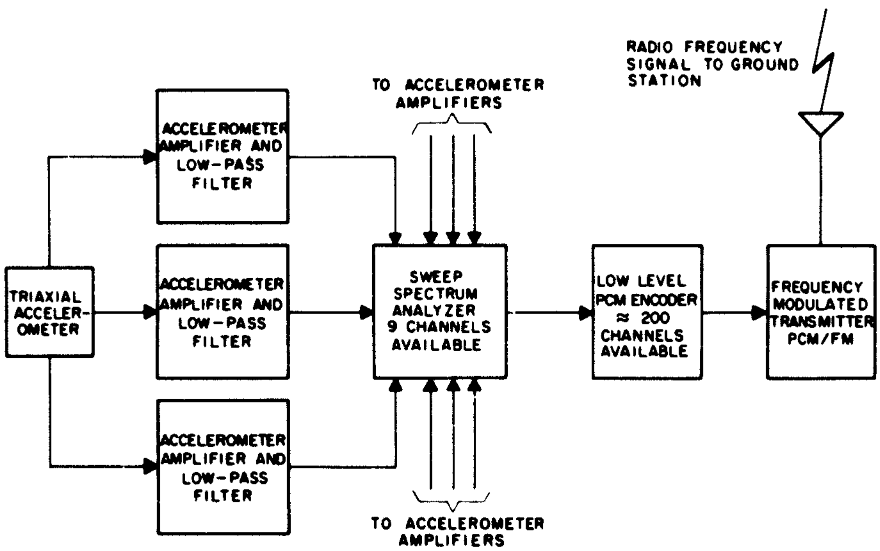

Fig. 1 - Block diagram of airborne vibration measurement system using airborne analyzer

*The description provided herein has been obtained through the courtesy of Martin-Marietta Corporation, Aerospace Division, Denver, Colorado.

†This instrument was designed and built by the Ortholog Division of Gulton Industries, Inc.

‡The sweep periods given here are the times required for the sweep to complete one cycle of operation, which includes the reset time for the sweep oscillator.

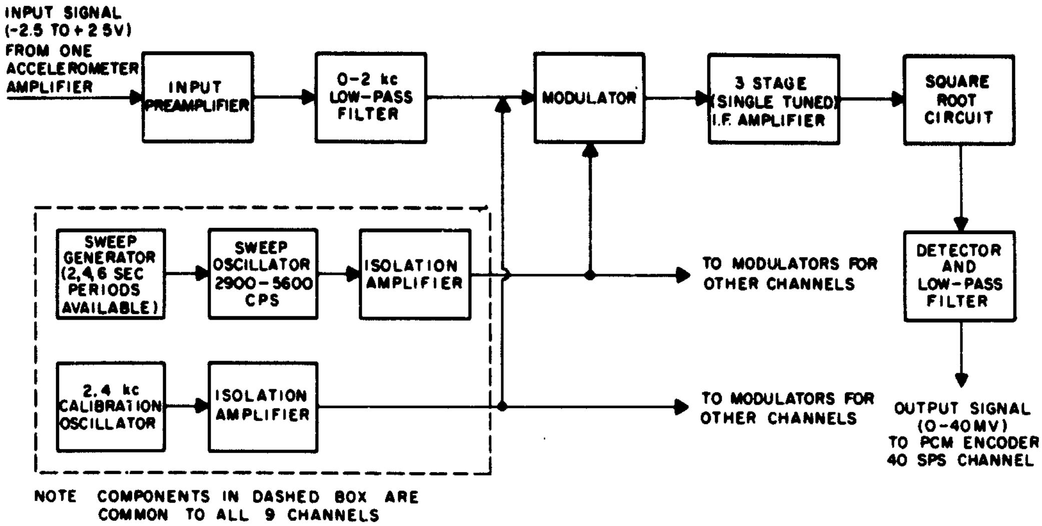

Fig. 2 - Block diagram of airborne, heterodyne, sweep-spectrum analyzer

and effectively scans the data frequencies in the lower sideband as the oscillator sweeps.

The output of the i-f is passed through a circuit having a square-root transfer function, before detection. The use of the square-root circuit reduces the dynamic range of the output of the instrument and produces an output approximately proportional to the 4th root of the power spectral density of the input signal. The signal from the square-root circuit is linearly detected and smoothed by a low-pass filter.

In addition to the spectrum of the data, the output of the i-f amplifier contains a residual carrier signal when the sweep oscillator coincides with the center frequency of the i-f amplifier. This residual carrier signal corresponds to zero frequency in the original data. (The effect of the carrier coming through the filter skirts will increase the spectrum level at low frequencies. The minimum frequency where valid data is obtained depends on the magnitude of the data spectrum. Most data reviewed to date are usable down to about 100 cps). A second known frequency is produced by a 2.4-kc oscillator included in the airborne analyzer. This 2.4-kc reference signal is applied to the input of the modulator along with the original data signal and results in a peak in the analyzer output above the 2-kc data range. By linear interpolation between this 2.4-kc mark and the zero frequency mark, it is possible to determine the frequency at any time during the sweep.

The amplitude of the 2.4-kc oscillator is regulated. Thus in addition to providing a frequency mark, it serves as reference to calibrate the gain of the analyzer once each sweep.

The final output of the airborne analyzer is thus a slowly varying dc-signal proportional to the 4th root of the power spectral density of the data, and containing peaks corresponding to zero and 2.4 kc. A typical airborne analyzer sweep is shown in Fig. 3.

The primary calibration of the analyzer is accomplished at the time of manufacture by applying a random input signal of known power spectral density and adjusting the output level to a specified value. This is then related to the maximum outputs produced by the internal 2.4-kc oscillator and by an externally applied 1-kc tone. A corresponding 1-kc signal is used for checkout of the instrument prior to a flight.

Ground Equipment and Data Processing Procedures

Reception and processing of the PCM signal from the TITAN II vehicle is handled in three distinct phases on the ground.

1. Reception, detection, and recording on analog magnetic tape.

2. Formating and re-recording on digital tape.

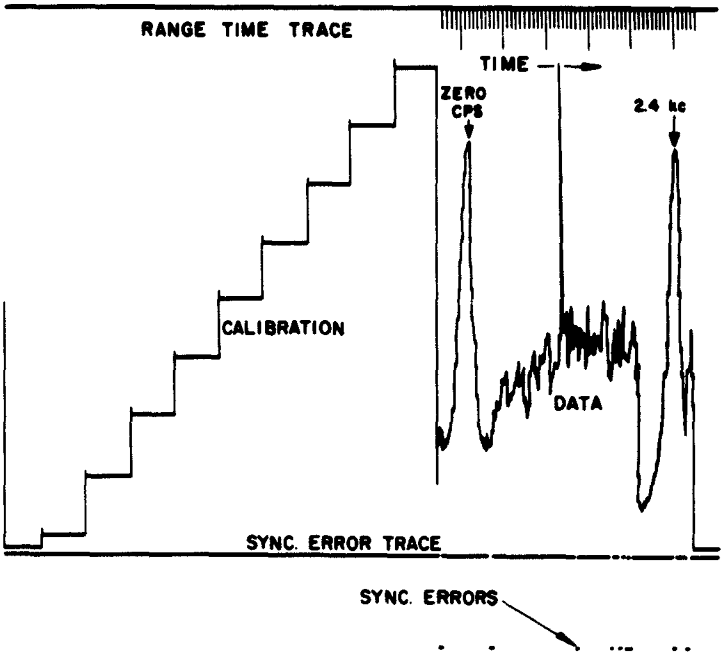

Fig. 3 - Typical airborne analyzer sweep from an oscillograph record

3. Decommutation, editing, calibration, and presentation of the data.

These three phases will be discussed separately.

PHASE 1. Reception and Initial Recording - The rf signal transmitted from the vehicle is initially received and handled at the test range. Ground reception and recording is begun a few seconds prior to launch. At the ground station, the detected serial PCM signal is recorded on one or more tracks of a magnetic tape machine. The range time at the receiving station is simultaneously recorded on another track.

PHASE 2. Formating and Recording on Digital Tape - The second step of the data-handling process consists of formating and re-recording the signal as distinct blocks of data on digital magnetic tape. This is shown schematically in Fig. 4. The tape so produced is usable as direct input to a digital computer. Special purpose format converters have been built to accomplish this step. The format converter must establish bit, word, and frame synchronization with the serial data on the analog tape and control the digital tape machine so as to record separate blocks of exactly one frame each on the digital tape. It also records range-time information extracted from the

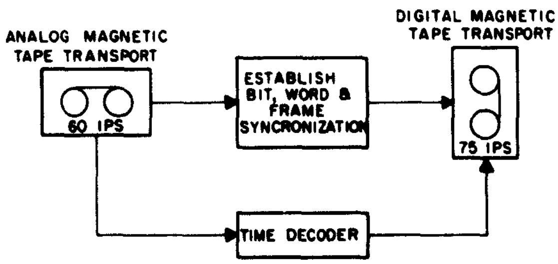

Fig. 4 - Block diagram of PCM formating system

analog tape and indications of any loss of synchronization for each word and frame during the formating process.

PHASE 3. Decommutation, Editing, Calibration, and Presentation - The digital magnetic tapes produced by the format converter in PHASE 2 are used as an input to a number of different digital computing centers. All of the data for all measurements during the flight are on these tapes, and the specific processing to be applied to individual measures varies greatly. The processing system* described here is that used by Bolt, Beranek, and Newman, Inc. to reduce the data on the channels assigned to the vibration spectrum analyzer aboard the vehicle. This system is illustrated in Fig. 5.

17-inch cathode-ray-tube (CRT) plotter, as in Figs. 6 through 11.

The display and output options available at this point in the processing provide a means for quick-look examination of the raw vibration data or of any other PCM data channel. In addition, copies of the raw data can be obtained in three ways - photographs of the CRT, x-y plots, and oscillograph records. Figures 6 through 11 are examples of raw vibration data photographed on the face of the CRT. Figure 3 is a tracing of an oscillograph record.

During quick-look examination of the raw data, erroneous data samples may be eliminated prior to subsequent processing. Figure 6

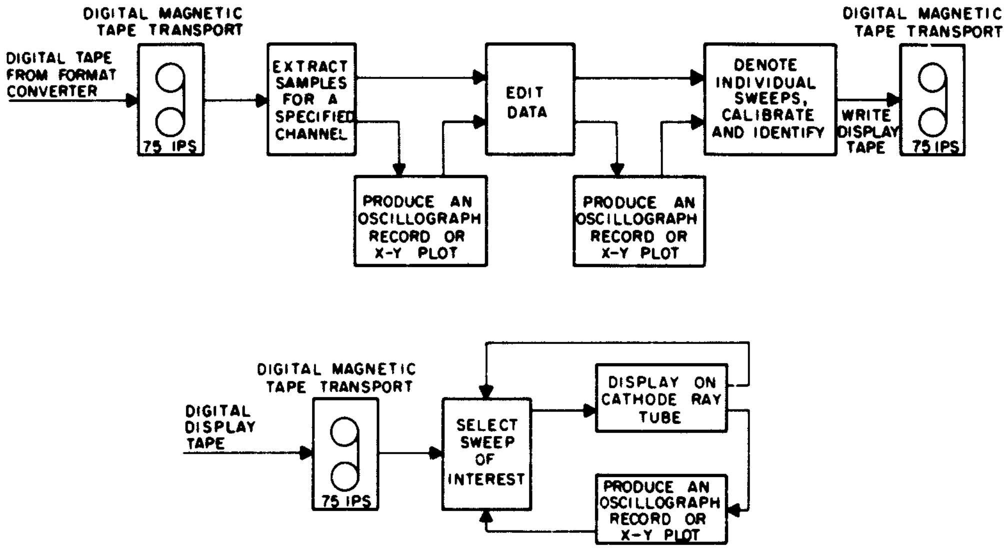

Fig. 5 - Block diagram of PCM decommutation and data processing system

In the initial phase of processing the digital tape, a particular data channel and starting time of interest are specified. The computer searches the tape to find the specified time and then proceeds to extract only the data samples for the specified channel. This extraction process continues until the available storage space in the computer memory is fully utilized. Time and synchronization information are also extracted. The data are then displayed on a

illustrates an extreme example of a vibration data channel with many errors. This example was taken prior to engine ignition. The bad samples are obvious since they do not follow the general line of the data. Figure 7 shows the same data with those samples which occurred during loss of synchronization by the format converter eliminated from the display. The remaining erroneous samples are eliminated by manually indicating them to the computer with a light pen.†

*A detailed paper describing the data processing system summarized here is to be presented at the 64th meeting of the Acoustical Society of America, November 1962.

†The light pen is a small pen-like device which, when pointed at a point on the CRT, will modify the computer program associated with that point in the display.

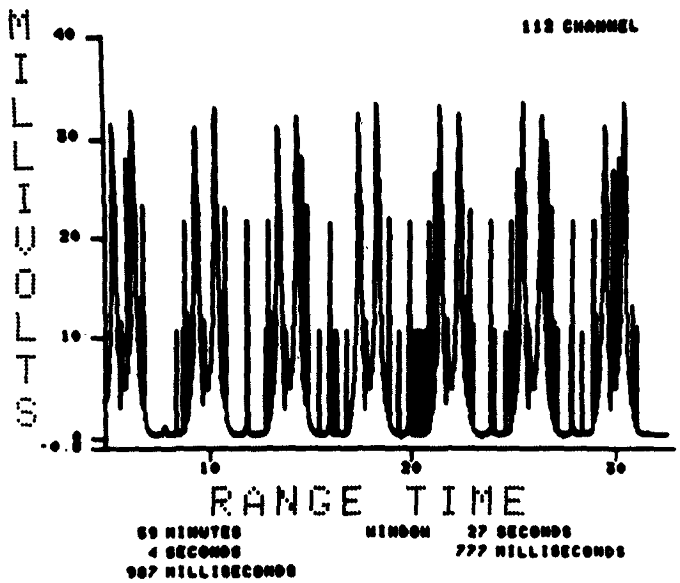

Fig. 6 - Quick-look display of vibration analyzer output containing many errors (illustration taken prior to engine ignition)

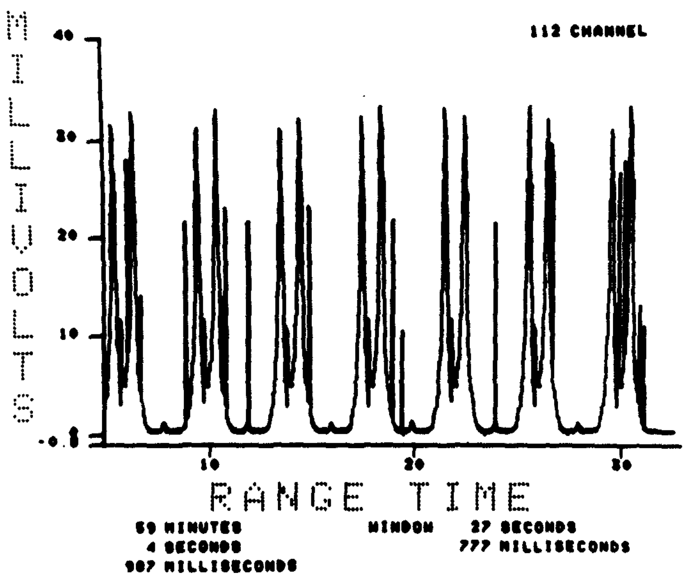

Fig. 7 - Data shown in Fig. 6 with synchronization errors deleted (illustration taken prior to engine ignition)

Figure 8 shows the data with all detectable errors eliminated. Individual samples eliminated from the display in this manner are adjusted to the average of adjacent valid samples so that the processing may continue. Without this editing, a single bad sample could, when raised to the fourth power, introduce a significant error into subsequent data computations.

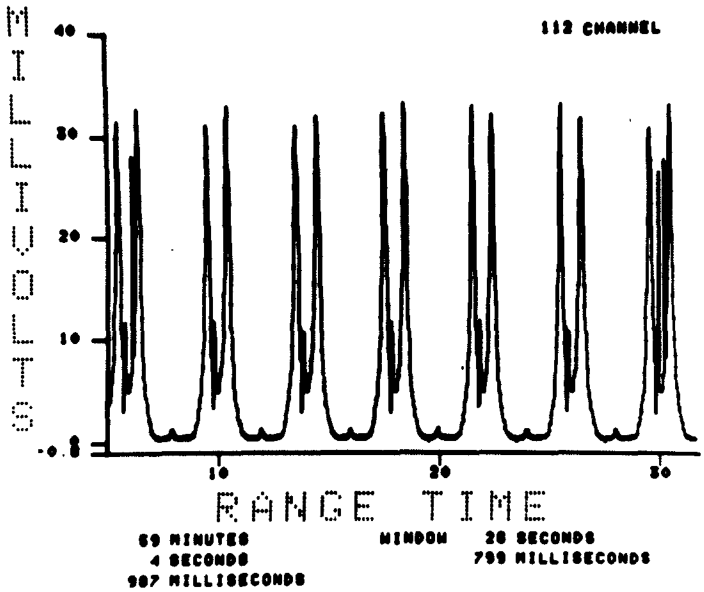

Fig. 8 - Data shown in Fig. 7 with remaining errors manually deleted (illustration taken prior to engine ignition)

The next step is to mark the beginnings and the ends of the analyzer sweeps. This is complicated by the fact that no synchronization exists between the analyzer sweep rate and PCM sampling rate. Two adjacent zero and 2.4-kc peaks are identified by the operator and pointed out to the computer utilizing the light pen. Subsequent peaks are identified by the computer. Errors occurring during this process may be corrected manually. However, computer determination of the zero cps point is necessary in those cases where the data obscure the peak. This has occurred and is illustrated in Fig. 9 around 50-second range time.

Calibration and identification information for the data channel is then entered into the computer. A program eliminates from each analyzer sweep all data samples except those between 100 and 2100 cps. The data are raised to the fourth power, scaled to power spectral density, and the overall rms level for each sweep is computed. Allowance is made for any difference between the actual value of the 2.4-kc calibration peak and the value recorded when the instrument was initially calibrated. A summary of the overall values for each successive sweep of the analyzer is provided.

A final display of a data spectrum derived from an individual sweep of the analyzer is shown in Figs. 10 and 11. The frequency and amplitude are logarithmic for ease of comparison of the data with that obtained from other sources. Copies of the data may be produced by an x-y plotter, or by photographing the CRT.

Data Properties

In addition to serving as descriptions of data processing procedures, Figs. 6 through 11 also illustrate some of the properties of the data from the airborne spectrum analyzer.

The form of the raw data from the vibration analyzer is best seen by examining Fig. 8.* This figure shows six sweeps of the analyzer prior to vehicle ignition. The pairs of peaks are, from left to right, the 2.4-kc calibration marker followed by the zero-cps marker. The output between these peaks occurs as the sweep oscillator resets and is of no interest. The data would appear between one zero-cps marker and the next 2.4-kc marker.

Nonstationarity of the vibration environment can be seen by examining the raw data in

*Figures illustrating the data are generally shown with lines connecting the individual data samples. This is done to aid viewing. The intersections of the lines are the only data points actually transmitted. (See Fig. 12).

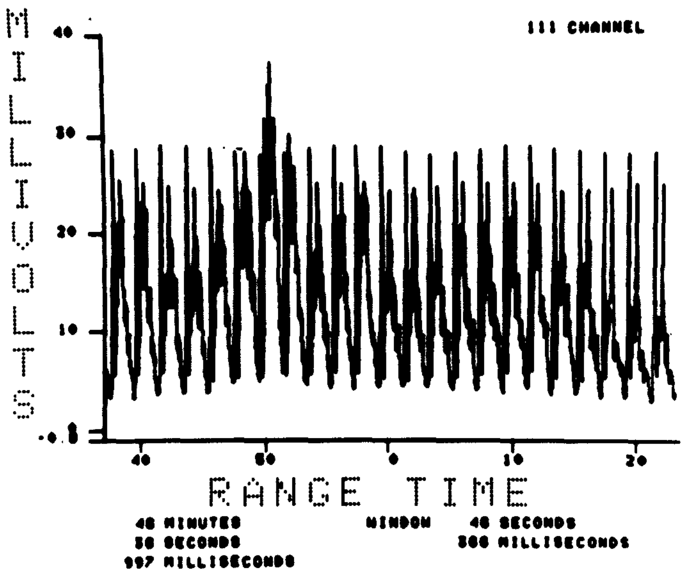

Fig. 9 - Quick-look display of vibration analyzer raw data showing many sweeps

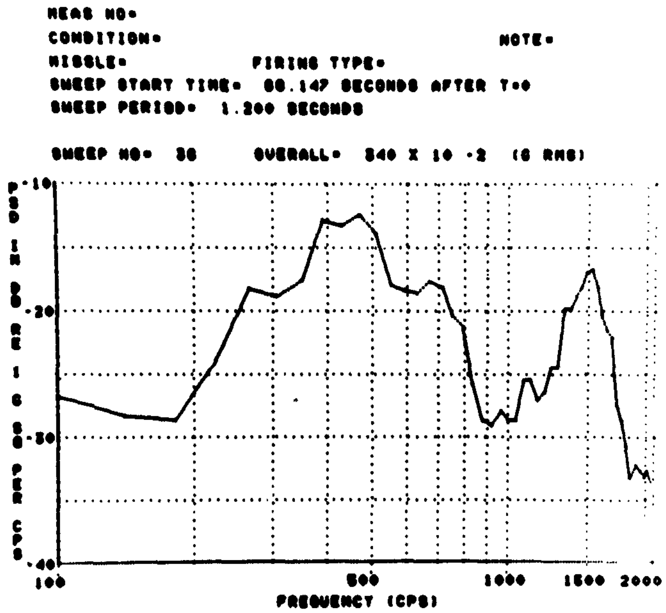

Fig. 10 - Reduced data for first of two consecutive sweeps illustration nonstationarity

Fig. 9. This shows a large number of sweeps, and it is evident that the mean spectrum level of the data varies considerably during the 45-second time period.

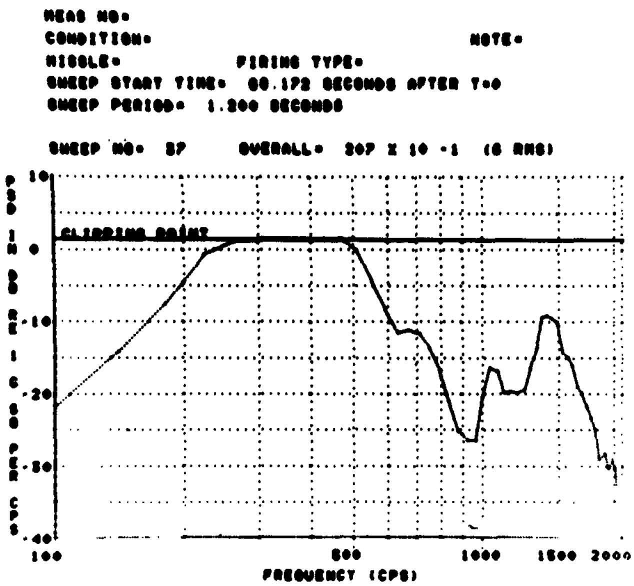

Fig. 11 - Reduced data for second of two consecutive sweeps illustration nonstationarity

An extreme case of nonstationarity is illustrated in Figs. 10 and 11 which show two consecutive sweeps in final form. The change in the apparent spectrum is considerable. The vertical scale in the two figures is different and must be noted to appreciate the full extent of the change.

Frequency resolution, due to sampling, is illustrated by Fig. 12. This shows the final

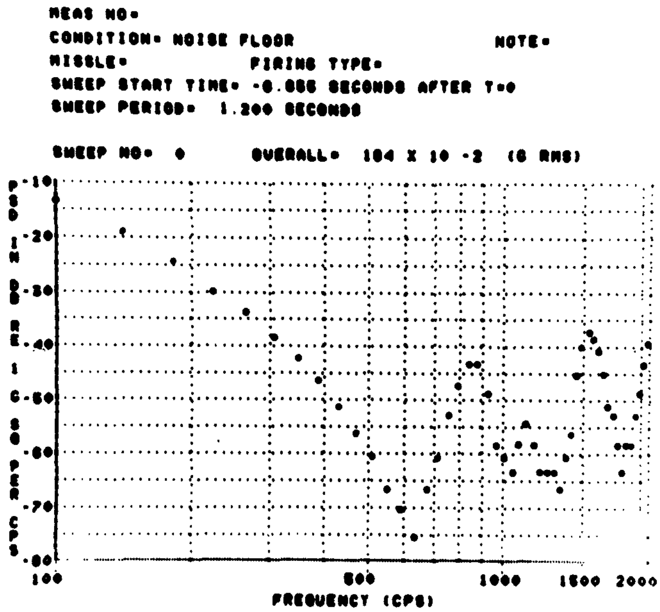

Fig. 12 - Reduced data for a single sweep showing only data points

presentation of a sweep in which the individual data points or samples have not been connected with lines.

Comparison of Example to Criteria

It is of interest to compare the TITAN II airborne spectrum analysis system with the previous listing of criteria for the selections of airborne spectrum analysis techniques. For the purposes of this discussion, pertinent properties of the analyzer are summarized in Table 1.

Transients which occur in less than one sweep time are probably not recognized as such. This is particularly true if the transients contain most of their energy in one frequency range. The analyzer spends only 0.09 second per sweep, or about 4.5 percent of its time examining any given point on the data spectrum. It is thus quite improbable that the filter will be tuned to a point in the spectrum when transient energy appears at that point. This transient problem is, of course, a difficulty with any real-time sweep-type, airborne, spectrum analyzer. In the case of the TITAN II vehicle, it is circumvented by having a few vibration measurements on wide-band telemetry which would report any serious transient excitation. These wide-band measurements are interchanged among the various measurement locations from test to test.

TABLE 1

Some Properties of the TITAN II Airborne Vibration Spectrum Analyzer

Properties	Characteristics
Type:	Heterodyne, sweeping oscillator
Nominal (3 db) Bandwidth[a]	100 cps (2-second sweep period)
Effective Rectangular Filter Bandwidth[a]	150 cps (2-second sweep period)
Data Bandwidth	100-2100 cps
Sweep Rate	1670 cps/sec
Time Per Data Sweep	1.2 second (2-second sweep period)
Effective Integration Time of Output Filter	0.11 second
Effective Analysis Time for Any Point on Data Spectrum	0.09 second (2-second sweep period)

[a]Both the nominal (3 db) bandwidth and the effective rectangular filter bandwidth are a function of sweep period, and tend to increase as the period decreases.

CRITERION 1: Minimize errors for the highest vibration levels. For data which is stationary over the time of a single sweep, the analyzer will tend to read low for narrow spectral peaks (and high for spectral dips). This is due to the relatively large filter bandwidth, and the fact that the analyzer is calibrated with a flat spectrum.

By reproducing a large number of representative data samples through both the airborne analyzer and a laboratory analyzer, the Martin Company has obtained an average difference of about 3 db. This correction may be added to data from the airborne analyzer to reduce errors.

CRITERION 2: Limit the time required to compute a single spectrum to the minimum expected period of stationarity. The analysis time of the analyzer over which stationarity is assumed, is 1.2 seconds. Overall rms levels computed from successive analyzer sweeps (Fig. 13) are usually constant within about 4 db during periods of maximum vibration. However, situations where this was not the case appear in the data (Figs. 10 and 11).

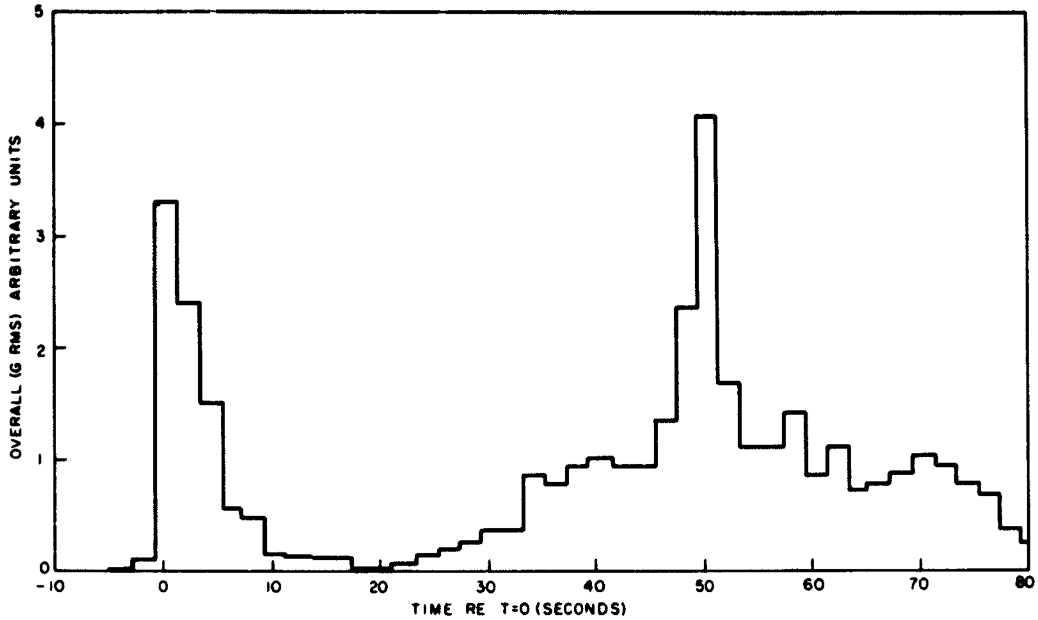

Fig. 13 - Overall rms vibration computed from consecutive sweeps as a function of time after vehicle launch

CRITERION 3: Maximize the number of degrees of freedom, k. For the TITAN II analyzer with a flat spectrum k = 27. This would mean that there is a 90-percent chance that the spectrum is within the range from 60 to 150 percent (-2.2 to +1.8 db) of the ensemble average. This should be adequate for the analysis of most airborne vibration measurements.

CRITERION 4: Minimize the analyzer bandwidth, Δf. The effective analyzer bandwidth is 150 cps. Although this is much too wide to resolve most spectral peaks, it is a compromise between the needs to minimize analysis time and to maximize k. Achieving an adequately narrow bandwidth to resolve power spectral density peaks is generally not practical in a real-time airborne vibration spectrum analyzer. Narrow band analyses are obtained from the continuous, wide-band measurements.

CRITERION 5: $T_i \approx \Delta f/\dot{f}$ (for heterodyne analyzers). Substitution of the values given in Table 1 yields $0.11 \approx 0.09$ and CRITERION 5 is satisfied.

CRITERION 6: $\dot{f}/\Delta f^2 << 1$ (for heterodyne analyzers). Substitution yields $0.074 << 1$ and CRITERION 6 is satisfied.

CRITERION 7: Maximize operational utility. One of the outstanding features of this analyzer is its operational utility. It is small, light in weight, and low in power consumption. It has performed satisfactorily in use, and is compatible with the instrumentation and data handling systems for the vehicle. Although the primary calibration procedure for the instrument could be improved, the flight calibration provided once every sweep by the 2.4-kc oscillator, is a very useful feature.

Conclusion

In conclusion, it appears that the TITAN II airborne spectrum analysis system represents a satisfactory compromise to the criteria for the selection of airborne spectrum analysis techniques, for the needs of the TITAN II program. This is particularly true because it is used in conjunction with a limited number of wide-band measurements which are interchanged from test to test with those handled by the airborne analyzer. Perhaps the major limitation of the airborne analysis system in this example is the inherent doubt about data stationarity during any given sweep.

OTHER AIRBORNE ANALYSIS TECHNIQUES

Modifications to TITAN II System

As a first step in the consideration of other airborne analysis techniques, we might examine some possible improvements to the system employed as an example in the previous section. One very desirable addition to the analyzer would be a broadband rms detector in parallel with each channel. The output of this detector could be sampled periodically during each sweep. The variation of the measured broadband level would be an indication of the data stationarity over a sweep. The overall level could also provide information on transient excitations.

A second useful addition would be a provision for synchronizing the analyzer sweep with the output sampling rate. In the present system, the analyzer output is sampled approximately once every 42 cps along the frequency scale. Because of slight variations in the analyzer sweep rate and the data sampling rate, an uncertainty of up to 21 cps in the data frequency represented by a sample may occur.* Asynchronization may also produce an apparent periodic variation in both the amplitude and location of the zero and 2.4-kc spectral peaks. This is evident in Fig. 9.

Asynchronism is no problem as far as spectrum resolution is concerned, for the sample spacing is much less than the filter bandwidth. If the asynchronism is fixed, however, a consistent error may be encountered in the amplitude of the 2.4-kc calibration signal output. If the sweep rate and sampling rate beat, as in Fig. 9, a difficulty arises which may be significant in some cases. This occurs in the calculation and scaling of the spectral data. In this calculation it is necessary to correct for the noise-floor output of the analyzer. A noise-floor sweep for this purpose is usually selected just prior to engine ignition. When any given data sweep is compared to the noise sweep, the data frequencies associated with the samples in the two sweeps may differ, and thus must be known with some precision. Slight inaccuracies in this knowledge can lead to large errors in spectra which are close to the noise level.

*The assumption is made that the data analysis program does no curve-fitting on the frequency markers. Curve-fitting programs may reduce the error in calculated frequencies, but cannot eliminate it completely.

Comb-Filter Analysis

Another approach to airborne vibration spectrum analysis employs a comb-filter analyzer. A comb filter, in this case, consists of a set of contiguous, bandpass filters which cover the whole data frequency range. All the filters are connected in parallel to a single data input, and their outputs are available simultaneously. Because more filters are examining the data at the same time, a comb-filter analyzer is inherently capable of analyzing a data spectrum more rapidly, with the same accuracy, than a sweep analyzer.

The comb-filter analyzer, which we would like to suggest for consideration, would employ peak-holding circuits at the output of the detector-smoother following each bandpass filter. The peak-holding circuits for a given set of filters would be sampled periodically. At each sampling a peak-holding circuit would report the highest level experienced by its associated filter since the last sampling time, and would then be discharged.

From the point of view of accuracy, such an analyzer approaches the best that can be achieved in real-time, spectrum analysis. All portions of the data spectrum are monitored at all times, and yet the spectrum may be sampled in time at an arbitrarily slow rate. From a practical point of view, though, it is seen that this accuracy is obtained through a loss in spectral resolution.

For the purposes of comparing with the criteria for the selections of airborne spectrum analysis techniques, let us postulate a comb-filter analyzer with 10 filter bands, each 200-cps wide. Because of the peak-holding circuits, CRITERION 1 is satisfied even for transient excitations (this is not true of spectral peaks, due to the wide bandwidth). The time over which stationarity must be assumed can be quite brief, and still provide large values of k. For instance, a spectrum could be observed in 0.25 second with 100 degrees of freedom.

On the other hand, the bandwidth is large, and the spectral resolution would be poor. A comb-filter analyzer would be fairly large and heavy if passive filters were employed in the data frequency range. This could be overcome, with some increase in complexity, by heterodyning the data to a frequency range where smaller filters could be used.

Playback Airborne Analysis

As mentioned in the second section of this paper, vibration data of major interest occurs only during certain portions of the flight of a test vehicle. These periods are fairly predictable, and occupy only a small percentage of the total flight time. This suggests the possibility of including a tape recorder on board the vehicle. This would record the broadband data during periods of interest, and reproduce it through an airborne analyzer during quiet periods. (This approach would not be practical for periods of intense vibration which occur very close to the end of a flight).

The use of an airborne tape recorder in this fashion would circumvent one of the fundamental limitations of current airborne spectrum analysis techniques - the necessity of processing the data in real time. The available analysis time could be increased by several orders of magnitude. This would provide a spectrum with statistical accuracy and bandwidth resolution approaching that of a laboratory analysis.

On the other hand, if a failure were to occur in flight prior to tape playback, no data at all would be obtained. Unexpected, high vibration levels occurring during tape playback would not be observed. The complete package consisting of a tape recorder, a spectrum analyzer, and a programmer for the record-playback operations would be larger, heavier, and more complex than an airborne analyzer alone. For these reasons, playback airborne analysis is probably the least advisable technique of any considered here.

CONCLUSIONS

The selection of an airborne vibration spectrum analysis technique requires compromises between various conflicting criteria. Because it is necessary, or at least, advisable to perform the airborne analysis in real time, it is not practical to obtain spectra as accurate as those produced by laboratory analysis. However, airborne spectrum analysis is quite useful when combined with telemetered wideband data and with other information about the properties of the original vibration signals, or both.

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DISCUSSION

R. Mustain (Douglas AC): Your paper was pretty complex and it took me quite a while to realize you were using PCM, I thought it was just ordinary telemetering, which we are used to. This brings up a lot of problems, for instance when you are recording, how many bits can you park and this sort of thing; also the sampling rate that you sample individual signals; the type of equipment you use to go from analog to digital and from that into the IBM system or whatever type of computer you use; also whose system were you using, is this a BB&N system or is this something that Martin has bought themselves and developed? There are a million questions that I would like to find out more about.

Mr. Keast: We tried not to complicate the paper too much by a detailed discussion of the digital telemetry system because it isn't of course the basic point of the paper. The telemetry system is the Martin Company's system. It is a pcm telemetry system that handles 202 channels, of these 196 are data channels, there are 8 bit samples, the bit rate I think is 316 mean bit rates, 316 kilabits per second or something like that. The data are transmitted PCM FM, received on the ground and recorded on a serial tape. The serial tape is put into a special purpose computer that is called a format converter. The format converter recognizes bit, word, and frame sync., and recompiles the data frame-by-frame, 20 frames coming along a second, and this is the rate at which the airborne encoder recycles and puts it on a digital magnetic tape. This digital magnetic tape can then be used by anyone who has a computer and programs to play it back. The play-back programs we showed here are our own. The rest of the system, the airborne system is Martin's, the format converter is Boeing's – this is a Minuteman format converter.

R. Kroeger (GE): I have some notes here that I took, I believe you said you had 27 degrees of freedom in your analysis.

Mr. Keast: Right.

Mr. Kroeger: Originally you remarked that this particular quantity was proportional to the bandwidth and the sample duration.

Mr. Keast: Proportional to this product, yes.

Mr. Kroeger: The bandwidth, I believe, was 150 cps, and the sample duration was 1.2 seconds.

Mr. Keast: No, in our case, the sample duration is the time required to sweep a spectrum with a 75-cycle, wide filter from 100 to 2100 cycles. The T in the formula can be the minimum of 3 values depending upon the way in which you are doing it. In this case it happens to be the time that the scanning filter dwells at a point on the spectrum. Because you're analyzing in real time you never examine that point again and this I think came out about 0.06 second.

Mr. Kroeger: In a paper [12] prepared for this meeting, I point out one error which is normally overlooked in connection with spectrum analysis of this type, wherein a linear detector is used. I believe you use a linear detector rather than a square-law detector.

Mr. Keast: Yes, as a matter of fact, this is a linear detector, an average detector operating on a square root of the input.

Mr. Kroeger: In setting up such an analyzer, normally, sinusoidal excitation is used to provide a scaling factor for the dc output; it turns out there is about a 13-percent error in setting it up on a sinusoidal basis and then providing it with gaussian random inputs. I notice that your overall error, however, was something like 2 db or you are 95 percent sure there was less than 2 db, so I doubt that 13 percent then would concern you.

Mr. Keast: That was a 95-percent confidence that I quoted. Actually this analyzer is not calibrated with a sine wave. It is calibrated by putting in a known power spectral density to the analyzer which gives you, in effect, a dc output.

A. Westneat (Gulton Industries): We of course are the company that developed these analyzers, though it was paid for by Martin. I have two comments: one on this gentleman's note on square-root detecting. The reason we went to this particular maneuver was that the dynamic range of the telemetering system did not equal the dynamic range required of the data itself and of the spectral analysis. Therefore, we had to go to the step of taking the square root of the data before going to pcm telemetry. Otherwise it would have very much extended the dynamic range of the system. On another problem we saw, between two slides, a very rapid change in the process; in other words, there was a spectrum on one sweep and then the next sweep it was well out of hand – way off scale. This is a basic problem with airborne spectrum analysis, but I might add it is a basic problem with any spectrum analysis. If your data has changed so rapidly in 1.2 seconds that it went from a process which was very much one scale, to one that is very much off, you've got problems, and I don't know really how you do it. It is too much of a job for a tape loop analyzer also.

INSTRUMENTATION AND ITS ROLE IN THE DEVELOPMENT OF A NEW VEHICLE

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Detroit Arsenal
Center Line, Michigan

The instrumentation and testing of a vehicle from concept to production is described in detail. The results of the testing are related to the establishment of overall vehicle reliability.

INTRODUCTION

The role of instrumentation in the development of a military vehicle is especially vital today. The Army has established the firm requirement that a new vehicle concept be taken through the development and testing stage and be ready for production within 4 years. This requires that the Arsenal's development program be improved so that the first pilot model becomes, substantially, the final vehicle. This can be done only if the design criteria are more realistic than in the past and if the development phase is more efficient.

This paper deals with the role of instrumentation in the development of a new vehicle from conception to production, and in the establishment of component, system, and overall vehicle reliability.

There was an urgent requirement for a fast, lightweight, unarmored, 105-mm, self-propelled howitzer. It had to have adequate cross-country mobility for use as direct support artillery for both infantry and airborne divisions. It was further required that it be suitable for transporting by and dropping from aircraft. These specifications prescribed a stable firing platform and a locomotion system with a total weight of approximately the same as a standard, towed, 105-mm fieldpiece and with rigid length and width requirements. In Figs. 1 and 2 a comparison is made of the towed 105-mm gun with the XM-104 vehicle tested.

Major problems were expected in vehicle stability because the heavy gun was carried rather high on the vehicle and also because of the short track and road contact. Also, problems were anticipated with hull stiffness and component performance because of the necessity of designing to minimum weight specifications. Maximum use was made of ATAC (Army Tank Automotive Command) computer facilities during the preliminary design stage. Also, maximum use was made of the laboratories' test facilities and instrumentation during the development and testing phases to monitor critical vehicle component performance.

Two test rigs were fabricated at ATAC: one for vehicle performance tests, the other for firing tests. This paper is mainly concerned with the test rig used to determine vehicular performance. This test rig was used to determine the adequacy of the power train during cross-country mobility and slope operation; performance of the suspension components such as track, road arms, road wheels, bearings, and so on; and stability and performance of the composite vehicle. Figure 3 shows the evolution of the XM-104.

Tests were planned to evaluate normal performance as well as to provide extreme loads with which to establish design limits. Details of the test program will not be discussed in this paper.

Instrumentation was planned to obtain maximum data on components without having an unmanageable system. The instrumentation was to serve a twofold purpose: first to measure loads, stresses, and motions which would allow improvement of the vehicle; and second, to establish criteria for the design of future vehicles of this type.

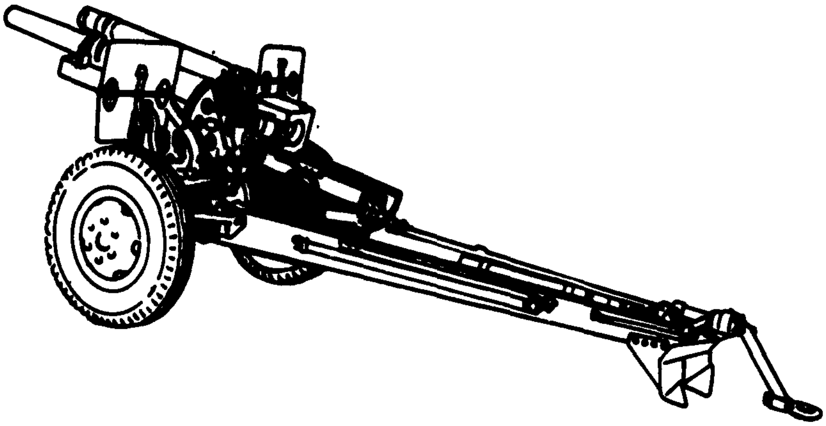


Fig. 1 - Towed gun carriage

Fig. 2 - Test rig no. 2

INSTRUMENTATION

Twenty-five channels of information were recorded (Figs. 4 and 5). A detailed discussion of the instrumentation used is given in the following sections.

Acceleration of Vehicle Center of Gravity

Three accelerometers were calibrated by vibrating them at ±5 g over the frequency range of 5 to 500 cps on an electromechanical shaker. The resultant signals were passed through a carrier amplifier into a light-beam, recording oscillograph. The amplitude of the recorded trace was plotted versus frequency, and later compared to the measured test values. The accelerometers were mounted on a steel block above the center of gravity of the vehicle to measure acceleration of the three mutually perpendicular planes.

Acceleration in Pitch, Roll and Yaw Planes

The output signals from two accelerometers were electrically connected so that like-direction accelerations would subtract and opposite-direction accelerations add. The two accelerometers were mounted side-by-side on the electromechanical shaker table and vibrated at ±5 g through the frequency range of 5 to 200 cps. They were then mounted on the vehicle a known distance apart to provide a method for measuring angular acceleration. Figure 6 shows the theory and calculations for making this measurement. Three of these systems were mounted on the test vehicle: one to measure pitch, another to measure roll, and the third to measure yaw.

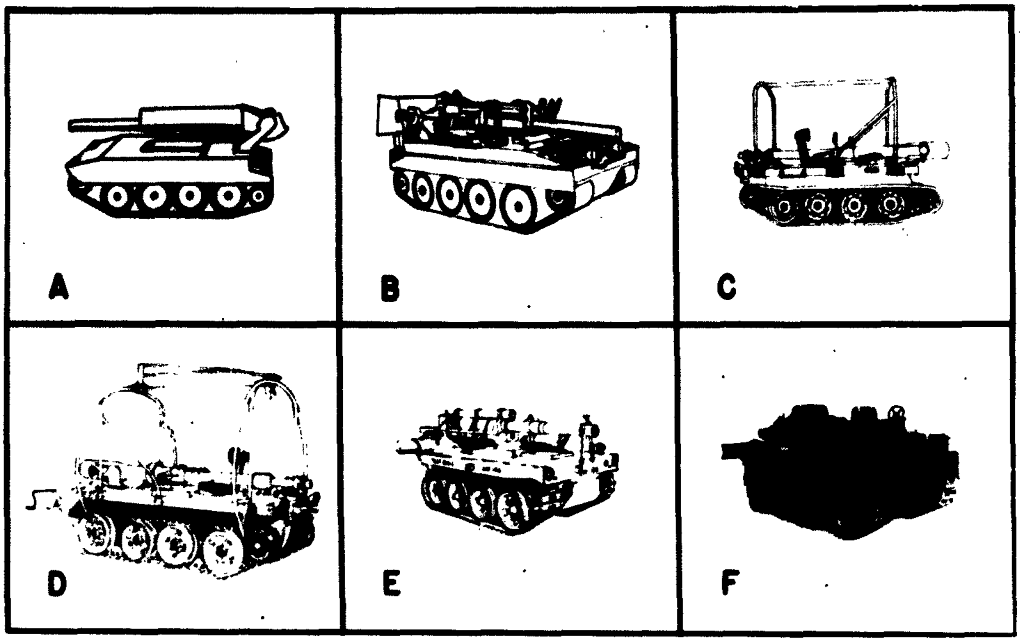


Fig. 3 - Evolution of the XM-104

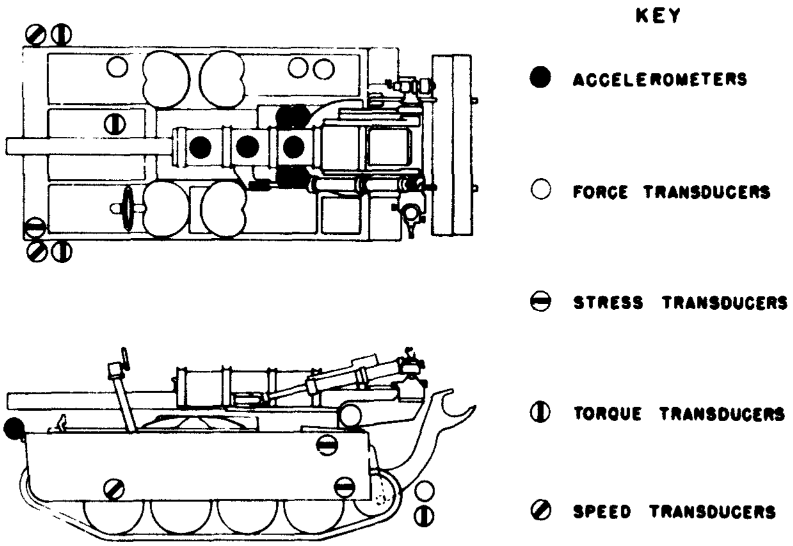


Fig. 4 - Transducer locations

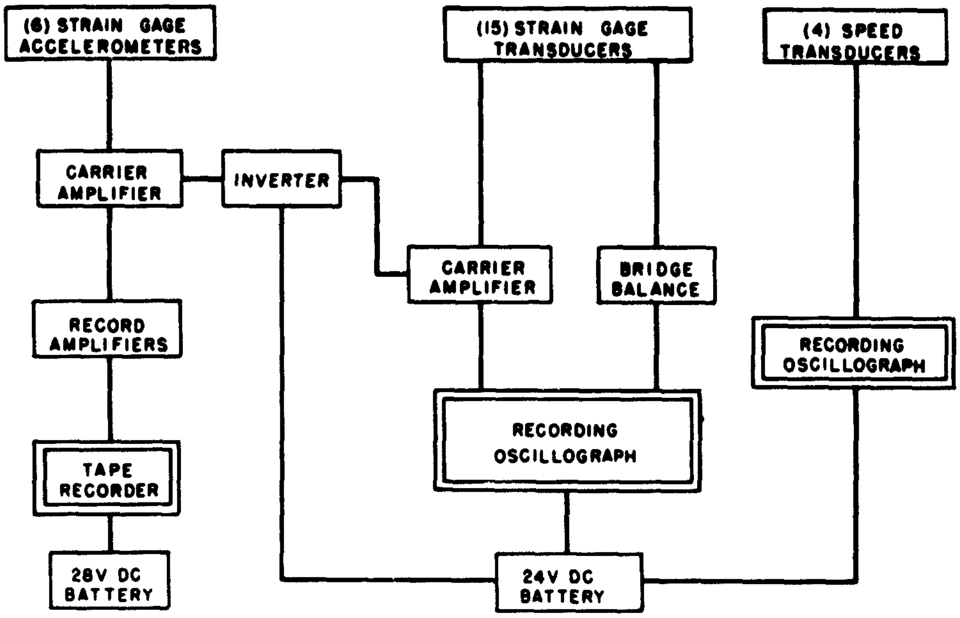
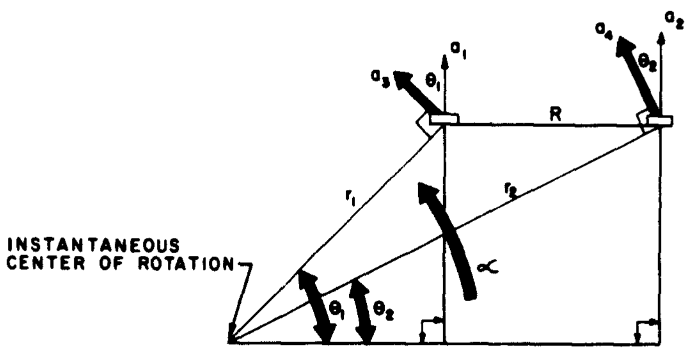


Fig. 5 - Instrumentation block diagram



a_3 AND a_4 ARE TANGENTIAL ACCELERATIONS

a_1 AND a_2 ARE LINEAR ACCELERATION MEASUREMENTS WHICH ARE A COMBINATION OF VEHICLE TRANSLATION AND ANGULAR ACCELERATION

α = ANGULAR ACCELERATION

$$\alpha r_1 = a_3 \qquad \alpha r_2 = a_4$$

$$a_1 = a_3 \cos\theta_1 \qquad a_2 = a_4 \cos\theta_2$$

$$\begin{aligned} a_1 - a_2 &= a_3 \cos\theta_1 - a_4 \cos\theta_2 \\ &= \alpha r_1 \cos\theta_1 - \alpha r_2 \cos\theta_2 \\ &= \alpha (r_1 \cos\theta_1 - r_2 \cos\theta_2) \\ &= \alpha R \end{aligned}$$

$$\alpha = \frac{a_2 - a_1}{R}$$

Fig. 6 - Theory of measurement of angular acceleration using two linear accelerometers

Drawbar Pull

A 10,000-pound load cell was mounted between the test vehicle and the loading device to measure drawbar pull. A 5000-pound load cell was mounted between the test vehicle and a tow truck to measure rolling resistance.

Drive Shaft Torque

Strain gages were applied to each sprocket drive shaft in a four-arm bridge configuration to measure vehicle drive-shaft torque (Fig. 7). A slip-ring assembly was fabricated and attached to each drive shaft to transmit strain signals from the rotating shafts to the recording instruments. A mechanical calibration was performed by applying torque up to 600 ft-lb to each drive shaft and recording the resultant strain. An electrical calibration was performed by using a calibrate resistor in the following manner:

$$\frac{16T}{\pi D^3} = \frac{E\,e}{1+U},$$

and

$$e = \frac{16T\,(1+u)}{\pi D^3\,E} = \frac{16T\,(1.285)}{\pi(1.51)^3\,30\,(10^6)} = 6.35T\,(10^{-8}).$$

$$^{*}e_a = 4e$$

and

$$e = \frac{R_g}{4\,(R_c + R_g)G}.$$

Then

$$6.35\,(10^{-8})T = \frac{R_g}{4\,(R_c + R_g)G}.$$

$$R_c + 120 = \frac{120\,(10^8)}{8\,(6.35)T} = \frac{2.36 \times 10^8}{T}$$

Fig. 7 - Suspension system test on rig no. 1 (strain gage attached)

Given,

Shaft diameter D = 1.51 inches
e_a = Apparent strain
u = Poisson's ratio
e = Actual strain
E = Tensile modulus
T = Torque
G = Gage factor
R_g = Bridge resistance
R_c = Calibrate resistor

then,

for, T = 12,000 lb-in.
R_c = 20,000 ohms

Vehicle Drive Shaft Speed

To obtain this measurement, two voltage pulses were recorded per shaft revolution (Fig. 7).

*There are four individual gages in the bridge which add to read four times the actual strain.

Bump Stop Load

The front and rear bump stops on the right side of the vehicle were replaced with strain-gaged bump stops (Figs. 8 and 9). Calibration was performed by applying known loads with a Tinius Olsen test machine and a calibrate resistor.

With a 50-pound force assumed, the strain is calculated as follows:

$$S = \frac{MC}{I} = \frac{4 \times 50\ (0.87)}{b\,\frac{h^3}{12}}$$

$$= \frac{200\ (0.87)}{0.244} = 712 \text{ psi.}$$

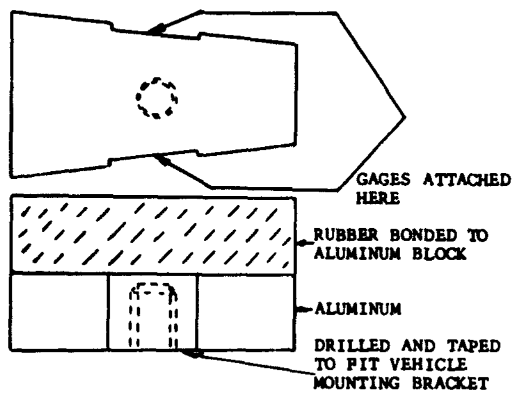


Fig. 8 - Fabricated bump stop (front and rear)

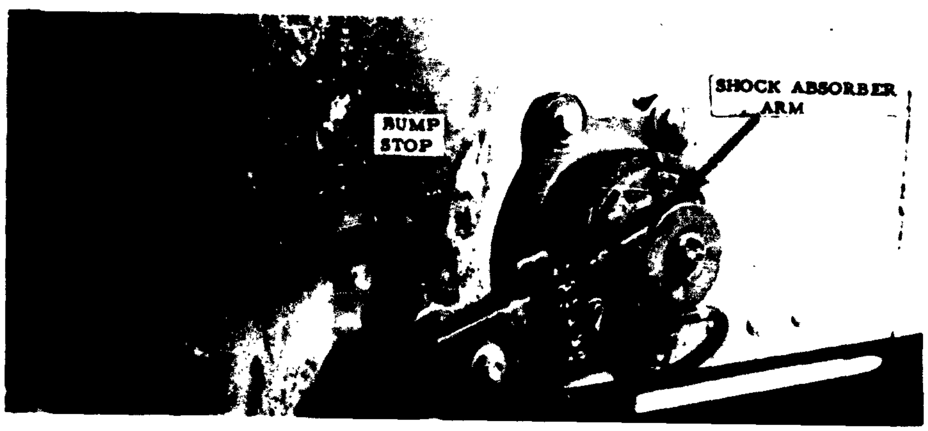


Fig. 9 - Suspension system test on rig no. 1 (front and rear bump stops replaced with strain-gaged bump stops)

Shock Absorber Load

Strain gages were applied to the arm of the rear shock absorber on the right side of the vehicle to measure force (Fig. 9). Parameters for this measurement system are shown in Fig. 10.

Also,

$$S = Ee = 30 \times 10^6 \times e$$

$$e = \frac{712}{30} \times 10^{-6} = 23.7 \text{ microinches}$$

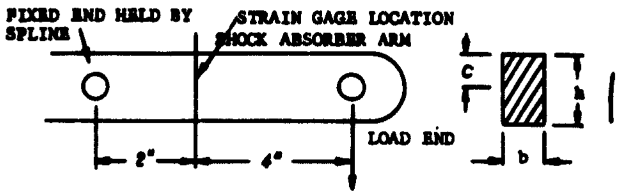


Fig. 10 - Measurement of shock absorber load

where

- S = Stress,
- M = Bending moment,
- I = Moment of inertia,
- e = Actual strain, and
- E = Modulus = 30×10^6 psi.

The gage factor was 2.6. The total bridge output = 2.6 x 23.7 = 61.7 microinches. The actual test with 50 pounds hanging from the arm gave an output of 61 microinches.

This calculation was used to extend the calibration by using a calibrate resistor as follows:

A 40,000-ohm resistor placed across one arm of the bridge gave 1525-microinches output.

$\frac{1525}{61} \times 50 = 1250$ pounds represented by the

40,000-ohm resistor.

Gun Mount Plate Stress

Strain gages were applied to the left and right sides of the gun-mount plate to measure bending stresses due to gun and hull motion (Fig. 11). Calculations for this measurement system are as follows:

Notation:

- S = Stress,
- E = Modulus of aluminum = 10×10^6,
- e = Actual strain,
- e_a = Apparent strain,
- u = Poisson's ratio = 0.285,
- G = Gage factor,
- R_g = Gage resistance, and
- R_c = Calibrate resistance.

An apparent strain of 750 microinches was measured when an 80,000-ohm resistor was placed across one arm of the bridge. Stress was calculated from this strain measurement as follows:

$$S = E\,e, \text{ where } e = \frac{e_a}{1.285};$$

therefore,

$$S = \frac{E\,e_a}{1.285} = \frac{750 \times 10^{-6} \times 10 \times 10^6}{1.285} = 5750 \text{ psi}.$$

Thus, an 80,000-ohm resistor placed across one arm of the bridge represents 5750 psi.

For any value of stress, a calibrate resistor can be calculated as follows:

$$e_a = \frac{R_g}{(R_c + R_g)G} = \frac{120}{(R_c + 120)G},$$

or, for small R_g,

$$e_a = \frac{60}{R_c}.$$

Then

$$e_a = \frac{60}{1.285\,R_c} \text{ and } S = E\,e_a = E\,\frac{60}{1.285\,R_c}.$$

Therefore,

$$R_c = \frac{60 \times 10 \times 10^6}{S\,(1.285)} = \frac{467 \times 10^6}{S}.$$

Acceleration of Front-End of Vehicle

One accelerometer was calibrated on an electromechanical shaker and mounted on the extreme front of the vehicle to measure acceleration in the vertical plane.

Idler-Arm Bending and Torsion

Strain gages were applied to the idler arm to measure vertical bending, crosswise bending, and torsion, Fig. 12. Sketches for these systems are shown in Fig. 13. Calculations are as follows:

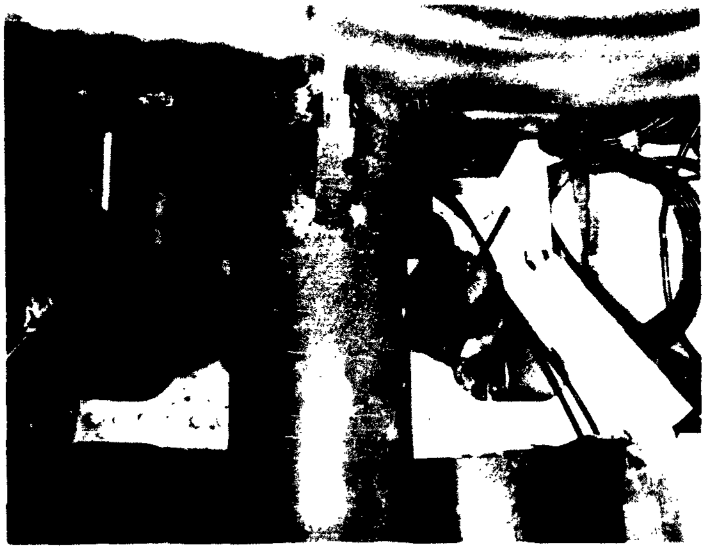


Fig. 11 - Suspension system test on rig no. 1 (strain gages on gun mount)

Fig. 12 - Suspension system test on rig no. 1 (strain gages on left idler wheel arm)

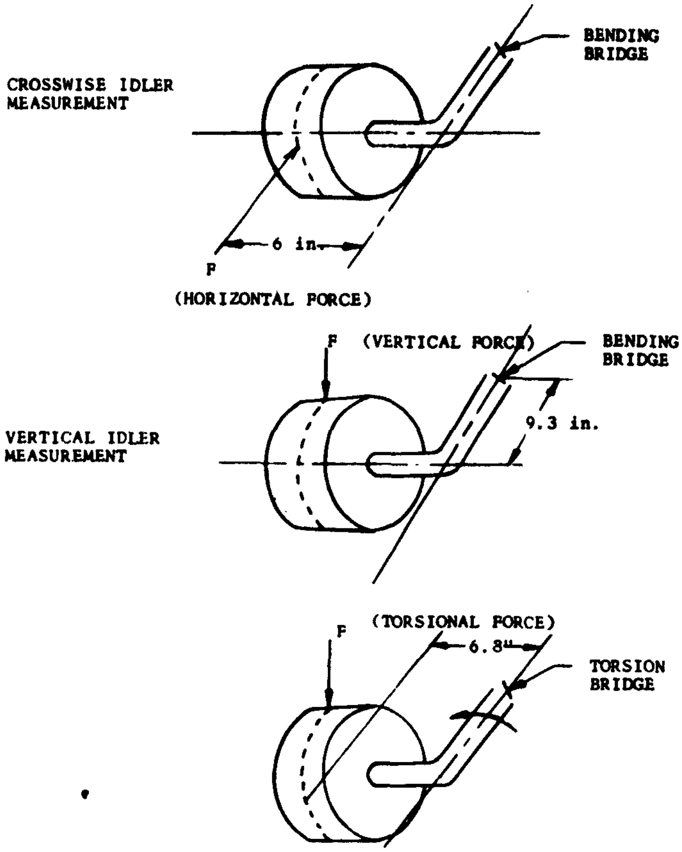


Fig. 13 - Forces on idler arm, measurement diagrams

Vertical Bending - The relationship between stress and applied load was calculated in the following manner:

$$S = \frac{MC}{I} = \frac{M\ d_o/2}{\pi\ (d_o^4 - d_1^4)}$$

$$S = \frac{M\ 64\ (2.75)}{2\pi\,(57.5 - 8.75)}$$

$$S = \frac{M\ (28)}{48.75} = .574\ M.$$

M = FL where F is assumed to be 300 pounds and L = 9.3 inches,

then

S = 0.574 (300) x (9.3),

or,

S = 1600 psi, calculated.

Notation:

e = Actual strain
e_a = Apparent strain
S = Stress
M = Bending moment
I = Moment of inertia
$= \frac{(D_o^4 - D_1^4)\ \pi}{64}$
d_o = Outside diameter of Idler arm = 2.75 inches
d_1 = Inside diameter of idler arm = 1.72 inches
C = $d_o/2$
= distance of applied force from gage area = 9.3 inches
F = Force applied
E = Modulus = 30 x 10^6 psi

When a 300-pound force was applied 9.3 inches from the gage area, a strain of 135 microinches was measured. The stress represented by this strain was calculated as follows:

$$S = E\,e$$

where

$$e = \frac{e_a}{2.47},$$

or

$$S = E\,\frac{e_a}{2.47} = \frac{30 \times 10^6\ (135)}{2.47}$$

= 1630 psi, actual.

This comparison was the basis for extending the calibration through use of a calibrate resistor as follows:

A 40,000-ohm resistor placed across one arm of the bridge gave 1530-microinch output.

$\frac{1530}{135}$ x 1630 = 18,500 psi, represented by

the 40,000-ohm resistor.

18,500 psi is equivalent to 3400-pounds applied force.

Crosswise Bending - The relationship between load and stress was determined in the following manner: Calibration was performed by subjecting the idler arm to a 915-pound lengthwise load. The output of the bridge was measured in terms of galvanometer deflection on a light-beam oscillograph. This deflection was 0.15 inch. Then, a 40,000-ohm resistor was placed across one arm of the bridge; the galvanometer deflected 1 inch. Therefore, the 40,000-ohm resistor represents the following load:

$$\frac{1.00}{0.15} \times 915 = 6100 \text{ lbs.}$$

By use of the same calculations as for vertical idler arm bending, it was established that a 6100-pound load is equivalent to 17,340 psi.

Torsion - The relationship between stress and applied load was calculated as follows:

$$S_s = \frac{16T}{\pi\, D_o^{\,3}(1 - D_1^{\,4}/D_o^{\,4})}$$

$$= \frac{16T}{\pi\ (20.9)\left(1 - \frac{8.75}{57.5}\right)} = 0.287T$$

T = 6.8 inches x vertical load.

Then, for a 300-pound vertical load and with T = 2040 in-lb, S_s = 0.287 (2040) = 585 psi, calculated

Notation:

S_s = Shear stress
E = Modulus = 30 x 10^6 psi
T = Torque
L = Length of torque arm = 6.8 inches
D_o = Outside diameter of idler arm = 2.75 inches
D_1 = Inside diameter of idler arm = 1.72 inches
e = Actual strain
e_a = Apparent strain
u = Poisson's ratio = 0.285.

Actual Calibration - When a 300-pound load was applied vertically on the idler wheel, a strain of 100 microinches was measured. The stress represented by this strain was calculated as follows:

$$S_s = \frac{E\,e}{1 + u} = 23.34e$$

$$e = e_{a/4}$$

$$S_s = 23.34\,\frac{e_a}{4} = \frac{23.34\ (100)}{4}$$

$$S_s = 584 \text{ psi, actual.}$$

The foregoing comparison was the basis for extending the calibration through use of a calibrate resistor as follows:

A 40,000-ohm resistor placed across one arm of bridge gave 1506-microinch output.

$\frac{1506}{100}$ x 584 = 8800 psi represented by the

40,000-ohm resistor. The equivalent to a 4500-pound applied force is 8800 psi.

Drive Sprocket Hub Stress

Strain gages were applied to the left drive sprocket hub to measure bending stresses in the vertical and lengthwise directions (Fig. 14). Calculations for this measurement system are as follows:

Notation:

S = Stress
E = Modulus = 30 x 10^6 psi
e_a = Apparent strain
e = Actual strain

Fig. 14 - Suspension system test on rig no. 1 (strain gages on drive sprocket hub)

When a 40,000-ohm resistor was placed across one arm of each bridge, the apparent measured strain was 1605 microinches in the vertical plane and 1477 microinches in the lengthwise plane.

The equivalent stress was calculated as follows:

$$S = E\ e, \text{ where } e = \frac{e_s}{1.285}$$

Vertical Plane S = 37,500 psi

Lengthwise Plane S = 34,500 psi

Torsion Bar Angular Motion

Strain gages were applied to the torsion bar of the left-front road wheel in a four-arm bridge configuration to measure torsional strain. The strain was related to angular motion of the torsion bar by calculation as follows:

Notation:

S_s = Shear stress
D = Diameter = 1.041 inches
u = Poisson's ratio = 0.285
e = Actual strain
E = Tensile modulus = 30 x 10^6 psi
T = Torque
G = Shear modulus = 12 x 10^6 psi.

The relationship between angular displacement and strain was calculated as follows:

A torque (T) of 12,000 in-lb was assumed.

$$S_s = \frac{16T}{\pi D^3} \text{ and } S_s = \frac{E\ e}{1+u} \quad \left(\begin{array}{l}\text{Effective length of} \\ \text{torsion bar = 30.8} \\ \text{inches}\end{array}\right)$$

$$S_s = \frac{16\ (12{,}000)}{\pi\ (1.041)^3} = 53{,}400 \text{ psi.}$$

$$53{,}400 \text{ psi} = \frac{E\ e}{1+u} = \frac{30 \times 10^6\ (e)}{1.285}.$$

Therefore,

$$e = \frac{53{,}400\ (1.285)}{30 \times 10^6} = \begin{array}{l}\text{2290 microinches,} \\ \text{for 12,000 in.-lb.}\end{array}$$

$$\theta \text{ (unit twist)} = \frac{2S_s}{G\ D}$$

$$= \frac{2\ (53{,}400)}{12\ (1.041) \times 10^6}$$

$$= 8.5 \times 10^{-3} \text{rad/in.,}$$

$$\text{or } \frac{360}{2\pi}(8.5 \times 10^{-3}) = 0.48 \text{ deg/in.}$$

$$\phi \text{ (total twist)} = 0.48 \text{ deg/in. } (30.8)$$
$$= 14.7 \text{ degrees for 12,000 in.-lb.}$$

Therefore, 14.7-degree twist will result in 2290-microinch actual strain.

The calibrate resistor which represents 2290 microinches is 6700 ohms.

Engine Speed

A contact was mounted on a fan belt pulley shaft and a magnetic pickup was mounted nearby to detect its passage. This system provided one voltage pulse per engine revolution.

Vehicle Speed

A fifth wheel was mounted on the test vehicle to measure vehicle speed.

Mat-Type Bump

Two load cells were mounted between two hinged steel plates to measure the impact of each track as the vehicle negotiated the bump, Fig. 15. A sketch of this system is shown in Fig. 16.

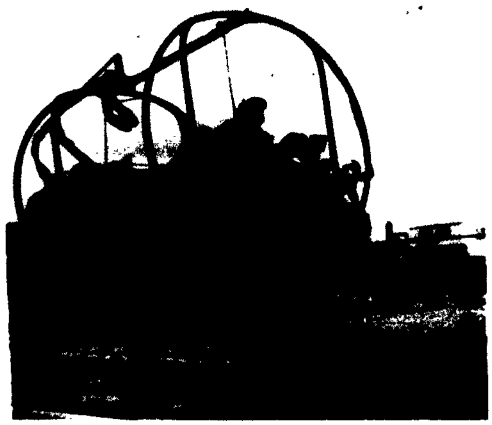


Fig. 15 - Evaluation of suspension system on test rig no. 1, second wheel about to go over steel obstacle

DISCUSSION OF MEASUREMENT SYSTEMS

The following is a discussion of the use of each measurement system in terms of vehicle development.

Accelerations About the Center of Gravity

Vertical, lengthwise, and crosswise pitch, roll, and yaw accelerations were measured to evaluate vehicle stability during the sine-wave and obstacle tests. These measurements showed that both suspension concepts were satisfactory and showed agreement with computer simulation results.

Drawbar Load and Rolling Resistance

The drawbar measurements showed that the vehicle had the necessary power. The rolling resistance measurements helped to determine the best combination of sprockets, track guides, and track tension.

Drive Shaft Torque

This measurement was used to evaluate engine power train performance. Together with sprocket speed, this measurement gave continuous readings of power available at the sprockets. In addition, this measurement was a good indicator of instantaneous track tension during the tests. During high-speed runs the information helped evaluate sprockets and track guides. That is, visual examination of oscillograms showed the pattern of torque transfer from sprocket to track on each side of the vehicle.

