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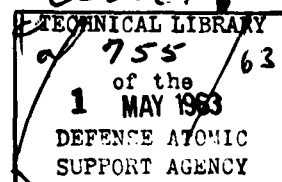
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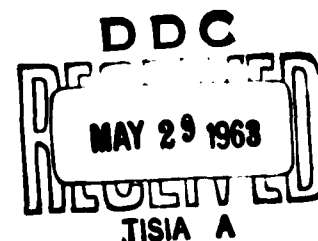
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THREE-DIMENSIONAL RAY TRACE COMPUTER PROGRAM FOR ELECTROMAGNETIC WAVE PROPAGATION STUDIES

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(6) THREE-DIMENSIONAL RAY TRACE COMPUTER PROGRAM
FOR ELECTROMAGNETIC WAVE PROPAGATION STUDIES,

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ABSTRACT

↙
A computer program is described for use on the IBM-704/7090 electronic data processing machine or any large computer accepting FORTRAN. The necessary modifications for use on these two computers are simple and self-evident. The computer program permits the computation of detailed ray patterns portraying the spatial distribution of rays emitted from a transmitter whose geographic coordinates with respect to the center of the earth are known. This program deals with the solution of the differential equations, given by Hamiltonian optics, for ray paths in non-isotropic, three-dimensionally nonhomogeneous media whose complex phase refractive index is given by the Appleton-Hartree formula.

~~This report is to be considered as a first attempt in presenting an account of the current status of the development of this program, which has yielded many useful results.* Presented also are sample calculations as well as some results that have been obtained by using this program.~~
are also presented.
↑

* not included.

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SECTION I

INTRODUCTION

What is customarily referred to as reflection in electromagnetic wave propagation, is actually the result of an integrated effect of a phenomenon of refraction (i. e., the gradual changing of the direction of the electromagnetic energy transport vector). Insofar as the phenomenon of refraction is concerned it is well established that the spatial gradient of electron density plays a crucial role in controlling the propagation behavior of an electromagnetic wave. Although this is well known, propagation studies so frequently incorporate a curious mixture of simple refraction phenomena (in the use of Snell's law) and of mirror-like reflection. Hence, in these studies they omit an accounting of these spatial gradients of electron density as well as their variation. As generally known, these oversimplified studies derive from Snell's law the condition for reflection, that being, that at the spatial point of reflection, the electron density must attain the value of $1.24 \times 10^4 f^2 \cos^2 I$ electron/cc, where f is the propagation frequency in Mc/sec and I is the angle of incidence of the ray upon the first layer of the assumed stratified ionosphere. Such a simplified approach is necessary because of the difficult task that leads to a numerical solution of a propagation problem which incorporates these electron density gradients in its solution.

With increasing use of transmitters in satellites, as well as, for the understanding of the behavior of wave propagation under severely abnormal atmospheric conditions, it becomes important to take a realistic account of the spatial gradient of the electron density in electromagnetic wave propagation studies. Only in this manner will it become possible to usefully utilize the new satellite propagation techniques in studies designed toward the understanding of the atmospheric ionization-deionization phenomena and through this the detailed structure of the ionosphere.

Within the last ten years some effort has been made in constructing analogue computers for the study of ray propagation which accounted for spatial electron density gradients (as for example D. F. Hartree, et al., Manchester, England; M. S. Wong, AFCRL, Bedford, Mass.).

Some of these analogue computers were and still are limited to spatial electron density gradients in a particular direction thus forcing the propagation problem into a two-dimensional plane, or to the study of the behavior of refraction on wave propagation. These constraints are built into the analogue computer and are not easily changed.

One approach that avoids these constraints is the use of a large electronic data processing system where the ordered logical flow controlling any calculation, is easily varied. Combining such a programmed computer with Hamiltonian optics, which give the desired ray tracing equations for a nonhomogeneous, non-isotropic medium, and the Appleton-Hartree formula for the complex refractive index, permits in addition to three dimensional ray tracing, the simultaneous study of numerous other variables of the propagation problem. Such an approach to the ray trace propagation problem is presented in its present state of development. A great deal of improvement in some of the routines is possible. As a result, the writer would like to encourage correspondence concerning these matters. In addition, it should be stated, that a computer program for solving the three-dimensional ray trace problem has also been written for the Ferranti Mercury Computer at Manchester University by C. B. Haselgrove and J. Haselgrove.²

SECTION II

COMPUTATIONAL PROCEDURE

A. RAY TRACE EQUATIONS

When electromagnetic waves are propagated through a medium whose dielectric constant or index of refraction is a varying function of the path, the waves undergo a change in direction, or refractive bending, and a retardation in the velocity of propagation. The spatial relationship expressing this angular bending of a ray of an electromagnetic wave can be determined by basing the theory of rays and waves on a variational principle (Fermat's) in space. By a ray is meant the path travelled by the transport vector of electromagnetic wave energy between the transmitter and an associated electromagnetic field-intensity point in space. This Hamilton Theory starts from the variational principle $\delta \int \mu ds = 0$, where μ is a medium function or index of refraction, depending on position and direction. From this principle the theory constructs the properties of systems of rays and the waves associated with them (i. e., extremals and transversals, in the language of the calculus of variations). Because of the stationarity in time, the theory may be regarded as a statical one, the rays being fixed curves in space and the waves fixed surfaces. Neither wave-length nor frequency is involved. Likewise the waves form a continuous set of surfaces, not distinguished as crests and troughs. This theory, whether in the form preferred by Hamilton or otherwise, has been the subject of many books under the general title "Geometric Optics".

Thus, applying Hamiltonian optics leads to the general Hamilton's Equations¹ for ray paths of electromagnetic waves in a three-dimensional non-isotropic nonhomogeneous medium. From them Haselgrove² has derived the following set of equations for ray paths in a spherical coordinate system in a format suitable for numerical integration on high speed computers:

$$\frac{dr}{d\tau} = \frac{1}{\mu^2} \left(\sigma_r - \mu \frac{\partial \mu}{\partial \sigma_r} \right) \quad (1)$$

$$\frac{d\theta}{d\tau} = \frac{1}{r\mu^2} \left(\sigma_\theta - \mu \frac{\partial\mu}{\partial\sigma_\theta} \right) \quad (2)$$

$$\frac{d\varphi}{d\tau} = \frac{1}{\mu^2 r \sin \theta} \left(\sigma_\varphi - \mu \frac{\partial\mu}{\partial\sigma_\varphi} \right) \quad (3)$$

$$\frac{d\sigma_r}{d\tau} = \frac{1}{\mu} \frac{\partial\mu}{\partial r} + \sigma_\theta \frac{d\theta}{d\tau} + \sin \theta \sigma_\varphi \frac{d\varphi}{d\tau} \quad (4)$$

$$\frac{d\sigma_\theta}{d\tau} = \frac{1}{r} \left[\frac{1}{\mu} \frac{\partial\mu}{\partial\theta} - \sigma_\theta \frac{dr}{d\tau} + r \cos \theta \sigma_\varphi \frac{d\varphi}{d\tau} \right] \quad (5)$$

$$\frac{d\sigma_\varphi}{d\tau} = \frac{1}{r \sin \theta} \left[\frac{1}{\mu} \frac{\partial\mu}{\partial\varphi} - \sin \theta \sigma_\varphi \frac{dr}{d\tau} - r \cos \theta \sigma_\varphi \frac{d\theta}{d\tau} \right] \quad (6)$$

In these equations r , θ , and φ are the spatial coordinates of a spherical system; μ is the arbitrary index of refraction; $\vec{\sigma}$ is a vector directed normal to the phase fronts of the ray of magnitude μ with σ_r , σ_θ , and σ_φ its respective components in the r , θ , and φ directions; τ is the time of phase travel along the ray path (i. e., $f\Delta\tau/c$ = the number of wavelengths in the medium along the ray path, where f is the electromagnetic wave frequency and c the velocity of light in vacuum).

It is noteworthy that the partial derivatives of μ appear explicitly in Equations 1 to 6, in accordance with the fact that the gradients of μ play crucial roles in determining the spatial ray paths.

This closed set of first-order partial differential equations which will describe the propagation behavior of an electromagnetic wave

under geometric optics conditions, can be integrated simultaneously by a point-by-point numerical process if expressions can be developed for the necessary derivatives of the phase refractive index μ . The quantity μ and its derivatives are obtained by using the Appleton-Hartree formula³ as the definition of the complex phase refractive index M .

The derivatives are derived under the conditions of ray optics, that is, that the imaginary part of M^2 is very much smaller than the real part. As an aid for computer use and comparison with published works of others^{2,4}, the Appleton-Hartree formula is written as:

$$M^2 = (\mu - j\kappa)^2 = 1 - \frac{2X(1 - X - jZ)}{D} \quad (7)$$

$$D = 2(1 - X - jZ)(1 - jZ) - Y^2 \sin^2 \psi + S \quad (8)$$

$$S = \pm \left[(Y \sin \psi)^4 + 4Y^2(1 - X - jZ)^2 \cos^2 \psi \right]^{1/2} \quad (9)$$

where

M = the complex phase refractive index

X = a scalar quantity representing the normalized electron density

$$\frac{4\pi N e^2}{m\omega^2} = \frac{\omega_p^2}{\omega^2}$$

ω_p = plasma frequency at the spatial point

ω = angular wave frequency = $2\pi f$

m, e = mass and charge of an electron

N = electron density at a spatial point

Y = normalized magnitude of the earth's magnetic field
vector $\vec{Y} =$

$$\frac{e\vec{H}}{mc\omega} = \frac{\vec{\omega}_c}{\omega}$$

ω_c = magnitude of the gyromagnetic frequency of the electron
about the earth's magnetic field

Z = normalized collision frequency = (ν/ω)

ν = collision frequency at a spatial point in collisions per
second

ψ = angle defined by the inner product of the vectors $\vec{\theta}$ and
 $\vec{Y} =$

$$\cos^{-1} \left[\frac{\sigma_r Y_r + \sigma_\theta Y_\theta + \sigma_\phi Y_\phi}{(\mu Y)} \right]$$

$\kappa = \frac{ck}{\omega} =$ imaginary part of the complex phase refractive
index

c = velocity of light in vacuum

k = absorption coefficient of the wave per unit length of
path (it is proportional to the conductivity of the medium)

It is noted that there are two possible values for the complex index of refraction M corresponding to the plus and minus sign on S which represent two different modes of ionospheric propagation. These are commonly called "ordinary" and "extraordinary" modes for the plus and minus sign respectively. Also, the Appleton-Hartree formula (Equations 7 to 9) is notable in that it presents μ , which actually is a spatial function of six variables, in the form containing purely algebraic operations on factors or terms each of which is a function of at most three variables, that is, either of r, θ, ϕ or $\sigma_r, \sigma_\theta, \sigma_\phi$. This reduces the representation of μ to a numerical problem, easily solvable to current computer techniques.

If i represents any one of the spatial spherical coordinates r , θ , and φ , then the partial derivatives of μ with respect to the components of the wave normal, $\vec{\sigma}$, can be easily shown to be

$$\frac{\partial \mu}{\partial \sigma_i} = \frac{\partial \mu}{\partial \psi} \frac{\partial \psi}{\partial \sigma_i} = \frac{\partial \mu}{\partial \psi} \left(\frac{\sigma_i Y \cos \psi - \sigma Y_i}{\sigma^2 Y \sin \psi} \right) \quad (10)$$

This useful transformation also enjoys the following property:

$$\text{When } \psi \rightarrow 0, \quad \frac{\partial \mu}{\partial \psi} \rightarrow 0, \quad \frac{\partial \psi}{\partial \sigma_i} \rightarrow \infty \quad \text{but} \quad \frac{\partial \mu}{\partial \sigma_i} \rightarrow 0 \quad (11)$$

To evaluate $\partial \mu / \partial \sigma_i$; the necessary partial derivative is:

$$\begin{aligned} \frac{\partial \mu}{\partial \psi} &= \text{Re} \frac{\partial M}{\partial \psi} = \text{Re} \left\{ \frac{(M^2 - 1)(Y^2 \sin \psi \cos \psi)}{MD} \left[1 - \frac{1}{S} [(Y \sin \psi)^2 - 2(1 - X - jZ)^2] \right] \right\} \\ \therefore \frac{\partial \mu}{\partial \psi} &= \left\{ -Y^2 \sin \psi \cos \psi \left[a_0(a_2 a_5 - b_2 b_5) - b_0(a_2 b_5 + b_2 a_5) \right] \right\} \quad (12) \end{aligned}$$

where a_0 , b_0 and all following a_j , b_j are defined in the List of Useful Formulae. The partial derivatives of the real part of the phase refractive index with respect to the spatial coordinates ($i = r, \theta, \text{ or } \varphi$) are similarly obtained by use of the relationship

$$\frac{\partial \mu}{\partial i} = \frac{\partial \mu}{\partial X} \frac{\partial X}{\partial i} + \frac{\partial \mu}{\partial Y} \frac{\partial Y}{\partial i} + \frac{\partial \mu}{\partial Z} \frac{\partial Z}{\partial i} + \frac{\partial \mu}{\partial \psi} \frac{\partial \psi}{\partial i} \quad (13)$$

where

$$\frac{\partial \mu}{\partial X} = \text{Re} \frac{\partial M}{\partial X} = \text{Re} \left\{ \frac{1}{MD} \left[2X - 1 + jZ + (M^2 - 1) \left(1 - jZ + \frac{2Y^2(1-X-jZ)\cos^2\psi}{S} \right) \right] \right\}$$

$$\therefore \frac{\partial \mu}{\partial X} = \left\{ a_o \left[(2X-1) - (a_4 a_5 - b_4 b_5) \right] + b_o \left[Z + a_5 b_4 + b_5 a_4 \right] \right\} \quad (14)$$

$$\frac{\partial \mu}{\partial Y} = \text{Re} \frac{\partial M}{\partial Y} = \text{Re} \left\{ \frac{(M^2 - 1)}{MD} Y \left[(\sin\psi)^2 - \frac{1}{S} \left[Y^2 \sin^4\psi + 2(1-X-jZ)^2 \cos^2\psi \right] \right] \right\}$$

$$\therefore \frac{\partial \mu}{\partial Y} = Y \left\{ (a_o a_5 - b_o b_5) \left[a_6 - (\sin\psi)^2 \right] - b_6 (a_o b_5 + b_o a_5) \right\} \quad (15)$$

$$\frac{\partial \mu}{\partial Z} = \text{Re} \frac{\partial M}{\partial Z} = -\text{Im} \left\{ \frac{1}{MD} \left[X + (M^2 - 1) \left(2 - X - 2jZ + \frac{2Y^2(1-X-jZ)}{S} \cos^2\psi \right) \right] \right\}$$

$$\frac{\partial \mu}{\partial Z} = \left[b_o (X - a_5 a_7 + b_5 b_7) - a_o (b_5 a_7 + a_5 b_7) \right] \quad (16)$$

and where Re and Im represent, respectively, the real and imaginary part of the complex expression. The partial derivative of the angle ψ (which is the angle defined by the inner product of the normalized geomagnetic field vector $\hat{Y}$ and the wave normal vector $\hat{\theta}$), with respect to the spatial coordinates r , θ , and φ , measures the change in spatial direction of the earth's magnetic field since the calculation is made holding $\hat{\theta}$ constant. The partial derivatives $\partial X/\partial i$, $\partial Y/\partial i$ and $\partial Z/\partial i$ are obtainable from the analytical expressions for the spatial variation of the electron density, geomagnetic field and collision frequency. Examples of these will be considered later.

In addition to Equations 1 to 6, which define the spatial ray path, it is usually desirable to calculate the optical path length s , the time of travel T , as well as, the one way absorption A , of the energy of an electromagnetic pulse. The equation describing the differential optical path is given by

$$\frac{ds}{d\tau} = \frac{1}{\mu} \left[\mu^2 + \left(\frac{\partial \mu}{\partial \psi} \right)^2 \right]^{1/2} \quad (17)$$

In determining the time of travel T , a distinction must be made between two electromagnetic wave velocities. The phase velocity, $v_p = c/\mu$, is defined as the spatial velocity with which a point of constant phase moves. Group velocity, $v_g = d\omega/d(\omega/v_p)$, is the spatial velocity of electromagnetic energy travel; put into other words, it is the velocity of a "Maxwell Demon" who remains at the same point on the envelope of the advancing wave. From these two definitions it can be easily shown that the time (in seconds) of energy pulse travel can be written as:

$$\frac{dT}{d\tau} = \frac{1}{c} \left[1 + \frac{\omega}{\mu} \frac{\partial \mu}{\partial \omega} \right]; \quad (18)$$

where

$$\begin{aligned} \frac{\partial \mu}{\partial \omega} &= \operatorname{Re} \frac{\partial M}{\partial \omega} = \\ &- \operatorname{Re} \left\{ \frac{1}{MD\omega} \left[X(2X + jZ) + (M^2 - 1) \left[2 - 2jZ - jZX + \frac{Z(Y \cos \psi)^2}{S} (1 - X - jZ)(1 + X) \right] \right] \right\} \\ \therefore \frac{\partial \mu}{\partial \omega} &= - \frac{1}{\omega} \left[a_0(2X^2 - a_5 a_8 + b_5 b_8) + b_0(XZ + b_5 a_8 + b_8 a_5) \right] \quad (19) \end{aligned}$$

The one-way absorption, A (in nepers), suffered by the energy of an electromagnetic pulse is determined by

$$\frac{dA}{d\tau} = - \frac{\omega}{c} \frac{\kappa}{\mu} A = - \frac{k}{\mu} A \quad (20)$$

where k (which is proportional to the spatial conductivity) is the absorption of the wave per unit length of path.

The solution of this set of first order partial differential equations will describe the propagation characteristics of an electromagnetic wave in a heterogeneous anisotropic medium.

B. LIST OF USEFUL FORMULAE FOR RAY TRACING

As an aid for the computer solution of these differential equations the following list of formulae are found to be very useful. As before, Re and Im respectively represent the real and imaginary parts of the complex quantity.

$$\text{ReS} = S_1 = R_S \cos \theta_S \quad (21)$$

$$\text{ImS} = S_2 = R_S \sin \theta_S \quad (22)$$

$$R_S = \left\{ \left[(Y \sin \psi)^4 + (2Y \cos \psi)^2 [(1-X)^2 - Z^2] \right]^2 + \left[(2Y \cos \psi)^2 [2(1-X)Z] \right]^2 \right\}^{1/4} \quad (23)$$

$$\theta_S = \frac{1}{2} \tan^{-1} \left\{ \frac{(2Y \cos \psi)^2 [2(X-1)Z]}{(Y \sin \psi)^4 + (2Y \cos \psi)^2 [(1-X)^2 - Z^2]} \right\} \quad (24)$$

$$\text{ReD} = d_1 = \left\{ 2 [(1-X) - Z^2] - (Y \sin \psi)^2 + S_1 \right\} \quad (25a)$$

$$\text{ImD} = d_2 = [S_2 - 2Z(2-X)] \quad (25b)$$

$$\text{ReM} = m_1 = \mu = R_m \cos \theta_m \quad (26)$$

$$\text{ImM} = m_2 = -\kappa = R_m \sin \theta_m \quad (27)$$

$$R_m \left\{ \left(1 - \frac{2X \left[(1 - X)d_1 - Zd_2 \right]^2}{d_1^2 + d_2^2} \right)^2 + \left(\frac{2X \left[Zd_1 + (1 - X)d_2 \right]^2}{d_1^2 + d_2^2} \right)^2 \right\}^{1/4} \quad (28)$$

$$\theta_m = \frac{1}{2} \tan^{-1} \left\{ \frac{2X' \left[Zd_1 + (1 - X)d_2 \right]}{d_1^2 + d_2^2 - 2X \left[(1 - X)d_1 - Zd_2 \right]} \right\} \quad (29)$$

$$a_o = \frac{(m_1 d_1 - m_2 d_2)}{(m_1 d_1 - m_2 d_2)^2 + (m_1 d_2 + m_2 d_1)^2} \quad (30)$$

$$b_o = \frac{(m_1 d_2 + m_2 d_1)}{(m_1 d_1 - m_2 d_2)^2 + (m_1 d_2 + m_2 d_1)^2} \quad (31)$$

$$a_1 = \left\{ 2 \left[(1 - X)^2 - Z^2 \right] - (Y \sin \psi)^2 \right\} \quad (32)$$

$$b_1 = 4Z(1 - X) \quad (33)$$

$$a_2 = \left[1 + \frac{(a_1 S_1 - b_1 S_2)}{S_1^2 + S_2^2} \right] \quad (34)$$

$$b_2 = \left[\frac{S_1 b_1 + a_1 S_2}{S_1^2 + S_2^2} \right] \quad (35)$$

$$a_4 = \left\{ 1 + \frac{2(Y \cos \psi)^2}{S_1^2 + S_2^2} [S_1(1 - X) - Z S_2] \right\} \quad (36)$$

$$b_4 = \left\{ Z + \frac{2(Y \cos \psi)^2}{S_1^2 + S_2^2} [S_2(1 - X) + Z S_1] \right\} \quad (37)$$

$$a_5 = \frac{2X[(1 - X)d_1 - Z d_2]}{d_1^2 + d_2^2} = (1 + m_2^2 - m_1^2) \quad (38)$$

$$b_5 = \frac{2X[Z d_1 + (1 - X)d_2]}{d_1^2 + d_2^2} = 2m_2 \quad (39)$$

$$a_6 = \frac{S_1 \left\{ (Y \sin^2 \psi)^2 + 2 \cos^2 \psi [(1 - X)^2 - Z^2] \right\} - S_2 [(2 \cos \psi)^2 Z (1 - X)]}{S_1^2 + S_2^2} \quad (40)$$

$$b_6 = \frac{S_2 \left\{ (Y \sin^2 \psi)^2 + 2 \cos^2 \psi [(1 - X)^2 - Z^2] \right\} + S_1 [(2 \cos \psi)^2 Z (1 - X)]}{S_1^2 + S_2^2} \quad (41)$$

$$a_7 = \left\{ (2 - X) + \frac{2(Y \cos \psi)^2}{S_1^2 + S_2^2} [S_1(1 - X) - S_2 Z] \right\} \quad (42)$$

$$b_7 = \left\{ 2Z + \frac{2(Y \cos \psi)^2}{S_1^2 + S_2^2} [S_1 Z + S_2(1 - X)] \right\} \quad (43)$$

$$a_8 = 2 \left\{ 1 + \frac{(Y \cos \psi)^2}{S_1^2 + S_2^2} [(1 - X^2)S_1 - S_2 Z(1 + X)] \right\} \quad (44)$$

$$b_8 = \left\{ Z(2 + X) + \frac{2(Y \cos \psi)^2}{S_1^2 + S_2^2} [S_1 Z(1 + X) + S_2(1 - X^2)] \right\} \quad (45)$$

Nomenclature Used in Ray Trace Equations

r, θ, φ	spatial coordinates of a spherical system
$\vec{\sigma}$	wave normal or refractive index vector
$\sigma_r, \sigma_\theta, \sigma_\varphi$	vector components ($\vec{\sigma}$)
τ	time of phase travel (in units of length)
μ	real part of complex phase refractive index
f	electromagnetic wave frequency
c	velocity of light in vacuum
m, e	mass and charge of an electron
Re	real part of the complex expression
Im	imaginary part of the complex expression
M	complex phase refractive index
X	scalar quantity representing normalized electron density
ω_p	plasma frequency at the spatial point
ω	angular wave frequency
N	electron density at spatial point
Y	normalized magnitude of the earth's magnetic field vector $\vec{Y}$
ω_c	magnitude of the gyromagnetic frequency of the electron about the earth's magnetic field
Z	normalized collision frequency (ν/ω)
ν	collision frequency at spatial point in collisions/sec.
ψ	angle defined by inner product of vectors $\vec{\sigma}$ and $\vec{Y}$

κ	imaginary part of the complex phase refractive index
k	absorption coefficient of the wave per unit length of path (proportional to the conductivity of the medium)
i	represents each of the spatial spherical coordinates r, θ, φ
T	time of travel (seconds)
s	optical path length (km)
A	one-way absorption (nepers)
v_p	spatial velocity with which a point of constant phase moves
v_g	group velocity - spatial velocity of electromagnetic energy travel

C. COORDINATE TRANSFORMATION

If one takes three orthogonal planes intersecting at a point, one knows that the position of any point S in space is uniquely determined by the three perpendiculars from S on these planes, each with its proper sign. However, the problem of selecting the most useful orientation of such an orthogonal system is difficult since the usefulness of a coordinate system partially depends on the problem definition and the application of its solution. This ray trace program is designed as a sub-set of a much larger computer programming effort⁷ where the earth's geomagnetic field plays an important part. To minimize the number of computer transformations in the design of the over-all program, an earth centered spherical coordinate system (r, θ, φ) was chosen, whose z-component is coincident with the magnetic dipole axis.

This selection permits the application of the computer program to a great many studies of ray path problems because it accounts for the earth curvature and accepts for solution any electromagnetic radiating source whose transmitter location specifications of elevation, E, and azimuth, A, angles, as well as, geographic latitude, φ_R , geographic longitude, λ_R , and position with respect to the surface of the earth are known. Because the usefulness of this computer program can be extended by modification to other coordinate systems, as for example, an earth centered geographic system, or a radar coordinate system, the necessary coordinate transformations from the radar to the earth centered geomagnetic coordinate system will be described in detail.

For the discussion of this coordinate transformation, it is assumed that the electromagnetic wave transmitter is earth bound (i. e., fixed to the surface of the earth) at a geographic latitude φ_R and a geographic longitude λ_R . It is assumed that the radar is positioned so that the transmitting direction is described by the elevation angle, E, with respect to the tangent plane to the earth surface at the radar location, and an azimuth angle, A, measured from the radar coordinate that is tangent to a great circle passing through the north geographic pole. The azimuth angle is plus when measured counter-clockwise from the coordinate axis, ζ , whose positive direction points in the direction of geographic north. It is further assumed that S is a spatial point on the non-deviated portion of the ray a distance, R, from the electromagnetic wave transmitter. This is the spatial starting

point at which the numerical methods necessary for solution of the differential equations, must be initialized.

A transformation is required from the spherical radar coordinate system to the magnetic coordinate system whose origin is at the center of the earth.

The necessary matrices which are required to transform R, A, E coordinates to r, θ , φ coordinates can be arrived at by a series of simple matrix transformations.

1. Let ϵ, η, ζ be a set of orthogonal coordinates with origin on the surface of the earth. Let ϵ -axis be perpendicular to the earth's surface while ζ is directed (geographically) northward and η to the east. As stated above R is the slant range; E is the elevation angle; and A is the azimuth angle. Hence going from R, E, A to ϵ, η, ζ

$$\begin{Bmatrix} \epsilon \\ \eta \\ \zeta \end{Bmatrix} = \begin{Bmatrix} R \sin E \\ R \cos E \sin A \\ R \cos E \cos A \end{Bmatrix} \quad (46)$$

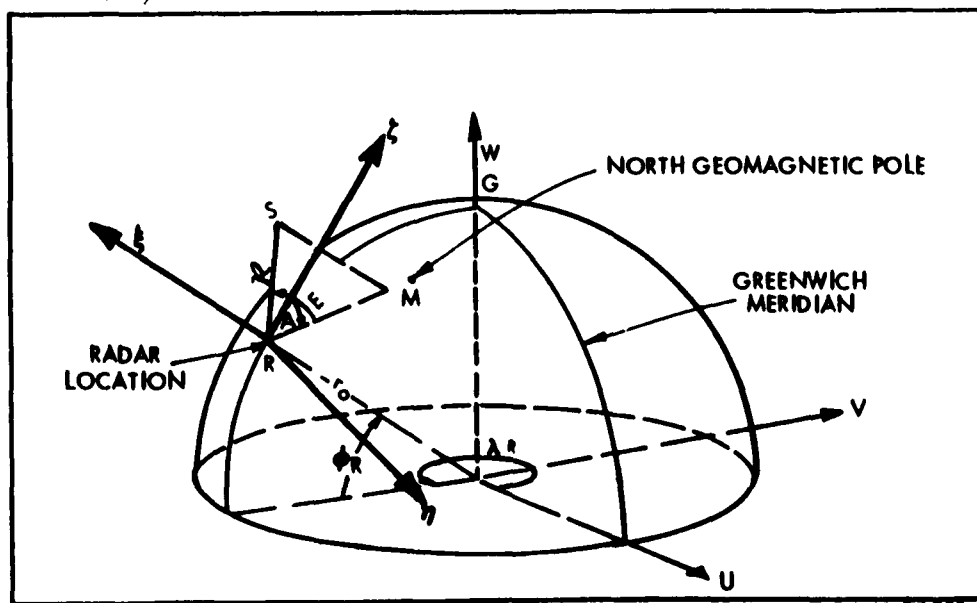


Figure 1a. Geometry of Coordinate Transformation

2. Let x_1, y_1, z_1 equal an orthogonal coordinate system with origin on the earth's axis of rotation. The x_1 -axis is in the latitude plane of the radar site and passes through the radar site. The z_1 -axis is coincident with the north geographic coordinate w . A translation and rotation is required in going from ϵ, η, ζ to x_1, y_1, z_1 .

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} \cos \varphi_R & 0 & -\sin \varphi_R \\ 0 & 1 & 0 \\ \sin \varphi_R & 0 & \cos \varphi_R \end{pmatrix} \begin{pmatrix} \epsilon \\ \eta \\ \zeta \end{pmatrix} + \begin{pmatrix} r_o \cos \varphi_R \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = (b_{ij}) \begin{pmatrix} \epsilon \\ \eta \\ \zeta \end{pmatrix} + (b_i) \quad (47)$$

where r_o equals the earth's radius.

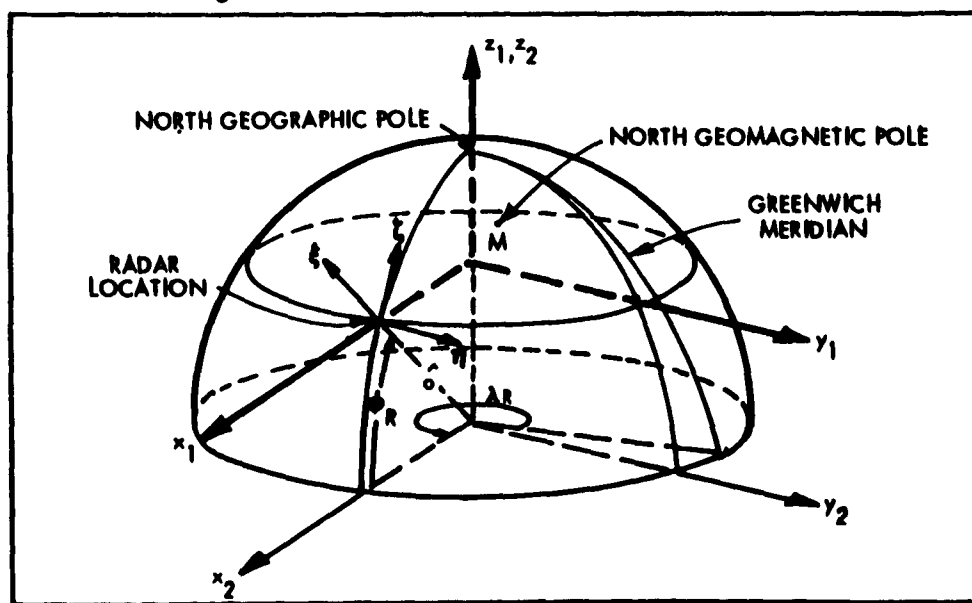


Figure 1b. Geometry of Coordinate Transformation

3. Let x_2, y_2, z_2 equal the orthogonal coordinate system with origin at the earth's center. Let the x_2 -axis be parallel to the x_1 -axis and z_2 coincide with w , hence also with z_1 . Then going from x_1, y_1, z_1 to x_2, y_2, z_2 by translation

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ r_o \sin \varphi_R \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + (c_i) \quad (48)$$

4. Let x_3, y_3, z_3 represent an orthogonal coordinate system with origin at the earth's center such that the x_3 -axis intersects the zero degree longitudinal geomagnetic meridian while the z_3 -axis coincides with w . Hence going from x_2, y_2, z_2 to x_3, y_3, z_3 by rotation about the z_2 -axis yields,

$$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \begin{pmatrix} \cos(\lambda_M - \lambda_R) & \sin(\lambda_M - \lambda_R) & 0 \\ -\sin(\lambda_M - \lambda_R) & \cos(\lambda_M - \lambda_R) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \quad (49)$$

$$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = (d_{ij}) \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

where as before λ_M and φ_M represent the geographic longitude and latitude of the geomagnetic north pole M.

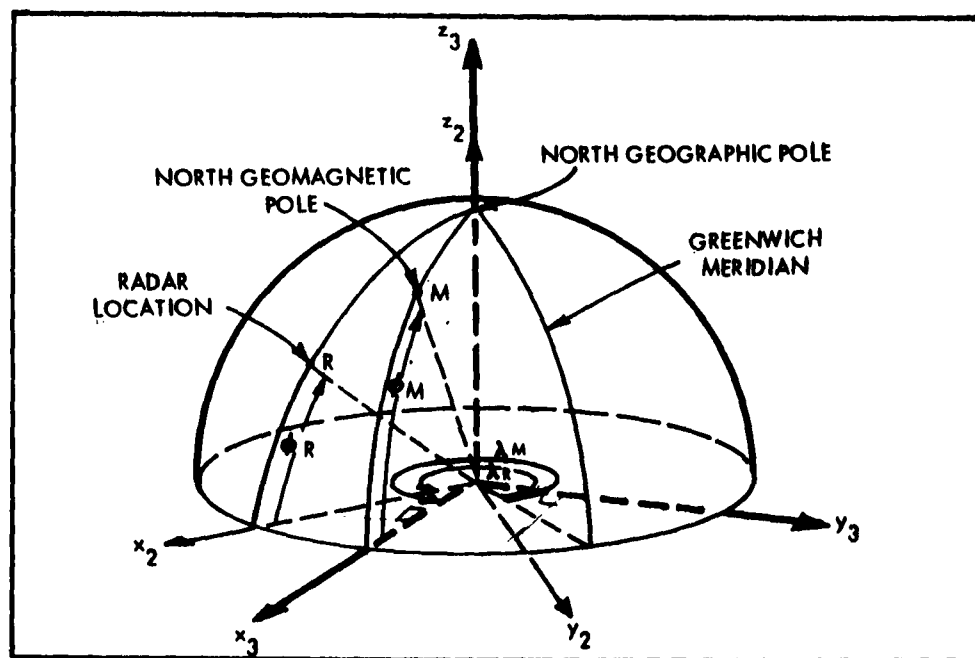


Figure 1c. Geometry of Coordinate Transformation

5. Let x, y, z represent an orthogonal coordinate system with origin at the earth's center. Let the x -axis pass through the great circle connecting the geographic and geomagnetic poles while the z -axis passes through the geomagnetic pole M . In going from x_3, y_3, z_3 to x, y, z by a rotation one obtains

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin\varphi_M & 0 & -\cos\varphi_M \\ 0 & 1 & 0 \\ \cos\varphi_M & 0 & \sin\varphi_M \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (e_{ij}) \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} \quad (50)$$

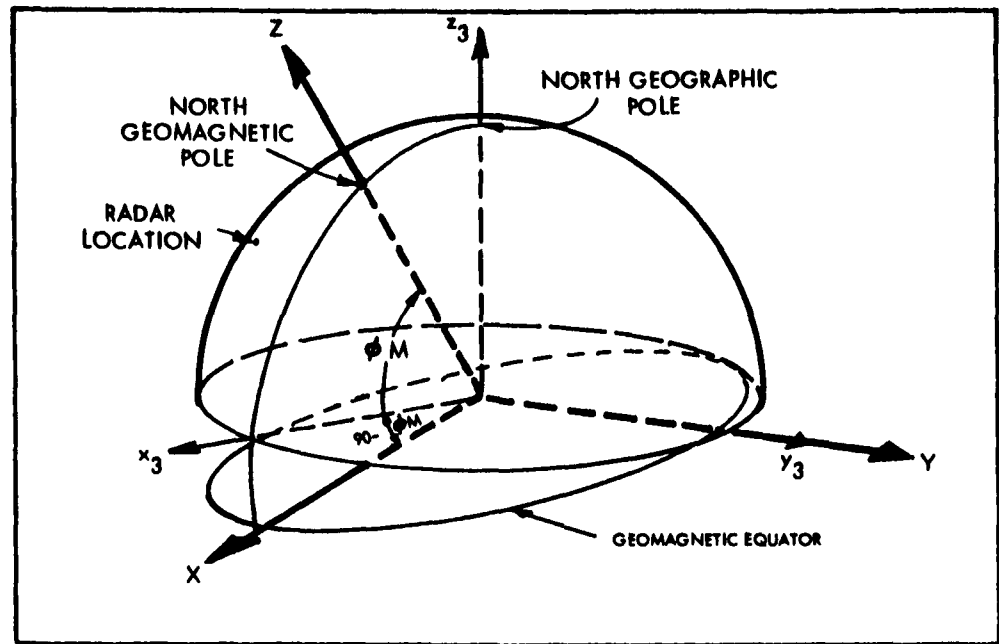


Figure 1d. Geometry of Coordinate Transformation

Hence by matrix multiplication one can transform from radar coordinates R, E, A to the earth centered coordinates x, y, z where the magnetic dipole axis of the earth coincides with the z-axis. From this coordinate system one can simply transform to the desired spherical coordinate system r, θ, φ.

$$\begin{aligned}
 \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= (e_{ij}) \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = (e_{ij}) (d_{ij}) \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = (e_{ij}) (d_{ij}) \left\{ \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + (c_i) \right\} \\
 &= (e_{ij}) (d_{ij}) \left\{ (b_{ij}) \begin{pmatrix} \epsilon \\ \eta \\ \zeta \end{pmatrix} + (b_i + c_i) \right\}
 \end{aligned} \tag{51}$$

$$(g_{ij}) = (e_{ij}) (d_{ij}) (b_{ij}) = (f_{ij}) (b_{ij}) \quad (52)$$

$$(f_{ij}) = \begin{pmatrix} \sin \varphi_M \cos(\lambda_M - \lambda_R) & \sin \varphi_M \sin(\lambda_M - \lambda_R) & -\cos \varphi_M \\ -\sin(\lambda_M - \lambda_R) & \cos(\lambda_M - \lambda_R) & 0 \\ \cos \varphi_M \cos(\lambda_M - \lambda_R) & \cos \varphi_M \sin(\lambda_M - \lambda_R) & \sin \varphi_M \end{pmatrix} \quad (53)$$

$$g_{ij} = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \quad (54)$$

$$g_{11} = (\cos \varphi_R \sin \varphi_M \cos(\lambda_M - \lambda_R) - \cos \varphi_M \sin \varphi_R)$$

$$g_{12} = \sin \varphi_M \sin(\lambda_M - \lambda_R)$$

$$g_{13} = (-\sin \varphi_R \sin \varphi_M \cos(\lambda_M - \lambda_R) - \cos \varphi_R \cos \varphi_M)$$

$$g_{21} = -\sin(\lambda_M - \lambda_R) \cos \varphi_R$$

$$g_{22} = \cos(\lambda_M - \lambda_R)$$

$$g_{23} = \sin \varphi_R \sin(\lambda_M - \lambda_R)$$

$$g_{31} = (\cos \varphi_R \cos \varphi_M \cos(\lambda_M - \lambda_R) + \sin \varphi_M \sin \varphi_R)$$

$$g_{32} = \cos \varphi_M \sin(\lambda_M - \lambda_R)$$

$$g_{33} = (-\sin \varphi_R \cos \varphi_M \cos(\lambda_M - \lambda_R) + \sin \varphi_M \cos \varphi_R)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R (g_{ij}) \begin{pmatrix} \sin E \\ \cos E \sin A \\ \cos E \cos A \end{pmatrix} + r_o (g_{il}) = r \begin{pmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{pmatrix} \quad (55)$$

By use of these matrix transformations the Cartesian coordinates, (x, y, z) , and from them the spherical coordinates, (r, θ, ϕ) , of the earth centered geomagnetic coordinate system can be determined for the spatial starting point S and the earth bound transmitter R. These can be expressed as:

$$\begin{aligned} x_S = & \left[\cos(\lambda_M - \lambda_R) \sin \phi_M \cos \phi_R - \cos \phi_M \sin \phi_R \right] (R \sin E + r_o) \\ & + \left[\sin(\lambda_M - \lambda_R) \sin \phi_M \right] R \cos E \sin A \\ & - \left[\cos(\lambda_M - \lambda_R) \sin \phi_M \sin \phi_R + \cos \phi_M \cos \phi_R \right] R \cos E \cos A \end{aligned} \quad (56)$$

$$\begin{aligned} y_S = & \left[-\sin(\lambda_M - \lambda_R) \cos \phi_R \right] (R \sin E + r_o) + \left[\cos(\lambda_M - \lambda_R) \right] R \cos E \sin A \\ & + \left[\sin(\lambda_M - \lambda_R) \sin \phi_R \right] R \cos E \cos A \end{aligned} \quad (57)$$

$$\begin{aligned} z_S = & \left[\cos(\lambda_M - \lambda_R) \cos \phi_M \cos \phi_R + \sin \phi_M \sin \phi_R \right] (R \sin E + r_o) \\ & + \left[\sin(\lambda_M - \lambda_R) \cos \phi_M \right] R \cos E \sin A \\ & - \left[\cos(\lambda_M - \lambda_R) \cos \phi_M \sin \phi_R - \sin \phi_M \cos \phi_R \right] R \cos E \cos A \end{aligned} \quad (58)$$

When $R = 0$, that is, for a point on the surface of the earth,

$$x_R = r_o \left[\cos(\lambda_M - \lambda_R) \sin \varphi_M \cos \varphi_R - \cos \varphi_M \sin \varphi_R \right] \quad (59)$$

$$y_R = -r_o \sin(\lambda_M - \lambda_R) \cos \varphi_R \quad (60)$$

$$z_R = r_o \left[\cos(\lambda_M - \lambda_R) \cos \varphi_M \cos \varphi_R + \sin \varphi_M \sin \varphi_R \right] \quad (61)$$

From simple trigonometric considerations (Figure 2a) it can be shown that the radar slant range, R , measured from the transmitter to the spatial starting point S is given by

$$R = -r_o \sin E + \sqrt{(r_o + h_S)^2 - r_o^2 \cos^2 E} \quad (62)$$

where h_S is the vertical height of the starting point above its projection, (point P) on the surface of the earth.

