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LANDING SITE COVERAGE FOR

ORBITAL LIFTING RE-ENTRY VEHICLES

R. G. Stern
S. T. Chu

1 April 1963

Prepared for

COMMANDER SPACE SYSTEMS DIVISION
UNITED STATES AIR FORCE
Inglewood, California

Contract No. AF04(695)-169

AEROSPACE CORPORATION

Systems Research and Planning Division

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Prepared by _R. G. Stern_

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ABSTRACT

The problem of landing site coverage of orbiting vehicles with maneuvering capability is analyzed in this report. The first part deals with the determination of the latitude coverage of a landing site per orbital period for the re-entry vehicle with a given maneuver capability defined by a rectangularly shaped "landing footprint." The second part relates the required latitude coverage to guarantee that within a specific waiting time in orbit the vehicle would attain a position from which it could be deboosted from orbit and subsequently land at a preselected location.

Extensive numerical results are obtained and presented in convenient graphical form. For example, the magnitude of the latitude coverage is presented in two ways: (1) as a function of orbital inclination for various cross-range maneuver capabilities and a given landing-site latitude of 35 degrees; (2) as a function of landing-site latitude for various cross-range maneuver capabilities and a given orbital inclination of 35 degrees. Similarly, the cross-range maneuver requirements for an orbiting vehicle with a permissible waiting time of one day and a fixed orbital period are presented for either a given landing-site latitude of 35 degrees, or a given orbital inclination of 35 degrees.

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I. INTRODUCTION

Space vehicles returning from orbit can use aerodynamic maneuver to reach preselected landing sites. The need of maneuver capability arises because:

    1. Imperfect control of the trajectory in space will result in deviations from the intended entry conditions. The atmospheric trajectory must then be modified by some form of aerodynamic maneuvering if the landing site is to be reached.

    2. Requirement of mission flexibility precludes fixed entry conditions to be selected prior to the completion of a mission. The orbiting vehicle, there- fore, should be considered as randomly placed in orbit. The locations of per- missible deboost points will depend upon the maneuver capability of the vehicle coupled with the allowed in-orbit waiting time.

The maneuvering required for correcting errors due to imperfect trajectory control is, in general, relatively small. The need of attaining mission flexibility, on the other hand, may require a sizable maneuver capability.

This report studies the problem of landing-site coverage for an orbiting lifting vehicle with mission flexibility in mind.

The landing-site coverage can be defined as the distance along the latitude of the landing site in which a potential landing may be made. This coverage will be referred to hereafter as the "latitude coverage." The latitude coverage per orbital period depends on the orbital inclination and period, heading and landing- site latitude, as well as the maneuver capability of the re-entry vehicle.

Two associated problems are considered within this study. The first part deals with the determination of the latitude coverage of a landing site per orbital period for the re-entry vehicle with a given maneuver capability defined by a rectangularly shaped "landing footprint." The second part relates the required latitude coverage to guarantee that, within a specific waiting time in orbit, the vehicle would attain a position from which it could be deboosted from orbit and subsequently land at a preselected location.

The purpose of this study is to assist in determining the maneuvering requirements of lifting re-entry vehicles returning from circular orbits. The maneuver capabilities of a re-entry vehicle, on the other hand, are essentially imposed by the re-entry conditions, the aerodynamic characteristics of the vehicle and the flight path control technique chosen for the re-entry trajectory.

II. ANALYSIS

A. Latitude Coverage of a Lifting Re-entry Vehicle

The latitude coverage per pass is defined as the distance along the latitude of the landing site in which the re-entry vehicle could land on any given orbital pass. The latitude coverage per pass depends upon the landing-site latitude and the vehicle orbital period, inclination and heading, along with the maneuver capability of the vehicle. The latter depends upon the re-entry conditions, flight path control and aerodynamic characteristics of the vehicle.

To facilitate analysis, a number of assumptions are made:

1. The vehicle is in a circular orbit prior to deboosting.

2. The landing footprint is assumed to be rectangularly shaped. This assumption simplifies the analysis but does not significantly degrade the usefulness of the result.

3. The effect of the earth's rotation or the point where re-entry occurs has no effect upon the shape of the landing footprint.

4. The flight times from deboost to landing for all points on the leading edge of the landing footprint are assumed to be the same. Also the flight times to the trailing edge of the landing footprint are assumed to be the same (but different from that of the leading edge). It is assumed that the leading edge flight times are greater than those of any other point within the landing footprint and the flight times to the trailing edge are less than any other point within the footprint.

5. The landing site was assumed to be in the northern hemisphere. The same results apply in the southern hemisphere except that when directions or orbital headings are involved in the southern hemisphere, the word south must be interchanged with north, and vice versa.

6. The effect of the average daily precession of the orbit plane is accounted for; however, oscillatory orbital precession terms are neglected.

7. Orbital periods less than twenty-four hours are assumed.

Two general modes of latitude coverage should be considered depending upon the cross-range maneuver capability, orbital inclination, and landing-site latitude. These modes are illustrated in Figure 2 by belts indicating where landings could be achieved on a given orbital pass. In Figure 2a, two separate belts of latitude coverage exist; while in Figure 2b, one continuous belt of latitude coverage is generated. The first of these coverage modes will be referred to as high orbital inclination, while the latter will be referred to as low orbital inclination. Figure 2c represents the bordering condition between high and low orbital inclinations. In this case, the sum of the landing-site latitude and the cross-range maneuver capability expressed in great circle arc length equals the orbital inclination. If the sum of the landing-site latitude and the cross-range maneuver capability is decreased, or the orbital inclination increased, the mode of coverage changes from a low to a high orbital inclination.

In passing, it is pointed out that a third class of orbital inclination exists where the landing-site latitude is greater than the sum of the orbital inclination and the cross-range maneuver capability of the vehicle. In this case it becomes impossible for the vehicle to land at the latitude of the landing site.

Because of the earth's rotation and the orbital precession, it is convenient to consider the projection of the geometry of the orbiting vehicle on a nonrotating sphere which coincides with the spherical earth surface, but is fixed with respect to the orbital plane. This sphere will be referred to as the reference sphere.

Since the problem at hand is to determine the coverage along the landing-site latitude, the coverage will be initially determined along the

projection of the landing-site latitude upon the reference sphere. This coverage on the reference sphere can be translated back to earth-fixed coordinates by accounting for the relative motion between the earth and the orbital plane while this coverage is being generated.

It is assumed that the time from deboost to landing is constant for a given point within the landing footprint. The center point on the leading edge of the footprint is arbitrarily chosen as a reference point. This point may be envisioned as passing over the surface of earth at orbital velocity. While this trace will be displaced from the actual trace of the orbiting vehicle, it will have the same projection upon the reference sphere which is rotating with the orbit plane. For convenience, the displacement will be neglected because it does not affect the magnitude of the latitude coverage. The orbit trace and the trace of the point at the center of the leading edge of the footprint will be considered identical. All points on the leading edge of the footprint are assumed to require the same time from deboost to landing. Therefore, all points along an arc on the reference sphere perpendicular to the orbit trace, within a distance from the orbit trace equal to the cross-range maneuver capability, can be reached at the same time.

1. **Low Orbital Inclination Mode**

Let us first consider the mode of low orbital inclinations. The latitude coverage via the low orbital inclination mode is determined by initially establishing the latitude coverage upon the reference sphere. This coverage extends over the landing-site latitude between two limiting points as indicated in Figure 3. The limiting points are defined by the condition when the arc length representing the cross-range maneuver capability is in contact with the reference latitude. From this figure it can be seen that the latitude coverage on the reference sphere is given by $2\Delta L_c$ degrees of longitude.

Indicated in Figure 3 are several spherical triangles which will be used to derive ΔL_c as a function of the orbital inclination, θ_o, landing-site latitude, ψ, and the cross-range maneuver capability, ℓ.

From spherical triangle II of Figure 3 we have,

$$\frac{\sin 90^{\circ}}{\sin y} = \frac{\sin (90^{\circ} - \alpha)}{\sin l} \quad \text{or,} \quad \cos \alpha = \frac{\sin l}{\sin y} \tag{1}$$

From triangle I of Figure 3, we may write,

$$\cos (90^{\circ} - \psi) = \cos y \cos (90^{\circ} - \theta) + \sin y \sin (90^{\circ} - \theta) \cos \alpha$$

or,

$$\sin \psi = \cos y \sin \theta_o + \sin y \cos \theta_o \cos \alpha \tag{2}$$

Substituting (1) into (2) we have,

$$\cos y = \frac{1}{\sin \theta_o} (\sin \psi - \cos \theta_o \sin l) \tag{3}$$

Also from triangle I, we have,

$$\cos y = \cos (90^{\circ} - \theta_o) \cos (90^{\circ} - \psi) + \sin (90^{\circ} - \theta_o) \sin (90^{\circ} - \psi) \cos \Delta L_c \tag{4}$$

Simplifying (4) and equating (3) and (4) we have

$$\cos \Delta L_c = \frac{1}{\cos \theta_o \cos \psi} \left(\frac{\sin \psi}{\sin \theta_o} - \frac{\sin l}{\tan \theta_o} - \sin \theta_o \sin \psi \right) \tag{5}$$

The above is a solution for ΔL_c in terms of l, ψ and θ_o and may be written in a simpler form as,

$$\cos \Delta L_c = \frac{\tan \psi}{\tan \theta_o} - \frac{\sin l}{\sin \theta_o \cos \psi} \tag{6}$$

To transfer the latitude coverage along the reference sphere to the earth it is necessary to know the time difference between the westernmost coverage and easternmost coverage. The arc $2X_c$ in Figure 3 is the orbital distance traveled between the two extreme points of latitude coverage.

The value of X_c will be determined now with the aid of Figure 3. From triangle II we have,

$$\cos y = \cos X_c \cos l \tag{7}$$

Equating (4) and (7) we have,

$$\cos X_c = \frac{1}{\cos l} (\sin \theta_o \sin \psi + \cos \psi \cos \theta_o \cos \Delta L_c) \tag{8}$$

Utilizing this distance and the orbital period, the time difference between the extreme points of coverage may be determined. Having determined the latitude coverage on the reference sphere it is now necessary to transform this coverage to earth coordinates. The latitude coverage on the reference sphere as given by equation (5) is a function of orbital inclination, landing site latitude, and cross-range maneuver capability. The latitude coverage along earth coordinates is, in addition, a function of orbital heading,[*] orbital period, orbital precession rate, the downrange maneuver capability and the arc length X_c. The relative motion between the reference sphere and fixed earth coordinates is given by the following relation:

$$\omega = \omega_e + \Omega \tag{9}[**]$$

where ω_e is the earth's rotational rate and Ω is the orbital precession rate with a rate in the westerly direction being taken as positive.

Consider first a limiting footprint of an easterly orbiting vehicle which has cross-range maneuver capability but no downrange maneuver capability. The landing-site latitude may be considered to have a rate of

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[*] For convenience the usual orbital inclination notation where the inclination is measured from that of an object headed in an easterly direction while in an equatorial orbit is dropped. The inclination in this case refers only to the magnitude of the acute angle between the orbit plane and the equatorial plane. Specification of the orbital heading is therefore required to determine if the vehicle is traveling in an easterly or westerly direction in the orbital plane.

[**] $\omega_e = 15.04$ deg/hr, $\Omega = -0.925 \, AT^{-7/3} \cos \theta_o$ where Ω is derived from Reference 1 and $A = +1$ for a westward orbital heading and -1 for an eastward orbital heading.

rotation, ω, in an easterly direction on the reference sphere. Since the vehicle is orbiting in an easterly direction the western boundary of the latitude coverage band will be established first (see Figure 4). This point will then travel eastward on the reference sphere at a rate equal to ω. At the same time, the leading edge of the landing footprint proceeds eastward along the orbit trace at the orbital rate. The arc length of the orbit trace between the establishment of the eastern and western boundaries is given by $2X_c$ which was determined previously. Given the orbital period (T) the time to traverse the distance $2X_c$ is given by the following:

$$\text{Time difference} = \frac{2X_c}{360} T, \text{ where } X_c \text{ is in degrees.} \tag{10}$$

The projected motion along the reference sphere of the point on the earth's latitude defining the western boundary between the time when it is established, and when the eastern boundary is established, is equal to $(2X_c/360)\omega T$. This distance represents the difference between the latitude coverage on the reference sphere and the actual latitude coverage in terms of earth-fixed coordinates. The latitude coverage achieved by the low orbital inclination mode with the vehicle heading in an easterly direction with no downrange maneuver capability is given by the following:

$$L_{LE} = 2 \Delta L_c - \frac{2X_c}{360} \omega T \tag{11}$$

If instead of orbiting in an easterly direction the vehicle is heading westward, the situation as depicted in Figure 5 exists. In this case the eastern boundary is established initially. This point on the earth then moves eastward on the reference sphere while the leading edge of the landing footprint proceeds westward to establish the western boundary. The latitude coverage in this case is given by the following relation:

$$L_{LW} = 2 \Delta L_c + \frac{2X_c}{360} \omega T \tag{12}$$

The latitude coverage relationships developed thus far have not
as yet given consideration to the downrange maneuver capability. For both
orbiting vehicles headed eastward or westward the downrange maneuver
capability can be used to increase the latitude coverage. In either case the
effect of downrange capability can be taken into account by establishing the
eastern boundary of the coverage at the first possible chance and establishing
the western boundary at the last possible chance.

Consider the two projections of the landing footprint upon the
reference sphere in Figure 6. The two footprints shown on the reference
sphere indicate the regions attainable from orbital positions A and B respec-
tively. A particular point (as indicated in Figure 6) on the reference sphere
at the landing-site latitude may be reached by deboosting at orbital position A
or B. When deboosting at orbital position A, the specific point on the reference
sphere at the landing site latitude is attained by utilizing the leading edge of the
footprint. The same point on the reference sphere at the landing-site latitude
is attainable by using the trailing edge of the footprint and deboosting at point B.
The footprint has a great circle arc length equal to ΔR.[*] The deboost points A
and B are separated by a range angle equal to ΔR. The flight time from deboost
at any given orbital position to the leading edge of the footprint will differ from
the flight time to the trailing edge. The time difference is here defined as Δt
with a positive value indicating that the flight time to the leading edge is larger
than to the trailing edge.[*] It is observed here that the only points of interest on
the footprint are on the leading and trailing edges as they determine the maximum
and minimum arrival times at a given point on the reference sphere. The time
difference between attaining a point on the reference sphere with the trailing edge
from that with the leading edge is established by the time it takes to move from
orbital position A to B less Δt. Therefore, for the leading edge of the foot-
print to arrive at a point on the reference sphere before the trailing edge it is
required that,

$$\frac{\Delta R}{360} T > \Delta t \qquad (13)$$

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[*]The quantity Δt and ΔR must be determined from studies concerning the
landing footprint.

If the above inequality is reversed then the trailing edge would arrive at the point before the leading edge.

To attain maximum utilization of the downrange maneuver capability for the low orbital inclination mode, the eastern boundary should be established as soon as possible on the reference sphere while the western boundary is established as late as possible. Therefore the latitude coverage should be determined from:

L. E. for eastern boundary, T. E. for western boundary when $[(\Delta R/360) T - \Delta t] > 0$.

T. E. for eastern boundary, L. E. for western boundary when $[(\Delta R/360) T - \Delta t] < 0$. It follows that the contribution of the downrange maneuverability to the latitude coverage depends only on the time difference $[(\Delta R/360) T - \Delta t]$, and is given by $|[(\Delta R/360) T - \Delta t]|$. The complete relationships for latitude coverage using the low orbital inclination mode are given below for east and west heading respectively.

$$L_{LE} = 2 \Delta L_c - \frac{2X_c}{360} \omega T + \left| \left(\frac{\Delta R}{360} T - \Delta t \right) \right| \omega \qquad (14)$$

$$L_{LW} = 2 \Delta L_c + \frac{2X_c}{360} \omega T + \left| \left(\frac{\Delta R}{360} T - \Delta t \right) \right| \omega \qquad (15)$$

In the preceding discussion it was taken for granted that the orbital period was low enough with respect to ω (approximately the earth's rotational rate) such that a projection upon the reference sphere of a point on the landing-site latitude did not travel as quickly along the latitude as the passing footprint. It can be shown that the above situation occurs at orbital periods in excess of 24 hours, the exact value depending on the cross-range maneuver capability, orbital inclination and landing-site latitude. Periods of this magnitude are excluded in this study.

2. High Orbital Inclination Mode

Latitude coverage via the high orbital inclination mode will be considered next. With the high orbital inclination mode two separate belts of latitude coverage are generated during each pass. These coverage belts will be considered separate. Figure 7 represents three typical classifications

of orbit trace intersections with the landing-site latitude projected upon the reference sphere. The orbit trace of a vehicle orbiting in an easterly direction will intersect the landing-site latitude twice during each orbital period, once traveling northeast and once traveling southeast. A vehicle orbiting in a westerly direction will also intersect the landing-site latitude twice, once in the northwesterly direction and once in the southwesterly direction. For both of these headings one of the orbit trace intersections with the landing-site latitude will be similiar to Figure 7a, and the other to Figure 7b. The boundaries of the latitude coverage belt on the reference sphere are established when an arc length normal to the orbit trace and equal to the cross-range maneuver capability exactly touches the latitude of the landing site. It should be noted that whether an orbiting vehicle is heading northeast or southeast the geometry and location of these perpendiculars will be a mirror image. The same is true for vehicles orbiting in northwest and southwest directions. Therefore it is convenient to define the geometry in a common manner. Let us define the term ΔL_a as the latitude coverage on the reference sphere from the point where the orbit trace crosses the landing-site latitude to the coverage boundary which is generated from the cross-range maneuver arc extending from the most northerly point along the orbit trace (see Figures 7a and 7b). Similiarly the term ΔL_b will be defined in the same manner except that the coverage boundary is generated from the cross-range maneuver arc extending from the most southerly point along the orbit trace. Since the two coverage boundaries always lie on opposite sides of the orbital trace, the sum of these two terms defines the latitude coverage on the reference sphere. In order to facilitate transform-ing this coverage into earth-fixed coordinates, a knowledge of the arc lengths along the orbit trace X_a and X_b indicated in Figures 6a and 7b, is required. The arc length X_a is measured upward from the landing-site latitude and is always considered positive. The value X_b is considered positive when it extends downward (southerly) from the landing-site latitude. For polar or very high orbital inclinations it is possible for this arc length to extend upward from the landing-site latitude. An example of this is indicated in Figure 7c in which X_a and X_b are equal in magnitude and X_b is considered negative.

The arc lengths ΔL_a and ΔL_b along the landing-site latitude and the great circle arc lengths X_a and X_b will now be determined as a function of orbital inclination (θ_o), cross-range maneuver capability (μ), and landing-site latitude (ψ).