Drive Shaft Speed

This measurement was used, in conjunction with drive shaft torque, to monitor power at the

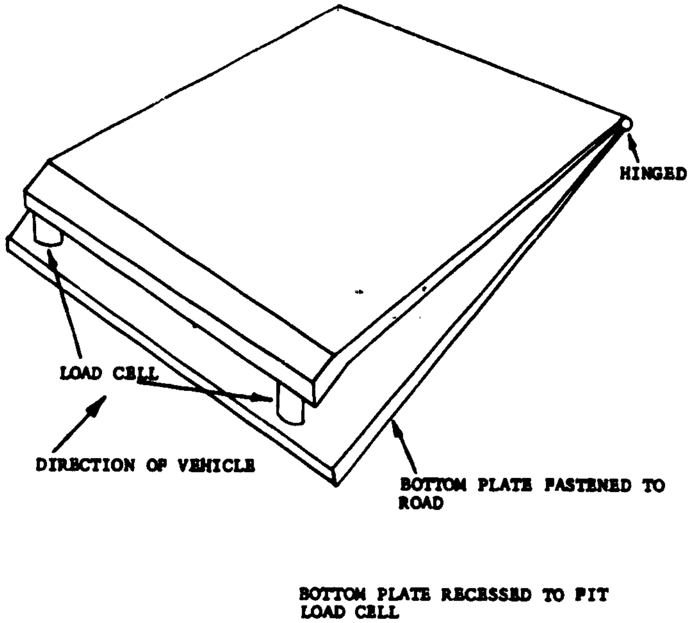


Fig. 16 - Impact loads recorded on consolidated oscillograph (Mat-type bump)

sprocket. It was also used to evaluate the steering system by providing a continuous reading of sprocket speed on each side of the vehicle.

Bump Stop Load

This measurement was used to monitor force transmitted to the hull through the suspension system. The bump stop mounting was broken while negotiating the 8-inch bump. The data obtained showed that the force necessary to cause this part to fail was greater than any we had expected to encounter. The results obtained with this particular measurement system were instrumental in changing the suspension and increasing the design limits by 50 percent.

Shock Absorber Load

This measurement was used to determine the suitability of this particular shock absorber and to tie-in with suspension reaction results obtained from other components.

Gun Mount Plate Stress

This plate is a major supporting member of the gun and is so located that it can be used to monitor twisting of the hull. Stress on this plate was much lower than expected. As a result of data obtained, plate dimensions were reduced and a large hull stiffening member was eliminated with a considerable savings in weight.

Idler Arm Bending and Torsion

These measurements were used to monitor stress encountered during the sine-wave and obstacle tests, and also during the track-throwing test in which side-loading was severe.

Torsion Bar

This measurement was used to monitor road-wheel loading and displacement. It is of interest to the instrument engineer that the total windup of the torsion bar caused an actual strain of approximately 6000 microinches. It was expected that the strain gage installation would not withstand many cycles of this high strain, but no difficulties arose during the 3 months of testing.

Vehicle Speed

There was a requirement that the vehicle must attain a speed of 35 mph. We were not able to reach this speed during our initial high-speed runs. Data showed that the horsepower at the final drives was low and that the engine speed necessary to provide the required vehicle speed was well beyond the peak of the engine power curve.

A mockup of the engine power train was being used in the power plant laboratory to determine engine cooling characteristics. This mockup was used to analyze horsepower consumed by the various engine power-train accessories. By making changes which had little effect on power-train performance, eight additional horsepower were made available at the final drives. Also, the engine-to-track gear ratio was changed so that a lower engine speed would result in a higher vehicle speed. As a result of these changes, a vehicle speed of 38 mph was attained.

Recording and Analysis

Vertical, crosswise, and lengthwise center-of-gravity accelerations, and pitch, roll, and yaw accelerations were recorded on magnetic tape. All other measurements were recorded on two light-beam type oscillograph systems. Recording instruments were mounted on another vehicle to protect personnel and instruments from the violent maneuvering of the test vehicle (Fig. 17).

Recording instruments were mounted on test vehicles during high-speed runs and slope climbing (Fig. 18).

Test data were analyzed immediately after each test run so that the maximum values encountered could be determined and transmitted to the vehicle design engineers.

All magnetic tape data were played back onto a light-beam oscillograph for analysis. Also, the tapes were available for further reduction by computer people. In addition, copies of pertinent portions of oscillograms, marked as shown in Figs. 19 through 21, were presented for use by interested engineers.

Further Development

After sine-wave and obstacle tests, the vehicle was subjected to a run (approximately 2000 miles) over secondary roads and cross-country terrain, during which there were only minor breakdowns. The reliability of the vehicle during these endurance tests demonstrated the value of the instrumentation in finding weak points in design, which could be corrected in the early stages of development.

The first test rig was followed by a second which was used to determine the stability of the XM-104 as a firing platform. This rig also was instrumented extensively. Instrument results, together with high-speed photographs, proved that the XM-104 provided a suitable firing platform for the 105-mm gun. The next phase was to build a pilot model incorporating all engineering results of the test rigs.

The excellent performance of the operational pilot model was the measure of instrumentation efficiency in a program requiring the cooperation of the various segments of research, design, testing, and instrumentation.

This first vehicle produced in the accelerated development cycle was approximately 2 years ahead of schedule.

Fig. 17 - Suspension system test with XM-104 test rig no. 1 and instrument vehicle

Fig. 18 - Test rig no. 1 going up a slope

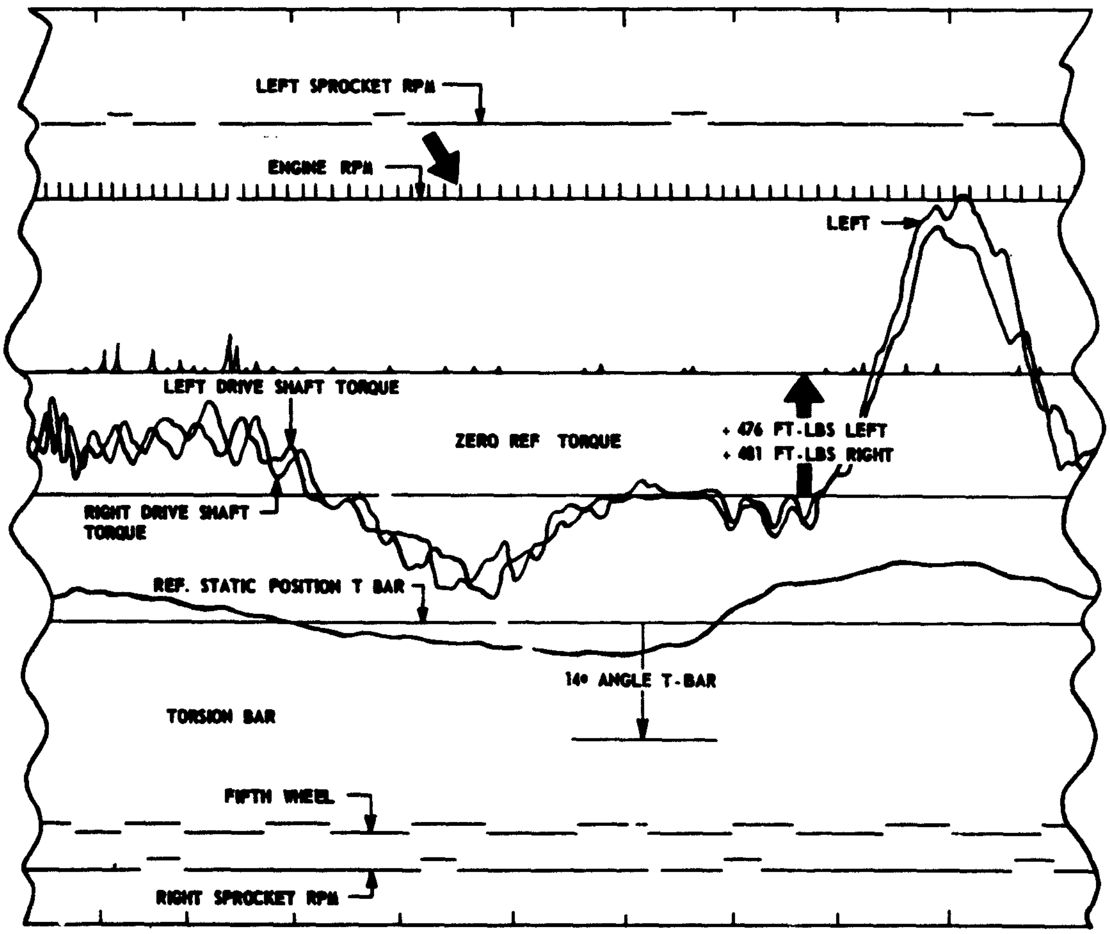


Fig. 19 - Representative oscillogram

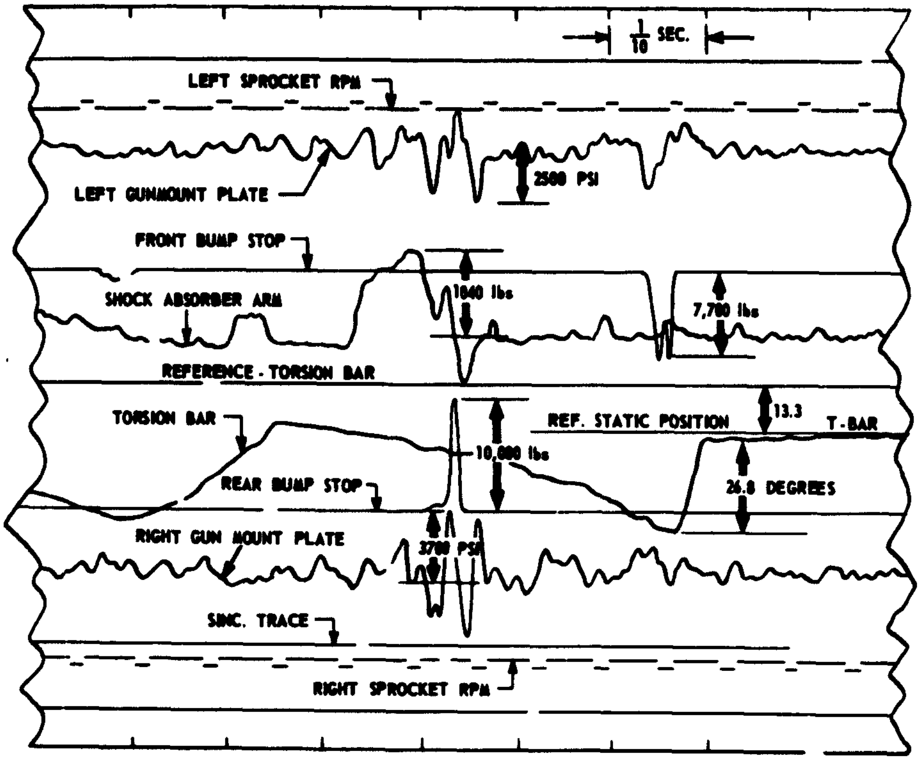


Fig. 20 - Representative oscillogram

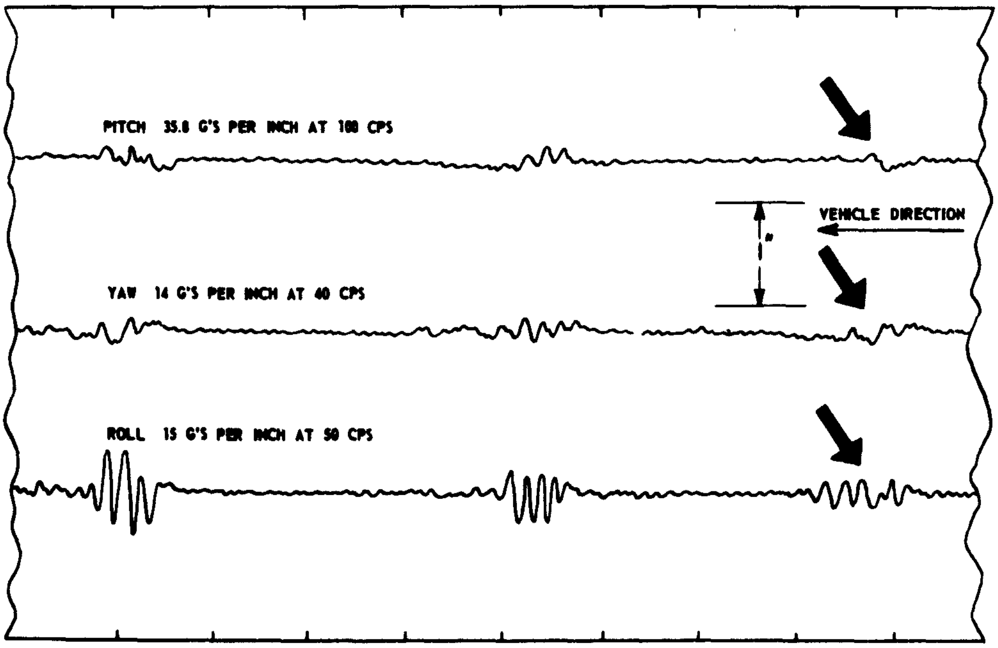


Fig. 21 - Representative oscillogram (pitch, yaw, and roll)

ACCELEROMETER SENSITIVITY TO DYNAMIC PRESSURE PULSES

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It is necessary to know the response to a dynamic pressure input when using accelerometers in pressure fields. In order to evaluate this effect, several different manufacturers' accelerometers were subjected to pressure pulses varying from 20 to 150 psi, approximately. This paper shows the experimental equipment used, presents the data obtained, and compares various accelerometers.

INTRODUCTION

During the MINUTEMAN flight and ground staging it was suspected that the crystal accelerometers mounted in the interstage area were responding to the buildup from engine ignition prior to separating the stages. Most of the information available in current literature was concerned with acoustic response involving a high-frequency, very low-pressure environment, and therefore was not very useful (Refs. 1 and 2).

Accordingly, to check this suspected pressure response it was decided that the subject transducer should be tested with pressure pulses similar to those encountered during the staging pressure transient; in addition, other manufacturers' models and types of accelerometers were included to provide additional data for future test programs.

This paper presents experimental data obtained by subjecting different types of accelerometers to pressure pulses having rise times equivalent to a 45-cps sinusoid, with peaks up to 120 psi. The case-compression, or isolated compression type, crystal accelerometers were found to have an external pressure sensitivity ranging from 0.012 to 0.1 g/psi. The shear and bender type crystal accelerometers and strain gauge type accelerometers were essentially insensitive to these pressure pulses (less than 0.004 g/psi), which was near the accuracy of the experimental test setup used.

The results of this study show that crystal accelerometers must be used with extreme care in a dynamic pressure environment. Any one of the following approaches is recommended under these conditions:

- Select a shear or bender mode crystal accelerometer or a strain gauge type accelerometer.

- Protect the accelerometer from the pressure pulse*.

- Know the pressure response in g per psi and, having measured the pressure pulse, subtract the pressure response from the acceleration response. This has been done on some MINUTEMAN flight data.

- Choose accelerometer-amplifier combinations which will not respond to the frequencies

[1] Wilson Bradley, Jr., "Effects of High Intensity Acoustic Fields on Crystal Vibration Pickups," prepared for the 26th Shock and Vibration Symposium, Endevco Corporation (revised February 6, 1959).

[2] Jack Fromkin, "A Study of Methods of Testing Vibration Transducers in an Acoustic Noise Environment," Report No. 1813, Rototest Laboratories, Inc., Lynwood, California (October 1959).

*This was done very successfully by Mr. J. Parker of Aerojet General Corporation, Sacramento, on several MINUTEMAN staging tests.

present in the pressure pulse. This is applicable for cases in which the pressure variations are at a low frequency compared with the acceleration frequency of interest.

Tabulated results are shown in Table 1.

pressure transducer in the base plate and wires for the accelerometer were potted into place, while still maintaining the pressure tight seal.

Operation of the pulse generator consisted of pressurizing the volume above the piston while holding the thrust rod, then closing valves to each chamber and releasing the thrust rod. The device thus becomes a spring mass system whose motion could be controlled by varying the initial position of the piston, the length of the chambers, the mass of the piston and thrust rod, and the initial pressure in the upper chamber. In the configuration used for the test, the length of the upper chamber was about 1 inch and the lower chamber (above the mounting block) 4 inches, and the piston and thrust rod weighed 2 pounds. With the upper chamber pressurized

TABLE 1

Test Results, Accelerometers Ranked in Order of Pressure Sensitivity

Rank	Manufacturer	Model	Serial Number	g/psi	Mode
1	Statham	A5-5-350	11147	0.000	Strain Gauge
2	Endevco	2221	AA18	0.00	Shear
3	Endevco	2226	DE09	0.002	Shear
4	Electra Scientific	ES6505	114	0.003	Shear
5	Clevite	5C1	3151	0.004	Shear
6	Glennite	AC1090	154	0.004	Bender
7	Endevco	2229	DA24	0.008	Shear
8	Endevco	2242M5A[a]	E-36	0.012	Isolated Compression
9	Endevco	2213-C	DD06	0.024	Single Ended Compression
10	Columbia	302-5	1433	0.052	(Compression)
11	Endevco	2242 M-5	BA15	0.100	Isolated Compression
12	Statham	AK105	33 & 39	1.8	Compression
13	Kistler	810	392	4.6	Compression (Exposed Top)

[a]This unit has superseded the 2242M5 and incorporated the modifications developed as a result of this test program.

EXPERIMENTAL SETUP

The pressure pulse generator consisted of a 3-inch diameter cylinder fitted with a double acting piston and thrust rod, as shown in Figs. 1 and 2. The thrust rod protruded through one end, where it was held by a quick release hook, as shown in Fig. 1. Sealing of the thrust rod, piston, and end caps was effected by O-rings. The base of the cylinder was rigidly attached to the reinforced concrete floor. Provisions were made for mounting a flush diaphragm

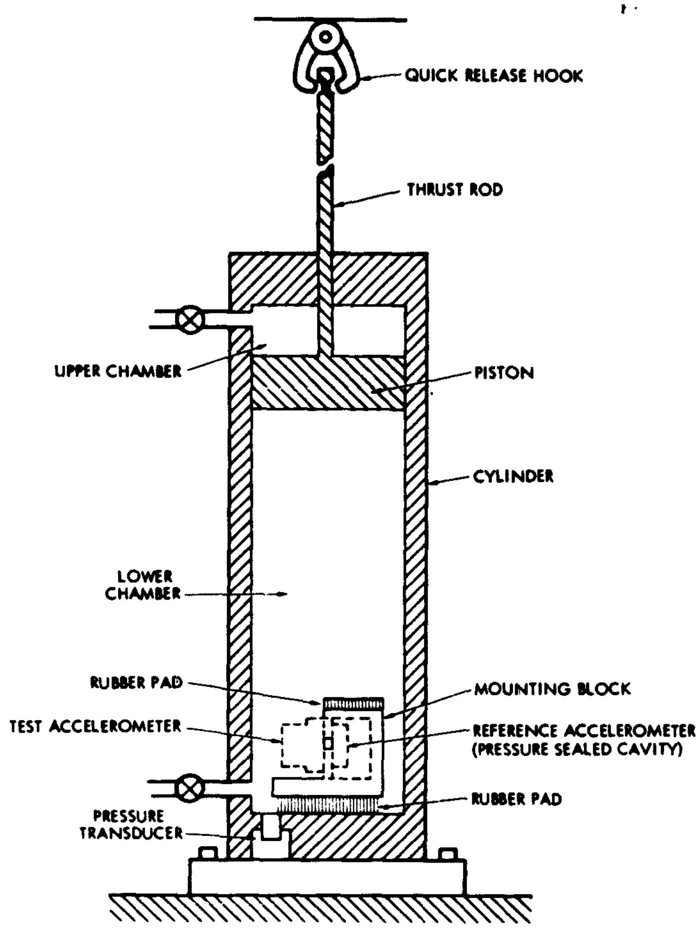


Fig. 1 - Pressure pulse generator

to 150 psi prior to releasing the piston, a pressure pulse of 0.022-second duration and 130-psi amplitude was produced in the lower chamber followed by several smaller pulses. Only the first pulse was used as a basis for collecting data. Because it was feared that vibration due to the release of the thrust rod would mechanically excite the transducer, the test accelerometer was mounted on a large brass block (Fig. 3) and isolated from the base by a 1/8-inch thick neoprene pad. A reference accelerometer was mounted directly behind the test accelerometer in a sealed cavity in the block. A further effort to reduce the mechanical transmission was instituted by orienting the test specimens transversely to the expected direction of motion of the block. To prevent mechanical damage to the accelerometers, should the piston overshoot, the mounting block was made high enough to prevent any part of the accelerometer from being struck and a 1/8-inch neoprene pad was glued to the top of the block to cushion any impact which might occur between the piston and block.

A block diagram of the instrumentation used in this study is shown in Fig. 4. The outputs of the test accelerometer, reference accelerometer, and the pressure transducer were recorded on an oscillograph after amplification. All sensitivities were based on the manufacturers' calibration information supplied with the accelerometers. As indicated in Table 2, the same equipment was not used for all of the tests. Various combinations were used as dictated by equipment availability. Prior to each test, the sensitivities of the amplifier and galvanometer were calibrated by inserting a known voltage at the input of the amplifier. The pressure calibration was performed with a Helicoid 150-psi gage.

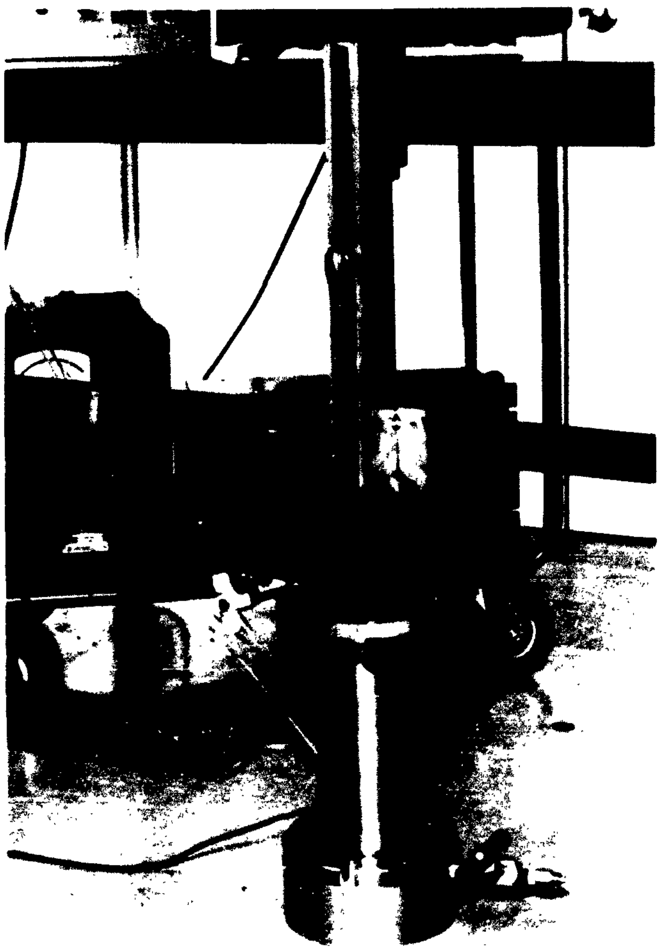


Fig. 2 - Test fixture

Certain test results led to the desirability of measuring the side load sensitivity of accelerometers (this will be discussed later). These side load tests were performed by placing a loop of string around the accelerometer being tested and applying a dynamic load with a spring scale. The spring scale had a mechanical stop such that the peak force could be read. Figures 5a and 5b show the procedure used. The accelerometer being tested is the ENDEVCO 2242M5 with the Autonetics proposed protective cap. This test technique was used to get qualitative data only.

PRESENTATION AND DISCUSSION OF DATA

Test on Standard Accelerometers

Initially it was planned to apply several different length pressure pulses to each accelerometer to determine the nature of the pressure sensitivity. After several accelerometers had been tested, an inspection of the data indicated that in the frequency range tested (which had a rise time corresponding to approximately 45 cps) no natural frequencies of the accelerometers

Fig. 3 - Disassembled test fixture

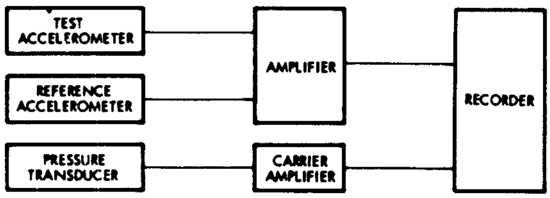
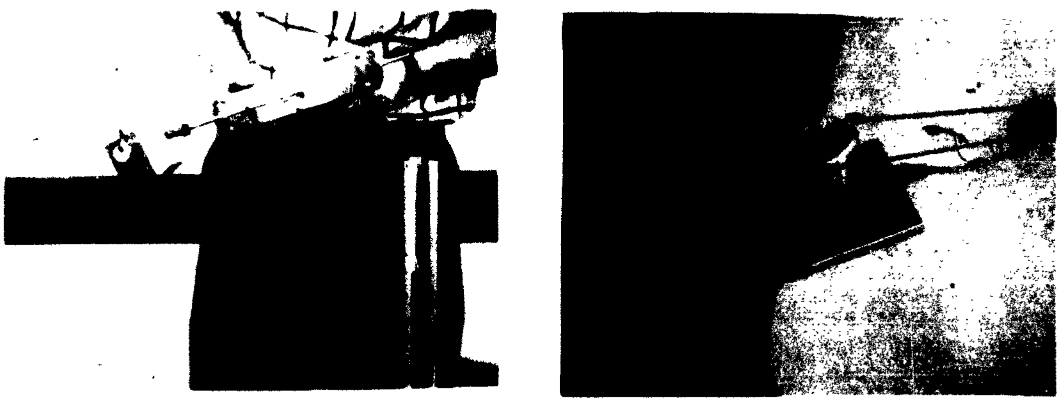


Fig. 4 - Instrumentation block diagram



(a) (b)

Fig. 5 - Sideloading test procedure

TABLE 2

Equipment List

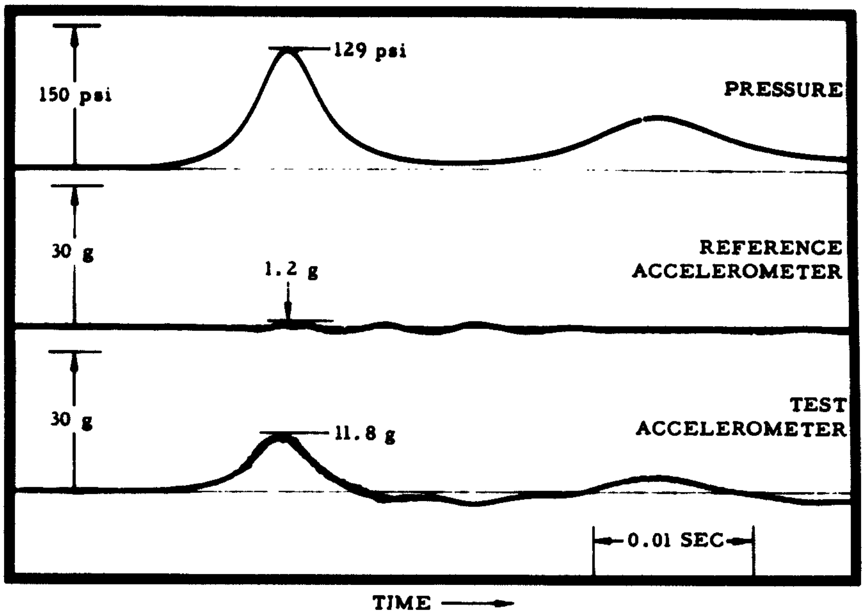
Test Equipment	
Pressure Transducer	Statham Type PA-130TC-150-350
Reference Accelerometer	Endevco 2229 or 2221
Carrier Amplifier	Consolidated Electrodynamics Corporation* 1-127
Amplifier	Endevco 2702-B Endevco 2702-C Kistler 568
Recorders	C. E. C. 5-124 C. E. C. 5-119 Midwestern 616
Galvanometers	C. E. C. 7-361 Midwestern 120B-3.5K (A Midwestern 102-1000 Galvanometer was used with the Kistler Amplifier)
Calibration Equipment	
Oscillator	Krohn-Hite MOD420-A
Voltmeter	J. F. Fuke MOD803
Pressure Gauge	Helicoid, 150 psi ±1/2 percent

were being encountered. The response was essentially a static response which was probably due to straining the case and then the crystal, producing a charge output. For these reasons, the basic pressure-time history was not modified during the test program. Slight timing variations were obtained due to the nature of the experimental setup caused by changing volume, friction, and pressure amplitude.

Since the pressure sensititivy is essentially a static response, it is possible to compute sensitivities as a function of pressure. These results are shown in Table 1. Only one strain gauge accelerometer was tested since it was felt that such accelerometers would be insensitive and also because they could easily be tested should doubt arise. For each test condition, the outputs of the test accelerometer, the reference accelerometer, and the pressure transducer were recorded on the oscillograph. Examples of the resulting data are shown in Figs. 6 through 9. It can be noted that the acceleration of the mounting block was small as indicated by the reference accelerometer.

As previously mentioned, the shear and bender models have the lowest sensitivity. Next in order are two Endevco compression models, 2242M5A and 2213C, the former being an improved 2242M5. Next is a Columbia Model 302-5 whose mode of operation is thought to be single ended compression. The last ranked accelerometer is the Kistler Model 810. This is a special model which can be converted to a load cell by exposing the crystal element. This no doubt accounts for its very high pressure sensitivity.

During the test program the Endevco 2242M5 was tested with elastic potting compound applied, Fig. 10(b). This configuration gave surprisingly high results until it was suspected that the accelerometer mounting block moved down under the pressure, side loading the accelerometer and producing an



DESCRIPTION

MANUFACTURER - ENDEVCO
MODEL NUMBER - 2242M5
SERIAL NUMBER - BA15

CALIBRATION INFORMATION

8.70 RMSMV/PEAK g
WITH 100 pf CABLE CAPACITY
AND 111 pf ACCELEROMETER CAPACITY
130.2 RMSMV/30 PEAK g's
WITH 297 pf CABLE + 15 pf
AMPLIFIER INPUT CAPACITY

PRESSURE, PEAK - 129 psi
TEST ACCELEROMETER - 11.8 g's
REFERENCE ACCELEROMETER - -1.2 g's
DIFFERENCE - 13.0 g's
PRESSURE RESPONSE - 0.101 g/psi

Fig. 6 - Typical data (ENDEVCO 2242M5)

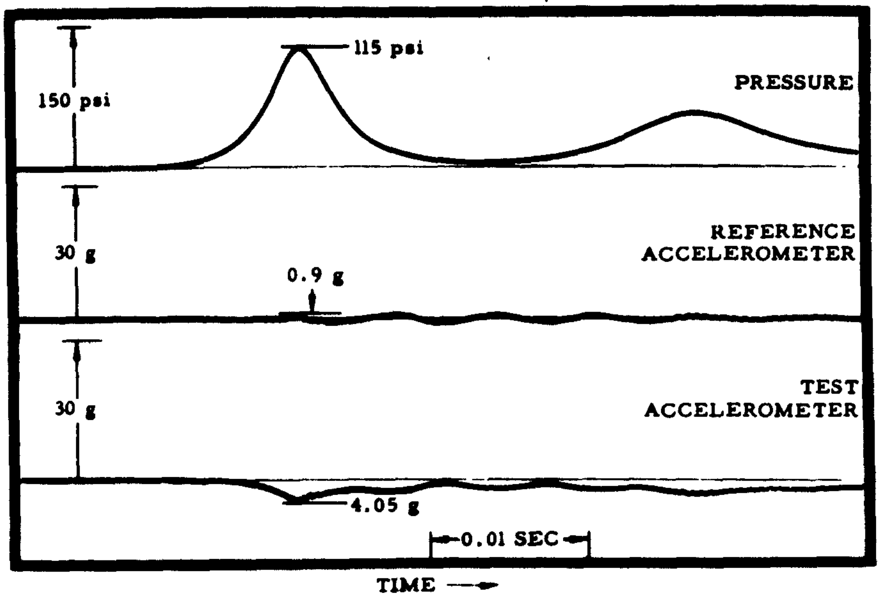
output. This was confirmed by testing the configuration shown in Fig. 10(c) which yielded data similar to that obtained when testing the original configuration, Fig. 10(a).

These data led to tests for side load sensitivity by the application of a dynamic transverse load to the accelerometer. Figure 11 shows the data for the two accelerometers so tested. There is considerable scatter in the data and it was not intended that it be quantitative. From this limited data, however, it is clear that care must be taken with potted accelerometers which may be subjected to side dynamic acceleration.

CRYSTAL ACCELEROMETERS

Low-Frequency Cutoff of Crystal Accelerometers

An inspection of certain bits of data reveals the presence of a slight, low-frequency discharge (undershoot) of the accelerometer during the pressure pulse. This is unfortunate and, in retrospect, a slightly faster pressure pulse would have eliminated most of this problem. In an attempt to eliminate this undershoot without altering the basic test setup, a Kistler charge amplifier was used for some of the tests.



DESCRIPTION

MANUFACTURER - ENDEVCO
MODEL NUMBER - 2213C
SERIAL NUMBER - DD06

●

CALIBRATION INFORMATION

33.5 RMSMV/PEAK g
WITH 100 pf CABLE CAPACITY
AND 804 pf ACCELEROMETER CAPACITY
814 RMSMV/30 PEAK g's
WITH 297 pf CABLE + 15 pf
AMPLIFIER INPUT CAPACITY

●

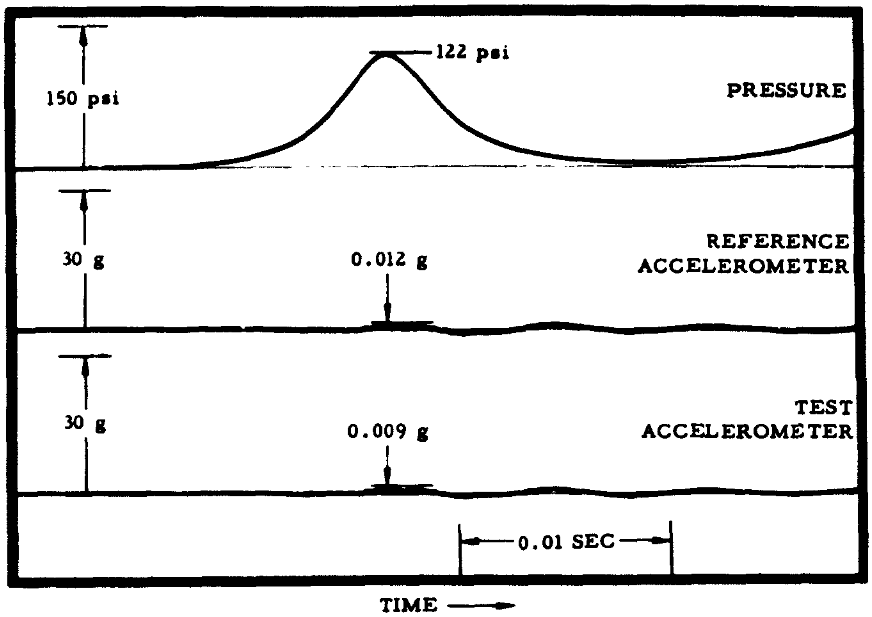
PRESSURE, PEAK - 115 psi
TEST ACCELEROMETER - 4.05 g's
REFERENCE ACCELEROMETER - +0.9 g's
DIFFERENCE - 3.15 g's
PRESSURE RESPONSE - 0.027 g/psi

Fig. 7 - Typical data (ENDEVCO 2213C)

While the low-frequency cutoff point of the Endevco accelerometer-amplifier combination made testing more difficult, it may be advantageous in some instances to tailor the low-frequency characteristics of a system such that a pressure pulse is below the system frequency response. Depending upon the severity of the pulse, several octaves of separation may be required. The input resistance of the amplifier offers a convenient point for this adjustment by eliminating the low-frequency components before they reach the amplifier where they might cause overload. This method obviously is not satisfactory when the acceleration being measured is produced by the pressure pulse and has similar frequency characteristics.

Effect of Sideload on Accelerometers

During the test program, it was suggested that a test accelerometer be potted to see if this had any effect on the response. The accelerometer was inadvertently potted on the mounting block and against the base plate, such that when the mounting block moved the accelerometer was sideloaded. The response results with the potted accelerometer were confusing until it was suspected that the major problem was the sideload on the accelerometer. The Autonetics capped, and the uncapped Endevco Model 2242M5 accelerometers were sideloaded using a spring scale and a light strap near the accelerometer's outer end. The



DESCRIPTION

MANUFACTURER - ENDEVCO
MODEL NUMBER - 2221
SERIAL NUMBER - AA18

●

CALIBRATION INFORMATION

6.40 RMSMV/PEAK g
WITH 100 pf CABLE CAPACITY
AND 287 pf ACCELEROMETER CAPACITY
124 RMSMV/30 PEAK g's
WITH 297 pf CABLE + 15 pf
AMPLIFIER INPUT CAPACITY

●

PRESSURE, PEAK - 122 psi
TEST ACCELEROMETER - 0.009 g's
REFERENCE ACCELEROMETER - 0.012 g's
DIFFERENCE - 0.003 g's
PRESSURE RESPONSE - 0.00 g/psi

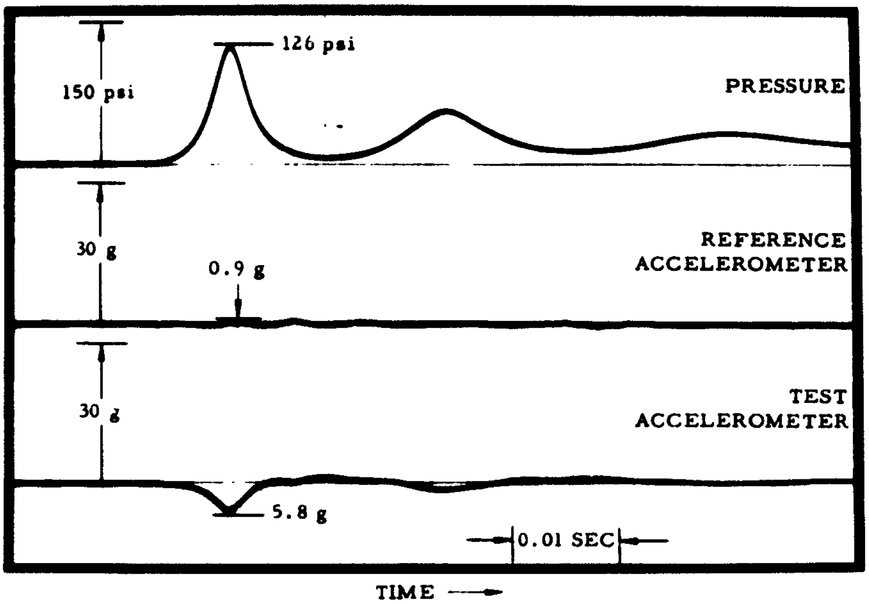
Fig. 8 - Typical data (ENDEVCO 2221)

resulting data are shown in Fig. 11 for these two accelerometers and are generally very scattered and inconsistent. However, the results do indicate the importance of avoiding sideloads on accelerometers. Sideloading conditions which might be encountered in practice are as follows:

- Axial missile acceleration on a potted lateral accelerometer which would produce sideloading. Staging transients contain dynamic loads to which the accelerometer-amplifier system will respond.

- Pressure environment on a potted accelerometer mounted near a restriction and either the unsymmetrical stress distribution or relative deflection of components near the mounting surface.

In general, to be absolutely certain that there is no sideload effect on accelerometers, it is recommended that the accelerometer not be potted, but enclosed in a small protective box. Additional work in this area may be helpful. Shock tests of potted accelerometers in the lateral direction would also be interesting.



DESCRIPTION

MANUFACTURER - COLUMBIA RESEARCH LABORATORIES
MODEL NUMBER - 302-5
SERIAL NUMBER - 1433

CALIBRATION INFORMATION

158 RMSMV/PEAK g
WITH 0 pf CABLE CAPACITY
AND 380 pf ACCELEROMETER CAPACITY
1840 RMSMV/30 PEAK g's
WITH 297 pf CABLE + 15 pf
AMPLIFIER INPUT CAPACITY

PRESSURE, PEAK - 126 psi
TEST ACCELEROMETER - 5.8 g's
REFERENCE ACCELEROMETER - -0.9 g's
DIFFERENCE - 6.7 g's
PRESSURE RESPONSE - 0.053 g/psi

Fig. 9 - Typical data (COLUMBIA 302-5)

Endevco Fixes and Tests

The 2242 M-5 accelerometer was tested and proved to be sensitive to the pressure pulses used, having a sensitivity of approximately 0.09 g/psi. To fix the accelerometer, Autonetics suggested enclosing it with an aluminum cap and applying Epon adhesive, as a sealant, near the intersection of cylindrical and hexagonal portions. This fix provided a great deal of improvement, but could degrade the frequency response and the cross-axis sensitivity performance below the specification requirements.

Endevco Corporation then instituted a program to outline certain fixes which would reduce the pressure sensitivity response for the 2242 M-5.

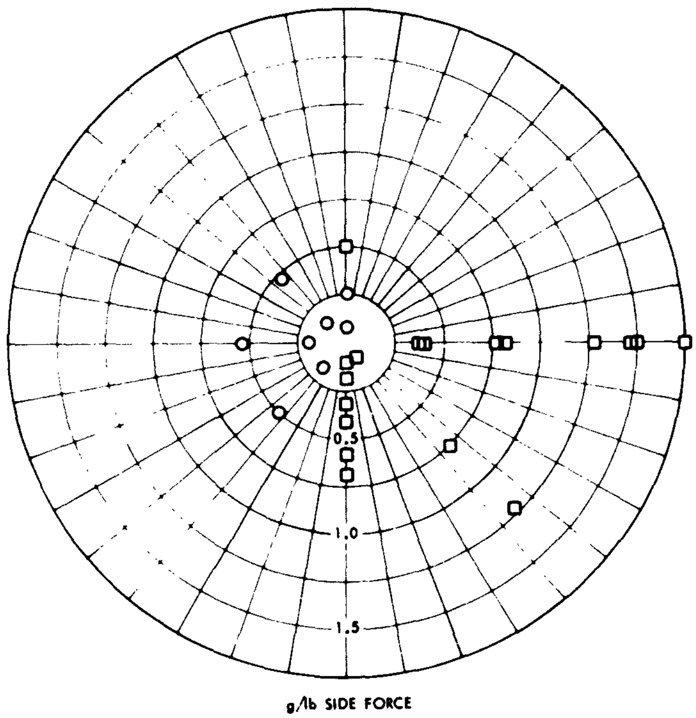
The second fix tried, which reduced the pressure response to 0.012 g/psi, was felt to be acceptable for our purposes. It is being incorporated by rework on those flight accelerometers subjected to pressure environment, as well as any new 2242 M-5 accelerometers supplied by Endevco. These new accelerometers will have the Model Number 2242M5A.

(a) Unpotted

(b) Potted

(c) Potted with potted compound cut away from accelerometer

Fig. 10 - Test mounting block



○ ENDEVCO 2242M5

□ ENDEVCO 2242M5 WITH AUTONETICS CAP

Fig. 11 - Sideload sensitivity, g/lb versus radial angle

EXPERIMENTAL VERIFICATION OF VIBRATION CHARACTERISTICS USING STATISTICAL TECHNIQUES*

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R W Division
Canoga Park, California

This paper discusses the application of statistical techniques to the analysis and interpretation of vibration data. Emphasis is placed on application of vibration measurements as estimators for future vibration environments. Theoretical considerations are presented which are supported by actual experimental results.

INTRODUCTION

When flight vehicle vibration data is gathered and analyzed, the usual objective is to obtain information concerning the vibration environment to be expected during future missions for that and all similar vehicles. Given a sample vibration response amplitude† time history record obtained during the flight of a given vehicle, one may readily measure various descriptive properties of the recorded vibration response, such as amplitude probability density functions, correlation functions, power spectral density functions, and so on. These measurements, however, describe only the vibration response in that vehicle for that interval of time in the past when the sample record was obtained. If the measurements are to be of value for predicting the vibration environment during future missions of that and other similar vehicles, certain fundamental characteristics of the vibration environment must be considered. Three such characteristics, to be discussed here, are randomness, normality, and stationarity.

IMPORTANT CHARACTERISTICS OF VIBRATION ENVIRONMENTS

Given the task of analyzing a flight vehicle vibration environment, the first question which should be answered is: Is the vibration response being measured random in nature as opposed to periodic? Perhaps both random and periodic components are present. The desired procedures for analyzing and describing the vibration environment may be quite different for each case. The techniques for applying the end data to design problems are also affected. Periodic vibrations can be described by explicit analytic functions while random vibrations must be described by statistical functions. As a result, the length of sample records to be gathered for analysis is more critical for random vibrations due to inherent statistical uncertainties or sampling errors in the resulting measurements.

The next fundamental characteristic to be considered is normality. For a random vibration, do the instantaneous vibration amplitudes

*The material presented in this paper is based in large part upon studies performed for the Dynamics Branch, Flight Dynamics Laboratory, Aeronautical Systems Division, USAF, under contract nos. AF33(616)-7434 and AF33(657)-7459.

†Vibration response amplitudes are usually obtained in the form of voltage signals from transducers, and may be a measure of any physical response parameter (acceleration, velocity, displacement, stress, and so on.) depending upon the nature of the transducer employed. The particular response parameter being measured is of no direct importance to the analysis procedures discussed herein.

have a Gaussian probability density function? If the amplitude distribution for a random vibration environment is found to be sufficiently Gaussian to justify the normality assumption the application of the resulting data to design problems is greatly simplified. Furthermore, the existence of normality will greatly increase the power of conclusions concerning the stationary characteristics of the vibration environment.

If a vibration environment is random in nature, the third fundamental characteristic of interest is stationarity. Is there reason to believe that the vibration response being measured will exist in the future under conditions identical to those which existed when a sample record was obtained for analysis? Clearly, the problem of predicting future events is directly dependent upon the stationary characteristics of the vibration environment being measured and analyzed.

During the measurement and analysis of a flight vehicle vibration environment, the characteristics of randomness, normality, or stationarity, or all three, are often assumed without quantitative justification. In many cases, strong qualitative arguments can be developed which tend to justify any or all of the assumptions. For example, if one can substantiate that the exciting forces to be expected during a mission are basically stochastic processes, such as aerodynamic boundary layer turbulence and jet exhaust gas mixing, it appears appropriate to assume that the vibration responses will be primarily random in nature. If the vibration environment is random, the practical implications of the Central Limit Theorem in statistics tend to support an assumption that the instantaneous vibration amplitudes will be distributed in a Gaussian manner. If it is known that at least certain phases of a mission will occur under reasonably constant conditions (airspeed, altitude, thrust, and so on), there appears to be no reason to question that the vibration responses will be stationary during such phases.

On the other hand, unknown or overlooked factors may destroy the validity of these assumptions. For example, a flight vehicle may carry equipments which generate intense periodic vibrations in local areas. Lack of normality in the exciting forces or nonlinear characteristics of the structure, or both, may result in vibration response amplitude distributions which deviate significantly from the ideal Gaussian form. Unexpected events, which produce extreme vibration levels for short intervals during a period of assumed stationary conditions, may occur during a mission.

It is important that fundamental assumptions such as randomness, normality, and stationarity be confirmed, when possible, by investigation of the sample vibration response records gathered for analysis. The detection of these characteristics from sample records has been considered theoretically in Ref. 1. Explicit analytical procedures for testing sample records for random, Gaussian, and stationary characteristics have been confirmed by laboratory experiments for a wide range of practical cases in Ref. 2. This paper summarizes some of the more important conclusions of those two references.

LIST OF SYMBOLS

B	equivalent ideal frequency bandwidth in cps
$G(f)$	power spectral density function in units of amplitude squared per cps
$H(f)$	frequency response function
H_o	null hypothesis
K	time constant in seconds
ms_x	mean square value in units of amplitude squared
N	number of records; sample size
$p(x)$	amplitude probability density function
$R(\tau)$	autocorrelation function in units of amplitude squared
s^2	sample variance
T	record length in seconds
T_a	averaging time in seconds

[1] J. S. Bendat, L. D. Enochson, G. H. Klein, and A. G. Piersol, "The Application of Statistics to the Flight Vehicle Vibration Problems," ASD TR 61-123, Aeronautical Systems Division, Air Force Systems Command, USAF, Wright-Patterson AFB, Ohio. (December 1961). (ASTIA AD 271 913).

[2] J. S. Bendat, L. D. Enochson, G. H. Klein, and A. G. Piersol, "Advanced Concepts of Stochastic Processes and Statistics for Flight Vehicle Estimation and Measurement," Unpublished Technical Report, Aeronautical Systems Division, Air Force Systems Command, USAF, Wright-Patterson AFB, Ohio. (Expected publication 1962).

Var( )	variance of ( )
$x(t)$	amplitude time history
$\bar{x}$	sample mean value
$\overline{x^2}$	sample mean square value
α	level of significance; probability of a Type I error
β	probability of a Type II error
σ_x^2	mean square value ($\mu_x = 0$)
σ_x	root mean square value ($\mu_x = 0$)
μ_x	mean value
$\wedge$	estimate of

DEFINITIONS OF FUNDAMENTAL CHARACTERISTICS

It is appropriate here to define the terms random, normal, and stationary as they are used in this paper. The definitions presented are limited and not necessarily exact. More detailed discussions and defintions of these fundamental characteristics are available in Ref 1.

Random Vibration

Random vibration is that type of time-varying motion which consists of randomly varying amplitudes and frequencies such that its behavior can be described only in statistical terms. No analytical representation for the motion is possible. The motion does not repeat itself in finite time periods. In this paper, a vibration response will be considered as random unless vibratory motion with a periodic form is present. There are, of course, other types of vibratory motions that are neither random nor periodic. Included would be those motions which can be represented by nonperiodic analytic functions, such as an exponential-sine function. However, it is usually more suitable to consider and analyze these motions as shock phenomena rather than as vibration.

Normal (Gaussian) Random Vibration

A random vibration response $X(t)$ is Gaussian if it has a specific instantaneous amplitude probability density function given by

$$p_o(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-(x-\mu_x)/2\sigma_x^2} \qquad (1)$$

Here, μ_x is the mean value for the response, and σ_x^2 is the mean square value about the mean (variance) for the response. Random vibration responses which are described by any other probability density function will be called non-Gaussian.

Stationary Random Vibration

The term stationary is often applied to random vibration data in a context that is slightly different from its classical statistical meaning. Because the concept of stationarity is so vitally important to vibration environment prediction problems, some general discussion of its meaning is included here.

Any particular random vibration response, whose amplitude time history has been recorded for analysis, represents a unique set of circumstances that are not likely to ever be repeated exactly. A single random vibration record, in actual practice, is merely a special example out of a large set of possible records that might have occurred. This collection (ensemble) of records may be thought of as a random process, as illustrated in Fig. 1.

Hypothetically, the number of records in a random process would be infinitely large and each record would exist over all time. The properties of the random process may be computed by taking averages over the collection of records at any given time t. Such averages, called ensemble averages, will be a function of time. For example, at time t, the mean value $\mu_x(t)$, the mean-square value $ms_x(t)$, and the autocorrelation function $R_x(t,\tau)$ would be given by

$$\mu_x(t) = \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} x_k(t), \qquad (2a)$$

$$ms_x(t) = \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} x_k^2(t), \qquad (2b)$$

$$R_x(t,\tau) = \lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} x_k(t)x_k(t+\tau). \qquad (2c)$$

For the general case where the properties defined in Eq. (2) vary with time, the random process is said to be nonstationary. For the special case where the properties defined in Eq. (2) do not vary with time (μ_x, ms_x, and $R_x(\tau)$ are time invariant), the random process is said to be weakly stationary. If all possible moments of the random process determined by ensemble averaging are time invariant, the random process is said to be strongly stationary. In its

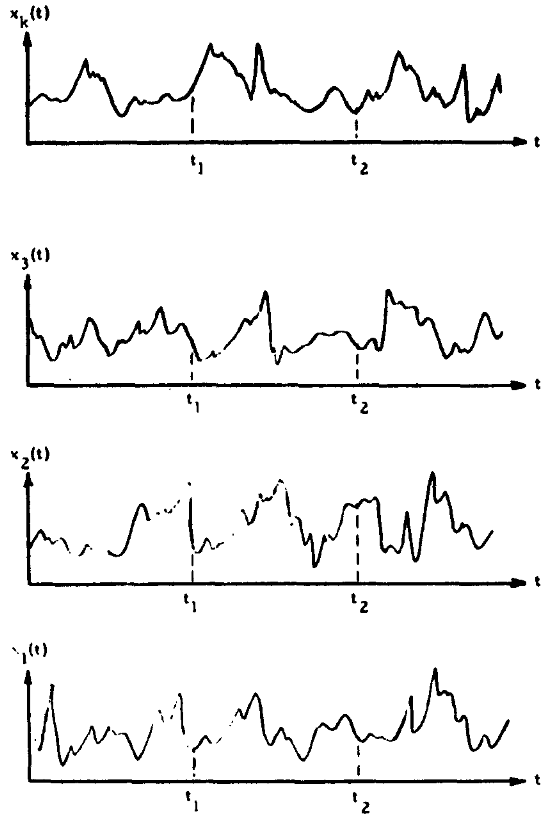

Fig. 1 - Vibration amplitude time history records

broadest meaning, stationarity is a characteristic of ensemble average properties.

The properties of individual records within the random process can also be computed by averaging over time. These time averages will be a function of the specific record in the collection. For the ith record, the mean value $\mu_x(i)$, the mean square value $ms_x(i)$, and the autocorrelation function $R_x(\tau, i)$, would be given by

$$\mu_x(i) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x_i(t)\, dt, \tag{3a}$$

$$ms_x(i) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x_i^2(t)\, dt, \tag{3b}$$

$$R_x(\tau, i) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x_i(t) x_i(t+\tau)\, dt. \tag{3c}$$

Let these time-averaged properties be determined for all possible records of a random process which is at least weakly stationary. If each property is the same for all records and equal to the corresponding property determined by ensemble averaging, the stationary random process is said to be weakly ergodic. If the random process is strongly stationary, and the previously mentioned requirements are met for all possible moments, the stationary random process is said to be strongly ergodic. Thus, ergodicity is a characteristic of time averaged properties for a stationary random process.

For the flight vehicle vibration problem, a single record could represent the vibration response at some point on the structure of a given flight vehicle. The collection of records (ensemble) would then represent the vibration responses at that point occurring during simultaneous flights of all vehicles of that type. The procedure of measuring descriptive properties of the vibration response by ensemble-averaging is clearly undesirable. In order to minimize the statistical uncertainty (sampling error) in the resulting measurements, a relatively large number of simultaneous flights of different vehicles (or repeated flights of the same vehicle) would be required. If data are reduced by analog instruments, the instrumentation required to ensemble-average a large number of sample records simultaneously is far more complex than is required to time-average the individual sample records. It is for these reasons, and others, that the vast majority of vibration data analysis, in actual practice, is accomplished by time-averaging individual sample records rather than by ensemble-averaging a collection of sample records. A slightly different concept for stationarity is required when one deals with vibration response data by time-averaging individual sample records.

An individual vibration response is often referred to as being stationary or nonstationary. In this case, the term is used to mean that the properties of the vibration response, determined by time-averaging sample records of finite length, do not change significantly for samples covering different time intervals. The word significantly means that variations in the sample values are greater than would be expected from sampling errors. This concept of stationarity will, for the moment, be referred to as self-stationarity to avoid confusion with the definition presented previously.

To clarify the idea of self-stationarity, consider an individual vibration response $x(t)$. Assume a sample record of length T is obtained at some starting time T_0. In general, the properties determined by time averaging over the sample will be a function of the starting time t_0. That is, for time t_0, the sample mean value $\bar{x}(t_0)$, the sample mean square value $\overline{x^2}(t_0)$, and the sample autocorrelation function $\hat{R}(\tau, t_0)$ would be given by

$$\bar{x}(t_0) = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\, dt, \tag{4a}$$

$$\overline{x^2}(t_0) = \frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)\, dt, \tag{4b}$$

$$\hat{R}(\tau, t_0) = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\, x(t+\tau)\, dt. \tag{4c}$$

For the general case where the sample properties defined in Eq. (4) vary significantly with the time t_0 that the sample was obtained, the individual vibration response is said to be self-nonstationary. For the special case where the properties defined in Eq. (4) do not vary significantly with time t_0, the vibration response is said to be weakly self-stationary. If this requirement is met for all possible moments, the vibration response is said to be strongly self-stationary.

It should be noted that if a vibration response is normally distributed, the first two moments (mean value and mean square value) will define all higher moments. Thus, if it can be shown that the vibration response is weakly self-stationary, this fact plus the existence of normality makes the response strongly self-stationary.

There are many possible associations between the self-stationarity of individual vibration responses, and the stationarity and ergodicity of a collect of vibration responses. These are discussed in some detail in section 6 of Ref. 1. The most important of these associations is as follows: Assume the vibration responses in many different vehicles of the same type are each found to be self-stationary, at least during some phase of a common mission. Also assume the properties of the vibration responses determined by time-averaging sample records obtained during a self-stationary phase are equivalent from vehicle to vehicle. Then, the vibration response during that phase is stationary and ergodic. The properties determined by time-averaging a short sample record from one vehicle, will be estimators for the vibration response to be expected in all vehicles of that type during that entire phase.

For simplicity in this paper and consistency with general usage, the word stationary will be used in the context defined here as self-stationarity.

Mean Square Vibration Response

For any random signal, the mean square value, ms_x, and the mean square value about the mean (variance), σ_x^2, are related by $\sigma_x^2 = ms_x - \mu_x^2$. Thus, if the mean value μ_x is zero, the two quantities are equal, and σ_x^2 defines the mean square value of the signal. The positive square root, σ_x, is the root mean square value of the signal.

It will be assumed in all discussions to follow that the mean value, μ_x, for the vibration response, is zero. The mean value of a vibration response is a measure of the steady-state magnitude for the physical parameter being measured (such as steady-state acceleration), and would appear as a dc voltage in the output of the vibration transducer. Because of the limited low-frequency response of most vibration data acquisition and analysis equipment, the mean value of the vibration response is usually rendered zero whether the mean value of the actual physical parameter being measured is zero or not. Hence, the term σ_x^2 will be used in this paper to denote the mean square value of a vibration response, as is done in Refs. 1, 2, where it is understood that the mean value of the vibration response is assumed to be zero.

BANDWIDTH AND AVERAGING TIME CONSIDERATIONS

Assume a sample record is obtained from a stationary random vibration response, $x(t)$. The properties determined from that sample record will be estimators for the true properties of $x(t)$. For example, the mean square value $\overline{x^2}$ measured from the sample record will be an estimate for σ_x^2. The uncertainty of this estimate is a function of the frequency bandwidth, B, for the response $x(t)$, and the averaging time, T_a, for the analysis.

The statistical procedures to be discussed in this paper require a knowledge of the uncertainty of sample record estimates. These uncertainties will be presented in terms of ideal bandwidths B and ideal averaging times T_a. Thus, the application of these ideal parameters to real data should be mentioned.

Bandwidth Considerations

If a random signal with a uniform power spectrum (white noise) is filtered with infinitely sharp cutoffs between two frequencies f_a and f_b, the bandwidth of the signal

$B = f_b - f_a$, is ideally defined. Given a constant parameter real filter with a linear frequency response function $H(f)$, an equivalent ideal bandwidth for the filter is given by

$$B = \frac{1}{|H_{max}|^2} \int_{-\infty}^{\infty} |H(f)|^2 \, df. \tag{5}$$

Furthermore, if the random signal is stationary, the power spectral density function for the response, $G_x(f)$, is associated with the power spectral density function for the excitation, $G_e(f)$, as follows.

$$G_x(f) = |H(f)|^2 G_e(f). \tag{6}$$

For most practical purposes, the power spectrum for a vibration response may be considered the result of linear filtering of an excitation with a uniform power spectrum of $G_e(f)$ = constant. Then, upon substitution of Eq. (6) into Eq. (5), the following result is obtained.

$$B = \frac{1}{G_{x\ max}} \int_{-\infty}^{\infty} G_x(f) \, df = \frac{\sigma_x^2}{G_{x\ max}}. \tag{7}$$

In words, the equivalent ideal bandwith for a vibration response $x(t)$, is given by the mean square value divided by the peak value of the power spectrum.

Record-Length and Averaging-Time Considerations

If the information available from a sample amplitude time history record is to be fully utilized, the properties of the sample record should be measured by averaging over the entire available record length of T seconds. When a sample record is analyzed by digital techniques or by analog instruments which average by true linear integration, no problems arise. The averaging time for the analysis, T_a seconds, is set equal to T, and a one number measurement of the desired property is obtained.

However, many analog instruments average by continuous smoothing with a low-pass RC filter having some time constant K. For example, various types of true rms voltmeters (not to be confused with conventional ac voltmeters) are widely used to measure the rms (or σ_x) level of random signals, including vibration response data. These instruments effectively square the input signal and then average the instantaneous squared amplitudes by RC filtering. The square root is obtained by an appropriate scale calibration. The result is a continuous rms level indication which, at any instant, represents a time-weighted average of the entire past history of the signal from the time it was first applied. For this case, after the signal has been applied for a time of $3K$ or $4K$, the rms indication at any instant will represent an rms measurement based on an equivalent averaging time of $T_a = 2K$.

Note that no additional information can be obtained by having an averaging time T_a that is longer than the available sample record length T. Then, for RC averaging,

$$\begin{aligned} T_a &= 2K \text{ for } K < (T/2) \\ T_a &= T \text{ for } (T/2) < K. \end{aligned} \tag{8}$$

TESTS FOR DETECTING FUNDAMENTAL CHARACTERISTICS

The analysis and prediction of a flight vehicle vibration environment starts, of course, with the gathering of sample records. These sample records may be a continuous record of an entire flight, a group of short sample records selected randomly throughout a flight, or a group of short sample records selected at predetermined times during a flight. Sample records may be available for one or more flights of one or more vehicles of the same type. These general matters are discussed in Refs. 1 and 2.

The techniques to be discussed here for detecting random, Gaussian, and stationary characteristics of sampled vibration responses are applicable to individual sample records. They are not directly dependent upon the number of sample records available or the sampling method employed. It should be mentioned that techniques for detecting stationary characteristics of a vibration environment can be greatly expanded if sample records are available from repeated flights of the same vehicle, or better yet, from different vehicles of the same type. The similarity of vibration responses from flight to flight can then be analyzed. These techniques for analyzing multiple flight data are, however, beyond the scope of this paper.

Test for Randomness

Consider the problem of detecting periodicities in an otherwise random vibration response. The first step is to take a close look at the properties of the vibration which would be measured from sample records as a normal part of the data analyses. These properties would probably include a power spectrum (or an autocorrelation function, or both) and an amplitude probability density function.

The most common property employed to describe a random vibration response is the power spectrum, $G(f)$. A periodic vibration response would appear in a properly resolved power spectral density measurement as one or more sharp peaks. However, a sharp peak in the power spectrum may also represent the narrow band random response of a lightly damped structural resonance. A narrow band random response would always have a finite bandwidth while a sinusoidal response would theoretically appear as a delta function with no bandwidth. Unfortunately, the resolution of actual measurement procedures is usually not sufficient to distinguish between these two cases, as illustrated in Fig. 2.

If an autocorrelation measurement is available, the problem of distinguishing between a sinusoidal vibration response and a narrow band random vibration response is greatly simplified. The autocorrelation function, $R(\tau)$, for a narrow band random vibration response will continually decay towards zero as the time displacement τ becomes large. However, if a sine wave is present, the autocorrelation function will approach a steady-state oscillation as the time displacement becomes large, as illustrated in Fig. 3.

Autocorrelation analysis presents a near foolproof method of detecting sinusoids in an otherwise random vibration response. However, autocorrelation functions are rarely measured when vibration data is analyzed using analog instruments. On the other hand, when vibration data is analyzed by digital techniques, autocorrelation functions are often computed as an intermediate step to obtain power spectra ($G(f)$ is the Fourier Transform of $R(\tau)$ for stationary random signals). In this case, an autocorrelation function can be made available for detection of periodicities.

The presence of periodic components in a vibration response will also be revealed by the amplitude probability density function for the

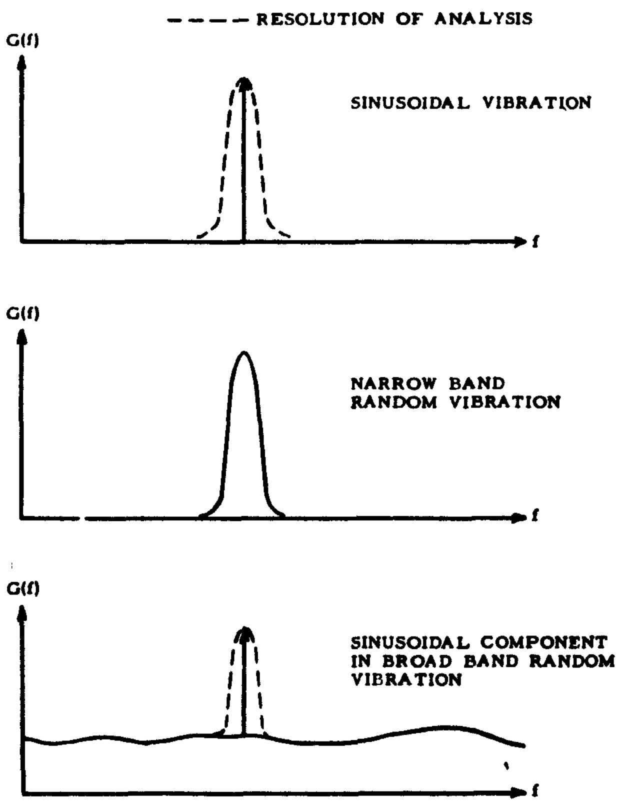

Fig. 2 - Power spectral density functions

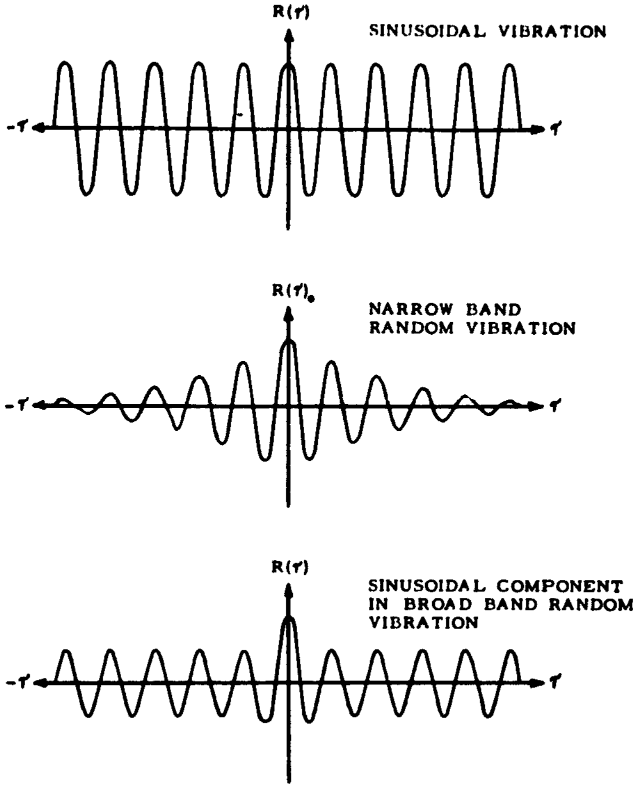

Fig. 3 - Autocorrelation functions

response. A random vibration response will, in most cases, have a probability density function which at least resembles the familiar bell-shaped Gaussian characteristic. This is particularly true for a narrow band random response (assuming the structural characteristics are reasonably linear), because linear narrow band filtering of random signals tends to suppress deviations from normality in the amplitude distribution characteristics. On the other hand, a sinusoidal vibration response will have a dish-shaped probability density function which is truncated at $\pm A = \sqrt{2}\,\sigma_x$, as illustrated in Fig. 4.

The above discussions illustrate how periodicities in an otherwise random vibration response may often be detected by simple observations from commonly analyzed data. However, these interpretations are fully effective only when sampling errors are small. If the available sample records (or analyses averaging times) are short, the uncertainty in the resulting measurements may mask the desired descriptive details in the various measured properties. For such cases, a statistical decision is often required. These matters are discussed in section 15 of Ref. 2. One statistical test for randomness studied in that reference will now be described.

Consider a sample record of length T obtained from a vibration response x(t) with an equivalent ideal bandwidth B. Assume x(t) is stationary over the sample record length T (to be verified by other procedures discussed later under Test for Stationarity, and that x(t) has an approximately Gaussian amplitude probability density function with a true mean square value of σ_x^2. If the vibration response is actually random, a measured mean square value $\overline{x^2}$ obtained from the sample record will have a sampling distribution with a variance given in normalized terms as follows:

$$\epsilon_o^2 = \frac{\mathrm{Var}\left(\overline{x^2}\right)}{\left(\sigma_x^2\right)^2} \approx \frac{1}{B\,T_a}\quad T_a \le T. \tag{9}$$

If a periodic component is present in the otherwise random vibration response, the normalized variance for a mean square measurement becomes

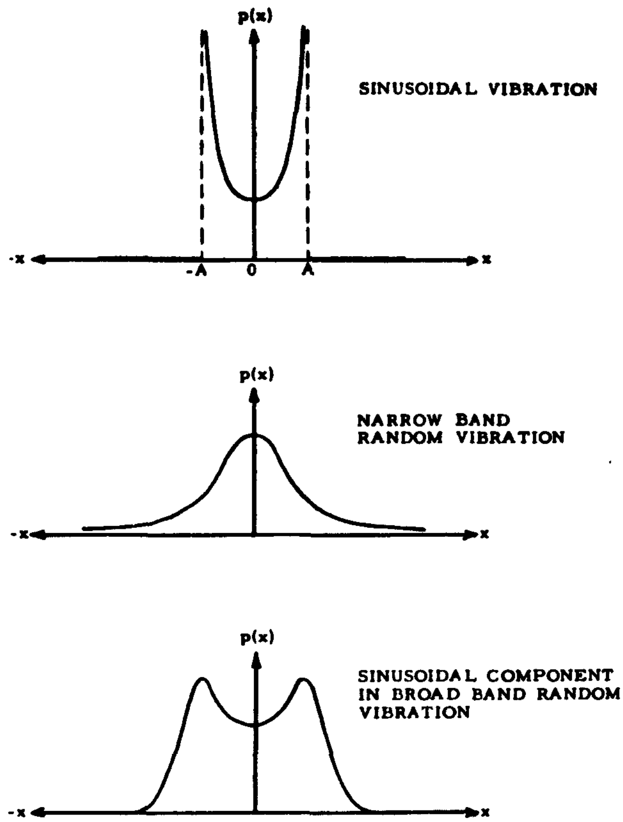

Fig. 4 - Amplitude probability density functions

$$\epsilon^2 = \frac{\mathrm{Var}\left(\overline{x^2}\right)}{\left(\sigma_x^2\right)^2} = \frac{\left(2\,\frac{p}{r} + 1\right)\epsilon_o^2}{\left(\frac{p}{r} + 1\right)^2}, \qquad (10)$$

Where

p = mean square value for periodic portion,

r = mean square value for random portion,

$\sigma_x^2 = p + r.$

From Eq. (10), the variance of a mean square measurement will be reduced as the level of the periodicity is increased relative to the random background (as p/r increases). This relationship lays the groundwork for a statistical hypothesis test for randomness.

Assume a collection of N independent mean square values, $\overline{x_i^2}$ ($i = 1,2,3,\ldots,N$), are measured from a stationary vibration response. This collection may be obtained from a single sample record by averaging over each of N equally long segments with an averaging time $T_s = T/N$. The variance of the mean square measurements can be estimated from the sample variance s^2 for the measured values $\overline{x_i^2}$ as follows:

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\overline{x_i^2} - \overline{x^2}\right)^2 = \frac{\sum_{i=1}^{N}\left(\overline{x_i^2}\right)^2}{N-1} - \frac{N\left(\overline{x^2}\right)^2}{N-1}, \qquad (11)^*$$

*The sample variance s^2 in Eq. (11) is an unbiased variance estimate and the statistical relationships to follow are consistent for an unbiased s^2. An unbiased s^2 is used in this paper so that OC curves in Ref. 3, which are developed for unbiased variance estimates, may be referred to directly. In Refs. 1, 2, a biased expression for s^2 is used where the denominator of Eq. (11) is N instead of $N-1$, and all statistical relationships in those references are consistent for this biased s^2.