Equations 1 through 6 point out that in addition to these transformations, the components of the directed normal to the phase fronts, $\vec{\sigma}$, at the starting point S are to be determined in this coordinate system. From spherical and plane trigonometric considerations, (Figure 2), it can be shown that these components are given by

$$\sigma_r = \sigma \cos e \quad (63)$$

$$\sigma_\theta = \sigma \sin e \cos \alpha \quad (64)$$

$$\sigma_\varphi = -\sigma \sin e \sin \alpha \quad (65)$$

Angle e can be evaluated directly by employing the law of sines. This yields

$$e = \sin^{-1} \left(\frac{r_o \cos E}{r_o + h_S} \right) \quad (66)$$

Angle α is the geomagnetic bearing angle (Figure 2) measured positive in a clockwise direction from geomagnetic north. By use of spherical trigonometry it is expressible by

$$\alpha = \tan^{-1} \left[\frac{\sin(\Phi_S - \Phi_R) \sin \theta_R}{\cos \theta_S \sin \theta_R \cos(\Phi_S - \Phi_R) - \sin \theta_S \cos \theta_R} \right] \quad (67)$$

where Φ_R , θ_R , and Φ_S , θ_S are the geomagnetic longitudes and co-latitudes, respectively, of the radar transmitter, R, and the spatial starting point, S. The geomagnetic angles are obtained from the following expressions

$$\begin{aligned} \Phi_R &= \tan^{-1} \left(\frac{y_R}{x_R} \right) = \tan^{-1} \left(\frac{g_{21}}{g_{11}} \right) \\ &= \tan^{-1} \left[\frac{-\sin(\lambda_M - \lambda_R) \cos \varphi_R}{\cos(\lambda_M - \lambda_R) \sin \varphi_M \cos \varphi_R - \cos \varphi_M \sin \varphi_R} \right] \end{aligned} \quad (68)$$

$$\begin{aligned} \theta_R &= \cos^{-1} \left(\frac{z_R}{r_R} \right) = \cos^{-1} (g_{31}) \\ &= \cos^{-1} \left[\cos(\lambda_M - \lambda_R) \cos \varphi_M \cos \varphi_R + \sin \varphi_M \sin \varphi_R \right] \end{aligned} \quad (69)$$

$$\phi_S = \tan^{-1} \left(\frac{y_S}{x_S} \right) \quad (70)$$

$$\theta_S = \cos^{-1} \frac{z_S}{r_o + h_S} \quad (71)$$

One additional useful expression can be obtained from these algebraic relations. The parameter is the angle ψ at the spatial starting point S which is defined by the inner product of the magnetic field vector and the wave normal. By the application of the sine and cosine laws to the geometry of Figure 2-a, it can be shown that

$$\psi = \cos^{-1} [-(\cos e \sin I + \sin e \cos I \cos \alpha)] \quad (72)$$

where angle I is the magnetic inclination angle. The inclination angle⁹ is only a function of the geomagnetic latitude at the particular point in question.

$$I = \tan^{-1} [2 \cotan \theta_S] \quad (73)$$

Expressions arising from the inverse coordinate transformation, that is, transformation from the geomagnetic coordinates (r, θ , ϕ) to the radar coordinates R, E, A can be easily developed from these formulae. Although used in the computer program they will not be presented here.

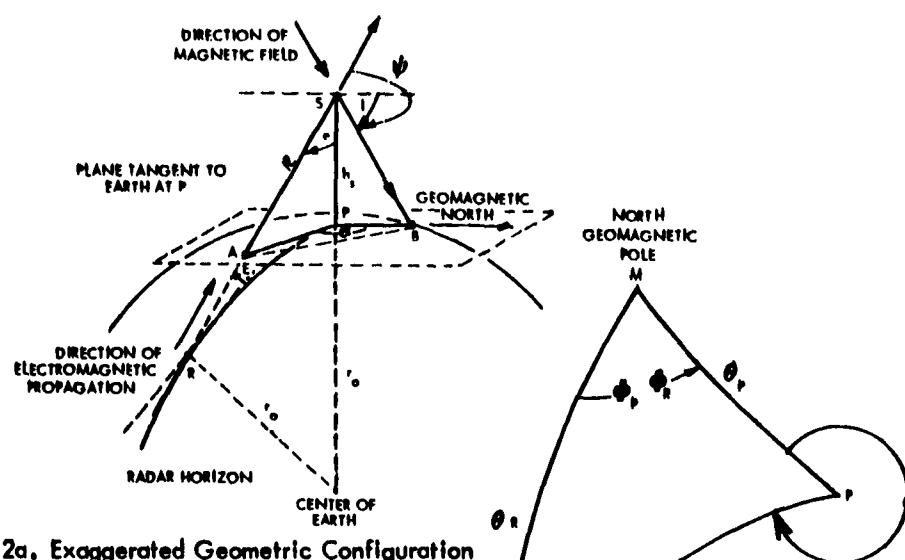


Figure 2a. Exaggerated Geometric Configuration for Starting Point Values

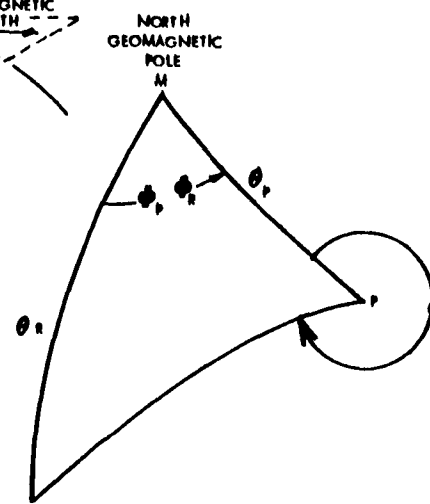


Figure 2b. Spherical Triangle Illustrating the Geomagnetic Bearing Angle

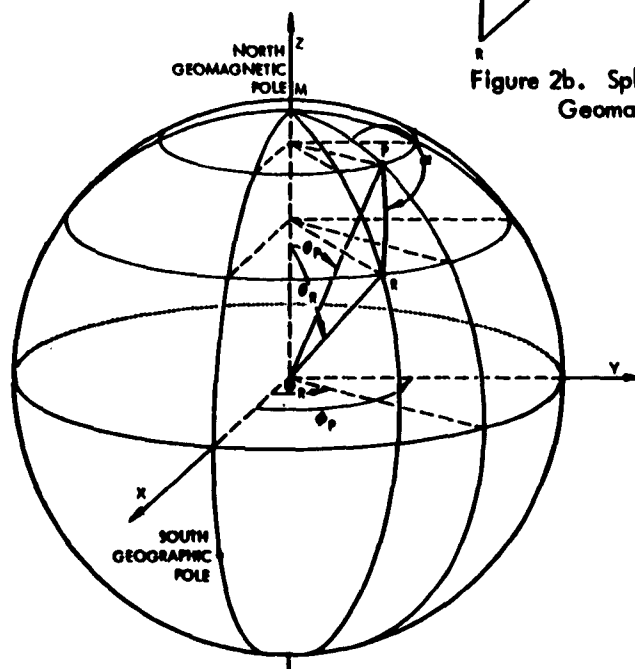


Figure 2c. Geomagnetic Coordinate System

Figure 2. Geometry for Starting Point and Geomagnetic Coordinate System

Nomenclature Used in Coordinate Transformation

r, θ, φ	spatial point in an earth centered spherical coordinate system
E	radar elevation angle
A	radar azimuth angle
$\varphi_{R, M}$	geographic latitude of point R or M, respectively
$\lambda_{R, M}$	geographic longitude of point R or M, respectively
S	spatial starting point on nondeviated portion of ray
R	distance from electromagnetic wave transmitter to starting point
ϵ, η, ζ	set of orthogonal coordinates with origin on the surface of the earth at radar site
x_1, y_1, z_1	orthogonal coordinate system with origin on the earth's axis of rotation
x_2, y_2, z_2	orthogonal coordinate system with origin at earth's center
x_3, y_3, z_3	orthogonal coordinate system with origin at earth's center
w	z component of the geographic coordinate system (u, v, w)
h_S	height of starting point above its projection on the surface of the earth
r_o	radius of earth
α	geomagnetic bearing angle
ϕ_R, ϕ_S	geomagnetic longitudes of points R and S

θ_R, θ_S	geomagnetic co-latitudes of points R and S
$\sigma_r, \sigma_\theta, \sigma_\varphi$	physical components of a vector of length μ , that is directed normal to the phase front
ψ	angle between magnetic field vector and the wave normal
I	angle of magnetic inclination

D. MODEL IONOSPHERE

As shown under Computational Procedure, the refractive index M and its spatial derivatives are dependent on the normalized density, X , and its spatial gradients, $\partial X/\partial r$, $\partial X/\partial \theta$, and $\partial X/\partial \varphi$. For an evaluation of these quantities an analytic model ionosphere can be chosen. One such ionospheric model that was found useful, is a spherical electron distribution as measured from a point in space. A reason for its selection is presented under Computational Results. Other uses as well as other ionospheric models are covered elsewhere^{7, 11}.

Let r_b , θ_b , φ_b represent the spatial location, B , of the center of the selected spherical ionosphere in the geomagnetic spherical coordinate system (r, θ, φ) . The values of these coordinates are obtainable from the specified geographic latitude, longitude and height above the earth surface of point B , in an analogous procedure as described for the transformation of coordinates of the spatial starting point S . Then the electron density at a spatial point r, θ, φ for an assumed spherical ionosphere can be written as:

$$N(r, \theta, \varphi, r_b, \theta_b, \varphi_b) = \frac{A}{\mathcal{L}^n} \quad (74)$$

where

$$\mathcal{L} = \left\{ (r_b \sin \theta_b \cos \varphi_b - r \sin \theta \cos \varphi)^2 + (r_b \sin \theta_b \sin \varphi_b - r \sin \theta \sin \varphi)^2 + (r_b \cos \theta_b - r \cos \theta)^2 \right\}^{1/2} = \left\{ XP^2 + YP^2 + ZP^2 \right\}^{1/2} \quad (75)$$

For this discussion A and n are appropriately chosen constants which give the desired electron density N (electrons/cc). By use of Equations 74 and 75, it is easily shown that the spatial electron density gradients can be expressed in the following manner,

$$\frac{\partial N}{\partial r} = \frac{nA}{\rho^{n+2}} \left[XP \sin \theta \cos \varphi + YP \sin \theta \sin \varphi + ZP \cos \theta \right] \quad (76)$$

$$\frac{\partial N}{\partial \theta} = \frac{nAr}{\rho^{n+2}} \left[XP \cos \theta \cos \varphi + YP \cos \theta \sin \varphi - ZP \sin \theta \right] \quad (77)$$

$$\frac{\partial N}{\partial \varphi} = \frac{nAr}{\rho^{n+2}} \left[-XP \sin \theta \sin \varphi + YP \sin \theta \cos \varphi \right] \quad (78)$$

Some results obtained by use of such a spherical ionospheric model will be discussed later.

E. MODEL OF EARTH'S MAGNETIC FIELD

Because magneto-ionic effects on the propagation of electromagnetic waves through an ionized medium are taken into account in the derivation of the equations under Computational Procedure, it is necessary to specify the normalized external magnetic field of the earth, $\vec{Y}$, its components Y_r , Y_θ , Y_ϕ and its spatial derivatives $\partial Y/\partial r$, $\partial Y/\partial \theta$, $\partial Y/\partial \phi$. It is known that the earth's magnetic field can be approximated by an earth centered magnetic dipole with its axis displaced such that the geographic longitude $\lambda_M = 70.1^\circ W$ and the geographic latitude $\Phi_M = 78.6^\circ N$. The magnetic potential, V , at a distant point from such a dipole is related to the magnetic moment, $\mathcal{M}$, by the expression

$$V(r, \theta) = -\frac{\mathcal{M} \cos \theta}{r^2} = -\frac{(Y_e r_0^3) \cos \theta}{r^2} \quad (79)$$

where r, θ are the geomagnetic coordinates of the spatial point irrespective of the coordinate ϕ and as before, r_0 = radius of the earth. In this equation Y_e is the magnitude of the normalized magnetic field at the earth's surface on the magnetic equator. By use of this algebraic equation all the desired quantities can be derived. They are

$$Y = Y_e \left(\frac{r_0}{r}\right)^3 (1 + 3 \cos^2 \theta)^{1/2} \quad (80)$$

where, as previously defined, Y is the normalized magnitude of the earth's magnetic field vector $\vec{Y} = (eH/mc\omega) = \omega_c/\omega$

$$Y_r = 2 Y_e \left(\frac{r_0}{r}\right)^3 \cos \theta = \frac{Y}{\sqrt{1 + \frac{1}{4} \tan^2 \theta}} \quad (81)$$

$$Y_\theta = Y_e \left(\frac{r_0}{r}\right)^3 \sin \theta = \frac{1}{2} Y_r \tan \theta \quad (82)$$

$$Y_{\phi} = \frac{\partial Y}{\partial \phi} = 0 \quad (83)$$

$$\frac{\partial Y}{\partial r} = - \frac{3Y}{r} \quad (84)$$

$$\frac{\partial Y}{\partial \theta} = - \frac{3Y \sin \theta \cos \theta}{[1 + 3 \cos^2 \theta]} \quad (85)$$

F. MODEL OF ATMOSPHERIC COLLISION FREQUENCY

For some of the trial calculations the atmospheric collision frequency was found from assumed exponential variations of collision frequency with height. The atmosphere was radially stratified and an approximate exponential equation was curve-fitted to measured experimental data for each stratified region. Hence, for each region the following relations were used to obtain Z and $\partial Z / \partial r$, $\partial Z / \partial \theta$, $\partial Z / \partial \varphi$ that are required by the ray trace equations.

$$Z = \frac{v}{\omega} = ae^{-b(r - r_o)} \quad (86)$$

$$\frac{\partial Z}{\partial r} = -bZ \quad (87)$$

$$\frac{\partial Z}{\partial \theta} = \frac{\partial Z}{\partial \varphi} = 0 \quad (88)$$

The dependence of collision frequency on a localized temperature distribution and degree of ionization⁷ complicates these simple relations. These complications (as derived by D. Archer) as well as their effects will not be discussed at this time.

G. COMPUTATIONAL RESULTS

The preceding equations are only a summary of the required set which will permit the detailed calculation of a ray path in three-dimensional space. Because of this, it becomes clear that the only realistic approach to the solution of this problem is the utilization of computer techniques, otherwise, the welter of data that must be handled through use of numerical methods, is beyond effective human handling capacity. However, the development of a computer program which can perform countless number of calculations, poses the very difficult task of determining the correctness of a computed result. To simplify this "debugging" task the classical idea of elastic collision between charged particles was borrowed from nuclear physics. It has been shown¹⁰ that if the electron density falls off as the inverse square of the distance from the center of a spherical electron distribution ($N = A/\mathcal{L}^2$), various exact expressions can be obtained, since the ray equations at zero azimuth are integrable.

The geometry of such a distribution, as well as, three ray paths computed by use of the computer program are shown in Figure 3. In the figure the center of the sphere is located by the fixed coordinates (R_0, β_0) with respect to the radar. The derivations are made in two dimensions, hence only the two-dimensional coordinate system (ξ, ζ) is used. The ray path has an initial elevation angle E_1 and its distance of closest approach to the center of the refracting sphere is denoted by $\mathcal{L}_0$. The coordinates of any point on the ray path with respect to the center of the sphere are ($\mathcal{L}, \beta$) and with respect to the radar (R, E). Angle δ represents the amount of ray bending experienced by the ray passing through the refractive medium. Under these conditions Archer¹⁰ has shown that the angle δ is given by

$$\delta = \pi - 2 \left[\gamma_0 + \mu (\mathcal{L}_0) \cos^{-1} \left(\frac{\mathcal{L}_0}{R_0} \right) \right] \quad (89)$$

The three plotted rays have actually a small third dimensional component. Because of this, the accuracy of these plotted rays is approximately (+3, -2) percent.

As shown by the tabulated results in Table 1, the computed deviation angle δ_c , arising from ray trace results, agrees very well with the calculated angle, δ , obtained by use of Equation 89. These computer

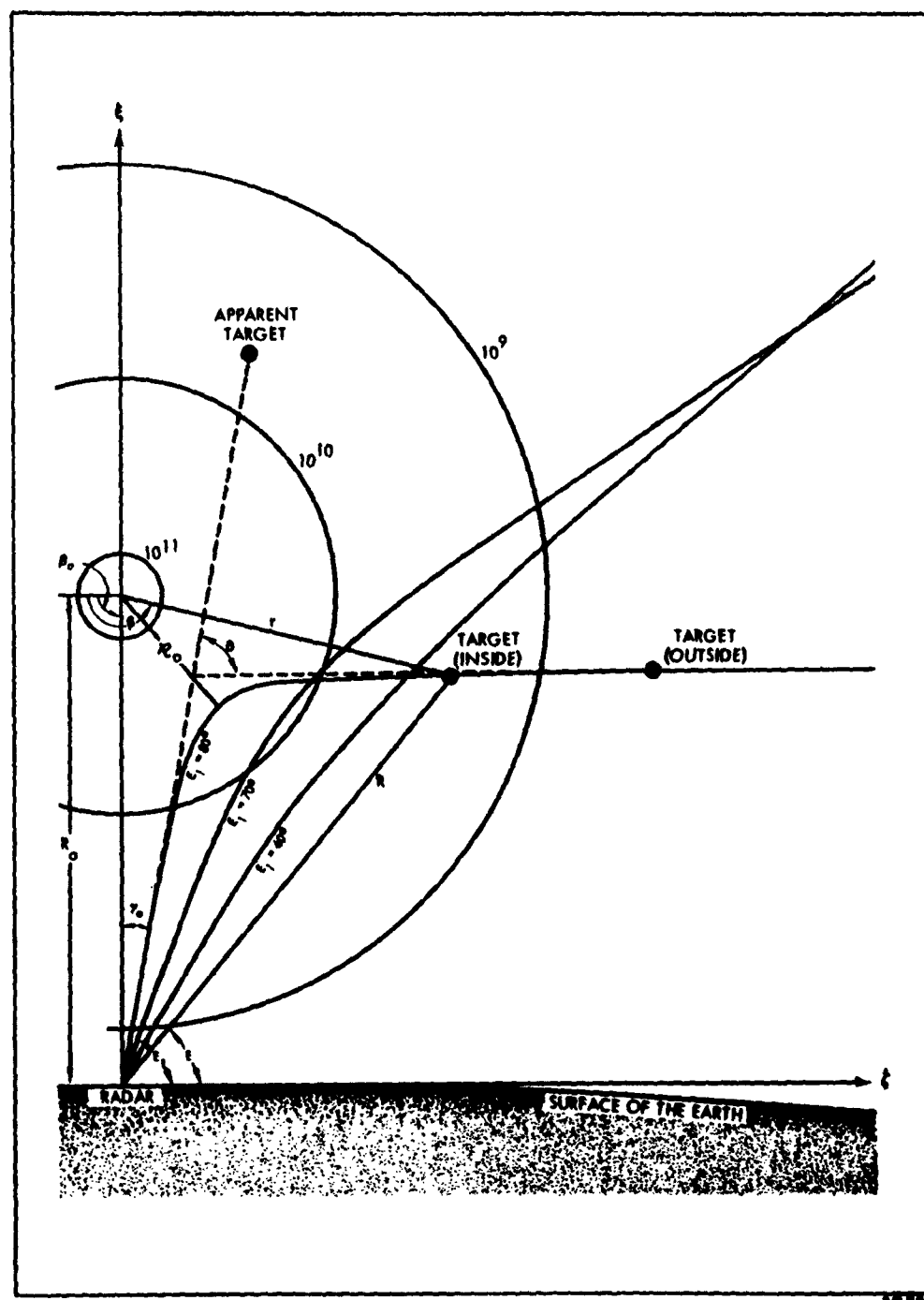


Figure 3. Geometry for Refraction by Spherically Stratified Region

calculations were performed under a large error upper bound condition ($\epsilon = 10^{-3}$, see description of Subroutine INT). The agreement is improved by a variation of this error condition. Additional results will not be presented here. Presented elsewhere¹² is the influence of ϵ on computed results using these numerical methods as applied to the study of ionization-deionization phenomena.

Radar Elevation Angle - E Degrees	Bending Angle - δ from Equation 89 Degrees	Bending Angle - δ_c Ray Trace Program Degrees
60	19.5	20
70	35.2	37
80	77.5	78

Table 1
Comparison of Total Ray Bending Angle

As an additional illustration, refractive errors through a particular spherical ionized region have been computed in detail to illustrate the concepts discussed and to indicate the kinds of refractive errors that could arise. The electron density contours of this ionization model are defined by Equation 74 where $A = 10^{33}$ and $n = 12$. The distance from the center of the spherical ionosphere is measured in kilometers. The center of the model is located at an elevation angle, E , of 30 degrees, zero degree azimuth angle, and 564 kilometers slant range as measured from the radar site.

Figure 4 shows the relation between the radar and ionization model in the plane of zero azimuth. Also shown are the ray paths for rays leaving the radar at several elevation angles. A frequency of one kilomegacycle was used in determining the refraction of the electromagnetic wave propagation vector. Because the electron density increases rapidly near the center of the model, there is significant bending of the ray path.

The refraction becomes so severe as the elevation angle of radar rays approach the elevation angle of the ionization center with respect to the radar, that there is a region (shown by half tones) into which no radar ray penetrates, hence, radar "blackout" is achieved. In three dimensions this blackout region is a cone in which a target is shielded from the radar. Because rays near this region intersect each other, two elevation angle paths to the same target exist, so multiple targets may be visible.

If the ray path is not in the zero azimuth plane, the amount of elevation error, or azimuth error, is a function of the location of the target. The elevation and azimuth errors for a target located at a slant range of 1200 kilometers have been computed as a function of radar elevation and azimuth angles. These are summarized in Figure 5, in which contours of constant elevation error, ΔE , in one quadrant and constant azimuth error, ΔA , in another quadrant as a function of the ray direction at the radar site are given. The contours have been terminated at a total bearing error of about 10 degrees. Due to symmetry the errors in the other quadrants are just the mirror image of the quadrants shown.

Nomenclature Used for Computational Results

E_1	initial elevation angle made by ray path
$\mathcal{L}_0$	distance of closest approach between ray path and center of sphere
$(\mathcal{L}, \beta)$	coordinates of any point on the ray path with respect to the center of the sphere
(R, E)	coordinates of any point on the ray path with respect to the radar
R_0	distance between the radar and center of the sphere
γ_0	apparent bearing angle

SECTION III

COMPUTER PROGRAM FOR THREE-DIMENSIONAL RAY-TRACING

Figure 6 schematically describes the computer program that was developed for three-dimensional ray tracing. As illustrated, the computer program is a composite of a group of subprograms. Because each subroutine is an entity in itself, the improvement of the entire program can be performed by the variation of each subprogram.

For the creation of this program the FORTRAN language¹³ was used wherever possible. FORTRAN is an automatic coding system for the IBM-704/709/7090 Data Processing Computer System that was designed for scientific application. Although there are limitations to FORTRAN, nevertheless, 1) it is at present the only language for scientific use, that is accepted by most existing large computer systems, and 2) it is simple and therefore without much effort, permits the elimination of the programmer, thus leaving the design of logical computer decisions, to the formulator of the scientific problem. The program has been written to operate "in or out" of the FORTRAN MONITOR CONTROL SYSTEM.

Except for the RINDEX subroutine the program has been divided into small, simple Functions and Subroutines to facilitate understanding. In the development of the program, concentration was mainly on obtaining a correct working program, as soon as possible, and not on optimization or clarity of output results. These tasks are left for future development.

The computer program consists of the following parts:

- | | |
|-----------------|-----------|
| 1) Main Program | RAY TRACE |
| 2) Function | SLANTR |
| 3) Function | QATAN |
| 4) Function | ARCOS |

5) Subroutine	COORD
6) Subroutine	DAUX
7) Subroutines	INT and INTM
8) Subroutine	RINDEX
9) Subroutine	ELECTX
10) Subroutine	BIGR
11) Subroutine	MAGY
12) Subroutine	COLFRZ
13) Subroutine	RCOORD
14) Subroutine	OUTONE
15) Subroutine	OUTPUT

These functions and subroutines are used to obtain numerical values for those variables which cannot be defined by only one arithmetic statement. In addition to these subprograms certain statements in the FORTRAN language cause the inclusion in the object program of the necessary input and output routines, as well as, various library functions and subroutines in relocatable binary form that are available on the FORTRAN MASTER LIBRARY TAPE. The names and locations of these necessary routines are given in the "storage map" of the arrangement of storage location in the object program that is compiled from a FORTRAN source program. These "maps" follow the listings of each source program. These added routines will not be discussed in this report. Following a brief description of the function of the main program and its associated subprograms, the necessary input data for a sample calculation is given with a description of the output. Some of the results listed in this output led to the graphical results presented in Figures 3, 4 and 5.

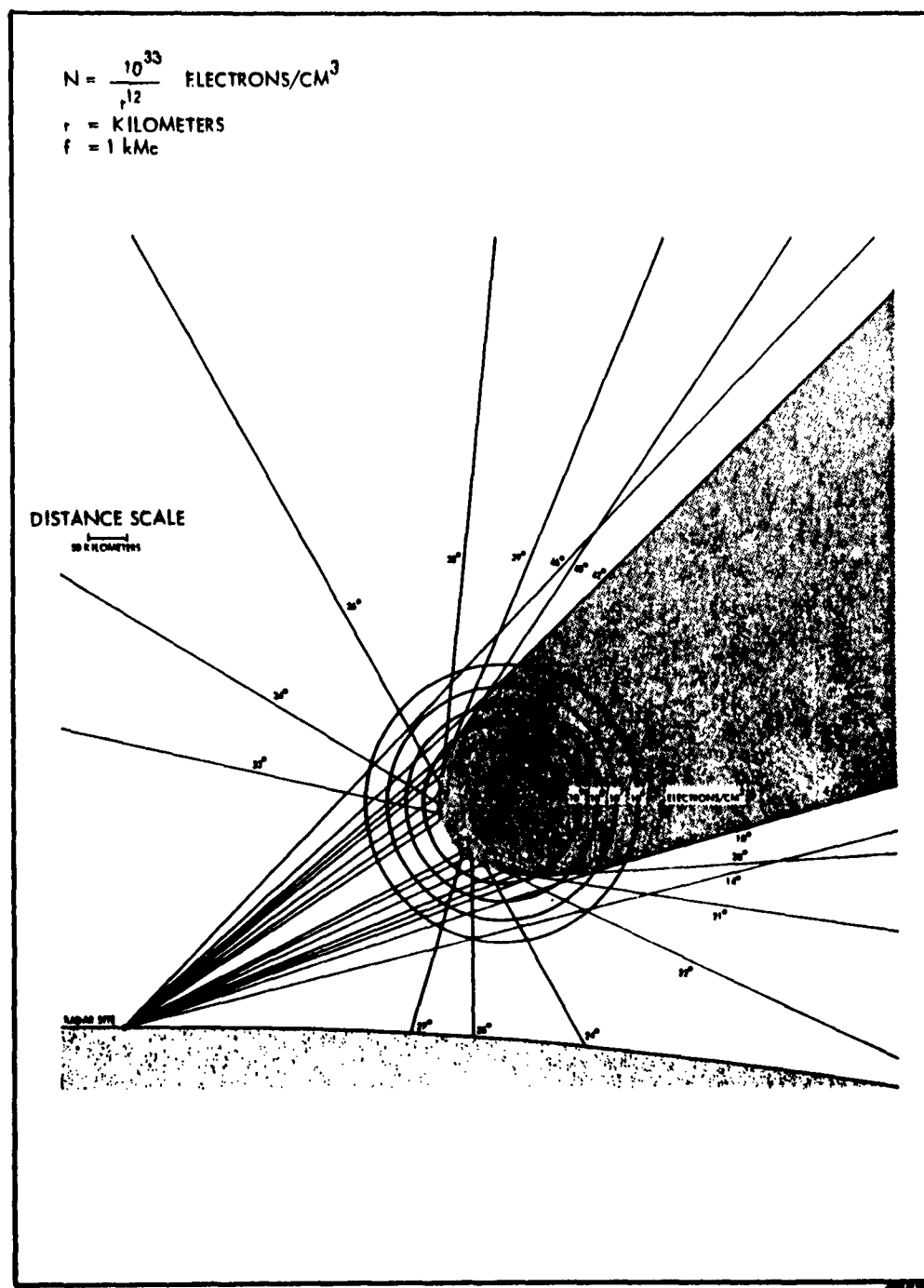


Figure 4. Radar Propagation Paths through Spherically Ionized Region

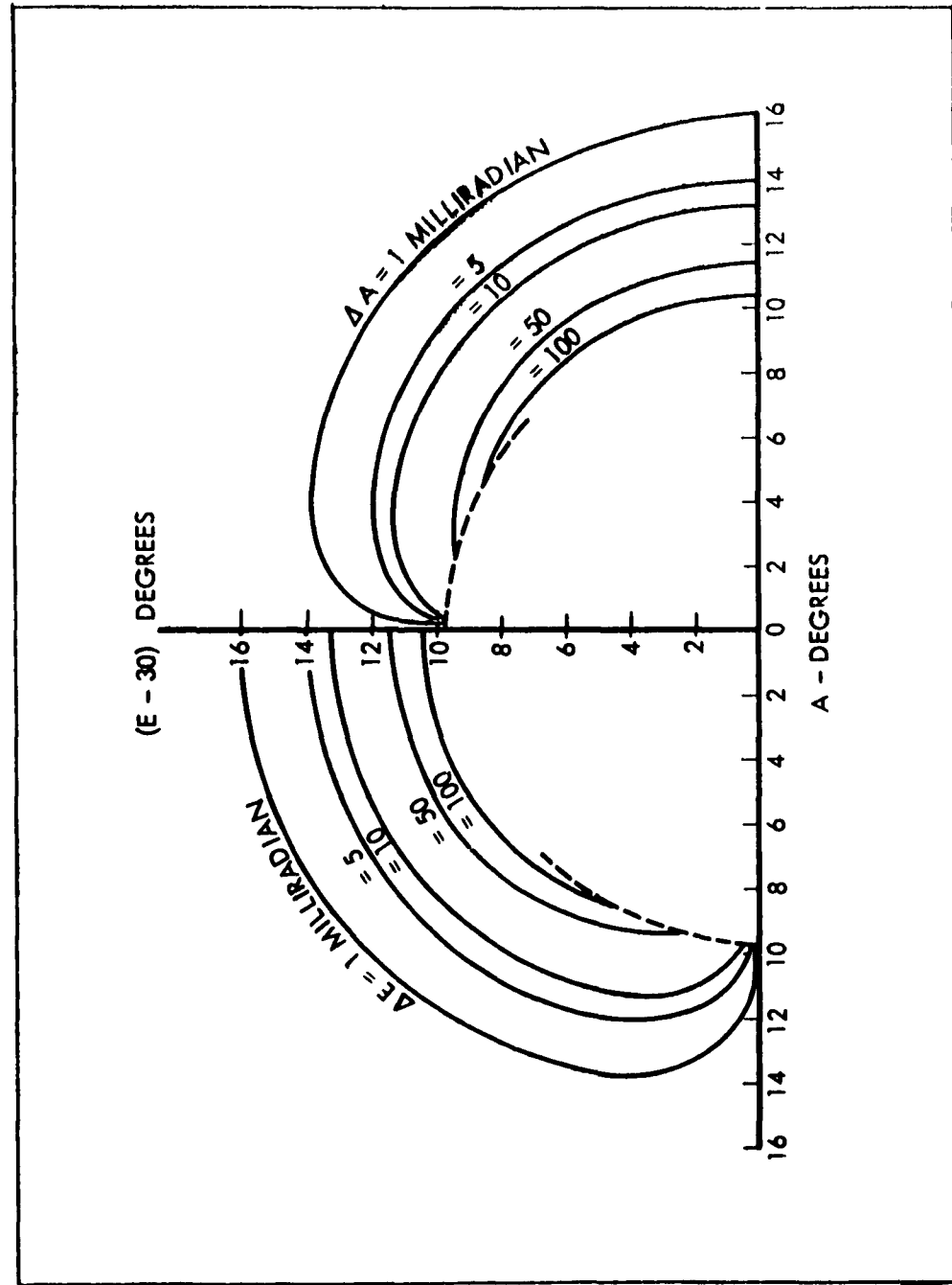


Figure 5. Elevation and Azimuth Errors for Propagation through a Spherical Model - $f = 1$ kMc

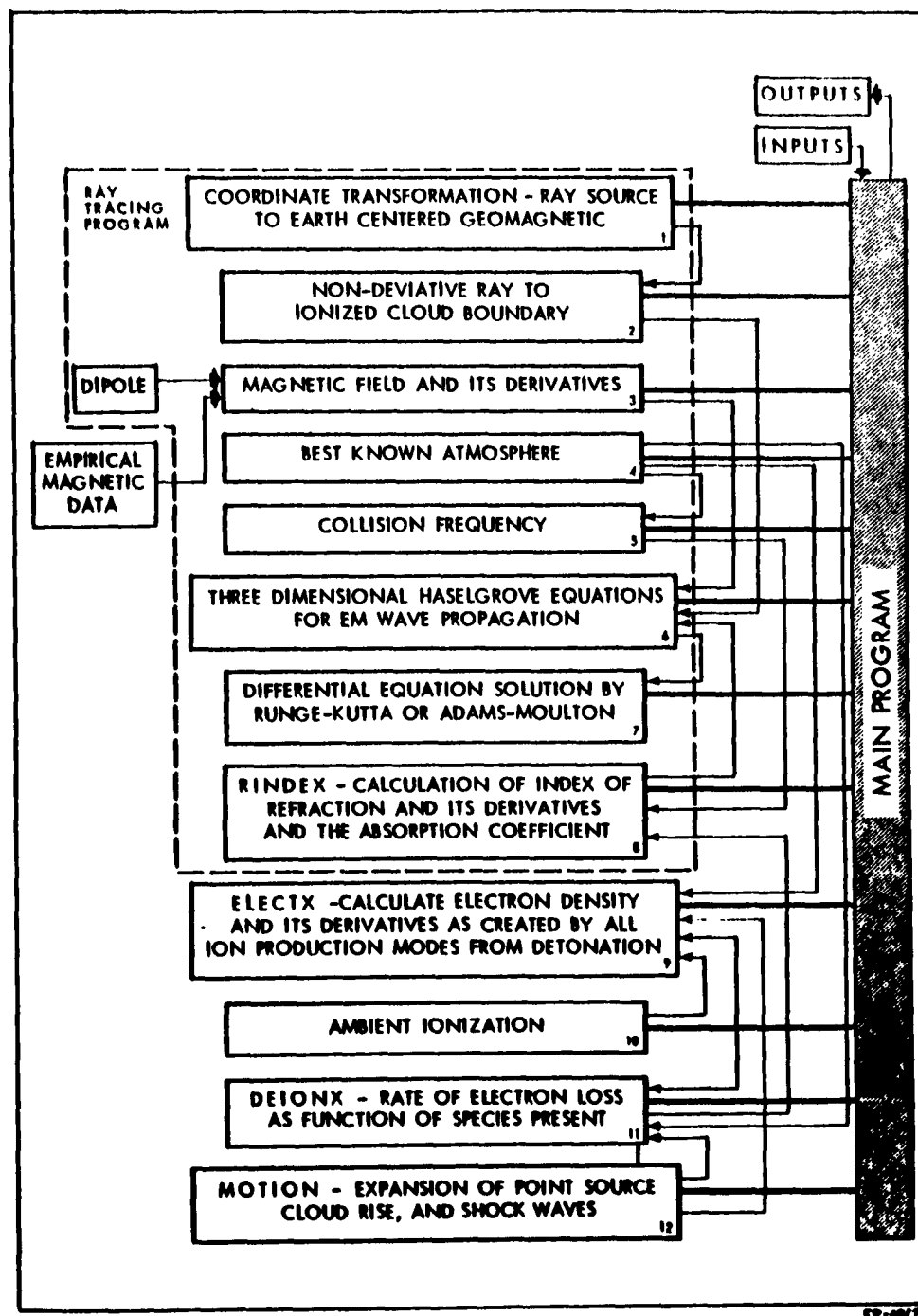


Figure 6. Block Diagram of a Possible Computer Program

A. MAIN PROGRAM RAY TRACE

The main program's function is to act as a master control of the logical flow necessary for the execution of the numerical methods. In addition, it is responsible for obtaining the necessary data, initializing the required starting conditions, performing the desired controlled printouts of computed results, and determining the condition for termination of the given computations. For initializing the starting conditions the main program requires the reading into storage of the following information in the format illustrated under INPUT, Table 4.