Before any of the desired arc lengths can be determined, it is necessary to first determine the angle between the orbital trace and the landing-site latitude. Figure 8 shows the orbit trace upon the reference sphere, the landing-site latitude, and the polar great circle arcs (longitudinal arcs) which pass through the intersections of the orbital trace and the landing-site latitude.

From the right spherical triangle of Figure 8 formed by the orbit trace, the equator and the longitudinal arc which passes through the intersection of the reference latitude and the orbit trace, we have

$$\sin \alpha = \frac{\cos \theta_o}{\cos \psi} \tag{16}$$

Further it can be seen that $\theta = (90 - \alpha)$, therefore

$$\cos \theta = \frac{\cos \theta_o}{\cos \psi} \tag{17}$$

With the aid of Figure 9, ΔL_a and X_a will be determined as functions of θ, μ, and ψ.

From spherical triangle I, we have,

$$\frac{\sin 90^\circ}{\sin \widetilde{L}} = \frac{\sin (\theta - 90 + \mu)}{\sin \mu} \quad \text{or,} \quad \frac{1}{\sin \widetilde{L}} = -\frac{\cos (\theta + \mu)}{\sin \mu} \tag{18}$$

From spherical triangle II, we have,

$$\frac{\sin \Delta L_a}{\sin \widetilde{L}} = \frac{\sin \mu}{\sin (90 - \psi)} = \frac{\sin \mu}{\cos \psi} \tag{19}$$

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Substituting (18) in (19), eliminating sin L and rearranging we have,

$$\sin \theta - \frac{\cos \theta}{\tan \mu} = \frac{\sin l}{\sin \Delta L_a \cos \psi} \tag{20}$$

From spherical triangle II, we also have,

$$\cos \mu = -\cos \mu \cos \Delta L_a + \sin \mu \sin \Delta L_a \cos (90 - \psi) \tag{21}$$

Rewriting equation (21), we have,

$$\tan \mu = \frac{1 + \cos \Delta L_a}{\sin \Delta L_a \sin \psi} \tag{22}$$

Substituting (22) in (20) we may write,

$$\frac{\sin^2 \Delta L_a}{1 + \cos \Delta L_a} (\sin \psi \cos \theta) - \sin \theta \sin \Delta L_a = \frac{\sin l}{\cos \psi} \tag{23}$$

Rewriting (23) we obtain,

$$\cos \Delta L_a \sin \psi \cos \theta + \sin \Delta L_a \sin \theta = \sin \psi \cos \theta + \frac{\sin l}{\cos \psi} \tag{24}$$

The above equation may be solved for ΔL_a, in terms of θ, ψ and l. Now we can determine the value of X_a, knowing ΔL_a. From the spherical triangle of Figure 9 we have,

$$\cos \tilde{L} = \cos X_a \cos l \tag{25}$$

From spherical triangle II we have,

$$\cos \tilde{L} = \cos^2 (90 - \psi) + \sin^2 (90 - \psi) \cos \Delta L_a \tag{26}$$

Equating (25) and (26) and rearranging, we have the solution for X_a in terms of ψ, ℓ, and ΔL_a,

$$\cos X_a = \frac{1 - \cos^2 \psi \, (1 - \cos \Delta L_a)}{\cos \ell} \tag{27}$$

Now ΔL_b will be determined with the aid of Figure 10. From spherical triangle I of Figure 10, we have,

$$\frac{\sin 90^{\circ}}{\sin \tilde{L}} = \frac{\sin (\theta + 90 - \mu)}{\sin \ell} \quad \text{or} \quad \frac{1}{\sin \tilde{L}} = \frac{\cos (\theta - \mu)}{\sin \ell} \tag{28}$$

From spherical triangle II of Figure 10, we have,

$$\frac{\sin \Delta L_b}{\sin \tilde{L}} = \frac{\sin \mu}{\sin (90 - \mu)} = \frac{\sin \mu}{\cos \psi} \tag{29}$$

Substituting (28) into (29) and rearranging we have,

$$\sin \theta + \frac{\cos \theta}{\tan \mu} = \frac{\sin \ell}{\sin \Delta L_b \cos \psi} \tag{30}$$

From the spherical triangle of Figure 10 we obtain the following equation in the same manner as equation (22),

$$\tan \mu = \frac{1 + \cos \Delta L_b}{\sin \Delta L_b \sin \psi} \tag{31}$$

Substituting (31) into (30) we have,

$$\frac{\sin^2 \Delta L_b \sin \psi \cos \theta}{1 + \cos \Delta L_b} + \cos \theta \sin \Delta L_b = \frac{\sin \ell}{\cos \psi} \tag{32}$$

and rewriting the above we obtain,

$$\cos \Delta L_b \sin \psi \cos \theta - \sin \Delta L_b \sin \theta = \sin \psi \cos \theta - \frac{\sin \theta}{\cos \psi} \tag{33}$$

The above equation may be solved for ΔL_b in terms of θ, ψ and l. The solution for X_b follows similarly to that of X_a (equation 27) and the result is

$$\cos |X_b| = \frac{1 - \cos^2 \psi (1 - \cos \Delta L_b)}{\cos l} \tag{34}$$

Equation (34) relates only the magnitude of X_b as this distance may lie in either a northerly (negative) or southerly (positive) direction from the landing-site latitude. This problem does not occur with X_a as it is always north of landing-site latitude and is defined as positive.

Now it remains to determine $(X_b / |X_b|)$. This is done with the aid of Figure 10. The value of θ will be determined at which point $X_b = 0$. This angle will be designated θ_x. It then follows that for

a. $\theta \geq \theta_x$ $\dfrac{X_b}{|X_b|} = -1$ (35)

b. $\theta \leq \theta_x$ $\dfrac{X_b}{|X_b|} = +1$ (36)

The angle subtended by the arc $\tilde{L}$ and the landing-site latitude is $(90 - \mu)$. When $X_b = 0$, $\tilde{L}$ and l coincide such that $(90 - \mu) + \theta_x = 90$; therefore, $\theta_x = \mu$.

From the spherical triangle II in Figure 10 and with the above conditions we have,

$$\frac{\sin \Delta L_b}{\sin l} = \frac{\sin \theta_x}{\sin (90 - \psi)} = \frac{\sin \theta_x}{\cos \psi} \tag{37}$$

and

$$\cos l = \cos^2 (90 - \psi) + \sin^2 (90 - \psi) \cos \Delta L_b \tag{38}$$

Rewriting (38) and substituting (37) into it to eliminate ΔL_b we have,

$$\sin^2 \theta_x = \frac{\cos^2 \psi}{\sin^2 l} - \frac{(\cos l - \sin^2 \psi)^2}{\cos^4 \psi \sin^2 l} \tag{39}$$

which enables θ_x and therefore $(X_b / |X_b|)$ to be determined.

With the preceding equations the coverage of latitude on the reference sphere can be obtained when the orbital inclination (θ_o), landing-site latitude (ψ), and cross-range maneuver capability (ℓ) are known. The next step is to transform this coverage on the reference sphere into fixed earth coordinates. For the high orbital inclination mode there are two separate belts of latitude coverage within one orbital period. Therefore it is not sufficient to determine the magnitude of coverage alone, but one must have a knowledge of the gap or spacing between the coverage belts generated during a given orbit. A convenient way to accomplish this is to reference the point on the landing-site latitude in the earth-fixed coordinates as the orbit trace crosses it, (recalling that the orbit trace refers to the center of the leading edge of the landing footprint). The latitude coverage to the east and west of this point will suffice in determining any gaps in the coverage as the distance between the orbit trace intersections with the landing-site latitude would be a known quantity.

Four general classes of passes exist (two passes per orbital period), depending on the orbit traces' heading. They are: (1) northeast heading, (2) southeast heading, (3) northwest heading, and (4) southwest heading. These classifications degenerate into either north or south headings for the case of polar orbits.

Consider first the coverage generated by a vehicle orbiting in a northeasterly direction. No downrange maneuver capability of the vehicle will be included at this time.

Figure 11 shows the projection of an orbit trace with a northeasterly heading on the reference sphere along with the projection of earth-fixed coordinates at various times upon the reference sphere. The reference point is established as the orbit trace (center of the leading edge of the footprint) crosses the landing-site latitude. The boundary of the latitude coverage to the west is established when the cross-range maneuver capability arc extends up perpendicularly from the orbit trace and exactly contacts the landing-site latitude. This occurs at a point along the orbit trace different from the reference point. The distance along the orbit trace between these two points

is denoted by X_b. The coverage of latitude upon the reference sphere is given by ΔL_b. Since the boundary of the latitude coverage to the west of the reference point occurs first (position 1 in Figure 11) this boundary will move eastward on the reference sphere as the leading edge of the footprint moves up to establish the reference point (position 2 in Figure 11). The latitude coverage to the west of the reference point, L_W for this orbit heading with no downrange maneuver capability is given by the following relation

$$L_W = \Delta L_b - \frac{X_b}{360} \omega T \tag{40}$$

The first term represents the coverage of latitude on the reference sphere while the second term represents the motion of the earth with respect to the reference sphere in the time it takes the orbiting vehicle to move from one position to another. The reference point and the western boundary continues eastward on the reference sphere as the orbiting vehicle moves to a new position (position 3 in Figure 11) to the east of the reference point establishing the eastern boundary of the latitude coverage.

The latitude coverage to the east of the reference point, L_E, is then given by a relationship which is similar to the one for L_W that is,

$$L_E = \Delta L_a - \frac{X_a}{360} \omega T \tag{41}$$

Expressions can be deduced for L_E and L_W in a similar manner for vehicles with no downrange maneuver capability with their orbit traces headed southeasterly, northwesterly or southwesterly.

If the heading were southeasterly the expressions for latitude coverage would be

$$L_E = \Delta L_b - \frac{X_b}{360} \omega T \tag{42}$$

and

$$L_W = \Delta L_a - \frac{X_a}{360} \omega T \tag{43}$$

If the orbital heading were northwesterly the expressions for the latitude coverage would be

$$L_E = \Delta L_b + \frac{X_b}{360} \omega T \tag{44}$$

and

$$L_W = \Delta L_a + \frac{X_a}{360} \omega T \tag{45}$$

If the orbital heading were southwesterly the expressions for the latitude coverage would be

$$L_E = \Delta L_a + \frac{X_a}{360} \omega T \tag{46}$$

and

$$L_W = \Delta L_b + \frac{X_b}{360} \omega T \tag{47}$$

The above expressions represent the latitude coverage for orbiting vehicles which possess no downrange maneuver capability. The existance of downrange maneuver capability allows the latitude coverage per orbital pass to be increased. The increase can be accounted for by establishing the western boundary of the latitude coverage at the latest possible time and by establishing the eastern boundary at the earliest possible time. This procedure, identical to that for the low orbital inclination mode, is independent of the direction in which the vehicle is headed.

The coverages which were determined for a vehicle with no downrange maneuver capability may be considered due to the utilization of the leading edge of the footprint alone. This is because the reference point on the landing site latitude is defined on the basis of the leading edge of the landing footprint.

It is recalled from inequality (13) that when $(\Delta R/360) T > \Delta t$ the leading edge arrives at a point before the trailing edge of the footprint. When this occurs the western boundaries of latitude coverage can be established by

the trailing edge of the footprint at the latest possible time. A time period difference which is equal to $(\Delta R/360)T - \Delta t$ results in an increase of the western latitude coverage over that accomplished by the leading edge of the footprint by an amount of $[(\Delta R/360)T - \Delta t]\omega$. No improvement may be made in increasing the eastern coverage as the leading edge arrives first. On the other hand, if $\Delta t > (\Delta R/360)T$, the trailing edge of the footprint arrives at a point before the leading edge; then the eastern latitude coverage may be increased by utilizing the trailing edge of the footprint. The increase of the coverage is equal to $[\Delta t - (\Delta R/360)T]\omega$. At the same time, no increase beyond that attained with the leading edge of the footprint can be made with the western latitude coverage. Table 1 summarizes the latitude coverage equations for the high orbital inclination mode. The equations are presented for the four different types of heading. Included for each of the headings are the relationships for attaining the latitude coverage to the east and to the west of the reference point along with the relationship for the total coverage during a pass (two complete passes occurring during each orbital period). Table 2 summarizes the latitude coverage equations for the low orbital inclination mode.

B. Required Latitude Coverage for Various Permissible Orbital Waiting Times

The permissible orbital waiting time is defined here as the maximum amount of time that a randomly-placed orbiting vehicle is allowed to attain a position from which it could be deboosted and subsequently land at a preselected landing site. This implies that an orbiting vehicle may perform the deboost maneuver for a return to a preselected landing site within this time period, regardless of its initial orbital position or the initial location of the landing site with respect to the orbital plane. Therefore, given a specific permissible orbital waiting time a return to a specific base is guaranteed within that time period plus the flight time of the vehicle from deboost to re-entry.

The permissible orbital waiting time becomes important when space systems are considered in which the time and place of orbital injection is

flexible. The need for flexibility of the injection point arises when one vehicle must rendezvous with another vehicle and subsequently make a return trip. The permissible orbital waiting time also becomes important when a return from an orbital position becomes desirable at unexpected or random times. Such instances could conceivably occur with reconnaissance vehicles or with manned space stations either with the development of an emergency or at the completion of a given task or mission.

The approach taken here will be to assume that the permissible waiting time is specified and then determine the required latitude coverage. For a vehicle to be assured of attaining a position from which it may be deboosted within a given waiting time, it is necessary that the total latitude coverage from all of the orbit traces generated within the permissible waiting time completely blanket 360° of longitude at the landing-site latitude.

Logical increments for permissible waiting times are sidereal days. The orbit traces generated within these time periods intersect the landing site latitude with spacings between them being repeated in a periodic fashion along the entire latitude. The orbit trace spacing along the landing-site latitude between two successive passes in the same direction is equal to the product of the orbital period and the rate at which the earth moves with respect to the reference sphere (ωT). The latitude coverage must be such that the coverage afforded by the orbit traces which are repeated periodically blanket the latitude. If the permissible waiting time is decreased to a value less than integer values of sidereal days, the latitude coverage requirements change in an abrupt manner. For example, if the permissible waiting time is reduced by a small amount of time, a complete envelopment of the landing-site latitude by periodic orbit trace intersections will not be accomplished on the final day. This causes a gap along the latitude in which the orbit trace spacing is equal to only those created in the time period prior to the entire final day. Therefore, the required latitude coverage will always be established by the orbit traces generated in the highest integer number of sidereal days closest to the permissible waiting time (for permissible waiting times of one day or more).

It will be assumed first that the orbital inclination is greater than the landing-site latitude and that the latitude coverage is attained by the high orbital inclination mode. The more trivial case for the low orbital inclination mode will be discussed later.

The minimum latitude coverage that a re-entry vehicle must have to be compatible with a given permissible waiting time depends upon the orbit trace spacings and the locations of the latitude coverage belts with respect to the orbit traces. The orbit trace spacings are a function of permissible waiting time, landing-site latitude, orbital inclination, orbital period, orbital precession rate and the earth's rotational rate. The relative location of the latitude coverage belts generated during an orbital period are dependent upon the same factors as the orbit trace spacing, with the exception of the permissible waiting time. In addition, the relative location of the coverage belts is a function of the vehicle's maneuver capability. Since the magnitude of the latitude coverage requirement is also a function of the vehicle's maneuver capability, the solution of the required latitude coverage must be based upon an iteration procedure dependent upon specific vehicle characteristics. Of perhaps more interest is the situation where a vehicle's maneuver capability is known and a permissible orbital waiting time selected. The restrictions upon the landing-site latitude and orbital missions (period, inclination, etc.) could then be established.

Figure 12 illustrates the orbit traces generated during a one-day period upon rectangular coordinates of latitude and longitude. The orbital inclination for this figure is 28.5 degrees, while the product of ωT for the orbit is 24 degrees. This figure illustrates the dependence of the orbit trace spacing upon the landing-site latitude. Similarly, for a specific landing-site location the relative orbit trace spacings will be dependent upon the orbital inclination. Figure 24 illustrates that the pattern in which the orbit traces intersect any latitude is repeated periodically along the latitude every ωT degrees (in this case 24 degrees).

To determine the maximum orbit trace spacing at a particular latitude, it is necessary to first determine the distance between the intersection of the ascending portion of the orbital trace with the landing-site

latitude and the intersection of the descending portion of the orbital trace with the landing-site latitude. With the aid of Figure 13, this distance will be determined and some new nomenclature introduced. The arc M measured along the landing-site latitude is the longitudinal separation of the ascending intersection to the descending intersection. The arc N measured in the same manner along the landing-site latitude is the longitudinal separation of the descending intersection to the ascending intersection. The arc length H measured in the orbit plane is the distance from the orbit trace intersection with the equator to its intersection with the landing-site latitude. From Figure 13 it can be seen that

$$\frac{\sin H}{\sin 90^\circ} = \frac{\sin \psi}{\sin \theta_o} = \sin H \tag{48}$$

and

$$\cos b = \frac{\cos H}{\cos \psi} \tag{49}$$

Alternative pictorial definitions of these quantities upon the reference sphere in spherical coordinates were given in Figure 8.