[3] A. H. Bowker and G. J. Lieberman, Engineering Statistics, (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1959).

where

$$\overline{x^2} = \frac{1}{N} \sum_{i=1}^{N} \overline{x_i^2} .$$

Here, $\overline{x^2}$ is the mean square value averaged over the collection length $T = N\,T_a$. The expected values for the sample variance s^2 and the sample mean $\overline{x^2}$ are as follows:

$$E\,[s^2] = \mathrm{Var}\,(\overline{x^2})$$
$$E\,[\overline{x^2}] = \sigma_x^2 \qquad (12)$$

Then, the normalized variance for a mean square measurement may be estimated by

$$\hat{\epsilon}^2 = \frac{s^2}{(\overline{x^2})^2}. \qquad (13)$$

In Eq. (13), both s^2 and $\overline{x^2}$ are random variables. However, it is shown in Ref. 2 that the variability of $\overline{x^2}$ is negligible compared to the variability of s^2, if the quantity $2\,B\,T > 40N$ or $2\,B\,T_a > 40$. If this requirement is met, it may be assumed that $\overline{x^2} = \sigma_x^2$ for the problem at hand.

The sample variance s^2 from Eq. (11) will have a distribution associated with the chi-squared distribution as follows:

$$\frac{s^2}{\mathrm{Var}(\overline{x^2})} \sim \frac{\chi^2_{(N-1)}}{N-1}, \qquad (14)$$

where $\sim$ means distributed as, and $\chi^2_{(N-1)}$ is a chi-squared distribution with $(N-1)$ degrees of freedom. From the relationships in Eqs. (10) and (13), it follows that

$$\frac{\hat{\epsilon}^2}{\epsilon^2} \sim \frac{\chi^2_{(N-1)}}{N-1}; \quad 2BT_a > 40. \qquad (15)$$

From Eq. (15), the following probability statement may be made:

$$\mathrm{Prob}\left[\frac{\hat{\epsilon}^2}{\epsilon^2} \geq \frac{\chi^2_{(N-1);(1-\alpha)}}{N-1}\right] = (1-\alpha), \qquad (16)$$

where α is the level of significance.

Let it be hypothesized that a sampled vibration response is random. If this is true $\epsilon^2 = \epsilon_o^2$, and the hypothesis H_o is

$$H_o : \hat{\epsilon}^2 = \epsilon_o^2. \qquad (17)$$

If a periodic component is present, $\hat{\epsilon}^2$ will be equivalent to some ϵ^2 less than ϵ_o^2 as defined in Eq. (10), which is why a one-sided probability statement is used in Eq. (16). Then, H_o may be tested by computing $\hat{\epsilon}^2$ from a collection of mean square measurements, computing ϵ_o^2 from the BT_a product for the measurements, and comparing the ratio of $\hat{\epsilon}^2/\epsilon_o^2$ to the χ^2 limit in Eq. (16) at any desired level of significance α. The region of acceptance for H_o is

$$\frac{\hat{\epsilon}^2}{\epsilon_o^2} \geq \frac{\chi^2_{(N-1);(1-\alpha)}}{N-1}. \qquad (18)$$

If $\hat{\epsilon}^2/\epsilon_o^2$ is greater than the above noted limit, H_o is accepted and vibration response is considered random. If $\hat{\epsilon}^2/\epsilon_o^2$ is less than the noted limit, H_o is rejected and there is reason to suspect that a periodic component is present in the vibration response. The appropriate values in Eq. (18), for any desired level of significance, can be obtained from most statistics books. A plot of Eq. (18) for $\alpha = 0.01$ and 0.05 is presented in Fig. 5.

The level of significance α is the probability of a Type I error for the test (the risk of rejecting H_o when in fact it is true). The risk of a Type I error may be reduced by testing with a smaller value of α. Now consider the probability of a Type II error denoted by β (the risk of accepting H_o when in fact it is false). β is a function of α, N, and the deviation of ϵ^2 from ϵ_o^2 which one wishes to detect. In general, both α and β can be reduced only by increasing the number of measurements N. The number N required for any value of α and β is given by the Operating Characteristic (OC) curve for Eq. (18). The OC curves for $\alpha = 0.05$ and 0.01 are available in Figs. 6.19 and 6.20 of Ref. 3.

In summary, the procedure for applying the test for randomness (detection of periodicities) is as follows:

1. Determine the magnitude of a periodic component in terms of a p/r ratio which you wish to detect with high probability.

2. Determine the ratio ϵ^2/ϵ_o^2 for that p/r ratio from Eq. (10).

3. Determine the desired values for α and β, the risk of making a Type I and Type II error.

4. Determine the number of measurements N_2 required from an OC curve for a one-sided (lower tail) χ^2 test.

5. Obtain the required number of mean square measurements, $\overline{x_i^2}$, and compute $\hat{\epsilon}^2$ from Eq. (13).

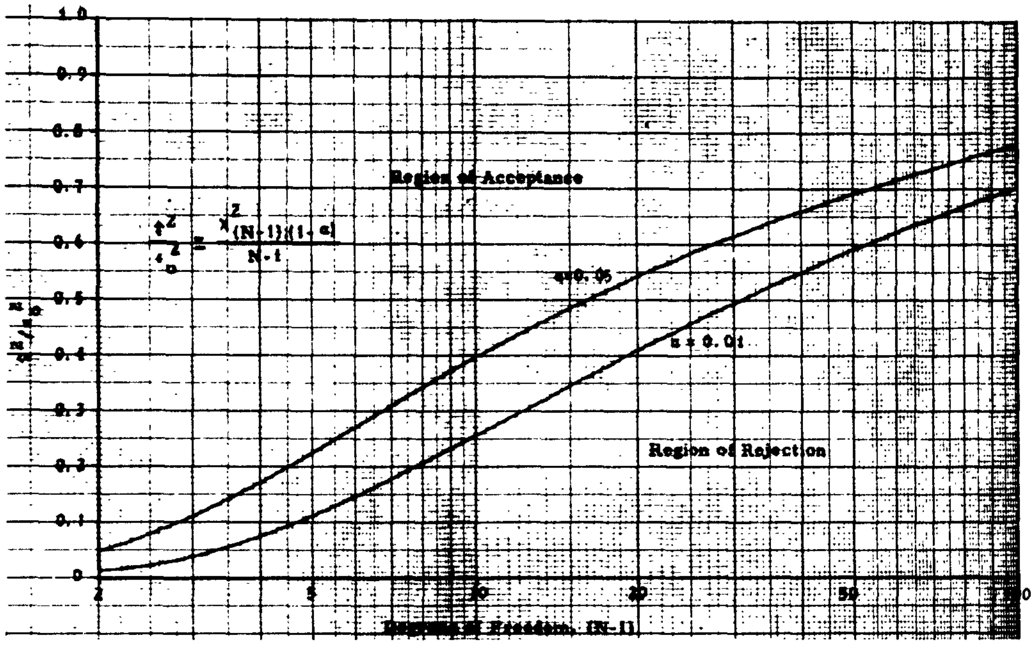

Fig. 5 - Acceptance regions for random hypothesis test

6. Compute ϵ_o^2 from Eq. (9).

7. Test the ratio $\hat{\epsilon}^2/\epsilon_o^2$ against $H_o: \hat{\epsilon}^2 = \epsilon_o^2$.

It should be noted that the effectiveness of the procedure will increase as the bandwidth B is made small. Thus, if a sharp peak is observed in the power spectrum, the frequency range of the peak should be isolated by a narrow band filter. If the peak represents a sine wave, the random portion within the narrow bandwidth B will be very small, causing a large p/r ratio. This not only improves the power of the test, but it helps to define the value of B needed to compute ϵ_o^2.

Example 1--The following example is based upon actual experimental data obtained in the laboratory and presented in Ref. 2.

Assume that a sample record of length $T = 7$ seconds, is obtained from a vibration response which is believed to be stationary. A frequency range of the response with a bandwidth $B = 56$ cps is to be tested for the presence of a periodic component. It is desired that a periodic component producing a p/r ratio of six be detected with a risk of making a Type I or Type II error of $\alpha = \beta = 0.05$. Then, the deviation to be detected with a 95 percent probability is

$$\epsilon^2 = \frac{\left(2\frac{p}{r} + 1\right)\epsilon_o^2}{\left(\frac{p}{r} + 1\right)^2} \approx \frac{1}{4}\epsilon_o^2 .$$

From Table 6.19 of Ref. 3, for $\epsilon^2/\epsilon_o^2 = 0.25$ ($\epsilon/\epsilon_o = 0.5$), the required number of measurements is $N = 13$.

Now, 13 mean square measurements are obtained from the sample record by averaging over 13 consecutive intervals of $T_a = 0.54$ seconds. Note that $2BT_a = 60 > 40$. The following results are obtained:

Mean Square Values, $\overline{x_i^2}$, in Volts^2
2.25
1.78
2.26
2.07
1.91
2.85
2.00
2.45
2.19
3.20
2.30
2.86
2.68

The required computations produce the following data:

$$s^2 = 0.176$$

$$\overline{x^2} = 2.369$$

$$\hat{\epsilon}^2 = \frac{s^2}{(\overline{x^2})^2} = 0.031$$

$$\epsilon_o^2 = \frac{1}{BT_a} = 0.033$$

$$\frac{\hat{\epsilon}^2}{\epsilon_o^2} = 0.94$$

$$\frac{\chi^2_{(N-1);(1-\alpha)}}{N-1} = \frac{\chi^2_{12;\ 0.95}}{12} = 0.436$$

$$\frac{\hat{\epsilon}^2}{\epsilon_o^2} = 0.94 \geq 0.436$$

Thus, the hypothesis, $H_o : \hat{\epsilon}^2 = \epsilon_o^2$, is accepted and there is no reason to suspect that a periodic component producing a (p/r) ratio of six or greater is present.

If it is desired that the test be performed with a more stringent Type II error limitation, more measurements will be required. For example, assume a p/r ratio of two is to be detected with a probability of $\beta = 0.05$. From the OC curve, the number of measurements required is $N \approx 70$ for a test at the $\alpha = 0.05$ level of significance. On the other hand, if the analyst is prepared to take a greater risk of making a Type II error, the required number of measurements will be reduced.

Tests for Normality

Now consider the problem of determining whether or not a random vibration response has a Gaussian amplitude probability density function, as given by Eq. (1). The procedures for testing a sample record for normality are straightforward if a sufficiently long sample record is available.

The simplest test for normality is the classical chi-squared goodness-of-fit test. The goodness-of-fit test is applicable to amplitude values from the sample record and, thus, requires no special measurements or instruments. It is necessary only to reduce the amplitude time history from the sample record to a digital form. The procedures for applying the goodness-of-fit test will not be discussed here since they are available from most statistics books. A detailed illustration of the test, for applications to vibration data, is presented in section 5.3.2 of Ref. 1.

If a probability density function is measured from a sample record using an analog instrument, other direct procedures for testing the vibration response for normality are available, as discussed in section 17 of Ref. 2. One statistical test studied in that reference will now be described.

Consider a sample records of length T obtained from a stationary vibration response $x(t)$ with an amplitude probability density function $p(x)$. Assume that a probability density function $\hat{p}(x)$ is measured from the sample record. The measurement $\hat{p}(x)$ for any given value of x will have a sampling distribution with a normalized variance as follows:

$$\epsilon^2 = \frac{\mathrm{Var}\ [\hat{p}(x)]}{[p(x)]^2} = \frac{0.06}{BT_a p(x)\ (\Delta x)}\ . \qquad (19)^*$$

Here, (Δx) is the width of the amplitude window used by the instrument to accomplish the analysis.

It is shown in Ref. 2 that $\hat{p}(x)$ may be considered normally distributed for most practical cases. Then, the measurement $\hat{p}$ for a specific value of x will have a distribution associated with a normal distribution as follows:

$$(\hat{p} - p) \sim \mathrm{Normal}\ (\epsilon p)\ , \qquad (20)$$

where Normal (ϵp) is a normal distribution with a standard deviation of ϵp, or a variance of $(\epsilon p)^2$. From Eq. (20), the following probability statement may be made

$$\mathrm{Prob}\ [-\epsilon p z_{\alpha/2} \leq (\hat{p} - p) \leq \epsilon p z_{\alpha/2}] = (1 - \alpha). \quad (21)$$

Here, z is the cumulative normal deviate for $\alpha/2$. A table of normal deviates will be found in any statistics book.

Let it be hypothesized that a sample random vibration response record has a Gaussian amplitude probability density function, $p_o(x)$. If this is true, $p(x) = p_o(x)$, and the hypothesis H_o is

$$H_o : \hat{p} = p_o \ \text{for any value of}\ x. \qquad (22)$$

If the vibration response is not Gaussian, $\hat{p}$ may be greater or less than p_o as defined in Eq. (1), which is why a two-sided probability statement is used in Eq. (21). Then, H_o is tested by computing ϵ in Eq. (19) using p_o, and comparing

*This equation assumes the lower cutoff of B is near zero. See section 14 of Ref. 2 for more general expressions.

the difference ($\hat{p} - p_o$) to the normal limits in Eq. (21). The region of acceptance for H_o is

$$-\epsilon p_o z_{\alpha/2} \le (\hat{p} - p_o) \le \epsilon p_o z_{\alpha/2}. \qquad (23)$$

If ($\hat{p} - p_o$) is within the noted limits, H_o is accepted and the vibration response is considered to have a Gaussian probability density for the specific amplitude x tested. If ($\hat{p} - p_o$) is outside the noted limits, H_o is rejected and there is reason to suspect that the vibration response probability density is not Gaussian for that amplitude x. The hypothesis H_o may be tested for as many different amplitudes x as desired.

As for the statistical test for randomness discussed earlier under Tests for Randomness, the risk of a Type I and Type II error, α and β, can be applied to the test for normality. For this test, however, the length of the sample record available fixes the values of α and β. If the test for normality is to be applied to sample records with a predetermined α and β risk, these matters must be considered to arrive at the necessary record length T to be obtained. Unfortunately, conventional OC curves are not applicable to this problem because the normalized variance ϵ^2 is a function of the property $p(x)$ being measured. For a normal sampling distribution, however, the risk of a Type II error for detecting a difference $(p - p_o) = \pm \epsilon p_o z_{\alpha/2}$ is always $\beta = 0.5$.

In summary, the procedure for applying the test for normality is as follows:

1. Determine the difference $(p - p_o)$ which you wish to detect with a probability of $\beta = 0.5$.

2. Determine the desired value for α, the risk of making a Type I error, and establish $z_{\alpha/2}$ from a table of cumulative normal deviates.

3. Determine the maximum amplitude which you wish to test, and establish the Gaussian probability density p_o for that amplitude from a table of normal ordinate values.

4. Determine the required value for ϵ^2 by letting $(p - p_o) = \pm \epsilon p_o z_{\alpha/2}$.

5. Solve for T_a in Eq. (19) using $p(x) = p_o$ and an estimated equivalent bandwidth B. The sample record to be obtained must have a length of $T \ge T_a$.

This test for normality poses two problems. First, if the test is applied repeatedly at N different amplitudes as part of a general test for normality of one sample record, αN Type I errors would be expected during the general test. Thus, the probability of a single Type I error occurring in N number of repeated applications of the procedure is high. There are methods for pooling the risk of Type I errors for repeated tests into a single α value as discussed in Ref. 2. However, for most applications where only a few amplitudes are checked and $\alpha N \ge 1$, the problem is minimized. Second, if extreme amplitudes ($x \ge 3\sigma_x$) are of interest, as is often the case, very long sample records are required to test for normality at these amplitudes. This point is illustrated by the following example:

Example 2—Assume a vibration response is to be sampled so that a test for normality may be performed for amplitudes out to ± 3 times the root mean square level of the response ($\pm 3\sigma_x$). The expected equivalent bandwidth for the response is $B = 500$ cps and the amplitude window of the instrument to be employed for the analysis is 0.1 volt for an input rms level of 1 volt. It is desired that a ± 10-percent deviation from normality be detected with a $\beta = 0.5$ probability of a Type II error and an $\alpha = 0.05$ probability of a Type I error.

From a table of normal ordinates, $p_o = 0.0044$ for $\pm 3\sigma_x$. For a ± 10-percent duration, $(p - p_o) = \pm 0.10\ p_o = \pm 0.00044$. From a table of cumulative normal deviates, $z_{\alpha/2} = 2.0$ for $\alpha = 0.05$. The following results are obtained:

$$\epsilon = \frac{(p - p_o)}{(\pm z_{\alpha/2})\, p_o} = \frac{0.00044}{2(0.0044)} = 0.05$$

$$T_a = \frac{0.06}{\epsilon^2 B p_o (\Delta x)}$$

$$= \frac{0.06}{(0.0025)(500)(0.0044)(0.1)} = 110 \text{ seconds.}$$

Thus, a record length of at least $T = 110$ seconds is required.

To actually perform the test for normality, one must now measure $\hat{p}(\pm 3\sigma_x)$ with $T_a = 110$ seconds, and calculate $(\hat{p} - p_o)$ where $p_o = 0.0044$. The region of acceptance will be the interval $\pm \epsilon p_o Z_{\alpha/2} = \pm 0.00044$. Hence, if $(\hat{p} - p_o)$ falls inside this interval, the hypothesis of normality is accepted for $x = \pm 3\sigma_x$. Otherwise, the hypothesis of normality is rejected.

If the analyst is prepared to take a greater risk of making a Type II error or relax his detection requirements, the record length can be greatly reduced. For example, if it is acceptable that a ± 50-percent deviation from normality be detected with a $\beta = 0.5$ probability, the required record length is reduced to about 22 seconds.

Tests for Stationarity

Finally, consider the problem of determining whether or not a vibration response is stationary in the context defined earlier under Stationary Random Vibration, as weakly self-stationary. In general, the requirement for stationarity is that the values for the mean level $\bar{x}$, the mean square level $\overline{x^2}$, and the autocorrelation function $\hat{R}(\tau)$, measured from a sample record covering the interval $(t_o, t_o + T)$ do not change significantly for different starting times t_o. In this paper, the vibration responses to be analyzed are considered to have a mean value $\mu_x = 0$, so sample record mean values $\bar{x}$ are of no interst. Furthermore, if the mean square level measurements from sample records are equivalent, it is unlikely that the autocorrelation functions would be significantly different. Thus, the procedures for establishing stationarity may be limited in most cases to a study of mean square measurements. Several procedures for testing a collection of sample records for stationarity are discussed in section 16 of Ref. 2. One statistical test studied in that reference will now be described. The procedure is identical in concept to the statistical test for randomness already described.

Assume that a collection of N independent mean square values, x_i^2 $(i = 1, 2, 3, \ldots, N)$, are measured from a random vibration response. This collection may be obtained from a single sample record by averaging over each of N equally long segments with an averaging time $T_a = (T/N)$. If the vibration response is stationary with a mean square value of σ_x^2, each measurement $\overline{x_i^2}$ will be an estimate of σ_x^2 and will have a sampling distribution with a normalized variance of ϵ_o^2, as defined in Eq. (9). For a collection of N mean square measurements, the actual normalized variance ϵ^2 may be estimated by $\hat{\epsilon}^2$, as defined in Eqs. (11) and (13). The ratio of $\hat{\epsilon}^2/\epsilon^2$ will be distributed as shown in Eq. (15).

The following probability statement may be made:

$$\text{Prob}\left[\frac{\hat{\epsilon}^2}{\epsilon^2} \le \frac{\chi^2_{(N-1);\,\alpha}}{N-1}\right] = (1-\alpha). \tag{24}$$

Let it be hypothesized that a sampled random vibration response is stationary. If this is true, $\epsilon^2 = \epsilon_o^2$ and the hypothesis e_o^2 is

$$H_o \cdot \hat{\epsilon}^2 = \epsilon_o^2 . \tag{25}$$

If the vibration response is nonstationary, $\hat{\epsilon}^2$ will be equivalent to some undefined ϵ^2 greater than ϵ_o^2, which is why a one-sided (upper tail) probability statement is used in Eq. (24). Then, H_o may be tested by computing $\hat{\epsilon}^2$ from a collection of mean square measurements, computing ϵ_o^2 from the BT_a product for the measurements, and comparing the ratio of $\hat{\epsilon}^2/\epsilon_o^2$ to the χ^2 limit in Eq. (24) at any desired level of significance α. The region of acceptance for H_o is

$$\frac{\hat{\epsilon}^2}{e_o^2} \le \frac{\chi^2_{(N-1);\,\alpha}}{N-1}. \tag{26}$$

If $\hat{\epsilon}^2/\epsilon_o^2$ is less than this noted limit, H_o is accepted and the vibration response is considered stationary. If $\hat{\epsilon}^2/\epsilon_o^2$ is greater than the noted limit, H_o is rejected and there is reason to suspect that the vibration response is nonstationary. A plot of Eq. (26) for $\alpha = 0.05$ and 0.01 is presented in Fig. 6.

The application of the test for any given risk of making a Type I or Type II error is exactly as outlined earlier under Tests for Randomness. The OC curves for $\alpha = 0.05$ and 0.01 are available in Figs. 6.17 and 6.18 of Ref. 3. It should be noted that the definition of a Type II error for this test is fictitious because there is no way to associate a nonstationary condition with a deviation of ϵ^2 from ϵ_o^2.

It is clear that this test can be combined with the test for randomness already discussed, by using a two-sided χ^2 test. If it is hypothesized that the vibration response is random and stationary, the region of acceptance for the double hypothesis H_o is

$$\frac{\chi^2_{(N-1);\,(1-\alpha/2)}}{N-1} \le \frac{\hat{\epsilon}_o^2}{\epsilon_o^2} \le \frac{\chi^2_{(N-1);\,\alpha/2}}{N-1}. \tag{27}$$

The OC curves for $\alpha = 0.05$ and $\alpha = 0.01$ are available in Tables 6.15 and 6.16 of Ref. 3.

It should be noted in Eq. (27) that since $(\alpha/2)$ replaces α, the lower limit in Eq. (27) is less than the limit for the randomness test alone, as given in Eq. (18). Similarly, the upper limit in Eq. (27) is higher than the limit for the stationarity test alone, as given in Eq. (26). Thus, a test for the double hypothesis for any probability α will involve a wider region of acceptance. This is to be expected since the probabilities of Type I errors for the individual tests are effectively pooled into a single Type I error probability for the combined test. The net result is that more measurements are required for the combined test than for each individual test if the same Type II error limitations are to be maintained.

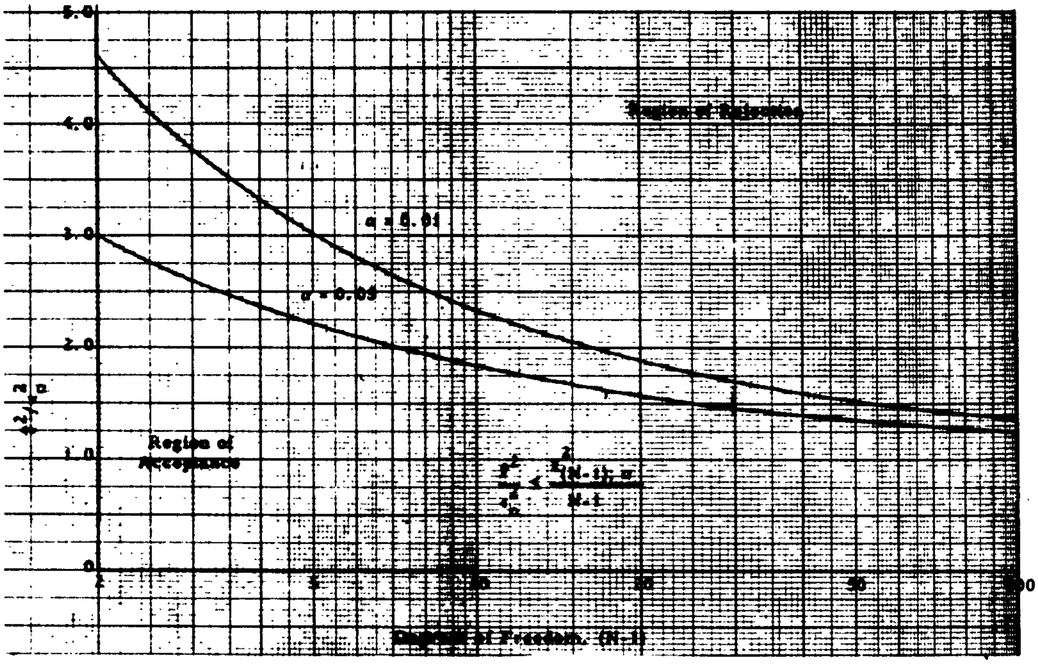

Fig. 6 - Acceptance regions for stationary hypothesis test

Example 3—Assume that a sample record of length $T = 7$ seconds is obtained from a vibration response which is believed to be random. A frequency range of the response with a bandwidth of $B = 56$ cps is to be tested for stationarity. It is desired that a condition of nonstationarity producing a deviation of $\epsilon^2/\epsilon_o^2 = 4$ be detected with a risk of making a Type I or Type II error of $\alpha = \beta = 0.05$. From Table 6.17 of Ref. 3, for $\epsilon^2/\epsilon_o^2 = 4$ ($\epsilon/\epsilon_o = 2$, the required number of measurements is $N = 13$.

The 13 mean square measurements are obtained from the sample record by averaging over 13 consecutive intervals of $T = 0.54$ seconds, as was done in Example 1. Assume the results obtained are those presented in Example 1. Then,

$$\hat{\epsilon}^2 = 0.031$$

$$\epsilon_o^2 = 0.033$$

$$\frac{\hat{\epsilon}^2}{\epsilon_o^2} = 0.94$$

$$\frac{\chi^2_{(N-1);\,\alpha}}{N-1} = \frac{\chi^2_{12;\,0.05}}{12} = 1.75$$

$$\frac{\hat{\epsilon}^2}{\epsilon_o^2} = 0.94 \le 1.75.$$

Thus, the hypothesis, $H_o = \hat{\epsilon}^2 = \epsilon_o^2$, is accepted and there is no reason to suspect that the vibration response is nonstationary over the length of the 7-second long sample record.

Now consider the case where the hypothesis of randomness and stationarity are to be tested simultaneously. Assume the combined test is to be performed with a risk of making a Type I or Type II error of $\alpha = \beta = 0.05$ for detecting $(\epsilon^2/\epsilon_o^2) = 0.25$ at one limit or 4.00 at the other limit. These are the same limitations for each of the two individual tests where $N = 13$ were required. From Fig. 6.15 of Ref. 3, the number of measurements required for the combined test is now $N = 16$. Of course the test could be performed using the $N = 13$ measurements, in which case the risk of a Type II error would be $\beta = 0.09$ instead of $\beta = 0.05$.

CONCLUSIONS

Statistical procedures for analyzing the fundamental characteristics of randomness, normality, and stationarity in flight vehicle vibration response data have been discussed. One test for each of these characteristics is presented.

The tests outlined are applicable to many practical situations. The scope of the general vibration analysis and measurement problem is far broader than the material presented in this paper, and is considered in greater depth in Refs. 1 and 2.

DISCUSSION

Mr. Kennard (ASD): Is the equipment necessary for making these tests for stationarity and so on, readily available to anyone, on the commerical market?

Mr. Piersol: Its a mean square estimating device which is, effectively, any commercial true rms voltmeter, so its readily available. It is also the squaring circuit available in your power spectral density analyzer.

Mr. Galef (NESCO): Can you tell us what the autocorrelation function would look like for two sinusoids which were present and separated by only a few cycles?

Mr. Piersol: The autocorrelation functions for anything other than a simple sine wave, of course, take on different characteristics. I am considering here only a simple sine wave which can always be obtained by tuning the filter over a peak in the power spectra; the other cases I can discuss with you later.

* * *

TECHNIQUES OF ANALYSIS OF RANDOM AND COMBINED RANDOM-SINUSOIDAL VIBRATION*

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This paper discusses the conversion from a voltage-frequency to a power spectral density curve; the determination of composite random acceleration level from a voltage-frequency curve; and determination of total rms acceleration of a sine + random-vibration test from a voltage-frequency curve.

INTRODUCTION

The parameters required to determine the input level of a random-vibration test are numerous. The test specification for a desired test will contain a power spectral density (PSD), which may vary with frequency, a bandwidth, and a vibration spectrum. The specification might also include a sinusoidal vibration to be superimposed upon the random vibration.

During the vibration test, the voltage output of an accelerometer, mounted on the test specimen, is recorded on a tape recorder.

The test data, which are used to analyze the vibration level, are obtained by feeding the recorded output voltage into an analyzer. If the analyzer inherently contains a squaring circuit, a PSD-frequency curve can be produced. However, if no squaring circuit is included, the analyzer will produce a voltage-frequency curve.

A voltage-frequency curve used to analyze random-vibration tests will be defined as the output voltage of an accelerometer at every frequency of the vibration spectrum.

The accelerometer signal is fed into a bandpass filter of narrow passband and a variable center frequency. Only components of the accelerometer signal that are within the passband are passed through the filter; thus, the output voltage is the voltage that is passed through the bandpass filter. The frequency of the output voltage represents the center frequency of the bandpass filter. When the center frequency is varied through the spectrum of the vibration test, the output voltage at each frequency is recorded onto an x-y plotter.

The conversion of a voltage-frequency curve to a power spectral density curve is presented in this report. Once the conversion has been established, analyses that require a power spectral density curve will be performed using a voltage-frequency curve. The analyses that are presented will include methods of determining acceleration of a random-vibration test of constant PSD, or a PSD that varies with frequency. An example of each method follows its presentation.

The final method of analysis of a voltage-frequency curve to be discussed will be to determine the acceleration of a random-vibration test upon which a sinusoidal vibration has been imposed. The sinusoidal and random components will be computed separately, then combined to produce the composite acceleration level of the test.

CONVERSION FROM VOLTAGE-FREQUENCY CURVE TO A PSD CURVE

Discussion

A method used to determine the composite vibration acceleration of a random test is to

*This paper was not presented at the Symposium.

indicate the vibration energy which exists at each frequency (PSD). The PSD is integrated over all frequencies in the spectrum to determine the composite vibration energy from which the composite rms acceleration of the vibration test can be calculated. The calculation of acceleration is simplified when the composite vibration energy is expressed in terms of mean squared acceleration. The voltage-frequency curve presents the output voltage of an accelerometer after it has passed through a bandpass filter. Therefore only the components of vibration at frequencies within the passband are included. The output voltage (usually measured in millivolts) is proportional to the magnitude of vibration contained in the incremental passband of the analyzing filter. The vibration magnitude can be determined by:

$$a_i = \frac{mv}{S} . \tag{1}$$

Where:

a_i = magnitude of root-mean-squared acceleration (g_{rms}) contained in the incremental bandwidth

mv = output of the bandpass filter

S = sensitivity of the accelerometer system (mv/g_{rms})

The incremental value of mean-squared acceleration, contained in the passband of the filter is:

$$P = (a_i)^2 . \tag{2}$$

To determine the incremental-mean-squared acceleration per cycle at the center frequency of the bandpass, the mean-squared acceleration contained in the passband is divided by the effective bandwidth:

$$G = \frac{(a_i)^2}{\Delta f_m} . \tag{3}$$

Where:

G = power spectral density (mean-squared acceleration per cycle)

Δf_m = effective bandwidth of the passband

Note: The effective bandwidth of the filter assumes a flat response through the passband and zero response at all other frequencies. Therefore, to determine the effective bandwidth, a preliminary calibration of the analyzing filter which compares a known PSD to an output voltage, might be required. Combining Eqs. (1) and (3) produces the relationship between voltage output and PSD:

$$G = \frac{(mv)^2}{S^2 \, (\Delta f_m)} . \tag{4}$$

The conversion from voltage to PSD can now be made on the voltage-frequency curve by applying Eq. (4) to the voltage scale of the curve (Fig. 1).

Example

If an analyzer is calibrated from 0-1.0 volt, what is the maximum PSD that can be analyzed?

Note: The maximum PSD that can be analyzed is dependent upon the sensitivity of the accelerometer system and the bandwidth of the filter used for the analysis.

Assume an effective bandwidth of 50 cycles.

$$G(1.0 \text{ volt}) = \frac{(1000)^2}{(10)^2 \, (50)} \; g^2/\text{cps},$$

$$G(1.0 \text{ volt}) = 200 \; g^2/\text{cps} .$$

By increasing the effective bandwidth of the filter to 100 cycles,

$$G(1.0 \text{ volt}) = \frac{(1000)^2}{(10)^2 \, (100)} ,$$

$$G(1.0 \text{ volt}) = 100 \; g^2/\text{cps} .$$

DETERMINING THE COMPOSITE RANDOM ACCELERATION LEVEL FROM A VOLTAGE-FREQUENCY CURVE

Discussion

The composite mean-squared acceleration of a random-vibration test is obtained by integrating the PSD through the spectrum of the vibration test:

$$a^2 = \left(\int_{f\ell}^{fh} G \, df \right) . \tag{5}$$

Where:

fh = high frequency of the vibration spectrum,

G = PSD of the vibration spectrum, and

$f\ell$ = low frequency of the vibration spectrum.

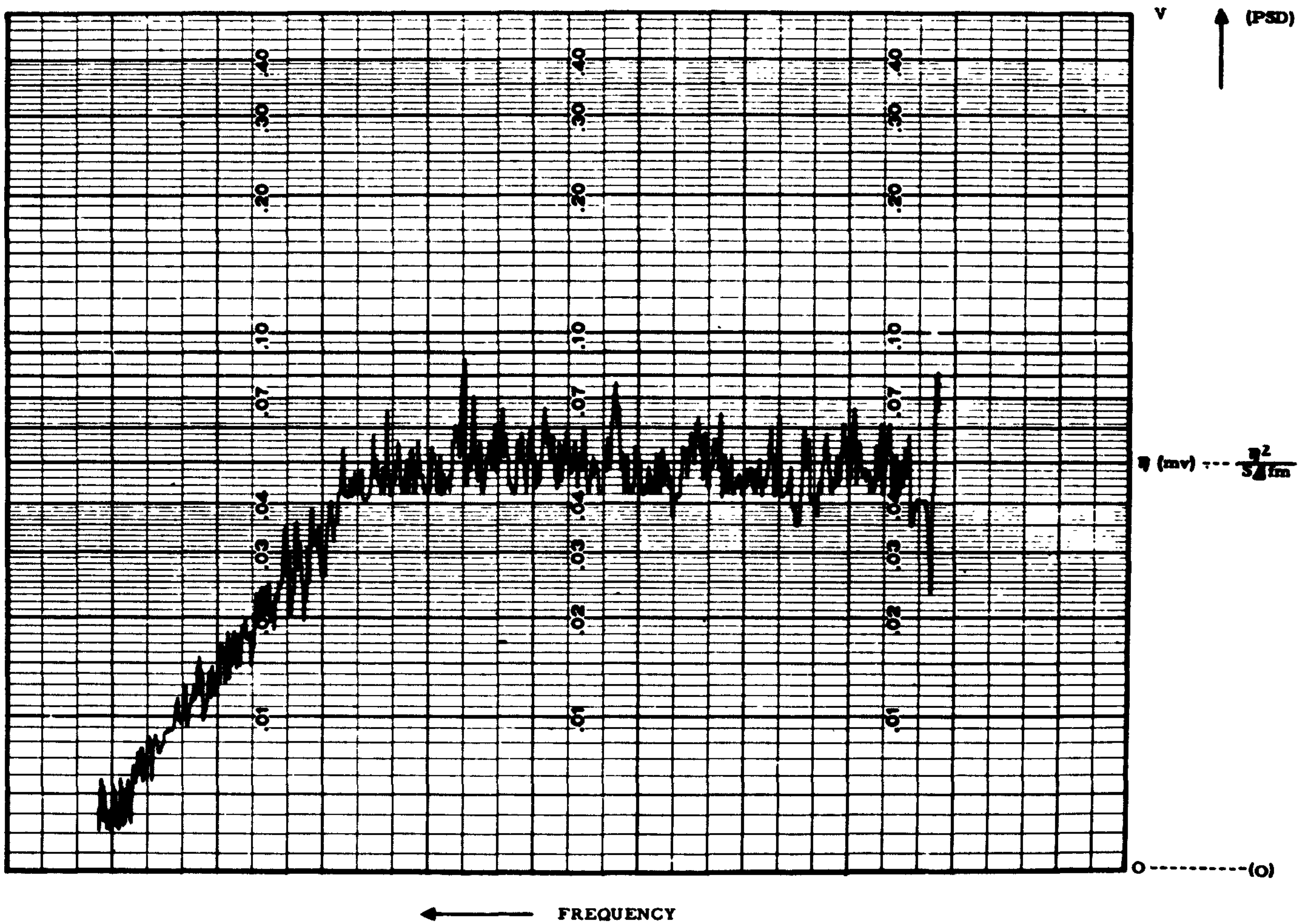

Fig. 1 - Voltage to PSD conversion

The composite rms acceleration is obtained by extracting the square root of the mean-squared acceleration:

$$a = \left(\int_{f\ell}^{fh} G \, df \right)^{1/2} . \qquad (6)$$

If the PSD is constant throughout the spectrum of the vibration test, the equation to determine the composite rms acceleration becomes:

$$a = (G \times BW)^{1/2} . \qquad (7)$$

Where:

BW = bandwidth of the vibration spectrum (fh - fℓ).

The composite mean-squared acceleration of a vibration test of constant PSD can be obtained directly from the output voltage of a voltage-frequency curve.

By applying the relationship of Eq. (4) to Eq. (7),

$$a^2 = \left(\frac{mv}{S}\right)^2 (BW/\Delta fm) . \qquad (8a)$$

The composite rms acceleration is obtained by extracting the square root of the mean-squared acceleration.

PSD which is constant throughout the frequency spectrum rarely occurs in a natural environment. Therefore, natural environments are more closely simulated if the PSD of vibration tests varies in the vibration spectrum. In general, the PSD can be varied in any way which simulates the environment, however, there are two methods of varying the PSD that are used most frequently for vibration tests. These include the following:

Step Function (Fig. 2)—The PSD is at one magnitude over a given bandwidth of the spectrum. It is then changed to assume a new magnitude over an adjacent bandwidth.

The composite mean-squared acceleration of the test is computed, using Eq. (7), by determining the components of mean-squared acceleration contained in each bandwidth of constant PSD, then summing all components. In general,

$$a^2 = \sum_{n=1}^{N} a_n^2 . \qquad (8b)$$

Where:

N = number of constant levels that PSD assumes in the vibration spectrum.

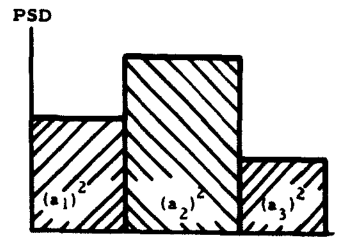

Fig. 2 - Step function

Variation (db) (Fig. 3)—The PSD varies r db with a frequency increase of one octave.

To determine the mean-squared acceleration of a vibration spectrum containing a db variation of PSD, the integration of Eq. (5) is required. The PSD, as a function of frequency, is integrated through the limits of the spectrum of variable PSD.

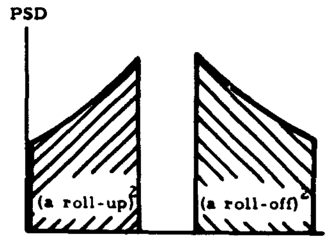

Fig. 3 - Variation (db)

The derivation of the PSD as a function of frequency and the required integration to determine mean-squared acceleration is presented in the appendixes of this report. The derivation and integration for a PSD that increases with frequency (roll-up) is presented in Appendix A. For the PSD that decreases with an increase of frequency (roll-off) the derivation and integration is presented in Appendix B.

The mean-squared acceleration of a vibration test of db variation can be obtained directly from a voltage-frequency curve. The rate of variation (db/oct) of the PSD can be determined from the parameters of the voltage-frequency curve. The method of deriving the rate of PSD variation from these parameters is presented in Appendix C.

Once the PSD variation rate (r) has been determined, a general expression which derives mean-squared acceleration directly from the parameters of a voltage-frequency curve, can be used. The general expression for the increasing PSD is presented as Eq. (10A) of Appendix A. The general expression for decreasing PSD is presented as Eq. (9B) of Appendix B. Using these equations, the need to convert voltage output to PSD and the integration procedure is not required to compute the mean-squared acceleration, but can be derived directly from a voltage-frequency curve.

Combination of PSD Variations (Fig. 4)—A vibration test is not limited to one type of PSD variation, but can include combinations of these in the vibration spectrum. When computing the composite mean-squared acceleration of the vibration test, the mean-squared acceleration of each segment of variable PSD is computed separately, then summed. In general,

$$a^2 = \underbrace{\sum_{n=1}^{N} a_n^2}_{\text{(step)}} + \underbrace{\sum_{m=1}^{M} a_m^2}_{\text{(variable)}} .$$

The composite rms acceleration of the vibration test is determined by extracting the square root of the mean-squared acceleration.

Example

The voltage-frequency curve of a random vibration test reveals the following information: The output voltage at 5 cps is 25 mv. The voltage then increases at a exponential rate from 5 to 80 cps at which point the voltage level reaches 100 mv. The voltage level is constant from 80 to 500 cps. At 500 cps the voltage rolls off at an exponential rate until it reaches a minimum level of 12.5 mv at 2000 cps (Fig. 5).

If the accelerometer system has a sensitivity of 10 mv/grms and the analyzing filter has an effective bandwidth of 100 cps, determine:

1. The roll-up rate from 5-80 cps.

2. The roll-off rate from 500-2000 cps.

3. The composite mean-squared acceleration.

4. The rms acceleration of the vibration test.

Roll-Up Rate—The general equation to determine the roll-up rate is presented as Eq. (4C) in Appendix C:

$$r_u = \frac{6 \log \frac{mv_2}{mv_1}}{\log \frac{f_2}{f_1}} .$$

By substituting the specific values of the example into Eq. (4C),

$$r_u = \frac{6 \log \frac{100}{25}}{\log \frac{80}{5}} .$$

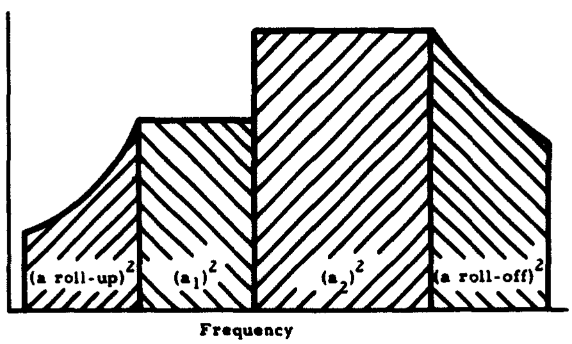

Fig. 4 - PSD curve

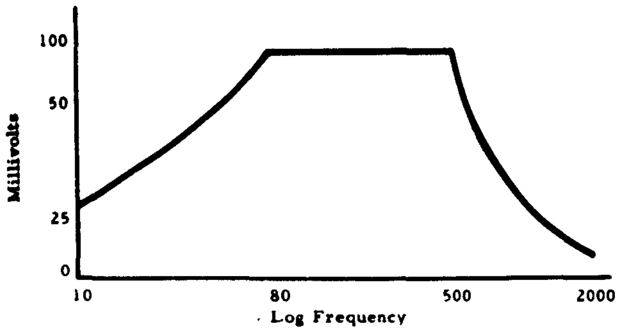

Fig. 5 - Voltage-frequency curve

$$r_u \text{ (roll-up)} = 3 \text{ db/oct.}$$

Roll-Off Rate—The general equation to determine the roll-off rate is presented as Eq. (8C) in Appendix C:

$$r_o = \frac{6 \log \frac{mv_1}{mv_2}}{\log \frac{f_2}{f_1}}.$$

By substituting the specific values of the example into Eq. (8C),

$$r_o \text{ (roll-off)} = \frac{6 \log \frac{100}{12.5}}{\log \frac{2000}{500}},$$

$$r_o \text{ (roll-off)} = 9 \text{ db/oct}.$$

Composite Mean-Squared Acceleration—The composite mean-squared acceleration is determined by computing the mean-squared acceleration of each segment, then summing. The mean-squared acceleration of the roll-up segment of the vibration spectrum is derived by using the general expression Eq. (10A),

$$a^2 \text{ (roll-up)} = \frac{3(mv_a)^2}{(r_u+3)(\Delta fm)(S^2)(f_a)^{r_u/3}} \times \left[f_c^{\left(\frac{r_u+3}{3}\right)} + f_a^{\left(\frac{r_u+3}{3}\right)} \right].$$

By substituting the values of the example and roll-up rate determined earlier,

$$a^2 \text{ (roll-up)} = \frac{3(25)^2}{(6)\ (100)\ (10)^2\ (5)} \times \left[(80)^2 - (5)^2 \right],$$

$$a^2 \text{ (roll-up)} = 40\ g^2.$$

The mean-squared acceleration of a segment that contains a constant PSD is determined by Eq. (8a):

$$a_1^2 = \left(\frac{mv}{S}\right)^2 \left(\frac{BW}{\Delta fm}\right).$$

Substituting the values of the example into Eq. 8,

$$a_1^2 \text{ (80-500 cps)} = \left(\frac{100}{10}\right)^2 \left(\frac{420}{100}\right).$$

$$a_1^2 \text{ (80-500 cps)} = 420\ g^2.$$

The mean-squared acceleration of the roll-off segment of the vibration spectrum is obtained from the general expression derived in Appendix B:

$$a^2 \text{ roll-off} = \frac{3}{(r_o - 3)} \frac{(mv_a)^2 (f_a)^{r_o/3}}{(\Delta fm)\ (S)^2} \left[\frac{1}{f_a^{\left(\frac{r_o-3}{3}\right)}} - \frac{1}{f_c^{\left(\frac{r_o-3}{3}\right)}} \right].$$

By substituting known values into the general expression,

$$a^2 \text{ (roll-off)} = \frac{3(100)^2\ (500)^3}{(9-3)\ (100)\ (10)^2} \times \left[\frac{1}{(500)^2} - \frac{1}{(2000)^2} \right],$$

$$a^2 \text{ (roll-off)} = 235\ g^2.$$

The composite mean squared acceleration is obtained by summing the mean-squared values,

$$a^2 = a^2 \text{(roll-up)} + a_1^2 \text{(80-500 cps)} + a^2 \text{(roll-off)}.$$

By substituting determined values,

$$a^2 = 40\ g^2 + 420\ g^2 + 235\ g^2,$$

$$a^2 = 695\ g^2.$$

Composite rms Acceleration—The composite rms acceleration is determined by extracting the square root of the mean-squared acceleration,

$$a = \sqrt{a^2}\ \text{grms},$$

$$a = 26.4\ \text{grms}.$$

DETERMINING THE TOTAL RMS ACCELERATION OF A SINE + RANDOM VIBRATION TEST FROM A VOLTAGE-FREQUENCY CURVE

Discussion

The total mean-squared acceleration of a random + sine vibration test is obtained by adding the composite random-mean-squared acceleration to the square of the rms sinusoidal acceleration,

$$a^2\,(\text{total}) = a^2\,(\text{sine}) + a^2\,(\text{random}). \qquad (9)$$

Where:

a^2 (total) = total mean-squared acceleration,

a^2 (sine) = sinusoidal rms acceleration squared, and

a^2 (random) = composite mean-squared acceleration of random vibration.

The total rms acceleration is determined by extracting the square root of the total mean-squared acceleration.

The total mean-squared acceleration of a random + sine vibration test can be determined directly from a voltage-frequency curve. The output voltage will present a continuous-vibration spectrum except one discrete frequency which will be represented by a peak voltage. The bandwidth of the peak will have the same bandwidth as the bandpass filter used to make the voltage-frequency curve (Fig. 6).

The discrete frequency of the peak is the frequency of the sinusoidal vibration present. The output voltage level contains the components of sine + random present at the frequency of the peak. The sinusoidal component of vibration is obtained by subtracting the random component present in the peak voltage output,

$$(\text{mv})^2\ \text{sine} = (\text{mv})^2\ \text{pk} - (\text{mv})^2\ \text{rand}. \qquad (10)$$

Where:

mv (sine) = output voltage of sinusoidal component,

mv (pk) = output voltage at the discrete peak frequency, and

mv (rand) = output voltage in the continuous-vibration spectrum.

The rms sinusoidal acceleration is proportional to the sinusoidal output voltage; the constant of proportionality is the sensitivity of the acceleration system:

$$a\,(\text{sine}) = \frac{\text{mv}\,(\text{sine})}{S}. \qquad (11)$$

The composite random-mean-squared acceleration is obtained by methods presented earlier in this report.

The rms sinusoidal acceleration and the composite random-mean-squared acceleration are combined, in accordance with Eq. (9), to determine the total mean-squared acceleration of the random + sine vibration test.

Example

A voltage-frequency curve of a random + sine vibration test reveals the following information: An output voltage of 50 mv from 5-500 cps except for a peak voltage of 100 mv at 400 cps. The voltage then rolls off at 6 db/oct from 500-2000 cps.

The analyzing filter has an effective bandwidth of 50 cps, and the sensitivity of the accelerometer system was 10 mv/grms. Determine:

1. The sinusoidal rms acceleration.

2. The composite random-mean-squared acceleration.

3. The total rms acceleration.

Sinusoidal rms Acceleration—The sinusoidal component is obtained by subtracting the random component at 400 cps. From Eq. (10):

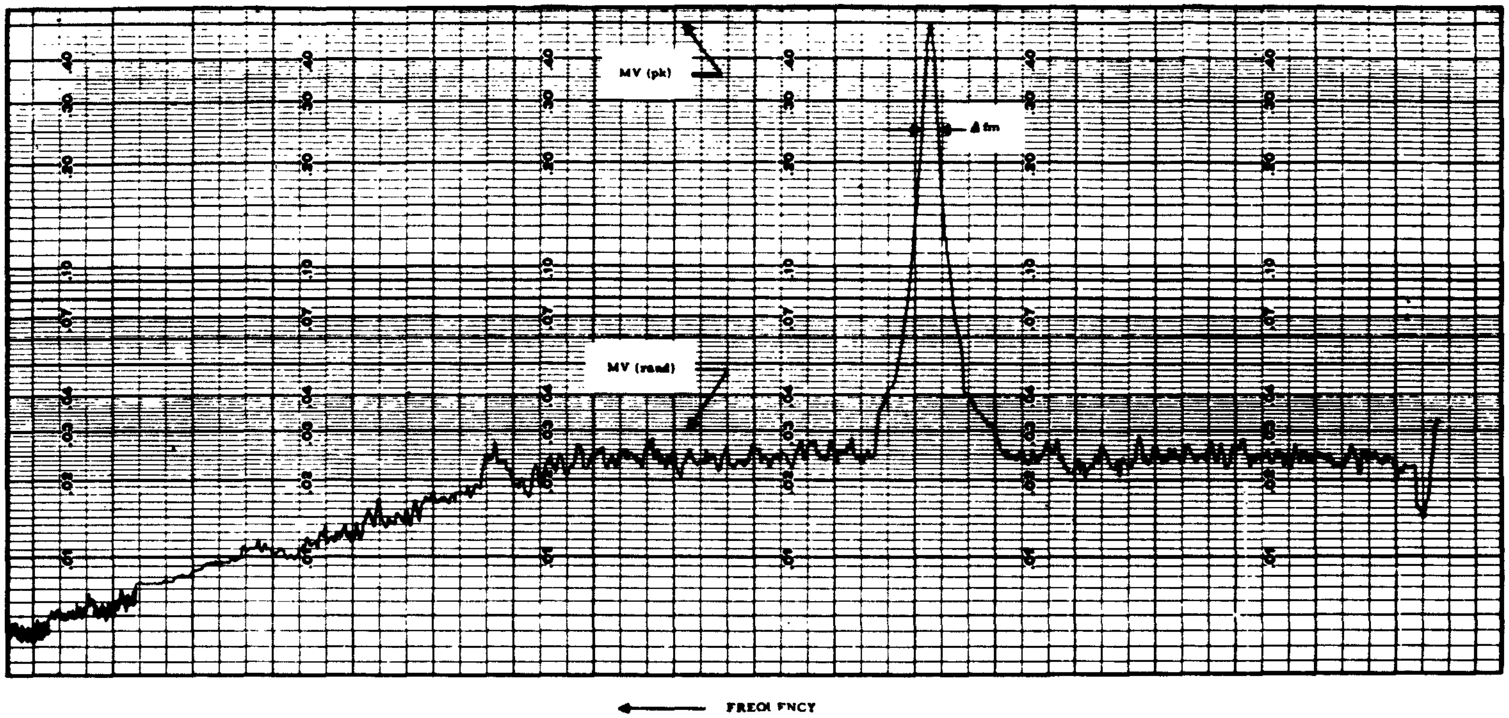

Fig. 6 - Random and sine voltage-frequency curve

$$(mv)^2 \text{ sine} = (mv)^2 \text{ pk} - (mv)^2 \text{ rand}.$$

By substituting known values,

$$(mv)^2 \text{ sine} = (100)^2 - (50)^2,$$

$$mv \text{ (sine)} = 86.6 \text{ mv}.$$

To determine the sinusoidal rms acceleration, the sinusoidal voltage is divided by the sensitivity: S = 10 mv/grms,

$$a \text{ (sine)} = 8.66 \text{ grms}.$$

Composite Random-Mean-Squared Acceleration—The composite random-mean-squared acceleration is determined by computing the mean-squared acceleration of each segment, then summing.

The mean-squared acceleration contained in the constant-voltage bandwidth from 5 to 500 cps is determined by Eq. (8):

$$a^2 \text{ (5-500)} = \left(\frac{mv}{S}\right)^2 \left(\frac{BW}{\Delta fm}\right).$$

By substituting known values,

$$a^2 \text{ (5-500)} = \left(\frac{50}{10}\right)^2 \left(\frac{495}{50}\right),$$

$$a^2 \text{ (5-500)} = 247.5 \text{ (g)}^2.$$

The mean-squared acceleration contained in the roll-off bandwidth from 500-2000 is determined by Eq. (9B):

$$a^2 \text{ (roll-off)} = \frac{3(mv_a)^2 \, f_a^{(r_o/3)}}{(r_o - 3)(\Delta fm)(S^2)} \times \left[\frac{1}{f_a^{\left(\frac{r_o-3}{3}\right)}} - \frac{1}{f_b^{\left(\frac{r_o-3}{3}\right)}}\right].$$

By substituting known values,

$$a^2 \text{ roll-off} = \frac{3}{(6-3)} \, \frac{(50)^2 \, (500)^2}{(50)\,(100)} \times \left[\frac{1}{(500)} - \frac{1}{(2000)}\right],$$

$$a^2 \text{ roll-off} = 187.5 \text{ (g)}^2.$$

The composite mean-squared acceleration is obtained by summing the mean-squared values,

$$a^2 \text{ rand} = 435 \text{ (grms)}^2.$$

Total rms Acceleration—The total mean-squared acceleration is obtained by adding the square of the rms sinusoidal acceleration to the composite random-mean-squared acceleration,

$$(a)^2 \text{ total} = (a)^2 \text{ sine} + (a)^2 \text{ random}.$$

By substituting in determined values,

$$(a)^2 \text{ total} = 75 + 435,$$

$$(a)^2 \text{ total} = 510 \text{ (g)}^2.$$

The total rms acceleration level of the test is derived by extracting the square root of the mean-squared acceleration,

$$a_{total} = 22.6 \text{ grms}.$$

Optional Method of Analysis—An optional analysis that could be performed would be to determine the peak voltage that would occur in the roll-off bandwidth if the sinusoidal component is swept through the vibration spectrum. For example, if the sinusoidal component of vibration in the preceding example is swept through the vibration spectrum, what would be the peak voltage that would be expected at 1500 cps?

The general expression which relates PSD to frequency in the roll-off spectrum was derived in Eq. (7B) in Appendix B:

$$Gx = Ga\left(\frac{fa}{f}\right)^{r_o/3}.$$

Thus the PSD at a frequency (f_1) in the roll-off spectrum is:

$$G_1 = Ga\left(\frac{fa}{f_1}\right)^{r_o/3}.$$

The PSD at a different frequency (f_2) in the roll-off spectrum is:

$$G_2 = Ga\left(\frac{fa}{f_2}\right)^{r_o/3}.$$

Dividing G_1 by G_2 gives the proportionality of the PSD's and frequencies in the roll-off spectrum:

$$\frac{G_1}{G_2} = \left(\frac{f_2}{f_1}\right)^{r_o/3}.$$

The relationship between voltage output and PSD is given by Eq. (4):

$$G = \frac{mv^2}{S^2 \Delta f_m} .$$

Substituting the voltage equivalent into the PSD ratio:

$$\left(\frac{mv_1}{mv_2}\right)^2 = \left(\frac{f_2}{f_1}\right)^{r_o/3} ,$$

$$\left(\frac{mv_1}{mv_2}\right) = \left(\frac{f_2}{f_1}\right)^{r_o/6} .$$

To determine the peak voltage that would occur if the sinusoidal component was at 1500 cps, the random component is computed, then added to the sinusoidal component.

The output voltage of the random component is derived from the expression:

$$\frac{mv_1}{mv_2} = \left(\frac{f_2}{f_1}\right)^{r_o/6} .$$

The initial conditions of the example were that the output voltage was 50 mv at 500 cps and the roll-off ratè was 6 db/oct. Therefore, the output voltage of the random component at 1500 cps is:

$$\frac{50}{mv_2} = \frac{1500}{500} ,$$

$$mv_2 = 16.7 \text{ mv} .$$

To determine the peak output voltage, the sinusoidal and random components are combined,

$$(mv)^2_{pk} = (mv)^2_{rand} + (mv)^2_{sine} .$$

Therefore:

$$(mv)^2_{pk} = 278 + 7500 ,$$

$$mv_{pk} = 88 \text{ mv} .$$

Appendix A

ROLL-UP

The PSD that increases at the rate of r_u db/oct is represented by the straight line of Fig. A-1.

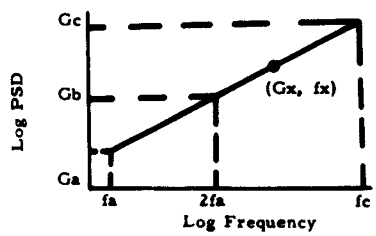

Fig. A-1 - Roll-up

The slope of the line of Fig. A-1 can be expressed as:

$$m = \frac{\log \frac{Gx}{Ga}}{\log \frac{fx}{fa}} , \qquad (1A)$$

where:

- m = slope of line in Fig. A-1,
- Ga = PSD at the beginning of roll-up,
- fa = frequency where roll-up begins,
- Gx = variable PSD in the spectrum of roll-up, and
- fx = variable frequency on roll-up spectrum.

If we consider only one octave in the roll-up spectrum, the slope becomes:

$$m = \frac{\log \frac{Gb}{Ga}}{\log 2} , \qquad (2A)$$

Gb = PSD after one octave of roll-up, thus:

$$\frac{Gb}{Ga} = 2^m . \qquad (3A)$$

The number of db that the PSD has increased after one octave:

$$r_u = 10 \log \frac{Gb}{Ga} , \qquad (4A)$$

r_u = rate of roll-up (db/oct).

By substituting Eq. (3A) into Eq. (4A),

$$r_u = 10 \log 2^m . \qquad (5A)$$

By approximating 3 db as the half-power level and solving for m,

$$m = r_u/3 \,. \tag{6A}$$

By equating (1A) and (2A),

$$\frac{\log \frac{Gx}{Ga}}{\log \frac{fx}{fa}} = \frac{\log \frac{Gb}{Ga}}{\log \frac{fb}{fa}} \,. \tag{7A}$$

By making the appropriate substitutions and solving for Gx,

$$Gx = Ga\left(\frac{fx}{fa}\right)^{r_u/3} \,. \tag{8A}$$

To obtain the composite mean-squared acceleration contained in the roll-up spectrum, the PSD is integrated through the limits of the spectrum.

$$a^2\,(\text{roll-up}) = \int_{fa}^{fc} Gx \, df \,,$$

$$= \frac{Ga}{f_a^{(r_u/3)}} \int_{fa}^{fc} f^{(r_u/3)} df \,, \tag{9A}$$

$$= \frac{3Ga}{(r_u+3)\,(fa)^{r_u/3}} \left[f_c^{\left(\frac{r_u+3}{3}\right)} - f_a^{\left(\frac{r_u+3}{3}\right)} \right] .$$

The composite mean-squared acceleration can be expressed in terms of output voltage of a voltage-frequency curve:

$$a^2\,(\text{roll-up}) = \frac{3(mv)_a^2}{(r_u+3)(\Delta fm)(S^2)(fa)^{r_u/3}} \times \left[f_c^{\left(\frac{r_u+3}{3}\right)} - f_a^{\left(\frac{r_u+3}{3}\right)} \right] . \tag{10A}$$

where:

mv_a = voltage output at the start of roll-up

Δfm = effective bandwidth of the analyzing filter, and

s = sensitivity of the accelerometer system (mv/grms).

The composite rms acceleration is extracted from the square root of the mean-squared acceleration.

Appendix B

ROLL-OFF

The PSD that decreases at the rate of r_o db/oct is represented by the straight line of Fig. B-1.

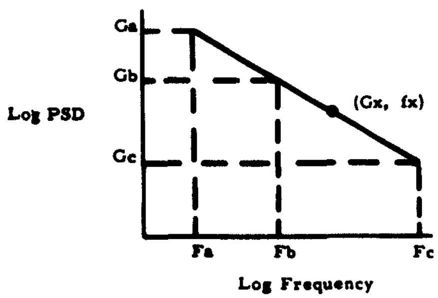

Fig. B-1 - Roll-off

The slope of the line of Fig. B-1 can be expressed as

$$-m = \frac{\log \frac{Ga}{Gx}}{\log \frac{fx}{fa}} \,, \tag{1B}$$

where,

m = slope,

Ga = PSD at the beginning of roll-off,

fa = frequency where roll-off begins,

Gx = variable PSD in the spectrum of roll-off, and

fx = variable frequency on roll-off spectrum.

If we consider one octave in the roll-off spectrum the slope becomes,

$$-m = \frac{\log \frac{Ga}{Gb}}{\log 2}, \tag{2B}$$

Gb = PSD after one octave of roll-off.

The number of db that the PSD has decreased after one octave,

$$r_o = 10 \log \frac{Ga}{Gb}, \tag{3B}$$

r_o = rate of roll-off db/oct.

By substituting (2B) into (3B),

$$r_o = -10 \log 2^{-m}. \tag{4B}$$

By approximating 3 db as the half power level and solving for m,

$$m = -r_o/3. \tag{5B}$$

By equating (1B) and (2B),

$$\frac{\log \frac{Ga}{Gx}}{\log \frac{fx}{fa}} = \frac{\log \frac{Ga}{Gb}}{\log 2}. \tag{6B}$$

By making the appropriate substitutions and solving for Gx,

$$Gx = Ga \left(\frac{fa}{fx}\right)^{r_o/3}. \tag{7B}$$

To obtain the composite mean-squared acceleration contained in the roll-off spectrum, the PSD is integrated through the limits of the spectrum:

$$a^2 \text{ roll-off} = \int_{fa}^{fc} Gx \, df ,$$

$$= Ga \, (fa)^{r_o/3} \left[\int_{fa}^{fc} f^{(-r_o/3)} df \right], \tag{8B}$$

$$= \frac{3 \, Ga \, (fa)^{r_o/3}}{(r_o - 3)} \times \left[\frac{1}{f_a^{\left(\frac{r_o - 3}{3}\right)}} - \frac{1}{f_c^{\left(\frac{r_o - 3}{3}\right)}} \right].$$

The composite mean-squared acceleration of the roll-off spectrum can be obtained directly from the output voltage of the voltage-frequency curve:

$$a^2 \text{ (roll-off)} = \frac{3(mv_a)^2 \, f_a^{(r_o/3)}}{(r_o - 3)(\Delta fm)(S^2)} \times \left[\frac{1}{f_a^{\left(\frac{r_o - 3}{3}\right)}} - \frac{1}{f_c^{\left(\frac{r_o - 3}{3}\right)}} \right]. \tag{9B}$$

The composite acceleration is obtained by extracting the square root of the mean-squared acceleration.

Appendix C

RATE OF PSD VARIATION

A voltage-frequency curve of a vibration test containing a spectrum of PSD roll-up might appear similar to Fig. C-1.

The number of db roll-up contained in the spectrum is determined by,

$$db = 20 \log \frac{mv_2}{mv_1}, \tag{1C}$$

where,

db = number of db roll-up,

mv_1 = voltage output at start of roll-up, and

mv_2 = voltage output at end of roll-up.

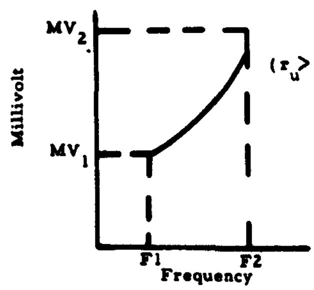

Fig. C-1 - Voltage-frequency roll-up curve

The number of octaves required to accomplish the roll-up,

$$n = \frac{\log \frac{f_2}{f_1}}{\log 2}, \qquad (2C)$$

where,

n = number of octaves,

f_1 = frequency beginning of roll-up, and

f_2 = frequency at end of roll-up.

Dividing Eq. (1C) by Eq. (2C) produces the roll-up rate:

$$r_u = (20)\,(\log 2)\,\frac{\log \frac{mv_2}{mv_1}}{\log \frac{f_2}{f_1}}. \qquad (3C)$$

r_u = rate of roll-up (db/oct).

By approximating $\log 2 = 0.3$,

$$r_u = 6\,\frac{\log \frac{mv_2}{mv_1}}{\log \frac{f_2}{f_1}}. \qquad (4C)$$

A voltage-frequency curve of a vibration test containing a spectrum of PSD roll-off, might appear similar to Fig. C-2.

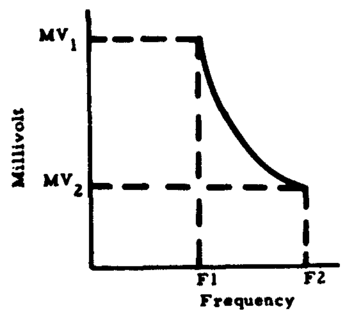

Fig. C-2 - Voltage-frequency roll-off curve

The number of db roll-off contained in the spectrum is determined by:

$$db = 20 \log \frac{mv_1}{mv_2}, \qquad (5C)$$

db = number of db roll-off mv,

mv_1 = voltage level at start of roll-off, and

mv_2 = voltage level at end of roll-off.

The number of octaves required to accomplish the roll-off:

$$n = \frac{\log \frac{f_2}{f_1}}{\log 2}, \qquad (6C)$$

where,

n = number of octaves,

f_1 = frequency at beginning of roll-off, and

f_2 = frequency at end of roll-off.

Dividing Eq. (5C) by Eq. (6C) produces the roll-off rate,

$$r_o = (20)\,(\log 2)\,\frac{\log \frac{mv_1}{mv_2}}{\log \frac{f_2}{f_1}}. \qquad (7C)$$

By approximating $\log 2 = 0.3$

$$r_o = 6\,\frac{\log \frac{mv_1}{mv_2}}{\log \frac{f_2}{f_1}}. \qquad (8C)$$

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* * *

THE APPLICATION OF DIGITAL ACQUISITION TECHNIQUES TO THE ANALYSIS OF SHOCK AND VIBRATION DATA*

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The analysis of shock and vibration data is becoming more and more difficult as tests become complex and space payloads require lighter and more efficient structures. This paper describes the evolution of a new, inexpensive, digital data system to help automate the laboratory and solve the more difficult problems. A unique multiplexing technique allows the sampling of many channels of 2000-cps data.

PURPOSE

The purpose of this paper is to describe a convenient, inexpensive, high-speed digital system for shock and vibration data acquisition, and to discuss some of its design considerations.

The paper will outline some problems involved in the digital sampling of 2000-cps data with a brief reference to some elements of communication theory. The parts of the data system will also be described with special emphasis on the programming variations possible with the unusual multiplexing techniques employed.

This digital invasion of a traditionally analog area is justified by the speed and quality of the analysis possible and because of the ease of implementing new and different digital data analysis techniques. This data system is quite general-purpose in its abilities. It can be, and is, used on a wide variety of typical laboratory data problems which require an above average amount of analysis. Most of the examples in this paper refer to shock and vibration uses, but the discussion applies to any data problem meeting certain theoretical qualifications and within the limits of 50 channels and 4-kc bandwidth, which meet the sampling of the following section.

*This paper was not presented at the symposium.

THE DESIGN PROBLEM

The major systems design problem involved in the digital acquisition of dynamic data follows directly from communication theory on the detection of a signal in the presence of noise for discrete data.[1-4] From the instrumentation standpoint this reduces to the problems of bandwidth, of channels, sampling rate, and time duration. In the design of any such system the solution to the design problem must encompass the whole problem; that is, not only should the data gathered be an accurate representation of the physical events being measured, but it should be gathered in such a way that the basic assumptions of subsequent analysis are satisfied. In this case, the amplifier linearity, drift, and bandwidth, the sample and hold aperture time, and the analog-to-digital converter accuracy are all concerned with the adequacy of measurement. The questions of sampling rate, time duration, and conversion resolution are concerned

[1] A. A. Samulon, "Spectrum Analysis of Transient Response Curves," Proc. Inst. Radio Engrs., 39:175-186 (1951).

[2] "Computer Controlled Systems," Scientific Data Systems Inc., Santa Monica, California (1962), pp. 3-1 to 3-6.

[3] Blackman and Tukey, The Measurement of Power Spectrum From the Point of View of the Communications Engineer (Dover Publications Inc., New York, N.Y., 1958) pp. 84-117.

[4] J. B. Scarborough, Numerical Mathematical Analysis (Johns Hopkins Press, Baltimore, Maryland, 1958).

with the adequacy of the analysis. Since the purpose of the data system is to produce analyzable data, analysis considerations will be discussed first.

ANALYSIS CONSIDERATIONS

The quality of the digital analysis techniques for dynamic data are intimately determined by how well the raw data represents what it is supposed to represent, and by the assumptions of the Fourier transform. The analysis techniques are based on the assumption that the input is an ensemble of sinusoids; therefore, the first design question is related to the sampling rate required to represent all of the relevant frequencies in the data. The ratio of the sampling rate to the highest frequencies of data that can be sampled legitimately is defined in a basic theorem evolved by Nyquist.[1] The highest input data frequency that can be uniquely defined by the digital system is commonly called the folding frequency or Nyquist frequency. This frequency limit is defined by the following equation:

$$f_n = \frac{1}{2\Delta t}, \qquad (1)$$

Where,

f_n = the folding or Nyquist frequency, and

Δt = the fixed time between digital samples in seconds.

If the sampling rate is in samples per second and the frequency in cycles per second, it can be seen that this equation can also be expressed as follows:

$$f_D = 2\ f_n, \qquad (2)$$

Where,

f_D = the digital sampling rate.

More simply, the folding frequency occurs at one-half the digital sampling rate. If information is digitized of a higher frequency than this limit, it could be analyzed incorrectly and no longer have a unique identity. Theoretically, if a filter with an infinitely sharp cutoff could be applied to the data before it was digitized, and that filter could be set just above the highest frequency of interest, the data system sampling rate would only have to sample the data 4000 times per second to be able to distinguish or extract any information from 0 to 1999 cps. Real-World filters are not infinitely sharp, therefore, we must raise the sampling rate of the data system sufficiently to take into account the following parameters:

- The highest frequency of interest in the data.

- The energy content of the information in the data spectrum above the highest frequency of interest. (This energy must be filtered out.)

- The shape or slop characteristics of the actual filters to be used in the data system.

- The stability of the data, or the rate of change of information in the data with time.

- The type of analysis to be performed on the data.

It must be remembered that too high a sampling rate, with respect to the frequency content of the data, will not improve accuracy and will cause excessive computer time to be used.

TYPICAL SOLUTIONS

The highest frequency of interest in most vibration or environmental laboratories is 2000 cps. This frequency is a common analysis cut-off point and is becoming a tradition in most labs. In the past, vibration testing stopped at 500 cps, then it crept up to 1000 cps, and now with most equipment the limit is 2000 cps. Whatever the particular circumstances behind this limit, we doubled it in order to be safe; we said that our maximum possible analysis limit would be 4000 cps.

Usually in lab tests there is not much energy left in the spectrum at 2000 cps. The most common reason for this is the low-pass filtering systems usually found in front of the vibration power amplifier. Not much energy can be present in real structures at 2000 cps because the wavelength at this frequency in metal is getting relatively short compared to specimen dimensions. At any rate, if there is any energy this high in the spectrum, it must be filtered to prevent ambiguity.