CARD 1

RECORD: This can be any desired information consisting of 72 alphanumeric characters that will serve to identify the calculations.

CARD 2

Contains the values of ID, KWIT. ID is an integer that can be used for identifying the calculation if the same CARD 1 is used. KWIT: On completion of the calculations in progress, the computer will check if additional problems are to be performed. Thus if KWIT=59 the computer will want to read a new W vector (CARD 3 --- onward); if KWIT=66, the computer will want to read a new CARD 1, CARD 2, and new W vector (CARD 3 --- onward); if KWIT equals any other integer the computer will PAUSE 44444.

CARD 3 onward

This card and all following cards describe the value of each component of the W vector that is not zero. As shown under INPUT, the first three columns of the card are for the integer that describes the W vector component. The next fourteen columns of the card are for the value of the W vector component. The number of these cards is variable since on completion of one calculation, often only one component of the W vector

CARD 3 (continued)	is to be changed for the next computation. The W vector can be read in any order.
LAST DATA CARD	This card follows the last card describing the W vector. It is any negative integer listed in the first three columns of a card. It transfers the computer out of the read mode to the location beginning the ray trace calculations.
SENSE SWITCH 1	The program is designed to calculate first the ordinary ray path and then the extraordinary ray path. SENSE SWITCH 1 DOWN will eliminate the calculation of the extraordinary ray path.
SENSE SWITCH 2	In DOWN position will permit the calculation of the extraordinary ray path and eliminate the calculation of the ordinary ray path.
SENSE SWITCH 3	Placing this SENSE SWITCH 3 DOWN will terminate the calculation on completion of the ray path calculations in progress.
SENSE SWITCH 4	Placing SENSE SWITCH 4 DOWN will cause the computer to check if SENSE SWITCH 6 is DOWN. If it is down the computer will terminate calculations immediately.
SENSE SWITCH 6	It is desirable to follow the course of any calculation on a computer. SENSE SWITCH 6 DOWN will print on-line, the total number of numerical integrations completed up to this point, integration mesh size, length of independent variable τ , height above surface of the earth (km), θ, φ (in degrees), $\sigma_r, \sigma_\theta, \sigma_\varphi, \mu, \kappa$, distance from the ion source center (km), value of the normalized electron density X.

SENSE LIGHT 2

When SENSE SWITCHES 1 and 2 are DOWN, then both ordinary and extraordinary ray paths are being calculated. When SENSE LIGHT 2 is ON, then the calculation is determining the ordinary ray path. When it is OFF, then the extraordinary ray path is being calculated.

PAUSE 17171

If the computer halts with this octal number in the address field of the STORAGE REGISTER it signifies that SENSE SWITCHES 1 and 2 are in the UP position and the problem is undefined. SENSE SWITCHES 1 or 2, or 1 and 2 are to be placed DOWN depending if only the ordinary, the extraordinary, or both ray paths are to be calculated. Following the definition of the problem, pushing START key will cause calculations to resume.

PAUSE 66666

The computer halts with this octal number in the STORAGE REGISTER just prior to beginning calculations. If the MONITOR system is used it permits the operator to know when it has left the MONITOR system and the SENSE SWITCHES can be changed as needed by the problem.

PAUSE 44444

The computer halts with this octal number in the STORAGE REGISTER on completion of all the necessary calculations specified by the INPUT data. It permits the operator to reset the desired sense switches for the MONITOR system. Pressing START will cause the computer to exit from the program to the MONITOR system.

Table 2 contains the nomenclature that describes some of the components of the V vector and the components of the W vector.

V(2)	independent variable τ
V(3)	initial step size input $\Delta\tau$
V(4)	radius from center of earth r
V(5)	variable angle θ
V(6)	variable angle φ
V(7)	σ_r
V(8)	σ_θ
V(9)	σ_φ
V(10)	optical path length one way s
V(11)	time one way T
V(12)	A absorption
V(13)	$dr/d\tau$
V(14)	$d\theta/d\tau$
V(15)	$d\varphi/d\tau$
V(16)	$d\sigma_r/d\tau$
V(17)	$d\sigma_\theta/d\tau$
V(18)	$d\sigma_\varphi/d\tau$
V(19)	$ds/d\tau$
V(20)	$dT/d\tau$
V(21)	$dA/d\tau$

Table 2. Nomenclature Describing the V and W Vectors.
(Page 1 of 6)

W(1)	refractive index μ
W(2)	imaginary part of complex phase refractive index κ
W(3)	radar transmitter angular frequency ω
W(4)	$\partial\mu/\partial\sigma_r$
W(5)	$\partial\mu/\partial\sigma_\theta$
W(6)	$\partial\mu/\partial\sigma_\phi$
W(7)	$\partial\mu/\partial r$
W(8)	$\partial\mu/\partial\theta$
W(9)	$\partial\mu/\partial\phi$
W(10)	$\partial\mu/\partial\psi$
W(11)	$\partial\mu/\partial\omega$
W(12)	unassigned for this program
W(13)	geographic longitudinal angle λ_M of geomagnetic north pole measured east of Greenwich Meridian (degrees)
W(14)	angle λ_R measured as W(13) in degrees
W(15)	angle ϕ_M geographic latitude of geomagnetic north-pole measured plus from geographic equator north
W(16)	angle ϕ_R geographic latitude of radar (degrees)
W(17)	radar elevation angle E (degrees)
W(18)	radar azimuth bearing angle angle A (degrees)
W(19)	r_0 radius of the earth (km)
W(20)	h_S height of starting point above surface of earth

Table 2. Nomenclature Describing the V and W Vectors.
(Page 2 of 6)

W(21)	angle φ_B of ionization source measured as W(15) (degrees)
W(22)	longitudinal angle λ_B of source measured as W(13)
W(23)	h_B height of ionization source center above earth surface
W(24)	$\Delta\tau$ initial mesh size of variable
W(25)	Y_e normalized equator magnetic field on earth's surface at the geomagnetic equator
W(26)	a constant determining collision frequency
W(27)	b constant in exponent determining collision frequency
W(28)	range (km) = distance from ionization source center to spatial point (r, θ, φ) =
W(29)	cosine of angle makes with the vertical through center of the ionizing source
W(30)	R_b radial distance from earth's center to center of ionizing source
W(31)	x geomagnetic coordinate of source
W(32)	y geomagnetic coordinate of source
W(33)	z geomagnetic coordinate of source
W(34)	A = constant in A/R^n determining electron density
W(35)	n = exponent in A/R^n equation
W(36)	unassigned for this program
W(37)	unassigned for this program
W(38)	plasma angular frequency cycles/sec

Table 2. Nomenclature Describing the V and W Vectors.
(Page 3 of 6)

W(39)	N_e in ion pairs/cc
W(40)	maximum $r = V(4)$ to be considered in this calculation
W(41)	$A1 =$ vector in INTM routine If $W(41) = 0$ routine will use predictor corrector with variable $V'(24)$ If $W(41) = 2$ will use Runge-Kutta with fixed $W(24)$ If $W(41) = 2$ will use predictor-corrector with fixed $W(24)$ If $W(41) = 1$ or 2 then $W(42)$ through $W(47)$ are ignored but must have some value. If $W(41) = 0$ they are not ignored
W(42)	$A2 = E$ upper bound on truncation error. See upper bound Equation (10) Appendix A in the INT and INTM subroutine
W(43)	$A3 = M$ is value from which lower bound E is calculated $LBE = UBE/M$ in subroutine INT
W(44)	$A4 = A$ as used in truncation error test EQ (10) in subroutine INT
W(45)	$A5 =$ upper bound on mesh size (If $= 0$ no upper bound as long as within error range)
W(46)	$A6 =$ lower bound on mesh size (If $= 0$ lower bound $= 0$)
W(47)	$A7 = \beta$, that is, 0 is less than β less than 1 . It is used to decrease or increase mesh size by dividing or multiplying current integration mesh being used
W(48)	smallest attenuation to be considered
W(49)	initial refraction index $= W(1)$
W(50)	initial absorption $\kappa = W(2)$

Table 2. Nomenclature Describing the V and W Vectors.
(Page 4 of 6)

W(51)	initial attenuation = A
W(52)	x_R (km) (Radar coordinate in geomagnetic coordinate system)
W(53)	y_R (km) (Radar coordinate in geomagnetic coordinate system)
W(54)	z_R (km) (Radar coordinate in geomagnetic coordinate system)
W(55)	$R = \sqrt{(x_R - x)^2 + (y_R - y)^2 + (z_R - z)^2}$ (km)
W(56)	$\Delta R = c\tau - W(55) = (2.99791 \times 10^5)[V(11)] - W(55)$ km
W(57)	new elevation angle E in degrees
W(58)	$\Delta E = W(57) - W(17)$ degrees
W(59)	$2[(W(1))(W(2))]/W(1)^2 - W(2)^2$
W(60)	slant range at r, θ, φ
W(61)	angle A at r, θ, φ
W(62)	elevation angle E at r, θ, φ
W(63)	assigned value to k; if 1 then control is on radius; if 2 then control is on range W(28); if 3 then control is on slant range W(60)
W(64)	value of Z
W(65)	value of Y
W(66)	value of X
W(67)	location of sign which determines the calculation for ordinary or extraordinary ray
W(68)	value of V(4) above which RINDEX is to print R vector

Table 2. Nomenclature Describing the V and W Vectors.
(Page 5 of 6)

W(69)	value W(1) below which R vector is printed if W(68) = 0
W(70)	number of performed integrations
W(71)	a in COLFRZ 100-200 km
W(72)	b in COLFRZ 100-200 km
W(73)	a in COLFRZ 200-300 km
W(74)	b in COLFRZ 200-300 km
W(75)	a in COLFRZ 300-400 km
W(76)	b in COLFRZ 300-400 km
W(77) to W(250) unassigned in this program	

Table 2. Nomenclature Describing the V and W Vectors.
(Page 6 of 6)

```

C MAIN PROGRAM - RAY TRACE JANUARY 20, 1961 IBM-7090
C SWITCH 1 DOWN CALCULATE ORDINARY RAY ONLY
C SWITCH 2 DOWN CALCULATE EXTRA-ORDINARY RAY
C SENSE SWITCH 3 DOWN WILL EXIT AFTER COMPLETING THIS RAY
C SENSE SWITCH 4 WITH SENSE SWITCH 6 DOWN WILL EXIT
C SENSE SWITCH 6 DOWN PERMITS ON-LINE PRINTING OF CALCULATIONS IN
C PROGRESS INT,VC3>,VC2>,VC4>-RO,VC5>,VC6>,VC7>,VC8>,VC9>,WC1>,WC2>,
C WC23>,WC66>
C PAUSE 66666 READY TO EXECUTE SET SENSE SWITCHES
C PAUSE 17171 FOR30T TO SET SS1 AND SS2 FOR PROPER RAY CALCULATION
C PAUSE 44444 READY TO EXIT SET SENSE SWITCHES FOR MONITOR
C COMMON RECORD,V,M,N,X,YN,ZN,G
C DIMENSION RECORD(12),VC(11),WC(250)
C DIMENSION XNC(7),YNC(7),ZNC(8),GC(3,3)
C PRINT 111
C PAUSE 66666
C PRINT 114
C READ INPUT TAPE 5,67,(RECORD(1),I=1,12)
C FORMAT (12F6.2)
C READ INPUT TAPE 5,113,10,KMIT
C READ INPUT TAPE 5,60,X,DATA
C FORMAT (13,E14.7)
C IF(X) 100,62,62
C IF (XK-13)*XK-14*(X-15)*(X-16)*(X-17)*(X-18)*(X-21)*(X-22) 64,
C 163,64
C DATA = DATA 57,28578
C NCK = DATA
C GO TO 59
C SENSE LIGHT 0
C IF SENSE SWITCH 1/ 102,106
C SENSE LIGHT 1
C IF SENSE SWITCH 2/ 104,1
C SENSE LIGHT 2
C GO TO 1
C IF SENSE SWITCH 2/ 104,107
C PRINT 110
C PAUSE 17171
C GO TO 100
C FORMAT (48H0 SET SS1 AND SS2 FOR PROBLEM BEING CONSIDERED)
C FORMAT (54H0 SET ALL SENSE SWITCHES IN POSITION DURING EXECUTED)
C FORMAT (48H0 SET SENSE SWITCHES FOR MONITOR - READY TO EXIT)
C FORMAT (216)
C OFORMAT (5H0 INT,ANDEL,5X,4HVC2),4X,7HVC4>-RO,3X,5HTHETA,5X,3HPHI,
C 15X,4HSGR,3X,8HSGTHETA,6X,6HSGPHI,5X,4HWC1),6X,4HWC2),5X,5HSHRANGE
C 2,7X,1HX)
C FORMAT (1H0,15,1P7E9,2,3H ,1PE9,2,1H ,1PE9,2,2H ,1P3E9,2)
C 1
C ARG1R = SINFC WC(13)-WC(40)
C ARG2R = COSFC WC(13)-WC(40)
C ARG3M = SINFC WC(15)
C ARG4M = COSFC WC(15)
C ARG5R = SINFC WC(16)
C ARG6R = COSFC WC(16)
C ARG7R = SINFC WC(17)
C ARG8R = COSFC WC(17)
C ARG9R = SINFC WC(18)
C ARG10R = COSFC WC(18)
C IM = WC(3)
C PI = 3.1415927

```

```

G(1,1) = ARG2R*ARG3M*ARG6R - ARG4M*ARG5R
G(1,2) = ARG1R*ARG3M
G(1,3) = - ARG5R*ARG3M*ARG2R - ARG6R*ARG4M
G(2,1) = - ARG1R*ARG6R
G(2,2) = ARG2R
G(2,3) = ARG5R*ARG1R
G(3,1) = ARG6R*ARG4M*ARG2R - ARG3M*ARG5R
G(3,2) = ARG4M*ARG1R
G(3,3) = - ARG5R*ARG4M*ARG2R + ARG3M*ARG6R
XHC(1) = G(2,2)*G(3,3)
XNC(2) = G(1,2)*G(2,3)
XNC(3) = G(1,3)*G(3,2)
XNC(4) = G(1,3)*G(2,2)
XNC(5) = G(1,2)*G(3,3)
XNC(6) = G(3,2)*G(2,3)
XNC(7) = W(19)*G(1,1)
VNC(1) = G(1,1)*G(3,3)
VNC(2) = G(2,3)*G(3,1)
VNC(3) = G(1,3)*G(2,1)
VNC(4) = G(1,3)*G(3,1)
VNC(5) = G(2,1)*G(3,3)
VNC(6) = G(1,1)*G(2,3)
VNC(7) = W(19)*G(2,1)
ZNC(1) = G(1,1)*G(2,2)
ZNC(2) = G(1,2)*G(3,1)
ZNC(3) = G(2,1)*G(3,2)
ZNC(4) = G(2,2)*G(3,1)
ZNC(5) = G(1,2)*G(2,1)
ZNC(6) = G(1,1)*G(3,2)
ZNC(7) = W(19)*G(3,1)
DENM = ZNC(1)*G(3,3) + ZNC(2)*G(2,3) + ZNC(3)*G(1,3) - G(1,3)*ZNC(4)
1 - ZNC(5)*G(3,3) - G(2,3)*ZNC(6)
ZNC(8) = DENM
2 SRSP = SLANTR (ARG7R,ARG9R,W(19),W(20))
N = 9
3 OCALL COORD CXSP,YSP,ZSP,ARG1R,ARG2R,ARG3M,ARG4M,ARG5R,ARG6R,ARG7R,
1 ARG8R,ARG9R,ARG10R,SRSP,W(19))
4 RSP = SORTF CXSP*2+YSP*2+ZSP*2)
5 OSP = ZSP/RSP
6 THETSP = ARCOS(OSP)
7 PHISP = DATAN(XSP,YSP)
8 ZERO = 0.0
9 OCALL COORD CXR,YR,ZR,ARG1R,ARG2R,ARG3M,ARG4M,ARG5R,ARG6R,ARG7R,
1 ARG8R,ARG9R,ARG10R,ZERO,W(19))
10 OR = ZR/W(19)
W(52) = XR
W(53) = YR
W(54) = ZR
11 THETR = ARCOS(OR)
12 PHIR = DATAN(XR,YR)
13 ARGSE = ((W(19))/RSP)*ARG8R)
14 TERM2 = (OSP*OSP)
15 ARG1B = SIN( W(13) - W(22))
ARG2B = COSF(W(13)-W(22))
ARG3B = SIN(W(21))
ARG6B = COSF(W(21))
16 OCALL COORD CXD,YD,ZD,ARG1B,ARG2B,ARG3M,ARG4M,ARG5B,ARG6B,ZERO,
1 ZERO,ZERO,ZERO,ZERO,W(19))

```

```

17 MC300=MC190*MC230
18 OB = 20/MC190
19 THET6 = ARCCOSQB
20 PHIB = DATAN (XO/MO)
21 MC130 = (20/MC190)*MC300
22 MC310 = (XO/MC190)*MC300
23 MC320 = (YO/MC190)*MC300
24 TERM4 = (PHISP-PHIR)
25 TERM5 = SORTF(1.0-OR*OR)
26 TERM6 = TERMS+SINF(CTERM4)
27 TERM7 = ASP+TERMS*COSE(CTERM4)- (SORTF(1.0-TERM2))*OR
28 ANALPH = DATAN(CTERM7,TERM6)
29 SIGP =MC1)* SORTF(1.0-ARGSE**2)
30 PI = 3.1415927
31 GAMMA = ANALPH - PI
32 SIGTHE = (MC1)* ARGSE)* COSF(GAMMA)
33 SIGPHI = -(MC1)*ARGSE)*SINF(GAMMA)
34 NA = MC41)
35 IF (SENSE LIGHT 1) 32, 34
36 SIGN = 1.0
37 GO TO 36
38 IF (SENSE LIGHT 2) 35,200
39 SIGN = -1.0
40 IF (SENSE SWITCH 3) 37,39
41 PRINT 112
42 PAUSE 4*444
43 CALL EXIT
44 MC10 = MC49)
45 MC670 = SIGN
46 MC700 = 0.0
47 VC20 = SRSP
48 VC30 = MC24)
49 VC40 = RSP
50 VC50 = THETSP
51 VC60 = PHISP
52 VC70 = SIGR
53 VC80 = SIGTHE
54 VC90 = SIGPHI
55 VC100 = SRSP
56 VC110 = CSRSP 3.0E 5)
57 VC120 = MC51)
58 MC20 = MC50)
59 IO = IO + 1
60 CALL OUTONE(SIGN,IO)
61 CALL INT CV, M, NA ,VC*20,MC*30,MC*40,MC*50,MC*60,MC*70)
62 IF (VC*40 - MC190) 31,31.5*
63 CALL INTM
64 IF (SENSE SWITCH 6) 116,117
65 IF (MC10) 118,118.56
66 PRINT 115
67 FORMAT(28H0 MC10 IS NEGATIVE OR ZERO
68 TE3 = VC*40- MC190
69 TE1 = VC50*57.29578
70 TE2 = VC60*57.29578
71 INT = MC700
72 PRINT 115,INT,VC30,VC20,TE3,TE1,TE2,VC70,VC80,VC90,MC10,MC10,MC20,MC28)
73 1,MC*40)
74 CALL OUTPUT

```


STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
G 32155 76645		N 32188 76674		RECORD 32561 77461		V 32549 77445	
XN 32187 76673		YN 32180 76664		ZN 32173 76655			

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
67	4	00000	60	5	00000	110	6	00000	111	7	00000
113	9	00000	114	10	00000	115	11	00000	119	12	00000
66	21	00047	59	24	00566	59	25	00073	61	26	00103
63	28	00170	64	29	00173	65	30	00175	100	31	00177
102	33	00203	103	34	00204	104	35	00206	105	36	00207
107	38	00214	107	38	00214	108	40	00220	1	41	00221
2	86	00510	3	88	00520	3	89	00520	4	90	00540
6	92	00557	6	93	00557	7	94	00562	7	95	00562
9	97	00570	9	98	00570	10	99	00610	11	103	00621
12	105	00624	12	106	00624	13	107	00630	14	108	00634
16	113	00655	16	114	00655	17	115	00675	18	121	00712
20	123	00722	22	124	00726	23	125	00731	24	126	00737
26	128	00763	26	129	00763	28	130	00767	29	133	01005
31	136	01035	32	137	01037	33	138	01041	34	139	01042
36	141	01047	37	142	01054	42	151	01075	43	152	01077
40	149	01071	46	155	01105	47	156	01107	48	157	01111
45	154	01103	53	163	01126	53	164	01126	51	165	01140
50	159	01116	118	169	01155	118	169	01155	116	170	01160
117	178	01242	202	179	01245	203	180	01252	204	181	01257
57	183	01271									

STORAGE NOT USED BY PROGRAM

DEC	OCT
32156	76634

LOCATIONS OF NAMES IN TRANSFER VECTOR

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
INT	14 00016	INTM	15 00017	EXIT	12 00014	COORD	8 00010
ARGOS	10 00012	OUTONE	13 00015	OUTPUT	16 00020	SIN	5 00005
SLANTR	7 00007	SCRT	9 00011	CFIL	2 00002	CRTN	4 00004
(SPH)	1 00001	(TSH)	3 00003				

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
IM	366 01542	ID	365 01541	GAMMA	864 01540	DENM	863 01537
ARGSE	861 01535	ARG9R	860 01534	ARG8R	859 01533	ARG7R	858 01532
ARGGB	856 01530	ARG5R	855 01527	ARG5B	854 01526	ARG4M	853 01525
ARG2R	851 01523	ARG2B	850 01522	ARG1R	849 01521	ARG1B	848 01520
ANALPH	845 01516	K	845 01515	KWIT	844 01514	NA	843 01513
PHIR	841 01511	PHISF	840 01510	PI	839 01507	QB	838 01506
OSP	836 01504	RSP	835 01503	SIGN	834 01502	STGPHI	833 01501
SIGTNE	831 01477	SPSP	830 01476	TEI	829 01475	TE3	828 01474
TERM2	826 01472	TERM4	825 01471	TERM5	824 01470	TERM6	823 01467
THETB	921 01465	THETR	920 01464	THETSP	819 01463	XO	818 01462
						SIGR	837 01505
						PHIB	842 01512
						ARG3M	852 01524
						ARG6P	857 01531
						DATA	862 01536

XSP 816 01460 VO 815 01437 YR 814 01456 YSP 813 01455 ZO 812 01454
 ZERO 811 01453 ZR 810 01452 ZSP 809 01451

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
E220J	553	01051	126	00176	E210	696	01270	E2R	673	01241
E2N	548	01044	139	00213	E29	130	00202	E26	119	00167
D240V	692	01264	28	00034	D210J	555	01053	C2G3	808	01450
823N	731	01333	740	01344	8231	763	01373	823H	764	01374
823F	783	01417	792	01430	8223	797	01435	8215	795	01433
2>	702	01276	716	01314	4>	32767	77777	6>	721	01321

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

	(FILE)	(CSPM)	(CTSH)	(CRIN)	SIN	SLANT	COORD	SOPT
ARCOS	EXIT	OUTONE	INT	INTM	OUTPUT			

00000	CFPT>	BCD 1CFPT>	00065	24A	CAL 2>	00160	ALS 17
00001	CSPND	BCD 1CSPND	00067		SXD 6>+4.4	00161	XCA
00002	CFIL>	BCD 1CFIL>	00070		TSX CTSND>4	00162	MPV 12+2
00003	CTSHD	BCD 1CTSHD	00071		PZE 8>15	00163	ALS 17
00004	CRND	BCD 1CRND	00072		LXD 6>+4.4	00164	TZE E>6
00005	SIN	BCD 1SIN	00073	25A	STR	00165	TPL 29A
00006	COS	BCD 1COS	00074		STR K	00166	TRA 29A
00007	SLANTR	BCD 1SLANTR	00075		STR DATA	00167	SXD C>G1.1
00010	COORD	BCD 1COORD	00076		STQ DATA	00170	CLA DATA
00011	ISRT	BCD 1ISRT	00077		SXD 6>+4.4	00171	FDP 3>
00012	ARCOS	BCD 1ARCOS	00100		TSX CRTND>4	00172	STQ DATA
00013	QATAN	BCD 1QATAN	00101		LXD 6>+4.4	00173	CLA DATA
00014	EXIT	BCD 1EXIT	00102		LXD K.1	00174	STO W+1.1
00015	OUTONE	BCD 1OUTONE	00103	26A	CLA K	00175	TRA 24A
00016	INT	BCD 1INT	00104	26A1	TZE 27A	00176	SXD C>G1.1
00017	INTM	BCD 1INTM	00105		TPL 27A	00177	PSE 96
00020	OUTPUT	BCD 1OUTPUT	00106		TRA E>108	00200	PSE 113
00021	SS	BCD 1CFPT>	00107	27A	CLA K	00201	TRA 37A
00022	SS	STO 8	00110		SUB 2>+8	00202	SXD C>G3.2
00023	14A	STC 4>-205	00111		STO 1>+1	00203	PSE 97
00024		BSS	00112		CLA K	00204	PSE 114
00025		TSX CSPND>4	00113		SUB 2>+7	00205	TRA 41A
		PZE 8>3F	00114		STO 1>+2	00206	PSE 98
00026		BSS	00115		CLA K	00207	TRA 41A
00027	15A	TSX CFIL>4	00116		SUB 2>+6	00210	PSE 114
	16A	MPR 28086	00117		STO 1>+3	00211	TRA EDC
00030		BSS	00120		CLA K	00212	TRA 35A
00031		TSX CSPND>4	00121		SUB 2>+5	00213	SXD C>G3.2
		PZE 8>31	00122		STO 1>+4	BSS	
		BSS	00123		CLA K	00214	TSX CSPND>4
00032		TSX CFIL>4	00124		SUB 2>+4	PZE 8>3E	
00033	D>601	LXD C>G3.2	00125		STO 1>+5	BSS	
00034	D>401	LXD C>G1.4	00126		CLA K	00216	TSX CFIL>4
00035	17A	CAL 2>	00127		SUB 2>+3	00217	MPR 7801
00036		SXD 6>+4.4	00130		STO 1>+6	00220	TRA 31A
00037		TSX CTSND>4	00131		CLA K	00221	CLA W-12
00040		PZE 8>23	00132		SUB 2>+2	00222	FSB W-13
00041		LXD 6>+4.4	00133		STO 1>+7	BSS	
00042	18A	LXD 2>+11.1	00134		CLA K	00223	TSX SIN.4
00043	19A	STR	00135		SUB 2>+1	00224	STO ARGIR
00044		STO RECORD+1.1	00136		STO 1>+8	00225	CLA W-12
00045	19A1	TXI 1+1.1.1	00137		LOR 1>+1	00226	FSB W-13
00046	19A2	TXL 19A.1.12	00140		MPV 12+8	BSS	
00047	21A	SXD 6>+4.4	00141		ALS 17	00227	TSX COS.4
00050		TSX CRTND>4	00142		XCA	00230	STO ARG2R
00051		LXD 6>+4.4	00143		MPV 12+7	00231	CLA W-14
00052	22A	CAL 2>	00144		ALS 17	BSS	
00053		SXD 6>+4.4	00145		XCA	00232	TSX SIN.4
00054		TSX CTSND>4	00146		MPV 12+6	00233	STO ARG3M
00055		PZE 8>3M	00147		ALS 17	00234	CLA W-14
00056		LXD 6>+4.4	00150		XCA	BSS	
00057	23A	STR	00151		MPV 12+5	00235	TSX COS.4
00060		STO 10	00152		ALS 17	00236	STO ARG4M
00061		STR	00153		XCA	00237	CLA W-15
00062		STO KNIT	00154		MPV 12+4	BSS	
00063		SXD 6>+4.4	00155		ALS 17	00240	TSX SIN.4
00064		TSX CRTND>4	00156		XCA	00241	STO ARG5R
00065		LXD 6>+4.4	00157		MPV 12+3	00242	CLA W-15

00243	BSS	00330	FMP ARG1R	00422	FMP G-8
00244	TSX COS.4	00331	STO G-7	00423	STO VN-4
00245	STO ARG6R	00332	LQ ARG3M	00424	LQ G
	CLA W-16	00333	FMP ARG5R	00425	FMP G-7
	BSS	00334	STO 1+1	00426	STO VN-5
00246	TSX SIN.4	00335	LQ ARG2R	00427	LQ W-18
00247	STO ARG7R	00336	FMP ARG6R	00430	FMP G-1
00250	CLA W-16	00337	XCA	00431	STO VN-6
	BSS	00340	FMP ARG4M	00432	LQ G
00251	TSX COS.4	00341	FAD 1+1	00433	FMP G-4
00252	STO ARG8R	00342	STO G-2	00434	STO ZN
00253	CLA W-17	00343	LQ ARG4M	00435	LQ G-3
	BSS	00344	FMP ARG1R	00436	FMP G-2
00254	TSX SIN.4	00345	STO G-5	00437	STO ZN-1
00255	STO ARG9R	00346	LQ ARG3M	00440	LQ G-1
00256	CLA W-17	00347	FMP ARG6R	00441	FMP G-5
	BSS	00350	STO 1+1	00442	STO ZN-2
00257	TSX COS.4	00351	LQ ARG2R	00443	LQ G-4
00260	STO ARG10R	00352	FMP ARG5R	00444	FMP G-2
00261	CLA W-62	00353	XCA	00445	STO ZN-3
00262	UFA 63	00354	FMP ARG4M	00446	LQ G-3
00263	LRS	00355	CHS	00447	FMP G-1
00264	ANA 63+1	00356	FAD 1+1	00450	STO ZN-4
00265	LLS	00357	LQ G-8	00451	LQ G
00266	ALS 18	00360	LQ G-4	00452	FMP G-5
00267	STO IM	00361	FMP G-8	00453	STO ZN-5
00270	LXD IM.2	00362	STO XM	00454	LQ W-18
00271	CLA 3+1	00363	LQ G-3	00455	FMP G-2
00272	STO PI	00364	FMP G-7	00456	STO ZN-6
00273	LQ ARG4M	00365	STO XM-1	00457	LQ G-7
00274	FMP ARG5R	00366	LQ G-6	00460	FMP ZN-5
00275	STO 1+1	00367	FMP G-5	00461	STO 1+1
00276	LQ ARG6R	00370	STO XM-2	00462	LQ ZN-4
00277	FMP ARG2R	00371	LQ G-6	00463	FMP G-8
00300	XCA	00372	FMP G-4	00464	STO 1+2
00301	FMP ARG3M	00373	STO XM-3	00465	LQ G-6
00302	FSP 1+1	00374	LQ G-3	00466	FMP ZN-3
00303	STO G	00375	FMP G-8	00467	STO 1+3
00304	LQ ARG1R	00376	STO XM-4	00470	LQ ZN-2
00305	FMP ARG3M	00377	LQ G-5	00471	FMP G-6
00306	STO G-3	00400	FMP G-7	00472	STO 1+4
00307	LQ ARG6R	00401	STO XM-5	00473	LQ ZN-1
00310	FMP ARG4M	00402	LQ W-18	00474	FMP G-7
00311	STO 1+1	00403	FMP G	00475	STO 1+5
00312	LQ ARG2R	00404	STO XM-6	00476	LQ ZN
00313	FMP ARG5R	00405	LQ G	00477	FMP G-8
00314	XCA	00406	FMP G-8	00500	FAD 1+5
00315	FMP ARG3M	00407	STO VN	00501	FAD 1+4
00316	CHS	00410	LQ G-7	00502	FSP 1+3
00317	FSP 1+1	00411	FMP G-2	00503	FSP 1+2
00320	STO G-6	00412	STO VN-1	00504	FSP 1+1
00321	LQ ARG1R	00413	LQ G-6	00505	STO DENH
00322	FMP ARG6R	00414	FMP G-1	00506	CLA DENH
00323	CHS	00415	STO VN-2	00507	STO ZN-7
00324	STO G-1	00416	LQ G-6		BSS
00325	CLA ARG2R	00417	FMP G-2	00510	TSX SLANTR.4
00326	STO G-4	00420	STO VN-3	00511	TSX ARG7R
00327	LQ ARG5R	00421	LQ G-1	00512	TSX ARG8R

00513	TSX W-18	00600	TSX ARG5R	00663	TSX ARG3M
00514	TSX W-19	00601	TSX ARG6R	00664	TSX ARG4M
00515	STO SRSP	00602	TSX ARG7R	00665	TSX ARG5B
00516	CLA 23+9	00603	TSX ARG8R	00666	TSX ARG6B
00517	STO N	00604	TSX ARG9R	00667	TSX ZERO
00520	BSS	00605	TSX ARG10R	00670	TSX ZERO
00521	TSX COORD.4	00606	TSX ZERO	00671	TSX ZERO
00522	TSX XSP	00607	TSX W-18	00672	TSX ZERO
00523	TSX YSP	00610	CLA 2R	00673	TSX ZERO
00524	TSX ARG1R	00611	FDP W-18	00674	TSX W-18
00525	TSX ARG2R	00612	STQ QR	00675	CLA W-18
00526	TSX ARG3M	00613	CLA XR	00676	FAO W-22
00527	TSX ARG4M	00614	STO W-51	00677	STO W-29
00530	TSX ARG5R	00615	CLA YR	00700	CLA 20
00531	TSX ARG6R	00616	STO W-52	00701	FDP W-18
00532	TSX ARG7R	00617	CLA 2R	00702	STO QB
00533	TSX ARG8R	00620	STO W-53	00703	BSS
00534	TSX ARG9R	00621	BSS	00703	TSX ARGOS.4
00535	TSX ARG10R	00622	TSX QR	00704	TSX QB
00536	TSX SRSP	00623	STO THETR	00705	STO THETB
00537	TSX W-18	00624	BSS	00706	119A BSS
00540	LDQ ZSP	00625	TSX QATAN.4	00707	TSX QATAN.4
00541	FMP ZSP	00626	TSX XR	00710	TSX XQ
00542	STO 13+1	00627	TSX YR	00711	TSX YQ
00543	LDQ YSP	00630	STO PHIR	00712	STO PHIB
00544	FMP YSP	00631	CLA W-18	00713	CLA 20
00545	STO 13+2	00632	FDP RSP	00714	FDP W-18
00546	LDQ XSP	00633	FMP ARG8R	00715	FMP W-29
00547	FMP XSP	00634	STO ARGSE	00716	STO W-32
00550	FAO 13+2	00635	LDQ QSP	00717	CLA XQ
00551	FAO 13+1	00636	FMP QSP	00720	FDP W-18
00552	BSS	00637	STO TERM2	00721	FMP W-29
00553	TSX SORT.4	00640	CLA W-12	00722	STO W-30
00554	STO RSP	00641	FSB W-21	00723	CLA YQ
00555	CLA ZSP	00642	BSS	00724	FDP W-18
00556	FDP RSP	00643	TSX SIN.4	00725	FMP W-29
00557	STO QSP	00644	STO ARG1B	00726	STO W-31
00557	BSS	00645	FSB W-21	00727	CLA PHISP
00560	TSX ARGOS.4	00646	BSS	00730	FSB PHIR
00561	TSX QSP	00647	TSX COS.4	00731	STO TERM4
00561	STO THETSP	00650	STO ARG2B	00732	LDQ QR
00562	BSS	00651	CLA W-20	00733	FMP QR
00563	TSX QATAN.4	00652	BSS	00734	CHS
00564	TSX XSP	00653	TSX SIN.4	00735	FAO 33+3
00565	TSX YSP	00654	STO ARG5B	00736	BSS
00565	STO PHISP	00655	CLA W-20	00737	TSX SORT.4
00566	CLA 33+2	00656	BSS	00740	STO JERMS
00567	STO ZERO	00657	TSX COORD.4	00741	CLA TERM4
00570	BSS	00658	TSX XQ	00742	BSS
00571	TSX COORD.4	00659	TSX YQ	00743	TSX SIN.4
00572	TSX XR	00660	TSX ZR	00744	STO 13+1
00573	TSX YR	00661	TSX ARG1R	00745	LDQ TERM
00574	TSX ARG1R	00662	TSX ARG2B	00746	FMP 13+1
00575	TSX ARG2R	00663	TSX COORD.4	00747	STO TERM
00576	TSX ARG3M	00664	TSX XQ	00748	CLA 33+3
00577	TSX ARG4M	00665	TSX YQ	00749	FSB TERM2
		00666	TSX ZR	00750	BSS
		00667	TSX ARG1B	00751	TSX SORT.4
		00668	TSX ARG2B	00752	

01210	LQ TE3	01300	OCT +000016000000	01372	BCD 1INT.4H
01211	STR	01301	OCT +000017000000	01373	BCD 1 C5H0
01212	LQ TE1	01302	OCT +000020000000	01374	BCD 1 C216
01213	STR	01303	OCT +000021000000	01375	BCD 1EXIT
01214	LQ TE2	01304	OCT +000022000000	01376	BCD 1DY TO
01215	STR	01305	OCT +000025000000	01377	BCD 1 - REA
01216	LQ V-6	01306	OCT +000026000000	01400	BCD 1ONITOR
01217	STR	01307	OCT +000011000000	01401	BCD 1 FOR M
01220	LQ V-7	01310	OCT +000002000000	01402	BCD 1ITCHES
01221	STR	01311	OCT +000001000000	01403	BCD 1NSE SW
01222	LQ V-8	01312	OCT +000073000000	01404	BCD 1SET SE
01223	STR	01313	OCT +000102000000	01405	BCD 1C48H0
01224	LQ W	01314	OCT +206712273407	01406	BCD 1CUTE
01225	STR	01315	OCT +202622077326	01407	BCD 1NG EXE
01226	LQ W-1	01316	OCT +000000000000	01410	BCD 1N DURI
01227	STR	01317	OCT +201400000000	01411	BCD 105110
01230	LQ W-27	01320	OCT +223444760000	01412	BCD 1N P
01231	STR	01321	OCT +233000000000	01413	BCD 1CHES 1
01232	LQ W-65	01322	OCT +000000077777	01414	BCD 1E SWIT
01233	STR	01323	OCT +000000000000	01415	BCD 1L SENS
01234	BSS	01324	OCT +000001000000	01416	BCD 1SET AL
01234	TSX (FIL) 4	01325	OCT +000000000000	01417	BCD 1C54H0
01235	BSS	01326	BCD 1RO	01420	BCD 1RED
01235	TSX OUTPUT, 4	01327	BCD 1 OR ZE	01421	BCD 1ONSIDE
01236	PSE 116	01330	BCD 1GATIVE	01422	BCD 1EING C
01237	TRA EXR	01331	BCD 1 IS NE	01423	BCD 1BLEM B
01240	TRA 142A	01332	BCD 1 M(C)	01424	BCD 1OR PRO
01241	EXR	01333	BCD 1C28H0	01425	BCD 1 S52 F
01242	TRA 178A	01334	BCD 1E9.22	01426	BCD 1S1 AND
01243	TRA 181A	01335	BCD 1 ,1P3	01427	BCD 1 SET S
01244	TRA 180A	01336	BCD 19.2.2H	01430	BCD 1 C46H0
01245	CLA V-3	01337	BCD 1H .1PE	01431	BCD 1
01246	FSB W-3	01340	BCD 1E9.2.1	01432	BCD 1E14.7
01247	TZE 165A	01341	BCD 1 ,1P	01433	BCD 1 C13.
01250	TPL 136A	01342	BCD 19.2.3H	01434	BCD 1
01251	TRA 165A	01343	BCD 15.1P7E	01435	BCD 1 C12A6
01252	CLA W-27	01344	BCD 1C1H0.1		
01253	FSB W-39	01345	BCD 1X		
01254	180A1	01346	BCD 1.7X.1H		
01255	TPL 136A	01347	BCD 1HRANGE		
01256	TRA 166A	01350	BCD 1.5X.5		
01257	CLA W-59	01351	BCD 1.4HW(C)		
01260	FSB W-39	01352	BCD 1C1.6X		
01261	TZE 166A	01353	BCD 1SX.4HW		
01262	TPL 136A	01354	BCD 1IGPHI.		
01263	TRA 166A	01355	BCD 16X.6HS		
01264	LXD C61.4	01356	BCD 1THETA.		
01265	CLA KWT	01357	BCD 1.8HSIG		
01266	SUB 2+12	01360	BCD 1IGR.3X		
01267	TZE 2+12	01361	BCD 1SX.4HS		
01270	EXR	01362	BCD 13MPhi.		
01271	183A	01363	BCD 1TA.5X.		
01272	SUB 2+13	01364	BCD 1.5HTHE		
01273	183A1	01365	BCD 1-RO.3X		
01274	TPL D10J	01366	BCD 17HV.4		
01275	TRA D10J	01367	BCD 12.4X.		
01276	OCT +000005000000	01370	BCD 1X.4HV		
01277	OCT +000015000000	01371	BCD 1DELV.5		

B. FUNCTION SLANTR

This function is used to calculate the non-deviated ray path between the transmitter, R, and the starting point, S, (See Figures 1 and 2) from the given input data describing the problem under consideration. The input data should be designed in a manner that this assumption is true.