The successive orbit traces over the earth's surface will move westward by ωT on each complete revolution. Therefore

$$N + M = (360 + A \omega_e T) \tag{50}$$

where

$$A = +1 \text{ for westward orbital heading}$$
$$A = -1 \text{ for eastward orbital heading}$$

Since a circular orbit has been assumed, the amount of time required to trace a portion of the orbital trace is proportional to the length of the arc. With this in mind, and with the aid of Figure 8 or Figure 13, we may write,

$$M = \left[180 - 2b + A\omega T \left(\frac{180 - 2H}{360} \right) \right] \tag{51}$$

and,

$$N = \left[180 + 2b + A\omega T \left(\frac{180 + 2H}{360} \right) \right] \tag{52}$$

It is noted, however, that the relative locations of the centers of the latitude coverage belts are of direct interest rather than the actual distance between the orbit trace intersections with the landing-site latitude. Recalling the derivations for the latitude coverage on the east and west side of an intersection between the orbital trace and the landing-site latitude, the offset distance of the centroid of the coverage from the orbital trace may be readily determined. Two new terms, δ_w and δ_e will now be introduced, which will reflect these distances. As indicated in Figure 13, the values δ_w and δ_e are the distances from the orbit trace intersections with the landing-site latitude to the centroids of the attendant latitude coverage belts. The positive directions of these values are indicated in Figure 13 such that they overlay the minor circle (along a latitude) arc length M. The distance δ_w is established at the west end of the arc length M while δ_e is established at the east end. These values can be determined utilizing the proper values of L_E and L_W corresponding to the orbit trace heading as it crosses the landing-site latitude.

$$\delta_e = L_W - \left(\frac{L_W + L_E}{2} \right) \tag{53}$$

$$\delta_w = L_E - \left(\frac{L_W + L_E}{2} \right) \tag{54}$$

The distance between the centers of latitude coverage generated during one orbital period will be denoted by M^*. Its value is then given by the following relationship

$$M^* = \left(M - \delta_w - \delta_e \right) \tag{55}$$

The maximum spacing between the centers of the latitude coverage belts for one complete orbit trace with the starting point of that trace lying on the landing-site latitude is M^*, or $(360 - M^*)$, whichever is larger. This distance is reduced by ωT with each successive orbital period. The maximum spacing is reduced to ωT once sufficient orbital passes have been completed. The number of complete orbital periods to accomplish this reduction is given by the following relation:

$$n_o = \left(\frac{M^*}{\omega T} + C_o\right) \quad \text{if} \quad M^* > 180^\circ \tag{56}$$

$$n_o = \left(\frac{360 - M^*}{\omega T} + C_o\right) \quad \text{if} \quad M^* < 180^\circ \tag{57}$$

where C_o is the minimum positive amount to make n_o an integer (may be zero). The maximum waiting time that would be required in the case where the latitude coverage were equal to ωT, would be given by

$$\text{maximum waiting time} = n_o T \tag{58}$$

or conversely, for the above waiting time the minimum lateral coverage required is

$$L_{R_o} = \omega T \tag{59}$$

As more waiting time is allowed, the subsequent westward moving latitude coverage belts begin to interlace with the initial latitude coverage belts, thereby reducing further the latitude coverage requirements. The inter-lacing occurs in two distinct modes. One situation is where the new latitude coverage belts generated by orbit traces headed in northerly directions pass between coverage belts generated previously by southerly-headed orbit traces. While interlacing of the coverage belts as described above is occurring, the opposite effect is progressing elsewhere along the landing-site latitude. The relative spacing between the interlaced latitude coverage belts will be similar and periodic along the latitude in the regions where the same mode of interlacing occurs. The relative spacing of the latitude coverage belts will generally be different in the two different regions. The maximum spacing between the centers of the interlacing latitude coverage belts over a portion of the latitude covered by one of the modes of interlacing becomes

$$L_{R1} = \omega T \, C_o \tag{60}$$

or

$$L_{R1} = \omega T \, (1 - C_o) \tag{61}$$

Equations similar to equations (56) and (57) which determine the number of orbital periods required to initiate the second mode of interlacing are given below

$$n_o' = \frac{M^*}{\omega T} + C_o' \quad \text{if} \quad M^* < 180 \tag{62}$$

$$n_o' = \frac{360 - M^*}{\omega T} + C_o' \quad \text{if} \quad M^* > 180 \tag{63}$$

It should be noted that this mode of interlacing begins sooner than, or during, the same orbital period, as does the interlacing of the first mode considered. The maximum spacing between the centers of the interlacing latitude coverage belts for the portion of the latitude over which the second mode of interlacing prevails is given by the following relations:

or

$$L_{R1} = \omega T\, C_o' \tag{64}$$

$$L_{R1} = \omega T\, (1 - C_o') \tag{65}$$

whichever is greater.

The largest value as determined by equations (60), (61), (64) or (65) becomes the maximum latitude coverage required if the permissible waiting time is one sidereal day. The required latitude coverage will be reduced for each additional day spent in orbit provided that the orbit traces of the first day are not duplicated. Duplication occurs when the quantity $(360/\omega t)$ is an integer. To illustrate this reduction we will refer to Figure 14 which shows the orbit traces in the vicinity of the first pass intersection with the landing-site latitude, where interlacing of the orbit traces is initiated (on pass $n_o + 1$). For convenience the orbit traces rather than the centers of latitude coverage are depicted. The same arguments apply to both the orbit traces and the centers. After an additional day's waiting period, further interlacing occurs between the first and second passes with the passes given by the following relation,

$$\text{pass numbers} = (n_1 + n_o + 1) \text{ and } (n_1 + 1) \tag{66}$$

where

$$n_1 = \left(\frac{360}{\omega T} + C_1\right) \tag{67}$$

and C_1 is the minimum positive value to make n_1 an integer.

The term, n_1, indicates the minimum number of complete orbital periods that are required for an orbit trace intersection with the landing site latitude to be displaced at least 360 degrees. It is noted that the pass $(n_1 + n_0 + 1)$ cuts the landing-site latitude to the west of pass 1 by the amount $\omega T(C_0 + C_1)$.

During the second day another orbit trace interlaces between pass 1 and 2 in the opposite direction of the aforementioned pass. This occurs on pass number $(n_1 + 1)$ and is a distance $C_1 \omega T$ west of pass 1. At the end of a two-day waiting period the traces and the spacing between them is represented by the above example which will include a maximum spacing. This maximum spacing is represented by the largest of the four spaces between adjacent orbit traces between number 1 and 2. From Figure 14 the magnitude of the spaces may be determined and are tabulated below, space 1 being the most westerly.

(a) Space 1 $= \omega T (1 - C_1 - C_0)$

(b) Space 2 $= \omega T (C_1)$ if $C_0 > C_1$

or Space 2 $= \omega T \, C_0$ if $C_1 > C_0$ $\tag{68}$

(c) Space 3 $= \omega T |(C_0 - C_1)|$

(d) Space 4 $= \omega T \, C_1$ if $C_0 > C_1$

or Space 4 $= \omega T \, C_0$ if $C_1 > C_0$

The above assumes that $C_1 + C_0 < 1$; had this been greater, then we would no longer have been interested in pass number $(n_1 + n_0 + 1)$ as it would lie to the west of pass 2. In this case, we would be concerned with pass number $(n_1 + n_0)$ which would lie between orbit traces 1 and 2.(see Figure 15). The distances to the west of trace 1 of $(n_1 + n_0)$ pass is given by $\omega T(C_0 + C_1 - 1)$.

The values of the four spacings are then given by the following
relation

(a) **Space 1** $= \omega T (1 - C_o)$ if $C_o > C_1$

or **Space 1** $= \omega T (1 - C_1)$ if $C_1 > C_o$

(b) **Space 2** $= \omega T \left| (C_o - C_1) \right|$

(c) **Space 3** $= \omega T (1 - C_o)$ if $C_o > C_1$ \hfill (69)

$= \omega T (1 - C_1)$ if $C_1 > C_o$

(d) **Space 4** $= (C_1 + C_o - 1) \omega T$

The above equations determine the orbit trace spacing existing at the
end of two days over a portion of the latitude in which the latitude coverage
belt interlacing of the first day was similar. The coverage spacing over the
remainder of the latitude is determined by substituting C_o' as determined by
equation (62) or (63) in place of C_o into the above equations. The maximum
spacing occurring in either of the two different regions along the latitude
becomes the minimum latitude coverage required for a permissible waiting
time of two days.

The above technique can be extended to greater permissible waiting
times. The general expression for the distances from the first pass and
$(n_o + 1)$ pass of the two additional orbit traces generated on the $(1 + k)^{th}$ day
can be derived in a similar manner.

The distance between the trace generated on pass $(n_k + 1)$ and that of
the first trace is given by

$$\Delta S_k = C_k \omega T \tag{70}$$

The values for n_k and C_k are determined from the following relationship.

$$n_k = \frac{360 \; k}{\omega T} + C_k \tag{71}$$

where C_k is the minimum nonzero value to make n_k an integer. These
two equations are also applicable for the distance between the trace generated
on pass $(n_k + n_o + 1)$ and that of the pass $(n_o + 1)$.

The minimum required latitude coverage required for a waiting period d days can be determined from the maximum spacing between adjacent orbit traces existing from passes

$$1,\ 2,\ 1 + n_1;\ 1 + n_2;\ + \ldots;\ 1 + n_{d-2};\ 1 + n_{d-1};$$

$$1 + n_o;\ 1 + n_o + n_1;\ 1 + n_o + n_2,\ \ldots,\ 1 + n_o + n_{d-1}$$

From these passes the values of n_k and C_k can be determined and the intersections with the landing-site latitude between the first and second traces determined. In doing so, it should be kept in mind that when the quantity $(C_o + C_k)$ is greater than unity it is replaced by $(C_o + C_k - 1)$.

For the case of low orbital inclination mode the determination of the coverage requirement is much simpler than it is for the case of high orbital inclination. In the low orbital inclination case only one continuous coverage belt will be created during each orbital period.

Within a one-day permissible waiting time the centers of the coverage belts along the landing-site latitude are spaced at a distance of ωT. Therefore the latitude coverage requirement with a permissible waiting time of one day for a low orbital inclination mode of coverage is given by the following relationship.

$$L_{R1} = \omega T \tag{72}$$

For a two-day permissible waiting time the centers created in the first day and the second day will interlace and produce two types of spacing along the landing-site latitude. One is given by $\omega T (C_1)$ and the other by $\omega T (1 - C_1)$. The value of C_1 was previously determined by equation (67). Therefore the latitude coverage requirement for a permissible waiting time of two days is given by the following relationship

$$L_{R2} = \omega T (C_1) \quad \text{or} \quad L_{R2} = \omega T (1 - C_1) \tag{73}$$

whichever is larger. For the above and greater permissible waiting times the same equations are valid for the high and low orbital inclination coverage

modes with the exception that in the latter case C_o is set at zero and the number of spaces existing will be halved. The latitude coverage requirement for the low orbital inclination mode is therefore independent of the landing-site latitude and orbital inclination.

The preceding analysis determines the latitude coverage required to guarantee that a deboost situation will arise within the permissible waiting time such that the vehicle may land at a preselected landing site. Figure 16 shows the latitude coverage required for permissible waiting times of one and two days as a function of landing-site latitude when the re-entry vehicle is orbiting in a circular polar orbit of 283-nautical mile altitude (period = 1.585 hours). It is assumed that the downrange maneuver capability has no effect upon contributing to offsetting the center of the coverage bands from the orbit trace. (The offset may be eliminated by changing the reference point from the leading edge of the landing footprint and considering it in the center of the footprint.) In the case of a polar orbit there are no other factors contributing to this offset.

Figure 17 shows the influence of the orbital period (or altitude) upon the latitude coverage required for permissible orbital waiting times of one and two days. As in Figure 16 polar orbits are considered with the landing-site latitude fixed at 35 degrees (that of Edwards AFB). To enable Figure 17 to be presented as a series of straight line segments and keep it fairly compact the graph is normalized with respect to the parameter ωT (orbital period).

Figure 17 contains many breaks in the coverage requirement curves. The number of breaks increases with greater permissible waiting time due to the increase in the number of orbit trace intersections with the landing-site latitude. The maximum spacing of the latitude coverage belts (which is equivalent to the orbit trace spacing) determines the latitude coverage requirement. As some coverage belt spacings increase as a function of orbital period or latitude, others are decreasing in size relative to them. The location of the spaces offering the maximum spacing between coverage belt centers will change occasionally as a function of the landing-site latitude and/or orbital period. Each time a change of this nature takes place a break in the curve occurs.

The latitude coverage requirements are unique for polar orbits and orbits which afford low orbital inclination modes of coverage. The coverage requirement is unique in these cases as it is not a function of the magnitude of the coverage or maneuver capability (linear solution). However, the latitude coverage requirement for orbital inclinations other than polar will fluctuate with variations in landing-site location and orbital period in a similar manner to that encountered with the polar orbit.

It had been shown that for allowable waiting times greater than one day that a reduction in the waiting time below an integer number of one-half day would increase the requirement to that of the requirement for the next lowest integer number of days. In the case of a one-day waiting time the preceding is not true. Consider the case where the orbital inclination is high and the allowable waiting time is one day. In this time period the landing-site latitude will be crisscrossed alternately by orbit traces headed in northerly and southerly directions. These orbits trace intersections with the latitude and their associated centers of the coverage belts divide the latitude into segments which are equal to or less than ωT. The latitude may be divided into segments no larger than ωT by the orbit traces in a time period which is less than a day. This time period varies from approximately one-half day for polar orbits, up to a full day (for low orbital inclinations). The required number of complete orbits for such a division of the latitude is given by n_o obtainable from equation (56) or (57). For allowable waiting times less than $n_o T$ the required latitude coverage equals the largest gap between the centers of latitude coverage generated by orbit traces headed in the opposite directions. A relatively small decrease in waiting time results in relatively large increases in latitude coverage requirements. Therefore, waiting time less than $n_o T$ will not be discussed in detail.

The latitude coverage requirement in the case of a single landing-site was given simply by the maximum spacing between the orbit traces generated at the end of a given time period. This implies that the landing site is located in the most unfavorable position possible. This landing-site position is equidistant from the most distantly spaced centers of latitude

coverage. In the case of multiple landing sites, the required latitude coverage is determined when the centers of the latitude coverage belts are oriented with respect to the landing sites such that the spacing between the closest orbit trace to either of the landing sites and that landing site is maximized. The complexity of the problem for determining the required latitude coverage is reduced somewhat if the landing sites are at the same latitude.

The orbit trace spacing at the end of n_o passes or at the end of any number of days is again repeated every ωT degrees. A convenient method of solving the multiple landing site (same latitude) problem is to translate the landing sites along the landing-site latitude distances which are integers of ωT such that all the sites are within a span of ωT. The location of the orbit traces is then shifted with respect to the translated landing sites until the closest approach to the centers of the coverage belts is maximized. This yields the latitude coverage requirement that occurs for a given allowable waiting time in orbit. Since the latitude coverage requirement for multiple landing sites is analytically cumbersome, it is best to consider specific missions in detail from the launch point to evaluate landing-site selection. Investigations of this sort are beyond the scope of the study made here.

III. DISCUSSION AND RESULTS

The magnitude of the latitude coverage of an orbiting re-entry system may be subdivided into three separate terms. The first of these represents the coverage of the projection of the landing-site latitude upon the reference sphere. The magnitude of this coverage (ΔL_c or ΔL_a and ΔL_b) is dependent upon the orbital inclination, landing-site latitude and the cross-range maneuver capability. The second term compensates for the earth rotation and orbital precession rate. This term is dependent upon all the factors that determine the first term as well as the earth rotation and orbital precession rate. The third term incorporates the coverage attainable with the downrange maneuver capability. This term is dependent upon the earth rotation and orbital precession rate, the length and the flight time interval between the leading and trailing edge of the footprint. Tables 1 and 2 relate these latitude coverage terms for the high and low orbital inclination modes respectively.

Figures 18 through 23 show the variation of ΔL_a, ΔL_b, ΔL_c, X_a, X_b and X_c with orbital inclination for various cross-range maneuver capabilities for a landing-site latitude of 35 degrees north. The terms $(\Delta L_a + \Delta L_b)$ and ΔL_c are plotted together in Figure 20. This provides the magnitude of the coverage upon the reference sphere for both the high and low orbital inclination. The maximum values are attained at the boundary of the high and low orbital inclination modes of coverage. For the low orbital inclination mode ΔL_c relates only one-half of the coverage afforded per orbital period. For the high orbital inclination mode $(\Delta L_a + \Delta L_b)$ relates the coverage upon the reference sphere on one pass (orbit trace intersection with the landing-site latitude); however, two complete passes occur during every orbital period. At the boundary between the high and low orbital inclination, the coverages from the two passes become continuous and the coverage belt upon the reference sphere is given by either $2(\Delta L_a + \Delta L_b)$ or $2\Delta L_c$.

Figure 23 indicates the effect of orbital inclination upon $(X_a + X_b)$ or X_c which are proportional to the effect upon the latitude coverage due to the earth rotation and the orbit plane precession rate. The maximum values of X_c and $(X_a + X_b)$ also occur at the boundary of the high and low orbital inclination modes of coverage.

Figures 24 through 29 relate the variation of ΔL_a, ΔL_b, ΔL_c, X_a, X_b and X_c with landing-site location for various cross-range maneuver capabilities in a manner similar to Figures 18 through 23. The orbital inclination for all of these figures is 35 degrees. The maximum values again occur at the boundary between the high and low inclination modes of coverage.

It has been pointed out that the maximum values of ΔL_a, ΔL_b, ΔL_c, X_a, X_b and X_c will always occur at the boundary between the high and low orbital inclination modes of coverage. Therefore for the cases of a vehicle orbiting in a westerly direction in which the ΔL and X terms of latitude coverage are additive, the maximum latitude coverage is attained when the landing site is situated at a latitude lower than the orbital inclination by an amount equal to the cross-range maneuver capability. The orbital inclination is therefore bordering between the high and low orbital inclination mode.

The second latitude coverage term (involving X_c or $X_a + X_b$) is negative for vehicles which are orbiting in an easterly direction. Conceivably the majority of orbital periods in which application of this study is of interest would be less than 12 hours (11,000-nautical mile altitude). For orbital periods of less than approximately 12 hours the second term of the latitude coverage will increase at a slower rate than the first latitude coverage term as the boundary between the high and low orbital inclination modes of coverage is approached. For this reason vehicles orbiting in easterly directions will generally have their maximum latitude coverage values at the boundary between the high and low orbital inclination modes of coverage; in spite of the fact that the latitude coverage is composed of the difference between the first and second terms.

Figure 30 relates the magnitude of the latitude coverage as a function of orbital inclination for various cross-range maneuver capabilities. The effect of the downrange maneuver capability is omitted. The data in this figure are based upon a vehicle having an eastward heading and a landing-site latitude of 35 degrees with successive spacing of orbit traces equal to 23.8 degrees. For the high orbital inclination mode the coverage is represented by $(L_E + L_W)$. It corresponds to the coverage generated in one pass (two passes occur per orbital period with a high orbital inclination) to the east (L_E) and to the west (L_W) of the orbit trace. For the low orbital inclinations mode the coverage is represented by $1/2\ L_L$. This corresponds to one-half the total coverage generated in the single pass per orbital period and is therefore on a comparable basis with the high orbital inclination mode where two passes occur per orbital period (one half of the coverage on each pass). The dashed line indicates the boundary between the high and low orbital inclination modes of coverage. As expected the maximum coverage occurs at this border.

In the determination of what waiting time is required to achieve complete coverage of the landing-site latitude when the high orbital inclination mode prevails, it is necessary to know the parameter $(\delta_W + \delta_E)$. This parameter represents the difference between the orbit trace spacing for one orbital period M, given by equation (51), and the spacing of the latitude coverage belts M^*, given by Equation (55). Values of $(\delta_W + \delta_E)$ are presented in Figure 31 for various orbital inclinations and cross-range maneuver capabilities for the same set of conditions which prevailed for Figure 30.

Figure 32 relates the magnitude of the latitude coverage in a manner similar to that of Figure 30. The coverage in this case is a function of landing-site latitude instead of orbital inclination. The orbital inclination is set at 35 degrees (eastward heading) with the parameter, ωT, equal to 23.8 degrees. Figure 33 presents the value of $(\delta_W + \delta_E)$ in a format similar to the manner that latitude coverage was presented in Figure 32. As expected, the maximum latitude coverage occurs at the boundary of the high and low orbital inclination modes of coverage.