The best kinds of filters to use are those that can be described mathematically and those that exhibit little change in phase lag with changing frequency. Filters of the Bessel family are probably most commonly used[2] because they are easy to make and exhibit sharp rolloff. Blackman and Tukey,[3] in their book on the measurement of power spectrum recommend that (as a general rule) if a sharp cut-off Bessel filter is used, the folding frequency can be set at

$$f_n = (3/2) f_{max}. \quad (3)$$

Where,

f_{max} = maximum frequency of interest and

f_n = Nyquist or folding frequency.

In our particular case, by assuming that

f_{max} = 4000 cps and $f_n = (3/2)(4000) =$ 6000 cps, the results of using Eq. (2) to find f_D would be

$f_D = 2f_n = (2)(6000) = 12{,}000$ samples per second,

or, the required digital sampling rate is 12,000 samples per second. By using similar logic, 6000 samples per second would be sufficient to define 2000-cps data if a sharp filter is used, starting right at the cutoff point. (Average slope of a sixth order Bessel filter is about 35 db per octave. See Figure 1 for typical curves.)

At this point a word of caution must be appended: each case should be examined carefully to see what errors will occur and how important they are. Avoid buying a system which will not meet the necessary requirements.

The actual sampling rates chosen for our data system were three in number—13,900, 6950, and 2500 samples per second. Why these particular numbers were picked is explained in the following section which describes the data system.

DATA SYSTEM OUTLINE

The digital data system purchased to meet the requirements outlined in the previous sections is similar to many systems now being used in the field to measure low-frequency data (less than 100 cps). The major contribution of this particular system involves the multiplexer. In order to spend more time on this item, the rest of the system will only be covered briefly.

The data system is an all solid-state system using the latest types of silicon transistors and diodes, exclusively. A block diagram, Fig. 2, illustrates the general connection of the subsystems. The system is highly portable and contained in two standard 10-inch racks; it requires no air conditioning.

Transducers

Piezoelectric accelerometers are used almost exclusively in high-frequency shock

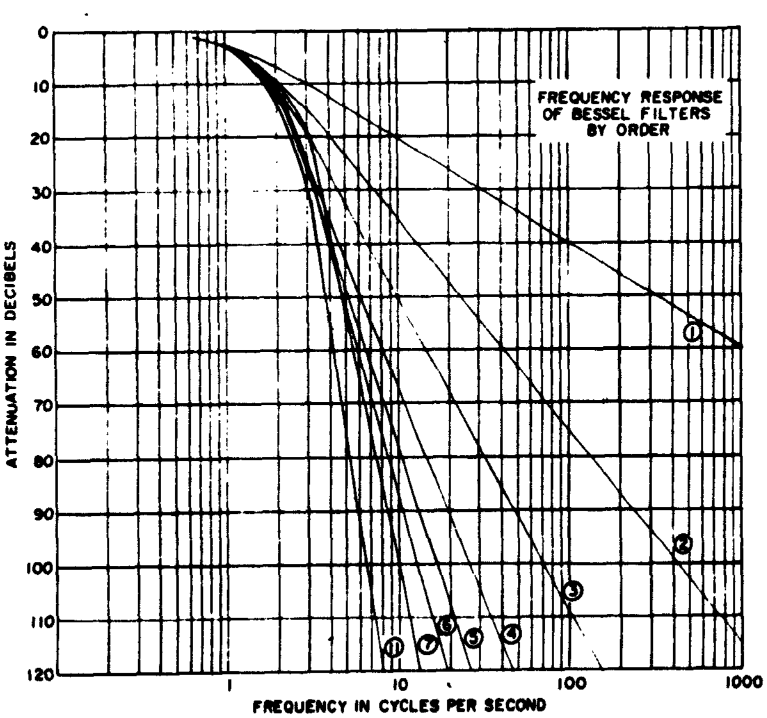

Fig. 1 - Frequency response of Bessel filters by order

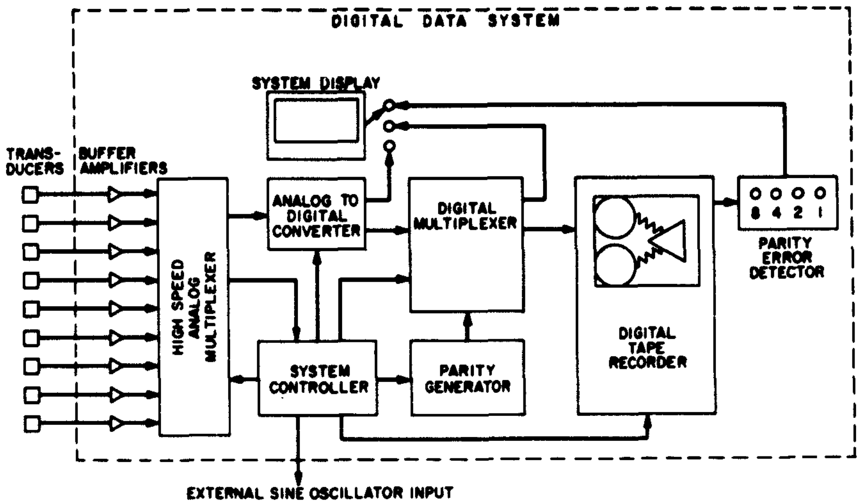

Fig. 2 - Block diagram of high-speed digital data system

and vibration work, mainly because of their light weight, high signal output, and resistance to damage at high g levels.

Buffer Amplifiers

The system is provided with 20 piezoelectric buffer amplifiers. They are all solid state and are mounted on plug-in cards, one to a card. The basic amplifier circuitry is that of an AC-amplifier; minimum input impedance is 300 megohms when connected to the transducer. This is sufficient impedance to give a maximum of ±1/2-percent error from 5 to 5000 cps due to all sources. There are two amplifier gains available (1 and 10).

Multiplexer

The output of each buffer amplifier is fed into the analog multiplexer. The job of the multiplexer is to connect one channel at a time to the analog-to-digital converter, upon command from the system control. The multiplexer (or scanner) in this system is again all solid state. It will switch the 0- to 10-volt outputs of the 20 buffer amplifiers with a maximum error of 0.01 percent of full scale at rates up to 20,000 channels per second. This particular multiplexer can be randomly programmed and it operates in a unique manner which will be explained later.

Analog-to-Digital Converter

After the multiplexer connects a particular channel, the analog-to-digital converter changes the analog voltage to an equivalent binary number. The ADC contains sample-and-hold circuitry in its input stage; this performs the function of freezing the rapidly moving AC voltages. The sample-and-hold is basically a capacitor charging process with electronic switches to expose the capacitor circuit to the slewing voltage for a precise amount of time. In this system, the charging circuitry is only exposed to the input for 4 microseconds, during which time it matches and tracks the moving voltage. When the input gate disconnects the sample-and-hold from the input signal, it stops changing and holds the voltage at the time the gate closes. The variance in gate closing times causes the major portion of the error here, which can be a maximum of 500 nanoseconds,total and can contribute as much as 0.3 percent of full-scale error when the signal is moving 2000 cps. After the sample-and-hold is operated, the analog-to-digital converter measures the voltage value frozen on the capacitor circuit. This process takes about 30 microseconds; at the end of this time, a 14-bit binary number with its plus or minus sign is gated into the format generator. In this particular system, the binary number is arranged so that each individual bit is equal to either 1 millivolt or 100 microvolts, depending on the settings of the buffer amplifiers.

Format Generator

The format generator (also often called digital multiplexer) must arrange the binary values in the correct format for digital recording. IBM format is used in this system. The format generator also counts the number of bits (ones or zeros) being gated out onto the tape and generates a parity bit which is added to the recording. This parity bit assures that the number of binary ones across the tape will always be even, that is either zero, two, four, or six ones. In this manner, the tape can be rechecked when it is read into the computer to make sure the data is being read back properly.

Digital Recorder

The recorder is a high-density, dual-speed transport equipped with read-after-write heads and electronics. All data recorded is immediately read back on a second head, and the parity is rechecked to make sure the recording process is proceeding properly. If parity errors are detected (some columns have an odd number of ones instead of an even number), a signal is sent back to the data system to tell the operator something is wrong. Tapes recorded can be analyzed directly on an IBM 7090 without prior editing, if desired. The format of the data recorded is shown in Fig. 3 with an explanation of the synch-bit information. The recorder used in this system operates at 75 and 37-1/2 ips. The recorder uses standard IBM packing densities of either 200 or 556 characters per inch of 1/2 tape. As explained in Fig. 3, it takes three character slots on the tape to describe one data word. Multiplying the tape speed by the standard

IBM TRACK
DESIGNATION

A		Parity	Parity	Parity	Parity	Parity	Parity	Parity	
B				A*	2048	32			
C				C*	1024	16			
8				C*	512	8			
4				+ or -	256	4			
2				8192	128	2			
1				4096	64	1			

ONE DATA WORD, THREE CHARACTER SLOTS.

← DIRECTION OF MOTION OF MAGNETIC TAPE

EXPLANATION OF CODES ON TAPE:

A* = WORD SYNCH. PULSE .... BINARY ONE IN SLOT WITH MOST SIGNIFICANT DIGITS OR BITS.

B* = CHANNEL SYNCH. PULSE .... BINARY ONE WHENEVER ANALOG MULTIPLEXER CHANGES CHANNELS, BINARY ZERO REST OF TIME.

C* = SCAN SYNCH PULSE .... BINARY ONE WHENEVER ANALOG MULTIPLEXER RETURNS TO CHANNEL ONE, BINARY ZERO REST OF TIME.

(WHEN "A" AND "B" ARE BOTH ONE'S YOU ARE READING THE FIRST DATA WORK ON A NEW CHANNEL. WHEN A, B, AND C ARE ALL ONES YOU ARE READING THE FIRST DATA WORD ON CHANNEL ONE.)

Fig. 3 - IBM tape format of high-speed digital data system

densities and dividing by three, produces the three sampling rates mentioned earlier (2500, 6950, and 13,900 samples per second). It can thus be seen that the recorder limitations are one of the first things to be looked into. IBM now has a higher density in operation: 800 characters per inch. This can be incorporated into the system later to give an ultimate speed of 20,000 samples per second.

DWELL MULTIPLEXING CONTROL

As mentioned previously, the problem confronting the designer of a digital system for high-frequency vibration data is primarily one of bandwidth and number of channels. As shown in the review of sampling theorem, 4000 samples per second would be, theoretically, the lowest sampling rate per channel, for 2000-cps data. We know that this rate, because of filter limitations, must be about 7000 to 10,000 samples per second. The method normally used to control a multiplexer can be described by the word sequential. The usual control involves digitizing channel one, then channel two, then three, four, five, and so on, until all channels in the system have been read once. The multiplexer then returns to channel one and starts through again. This type of multiplexer control is common to most digital data systems including missile and satellite telemetry systems. The preferential sampling of certain channels containing higher frequency information involves about the only variation of this technique.

If this technique were used to sample 2000-cps data, and we were to use the criteria developed earlier, we could only sample two channels. (The system tape recorder is limited to about 14,000 three-character words per second in IBM format.) The fastest digital recorder available, which records IBM format directly, runs at a tape speed of 150 ips. This is still only twice as fast as the recorder in our system. When this machine is used, 4 channels, or possibly 5 if the criteria is squeezed, can be multiplexed.

Clearly, two channels are better than none, but they are not very handy for recording 20 or 30 accelerometers in real time during routine tests.

A new system or method of multiplexing has been developed which allows the simultaneous, real time, recording of 20 or more channels of 2000-cps sinusoidal vibration data. This method is applicable to any system which has sufficient speed to digitize one channel within the previously developed criteria.

We sample the 20 channels by cheating, legitimately. That is to say, we know something about the data we are supposed to be independently measuring. We know the sinusoidal input or driving frequency of the testing vibrator, and we know that the vibration is almost stationary in nature, or at least we know its rate of change from a true stationary process. Stationary means that successive, statistically correct samples of data, when analyzed, will not show any change in a given period of time.

In most flight telemetry systems and other digital systems recording field phenomena, very little is known about the frequency content of the data at any given time (except possibly the highest frequency to expect). Under these conditions one has no choice but to sample sequentially, and as frequency goes up, the number of channels must come down. The basic processes of flight are not stationary as is a vibration laboratory test. Knowing this fact alone requires more frequent sampling.

The multiplexer in this system can function in the normal sequential manner is desired and provided the frequencies are low enough. This type of sampling will be called DC. A second mode of operation of this multiplexer will be called AC to signify its adaptability for sampling high frequencies. In the AC mode, the multiplexer starts on channel one as in the DC mode. Multiple readings are made on channel one, instead of just a single reading. When the multiplexer has dwelled long enough to gather a sufficient number of readings to define that channel completely, it switches to the next, and so on, until all channels have been measured. The multiplexer then returns to channel one and starts through again. As long as the sampling rate is fast enough to sample the highest frequency of interest in one channel and enough samples are taken before switching to the next, all the criteria are satisfied. The number of samples required on a given channel before switching depends on the degree of confidence required and the type of analysis made on the data. The correlation between channels measured in this manner depends on two factors—(1) The time required to sample all channels connected to the input, and (2) The rate of change of the almost stationary process.

The controls of this system allow three different analog to digital converter rates. These are set according to the highest frequency of interest desired. Additional controls allow the counting of zero crossings of the fundamental sinusoid in the vibration system and cause the changing of multiplexer channels every 1, 5, or 10 vibration cycles.

Any known analysis can be performed on sinusoidal data if it is sampled at a rate 5 times the highest frequency of interest, and if 10 sequential waves are sampled. At 1000 cps, if set on 10 waves, the multiplexer can scan 20 channels in 200 milliseconds, or define each channel 5 times per second. At 2000 cps it will read every channel 10 times per second. If it is desired to sweep from low to high frequencies without changing any controls, this can be done, but an excessive number of samples will be taken at the lower frequencies. In our system the computer program will throw out the extra readings and only compute on the required number, thus making the system completely automatic.

SUMMARY

The major contribution made by the digital system described here is that both the laboratory and the customers are now required to think about the test before it happens. Automation in the data gathering processes for shock and vibration has been slow in coming. The most popular analysis techniques for this type of data include crystal balls, hokus-pokus, visual inspection, black magic, and guesswork. All of these methods are in the same boat.

A digital data gathering system for this type of dynamic test and analysis encourages the engineer to think about the type of analysis he wants before the money is spent on the test. A system such as this can turn out labeled graphs or compute the most complex correlation factors, draw pictures of node lines, or look for failures automatically. Dynamic systems are very difficult systems to understand, which is all the more reason to get analytical when analyzing them.

One additional factor is very important in this day of competition—more personnel in the laboratory cannot solve the problems of increased complexity and sophistication in tests and testing systems. Our system was designed in conjunction with a four-thruster vibration system. One trained man can operate the entire system while testing very large specimens with many instrumentation points. The complete data system, including digital recorder, costs less than $50,000 delivered.

* * *

REAL-TIME ANALYSIS OF RANDOM VIBRATION POWER DENSITY SPECTRA*

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This paper describes an analysis method which, using a multiple-filter spectrum analyzer operating in real time, is capable of providing power density spectra, Fourier analysis, and a bandpass filter analysis.

INTRODUCTION

The large quantities of random vibration data that Sandia Corporation Livermore Laboratory (SCLL) has handled required finding a means of obtaining power density spectra that was considerably faster than the usual swept-filter techniques. An acceptable system would have to be able to analyze both stationary signals from laboratory tests and nonstationary signals from missile flight environments.

Swept-filter techniques have the advantages of selectable filter bandwidths and only one channel to calibrate; however, analysis time is too long for practical purposes when the amount and type of data are considered. For example, 30 or more accelerometers are not uncommon on a vibration table test; missile flight vibration is, in general, nonstationary, requiring simultaneous analysis in both frequency and time.

ANALYZER

The analysis method adopted uses a multiple-filter spectrum analyzer, operating in real time, that is capable of providing power density spectra, Fourier analysis, or bandpass filter analysis. Figure 1 shows a functional block diagram of one channel of the analyzer. This analyzer was built to SCLL specifications by Technical Products Company.

For power density spectrum analysis, the signal is filtered, squared, averaged, and normalized for the bandwidth of the filter. The averaging and normalizing functions are provided

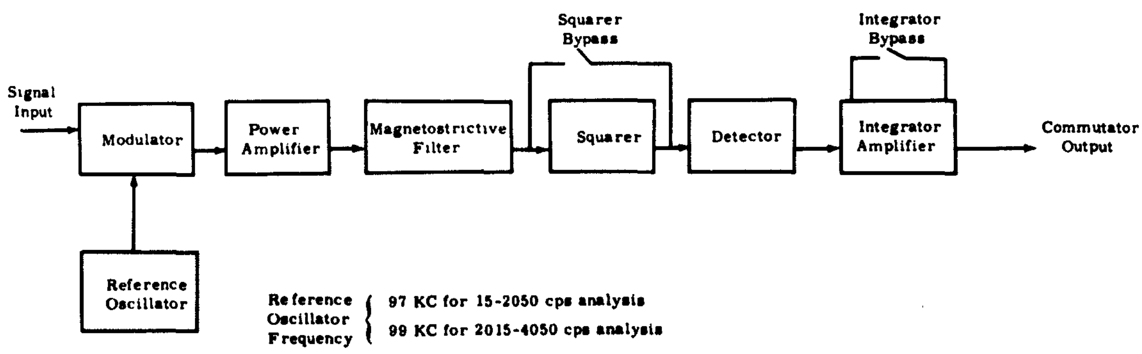

Fig. 1 - Functional block diagram of one channel of analyzer

*This paper was not presented at the Symposium.

by the integrator amplifier. For the Fourier analysis, the squaring circuit is bypassed (Fig. 2). For bandpass filter analysis, each channel acts as a real-time filter (Fig. 3).

The actual filters are magnetostrictive rod filters with center frequencies set from 96,980 to 95,000 cps. The input signal is heterodyned with 97 kc for 15- to 2050-cps analysis and with 99 kc for 2015- to 4050-cps analysis. The lower sideband is applied to the filter set.

The nominal filter characteristics for analysis of data in the 15- to 2050-cps frequency range are given in Table 1. For the analysis of 2015- to 4050-cps data, 2000 cps is added to the

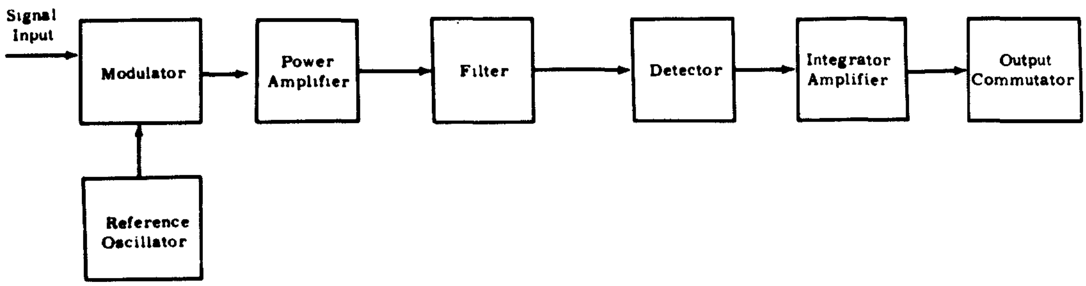

Fig. 2 - Functional block diagram of one channel of analyzer for Fourier analysis

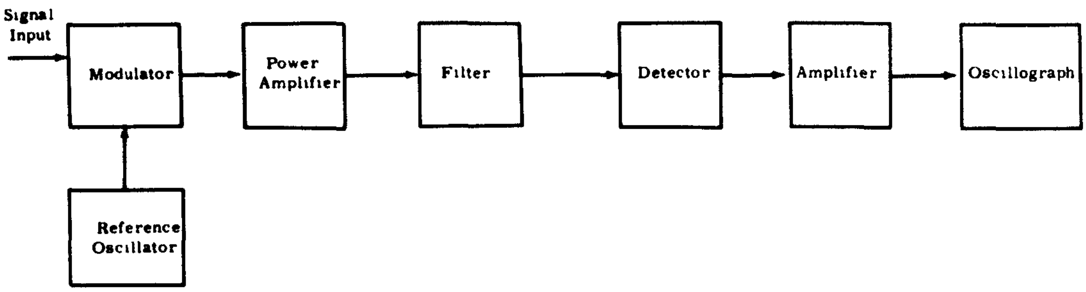

Fig. 3 - Functional block diagram of one channel of analyzer for bandpass filter analysis

TABLE 1
Random Vibration Analyzer Channels for 15- to 2050-cps Analysis

Channel No.	Center Frequency	Bandwidth (cps)	Channel No.	Center Frequency	Bandwidth (cps)
1	20	10	12	130	10
2	30	10	13	140	10
3	40	10	14	150	10
4	50	10	15	160	10
5	60	10	16	175	25
6	70	10	17	200	25
7	80	10	18	225	25
8	90	10	19	250	25
9	100	10	20	275	25
10	110	10	21	300	25
11	120	10	22	325	25

TABLE 1 (Cont.)
Random Vibration Analyzer Channels for 15- to 2050-cps Analysis

Channel No.	Center Frequency	Bandwidth (cps)	Channel No.	Center Frequency	Bandwidth (cps)
23	350	25	37	900	50
24	375	25	38	950	50
25	400	25	39	1000	50
26	425	25	40	1050	50
27	450	25	41	1100	100
28	475	25	42	1200	100
29	500	50	43	1300	100
30	550	50	44	1400	100
31	600	50	45	1500	100
32	650	50	46	1600	100
33	700	50	47	1700	100
34	750	50	48	1800	100
35	800	50	49	1900	100
36	850	50	50	2000	100

center frequencies. This is accomplished merely by changing the heterodyne reference frequency by 2000 cps. Provisions are included for use of an external oscillator so that other 2-kc frequency bands of interest may be analyzed.

The integration time for power density spectra analysis or Fourier analysis is variable in unit steps from 1 to 10 seconds. It can be shown[1,2] that, for stationary random signals, the variance of an estimate of the power density spectrum about the true spectrum decreases with increase in sampling time. That is, for a phenomenon with unchanging statistical characteristics, an estimate is more reliable if it is obtained from a large sample. For such phenomena, long averaging times are desirable. For nonstationary phenomena, such as are encountered in many missile flight environments, the statistical properties change in time, and long samples are relatively meaningless. In such cases, shorter samples that are contiguous in time, yield more meaningful data.[3]

[1]C. T. Morrow, "Averaging Time and Data Reduction Time for Random Vibration Spectra," J. Acoust. Soc. Am., Vol. 30, Nos. 5 and 6 (June 1958).

[2]R. B. Blackman and J. W. Tukey, "The Measurement of Power Spectra from the Point of View of Communications Engineering," Bell System Technical Journal, Part I, Vol. 37 (January 1958) and Part II, Vol. 37 (March 1958).

[3]P. T. Schoenemann, "Measurement of Power Spectra for Nonstationary Random Signals," Sandia Corporation Livermore Laboratory, SCTM 311-62-81 (April 1962). (Available from the Office of Technical Services, Department of Commerce, Washington 25, D. C., Price $.50.)

The analyzer can be operated in various modes, depending upon the type of data to be treated. For signals of long duration, the analyzer is operated in a continuous mode in which the system sequentially samples the spectrum for the number of seconds chosen (1 to 10, as mentioned previously) and reads out the data through a commutator. The readout operation takes 1 second. This sequence repeats continuously. In this mode, samples of the spectrum are obtained throughout the duration of the signal being analyzed.

For short duration data (but not less than 1 second), the system is capable of sampling and reading out once. Initiation of this mode is controlled either by a pushbutton on the front panel or by an externally applied signal triggered from the actual flight data tape.

Data Output

The outputs of the individual channels are commutated at the end of each sampling period. An output proportional to the center frequencies of the filters is provided coincidentally to yield semilog x-y plots on oscilloscopes. A Hughes model 104 memoscope is used for monitoring and system alignment. A Tektronix model 536 oscilloscope is available for photographic data recording using either a Beattie-Coleman 35-mm camera controlled by the analyzer, or a Polaroid camera.

The commutated output of the power density spectrum analyzer is displayed on oscillograph recordings along with the time record of the signal being analyzed (Fig. 4). The sequence of data frames is usually in the following order:

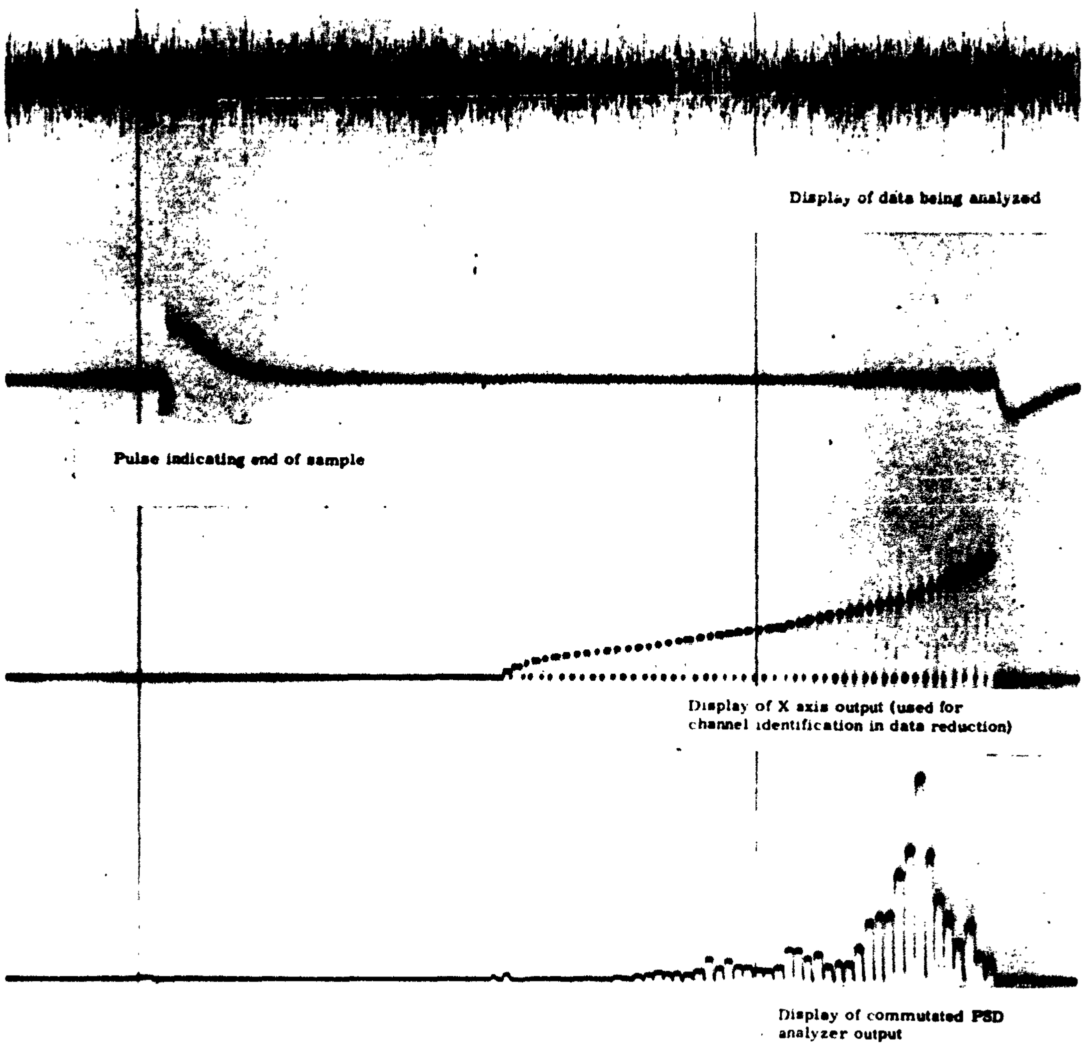

Fig. 4 - Oscillograph presentation of analyzer output for flight data

1. At least three 10-second samples of white-noise calibration.

2. Analysis of the data for the 15- to 2050-cps frequency range.

3. Analysis of the data for the 2015- to 4050-cps frequency range (if needed).

The calibration frames apply to both the 15- to 2050-cps and 2015- to 4050-cps data. The calibration for one channel of data is obtained by averaging at least 3 of the 10-second calibration values for the channel. For example, the calibration for Channel 50 of any data frame is obtained by averaging the values of Channel 50 for three separate calibration frames. This method of calibration removes the necessity of having all 50 channel gains set for the same output for white-noise input.

Figure 5 shows oscillograph records of calibration, input spectra, and response spectra produced by the analyzer for a vibration table test.

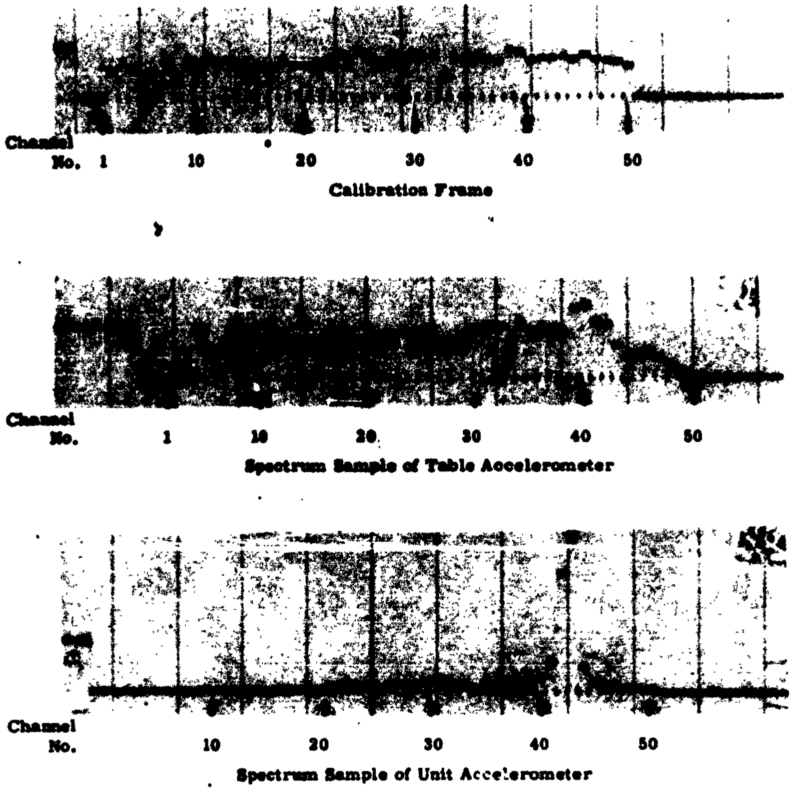

Fig. 5 - Examples of oscillograph outputs for shaker random vibration data

Accuracy

The analyzer yields data that is accurate to within 10 percent. This means that a given signal sample repeatedly analyzed will yield spectrum samples within 10 percent of the actual spectrum of the sample.

The statistical reliability of the sample is an additional consideration. The spectrum of a stationary signal is defined as the average overall time of the $(\text{signal})^2/\text{cps}$. A finite sample will, in general, yield only an estimate of this true spectrum. As is shown in Ref. 2, the estimate for stationary signals becomes more reliable as the product of filter bandwidth times the sampling time, increases.

For nonstationary phenomena, such as most missile flight environments, the above argument is not true. The analyzer then yields spectral samples that are accurate to within 10 percent. Determination of the statistical reliability of the samples necessitates a knowledge of the detailed nature of the nonstationary characteristics of the signal.

DATA ANALYSIS

The output of the analyzer is converted to suitable form for analysis on an IBM 1401 digital computer. The following finished data is obtained:

- Power density spectra plots.

- RMS values corresponding to the spectral plots.

- Estimates of transmissibility functions (for vibration table tests).

The power density spectra plots are obtained merely by comparing the raw data to the calibration. The rms values are determined by taking the square root of the area under the power density plots. This provides a check when compared with true rms meter readings taken during the actual tests. Estimates of transmissibility functions are obtained using the following formula:

$$H(f) = \sqrt{\frac{P(f)\ out}{P(f)\ in}}$$

where

$H(f)$ is the transmissibility function,

$P(f)$ out is the power density spectrum of the response signal, and

$P(f)$ in is the power density spectrum of the input signal.

In this calculation, the raw data may be used because the calibration applies to both the numerator and the denominator in the expression and, therefore, is cancelled out.

APPLICATIONS TO NONSTATIONARY PHENOMENA

In many practical engineering applications, the fact that the process measured may be nonstationary is ignored, sometimes with unfortunate consequences. A succession of spectrum samples adjacent in time is needed to completely describe the power density spectrum of a nonstationary signal. For an item that fractures on a shaker, the spectrum samples before fracture yield information concerning resonances of the item, and the transition of the spectrum after fracture indicates at what frequencies and at what time the trouble occurred. Complex situations abound where the changes in the spectrum are continuous. One such condition is the vibration environment experienced by a missile in flight. Figures 6, 7, and 8 show successive spectrum samples, each averaged for 1 second, taken from telemetered missile vibration data. Figure 9 shows the same phenomenon, but averaged over the total interval of Figs. 6, 7, and 8. The time-varying characteristics of the spectrum are lost when too long an averaging time is used.

An analyzer suitable for the spectral analysis of nonstationary random signals must measure all the spectrum all of the time. The amount of reduced data is necessarily more voluminous than for stationary processes because a function

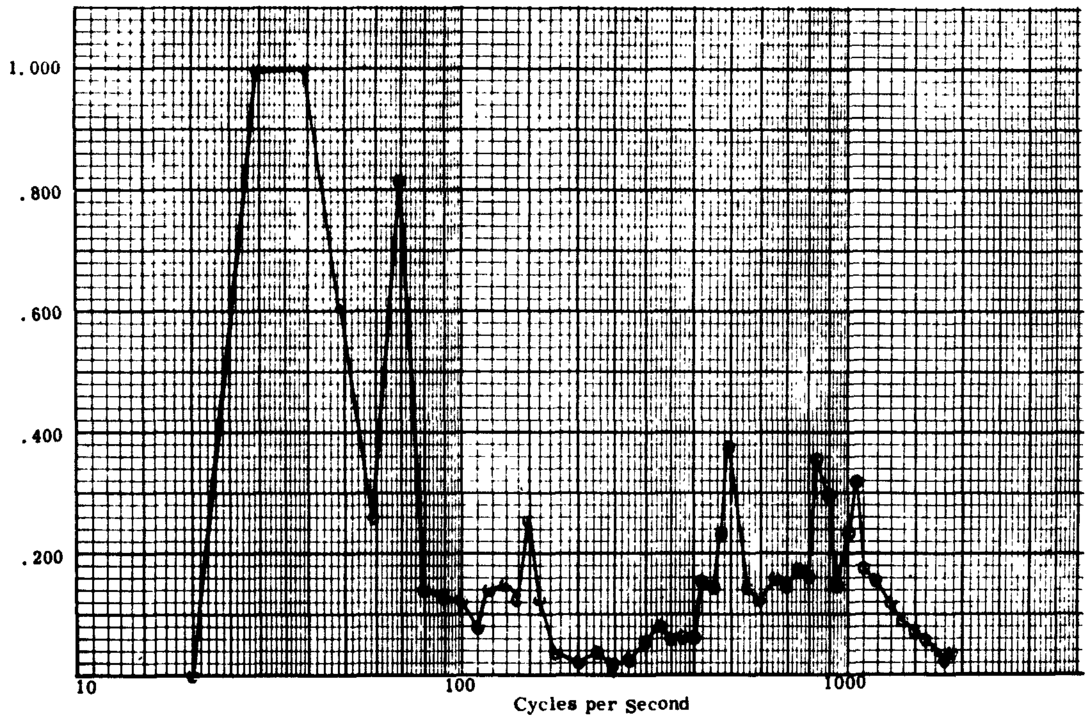

Fig. 6 - Missile vibration power spectrum (sample 1)

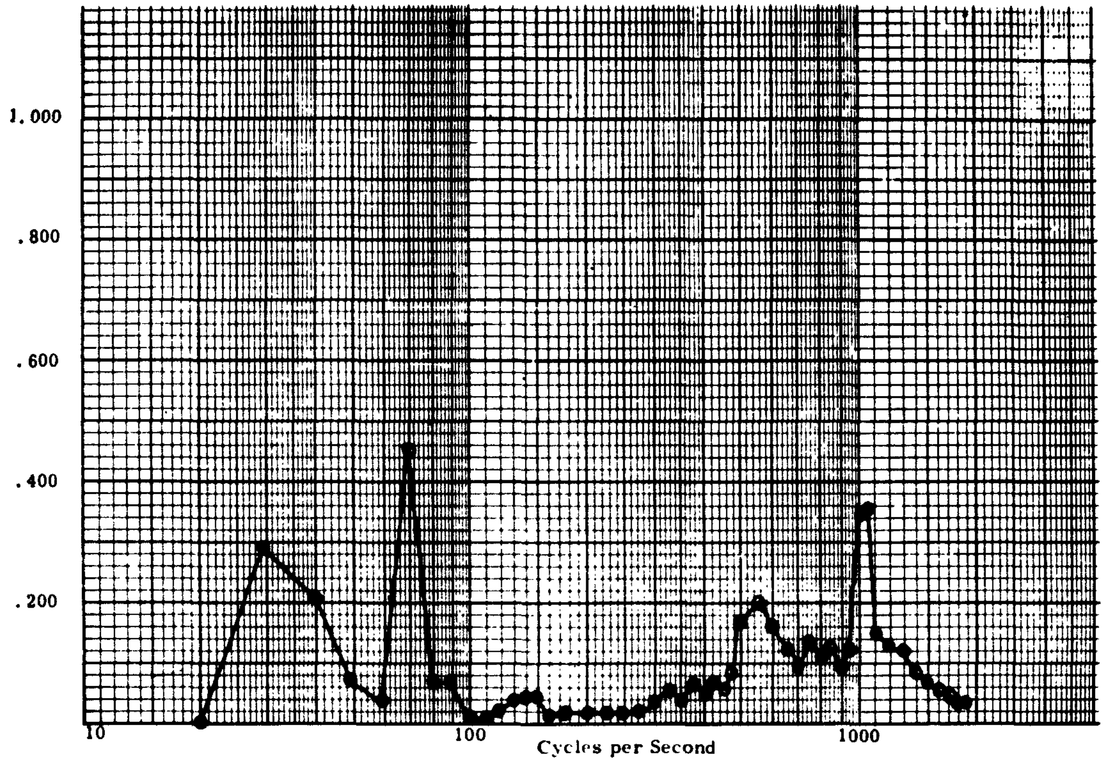

Fig. 7 - Missile vibration power spectrum (sample 2)

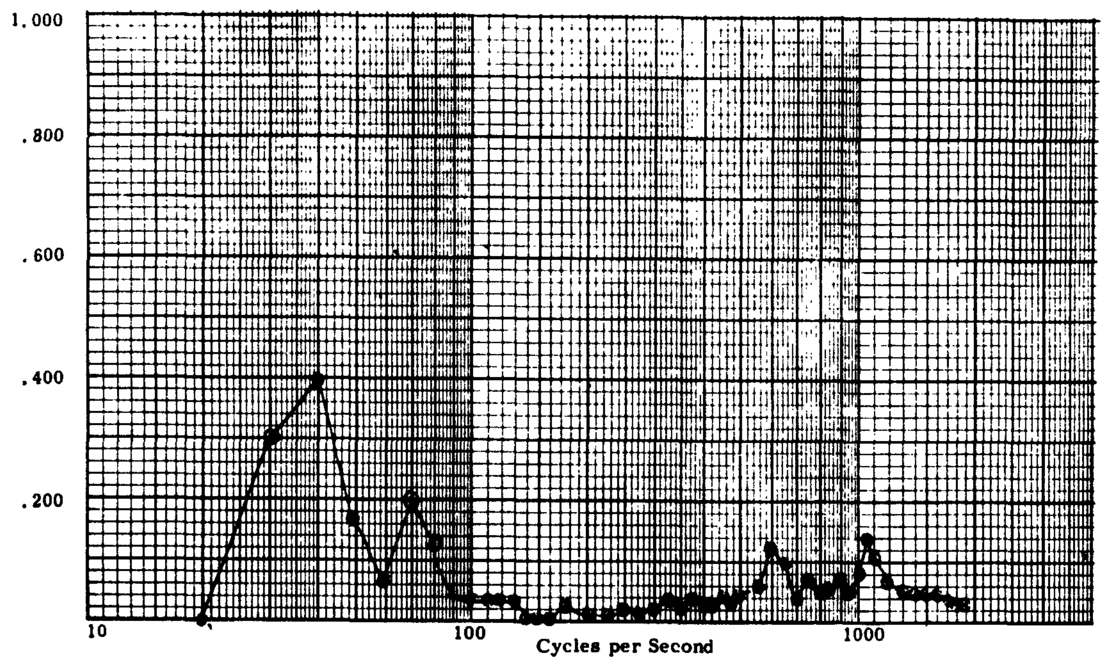

Fig. 8 - Missile vibration power spectrum (sample 3)

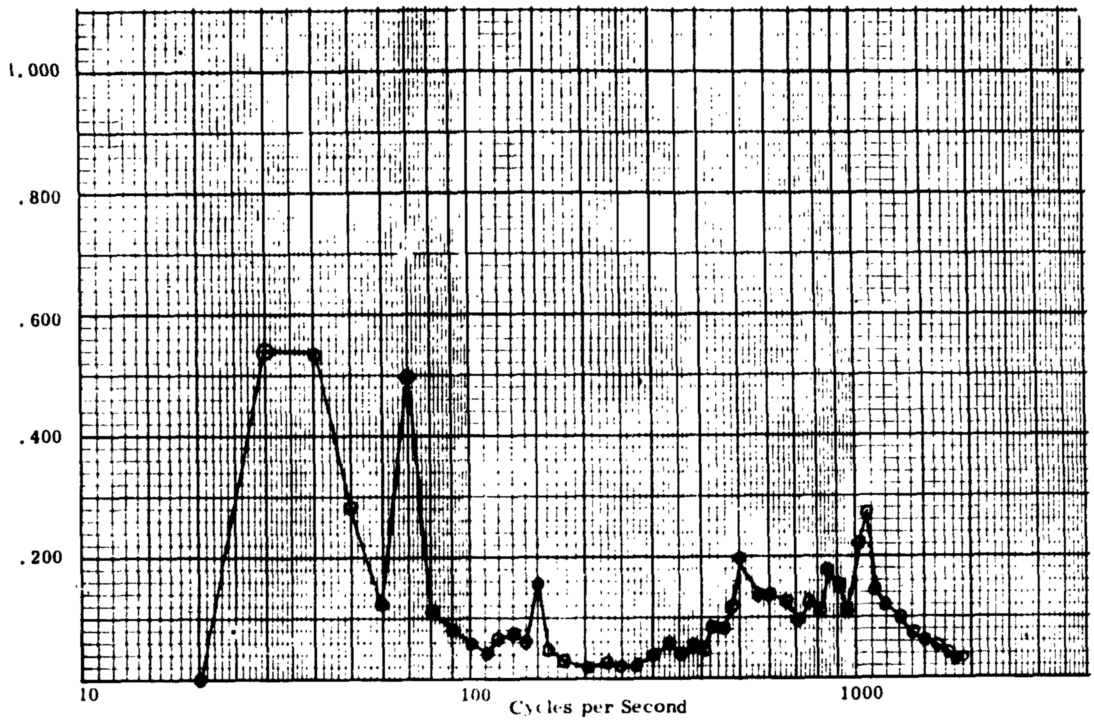

Fig. 9 - Average of Figs. 6, 7, and 8

of two variables, frequency and time, is being measured and another order of complexity is introduced.

CONCLUSION

The use of a multiple-filter spectrum analyzer allows real-time reduction of random vibration data. Because all of the spectrum is observed for all of the time, the analysis of nonstationary phenomena is also facilitated.

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TWO NEW SYSTEMS FOR MEASURING VIBRATION DATA IN THE FREQUENCY DOMAIN*

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Minneapolis-Honeywell

Two analysis systems—a 1/3-octave-band true-rms spectral analyzer and a narrow-band heterodyne true-rms spectral analyzer—have been developed. These systems were developed for the Aeronautical System Division's new sonic test facility.

INTRODUCTION

Despite recent interest in statistical and time domain analysis of random vibration and noise phenomena, frequency or spectral analysis continues to provide the structural design and test engineer with meaningful answers in usable form. Unfortunately, the answers have often been inadequate because the available analysis equipment lacked the required dynamic range and accuracy, or excessively compromised the underlying mathematical principles. This was the situation which existed when design specifications were written by Bolt, Beranek, and Newman for the new sonic test facility at Wright Air Development Division (WADD). Because this facility would provide new measurement capabilities, it became necessary to specify new analysis equipment commensurate with those capabilities. In response to those specifications, Minneapolis-Honeywell undertook development of two analysis systems, for the WADD facility, which would provide substantial improvement over commercially available equipment. The result of this development is a 1/3-octave-band, true-rms spectral analyzer and a narrow-band heterodyne, true-rms spectral analyzer. Both of these systems provide high accuracy and range, as well as design techniques which substantially decrease the compromise often necessary when electronic devices are used to implement mathematical relationships.

GENERAL

To simplify the description and better illustrate the design objectives of the subject spectrum analyzers, a brief review of basic techniques and assumptions is in order. First, it is assumed that there is available a finite data sample of a stationary process and that this is made repetitive by recording it on a loop of magnetic tape. The analysis techniques discussed here are also applicable to continuous samples of an ergodic stationary process or, in some cases, to real time or on-line analysis. However, the tape loop input is most common and best serves to illustrate the principle.

Analog spectral analysis of this type of data usually takes one of two forms. For a quick-look, parallel octave-band analysis is often employed through the use of a device similar to the 1/3-octave system to be described later. For detailed analysis, a narrow-band approach is employed, through the use of a heterodyne analyzer. In either case, the basic techniques are rather straightforward and do not require detailed explanation. But, a quick review of these basic devices will serve to illustrate the problems which had to be solved in order to provide the sonic facility with the needed spectral analysis capability.

In the parallel filter device, each filter has a different center frequency passing only a

*This paper was not presented at the Symposium.

restricted frequency band. The output of the filters is usually commutated so that only one measuring circuit is required. The amplifier and detector must handle the full frequency range of the system as well as the full dynamic range.

In the heterodyne device (Fig. 1), a single narrow filter with a fixed center frequency is employed, the data frequency being translated into an intermediate frequency by the heterodyne action. Here, only the input amplifier and modulator must accommodate the full frequency range; the detector operates only on the i-f. All circuits, however, must handle the full dynamic range desired for the system. Furthermore, if the power spectrum or the true rms is required, the dynamic range at the detector (squarer) output is twice that of its input. Since power or true rms is usually the desired quantity, and since with the present state-of-the-art squaring devices are generally limited to 30 db or less at the input, it follows that most spectral analyses are limited to a 30-db (referred to the input) range. But, magnetic tape can provide a 50-db signal-to-noise ratio on a wide-band basis. On a narrow-band basis, which is what one achieves with spectral analysis, a tape system is capable of 70 db or more. Also transducers are capable of far more than 30 db. Consequently, with typical techniques the spectrum analyzer severely limits the inherent dynamic range whenever power spectral or true-rms readings are required.

In addition to the problem of dynamic range, there is usually a problem of frequency accuracy, particularly with the heterodyne analyzer. The typical heterodyne device employs a variable LC or voltage controlled type oscillator which sweeps through a range of 10 or 20 kc. Even under the best conditions, the oscillators usually provide an accuracy that is perhaps no better than 1 percent of full scale. Consequently, it is usual to provide several full-scale frequency ranges so that with several sequential analyses, the full spectrum can be obtained with a frequency accuracy that is a reasonable percentage of the reading. Thus, capability and accuracy are further limited. To overcome such problems and provide needed accuracy and capability in the spectral analysis section of the test facility, the subject equipment was developed.

NARROW-BAND ANALYZER TECHNIQUES

The narrow-band system developed for WADD uses the basic heterodyne principle, but radically improves the method of adapting that principle to spectral analysis. For example, a stepped, rather than swept, frequency scan is combined with a true integration (in the RMS Computer) to provide improved accuracy and resolution while reducing analysis time. Further, a unique method of dynamic range extension allows true-rms measurements over 70 db referred to system input. In keeping with these advanced techniques, complete system automation is accomplished with digital logic programming. Finally, these features are realized with modular packaging of solid-state circuitry (Fig. 2).

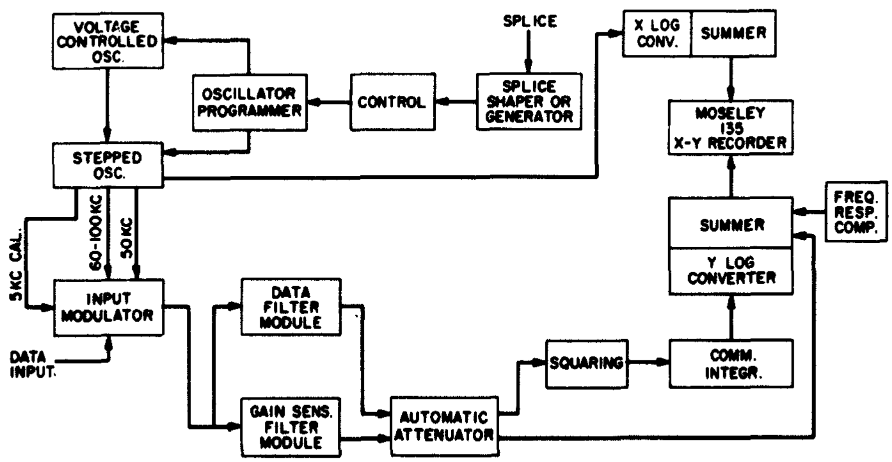

Fig. 1 - Block diagram of narrow-band spectrum analyzer

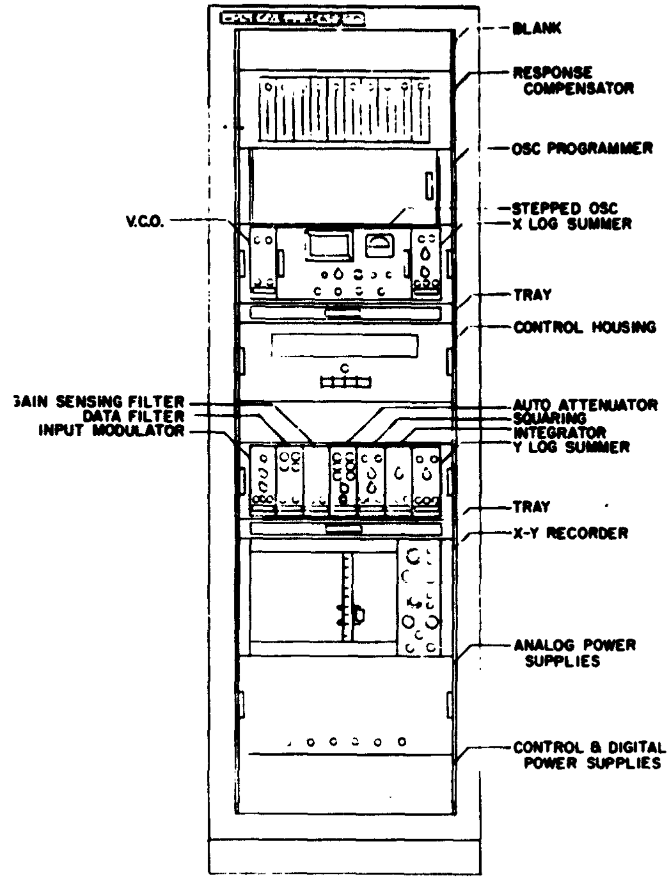

Fig. 2 - Rack layout of narrow-band spectrum analyzer

Graphic results of this design are illustrated in the sample plots of Figs. 3 and 4. Figure 3 displays the high resolution and dynamic range capabilities by analysis of a 5-cps square wave. Analysis of a perfect square wave should reveal no even harmonics, and odd harmonics should fall off at a rate of 20 db per decade. The plot represents a 1.414-v rms square wave so that the 5-cps component is 1 v rms (0 db). The imperfection of the square wave is shown by the even harmonics which may be seen in the region of -60 db. The amplitudes of the odd harmonics closely follow the ideal 20 db per decade rolloff. The partial loss of resolution is immediately evident when the system switches to 5-cps bandwidth as programmed. An additional feature of the system is scale expansion which allows close investigation at high frequencies. In this mode of operation, the frequency axis is changed to linear and the x voltage is expanded. By use of this mode, the 49th, 50th, and 51st harmonics of a 5-cps square wave have been plotted with a 1-cps bandwidth by x-axis linear expansion, thereby providing the necessary horizontal spread of points for readability.

It should be noted that the spectrum mode (sine-wave filter weighting) was used on this plot. Connecting lines were added to this graph to eliminate confusion due to losses in reproduction techniques.

Figure 4 is a plot of the white random noise output of a GR Model 1390-A random noise generator whose output is low-pass filtered at approximately 10 kc. The filtered noise is adjusted to 1 v rms. The analyzer is in spectral density mode such that all points represent rms, or (energy/cps).

If the input were precisely 10-kc bandwidth at the system input, the plot should be flat at -40 db. This assumes equal energy per cycle

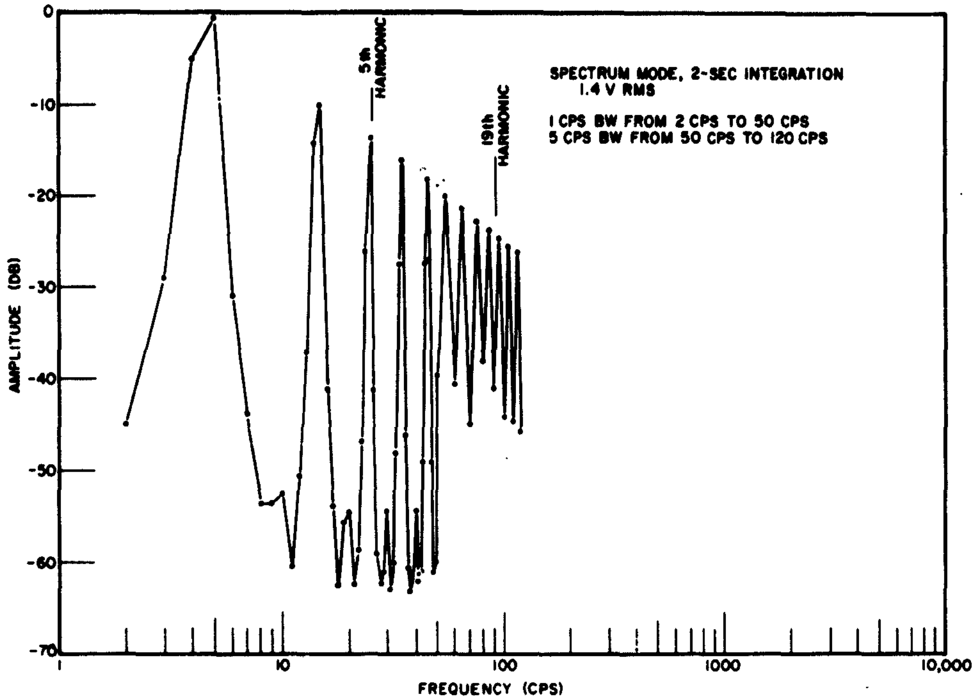

Fig. 3 - Heterodyne analyzer sample plot (5-cps square-wave input)

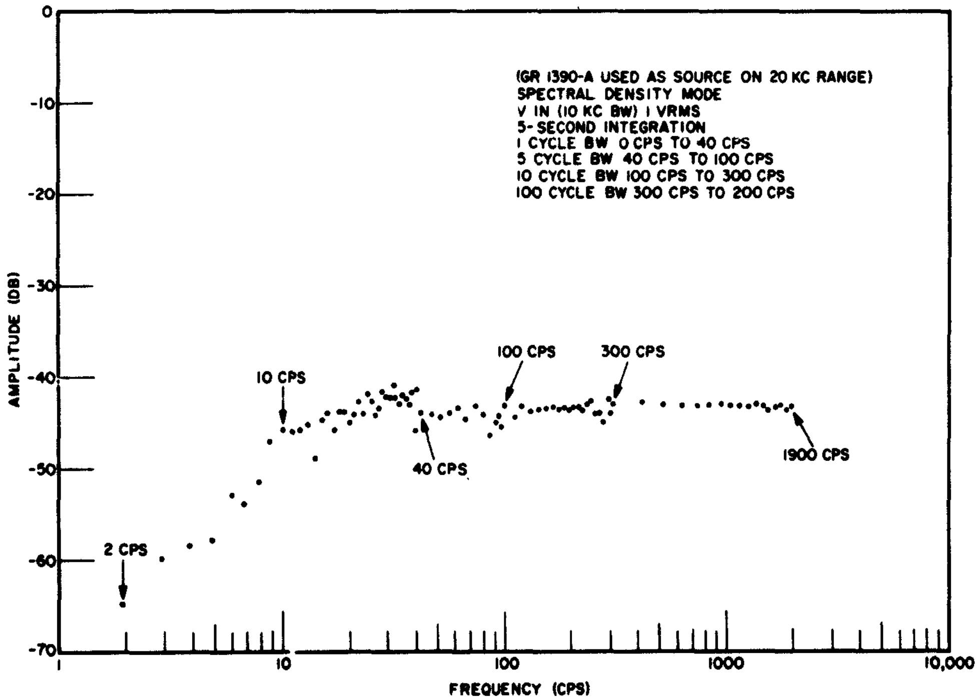

Fig. 4 - Heterodyne analyzer sample plot (white noise input)

and a bandwidth reduction of 10,000 to 1 where the voltage ratio should be $\sqrt{bw_1/bw_2}$ or 100 to 1 (-40 db).

That the plot appears at about -43 db bears out the fact that the input bandwidth is greater than 10 kc. The input low-pass filter is down 3 db at 15 kc.

The plotted curve of Fig. 4 follows the GR Model's published response curve, in that it begins to depart from a straight line at about 20 cps. The results, however, show a slightly faster fall-off below 10 cps for this particular unit.

All four bandwidths are presented in Fig. 4, demonstrating bandwidth normalization by the flat response. Further, one can quickly discern the range of each bandwidth by the spread of the plotted points. Integration time is 5 seconds in every case. In one-cycle bandwidth, point spread may be as great as ±2.5 db due to the statistical uncertainty of the narrow band and relatively short integration time. In the 5-cps mode uncertainty is reduced to about ±1.5 db, in the 10-cps mode to ±1 db, and in the 100-cps bandwidth to about ±0.5 db. The actual figures are biased due to the ±1-db frequency response of the GR noise generator, but it is quickly apparent that wider bandwidth provides greater statistical certainty just as longer integration time would have done.

The techniques which allow plots of this detail are discussed in the following paragraphs.

Stepped Frequency Scan

The most widely used method of automatic spectral analysis is that of the continuously swept frequency scan. A basic requirement of the swept frequency scan analysis system is that the sweep speed must be adjusted so that the analyzer scans less than one filter bandwidth for each loop time. This allows the approximate coverage of an entire data sample for each center frequency of analysis. The normal method of integration for this approach is an approximate integration or averaging network. As a result of these approximations, the analyzed result at a particular point is not truly the sum of the total energy in a frequency band over the total sample, but is a recent-past-history weighted estimate of the energy contained in the band over some portion of the data sample. Bendat[1] has shown that averaging requires considerably more data reduction time than integrating for 2-percent accuracy (referred to the integrated value) even when the method of frequency scan is not considered.

The stepped scan approach permits determination of the precise summation of energy in a filter band because the analysis window (frequency band) remains stationary over the total sample while the energy in that band is integrated (true Miller integrator method). Then during the loop dead time (when the tape splice is passing over the head) the oscillator is stepped one bandwidth and a new integration is begun.

Not only does this method give the best possible energy estimate with maximum frequency resolution, but optimum analysis time is also achieved because the maximum sweep speed of one bandwidth per data sample time is always accomplished.

Dynamic Range

Most available automatic laboratory analyzers offer a dynamic range of 60 db for straight amplitude measurements and a dynamic range of 30 db true-rms or power when referred to the system input. With such a device, the power spectrum in Fig. 4 could not have been obtained in a single analysis since it is 40 db below the wide-band noise input level. This is of more than academic interest because it is common to find components of a typical data sample which at least are this far below the wide-band level. Consequently, the subject narrow-band analyzer was designed for 70 db true-rms at the system input.

The basic problem in achieving a wide range true-rms measurement is finding a wide range method of squaring. The squaring-dynamic-range requirement for random signals is discussed in some detail later in the section on the RMS Computer. Suffice it here to state that the necessary squaring accuracy (approximately ±0.3 db or ±3 percent of reading referred to system input) was achieved over a dynamic range greater than 15 db rms plus peak factor. Thus, a method was required to place the rms of each component within this range. This was accomplished by breaking the range into seven 10-db increments with six threshold sensing circuits. The threshold sensors select the 10-db increment in which the particular component falls and place this component in the acceptable squaring input range. Because the threshold sensor accuracy is better than ±2.5 db, the total rms input dynamic

[1]Bendat, et al., "The Application of Statistics to the Flight Vehicle Vibration Problem," ASD Technical Report 61-123 (1961), section 6.

range that must be handled by the squaring circuit is held to 15 db; the 10-db increment plus 2.5-db worst case threshold-sensor error at either end.

Now that a method of accommodating dynamic range has been determined, it merely remains to find a way of making the increment selection prior to application of the data to the squaring device. This was accomplished by stepping a gain sensing spectral window, one bandwidth ahead of the window providing filtered data for the squaring amplifier. In every respect, except center frequency, this gain sensing filter is identical to the data filter. Referring to the discussion of stepped frequency scan, if the gain sensing operation is performed on the gain sensing filter output for one data sample time and the proper attenuation is placed in series with the data filter when the system is stepped by one bandwidth, then the data filter output has been shifted to the dynamic range of the squaring circuit. To restore the level removed by attenuation, a level proportional to its rms is reapplied at the system output. Gain sensing is performed in parallel with the data filter channel with separate circuitry so that no analysis time is lost.

System Automation

The automatic operations of frequency stepping and gain sensing are discussed in other portions of this paper. To provide full system usability, automatic features were also incorporated to program selection of start and stop limits and bandwidth switch points prior to the actual analysis.

When it is not necessary to scan the entire range of 2 cps to 10 kc, it is highly advantageous to employ start and stop limit programs (front panel selected). When the system receives a start command, from a remote or front panel source, the system fast-scans to the start frequency at a 5 kc/sec rate. At this point normal analysis procedure is employed until the system stops at the preset upper limit.

It is often advantageous to sacrifice unnecessary frequency resolution to reduce analysis time at the higher data frequencies. For this reason 1, 5, 10, and 100-cps bandwidths are available for use almost anywhere in the range. Change of bandwidth is automatically accomplished by comparison of frequency in use with a front panel programmed bandwidth change frequency. As soon as the bandwidth change takes place, the frequency count is increased to the new value increasing system scan rate the same amount.

ONE-THIRD-OCTAVE-BAND ANALYZER TECHNIQUES

This system (Figs. 5 and 6) was provided to measure random data accurately and quickly for relatively wide (octave or 1/3 octave) bandwidths. Tried and proved B & K filters were used for the parallel filter bank. Because of the unique method of dynamic range enlargement already described, it was necessary to provide a second filter output which could be programmed one filter ahead of the normally selected data filter for the purpose of gain sensing.

The selected data and gain sensing outputs are fed to the remaining circuits (Fig. 6) for true-rms computation and plotting on an X-Y Recorder. The output is presented as a horizontal bar graph of true-rms amplitude in db versus a log-frequency scale. Sample plots are included which show a plot of white noise (Fig. 7) and a plot of 12.5-cps sine wave (Fig. 8).

Figure 7 is included to illustrate system operation with random noise input. Once again the GR 1390-A white noise generator was the source. The use of octave filters provides a linear ramp through the filter center frequencies on the log-amplitude scale. The reason for this is the increasing bandwidth (logarithmic) and therefore increasing rms output, as these filters are not normalized for bandwidth. Again the relative certainty is displayed in that the higher bandwidths provide a more linear ramp. The plot is in octave-1/3 octave overlay mode.

Figure 8 displays operation with a sinewave (12.5 cps) input. In this case the plot is in 1/3 octave only mode.

The 1/3-octave-band analyzer has a frequency range of 10 cps to 10 kc. The system may be programmed by front panel control to plot in octave band, in 1/3-octave band, or both (overlaying plots). System operation is accomplished by selection of appropriate integration time (front panel), selection of the desired program, and by pushing the start button.

The RMS Computer section of the 1/3-octave-band analyzer is identical to that employed on the narrow-band system. Further, operation and capabilities of the 1/3-octave-type analyzer are widely understood on a manual basis and one merely needs to consider the automation and the rms measurement to compare the system to a manual parallel filter type system. For these reasons, the bulk of this

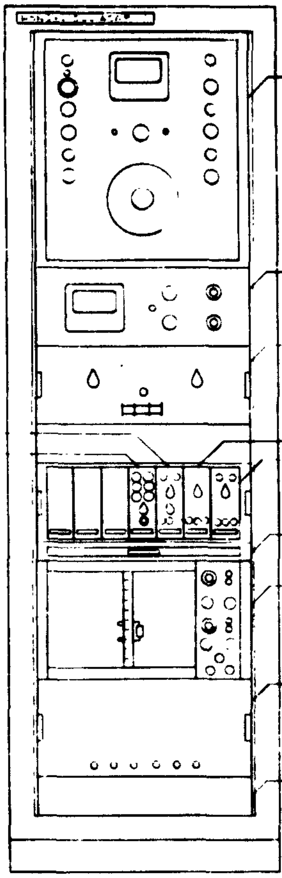

Fig. 5 - Rack layout of 1/3-octave-band analyzer

paper has been devoted to a discussion of the heterodyne analyzer.

NARROW-BAND ANALYZER CIRCUIT DESCRIPTION

The heterodyne system is composed of three basic parts—Frequency Synthesizer and Programmer, Basic Analyzer, and RMS Computer. The Frequency Synthesizer and Programmer combination provides an accurately controlled, stepped, oscillator signal to the Basic Analyzer. The precise frequency is mixed with the input function in the Basic Analyzer and the resultant signal is filtered. This filtered output is fed to the RMS Computer which performs an rms calculation and provides the output to the plotter. Operation of each of these sections is discussed below.

Frequency Synthesizer and Programmer

In order that the frequency stability and accuracy requirements of the heterodyne system could be met, a highly sophisticated frequency synthesizer was designed. This device generates an automatically stepped frequency scan from 60 to 70 kc (0 to 10 kc referred to system input). Frequency accuracy, including stability is better than ±1 cps between 60 and 61 kc, and better than ±2 cps between 61 and 70 kc. The oscillator frequency is generated by a series of suppressed carrier modulators and precise frequency sources (Fig. 9).

One- and 10-cps steps from a voltage controlled oscillator are first added to 1 of 10 100-cps steps from a bank of crystal oscillators. This sum, which may be anywhere between 0 and 1 kc, is then added to 1 of 10 1-kc steps

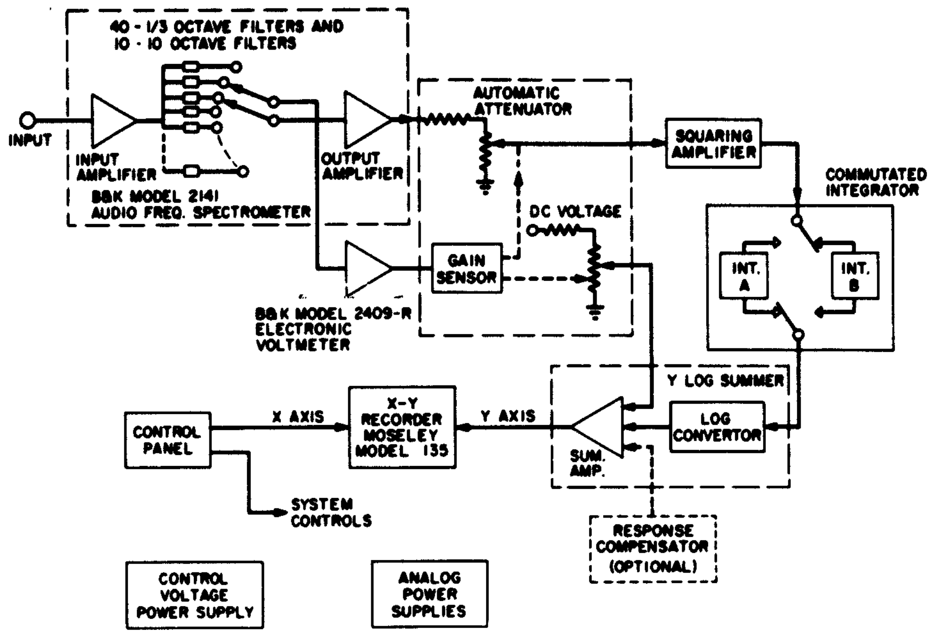

Fig. 6 - Block diagram of 1/3-octave-band analyzer

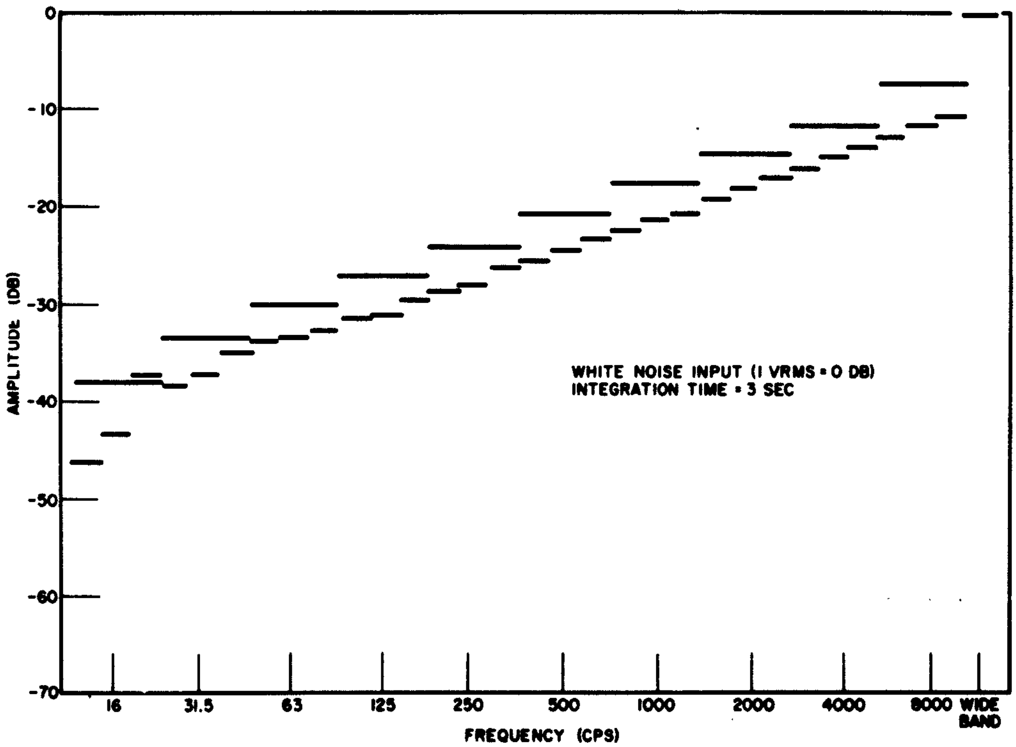

Fig. 7 - One-third-octave analyzer sample plot (octave-1/3-octave overlay)

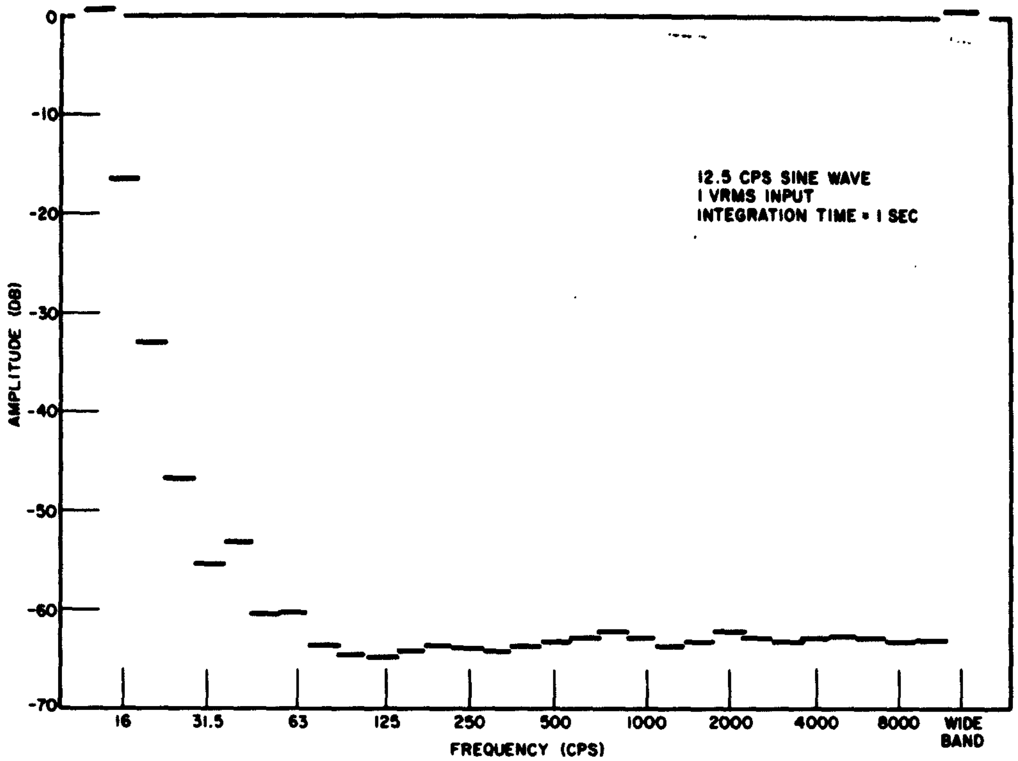

Fig. 8 - One-third-octave analyzer sample plot (1/3-octave only)

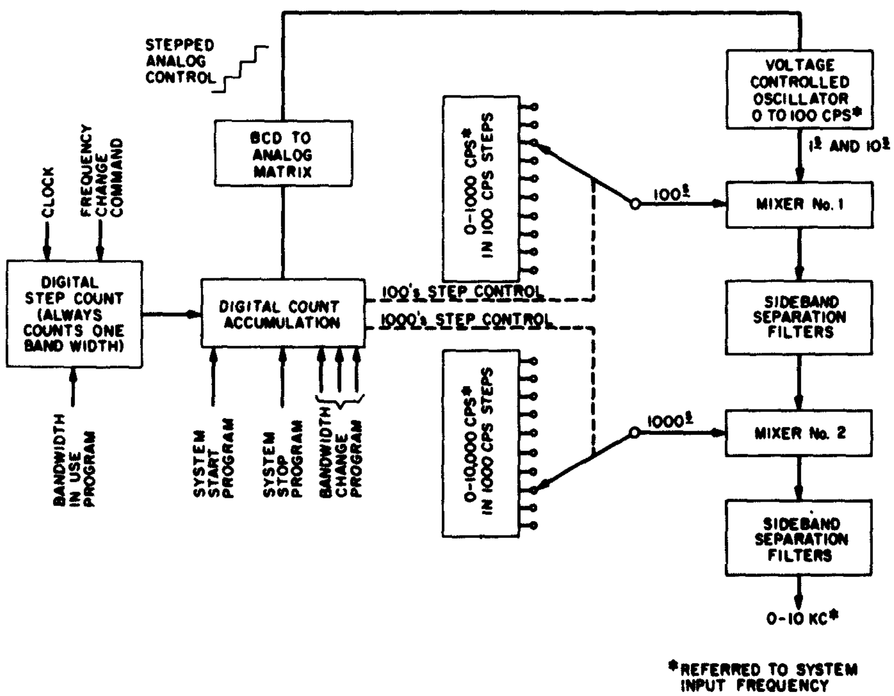

Fig. 9 - Block diagram of Frequency Synthesizer and Programmer

from a second bank of crystal oscillators. In this manner, any desired frequency between 0 and 10 kc (referred to system input) is automatically selected.