FUNCTION SLANT# DECEMBER 22, 1960 13M-7090
FUNCTION SLANTRCARGT ARG12+ ARG13
TERM= SATE C(0)+CARG12+CARG13**2 - ARG8**2
SLANT# = ARG12+ C-ARG7+ TERM1
ENDJOB
REQUIT

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	00000	2	3	00022	4	00040		

STORAGE NOT USED BY PROGRAM

DEC	OCT
32531	77461

LOCATIONS OF NAMES IN TRANSFER VECTOR

DEC	OCT	DEC	OCT	DEC	OCT
0	00000				

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE, OR COMMON SENTENCES

DEC	OCT	TERM	DEC	OCT	DEC	OCT
54	03066		52	00065		

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
50	00062	20	43 01053	30	44 00054	60	45 00055

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

3087

00000	BCD SORT
00001	HTR
00002	HTR
00003	HTR
00004	BCD SLANTR
00005	SKD 5+1
00006	SKD 5+1.2
00007	SKD 5+2.4
00010	CLA 1+4
00011	STA 3A+14
00012	CLA 5+4
00013	STA 3A
00014	STA 3A+1
00015	CLA 3+4
00016	STA 3A+4
00017	STA 3A+17
00020	CLA 9+4
00021	STA 3A+3
00022	STA 3A+3
00023	FMP ARG8
00024	STA 12+1
00025	CLA ARG13
00026	FOP ARG12
00027	STA 12+2
00030	CLA 32
00031	FAO 12+2
00032	STA 12
00033	LSO 12
00034	FMP 12
00035	FSS 12+1
00036	TSX SORT.4
00037	STA JERN1
00040	CLS ARG7
00041	FAO JERN1
00042	STA 12+1
00043	LSO ARG12
00044	FMP 12+1
00045	STA SLANTR
00046	CLA SLANTR
00047	LSO 1+1
00050	LSO 5+1.2
00051	LSO 5+2.4
00052	TFA 5+4
00053	OCI +000002002000
00054	OCI +201400000000
00055	OCI +233000000000
00056	OCI +000000077777
00057	OCI +000000000000
00060	OCI +000001000000
00061	OCI +000000000000

C. FUNCTION QATAN

Function QATAN permits the evaluation of the value of the arctangent of an angle with proper quadrant allocation.

```

1  FUNCTION DATAN, DECEMBER 22, 1980, 181-7090
2  FUNCTION DATAN(X,Y)
3  PI = 3.1415927
4  IF (ABS(X) - 1.0E-18) 9.2, 2
5  IF (ABS(Y) - 1.0E-18) 2, 17
6  DATAN = PI/2.0
7  GO TO 6
8  QRS = 0.0
9  IF (ABS(X) - 4.13E-9)
10  QRS = -QRS
11  DATAN = DATAN(QRS)
12  IF (QRS - 8.7E-7)
13  IF (QRS - 10.18E-13)
14  IF (QRS - 12.11E-11)
15  DATAN = QRS * PI/2.0
16  GO TO 13
17  DATAN = PI - DATAN
18  GO TO 18
19  DATAN = PI + DATAN
20  GO TO 19
21  IF (QRS - 14.18E-16)
22  DATAN = PI
23  GO TO 18
24  DATAN = 0.0
25  RETURN
26  ENDD, 1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0

```

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	LOC	IFN	LOC	EFN	IFN	LOC	IFN	LOC
1	1	00000	5	00031	17	6	00037	2	8	00043
2	10	00051	5	11	00053	6	12	00056	7	13
10	15	00072	11	17	00077	12	19	00103	13	21
15	23	00114	16	24	00115	18	25	00117	14	22

STORAGE NOT USED BY PROGRAM

DEC	OCT
39	00142
	32561 77461

LOCATIONS OF NAMES IN TRANSFER VECTOR

ATAN	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
	0	00000						

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

ARG	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
	37	00141	P1	96	00140	321AN	25	00137

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

23	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
	34	00124	35	85	00125	65	90	00132

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

ATAN

00000	ATAN	BCQ 181AN	00071	TRA 19A
00001	S	HR	00072 15A	LDO 32+3
00002		HR	00073	FMP PI
00003		HR	00074	FSB QATAN
00004		BCQ 181AN	00075	STO QATAN
00005		SXD S.1	00076 13A	TRA 25A
00006		SXD S+1-2	00077 17A	CLA PI
00007		SXD S+2-4	00100	FSB QATAN
00008		CLA 1-4	00101	STO QATAN
00009		STA 3A+2	00102 13A	TRA 25A
00010		STA 3A+18	00103 19A	CLA PI
00011		STA 3A+28	00104	FAD QATAN
00012		STA 3A+53	00105	STO QATAN
00013		CLA 2-4	00106 20A	TRA 25A
00014		STA 3A+7	00107 21A	CLA X
00015		STA 3A+17	00110 21A1	TZE 24A
00016		STA 3A+32	00111	TPL 24A
00017		STA 3A+36	00112 22A	CLA PI
00018		CLA 32	00113	STO QATAN
00019		STO PI	00114 23A	TRA 25A
00020		CLA X	00115 24A	CLA 32+4
00021		SSP	00116	STO QATAN
00022		FSB 32+1	00117 25A	CLA QATAN
00023		TZE 8A	00120	LXD S.1
00024		TPL 8A	00121	LXD S+1-2
00025		CLA Y	00122	LXD S+2-4
00026		SSP	00123	TRA 32-4
00027		FSB 32+2	00124 20	OCT +000000000000
00028		TZE 8A	00125 30	OCT +20262207326
00029		TPL 6A	00126	OCT +103447113564
00030		TRA 8A	00127	OCT +274627403531
00031		CLA PI	00130	OCT +202400000000
00032		FDP 32+3	00131	OCT +000000000000
00033		STO QATAN	00132 60	OCT +233000000000
00034		TRA 12A	00133	OCT +000000077777
00035		CLA Y	00134	OCT +000000000000
00036		FDP X	00135	OCT +000001000000
00037		STQ ARG	00136	OCT +000000000000
00038		CLA ARG		
00039		TZE 21A		
00040		TPL 11A		
00041		CLS ARG		
00042		STO ARG		
00043		CLA ARG		
00044		SSS		
00045		TSX ATAN.4		
00046		STO QATAN		
00047		CLA X		
00048		TZE 13A		
00049		TPL 13A		
00050		TRA 14A		
00051		CLA Y		
00052		TZE 25A		
00053		TPL 25A		
00054		TRA 15A		
00055		CLA Y		
00056		TZE 17A		
00057		TPL 17A		

D. FUNCTION ARCOS

Function ARCOS permits the evaluation of the value of the arccosine of an angle. Presently no quadrant allocation is made.

```

0  FUNCTION ARCOS  DECEMBER 22, 1960  13:17090
1  FUNCTION ARCOS00
2  IF ABSFC00-1.0E-13>2.2E-3
3  ARCOS = 1.5707963
4  GO TO 8
5  B1 = 0.2
6  B2 = SORTFC1.0/B1 -1.02
7  ARCOS = ATANFCB2
8  IF 0.07>B1
9  ARCOS = 3.1415927 - ARCOS
10 RETURN
11 ENDC00,1.0,1.0,0.0,0.0,0.0,0.0

```

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	3 00016	02	3	00028	3	6 00026	4	7 00031	5	8 00037	
2	9 00042	7	10 00145	3	11 00050						

STORAGE NOT USED BY PROGRAM

DEC	OCT
58 00072	32561 77491

LOCATIONS OF NAMES IN TRANSFER VECTOR

ATAN	DEC	OCT	SOPT	DEC	OCT	DEC	OCT	DEC	OCT
1	00001	00000							

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
82	57 00071	81	56 00070	ARCOS	55 00067		

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
20	45 00059	30	46 00056	60	50 00062		

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

SOPT ATAN

00000	SPT	BCC	ISORT
00001	ATAN	BCC	ATAN
00002	S	HTR	
00003		HTR	
00004		HTR	
00005		BCC	ARCOS
00006		SXD	3+1
00007		SXD	3+1.2
00010		SXD	3+2.4
00011		CLA	1+4
00012		STA	3A
00013		STA	3A+8
00014		STA	3A+9
00015		STA	3A+20
00016	32	CLA	0
00017		SSP	
00018		F58	32
00019		F58	4A
00021	3A	F2L	3A
00022	4A	CLA	3+1
00023		STA	ARCOS
00024	9A	T2L	11A
00025	9A	T2L	0
00026	9A	T2L	0
00027		T2L	0
00028		STA	81
00030	7A	CLA	3+2
00032		F58	81
00033		XCA	
00034		F58	3+2
00035		F58	
00036		T2L	SPT+4
00037		STA	0
00038	9A	CLA	0
00039		F58	
00040		T2L	ATAN+4
00041	9A	STA	ARCOS
00042	9A	CLA	0
00043	9A	T2L	11A
00044		T2L	11A
00045	10A	CLA	3+3
00046		F58	ARCOS
00047		STA	ARCOS
00050	11A	CLA	ARCOS
00051		LXD	3+1
00052		LXD	3+1.2
00053		LXD	3+2.4
00054		T2L	2+4
00055	20	OCT	+000000000000
00056	32	OCT	+113715136246
00057		OCT	+201622077323
00058		OCT	+201400000000
00059		OCT	+202622077326
00061		OCT	+233000000000
00062	92	OCT	+233000000000
00063		OCT	+000000077777
00064		OCT	+000000000000
00065		OCT	+000001000000
00066		OCT	+000000000000

E. SUBROUTINE COORD

This subroutine is used to transform the problem from the radar system to an earth centered geomagnetic spherical coordinate system.

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EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EF	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	00000		2	00123		3	4	00200	4	5	00240
									5	6	00316

STORAGE NOT USED BY PROGRAM

DEC OCT
224 00340 3 361 77 61

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC OCT	DEC OCT	DEC OCT	DEC OCT
215 00330	20	310 00322	62
		211 00323	

00000	3	HTF	00074	STA 3A-94	00194	FMP ARG11
00001		HTF	00075	STA 3A-104	00195	XCA
00002		HTF	00076	STA 3A-113	00196	FMP ARG8
00003		BOD 100000	00077	CLA 12-4	00197	STO 12+7
00004		END 3-1	00078	STA 3A-28	00170	LQD ARG7
00005		XAD 3+1-2	00079	STA 3A-59	00171	FMP 12+2
00006		XAD 3+2-4	00100	STA 3A+105	00172	XCA
00007		CLA 1-4	00101	CLA 13-4	00173	FMP ARG11
00010		STA 3A+44	00102	STA 3A+21	00174	FAD 12+7
00011		CLA 2-4	00103	STA 3A+50	00175	FSB 12+6
00012		STA 3A+76	00104	STA 3A+99	00176	FAD 12+3
00013		CLA 3-4	00105	CLA 14-4	00177	STO X
00014		STA 3A+122	00106	STA 3A+24	00200 4A	LQD ARG6
00015		CLA 4-4	00107	STA 3A+33	00201	FMP ARG12
00016		STA 3A+26	00110	STA 3A+40	00202	XCA
00017		STA 3A+48	00111	STA 3A-58	00203	FMP ARG1
00020		STA 3A+51	00112	STA 3A-52	00204	STO 12+1
00021		STA 3A+67	00113	STA 3A+71	00205	LQD ARG10
00022		STA 3A+107	00114	STA 3A+102	00206	FMP ARG1
00023		CLA 5-4	00115	STA 3A-111	00207	XCA
00024		STA 3A+4	00116	STA 3A-118	00210	FMP ARG5
00025		STA 3A+16	00117	CLA 15-4	00211	XCA
00026		STA 3A+60	00120	STA 3A+9	00212	FMP ARG11
00027		STA 3A+91	00121	STA 3A+46	00213	XCA
00030		STA 3A+93	00122	STA 3A+86	00214	FMP ARG8
00031		CLA 6-4	00123	LQD ARG4	00215	STO 12+2
00032		STA 3A+6	00124	FMP ARG5	00216	LQD ARG9
00033		STA 3A+13	00125	STO 12+1	00217	FMP ARG2
00034		STA 3A+31	00126	LQD ARG8	00220	XCA
00035		STA 3A+77	00127	FMP ARG2	00221	FMP ARG11
00036		STA 3A+89	00128	XCA	00222	XCA
00037		CLA 7-4	00131	FMP ARG3	00223	FMP ARG6
00040		STA 3A	00132	FSB 12+1	00224	STO 12+3
00041		STA 3A+12	00133	STO 12+2	00225	LQD ARG7
00042		STA 3A+32	00134	LQD ARG12	00226	FMP ARG1
00043		STA 3A+95	00135	FMP 12+2	00227	XCA
00044		STA 3A+109	00136	STO 12+3	00230	FMP ARG6
00045		CLA 8-4	00137	LQD ARG4	00231	XCA
00046		STA 3A+1	00138	FMP ARG8	00232	FMP ARG11
00047		STA 3A+15	00139	STO 12+4	00233	CHS
00050		STA 3A+53	00142	LQD ARG5	00234	FAD 12+3
00051		STA 3A+78	00143	FMP ARG2	00235	FAD 12+2
00052		STA 3A+92	00144	XCA	00236	FSB 12+1
00053		CLA 9-4	00145	FMP ARG3	00237	STO Y
00054		STA 3A+3	00146	FAD 12-4	00240 5A	LQD ARG3
00055		STA 3A+13	00147	STO 12+5	00241	FMP ARG5
00056		STA 3A+45	00150	LQD ARG10	00242	STO 12+1
00057		STA 3A+69	00151	FMP 12+5	00243	LQD ARG6
00060		STA 3A+80	00152	XCA	00244	FMP ARG2
00061		STA 3A+90	00153	FMP ARG11	00245	XCA
00062		CLA 10-4	00154	XCA	00246	FMP ARG4
00063		STA 3A+37	00155	FMP ARG3	00247	FAD 12+1
00064		STA 3A+66	00156	STO 12+6	00250	STO 12+2
00065		STA 3A+115	00157	LQD ARG9	00251	LQD ARG12
00066		CLA 11-4	00160	FMP ARG1	00252	FMP 12+2
00067		STA 3A+26	00161	XCA	00253	STO 12+3
00070		STA 3A+35	00162	FMP ARG3	00254	LQD ARG3
00071		STA 3A+57	00163	XCA	00255	FMP ARG5

00286	STO 12+4
00287	LDR ARG5
00288	FMP ARG2
00289	XOR
00290	FMP ARG4
00291	CHS
00292	FAD 12+4
00293	STO 12+5
00294	LDR ARG10
00295	FMP 12+5
00296	XOR
00297	FMP ARG11
00298	XOR
00299	FMP ARG3
00300	STO 12+6
00301	LDR ARG9
00302	FMP ARG1
00303	XOR
00304	FMP ARG8
00305	STO 12+7
00306	LDR ARG7
00307	FMP 12+2
00308	XOR
00309	FMP ARG11
00310	FAD 12+7
00311	FAD 12+6
00312	FAD 12+3
00313	STO 2
00314	LDR \$1
00315	LDR \$1+2
00316	LDR \$2+4
00317	TRA 16+4
00318	OCT +000000000000
00319	OCT +330000000000
00320	OCT +000000077777
00321	OCT +000000000000
00322	OCT +000001000000
00323	OCT +000000000000
00324	OCT +000000000000
00325	OCT +000000000000
00326	OCT +000000000000
00327	OCT +000000000000

RM 61TMP-32

F. SUBROUTINE DAUX

Subroutine DAUX is used to define the differential equations that are to be numerically integrated. As a result the previously described ray trace equations are defined in this subroutine.

```

C SUBROUTINE DAUX DECEMBER 22, 1960 13M-7020
1 SUBROUTINE DAUX
  DIMENSION RECORD(12), WC(11), WC2(2)
  COMMON RECORD, U, M, N
  CALL FINDEN
  * TER1 = WC(1)*WC(2)
  * TER2 = SIN(WC(3))
  * TER3 = COS(WC(5))
  7 IF(CABS(CTER2)-1.0E-30)17,8,8
  17 TER2 = 1.0E-3
  8 WC(12) = C1.0/TER12 * WC72 = WC(12)*WC(22)
  9 WC(14) = C1.0/CV(43)*TER13 * WC(8) = WC(13)*WC(5)
  10 WC(15) = C1.0/CV(43)*TER1*TER22 * WC(9) = WC(13)*WC(22)
  11 WC(16) = C1.0/WC(13)*WC7 * WC(3) * WC(14) * WC(9) * WC(15) * TER2
  12 WC(17) = C1.0/CV(43)*C1.0/WC(12) * WC(8) = WC(8)*WC(12) * WC(42) * WC(22)
  13 WC(18) = C1.0/CV(43)*TER22 * C1.0/WC(12) * WC(9) = WC(9)*WC(12)*TER2
  14 WC(19) = C1.0/TER12 * C30RIF * TER1 + WC(19)*2.22
  15 WC(20) = C1.0/3.0E5 * C1.0 * WC(3)/WC(13)*WC(11)
  16 WC(21) = -WC(32)/3.0E5 * C1.0 * WC(2)/WC(13)
  20 RETURN
  END DAUX

```

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	
N 32129	76374	RECORD	32161	77461	U 32545	77445	W 32438	77256

EXTERNAL FORMULA NUMBERS 1-4 CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	00000	4	00014	5	7 00017	6	8 00022	9 00028
17	00032	3	00034	9	12 00045	10	13 00061	11 14 00072
12	00015	12	00141	14	17 00172	15	18 00203	16 19 00214
20	00224							

STORAGE NOT USED BY PROGRAM

DEC	OCT
32187	76573

LOCATIONS OF NAMES IN TRANSFER VECTOR

DEC	OCT	RINDEX	DEC	OCT	SIN	DEC	OCT	SORT	DEC	OCT
105	00000		0 00000		1 00001		3 00003			

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

DEC	OCT	DEC	OCT	DEC	OCT
TER1	166 00250	TER2	167 00247	TER3	166 00246

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC	OCT	DEC	OCT	DEC	OCT
10	161 00241	20	152 00230	30	153 00231
				60	156 00234

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

RINDEX	SIN	SWRT
CO5		

00000	RINDEX	BCD	RINDEX	00067	FMP V-3	00161	ESB 12+1
00001	SIN	BCD	ISIN	00070	XCR	00162	STO 12+3
00002	COS	BCD	ICOS	00071	FMP TER1	00163	LQ V-3
00003	SORT	BCD	ISORT	00072	STO 12+2	00164	FMP TER2
00004	S	STR		00073	CLA 32+1	00165	STO 12+4
00005	STR			00074	FDP 12+2	00166	CLA 32+1
00006	STR			00075	FMP 12+1	00167	FDP 12+4
00007	BCD	10AUX		00076	STO V-14	00170	FMP 12+3
00010	SXD	S+1		00077	LQ TER2	00171	STO V-17
00011	SXD	S+1.2		00100	FMP V-8	00172	LQ M-9
00012	SXD	S+2.4		00101	XCR	00173	FMP M-9
00013	SA	BSS		00102	FMP V-14	00174	FQD TER1
00013	TSX	RINDEX.4		00103	STO 12+1		BSS
00014	6A	LQ M		00104	LQ V-7	00175	TSX SORT.4
00015	FMP	M		00105	FMP V-13	00176	STO 12+1
00016	STO	TER1		00106	STO 12+2	00177	CLA 32+1
00017	7A	CLA	V-4	00107	CLA 32+1	00200	FDP TER1
00018	BSS			00110	FDP M	00201	FMP 12+1
00020	TSX	SIN.4		00111	FMP M-6	00202	STO V-18
00021	STO	TER2		00112	FQD 12+2	00203	CLA M-2
00022	8A	CLA	V-4	00113	FQD 12+1	00204	FDP M
00023	BSS			00114	STO V-15	00205	FMP M-10
00023	TSX	COS.4		00115	LQ TER3	00206	FQD 32+1
00024	STO	TER3		00116	FMP V-3	00207	STO 12+1
00025	9A	CLA	TER2	00117	XCR	00210	CLA 32+1
00026	SFP			00120	FMP V-8	00211	FDP 32+2
00027	FSD	32		00121	XCR	00212	FMP 12+1
00030	9A1	TZE	11A	00122	FMP V-14	00213	STO V-19
00031	11A	TPL	11B	00123	STO 12+1	00214	CLA M-1
00032	10A	CLA	32	00124	LQ V-7	00215	FDP M
00033	STO	TER2		00125	FMP V-12	00216	STO 12+1
00034	11A	LQ	M	00126	STO 12+2	00217	CLA M-2
00035	FMP	M-3		00127	CLA 32+1	00220	FDP 32+2
00036	CMS			00130	FDP M	00221	FMP 12+1
00037	FQD	V-6		00131	FMP M-7	00222	CMS
00040	STO	12+1		00132	FSD 12+2	00223	STO V-20
00041	CLA	32+1		00133	FQD 12+1	00224	LQ S.1
00042	FDP	TER1		00134	STO 12+3	00225	LQ S+1.2
00043	FMP	12+1		00135	CLA 32+1	00226	LQ S+2.4
00044	STO	V-12		00136	FDP V-3	00227	TRA 1.4
00045	12A	LQ	M	00137	FMP 12+3	00230	OCI +000002000000
00046	FMP	M-4		00140	STO V-16	00231	OCI +146527461670
00047	CMS			00141	LQ TER3	00232	OCI +201400000000
00050	FQD	V-7		00142	FMP V-3	00233	OCI +223444760000
00051	STO	12+1		00143	XCR	00234	OCI +233000000000
00052	LQ	V-3		00144	FMP V-8	00235	OCI +000000077777
00053	FMP	TER1		00145	XCR	00236	OCI +000000000000
00054	STO	12+2		00146	FMP V-13	00237	OCI +000001000000
00055	CLA	32+1		00147	STO 12+1	00240	OCI +000000000000
00056	FDP	12+2		00150	LQ TER2		
00057	FMP	12+1		00151	FMP V-8		
00060	STO	V-13		00152	XCR		
00061	12A	LQ	M	00153	FMP V-12		
00062	FMP	M-5		00154	STO 12+2		
00063	CMS			00155	CLA 32+1		
00064	FQD	V-8		00156	FDP M		
00065	STO	12+1		00157	FMP V-8		
00066	LQ	TER2		00160	FSD 12+2		

G. SUBROUTINES INT AND INTM

This is a generally available SHARE program which permits the numerical integration of a chosen set of first order non-linear differential equations. It can be operated in three possible numerical integration modes (a) Runge-Kutta with a fixed integration mesh size, (b) fourth order Adams-Moulton with a fixed integration mesh size, and (c) fourth order Adams-Moulton with a variable integration mesh size that is controlled by an error sensing routine. Because no FORTRAN source program listing is available, a SHARE description of this routine is given along with a SAP listing.

IDENTIFICATION

RW INT, Adams-Moulton, Runge-Kutta Integration
704 - FORTRAN SAP Language Subroutine
Space Technology Laboratories,
Robert Causey and Werner L. Frank,
November 30, 1958

ABSTRACT

FORTRAN version of RW-DE2F which integrates a system of N simultaneous, first order, ordinary differential equations. Option of using either 4th order Runge-Kutta method or 4th order predictor-corrector method (Adams-Moulton) is provided. Also option of automatic error control with variable step-size is provided. Input and output are single precision but double precision is used internally to control round-off errors. Requires $12N + 3$ cells for data and 693 words for program.

PURPOSE

This FORTRAN subprogram integrates a set of N simultaneous, first order differential equations. It is the FORTRAN version of the standard subroutine RW-DE2F.

RESTRICTIONS

This program has two distinct entries, one for set up and the second for performing the integration steps. The user must supply a FORTRAN subprogram (with the name DAUX) which evaluates the derivatives y' .

METHOD

The user has the option of using either a fourth order Runge-Kutta method or the fourth order Adams-Moulton method with a fixed step-size. There is also a variable step-size mode.

While input and output to this routine are single precision, double precision is used internally to control round-off errors. Truncation error is controlled either by choosing an appropriate step-size, or by using the variable step-size mode of operation.

For details of the method see RW-DE2F.

USAGE

a. Calling Sequence for set up (performed prior to initiating the integration).

CALL INT (V, N, A1, A2, A3, A4, A5, A6, A7)

Where V is a region of at least dimension $12N + 3$

N is the number of equations

A1 is the option word

A2 is $\overline{E}$

A3 is M

A4 is A

A5 is $h_{\max}$

A6 is $h_{\min}$

A7 is β

For meaning of A1 - A7 see Appendix A and B of RW-DE2F.

Region V contains the following information prior to Set Up entry.

V(2) = x, initial value of independent variable

V(3) = h, value of step-size

V(4) = y_1

$\vdots$

V(3+N) = y_N

} values of dependent variables y_i

$$\left. \begin{array}{l} V(4+N) = y_1' \\ \vdots \\ V(3+2N) = y_N' \end{array} \right\} \text{values of the derivatives } y_i' \text{ to be supplied by} \\ \text{the auxiliary DAUX.}$$

Note: This region and the parameter N should be placed in COMMON since it is necessarily referred to in the main program and in the auxiliary. The cell V (1) is set up by the subprogram RW INT and will contain N scaled at 35.

b. Calling Sequence for integrating one step.

CALL INTM

No arguments are required for this statement.

SPACE REQUIRED

693 cells

CHECKOUT

This routine has been extensively tested on several check problems. In all cases the errors were approximately equal to their expected values, and there were no indications that round-off errors accumulate rapidly.

METHOD

References:

1. S. D. Conte and J. Titus, An interpretive floating point sub-routine for the solution of systems of ordinary differential equations, Appendix I, Proc. Math. Committee of Univac Scientific Exchange Meeting, Nov. 21-22, 1957 (Obtainable from Remington Rand Univac, St. Paul, Minnesota).
2. E. K. Blum, A modification of the Runge-Kutta fourth-order method, Appendix H, Proc. Math. Committee of Univac Scientific Exchange Meeting, Nov. 21-22, 1957.

In this routine the user is allowed an option of using either the Runge-Kutta classical fourth-order method as modified by E. K. Blum [Ref. (2)] or the Adams-Moulton predictor-corrector method using the Runge-Kutta method for starting the process. Let the system of equations to be solved be given in the form

$$(1) \quad \left. \begin{aligned} y'_i &= f_i(x, y_1, y_2, \dots, y_N) \\ y_i(x_0) &= y_{i0} \end{aligned} \right\} \quad i = 1, 2, \dots, N.$$

Let y_{in} be the value of y_i at $x = x_n$ and f_{in} the derivation of y_i at $x = x_n$ and let h be the increment (step-size) of the independent variable x . The classical Runge-Kutta fourth-order method uses the formulas

$$\begin{aligned} k_{i1} &= h f_i(x_n, y_{in}) , \\ k_{i2} &= h f_i\left(x_n + \frac{1}{2}h, y_{in} + \frac{1}{2}k_{i1}\right) , \\ k_{i3} &= h f_i\left(x_n + \frac{1}{2}h, y_{in} + \frac{1}{2}k_{i2}\right) , \end{aligned}$$

$$\begin{aligned}
 k_{i4} &= hf_i(x_n + h, y_{in} + k_{i3}) , \\
 (2) \quad y_{i,n+1} &= y_n + \frac{1}{6} (k_{i1} + 2k_{i2} + 2k_{i3} + k_{i4}) ,
 \end{aligned}$$

The following formulas (we omit the subscript i for notational simplicity) were derived by E. K. Blum to control the growth of round-off errors.

$$\begin{aligned}
 (3) \quad & \begin{cases} z_0 = y_n , \\ q_0 = q_{4n} \end{cases} \\
 (4) \quad & \begin{cases} P_0 = hf(x_n, z_0) \\ r_1 = L^{(1)}R^{(1)} \left[\frac{1}{2}P_0 - q_0 \right] , \\ z_1 = z_0 + r_1 , \\ q_1 = 3r_1 - \left[\frac{1}{2}P_0 - q_0 \right] , \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad & \begin{cases} P_1 = hf(x_n + \frac{1}{2}h, z_1) , \\ r_2 = L^{(2)}R^{(2)} \left[\frac{1}{2}P_1 - \frac{1}{2}q_1 \right] , \\ z_2 = z_1 + r_2 , \\ q_2 = -r_2 - \frac{1}{3}q_1 + \frac{1}{2}P_1 , \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad & \begin{cases} P_2 = hf(x_n + \frac{1}{2}h, z_2) , \\ r_3 = L^{(3)}R^{(3)} [P_2] , \\ z_3 = z_2 + r_3 , \\ q_3 = -r_3 + q_2 , \end{cases}
 \end{aligned}$$

$$(7) \quad \begin{cases} P_3 &= hf(x_n + h, z_3) + 2P_2, \\ r_4 &= L^{(4)}R^{(4)}\left[\frac{1}{6}P_3 + q_3\right], \\ y_{n+1} &= z_4 = z_3 + r_4, \\ q_{4,n+1} &= 3\left[r_4 - \left(\frac{1}{6}P_3 + q_3\right)\right], \end{cases}$$

where $R^{(m)}$, $L^{(m)}$ denote operators which shift right m places or left m places respectively and q_{40} is taken to be zero to start the computation. (See Ref (2) for a complete description of this method.) Formulas (3) - (7) are those used in this routine.

The Adams-Moulton predictor-corrector formulas for the system (1) are

$$(8) \quad y_{i,n+1}^{(p)} = y_{in} + \frac{h}{24} (55f_{in} - 59f_{i,n-1} + 37f_{i,n-2} - 9f_{i,n-3})$$

$$(9) \quad y_{i,n+1}^{(c)} = y_{in} + \frac{h}{24} (9f_{i,n+1}^{(p)} + 19f_{in} - 5f_{i,n-1} + f_{i,n-2})$$

The corrector formula (9) is applied only once so that only two derivative evaluations are needed for each Adams-Moulton integration step. The starting values needed in (8) are obtained using the Runge-Kutta-Blum (RKB) method.

The Adams-Moulton method may be used either with a fixed step-size or with a variable step-size. The step-size to be used in the variable mode is determined as follows. Let

$$(10) \quad \begin{aligned} E_{n+1} &= \max_i \frac{|y_{i,n+1}^{(p)} - y_{i,n+1}^{(c)}|}{14D_i}, \\ D_i &= \max \left\{ |y_{i,n+1}^{(c)}|, A \right\}, \end{aligned}$$

where $A > 0$. The user will specify an upper bound $\bar{E}$ on the truncation error estimate E_{n+1} . This is equivalent to specifying the number of significant figures which the user desires to preserve locally throughout the integration. There must also be specified a constant $M > 0$

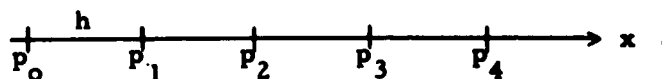
from which a lower bound $\underline{E} = M^{-1} \bar{E}$ is obtained. M should normally range from 50 to 150. The interval will then be decreased, left as it is, or increased according as the following inequalities hold:

- (11a) If $E_{n+1} \geq \bar{E}$, the interval is reduced to βh ($0 < \beta < 1$)
- (11b) If $\underline{E} \leq E_{n+1} < \bar{E}$, the interval size is kept fixed.
- (11c) If $E_{n+1} < \underline{E}$ for 3 successive steps, the step-size is increased to $\frac{1}{\beta} h$.

Normally, the routine will take $\beta = 1/2$, unless β is otherwise specified. The constant A in (10) is used to prevent unnecessary reductions in $|h|$ whenever $|y_{i,n+1}|$ is small. Normally the routine will set $A = 1$. However, some other value for A may be specified by the user if he desires to use some other characteristic length for A . While the test based on (10) will guarantee that the local error does not exceed $\bar{E}$, the cumulative error will usually exceed $\bar{E}$. Hence, $\bar{E}$ should be chosen small enough to allow for an accumulation of truncation error. Normally $\bar{E}$ should be in the range $10^{-8} < \bar{E} < 10^{-3}$.

After an interval is increased, the program prevents increasing again until 6 more points have been completed. However, the program may decrease the interval as often as necessary.

Starting values for the Adams-Moulton formulas are always obtained using the RKB method whenever the interval size is changed, just as at the beginning of an integration. Consider the following diagram of the axis of the independent variable x



If the values of the y_i are computed at the points p_1 , p_2 , and p_3 using the RKB method and the truncation error test (11) calls for decreasing $|h|$ at the point p_4 , then the routine returns to the point p_0 and again computes three new points with the RKB method using the decreased value of $|h|$. If on the other hand (11a) holds at p_4 and the y_i at p_3 had been computed using the AM method, then the routine returns to the point p_3 for a new start. If the inequality in

(11c) is not satisfied at p_1 , but is satisfied at p_2 , p_3 , and p_4 , then a new start is initiated at p_4 with the increased value of $|h|$.

The user must provide a starting value for h and he may, if desired, specify a maximum value of $|h|$ beyond which the routine will not increase $|h|$ and a minimum value of $|h|$ below which it will not decrease $|h|$. Negative values of h may be supplied for backward integration.

Both the RKB method and the AM method incorporate round-off control features. This is performed in the RKB method by carrying the q 's in formula (3)-(7). In the AM method this is done by keeping the y_{in} in double precision forming the sums $y_{in} + \nabla y_{in}$ in (8) and (9) in double precision. The derivative calculations are all performed in single precision. Both procedures have shown to be very effective in controlling the growth of round-off errors.

USAGE AND CODING INFORMATION (APPENDIX B)

There are two entries to this routine. The first must be used once at the beginning to set up the routine for integration of a given set of N differential equations. The second entry may be used any number of times after the first to integrate all y_i from x to $x + h$. The first entry has the following calling sequence.

Loc.	Instruction	Comment
A1-2	TSX DE2F, 4	Setup entry
A1-1	PZE T, 0, V	Parameter word with addresses
A1	(Binary integer)	Option word (= 0 or 1 or 2)
A2	(Floating-point number)	$\overline{E}$ } Truncation error testing M } information
A3	(Floating-point number)	
A4	(Floating-point number)	A
A5	(Floating-point number)	$h_{\max}$ } $h_{\min}$ } Bounds on h , if any
A6	(Floating-point number)	
A7	(Floating-point number)	β - Increase or decrease h factor
A7 + 1	Return	

The eight parameter words have the following meaning, (A1-1): V is the address of the first word of a block of $12N + 3$ cells, reserved by the user, with the arrangement

Loc.	Contents	
V	N	Fixed point binary integer, point at right
V + 1	x	Value of independent variable in floating point
V + 2	h	Value of step size in floating point
V + 3	y_1	Values of the y_i in floating point
V + 4	y_2	
.	.	
.	.	
.	.	
V + N + 2	y_N	

<u>Loc.</u>	<u>Contents</u>	
$V + N + 3$	y_1'	Locations where the user's auxiliary subroutine must place the derivatives y_1' .
$V + N + 4$	y_2'	
.		
.		
$V + 2N + 2$	y_N'	10 N cells of temporary storage
$V + 2N + 3$		
etc.		

[Note: If the Runge-Kutta only option (see under A1 below) is used, it is only necessary to reserve a block of $4N + 3$ cells].

Before executing the setup entry, the user must have already placed the appropriate numbers in cells V through $V + N + 2$.

The address V in the entry point of an auxiliary subroutine which the user must provide to evaluate the derivatives y_1' and store them in cells $V + N + 3$ through $V + 2N + 2$ as shown above. This auxiliary subroutine is entered by the calling sequence

<u>Loc.</u>	<u>Instruction</u>
A1-2	TSX V, 4
A1-1	Return

The setup entry uses the auxiliary subroutine to evaluate the derivatives for the initial data.

(A1): The option word may have any one of three values which designate three different modes of operation for RWINT

A1 = 0 designates the predictor-corrector variable h mode

A1 = 1 designates the fixed h Runge-Kutta only mode

A1 = 2 designates the fixed h predictor-corrector mode

For A1 = 1 or 2, the contents of A2 through A7 may be arbitrary.

(A2): This cell contains the upper bound $\bar{E} > 0$ for the truncation error testing done in the predictor-corrector variable h mode. ($10^{-8} \leq E \leq 10^{-3}$)

(A3): This cell contains the number $M > 0$ from which the lower bound $\underline{E}$ is calculated. If $A3 = 0$, M is set equal to 100.