With the orbital inclination equal to 35 degrees and the successive orbit trace spacing, ωT, equal to 23.8 degrees, other orbital characteristics would be an orbit plane's precession rate of 6.45 degrees per day in the westerly direction and an orbital period of 1.55 hours (altitude = 240 nautical miles). Had a polar orbit been selected along with the parameter ωT again equal to 23.8 degrees the precession rate of the orbit plane would be zero and the period of the orbit would be 1.585 hours (altitude = 283 nautical miles). The periods differ because the relative angular motion between the earth and the reference sphere is equal to the sum of the earth rotational rate and the average precession rate of the orbit plane about the earth polar axis.

A comparison of Figure 20 with Figure 30, and Figure 26 with Figure 32 relates the difference between the coverage of latitude on the earth surface and that on the reference sphere. The difference in this coverage is afforded by the second latitude coverage term. It is clearly evident that the second term makes a secondary contribution to the final result. For the value of ωT utilized in Figures 30 and 32, the second term affected the result by approximately 5 per cent. If the value of ωT is increased (the orbital altitude also increases) then the effect of the second term of latitude coverage is increased proportionally. When the vehicle is orbiting in an easterly direction then the second term is negative and subtracts from the reference sphere coverage. On the other hand, when the heading is westerly then the second term is positive and tends to increase the latitude coverage. The orbital precession rate varies with orbital inclination; therefore the given parameter ωT will vary slightly with orbital inclination. However, a constant value of ω equal

to the earth rotation rate (15.04 degrees per hour) can be used in the deter-
mination of the second latitude coverage term. This approximation incurs a
few per cent error in a term which itself contributes only a few per cent to
the total latitude coverage.

The third latitude coverage term is dependent only upon the length of the
landing footprint, the flight time differen   between the leading and trailing
edge of the footprint along with the parameter ωT. The downrange maneuver-
ability has no direct bearing on the distance M^*. The reason for this is because
the coverage will be added to the same side (i. e. , either east or west) of both
latitude coverage belts generated during a pass and since they are otherwise
of the same magnitude the arc M^* is merely displaced with no change in its
magnitude. The latitude coverage requirements for any permissible orbital
waiting time is independent of this term since it has no effect upon the
magnitude of M.

Figure 34 shows the increase in latitude coverage due to the downrange
maneuver capability when the parameter, ωT, equals 23. 8 degrees. The
increase is plotted versus the absolute value of the parameter $[\Delta R - 360 (\Delta t/T)]$.
The latitude coverage increase due to downrange maneuver capability for a value
of ωT other than 23.8 degrees is directly proportional to the difference. The
flight times for longer ranges will generally be greater than for shorter ranges
whether the range increment is achieved through the deboost maneuver outside
the atmosphere or by the use of aerodynamic lift within the atmosphere.
Because of this the terms in the quantity $[\Delta R - 360 (\Delta t/T)]$ will always tend
to negate each other. The sign of the quantity within the brackets will be positive
when the arc length upon the reference sphere to the landing point is traversed
more rapidly after the deboost maneuver than it would be traversed at the
orbital rate. The sign will be negative when the reverse is true. It has been
pointed out that the contribution to latitude coverage due to the downrange
maneuver capability has a greater effect as the orbital altitude increases.
Also at higher altitudes ΔR will be increased due to an increase in re-entry
velocity. At the lower orbital altitudes the contribution of this third latitude
coverage term is on the order of one or two degrees of latitude for a maximum

hypersonic lift-to-drag ratio of 1.0. This additional latitude coverage is based upon aerodynamic maneuverability obtainable by following equilibrium glide flight paths. Furthermore, this contribution varies linearly with the lift-drag ratio. When the landing-site, orbital inclination and cross-range maneuver capability are such that the mode of latitude coverage is near the borderline situation between the high and low orbital inclination modes of coverage, the latitude coverage contribution from downrange capability will become insignificant in comparison with the other latitude coverage terms. In this case the term may be neglected; however, for orbital inclinations which approach that of the landing-site latitude less the cross-range maneuverability, the effect of the third term should be included.

To insure that a vehicle may be deboosted from orbit within a given waiting period (permissible waiting time) the vehicle must have certain maneuver capabilities. Both the latitude coverage for a given maneuver capability and the required coverage vary with the latitude of the landing site and the orbital inclination. Therefore the maneuver requirements (regarding the landing footprint) will vary with landing-site latitude and orbital inclination for a given permissible waiting time.

Figures 35 and 36 demonstrate the variation of the cross-range maneuver requirements with orbital inclination and landing-site latitude respectively. The cross-range maneuver requirement of Figures 35 and 36 is based upon zero downrange maneuver capability. In both Figures 35 and 36 the permissible waiting time is one day and the parameter ωT is fixed at 23.8 degrees. In Figure 35, the landing-site latitude was fixed at 35 degrees while the orbital inclination was varied (easterly headings only). In Figure 36, the orbital inclination was fixed at 35 degrees while the landing-site latitude was varied.

In Figures 35 and 36 a region is designated, within the low orbital mode of coverage region, where a landing at the landing-site latitude is impossible. A landing is impossible when the landing-site latitude is greater than the orbital inclination plus the cross-range maneuver capability expressed in great circle arc length.

Figure 37 relates the variation in the cross-range maneuver requirement as the parameter ωT varies. The landing-site latitude was selected as $35^\circ N$. Two orbital inclinations are indicated in the figure representing both the high orbital inclination mode of coverage and the low orbital inclination mode of coverage. The high mode is represented by an orbital inclination of 90 degrees (polar orbit) while the low mode is represented by an orbital inclination of 35 degrees. The permissible waiting time is taken as one day and again the cross-range maneuver requirement is based upon zero downrange maneuver capability.

The cross-range maneuver requirement for the low orbital inclination mode of Figure 37 is that amount required to make the latitude coverage equal to the spacing of successive orbit traces. This spacing is given by the parameter ωT.

The cross-range maneuver requirement for the high orbital inclination mode takes on a sawtoothed shape as it did on Figures 35 and 36. The coverage requirement is equal to the maximum distance between the centers of the latitude coverage belts generated during one day. This spacing between the coverage belts generated as the orbiting vehicle heads northward and the coverage belts generated as the vehicle heads southward varies with ωT in a cyclic fashion. (also varies with orbital inclination and landing-site latitude). When the belts generated on a northern heading coincide with those of the southerly heading the spacing of the centers of coverage will be equal to the successive orbit trace spacing ωT.

The cross-range maneuver requirements which provide a latitude coverage equal to ωT define an envelope containing all of the maximum values relative to the neighboring points for a one-day permissible orbital waiting time. The envelope of these relative maximum points for a one-day permissible waiting time is indicated in Figures 35, 36, and 37.

The appearance of the cross-range maneuver requirement in the low orbital inclination region of Figures 35, 36, and 37 is a well-behaved smooth function of the variable under consideration. The centers of the latitude coverage belts generated in a low orbital inclination mode on successive

orbit traces will completely envelop the landing-site latitude only after a one-day period. The required latitude coverage in this case is simply the orbit trace spacing ωT. The cross-range maneuver requirement is therefore a well-behaved function of θ_0, ψ or ωT. If the permissible waiting time is increased an additional day, the centers of the latitude coverage belt generated in this time period fall between, or on those generated in the first day. The relative location of the second day's centers with respect to the first day's centers will depend upon ωT, the landing-site latitude ψ, and the orbital inclination θ_0. Therefore the cross-range maneuver requirement expressed as a function of ωT, θ_0 or ψ for low-orbital inclinations and a permissible orbital waiting time of two days will be sawtooth-shaped, reflecting the alternately increasing and decreasing spacings between latitude coverage centers.

If a permissible waiting time of n_0 orbital periods (determined by equations (56) or (57)) is established, the problem of determining the required maneuverability is somewhat simplified due to the simple form of the latitude coverage requirement. It is recalled that n_0 is the number of orbital periods required to completely encircle the landing-site latitude with latitude coverage belts such that the maximum spacings between their centers equal ωT. Complete coverage of the landing-site latitude in n_0 orbital periods affords the lowest permissible waiting time without requiring excessive maneuver capability. Therefore the maneuver capability and time required to achieve this coverage is of interest. For a landing site located at $35^\circ N$ latitude (Edwards AFB) Figures 20 and 23 may be readily used with the equations of Table 1 or 2 to establish the required cross-range maneuver capability for a given orbital inclination and orbital period.

The analysis presented considers the landing-site latitude coverage attainable for given maneuver capabilities of orbiting vehicles. Furthermore, it considers the cross-range requirement which insures that a vehicle may be deboosted from orbit within a given waiting time. The maneuver capability can either be achieved within the atmosphere or by variation of the deboost conditions. The results are therefore adaptable to orbital bombardment systems as well as lifting re-entry vehicles provided that the meaning of the footprint is properly interpreted.

Table 1. Latitude Coverage for High Orbital Inclinations.

Orbital Heading	Latitude Coverage (1) + (2) + (3)	(1)	(2)	(3) $\frac{\Delta R}{360} T > \Delta T$	$\Delta T > \frac{\Delta R}{360} T$
NE	L_E	ΔL_a	$- X_a \frac{\omega T}{360}$	0	$\omega \left(\Delta t - \frac{\Delta R}{360} T \right)$
	L_W	ΔL_b	$- X_b \frac{\omega T}{360}$	$\omega \left(\frac{\Delta R}{360} T - \Delta t \right)$	0
	$L_{(E+W)}$*	$\Delta L_a + \Delta L_b$	$- (X_a + X_b) \frac{\omega T}{360}$	$\omega \left(\frac{\Delta R}{360} T - \Delta t \right)$	$\omega \left(\Delta t - \frac{\Delta R}{360} T \right)$
SE	L_E	ΔL_b	$- X_b \frac{\omega T}{360}$	0	$\omega \left(\Delta t - \frac{\Delta R}{360} T \right)$
	L_W	ΔL_a	$- X_a \frac{\omega T}{360}$	$\omega \left(\frac{\Delta R}{360} T - \Delta t \right)$	0
	$L_{(E+W)}$*	$\Delta L_a + \Delta L_b$	$- (X_a + X_b) \frac{\omega T}{360}$	$\omega \left(\frac{\Delta R}{360} T - \Delta t \right)$	$\omega \left(\Delta t - \frac{\Delta R}{360} T \right)$
NW	L_E	ΔL_b	$X_b \frac{\omega T}{360}$	0	$\omega \left(\Delta t - \frac{\Delta R}{360} T \right)$
	L_W	ΔL_a	$X_a \frac{\omega T}{360}$	$\omega \left(\frac{\Delta R}{360} T - \Delta t \right)$	0
	$L_{(E+W)}$*	$\Delta L_a + \Delta L_b$	$(X_a + X_b) \frac{\omega T}{360}$	$\omega \left(\frac{\Delta R}{360} T - \Delta t \right)$	$\omega \left(\Delta t - \frac{\Delta R}{360} T \right)$
SW	L_E	ΔL_a	$X_a \frac{\omega T}{360}$	0	$\omega \left(\Delta t - \frac{\Delta R}{360} T \right)$
	L_W	ΔL_b	$X_b \frac{\omega T}{360}$	$\omega \left(\frac{\Delta R}{360} T - \Delta t \right)$	0
	$L_{(E+W)}$*	$\Delta L_a + \Delta L_b$	$(X_a + X_b) \frac{\omega T}{360}$	$\omega \left(\frac{\Delta R}{360} T - \Delta t \right)$	$\omega \left(\Delta t - \frac{\Delta R}{360} T \right)$

*Magnitude of latitude coverage per pass.

Equation (17) $\cos \theta = \frac{\cos \theta_o}{\cos \psi}$

Equation (24) $\cos \Delta L_a \sin \psi \cos \theta + \sin \Delta L_a \sin \theta = \sin \psi \cos \theta + \frac{\sin l}{\cos \psi}$

Equation (33) $\cos \Delta L_b \sin \psi \cos \theta - \sin \Delta L_b \sin \theta = \sin \psi \cos \theta - \frac{\sin l}{\cos \psi}$

Equation (27) $\cos X_a = \frac{1 - \cos^2 \psi (1 - \cos \Delta L_a)}{\cos l}$

Equation (34) $\cos |X_b| = \frac{1 - \cos^2 \psi (1 - \cos \Delta L_b)}{\cos l}$

   $c = -1$ if $\theta > \theta_x$ $c = +1$ if $\theta \leq \theta_x$ $c = \frac{X_b}{|X_b|}$

Equation (39) $\sin^2 \theta_x = \frac{\cos^2 \psi}{\sin^2 l} - \frac{(\cos l - \sin^2 \psi)^2}{\cos^2 \psi \sin^2 l}$

Table 2. Latitude Coverage for Low Orbital Inclinations.

L_L = (1) + (2) + (3) (The Values of (1), (2) and (3) are Given Below)

Orbital Heading	(1)	(2)	(3)	
			$\frac{\Delta R}{360} T > \Delta t$	$\Delta t > \frac{\Delta R}{360} T$
East	$2 \Delta L_c$	$-2X_c \frac{\omega T}{360}$	$\omega\left(\frac{\Delta R}{360} T - \Delta t\right)$	$\omega\left(\Delta t - \frac{\Delta R}{360} T\right)$
West	$2 \Delta L_c$	$+2X_c \frac{\omega T}{360}$	$\omega\left(\frac{\Delta R}{360} T - \Delta t\right)$	$\omega\left(\Delta t - \frac{\Delta R}{360} T\right)$

Equation (6) $\quad \cos \Delta L_c = \frac{\tan \psi}{\tan \theta_o} - \frac{\sin \ell}{\sin \theta_o \cos \psi}$

Equation (8) $\quad \cos X_c = \frac{1}{\cos \ell} (\sin \theta_o \sin \psi + \cos \psi \cos \theta_o \cos \Delta L_c)$
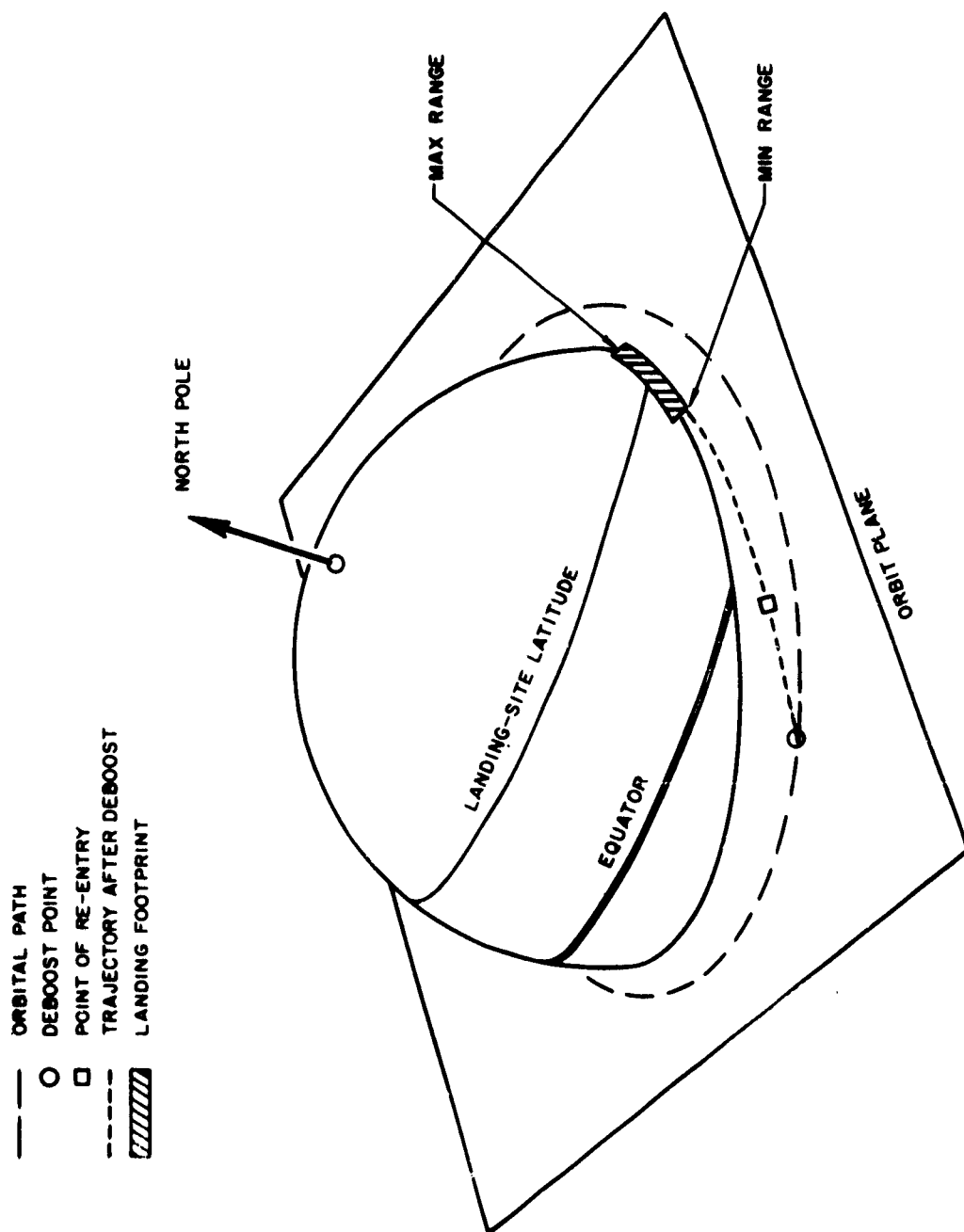
Figure 1.   Schematics for Landing Site Coverage.

2 a. HIGH ORBITAL INCLINATION LATITUDE COVERAGE.

2 b LOW ORBITAL INCLINATION LATITUDE COVERAGE.