Basic Analyzer

The heart of the heterodyne analyzer is that portion called the Basic Analyzer (Fig. 10). This group of modules accepts the frequency scan of 60 to 70 kc and uses it to move a spectral window through the 2-cps to 10-kc data frequency range. This is done by first selecting the frequency band to be analyzed and translating it to an intermediate frequency and then filtering the i-f.

A dual modulator scheme allows minimum spurious interference while also providing optimum center frequency for crystal filter application. The first i-f of 60 kc is used to eliminate the possibility of data signals and their harmonics feeding directly through the first modulator at the i-f frequency. The second i-f of 10-kc is selected for crystal filtering.

The filters employed in the Basic Analyzer are of an extremely stable crystal type. A sample filter curve (1-cps bandwidth) is given in Fig. 11. All filters have 70-db ultimate rejection in the stop band. Other than ultimate rejection and a requirement for ±0.5-db passband ripple, the primary design parameter for the filter is that the effective noise bandwidth be within 10 percent of the bandwidth as determined by the half power points. The effective noise bandwidth of a filter represents the width of a rectangle whose top passes through the square of the peak of the actual filter curve and whose area is equal to the area under the squared filter curve. For example, the 3-db bandwidth of the 1-cycle filter in Fig. 11 is 1.01 cps while its effective noise bandwidth is 0.99 cps. Effective noise bandwidth may be written as:

$$BW_n = \frac{1}{e_{max}^2} \int_0^\infty e^2(f)\, df ,$$

where e = output voltage to a fixed load.

The usual method of filter normalization is to provide equal outputs from all filters for equal sine-wave inputs. This mode (spectrum) is provided on the heterodyne system for the case where it is desired to measure data consisting of pure sinusoids. However, a second mode of operation (spectral density) is provided such that with white noise into all filters, the output levels will be equal. In this mode the effective noise bandwidth is normalized to a 1-cps bandwidth so that the filter output is a density (energy/cps).

RMS Computer

The root-mean-square amplitude of a data sample is the measure of the energy contained in that sample:

$$RMS = \sqrt{\int_0^t e^2\, dt} ,$$

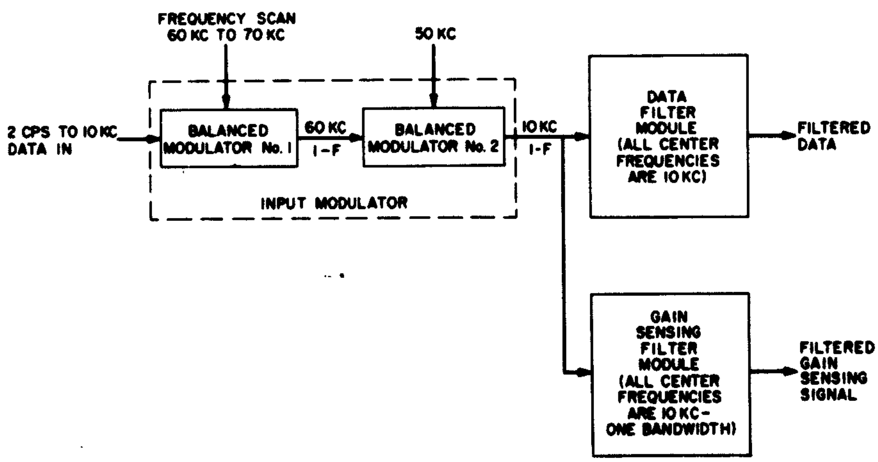

Fig. 10 - Block diagram of Basic Analyzer

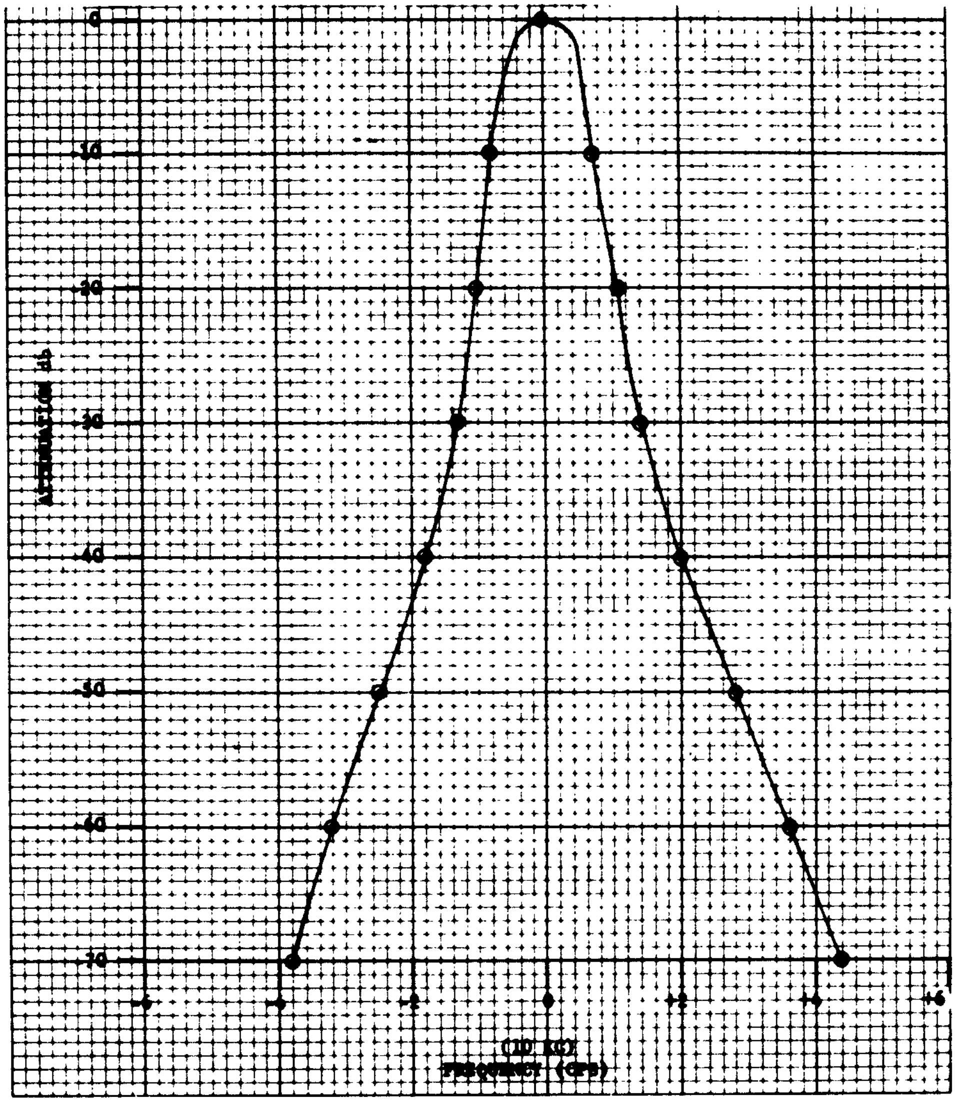

Fig. 11 - Typical filter response curve (bandwidth = 1 cps)

where

t = the sample length, and

e = the instantaneous data amplitude.

For the case of sine wave data

$$\int_0^t edt ,$$

or the average of the sample, conveys the same amount of information as the rms because there is a constant relation between the two. However, as the data waveform becomes less predictable, it is necessary to determine rms in a more rigorous manner. The requirements of accuracy placed on these analyzers made it necessary to perform the actual squaring and integration functions. Further, the circuitry was required to operate compatibly with the statistical properties of random data of Gaussian distribution.

The RMS Computer is composed of an automatic attenuator, a squarer, an integrator, and a log summer (Fig. 12). Its output is

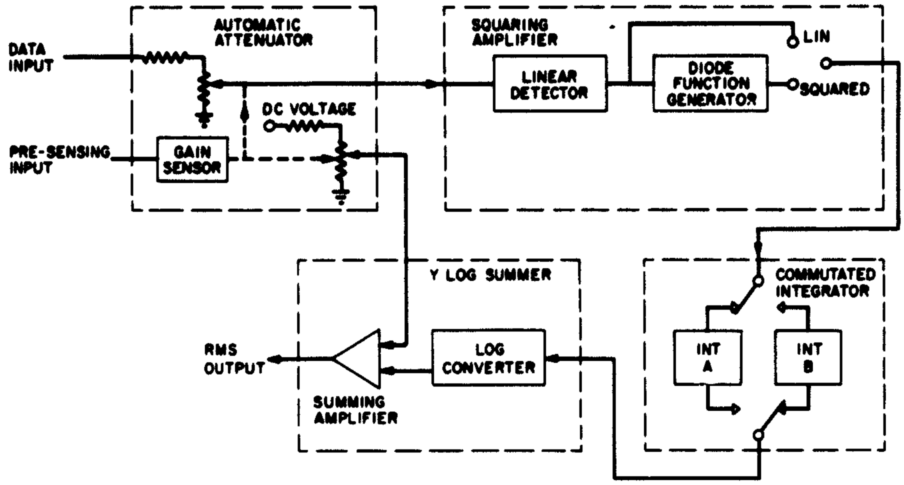

Fig. 12 - Block diagram of RMS Computer

$$1/2 \log \int^{t} e^{2}dt \; .$$

The plot then may be considered the log of the rms.

The critical element in the design of the RMS Computer was that of the squaring amplifier where dynamic range requirements are quite severe. As mentioned previously, the input rms dynamic range of 15 db must be squared with an accuracy of ±0.3 db rms. The instantaneous dynamic range of the input extends 15 db on either side of the 15-db rms range so that instantaneous peaks up to and including 99.9 percent of the peaks occurring in a Gaussian or Rayleigh probability distribution are considered in the energy estimate. Thus, the instantaneous input dynamic range was 45 db requiring an instantaneous squared output dynamic range of 90 db. Squaring accuracy is discussed in the appendix.

The squaring method used was the straightforward diode function generator. The input to the function generator is electronically rectified so that it may be operated in one polarity only. The rectified signal is amplified to the point where the required dynamic range may be realized.

In normal automatic operation, it is important that no averaging take place prior to the integrator so that integration time is well defined. In manual mode, however, true integration is impractical so averaging is provided for manual analysis and system calibration.

The method of integration employed is that of a high-gain chopper-stabilized amplifier witl a precise capacitance feedback component. Two integrators are commutated in such a way that while one integrator is charging (with open output), the other integrator is storing previously read data (with open input) for plotting. The two circuits alternate so that no analysis time is wasted. As each integrator cycles from the readout position to the charging position, it is clamped to zero so that the new integration may start with negligible residual error.

In order that analyzer output will be independent of sample length, normalization of the integration time is performed. The integrator output is in volt-seconds rather than volts. Therefore, a 1-volt input for 1 second provides the same output as 0.1-volt input for 10 seconds. This approach makes it possible to compare directly two analyses of different sample length without introducing proportionality constants.

The log-converter portion of the log summer is a unique analog device with a thermionic log diode in the feedback loop of an operational amplifier. The output of the log converter provides one of the inputs to the summer. Another input to the summer is a gain code from the automatic attenuator. This input restores the db output by precisely the amount that was removed prior to the squaring input. The summer output to the X-Y plotter is 0 to 10 volts dc, where 10 volts corresponds to 0 db, or full scale, and 0 volt corresponds to -70 db. Another log summer, operating on the analog voltage (volts/cps) from the Frequency Synthesizer, provides a log-frequency axis.

SUMMARY

Design and development of two spectral analysis systems was recently completed for the Sonic Test Facility at Wright Air Development Division. These systems, a one-third-octave, parallel-filter type and a narrow-band heterodyne type, provide extended measurement capability to the vibration test and design engineer.

In the case of the heterodyne analyzer, several valuable features are incorporated. The use of stepped frequency scan and a true Miller Integrator provides optimum accuracy, resolution, and data reduction time. A unique method of dynamic range extension provides the ability to measure true-rms over a 70-db dynamic range referred to system input. The use of spectral density filter weighting, and integration time normalization, allows an analysis result to be presented in the standard form of energy/cps.

The automatic features of the systems permit a minimum amount of operator time and provide flexibility not previously obtainable in commercial equipment.

Finally, the system designs include allowance for the proper handling of the instantaneous amplitude characteristics of random signals, as well as eliminating unnecessary approximations of the mathematical principles involved in spectral analysis.

Appendix

DYNAMIC RANGE VERSUS INSTANTANEOUS SQUARING ACCURACY IN THE CASE OF OPERATION ON A NARROW-BAND RANDOM SIGNAL

If it is assumed that the analysis system input is a wide-band random noise with Gaussian distribution, it is represented by:

$$P(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-x^2/2\sigma^2}, \qquad (1)$$

where

0 = mean

σ = standard deviation.

This wide-band noise is then modulated and filtered such that the signal of interest is the envelope of the carrier, and the modulating frequency bandwidth is small with respect to the carrier frequency. The distribution of the envelope may be shown to be Rayleigh,[2]

$$P(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, \qquad R \geq 0$$

$$= 0, \qquad R < 0,$$

where $1/2\ \sigma^2$ = the mean square value of x.

The noise modulated carrier is now passed through a nonlinear device such that the output consists of a number of frequency bands which are located at multiples of the input band.[3] If the output circuit selects all of one band, the distribution may be shown to be exponential,

$$P(x) = \frac{1}{k\sigma^2} e^{-x^2/2\sigma^2} \qquad (2)$$

for the particular case of squaring circuit with a low-pass filter output (integrator).

To arrive at Eq. (2), assume the narrow-band noise is of the form

$$V_1 = x \cos (Wmt + \phi),$$

where x and ϕ are functions of time, but vary at a slow rate compared with Wm. Utilizing the zero frequency multiple of the output:

$$Ao(x) = \frac{Kx^2}{2}, \qquad (3)$$

where K is the gain of the squaring circuit.

Rice[4] has shown that the probability distribution of the output envelope $An(x)$ is of the form:

[2]Davenport and Root, Random Signals and Noise (McGraw-Hill Book Company, Inc., New York, 1958), Chap. 8.

[3]Bell, Electrical Noise (Van Nostrand, 1960), Chap. 2.

[4]Rice, "Mathematical Analysis of Random Noise," Bell System Technical Journal 23, 282, and 24 (1946).

$$P[An(x)] = \frac{P(x)}{\frac{d[An(x)]}{dx}}, \qquad (4)$$

where $P(x)$ = the probability distribution of the input envelope.

By assuming

$$An(x) = Ao(x) = \frac{K}{2}x^2,$$

$$P[Ao(x)] = \frac{P(x)}{\frac{d[Ao(x)]}{dx}}$$

$$= \frac{\frac{x}{\sigma^2} e^{-x^2/2\sigma^2}}{\frac{d\left[\frac{k}{2}x^2\right]}{dx}}$$

$$= \frac{1}{k\sigma^2} e^{-x^2/2\sigma^2}, \text{ or Eq. (2).}$$

Now from the theoretically determined distribution of the squared output, we may evaluate instantaneous amplitude accuracy requirements in the squaring operation on a graphical basis. This was done by dividing the dynamic range into subsections and determining the area contributions of each interval for the ideal case. From these values, accuracy figures were assigned to each interval in the most advantageous way for the electronic squaring device employed, with the restriction that the sum of the errors (areas) must be less than 3.5 percent of the total when referred to the analysis system input.

The area contribution of each 5-db interval of instantaneous variation is shown in Table 1. These values assume a stationary rms signal whose rms value referred to squaring input is 0 db.

TABLE 1
Area Contribution to Squaring Inaccuracy

Input Level (db)	Interval Sum (in % of total sum)
-40 db	
	0.03%
-35	
	0.05
-30	
	0.06
-25	
	0.3
-20	
	1.0
-15	
	2.7
-10	
	6.9
-5	
	15.5
0	
	26.5
+5	
	29.5
+10	
	15.0
+15	
	2.4
+20	
	0.5
+25	
	--
+30	
	--
+35	
	--
+40	
	99.99

* * *

ACOUSTIC AND VIBRATION STANDARD ENVIRONMENTAL DATA ACQUISITION PROCEDURES*

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Denver, Colorado

This paper discusses requirements necessary for obtaining good quality acoustic and vibration data. Included are frequency response, dynamic range, linearity, inter-channel phase shift, response to environments, calibration, and accuracy. Techniques are not discussed, only those factors which can be controlled by the person requesting data.

INTRODUCTION

Acoustic and vibration test specifications are extensively derived from measured data. Early in the development of a missile or space vehicle system, preliminary test specifications are generally a product of analytical and laboratory studies combined with data acquired from other programs. Such data must be extrapolated to the existing situation considering both rocket engine noise parameters and structural transfer functions. Measured data from scale models may also be included. Final test specifications are then derived from measured data which may be acquired from static firings or test flights or both. Thus, the quality of the test specification and, subsequently, the reliability of the flight system become, in part, functions of data quality. In the past, it has been possible to obtain large samples of data through extensive research and development programs. The future employment of multi-million-pound boosters will probably preclude the use of extensive static firing programs, however, and statistical evaluation of large amounts of data will not be possible. Thus, test specifications will become dependent, in part, upon a few accurately measured data points. The importance of accurate and reliable data acquisition is therefore obvious.

It would appear that the measurement of acoustical and vibrational data would be a relatively simple process. Data acquisition is often complicated, however, because remote systems are required. Also, the very nature of the random phenomenon and the requirement for placing measurement system components in the presence of adverse environments, places further burdens upon the measurement system.[1] Perhaps the most serious burden arises from the complexity of integrating the data acquisition system into the total missile and space vehicle program. In most military and civilian organizations many different departments share a function between the initial request and final presentation, providing many opportunities for error. The data user is then dependent upon measurement system specifications and procedures to insure data accuracy.

The following discussion is concerned with those requirements that are necessary for good acoustical and vibrational data quality. These requirements include frequency response, dynamic range, linearity, inter-channel phase shift, response to environments, calibration, and accuracy. The discussion does not include techniques, but is limited to those factors which can be controlled by the data requester.

FREQUENCY RESPONSE

It is most desirable that the frequency response within the measurement spectrum be as flat as possible. Often, however, this is not economically feasible because of equipment

*This paper was not presented at the Symposium.

[1] D. N. Keast, "Measurement of Rocket Engine Noise," Noise Control, 7:25-36 (March 1961).

limitations. For this case, the frequency response must be well defined and within reasonable limits. An example of measured frequency response for four different types of landline channels is shown in Fig. 1. Curve A shows a highly undesirable situation which was corrected as shown in Curve B. Obviously common source data acquired by these four channels would exhibit different results, if frequency response were not considered. End-to-end measurements of the overall system response should be presented with the data. It is most desirable that stimulation be supplied through the transducer. If such a requirement places a burden on the measurement program, the transducer and the system may undergo frequency response verification, separately. Frequency calibration of microphones to be used near, or flush with, a rigid surface such as the vehicle skin should include a frequency calibration in their mounting configuration. An example of the effect of mounting configuration upon frequency response is shown in Fig. 2.

DYNAMIC RANGE

Dynamic range can be described as a measure of the distance between the electrical noise floor and the maximum recording level. Each data channel, therefore, must possess a sufficient dynamic range to insure that the electrical noise floor has a minimum effect on the measured data. For the purpose of specifying system requirements, the following guide can be used:

> A noise floor 10 db (voltage ratio) below the data would result in a total level 0.4 db in error. This 10-db spread must therefore be considered a minimum. Since acoustic and vibration spectrum levels may range through 30 db in amplitude, this spread must also be specified. In addition, a 10-db spread must be provided between the total band rms level and the maximum recording level to prevent clipping. Thus, a dynamic range of 50 db in 1/3-octave bands or 45 db in octave bands is recommended.

Dynamic range must be verified by measurement. Noise floor data acquired during periods before a firing will not always be indicative of the electrical noise on a channel during a firing. Often the noise floor level is reduced as the vehicle gantry is moved away prior to the firing. The noise floor levels, however, are often increased after ignition because of the presence of higher excitation levels and because vehicle electrical systems are operating at maximum capacity. One satisfactory procedure for analyzing electrical noise problem areas is to record electrical noise only, on a data channel

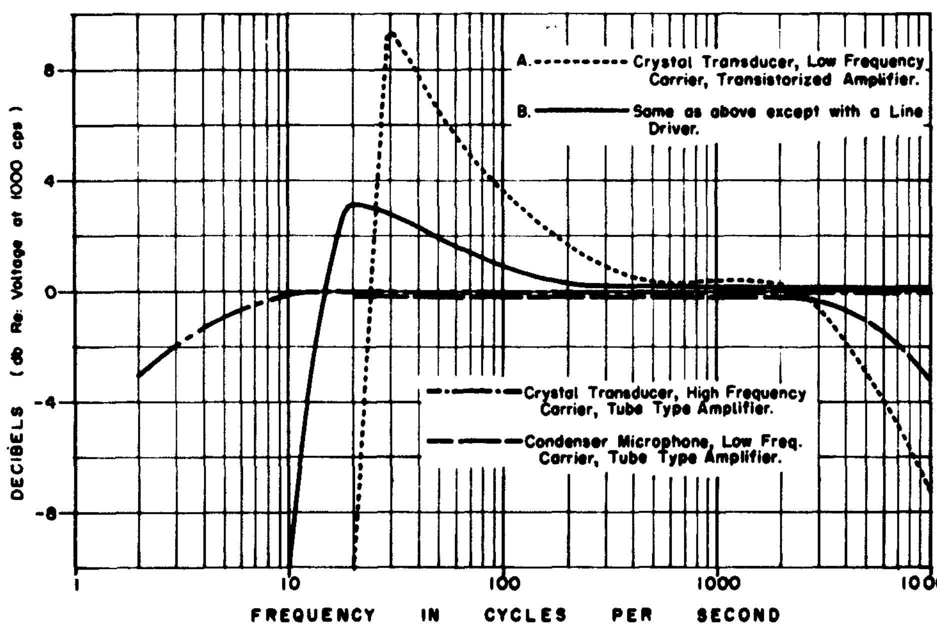

Fig. 1 - Comparative frequency response of four different acoustic data acquisition channels

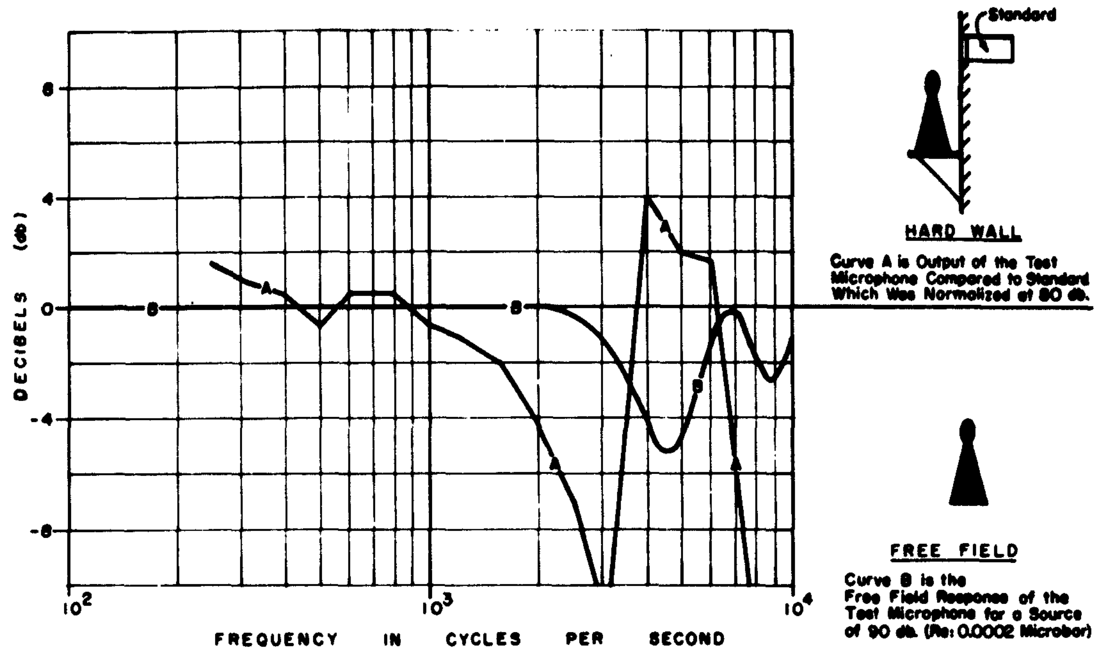

Fig. 2 - Frequency response curves for a high-intensity crystal microphone

before and during a firing, with shorted leads in place of the transducer. By observing levels immediately preceding and following ignition, one can ascertain the change in the noise floor that has been induced by the vehicle electrical systems during engine operation. Estimates of electrical noise levels on data channels can then be made by observation of the noise floor just prior to ignition. An example of the electrical noise obtained in this manner is shown in Fig. 3. The noise levels of Fig. 3 measurement A are sufficiently lower than the data, while a definite effect is observed in Fig. 3 measurement B. Although the data are still useful, corrective action would be required before additional data could be recorded on these channels.

LINEARITY

Each data channel must exhibit amplitude linearity throughout its dynamic range. The high quality of available components generally provides easy compliance with this requirement. Amplitude linearity and linearity tests must be specified, however, to assure correct mating of components within the system and correct gain settings. Linearity measurements should be made through the entire system, including the transducer, with the system gain set at the measurement level.

INTER-CHANNEL PHASE SHIFT

The increased demand for cross-correlation and cross-spectrum analyses has made it necessary to impose a requirement on inter-channel phase shift. Phase differences generally result from a misalignment of tape recorder heads, which produces a time delay between tracks. Thus, any phase shift between tracks will produce a time error in the correlation function.[2]

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[2]The cross-correlation and cross spectrum functions may be expressed as follows:

Cross-Correlation:[3]

$$R_{xy}(t) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(T)\, y(T-t)\, dT\,;$$

Cross-Spectrum:[4]

$$S_{xy}(\omega) = \int_{-\infty}^{\infty} R_{xy}(t)\, e^{-i\omega T}\, dt\,.$$

Thus, any time delay in the recording system will result in an error in t in these equations.

[3]K. Eldred, et al., "Structural Vibration in Space Vehicles," WADD TR 61-62 (December 1961).

[4]J. Bendat, Principles and Applications of Random Noise Theory, (John Wiley & Sons, Inc., 1958).

The amount of phase shift which can be tolerated is dependent upon the analysis system used. It is important, therefore, that the allowable inter-channel phase shift be specified and test data presented for verification.

RESPONSE TO ENVIRONMENTS

Because many of the measurement system components must be located on the vehicle, be careful that changes in system gain and unwanted signals are not introduced by environmental effects. Those environments of greatest concern are temperature, ignition transients, humidity, and acoustic and vibration excitation. Large changes in temperature can induce changes in sensitivity of accelerometers and microphones, particularly those employing crystal sensing elements. Amplifier gain can also be affected. Thus, components near cryogenic fuel tanks or in the thermal radiation path of the jet exhaust must be tested to assure minimum degradation of performance. Ignition transients have the effect of inducing transient overloads into the system. The ability to recover rapidly and without noticeable drift must be specified and proven by laboratory test. Because of the wide use of deflector cooling water, large amounts of steam are normally generated. All system components may therefore be required to function in an atmosphere that contains large amounts of water vapor. Perhaps the most serious environmental problem results from acoustical and vibratory excitation of components. Amplifiers, preamplifiers, cables, and so on, can generate spurious signals in the presence of both high-amplitude acoustic and vibration environments. Microphones and accelerometers are sensitive to vibrational and acoustical environments, in that order. An example of microphone sensitivity to vibration[5] is shown in Fig. 4A. The effect of this sensitivity upon measured data is shown in Fig. 4B for microphone A.

The excitation levels have not caused degradation of the data. It should be obvious, however, that increases in the excitation environment or frequency changes in excitation sensitivity, could result in masking of data. Prevention of environmental problems is relatively simple and generally requires proper

[5]R. Peverley, "Vibration Problems with Microphones for Rocket Engine Noise Measurements," Sound 1-6, (December 1962) (to be published).

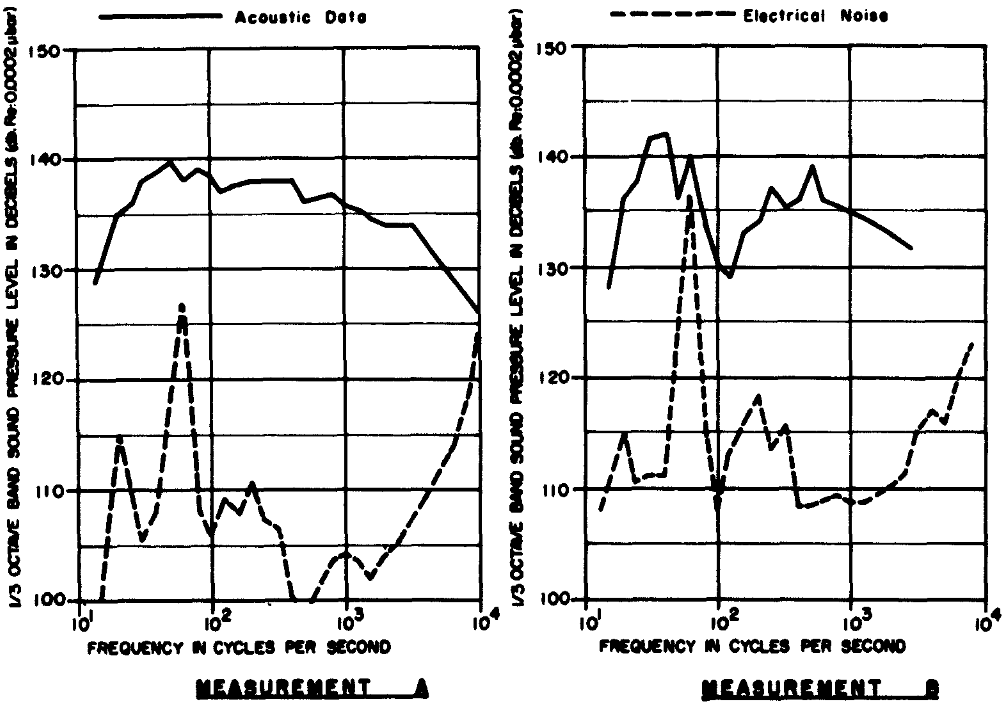
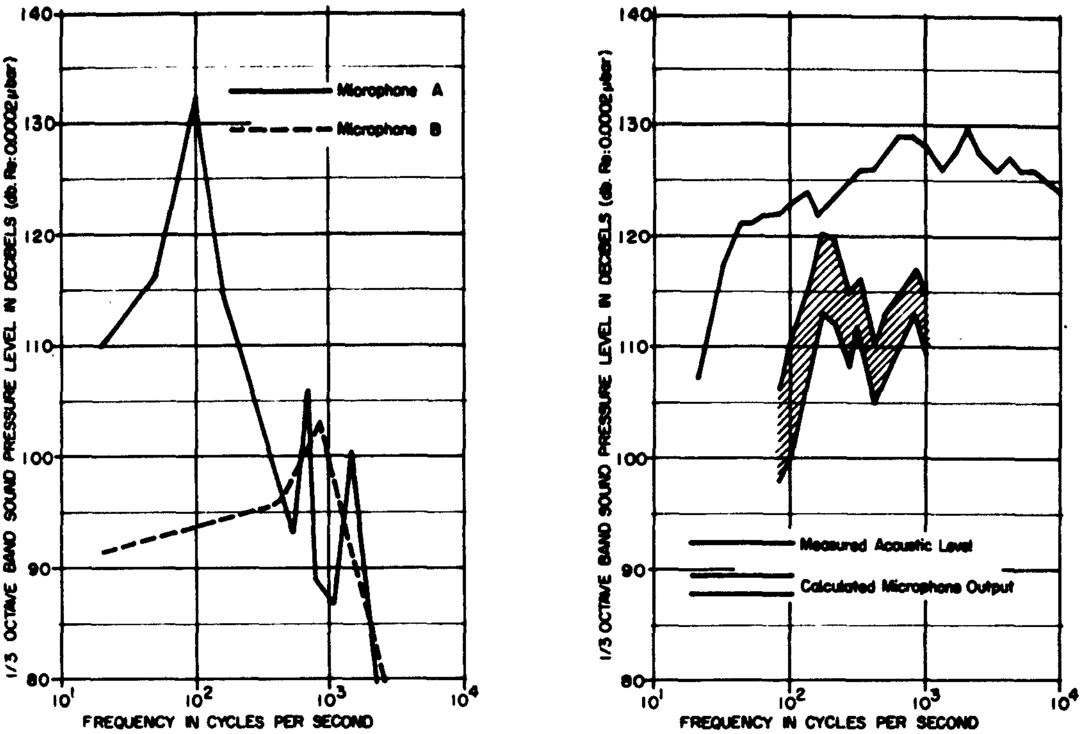


Fig. 3 - Measured acoustic spectra and electrical noise floor



(A) Microphone output to 1-g sinusoidal vibration

(B) Microphone vibratory output due to noise and vibration

Figure 4

consideration early in the program to assure selection of good equipment and the use of optimum locations and proper bracketry. Requirements should be explicit and specify allowable response as a function of environmental level.

CALIBRATION

System gain calibration should occur preceding and immediately following the firing or flight, or both, if possible. The requirements should establish the maximum allowable time before recalibration, in the event delays in a firing schedule occur. In addition, accurate logs and data sheets should be required. These items help to insure compliance with procedures, and will also be valuable in the evaluation of what appears to be bad data.

ACCURACY

It is a general practice for a data user to specify accuracy requirements in his initial request. This accuracy establishes the required quality of the equipment, the standard of calibration, and so on. In the case of missile and space vehicle acoustic and vibration measurements, the best available equipment must always be used, and the greatest care exercised in calibrating. Requiring extreme accuracies, however, will not yield large benefits because of the severe measurement boundary conditions. An accuracy of ±2 db has been found to be practical.

EXAMPLE

An example of missile landline acoustic measurement requirements is offered in Table 1:

TABLE 1
Example of Requirements for Missile Landline Acoustic Measurements

Item	Requirements
System Parameters:	
Amplitude Range	105 to 165 db
Frequency Responses	+1, -3 db from 2 to 20 cps, ±1 db from 20 to 10,000 cps
Dynamic Range	50 db in 1/3-octave bands
Linearity	±5%
Interchannel Phase Shift	Maximum 10 degrees at any frequency
Accuracy	±2 db
Environmental:	
Temperature	10-percent drift from -100° to +120°F
Ignition Pulse	Less than 10-percent drift due to transient overload
Humidity	Operation for 30 minutes in a relative humidity of 95 percent
Acoustic	Less than 10 mv at a random excitation level of 165 db
Vibration	Less than 10 mv at a random excitation level of 8 g^2/cps
Verification:	All system parameters will be verified by tests of final system, and data sheets will be forwarded to requester.
Calibration:	
Time	Precalibration - system gain will be re-established every 24 hours. Post calibration - system gain will be re-established as soon after the firing as safety permits.
Logs	Copies of calibration logs will be forwarded to the requester.

CONCLUSION

Those persons who are assigned the responsibility for the specification accuracy are, unfortunately, seldom assigned a parallel responsibility for the measurement of these data. Generally, the data user may exercise control of the data acquisition system only through his initial requirements. Thus, it is imperative that these requirements and specifications be written in such a manner as to assure good data quality, and to provide enough information to evaluate the data intelligently. The preceding requirements have been found to satisfy this objective. None of the factors presented herein are technically new, but are, instead, laboratory methods that have been adapted for field use.

* * *

Section 4
ENVIRONMENTAL PROGRAMS

DATA EXCHANGE PROGRAMS CONDUCTED BY U.S. NAVAL ORDNANCE LABORATORY CORONA, CALIFORNIA*

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The purpose of this paper is to describe four separate and distinct Reliability Data Exchange Programs being conducted by the Naval Ordnance Laboratory (NOL), Corona, California, under the sponsorship of the Bureau of Naval Weapons (BUWEPS) and the Special Projects Office (SPO).[1] These are: The Fleet Ballistic Missile Weapon System (FBMWS) (POLARIS) Component Reliability History Survey (CRHS); the Interservice Data Exchange Program (IDEP); the Bureau of Naval Weapons Guided Missile Data Exchange Program (GMDEP); and the Failure Rate Data Exchange Program (FARADA).

OBJECTIVE

One of the major problems in achieving the reliability of a missile subsystem, or of a complete weapon system, expressed as a goal or an objective, or stipulated quantitatively, lies in the selection of sufficiently reliable parts and components. The design engineer, working on a new missile program, is greatly in need of part[2] information that is as complete and up-to-date as possible. Such information may be produced by a test program, running concurrently with the development of a new missile. However, the selection of parts and components is a function of time, money, quantity, and priority of effort.

The problem would not be paramount if the designer had ample time to base his selection on sufficient quantities of test data derived from statistically significant quantities of parts and components. While the parts are in themselves costly, the testing is many times more costly. Often, because of work priorities, a testing laboratory does not have sufficient time to perform a complete or exhaustive test, and, more often, the limited quantities of parts furnished for testing are insufficient to provide decisive data.

Therefore, the selection of a part from a reliability standpoint, is often a compromise because of the limitation of known and available reliability data that the designer has at his disposal, despite the representations made by the manufacturer of parts and components attesting to the reliability of his products.

Often, the designer may use information that he personally acquired in other programs,

*This paper was not presented at the Symposium.
[1] Special Projects Office (SPO) established within BUORD (now BUWEPS) for the development of the Fleet Ballistic Missile (POLARIS) Weapon System (FBMWS).
[2] For the purposes of this paper, the word "part or parts" means "part of component, or both."

but it is almost impossible for him to keep abreast of the latest reliability information on either old or new parts.

DISCUSSION

FBMWS (POLARIS) Component Reliability History Survey

In the early part of 1958, NOL, Corona was assigned a component reliability task by the SPO of the Bureau of Ordnance (BUORD), Department of the Navy. The purpose of this task was to provide assistance to the prime contractor's designers in the selection of reliable parts for the POLARIS missile. The actual implementation was to be accomplished by the acquisition of applicable part reliability information that had been, or was being, generated in other missile programs, and the condensation of such data into a quick and useful form for presentation and utilization by design engineers.

It was recognized that a large number of corporations, involved in research and development programs as well as in the production phase of various missile programs, had acquired a considerable amount of data pertaining to the reliability of various parts. Further, it was known that this information rarely went further and was ultimately filed after use by the activity which generated or collected it.

It was therefore believed that both time and money could be saved in the POLARIS program if all available information could be obtained, analyzed, evaluated, summarized, and made available to the design engineer to assist him in the selection of reliable parts and components. In particular, reliability information was desired on parts and components categorized as electronic, electrical, mechanical, electromechanical, hydraulic, electrohydraulic, and pyrotechnic.

In order to obtain as complete and valid reliability information as possible on the parts tentatively selected, it was decided to solicit information from many varied sources. These included missile manufacturers under contract to the U.S. Army, Navy, and Air Force; miscellaneous testing laboratories and similar facilities both private and government; field testing and firing facilities; and vendor or part manufacturers.

The type of information sought included not only engineering and evaluation tests performed on the part or components, but also additional information that could be obtained from the incoming receiving inspection level, from various stages of the manufacturing process, and from final acceptance tests. Figure 1 illustrates the flow of data from originating source to NOL, Corona, and to the prime and major subcontractors in the FBMWS Program.

In acquiring data for this survey, every effort was made to include only the reliability information on similar parts experiencing similar conditions of shock, vibration, acceleration, temperature, pressure, and humidity as are specified for the FBMWS.

The total amount of information desired includes:

Total quantity of a particular part for which data are available,

Number of failures of this part over a specified period of time,

Time to failure (operating time),

Mode of failure (electrical, mechanical, thermal), and

Stress on the part at the time of failure (the load conditions at the time of failure, i.e., voltage, pressure, temperature, vibration, and so on).

It is to be emphasized that the reliability history survey program being conducted by NOL, Corona, is not intended to replace any evaluation testing normally performed by the prime contractor; however, it is intended to augment his testing information and to identify peculiar or critical characteristics to aid in test planning and to minimize actual testing of parts and components for certification or qualification. However, the material in the reports may be used to serve as the primary basis for selecting the most reliable parts for the intended application.

Further, reports may also be used to substantiate the initial selection of parts which are often chosen with limited reliability background data. They may be used to indicate or predict possible causes of failure that might be experienced, either during the research and development phase, the production phase, or the in-use or service phase. In addition, they provide a reference for steps that could be taken to minimize failures by employing the principles of derating, substitution circuits, or redundance.

Without this information, reliability all too frequently becomes a post-mortem process

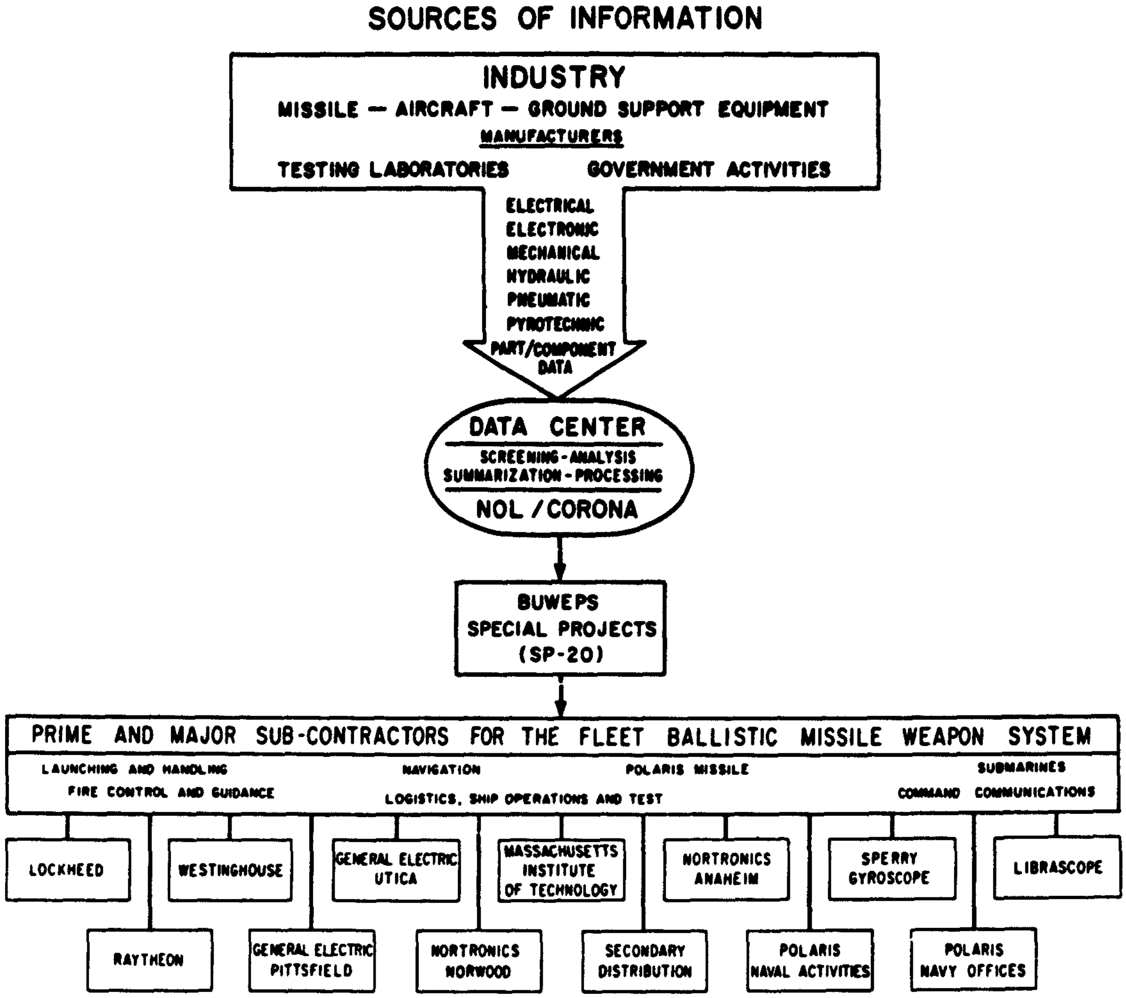

Fig. 1 - FBMWS Program; sources of information and flow of data to and from NOL, Corona

whereby a great amount of effort and money are required to correct unsatisfactory weapon performance in tactical use. However, it is desired to stress the fact that the reports prepared by NOL, Corona, do not include any recommendation or endorsement pertaining to the use of any particular part or component by brand or trademark name.

In the beginning of the survey, NOL, Corona, prepared formal reliability reports for the more critical parts. These were intended to present a brief history of the development of the part being investigated, the causes of unreliability as discovered from the experience of other users, preventive measures recommended to minimize or eliminate deficiencies, and a statement covering the intended use of the part within the FBMWS and including the normal stresses that might be experienced by these parts.

At present, the majority of the reliability reports are submitted in the card form illustrated in Fig. 2. This report form was developed to replace the informal or letter type of reliability report which also was employed at the beginning of the survey. Information is presented in two ways. In the upper part of the form, data is placed in such a manner as to identify quickly the part or component that is being investigated according to the following items:

Name,

Digital generic code,

COMPONENT	NUMBER
Resolvers, Electrical; Data Transmission Marine	[illegible].00.00

COMPONENT RELIABILITY HISTORY SURVEY SUMMARY REPORT

11ND-NOLC 318 (9-61) SC [illegible]

ADDITIONAL MAJOR CHARACTERISTICS

Metal and Glass Plate Inductosyns

1. VENDOR	2. VENDOR TYPE	3. TEST ACTIVITY	4. PROGRAM
"A"		"A-A"	"1"
5. TEST ACTIVITY REPORT NO.	6. TEST TYPE	7. QUANTITY TESTED	8. TEST DATE
"a-a"	Environmental	2 Metal, 2 Glass	Oct 1961

9. OVERALL PERFORMANCE FOR ENVIRONMENTS TESTED PER TEST ACTIVITY

☐ SATISFACTORY ☐ UNSATISFACTORY ☐ UNKNOWN

10. SPECIFICATIONS OR PROCEDURES NECESSARY FOR PROPER INTERPRETATION OF REPORT

"2", "3", and "4"

11. REMARKS

According to Test Activity "A-A' it is recommended that glass plate inductosyns in the field and those presently procured for Program "1" be retrofitted to metal plates, and that metal plates be specified in future orders for inductosyns.

FBMWS REPORT NUMBER	COMPONENT FILE NUMBER	REPORT DATE
992	GIA	

ORDER OF TEST ENVIRONMENTS	MODE PERFORMANCE SAT	MODE PERFORMANCE UNSAT	TEST LEVELS, DURATION, DETAILS, SPECIFICATIONS, FAILURE RATES, ETC.
TEMP & HUMIDITY			(2 metal tested) MIL-F-18870B (WEP) Para 4.11.15.2.1 resistance measurements of stator.
THERMAL SHOCK		X	(2 glass tested) 24 hrs @ -54°C; increase to +65°C & hold for 4 hrs. Check operation at both temp.
MECHANICAL SHOCK			(1 metal, 1 glass) 3 shocks normal to axis at 20, 40, & 60 g, check operation after each series.
VIBRATION	X		(1 metal, 1 glass) 5 - 33 cps to check resonance - vibrated in 3 axis for total of 2 hrs.

COMPONENT: Resolvers, Electrical

INDEX NUMBER: 675.60.00.00

REPORT NO.: 992

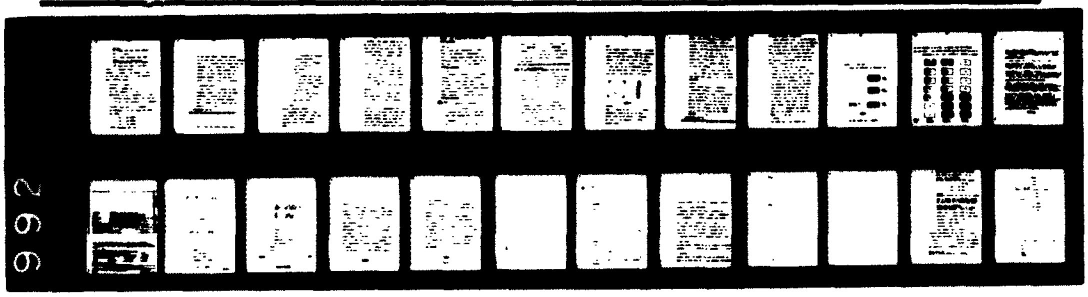

Fig. 2 - FBMWS Program; card form for reliability reports

Part number,

Manufacturer model and type,

Test activity that performed the tests,

The quantity tested,

For what programs,

Type of tests performed,

Results of the tests performed,

Any unusual or mitigating circumstances (such as being tested above or beyond the vendor's specifications), and

Special instructions or specifications required in order to fully interpret the method of testing and results.

At the lower edge of the card form, a microfilm strip is added. This contains the complete test report, or significant portions of the report, reduced at the approximate ratio of 22-to-1. The design engineer, if interested in more detailed data than briefly stated in the upper and center portion, may then view the microfilmed data for the complete test results. By utilizing an appropriate viewer, the data on the microfilm is presented for easy reading on a large ground-glass screen. A special feature of the viewer employed at NOL, Corona, is an integral printer which enables the operator, or design engineer, to acquire immediately a letter size print of the data on the screen. This print then provides a ready reference for the design engineer to use at his convenience.

NOL, Corona, has employed this form for approximately 2-1/2 years and has found it to be extremely effective. It permits the acquisition, processing, and dissemination of data in a comparatively short period of time. Further, it permits the design engineer, who ultimately receives the report, to have as complete a record as possible of the tests actually performed, and eliminates the possibility that someone, in the process of screening, analyzing, and evaluating the data, has either omitted important details or has permitted an interpolation or interpretation of data which may influence the results. For ready reference by engineers, many microfilmed reports can be kept in loose-leaf notebook binders and, when no longer pertinent to a current study, they require a minimum of storage space.

In addition, a Generic Part and Component Classification Code was prepared for the CRHS program and a Bibliography of Reports is published at periodic intervals to aid the design engineer in utilizing these reports. As of June 30, 1962, a total of 824 reports have been disseminated to FBMWS prime contractors engaged in developing and producing the Missile, Fire Control, Guidance, Navigation, and Launcher Subsystems.

Interservice Data Exchange
Program (IDEP)

Because of the favorable reaction resulting from an informal survey conducted among various ballistic missile contractors during the latter part of 1958, and because of the encouraging results being obtained and manifested in the Navy's FBMWS (POLARIS) CRHS and the Air Force's TITAN Reliability Exchange Program, representatives of the three Services proposed that a similar program be initiated for the free interchange of parts data among the Department of Defense ballistic missile activities. This proposal was accepted by the three Services and has resulted in the establishment of what is known as IDEP which is the Interservice Data Exchange Program for ballistic missiles and space systems. Figure 3 shows the sources of information and flow of data in this exchange program.

During the calendar year 1959, and the early part of 1960, much effort was exerted by representatives of the three Services, including NOLC, toward establishing the administrative details of this program, which included the format to be used by the participating contractors, and by the Data Distribution Centers (DDC) for the purpose of transmitting and receiving test reports information in order to result in the maximum utilization thereof by the recipients; also, a comprehensive Part/Component Digital Generic Code, acceptable to all three Services, was agreed upon, which was similar to that employed in the Component Reliability History Survey Program. This work was documented as IDEP I, "Procedures for Participants," and IDEP II, "Codes for Establishing Index Number" (Figs. 4 and 5). In October and November 1959, representatives of the Services agreed to the principles of IDEP, and the approval to implement it in the respective commands was obtained early in 1960.

Because of the participation of NOLC in the FBMWS CRHS Program, it was designated to be one of the three Data Distribution Centers. The Army has designated Huntsville, Alabama, as its center; the Air Force is utilizing Aerospace, Inc., located at Los Angeles, California, as its center.

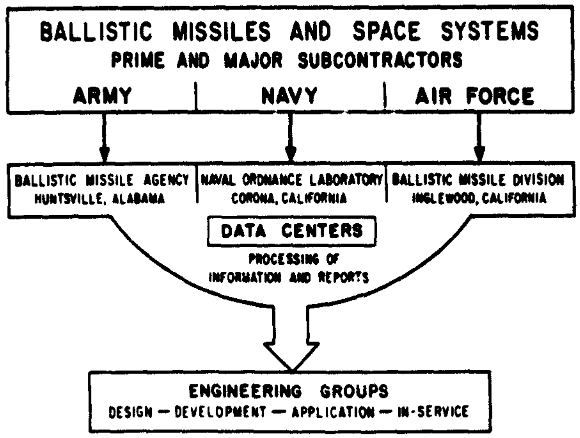

Fig. 3 - Interservice Data Exchange Program (IDEP); sources of information and flow of data

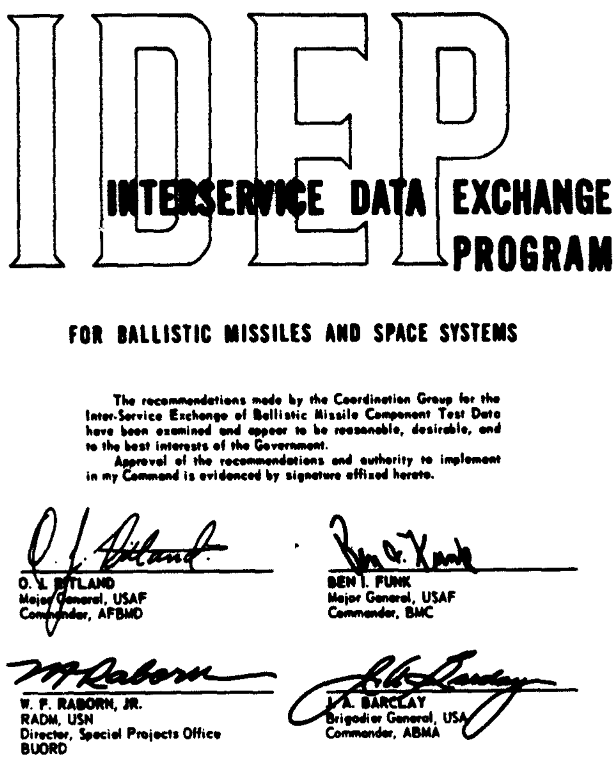

Fig. 4 - Cover for document IDEP-I; procedures for participants

Fig. 5 - Cover for document IDEP-II; codes for establishing index number

A few of the objectives of this program are:

To reduce duplicate expenditures for parts testing,

To avoid repetition of tests already accomplished and provide prompt indication of failure modes, thereby speeding up new projects,

To encourage standardization of test methods, levels, reports, and specifications as an incidental result of widespread observation of varied techniques,

To lead eventually into a voluntary intercontractor preplanning of complementary test programs. The data required include, but are not limited to, test results such as: Qualification or Certification Tests; Production Acceptance Tests (if particularly significant); Diagnostic or Design and Development Tests; General or Comparative Evaluation Tests; and Reliability, Exaggerated Stress, and Life Tests.

Each Service has stated that reports of tests conducted on parts by their contractors engaged in the development and production phases of ballistic missiles and space systems shall be transmitted to the DDC specified by the cognizant service. For example, those Navy contractors engaged in the development of the ballistic missile weapon system which have been selected to participate in this program forward their data to NOL, Corona.

Each participating contractor summarizes the results of his test on a Standard Report Summary Sheet which is forwarded, with the complete test results, to the DDC. By multilith process, the information on the Standard Report Summary Sheet is transferred to an 8 x 10-1/2-inch Report Summary Card; the test data itself is microfilmed at a reduction ratio of approximately 22 to 1 and attached to the card (Fig. 6). The cards are also color coded, by date, to facilitate purging obsolete data from the files. One set of cards with microfilm attached is transmitted to each of the other two DDCs to establish a complete library of cards for test reports at each center and for their further duplication and transmittal to cognizant contractors.

This exchange program was actually implemented at NOL, Corona, for Navy contractors engaged in the FBMWS program and certain Government agencies during July 1960. At the present time the number of contractors and Government agencies that participate in this program are Navy — 29; Army — 26; and Air Force — 49.

As the program developed, it appeared desirable to exchange other types of intelligence, such as specifications and information on tests planned or in-process by the contractor. These proposals were approved and today such information is also being interchanged through the medium of the IDEP Offices. The specifications are of two kinds: those pertaining to the tests to be performed and necessary for a complete understanding of the tests, and those referred to as Hi-Rel (High-Reliability) and prepared by contractors for the purpose of obtaining high-reliability parts and components.

The information pertaining to tests planned or in-process are placed on cards and identified as PIDEP, which means Pre-IDEP. In other words, information on tests planned is disseminated. The actual results of these tests, when completed, are then formally documented and distributed through IDEP at a later date.

An Index of Reports is published on a quarterly basis, but kept up-to-date on a monthly basis. This report is illustrated by Fig. 7.

BUWEPS Guided Missile Data Exchange Program

A program similar in purpose and intent to both the FBMWS CRHS and the IDEP is also being conducted at NOL, Corona, for the Navy's guided missile programs. The significant difference in this program is that its purpose is to exchange data generated by those Navy contractors engaged in the research, development, and production of guided missiles, whereas IDEP is

Q-206

REPORT SUMMARY SHEET

INTEREST CATEGORY	WHITE
1. COMPONENT/PART CLASS NAME COMPLETE	Switch, Manual, Rotary, Multiple Pole, Multiple Throw
3A REASON FOR TEST	Qualification to Program requirements.
CARD	1 OF 1
2. REPORT NO.	721.70.55.95-T1-01
3. ORIGINATORS REPORT TITLE	Non-Standard Part Qualification Test Report (P/N 400749)
4. ORIGINATORS REPORT NO.	C152-192
5. TEST COMPL. (DAY MO YR)	10 4 62
REPT. COMPL.	16 4 62
6. PROGRAM OR WEP. SYST.	Polaris Transit
7. TITLE UNCL [X] CONF [ ] SECRET [ ]	8. TEST/DOC. TYPE 2

10.	10A. PART TYPE, SIZE, RATING	11. VENDOR	12. VENDOR PART NO.	13. TOTAL NO. TESTED
1	400749	Chicago Dynamic Industries	TSB-P-8C	4
2				
3				
4				

14. INTERNAL SPECS ETC REQ'D TO UTILIZE REPT	ENCL	SENT WITH REPORT NO	15. MIL-SPECS OR STDS. REF'D BELOW
A 400749	X		D MIL-STD-202A
B			E
C			F

16. ASSOCIATED IDEP TESTS NO & BRIEF NAME

18. TEST OR ENVIRONMENT	C NO. PASS	D NO. FAIL	E INSP DATA	F PER SPEC	G TEST LEVELS, DURATION, PART PERFORMANCE AND OTHER DETAILS
1 Insulation Resistance	4	0		A	200 megohms min. at 500 VDC, 1 minute
1 Dielectric Strength	4	0		A	1000 Vrms, 1 minute, between non-conductors and conductors. No discharge.
1 Resistance	4	0		A	Rotor to all stator points. 0.023 ohm max.
1 a Shock	1	0		A	50 g drop, 3 axes.
1 a Vibration	1	0		D	Method 201A, 1 hour and 6 minutes in each of the 10 sw. pos. Monitor for contact opening.
1 b Temp.	1	0		D	Method 102A, Cond. C. 400°C/122°C, 5 cycles. No physical degradation.
1 b Humidity	1	0		D	Method 103A, Cond. B, 100 volts. No surface discharge
1 c Cur. Breaking Capacity	1	0		A	0.023 ohm max. after 10,000 cycles, 125 ma, 115 VAC
					(OVER)

19. SUMMARY OF REPORT, NATURE OF FAILURES AND CORRECTIVE ACTIONS TAKEN:

No perceptible degradation on any sample from any test.

(OVER)

20. WAS TEST BEYOND VENDOR SPEC?	21. VENDOR INFORMED OF TEST RESULTS BY	22. SIGNED	23. IDEP CONTRACTOR	SUBCONTRACTOR
YES [X] NO [ ] NONE [ ]	LETTER [ ] CY OF REPT [X] VERBAL [ ] NOT [ ]	I. R. Hixson	Ramo Wooldridge	

IDEP FORM NO 10-341 — TO BE COMPLETED BY DOC ONLY

1. COMPONENT/PART CLASS NAME COMPLETE: Switch, Manual, Rotary, Multiple Pole, Multiple Throw

CARD 1 OF 1 — 2. REPT. NO. 721.70.55.95-T1-01

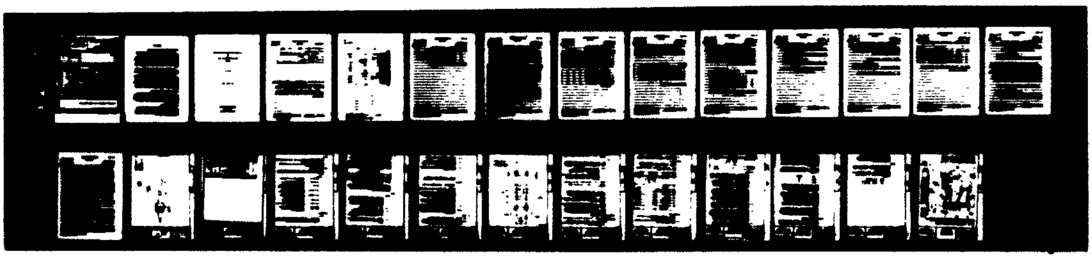

Fig. 6 - IDEP report summary card

VOLUME II, NUMBER 2
1 AUGUST 1962

IDEP

Interservice Data Exchange Program REPORT LISTING

EFFECTIVITY: This is a cumulative listing, covering the period from the cumulative index dated 1 January 1962 (Vol. I).

This plus Volume I comprise a total listing.

For copies of reports listed herein, or further information concerning IDEP, contact your IDEP Contractor Data Coordinator:	Mr. ______ Building ______ Dep't. and Group ______ Mail Station ______

Fig. 7 - Cover for IDEP Index of Reports

to cover the interchange of ballistic missile and space system information. In addition to test reports on and specifications for parts, application data sheets are also exchanged.

The development of this program was as follows:

In June 1959, NOL, Corona, because of its experience in the previously mentioned data exchange programs was assigned the task of developing a method and means by which design engineers might have readily available to them current information on the reliability and performance experience of certain parts and components proposed for use in BUORD (now BUWEPS) guided missiles.

To accomplish this task, a feasibility study was made which took into consideration several factors: types and kinds of data desired by reliability groups and design engineers, types and kinds of data that are presently being generated within each prime contractor's establishment, method and format to be employed by each of the prime contractors participating in this program in the transmittal of data, and method and format to be used by a center, designated as the Data Collection and Distribution Center, (DC and DC) in transmitting data to each of the other participating contractors.

As a result of the study, it was decided that the design engineer would gain the greatest amount of information from three principal sources. These are: the specification data sheets, the application data sheets, and the results and reports of tests performed on parts and components. All this information is to be transmitted by each prime contractor to NOL, Corona (DC and DC) for processing and dissemination. If this procedure is followed, the method of presentation would be more nearly uniform and standard. It is thus possible to place each type of data in the same kind of binder at each of the participating contractor establishments. Figure 8 is a sample of the Part/Component Test Report Summary Card with microfilm. This card is similar in appearance, purpose, and intent to the Summary Report Card employed by NOL, Corona, in the FBMWS (POLARIS) CRHS and to the IDEP Summary Report.

In the early part of June 1960, a meeting was held between representatives of BUWEPS, NOL, Corona, and four prime contractors engaged in producing guided missiles for BUWEPS, in accordance with the provisions of Task Assignment Qca-1-60. At that time, the procedures for this data exchange program were discussed and agreed upon. The implementation of the program was authorized and effected by Task Assignment FQ-1-61 of June 30, 1960.

Under the provisions of this program, data and test results pertaining to certain parts and components shall be made readily available to design engineers to enhance the selection of items having high reliability and performance for use in the BUWEPS guided missile programs.

Figure 9 shows the present prime sources of information in this program and the types of data interchanged through the Navy GMDEP office at NOL, Corona.

As a result of discussions between personnel of BUWEPS, the Special Projects Office, and NOL, Corona, arrangements were made to:

Provide GMDEP data to those Navy prime contractors and cognizant Navy (Government) offices engaged in producing the FBMWS (POLARIS).

Provide IDEP data generated by Air Force, Army, and Navy contractors to the Navy prime contractors and cognizant Navy (Government) offices engaged in producing Navy guided missiles.

In addition, certain types of information generated in GMDEP will be transmitted by the GMDEP Office at NOL, Corona, to the Air Force and Army IDEP Office for their distribution to Air Force and Army contractors engaged in the production of ballistic missiles and space systems.

BUWEPS Failure Rate Data (FARADA) Program

The Bureau of Naval Weapons' Failure Rate Data (FARADA) Program is a Navy sponsored service effort to provide reliability design data to prime contractors and major subcontractors engaged in the design, development, and production of hardware for various submarine, ship, aircraft, guided and ballistic missile, satellite, and ground support equipment programs.

The formation of the FARADA Program was fostered by BUWEPS and is based primarily on the findings and recommendations of the Department of Defense Ad Hoc Study Group on Parts Specification Management for Reliability. That study group was established by a memorandum of agreement dated July 14, 1958 under the joint sponsorship of the Office of the Assistant Secretary of Defense (Research and Engineering) and the Office of the Assistant Secretary of Defense (Supply and Logistics). The study group (commonly known as the Darnell Committee) issued a report (PSMR-1, Volumes I and II in May 1960) which contains their findings and recommendations.

One of the Department of Defense problems evaluated by the Study Group is stated in PSMR-1 as follows:

> "To enable designers to develop equipments which will meet quantitative specification of reliability for equipments and systems, it is essential that design guidance and application data such as component failure rates as a function of time and environment be made available to engineering groups. There is a great need to obtain and disseminate these technical characteristics of components as quickly as possible."

The report also contains detailed recommendations and supporting data for (1) the collection of test data (including failure rate data)

PART/COMPONENT TEST REPORT SUMMARY
NAV-NOLC 418 (5-61) SC #831

VOLUME IV PART/COMPONENT TEST REPORTS
BUWEPS GUIDED MISSILE DATA EXCHANGE PROGRAM

PART/COMPONENT: SEMICONDUCTORS, DIODES	DIGITAL GENERIC CODE: 751.40.10.09

ADDITIONAL IDENTIFICATION (Major characteristics): 5.9 to 6.5V 250 MW Silicon, Zener, Single Unit

1. TYPE OF TEST	2. QUANTITY TESTED	3. VENDOR	4. VENDOR TYPE
QAET	23	Transitron	1N827

5. PART/COMPONENT TESTED ABOVE VENDOR'S SPECS.	6. PAGE 2.66, PARA. 16. COMPLIED WITH
☒ YES ☐ NO	☒ YES ☐ NO

7. TEST ACTIVITY	8. PROGRAM	9. SIGNIFICANT MODE(S) OF FAILURE (Identify)
Bendix Mishawaka	TALOS	Initial Electrical Intermittent Life
10. TEST ACTIVITY REPORT NO.: EER 906	11. DATE OF TEST: 31 January 1962	

12. SPECIFICATIONS, PROCEDURES OR STANDARDS NECESSARY FOR PROPER INTERPRETATION OF REPORT AND THEIR SOURCE.
1642742 (1561389-101) OS-9596 (JS-1617)

TYPE OF ENVIRONMENTAL TEST PERFORMED (In order of test)	TEST LEVELS, DURATION, DETAILS, SPECIFICATIONS, ETC.	TEST RESULTS PASSED (NO.)	TEST RESULTS FAILED (NO.) *
Visual	Inspected per procurement specification	23	0
Initial Elec. Measurements	B_V (Breakdown Voltage) at I_R = 7.5 mA; 5.9V to 6.5V; bz (Breakdown Impedance) at I_R = 7.5 mA; 15 ohms max.;	19	4
	I_S (Saturation Current) at V_R (Reverse Voltage) = 4.7 Vdc at 25° and 150°C		
Life Test	Operated 20 minutes on, 10 minutes off at ambient 25°C for total on time of 250 hours	18	5
Temperature Cycling	-65°C to +150°C for 10 cycles and 20 minutes each end point	23	0
Shock	Three impacts of 500 G's in X_1 and Y_1 planes; monitored I_S at 4.7 Vdc	23	0
Vibration	From 100 - 2000 - 100 cps sweep for 10 minutes in X_1 and Y_1 planes; monitored IS at V_R = 4.7 Vdc		
Storage Life	100 hours at 150°C non-operating	23	0

REMARKS

The parts are acceptable for Talos use. The results of this test (the variation of I_S at both 25°C and 150°C) indicate an unstable surface condition exists which does not necessarily affect the B_V at 7.5 mA.

*The failures indicated are all I_S at 25°C and/or 150°C and varied throughout the environmental tests.