(A4): This cell contains the number $A > 0$ used to designate a fixed-point truncation error test as described in Appendix A. If $A4 = 0$, A is set equal to 1.

(A5): This cell may contain the upper bound $h_{\max} > 0$ for $|h|$. If $A5 = 0$, this means that there is to be no upper bound for $|h|$.

(A6): This cell may contain the lower bound $h_{\min} > 0$ for $|h|$. If $A6 = 0$, this means that there is to be no lower bound for $|h|$.

(A7): This cell may contain the factor $1 > \beta > 0$ used to increase or decrease $|h|$. If $A7 = 0$, β is set equal to $1/2$.

The integration entry is quite simple and has the calling sequence

<u>Loc.</u>	<u>Instruction</u>
A1-2	TSX DE2F + 1, 4 Integration entry
A1-1	Return

Ordinarily, after execution of the integration entry, all y_i assume new values, x will have been advanced to the value $x + h$ and h will be unchanged. However, in the variable h mode, three other things can happen. (1) If the truncation error test indicates that $|h|$ should be increased, h will have been changed to $\beta^{-1}h$ unless $|\beta^{-1}h| > h_{\max}$. If the truncation error test indicates that $|h|$ should be decreased, then h will have been changed to βh unless $|\beta h| < h_{\min}$ and either (2) y_i and x will remain as they were before entry or (3) x will be changed to $x - 3h$ and the corresponding y_i values will occupy cells $V + 3$ through $V + N + 2$. Case (3) can only happen when successive decreases in $|h|$ are called for. On exit the values y_i in $V + N + 3$ etc. are always those which correspond to the x and y_i in $V + 1$ and $V + 3$ etc.

The integration entry must be used for each integration step. In the variable h mode, a particular integration step may involve

either AM or RKB integration but not both. In the fixed h predictor-corrector mode, the first three integration entries involve RKB integration and all subsequent ones involve AM integration.

Whenever an integration step involves AM integration, the truncation error estimate E_{n+1} is in the accumulator on exit. Zero is always placed in the accumulator if the step involved RKB integration.

The setup entry may be used again at any time to set up another problem or to change the mode of operation.

In addition to the auxiliary subroutine for derivative evaluation and the $12N + 3$ cells for data storage, the storage requirements are 693 words for RWINT plus 2 words of COMMON.

	ORG 0		INT 0001
DAUX	BCD 1DAUX		INT 0002
	HTR		INT 0003
	HTR		INT 0004
	HTR		INT 0005
INT	SXD INT-3,4		INT 0006
	SXD INT-2,2		INT 0007
	SXD INT-1,1		INT 0008
	CLA 1,4		INT 0009
	STA REV1+1		INT 0010
	STA REV1+2		INT 0011
	STA A2		INT 0012
	CLA 2,4		INT 0013
	STA A1		INT 0014
A1	CLA	N	INT 0015
	ARS 18		INT 0016
A2	STO		INT 0017
	ALS 2		INT 0018
	STO C		INT 0019
	ALS 1		INT 0020
	ADD C	-12N	INT 0021
	ADD C1	12N+2	INT 0022
	STO C		INT 0023
	CLA 1,4		INT 0024
	SUB C		INT 0025
	STA PAR1		INT 0026
	CLA C		INT 0027
	ARS 1		INT 0028
	STO C		INT 0029
	ADD PAR1		INT 0030
	STA REV1		INT 0031
	STA REV1+3		INT 0032
	LXA C2,1		INT 0033
A4	CLA 3,4		INT 0034
	STA A5		INT 0035
A5	CLA		INT 0036
	STO PAR8+1,1		INT 0037
	TXI A6,4,-1		INT 0038
A6	TIX A4,1,1		INT 0039
	CLA PAR2		INT 0040
	LRS 18		INT 0041
	STO PAR2		INT 0042
	CLA DAUX		INT 0043
	STA AUX+2		INT 0044
	TSX REV,4		INT 0045
	TSX DE2F,4		INT 0046
PAR1	PZE 0,0,AUX		INT 0047
PAR2	PZE		INT 0048
PAR3	PZE		INT 0049
PAR4	PZE		INT 0050
PAR5	PZE		INT 0051
PAR6	PZE		INT 0052
PAR7	PZE		INT 0053

PARB	PZE		INT 0054
	TSX REV,4		INT 0055
	LXD INT-2,2		INT 0056
	LXD INT-1,1		INT 0057
	LXD INT-3,4		INT 0058
	TRA 10,4		INT 0059
INTM	SXD INT-3,4		INT 0060
	SXD INT-2,2		INT 0061
	SXD INT-1,1		INT 0062
	TSX REV,4		INT 0063
	TSX DE2F+1,4		INT 0064
	TSX REV,4		INT 0065
	LXD INT-3,4		INT 0066
	LXD INT-2,2		INT 0067
	LXD INT-1,1		INT 0068
	TRA 1,4		INT 0069
REV	LXA C,1		INT 0070
	LXD C1,2		INT 0071
REV1	CLA 0,1		INT 0072
	LDQ 0,2		INT 0073
	STO 0,2		INT 0074
	STQ 0,1		INT 0075
	TXI REV3,2,1		INT 0076
REV3	TIK REV1,1,1		INT 0077
	TRA 1,4		INT 0078
AUX	SXD C3,4		INT 0079
	TSX REV,4		INT 0080
	TSX 0,4		INT 0081
	TSX REV,4		INT 0082
	LXD C3,4		INT 0083
	TRA 1,4		INT 0084
C	PZE		INT 0085
C1	DEC 2		INT 0086
C2	DEC 7		INT 0087
C3			INT 0088
	REM FLOATING POINT ADAMS-MOULTON, RUNGE-KUTTA INTEGRATION		INT 0089
DE2F	TRA DE2F+0293	SETUP ENTRY	INT 0090
	SXD DE2F+0240,1	INTEGRATION ENTRY	INT 0091
	SXD DE2F+0241,2		INT 0092
	SXD DE2F+0242,4		INT 0093
	TRA DE2F+0003	SWITCH 1	INT 0094
	CLA DE2F+0285	3 TO ACC	INT 0095
	CAS DE2F+0228	TEST ALPHA	INT 0096
	TRA DE2F+0186		INT 0097
	TRA DE2F+0175		INT 0098
	LXA DE2F+0229,1	Y PRIMES TO D	INT 0099
	CLA *,1		INT 0100
	STO *,1	AND	INT 0101
	CLA *,1		INT 0102
	STO *,1	DOUBLE PRECISION	INT 0103
	CLA *,1		INT 0104
	STO *,1	YS TO TS2	INT 0105
	TIK DE2F+0010,1,1	END 3F LOOP	INT 0106

LXA DE2F+0229.1		INT 0107
LDQ DE2F+0223		INT 0108
FMP *.1	D3 SUB 1	INT 0109
STO COMMON+000		INT 0110
CLA *.1	PLANT Y SUB 1	INT 0111
STO DE2F+0267	IN SFA	INT 0112
CLA *.1	SUBROUTINE	INT 0113
STO DE2F+0268		INT 0114
LDQ DE2F+0222	D2 SUB 1	INT 0115
FMP *.1		INT 0116
FAD COMMON+000		INT 0117
STO COMMON+000		INT 0118
LDQ DE2F+0221		INT 0119
FMP *.1	D1 SUB 1	INT 0120
FAD COMMON+000		INT 0121
STO COMMON+000		INT 0122
LDQ DE2F+0220		INT 0123
FMP *.1	D SUB 1	INT 0124
FAD COMMON+000		INT 0125
STO COMMON+000		INT 0126
LDQ *	LOAD H	INT 0127
FMP COMMON+000	DELTA Y1 UPPER P	INT 0128
TSX DE2F+0269.2	ADD TO Y1	INT 0129
STO *.1	STORE IN TS1	INT 0130
TIX DE2F+0018.1.1	END OF LOOP	INT 0131
CLA *		INT 0132
FAD *	X+H	INT 0133
STO *		INT 0134
LXA DE2F+0229.1		INT 0135
CLA *.1		INT 0136
STO *.1		INT 0137
TIX DE2F+0046.1.1	END OF LOOP	INT 0138
TSX 0.4	EVALUATE DERIVATIVES	INT 0139
LXA DE2F+0229.1		INT 0140
LDQ DE2F+0227		INT 0141
FMP *.1	D2 SUB 1	INT 0142
STO COMMON+000		INT 0143
CLA *.1	Y1 FROM TS2	INT 0144
STO DE2F+0267		INT 0145
CLA *.1	DP EXT.	INT 0146
STO DE2F+0268		INT 0147
LDQ DE2F+0226		INT 0148
FMP *.1	D1 SUB1	INT 0149
FAD COMMON+000		INT 0150
STO COMMON+000		INT 0151
LDQ DE2F+0225		INT 0152
FMP *.1	D SUB1	INT 0153
FAD COMMON+000		INT 0154
STO COMMON+000		INT 0155
LDQ DE2F+0224		INT 0156
FMP *.1	Y PRIME SUB1	INT 0157
FAD COMMON+000		INT 0158
STO COMMON+000		INT 0159

LDO *	LOAD H	INT 0160
FMP COMMON+000	DELTA YI UPPER C	INT 0161
TSX DE2F+0269,2	ADD TO Y I	INT 0162
STO *,1		INT 0163
STO *,1		INT 0164
TIX DE2F+0051,1,1	END OF LOOP	INT 0165
STZ DE2F+0230		INT 0166
LXA DE2F+0229,1		INT 0167
CLA *,1		INT 0168
STO DE2F+0243		INT 0169
CLA *,1		INT 0170
STO DE2F+0244		INT 0171
TSX DE2F+0246,2		INT 0172
TIX DE2F+0078,1,1	END OF LOOP	INT 0173
TRA DE2F+0085	SWITCH 2	INT 0174
CLA DE2F+0234		INT 0175
CAS DE2F+0230		INT 0176
TRA DE2F+0090		INT 0177
TRA DE2F+0089		INT 0178
TRA DE2F+0115	DECREASE H SWITCH	INT 0179
CLA DE2F+0235		INT 0180
CAS DE2F+0230		INT 0181
TRA DE2F+0164	INCREASE H SWITCH	INT 0182
TRA DE2F+0211		INT 0183
TRA DE2F+0211		INT 0184
LXA DE2F+0229,1		INT 0185
CLA *,1		INT 0186
STO *,1	D2 TO D3	INT 0187
CLA *,1		INT 0188
STO *,1	D1 TO D2	INT 0189
CLA *,1		INT 0190
STO *,1	D TO D1	INT 0191
TIX DE2F+0096,1,1	END OF LOOP	INT 0192
CLA DE2F+0228		INT 0193
ADD DE2F+0284	ALPHA PLUS ONE	INT 0194
STO DE2F+0228		INT 0195
TSX 0.4	EVALUATE DERIVATIVES	INT 0196
CLA DE2F+0230		INT 0197
FDP DE2F+0290	E	INT 0198
STQ COMMON+000		INT 0199
CLA COMMON+000	GET E IN ACC	INT 0200
LXD DE2F+0240,1		INT 0201
LXD DE2F+0241,2		INT 0202
LXD DE2F+0242,4		INT 0203
TRA 1,4	EXIT	INT 0204
CLA *		INT 0205
SSP		INT 0206
CAS DE2F+0238	TEST H WITH HMIN	INT 0207
TRA DE2F+0121		INT 0208
TRA DE2F+0121		INT 0209
TRA DE2F+0211		INT 0210
LDO *		INT 0211
STQ DE2F+0233	STORE OLD H	INT 0212

FMP DE2F+0239	BETA TIMES H	INT 0213
STO *		INT 0214
CLA DE2F+0285		INT 0215
CAS DE2F+0228	TEST ALPHA	INT 0216
HTR DE2F+0127		INT 0217
TRA DE2F+0130		INT 0218
TRA DE2F+0144		INT 0219
LDO DE2F+0289	4H	INT 0220
FMP DE2F+0233		INT 0221
CHS		INT 0222
FAD *	X-4H	INT 0223
STO *		INT 0224
LXA DE2F+0229,1		INT 0225
CLA *,1		INT 0226
STO *,1		INT 0227
CLA *,1		INT 0228
STO *,1		INT 0229
CLA *,1		INT 0230
STO *,1		INT 0231
TIX DE2F+0136,1,1	END OF LOOP	INT 0232
TRA DE2F+0161	JUMP TO SET ALPHA	INT 0233
CLA DE2F+0233		INT 0234
CHS		INT 0235
FAD *	X-H	INT 0236
STO *		INT 0237
LXA DE2F+0229,1		INT 0238
CLA *,1		INT 0239
STO *,1		INT 0240
CLA *,1	D TO YI PRIME	INT 0241
STO *,1	TS2 TO YI	INT 0242
CLA *,1		INT 0243
STO *,1		INT 0244
TIX DE2F+0149,1,1		INT 0245
LXA DE2F+0229,1		INT 0246
LDO DE2F+0291	CONVERT DP YI	INT 0247
FMP *,1	TO YI, QI	INT 0248
STO *,1		INT 0249
TIX DE2F+0157,1,1	END OF LOOP	INT 0250
STZ DE2F+0228		INT 0251
STZ DE2F+0231		INT 0252
TRA DE2F+0107		INT 0253
CLA *		INT 0254
SSP		INT 0255
CAS DE2F+0237	TEST H WITH HMAX	INT 0256
TRA DE2F+0095	GO TO SHIFT DS	INT 0257
TRA DE2F+0213		INT 0258
TRA DE2F+0213		INT 0259
CLA *		INT 0260
FDP DE2F+0239	H DIVIDED BY BETA	INT 0261
STO *		INT 0262
TSX 0,4	EVALUATE DERIVATIVES	INT 0263
TRA DE2F+0156		INT 0264
LXA DE2F+0229,1		INT 0265

CLA *,1		INT 0266
FDP DE2F+0291	Q1 DIVIDED BY -3	INT 0267
CLA *,1		INT 0268
STQ COMMON+000		INT 0269
FAD COMMON+000		INT 0270
STO *,1		INT 0271
STQ *,1		INT 0272
TIX DE2F+0176,1,1		INT 0273
TRA DE2F+0009		INT 0274
STO *,1	DUMMY ORDER	INT 0275
LDQ DE2F+0229		INT 0276
MPY DE2F+0228		INT 0277
STQ COMMON+000		INT 0278
CLA COMMON+000		INT 0279
ADD DE2F+0185		INT 0280
STO DE2F+0194		INT 0281
LXA DE2F+0229,1		INT 0282
CLA *,1		INT 0283
STO 0	STORE DERIVATIVES	INT 0284
TIX DE2F+0193,1,1		INT 0285
CLA DE2F+0228		INT 0286
TNZ DE2F+0204		INT 0287
LXA DE2F+0229,1		INT 0288
CLA *,1		INT 0289
STO *,1		INT 0290
CLA *,1		INT 0291
STO *,1		INT 0292
TIX DE2F+0199,1,1		INT 0293
CLA DE2F+0228		INT 0294
ADD DE2F+0284	ALPHA PLUS ONE	INT 0295
STO DE2F+0228		INT 0296
TSX DE2F+0454,4	RUNGA-KUTTA ENTRY	INT 0297
STZ COMMON+000		INT 0298
CLA COMMON+000	ZERO TO ACC	INT 0299
TRA DE2F+0111		INT 0300
STZ DE2F+0231		INT 0301
TRA DE2F+0095		INT 0302
CLA DE2F+0231		INT 0303
ADD DE2F+0284		INT 0304
STO DE2F+0231	R+1	INT 0305
CAS DE2F+0292		INT 0306
HTR DE2F+0217		INT 0307
TRA DE2F+0170		INT 0308
TRA DE2F+0095		INT 0309
DEC 2.291666667		INT 0310
DEC -2.458333333		INT 0311
DEC 1.541666667		INT 0312
DEC -3.75E-1		INT 0313
DEC 3.75E-1		INT 0314
DEC 7.916666667E-1		INT 0315
DEC -2.083333333E-1		INT 0316
DEC 4.166666667E-2		INT 0317
BSS 4		INT 0318

BSS 8		INT 0319
BSS 3		INT 0320
HTR	HI UPPER C	INT 0321
HTR	YI UPPER P	INT 0322
HTR	DIVISOR	INT 0323
CLA DE2F+0243	ENTRY	INT 0324
SSP		INT 0325
CAS DE2F+0236	TEST FOR DIVISOR	INT 0326
TRA DE2F+0254		INT 0327
TRA DE2F+0254		INT 0328
CLA DE2F+0236		INT 0329
STO DE2F+0245		INT 0330
TRA DE2F+0255		INT 0331
STO DE2F+0245		INT 0332
CLA DE2F+0244		INT 0333
FSB DE2F+0243		INT 0334
SSP		INT 0335
FDP DE2F+0245		INT 0336
STQ COMMON+000		INT 0337
CLA DE2F+0230		INT 0338
CAS COMMON+000		INT 0339
TRA 1,2		INT 0340
TRA 1,2		INT 0341
CLA COMMON+000		INT 0342
STO DE2F+0230		INT 0343
TRA 1,2		INT 0344
HTR	A1	INT 0345
HTR	A2	INT 0346
UFA DE2F+0267	ENTRY.	INT 0347
STO DE2F+0267		INT 0348
STO COMMON+000	SPECIAL	INT 0349
CLA COMMON+000	FLOATING	INT 0350
UFA DE2F+0268	ADDITION	INT 0351
FAD DE2F+0267	SUBROUTINE	INT 0352
TRA 1,2		INT 0353
TRA DE2F+0005	SWITCH 1, A LEG	INT 0354
TRA DE2F+0207	SWITCH 1, B LEG	INT 0355
TRA DE2F+0085	SWITCH 2, A LEG	INT 0356
TRA DE2F+0095	SWITCH 2, B LEG	INT 0357
TRA DE2F+0115	DECREASE H SWITCH, TEST LEG	INT 0358
TRA DE2F+0121	DECREASE H SWITCH, NO TEST LEG	INT 0359
TRA DE2F+0164	INCREASE H SWITCH, TEST LEG	INT 0360
TRA DE2F+0213	INCREASE H SWITCH, NO TEST LEG	INT 0361
DEC 1		INT 0362
DEC 3		INT 0363
DEC 1.		INT 0364
DEC 5E-1	ONE HALF	INT 0365
DEC 100.		INT 0366
DEC 4.		INT 0367
DEC 14.		INT 0368
DEC -3.		INT 0369
DEC 3	SPECIAL TEST NO. FOR R	INT 0370
SXD DE2F+0241,2		INT 0371

SXD DE2F+0242,4
 CLA DE2F+0276
 STO DE2F+0004
 CLA DE2F+0278
 STO DE2F+0084
 CLA DE2F+0280
 STO DE2F+0089
 CLA DE2F+0282
 STO DE2F+0092
 CLA 2,4
 PAX 0,2
 TRA DE2F+0308,2
 TRA DE2F+0346
 TRA DE2F+0343
 LDQ 3,4
 FMP DE2F+0290
 STO DE2F+0234
 CLA 8,4
 TNZ DE2F+0314
 CLA DE2F+0287
 STO DE2F+0239
 CLA 4,4
 TNZ DE2F+0318
 CLA DE2F+0288
 STO DE2F+0235
 CLA DE2F+0234
 FDP DE2F+0235
 STQ DE2F+0235
 CLA 5,4
 TNZ DE2F+0325
 CLA DE2F+0286
 STO DE2F+0236
 CLA 6,4
 TNZ DE2F+0331
 CLA DE2F+0283
 STO DE2F+0092
 TRA DE2F+0335
 STO DE2F+0237
 LDQ DE2F+0239
 FMP DE2F+0237
 STO DE2F+0237
 CLA 7,4
 TNZ DE2F+0340
 CLA DE2F+0281
 STO DE2F+0089
 TRA DE2F+0348
 FDP DE2F+0239
 STO DE2F+0238
 TRA DE2F+0348
 CLA DE2F+0277
 STO DE2F+0004
 TRA DE2F+0348
 CLA DE2F+0279

SET
 SWITCHES
 TO
 NORMAL
 POSITIONS

SELECT OPTION
 OPTION 2
 OPTION 1
 OPTION 0
 14E (UPPER)

TEST BETA
 STORE BETA

STORE M

STORE 14E (LOWER)

TEST A

STORE A

TEST H MAX

STORE BETA (H MAX)

TEST H MIN

INT 0372
 INT 0373
 INT 0374
 INT 0375
 INT 0376
 INT 0377
 INT 0378
 INT 0379
 INT 0380
 INT 0381
 INT 0382
 INT 0383
 INT 0384
 INT 0385
 INT 0386
 INT 0387
 INT 0388
 INT 0389
 INT 0390
 INT 0391
 INT 0392
 INT 0393
 INT 0394
 INT 0395
 INT 0396
 INT 0397
 INT 0398
 INT 0399
 INT 0400
 INT 0401
 INT 0402
 INT 0403
 INT 0404
 INT 0405
 INT 0406
 INT 0407
 INT 0408
 INT 0409
 INT 0410
 INT 0411
 INT 0412
 INT 0413
 INT 0414
 INT 0415
 INT 0416
 INT 0417
 INT 0418
 INT 0419
 INT 0420
 INT 0421
 INT 0422
 INT 0423
 INT 0424

STO DE2F+0084
 CLA 1,4
 STO DE2F+0232
 STO DE2F+0352
 TSX DE2F+0453,4
 PZE
 STZ DE2F+0228
 STZ DE2F+0231
 CLA DE2F+0232
 STA DE2F+0357
 CLA 0
 STO DE2F+0229
 CLA DE2F+0232
 ARS 18
 STA DE2F+0049
 STA DE2F+0106
 STA DE2F+0173
 CLA DE2F+0232
 ADD DE2F+0284
 STA DE2F+0042
 STA DE2F+0044
 STA DE2F+0133
 STA DE2F+0134
 STA DE2F+0146
 STA DE2F+0147
 ADD DE2F+0284
 STA DE2F+0037
 STA DE2F+0043
 STA DE2F+0070
 STA DE2F+0115
 STA DE2F+0121
 STA DE2F+0124
 STA DE2F+0164
 STA DE2F+0170
 STA DE2F+0172
 ADD DE2F+0284
 ADD DE2F+0229
 STA DE2F+0010
 STA DE2F+0021
 STA DE2F+0047
 STA DE2F+0073
 STA DE2F+0078
 STA DE2F+0139
 STA DE2F+0152
 STA DE2F+0178
 STA DE2F+0181
 STA DE2F+0199
 ADD DE2F+0229
 STA DE2F+0012
 STA DE2F+0067
 STA DE2F+0137
 STA DE2F+0150
 STA DE2F+0193

SETUP RK SUBROUTINE
 PARAMETER WORD
 SET ALPHA TO ZERO
 R 0

STORE N

SETUP
 DERIVATIVE
 EVALUATIONS

T=1

T=2

T=3 EQUALS 0
 D=N

D+2N

INT 0425
 INT 0426
 INT 0427
 INT 0428
 INT 0429
 INT 0430
 INT 0431
 INT 0432
 INT 0433
 INT 0434
 INT 0435
 INT 0436
 INT 0437
 INT 0438
 INT 0439
 INT 0440
 INT 0441
 INT 0442
 INT 0443
 INT 0444
 INT 0445
 INT 0446
 INT 0447
 INT 0448
 INT 0449
 INT 0450
 INT 0451
 INT 0452
 INT 0453
 INT 0454
 INT 0455
 INT 0456
 INT 0457
 INT 0458
 INT 0459
 INT 0460
 INT 0461
 INT 0462
 INT 0463
 INT 0464
 INT 0465
 INT 0466
 INT 0467
 INT 0468
 INT 0469
 INT 0470
 INT 0471
 INT 0472
 INT 0473
 INT 0474
 INT 0475
 INT 0476
 INT 0477

RM 61TMP-32

ADD DE2F+0229	D+3N	INT 0478
STA DE2F+0014		INT 0479
STA DE2F+0023		INT 0480
STA DE2F+0056		INT 0481
STA DE2F+0074		INT 0482
STA DE2F+0141		INT 0483
STA DE2F+0154		INT 0484
STA DE2F+0158		INT 0485
STA DE2F+0159		INT 0486
STA DE2F+0176		INT 0487
STA DE2F+0182		INT 0488
STA DE2F+0201		INT 0489
ADD DE2F+0229	D+4N	INT 0490
STA DE2F+0040		INT 0491
STA DE2F+0046		INT 0492
STA DE2F+0080		INT 0493
ADD DE2F+0229	D+5N	INT 0494
STA DE2F+0011		INT 0495
STA DE2F+0054		INT 0496
STA DE2F+0151		INT 0497
ADD DE2F+0229	D+6N	INT 0498
STA DE2F+0015		INT 0499
STA DE2F+0153		INT 0500
ADD DE2F+0229	D+7N	INT 0501
STA DE2F+0138		INT 0502
STA DE2F+0200		INT 0503
ADD DE2F+0229	D+8N	INT 0504
STA DE2F+0140		INT 0505
STA DE2F+0202		INT 0506
ADD DE2F+0229	D+9N	INT 0507
STA DE2F+0019		INT 0508
STA DE2F+0097		INT 0509
STA DE2F+0136		INT 0510
STA DE2F+0185		INT 0511
ADD DE2F+0229	D+10N	INT 0512
STA DE2F+0026		INT 0513
STA DE2F+0052		INT 0514
STA DE2F+0096		INT 0515
STA DE2F+0099		INT 0516
ADD DE2F+0229	D+11N	INT 0517
STA DE2F+0030		INT 0518
STA DE2F+0059		INT 0519
STA DE2F+0098		INT 0520
STA DE2F+0101		INT 0521
ADD DE2F+0229	D+12N	INT 0522
STA DE2F+0013		INT 0523
STA DE2F+0034		INT 0524
STA DE2F+0063		INT 0525
STA DE2F+0100		INT 0526
STA DE2F+0149		INT 0527
LXD DE2F+0241,2		INT 0528
LXD DE2F+0242,4		INT 0529
TRA 9,4	EXIT	INT 0530

TRA DE2F+0562
 SXD DE2F+0553,1
 SXD DE2F+0554,2
 SXD DE2F+0555,4
 CLA
 FDP DE2F+0520
 STQ DE2F+0525
 STQ DE2F+0541
 LXA DE2F+0551,2
 LXA DE2F+0556,1
 LDQ ,1
 FMP DE2F+0551,2
 STO ,1
 LDQ
 FMP ,1
 FAD ,1
 STO ,1
 FDP DE2F+0552,2
 STQ COMMON+000
 LDQ ,1
 FMP DE2F+0553,2
 FAD COMMON+000
 STO COMMON+000
 TZE DE2F+0492
 ARS 27
 STO COMMON+001
 CLA ,1
 TZE DE2F+0492
 SSP
 ARS 27
 SBM COMMON+001
 TMI DE2F+0492
 TZE DE2F+0492
 STA DE2F+0489
 STA DE2F+0490
 CLA COMMON+000
 ARS
 ALS
 STO COMMON+000
 CLA ,1
 FAD COMMON+000
 STO ,1
 TRA DE2F+0558,2
 LDQ ,1
 FMP DE2F+0556,2
 STO ,1
 LDQ ,1
 FMP DE2F+0555,2
 FAD ,1
 STO ,1
 LDQ COMMON+000
 FMP DE2F+0554,2
 FAD ,1

TO SETUP REGION
 SAVE INDEX REGISTERS
 FROM
 MAIN PROGRAM

CALCULATE
 H DIVIDED BY 2

SET PARAMETER INDEX
 SET N INDEX.

CALCULATE NEW
 VALUE OF P

CALCULATE NEW
 VALUE OF R

TEST VALUES

TO

DETERMINE

IF SHIFTING

IS NECESSARY

CALCULATE NEW
 VALUE OF Z

CALCULATE

NEW VALUE

OF Q

INT 0531
 INT 0532
 INT 0533
 INT 0534
 INT 0535
 INT 0536
 INT 0537
 INT 0538
 INT 0539
 INT 0540
 INT 0541
 INT 0542
 INT 0543
 INT 0544
 INT 0545
 INT 0546
 INT 0547
 INT 0548
 INT 0549
 INT 0550
 INT 0551
 INT 0552
 INT 0553
 INT 0554
 INT 0555
 INT 0556
 INT 0557
 INT 0558
 INT 0559
 INT 0560
 INT 0561
 INT 0562
 INT 0563
 INT 0564
 INT 0565
 INT 0566
 INT 0567
 INT 0568
 INT 0569
 INT 0570
 INT 0571
 INT 0572
 INT 0573
 INT 0574
 INT 0575
 INT 0576
 INT 0577
 INT 0578
 INT 0579
 INT 0580
 INT 0581
 INT 0582
 INT 0583

STO ,1
TIX DE2F+0463,1,1
CLA
FAD DE2F+0557,2
STO
SXD DE2F+0557,2
TSX ,4
LXD DE2F+0557,2
TIX DE2F+0462,2,8
LXD DE2F+0553,1
LXD DE2F+0554,2
LXD DE2F+0555,4
TRA 1,4
DEC 0
DEC 2.
DEC -1.
DEC 3.
DEC -.5
DEC 1.
DEC
TRA DE2F+0496
DEC 0
DEC 2.
DEC -.5
DEC -1.
DEC .5
DEC -3.
DEC 0
TRA DE2F+0558
DEC -.5
DEC 1.
DEC 0
DEC -1.
DEC 0
DEC 1.
DEC
TRA DE2F+0496
DEC 2.
DEC 6.
DEC 1.
DEC 3.
DEC -.5
DEC -3.
DEC 0
TRA DE2F+0496
PZE 32.,
PZE 1.,
BSS 5
CLA ,1
FDP DE2F+0556,2
STO ,1
TRA DE2F+0499
SXD DE2F+0553,1

TEST N

INCREASE X

FIND DERIVITIVES

TEST PASS NO.

RESTORE

INDEX

REGISTERS

SAVE INDEX REGISTERS

INT 0584
INT 0585
INT 0586
INT 0587
INT 0588
INT 0589
INT 0590
INT 0591
INT 0592
INT 0593
INT 0594
INT 0595
INT 0596
INT 0597
INT 0598
INT 0599
INT 0600
INT 0601
INT 0602
INT 0603
INT 0604
INT 0605
INT 0606
INT 0607
INT 0608
INT 0609
INT 0610
INT 0611
INT 0612
INT 0613
INT 0614
INT 0615
INT 0616
INT 0617
INT 0618
INT 0619
INT 0620
INT 0621
INT 0622
INT 0623
INT 0624
INT 0625
INT 0626
INT 0627
INT 0628
INT 0629
INT 0630
INT 0631
INT 0632
INT 0633
INT 0634
INT 0635
INT 0636

```

SKD DE2F+0555.4
CLA 1.4
STA DE2F+0553
STA DE2F+0570
ARS 18
STA DE2F+0512
STA DE2F+0603
CLA
STO DE2F+0556
CLA DE2F+0553
ADD DE2F+0552
STA DE2F+0508
STA DE2F+0510
ADD DE2F+0552
STA DE2F+0466
STA DE2F+0457
ADD DE2F+0552
ADD DE2F+0556
STA DE2F+0479
STA DE2F+0492
STA DE2F+0494
ADD DE2F+0556
STA DE2F+0467
ADD DE2F+0556
STA DE2F+0605
STA DE2F+0560
STA DE2F+0558
STA DE2F+0472
STA DE2F+0496
STA DE2F+0498
STA DE2F+0501
STA DE2F+0502
STA DE2F+0505
STA DE2F+0506
ADD DE2F+0556
STA DE2F+0463
STA DE2F+0465
STA DE2F+0468
STA DE2F+0469
STA DE2F+0499
TSX .4
LXA DE2F+0556.1
STZ .1
TIX DE2F+0605.1.1
LXD DE2F+0553.1
LXD DE2F+0555.4
TRA 2.4
COMMON BSS 2
R      END

```

FROM MAIN PROGRAM

```

SET ADDRESS OF
DERIVATIVE ROUTINE
STORE VALUE
OF N

```

```

STORE ADDRESS
OF X

```

```

STORE ADDRESS
OF H

```

```

STORE ADDRESS
OF Y

```

```

STORE ADDRESS
OF DERIVATIVE

```

```

STORE

```

```

ADDRESS

```

```

OF

```

```

Q

```

```

STORE ADDRESS

```

```

OF P

```

```

FIND INITIAL DERIVATIVES

```

```

SET ORIGINAL Q
TO ZERO
RESTORE INDEX
REGISTERS

```

```

INT 0637
INT 0638
INT 0639
INT 0640
INT 0641
INT 0642
INT 0643
INT 0644
INT 0645
INT 0646
INT 0647
INT 0648
INT 0649
INT 0650
INT 0651
INT 0652
INT 0653
INT 0654
INT 0655
INT 0656
INT 0657
INT 0658
INT 0659
INT 0660
INT 0661
INT 0662
INT 0663
INT 0664
INT 0665
INT 0666
INT 0667
INT 0668
INT 0669
INT 0670
INT 0671
INT 0672
INT 0673
INT 0674
INT 0675
INT 0676
INT 0677
INT 0678
INT 0679
INT 0680
INT 0681
INT 0682
INT 0683
INT 0684
INT 0685

```

H. SUBROUTINE RINDEX

This subroutine permits the calculation of the refractive index, its spatial derivatives, and the absorption coefficient, all as functions of the local atmosphere and its state of ionization. This subroutine also permits the output of "debugging data" which is called the R vector whose components are defined in Table 3. See subroutine OUTPUT for additional information.

$$\begin{aligned}
R(1) &= X \\
R(2) &= PXR = \frac{\partial X}{\partial r} \\
R(3) &= PXTHET = \frac{\partial X}{\partial \theta} \\
R(4) &= PXPFI = \frac{\partial X}{\partial \varphi} \\
R(5) &= Y \\
R(6) &= YR = Y_r \\
R(7) &= YTHETA = Y_\theta \\
R(8) &= YPFI = Y_\varphi \\
R(9) &= PYR = \partial Y / \partial r \\
R(10) &= PYTHET = \partial Y / \partial \theta \\
R(11) &= PYPFI = \partial Y / \partial \varphi \\
R(12) &= Z \\
R(13) &= PZR = \partial Z / \partial r \\
R(14) &= PZTHET = \partial Z / \partial \theta \\
R(15) &= PZPFI = \partial Z / \partial \varphi \\
R(16) &= COSPSI = \cos \psi \\
R(17) &= SINPSI = \sin \psi \\
R(18) &= YSI = Y \sin \psi \\
R(19) &= YCI = Y \cos \psi \\
R(20) &= TE1 = (1 - X)^2 - Z^2
\end{aligned}$$

Table 3. Nomenclature Describing the R Vector.
(Page 1 of 4)

$$\begin{aligned}
 R(21) &= TE2 = (YSI)^4 + 4TE1(YCI)^2 \\
 R(22) &= TE3 = 8(YCI)^2 Z(1 - X) \\
 R(23) &= R2S = R_S^2 \\
 R(24) &= R1S = R_S \\
 R(25) &= THET2S = 2\theta_S \\
 R(26) &= THET1s = \theta_S \\
 R(27) &= S1 = S_1 \\
 R(28) &= S2 = S_2 \\
 R(29) &= D1 = d_1 \\
 R(30) &= D2 = d_2 \\
 R(31) &= TE4 = d_1^2 + d_2^2 \\
 R(32) &= TE5 = 2X[Zd_1 + (1 - X)d_2]/TE4 \\
 R(33) &= TE6 = 1 - [2X(1 - X)d_1 - Zd_2]/TE4 \\
 R(34) &= R2M = R_M^2 \\
 R(35) &= R1M = R_M \\
 R(36) &= THET2M = 2\theta_M \\
 R(37) &= THET1M = \theta_M \\
 R(38) &= AM1 = M_1 \\
 R(39) &= AM2 = M_2 \\
 R(40) &= TE7 = M_1d_1 - M_2d_2
 \end{aligned}$$

Table 3. Nomenclature Describing the R Vector.
(Page 2 of 4)

$$R(41) = TE8 = M_1 d_2 + M_2 d_1$$

$$R(42) = AO = a_o$$

$$R(43) = BO = b_o$$

$$R(44) = TE9 = S_1^2 + S_2^2$$

$$R(45) = A4 = a_4$$

$$R(46) = B4 = b_4$$

$$R(47) = A5 = a_5$$

$$R(48) = B5 = b_5$$

$$R(49) = PNPX = \partial\mu/\partial X$$

$$R(50) = TE10 = (\sin\psi)^2 (YSI)^2 + 2TE1 \cos^2\psi$$

$$R(51) = TE11 = 4Z(1 - X)\cos^2\psi$$

$$R(52) = A6 = a_6$$

$$R(53) = B6 = b_6$$

$$R(54) = PNPY = \partial\mu/\partial Y$$

$$R(55) = A7 = a_7$$

$$R(56) = B7 = b_7$$

$$R(57) = PNPZ = \partial\mu/\partial Z$$

$$R(58) = A8 = a_8$$

$$R(59) = B8 = b_8$$

$$R(60) = TE13 = 1/(2 \cos^2\theta + \frac{1}{2} \sin^2\theta)$$

Table 3. Nomenclature Describing the R Vector.
(Page 3 of 4)

$$R(61) = A1 = a_1$$

$$R(62) = B1 = b_1$$

$$R(63) = A2 = a_2$$

$$R(64) = B2 = b_2$$

$$R(65) = TE12 = (W_1)^2 YSI$$

$$R(66) = PPSIPT = \partial\psi/\partial\theta$$

$$R(67) = PPSIPR = \partial\psi/\partial r$$

$$R(68) = PPSIPP = \partial\psi/\partial\phi$$

$$R(69) = TE14 = \sqrt{\sigma_r^2 + \sigma_\theta^2 + \sigma_\phi^2}$$

Table 3. Nomenclature Describing the R Vector.
(Page 4 of 4)

C INDEX WITH MAGNETIC FIELD DECEMBER 22, 1960 IBM-7090

1 SUBROUTINE RINDEX

COMMON RECORD-V,M,N

DIMENSION RECORD(12),VC(110),VC(500),RC(750)

PI = 3.1415927

CALL ELECTX CX,PXR,PXTHET,PXPHI

RC(2) = X

RC(3) = PXR

RC(3) = PXTHET

RC(4) = PXPHI

SIG = VC(70)

CALL MAGY CV,YR,VTHETA,VPHI,P,F,PVTHET,PVPHI

RC(5) = Y

RC(6) = VR

RC(7) = VTHETA

RC(8) = VPHI

RC(9) = PVR

RC(10) = PVTHET

RC(11) = PVPHI

CALL COLFRZC Z,PZR,PZTHET,PZPHI

RC(12) = Z

RC(13) = PZR

RC(14) = PZTHET

RC(15) = PZPHI

VC(60) = X

VC(80) = Y

VC(80) = Z

VC(51) = VC(70)*VR + VC(80)*VTHETA + VC(90)*VPHI / CV*VC(100)