2 c. BORDERLINE BETWEEN HIGH AND LOW ORBITAL INCLINATION LATITUDE COVERAGE.
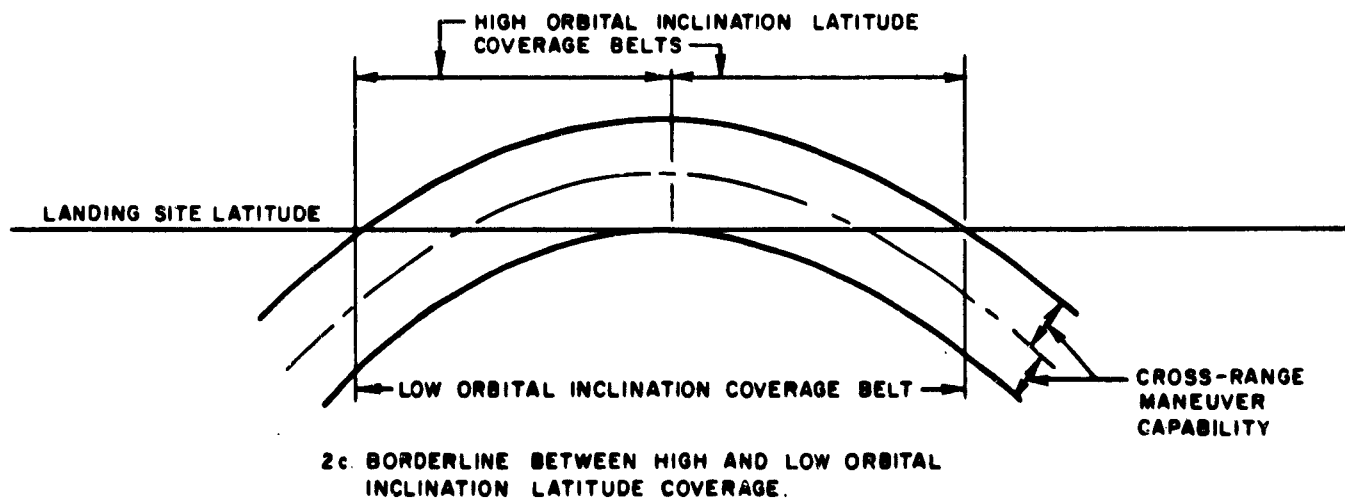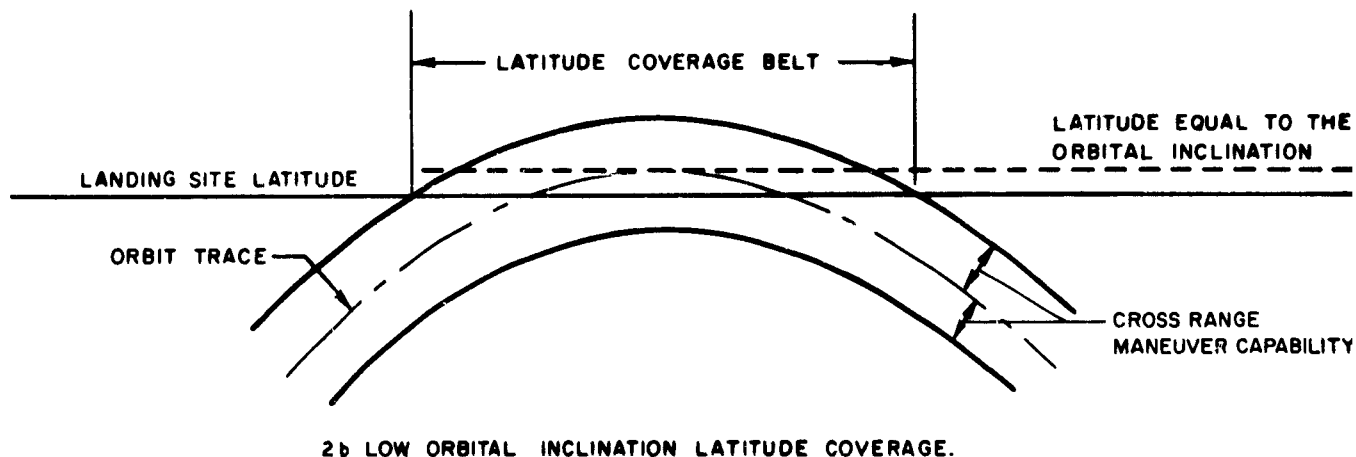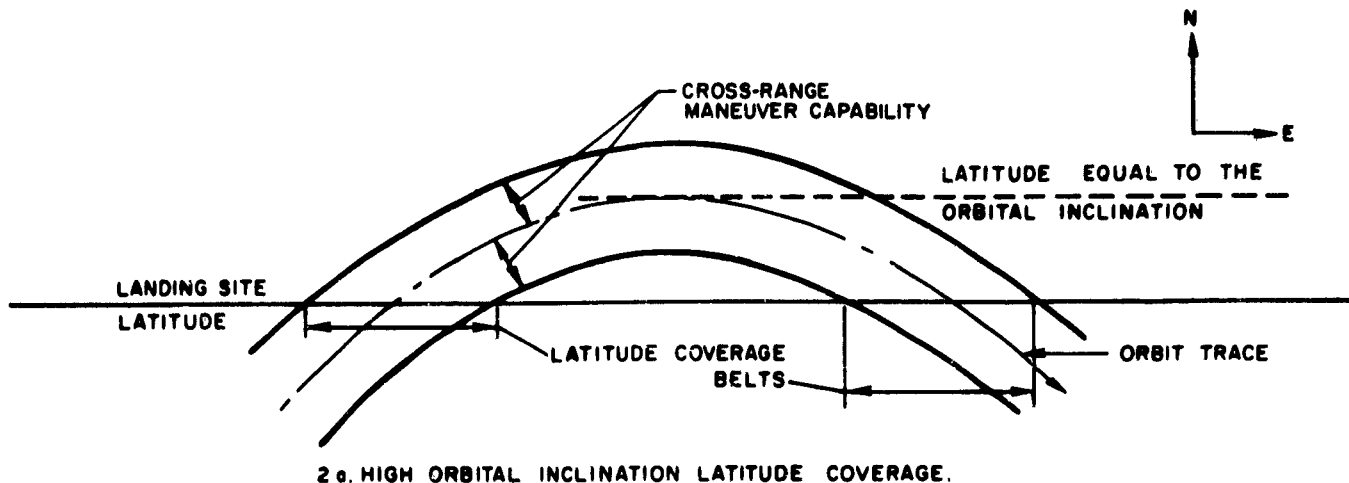
Figure 2. Modes of Latitude Coverage.
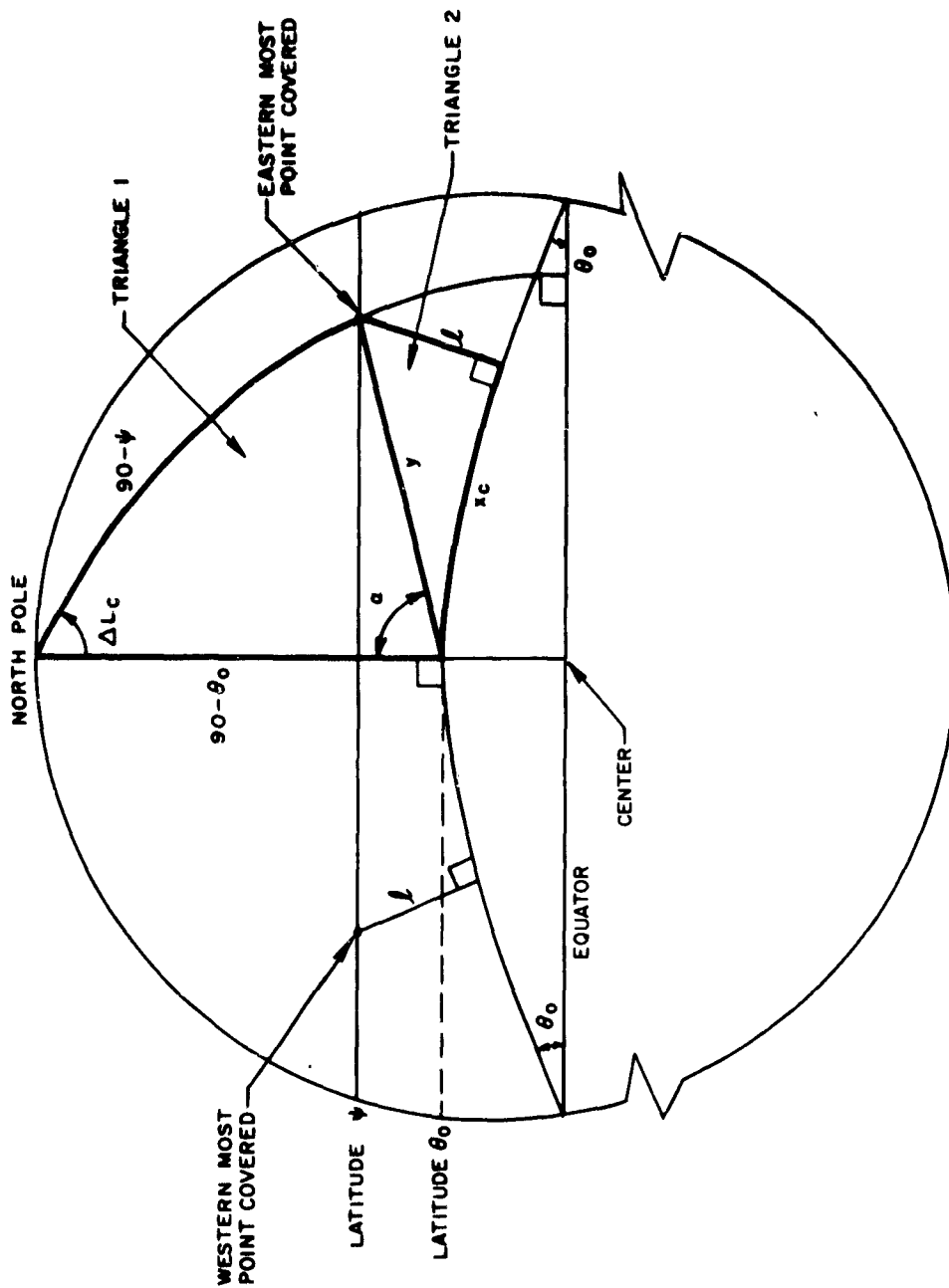
Figure 3. Reference Sphere Geometry for a Low Orbital Inclination
Mode of Coverage.
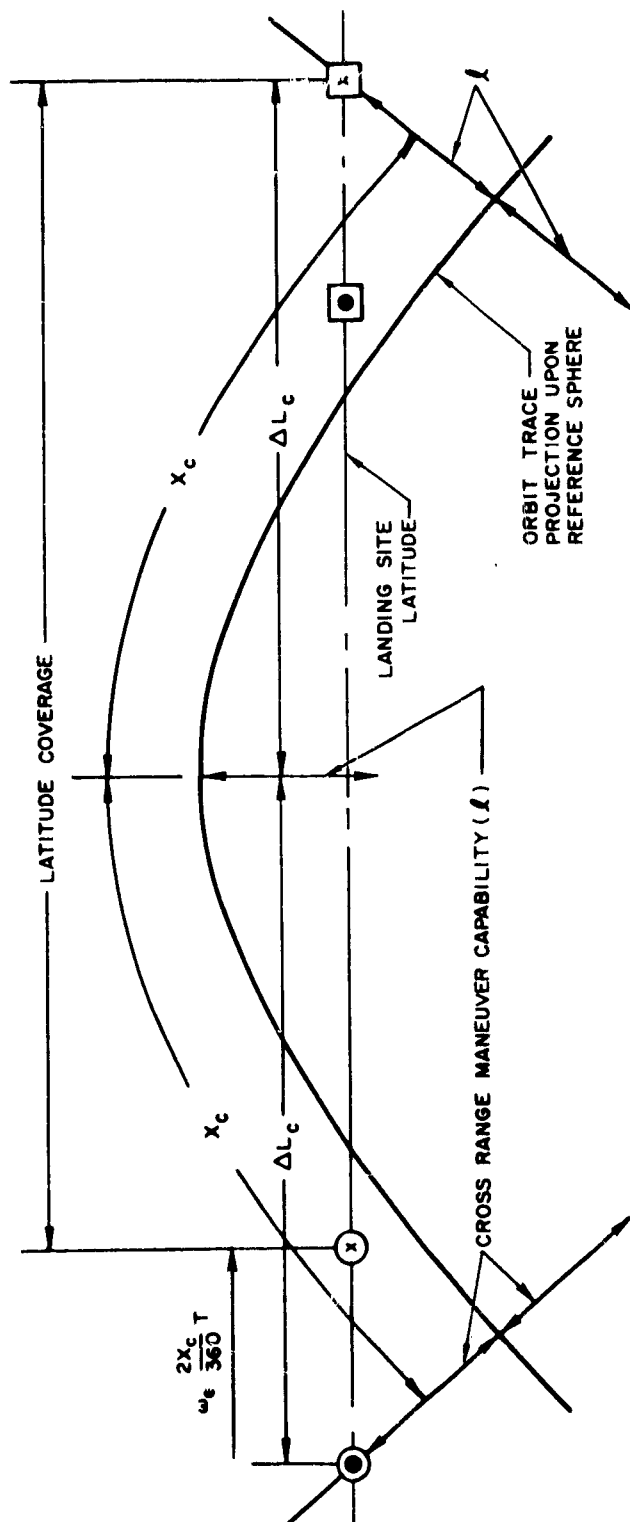
KEY TO THE LOCATION OF POINTS OF INTEREST ON THE EARTH'S SURFACE PROJECTED UPON THE REFERENCE SPHERE AT VARIOUS TIMES. (NOTE THE SYMBOLS UTILIZED CONSIST OF TWO PARTS, ONE REFERS TO TIME AND THE OTHER TO THE POINT OF INTEREST).

TIME

x   TIME AT WHICH THE EASTERN BOUNDARY OF THE LATITUDE COVERAGE BELT IS ESTABLISHED.

●   TIME AT WHICH THE WESTERN BOUNDARY OF THE LATITUDE COVERAGE BELT IS ESTABLISHED.

POINT OF INTEREST

▢   EASTERN BOUNDARY OF THE LATITUDE COVERAGE BELT.

◯   WESTERN BOUNDARY OF THE LATITUDE COVERAGE BELT.

Figure 4. Low Orbital Inclination Latitude Coverage Displayed Upon the Reference Sphere (Orbital Heading Easterly).
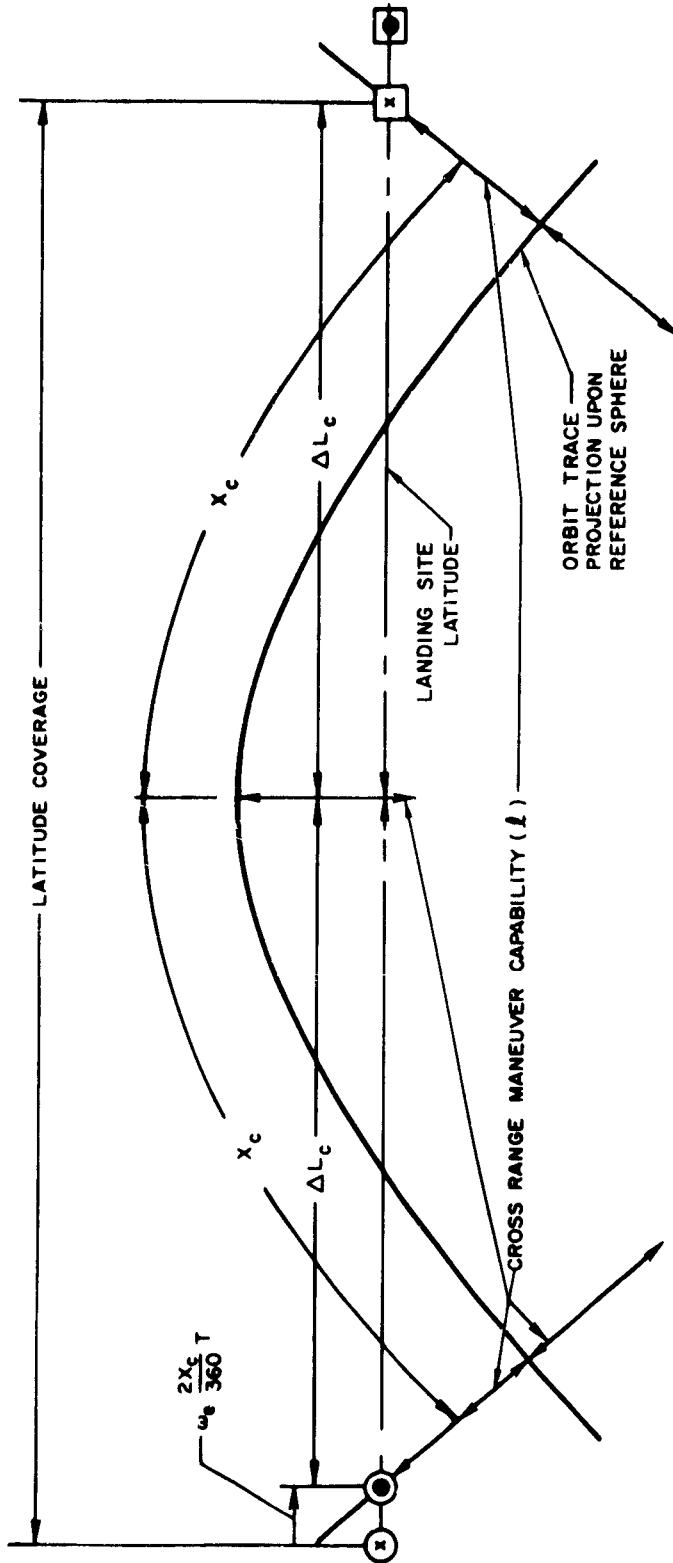
KEY TO THE LOCATION OF POINTS OF INTEREST ON THE EARTH'S SURFACE PROJECTED UPON THE REFERENCE SPHERE AT VARIOUS TIMES. (NOTE THE SYMBOLS UTILIZED CONSIST OF TWO PARTS, ONE REFERS TO TIME AND THE OTHER TO THE POINT OF INTEREST).

TIME

x   TIME AT WHICH THE EASTERN BOUNDARY OF THE LATITUDE COVERAGE BELT IS ESTABLISHED.

●   TIME AT WHICH THE WESTERN BOUNDARY OF THE LATITUDE COVERAGE BELT IS ESTABLISHED.

POINT OF INTEREST

☐ EASTERN BOUNDARY OF THE LATITUDE COVERAGE BELT.

◯ WESTERN BOUNDARY OF THE LATITUDE COVERAGE BELT.

Figure 5.   Low Orbital Inclination Latitude Coverage Displayed Upon the Reference Sphere (Orbital Heading Westerly).

ORBITAL PATH

LANDING SITE LATITUDE

ORBIT TRACE PROJECTION
UPON REFERENCE SPHERE

EQUATOR

① FOOTPRINT LOCATION FOR ORBITAL POSITION A
② FOOTPRINT LOCATION FOR ORBITAL POSITION B
—— TRAILING EDGE OF FOOTPRINT
---- LEADING EDGE OF FOOTPRINT
✱ A POINT ON THE REFERENCE SPHERE AT THE
LANDING SITE LATITUDE

Figure 6. Projections of the Landing Footprint upon
the Reference Sphere.