BUWEPS (FG) REPORT NO.: G 791

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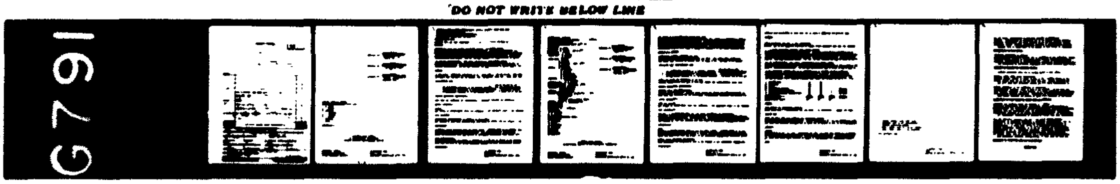

Fig. 8 - BUWEPS Guided Missile Data Exchange Program (GMDEP); Part/Component Test Report Summary Card

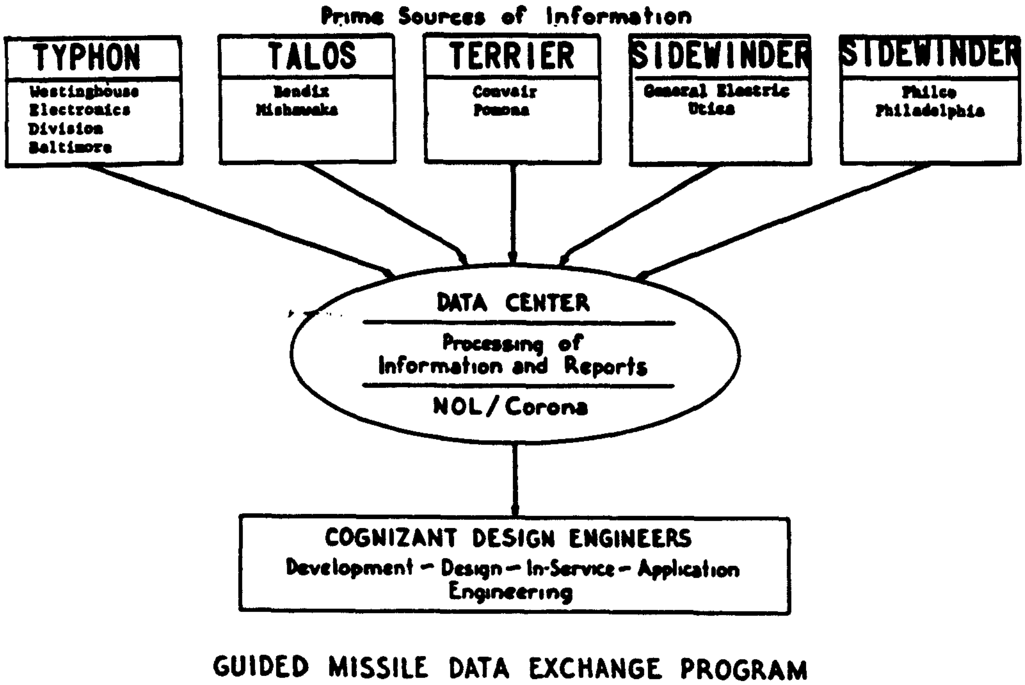

Fig. 9 - BUWEPS GMDEP; information sources and data flow

from all military service contractors and subcontractors, (2) the use of such data for preparing revised and new military specifications, and (3) the establishment of a program to obtain and publish failure rate data for designers of electronic equipment. Specifically, it was recommended that:

"1. New and controlled programs, manned by expert technicians whose sole responsibility is to obtain and analyze failure rate data, be established to obtain observed failure rates for typical air, shipboard, and ground applications.

"2. Observed failure rates obtained from the programs recommended in paragraph (1) be compared with failure rates obtained from an improved Electronic Equipment Field Data Reporting System to determine whether a correlation exists.

"3. The military services establish a more comprehensive program to evaluate electronic parts to obtain data on failure rate as a function of circuit and environmental stress.

"4. Failure rate data be analyzed and published in a suitable form, such as Department of Defense handbooks, for contractor and subcontractor use in the prediction of electronic systems reliability."

The PSMR-1 report indicated that the time schedule was critical and that maximum benefits could be obtained only if the recommendations were to be carried out immediately.

BUWEPS, being aware of the existing problem and recognizing the need for such data, early in 1960, assigned to NOL, Corona, a subtask to the Guided Missile Data Exchange Program (GMDEP) to conduct a study pertaining to the collection, summarization, analyses, compilation, and distribution of failure rate data. In July 1961, after its feasibility was established, a Task Assignment (RREN-08/004/211/F008.01/013) was provided NOL, Corona, by BUWEPS for the collection, analysis, compilation, and distribution of failure rate data on mechanical, electrical, and electronic component parts. This Program became known as FARADA, i.e., Failure Rate Data.

BUWEPS then formally queried contractors (both in Navy programs and in other service sponsored programs) as to their willingness to participate in a failure rate data collection and analysis program. The response was overwhelmingly favorable, indicating an urgent need for such information. It was also recognized that the benefits to be gained by participating organizations would be many times more than their original contribution to the program. In addition, the integration and analysis of data by the FARADA Office would also result in the availability of more realistic failure rate data pertaining to laboratory, ground, submarine, shipboard, aircraft, missile, and satellite environments.

Plans were then formulated to include as many organizations as were willing to forward a resume of their failure rate experiences in their various in-house programs to the FARADA Office established at NOL, Corona.

The increase in system complexity, resulting from the evolution of shipboard, aircraft, missile, and satellite weapon systems, has emphasized the need for increased reliability of parts and their use in circuits. Consequently, the parameters and tolerances to which parts and circuits are manufactured must be changed as often as necessary to meet the rigid requirements of specific weapon systems. The FARADA Program provides failure rate data which should aid in the design and development of more reliable military equipment. Without these basic failure rate data, which provide background knowledge of the probabilities of the proper functioning of individual component parts, it is impossible to estimate the reliability potential of a complex system design.

In detail, the program covers:

Stress Analysis to assist designers in performing quantitative reliability stress analyses by providing operational stress data on component parts.

Specifications to provide failure rate data for use in developing reliability requirements for component part specifications, standards, and drawings.

Reliability estimation and growth prediction to provide accurate failure rate information, so that realistic quantitative estimates of product reliability can be made as soon as sufficient design specifications exist.

Environmental factors to provide data on various operating modes influencing failure rates and highlighting the critical functional environmental stresses of each mode.

Application factors to provide data which modify the basic failure rate in order to allow for different applications for the component part.

Performance degradation to provide data on stability or degradation of component part operating parameters or characteristics as a function of time and stress.

Observance of test data presently being circulated in the current data exchange programs (IDEP and GMDEP) indicated a need for more sophisticated methods of data analysis and summarization to facilitate the extraction of failure rates in respect to time and environment. It was determined from the previously mentioned study that modified or new forms and procedures would be required to obtain this information and to present it uniformly. As a result, a Standard Operating Procedure (SOP) has been developed by the FARADA Office at NOL, Corona, for the collection of the failure rate data. This will enable better correlation of the data obtained from the various programs (Fig. 10).

Program output is to be published in the form of a component part "Failure Rate Data Handbook" (Fig. 11), which will be available to each program participant. The handbook will be kept current with newly generated failure rate data. This failure rate data (to be distributed by the FARADA Office at NOL, Corona) will provide an up-dating of MIL-HDBK-217, "Reliability Stress Analysis for Electronic Equipment." These data will also be used as a basis for reliability prediction as outlined in MIL-STD 756(Wep) of October 3, 1961, "Reliability of Weapons Systems, Procedures for Prediction and Reporting Prediction Of."

The purpose of this handbook is to provide design engineers with failure rate information in convenient form for use during the design phase. If properly applied, the information should provide a means of numerically assessing the probability of survival (reliability) of a subsystem or equipment prior to, or simultaneously with, the construction of hardware. After sufficient hardware has been manufactured and a designated number of systems have been operated for a specified length of time, the accuracy of survival estimates made on the basis of failure history can be checked. As experience in the use of this method is gained, refinements can be made and improvements in design should result.

It must be emphasized that these failure rates are obtained from specific engineering data and test results. These data are compiled from long histories of component part failures based upon material gathered by the FARADA Office at NOL, Corona. The validity of these data has been and will continue to be carefully examined.

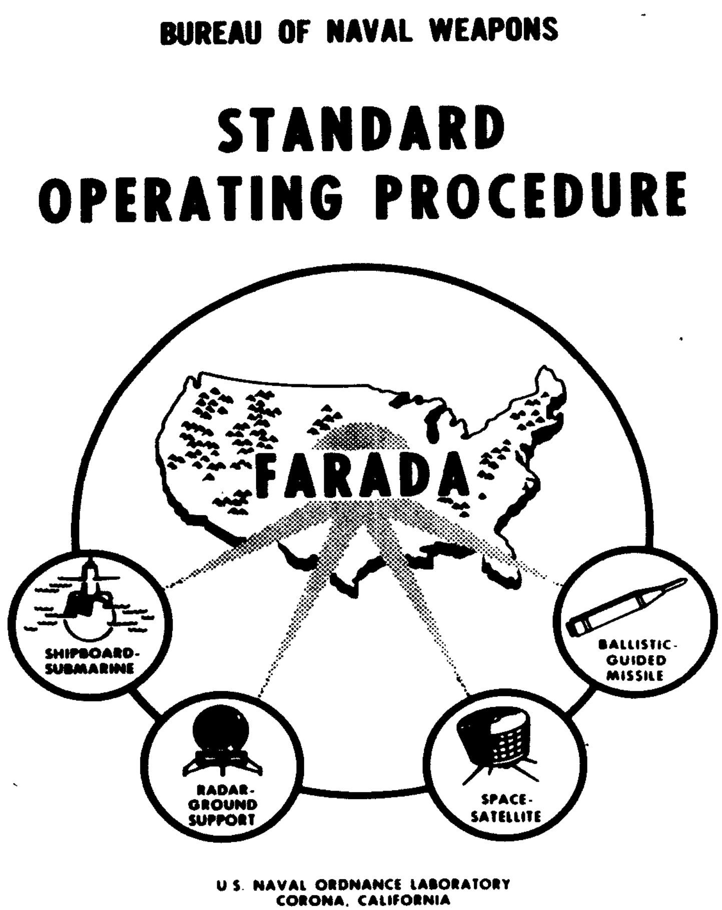

Fig. 10 - Cover for FARADA document;
Standard Operating Procedure

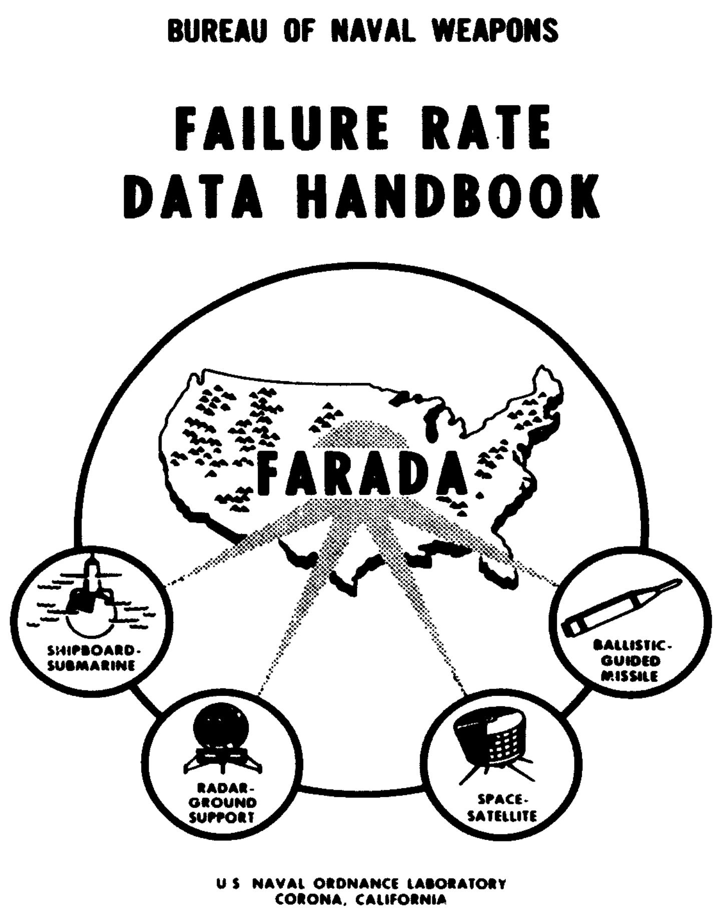

Fig. 11 - Cover for FARADA document;
Failure Rate Data Handbook

As of August 15, 1962, a total of 75 organizations and activities have documented their intention (by letter) to participate in the FARADA Program and, in keeping therewith, FARADA Coordinators have been designated at each.

Failure rate data received at NOL, Corona, from these participants, representing an initial contribution to this program was compiled and included in the Failure Rate Data Handbook, SP 63-470, which was distributed early in June 1962. The first revisions/additions to this Handbook were issued on August 1, 1962; beginning in September 1962, revisions/additions are to be made on a quarterly basis.

By recent correspondence to BUWEPS, the Headquarters, United States Air Force, Washington, D. C., indicated that the Air Force will participate jointly with the Navy in the operation of the FARADA Program, and has designated Rome Air Development Center (RADC) to be the Air Force Coordinator on FARADA; it is hoped that a similar expression may be received from the Department of the Army and the National Aeronautical and Space Administration.

* * *

INFORMATION ON DATA EXCHANGE PROGRAM—ENVANAL

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The word ENVANAL stems from the two words ENVironment and ANALysis. It describes a program which has been active since about 1950. It began actually as an Army-wide program, but only that portion which concerns ordnance materiel will be dealt with in this paper. Therefore, the title of this paper might more accurately read Ordnance ENVANAL. As implied by its name, the program deals exclusively with the environment and is in actuality restricted to engineering tests which have been conducted by the former U.S. Army Ordnance Corps in both the arctic and the desert.

In brief, the concept of ordnance ENVANAL is to develop a system which summarizes the extent of available data pertaining to the performance of ordnance equipment under arctic and desert conditions.

As can be easily realized, there are a number of customers for such data and it is necessary that the date be presented in a format suitable to each class or type of customer. First of all, the Army staff desires guidance in order to select the best available equipment for use in the various potential theaters of war. Staff personnel have no time to review great masses of detail data. Consequently, the ENVANAL system must present, in a concise form, the expected capabilities of the equipment under severe environmental stresses. Secondly, environmental program planners must have a method which will enable them to formulate the most effective environmental program for the amount of funds available. The numerous items within the cognizance of the former Army Ordnance Corps creates an appreciable problem when one is attempting to establish some priority as to which items should be tested immediately. Naturally, there are never enough funds to test everything. Consequently, effective program planning can be and is assisted by the ENVANAL system which gives the program planner the ability for a quick review of what has gone on before, and which can also quickly establish those items which require additional testing.

The third benefit of the ENVANAL system is to aid the environmental or research and development activities through analysis. The ENVANAL system will indicate the degree of success with which a particular item meets its military requirements. It can also be used to show generalized trends where research is required. As an example, the ENVANAL system can be used to indicate how many of the ordnance vehicles have deficient cooling systems. If a majority of the vehicles have sub-par cooling systems, the basis for additional research and development in this area has thus been established.

The mechanics of the ENVANAL system are rather complex, but basically, they involve the following steps: First, a review of the technical report; secondly, an analysis of this report and the item performance; thirdly, a conference between the ENVANAL staff and the project engineer to establish that the ENVANAL analysis accurately represents the item performance; and fourth, publication of the ENVANAL data in chart form. The heart of the system is the rating scale running from 1-5. This, in essence, describes the degree to which the item successfully meets its military requirements.

These ratings are then listed on a chart known as the Record of System Performance. The chart is rather complex and contains a great deal of information, all of which is probably of interest to only a few people; however, each of what we consider the ENVANAL customers is interested in some portion of the chart. The top portion of the chart describes the test item along with the associated components which are used to support the test item. The chart also contains a summary of performance which is a short generalized description of the current performance capability of the item when operating in the environment indicated on the chart. The remainder of the chart consists of performance factors associated with the environmental factor under which the performance was measured. Insofar as possible, every element of the performance is separated and an associated evaluation is made of each of the many combinations. Any rating which is less than satisfactory is explained by means of notes contained on the bottom of the sheet.

These charts are published periodically and at the moment or at least by January 1963, there will be four volumes covering Arms and Ammunition in the Arctic and Desert, and Automotive Equipment in the Arctic and Desert. Some 1500 items are currently on the charts and, in the future, more will be added as they are tested.

Certainly the system is not perfect. It has been undergoing changes since its concept and will continue to do so. At the present the charts are probably misleading in that rarely does a single environment influence a single performance factor. In reality, the environment is a complex of many factors, each of which influence the intermediate performance of an item which in turn influences the end performance of the item. Techniques have been developed which will probably help the situation, but it is doubted that a chart will ever accurately represent the combined environmental effect. When you consider, for instance, that some seven environmental factors are pertinent to the performance of a gun when an accuracy test is being conducted, and that probably never again will these seven conditions occur with the same precise limits, it may be seen that extreme care must be exercised in the interpretation and prediction of the weapon performance. We are currently attempting to categorize those conditions.[1] It is believed that, if the environmental factors can be properly grouped as to their probability of occurrence in combination, then the ENVANAL chart capacity to predict performance will be improved.

The system has also been investigated regarding the feasibility of converting it to punched cards.[2] Such a technique has been perfected and its effect will be to improve the rapid assimilation of the information contained on the performance charts. This system is such that practically any combination of events recorded on the charts can be simply and quickly analyzed.

ENVANAL data is believed to be extremely indicative of the efficiency of the Ordnance Corps in meeting its environmental obligations. The areas which need further development or emphasis can be quickly ascertained. While the ENVANAL system does not do as much as we would like and does have limitations, it does fulfill the need for a rapid review of what has gone on before, and it does have an ability to forecast environmental performance of equipment.

As mentioned earlier, the system presently contains only information pertaining to engineering tests on ordnance items in the Arctic and the Desert. As an addition to the program, another publication entitled, "Ordnance Technical Index of Environmental Factors" is also published as a part of the ENVANAL Program. The index by and large consists of abstracts of reports and other technical data pertaining to the terrestrial environments of cold, hot, tropical, and altitude. This is a three volume publication which is periodically supplemented and which serves as a source of reference information. It is not intended in any way to replace the ENVANAL system but more or less to supplement the ENVANAL data with generalized types of information on the terrestrial environments. By and large, this index is also restricted to equipment performance in an environment; the equipment considered is that which is of interest to the former U.S. Army Ordnance Corps.

This program is under the technical supervision of the Field Liaison Office, Development and Proof Services, Aberdeen Proving Ground, Maryland. Further information on the activities and publications pertaining to the ordnance ENVANAL system is available through this office.

[1] "Information and Coding Handbook for Arms and Ammunition Field-Type ENVANAL Charts," Report No. AR-468, Southwest Research Institute (July 1962).

[2] C. G. Robinson, "An Exploratory Mathematical Analysis of the ENVANAL Rating System," Southwest Research Institute (October 1960).

* * *

THE ARMY QUARTERMASTER CORPS* PROJECTS COVERING THE ANALYSIS AND EVALUATION OF ENVIRONMENTAL FACTORS

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The Earth Sciences Program of the Army Quartermaster Corps provides data on the natural environments which are of primary importance to design, development, testing, and storage of military equipment and materiel. Research findings of this program are presented here together with some of the problems inherent in their establishment and use.

INTRODUCTION

In keeping with the theme of the Symposium – the application of environmental data to specifications and design criteria – this paper discusses (1) the type of environmental data being made available at the U.S. Army Quartermaster R&E Laboratory at Natick, Mass.; (2) some of the results obtained from these studies over the past few years; and (3) some of the technical difficulties involved in interpreting the data for use in developing environmental criteria for design and testing of Army materiel. Specific examples will be drawn from reports published by the agency.

For the past decade, the Quartermaster Corps has been responsible for reporting the Applied Environmental Research Program to the Department of the Army.

Applied Environmental Research as defined by the Army refers to the collation of statistical, meteorological, climatic, and geographic data; the interpretation of these data; and the presentation of evaluated information in suitable form for application to logistics problems of equipment, personnel, and operational functions. The QMC also has a specific assignment from the General Staff for recommending environmental criteria for design and testing and coordinates the regulatory documents within the Department of the Army.

The primary objectives of the Earth Science Research Program at Natick is to discover the true nature of physical environments and to assist in the development of materiel that provide proper protection to the soldier under all stresses of the environment.

At the Quartermaster Laboratory this assignment refers principally to projects in the fields of climatology and geography, although research is also conducted in related fields. The research is carried out within the Earth Sciences Division (Fig. 1).

The Earth Sciences Division is composed of two branches – Regional Environments and General Environments – which are further divided into the Polar and Mountain, Tropic and Desert, Global Environments, and Logistics Applications Sections. These sections, or laboratories, are supported by a Cartographic Branch and a library data bank. There are about 40 people in the Division, of whom two-thirds are professional.

TYPE OF STUDIES AVAILABLE (Fig. 2)

For purposes of simplification, the subjects covered by the Earth Sciences Research Program can be grouped into the following categories (Table 1):

*On August 1, 1962, the Quartermaster Corps was dissolved and the QM R and E Center became the U.S. Army Quartermaster R and E Laboratory, a part of the Army Materiel Command.

• Systematic or Topical Studies which are concerned with basic concepts and principles governing the measurement, occurrence, persistence, and distribution of specific terrain, climatic and biogeographic elements of the natural environment, and the variable processes which govern their interrelationships.

• Regional Environmental Studies which cover analyses of environmental complexes and their characteristic patterns.

• Testing Site Studies which provide geographical information on Army testing sites and analogous areas in the world where corresponding environmental conditions exist.

• Applied Research Studies and Applications of research findings which include the preparation of environmental criteria for design and testing, contributions to the Department of the Army Envanal System, preparation of logistics aids and guides for food, clothing, shelter, fuel, and the coordination of regulatory documents covering the environmental field within the Department of the Army.

• Army Support to Non-Government Scientific Expeditions which provides for granting funds or lending equipment and materiel to private expeditions that will agree to provide data of value to science.

EXAMPLES OF THE TYPE OF STUDIES PRODUCED

Many of the studies completed to date qualify for inclusion into one or more of the categories listed. For the sake of simplicity, they are shown within the category which more nearly covers the original intent of the study.

Systematic or Topical Research

Terrain — Despite the advances made in science since the turn of the century, the surface landforms and associated terrain elements and the erosional and gradational processes which have developed them, still escape definition by quantitative means. Surfaces continue to be defined qualitatively by such terms as

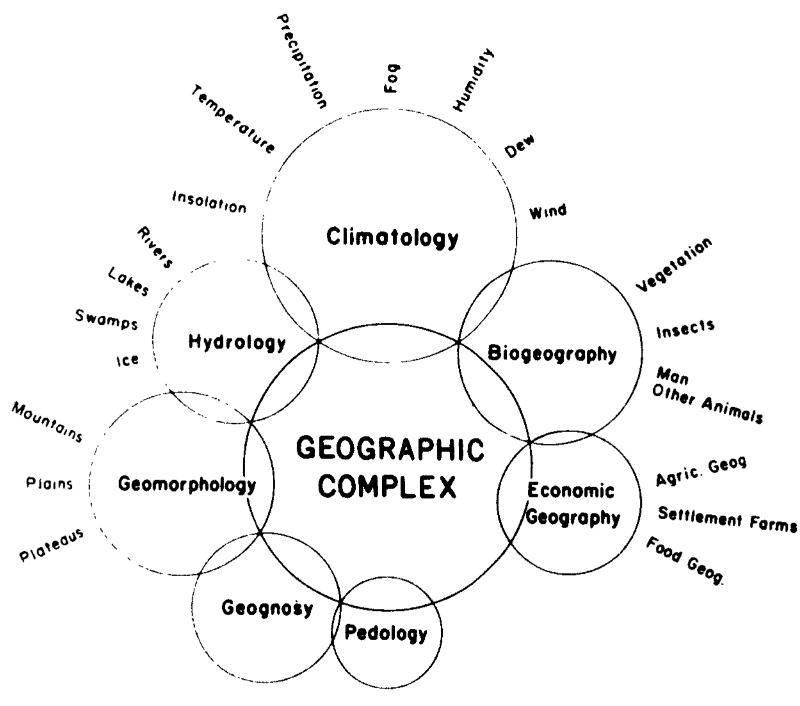
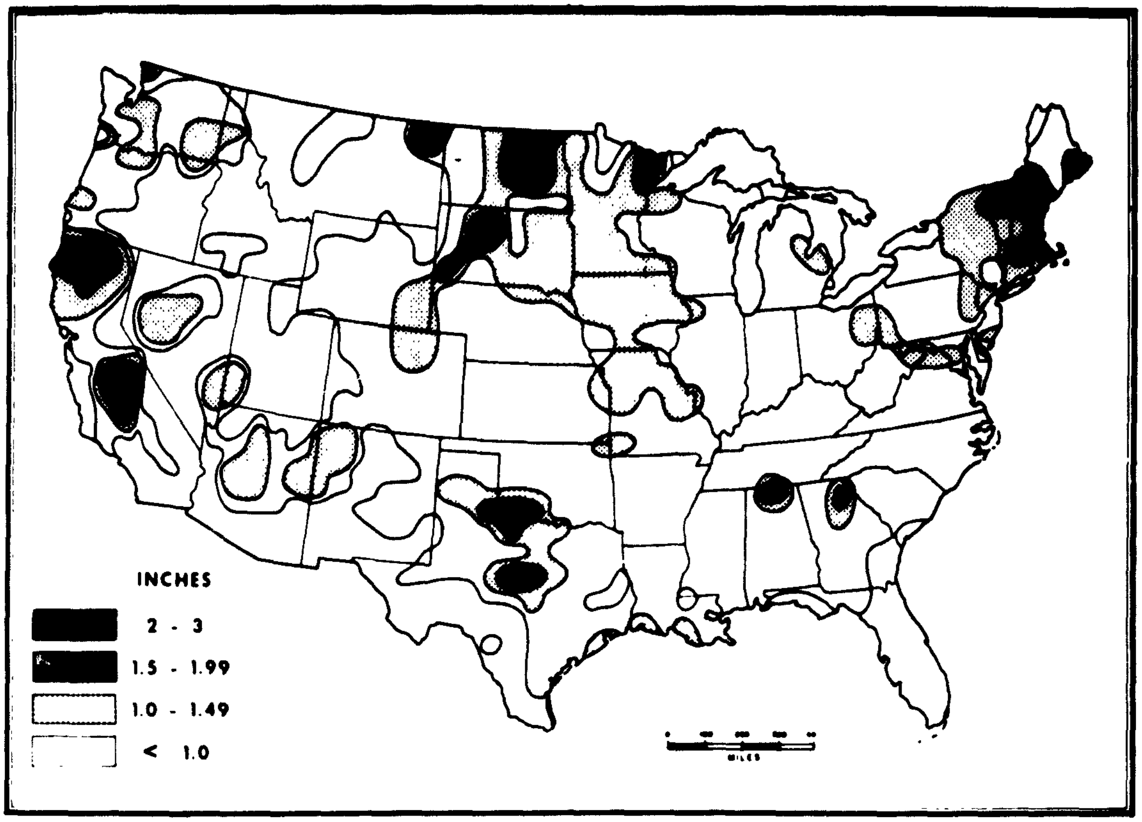


Fig. 1 - The fields of science covered by the earth sciences program at the U. S. Army Natick Laboratories. The size of the circles are roughly indicative of the emphasis being placed on the subjects.



Source: Association of American Railroads

Fig. 2 - The maximum radial thickness of glaze on utility wires in the United States (1928-29 and 1936-37)

youthful, mature, old age, steep, rough, rolling, and flat, which are frequently subject to misinterpretation owing to the individual interpreter's background. Since the military primarily operates on and in intimate contact with the surface of the earth, there is a pressing need for terrain information expressed in quantitative terms. This is one of the problems that has occupied the attention of several U.S. Army Laboratories. Research advances to date have been quite fruitful. A landform classification system based on quantified terrain data has been devised and tests of the system utilizing topographic maps of Central Europe have been made with gratifying results. Measurements for six terrain factors – grain, relief, average elevation, elevation-relief ratio, average slope, and slope direction changes – were taken from topographic maps for 413 sample areas within an area of over 100,000 square miles. Individual samples were grouped into 25 homogeneous regions in accordance with the similarity of the six terrain factor measurements. A map of Central Europe delineating these 25 regions was prepared, and it was found to compare favorably with physiographic maps of the same area based upon field studies.

Another aspect of the quantified terrain research is the application of line-of-sight research to nap-of-the earth flying. The amount of protection that the terrain offers to light aircraft operating in an environment of high-performance short-range missiles, can be predicted by utilizing a mathematical model for predicting the availability of unobstructed lines-of-sight from high points out into valleys as far as 20 miles from the viewing point. A by-product of this research has produced a mathematical model for evaluating vegetation and terrain dimensions in terms of aerial observer efficiency and on the vulnerability of light aircraft to fire from short and medium range weapons located on the ground. From these projects, recommendations can be made on tactical doctrine, systems for protecting the pilot and aircraft,

TABLE 1
Subjects Covered in the Earth Sciences Program and upon which Reports have been Published

Systematic or Topical Studies	Regional Environmental Studies	Testing Site Studies and their Analogs	Other Applied Research Studies and Application of Findings	Army Support to Non-Government Scientific Expeditions
Climatic	Cold Regions	Cold Regions	Design Criteria	Cold Regions
Temperature	Alaska	Thule, Greenland	AR 705-15	Devon Island, Canada (Arctic Institute)
Extremes-hot and cold	Greenland	Ft. Churchill, Canada	MIL-STD 210A	Mt. St. Elias Mts., Canada (Arctic Institute)
Mean minimum	Mountain	Ft. Greely, Alaska	ABC Army Std #14	King Karl Island, Svalbard (Am. Inst. Biol. Sc. thru Army Med. Serv.)
Mean maximum	Appalachians	Ft. Wainwright, Alaska	Evaluation of Climatic Extremes	Sukkertoppen Area, Greenland (Ohio State)
Mean monthly	Mt. Glaciers, Northern Hemisphere	N. America and Eurasia	Climatic Extremes for Mil. Eq.	Mountain Regions
Predicting Jan. daily min.	White Mountains	Mountain	D/A Applied Environmental Research	Climate and Related Phenomena in Central Peru (Syracuse Univ. Res. Inst.
Predicting hourly temperatures	Temperate	Camp Hale, Colorado	(Annual Report on the Field)	Temperate Region
Precipitation	Cold-Wet environments	Mt. Washington, N.H.	D/A Envanal System	Soil Creep in the Presidential Range, N.H. (Harvard University
Probabilities of Extreme Rain Storms	Winter weather types	Temperate	Logistics Guides	Tropical Region
Humidity	Korean environment	Devils Lake, Wisconsin	Food Storage Guide S.E. Asia	Kon-Tiki
Influence of irrigation at Yuma on high humidities at high temperature	Tropical	Maynard, Mass.	Staple Subsistence Crops-Africa	British Guiana, S. America (McGill University)
Winds	Wet-tropics	Ft. Lee, Virginia	World Clothing Requirement Areas	Goyaz, Brazil
Freq. and Dist. low level winds	Desert	Tropical	Tent Heating Guides	Desert
Dew	Death Valley	Ft. Sherman-Ft. Gulick, C.Z.	Storage Environments	Physiology of the Camel (Dr. Schmidt-Nielson)
Measurement	U.S. Desert Surface Conditions	W. Central Africa	Box-car at Yuma	
Hail	Other Areas	S. Central Africa	Open-Dump at Yuma	
Size and Distribution	S.E. Asia	Madagascar	Leather in Warehouses	
Glaze	S.W. Asia	India and S.E. Asia	Temperatures in Army Warehouses	
Freeze-Thaw	World color regions	South America		
Terrain	Thailand	Desert		
Quantitative classification of landforms	Hawaii	Yuma, Arizona		
Analysis of Line-of-Sight		Dugway Proving Ground		
Visibility in U.S. Forest Stands		Middle East (Asia)		
Muskeg		Soviet Middle Asia		
Soil Temperatures		Chinese Inner Asia		
		E. Central Africa		
		N. America		

and design specifications for new aircraft embodying protective devices.

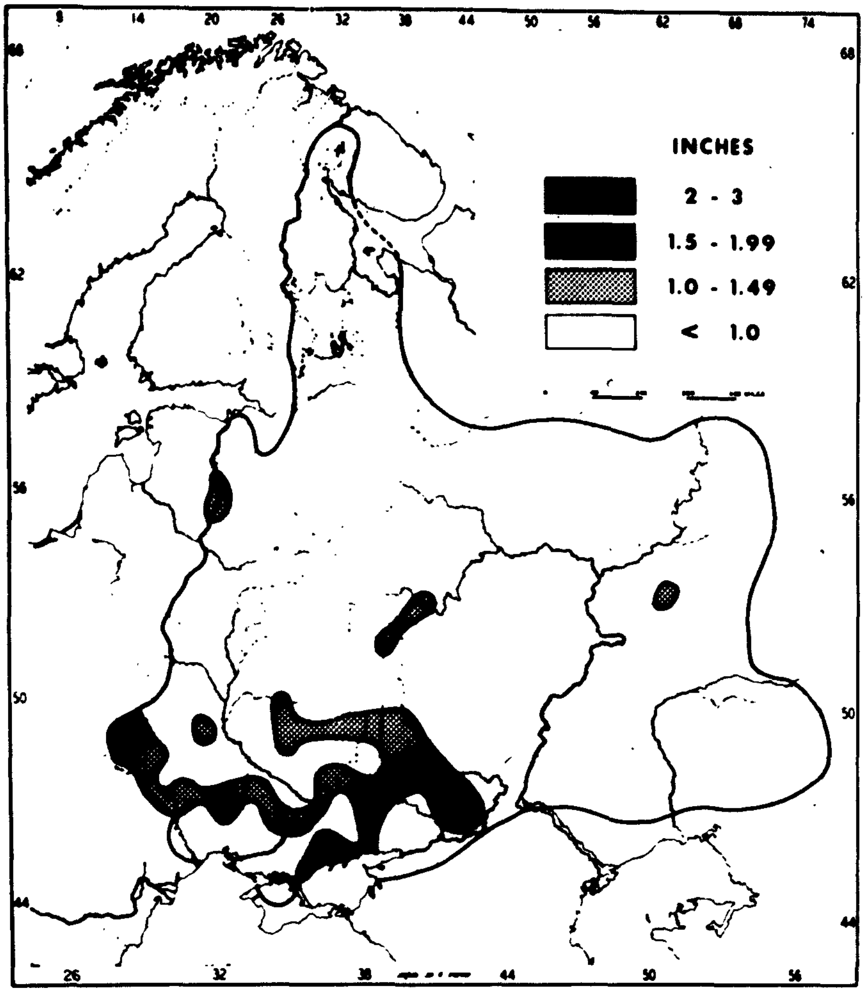
Climate — The interpretation of world climatological records for application to Department of the Army research and development activities is of vital concern to the Earth Sciences research. Many countries of the world do not report their data in sufficient detail to be used for environmental design criteria. Frequently there is need for interpolating or predicting the environmental conditions that occur in areas for which data is meager or just not available. Research has indicated that by knowledge of certain basic data, such as, mean daily or monthly temperatures, the absolute temperature extremes, and the length of the record, a reasonable approximation of the existing hourly temperatures at a given station may be achieved. Techniques or methods have been developed for assessing the frequency of daily minimum temperatures, the longest period (duration) during any one month when the temperature will lie at, or above, a given level, and the probabilities of extreme rain storms.

Terrain and Climate — Studies of mountain environments involving non-instrumental photo-interpretive climatology, based largely on clima-geomorphic features and processes, are being carried out in North American mid-latitude mountains, with a view toward world application. Inter-regional climatic and environmental comparisons and the determination of regional environmental and climatic analogy permit transfer of military and related experience from well-known to little-known mountain areas. Correlation of site environment with site climate, as determined by instruments, is expected to permit eventual quantitative study of mountain climates over extensive regions.

Glaze — The amount of ice that forms on radomes, antennas, and utility wires, under winds of certain high velocities, are questions that have plagued equipment designers and specification writers for some time. Repeated requests for information of this type have occasioned the inclusion of Figs. 2 and 3, which were taken from a glaze report and showing the radial thickness of ice on utility wires in the United States and in the European part of the U.S.S.R. The glaze belt of the United States includes almost all of the nation east of the Rocky Mountains, with the exception of the northern and southern sections of the High Plain, the Gulf Coast, the Atlantic Coast south of Virginia, and Northern Maine. Many areas within the glaze belt have received as many as two storms in 3 years in which ice reached 1/4 inch or more in thickness, and one storm in 3 years in which ice thickness was 1/2 inch or more. Thicknesses greater than 1 inch have been experienced in almost all parts of the belt, and also in Louisiana. Thicknesses close to 2 inches have been experienced in New England, Northern Texas, the Dakotas, and Michigan. Thicknesses of 3 inches have been observed in Montana. The area affected by one glaze storm varies in size from a few square miles to distances that extend from the central part of Texas to southern New England. The average storm of more than local size probably deposits ice on an area 200 to 600 miles in length. Locations exposed to strong winds and in which temperatures are apt to be low during storms are most likely to receive heavy deposits. Wind data available during the period of glaze suggests that moderate velocities prevail. However, speeds of 25 mph or more are not unusual and there have been cases of winds in excess of 40 mph occurring with ice deposits of 1/2 inch or more in thickness. Strong winds with glaze appear to be more common in the United States than in other parts of the world.

Glaze can occur in Europe from the British Isles to the Urals (and beyond into Central Asia), from as far North as Scandinavia and Murmansk, U.S.S.R., to as far south as Greece and Italy, and at least one case has been reported from Algiers in North Africa. South of the Cantabrian Mountains (Spain), the Pyrenees, the French, Swiss, Austrian, and Dinaric Alps to the Balkan Mountains (Bulgaria), cases of glaze formation are rare in the lowlands and only slightly less uncommon in the mountains. The reason for this low frequency in the Mediterranean region is the rare appearance, at the surface, of continental air masses with sub-freezing temperatures. Though glaze is fairly common in western Europe, frequencies are probably nowhere as high as in the major portion of the glaze belt of the central and eastern United States. Figure 3 shows the maximum radial thickness of ice on utility wires in the European part of the U.S.S.R. Data show at least one storm per year during a 10-year period in the area extending from Leningrad to the southern Ukraine, eastward past Stalingrad, and as far north as Moscow. Frequencies of two to three storms per year are common north of the Black Sea and Sea of Azov. It should be pointed out that only one station (north of the Sea of Azov) reports 2 inches of glaze.

South Pole Data — Results of research carried out in extreme cold areas are provided by the South Pole Micrometeorology Program conducted by a member of the Division in conjunction with the IGY. On this coldest of all continents where temperatures as low as -127°F



Source: N.T. Zikeev.

Fig. 3 - The maximum radial thickness of glaze on utility wires in the European part of the U.S.S.R. (1923-24 and 1932-33)

have been recorded, high velocity winds have been observed in conjunction with very cold temperatures. Table 2 shows the mean temperature for various Beaufort wind forces for the sunless period at the South Pole Station in 1958. The last Beaufort category represents wind speeds of from 32 to 38 mph at a mean temperature of -46.7°F. The table also shows that wind speeds of from 13 to 18 mph were observed at -75.7°F (-59.80°C) for 54 percent of the time during the sunless period.

Figure 4 shows the mean temperature and wind direction frequency for the same sunless period at the South Pole. Coldest temperatures are experienced with winds blowing from the direction of the highest elevation in East Antarctica, and warmest temperatures are associated with up-slope winds blowing from West Antarctica. The frequency of winds are shown by the cross lined pattern about the pole. Ninety percent of the winds come from the 0° to 90° East Longitude Quadrant.

Figure 5 shows some of the selected summer wind speed profiles measured at the South Pole in 1958. These profiles are logarithmic to the top of the 8 meter mast, and are representative of midsummer wind profiles at the South Pole.

Rapid changes in temperature also take place at the South Pole. Figure 6 shows an

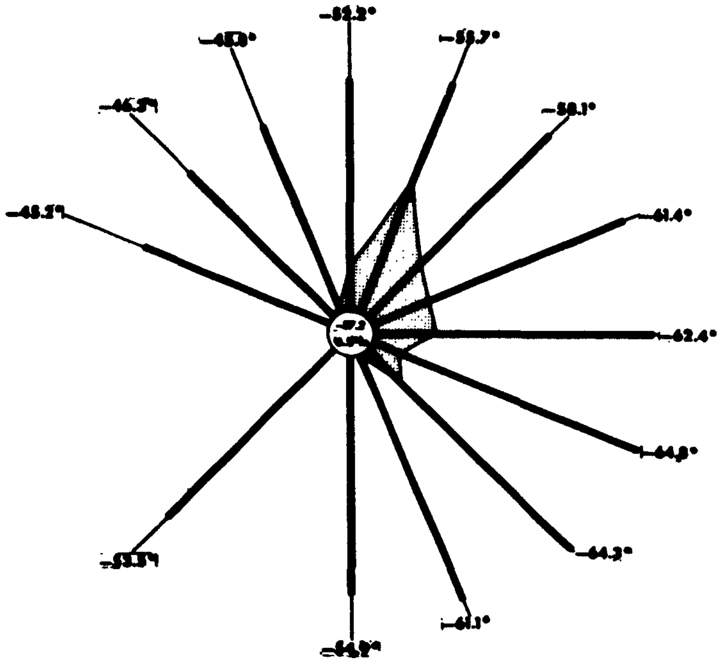


Fig. 4 - Mean temperature (-0°C) and wind direction frequency (%) for the sunless period at the South Pole Station (Mar.-Sept. 1958)

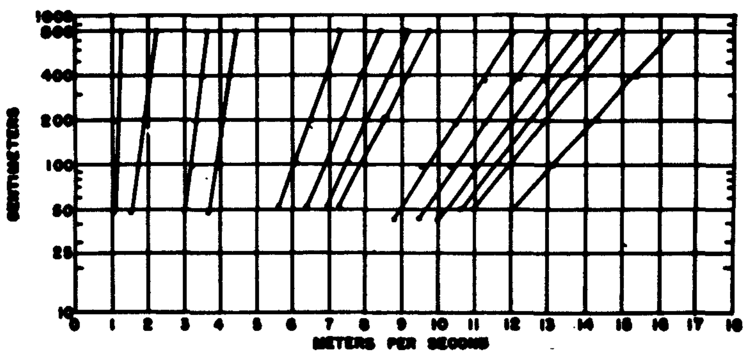


Fig. 5 - Selected summer wind speed profiles measured at the South Pole in 1958

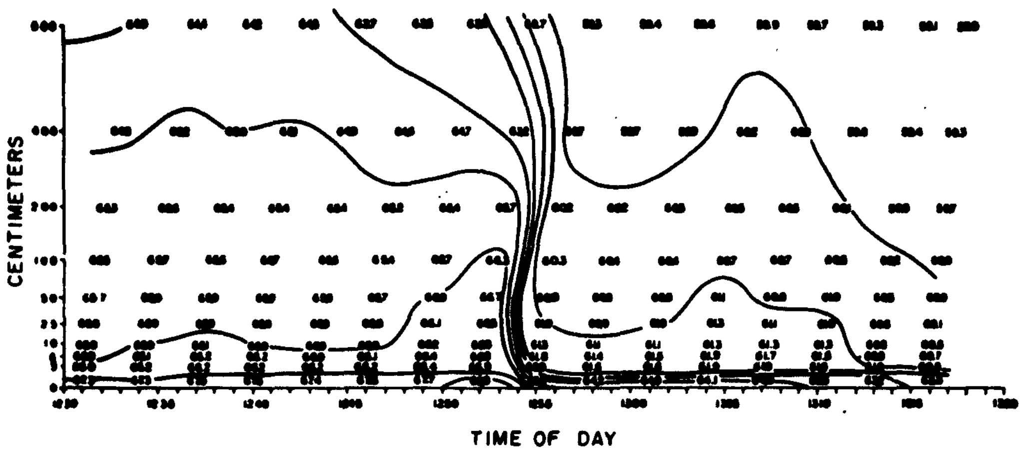


Fig. 6 - Example of rapid temperature change at the South Pole on May 8, 1958

TABLE 2
Mean Temperatures and Beaufort Wind Forces for the Sunless Period at the South Pole Station, 1958

Wind Speed (m/s)	Frequency (No. of Hourly Observations)	Mean Temp. (°C)
0.0 to 0.2	20	-59.0
0.3 to 1.5	28	-61.7
1.6 to 3.3	133	-62.2
3.4 to 5.4	733	-62.3
5.5 to 7.9	2355	-59.8
8.0 to 10.7	807	-55.1
10.8 to 13.8	237	-54.2
13.9 to 17.1	9	-43.7

outstanding example of rapid temperature change due to subsidence of warm air on May 8, 1958. A rapid warming of approximately 10°F occurred between 50 centimeters and 2 meters in a 3-minute interval. This rapid temperature change was not accompanied by any perceptible change in pressure, wind speed, wind direction, or in sky conditions, and is typical of cases of subsidence which occur when warm air from aloft is brought to the surface.

Regional Environmental Studies

This category includes studies of the elements of terrain, climate, and biogeography. The studies consider the character, distribution, and interrelationships of the elements within a given part of the earth's surface. The studies are made for the purpose of improving our understanding of the geographic complexes in which Army men and materiel must be able to operate. These areas may be either homogeneous or heterogeneous in composition. They may be established on the basis of a political entity such as Thailand, Alaska, or Korea; a portion of a continent such as Southeast Asia; or, on the basis of areas possessing some degree of homogeneity, such as aridity, wetness, or coldness.

Deserts — One of the extreme environments which has received a considerable amount of interest is hot desert environments. These are environments in which both men and equipment are subjected to extreme heat stress coupled with the absence of moisture in all its forms. Some of our problems are: What constitutes a desert? Where are they found? What are their characteristics? In attempting to answer these questions, the following figures which were taken from reports prepared on various aspects of desert geography, are pertinent.

Approximately 19 percent of the land surface of the earth is desert or marginal desert-like land; these deserts are arranged on the continents in a definite pattern related to the earth's wind systems. Major world hot-deserts are associated with the warming descending winds of the subtropical high-pressure ridges. Figure 7 shows the highest shade temperatures recorded in Stevenson shelters from 4 to 6 feet above the ground. Your attention is directed to El Azizia, Libya where the world's highest

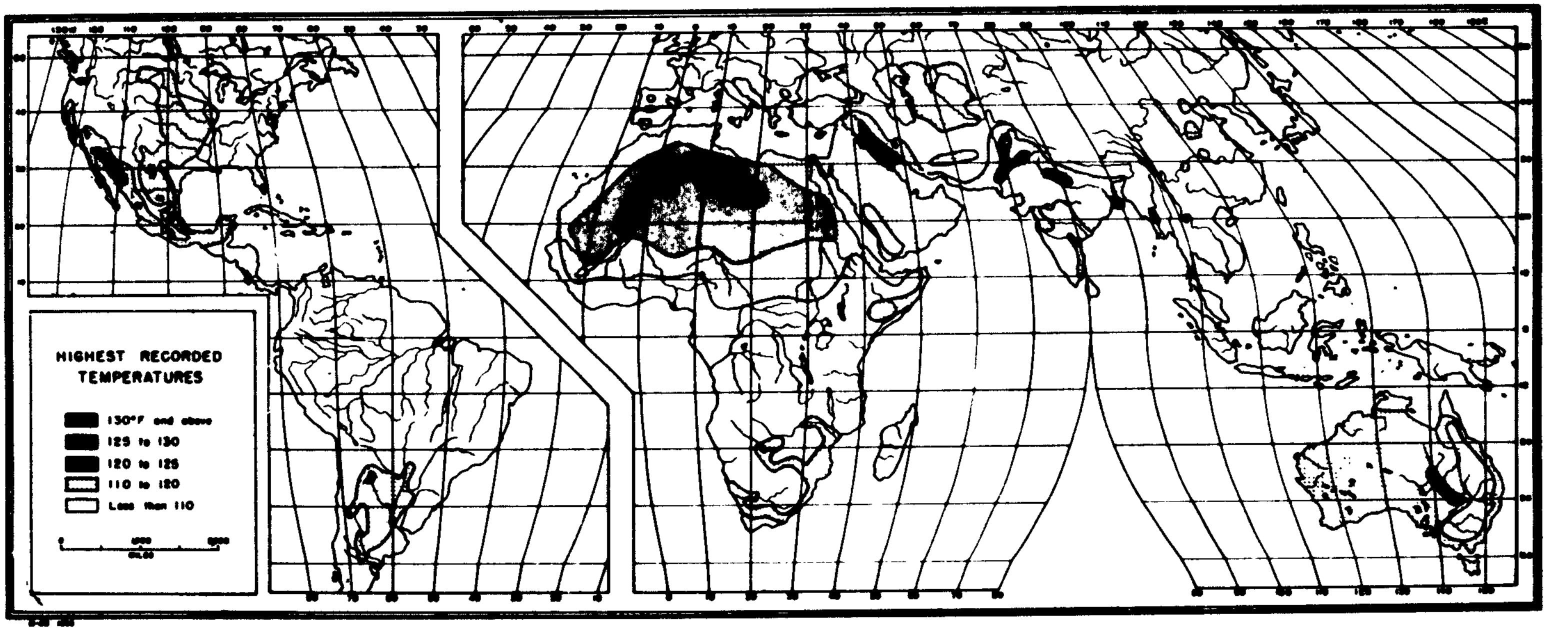


Fig. 7 - Highest shade temperatures recorded in Stevenson shelters from 4 to 6 feet above the ground

temperature 136.2°F was reached in 1922 and to Death Valley, U.S.A. where 134°F was reached in 1913. It should be explained at this point that the temperature on the ground surface or surface of equipment exposed to direct sunlight reaches much higher figures and that records are available of soil temperatures as high as 148° to 171°F and equipment surface temperatures from 152° to 250°F taken under variable circumstances.

Desert Surface Features — One element of desert environment that is important to equipment operation is relief. Owing to the dominance of mechanical decomposition processes, the surface features of deserts present their own distinctiveness. Figure 8 shows an empirical classification of desert terrain surface features of this very area prepared by a Quartermaster Corps Contractor. Phoenix is located at the lower right, Tucson in the lower right corner, Yuma, the lower left corner and Mt. Whitney at 14,495 feet in the upper left corner. Surface features covered include desert flats, playas, alluvial fans and bajadas, dunes, bedrock fields, bad lands, volcanic cones, desert mountains distinguished as to origin, dry washes, and features bordering thru-flowing rivers. Basic studies such as this are essential to field studies of equipment performance, and when quantified, will permit valid evaluation of field performance.

Desert Storage Temperatures — Specific micro-environments within the desert have occupied the attention of Quartermaster researchers, particularly those environmental conditions experienced during shipping and storage. Preliminary investigations have shown that steel box cars and tarpaulin covered boxes provide conditions favoring maximum heat accumulation in stored items. A steel box car (Fig. 9) loaded

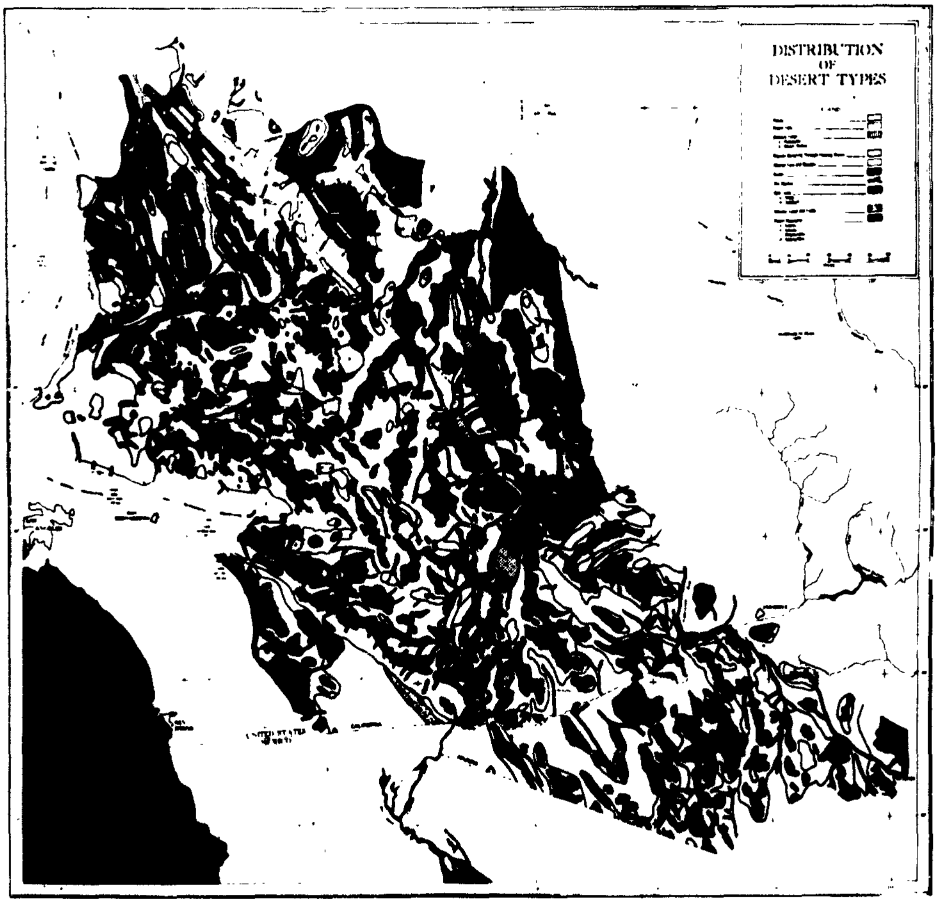


Fig. 8 - Classification of desert surface features

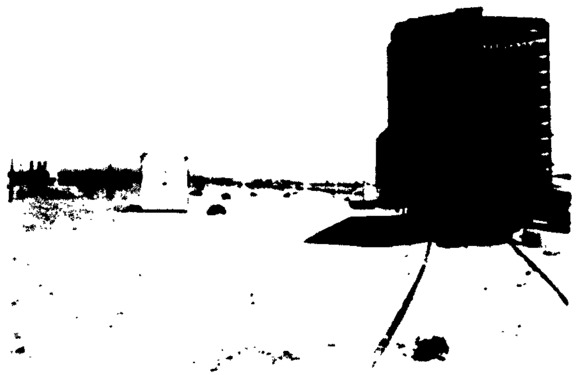


Fig. 9 - Weather bureau instrument shelter and steel boxcars (facing N-S) at the Yuma, Arizona site

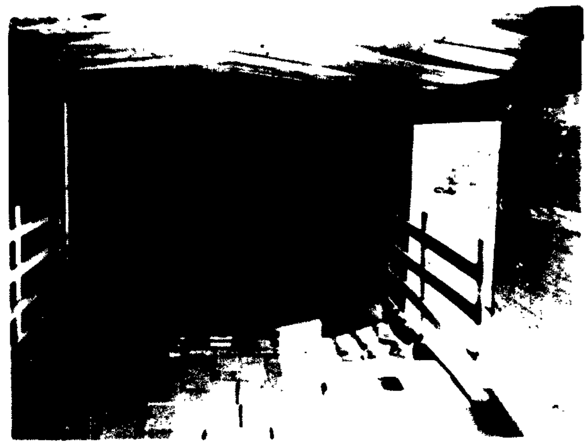


Fig. 10 - Interior of the steel boxcar (facing south) at Yuma, Arizona site

to a depth of 5 feet with fiberboard cartons containing canned string beans and test cans of Army C-rations spaced strategically throughout the load, were thermocoupled (Figs. 10 and 11).

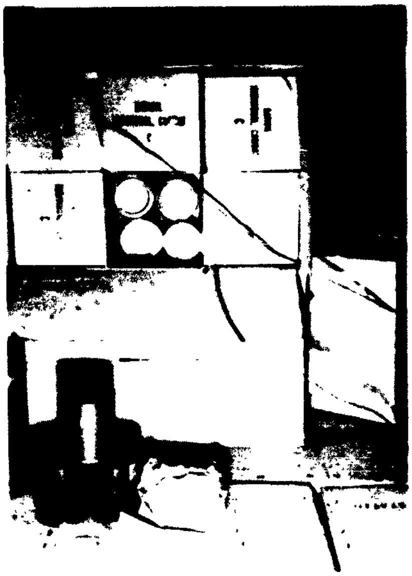


Fig. 11 - Thermocouples in C-rations within the boxcar at Yuma, Arizona site

With an ambient temperature at 112°F, the highest temperature recorded on the outside surface of the door of the steel box car was 172°F. Box car free-air temperature taken at the inside top-center of the car was 152°F, air in the top-center carton was 119°F, and food temperature within this carton was 113°F. In the buried load, the maximum food temperature recorded was 105°F. Inside roof-air temperature and air and food temperatures of the top-center carton were lowered to 147°, 102°, and 101°F, respectively, by use of a layer of foil laid on top of the center carton.

High temperatures were also recorded in four differently protected open-dump storage stacks at Yuma during a 43-day test period. Carton air temperatures and food temperatures were recorded by thermocouples at critical locations in cartons in the stacks which were exposed as follows: open and unprotected, protected by a tarpaulin tightly lashed about the stack, protected by a raised tarpaulin fly, and protected by a raised tarpaulin fly with the addition of reflective foil laid on the stack surface.

Each stack was composed of 96 cartons of Army C-rations in approximately a 5-foot cube. The ambient temperature, relative humidity, windspeed, wind direction, and solar radiation were recorded. Results of this study show the top-center maximum food temperature reached 118°F in the tarpaulin-bound stack, 107°F in the open stack, 104°F in the stack with raised fly, and only 98°F in the stack with raised fly and foil. The highest temperature recorded was 171°F on the inside top surface of the tarpaulin. Maximum ambient temperature was 112°F.

Site Studies and their Analogs

Within the western hemisphere, stations have been established to carry out engineering or field tests of military materiel prior to adoption of the materiel by the Army. In order to enable test planners to select optimal areas and conditions for scheduling performance tests, a series of studies designed to provide pertinent background information concerning environmental conditions at each site has been undertaken. The location and status of these studies are indicated in Fig. 12. In order to determine how representative these test stations are, of the type of environment that can be expected to be encountered in other parts of the world and

Fig. 12 - Location and status of the site studies

to provide a basis for estimating regional suitability of end items based upon performance trials, a companion series of studies known as analogs has been established. The analogs contain comparisons of environmental conditions existing at the test station with environmental conditions prevailing on this and other continents. Terrain data are provided by the U.S. Army Engineer Waterways Experiment Station at Vicksburg, Mississippi. The climatic analogs, which include data on temperature, windchill, cloudiness, snow, depth, precipitation, humidity, days with fog, and other pertinent data, are prepared by the Quartermaster Corps. The content of these studies varies with the area under consideration. Figure 13 shows a comparison of one of the elements, the mean daily minimum temperature of the coldest month as found at Fort Greely, Alaska and Fort Churchill, Manitoba, compared with that of Siberia. It will be noticed that the field testing temperatures encountered at the test stations in North America are not cold enough to provide field tests for equipment that will assure operation in much of Siberia.

Other Applied Research and Applications

This category of studies is primarily devoted to finding solutions to specific problems facing the military. These studies are concerned with research on the elements of terrain, climate, or biogeography that contribute to a solution of a given problem or to the modification of its more critical characteristics. The category includes such important items as the preparation of environmental design criteria for the Department of the Army, responsibility for preparing an annual report on applied environmental research for the Army Research Office, and preparation of windchill reports for protection of troops. The preparation of logistics guides designed to reduce complex logistical problems to their simplest form by graphic and tabular presentation, has been undertaken for each major mission of the Quartermaster Corps — food, clothing, shelter, POL (petroleum, oil and lubrication) supplies, and aerial delivery. Examples of some of the studies will be discussed.

Environmental Design Criteria — In order to attain the capability of conducting military operations in any area of the world, the Department of the Army policy is to insure that all Army combat and combat support materiel is capable of satisfactory performance at all times under the basic operating conditions (areas ranging in temperature between -25° and 115°F)* and that essential Army combat and combat support materiel is capable of satisfactory performance under extreme operating conditions (temperature from -65°F to 125°F).*

To meet these requirements, the environmental conditions must be carefully analyzed and evaluated so that costly over-design or under-design may be avoided and adequate performance assured. As an example of some of the work covered in analyzing just one of the elements of the natural physical environment, I would like to demonstrate some of the steps taken to assure optimum temperature criteria.

Temperature values from official stations always refer to free-air temperatures taken within the shaded interior of a slatted Stevenson shelter from 4 to 6 feet above a grassy ground surface. We know from experience in some parts of the world that the temperature at the ground surface may be far in excess of that experienced within the shelter. At Yuma, Arizona, in July, ground surface temperatures as high as 153°F have been observed directly under a shelter recording an ambient temperature of 115°F. The temperature that a given component or system may be exposed to in areas of cloudless skies, is one of the conditions a design engineer has to worry about and rightfully so, for temperatures of between 171° and 250°F have been observed on the surface or in compartments of equipment exposed to the direct rays of the sun.

Variation in type of soil, soil moisture conditions, vegetative cover, relief, and other related environmental characteristics of the environment cause significant variations in temperature from place to place. Consequently, the air shade temperature of a Stevenson shelter may not be typical of a given area even though it be located in the most suitable site from a meteorological viewpoint. For example, in the Fort Greely Test Area, Alaska, temperature departures in 27 shelters within a 9-mile radius of the FAA station at the Big Delta Airport showed variations of from 15 degrees colder to 20 degrees warmer during the normally cold period. At the temperatures experienced in this area during the cold season, such departures may be quite critical to equipment performance. Further analysis of station

*These temperature limits are presently under review.

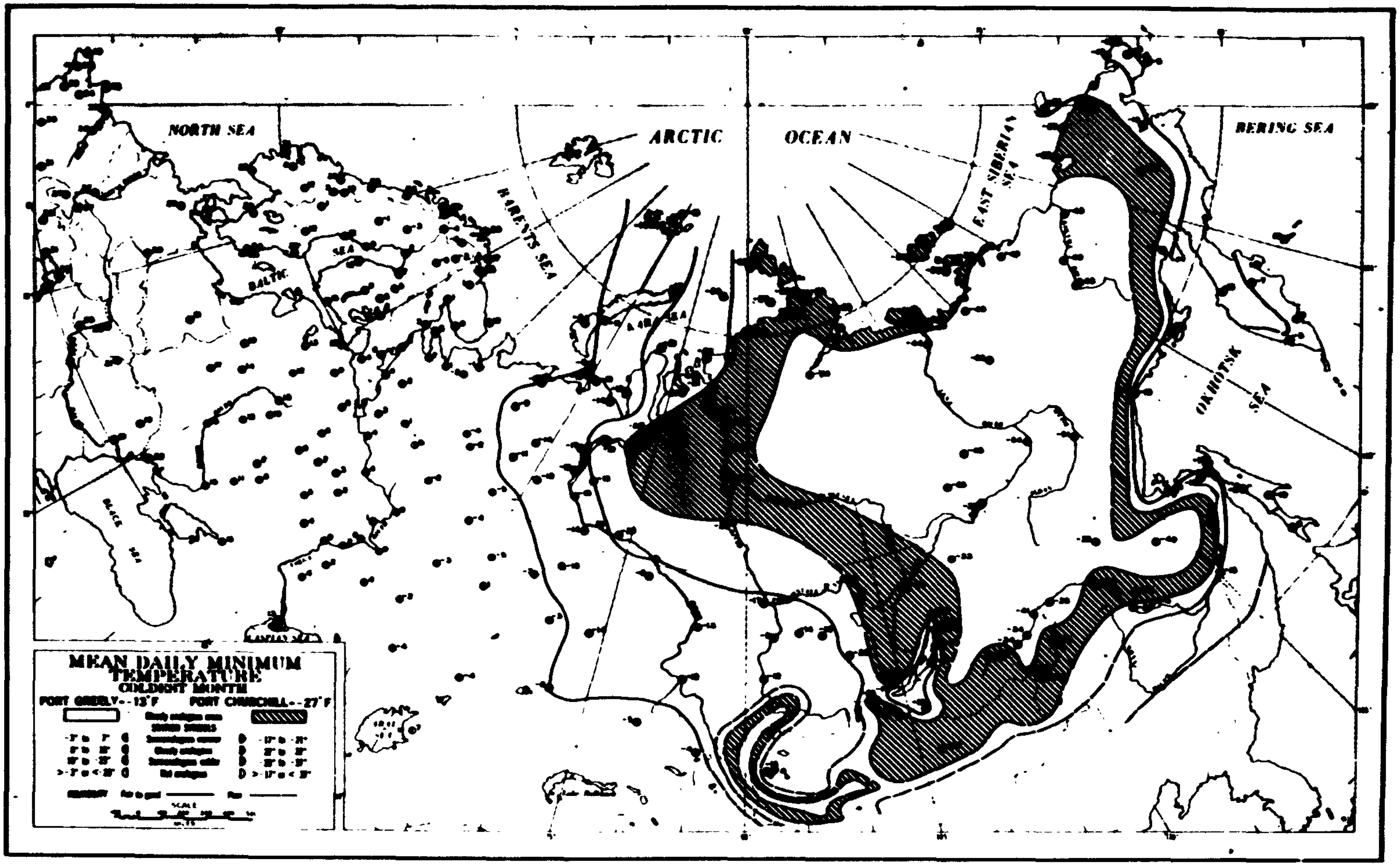


Fig. 13 - Areas in Siberia having mean daily minimum temperatures in the coldest month analogous to Forts Greely and Churchill in North America

data reveals that in some areas of the world, temperatures considered critical to certain materials may occur only once in 5 or 10 years. Such occurrences may require some means of assessing the risk of operation and establish this risk factor for application to design of equipment. In making an assessment of the conditions experienced not only is an appraisal needed of the frequency with which these conditions prevail, but how long do these conditions last and how often do they recur. By studying temperature patterns of frequency, duration, and areal distribution, much more realistic criteria can be established for use in design.

Some of the weather extremes that are considered in the establishment of environmental design criteria are shown in Fig. 14.

Application to Clothing Requirements — One of the more successful applications of environmental research has been toward the preparation of a world guide to field clothing requirements (Fig. 15). Data from field installations throughout the world and environmental data provided by the researchers are combined to produce a nine-zone map of the world to which all pertinent clothing requirements have been keyed. By reference to the map and to a few tables, the staff planner or field commander can receive in a few moments: (1) a brief description of the environment in which he expects to operate; (2) the clothing ensemble prescribed for this area; (3) the constituents of this ensemble; (4) the time to change the issue with season; and, (5) the camouflage color requirement for each area of the world.

Food Storage Guides — Based upon the thermal degradation rates established by the storage dump study at Yuma, Arizona and other research which has shown that under constant temperature conditions the deterioration rate doubles for each 18°F-increase in food temperature, it is possible by use of a weighted mean temperature to show on a map the relative food storage severity of different areas. Figure 16, showing long-term food storage life areas in the Near East, represents an application of this data to a specific analogous area. This guide consists of 16 pages of tables and 2 maps — one for short-term and the other for long-term (over 6 months) — on which is indicated the safe keeping time in months by area for every subsistence item handled by an Army Quartermaster. For example, canned pea soup lasts 7 months in Area I, 8 months in Area II, 12 months in Area III, and 16 months in Area IV. A device such as this permits more effective shipment logistics at a time when shipping space may be at a premium and more realistic storage discipline that should obtain the maximum useful storage life from each subsistence item and reduce spoilage from thermal degradation to a minimum.

Equipment Performance Maps — There is a great need for more and more precise information relating to the performance of materials and materiel to environmental conditions. Some equipment may fail when it operates above or below a critical temperature, but be completely effective when the temperature returns to acceptable levels. For such equipment, a map showing frequency of occurrence of critical temperatures becomes an indicator of probability of equipment difficulty. Figure 17 shows the percentage of time during January that the bazooka can be expected to perform satisfactorily in Northern North America. The distributional pattern indicates that this weapon should not be relied upon in Arctic winters when low temperatures cause the double base propellant to break up into many burning surfaces providing conditions for a buildup of high pressures which cannot be contained within the 3.5 inch rocket launcher. The black areas on the accompanying map indicate that this weapon cannot be used 75 percent of the time during January.

Gasoline Storage Life — Gasoline deteriorates through formation of gum which causes filter clogging and lowering of octane number. Although the initial gum content and the rate of gum formation differ widely, the effect of varying temperature on this rate is quite similar for all gasolines. The rate approximately quadruples for each 20°F rise in temperature. For a typical gasoline with inhibitors added, 5 mg of gum per 100 ml of gasoline might form in 12 months at 100°F gasoline temperature. Since this amount is enough gum to cause rejection for vehicle use, its storage life is estimated to be 12 months for this particular gasoline. Figure 18 shows effective gasoline storage temperatures in the Near East. The map has been prepared by weighting the effective storage temperatures so that they represent in gum degradation effect, the whole series of cycling temperatures experienced in a field storage season in 55-gallon drums at various places, which have been converted to storage life in months.