RC(16) = COSPSI

SINPSI = SORTFC 1.0 - COSPSI*0.05PSI

RC(17) = SINPSI

VSI = V*SINPSI

RC(18) = VSI

VCI = V*COSPSI

RC(19) = VCI

IF(Z-1.0E-18) 9,10,10

T = 0.0

G TO 9

10 T1 = Z*Z

11 TE1 = CC(1.0 - XD*(1.0 - T)) -

RC(20) = TE1

TEE = VSI*VSI

TEA = VCI*VCI

12 TE2 = TEE*TEE + 4.0*TEA*TE1

RC(21) = TE2

TEB = Z*(1.0 - XD)

13 TE3 = 8.0*TEA*TEB

RC(22) = TE3

14 RA5 = TE2*IE2 + TE3*IE3

RC(23) = SORTFC(RA5)

RC(23) = R2

15 R15 = SORTFC(RA25)

16 THET25 = QATANCTE2, -TE3

IF(CPI - THET25) 200,200,17

200 R15 = -R15

17 THET15 = 0.5*THET25

RC(24) = R15

RC(25) = THET25

```

308 RC265 = THET15
309 S1 = R15+ COSF(THET15)*SIGN
310 RC275 = S1
311 S2 = R15+ SINFC(THET15)*SIGN
312 RC285 = S2
313 O1 = 2.0*(1.0-X-Z*Z) - TEE + S1
314 RC295 = O1
315 O2 = S2 + 2.0*(1.0-X*Z)
316 RC305 = O2
317 TE9 = O1*O1 + O2*O2
318 PC315 = TE9
319 TEE = 2.0*(X*Z*O1 + O1.0-X*O2*O2)/TE9
320 RC325 = TEE
321 TEC = O1.0-X*O1
322 TEE = 1.0-X*(TEC-Z*O2)/TE9
323 RC335 = TEC
324 R+M = TEE*TE9 + TEE*TEE
325 R2M = SORTFCR4ND
326 RC3+5 = R2M
327 R1M = SORTFCR2M2
328 THET2M = CATAN(C1E8, TEE5)
329 IF(CPI - THET2M)*2.0,1.2
330 R1M = - R1M
331 THET1M = 0.5 * THET2M
332 RC355 = R1M
333 FC355 = THET1M
334 RC375 = THET1M
335 R+M = R1M + COSF(THET1M)
336 RC385 = R+M
337 R+M2 = R1M *SINF(THET1M)
338 FC385 = R+M2
339 UO15 = R+1
340 W25 = T+M2
341 TE9 = R+1*O1 - R+M2*O2
342 RC405 = TE9
343 TEE = R+1*O2 + R+M2*O1
344 RC415 = TEE
345 TE9 = TEE*TE9
346 R+ = TE9/C1ED + TE9
347 R+*25 = R+
348 R+ = TEE*AO/TE9
349 RC435 = R+
350 TEE = S1*S1 +S2*S2
351 RC445 = TEE9
352 O5 = 2.0*TE9
353 TEE = O1.0-X*O1 - Z*O2
354 R+ = 1.0 + O5*TE9/TE9
355 RC455 = R+
356 TEM = O2*S1 + O1.0-X*O2*O2/TE9
357 R+ = Z + TEM*O5
358 RC465 = R+
359 R+ = 2.0*(X*(TEC-Z*O2)/TE9
360 RC475 = R+
361 R5 = 2.0*(X*O2*O1 + O1.0-X*O2*O2)/TE9
362 TEU = 2.0*(X-1.0 - R+*O5 + R+*R5
363 TEK = Z + R+*O5 + R+*R5

```

$$42 \quad \text{PNPX} = \text{A0*TEJ} + \text{B0*TEK}$$

$$\text{RC49} = \text{PNPX}$$

$$\text{Q5} = \text{SINPSI**2}$$

$$\text{Q6} = \text{COSPSI**2} * 2.0$$

$$43 \quad \text{TE10} = \text{TEE*Q5} + \text{Q6*TE1}$$

$$\text{RC50} = \text{TE10}$$

$$\text{TE82} = 2.0*TE8$$

$$44 \quad \text{TE11} = \text{TEE2*Q6}$$

$$\text{RC51} = \text{TE11}$$

$$45 \quad \text{A6} = (\text{TE10*S1} - \text{TE11*S2}) / \text{TE9}$$

$$\text{RC52} = \text{A6}$$

$$46 \quad \text{B6} = (\text{TE10*S2} + \text{TE11*S1}) / \text{TE9}$$

$$\text{RC53} = \text{B6}$$

$$\text{Q1} = \text{B0*A5} + \text{B5*A0}$$

$$\text{Q2} = \text{A0*A5} - \text{B0*B5}$$

$$\text{Q3} = \text{A6} - \text{SINPSI*SINPSI}$$

$$\text{Q4} = 32*Q2$$

$$47 \quad \text{PNPV} = \text{V*Q4} - \text{Q1*B6*V}$$

$$\text{RC54} = \text{PNPV}$$

$$\text{Q5} = 2.0*TEA$$

$$48 \quad \text{A7} = (\text{Q2,0-X}) + \text{Q5*TEG/TE9}$$

$$\text{RC55} = \text{A7}$$

$$49 \quad \text{B7} = 2.0*Q2 + \text{TEA*TEA2}$$

$$\text{RC56} = \text{B7}$$

$$\text{Q1} = \text{X} - \text{B5*A7} + \text{B5*B7}$$

$$\text{Q2} = \text{A5*B7} + \text{B5*A7}$$

$$50 \quad \text{PNPZ} = \text{B0*Q1} - \text{A0*Q2}$$

$$\text{RC57} = \text{PNPZ}$$

$$\text{Q3} = 1.0 - \text{X*X}$$

$$51 \quad \text{Q4} = \text{S1*Q3} - \text{S2*Q1,0-X}$$

$$\text{Q5} = 2.0*Q1,0 + \text{TEA*Q4/TE9}$$

$$\text{RC58} = \text{A}$$

$$52 \quad \text{Q6} = \text{S2*Q3} + \text{Q5*Q1,0*Y}$$

$$\text{Q7} = 2*(2.0*X) + \text{Q5*Q6/TE9}$$

$$\text{RC59} = \text{B8}$$

$$\text{Q1} = 2.0*X*X - \text{Q5*A8} + \text{B5*B8}$$

$$\text{Q2} = 2.0*Q6*Q4*X2 - \text{Q6*Q5*A8} + \text{Q6*Q5*B82}$$

$$\text{Q3} = X*2 + \text{A5*B8} + \text{B5*A8}$$

$$53 \quad \text{WC112} = -\text{Q0*Q1} + \text{B0*Q22/RC3}$$

$$\text{Q1} = \text{COSFCV(C32)}$$

$$\text{Q2} = \text{SINFCV(C32)}$$

$$54 \quad \text{Q3} = 2.0*Q1*Q1 + 0.5*Q2*Q2$$

$$\text{TE13} = 1.0/Q1$$

$$\text{RC60} = \text{TE13}$$

$$55 \quad \text{IF } \text{COSFCV(S12)} = 1, \text{DE} = 162.100, 100, 60$$

$$60 \quad \text{A1} = 2.0* \text{TE1} - \text{TEE}$$

$$\text{RC612} = \text{A1}$$

$$61 \quad \text{B1} = 4.0* \text{TE8}$$

$$\text{RC622} = \text{B1}$$

$$62 \quad \text{A2} = \text{C1,0} + (\text{A1*S1} - \text{B1*S2}) / \text{TE9}$$

$$\text{RC632} = \text{A2}$$

$$63 \quad \text{B2} = \text{C1*S1} + \text{A1*S2} / \text{TE9}$$

$$\text{RC642} = \text{B2}$$

$$\text{TAA} = \text{A2*A3}$$

$$\text{TBB} = \text{B2*B3}$$

$$\text{TAC} = \text{A2*B3}$$

$$\text{TAD} = \text{B2*A3}$$

$$\text{TAE} = \text{A0*CTAA-TAB}$$

```

TAF = 80*CTAC+TAD
64 Q1 = VCI*VSI
   WC10 = - Q1*CTAE-TAFD
   TAG = WC10*WC10
65 TE12 = TAG*VSI
   RC65 = TE12
   X1 = UC70*VCI
   X2 = WC10*VR
   X3 = X1 - X2
66 WC40 = WC100*X3/TE12
   X1 = UC80*VCI
   X2 = WC10*VTHETA
   X3 = X1 - X2
67 WC50 = WC100*X3/TE12
   X1 = UC90*VCI
   X2 = WC10*VPHI
   X3 = X1 - X2
68 WC60 = WC100*X3/TE12
   X1 = UC70*VTHETA - VC90*VR
   X2 = WC10*VSI
   X3 = X1/ X2
69 PPSIPT = X3*TE13
   RC60 = PPSIPT
70 PPSIPK = 0.0
71 PPSIPP = 0.0
72 WC70 = FAPX*PXR + PNEU*PVR + INFZ*PZR + WC100*PPSIPP
73 WC80 = PIPX*PXTHEI + PNPV*PYTHET + PNPZ*PZTHET + WC100*PPSIPT
74 WC90 = FNPX*PXPPI + FNPV*PYPI + PNPZ*PZPI + WC100*PPSIPP
75 GO TO 104
100 MC40 = 0.0
101 MC50 = 0.0
102 MC60 = 0.0
103 PPSIPT = TE13
   GO TO 70
104 X1 = UC70*UC70
   X2 = UC80*UC80
   X3 = UC90*UC90
   TE14 = SORTFC X1 + X2 + X3
   RC67 = PPSIPP
   RC68 = PPSIPP
   RC69 = TE14
105 UC70 = UC70*WC10/TE14
106 UC80 = UC80*WC10/TE14
107 UC90 = UC90*WC10/TE14
125 IF (WC10 - WC60) 127, 108, 108
126 IF (WC40 - WC60) 108, 127, 127
131 IF (WC70 - WTST) 108, 132, 132
132 WTST = WTST + WC60
127 WRITE OUTPUT TAPE 6, 128, (WC10, 1=1, 60)
128 FORMAT(1H0, 1P0E14, 6)
   GO TO 108
129 IF (WC70 - 1.0) 130, 131, 131
130 WTST = WC60
   GO TO 127
108 RETURN
ENDC0, 1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0

```

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
1	32139 73674	RECORD	32531 77461	U	32549 77445	M	32438 77266
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STORAGE NOT USED BY PROGRAM

DEC OCT
1197 02558
10127 73673

LOCATIONS OF NAMES IN TRANSFER VECTOR

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
1	00000	2	00002	3	00004	4	00006
5	00007	6	00009	7	00011	8	00013
9	00015	10	00017	11	00019	12	00021
13	00023	14	00025	15	00027	16	00029
17	00031	18	00033	19	00035	20	00037
21	00039	22	00041	23	00043	24	00045
25	00047	26	00049	27	00051	28	00053
29	00055	30	00057	31	00059	32	00061
33	00063	34	00065	35	00067	36	00069
37	00071	38	00073	39	00075	40	00077
41	00079	42	00081	43	00083	44	00085
45	00087	46	00089	47	00091	48	00093
49	00095	50	00097	51	00099	52	00101
53	00103	54	00105	55	00107	56	00109
57	00111	58	00113	59	00115	60	00117
61	00119	62	00121	63	00123	64	00125
65	00127	66	00129	67	00131	68	00133
69	00135	70	00137	71	00139	72	00141
73	00143	74	00145	75	00147	76	00149
77	00151	78	00153	79	00155	80	00157
81	00159	82	00161	83	00163	84	00165
85	00167	86	00169	87	00171	88	00173
89	00175	90	00177	91	00179	92	00181
93	00183	94	00185	95	00187	96	00189
97	00191	98	00193	99	00195	100	00197
101	00199	102	00201	103	00203	104	00205
105	00207	106	00209	107	00211	108	00213
109	00215	110	00217	111	00219	112	00221
113	00223	114	00225	115	00227	116	00229
117	00231	118	00233	119	00235	120	00237
121	00239	122	00241	123	00243	124	00245
125	00247	126	00249	127	00251	128	00253
129	00255	130	00257	131	00259	132	00261
133	00263	134	00265	135	00267	136	00269
137	00271	138	00273	139	00275	140	00277
141	00279	142	00281	143	00283	144	00285
145	00287	146	00289	147	00291	148	00293
149	00295	150	00297	151	00299	152	00301
153	00303	154	00305	155	00307	156	00309
157	00311	158	00313	159	00315	160	00317
161	00319	162	00321	163	00323	164	00325
165	00327	166	00329	167	00331	168	00333
169	00335	170	00337	171	00339	172	00341
173	00343	174	00345	175	00347	176	00349
177	00351	178	00353	179	00355	180	00357
181	00359	182	00361	183	00363	184	00365
185	00367	186	00369	187	00371	188	00373
189	00375	190	00377	19			

SIGN	1070 02056	SINPSI	1065 02055	T1	1068 02054	TAR	1067 02053
TAC	1065 02051	TAD	1064 02050	TRE	1063 02047	TAF	1062 02046
TE10	1060 02044	TE11	1059 02043	TE12	1058 02042	TE13	1057 02041
TE1	1055 02037	TE2	1054 02036	TE3	1053 02035	TE4	1052 02034
TE8	1050 02032	TE7	1049 02031	TE8	1048 02030	TE9	1047 02027
TE8	1045 02025	TE8	1044 02024	TEC	1043 02023	TEC	1042 02022
TE8	1040 02020	TEG	1039 02017	TEH	1038 02016	TEJ	1037 02015
THETM	1035 02013	THETIS	1034 02012	THET2M	1033 02011	THET2S	1032 02010
X1	1030 02006	X2	1029 02005	X3	1028 02004	X4	1027 02003
Y1	1025 02001	YR	1024 02000	Y	1023 01777	Y51	1022 01776
Z	1020 01774						

PLEASE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC	OCT	DEC	OCT	DEC	OCT
1010 01770	20	996 01744	30	999 01747	60

ENTER PRINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

QATAN	COS	SIN	QW	CSTH2	CFIL2
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00000	ELECTX	BCD	IELECTX	23A	BSS	00160	40A1	IZE	43A
00001	MAGV	BCD	IMAGV	00070 24A	TSX COLFRZ.4	00161		TPL	43A
00002	COLFRZ	BCD	COLFRZ	00071	TSX Z	00162	41A	CLA	32+3
00003	BCD	BCD	ISORT	00072	TSX PZR	00163		STO	T1
00004	QATAM	BCD	QATAM	00073	TSX PZTHET	00164	42A	TRA	44A
00005	COS	BCD	ICOS	00074	TSX PZPHI	00165	43A	LQZ	Z
00006	SIN	BCD	ISIN	00075 25A	CLA Z	00166		EMP	Z
00007	OM	BCD	IQM	00076	STO R-11	00167		STO	T1
00008	STW2	BCD	ICSTW2	00077 26A	CLA PZR	00170	44A	CLA	32+1
00009	CFIL	BCD	ICFIL	00100	STO R-12	00171		FSB	X
00010	STW2	BCD	ICSTW2	00101 27A	CLA PZTHET	00172		STO	12+1
00011	STW2	BCD	ICSTW2	00102	STO R-13	00173		LQZ	12+1
00012	STW2	BCD	ICSTW2	00103 28A	CLA PZPHI	00174		EMP	12+1
00013	STW2	BCD	ICSTW2	00104	STO R-14	00175		FSB	T1
00014	STW2	BCD	ICSTW2	00105 29A	CLA X	00176		STO	TE1
00015	STW2	BCD	ICSTW2	00106	STO W-5	00177	45A	CLA	TE1
00016	STW2	BCD	ICSTW2	00107 30A	CLA Y	00200		STO	R-12
00017	STW2	BCD	ICSTW2	00110	STO W-64	00201	46A	LQZ	Y51
00018	STW2	BCD	ICSTW2	00111 31A	CLA Z	00202		EMP	Y51
00019	STW2	BCD	ICSTW2	00112	STO W-63	00203		STO	TEE
00020	STW2	BCD	ICSTW2	00113 32A	LQZ Y	00204	47A	LQZ	Y51
00021	STW2	BCD	ICSTW2	00114	EMP W	00205		EMP	Y51
00022	STW2	BCD	ICSTW2	00115	STO 12+1	00206		STO	TEA
00023	STW2	BCD	ICSTW2	00116	Q V-8	00207	48A	LQZ	TE1
00024	STW2	BCD	ICSTW2	00117	EMP Y-HI	00210		EMP	32+4
00025	STW2	BCD	ICSTW2	00120	STO 12+2	00211		XCA	
00026	STW2	BCD	ICSTW2	00121	LQZ V-7	00212		F-P	TER
00027	STW2	BCD	ICSTW2	00122	EMP VTHETA	00213		STO	12+1
00028	STW2	BCD	ICSTW2	00123	STO 12+3	00214		LQZ	TEE
00029	STW2	BCD	ICSTW2	00124	LQZ V-6	00215		EMP	TEE
00030	STW2	BCD	ICSTW2	00125	EMP YR	00216		FAO	12+1
00031	STW2	BCD	ICSTW2	00126	FAO 12+3	00217		STO	TE2
00032	STW2	BCD	ICSTW2	00127	FAO 12+2	00220	49A	CLA	TE2
00033	STW2	BCD	ICSTW2	00130	FAO 12+1	00221		STO	R-20
00034	STW2	BCD	ICSTW2	00131	STQ COSPSI	00222	50A	CLA	32+1
00035	STW2	BCD	ICSTW2	00132 33A	CLA COSPSI	00223		FSB	X
00036	STW2	BCD	ICSTW2	00133	STO R-15	00224		STO	12+1
00037	STW2	BCD	ICSTW2	00134 34A	LQZ COSPSI	00225		LQZ	Z
00038	STW2	BCD	ICSTW2	00135	EMP COSPSI	00226		EMP	12+1
00039	STW2	BCD	ICSTW2	00136	CHS	00227		STO	TEB
00040	STW2	BCD	ICSTW2	00137	FAO 32+1	00230	51A	LQZ	TEB
00041	STW2	BCD	ICSTW2	00140	BSS	00231		EMP	32+5
00042	STW2	BCD	ICSTW2	00141	TSX SORT.4	00232		XCA	
00043	STW2	BCD	ICSTW2	00142 35A	STO SINPSI	00233		EMP	TEA
00044	STW2	BCD	ICSTW2	00143	CLA SINPSI	00234		STO	TE3
00045	STW2	BCD	ICSTW2	00144	STO R-16	00235	52A	CLA	TE3
00046	STW2	BCD	ICSTW2	00145	LQZ Y	00236		STO	R-21
00047	STW2	BCD	ICSTW2	00146	EMP SINPSI	00237	53A	LQZ	TE3
00048	STW2	BCD	ICSTW2	00147 37A	STO Y51	00240		EMP	TE3
00049	STW2	BCD	ICSTW2	00148	CLA Y51	00241		STO	12+1
00050	STW2	BCD	ICSTW2	00149	STO R-17	00242		LQZ	TE2
00051	STW2	BCD	ICSTW2	00150	LQZ V	00243		EMP	TE2
00052	STW2	BCD	ICSTW2	00151 38A	EMP COSPSI	00244		EMP	12+1
00053	STW2	BCD	ICSTW2	00152	STO Y51	00245		STO	R45
00054	STW2	BCD	ICSTW2	00153	CLA Y51	00246	54A	CLA	R45
00055	STW2	BCD	ICSTW2	00154 39A	STO R-18	00247		BSS	
00056	STW2	BCD	ICSTW2	00155	CLA Z	00248		TSX	SORT.4
00057	STW2	BCD	ICSTW2	00156 40A	FSB 32+2	00250		STO	R25
00058	STW2	BCD	ICSTW2	00157					

00251 55A	CLA R25	00337	F58 TEE	00431 79A	CLA TE6
00252	STO R-22	00340	FAD S1	00432	STO R-32
00253 55B	CLA R25	00341	STO D1	00433 80A	LQD TES
	BSS	00342 70A	CLA D1	00434	FMP TEC
00254	TSX SORT, 4	00343	STO R-28	00435	STO 12+1
00255	STO R15	00344 71A	CLA X	00436	LQD TE6
00256 57A	CLS TE3	00345	F58 32+7	00437	FMP TE6
00257	STO 12+1	00346	STO 12+1	00440	FAD 12+1
	BSS	00347	LQD 32+7	00441	STO R4M
00260 59A	TSX QATAN, 4	00350	FMP Z	00442 81A	CLA R4M
00261	TSX TE2	00351	XCA		BSS
00262	TSX 12+1	00352	FMP 12+1	00443	TSX SORT, 4
00263	STO THET2S	00353	FAD S2	00444	STO R2M
00264 59A	CLA PI	00354	STO D2	00445 82A	CLA R2M
00265	F58 THET2S	00355 72A	CLA D2	00446	STO R-33
00266 59A1	TZE 60A	00356	STO R-29	00447 83A	CLA R2M
00267	TPL 61A	00357 73A	LQD D2		BSS
00270 60A	CLS R15	00358	FMP D2	00450	TSX SORT, 4
00271	STO R15	00361	STO 12+1	00451	STO R1M
00272 61A	LQD 32+6	00362	LQD D1		BSS
00273	FMP THET2S	00363	FMP D1	00452 85A	TSX QATAN, 4
00274	STO THET1S	00364	FAD 12+1	00453	TSX TE6
00275 62A	CLA R15	00365	STO TE4	00454	TSX TES
00276	STO R-33	00366 74A	CLA TE4	00455	STO THET2M
00277 63A	CLA THET2S	00367	STO R-30	00456 86A	CLA PI
00300	STO R-24	00370 75A	CLA 32+1	00457	F58 THET2M
00301 64A	CLA THET1S	00371	F58 X	00460 86A1	TZE 87A
00302	STO R-25	00372	XCA	00461	TPL 88A
00303 65A	CLA THET1S	00373	FMP D2	00462 87A	CLS R1M
	BSS	00374	STO 12+2	00463	STO R1M
00304	TSX COS, 4	00375	LQD Z	00464 88A	LQD 32+6
00305	STO 12+1	00376	FMP D1	00465	FMP THET2M
00306	LQD SIGN	00377	FAD 12+2	00466	STO THET1M
00307	FMP R15	00400	STO 12+3	00467 89A	CLA R1M
00310	XCA	00401	LQD X	00470	STO R-34
00311	FMP 12+1	00402	FMP 32+7	00471 90A	CLA THET2M
00312	STO S1	00403	FDP TE4	00472	STO R-35
00313 66A	CLA S1	00404	FMP 12+3	00473 91A	CLA THET1M
00314	STO R-26	00405	STO TES	00474	STO R-36
00315 67A	CLA THET1S	00406 76A	CLA TES	00475 92A	CLA THET1M
	BSS	00407	STO R-31		BSS
00316	TSX SIN, 4	00410 77A	CLA 32+1	00476	TSX COS, 4
00317	STO 12+1	00411	F58 X	00477	STO 12+1
00320	LQD SIGN	00412	XCA	00500	LQD R1M
00321	FMP R15	00413	FMP D1	00501	FMP 12+1
00322	XCA	00414	STO TEC	00502	STO AM1
00323	FMP 12+1	00415 78A	LQD Z	00503 93A	CLA AM1
00324	STO S2	00416	FMP D2	00504	STO R-37
00325 68A	CLA S2	00417	CHS	00505 94A	CLA THET1M
	BSS	00420	FAD TEC		BSS
00326	STO R-27	00421	STO 12+1	00506	TSX SIN, 4
00327 69A	LQD Z	00422	LQD X	00507	STO 12+1
00330	FMP Z	00423	FMP 32+7	00510	LQD R1M
00331	CHS	00424	FDP TE4	00511	FMP 12+1
00332	F58 X	00425	FMP 12+1	00512	STO AM2
00333	STO 12+1	00426	CHS	00513 95A	CLA AM2
00334	LQD 32+7	00427	FAD 32+1	00514	STO R-38
00335	FMP 12+1	00430	STO TE6	00515 96A	CLA AM1
00336					

00516	00516	STO M	00610	XCR	00702	L00 R4
00517	97A	CLS RM2	00611	FMP S1	00703	FMP R5
00520	STO M-1	00612	F5B 12+1	00704	STO 12+2	
00521	98A	L00 RM2	00613	STO TEG	00705	L00 32+7
00522	FMP D2	00614	112A	CLR R5	00706	FMP X
00523	STO 12+1	00615	FMP TE9	00707	F5B 32+1	
00524	L00 RM1	00616	FMP TEG	00710	F5B 12+2	
00525	FMP D1	00617	FAD 32+1	00711	FAD 12+1	
00526	F5B 12+1	00619	STO R4	00712	STO TEJ	
00527	STO TE7	00621	113A	CLR R4	00713	L00 R4
00530	99A	CLR TE7	00622	STO R-44	00714	FMP R5
00531	STO R-39	00623	114A	CLR 32+1	00715	STO 12+1
00532	L00 RM2	00624	F5B X	00716	L00 R4	
00533	FMP D1	00625	XCR	00717	FMP R5	
00534	STO 12+1	00626	FMP S2	00720	FAD Z	
00535	L00 RM1	00627	STO 12+2	00721	FAD 12+1	
00536	FMP D2	00630	L00 Z	00722	STO TEK	
00537	FAD 12+1	00631	FMP S1	00723	123A	L00 R0
00540	STO TE8	00632	FAD 12+2	00724	FMP TEK	
00541	101A	00633	FMP TE9	00725	STO 12+1	
00542	STO R-40	00634	STO TEH	00726	L00 R0	
00543	102A	00635	115A	L00 TEH	00727	FMP TEJ
00544	FMP TE7	00636	FMP R5	00730	FAD 12+1	
00545	STO TE0	00637	FAD Z	00731	STO PMPX	
00546	103A	00638	STO R4	00732	124A	CLR PMPX
00547	FMP TE0	00641	116A	CLR R4	00733	STO R-48
00550	STO TE6	00642	STO R-43	00734	125A	L00 SINPSI
00551	104A	00643	117A	L00 Z	00735	FMP SINPSI
00552	FAD TE6	00644	FMP D2	00736	STO R5	
00553	STO 12+1	00645	CMS	00737	126A	L00 COSPSI
00554	CLR TE7	00646	FAD TEC	00740	FMP COSPSI	
00555	FMP 12+1	00647	STO 12+1	00741	XCR	
00556	STO R0	00650	L00 X	00742	FMP 32+7	
00557	105A	00651	FMP 32+7	00743	STO R6	
00560	STO R-41	00652	FMP TE4	00744	127A	L00 R6
00561	106A	00653	FMP 12+1	00745	FMP TEI	
00562	FMP TE7	00654	STO R5	00746	STO 12+1	
00563	FMP R0	00655	118A	CLR R5	00747	L00 TEE
00564	STO R0	00656	STO R-46	00750	FMP R5	
00565	107A	00657	119A	CLR 32+1	00751	FAD 12+1
00566	CLR R0	00660	F5B X	00752	STO TEI0	
00567	108A	00661	XCR	00753	128A	CLR TEI0
00570	FMP S2	00662	FMP D2	00754	STO R-49	
00571	STO 12+1	00663	STO 12+2	00755	129A	L00 32+7
00572	L00 S1	00664	L00 Z	00756	FMP TEB	
00573	FMP S1	00665	FMP D1	00757	STO TEB2	
00574	FAD 12+1	00666	FAD 12+2	00760	130A	L00 TEB2
00575	STO TE9	00667	STO 12+3	00761	FMP R6	
00576	109A	00670	L00 X	00762	STO TEI1	
00577	STO R-43	00671	FMP 32+7	00763	131A	CLR TEI1
00600	110A	00672	FMP TE4	00764	STO R-50	
00601	FMP TEA	00673	FMP 12+3	00765	132A	L00 TEI1
00602	STO R5	00674	STO R5	00766	FMP S2	
00603	111A	00675	120A	CLR R5	STO 12+1	
00604	FMP S2	00676	STO R-47	00767	L00 TEI0	
00605	STO 12+1	00677	121A	L00 R4	00771	FMP S1
00606	CLR 32+1	00700	FMP R5	00772	F5B 12+1	
00607	F5B X	00701	STO 12+1	00773	FMP TE9	

00774	133A	STO A6	01066	STO R-54	01160	STO A8
00775	133A	CLA A6	01067	LQ TEH	01161	CLA A8
00776	134A	STO R-51	01067	FMP TER	01162	STO R-57
00777	134A	LQ TE11	01071	FQD Z	01163	CLA 33+1
01000		FMP 51	01072	STO 12+1	01164	FQD X
01001		STO 12+1	01073	LQ 32+7	01165	XCA Z
01002		LQ TE10	01074	FMP 12+1	01166	FMP Z
01003		FMP 52	01075	STO B7	01167	XCA
01004		FQD 12+1	01076	CLA B7	01170	FMP 51
01005		FQD TE9	01077	STO R-55	01171	STO 12+2
01006		STO B6	01100	LQ B5	01172	LQ 52
01007	135A	CLA B6	01101	FMP B7	01173	FMP B3
01010		STO R-52	01102	STO 12+1	01174	FQD 12+2
01011	136A	LQ B5	01103	LQ A5	01175	STO Q6
01012		FMP A0	01104	FMP A7	01176	CLA Q5
01013		STO 12+1	01105	CHS	01177	FQD TE9
01014		LQ B0	01106	FQD X	01200	FMP Q6
01015		FMP A5	01107	FQD 12+1	01201	STO 12+1
01016		FQD 12+1	01110	STO Q1	01202	CLA 32+7
01017		STO Q1	01111	LQ B5	01203	FQD X
01020	137A	LQ B0	01112	FMP A7	01204	STO 12+2
01021		FMP B5	01113	STO 12+1	01205	LQ Z
01022		STO 12+1	01114	LQ A5	01206	FMP 12+2
01023		LQ A0	01115	FMP B7	01207	FQD 12+1
01024		FMP A5	01116	FQD 12+1	01210	STO B8
01025		FQD 12+1	01117	STO Q2	01211	CLA B8
01026		STO Q2	01120	LQ A0	01212	STO R-58
01027	138A	LQ 5IMP51	01121	FMP Q2	01213	LQ B5
01030		FMP 5IMP51	01122	STO 12+1	01214	FMP B8
01031		CHS	01123	LQ B0	01215	STO 12+1
01032		FQD A6	01124	FMP Q1	01216	LQ B5
01033		STO Q3	01125	FQD 12+1	01217	FMP A8
01034	139A	LQ Q2	01125	STO PNPZ	01220	STO 12+2
01035		FMP Q3	01127	CLA PNPZ	01221	LQ X
01036		STO Q4	01130	STO R-56	01222	FMP 32+7
01037	140A	LQ Q Y	01131	LQ X	01223	XCA
01040		FMP Q1	01132	FMP X	01224	FMP X
01041		XCA	01133	CHS	01225	FQD 12+2
01042		FQD B6	01134	FQD 32+1	01226	FQD 12+1
01043		STO 12+1	01135	STO Q3	01227	STO Q1
01044		LQ Q Y	01136	CLA 32+1	01227	159A
01045		FMP Q4	01137	FQD X	01230	160A
01046		FQD 12+1	01140	XCA	01231	15X B5
01047		STO PNPV	01141	FMP 52	01232	15X B8
01050	141A	CLA PNPV	01142	XCA	01233	STO 12+1
01051		STO R-53	01143	FMP Z	01234	161A
01052	142A	LQ 32+7	01144	STO 12+2	01235	15X A5
01053		FMP TEA	01145	LQ 51	01236	15X B8
01054		STO B5	01146	FMP Q3	01237	STO 12+2
01055	143A	CLA B5	01147	FQD 12+2	01240	162A
01056		FQD TE9	01150	STO Q4	01241	15X QM+4
01057		FMP TE6	01151	CLA TEA	01242	15X X
01060		STO 12+1	01152	FQD TE2	01243	15X X
01061		CLA 32+7	01153	FMP Q4	01244	LQ 32+7
01062		FQD X	01154	FQD 32+1	01245	FMP 12+3
01063		FQD 12+1	01155	STO 12+1	01246	FQD 12+2
01064		STO A7	01156	LQ 32+7		
01065	144A	CLA A7	01157	FMP 12+1		

01247	F80 12+1	01337	STO R-60	01431	STO 12+1
01250	STO 01	01340 173A	L00 32+4	01432	L00 01
01251 163A	L00 B5	01341	FMP TE8	01433	FMP 12+1
01252	FMP B8	01342	STO B1	01434	CH5
01253	STO 12+1	01343 174A	CLA B1	01435	STO M-2
01254	L00 A5	01344	STO R-61	01436 187A	L00 M
01255	FMP B8	01345 175A	L00 B1	01437	FMP M
01256	STO 12+2	01346	FMP S2	01440	STO TAG
01257	L00 X	01347	STO 12+1	01441 188A	L00 TAG
01260	FMP 2	01350	L00 A1	01442	FMP V51
01261	F80 12+2	01351	FMP S1	01443	STO TE12
01262	F80 12+1	01352	F80 12+1	01444 189A	CLA TE12
01263	STO 02	01353	FDP TE9	01445	STO R-64
01264 164A	L00 B0	01354	STO 12+2	01446 190A	L00 V-6
01265	FMP 02	01355	CLA 32+1	01447	FMP YC1
01266	STO 12+1	01356	F80 12+2	01450	STO X1
01267	L00 A0	01357	STO A2	01451 191A	L00 M
01270	FMP 01	01360 176A	CLA A2	01452	FMP VR
01271	F80 12+1	01361	STO R-62	01453	STO X2
01272	FDP M-2	01362 177A	L00 A1	01454 192A	CLA X1
01273	X58	01363	FMP S2	01455	F80 X2
01274	CH5	01364	STO 12+1	01456	STO X3
01275	STO M-10	01365	L00 S1	01457 193A	CLA M-9
01276 165A	CLA V-4	01366	FMP B1	01460	FDP TE12
	B55	01367	F80 12+1	01461	FMP X3
01277	TSX COS-4	01370	FDP TE9	01462	STO M-3
01300	STO 01	01371	STO B2	01463 194A	L00 V-7
01301 166A	CLA V-4	01372 178A	CLA B2	01464	FMP YC1
	B55	01373	STO R-63	01465	STO X1
01302	TSX SIN-4	01374 179A	L00 A2	01466 195A	L00 M
01303	STO 02	01375	FMP B5	01467	FMP YTHETA
01304 167A	L00 02	01376	STO TAA	01470	STO X2
01305	FMP 32+6	01377 180A	L00 B2	01471 196A	CLA X1
01306	X58	01400	FMP B5	01472	F80 X2
01307	FMP 02	01401	STO TAA	01473	STO X3
01310	STO 12+1	01402 181A	L00 A2	01474 197A	CLA M-9
01311	L00 01	01403	FMP B5	01475	FDP TE12
01312	FMP 32+7	01404	STO TAC	01476	FMP X3
01313	X58	01405 182A	L00 B2	01477	STO M-4
01314	FMP 01	01406	FMP B5	01500 198A	L00 V-8
01315	F80 12+1	01407	STO TAA	01501	FMP YC1
01316	STO 03	01410 183A	CLA TAA	01502	STO X1
01317 168A	CLA 32+1	01411	F80 TAA	01503 199A	L00 M
01320	FDP 03	01412	STO 12+1	01504	FMP VPHI
01321	STO TE12	01413	L00 A0	01505	STO X2
01322 169A	CLA TE13	01414	FMP 12+1	01506 200A	CLA X1
01323	STO R-57	01415	STO TAE	01507	F80 X2
01324 170A	CLA V51	01416 184A	CLA TAC	01510	STO X3
01325	S52	01417	F80 TAA	01511 201A	CLA M-9
01326	F80 32+8	01420	STO 12+1	01512	FDP TE12
01327 17081	TAE 2120	01421	L00 B0	01513	FMP X3
01330	TAE 171A	01422	FMP 12+1	01514	STO M-5
01331	L00 2130	01423	STO TAF	01515 202A	L00 V-7
01332 171A	L00 32+7	01424 185A	L00 YC1	01516	FMP VR
01333	FMP TE1	01425	FMP Y51	01517	STO 12+1
01334	F80 TEE	01426	STO 01	01520	L00 V-6
01335	STO 01	01427 186A	CLA TAE	01521	FMP YTHETA
01336 172A	CLA 01	01430	F80 TAF	01522	F80 12+1

01523	STO X1	01615	FAD 12+2	01706	TRA 242A
01524	LDO U	01616	FAD 13+1	01707	CLA W-69
01525	FMP Y51	01617	STO W-8	01710	FSB WTSI
01526	STO X2	01620	TRA 218A	01711	TZE 232A
01527	CLA X1	01621	CLA 32+3	01712	TPL 232A
01530	FMP X2	01622	STO W-3	01713	TRA 242A
01531	STO X3	01623	CLA 32+3	01714	CLA WTSI
01532	LDO X3	01624	STO W-4	01715	FAD W-62
01533	FMP TE13	01625	CLA 32+3	01716	STO WTSI
01534	STO PPSIPT	01626	STO W-5	01717	CAL 2D+1
01535	CLA PPSIPT	01627	CLA TE13		BSS
01536	STO R-65	01630	STO PPSIPT	01720	TSX C5THD.4
01537	CLA 32+3	01631	TRA 207A	01721	PZE 8240
01540	STO PPSIPT	01632	LDO V-6	01722	LXD 2D+2.1
01541	CLA 32+3	01633	FMP V-6	01723	LDO R+1.1
01542	STO PPSIPT	01634	STO X1	01724	STR
01543	LDO W-9	01635	LDO V-7	01725	TXI **1.1.1
01544	FMP PPSIPT	01636	FMP V-7	01726	TXL 235A.1.69
01545	STO 12+1	01637	STO X2		BSS
01546	LDO PNP2	01640	LDO V-8	01727	TSX (FIL).4
01547	FMP P2R	01641	FMP V-8	01730	TRA 242A
01550	STO 13+2	01642	STO X3	01731	CLA W-69
01551	LDO PNPV	01643	CLA X1	01732	FSB 3D+1
01552	FMP PVR	01644	FAO X2	01733	TZE 231A
01553	STO 12+3	01645	FAO X3	01734	TPL 231A
01554	LDO PNPX		BSS	01735	CLA W-62
01555	FMP PXR	01646	TSX SART.4	01736	STO WTSI
01556	FAO 13+3	01647	STO TE14	01737	TRA 233A
01557	FAO 12+2	01650	CLA PPSIPT	01740	LXD 5+1
01560	FAO 13+1	01651	STO R-66	01741	LXD 5+1.2
01561	STO W-6	01652	CLA PPSIPT	01742	LXD 5+2.4
01562	LDO W-9	01653	STO R-67	01743	TRA 1.4
01563	FMP PPSIPT	01654	CLA TE14	01744	OCT +000002000000
01564	STO 13+1	01655	STO R-68	01745	OCT +000006000000
01565	LDO PNP2	01656	CLA V-5	01746	OCT +000001000000
01566	FMP P2THET	01657	FDP TE14	01747	OCT +202622077326
01567	STO 12+2	01660	FMP W	01750	OCT +201400030000
01570	LDO PNPV	01661	STO V-6	01751	OCT +105447113564
01571	FMP P2THET	01662	CLA V-7	01752	OCT +000000000000
01572	STO 13+3	01663	FDP TE14	01753	OCT +203400000000
01573	LDO PNP2	01664	FMP W	01754	OCT +204400000000
01574	FMP P2THET	01665	STO V-7	01755	OCT +200400000000
01575	FAO 12+3	01666	CLA V-8	01756	OCT +202400000000
01576	FAO 13+2	01667	FDP TE14	01757	OCT +113715126246
01577	FAO 13+1	01670	FMP W	01760	OCT +233000000000
01600	STO W-7	01671	STO V-8	01761	OCT +000000077777
01601	LDO W-9	01672	CLA W-67	01762	OCT +000000000000
01602	FMP PPSIPT	01673	TZE 239A	01763	OCT +000001000000
01603	STO 12+1	01674	TPL 230A	01764	OCT +000000000000
01604	LDO PNP2	01675	CLA W		BCD 163
01605	FMP P2THET	01676	FSB W-68	01765	BCD 1P8E1.4
01606	STO 13+2	01677	TZE 242A	01766	
01607	LDO PNPV	01700	TPL 242A	01767	8D+0 BCD 1C1HO.1
01610	FMP P2THET	01701	TRA 233A		
01611	STO 12+3	01702	CLA V-3		
01612	LDO PNPX	01703	FSB W-67		
01613	FMP P2THET	01704	TZE 233A		
01614	FAO 13+3	01705	TPL 233A		