7a. ORBITAL HEADING NORTHEASTERLY OR SOUTHWESTERLY.
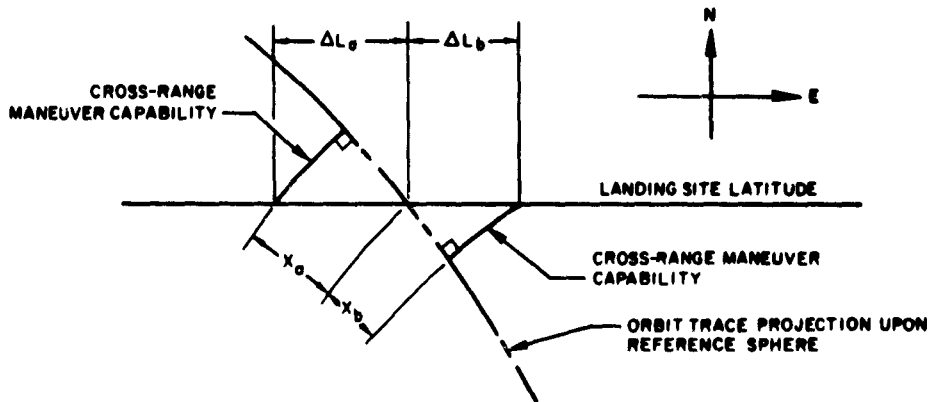
7b. ORBITAL HEADING NORTHWESTERLY OR SOUTHEASTERLY.
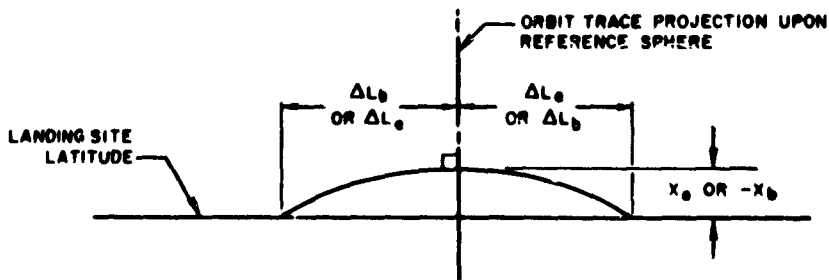
7c. ORBITAL HEADING — POLAR.

Figure 7. General Types of Orbit Trace Intersections with the Landing Site Latitude Projected Upon the Reference Sphere - High Orbital Inclination.
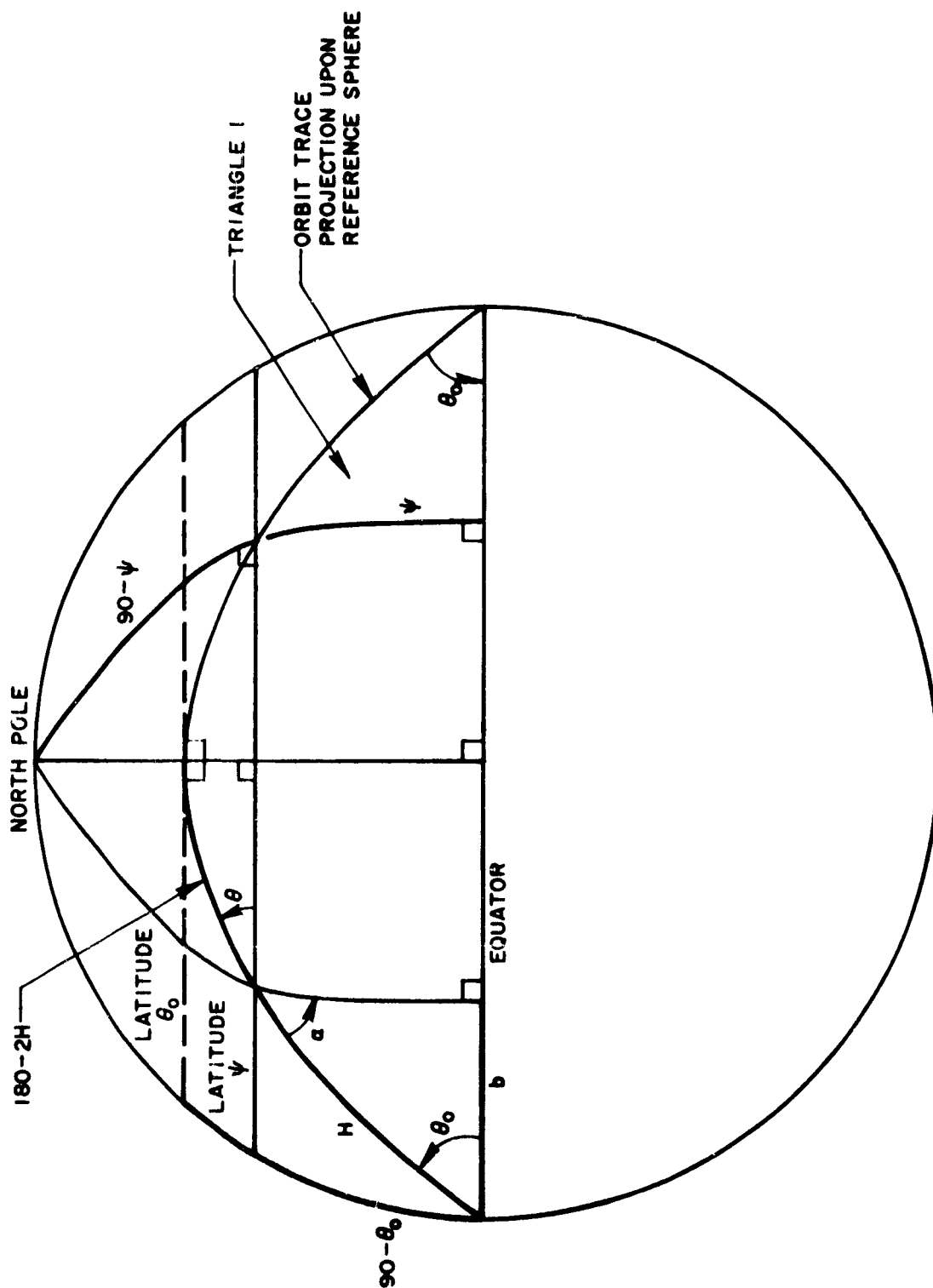
Figure.8. Reference Sphere Geometry for a High Orbital Inclination Mode of Coverage.
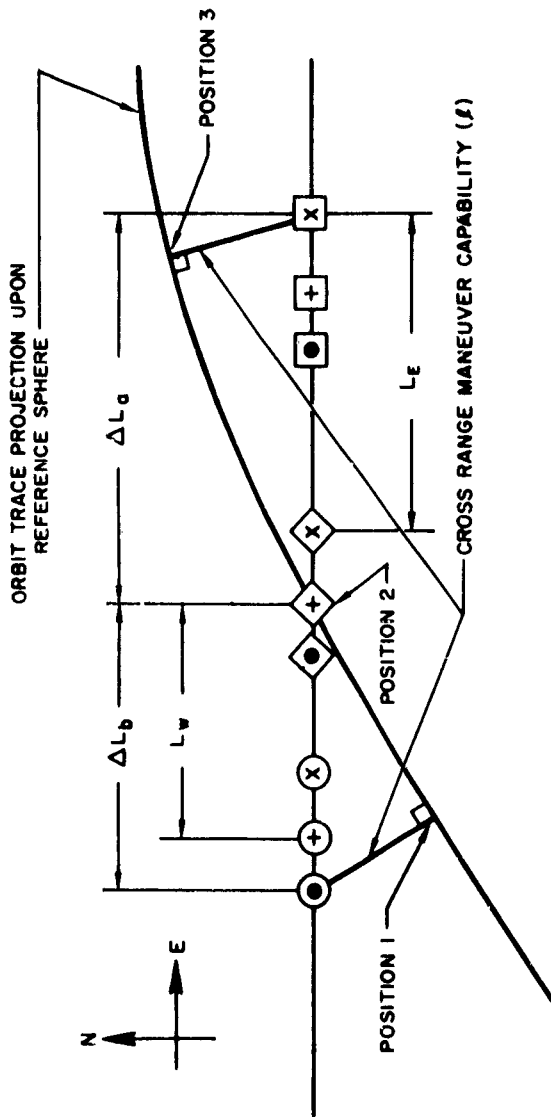
Figure 9. Definition for Parameters ΔL_a and X_a.

Figure 10. Definition for Parameters ΔL_b and X_b.