Atlases — A considerable variety of useful atlases are produced by the Quartermaster Corps — Atlas of Arctic Environment, Atlas of daily minimum temperatures, Atlas of surface temperature frequencies for North America and Eurasia, and so on. Figure 19, depicting the Glaciers of the U.S.S.R. Pamir, is a page out of

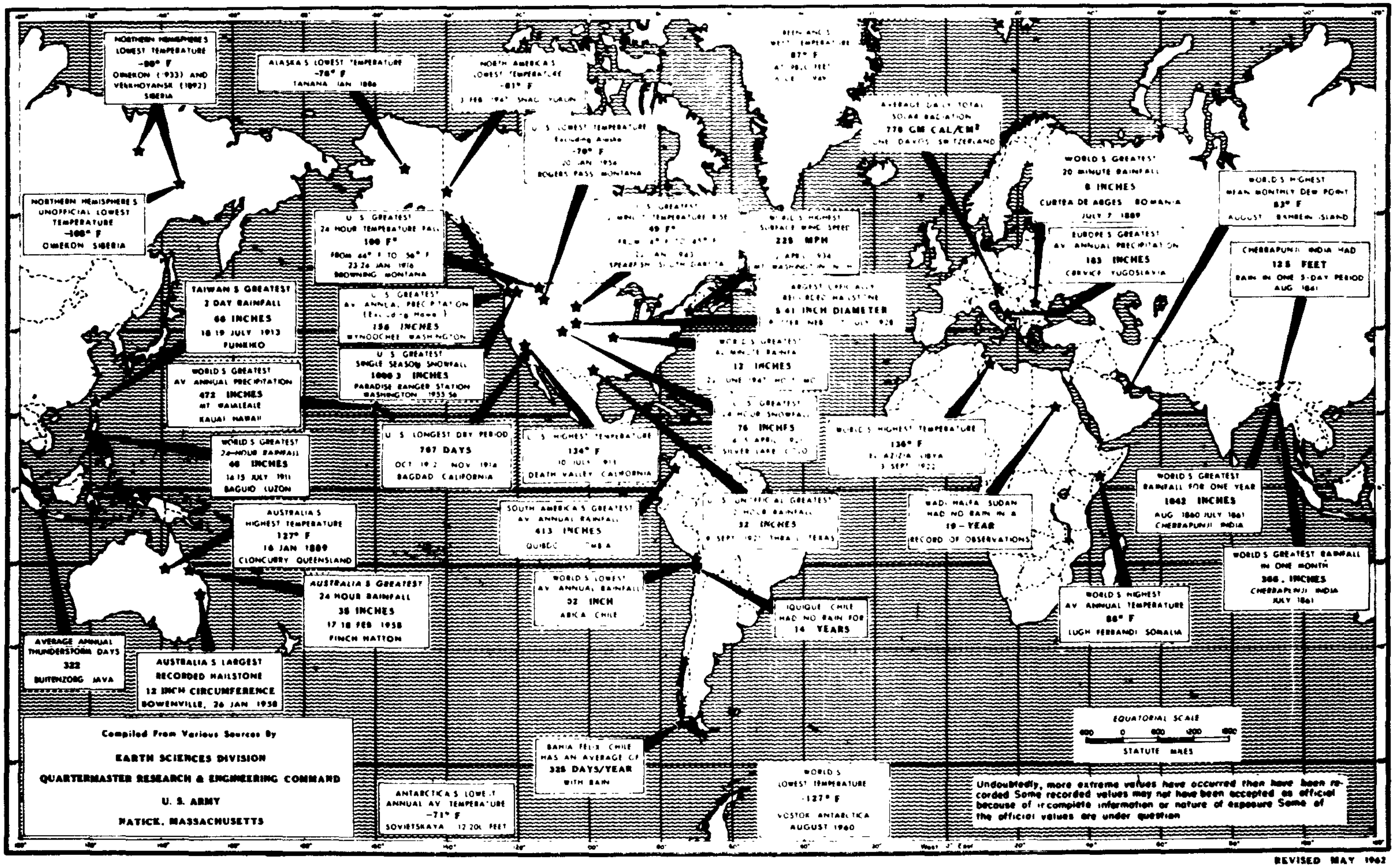


Fig. 14 - World weather extremes considered in the establishment of environmental design criteria

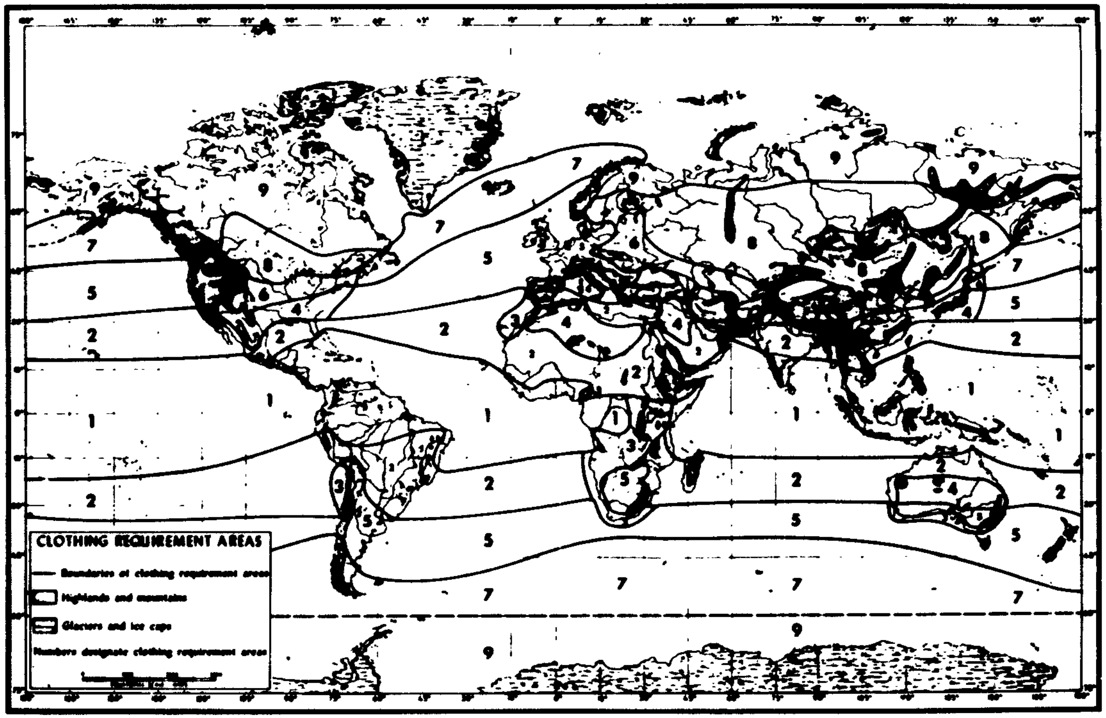


Fig. 15 - World guide to field clothing requirements

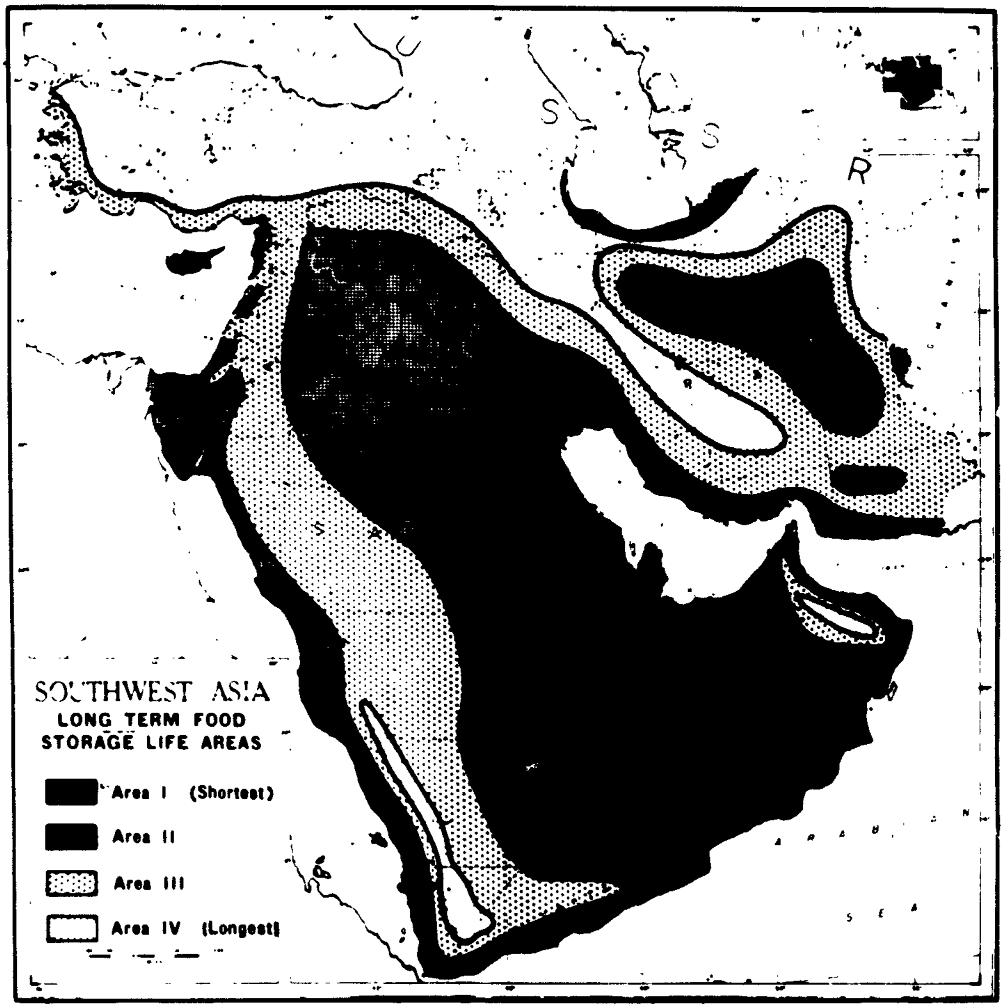


Fig. 16 - Long-term food storage-life areas in Southwest Asia

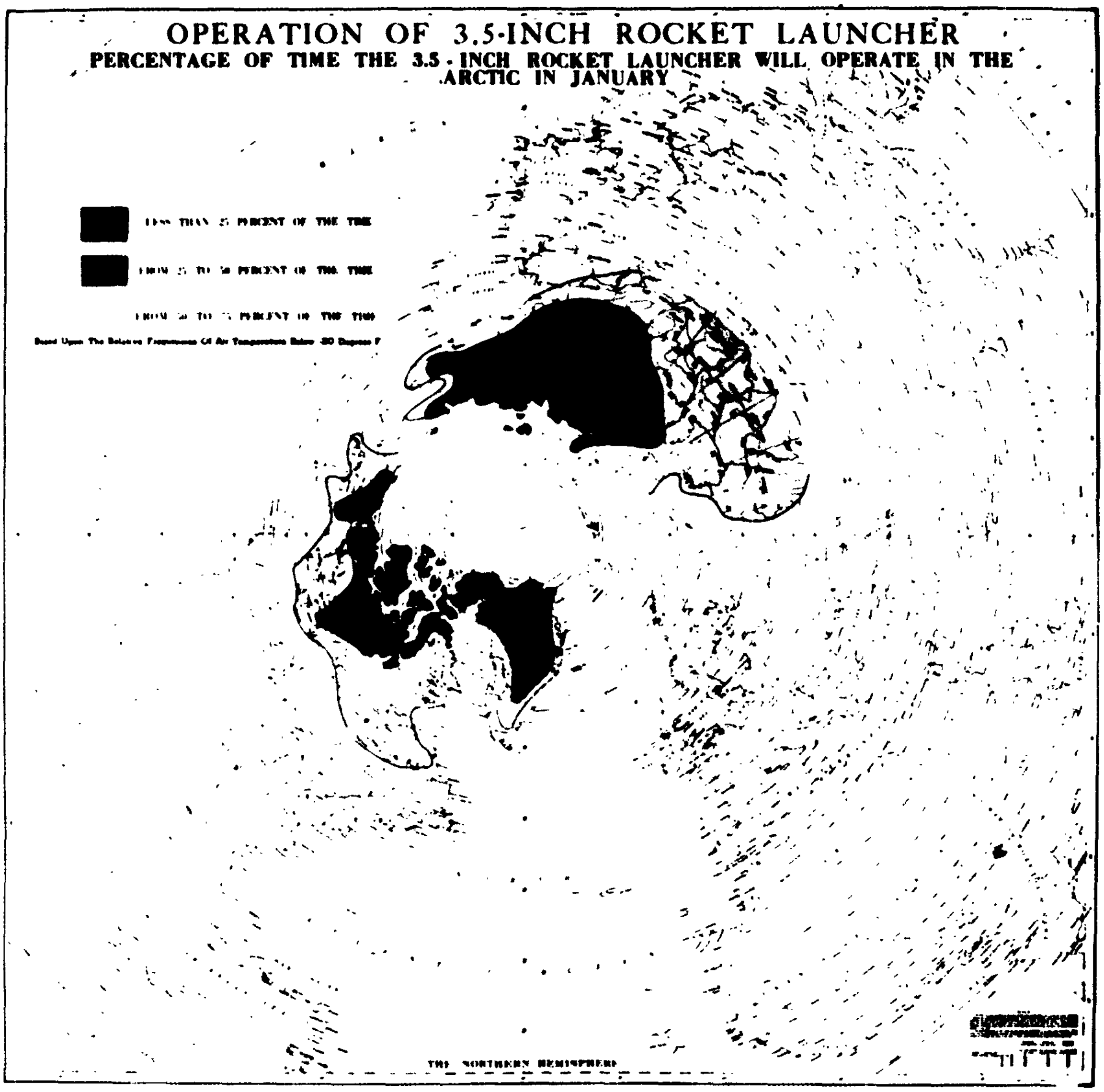


Fig. 17 - Operation of the 3.5-inch rocket launcher

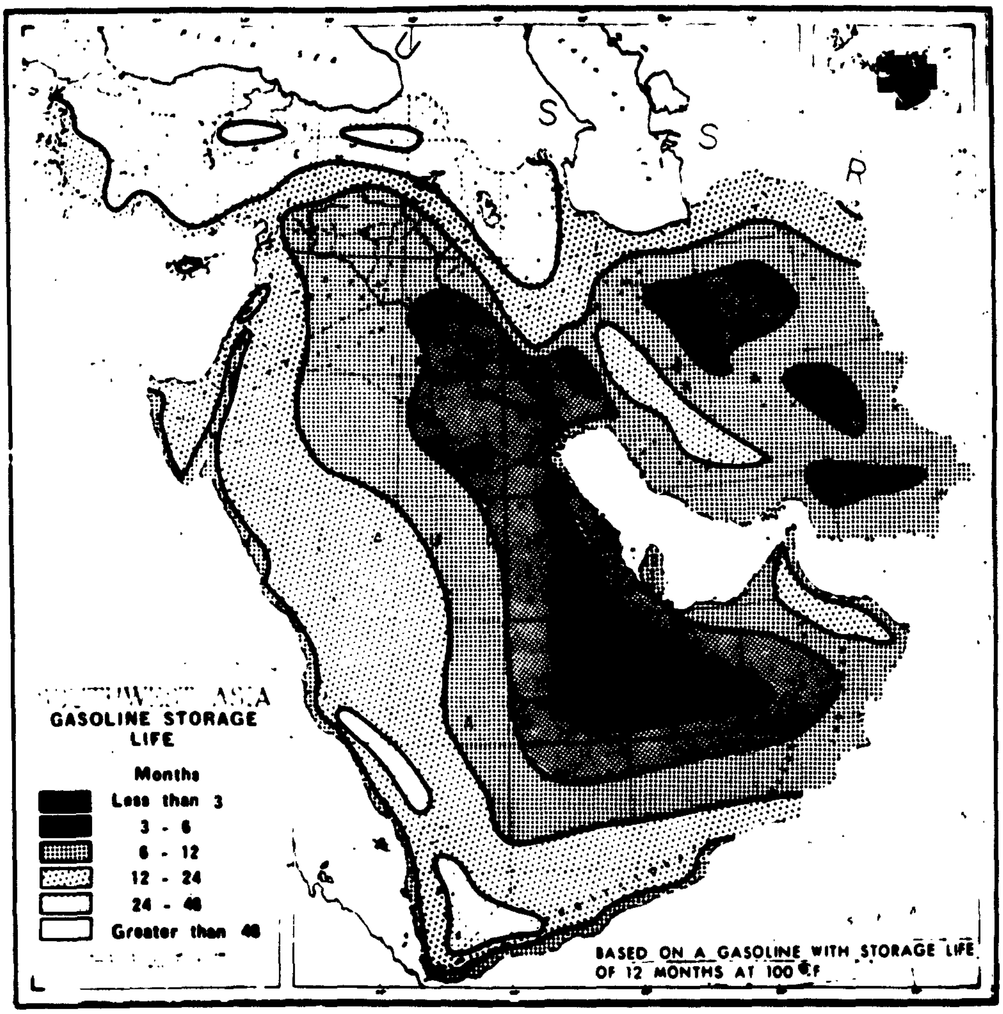


Fig. 18 - Gasoline storage life in Southwest Asia

the Atlas of Glaciers of the Northern Hemisphere produced by the American Geographical Society under contract to the Quartermaster Corps.

Army Support to Scientific Expeditions

In order to obtain technical information not otherwise available, the Army provides financial or logistical support, or both, to scientific expeditions sponsored by agencies, institutions, or groups outside of the Department of the Army. The Quartermaster Corps, acting as the responsible agent for the Army has provided both financial and logistical support to a considerable number of expeditions. This support has already yielded an abundance of valuable information for the Army from every continent, covering the subjects of glaciology, geology, geomorphology, entomology, transportation, meteorology, botany, and geography. Some of the groups which received or are receiving this support include: The Kon-tiki Expedition, The American Geographical Society, Arctic Institute of North America, and the following educational institutions — Duke, Minnesota, Wisconsin, and Washington.

SUMMARY

This paper has attempted to describe, in a general way, the comprehensive program in environmental research being carried out at the

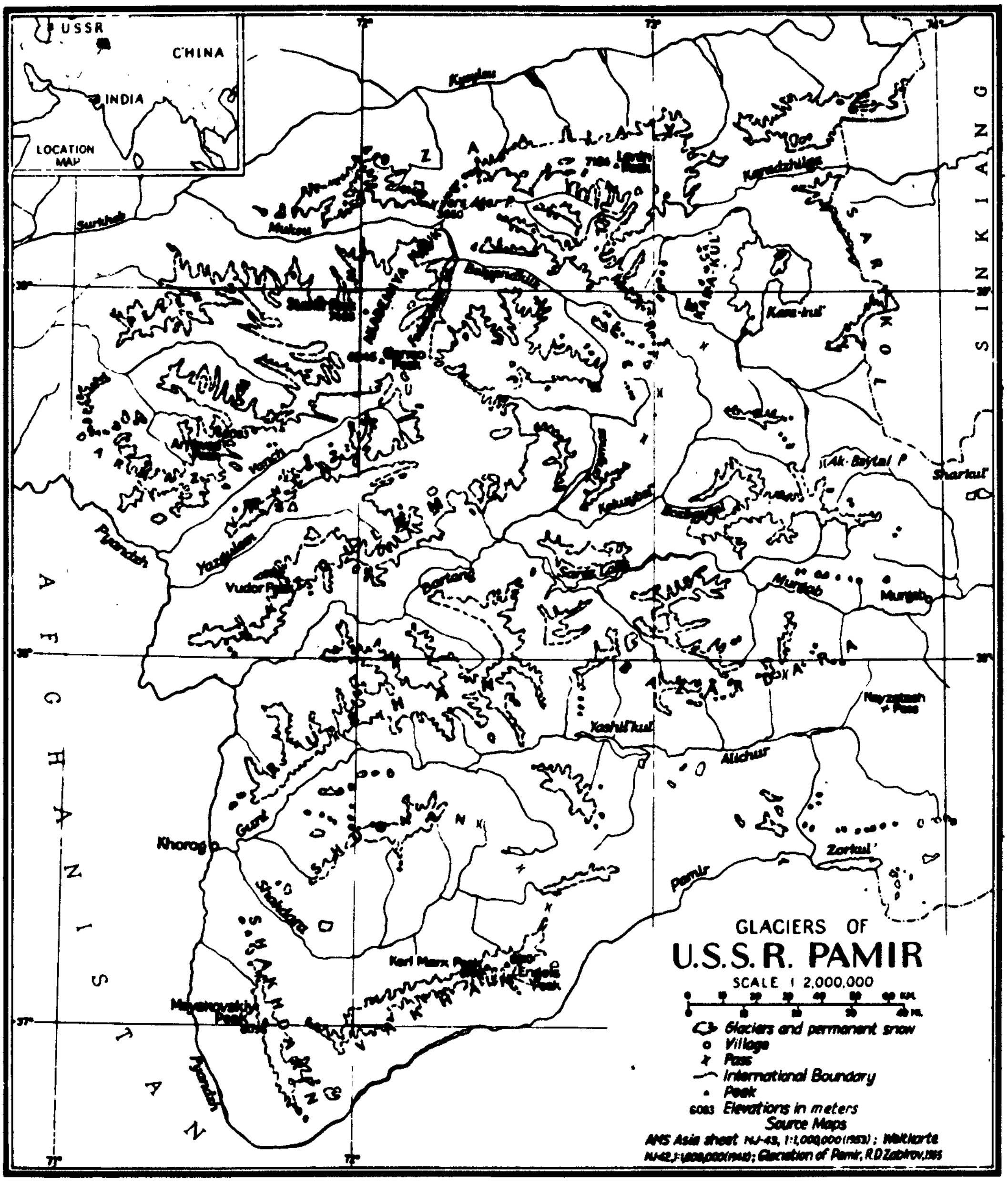


Fig. 19 - Glaciers of the U.S.S.R. Pamir

U.S. Army Quartermaster Research and Engineering Laboratories at Natick, Mass.

Since the Quartermaster laboratories has the responsibility for recommending environmental design criteria to the U.S. Army for adoption, it is requested that you (the readers) let your immediate or future needs for environmental design criteria be known to this laboratory in order that closer coordination between the three military services and industry can be carried out more effectively.

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Section 5
PANEL SESSIONS I AND IV

The discussions which took place during Panel Sessions I and IV of the 31st Symposium are printed in this section. These discussions have been edited for clarity. A few remarks have been omitted usually because they repeated points already brought out earlier in the discussion. For the most part the comments appear in the order in which they were made.

The Centralizing Activity would welcome suggestions regarding sessions of this type, particularly concerning subjects to be covered, the number of panel discussions that should be held at each Symposium, and organizational techniques that will permit the most useful exchange of information. All comments or suggestions should be addressed to Code 4021, U. S. Naval Research Laboratory, Washington 25, D. C.

PANEL SESSION I

THE PROBLEM OF DETERMINING THE SHOCK AND VIBRATION ENVIRONMENT IN VEHICLES

Moderator: Professor C. E. Crede
California Institute of Technology

Panelists: M. Gertel, MITRON
R. W. Hager, Boeing
E. R. Mullen, NADC
C. S. O'Hearne, Martin
C. Thomas, ASD

OPENING REMARKS BY PROFESSOR CREDE

"The panelists, in a preliminary meeting last evening on the problem of determining the shock and vibration environment in vehicles, were able, essentially, to agree on only one thing, that there are problems. We were not sure that we could even agree on what the words 'environment' and 'determining' mean in the context of the panel topic. If you have different notions of the meaning of these words, you are perfectly at liberty to correct the panelists' interpretation of them. We're not sure whether 'determining' is intended to mean measuring the environment or whether it is intended to mean the application of some interpretation to the results of the measurements so that the results could then be used for further work of one kind or another.

"We did ask ourselves a question — in our context of the words 'determine' and 'environment,' why would we want to determine an environment? There seemed to be two answers to that question which are more or less obvious. First, if we were able to determine the environment well, we should be in a better position to design equipment that would ultimately be used in such an environment. The other reason for wishing to know the environment is concerned with the problem of laboratory testing. It's said widely, and conceded by essentially everyone, that the purpose of a laboratory test is to simulate 'the actual environment' so that the test can be used to disqualify equipment that would otherwise fail to meet the required service conditions.

"Those of us who have been in this business of shock and vibration for a number of years recall one of the earliest efforts to devise a test to aid in the design of equipment to meet certain environmental conditions. This concerns the experience of the British Navy in the early days of World War II. It was characteristic of British ships, and perhaps other ships at that time, that large quantities of cast iron machinery were used in them. This was done without a very deep knowledge of the effect of shock upon cast iron. We think we are a little more sophisticated in this respect now, but back in the late 30's this degree of sophistication had not developed, at least insofar as cast iron was concerned.

"The Germans had, very extensively, mined the waters in the vicinity of the British Isles with noncontact mines which turned out to be extremely effective in wrecking the cast iron machinery, but not doing much visible damage to the ship as a whole. The Navy ended up with a large quantity of ships that would float, but which had equally large quantities of inoperative machinery in them. This was an emergency that required a very drastic effort in order to solve the problem.

"So they set out to develop a shock testing machine that could be used for the purpose of qualifying redesigned and newly designed items of equipment for use in this environment of a ship subjected to underwater explosions. They did it in a very practical way by purely empirical methods and succeeded in building a shock testing machine which, when used to test a piece of equipment similar to that which had been damaged in service, would break it up into the same number of pieces of the same size as resulted from the underwater explosion. This empirical method was actually the only method available to the British at that time. They had not the instruments that are available today nor did they have the time to use such instruments if the instruments had been available.

"Today many of us would not approach a similar problem in the same way. We would gather up a few hundred channels of recording equipment and a few hundred accelerometers, strain gages, and so on, and set out to measure the environment so we would know how to design our equipment and to devise a test. There is something to be said on both sides of the question. The British were unique in that they had large quantities of damaged equipment, but few instruments, whereas we have large quantities of instruments but small accumulations of damaged equipment in the same sense that it was available at that time. However, it is not difficult to envision that, if the empirical method were considered to be a good approach to the problem, we might be able to find ways of collecting the damaged equipment. This still might be cheaper than some of the other things that are being done.

"This was one of the problems that the panel discussed; we found that both viewpoints had some support. Mr. Gertel is a supporter of the empirical approach and he will tell you his views."

EMPIRICAL VERSUS MEASUREMENT TECHNIQUES

Mr. Gertel said, "We have to look at the shock and vibration environment in vehicles as one aspect of a three-phase problem. The total problem involves, first, defining the environment which, in a sense, defines goals for designing the equipment; second, understanding the mechanism of failure or damage to the equipment that will be exposed to this environment; and third, some analytical technique for predicting the damage, which will occur in equipment, from a knowledge of the environmental design goal and the equipment itself. The empirical approach, in a sense, lumps together the first two, namely, defining the environment and defining the damage.

"The rather classic example that Charlie gave of the empirical approach, wherein damaged equipments were available from the field for correlation with equipment damaged in the laboratory, represents an ideal and unusual situation. The empirical approach involves synthesizing the laboratory conditions which will produce the same damage as experienced by equipment under actual service conditions. The resulting synthesis of the environment may or may not look anything like the real environment. Nevertheless, once it has been established in the form of a laboratory test, and usually a specification and a machine, it then represents the design goal for people designing equipment.

"There are still examples of the empirical approach very much prevalent in today's testing and environmental determination. I think the most spectacular application of this approach involves some of these full scale field tests that the Navy undertakes on almost a regular basis. They subject actual vessels to nearby underwater explosions so as to observe the effects of these real environments on equipments and, if damage results, they compare this with what can be produced in the laboratory. In a less spectacular sense, the requirements that we have in MIL SPECS define the environment for us and these are all rather empirical. There aren't too many of them that look exactly like the environment. In a sense the sweep frequency test is a rather empirical test. There is no exact environmental condition that this represents, yet the principal virtue of a test of this nature is that it can successfully screen out equipment in the laboratory and insure that if it passes this test, its likelihood of surviving in natural service is very good. I think I will hold on that point and wait for some rebuttal before I throw in some more."

Professor Crede remarked, "We, on the panel, want to follow the practice of one business reporting service that attempts to size up the condition of the economy each week by reporting favorable factors and unfavorable factors. They are always very careful to have three favorable factors and three unfavorable factors so that no one will get the impression that the economy is either on the way up or the way down. In order to keep our panel equally balanced, I think it is time that we hear from the other side of the picture and Mr. O'Hearne."

Mr. O'Hearne said, "When I was asked to take one position or another on this question it was very easy for me to take this so-called measurement approach as opposed to the empirical approach. In fact, until dinner time last night I was ignorant of the existence of a so-called empirical approach, at least by that designation, though some of the other members of the panel seemed to have heard of it as a concept. So in taking the side of the measurement approach I'm particularly going to attack the empirical approach because, in this difficult work we are doing, it is so much easier to criticize than to be creative.

"When the empirical approach was first described to me I felt a natural antipathy to it, primarily because it struck me as one of these simplistic ideas which some people tend to grab for. In their own terms these people are striving for the simple, practical, unsophisticated, and direct method. Now, superficially, you can't criticize that, yet we are really dealing with complicated situations. These people have very simple guidelines. They say, 'If it waddles like a duck, has feathers like a duck, and quacks like a duck, it is a duck!' What could be simpler than that? That is a very excellent guideline until someone comes in with a goose or some other water fowl. In the problems that I've seen, these geese keep flocking in; there are always other questions coming up which the simple approach isn't prepared to answer.

"The empirical approach, as it has been called, is that you find a failure in a piece of equipment and try to duplicate it in the laboratory. Let's say that you have some criterion for saying what is a duplication of the failure. You duplicate the failure and then you have something with which you can test your fix for this failure. You modify the particular piece of equipment so that it doesn't fail under these circumstances and then you test it again and, if it doesn't fail, you have the fix. But, you may have tested it with one hammer blow and now you test it with two, three, or four and it does fail, only it fails some place else. Now you don't know whether it would have failed under four blows of the original environment if it failed under four blows of this laboratory environment. Saying there is no variance in your damage from the four blows is oversimplifying the question, a simple example of another goose that can come walking in or another question that can be raised for which these simple approaches have no answer.

"I have lately come to believe that we have to get more sophisticated in our approaches, and we have to do this in a more rigorous way. The thing that brought me to this belief is that I've been thoroughly confused by the literature. Practically everything I read leads me to ask more questions than the author has answered, or else I'm asking questions to which he hasn't directed himself at all. I think the reason for this is that we don't have, as one does in mathematics, an axiomatic structure to which everything can be referred. We are flying by the seat of the pants and I think that is something that has to be improved."

Mr. P. Marnell of Technik, Inc. asked Mr. Gertel, "Is the empirical approach a trial and error procedure to develop a method of breaking up the equipment in a manner directly analogous to the equipment breakup under the actual environments? If this is so, how can one be sure that there is a uniqueness between what you have been able to derive as an equivalent laboratory test? How do you know for sure that your laboratory test will always be in direct correspondence with the environment?"

Mr. Gertel responded, "In a sense the empirical approach is a cut and try method. It's really a synthesis of some potentially arbitrary condition which will produce a certain damage. There are semianalytical techniques that go along with this approach such as considering the response of systems to the empirical test. The empirical test, after it has been defined or resolved, then represents our environment. We can now subject this empirical test environment to all kinds of sophisticated analyses and attempt to design our equipments to this environment. We could subject many single degree of freedom systems of different natural frequencies to the input defined by the test environment and the resulting spectrum will give us some indication of the damaging characteristics in the sense that we will get the maximum responses of these individual systems. If we also had the corresponding shock spectrum of the real environment, we would have a really good technique for comparing the two environments.

"We spoke about having damaged equipment, but we have to visualize the damaged equipment in terms of a shock spectrum which might have produced that damage. In the more common cases that we run across today, we don't really have the damaged equipment for correlation in the laboratory. That circumstance, the derivation of the Navy high-impact shock machine test, was rather unusual. We do attempt to correlate effects of environments in the field with effects of environments in the laboratory."

Mr. Mullen cast his vote as being strongly in favor of the empirical approach. He cited as his reasons the fact that, in measuring the environment, we frequently do not know what we have measured and, even if the measurement were perfect, it could not be duplicated in the laboratory. In attempting to duplicate a long term environment with a short term laboratory test, he said that the damage produced in one case must be correlated with that produced in the other. This is what the Navy has been trying to do at Johnsville.

Mr. R. Volin of NOL asked Mr. Gertel whether the empirical approach was not a negative approach, since testing is to assure that equipment will survive the environment, not to produce damage. He also asked how we would know the equivalent damage under combined environments.

Mr. Gertel answered, "In a sense, the damage approach is a negative approach. Ideally, we would like to have a laboratory environment which would produce the same kinds of damage and malfunctions that would occur in service, but at the same time not be so severe that we have to overdesign our equipment. This makes the process of design extremely difficult. It's impossible to achieve the ideal of having the one horse shay that lasted one hundred years, the perfect test that will exactly qualify an equipment for an environment. There is no such thing as a single environment for which we are striving to design. Equipment, in general, probably has to withstand environments of several different kinds. The same equipment potentially can be used on many different aircraft, in missiles, in ground vehicles, or even in ships. Consequently, we can't get this optimum of having equipment which will exactly meet an environment but not be overdesigned.

"Insofar as the combined environments are concerned, this is a difficult question to answer. It is easier to answer in retrospect than when you are looking ahead into a combined environment. After you've been through a combined environmental program and have seen the results of it, you can distinguish which combinations of environments really mattered. For example, did the temperature in combination with vibration really make the failure occur sooner or later? We don't have enough information on the effects of combined environments really to speak with much authority on this. I think this is a perfect setup for the empirical approach in that we have to approach this thing in the laboratory, see what happens when we combine different types of environments, and then make decisions as to which combinations are important."

Professor Crede remarked, "Perhaps the most significant thing that has been said so far this morning is a comment by Mr. O'Hearne that there seem to be many more questions than answers. In this last exchange of information there were two complicating problems referred to. One was the matter of the combined environments; the other was Mr. Gertel's statement that very often an equipment has to operate in several different vehicles and it is necessary to design it so it will withstand each vehicle's environment, which means that it must be designed so that it withstands all of them together. In my mind these are some sort of higher order problems. If we had to design one equipment to withstand only one environment in one vehicle, I'm afraid the state of the art still falls far short of telling us how to do that. I think that the discussion will remain more to the point if we stick to the type of questions that we know a little bit about rather than drift off to those about which we know practically nothing, such as the effect of combined environments."

Mr. C. Colaluca of Republic commented, "I certainly agree that there are two approaches to the problem, one mentioned by Mr. Gertel and one by Mr. O'Hearne. In both of these instances you assume that you have the actual vehicle to run field trials and you know what failures have occurred, even in the measurement aspect. In order to make measurements you must have the actual hardware vehicle. I think there is a third problem, when you don't yet have the actual vehicle; you just have scattered components that you want to qualify to a specific environment. What do you do in this case when you can't take measurements or you can't duplicate field failures? You are in the R&D stage now and you don't have hardware.

"In defense of Mr. Gertel, at Republic Aviation we have used both approaches. When we have used the approach propounded by Mr. O'Hearne, we have simulated the actual environment, but we have not caused the failure that has taken place in the field. Invariably the designer has come to us and said that the item didn't fail as it did in the field, therefore we have run our test incorrectly. Even when we have had actual measurements, we have been criticized for not creating the actual field failure. I don't know what the solution is, but I think both solutions should be considered, that is, trying to duplicate the failure and using the actual recorded inputs or maybe modifying the recorded inputs slightly to create the field failure."

Professor Crede said, "The first point that the speaker made concerning the prediction of the environment in vehicles that have not yet been built seems to be a very important point in this entire discussion. Perhaps Mr. Hager has some thoughts on that point he would like to present."

Mr. Hager commented, "I think we are all probably faced with this requirement with the tremendous advances that are made from one system to another. I know that, particularly with the aerospace companies, we seem to hop from one program to another, and there are some intermediate steps that are missing. However, the knowledge that has been gained from measurements, and I'm a very strong advocate of this approach, can best be used when you also know the changes that are going to be made in your system relative to the original system or some other comparable system. The only way that you can use this approach is by having the measurements and by having sufficient analytical capability to predict the changes that will occur from one system environment to the other. The problem is not one of a bunch of independent components that you have for a system from the R&D standpoint. You still have a system which has an immediate mission or an immediate environment. Our problem here is to establish the environment under which the total system will operate and then, by analysis, break it down into the local environments that each component will see."

Mr. Schwabe of Lockheed asked Mr. Gertel, "Does not the empirical method, which you propose, have the tendency of overdesigning any missile components in weight and size? Doesn't this method tend to cause the designer to design any package or system to pass a test rather than to comply with the design criteria or requirement?"

Mr. Gertel answered, "The empirical approach, as applied to your particular types of problems in missiles, is perhaps a little more difficult to apply than in other instances where the vehicle comes back. There is a tendency for overdesign, and I don't know of any way around it other than to say that, with that overdesign, if the system performs successfully we at least have something on the plus side of the ledger.

"I think your second question pertained to specifications. It is very difficult to tell anybody to design a piece of equipment to withstand a flight to the moon, or something of that nature. We have to provide the equipment designer with something tangible which means giving him amplitudes, frequencies, or some quantitative definition of the projected environment. The easy way to do this is to give the environment in terms of a laboratory test specification because this gives some substance to the flight to the moon, or whatever the ultimate mission is. True enough, once it's in a laboratory specification, the problems become very legalistic as to whether the system withstands the test spec input or not. Sometimes we get carried away in meeting test specifications, losing sight of the real, overall objective and of the fact that there are certain empiricisms involved in some of these specification test procedures."

Professor Crede said, "It seems appropriate at this point to get straight what we mean when we talk about empiricism. I think we are all aware that the empirical approach to the design of equipment to withstand laboratory tests is widely used. I'm sure that you've all had the experience of hauling your pet equipment into the laboratory and seeing it busted up on a vibration or shock machine; and then, largely by cut and try, of remedying the defects so that the equipment ultimately passes the laboratory test. This begs the issue in a very fundamental way, because it only raises the question of what is the relationship of laboratory test to the actual field environment. This is what is so difficult to determine. As Mike says, the missile doesn't come back and thereby we lose what might be a very vital piece of information in evaluating the performance of the equipment. We can only infer what happened from what we can determine from telemetered data, which sometimes it not what we would like to know."

Mr. T. Connare of Erie Army Depot asked, "Aren't we really talking about two different types of fundamental testing here? Probably empirical testing would be all very well for something in production, but if we've only got a pilot model we certainly can't stand to break it to the extent inferred by the empirical approach. Once we get into production I think it might be the better of the two methods, but while we're still in the R&D phase I don't see, in our own case, that we could ever adapt to it."

Dr. J. Bogdonoff of Midwest Applied Science commented, "It seems to me we are confusing two points here. I don't think there is an argument. We have a problem which is to design a piece of gear which will survive a certain kind of environment. It's rather obvious that the environment is built up largely of chance occurrences. It doesn't happen to be something we can be sure is going to occur this time or the next time. One of the panel members spoke of subjecting a piece of equipment to a specific environment. There is no specific environment; there is a whole class of environments. You don't know what the next one will be. We have two approaches, but isn't it really the point that we have to use one in one situation and one in another? They apply in different sets of circumstances. If I have to come up with an answer very quickly to a rather specific question, I would be inclined to use the empirical method right off. I would not fool around and try to develop a class of information about a stochastic process. It is known that this is going to cost a fortune. On the other hand, if I were interested in a fairly long range point of view, had the time and money to accumulate data, and would like to make statistical predictions, then I think I would favor the other point of view. I don't think there is any argument. They apply in different circumstances. We should accept this and merely discuss the advantages of one versus the other, rather than one's no good and one's good."

Mr. Lynn of Lockheed commented, "I'm completely in agreement with the empirical approach and we are more or less forced to operate in that manner. We don't get the things back and sometimes the measurements that are sent back are not so complete. One of the troubles is that designers don't have any feedback from the laboratory nor from the services as to what constitutes rough handling in transport, manufacturing, storage, or even testing. We have seen a number of cases of testing which destroyed material in advance of its use or caused damages which resulted in eventual failure in operational phases. Somewhere out of the searching that we are doing we've got to come to a set of numbers for handling phases. Until that time the designer is just going it blind and he's got to overspecify. When he overspecifies, as a general rule, he will take us into expensive areas of testing which are complicated further by the fact that the manufacturer, in designing, is going to have to find ways of overbuilding products so that they will meet the test. So that what our plea would be to testing engineers is to give us general numbers. It doesn't matter whether the numbers that we get are too big; we can always back off from them if we have to. We are faced every day with the fact that a designer doesn't know what numbers constitute a set of criteria that should be given for any given phase of the life of any given product. I wonder if the panel could help to direct the thought toward getting some general numbers rather than methods of testing?"

Mr. Thomas said, "Mr. Lynn has covered two points here that are a bit touchy as far as I'm concerned. In particular, he says that the designer doesn't get the feedback from the services which he needs to establish these conditions in the laboratory. I would like to remind him and everyone else that this is a problem which cannot be met in its entirety by the services. In particular, we are trying to accomplish something in this general area and giving general guidance as he has indicated here. One of the latest attempts in this area is the U.S. Air Force Document MIL STD 810. The general purpose in this case is to give design criteria which we feel are goals for this sort of operation. This particular type of effort is only as good as the inputs to it, and we need a great deal more input from people such as you to make this, or any similar document, fulfill the purpose for which it was intended.

"I feel, as a couple of these fellows have indicated, that there are two sides to this, and we can't get along with only one of them. So, before you take such violent exceptions to some of the specification requirements, you might go back to the guy who is trying to come up with this requirement. What he is really trying to do is give general guidance in designing equipment. We feel that some general requirement is better than none at all."

Professor Crede said, "The last two speakers inferred that if they had some kind of general numbers available to assist them, it would be useful for design purposes and they would then have the situation well in hand. I've been in this business for a long time and I'm still looking for the right parameters to use, that would serve as these general numbers, that could be the basis for design and testing procedures. I wonder if either of the last two speakers from the floor would be willing to amplify their comments in that respect. I think the discussion might become a little more pointed and perhaps the panel could make some comments that might be a little more useful."

Mr. A. Paladino of BUSHIPS spoke for the Navy pertaining to standards and specifications. Against the arguments that specifications were not realistic and it was too expensive to break up equipment, he said that it was too late to find out that the equipment did not work after it was installed on a ship. Referring to vibration specs and MIL-STD-167 in particular, he stated that equipment resonances were primarily the cause of failure. For example, for 167, any equipment with no resonance below 25 cps would pass the test, but with a resonance in this range it would likely not pass.

Referring to the statement that the government did not give forth data, Mr. Paladino said that in the specification they had attempted to give levels representative of shipboard environment. This did not mean that these levels were representative of any special location on the ship since they could not specify location. He concluded that a missile designed mainly to survive a flight, for example, was likely to run into difficulty while waiting on a ship for the time of launch.

In answering Professor Crede's request for amplification of his remarks, Mr. Lynn spoke as follows: "In one particular incident we went to at least 25 different manufacturers of test equipment which we use a great deal and asked them for numbers that they would consider would surely destroy the material that they were delivering to us. No one would give us the numbers that would damage the equipment; not in terms of g, or cycles per second, or anything else. We went to the vehicle manufacturers and asked them if they would give us some numbers which we could use as a satisfactory range for transport vehicles, those which we could expect in the course of transport phases. We got no results at all. They said that they knew lots of ways of measuring it but they couldn't give us any numbers. We've also gone to the railroads and asked them what constitutes a set of numbers for transportation phases. They were a little better and gave us the number of miles per hour, but that was about the end of it. We found that if we made measurements we didn't get any numbers because there were too many variables. We got little more from package manufacturers and others who were supposed to know about those things.

"We went then to the services. I think your air people have gone as far as they possibly can and have sincerely and seriously for the last 8 or 9 years tried to find some answers to this question. I also think that, in spots, they don't even know what their instruments are doing and are trying to find a means of righting the instrument so that it will give some repeated readings in the same types of environment. We are now in a reliability phase of this missile business. Customers are pressing us for a year of life in the air in an environment that we don't know very much about. We have to assemble millions of components into these missiles and they've got to be 100 percent reliable. We've got to build these systems and we've got to have numbers. We haven't been able to get them by any route that we've taken so far, so what I'm asking for is some advice, really, about the route to go. I don't know what route to advise my people to take without building up a lot of expense. We don't want to do that if it is possible to get the answers at some one place. Of all the places I could think of, this would be the best."

Professor Crede asked Dr. Bogdonoff, "Do you have any words of wisdom as to where we might get some numbers and what kind of numbers we are looking for?"

Dr. Bogdonoff said, "To answer each and every one of your questions, the answer is no. Let me point out that anyone who seeks a number in a situation which is governed largely by chance is looking for something which he isn't going to find. This just happens to be a view which is shared by a few of us who have been working rather seriously in this area for the past few years.

"I am reminded of a meeting in New York City, the annual ASME meeting last November. A gentleman gave a long and learned paper on the response of beams to random excitations of various kinds. He arrived at variances for hosts of beams, models such as the Bernoulli, Timoshenko, the modified Timoshenko, and so on. He ended up with variances for maximum stress. The problem is trivial since what he was talking about wasn't what anyone was interested in — this is the difficulty.

"What is the criteria by which you can describe a failure? As far as I know, under no condition in a beam case, except for one rather simple situation, fatigue, which unfortunately happens very rarely, is this practical. Variance is a description of one statistic. If the material only knew that it had to fail according to that statistic, we would be in, but it doesn't. We haven't found a single statistic yet, and I doubt there is one in this particular case. I do feel that by a fairly long range program such as we are endeavoring to carry out at a center we have established at Purdue, we may be able to guess those statistical aspects of a random process or function which contribute to failure. Again, there is no number that we know of and I doubt we'll find one."

Dr. J. Bendat, an independent consulting mathematician, spoke. "The problem of determining the shock and vibration environment in vehicles, which is the subject of this panel, lists a number of questions that they are supposed to be discussing, all involving the problems of measurement and analysis. I disagree rather strongly with one of the panel members, I think it was Mr. Mullen, about the fact that we do not know what we are measuring. I also disagree, not so strongly, with one of the speakers from the floor about the expense that's involved in carrying out a rather extensive analysis of random processes. What is required, of course, is a great deal of thought in advance on the design of the experimental program that is to be carried out so that there is proper weighting given to all of the different measurements that might be conducted. Power spectra is not given any special elevated role but takes its place as only one of several conditions. One should be concerned about the location of the various transducers and the correlation of information from point to point so as to come up with an understanding of the environment over an extended structure as opposed to what is merely going on at a single point. One must be concerned about measurement uncertainties as a function of the various parameters that are available in the data. One must be concerned about trying to verify, if possible, any assumptions that one makes which are necessary for later predictions. If these considerations are borne in mind, then it is possible to make a great deal of sense out of measurements and it is possible to be able to predict the shock and vibration environment in a proper scientific way where you put some confidence limits on the measurements. These are steps that are being taken today by many enlightened groups throughout the country, certainly there is a great deal more to be done, but in the words of our illustrious President I say, let us begin."

Mr. S. Grabowski of Detroit Arsenal commented about the two approaches. "With the empirical approach we can probably describe damage qualitatively, whereas, with the laboratory approach we can very readily describe the damage quantitatively. Going further in laboratory work we can probably determine not only how good or how bad something is, but we can start putting in upper and lower limits by using the statistical approach."

Mr. Galef of NESCO said, "I would like to take mild issue with some of the remarks of Professor Crede, who said that bringing in combined environments is just complicating things hopelessly. Actually we always have a combined environment. We have vibration in the y direction combined with vibration in the x direction and in the z direction. Sometimes there are certain correlations among them; sometimes there's no correlation. I don't think you're ever going to reproduce these on a machine even if you knew them. Therefore, we're necessarily reduced to the problem of defining some kind of a test which will do something like this combined environment. We can never reproduce it."

Mr. M. Stansbury of Mare Island Naval Shipyard commented, "Mr. Lynn of Lockheed did indicate one phase of my particular problem. The Navy has MIL-S-901B and very shortly there will be a C modification which will 'up' shipboard environment for shock. However, before the machinery gets on the ship, in our case submarines, we are getting delivery of equipment in a damaged condition. We have put Vidot indicators on equipment for transport from wherever the vendor may be. We can set this for a given g value and, if the little balls are upset, we know that the input has exceeded this. I have several questions, all integrated. Do we have any packaging criteria? Do we have any transport inputs whether by truck or by train? How can we assure ourselves as time goes on that we do not receive damaged equipment, have to retest it, and perhaps return it to the vendor? I would appreciate some inputs there because it is a very real problem to us."

Mr. Hager answered both Mr. Lynn and Mr. Stansbury. "We have been subject to the same problems in the design of the MINUTEMAN Program. I would like to restrict my comments, not to ships, but to the road and rail transportation. We asked the same questions that Mr. Lynn asked. We got the same answers.

We had to solve the problem with our particular transport requirement, so we started where these people left off. They could tell us the speed of impact, say for rail car. At this point we had an input with which to start; however this requires, as one other speaker mentioned, a very complex analytical approach to resolve the loading problems. In most cases your problems are the result of load or force which is what damages your equipment, not necessarily frequency alone. I only know of a few instances where the frequency content alone is responsible for the failure. It's the response of the system to the input and the force which it develops.

"What we have done is to produce an analytical model comparable to our complete system including rail car, coupling mechanism, cargo, and right down to the important component with which we are concerned. After we have modeled the system adequately, we conduct an analysis whereby we effectively impact the system into what is a realistic set of rail cars that you might find on a siding. From this we can adjust the isolation characteristics to keep our loads within limitations. We have gone through our program far enough in this respect that we've actually conducted coupling tests and verified our analytical approach. In regard to rail travel, we are using what we consider a basic input environment which, in this case, is the axle motion. In this particular instance we then have to duplicate the rail car characteristics from the axle on up to the particular component with which we are concerned. Today we have both the digital and analog computer facilities with which to conduct this type of analysis and it is high time that we did so.

"In road transport the problem becomes even more complex. Here we can't use the axle environment since the axles are free to bounce from the ground. What we can do is to go to the basic input environment which, in our opinion, is the road roughness. There are, of course, problems in determining road roughness, but I think our efforts should be directed toward determining more of these basic environments and making better use of the analytical capabilities that we have today."

Professor Crede asked, "That's an interesting comment, Mr. Hager. I wonder if I could ask you if your mathematical model is a lumped parameter system?"

Mr. Hager replied, "Our mathematical model is, in general, a lumped parameter system; however, for certain beams we are able to use a distributed characteristic. We attempt to lump our problem in a manner which would be the best from the standpoint of the particular component or portion of the system for which we are interested in solving the load problem. If it requires a distribution, say in a beam, we can do this. If a lumped parameter approach is adequate we will use this approach."

Professor Crede asked, "How many degrees of freedom do you find that it takes to accomplish this purpose?"

Mr. Hager said, "Our approach so far has been to use the CEA analog, wherein, we can duplicate various portions of the system; various mechanical and structural elements can be replaced by analogous electrical elements. In this system, the number of degrees of freedom could approach as many as 40. Of the range that we're most concerned with, something on the order of 6 to 8 modes is sufficient to describe our main structural loadings, say in our missile or in our support structure."

Mr. A. Cohen of Sylvania, referring to the discussion on empirical versus measurement techniques, felt that both were used. In some cases the test method was determined by an analytical technique, then the testing was done to empirical criteria.

Mr. Cohen reminded designers who expected to have a set of conditions handed to them on a silver platter that there were several useful documents available. He mentioned the Handbook of Environmental Engineering and MIL-STD-810 as examples and suggested that 90 percent of the problems did not involve extreme or unusual environments, therefore the information in these documents could be used as guidelines.

Mr. Cohen also suggested that the panel return to the stated purpose of the session, and said, "I would like to get some answers to some questions on impedance measurements and on determining the number of measurements on, let's say, a tracked vehicle which will give us reasonable inputs to the equipment on this vehicle. We should go into the problem of selection of transducers and where we should place these transducers, in particular on a tracked vehicle to find out what excitation electronic equipment will see in a tracked vehicle application."

Mr. O'Hearne said, "This gentleman just pointed out that there are sources of data to these packaging and transport environments: the Handbook of Environmental Engineering, the Shock and Vibration Handbook, and all the reports available through the ASTIA library, for example. The difficulty with this data is the definition of it. This goes back to what I was pointing out earlier. We haven't got a rigorous system of definitions and relations for defining our environment. I think that what we have to do is to understand the foundations of our physical phenomena in probability theory. It seems to me it is going to be the only way we're really going to know what we are talking about. The environment is a random process and we have to think in probabilistic terms. Even when we're limited by economic considerations, we have to have an idea of what we are doing when we simplify and how much information we've lost in the process of analysis."

Professor Crede asked Mr. Cohen, "Did you wish to pursue the subject of mechanical impedance? You included that among many things that you said. Could you be a little more specific since it's a broad subject?"

Mr. Cohen said, "No, I simply included it as one of the items which was listed on the agenda. In a discussion such as this, or any discussion at a symposium, we cannot hope to cover all problems in detail. We must generalize, naturally. But I think at this point we are generalizing on a particular subject. We should be generalizing more on instrumentation and on determination of the environmental conditions in vehicles, rather than determining the principles of how we should approach this instrumentation in the first place. I think we've agreed, as I said, that we do require instrumentation and that we do require analysis of data. I now propose that we investigate the methods of determining how to acquire this data and perhaps what we do with the data when we get it."

Professor Crede said, "Presumably you must make a distinction between the words measurement and determination. I would infer that the end result of a measurement is perhaps a plot of acceleration as a function of time at some particular point in the vehicle. Whether that constitutes a determination of the environment or whether that constitutes only the first step in determining the environment I think depends upon the interpretation which you give the words. I don't want to drop the subject of impedance. Does any member of the panel or any member of the audience have some comments along those lines that might be relevant in the present context?"

MECHANICAL IMPEDANCE

Mr. S. Burgwin of Minneapolis-Honeywell commented, "I'm interested in inertial guidance and I'm an electrical engineer. I rather suspect I'm in the wrong pew, but I'm not sure. With respect to impedance, going back to the electrical theory, we are very much interested in power generators and in transmission lines. We're interested in the characteristics of the equipment that we want to operate. Now, if I understand what we are after, we'd like to know the impedance characteristics of our transmission lines. In other words, by means of a shock or vibration generator, at some point in our equipment we set up a train of events which are transmitted through the transmission lines to a particular component or subassembly that we are interested in. I think this presents a very difficult problem because these transmission lines are not as easily defined as they are in electrical theory; they also are nonlinear. This makes it quite a problem in measuring impedance between the source of the pulse or power and the place where you are interested in seeing what happens. I would be interested in any comments on how you define these transmission lines and their characteristics."

Professor Crede asked, "Would anybody like to define a transmission line or its characteristics in the context of a mechanical system?"

Mr. Gertel said that impedance is a very convenient technique for taking into account the interaction effect between equipment and its supporting structure. At the present time, impedance cannot be measured properly on very light structures such as missiles and aircraft, but several people are actively studying techniques of doing this.

Mr. Burgwin said, "I think the difficulty in trying to define impedance is that when you talk about impedance you assume that it is linear and this allows us to use the principle of superposition. The impedances we are talking about, mechanical impedances, are not linear. You can't use the time-honored schemes of superposition and thus are thrown into something entirely new. I think this may be the reason that you are having difficulty in dealing with mechanical impedances."

Professor Crede commented, "It may be of interest to this group to hear about the scheme that has been devised by Dr. Belsheim of the Naval Research Laboratory. As a member of the Committee on Shock and Vibration of the Acoustical Society, he is attempting to determine whether it is possible to measure mechanical impedance and obtain consistent results. Dr. Belsheim proposes to build a typical standard structure and to ship it around the country from one laboratory to another. Any laboratory that thinks it has capabilities for measuring mechanical impedance can request that this structure be routed through it. After the structure has made its route around the laboratories there will be a conference at which the results of the various attempts to measure mechanical impedance will be compared. This is a very interesting experiment because some of the results that have been received from a less orderly attempt to reproduce impedance measurements indicate, as Mike says, that there is a lot more to it than meets the eye. A lot of us are eagerly awaiting the results of this experiment to see how good the reproduction is. I see that Mr. Blake wants to make a comment. I think he may know more about this program than I do."

Mr. R. Blake, Lockheed said, "No, I don't know about that particular program. I would like to suggest that the program might be supplemented by shipping around sets of blueprints and have different laboratories, themselves, make the structure that is going to be measured. This is an experiment that was run for us at Burbank in which the impedance of several so-called typical aircraft structures, all made to the same drawings, were measured, and the impedances differed very widely. So that even though you may measure the impedance of a given structure, you have not made a prediction of what the structures in the field are going to be like."

Mr. Gertel added something from the empirical point of view. "The subject of impedance gets into the act when we are concerned with the interaction effect of heavy equipments mounted on structures and the effect that these have in modifying the so-called input environment to the equipment. There have been empirical approaches which attempted to take into account how heavy equipments would modify the input environment by decreasing it. For example, in some of the Navy specifications there is a chart which calls out some correction factor for input amplitude as a function of the mass of equipment under consideration. I think this is an empirical approach based on the realization that there are such things as impedance interaction effects which cause heavy equipments to lower the inputs."

Mr. Thomas commented, "At ASD we have done some work in the past in a rough cut approach to determining these effects of weight. I think those of you who are familiar with MIL-E-4970 will remember that there is a curve in that particular specification which takes into account the weight of the item which you are testing. It gives a great deal of emphasis to the reduction in frequency that's involved as the weight of the specimen goes up. In addition to this, we have done some more work on basic aircraft structure to measure the effects of loading it. This is again a rough attempt to get at a problem which we all know to exist. Knowing no easier approach to it, we simply took dead-weight specimens and loaded them on structures to determine the effects that this would have on the inputs to them. We are thinking very strongly of incorporating such a requirement, a reduction in frequency limits and amplitudes, in MIL-STD-810, the document I mentioned before. If any of you have better methods of doing this, we would certainly be open to suggestion. In lieu of anything different, we have adopted the approach of making actual in-flight measurements to determine what these effects of weight are and simply going at this by the rule of thumb approach. You may or may not agree with this, but at this stage of the game we think it at least represents a starting point."

Mr. Paladino remarked on the question of mechanical impedance. "We have done quite a bit of work in the ship silencing area. In the early days we thought that mass loading and damping were the answer to the maiden's prayer, but it doesn't work out that way in all cases. One of the biggest things that plagued us was the structure that the equipment was mounted on. If you have one or more resonances within the mounting structure, this can really kill you. I think, if you want to treat the area of impedance, you should try to eliminate all the resonances in this structure that you can and then eliminate these frequencies in the operating equipment, such as motors, turbines, and so on. Also, if you make measurements at the point of attachment, this is not the whole story because, if you remove yourself from the point of attachment, particularly in submarines, you get into other types of excitation like lobar, radial, and so on. This has been our experience. We haven't got all the answers but this is what we're working on."

Mr. R. Mustain of Douglas Aircraft said, "I'd like to put in my two cents on mechanical impedance because for several years it has been fairly dear to the hearts of the SAE G5 Committee from Los Angeles. Dr. Sheldon Rubin is here and he has been one of the hardest workers on this. My impression from the last few people who spoke on this subject was that maybe we ought to give it up and forget making impedance measurements. I'd say that you can take even a vibration measurement, which we feel we know quite a lot about, and send the same piece of magnetic tape around to several places and you would get different answers from your analysts. So let's not give up on this mechanical impedance. I think it is just getting to the point where people are getting interested, and possibly we can begin to make something out of it. Certainly you have complexities, such as a phase measurement between the velocity and the force, but these are things that can all be worked out. One beautiful thing about it is that you can make measurements on a structure before the thing is fired, before it is loaded down. Maybe you have a piece of structure around that is not going to be fired and you can use this continuously to make many measurements at various points so that maybe, in years to come, you may load the same structure down with a different type of mass and you will have some idea what is going on. Certainly you have resonances at the various local inputs, but this is what mechanical impedance tries to help you do. It tries to tell you what your structure is doing. It gives you some idea of what will happen when you have more information, such as fragility levels of your black boxes and vibrations measurements that are made in flight. You can combine all these things together and make a better analysis of what your box will see in the future. I'm particularly interested in the SATURN project. It runs through something like 10 years or so, and I know for sure we'll be changing our black box, brackets, and local structures from time to time. I think in our case it would be very good for us to be able to predict what will happen in the future."

Mr. Stansbury, Mare Island Naval Shipyard commented, "In answer to Mr. Paladino's comment about mass loading and damping, don't forget the other half of the problem is stiffness. We did do some preliminary work on one ship that was in overhaul. You can follow a mass line; you can follow a stiffness line. Though I'm not familiar with missile work, it is quite obvious that you cannot tolerate excessive weight. As a consequence, I think research should be directed more towards stiffness because, in all fields of endeavor, we want to reduce weight. On this particular ship, after analysis of the "as is" conditions of resonance and with due respect to the forcing frequencies in the machinery, we did a combination of stiffening as well as damping and moved the frequencies away from the forcing ones. Insofar as our evaluation was concerned, we had some success. I'm sure many of you are aware of this, but we consider the stiffness approach as well as the mass approach."

Mr. J. Barrett of Watervliet Arsenal said, "It seems to me that mechanical engineers certainly can borrow a page out of the electrical engineers' book and describe impedance functions in terms of transfer functions as they do with electrical quantities. I'm sure you've done this with your analog. I don't propose any quick answers, but weight is something we don't like to add to anything if we don't have to. Sometimes a clever application of some fairly simple piece of material can help your shock problem quite a bit. I think it is possible to measure impedance in nonlinear elements such as fiberglass insulation or a piece of polyethylene insulation. You can measure spring rate; you can make a guess at distributed mass and things like this. It would seem to me that the electrical engineer has dealt with throughputs and impedances since around 1932 and has done some real fine work in nonlinear analysis. Certainly the people who are trying to determine the cleverest way to hang a piece of equipment on a wall or what have you could use the same techniques. Maybe this is done and I'm just being naive."

Dr. R. Bouche of Endevco commented on the subject of mechanical impedance. "The direction that we're going in at this time is the laboratory experimental measurement of mechanical impedance of actual structures. Work with the analogy, analytical methods of predicting mechanical impedance, using electrical concepts or using mechanical concepts directly, is being done and has been done. In starting into the field of laboratory measurement of mechanical impedance on structures, I think we are in a rather ideal situation right now because the instrumentation for doing this has just been developed to the stage where we can start to hope to get some correlation in laboratory measurements from laboratory to laboratory. So I look with favor upon this program that Bob Belsheim is starting with the Acoustical Society. One of the advantages of having the same structure being routed to several laboratories is that it will be more a procedure of proving that the instruments are accurately measuring the mechanical impedance. I like Ralph Blake's suggestion also of each laboratory, from blueprints, constructing its own structure. Maybe they both should be done simultaneously. The reason for this is that, in round-robin programs that I've been aware of in the past, they never seem to get completed. If all laboratories start at the same time, they at least will measure the impedance of the same structure. Then when the round-robin one catches up they'll see if they get the same impedance with their own instrumentation on the second structure made from the same blueprint."

Mr. W. DuBois of Boeing offered, "There is one valuable use of this mechanical impedance concept and that is separating coupled flutter modes of structures, especially closely spaced flutter modes. This is written up in the February, 1959, Product Engineering Magazine, if anyone is interested."

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Mr. DuBois said, "I'm going to be real brazen and disagree with Dr. Bendat very slightly. I feel that we often don't know what our measurements mean. Take, for example, accelerometers that are sensitive to pressure and sensitive to transient heat pulses. Every test is an environmental test. The only real choice that the test engineer has is whether or not to measure these other environments."

Mr. Thomas commented, "In any measurement program you are faced with variables such as these but, by and large, if you make a judicious choice of the instrumentation available to you and use accepted principles in accomplishing a program such as this, taking into account such things as temperature transients and things of this type, generally they can be handled to an extent that they don't completely negate the data which you are acquiring. We've had for a long, long time the effect of acoustics, for example, on the accelerometers with which we were trying to measure structural vibration. We at ASD just finished an evaluation program on a large number of accelerometers and, in varying degrees, this is still a problem. However, by proper choice of instrumentation, by evaluating the instrumentation which you are using in any of these programs, and by just using a bit of common sense the difficulties are eased. Generally, temperature measurements are being made and you have some indication of the acoustic levels you're going to be facing. A great deal of good data can be gleaned from vibration measurements of this type."

Mr. J. Barrett made a plea for the instrumentation engineer. He suggested that, if more people would come to the instrumentation groups and state their problem, they just might get some answers. As an example, he stated that the careful instrumentation of a missile container in a truck over selected routes might produce some of the answers to the transportation environmental problem for that particular missile.

Professor Crede asked Mr. Barrett, "I'd be interested in knowing whether you consider that last account to be an example of an analytical method or an empirical method?"

Mr. Barrett answered, "I don't propose to make it either one. I'm not particularly concerned about which method I'm using. All I would like to do is try to answer a question for somebody if they have one. If they come and ask me I'll do my darndest to answer it. I think instrumentation people in general will try to do this."

Professor Crede further asked, "Would you infer that, if you haul this missile in a box across country in a truck and have several accelerometers mounted some place, you have some means of determining from the acceleration records, in the absence of further data reduction, a time history of acceleration? Does this define the environment well enough that you can use it for designing the missile or perhaps testing the missile in some way other than hauling it across country on a truck?"

Mr. Barrett replied, "I think it does a better job than sitting in my chair at a desk and trying to guess what happens. I'm not proposing that you turn this thing on for 6 months and drive the truck all over and that would be it. You drive the truck for a while on a good four-lane concrete throughway for 15 minutes or 1/2 hour and take data. Then the truck goes off of this road onto some kind of a secondary county road which might be a two-lane tar road. You turn the thing on for another 1/2 hour and see what happens, then you drive it on a dirt road and do the same thing. Finally, you measure cross country conditions for the same period. At least you have 2 hours worth of data to show you what's going on. You can argue with it if you want to, but there it is."

Mr. Hager said, "I think I would say that the statistical sample that you generate by this technique is not adequate for the design purposes that most of us will have for equipment with a long life."

Mr. Barrett asked if he had something better.

Mr. Hager said, "No, that's what I'm hoping to find out here if I can get some answers to my questions."

Mr. Gertel objected to Mr. Barrett's comment that more progress can be made by putting an accelerometer in a box and shipping it around the country than by the man who sits behind his desk and thinks about it. "I object because we mustn't confuse action with progress. I believe the man who sits intelligently behind a desk, and thinks carefully about the problem, and understands it can possibly make more progress than the man who runs around making measurements. Having the measurements, he may not necessarily know what to do with them. It would be much simpler to make measurements if, beforehand, we had some knowledge of what it is about the environment that produces the damage in the equipments that we are so concerned about. If we approach our problem from the point of view of what it is in the environment that produces the damage, I think the measurements made might then be a lot more meaningful than merely sticking accelerometers on and collecting data."

Mr. Barrett said he was not proposing just a buckshot approach to answering a problem. "I would expect a certain amount of analytical thought to go into this thing before you go ahead and do it. I'm certainly not proposing that you do this with the complete absence of sitting at a desk and thinking about it. It certainly will take a great deal of analysis to do a good job."

Professor Crede added, "I think a reasonable summary of the recent exchange is that we need both action and thinking to solve this problem."

Mr. Paladino mentioned earlier symposia on shock and vibration at which the theme was transportation and said that there was good information in the proceedings (Symposia Nos. 15, 16, and 21).

Mr. Hollings of the Centralizing Activity added that the very last Symposium (No. 30) was devoted partially to the subject of transportation environments. A lot more information is contained in the proceedings of that meeting.

Mr. A. Cohen of Sylvania said that in the Environmental Handbook there was a summary of a test in which a box was shipped to all the major cities by air. The results were of value to his group because they had a piece of equipment which was to be airborne. It turned out that the environments which would fail this equipment were not at all in the airborne environment, but in the handling of the box from the airplane to the truck and from the truck to a shipping platform and then back onto a truck. The worst case of this handling was actually in Boston. Mr. Cohen suggested that it was not entirely a fallacy to suggest such a proposition as shipping a box around the country. In answer to Professor Crede's question he felt that this was both an analytical and an empirical solution to a problem.

Mr. Gertel said, "I happen to be participating now in a program sponsored by the Quartermaster Corps which will involve, essentially, disguising appropriate instrumentation in boxes of one kind or another, shipping them around the country, and collecting appropriate data on a statistical basis. In this particular case we consider that the velocity change involved in the drop handling shock of packages appears to be the most significant parameter for the shipping condition. We will be collecting data of this drop shock on a statistical basis and perhaps a year from now there'll be more information available on this subject."

Mr. J. Elsenheimer of the Army Tank-Automotive Command offered these comments. "I think everyone agrees here that great progress in vibration and environmental work has come since the missile. I'm still in land vehicles, tracked and wheeled, and this problem has been brought up about not having numbers. What has really happened, as far as I'm concerned, is that the concentration of effort has been on the sophisticated systems. The ground vehicles have always been considered a minor problem; add more weight, make it stiffer. Recently we have begun to realize the fact that this is not true. You have to consider vibration and shock. I think it is just a problem of concentrating a little more money, time, and brains on the land vehicles and you will get the information that you need.

"With regard to the empirical approach, it seems to me that too much has been done along this line already with land vehicles. I don't want to get in a fight with anybody but, in the packaging field, packaging recorders are, in my opinion, not reliable at all. In those that I have looked at it is very, very difficult to define exactly what you have when you look at the record. It comes down to the fact that you'll have to spend time and money if you want to come up with these answers."

Mr. O'Hearne responded to Mr. Elsenheimer's remarks. "I think that I appreciate very much his remarks about measurements on land vehicles. It strikes me that land vehicles represent a great opportunity to establish valid techniques because we have such large populations of standard vehicles and they are relatively inexpensive to operate. So if we can find the statistic we want to get, this would really be an opportunity to validate our methods — an opportunity that we don't have with the missile. If we're just going to gather data and reduce it to the g frequency scatter plots or density plots, we may as well not bother."

Mr. T. Soo-Hoo of CNO commented, "I was present at the first Symposium and it looks as if we haven't licked the problems that we had then. It seems to me that a lot of people who have different interests, are dealing with an information system. We are interested in a large number of different vehicles. The shocks that would be significant on all of these different vehicles are not particularly comparable. Some of these vehicles probably have to worry less about shocks than other vehicles. For example, in a ship or in an aircraft we have to worry about shocks from all directions.

"It seems pretty easy for the people who criticize to say the services won't furnish data. I was with Russ Oliver on the NIAGARA when they were trying to measure shock on boiler foundations. Not being an expert in this field, I was greatly amazed at the tremendous difficulty of taking measurements at one point. At that one point the measurement was in one direction and, as you all know, shocks on ships are not in one direction. This is an information system that we are looking into. Each of us, perhaps, is trying to reach into this information system and get a read-out that is particularly of interest to him in his business.

"If we had a blackboard we could draw some white boxes, not black boxes, and then write the names of these boxes. Suppose we start at any part of this closed loop. Maybe you are in R&D and you don't have a piece of hardware so you have to make predictions. Well, let's start there, in the left-hand white box. Let's call it 'hardware design and fabrication.' In the box to the right of it, say 'test.' In the box to the right of that, say 'after you test the hardware in the laboratory, install and test in your vehicle.' To the right of that have another box that says 'start a war and lose a lot of these vehicles by various attacks from all directions.' All of these stages are necessary. Some of them, as you will recognize, are much more costly than others and some of them are not particularly practicable.

"Let's look at all of these boxes. In testing, look at the problems that we have; define failure. In the question of installation and test, let's think about the meaning of measurement. In the question of hardware design and fabrication, let's look at the problem of what do you have to give to the poor old engineer who has to design this thing, particularly the small parts. In other words, what form do these criteria have? Are they just static design criteria? Do we give them the strength of the holding down bolts only, or what do we do? You can see in looking at this whole system with all these loops that each one feeds back to the one to its left and you have all of this stuff circulating around.

"Perhaps somebody drew this diagram at an earlier symposium. Perhaps we had a committee or something looking at this diagram and putting in all of these various pertinent comments to see what effect they had on the information system, this loop that we have. It seems to me that we have to cross off certain of these things and conclude that we just can't do it; it's too expensive. Then assess the value of doing other things. If this particular method of accumulating our experience and our opinions hasn't been done, and therefore isn't in the process of continuing, it probably should be done."

Mr. O'Hearne observed, "I think that some of the difficulties we've had this morning in trying to keep to a point demonstrate how difficult it is to articulate our subject. I simply wanted to point out that there is a section in the Handbook of Environmental Engineering which I consider rather excellent. It was written by Dr. G. Chernowitz of American Power Jet on applying operational analysis techniques to the problem."

Mr. Shackson of NY Central RR commented as a member of the transportation industry. "I believe that the past use of the empirical method without coupling it with suitable analysis has been, to a great extent, responsible for Mr. Hager's and Mr. Lynn's inability to receive valid data from our industry. We used to take great pains to make up a load of 100,000 pounds of water, canned in No. 10 cans, and packaged in cartons. After a great many impacts and the production of a significant amount of data, we realized that nobody was interested in shipping water in No. 10 cans. The data was, of course, invalid for loads of different impedances. The second point, a more serious one, is the fact that we somehow don't communicate. The gentlemen who have attempted to obtain information on the rail transport environment have gone to an industry group which is apparently still thinking in terms of miles per hour as being significant. There are railroads, ourselves, and several others, who have done, I believe, significant work in translating the environment into terms of significance to the packaging engineer. I hope to have more to say about that tomorrow."

Mr. F. Kozin of Midwest Applied Science said, "I think a number of us are perhaps willing to accept the fact that a stochastic approach may yield answers to this problem of the systems in the field. However, my gripe is that we've actually had very little support from the so-called theoreticians who are working in these areas and who are apparently saying they are giving us the answers to such problems as the response of nonlinear systems to random inputs and things of that nature. There are many problems that still exist. People are proposing things like linearization techniques and stability of systems under random inputs, but, as of now, they've not said anything with respect to what we can do with these relative to the problems that we're here to try to answer. I would like to say, however, that we are now trying to do these things at the Center of Applied Stochastics at Purdue University. We are doing this at a theoretical level, an analytical level, and also an experimental level in our Stochastics Environments Laboratories. We had a symposium on the Applications of Stochastic Processes to the Engineering Sciences. The proceedings, which will come out shortly, are being published by Wyle."

Professor Crede requested, "I wonder if you could elaborate a little bit, at least for my benefit and perhaps for the benefit of some of the others here, on what your experimental program consists of. I think we can envision what the theoretical program consists of but we are interested in the experimental part."

Mr. Kozin replied, "We are just instituting experiments using a large shaker with which we are going to be studying the stability of systems under random vibrations. It will be the first part. The next project we are going to tackle will be the response of brittle systems to various shocks and vibrations, earthquake type inputs."

Professor Crede suggested, "The idea is, I gather, that you want to eliminate as many nonessential variables as possible so you can relate response to damage."

Mr. Kozin said that he wanted to find out what the essential factors are that are causing all these things.

Professor Crede continued, "Right, and then eliminate those that aren't essential."

Mr. Kozin agreed.

Mr. Schwabe of Lockheed addressed the audience or the panel. "Since, for missiles, we have a very limited number of channels available to measure shock and vibration, generally I think it's a practice to place these pickups at a basic reference structure of the missile. Now, the problem is how to predict transmissibility to any black box? Has anybody found any approach for predicting transmissibility from any point of basic reference structure to any black box for any type of environment, particularly shock and vibration?"

Mr. Hager answered from the Boeing standpoint. "The locations that we have for our pickups are not on what we could consider basic reference structure by which, I assume, you mean the shell of the missile. Our approach has been to put the pickups where they are meaningful to the hardware that are developing our major problems, or that we suspect will develop major problems. This means that we are generally locating our pickups, and we're quite limited in the number that we can have, on the responsive structure close to our equipment. Then we take the major sections, as large a section as we can handle, into the laboratory and reproduce these responses as best we can on the shaker facility. We then go into a more detailed program of evaluating local responses for hardware. In other words, it is not an analytical, but actually more of an empirical, approach to evaluating the response levels at the various critical points. This technique is also good for locating the most meaningful positions for measuring vibration environment during the flight."

Mr. O'Hearne suggested that the cross-correlation techniques which Dr. Bendat had mentioned were applicable to this and that perhaps he would comment further.