I. SUBROUTINE ELECTX

The whole process of determining the free normalized electron density and its spatial derivatives at any point in space is explored in this subroutine. For the case under consideration this is a simple sphere where the values in Equation 74 are represented by the following components of the W vector

$$W(28) = R; W(34) = A; W(35) = n$$

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC OCT DEC OCT DEC OCT DEC OCT
M 32188 76674 RECORD 32561 77461 U 32549 77445 W 32438 77266

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	1	00000	3	6	00034	5	7	00042	6	8	00046
9	10	00057	9	11	00061	11	13	00064	12	14	00067
14	16	00075	15	17	00100	16	18	00103	17	19	00106
19	21	00114	20	22	00117	21	23	00122	22	24	00125
24	26	00170	26	27	00202	27	28	00206	28	29	00211
30	31	00222	31	32	00225	32	33	00230	33	34	00236

STORAGE NOT USED BY PROGRAM

DEC OCT
198 00306 32187 76673

LOCATIONS OF NAMES IN TRANSFER VECTOR

DEC OCT	DEC OCT	DEC OCT	DEC OCT	DEC OCT	DEC OCT
EXPC3	1 00001	COS	3 00003	BIGR	0 00000
				SIN	2 00002
				SGRT	4 00004

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSIONAL EQUIVALENCE OR COMMON SENTENCES

DEC OCT	DEC OCT	DEC OCT	DEC OCT	DEC OCT	DEC OCT
CT	197 00309	CP	196 00304	PARTP	195 00303
ECT	192 00300	ESICP	191 00277	EST	190 00276
ST	187 00273	TER1	186 00272	TER2	185 00271
Y882	182 00264	Z882	181 00263		
				TR3	184 00270
				SP	183 00267
				XBRP	183 00267

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC OCT	DEC OCT	DEC OCT	DEC OCT	DEC OCT	DEC OCT
13	176 00260	23	168 00250	33	169 00251
				63	171 00253

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

BIGR	EXPC3	SIN	COS	SGRT

00000	B1GR	BCD 1B1GR	00067	14A	CLA V-4	00156	FMP CP
00001	EXPC3	BCD 1EXPC3			BSS	00157	FAD 12+2
00002	SIN	BCD 1SIN	00070		TSX C05.4	00160	STO 12+3
00003	COS	BCD 1COS	00071		STO CT	00161	LQD RCT
00004	SORT	BCD 1SORT	00072	15A	CLA V-5	00162	FMP 12+3
00005	S	HTR			BSS	00163	FSB 12+1
00006	HTR	HTR	00073		TSX SIN.4	00164	STO 12+4
00007	HTR	HTR	00074		STO SP	00165	LQD M-34
00010	BCD 1ELECTX	BCD 1ELECTX	00075	16A	CLA V-5	00166	FMP 12+4
00011	SXD \$+1.2	SXD \$+1.2			BSS	00167	STO PARTT
00012	SXD \$+2.4	SXD \$+2.4	00076		TSX C05.4	00170	LQD XBRP
00013	SXD \$+2.4	SXD \$+2.4	00077		STO CP	00171	FMP RSTSP
00014	CLA 1-4	CLA 1-4	00100	17A	LQD V-3	00172	STO 12+1
00015	STA SA+17	STA SA+17	00101		FMP ST	00173	LQD VBRP
00016	STA SA+109	STA SA+109	00102		STO RST	00174	FMP RSTCP
00017	STA SA+113	STA SA+113	00103	18A	LQD RST	00175	FSB 12+1
00020	STA SA+125	STA SA+125	00104		FMP CP	00176	STO 12+2
00021	STA SA+131	STA SA+131	00105		STO RSTCP	00177	LQD M-34
00022	CLA 2-4	CLA 2-4	00106	19A	LQD RST	00200	FMP 12+2
00023	STA SA+19	STA SA+19	00107		FMP SP	00201	STO PARTP
00024	STA SA+118	STA SA+118	00110		STO RSTSP	00202	CLA M-27
00025	CLA 3-4	CLA 3-4	00111	20A	LQD V-3	00203	LQD M-34
00026	STA SA+21	STA SA+21	00112		FMP CT		BSS
00027	STA SA+121	STA SA+121	00113		STO RCT	00204	TSX EXPC3.4
00030	CLA 4-4	CLA 4-4	00114	21A	CLA M-30	00205	STO TER2
00031	STA SA+23	STA SA+23	00115		FSB RSTCP	00206	CLA TER1
00032	STA SA+124	STA SA+124	00116		STO XBRP	00207	FDP TER2
00033	BSS	BSS	00117	22A	CLA M-31	00210	STQ X
00034	CLA M-33	CLA M-33	00120		FSB RSTSP	00211	LQD M-27
00035	FDP M-2	FDP M-2	00121		STO VBRP	00212	FMP M-27
00036	XCA	XCA	00122	23A	CLA M-32	00213	STO 12+1
00037	FDP M-2	FDP M-2	00123		FSB RCT	00214	CLA X
00040	FMP 30	FMP 30	00124		STO ZBRP	00215	FDP 12+1
00041	STO TER1	STO TER1	00125	24A	LQD CT	00216	STQ TR3
00042	CLA M-27	CLA M-27	00126		FMP ZBRP	00217	LQD TR3
00043	FSB M-35	FSB M-35	00127		STO 12+1	00220	FMP PARTP
00044	TZE 13A	TZE 13A	00130		LQD VBRP	00221	STQ PXR
00045	JPL 13A	JPL 13A	00131		FMP ST	00222	LQD TR3
00046	CLA M-35	CLA M-35	00132		XCA	00223	FMP PARTT
00047	LQD M-34	LQD M-34	00133		FMP SP	00224	STO PXTHET
00050	TSX EXPC3.4	TSX EXPC3.4	00134		STO 12+2	00225	LQD TR3
00051	STO 12+1	STO 12+1	00135		LQD XBRP	00226	FMP PARTP
00052	CLA TER1	CLA TER1	00136		FMP ST	00227	STQ PXPHI
00053	FDP 12+1	FDP 12+1	00137		XCA	00230	CLA X
00054	STQ X	STQ X	00140		FMP CP	00231	BSS
00055	CLA 3D+1	CLA 3D+1	00141		FAD 12+2	00232	TSX SQR1.4
00056	STO PXR	STO PXR	00142		FAD 12+1	00233	STO 12+1
00057	CLA 3D+1	CLA 3D+1	00143		STO 12+3	00234	LQD M-2
00058	STO PXTHET	STO PXTHET	00144		LQD M-34	00235	FMP 12+1
00059	STA SA+109	STA SA+109	00145		FMP 12+3	00236	STO M-37
00060	CLA 3D+1	CLA 3D+1	00146		STO PARTP	00237	CLA X
00061	STO PXTHET	STO PXTHET	00147	25A	LQD RST	00240	FDP 30
00062	STA SA+131	STA SA+131	00150		FMP ZBRP	00241	FMP M-2
00063	TAA 33A	TAA 33A	00151		STO 12+1	00242	XCA
00064	CLA V-4	CLA V-4	00152		LQD VBRP	00243	FMP M-2
00065	BSS	BSS	00153		FMP SP	00244	STO M-38
00066	TSX SIN.4	TSX SIN.4	00154		STO 12+2	00245	LQD X.1
	STO ST	STO ST	00155		LQD XBRP		LQD \$+1.2

J. SUBROUTINE BIGR

This subroutine is used to calculate the distance from the center of the ionizing source to the spatial point r , θ , φ at which the electron density and spatial gradients are desired. For the simple sphere this distance is $W(2\theta) = \mathcal{L}$. In addition to this it calculates the angle that $\mathcal{L}$ makes with respect to the vertical distance passing through the center of the ionizing source.

```

C 1 SUBROUTINE B1GR DECEMBER 22, 1960 134-7090
2 SUBROUTINE B1GR
3 COMMON RECORD,V,M,N
4 DIMENSION RECORD(12),VC(11),WC(50)
5 PST=VC(4)*SINF(VCS)
6 RSTCP=ST+COSF(VCS)
7 RSTSP=ST+SINF(VCS)
8 RCT=VC(4)*COSF(VCS)
9 NC28=SQRTF(CWC312-RSTCP)**2+CWC322-RSTSP)**2+CWC332-RCTD**2
10 WC55=SQRTF(CWC322-RSTCP)**2+CWC332-RSTSP)**2+CWC342-RCTD**2
11 TERM1=WC302**2+CWC282**2-CWC402**2/(2.0*WC282*WC302)
12 IF (ABS(CTERM1)-1.0E-7)12,12,14
13 NC29=2.0
14 GO TO 15
15 NC29= -CTERM1
16 TE1 =CWC42)**2-CWC192)**2-CWC552)**2/(2.0*WC192)*WC552
17 TE2 = SQRTF(TE1-TE2**2)
18 NC37 = 4PCOS(TE3)
19 RETURN
20 ENDC

```

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC OCT DEC OCT DEC OCT DEC OCT DEC OCT
N 32138 76674 RECORD 32561 77461 V 32549 77445 W 32438 77266

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	100000	4	100013	5	6 00021	6	7 00027	7	8 00035		
2	100043	9	100070	10	11 00115	11	12 00136	12	13 00143		
13	14 00145	14	15 00146	15	16 00150	16	18 00177	16	19 00177		
17	20 00202										

STORAGE NOT USED BY PROGRAM

DEC OCT
155 00233
32187 76673

LOCATIONS OF VALUES IN TRANSFER VECTOR

COS	DEC	OCT	AR205	DEC	OCT	SIN	DEC	OCT	AR205	DEC	OCT
1	00201		1 00003		0 00000		2 00002				

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE, OR COMMON SENTENCES

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
154 00232		RSTP	151 00227	TE2	150 00226		
149 00225		TERM1	148 00224				

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
144 00220	20	134 00206	30	135 00207	60	139 00213	

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

SIN	COS	AR205

00000	SIN	BOD 15IN	BSS	00156	FMP W-54
00001	COS	BOD 1005	TSX SORT,4	00157	STO 12+2
00002	SORT	BOD 150RT	STO W-27	00160	LDD W-18
00003	ARCOS	CLM W-53	CLM W-53	00161	FMP W-18
00004	S	HTR	F3B RCT	00162	STO 12+3
00005		HTR	STO 12	00163	LDD V-3
00006		HTR	LDD 12	00164	FMP V-3
00007		BOD 181GR	FMP 12	00165	F3B 12+3
00010		SXD 1+1	STO 12+1	00166	F3B 12+2
00011		SXD 1+1+2	CLM W-52	00167	FDP 12+1
00012		SXD 1+2+4	F3B RSTSP	00170	STQ TE2
00013	SA	CLM W-4	STO 12	00171	17A LDD TE2
00014		BSS	LDD 12	00172	FMP TE2
00015		TSX SIN,4	FMP 12	00173	CH5
00016		STO 12+1	STO 12+2	00174	FAD 32+3
00017		LDD V-3	CLM W-51		BSS
00018		FMP 12+1	F3B RSTCP	00175	TSX SORT,4
00019		STO RST	STO 12	00176	STO TE3
00020		CLM W-5	LDD 12		18A BSS
00021	SA	F3B	FMP 12	00177	19A TSX ARCOS,4
00022		TSX COS,4	FAD 12+2	00200	TSX TE3
00023		STO 12+1	FAD 12+1	00201	STO W-56
00024		LDD RST	BSS	00202	20A LDD \$
00025		FMP 12+1	TSX SORT,4	00 03	LX 1+1+2
00026		STO RSTCP	ST W-5	00204	LDD 1+2+4
00027	7A	CLM W-5	0 11 111	00205	TRA 1+4
00028		BSS	FMP 32	00206	22 OCT +000002000000
00029		TSX SIN,4	XCA	00207	30 OCT +202400000000
00030		STO 12+1	FMP W-27	00210	OCT +151655376246
00031		LDD RST	STO 12+1	00211	OCT +000000000000
00032		FMP 12+1	LDD V-3	00212	OCT +201400000000
00033		STO RSTCP	FMP V-3	00213	60 OCT +233000000000
00034	8A	CLM W-4	STO 12+2	00214	OCT +000000077777
00035		BSS	LDD W-17	00215	OCT +000000000000
00036		TSX COS,4	FMP W-27	00216	OCT +200000100000
00037		STO 12+1	STO 12+3	00217	OCT +000000000000
00038		LDD V-3	LDD W-29		
00039		FMP 12+1	FMP W-29		
00040		STO RCT	FAD 12+3		
00041		CLM W-32	F3B 12+2		
00042	9A	F3B RCT	FDP 12+1		
00043		STO 12	STQ TERM1		
00044		LDD 12	CLM TERM1		
00045		FMP 12	S3P		
00046		STO 12+1	F3B 32+1		
00047		CLM W-31	TZE 13A		
00048		F3B RSTSP	TPL 15A		
00049		STO 12	CLM 32+2		
00050		LDD 12	STO W-28		
00051		FMP 12	TRA 16A		
00052		STO 12+2	CLS TERM1		
00053		CLM W-30	STO W-28		
00054		F3B RSTCP	LDD W-54		
00055		STO 12	FMP 32		
00056		LDD 12	XCA		
00057		FMP 12	FMP W-18		
00058		FAD 12+2	STO 12+1		
00059		FAD 12+1	LDD W-54		

K. SUBROUTINE MAGY

The normalized earth's magnetic field and its spatial derivatives are calculated in this subroutine. In this program it is represented by an assumed magnetic dipole field with the dipole located at the center of the earth.

```

C      SUBROUTINE MAGV  DECEMBER 22,1960  IBM-7090
      SUBROUTINE MAGV C V,VR,VTHETA,VPHI,DVDR,DVDTHE,DVDPHI>
      DIMENSION RECORD(12),VC(112),WC(250)
      COMMON  RECORD,V,W,N
      TERM7 = SINFC VC(5)
      TERM8 = COSFC VC(5)
      TERM9 = SORTF C(1.0 + 3.0* TERM8**2 )
      TERM10=(WC(19)/VC(4))**3
      T1 = WC(5)+TERM10*WC(3)
      V = T1+ TERM9
      VR = - 2.0 + T1 + TERM8
      VTHETA = - T1 + TERM7
      VPHI = 0.0
      DVDR = C-3.0*V D/VC(4)
      DVDTHE = C-3.0*V*TERM7+TERM9*(C*TERM9**2)
      DVDPHI = 0.0
      RETURN
      END C(1.0,1.0,0.0,0.0,0.0,0.0,0.0,0.0)

```

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC OCT DEC OCT DEC OCT DEC OCT
N 32188 76674 RECORD 32561 77461 V 32549 77445 W 32438 77266 DEC OCT

STORAGE NOT USED BY PROGRAM

DEC OCT
107 00153 32187 76673

LOCATIONS OF NAMES IN TRANSFER VECTOR

DEC OCT DEC OCT DEC OCT DEC OCT
COS 1 00001 SIN 0 00000 SORT 2 00002 DEC OCT DEC OCT

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

DEC OCT DEC OCT DEC OCT DEC OCT
Y1 106 00152 YERN10 105 00151 YERN7 104 00150 YERN8 103 00147 YERN9 102 00146

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC OCT DEC OCT DEC OCT DEC OCT
1> 100 00144 2> 89 00131 3> 91 00133 6> 95 00137 DEC OCT

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

SIN COS
SORT

00000	SIN	BCD 15IN	00067	FMP 3D+2
00001	COS	BCD 1COS	00070	XCR
00002	SORT	BCD 1SORT	00071	FMP T1
00003	\$	STR	00072	CHS
00004		STR	00073	STO YR
00005		STR	00074	LDQ T1
00006		BCD 1ARGY	00075	FMP TERM7
00007		SXD \$1	00076	CHS
00010		SXD \$+1.2	00077	STO YTHEIR
00011		SXD \$+2.4	00100	CLA 3D+3
00012		CLA 1.4	00101	STO YPHI
00013		STA SA+27	00102	LDQ 3D+1
00014		STA SA+41	00103	FMP Y
00015		STA SA+31	00104	CHS
00016		CLA 2.4	00105	FDP U-3
00017		STA SA+33	00106	STQ DVDR
00020		CLA 3.4	00107	LDQ TERM9
00021		STA SA+37	00110	FMP TERM9
00022		CLA 4.4	00111	STO 1D+1
00023		STA SA+39	00112	LDQ TERM8
00024		CLA 5.4	00113	FMP 3D+1
00025		STA SA+44	00114	XCR
00026		CLA 6.4	00115	FMP Y
00027		STA SA+56	00116	XCR
00030		CLA 7.4	00117	FMP TERM7
00031		STA SA+58	00120	CHS
00032	9A	CLA V-4	00121	FDP 1D+1
00033		BSS	00122	STQ DVDTHE
00034		TSX SIN.4	00123	CLA 3D+3
00035	6A	STO TERM7	00124	STO DVDPHT
00036		CLA V-4	00125	LDQ \$1.1
00037		BSS	00126	LDQ \$+1.2
00038		TSX COS.4	00127	LDQ \$+2.4
00039		STO TERM8	00130	TRA 8.4
00040	7A	LDQ TERM8	00131	OCT +000002000000
00041		FMP TERM8	00132	OCT +000003000000
00042		STO 1D+1	00133	OCT +201400000000
00043		LDQ 3D+1	00134	OCT +202600000000
00044		FMP 1D+1	00135	OCT +202400000000
00045		FAD 3D	00136	OCT +000000000000
00046		BSS	00137	OCT +233000000000
00047		TSX SORT.4	00140	OCT +000000077777
00048		STO TERM9	00141	OCT +000000000000
00049	8A	CLA W-18	00142	OCT +000001000000
00050		FMP V-3	00143	OCT +000000000000
00051		STO 1D		
00052		FMP 1D		
00053		LRS 35		
00054		FMP 1D		
00055		FMP 1D		
00056		STO TERM10		
00057	9A	CLA W-24		
00058		FAD V-3		
00059		FMP TERM10		
00060		STO Y1		
00061		LDQ T1		
00062	10A	FMP TERM9		
00063		STO Y		
00064		LDQ TERM8		
00065				
00066	11A			

L. SUBROUTINE COLFRZ

The normalized collision frequency and its derivatives at a spatial point are determined in this subroutine. In this presentation it is assumed that collision frequency varies exponentially with height. One should refer to the definition of the W vector where the required coefficients for the height stratifications are defined.

RETURN

ENDC0,1-0,1-0,0-0,0-0,0-0,0-0,0-0,0-0

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
N 32198	76674	RECORD 32561	77461	U 32549	77445	W 32438	77266

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	1	00000	2	5	00027	3	8	00033
6	12	00059	9	14	00060	12	15	00064
11	22	00102	14	23	00106	15	26	00113

STORAGE NOT USED BY PROGRAM

DEC	OCT
32187	76673

LOCATIONS OF NAMES IN TRANSFER VECTOR

DEC	OCT	DEC	OCT	DEC	OCT
EXP	0 00000				

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

DEC	OCT	DEC	OCT	DEC	OCT
8	99 00153	2	98 00142	TERM1	97 00141

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC	OCT	DEC	OCT	DEC	OCT
10	95 00137	20	84 00124	30	85 00125
				60	90 00132

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

EXP

00000	EXP	BCD 1EXP	00071	13A	CLA V-3
00001	5	HTR	00072	13A1	F5B 3D+3
00002		HTR	00073	13A1	TZE 19A
00003			00074		TPL 22A
00004		BCD 1COLFRZ	00075	19A	CLA M-72
00005		SXD 3+1.2	00076	20A	STO A
00006		SXD 3+2.4	00077	20A	CLA M-73
00007		SXD 1.4	00100		STO B
00010		CLA 1.4	00101	21A	TRA 8A
00011		STA 50+19	00102	22A	CLA V-3
00012		STA 5A+21	00103		F5B 3D+4
00013		STA 5A+57	00104	22A1	TZE 23A
00014		CLA 2.4	00105		TPL 26A
00015		STA 5A+23	00106	27A	CLA M-74
00016		STA 5A+59	00107		STO A
00017		CLA 3.4	00110	24A	CLA M-75
00020		STA 5A+25	00111		STO B
00021		CLA 4.4	00112	25A	TRA 8A
00022		STA 5A+27	00113	15A	CLA 3D+1
00023	5A	CLA V-3	00114		STO Z
00024		F5B 3D	00115	27A	CLA 3D+1
00025	7A1	TZE 5A	00116		STO PZP
00026		TPL 14A	00117	43A	TRA 11A
00027	5A	CLA M-26	00120	29A	LXD 3+1
00030		STO A	00121		LXD 3+1.2
00031	7A	CLA M-25	00122		LXD 3+2.4
00032		STO B	00123		TRA 5.4
00033	8A	CLA M-3	00124	4D	OCT +000000000000
00034		F5B M-19	00125	3D	OCT +21533435121
00035		STO 1D+1	00126		OCT +000000000000
00036		LQD A	00127		OCT +215631635121
00037		FMP 1D+1	00130		OCT +215846235121
00040		CHS	00131		OCT +215846235121
00041		STO TERM1	00132	6D	OCT +233000000000
00042	9A	CLA TERM1	00133		OCT +000000077777
		B35	00134		OCT +000000000000
00043		TSX EXP.4	00135		OCT +000001000000
00044		FOP M-2	00136		OCT +000000000000
00045		FMP B			
00046		STO Z			
00047	10A	LQD A			
00050		FMP Z			
00051		CHS			
00052		STO PZP			
00053	11A	CLA 3D+1			
00054		STO PZTHET			
00055	12A	CLA 3D+1			
00056		STO PZPHI			
00057	13A	TRA 29A			
00060	14A	CLA V-3			
00061		F5B 3D+2			
00062	14A1	TZE 15A			
00063		TPL 18A			
00064	15A	CLA M-70			
00065		STO A			
00066	16A	CLA M-71			
00067		STO B			
00070	17A	TRA 8A			

M. SUBROUTINE RCOORD

This subroutine permits the transformation from the earth centered geomagnetic spherical coordinate system, to the radar coordinate system.

ENDC(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
G 32165 76645	N 32188 76674	RECORD 32561 77461	V 32549 77445	W 32438 77266			
XN 32187 76673	YN 32180 76664	ZN 32173 76655					

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
10	20	00205	10	21	00210	20	23	00215
						20	24	00220
						40	31	00252

STORAGE NOT USED BY PROGRAM

DEC	OCT
202 00312	32156 76634

LOCATIONS OF NAMES IN TRANSFER VECTOR

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
COS 1 00001	ARCOS 3 00003	SIN 0 00000	SQRT 2 00002				

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
ETA 201 00311	ETAN 200 00310	EPS 199 00307	EPSN 198 00306	ANGE 197 00305			
ANGA 196 00304	RCE2 195 00303	RCE 194 00302	RST 193 00301	XP 192 00300			
VP 191 00277	ZETAN 190 00276	ZETA 189 00275	ZP 188 00274				

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
1) 182 00266	2) 174 00256	3) 175 00257	6) 177 00261				

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

SIN	COS	SQRT	ARCOS
-----	-----	------	-------

00000	SIN	BCD	1SIN	00066	STO 1>5	00160	STO ETA
00001	COS	BCD	1COS	00067	LQD XP	00161	CLA ZETAN
00002	SQRT	BCD	1SQRT	00070	FMP XN	00162	FDP ZN-7
00003	ARCOS	BCD	1ARCOS	00071	FAD 1>5	00163	STQ ZETA
00004	S	HTR		00072	FAD 1>4	00164	LQD ZETA
00005		HTR		00073	FEB 1>3	00165	FMP ZETA
00006		HTR		00074	STO EPSN	00166	STO 1>1
00007		BCD	1RC00R0	00075	LQD ZP	00167	LQD ETA
00010		SXD	S,1	00076	FMP VN-5	00170	FMP ETA
00011		SXD	S+1,2	00077	STO 1>1	00171	FAD 1>1
00012		SXD	S+2,4	00100	LQD XP	00172	STO RCE2
00013	6A	CLA	V-4	00101	FMP VN-4	00173	CLA RCE2
00014		BSS		00102	STO 1>2		BSS
00015		TSX	SIN,4	00103	LQD VP	00174	TSX SQRT,4
00016		STO	1>1	00104	FMP VN-3	00175	STO RCE
00017		LQD	V-3	00105	FAD 1>2	00176	LQD EPS
00018		FMP	1>1	00106	FAD 1>1	00177	FMP EPS
00020		STO	RST	00107	STO 1>3	00200	FAD RCE2
00021	7A	CLA	V-5	00110	LQD ZP		BSS
00022		BSS		00111	FMP VN-2	00201	TSX SQRT,4
00023		TSX	COS,4	00112	STO 1>4	00202	STO W-5,9
00024		STO	1>1	00113	LQD XP	00203	CLA W-59
00025		LQD	RST	00114	FMP VN-1	00204	TZE 31A
00026		FMP	1>1	00115	STO 1>5	00205	CLA RCE
00027		FEB	XN-6	00116	LQD VP	00206	FDP W-59
00028		STO	XP	00117	FMP VN	00207	STQ 1>1
00030	8A	CLA	V-5	00120	FAD 1>5		BSS
00031		BSS		00121	FAD 1>4	00210	21A TSX ARCOS,4
00032		TSX	SIN,4	00122	FEB 1>3	00211	TSX 1>1
00033		STO	1>1	00123	STO ETAN	00212	STO ANGE
00034		LQD	RST	00124	LQD VP	00213	CLA RCE
00035		FMP	1>1	00125	FMP ZN-5	00214	TZE 31A
00036		FEB	VN-6	00126	STO 1>1	00215	CLA ZETA
00037	9A	STO	VP	00127	LQD ZP	00216	FDP RCE
00038		CLA	V-4	00130	FMP ZN-4	00217	STQ 1>1
00039		BSS		00131	STO 1>2		BSS
00040		TSX	COS,4	00132	LQD XP	00220	24A TSX ARCOS,4
00041		STO	1>1	00133	FMP ZN-3	00221	TSX 1>1
00042		LQD	V-3	00134	FAD 1>2	00222	STO ANGA
00043		FMP	1>1	00135	FAD 1>1	00223	LQD ANGA
00044		FEB	ZN-6	00136	STO 1>3	00224	FMP 3D
00045		STO	ZP	00137	LQD XP	00225	STO W-60
00046	10A	LQD	XP	00140	FMP ZN-2	00226	LQD ANGE
00047		FMP	XN-5	00141	STO 1>4	00227	FMP 3D
00048		STO	1>1	00142	LQD VP	00230	STO W-61
00049		LQD	VP	00143	FMP ZN-1	00231	CLA W-17
00050		FMP	XN-4	00144	STO 1>5	00232	FEB ANGA
00051		STO	1>2	00145	LQD ZP	00233	XCA
00052		LQD	ZP	00146	FMP ZN	00234	FMP 3D
00053		FAD	1>2	00147	FAD 1>5	00235	STO W-79
00054		FAD	1>1	00150	FAD 1>4	00236	CLA W-16
00055		FAD	1>1	00151	FEB 1>3	00237	FEB ANGE
00056		STQ	1>3	00152	STO ZETAN	00240	XCA
00057		LQD	VP	00153	CLA EPSN	00241	FMP 3D
00058		FMP	XN-2	00154	FDP ZN-7	00242	STO W-80
00059		STO	1>4	00155	STQ EPS	00243	CLA V-9
00060		LQD	ZP	00156	CLA ETAN	00244	FEB W-59
00061		FMP	XN-1	00157	FDP ZN-7	00245	STO W-81

00246	30A	LX0 3)+1
00247		FMP U-10
00250		FSB W-59
00251		STO W-82
00252	31A	LX0 S-1
00253		LX0 S+1,2
00254		LX0 S+2,4
00255		TMA 1,4
00256	2)	OCT +00000200000
00257	3)	OCT +206712273407
00260		OCT +22344607400
00261	6)	OCT +233000000000
00262		OCT +00C000077777
00263		OCT +000000000000
00264		OCT +000001000000
00265		OCT +000000000000

N. SUBROUTINE OUTONE

In this presentation this subroutine is used to list the input data and the initial conditions that define the ray tracing problem on Tape Unit 6. Table 5-1 illustrates this output by the statement of the input RECORD and the next ninety-one words of data which define in order the first 70 components of the W vector, followed by the first 21 components of the V vector. This subroutine can be greatly improved.

```

C  SUBROUTINE OUTONE DECEMBER 22,1960 IBM-7090
1  SUBROUTINE OUTONE (SIGN,ID)
COMMON RECORD,V,M,N
DIMENSION RECORD(12),VC(11),WC(250)
2  IF (SIGN) 3,3,6
3  WRITE OUTPUT TAPE 6,4,10,CRECORD(1),1=1,12)
4  FORMAT(25H1 EXTRA-ORDINARY RAY ,16,12A6)
5  GO TO 8
6  WRITE OUTPUT TAPE 6,7,10,CRECORD(1),1=1,12)
7  FORMAT(22H1 ORDINARY RAY ,16,12A6)
8  WRITE OUTPUT TAPE 6,9,CVC(1),1=1,70),CVC(1),1=1,21)
9  FORMAT(14H0,1P8E14,7)
RETURN
ENDC0,1-0,1-0,0-0,0-0,0-0,0-0,0-0

```

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC OCT DEC OCT DEC OCT DEC OCT
N 32188 76674 RECORD 32361 77461 V 32549 77445 W 32438 77266

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	1	00000	4	4	00000	7	5	00000	9	6	00000
3	9	00021	3	14	00033	5	15	00034	6	16	00035
8	22	00050	8	29	00065						

STORAGE NOT USED BY PROGRAM

DEC OCT
81 00121
32187 76673

LOCATIONS OF NAMES IN TRANSFER VECTOR

DEC OCT DEC OCT DEC OCT DEC OCT
(FILS) 1 00001 (STAS) 0 00000

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
829	67 00103	827	73 00111	824	80 00120	22	58 00072
							60 00074

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY

(FILS) (FILS)

00000	CSTHD	BCD 1CSTHD	29A	BSS
00001	CFILS	BCD 1CFILS	00065	TSX CFILS.4
00002	\$	HTR	00066	LXD \$+1
00003		HTR	00067	LXD \$+1.2
00004		HTR	00070	LXD \$+2.4
00005		BCD 1OUTONE	00071	TBA 3.4
00006		SXD \$+1	00072	2> OCT +000006000000
00007		SXD \$+1.2	00073	OCT +000011000000
00010		SXD \$+2.4	00074	6> OCT +233000000000
00011		CLA 1.4	00075	OCT +000000077777
00012		STA 8A	00076	OCT +000000000000
00013		CLA 2.4	00077	OCT +000001000000
00014		STA 8A+6	00100	OCT +000000000000
00015		STA 8A+18	00101	BCD 14.7>
00016	8A	CLA SIGN	00102	BCD 1 1P8E1
00017	8A1	TZE 9A	00103	8>9 BCD 1 C1H0.
00020		TPL 16A	00104	BCD 1.12A6>
00021	9A	CAL 2>	00105	BCD 1 .16
00022		BSS	00106	BCD 1 RAY
00023		TSX CSTHD.4	00107	BCD 1DINARY
00024	10A	PZE 82.4	00110	BCD 1 OR
00025		LDD 10	00111	8>7 BCD 1C22H1
00026	11A	STR	00112	BCD 16>
00027	12A	LXD 2>+1.1	00113	BCD 116.12A
00030		LDD RECORD+1.1	00114	BCD 1AY
00031	12A1	STR	00115	BCD 1NARY R
00032	12A2	TXI +1.1.1.1	00116	BCD 1A-ORDI
00033	14A	TXL 12A.1.12	00117	BCD 1 EXTR
00034	15A	BSS	00120	82.4 BCD 1C25H1
00035	16A	TSX CFILS.4		
00036		TBA 22A		
00037		CAL 2>		
00040	17A	BSS		
00041		TSX CSTHD.4		
00042	18A	PZE 82.7		
00043	19A	LDD 10		
00044		STR		
00045	19A1	LXD 2>+1.1		
00046	19A2	LDD RECORD+1.1		
00047	21A	TXI +1.1.1.1		
00048	21A	TXL 19A.1.12		
00049		BSS		
00050	22A	TSX CFILS.4		
00051		CAL 2>		
00052		BSS		
00053	23A	TSX CSTHD.4		
00054	24A	PZE 82.9		
00055	24A	LXD 2>+1.1		
00056	24A1	LDD 8+1.1		
00057	24A2	STR		
00058	24A1	TXI +1.1.1.1		
00059	24A2	TXL 24A.1.12		
00060	26A	LXD 2>+1.1		
00061	27A	LDD 8+1.1		
00062	27A	STR		
00063	27A1	TXI +1.1.1.1		
00064	27A2	TXL 27A.1.12		

O. SUBROUTINE OUTPUT

This subroutine will presently yield the output data on Tape Unit 6 that is summarized in Tables 5 and 6. Following the information that is the result of subroutine OUTONE, Tables 5-1 and 5-2 illustrate the information obtainable after each integration. Beginning with the first word this information has the following meaning:

- Integration number W(70)
- Vector components V(2), V(3)
- Height above earth surface (km)
- Angle θ in degrees
- Angle φ in degrees
- Vector components V(7) through V(21)
- Vector components W(1) through W(11)
- Vector components W(28), W(29), W(38), W(39)
- Vector components W(55) through W(70)
- Vector components W(80) through W(90)

Under certain conditions of vector components W(68) and W(69) the R vector described in subroutine RINDEX will be listed between each of these sets of data for each time that the RINDEX subroutine is entered.

In addition to this data, the data summarized in Table 6 is listed on Tape Unit 10.

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
G 32165	76674	N 32188	76674	RECORD 32561	77461	V 32549	77445
W 32187	76673	Y 32180	76664	Z 32173	76655		

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	1	00000	5	5	00000	50	6	00000	11	7	00000
32	9	00000	3	13	00016	4	14	00020	5	17	00047
7	23	00074	7	24	00077	9	26	00124	9	40	00163
10	46	00177	30	53	00215	31	54	00220	31	55	00223
40	56	00255									

STORAGE NOT USED BY PROGRAM

DEC	OCT
234	00352
	32156 76634

LOCATIONS OF NAMES IN TRANSFER VECTOR

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
RECORD	0 00000	(FIL)	2 00002	CSTH	1 00001		

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
IN	233 00351	11	232 00350	TE1	231 00347	TE2	230 00346
							TE3 229 00345

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
8511	222 00346	8510	197 00305	85C	218 00332	85B	221 00335
15	226 00342	25	177 00261	35	186 00272	65	190 00276

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY

RECORD	CSTH	(FIL)

00000	RCOORD	BCD	IRCOORD	00071	22A	LDR M-56	00156	37A	LXD 23+8,1
00001	CSHD	BCD	ICSHD	00072		FMP 3D+2	00157	38A	LDR M+1,1
00002	CFIL	BCD	ICFIL	00073		STO M-56	00160		STR
00003	S	MTR		00074	23A	CAL 2D+2	00161	38A1	TXI +1,1,1
00004		MTR				BSS	00162	38A2	TXL 38A,1,90
00005		MTR		00075		TSX CSHD,4	00163	40A	BSS
00006		BCD	IOUPT	00076		PZE 8D	00164	41A	TSX CFIL,4
00007		SXD S,1		00077	24A	LDR M-69	00165	41A1	CLA II
00010		SXD S+1,2		00100		STR	00166	42A	TZE 42A
00011		SXD S+2,4		00101		LDR V-1	00166	53A	TPL 53A
00012	11A	BSS		00102		STR	00167	42A	CAL 23+3
00013	12A	TSX RCOORD,4		00103		LDR V-2	00170		BSS
00014	12A1	CLA M-69		00104		STR	00171		TSX CSHD,4
00015		TZE 13A		00105		LDR TE3	00172	43A	PZE 8D8
00016	13A	TPL 14A		00106		STR	00173	44A	LXD 23+4,1
00017		STO 11		00107		LDR TE1	00174		LDR RECORD+1,1
00020	14A	CLA M-69		00110		STR	00175	44A1	STR
00021		FAD 3D		00111		LDR TE2	00176	44A2	TXI +1,1,1
00022		STO M-69		00112		STR	00176	46A	TXL 44A,1,12
00023	15A	CLA M-69		00113		LDR V-6	00177		BSS
00024		UFA 62		00114		STR	00177		TSX CFIL,4
00025		LRS		00115		LDR V-7	00200	47A	CAL 23+2
00026		ANA 6D+1		00116		STR	00201		BSS
00027		LLS		00117		BSS	00202		TSX CSHD,4
00030		ALS 18		00120	25A	TSX CFIL,4	00203	48A	PZE 8D8
00031		STO 1M				CAL 23+2	00204	49A	LXD 23+4,1
00032	16A	LDR M-1		00121		BSS	00205		LDR RECORD+1,1
00033		FMP M-1		00122		TSX CSHD,4	00206	49A1	STR
00034		STO 1D+1				PZE 8D1	00207	49A2	TXI +1,1,1
00035		LDR M		00123		BSS	00210	51A	TXL 49A,1,12
00036		FMP M		00124	26A	TSX CFIL,4	00211	52A	BSS
00037		FSD 1D+1				CAL 23+2	00212		TSX CFIL,4
00040		STO 1D+2		00125		BSS	00213		CAL 23+3
00041		LDR M-1		00126		TSX CSHD,4	00214		BSS
00042		FMP 3D+1		00127	27A	PZE 8D8	00215	53A	TSX CSHD,4
00043		XCA		00130	28A	LXD 23+6,1	00216		PZE 8D8
00044		FMP M		00131		LDR V+1,1	00217		BSS
00045		FMP 1D+2		00132	28A1	STR	00220	54A	TSX CFIL,4
00046		STO M-58		00133	28A2	TXI +1,1,1	00221		CLA II
00047	17A	LDR V-4		00134	30A	TXL 28A,1,21	00222		CLD 11
00050		FMP 3D+2		00135	31A	LXD 23+4,1	00223	55A	ADD 23+4
00051		STO TE1		00136		LDR M+1,1	00224		STO 11
00052	18A	LDR V-5		00137	31A1	STR	00225		STR
00053		FMP 3D+2		00140	31A2	TXI +1,1,1	00226		LDR M-59
00054		STO TE2		00141	33A	TXL 31A,1,11	00227		STR
00055	19A	CLA V-3		00142		LDR M-27	00230		LDR V-1
00056		FSD M-18		00143		STR	00231		STR
00057		STO TE3		00144		LDR M-28	00232		LDR M-82
00061		LDR 3D+3		00145		STR	00233		STR
00062		FMP V-10		00146		LDR M-37	00234		LDR M-81
00063		FSD M-54		00147		STR	00235		STR
00064	21A	STO M-55		00150		LDR M-38	00236		LDR M-61
00065		CLA M-16		00151	34A	LXD 23+7,1	00237		STR
00066		FSD M-56		00152	35A	LDR M+1,1			
00067		XCA		00153		STR			
		FMP 3D+2		00154	35A1	TXI +1,1,1			
		STO M-57		00155	35A2	TXL 35A,1,70			

00240	STR		00331	BCD 1 IN-5X
00241	LQ	W-60	00332	BCD 1 CSH0
00242	STR		00333	BCD 16
00243	LQ	W-79	00334	BCD 13X.128
00244	STR		00335	BCD 1C1H1.2
	BSS	CFILJ.4	00336	BCD 1C1H.2
00245	TSX	CL11	00337	BCD 114.75
00246	CLA	11	00340	BCD 1 1P8E
00247	SUB	25+5	00341	BCD 1 C1H.
00250	TZE	57A		
00251	TPL	57A		
00252	TIR	58A		
00253	CLA	25		
00254	STO	11		
00255	LXD	S.1		
00256	LXD	S+1.2		
00257	LXD	S+2.4		
00260	TIR	1.4		
00261	ACT	+000000000000		
00262	ACT	+000002000000		
00263	ACT	+000006000000		
00264	ACT	+000012000000		
00265	ACT	+000001000000		
00266	ACT	+000062000000		
00267	ACT	+000011000000		
00270	ACT	+000067000000		
00271	ACT	+000120000000		
00272	ACT	+201400000000		
00273	ACT	+202400000000		
00274	ACT	+206712273407		
00275	ACT	+223444607400		
00276	ACT	+233000000000		
00277	ACT	+000000777777		
00300	ACT	+000000000000		
00301	ACT	+000001000000		
00302	ACT	+000000000000		
00303	BCD	114.75		
00304	BCD	14.1P8E		
00305	BCD	1C1H.1		
00306	BCD	1LT-RD		
00307	BCD	1X.6H0E		
00310	BCD	1LE R.7		
00311	BCD	1.7H0A.6		
00312	BCD	1T-E.7X		
00313	BCD	1.6H0EL		
00314	BCD	1E E.8X		
00315	BCD	17H0A.6L		
00316	BCD	105.6X.		
00317	BCD	105-9C6		
00320	BCD	111HVC1		
00321	BCD	1X.		
00322	BCD	1C602.4		
00323	BCD	112+C.4		
00324	BCD	113HVC1		
00325	BCD	125.6X.		
00326	BCD	1X.4HVC		
00327	BCD	INTR. 6		
00330	BCD	1.6HSLQ		

P. INPUT-OUTPUT

Table 4 illustrates the format that is necessary for the input data. Tables 5 and 6 illustrate the format of the output data that is currently obtained by use of Subroutines OUTONE and OUTPUT. These two subroutines can be easily modified. As they are presented in this report their primary purpose was to collect "debugging" data.