Figure 11. High Orbital Inclination Latitude Coverage Projected Upon the Reference Sphere (Orbital Heading Northeasterly).
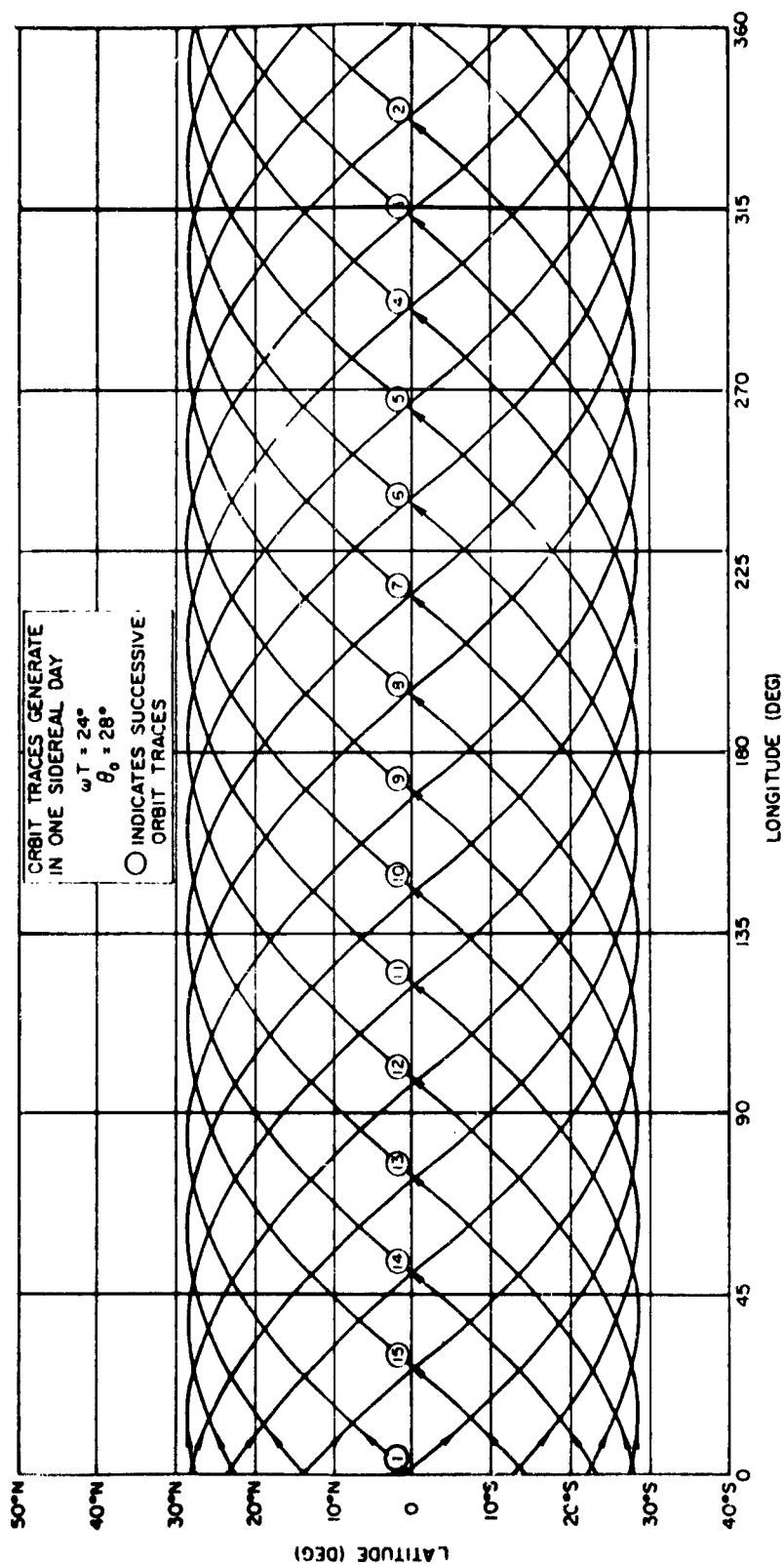
Figure 12. Typical Orbital Traces Generated in One Sidereal Day.
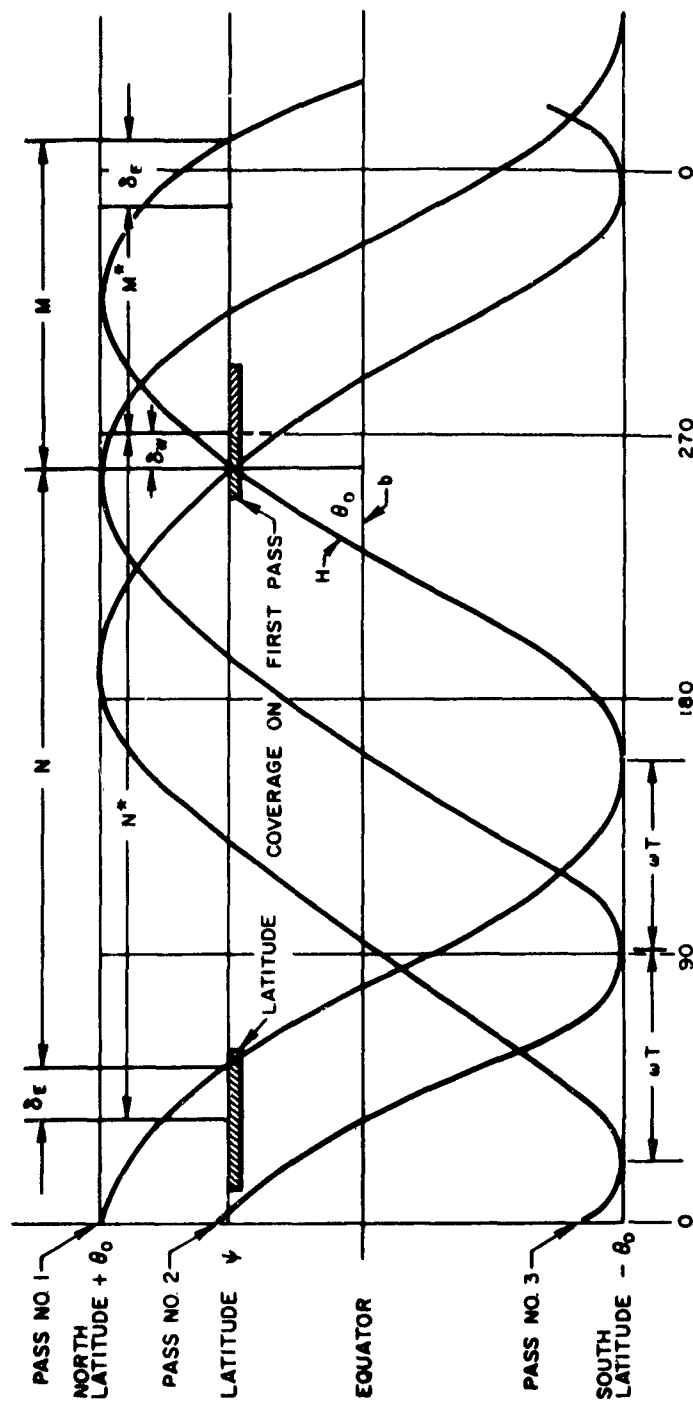
Figure 13.   Orbit Traces in Cartesian Coordinates.
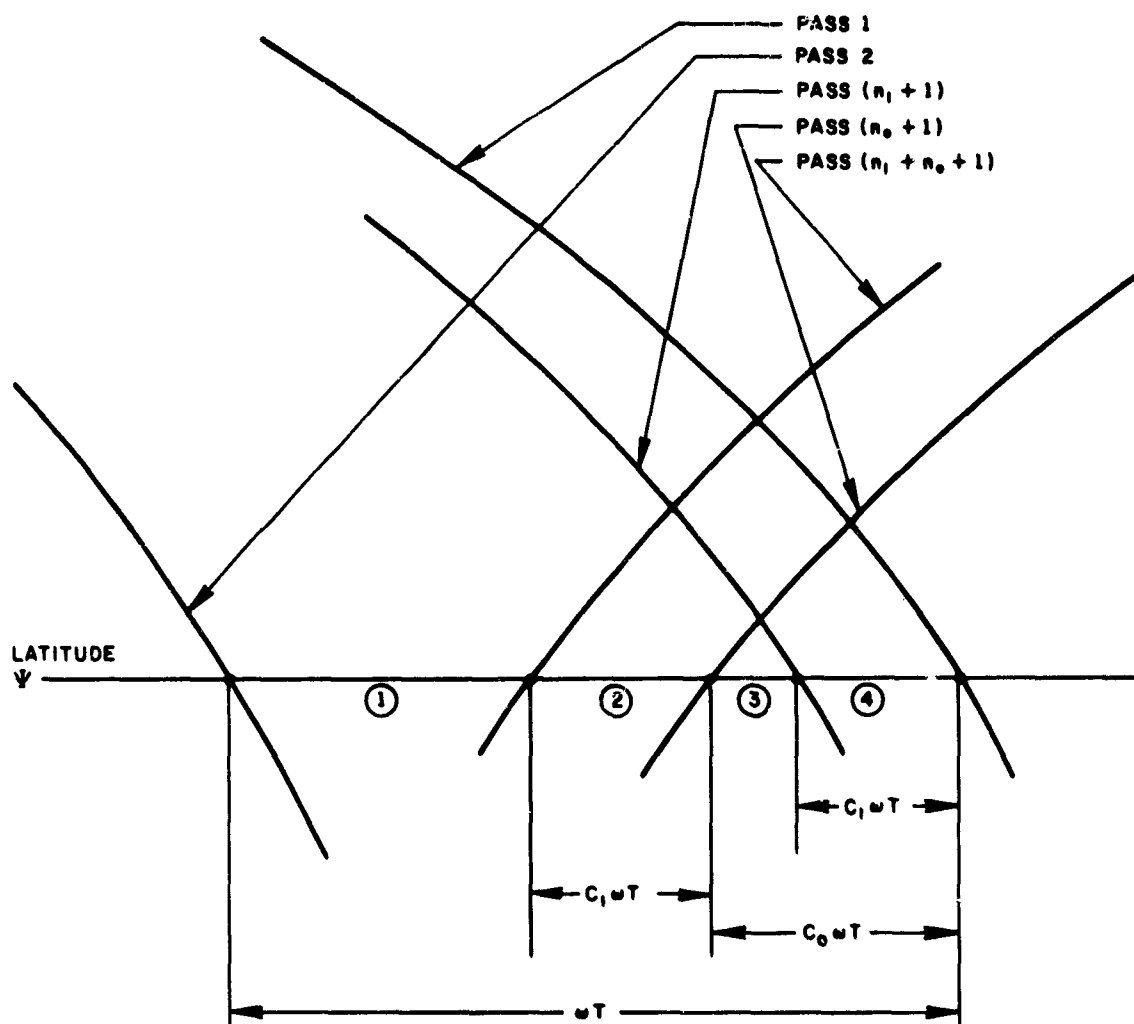
Figure 14. Orbit Trace Spacing in the Vicinity of the First Orbit Trace $(n_o + n_1 > 1)$.
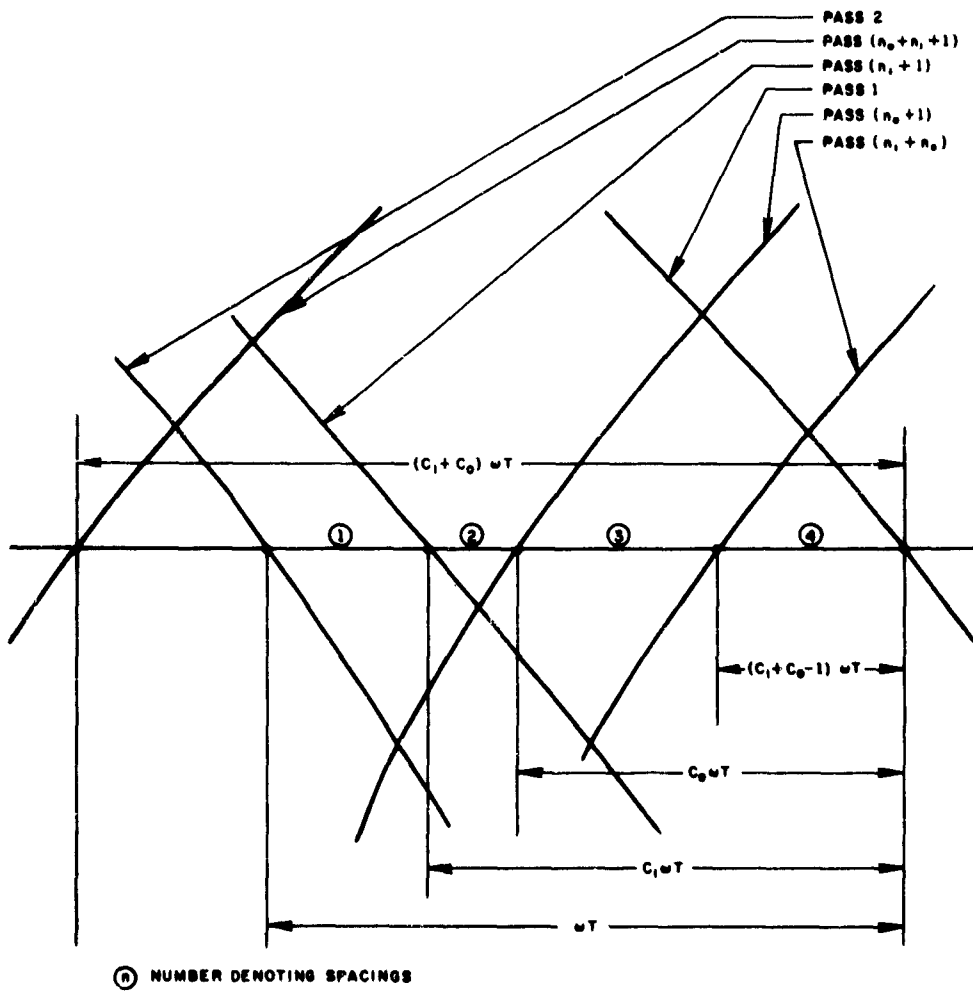
Figure 15. Orbit Trace Spacing in the Vicinity of the First Orbit Trace $(C_0 + C_1 \leqslant 1)$.
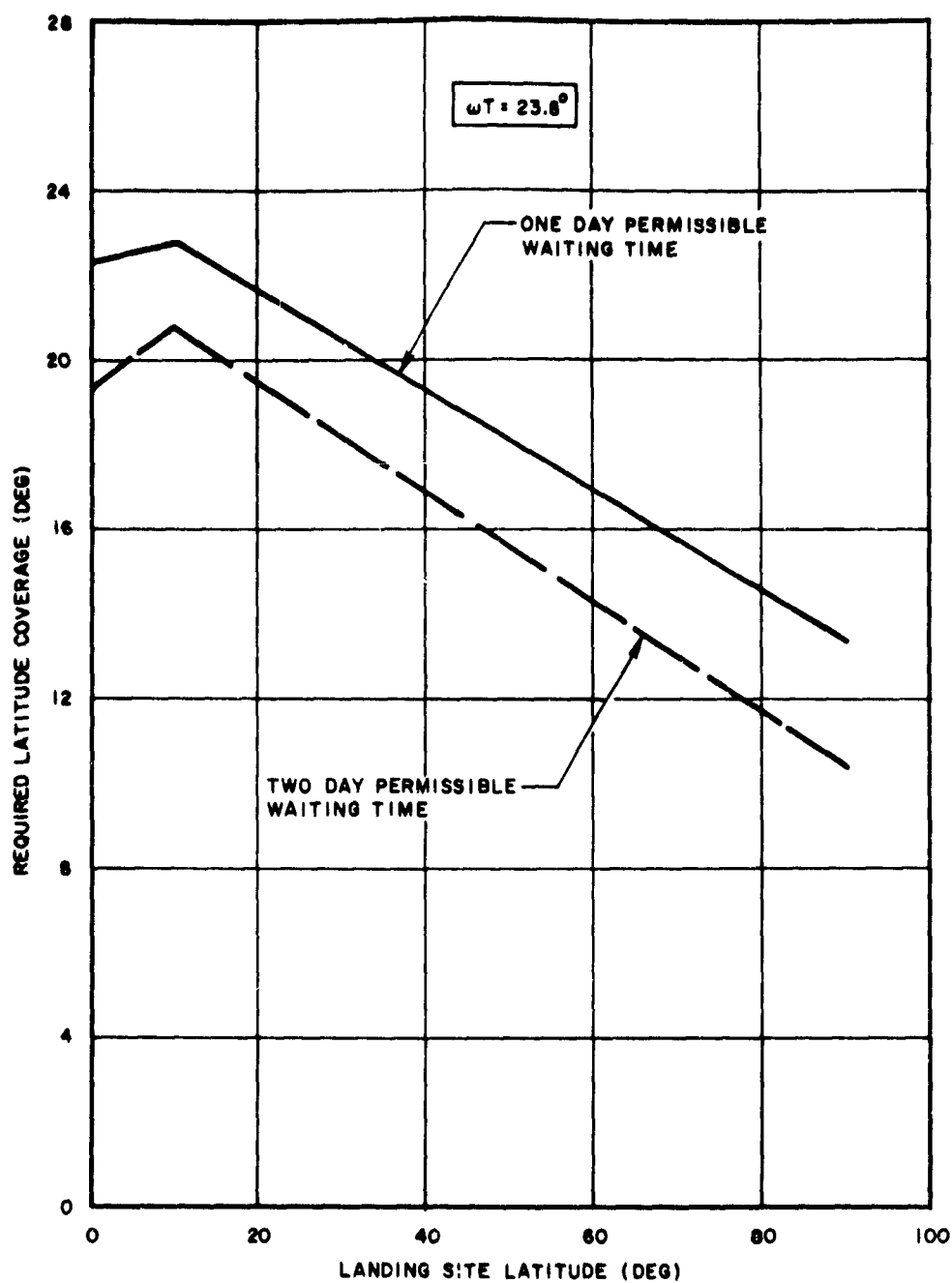
Figure 16. Latitude Coverage Required for Polar Orbits Versus Landing Site Latitude.
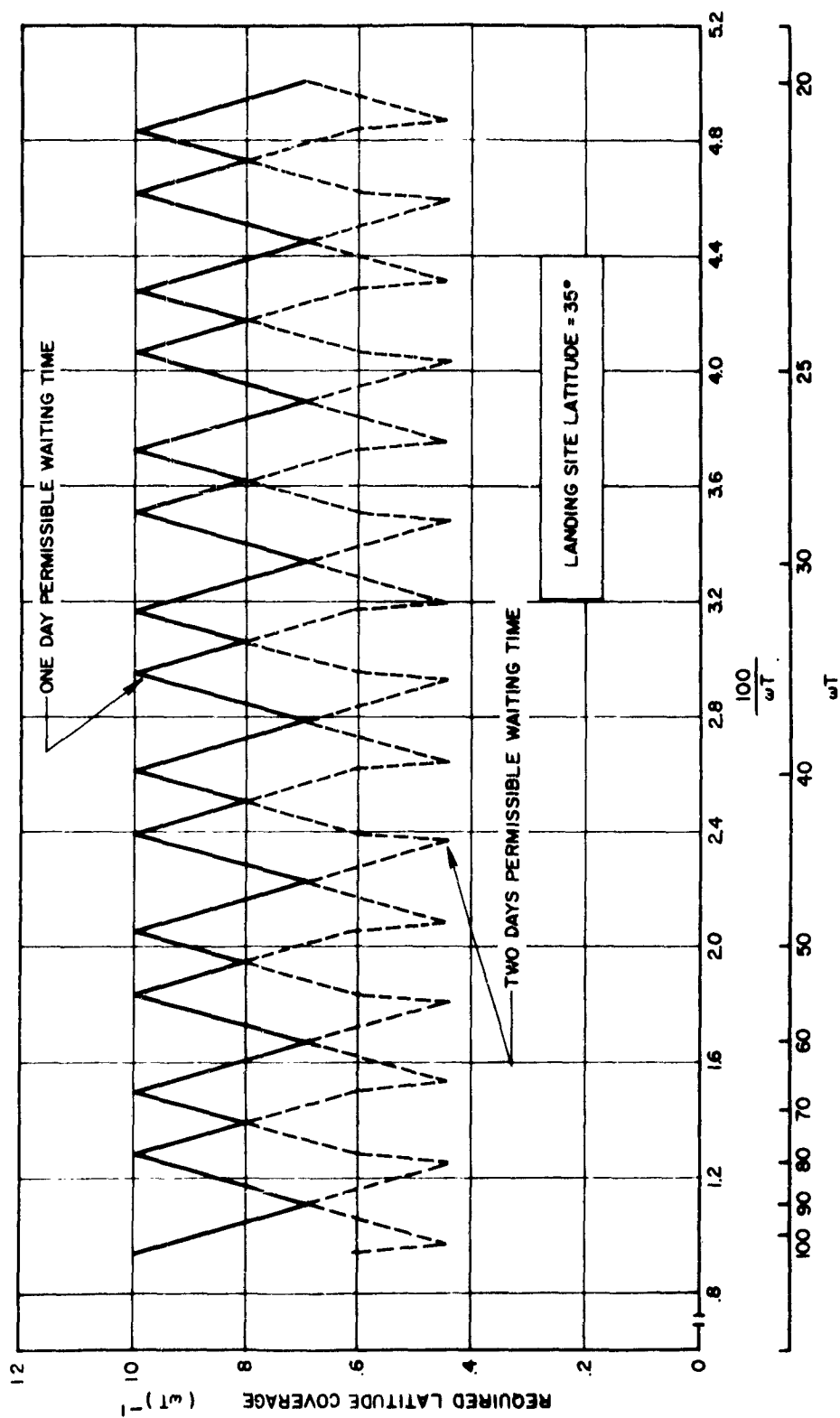
Figure 17. Latitude Coverage Required for Polar Orbits as a Function of the Parameter ωT.
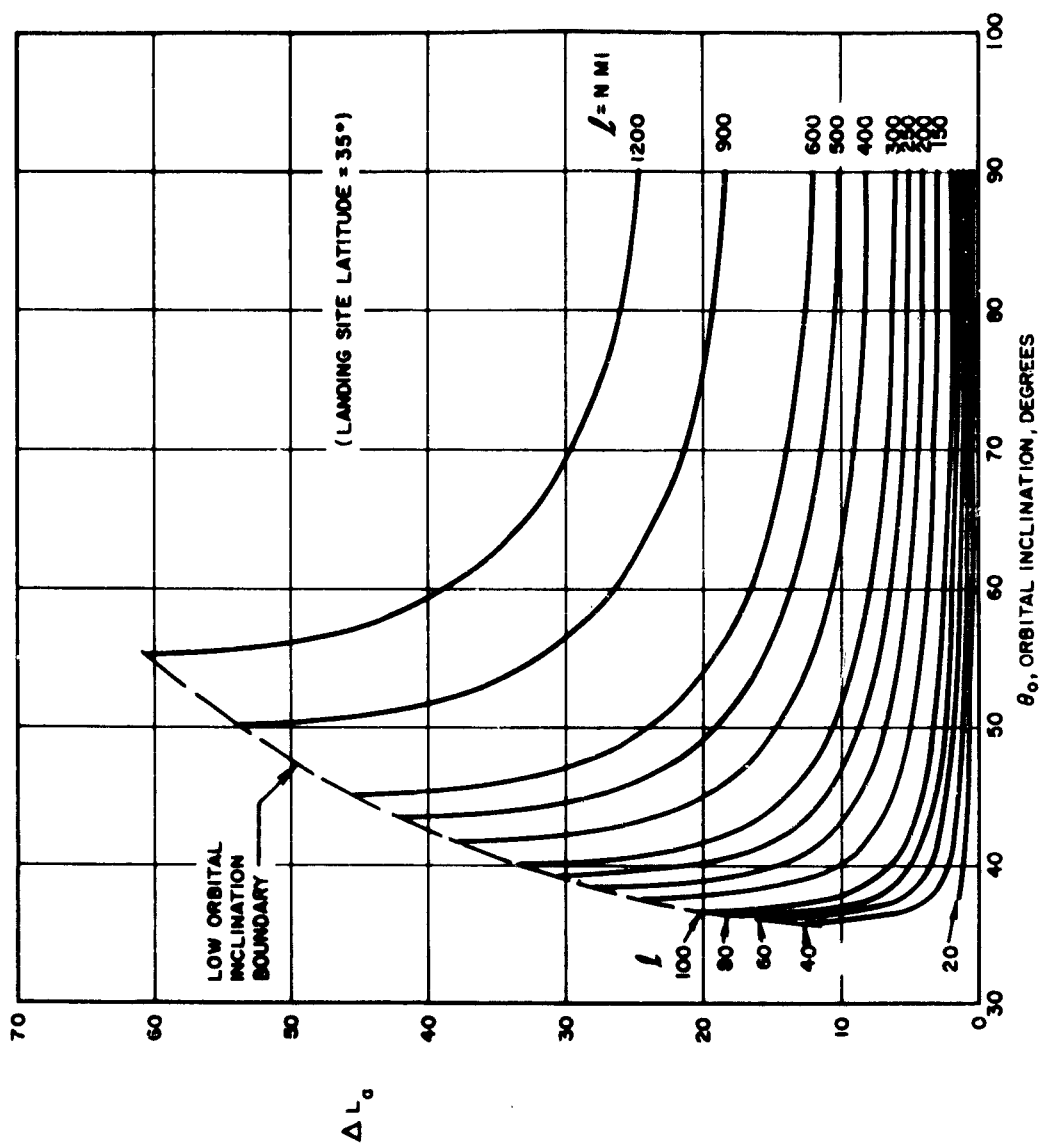