Mr. K. Johnson of K. W. Johnson & Co. stated, "The many comments that are being made from the floor imply that there has been no activity over the last 15 years in the field of shock and vibration. There has been activity and there has been progress. If you'll stop long enough to go back home and read the Shock and Vibration Bulletins for the past 15 years, you will find many of these subjects have been covered; they've been talked about; they have been worked on and there has been progress made. There have been things shipped over the country. Measurements have been made on aircraft. The Air Force has a report some 5 or 6 years old covering possibly 7000 test points having to do with shock and vibration measurements in aircraft. The Navy has been busy for the past 20 years in evaluating many ships under many circumstances. That data is available. It has been published in the many reports the Navy has put out. The Signal Corps has likewise run a great deal of data upon many land vehicles. They have shipped things over the country. Many of the test procedures that are presently in military specifications are the result of many of those tests.

"The thing that is lacking is the conversion of a lot of this data and information so that it is reflected in actual specifications that are up-to-date and which continue to be kept up-to-date all the time. We speak of empirical tests first and a more analytical method later. There were empirical tests established, and much data has been gathered since that time to establish and to build those tests up to a better point than they are today. I think I can name many people who are in this room that had a lot to do with establishing a lot of tests that are in military specifications, yet, with all of our data, we are still using many of the original tests that were established a good 15 or 20 years ago. That does not make them right, but the point is that a lot of the data that we have gathered has not been converted into workable specifications so that we have something better to design to today than we had yesterday. I think our biggest problem is to put our minds together and start converting a lot of the data that we have available into up-to-date specifications that tell the stories a little better than they have in the past."

Dr. Bendat make the following remarks: "I would like to tell you about two pieces of work that have been done in the last couple of years, sponsored by the Aeronautical Systems Division at Wright Field. This work was accomplished by four men at Ramo-Wooldridge, including myself, and it will answer one of the questions that was raised a short time ago about what theoreticians are trying to do to handle some of the problems of making random process theory applicable to the shock and vibration field. This particular work was called the 'Application of Statistics to Flight Vehicle Vibration Problems' and it concerned not only classical statistical techniques, but also current ideas in random process theory. At the conclusion of this contract, a second contract was given which deals with the advanced concepts of the stochastic process for random signal estimation and measurement.

"The first report which was published at the beginning of this year is No. ASD 61-123 and contains a very comprehensive review of the state of the art on various measurement techniques and on analytical techniques that people were using to analyze response vibration data from arbitrary structures at arbitrary points. In this particular study we looked at various types of flight vehicles from aircraft to high flying missiles and space probes. We investigated the types of expected vibration environments. We looked at the fundamental assumptions that people were making in trying to predict environments from their measured data and found there were many holes in the analytical techniques. There were many assumptions being made which weren't subjected to proper tests. There was a lack of understanding about the fundamental concepts of random processes, concepts of stationarity versus nonstationarity and concepts of randomness. All of these matters were covered quite thoroughly in this first report. We looked at the response of various types of systems, both linear and nonlinear, to random excitation and made certain predictions about what would be encountered under various operating conditions. There was a survey made of practical instrumentation equipments that are available in the field and some of their limitations. A complete experimental program was outlined in this report for actually carrying out some of these desired measurements.

"Tomorrow Mr. Alan Piersol is going to give a paper here which is devoted to some of the experimental work that was carried out in a continuation contract where various measurement uncertainties, amplitude probability density measurements, zero crossing measurements, as well as power spectrum measurements, were subjected to a wide range of operating conditions. We considered many different frequencies of interest to vibration and acoustic engineers, many different bandwidths, many different sample record lengths, and, in all cases, determined what the appropriate measurement uncertainty would be in making certain desired measurements. From these measurement uncertainties it was possible actually to develop some appropriate practical tests for randomness, stationarity, and normality, so that with a limited amount of data one could come up with rather significant conclusions about what was actually contained in that data. This is as opposed to waving one's hand blindly in the air and saying that he doesn't understand these because he doesn't understand the appropriate background. This particular work represented an experimental verification of many of the analytical results that were published in the first report.

"In this second report, which, incidentally, is now being written up for final submission to Wright Field, we also went deeply into certain theoretical matters. The first report was confined largely to the analysis of stationary data, but a major unsolved problem which still has lots of openings is the problem of trying to develop appropriate techniques for analyzing nonstationary data where the statistics of the data definitely change with time. Therefore, when you analyze a single record you get a lot of information about that one record, but it doesn't represent anything about the overall environment that you are interested in. So, new techniques were explored for analyzing appropriate properties of nonstationary data, and, in the panel discussion which I'm participating in on Thursday, we may have occasion to go into some of those matters. The fact remains that there has been a great deal of significant work done, not only by our group, but also by other groups. We have a problem here that goes throughout the industry to find somebody that wants to see this work done. We have the problem of doing the work, then we have the problem letting the people know it has been done."

Mr. R. Johnson of Atomics International addressed either Mr. O'Hearne or Mr. Hager. "We are designing a satellite or a payload which, I hope, does not come back, so we won't be able to find out what the environment did to it. The trouble we're having is a lack of adequate instrumentation. From the earlier discussion, I'm wondering if I could get some comments on the benefit of measuring shock and vibration on the basic structure as opposed to measuring the response of various weight components. The acoustic nature of the input has probably more effect on these components and maybe we should be measuring the response of a series of components having different weights rather than the basic structure."

Mr. Hager answered, "I think about all I can do is to repeat my original statement. I feel that, with the decided lack of instrumentation that is usually the case with missile flights, you have to be very judicious where you place the pickups. You should not be looking for the generalized environment as such, but you should be looking for the response in the key critical locations. My recommendation would be that, if you know from previous laboratory tests where your particular problem lies, you should instrument for your flight in that area."

Mr. O'Hearne's views differed somewhat. "In line with the question that the gentleman from Lockheed raised on the uses of transmissibility, you are going to have a lot of components in there. If you only have a few points you can instrument, you don't want just to choose one or two components. The idea usually is to try to instrument basic structure on the theory that the basic structure is driving the components. If you can find out what the basic structure is doing and know the transmissibility, you can design a component test or perhaps you can test entire assemblies. A real difficulty arises in these space structures because finding a point that you can say represents what's driving the secondary structure, is rather difficult. This is one reason I had hoped Dr. Bendat would say something about cross correlation because it has been suggested to me that it would be applicable in answering some of these questions.

Mr. Cohen of Sylvania put in a plug for MIL-STD-810 as a document that covered everything in the way of experience and testing procedures in the environmental field to date. He also raised the question as to whether anyone had a better way to write a design specification than referring to a series of definite environmental tests to be performed on an object.

CONCLUDING REMARKS BY THE PANEL

Mr. Thomas: "At ASD for several years now we've been faced with the problem of coming up with some input to specifications like MIL-STD-810 and others. We've been accused of using the shotgun approach, and I prefer it that way. We feel it has worked out rather well. With regard to points of measurement, we have long ago found that by taking measurements on the basic structure we were pretty well able to define the inputs to items of equipment which will come down the road a little bit later on. We were not always limited by the number of pickups and usually we had an entire vehicle at our disposal. I'm talking about aircraft in particular here now. We had an almost unlimited number of channels available to us if we wished to use them.

"I feel that one of the real problems we are facing is that of communication. In particular, we have a capability for gathering large quantities of data and we are actually faced with the problem of having too much as opposed to having too little. We're not able to get, in detail, the information out of these great quantities of data that are required to give them the fullest meaning. Because of this, a great deal of cooperation is required between ourselves and other governmental agencies and industry in coming up with some generalized requirements. Dr. Bendat has pointed out that a good deal of work has been done to define the things that we need to come up with in any measurement program. In the past 5 or 6 years we've gathered something over a million data points from about 30 different aircraft through rather comprehensive tests, yet we still feel that we have fallen short of the goal that we initially set out to reach. We're able to define, in a shotgun approach, the overall levels, the frequencies, the time durations and things of this type, but we still feel there is a great deal to be done in this area. I think it comes down to a cooperative approach, using data that has been acquired by somebody else to fulfill your needs."

Mr. O'Hearne: "I think that we have to make a greater effort to understand the probabilistic and theoretical basis of what we are doing and have more engineers who are working in this field understand it in a rigorous way. One of the biggest problems we come up against is trying to hold discourse when we don't know what we are talking about. In making this statement, I don't put myself in any better category than anyone else, perhaps worse. We have also to find formal ideas in our management of the problem along the lines of operations analysis as suggested by Dr. Chernowitz in the Handbook of Environmental Engineering and in the paper that Ralph Blake is going to present at this meeting on decision theory."

Mr. Gertel: "I would like to emphasize in closing that, whether we are talking about empirical approaches to determining environments or the measurement approach to determining environments, the objective of both approaches is the same. Both approaches have the objective of establishing design goals for equipment which must withstand these environments."

Mr. Mullen: "I strongly feel that we should know a little bit more about damage criteria. What causes things to fail? If we know this then we can better determine what stress levels or what type of environmental test we should subject equipment or components to. This is a rather empirical type of approach. It is, I think, the best way to establish new test techniques. I had an experience recently involving a failure of a piece of micro electronics equipment. The failure was a rather odd one, produced by many environments, but basically the fault was disclosed by inspection to determine what the mechanics of the failure were. After this was established then we were able to devise tests which would cause these failures. These were failures that had been occurring in equipment and by adding a rather severe test we were able to uncover weaknesses and actually get into the mechanics of the failure. I think we should do a little more thinking about the mechanics of failure because, although I do think the stresses or the knowledge of stresses are important, the mechanics of the failure are also important."

Mr. Hager: "I think that I would like to leave one statement with you in regard to measurements. There are many of them made. A lot of people have talked about all the data that has been measured in the past and when you attempt to use this you find that there is insufficient information on where it was obtained, frequency content in many cases and things like this. What you find then is that the specification people will tend to draw an envelope over all this measured data so that it loses entirely its dynamic significance at various frequency ranges. It does not allow the designer any capability to utilize the basic vehicle structure as a device to protect his equipment."

Professor Crede: "In closing I'd merely like to say that I've been to many symposia and have heard many discussions of this type. For the first time I think I can truly say that I feel encouraged, not at what has been done, but what is being done or what will be done if the present programs that have been described here are carried through to completion. It's quite obvious that we as a whole have an enormous capacity to make measurements. We can put hundreds of channels of acceleration measurements, strain measurements, and many other types of measurements into any project where it looks important. But most of these measurements in the past have been limited in their usefulness, either for the design of equipment or for the testing of equipment in the laboratory. The simple reason is that there have never been anything more than a few simple hypotheses that could be used to relate the measured quantities to the damage that might occur to equipment.

"The things that encourage me are two or three projects. One of them is the work that Dr. Bendat described essentially in the areas of data analysis or data reduction which will weed out, from what otherwise might look like a mass of incomprehensible data, some parameters that might be significant. A second thing is the work that the people from Purdue have referred to on a couple of occasions in which they are actually making physical tests in an attempt to determine the characteristics of an environment, whether it be a vibration of some kind or a shock of some kind, that are significant in determining whether failure of the equipment will occur or not. We at Cal Tech are getting underway on a program that has something in common with that described by the people at Purdue.

"It seems to me that a start is being made now toward opening up this blind alley so that sooner or later we will be able to determine what are the significant parameters. Everyone talks about wanting some numbers but the discussion becomes vague at that point, for what are the parameters in which these numbers are expressed? This is what has to come out of this practical type of research. We must find out what kind of numbers, not numerical values, but what are the dimensions of these numbers that can be related to the design of equipment. Then, we must have data reduction procedures for extracting these numbers from the raw data that come from the measurement system. So, I feel encouraged that we are on the right track and I hope that this work, which is going to take a long time, I assure you, carries on to the point where it will be useful in a practical sense. I think it is headed in the right direction and I hope that a great deal more of it will be done in various places as the years go by."

PANEL SESSION IV

THE ANALYSIS AND PRESENTATION OF ENVIRONMENTAL DATA FOR DESIGN PURPOSES

Moderator: Mr. W. A. Gardner, Sandia Corporation

Panelists: Dr. Julius Bendat, Ramo-Wooldridge
Dr. R. B. Muchmore, Space Technology Laboratories
Mr. G. L. Getline, General Dynamics/Convair
Dr. A. J. Curtis, Hughes Aircraft Company
Mr. J. T. Foley, Jr., Sandia Corporation

Mr. Gardner opened the session by asking each of the panelists to tell something of his background and interests.

OPENING REMARKS BY THE PANEL

Dr. J. Bendat: "Two years ago I attended my first Shock and Vibration Symposium and, at that time, I sat in the audience as you are doing and listened to a panel discussion. The panel members were Dr. Curtis, who is on this panel, Mike Gertel, Professor Crede as Chairman, and others. I listened very patiently and tried to learn as much as I could from the discussion. At the end of the session I finally got the microphone and asked a very detailed question to each and every member of the panel. Some of you may have been present at that time and you will recall that the first point that I raised was the fact that they hadn't been talking on the session topic and, where they were, they were frequently skirting the main issues. Most of the panel members were very amused and receptive to what I had to say and they answered my questions in a straightforward manner. There was one exception, however, which is one of the reasons why I think we have the present panel discussion. There is a need, I feel, to update the fundamental understanding of exactly what kind of data and presentation is important for vibration analysis and, actually, for analysis of many other physical problems as well. The interrelationship between proper mathematical concepts, statistical techniques, computer techniques, the role of different types of engineering talents, all that must be brought to bear if a person is to do a proper job on his particular problem.

"My own background and interests are in mathematical analysis. I specialize to a great extent in the area of stochastic processes, both in the theory and the applications. The applications have been not only in the vibration field, but also in the general area of prediction and filtering, using different types of electrical networks, and in the analysis of ocean waves. I'm getting involved now in some biomedical data analysis. There is an underlying and common thread in all of these problems where some of the same ideas go across the board. In many cases you must, at great length, spend time formulating the problem, seeing what the fundamental considerations are, and trying to define as precisely as you can exactly what you should do before you go out and do anything. I feel that there is a great deal of data being gathered in this field, for design purposes or for testing purposes, where the people who are involved have not taken into account all of the factors that they should in order to do a proper job. They frequently go out and gather a lot of data, thinking that they are going to analyze it later in some mysterious way yet to be developed. This creates all kinds of difficulties. The discussion today, I'm sure, will go into many facets of this area.

"As I said, my main interests are in the theoretical aspects, although, because I work with engineers all the time, I try to understand their needs. I try to have them frame their questions in proper ways so that we can see whether or not they are really asking all the questions that they should. I think that in many cases we have more advanced mathematical and statistical techniques available than are being utilized properly. I feel that there are many opportunities today for people who want to take the time and trouble to understand these matters, to make some contributions."

Dr. R. Muchmore: "My interests have been rather wide. At the present time, as Director of Physical Research at STL, I have no direct connection with vibration and shock problems, so I think you have to count me today as an interested amateur. In the past I've had the responsibility for data analysis and vibration problems and have been interested in the field for a long time. My interests, rather close to those of Dr. Bendat, lie primarily in the theory of stochastic processes and their applications in this field.

"The chairman told me that I had 5 minutes and, since he told a joke, I can tell a joke too. This is one that I think has some application to the subject of shock and vibration and loading in general. It concerns a man who was driving a truck down the road. Every once in a while he would stop the truck, get out, run around behind and beat on the sides of it with a baseball bat. Then he would get back in and drive the truck down the road a little further, again stop, get out and beat on the sides with the bat. As he repeated this procedure, naturally his progress wasn't too rapid and pretty soon he had a long line of traffic behind him. A policeman finally took notice of this, ran up to the head of the line and asked, 'Well, what in the world are you doing?' The man replied, 'Well, I have a two-ton truck and I've got four tons of pigeons in it. I have to keep half of them in the air at all times.' Actually, that's rather poor physics but perhaps some of us can think of a way to apply this to a missile system."

Mr. J. Foley: "I will just state my position at Sandia and what I am concerned with, then leave it up to the audience, should they be interested in similar things. At the present time I'm a member of the Environmental Analysis Group. This Group is also known as the Environmental Data Bank. It is administratively attached to the Dynamics Section of the Structural Analysis Division and has three basic functions. We screen and analyze field data for storage in the data bank. Upon request, we furnish this data to designers for use in their design programs. Finally, we use this collected data ourselves to establish standard test levels.

"In the area of the panel discussion topic — the analysis and presentation of environmental data — my interests lie in what may be called past history data. I'm concerned with the means by which it may be recovered and presented to satisfy the questions of designers concerning environmental levels, and how this data may be employed as one of the bases for the derivation of standard test levels. This can encompass many problems. I might mention two on which I have been working quite recently. One is a slightly different method of using environmental envelopes to come up with a standard test. Another is associated with uniform formats for presenting environmental data for storage and for use by designers. I think I will just leave it there and see what we can generate."

Mr. G. Getline: "I am with the technical groups at Convair, consisting of dynamics, thermo, hydro, aero, and so on. Included in these technical groups is the predesign group. All design work starts in predesign, and we provide the parametric inputs to the predesign effort. At the same time, I prepare vibration and acoustic test specifications for vendors use; I monitor vendors' work in regard to testing; I set up flight test programs, stipulate the way the data is supposed to be reduced, and interpret the data to the design groups. I'm a sort of human filter. I might point out that the relationship which I, and our groups as a whole, have with the design groups is a common law marriage. They are not required to take any advice that we give; they have ultimate design responsibility. Yet, over a period of years which began before I came to Convair, the relationship has become more closely knit, especially since I've been there. The design groups rely heavily on the technical groups for help. This help may reach the point where you will stand over the drafting board of a man who is laying down lines on paper and suggest to him that a 2-pound transformer on a piece of 0.016-inch skin in the middle of a 1-sq ft panel is not quite the right way to do something. I think this will give you an idea of my work."

Dr. A. Curtis: "At Hughes, I've been associated with the environmental laboratory in Culver City and primarily with the missile end of the Hughes efforts. We have tried to obtain data in the field regarding the dynamic environments of the missiles and to reduce this data by various and sundry means. I guess primarily I've been interested in translating the data to adequate laboratory tests, but we have also had a hand in trying to establish design criteria for the design groups. I think that, probably, this is where people such as ourselves have not done as good a job as perhaps is necessary. I say this not so much with respect to the large pieces, the structural problems, so much as the black box problems. I wonder, if I tell a designer that his black box is going to have to withstand a given power spectral density for so many hours or minutes whether he really knows how to use that data. We convince him sometimes that he doesn't when he brings it down to the lab and we shake it apart for him, but this is somewhat late. I think we need to get to the designer earlier and provide him data that will improve his chances of coming up with a satisfactory design the first time. If there are people in the audience who are more intimately connected with the design area than I, I would like them to tell me in what form they want this data; what is wrong with the way we present the data at this point? My other area of interest is in establishing adequate means of evaluating system performance under environmental test. In other words, how good is our laboratory simulation?"

Mr. W. Gardner: "A panel is somewhat like a committee and you will remember the old story about the committee which designed a horse and it came out looking like a camel. If you expect to get precise information on the right way to go about design, then you are wasting your time attending this session. We are, and have been for quite a few years, in a period of very rapid change. Many things are being done around the country in which each of you has a part. If we can generate discussion and ideas, then what we should expect from the session is that, over the years, our efforts will help to establish and confirm design methods.

"Let us look at design in a hypothetical, but ideal, situation. We would have a handbook giving data on the complete physical properties of all materials; we would know their responses under any environmental condition. We could then easily lay out a complete model of the environments and the expected reactions, put this information all in a computer and come up with a design. Perhaps the computer would produce a tape that went directly to tape controlled machines thus eliminating the possibility of human error in the drafting room and so on.

"We are making steps in this direction. Better understanding of materials and more handbook data are improving our techniques for setting up mathematical models, but we are still a long way from the ideal. Here, we are not going to cover all facets of the problem. We know that in the area of data analysis and presentation we have made some gains but we wonder if we are providing the right analysis, if the reduced information can be used by the designer. Not only do we have to turn out an end product that is reliable, but we have to turn it out in a short-time period. Because of this, we sometimes use techniques we know are not the best, but if we could improve other techniques we would use them. I will ask one question: What working relationship should exist between the vibration engineer and the design engineer?"

DISCUSSION

Dr. Soechting of Picatinny Arsenal, expanding on Mr. Gardner's question, asked what could be done to improve the line of communication between the mathematician, the test engineer, and the designer, so that each would give the other what he needed.

Mr. Getline said, "When we talk about contributing to design we have to look at the time scale. You can start providing input into design in the predesign stage when you know the final product is still a long way off. Then, you can go along with Dr. Bendat and analyze everything that you have in great detail, generate numbers as precisely and accurately as you can, and feed them ultimately into the design groups or into the test areas, wherever they are needed. However, there is often a very short time scale for design input; I expect a lot of you have met with this when you have decided to prepare a proposal for some government requirement. There is a due date on it which causes a tremendous flurry of activity in predesign. About 2 days before this proposal has to be in the mail, predesign comes up with half a dozen different configurations and these are up for grabs. They will send them around to the technical groups who will be in there en masse to criticize them. This will be a last minute critique. In this case it isn't a question of sophisticated techniques, rather, you use the accumulated knowledge and experience that you have, as little or as great as it may be. Because you have specialized in a particular area, you have a better feel for the problem than these other people and will recommend on that basis. I think these are the two extremes and the questions have to be asked in that context."

Dr. Bendat commented, "I agree that one should not become overly sophisticated when a simple procedure will give substantially the same desired results. There are too many people, and mathematicians are among them, who hide behind mathematical symbolism. It isn't enough to set down a very rigorous and abstract point of view and then say that this particular wiggly line in this integration proves whatever we want it to prove. There have been many people who have made careers out of hiding behind mathematical symbols, and there are many people today who are making careers out of hiding behind computers.

"The problem that a mathematician faces in this kind of an environment is to guide the engineers, with whom he is working, into the fundamental areas of concern. If they are analyzing certain data and trying to predict future properties of the data because they want to design to some future environment, then it is important for them to take steps in their analysis to try to verify that the data they are analyzing is likely to be the data that they will encounter in the future. This is the concept of stationarity as many of you know, but many of you don't know it and, therefore, should be spending some time trying to understand.

"Also, there are a number of terms that are loosely used in this field which refer to very idealized mathematical models. One example would be the concept of a transfer function. There was a paper given here that talked about measuring amplitude and phase characteristics of transfer functions in which the overriding assumption in the whole procedure was that the system was linear. If the system is not linear then one is not measuring a transfer function. He is measuring something which he is calling a transfer function. The mathematician's role is to say, 'Let us determine, as precisely as we can, whether or not the system under consideration is linear and let us devise proper procedures and tests for the linearity before you go through the measurement of what you call the transfer function.'

"The fundamental assumption of linearity is frequently overlooked. There is equipment on the market today with which it is claimed that by twisting a dial you can measure a transfer function. I would assert that probably 80 percent of the time the people who are using this equipment are not measuring transfer functions. Not only does the system have to be linear, it has to be a constant lumped parameter system where the output is not dependent on the time at which the input occurs. If you have a linear time bearing system, you get a different type of transfer characteristic. If you have a nonlinear system the whole area is still wide open. There are many such fundamental considerations of which one must be aware in order to do a more competent job in presenting the right kind of information to the design person."

Mr. E. Mogil of Rocketdyne asked, "When you have a power spectra in terms of so many g squared per cps, how do you take this data and tell a designer how to design a package? If you have a predominate sinusoid, it is fairly easy to determine what deflections you may have or to calculate resonances of the system and see if you are in trouble. In the case of a power spectral density, how do you tell a designer what to do? He is completely confused and we usually tell him, 'We shall see whether your package will live when we get it on the shake table.' This doesn't help him in his original design. I'd like to hear what some people here do with power spectral data to assist a design engineer in designing his prototype packages?"

Mr. Gertel of MITRON posed a similar problem. "The problem in analyzing and presenting data for design purposes is that, as yet, we have no really solid criteria on what constitutes failure. In designing something it is implied that we're putting on paper something that will withstand an environment. In the analysis and presentation of the data we've got to know what it is that causes failure in the thing that we are trying to make so that we can intelligently present the data, at least to the designer. As this gentleman just said, g squared per cps doesn't really tell him anything. It defines the environment in a way, but in this g squared per cps, or other numbers that might quantitatively be used to specify the environment, what parameters create the failure? Until we know that we just keep on going around in circles."

Mr. Bort of DTMB commented, "I think this boils down partly to the fact that the designer wants a design formula. He wants something which he can work out with pencil and paper and may be a slide rule and a desk computer. He is not particularly interested in where the data came from or what was the general background of the data analyst. He has to know when the formula is applicable, what its errors are likely to be, and where its limits of applicability fall. It is, I think, one of the responsibilities of the analysts and the data reduction people to connect up their data, not with general theoretical concepts, but with some kind of a design formula that the designer can use immediately."

Dr. Muchmore said, "I don't pretend to be able to answer all of it, but I'd like to say a few words about the fundamentals and then maybe we can work from the fundamentals on to the final answer of the problem. It appears to me that there is some question as to why we use power spectra at all and I think that this is a question that does need answering. It goes back to the fact that almost all data reduction methods are founded upon what is needed in analytic treatment of problems. Now, why a power spectrum? The reason is very simple but is not always clearly stated or understood. It lies in the fact that for a stationary process, and they are never actually quite stationary, a Fourier series representation evolves into a sum of orthogonal components. In other words, every frequency component is independent of every other frequency component. Now you may ask, 'So what and who cares?' The man who cares is the analyst who is going to solve the problem because, if every frequency component is independent of every other frequency component, it is an easy problem to solve and he can get a design formula. The man who reduces the data, I think, really tacitly assumes that the analyst has given the designer the formula into which the number goes and this may be the missing link in the chain.

"The point I really wanted to make is that there is a very good reason for using the power spectrum which is to make easy the analysis that would otherwise be very difficult. We put the data in this form because the analyst then knows what to do with it; it is possible then for him to give a formula to a designer."

Mr. Mustain of Douglas discussed some of the previous questions. He said, "I want to go back first to the question that was presented by Mr. Gardner concerning the relationship between a vibration engineer and a designer. I've worked for several years in this type of situation where I've tried to handle vibration and work with designers. Admittedly, it is a hard thing to follow up. For one thing there are hundreds of designers with whom you have to try to keep in contact. So you do things such as Mr. Getline suggested; you look over a drawing and see something obvious — a transformer in the middle of a panel — and you move it; or, you see something that is mounted as a cantilever and you know it is going to vibrate like mad under high excitation, so you try to fix it. This is primarily the experience of a vibration engineer, somebody who has been around designs a lot, who has seen lots of shaker tests, and who has some feeling for them. Yet you can't put this down in black and white for these thousands of engineers. You have to be able to do a tremendous job of covering the field, which is really what the vibration engineer should be trying to do.

"Let's assume we have a random environment and a complex black box with many elements in it. You don't know for sure what its response is going to be, especially during design stages. You can't tell the design engineer 'This will see 0.25 g squared per cps,' and expect him to know what to do with his black box. People have gotten used to the sinusoidal test and many engineers feel that they know how to design to sinusoids because they know that there is going to be one major resonance which they will try to avoid. We have to convince them, however, that we have a random environment requiring a new approach.

"The only real way to handle the problem is by having some feeling for vibration tests, which gets us right back to testing black boxes. You have to know whether a black box will take a 0.07 g squared per cps; my feeling is that most black boxes nowadays will take that. At 0.1 g squared per cps they start getting into trouble; at 0.25 g squared per cps they are pretty bad and so on. I feel, along with Mr. Mogil, that you just don't have a good answer for the design engineer and it's a tremendous task to cover. It would be nice if we could have a book that said how you design to random environments."

Mr. W. DuBois of the Boeing Company commented, "We have always had random environments with us and we are still having trouble coping with them. Our object in life in the environmental lab is to provide whatever environment is necessary to solve problems. Suppose you have a problem with a telemetering system. You bring it down and we will provide the missile environment so that you, the designer, can locate and solve your problem. This is all we can do. We can tell the designer what the environment is in the missile, and then it is up to him."

Mr. Cohen of Sylvania said that, like many others in the room, his company was faced with the problems involved in designing black boxes to meet military specifications. Referring to Mr. Getline's remarks about advising designers when they're in trouble, Mr. Cohen suggested that one of the big problems is that most design engineers do not get a chance to watch their equipment under test and therefore may not be convinced that it will fail when the only proof is the experience of the vibration engineer. Mr. Cohen went on to say, "We haven't given the designer a formula to go by because this is something that we have learned from experience.

"Along the lines of formula, I did work for a company as a mechanical engineer in the design section. I was given a large heavy document and told, 'Here is your bunch of formulas;

here's your IBM program; here are your in-conditions. All you have to do when you have a new condition is to plug in a new in-condition, give a new temperature, and so on, and you come out with a cross-sectional area of the part from the IBM 704 or the IBM 650. This is the way you design your parts here.' When I would go into the test cell, for example, and look at this jet engine running, I still couldn't get an idea of what my parts were doing because there were more pressing problems. They were aerodynamic problems rather than mechanical problems. I felt that we were hampered because we were not allowed, essentially, to go in and investigate what our parts were doing under the design conditions.

"Either it is a question of communications between the design engineer and the test engineer or it is a matter of policy of the company that one is a design engineer, another is a test engineer, and they must not talk to each other. It is also a question of whether the design engineer is too lazy to get up from his board and go down and look at a test being run, and actually get some information from it so he'll know what to do. I don't think that we should be supplying design engineers with a given set formula and say this is the way to design to it because it has been proven analytically. We can't prove it reliably that way. If we do we are way overdesigning. We have to tell him the way to design a part based on our experience. Usually, what we do is to overdesign the part for the man because we want to be darn sure it isn't going to fail in the lab, otherwise our necks get chopped off.

"So, I would like to ask Mr. Getline, who do you consider should have the final say in a problem involving, for the sake of an argument, vibration or shock? Should it be the design engineer, the test engineer, or a design engineer with test experience who knows what he is talking about when he actually does the design?"

Mr. Getline explained how it worked at his plant. "The design groups have ultimate responsibility. They have no limitations imposed on them as to where or how or by what means they can get advice. The input that we provide is at their request. We can go over to the design groups, look at the boards, and suggest things, but suggestion is what it remains. However, once you have built up some confidence in your efforts, you will find they'll be coming over in droves. In fact, every time they want to change the size of a rivet they will be over and want you to put your blessing on it. The main thing is to build up the confidence of the design groups in the fact that you can do a job for them.

"I remember when we started our commercial jet program, at which time sonic fatigue had been around to a very small extent on a few airplanes. Our design people were only going to scale up sheet metal airplanes for a large jet airplane. I had to convince our people that we had a problem. I wrote memo after memo after memo pointing out this problem area and, in general, describing what could be done. I got together 'a dog and pony show' for design groups right up to the chief engineer and I had a 'chamber of horrors' which consisted of photographs of some of the early B52 structures. If you've ever seen those you will understand what I mean. All this was used as a sales pitch to get the design groups to understand that there was a problem. They bought it and I had the whole thing dumped in my lap.

"There was no problem on our part of setting up ground rules. These ground rules were nothing exotic; you can find them in any handbook on fatigue resistant design. Using them as a basis, we worked with various types of sample structures in our siren facility well in advance of the final detailed design. When we came down to the detailed design stages we were pretty well sure of what we were going to have. I think this is a good example of how you can provide input in this area.

"As far as the black boxes go, our design engineers who sign off their drawings are responsible for the hardware. In fact they will go right down to the shop and see it installed in the vehicle. In assuming this responsibility they'll go down to the test lab and watch tests. They'll go to a vendor's facility and watch what he is doing. When I say they have design responsibility, they really have it. They assume it, they are stuck with it, and all we can do is help them. And we help them because they come and ask us."

Dr. Burgess of United Technology Corporation said, "I wonder, are we not saying here that the topic which is the subject of this session is an abstraction — the analysis and presentation of data for design purposes. We can analyze it, we can present it in various forms, but finally it has to go through the filter of an individual's mind; this individual is able to look at all of the details and use his judgment. Aren't we trying to bypass that word judgment in the present discussion?"

Mr. Foley responded, "I don't think we are trying to avoid the term judgment; I think we are trying to paraphrase it. We are trying to define what this judgment is. I have a little blurb here that defines a basic analysis technique used by a designer as being the exercise of judgment based on what he can gather at a particular time in the way of past history, experience, direct test results and theoretical studies."

Mr. Gertel commented, "It would appear that judgment or personal experience is the only expedient for coming up with a successful design. It is unfortunate that this is true. What we really need to do is take this experience which people have and put it down in some formula, as was suggested earlier. The problem is old and I'm going to go back to the analogy of fatigue of metals because I think this provides a platform for extending into failures of more complicated systems. We have now got to the point where there is enough analytical equipment so that we can really define our environment. We can determine whether it is stationary. We can come up with statistical descriptions of the various individual peak loadings comprising a particular environment, whether we call it in g squared per cps or choose to re-create the statistical distribution that the g squared per cps implies. What now has to be done is to take these inputs and apply them to a criterion of failure. It would appear that the way to establish this kind of information would be through an experimental program, at least this is the experience that exists in fatigue of metals. There are certain very sophisticated theories about why metals fatigue, but the designer still resorts to the use of experimental SN curves. We need something very similar to that from the vibrations point of view."

Mr. Hager of the Boeing Company returned to a question that had been asked earlier concerning how to interpret to the designer how he should build his equipment in order to meet certain requirements. "We are fortunate in that, once we get to the specification level, we have a test requirement which is well defined, not some arbitrary environment that we don't know much about. With the problems that we have with the hardware, the predominant one is the loading of all the parts in the system. It will tend to fail, and I could safely say 99 percent of the time, as a result of the maximum load. There are a few cases where the frequency alone is important. We interpret the environment into an equivalent static load or g's that the designer can then apply, based on his individual weight items, to the structure and pick his structure on the basis of stress. In addition to this, we will give to him certain stiffness requirements. By the combination of stiffness control and load he will design his structure which will fit the analysis that you use to give him the load. It is not guaranteed that the design will be completely adequate under test, but we can guarantee that the designer will be very close and need to make only minor revisions in his design. If you look at most structures, the black box is attached through some form of structure, either to a secondary structure or to a primary structure. This system will exhibit a primary mode. In the majority of cases this will be the mode in which your highest loadings will occur in the system, whether it be the basic structure or whether it will be an internal component."

Dr. Curtis said, "Listening to the discussion over the last few minutes I think we seem to be in need of classifying our problems a little bit, both in terms of scale and in terms of failure classifications. If you start talking about solutions of the structural problem to the electronic designer of a black box, he won't know what you are talking about in the first place and in the second place he doesn't have time to conduct a rigorous structural analysis of a small black box. Basically it's cheaper and quicker for him to build one, pretty much by the seat of his pants, and bring it down to the lab and let us have at it. So, this is one extreme, the small item. At the other extreme you have a complete airplane or missile which, basically, is a structural problem. I think we have enough knowledge here, and the structural dynamicist can take the time and the effort to understand and translate a random or a stochastic process into some kind of an equivalent static load and design the structure to this. It's so big that you can't test it in a meaningful way anyhow, so you are forced to employ analysis.

"I think our problems are in the ill defined area in between the two extremes. Here, we may have a significant piece of structure which may weigh up to a ton or so and perhaps even more in the future. We have an item which is getting to be big enough on which to conduct a rigorous structural analysis if we knew the loads. The trouble is often we don't know the loads. Somebody has specified a test condition to us in the spec as input acceleration, in the same way we have specified inputs to black boxes in years past. They've written an environmental spec for this thing which is just not sophisticated enough to define the input conditions. It must take account of the response of the system. Furthermore, when you get a 1-ton test object, I hope most of us are beginning to ask ourselves when an item is too big to test. Oh, no matter how big it is, you can run some sort of a test, but where do we stop kidding ourselves that we can really qualify an item for a dynamic environment merely by shoving on it at one place with a shaker. I hope I've outlined this problem of the scale factor that very much affects our problem.

"My second point is the kind of failure. We've been talking here about fatigue failures, things falling apart and so on. Fatigue failures are something for which I think we've adopted the phraseology of a catastrophic failure. The other classification is a functional failure which is a much more insidious type of a failure, much more difficult to design against. It required much more sophisticated test methods and much more accurate simulation of the environment to assure ourselves that this functional type of failure is real. In this case, I'm not sure the designer has any way, even through past experience and so on, of knowing how to avoid these kinds of failures until he gets down into the lab or, maybe, into the flight test program. Therefore, I think that we might do well, when we start talking about these problems, to classify them a little bit and not try to seek a general answer to the question."

Mr. Stern of General Electric Company commented, "I think one of the things I've learned this morning is how to be more evasive when the designers come to me. Actually, the function I serve is not only to provide services to designers, but also to the test group. Usually, one of the things designers enjoy doing, after they've had a box tested, is to bring it to me and say, 'Well, it failed, point to the part that failed.' This is the black box type that is full of a lot of resistors, capacitors, and transistors. I've never been able to do it yet and I don't think anyone on this panel or in this room would be able to do it.

"From my standpoint, it is most effective to have designs pass vibration before they are built, because that's when it counts; that's when you save money. I'm talking of the case now in which we take a contract at a fixed price and have to deliver on a fixed date. The designer who designs the electronic part has enough trouble just getting the circuit to work let alone putting it in the structure and having it pass a vibration test. With this sort of environment I find the most effective way is through packaging. For example, in ground support equipment I try to get the metal outside and use a rigid skeleton so that when they do get into trouble, major overhauls are not required or major mod. kits do not have to be sent out to the field. These are rather expensive and the product is unsightly. For smaller equipment, I try to get the customer to buy potted modules, throw away modules. It is extremely effective. I think, in line with this, the evolution from vacuum tubes to transistors had a far more positive effect on successful vibration design than all the symposia and all these people here. As we go to other new type components, say thin film deposition and even smaller lumps, we'll be even more successful.

"As for printed boards or panels, I find that if you can have all the structures with natural frequencies of 150 cycles or better, generally you won't get into serious difficulty with sine or random testing. These are the sort of general criteria that I find quite effective, but I would appreciate any other methods of this type or approaches that people use rather than the sort of evasive and general responses that I've heard so far. There was one good question asked. How do you design to random? What do you use specifically as against designing for sine? I think these are the sort of questions that should be answered."

Mr. Gardner said, "I think we will agree. We've been searching; we've been talking a lot of philosophy. We have to get a little specific if we are going to do any good, and we might pose a question. Is there anyone here who would like to give an example of how data analysis has been used in translating data and getting it into a design? Do we have a good example?"

Dr. Bendat offered these comments. "The mathematical theory, it is a theory, that's devoted to the response of different types of systems to random excitation usually idealizes the system. There has been practical work which has led to practical designs of optimum systems to handle various types of random excitation when there was sufficient information about the nature of the random excitation. If you were building a system to try to pass desired signal information in the presence of noise and if you were going to do this with a linear system, you could use certain theories developed by Norbert Wiener and others. From a knowledge of the input signal to noise properties and of the desired output properties, one could say what the transfer characteristics of this black box should be in order to pass the desired signal and, in this sense, filter out the interfering noise. This has been done in electric problems and there are certain mechanical analogies. I've heard of certain shipboard and submarine design problems which, in large scale theoretical form, have been approached in this fashion, but I've never personally participated in those problems.

"The failing, of course, in these techniques is that it is an idealized mathematical situation and you have a nice input-output relation to consider. Therefore, if you know the input power spectra and if you know what is really the transfer characteristics of the system, you can determine the output power spectrum and from that what its mean square history will be.

You can make other statistical measurements of amplitude probability density distributions or peak distributions. When the system is more complex, and we are now talking mainly of mechanical structural types of systems, the transfer characteristics are not so simple to define. You usually don't have a linear constant parameter situation. It's time varying or it has nonlinear feedback loops, thus there isn't any simple mathematical relationship anymore that is appropriate.

"What we are doing on the theoretical side is trying to extend some of these simpler formulas to these more general situations. I don't believe there is any simple formula that will become available in this field now or in the near future. I don't think that we have to have any simple formulas. The world in which we live is quite complicated. The types of environments that people measure show great variances from one set of measurements to another and you don't get a unique curve that describes the environment, nor should you ever expect to find one. There are these changes that do take place and these are the changes that we have to understand and to which we have to develop our designs.

"I have, in the last 6 months, spent considerable effort trying to develop some new techniques for analyzing nonstationary data and trying to develop some appropriate theoretical formulas that will indicate what the nature of the transfer properties are when nonstationary data is passed to different types of systems. Many people here think they understand the problem of passing stationary processes through linear systems. This involves Fourier transforms and certain simple one-dimensional types of power spectra or correlation functions. When you get into the nonstationary case, and this is the case that is of concern, you have functions of two parameters. No longer is it true that the statistical properties of interest are invariant with respect to time translations. They depend on the particular time at which you examine the process. You don't have correlation functions which are functions merely of the time difference. You have correlation functions which are functions of the exact two times that you are looking at. Nor do you have simple power spectral density functions which are functions of a single frequency parameter. When you get into the nonstationary case, the proper power spectral density function is a function of two frequency parameters. You don't have a single straight line representation; you have a plane area representation. Instead of simple Fourier transforms, we get into double Fourier transforms. This is what the theoretical world requires us to do.

"Obviously, it is going to be a long time before some of these theoretical ideas are translated down into the practical area and it will be a long time before certain of the equipment manufacturers make equipment that will give you the kind of information that you need. But we are moving in this direction and certainly, by a concentrated team effort, those of us who are theoretically inclined will be able to understand some of the practical problems. Together, I feel that there will be considerable progress made in the next few years."

Mr. Chaplin of Farnborough, England remarked, "I'm not really speaking for Great Britain but for myself. I'm largely concerned with the dissemination of environmental information, but occasionally the designer asks me for some advice. When he does I give him what advice and help I can. I take my g squared per cps analysis and present to the designer the same information analyzed over a variety of percentage bandwidths, so that, in effect, I am giving him the rms sine for a variety of q's. He can then pick out those which he thinks are applicable to his particular circumstances. Now, if he should ask me for further advice regarding structural fatigue, I would re-analyze my information on a v squared per cps basis. If over a reasonable bandwidth my rms v squared was less than 1 inch per second, I would say that with standard commercial practices he should find no difficulty with that particular part. If, however, it was greater than this, I would say he could have a structural problem."

Mr. Kfoury of Korfund Dynamics said, "I would like to hide my comment behind an analogy of Dr. Bendat's comment before. Assume that an operation is going to be performed in a hospital and the surgeon didn't show up. You wouldn't take one of the nurses in the operating room and tell her to perform the operation because she has seen quite a few of them. By the same token, I realize that we can't have a vibration engineer at each and every design engineer's board. However, I think that there is a basic underlying lack in the education and the background of most of our designers that should be remedied. I do believe that, if their training were improved, it would go a long way towards helping the environmental laboratory and the systems engineer get together with the designer on what has to be done. I'm sure that most of the designers today are capable of analyzing what stresses there will be in the welded sections or in rivet patterns and of calculating buckling stresses on bulkheads and skins. However, there are not too many of them that have at their fingertips and in a machinery handbook all of the information that I'm sure most of the gentlemen in this room have, with regard to fatigue stresses, the normal q's for certain structures or the new techniques and the application of damping materials that have come to the fore in the past few years. I believe that the colleges and the institutions that are preparing designers for their careers should take a real hard look at their curriculums and, perhaps, should introduce a few more courses in the dynamics of structures rather than so many on the statics of structures, such as we have today."

Mr. C. Lutz of General Electric Company, after listening to earlier comments, said, "I have come to the conclusion from what I've heard that design engineering is one specific talent and test engineering is another specific talent. I happen to be from the data reduction area, so I'm wondering whether data reduction is a third specific talent or whether we take a data-oriented person and convert him into a design engineer or a test engineer or whether we take a test engineer or a design engineer and orient him to data reduction?"

Dr. Curtis commented, "I seem to recall agreeing with the gentleman from General Electric at another symposium, where I went out on a limb somewhat and blamed most of our troubles on the organization of the way in which we do business and suggested that compartmentation of disciplines is probably one of the more significant road blocks in the way of progress in achieving good design. I second your comments very much. I have to admit that we have not found the solution for this out in California. It seems to be a problem of the size and age of the organization that we happen to work for. I have been at Hughes long enough to know better, but I can remember, some years ago when we were much smaller, the contact and sort of team spirit of all these various areas of design, evaluation, data reduction, and measurement. Everybody got their hands dirty and their heads knocked in all the various disciplines which gave each of us a fairly broad outlook and appreciation for each other's problems.

"As an organization gets older and larger, for some reason procedures and responsibilities have to be defined very neatly in such a way that they discourage this team work. They discourage this interchange of information between people in different activities and different primary responsibilities. I once heard a statement that, if you have good people and a poor organization you will get the job done; if you have good people and a good organization it makes the job a little easier; if you have a good organization but your people aren't any good, you might as well quit because you will never get the job done. What we are really talking about this morning, I think, are the personalities rather than the technical problems. All of us are human in one way or another, and there are no formulas that describe humans at this point. I think we should look for them. Of course we should keep trying to solve the problems in a technical sense, but if we look for that only without looking for better ways of organizing our work, and if we don't recognize this human factor for what it is, we'll be tilting at a windmill."

Mr. Johnson of United Kingdom Defense Research Staff endorsed the remarks of Mr. Lutz and Dr. Curtis. He said, "In fact, it's the degree of compartmentation that appears to exist between designers, testers, and data gatherers which causes many of our difficulties. I can illustrate this by attempting to answer the very first question that was asked, namely, how do we advise the design engineer to deal with a given g squared per cps? In my experience, if you take the value of g squared per cps to the design engineer, he barely knows what it is and, at least, one must translate it into an equivalent sine wave of some type. In fact, for 90 percent of the design jobs, even for dynamic environments, the design engineer designs on static principles merely multiplying his dynamic g by some factor which is probably around 10. I think the reason for this is that, if you attempt to do a complete dynamic analysis on any complicated structure, it becomes nearly impossible and certainly extremely long winded. Therefore, what one has to do is to simplify it down and pick out the particular modes of oscillation which are important. Generally, the design engineer has little or no experience as to which are the important modes of oscillation in any particular structure he has happened to design. In summarizing, I think what we want is a greater degree of cross-fertilization between design engineers, test engineers, and the people who gather the data in the first place."

Mr. R. Mustain of Douglas had questions for Mr. Hager of Boeing and Mr. Chaplin of the UK. He asked Mr. Hager, "Since you proposed an rms value, are you primarily dealing in sinusoidal environments or random environments?"

Mr. Hager answered, "We have worked this with both random and sinusoidal environments. Here again, you have to make some estimation of the q that you expect with the particular structure or the particular device concerned. We have techniques for using both random and sinusoidal."

Mr. Mustain then gave Mr. Chaplin a practical problem. "I have a small black box which can be easily vibrated. It is a new design and we do have a random environment that has been measured. It is flat from approximately the 10-cycle range up to 2000 cycles at a value of 0.1 g squared per cps. There are things like klystrons, a waveguide, and a few other things in there that are rather delicate and respond to quite a few different complex frequency modes. I would like to ask Mr. Chaplin to guide me through this problem and tell me what he would do with it."

Mr. Chaplin responded, "I think Mr. Mustain said that he had an environment of 0.1 g squared per cps from 10 cps up to 2 kc. In the first place, I would go back to the one who measured this environment and ask him to look again at the bottom end, because I should doubt it. However, let us assume that the man came back with the same answer and we have this 0.1 g squared per cps from 10 cps upwards. Now, I'm not very good at mental arithmetic but, going back to my experience, I would do a little algebra or integration with a planimeter to convert this into the rms values for percentage bandwidth values, say, of 10 and 20 percent. I'd look particularly at the bottom end where this g squared per cps seemed to me to be unreasonably high. If the values which I so obtained at 10 cps came out at greater than 5 or 7 percent of my ultimate static design criteria, I would say that the design was inadequate and would ultimately fail structurally.

"Looking now towards the middle of this environment, I would convert the figures concerned into v squared per cps and look at the rms value of that figure. If that figure exceeded 1, I would say that, if the environment was prolonged, you would again find the structural integrity not being maintained. This inference I would draw from my experience with engines, where one would anticipate failures of structural members with these sorts of rms velocities. Now, the upper end of my friend's spectrum is unlikely to produce a structural problem at all. The question of malfunction will arise. I know of no method or no advice that the environmental engineer can offer to the designer on the question of malfunctioning. This is concerned with the design as a design."

Mr. Hillyer of the Sandia Corporation addressed Dr. Bendat. "Already, according to the Symposium papers given, the experimentalist has started his damage criteria testing. Quite obviously, at this point he doesn't have his parameters and really, in fact, he doesn't have his test method. Now, the 2 or 3 years that you are talking about to come up with the parameters means that the experimentalist will probably have the biggest backlog of answers in the history of the world, for which there are no questions. Another little item of concern is the tax bill by which we duplicate in every lab around the country the same information without too much correlation. What can you do about giving us the parameters and the test method in a hurry, because we have already started the ball rolling in the lab?"

Dr. Bendat said the question was a very good one and one of great concern to him. "I have had occasion during the last couple of years, because of work that I have been doing for the Aeronautical Systems Division at Wright Field, to visit a number of both military and industrial establishments throughout the country where people are spending tremendous amounts of money for more advanced and complicated environmental testing chambers. They want to do all kinds of special tests, combined environments, and the like. Also, there are hundreds of thousands of dollars being spent on equipment that is going to assist them in evaluating the data that they expect to collect in these environmental chambers. Certainly what is being done, and it is being duplicated tremendously, has to be done. The bottleneck in the whole problem, though, is going to be in staffing these establishments with qualified people to really make proper use of the facilities and the equipment. In my opinion, this is a difficult area that is not going to be well handled in the next few years.

"I think that there is going to be a lot of unnecessary work done. I think that there is going to be a lot of data gathered that will not be worth the paper it will be written on because these people haven't taken the time or don't have the ability to design the necessary experiments properly. They don't know precisely what measurements to make or how to analyze some of these measurements. I don't know the answers to many of the questions either, of course. There is a problem of trying to develop from mathematical and statistical considerations and, whether you like it or not, you've got to try to understand these techniques. I'm not speaking as a statistician and I'm not a statistician. I consider myself to be an applied mathematician who will use statistical techniques if they seem to be appropriate.

"I've been accused of being one of the pioneers in power spectral methods of analysis, but I have no desire to be an advocate of this method of analysis alone. This is a useful tool for certain types of problems and it should be used properly when it is justified, but there are other types of statistical and probabilistic analysis which have not had the play that they deserve. One of these big areas is that of probability density analysis, not only first order, but joint probability density measurements and the applications one would be able to make with this information if he had it and knew it was accurate.

"I feel that one of the big areas you've raised is the need for developing better procedures for decreasing the amount of data that should be collected in the first place, while still knowing in advance and taking certain risk factors into consideration, that you will be able to measure those parameters that you want to measure, that you will be able to predict the range of expected values that would occur under continuous sampling which you didn't do, that you would detect unexpected events, and that you would have the right kinds of data so that when you make desired measurements they would be meaningful.

"You would be able to present the designer certain information, with confidence bands about it, on the type of environment in which his equipment is going to have to operate. You don't give him any simple curve because there isn't any, but you tell him that his equipment should be expected to operate over this range of conditions. The designer now has to upgrade his manner of thinking. He can no longer design to any simple formula based on any text book work that he learned in college, but he has to extend his thinking to where he appreciates the fact that his equipment will have to operate in this nondeterministic type of environment. He recognizes the fact that the equipment which he has designed for this application may not be adequate for another application unless he is aware of the tremendous range of variables that might be pertinent.

"As I say, I think that the area of making proper use of the facilities that are being built by the military and by commercial companies is a big one. I'm active in several groups that are trying to develop some proper procedures and outlines to guide some of this work and I'm sure that there are lots of other people doing the same thing. I'm learning all the time from them as well."

Mr. Stansbury of Mare Island Naval Shipyard said his remarks were based on experience. "We have reached, in my opinion, the stage of overspecialization. We have people who are purely concerned with the taking of data and its analysis; this data are then turned over to the designer. Whether it be missiles or the more prosaic case of submarines, which is my business, the man to interpret the data properly is the man who knows the forcing frequencies of the machinery. We need to return, to some degree, to the general practicioner who then can submit his problem to the specialist. I'm not saying that we don't need the theorist or theoretician, but we do need a midpoint. The design engineer should be required to be on the test floor to witness the measurements taken by the acoustician, for example. If you look at this thing purely theoretically, not recognizing the forcing frequencies or the cancellations that may occur in a machine or as a function of damping in a missile shell, you lose much of the point.

"I fortunately had the benefit of many years laboratory experience before I went into design, so I can correlate these. I have also seen occasions where the acoustician presents data which he has measured and it's professionally correct, but he cannot interpret it because he doesn't know what is causing these frequencies. Secondarily, the design engineer who sees the data cannot think along vibration requirements and, as a consequence, he loses much of the point. Much time, effort, and money is expended because there is no interrelation between the two. However large and unwieldly the corporation, I do think that before we can come to an answer more quickly for a specific problem, we have to have people working together as a team so each begins to think in the other's medium. One doesn't become proficient in it, but there is a better communication and freer exchange of opinion. You can come to a conclusion. Without this exchange you go around in circles and spend a tremendous amount of money testing, failing, and finally achieving the result. It can be short-circuited."

Mr. W. Metzgar of United Aircraft commented, "I'm particularly interested in the comment of a few minutes ago that the test engineer supplies data to the designer who takes this environment and multiplies by a factor of 10, then based on this, assuming the primary mode, he calculates stresses. I find it a little bit disturbing that this procedure, which is so frequently followed, brought a ripple of laughter and no additional specific comment. I think it is directly related to the last comment, however, that the test engineer does do a very good job of defining the environment. I've also been very favorably impressed by the talent of designers in analyzing stresses based on the load and in being able to calculate the primary mode frequency. It seems that the big unknown here is this factor of 10, at least in designing relatively simple structural elements. In using this common factor of 10, many designers have gotten into very serious trouble. Anyone with experience in designing structural parts like this will realize that the factor of 10 can just as easily be any place from 2 to 50. In fact, I recall one of the papers yesterday given by a Mr. Shantz, I believe, where the analysis group had predicted a 2 percent critical damping and, in testing the part, they actually found 20 percent.

"I would address my question to the test engineers and to the practitioners. In terms of magnification factor or damping inherent in certain types of structures, does anyone have a specific method of collecting this sort of data from tests and presenting it systematically to the designer? I do feel confident that the designer will be able to use it in determining what load he should use in his admittedly simple analysis from which he will produce dimensions of hardware."

Mr. Johnson of the UK answered, "I gave rise to this factor of 10, although I only used it as an example. I think it is a fairly typical one, although I quite agree it is very frequently exceeded, certainly in the field in which I work. On the question of its actual measurement I'm afraid I'm not allowed to say too much about the kind of work I do. However, we have been doing a statistical analysis of q-values based on the bandwidths and on the percentage frequency separation of the half power points on any resonance curve we can find anywhere in any of our test data. We have surfaced with the fact that the logarithms of the q's are normally distributed and that the antilog of the average log is 11 in all the data which we've taken, which was some 1200 points. So, we consider that the most typical value, certainly for our data, is a q of 11, but there is considerable scatter on this. We ignored q's below 3; some high q's are nearly 100."

Mr. Stern of General Electric, returning to his electronic black boxes, said that their measurements showed q's of about 15 for printed boards and about 40 for 060-035 aluminum or magnesium. This provided their designers with information so that, as they went to a later design, they knew what to expect.

Mr. Stern suggested that, in all fairness, the designers don't care about nonlinearities of mounts. They don't care whether the q is 10 or 15, or 40 or 50, because they wouldn't apply it any differently in one case than in the other. His point was that, for electronic black boxes, an accuracy of 20 percent in data presentation was good enough. Mr. Stern conceded that this was not necessarily true in large structures.

Mr. Getline commented on the problem. "I'll restrict my thoughts to structures which are put together by rivets, nuts and bolts, spot welds and so on. It has been our experience that, for an aerospace vehicle, there are at least 2 values of q with which you have to deal. At very small response levels of the structure you may be dealing with very very low damping in the structure. Once the forcing function gets to a certain magnitude and breaks through this so-called static friction, then you are in another regime which may vary by 10 to 1 or more. There is a lot to be gained design-wise if you can provide yourself with reasonable assurance that you are going to stay down at this low value. This is something that can be done, in certain cases, without the addition of a lot of weight in the structure. The use of laminates or slight stiffening of the structure might be the difference between coming in under a weight bogie or going up by a factor of 10 or 15 percent."

Mr. Foley commented on a method of collecting information and furnishing it to design people. "In a sense we do have a system established for doing this within the data bank area, in which we collect inputs and responses which can give a designer some guidance. These are inputs and responses that have been measured previously on everything we can find that may be applicable to our own particular problems. There is still an awful lot of work to be done on this; however, this is the sort of information that we feel we can deliver to a designer which may be of some help before his black box flies. In the beginning of a design it will give him an idea, not necessarily of exactly what response he may get, but at least what sort of a ballpark he is in."

Dr. C. Morrow of the Aerospace Corporation commented, "The general trend of the discussion this morning has had many points in common with comments I made in my book, in which I have not tried to eliminate all confusion. I was just taking what I think is a more modest objective of trying to eliminate some of it and, hopefully, leave us understanding the remaining confusion a little better.

"There was a question a little while ago as to how one designs to random vibration as opposed to periodic. I think if our design procedures were a little bit more sophisticated, there might be some differences of approach. For example, you don't gain as much by damping the resonance when the excitation is random as when it is periodic. In the case of periodic excitation, if you reduce the q by a factor of 4, the response goes down by a factor of 4, whereas if it is random excitation and you decrease the q by a factor of 4 then the most probable gain you make is a factor of 2 in response. So, if our techniques were a little bit more sophisticated than they are now, then for the more severe environments we might find ourselves leaning a little more on detuning of resonances for random excitation and a little bit more in the direction of damping for periodic excitation. I don't make this as a final prediction, but it perhaps illustrates some differences of approach one might take.

"There is a strong temptation for people who are gathering data to assume that the designers are going to use this in great detail and proceed to compute the responses of everything inside a black box. I wonder if it is not a reasonable assessment of the present situation to say that there is virtually no detailed quantitative relationship between the black box design and the data prescribed until after the first test. If this is so, there is a related question of just how much correlation would we like to have. Do we really want everything calculated out in detail or is it possible that there are a few critical areas that need special attention? If we could identify these, then perhaps we could get the designer to give them some priority and get him to understand how to accomplish these specific tasks in the time he has available. In short, if we want to make the use of our data more sophisticated, couldn't we profitably spend a little time identifying some special problem areas that stand out as ready for this sort of work with a modest enough program for the typical black box so that the designer has time to do it. If this should be reasonable, it is clear that the bulk of the designing of the black box will proceed in principle as it is now with, hopefully, a little more sophistication. That is, the bulk of the design will be done using simple design rules without quantitative reference to the environments specified. I wonder if we could not perfect that process a little bit.

"We were discussing a little while ago the idea of taking a number out of the specifications, multiplying it by a factor of 10, and using this for an equivalent static design. It was immediately pointed out that this is not an infallible way to design equipment, nevertheless the process can be helpful. Regardless of what the data are and what the facts are, is this one design rule which can help improve a black box? We need to give a little more attention to the buildup of the resonances as well. Can't we put a little more effort into how to select the dynamics that the black box should have as opposed to taking this for granted and hoping that eventually we will get so sophisticated we can calculate it all out in detail after the fact? Wouldn't it be helpful if we could gather a few more bits of data on the resonances of the standard parts we use such as vacuum tubes, klystrons, relays, and so on, so that we can actually get an intuitive feeling for the dynamics of these things? Maybe we don't use this data for intensive calculation of random responses throughout the black box, but perhaps we can design in a little more orderly manner. I'd like to throw these questions on the floor in the hopes that they will stimulate a little helpful discussion."

Mr. R. Seely of NOL White Oak said, "If I understand the problem of the panel, it is to discover what piece of paper we can hand to the designers to help them out. Apparently the description of the environment is not enough. They were given g squared per cps power spectra and that sort of thing and, as somebody suggested, the designers don't know which end to put in their mouth. I submit that we have to take just one more step. We have to give the designers the response to the environment. Furthermore, as a practical matter, this can be done on an analog computer by setting up analogs and feeding in the environment so that the designer himself can sit there and twiddle knobs to find out the effect of damping, changing frequency, and the like. This is something he can do which is of practical benefit to him."

Mr. DuBois of Boeing said, "This problem of how to present data to the designer and what is my responsibility as the lab manager toward the final product hardware has been bothering me very much for a year or more. In fact, that is the main reason that I came to this Symposium. I now have this question answered somewhat in my own mind and this is the direction that I'm going to go unless somebody can give me some better advice. If and when this designer comes down to the lab and says that he has to design a black box or something to go in the second interstage of a given missile, I'm going to give him plots of the environments. I'll say, 'Here is your temperature profile for the flight; here are your g squared per cps plots for the various phases of the flight; here is your sound pressure profile. You take these and go have at a design and, when you get your prototype, bring it back down to the lab and we'll subject it to these environments and see how it goes. If it doesn't work we will both look at it and try to figure out why.' What this means to me in dollars is that I am going to concentrate my attention on having these environments readily available. If the design group calls up and wants to run a test on some gadget at 0.1 g squared per cps and 2000°F at 2:00 o'clock tomorrow afternoon, I'll have it readily available at minimum cost. Can I have some comments on this?"

Mr. Foley suggested that, in addition to giving the environmental data to the designer, he should also be advised to take this data to specialists in each environmental area and work out the best design.

Mr. Kfoury of Korfund Dynamics commented, "One of the things we haven't touched on this morning, and I wonder if it isn't an important point, is that of concurrent examination of typical structures and typical designs along with the designer's effort. In other words, if a designer had an environmental spec for a black box that he was designing, why couldn't the environmental lab concurrently run some tests on typical circuit boards or on components such as the klystron or the gyro that will go into this black box, and parallel to the designer's efforts, feed back some information on typical shapes, end fixity conditions, and possible use of damped materials. At the end of the designer's initial effort I think that he should have a pretty good picture of just what that structure might do before he goes down to the lab with his prototype. I think schedules are probably important in most of the programs we are all working on and I can't see the tandem approach to this problem. I think the concurrent approach should be used. May I have some comments from the panel please?"

Mr. Cohen of Sylvania directed his remarks to Mr. DuBois. "I have the position at Sylvania corresponding to yours at Boeing. I feel that we are forgetting that the majority of the design engineers that we are getting in now are in the lower age groups, just out of engineering schools. Our older men perhaps are going to supervisory positions where they have less to do with actual design. So, it doesn't become simply the job of the shock and vibration test engineer to say that he will have the environment ready. His job is also to help the company make money and to save money for the government, you might say. It is his job, therefore, to bring the design engineers down to the test lab and tell them, 'Perhaps we won't give you the answer, but let us try. Let us look at your problems. Here is what we think will happen. Listen to us, hear us out. Don't just shut us out of your minds, but let us into your design means with some authority and some responsibility.'

"You have one man who is responsible for the design, but calling upon his different technicians or his different specialists. The environmental engineer certainly should be one of these design specialists. My own personal experience is, as one man on the panel said, that if we consistently show the designers that we do have a point and demonstrate that we can help them, they will flock to our door. We have doubled our staff in the past 6 months just because of this. We put notices on the bulletin board that we are going to run a vibration test on box number 105. We schedule the vibration test on this box at a certain time. The gallery will be limited to 10 or 15 people and we will have 20 or 30 people lined up at the door, because they want to see the test. Most of these people will be the younger men, designers who are just out of school. The managers either feel they know too much about it or they aren't directly involved in the design of the part.

"I think the man from Boeing is sadly mistaken in feeling that his responsibility ends with simply providing the environment. I think that we've shown in this discussion that we cannot technically and specifically help the design engineer with a formula. We cannot give this to him because we don't have it ourselves as yet. The first discussion at the Symposium involved the analytical versus the empirical method and I say that it has to be a balance of both. We've got to give the man a test and we've got to show him what can happen to his equipment if he doesn't consider vibration."

Mr. Schwabe of Lockheed spoke with regard to the human element. "We have to consider, being myself an environmental engineer, that we disseminate information both to the designer and to the test engineer. There is a tendency of some environmental engineers to put a halo on top of their heads and shroud themselves in secrecy and magic without properly disseminating information. I think our basic task is to teach both the test engineer and the designer and to establish a human relationship of understanding and proper communication. I think it is most important to teach and to pass on the information, to establish communication."

Mr. J. White of Bell Laboratories said, "I've heard so much about what we're going to do in the next ten years and, it seems to me, we'll be quite successful in obtaining field data and building large shakers and shock machines. I'd like to propose this to the panel for their comments. I wonder if it would be possible for industry to get together in some organized committee and develop what I might call a dictionary that would bring together all the basic techniques that exist in the field today. It should include the normal mode approach and some of the statistical approaches, so that in some way in the future we might all have techniques for breaking a system down into basic components. Say you had a complicated missile structure that you could break down into cantilever beams, plates, and cylindrical shells. You could apply certain techniques to determine the response at different points on the structure to a certain random, sinusoidal, or shock input. From this maybe we could generate some tools that the designer could use, and combine these with statistical tools for predicting how things will fail, when they will fail and how to eliminate failure. I wonder if such a dictionary could be generated by industry?"

Dr. Muchmore answered, "We've had a number of handbooks come through our organization that attempt to do this in some way, shape, or form, and they have been sent to designers. Some of them are dictionaries; some of them are how-to-do-it books; some of them are essentially comic books that attempt to simplify this in the best way possible. Too often I hear a reaction, 'What, another handbook! Who needs it?' It's beginning to look to me that we are being inundated with handbooks and such things as this already, and a strong antagonism towards them is developing. Perhaps there is some other way of disseminating this, a sugar pill system."

Mr. Sanders of Rocketdyne commented, "I think that we might present the data that we get in such a manner that the designer would accept them and use them. Too often we find the designer making the same mistake a few months later that you had told him about previously. I think that is brought on by the antagonism we build up in him. We talked about inviting him down into the laboratory; let's get our test engineers up into the design area occasionally and make this a two-way street. You've got to have two-way communication, not one-way. It is not true that you know it all and he knows nothing. I happen to be responsible for test people, not design people, if you're wondering.

"Concerning the presentation of data in internal ballistics, which is also my responsibility in data reduction, many times we present data in a number of different ways. Quite often a problem is solved just by looking at one of these presentations. I think that we need to do a lot more of this. I hope soon to be able to be doing some of this from a digital computer output. If any of you have some programs or approaches on that, I would like to talk with you after the session is over."

Dr. Brierly of Quartermaster R&E Center mentioned two efforts underway that had a bearing on the subject. "One of them is an American Standards Association Committee for defining terms in the field. At least this will start the effort to get out some kind of a handbook of terms for the field of environmental engineering. This is a rather comprehensive attempt to define these terms for the entire field. I happen to be handling the earth sciences portion of that American Standards Association Committee, Z-84, and Roger Amorosi of the Institute of Environmental Sciences is chairman. We have it broken up into a number of subcommittees and we need your assistance in putting it together. For further information, I would suggest that you see me or Dave Askin of Frankford Arsenal. This is a very important effort and we need your help.

"The second effort is by the Institute of Environmental Sciences to turn out an environmental handbook. I believe that it was planned in such a way that it would take care of some of the needs of the previous speakers. I've been asked to handle portions of the geographical data end of it, but I think there are others that probably know more about it. If you don't know about the effort I think you should investigate."

Mr. Callahan of McDonnell Aircraft remarked, "I've been listening here for a couple of hours and I've been sort of appalled by some of the comments indicating that we have only lab people and we have only designers. I certainly know we at McDonnell maintain on-the-project technical groups who have full cognizance of the technical areas on that project. It is not a problem of hauling a designer down to the laboratory or hauling a test engineer up to the project, but of the structural dynamics engineer or the thermodynamics engineer who has responsibility in those areas to know what is going on and to see that the correct thing is done. I fail to see where there are just data people, just laboratory people, and just designers. There are people with much broader backgrounds and they certainly work with the full cross section of all three areas. If it doesn't work like this in other companies, it surprises me. I don't see, personally, that a man who has the duties of maintaining the laboratory, with all its problems, is in a position to do studies and furnish environmental data, and so on."

CLOSING REMARKS BY THE PANEL

Dr. Curtis: "We've had rather a hodge podge of questions and answers or not-answers, this morning and I think, basically, what we've all been saying is that our biggest problem is communications between people of different disciplines and the communication of knowledge and past experience to the people who need to use it today. I would like to suggest three efforts which we seem to need to undertake that may help the communication problem. This is certainly not a technical answer; it is just a communications answer.

"One thing that we have found useful, when we can conduct them in the proper way, is the holding of periodic and early design reviews to which we bring people with all these different backgrounds and experience to look over the design, find out what is troubling the designer, and give him a chance to ask questions and to define his problems also. The second thing which seems to pay off is to conduct early design evaluation testing, not to any spec or formalized procedure, but just to get the early items down in the laboratory with the designer present and ring out the system. The obvious weak points of any design will become readily apparent early enough to be of some use. The third thing I would suggest is that we, not for any fancy exotic reliability purposes, just do more capability testing so that, not only do we have a background of experience of what kind of design failed, but we also know what capability a certain design has. Knowing the design margins of past designs, we shall have a feel for how much extra design work will be necessary so that the next design can be produced to withstand a more severe environment."

Mr. Gertline: "I want to concur with Allen that it is a problem of communication. We are dealing with ideas and we are trying to communicate with people who are dealing with hardware. I think one of the basic things we have to understand is that the man on the board, whether he is designing a structure or putting together an electronic package from a schematic, can manipulate only two things, stresses and forces. In whatever form we get our environmental data, I think it is up to us, the go-betweens between the technical and the design areas, to interpret these data in the stress and load form that the designer can use. In closing, since we are actually a part of reliability programs, I would just like to requote what Colonel Glenn said. 'The way to build more reliability into missiles is to build canvas covered blockhouses.'"

Mr. Foley: "We seem to have covered quite a bit of discussion on obtaining and analyzing environmental information. I thought I might close with something about a survey that was conducted among designers in which the question was asked, 'In what form would you most like to see your data presented to you?' The majority thought the presentation should be graphical, statistical, and must contain only those parameters which can be measured in the test laboratory and verified in the field. Data that are directly applicable to the particular project must meet two other requirements. The data must be recorded in a manner which permits their presentation through an automated system and they must be recorded in a manner which permits their analysis in an automated system."

Dr. Muchmore: "I think we should give those designers in the audience a real hand for sitting there quietly and not objecting to the malignment that we have given to their profession. I would like also to close on a word of defense for philosophy. I feel that we shouldn't forget that one of the greatest natural philosophers of all time was the man who enunciated the very deep sophisticated fact that force is the rate of change of momentum. Now I ask you, where would a shock and vibration symposium be without $f = ma$? It is one of the fundamental laws of physics and if you stop and think about it, it is a very deep and a very sophisticated thing. Mass is an abstraction. It took a real genius to realize this. I think that we have all come to think of $f = ma$ as just part of the folklore that everyone knows, whereas some of these modern things such as power spectral density are too deep for us. I think it is really the other way around. If we ask ourselves which is the deepest most sophisticated idea, it is $f = ma$ and not that the rms output is integral with respect to omega squared of the transfer function times the power spectral density. That's really a simple idea. This is one that was enunciated by a couple of modern-day philosophers, Kinshein and Wiener.

"I think that designers really are not quite the unthinking persons we seem to have made them out this morning. They are going to use their heads and, really, what we have to do is use our heads. We use the tools that are available to us, one of which is $f = ma$, another is that the mean square output is the integral over the spectrum. They are both just simple applications of calculus which, incidentally, was also invented by Newton. The real thing that we have to do is use what tools we have to solve the problems that we have and not close our minds to the fact that new tools are being invented. We should use the new ones as well as the old ones and not always regard the new ones as necessarily more difficult than the old ones. Often times it is the other way around."

Dr. Bendat: "I have enjoyed being on this panel because I think we have stuck pretty close to the subject of the panel. We've covered it from many different aspects and I think we have demonstrated the fact that, here in this particular area of work, you violate a common mathematical linear law, namely that a combined effort can be greater than the sum of its parts. We need to do this in order to advance the field. I would also like to note the fact that mathematics is an art and a philosophy, as well as a science. There is a human element involved; there is a matter of presentation; and there is a matter of being able to think in terms of the mathematical concepts, to be guided by them, and to interpret them. When people put the proper emphasis on the mathematics and recognize it as a guideline and not as a solution to all the other practical problems that have been mentioned here today, it can take its rightful role."

Mr. Gardner: "I think we've discussed the subject pretty well from a few basic points in design to well into the philosophy area. We've talked not only about the analytical and the practical, the hardware problems, but also about the human problem. Any further comments I could add would be redundant. I want to thank the panel and the audience for a very good session.

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