Table 4. Input Data for a Spherical Ionosphere.

ORDINARY RAY TOTSIMPLE SPHERE 10**33/R**12 Z = 0.0 V VERY SMALL FEB. 3.1961 E=260G

1.000000E 00 0.	5.280000E 09 0.	0.	0.	0.	0.
0.	0.	0.	0.	4.6059238E 00	1.5707963E 00 7.2082097E-01
+5.578560E-1 0.	6.3569112E 03 1.000000E 00	7.9412480E-01	4.6059238E 00	3.000000E 02	1.0000000E 00
5.4377431E 01 0.	0.	0.	6.6569111E 03-4.9581701E 02	-4.6394721E 03	
+7.480446E 03 1.000000E 00	33 1.2000000E 01 0.	0.	0.	1.2000000E 03	
0.	5.9999999E-06 5.0000000E 01	1.000000E 00 0.	0.	5.0000000E-01 0.	
1.000000E 00 0.	0.	-5.0743736E 02-4.7486789E 03	4.1955717E 03 0.	0.	
0.	0.	0.	0.	3.0000000E 00 0.	
0.	1.000000E 00-1.000000E 00	9.9999999E-05 0.	0.	2.2802051E 00	
1.000000E 00 0.3579108E 03	8.4965302E-01	4.6059238E 00	4.3866078E-01-3.9865271E-01	-2.3434196E-08	2.2802051E 00
5.000000E-06 0.	0.	0.	0.	0.	
0.	0.	0.	0.	0.	
1.000000E 00 3.2802051E 00	1.000000E 00	1.4383545E 00	4.8673434E 01	2.6389999E 02	4.3278772E-01-8.9859070E-01
-2.2423490E-08	3.2802051E 00	1.00339017E-05 0.	4.3878772E-01	-1.4132452E-04	-4.9081066E-12 1.2699290E-04
0.2011474E-05	-1.2951176E-12	1.000000E 00	3.3333333E-06 0.	9.999999E-01 0.	6.2800000E 09
-5.0457822E-19	-2.4658421E-19	8.2774176E-28	-4.5618969E-13	5.0024484E-09	-1.2781570E-15-5.8136627E-19
5.6001182E 02	-3.6324143E-01	5.7859963E 04	1.0510761E 00	3.2802163E 00-2.2964180E-03	2.5994452E 01 5.5473943E-03
-0.	3.2802163E 00	2.5994452E 01	3.000000E 00 0.	1.3267062E-08	8.4883252E-11
1.000000E 00-1.000000E 00	9.9999999E-05	1.000000E 00 0.	0.	5.1358793E-03-1.1175871E-05	-2.2964180E-03
0.	0.	0.	0.	0.	

Table 5-1. Output Data for "Debugging" Purposes from Tape 6.

SAMPLE SPHERE 10**33/R**12 Z = 0.0 V VERY SMALL FEB 3 1961 E=2600									
2.0000000E-00	4.2802051E-00	1.0000000E-00	1.8771373E-00	4.8665335E-01	2.6389999E-02	4.3891476E-01	8.9852868E-01		
-2.3436785E-08	4.2802051E-00	1.4267350E-05	0.	4.3991478E-01	1.4130502E-04	4.9086421E-12	1.2696661E-04		
6.2020858E-05	1.2952606E-12	1.0000000E-00	3.3333333E-06	0.	9.9999998E-01	0.	6.2800000E-09		
-5.1581012E-19	2.5196378E-19	8.4201920E-28	4.6710138E-13	5.1192869E-09	1.3804648E-15	5.7406062E-19	1.3808698E-20		
5.5901442E-02	5.6334660E-01	5.8481141E-04	1.0738028E-00	4.2801423E-00	2.5190779E-03	2.5996310E-01	3.6895828E-03		
-1.0000000E-00	4.2801423E-00	0.	2.5996299E-01	3.0000000E-00	1.3265523E-08	8.6718626E-11			
0.	0.	0.	0.	0.	0.	0.	0.		
3.0000000E-00	5.2802051E-00	1.0000000E-00	2.3161621E-00	4.8657241E-01	2.6389999E-02	4.3904172E-01	8.9846666E-01		
-2.3438080E-08	5.2802051E-00	1.7600684E-05	0.	4.3904173E-01	1.4128551E-04	4.9091917E-12	1.2694032E-04		
6.2030233E-19	2.5747120E-19	8.5658406E-28	4.7829354E-13	5.2390857E-09	1.4420931E-15	5.8643903E-19	1.4707853E-20		
5.5801687E-02	5.6345218E-01	5.9111220E-04	1.0970659E-00	5.2802448E-00	2.5996329E-03	2.5996329E-03	3.4701630E-03		
0.	5.2802448E-00	0.	2.5996639E-01	3.0000000E-00	1.3263984E-08	8.8597317E-11			
1.0000000E-00	1.0000000E-00	9.9999999E-05	3.0000000E-00	0.	3.3604530E-03	3.9637089E-05	3.7182570E-03		
0.	0.	0.	0.	0.	0.	0.	0.		
4.0000000E-00	6.2802051E-00	1.0000000E-00	2.7552430E-00	4.8649147E-01	2.6389999E-02	4.3916864E-01	8.9840461E-01		
-2.3439376E-08	6.2802051E-00	2.0934017E-05	0.	4.3916866E-01	1.4126600E-04	4.9097344E-12	1.2691402E-04		
6.2039598E-05	1.2955476E-12	1.0000000E-00	3.3333333E-06	0.	9.9999998E-01	0.	6.2800000E-09		
-5.3623936E-19	2.6310846E-19	8.7141767E-28	4.8977471E-13	5.3618962E-09	3.9206091E-15	5.9910571E-19	1.4414011E-20		
5.5701944E-02	5.6355817E-01	5.9749170E-04	1.1208736E-00	6.2802376E-00	4.4077635E-03	2.5996768E-01	3.2315332E-03		
0.	5.2802376E-00	0.	2.5996760E-01	3.0000000E-00	1.3262444E-08	9.0519986E-11			
1.0000000E-00	1.0000000E-00	9.9999999E-05	4.0000000E-00	0.	3.2392172E-03	3.2484531E-05	4.4077635E-03		
0.	0.	0.	0.	0.	0.	0.	0.		
5.0000000E-00	7.2802051E-00	1.0000000E-00	3.1945190E-00	4.8641053E-01	2.6389999E-02	4.3929355E-01	8.9834257E-01		
-2.3440671E-08	7.2802051E-00	2.4267350E-05	0.	4.3929356E-01	1.4126649E-04	4.9102771E-12	1.2688773E-04		
6.2048950E-05	1.2956908E-12	1.0000000E-00	3.3333333E-06	0.	9.9999998E-01	0.	6.2800000E-09		
-5.4834790E-19	2.6887931E-19	8.8652668E-28	5.0155580E-13	5.4878119E-09	3.8703605E-15	6.1206927E-19	1.4727375E-20		
5.5802198E-02	5.6369439E-01	6.0395159E-04	1.1452417E-00	7.2802491E-00	5.1159263E-03	2.5997538E-01	2.4420060E-03		
0.	7.2802491E-00	0.	2.5998136E-01	3.0000000E-00	1.3260903E-08	9.2487912E-11			
1.0000000E-00	1.0000000E-00	9.9999999E-05	5.0000000E-00	0.	1.8640013E-03	4.3928623E-05	5.1159263E-03		
0.	0.	0.	0.	0.	0.	0.	0.		
6.0000000E-00	8.2802051E-00	2.0000000E-00	3.6338501E-00	4.8632261E-01	2.6389999E-02	4.3942242E-01	8.9828051E-01		
-2.3441967E-08	8.2802051E-00	2.7600683E-05	0.	4.3942243E-01	1.4126698E-04	4.9108201E-12	1.2686144E-04		
6.2058299E-05	1.2958342E-12	1.0000000E-00	3.3333333E-06	0.	9.9999998E-01	0.	6.2800000E-09		
-5.6172867E-19	2.7478741E-19	9.0191721E-28	5.1363971E-13	5.6169183E-09	4.0111253E-15	6.2533763E-19	1.5048139E-20		
5.5502456E-02	5.6377108E-01	6.1049324E-04	1.1701852E-00	8.2802662E-00	5.8296919E-03	2.5997537E-01	2.4691134E-03		
0.	8.2802662E-00	0.	2.5997435E-01	3.0000000E-00	1.3259361E-08	9.4502312E-11			
1.0000000E-00	1.0000000E-00	9.9999999E-05	6.0000000E-00	0.	2.5632419E-03	6.1035156E-05	5.8296919E-03		
0.	0.	0.	0.	0.	0.	0.	0.		
7.0000000E-00	1.0280205E-01	2.0000000E-00	4.5129395E-00	4.8616781E-01	2.6389999E-02	4.3967609E-01	8.9815336E-01		
-2.3444559E-08	1.0280205E-01	3.4267350E-05	0.	4.3967610E-01	1.4118795E-04	4.9119061E-12	1.2680885E-04		
6.2078964E-05	1.2961202E-12	1.0000000E-00	3.3333333E-06	0.	9.9999998E-01	0.	6.2800000E-09		
-5.8633108E-19	2.7602769E-19	9.3359688E-28	5.3875341E-13	5.8850231E-09	1.5936303E-15	6.5281622E-19	1.5712533E-20		
5.5302984E-02	5.6398542E-01	6.2382469E-04	1.2218503E-00	1.0280153E-01	7.1092036E-03	2.5998337E-01	1.6123515E-03		
0.	1.0280153E-01	0.	2.5998160E-01	3.0000000E-00	1.3256276E-08	9.8674704E-11			
1.0000000E-00	1.0000000E-00	9.9999999E-05	7.0000000E-00	0.	1.8394553E-03	5.2571297E-05	7.1094036E-03		
0.	0.	0.	0.	0.	0.	0.	0.		
8.0000000E-00	1.2380205E-01	2.0000000E-00	5.3925781E-00	4.8600603E-01	2.6389999E-02	4.3992965E-01	8.9803221E-01		
-2.3447131E-08	1.2380205E-01	4.0934017E-05	0.	4.3992966E-01	1.4114891E-04	4.9129924E-12	1.2675626E-04		
6.2095990E-05	1.2964069E-12	1.0000000E-00	3.3333333E-06	0.	9.9999998E-01	0.	6.2800000E-09		
-6.1210802E-19	2.946062E-19	9.4663290E-28	5.6519361E-13	6.1669803E-09	1.6783162E-15	6.8161036E-19	1.6408873E-20		
5.5103499E-02	5.6420140E-01	6.3749802E-04	1.2759997E-00	1.2280224E-01	8.5737705E-03	2.5998542E-01	1.4576050E-03		
0.	1.2280224E-01	0.	2.5998738E-01	3.0000000E-00	1.3253187E-08	1.0304772E-10			

Table 5-2. Output Data for "Debugging" Purposes from Tape 6.

SIMPLE SPHERE 10**33/R**12 Z = 0.0 V VERY SMALL FEB. 3, 1961 E2603

IN	SCANT#	VC25	VC11**C**C60	VC105**C60	ANGLE E	DELTA E	ANGLE A	DELTA A
1	3.2802163E	00	3.2802031E	00-2.2964180E-03	1.1175871E-05	2.5994864E	01	5.1358753E-03
2	4.2801423E	00	4.2802031E	00-2.9190779E-03	6.2832296E-05	2.5995629E	01	4.3706807E-03
3	5.2802448E	00	5.2802031E	00-3.7182570E-03	3.9637089E-05	2.5996639E	01	3.3604530E-03
4	6.2802376E	00	6.2802031E	00-4.4077635E-03	3.2484531E-05	2.5996760E	01	3.2392172E-03
5	7.2802491E	00	7.2802031E	00-5.1159263E-03	4.3928623E-05	2.5998135E	01	1.8640013E-03
6	8.2802662E	00	8.2802031E	00-5.8296519E-03	6.1035156E-05	2.5997436E	01	2.5632419E-03
7	1.0280153E	01	1.0280205E	01-7.1094036E-03	5.2571297E-05	2.5998160E	01	1.8394553E-03
8	1.2380224E	01	1.2380205E	01-8.5737705E-03	1.8477440E-05	2.5998738E	01	1.2614505E-03
9	1.4280193E	01	1.4280205E	01-9.9369287E-03	1.1801720E-05	2.5998878E	01	1.2123586E-03
10	1.6280242E	01	1.6280205E	01-1.1350632E-02	8.5830688E-06	2.5998986E	01	1.0138562E-03
11	1.8280242E	01	1.8280205E	01-1.2772560E-02	3.7192298E-05	2.5999053E	01	9.4662153E-04
12	2.0280136E	01	2.0280205E	01-1.4060602E-02	6.864551E-05	2.5999174E	01	8.2559911E-04
13	2.2280244E	01	2.2280205E	01-1.6954660E-02	3.8862228E-05	2.5999345E	01	6.5377716E-04
14	2.8280242E	01	2.8280205E	01-1.9739389E-02	3.6954880E-05	2.5999345E	01	6.5527127E-04
15	3.2280224E	01	3.2280205E	01-2.2507668E-02	1.8596649E-05	2.5999445E	01	5.5452598E-04
16	3.5280154E	01	3.5280205E	01-2.5224686E-02	5.0544739E-05	2.5999454E	01	5.4556136E-04
17	4.0280238E	01	4.0280205E	01-2.8095245E-02	3.3378601E-05	2.5999654E	01	3.4556488E-04
18	4.4280252E	01	4.4280205E	01-3.0897141E-02	4.8160553E-05	2.5999548E	01	4.5143281E-04
19	5.2280149E	01	5.2280204E	01-3.6366940E-02	5.789948E-05	2.5999658E	01	3.4150946E-04
20	6.0280247E	01	6.0280204E	01-4.2037487E-02	4.1484833E-05	2.5999701E	01	2.9903422E-04
21	6.8280211E	01	6.8280205E	01-4.7575951E-02	6.6757202E-06	2.5999759E	01	2.4097761E-04
22	7.6280181E	01	7.6280205E	01-5.3120613E-02	2.2881844E-05	2.5999778E	01	2.2176770E-04
23	8.4280162E	01	8.4280205E	01-5.8672905E-02	4.2915344E-05	2.5999774E	01	2.2560968E-04
24	9.2280204E	01	9.2280204E	01-6.4210892E-02	7.913568E-05	2.5999810E	01	1.8932431E-04
25	1.0828020E	02	1.0828020E	02-7.5426102E-02	1.144092E-05	2.5999834E	01	1.6605897E-04
26	1.2428027E	02	1.2428020E	02-8.6650848E-02	6.6757202E-05	2.5999837E	01	1.6243043E-04
27	1.4028022E	02	1.4028020E	02-9.7742081E-02	1.144092E-05	2.5999853E	01	1.4684907E-04
28	1.5628020E	02	1.5628020E	02-1.0894012E-01	6.2942505E-05	2.5999884E	01	1.1589977E-04
29	1.7228023E	02	1.7228020E	02-1.2004662E-01	2.479532E-05	2.5999872E	01	1.2742577E-04
30	1.8828029E	02	1.8828020E	02-1.3125420E-01	8.3923340E-05	2.5999889E	01	1.107713E-04
31	2.028024E	02	2.028020E	02-1.5349960E-01	3.6239624E-05	2.5999897E	01	1.0287972E-04
32	2.528023E	02	2.528020E	02-1.7578315E-01	2.6702881E-05	2.5999925E	01	7.4705193E-05
33	2.8428025E	02	2.8428020E	02-2.034073E-01	1.144092E-05	2.5999950E	01	5.0159202E-05
34	3.1628023E	02	3.1628020E	02-2.4254608E-01	1.5258789E-05	2.5999936E	01	6.4246466E-05
35	3.4828026E	02	3.4828020E	02-2.6443481E-01	4.9591064E-05	2.5999938E	01	6.1685145E-05
36	3.8028060E	02	3.8028020E	02-2.8361130E-01	3.4332275E-05	2.5999895E	01	1.1461911E-04
37	4.1228375E	02	4.1228020E	02-2.8361130E-01	3.4332275E-05	2.5999885E	01	1.1461911E-04
38	4.1228375E	02	4.1228020E	02-2.8361130E-01	3.4332275E-05	2.5999885E	01	1.1461911E-04
39	4.2829058E	02	4.2828020E	02-2.8804398E-01	2.2888194E-05	2.5999618E	01	3.8163682E-04
40	4.4431643E	02	4.4428020E	02-2.7328873E-01	3.8146973E-06	2.5998635E	01	1.3647572E-03
41	4.6043275E	02	4.6028020E	02-1.6714095E-01	4.5776367E-05	2.5993755E	01	6.2440735E-03
42	4.1228375E	02	4.1228020E	02-2.8361130E-01	3.4332275E-05	2.5999885E	01	1.1461911E-04
43	4.2028614E	02	4.2028020E	02-2.8687668E-01	3.8146973E-06	2.5999787E	01	2.1258964E-04
44	4.2829058E	02	4.2828020E	02-2.8806305E-01	3.0517578E-05	2.5999618E	01	3.8163682E-04
45	4.3629912E	02	4.3629020E	02-2.8503036E-01	3.8146973E-06	2.5999311E	01	6.8978189E-04
46	4.4431643E	02	4.4428020E	02-2.7322006E-01	2.6702881E-05	2.5998628E	01	1.3711605E-03
47	4.4431643E	02	4.4428020E	02-2.7322006E-01	2.6702881E-05	2.5998628E	01	1.3711605E-03
48	4.4833113E	02	4.4828020E	02-2.6126862E-01	3.4332275E-05	2.5998039E	01	1.9606912E-03
49	4.5235265E	02	4.5228020E	02-2.4251938E-01	1.9073486E-05	2.5997151E	01	2.8484023E-03
50	4.5638447E	02	4.5628020E	02-2.1312332E-01	6.8664455E-05	2.5995794E	01	4.2054754E-03

Table 6-1. Output Data from Tape 10.

SIMPLE SPHERE 10**33/R**12 Z = 0.0 V VERY SMALL FEB. 3, 1961 E=260G

IN	SLANT R	V23	VCT13=C-WC60	VCT03=W60	ANGLE E	DELTA E	ANGLE R	DELTA R
51	4.6043246E 02	4.6028020E 02	1.6741943E-01	6.4849854E-05	2.5993686E 01	6.3136561E-03	0.	0.
52	4.6450379E 02	4.6428020E 02	9.5439911E-02	1.4495850E-04	2.5990346E 01	9.6538320E-03	0.	0.
53	4.6450379E 02	4.6428020E 02	9.5439911E-02	1.4495850E-04	2.5990346E 01	9.6538320E-03	0.	0.
54	4.6655646E 02	4.6628020E 02	4.4773102E-02	2.2888184E-04	2.5987965E 01	1.2034366E-02	0.	0.
55	4.6862007E 02	4.6828020E 02	1.9802094E-02	4.2724609E-04	2.5984883E 01	1.3116489E-02	0.	0.
56	4.7070039E 02	4.7028020E 02	1.0268402E-01	7.2479248E-04	2.5980896E 01	1.9103399E-02	0.	0.
57	4.7280349E 02	4.7228020E 02	2.1018982E-01	1.177063E-03	2.5975617E 01	2.4382921E-02	0.	0.
58	4.7493609E 02	4.7428020E 02	3.5182571E-01	1.9416809E-03	2.5968559E 01	3.1440641E-02	0.	0.
59	4.7710944E 02	4.7628020E 02	5.4186630E-01	3.4027100E-03	2.5958918E 01	4.1081667E-02	0.	0.
60	4.7710944E 02	4.7628020E 02	5.4186630E-01	3.4027100E-03	2.5958918E 01	4.1081667E-02	0.	0.
61	4.7821506E 02	4.7728020E 02	6.6185760E-01	4.6882629E-03	2.5952733E 01	4.7266189E-02	0.	0.
62	4.7933971E 02	4.7828020E 02	8.0370712E-01	6.3629150E-03	2.5945338E 01	5.4661363E-02	0.	0.
63	4.8043876E 02	4.7928020E 02	9.7332001E-01	8.776184E-03	2.5936385E 01	6.3614247E-02	0.	0.
64	4.8165315E 02	4.8028020E 02	1.1788902E 00	1.2409210E-02	2.5925401E 01	7.4598475E-02	0.	0.
65	4.8285357E 02	4.8128020E 02	1.4322815E 00	1.8035889E-02	2.5911684E 01	8.9315413E-02	0.	0.
66	4.8409366E 02	4.8228020E 02	1.7513924E 00	2.6840210E-02	2.5894148E 01	1.0585171E-01	0.	0.
67	4.8409366E 02	4.8228020E 02	1.7513924E 00	2.6840210E-02	2.5894148E 01	1.0585171E-01	0.	0.
68	4.853843E 02	4.8378020E 02	1.9440346E 00	3.322198E-02	2.5883437E 01	1.1656273E-01	0.	0.
69	4.853843E 02	4.8378020E 02	1.9440346E 00	3.322198E-02	2.5883437E 01	1.1656273E-01	0.	0.
70	4.8674426E 02	4.8428020E 02	2.2655302E 00	4.1664124E-02	2.5871027E 01	1.2897275E-01	0.	0.
71	4.8674426E 02	4.8428020E 02	2.2655302E 00	4.1664124E-02	2.5871027E 01	1.2897275E-01	0.	0.
72	4.8819143E 02	4.8578020E 02	2.7265302E 00	5.2982330E-02	2.5856449E 01	1.4355073E-01	0.	0.
73	4.8819143E 02	4.8578020E 02	2.7265302E 00	5.2982330E-02	2.5856449E 01	1.4355073E-01	0.	0.
74	4.8953599E 02	4.8728020E 02	3.0903854E 00	6.8634033E-02	2.5839092E 01	1.6090730E-01	0.	0.
75	4.8953599E 02	4.8728020E 02	3.0903854E 00	6.8634033E-02	2.5839092E 01	1.6090730E-01	0.	0.
76	4.9096599E 02	4.8878020E 02	3.5360030E 00	9.0740204E-02	2.5818072E 01	1.8192742E-01	0.	0.
77	4.9096599E 02	4.8878020E 02	3.5360030E 00	9.0740204E-02	2.5818072E 01	1.8192742E-01	0.	0.
78	4.9237373E 02	4.8978020E 02	4.0955638E 00	1.2361526E-01	2.5792013E 01	2.0798294E-01	0.	0.
79	4.9237373E 02	4.8978020E 02	4.0955638E 00	1.2361526E-01	2.5792013E 01	2.0798294E-01	0.	0.
80	4.9373798E 02	4.9078020E 02	4.8191681E 00	1.7445755E-01	2.5758884E 01	2.4111571E-01	0.	0.
81	4.9373798E 02	4.9078020E 02	4.8191681E 00	1.7445755E-01	2.5758884E 01	2.4111571E-01	0.	0.
82	4.951373E 02	4.9228020E 02	5.2637598E 00	2.5856400E-01	2.5715396E 01	2.8460416E-01	0.	0.
83	4.951373E 02	4.9228020E 02	5.2637598E 00	2.5856400E-01	2.5715396E 01	2.8460416E-01	0.	0.
84	4.9653556E 02	4.9378020E 02	5.7776680E 00	3.2129669E-01	2.5688341E 01	3.1165854E-01	0.	0.
85	4.9653556E 02	4.9378020E 02	5.7776680E 00	3.2129669E-01	2.5688341E 01	3.1165854E-01	0.	0.
86	4.9796599E 02	4.9528020E 02	6.3705139E 00	4.0545654E-01	2.5656798E 01	3.4320142E-01	0.	0.
87	4.9796599E 02	4.9528020E 02	6.3705139E 00	4.0545654E-01	2.5656798E 01	3.4320142E-01	0.	0.
88	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
89	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
90	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
91	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
92	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
93	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
94	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
95	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
96	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
97	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
98	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
99	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.
100	4.993688E 02	4.9678020E 02	7.0464899E 00	5.1227185E-01	2.5620041E 01	3.7925808E-01	0.	0.

Table 6-2. Output Data from Tape 10.

SIMPLE SPHERE 10*33/R**12 Z = 0.0 V VEY SMALL FEB. 3. 1961 E=2605

IN	SCNTR	UC23	UC13*6-UC60	UC100-UC60	ANGLE E	DELTA E	ANGLE R	DELTA R
101	4.9214146E 02	4.9065520E 02	1.7130016E 01	5.3347359E 00	2.406487E 01	1.0935124E 00	0.	0.
102	4.9207153E 02	4.9090520E 02	1.7724288E 01	6.2670479E 00	2.4865093E 01	1.1349088E 00	0.	0.
103	4.9199687E 02	4.9114052E 02	1.8304085E 01	7.204085E 00	2.4822463E 01	1.1753520E 00	0.	0.
104	4.9183705E 02	4.9165520E 02	1.8940559E 01	8.1566183E 00	2.4746152E 01	1.2537408E 00	0.	0.
105	4.9166636E 02	4.9215520E 02	2.0489605E 01	9.1397247E 00	2.4670725E 01	1.3292748E 00	0.	0.
106	4.9148904E 02	4.9285520E 02	2.1518730E 01	10.1204819E 00	2.4597570E 01	1.4024301E 00	0.	0.
107	4.9130426E 02	4.9315520E 02	2.2313950E 01	1.0041428E 01	2.4526440E 01	1.4735602E 00	0.	0.
108	4.9111628E 02	4.9365520E 02	2.3480166E 01	1.0853451E 01	2.4457020E 01	1.5429799E 00	0.	0.
109	4.9092530E 02	4.9415520E 02	2.4421440E 01	1.1657089E 01	2.4389100E 01	1.6108933E 00	0.	0.
110	4.9073188E 02	4.9465520E 02	2.5340874E 01	1.2453209E 01	2.4322460E 01	1.6775333E 00	0.	0.
111	4.9053696E 02	4.9515520E 02	2.6240871E 01	1.3242081E 01	2.4256943E 01	1.7430547E 00	0.	0.
112	4.9034063E 02	4.9565520E 02	2.7123555E 01	1.4024597E 01	2.4192421E 01	1.8075734E 00	0.	0.
113	4.9014326E 02	4.9615520E 02	2.7991978E 01	1.4801239E 01	2.4128764E 01	1.8712355E 00	0.	0.
114	4.8994338E 02	4.9665520E 02	2.8846248E 01	1.5572197E 01	2.4065884E 01	1.9341151E 00	0.	0.
115	4.8974698E 02	4.9715520E 02	2.9688354E 01	1.6338116E 01	2.4003682E 01	1.9963179E 00	0.	0.
116	4.8954836E 02	4.9765520E 02	3.051959E 01	1.7099239E 01	2.3942113E 01	2.0578862E 00	0.	0.
117	4.8915055E 02	4.9865520E 02	3.2152355E 01	1.8608814E 01	2.3820574E 01	2.1794254E 00	0.	0.
118	4.8875388E 02	4.9965520E 02	3.3751873E 01	2.0103142E 01	2.3700806E 01	2.2991537E 00	0.	0.
119	4.8835980E 02	5.0065520E 02	3.523261E 01	2.1584251E 01	2.3582515E 01	2.4174844E 00	0.	0.
120	4.8795968E 02	5.0165520E 02	3.6870712E 01	2.3053741E 01	2.3465406E 01	2.5345933E 00	0.	0.
121	4.8756485E 02	5.0265520E 02	3.8397495E 01	2.4512874E 01	2.3349311E 01	2.6505866E 00	0.	0.
122	4.8717145E 02	5.0365520E 02	3.9906414E 01	2.5962818E 01	2.3234048E 01	2.7659519E 00	0.	0.
123	4.8677946E 02	5.0465520E 02	4.1399868E 01	2.7404682E 01	2.3119443E 01	2.8805563E 00	0.	0.
124	4.8638915E 02	5.0565520E 02	4.2879550E 01	2.8839089E 01	2.3005413E 01	2.9945872E 00	0.	0.
125	4.8600050E 02	5.0665520E 02	4.437145E 01	3.0266825E 01	2.2891845E 01	3.1081551E 00	0.	0.
126	4.8561355E 02	5.0765520E 02	4.5803897E 01	3.1688389E 01	2.2778679E 01	3.2213204E 00	0.	0.
127	4.8484813E 02	5.0865520E 02	4.8689540E 01	3.4513350E 01	2.2553197E 01	3.4468029E 00	0.	0.
128	4.8408381E 02	5.0965520E 02	5.1543705E 01	3.732376E 01	2.2328518E 01	3.6714813E 00	0.	0.
129	4.8332977E 02	5.1065520E 02	5.4371769E 01	4.0114695E 01	2.2104332E 01	3.8956767E 00	0.	0.
130	4.8258300E 02	5.1165520E 02	5.7177875E 01	4.2891628E 01	2.1880354E 01	4.1196459E 00	0.	0.
131	4.8184375E 02	5.1265520E 02	5.9964867E 01	4.5655311E 01	2.1656430E 01	4.3435693E 00	0.	0.
132	4.8111172E 02	5.1365520E 02	6.2735428E 01	4.8407218E 01	2.1432406E 01	4.5675936E 00	0.	0.
133	4.8038703E 02	5.1465520E 02	6.5491314E 01	5.1148151E 01	2.1208151E 01	4.7919387E 00	0.	0.
134	4.7966989E 02	5.1565520E 02	6.8233753E 01	5.3878605E 01	2.0983610E 01	5.0163895E 00	0.	0.
135	4.7896019E 02	5.1665520E 02	7.0964004E 01	5.6332230E 01	2.0758705E 01	5.2412328E 00	0.	0.
136	4.7825773E 02	5.1765520E 02	7.3683174E 01	5.9310753E 01	2.0533375E 01	5.4666247E 00	0.	0.
137	4.7687338E 02	5.1865520E 02	7.9090130E 01	6.4706783E 01	2.0081246E 01	5.9187036E 00	0.	0.
138	4.7552398E 02	5.1965520E 02	8.4456717E 01	7.0068665E 01	1.9627151E 01	6.3728482E 00	0.	0.
139	4.7420058E 02	5.2065520E 02	8.9791607E 01	7.5397701E 01	1.9170812E 01	6.8291877E 00	0.	0.
140	4.7290890E 02	5.2165520E 02	9.5090282E 01	8.0694469E 01	1.8712172E 01	7.2878274E 00	9.8911701E-03	0.
141	4.7165694E 02	5.2265520E 02	1.0035575E 02	8.5959381E 01	1.8251248E 01	7.7467511E 00	0.	0.
142	4.7041619E 02	5.2365520E 02	1.0558853E 02	9.1192539E 01	1.7787991E 01	8.2120034E 00	0.	0.
143	4.6921613E 02	5.2465520E 02	1.078929E 02	9.6394352E 01	1.7322420E 01	8.6773501E 00	0.	0.
144	4.680439E 02	5.2565520E 02	1.1595780E 02	1.0156436E 02	1.6854547E 01	9.1454523E 00	0.	0.
145	4.6691003E 02	5.2665520E 02	1.2109425E 02	1.0670266E 02	1.6364413E 01	9.6155864E 00	0.	0.
146	4.6473008E 02	5.2765520E 02	1.3127106E 02	1.1688388E 02	1.5437478E 01	1.0562522E 00	0.	0.
147	4.6267830E 02	5.2865520E 02	1.4131864E 02	1.2693360E 02	1.4461931E 01	1.1518063E 00	0.	0.
148	4.6075640E 02	5.2965520E 02	1.5123575E 02	1.3685859E 02	1.3518227E 01	1.2481773E 00	0.	0.
149	4.5896391E 02	5.3065520E 02	1.6102116E 02	1.4664932E 02	1.2546701E 01	1.3451245E 00	9.8911701E-03	0.
150	4.5730942E 02	5.3165520E 02	1.7067335E 02	1.5670695E 02	1.1567836E 01	1.443113E 00	0.	0.

SIMPLE SPHERE 10**33/R**12 Z = 0.0 V VERY SMALL FEB. 3, 1961 E=2605

IN	SLANT R	VR23	VC113-C-WC60	VC103-WC60	ANGLE E	DELTA E	ANGLE R	DELTA R					
151	4.557819E	02	6.1165520E	02	1.8019119E	02	1.6583028E	02	1.0582244E	01	1.5417756E	01	0.
152	4.5314751E	02	6.2765520E	02	1.9881790E	02	1.846804E	02	8.5925636E	00	1.7407436E	01	0.
153	4.5106232E	02	6.4365520E	02	2.1689203E	02	2.025327E	02	6.5820493E	00	1.9417950E	01	0.
154	4.4953725E	02	6.5965520E	02	2.3440598E	02	2.2007836E	02	4.953746E	00	2.1444625E	01	0.
155	4.4857310E	02	6.7565520E	02	2.5135401E	02	2.3703752E	02	2.5174736E	00	2.3482526E	01	0.
156	4.4818835E	02	6.9165520E	02	2.6773260E	02	2.5342726E	02	4.7352738E	01	2.5526472E	01	0.
157	4.4836954E	02	7.0765520E	02	2.8354037E	02	2.6924607E	02	1.5715396E	00	2.4428460E	01	0.
158	4.5044008E	02	7.3655520E	02	3.1344744E	02	2.9917554E	02	5.6436183E	00	2.0356381E	01	0.
159	4.5479845E	02	7.7165520E	02	3.4110675E	02	3.2683715E	02	9.6583892E	00	1.6341610E	01	0.
160	4.6120162E	02	8.0365520E	02	3.6658133E	02	3.5235401E	02	1.3578833E	01	1.2421166E	01	0.
161	4.6983384E	02	8.3365520E	02	3.8996181E	02	3.7575679E	02	1.7373080E	01	8.6269197E	00	0.
162	4.8043752E	02	8.6765519E	02	4.1136072E	02	3.9717800E	02	2.1015699E	01	4.9843011E	00	0.
163	4.9287056E	02	8.9655520E	02	4.3090550E	02	4.1674507E	02	2.4468013E	01	1.5119870E	00	9.8911701E-03
164	5.074979E	02	9.2655520E	02	4.498168E	02	4.5036583E	02	3.0879896E	01	4.8798959E	00	0.
165	5.2833539E	02	1.076552E	03	4.9325149E	02	4.7922024E	02	3.651911E	01	1.051911E	01	0.
166	5.4877833E	02	1.0916552E	03	5.1686395E	02	5.0283729E	02	4.1440417E	01	1.5440418E	01	0.
167	5.700655E	02	1.1956552E	03	5.3659111E	02	5.2260904E	02	4.5712849E	01	1.9712850E	01	0.
168	5.9038150E	02	1.296552E	03	5.5321150E	02	5.3927402E	02	4.9417751E	01	2.3417752E	01	0.
169	6.1019750E	02	1.386552E	03	5.6732103E	02	5.5342812E	02	5.2635686E	01	2.6635687E	01	0.
170	6.301546E	02	1.411655E	03	5.8930389E	02	5.7600017E	02	5.7894388E	01	3.1894388E	01	0.
171	6.5051533E	02	1.5396552E	03	6.0676255E	02	5.9304810E	02	6.1961260E	01	3.5961260E	01	0.
172	6.713438E	03	1.6673552E	03	6.1991215E	02	6.0628677E	02	6.5170284E	01	3.9170285E	01	0.
173	6.923667E	03	1.7936552E	03	6.3035513E	02	6.1631892E	02	6.7751948E	01	4.1751949E	01	0.
174	7.132442E	03	1.9336552E	03	6.3982043E	02	6.2537339E	02	6.9865861E	01	4.3865861E	01	0.

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