Figure 18. ΔL_a Versus Orbital Inclination for Various Crossrange Maneuver Capabilities.
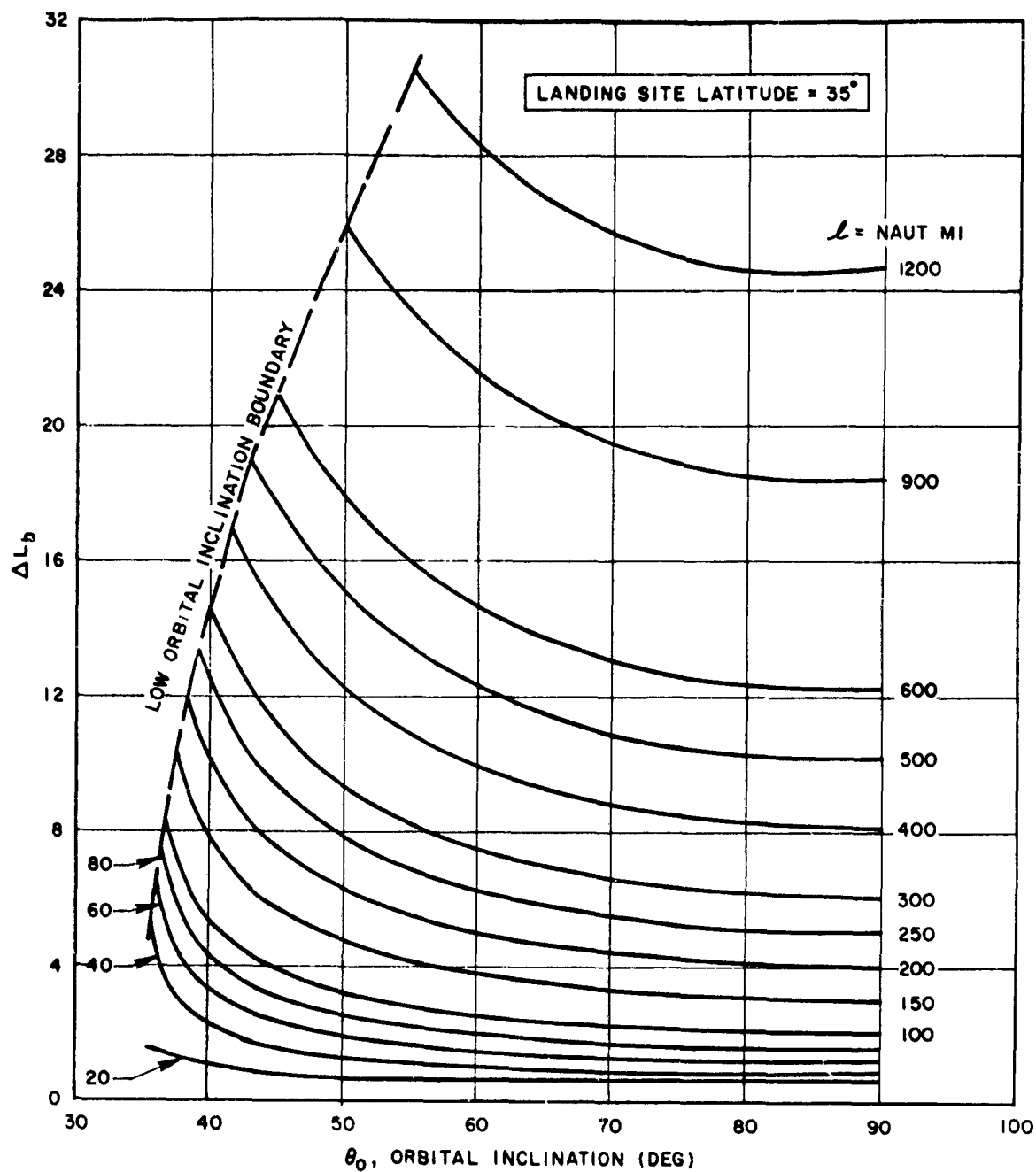
Figure 19. ΔL_b Versus Orbital Inclination for Various Crossrange Maneuver Capabilities.
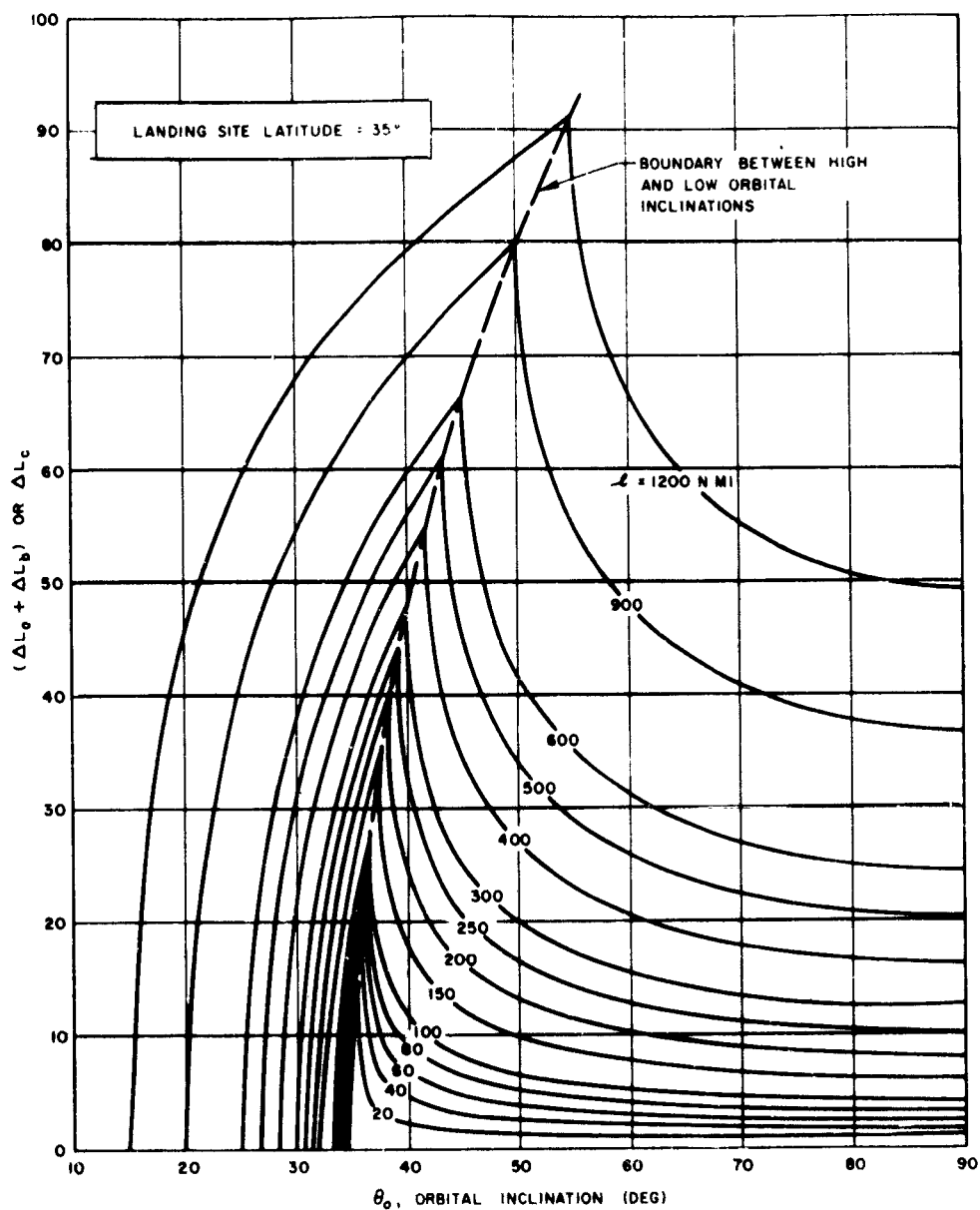
Figure 20. $(\Delta L_a + \Delta L_b)$ or ΔL_c Versus Orbital Inclination for Various Crossrange Maneuver Capabilities
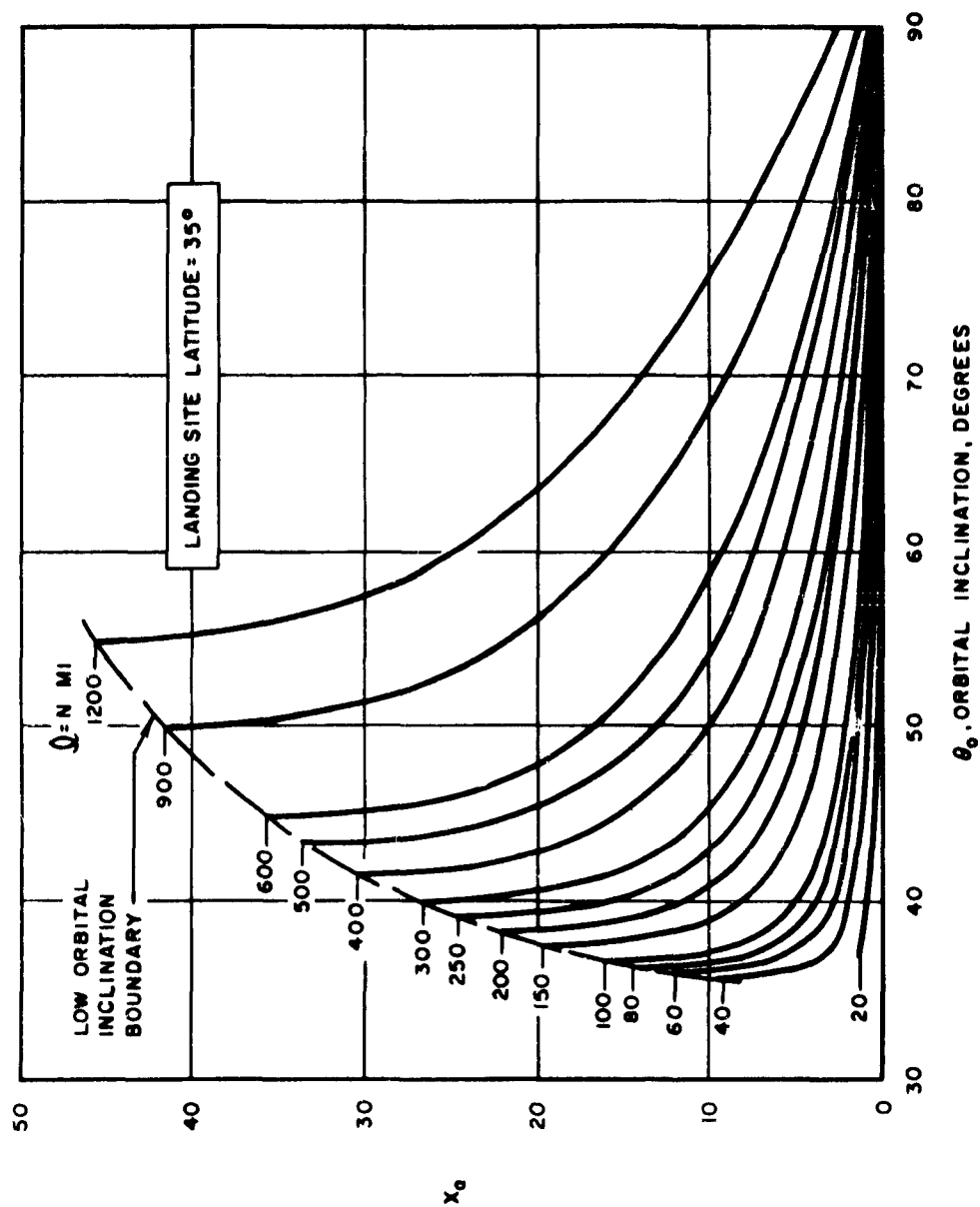
Figure 21. X_a Versus Orbital Inclination for Various Crossrange
Maneuver Capabilities.
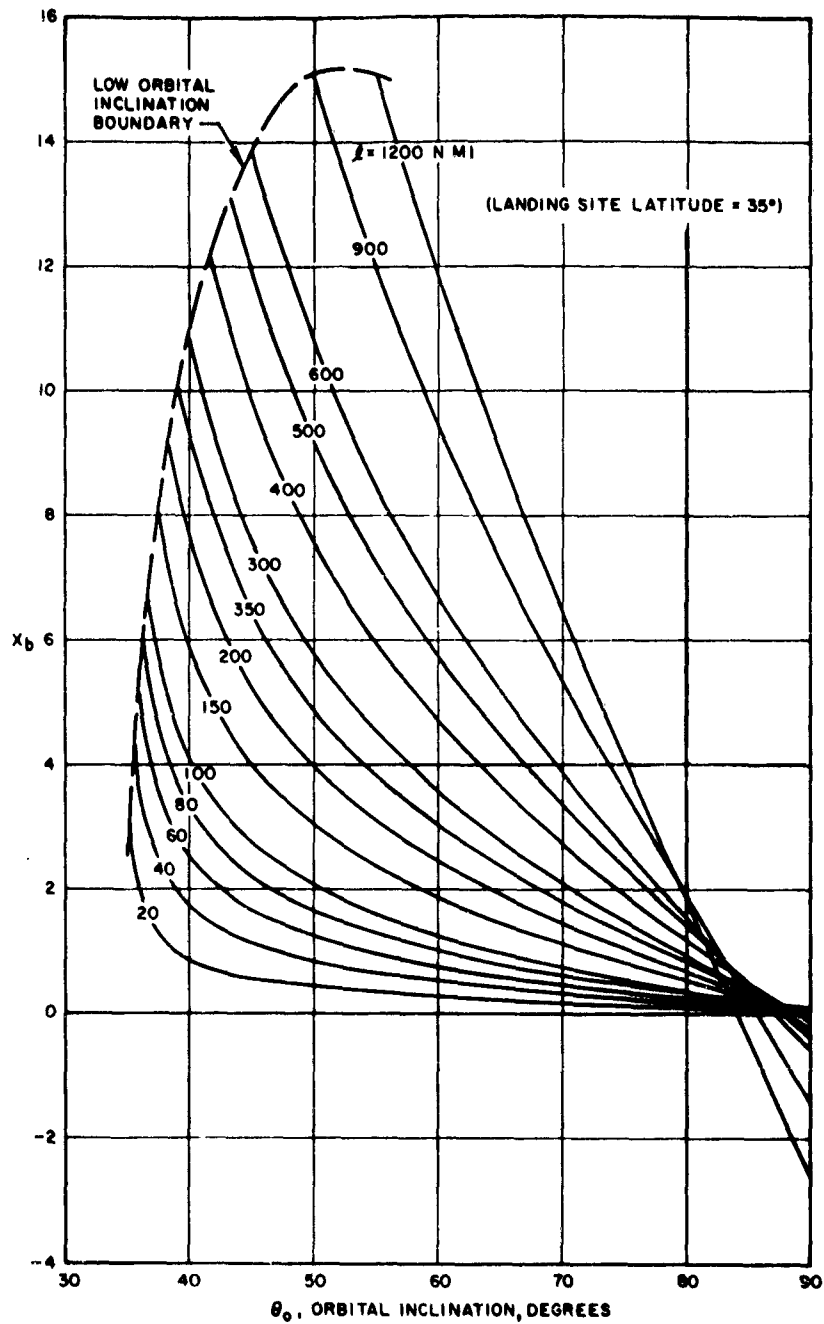
Figure 22. X_b Versus Orbital Inclination for Various Crossrange Maneuver Capabilities.
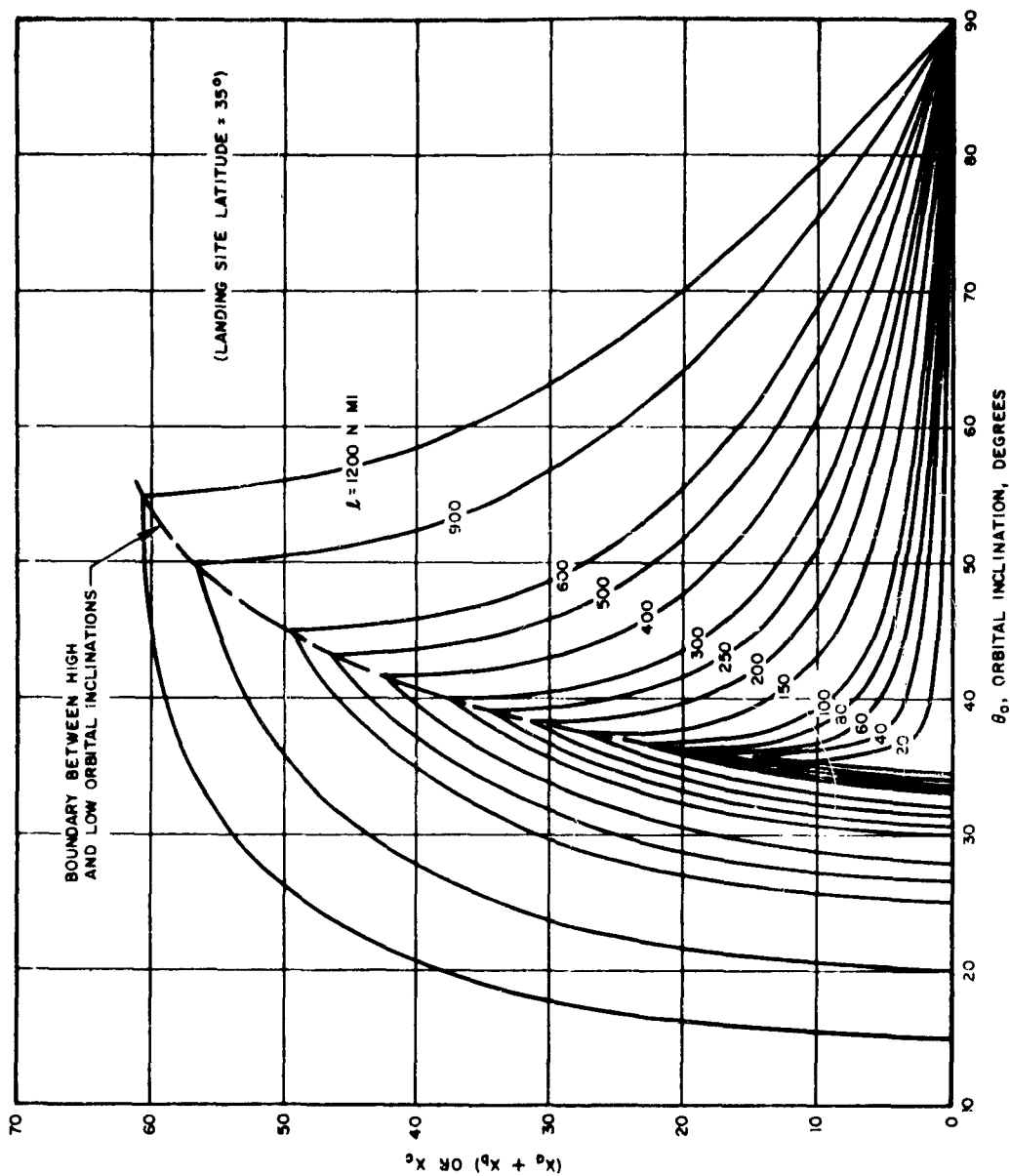
Figure 23. $(X_a + X_b)$ or X_c Versus Orbital Inclination for Various Crossrange Maneuver Capabilities.
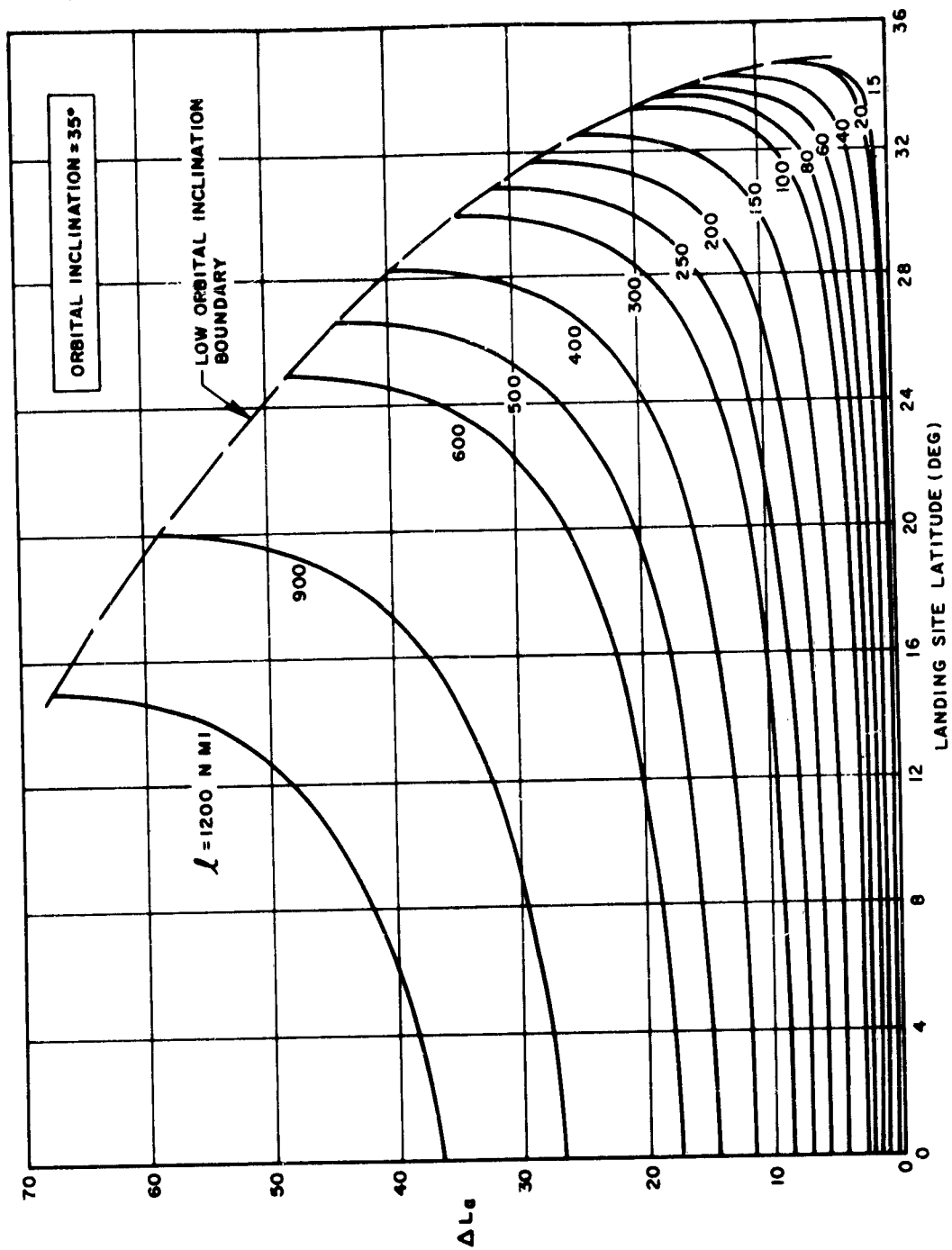
Figure 24. ΔL_a Versus Landing Site Latitude for Various Crossrange Maneuver Capabilities.
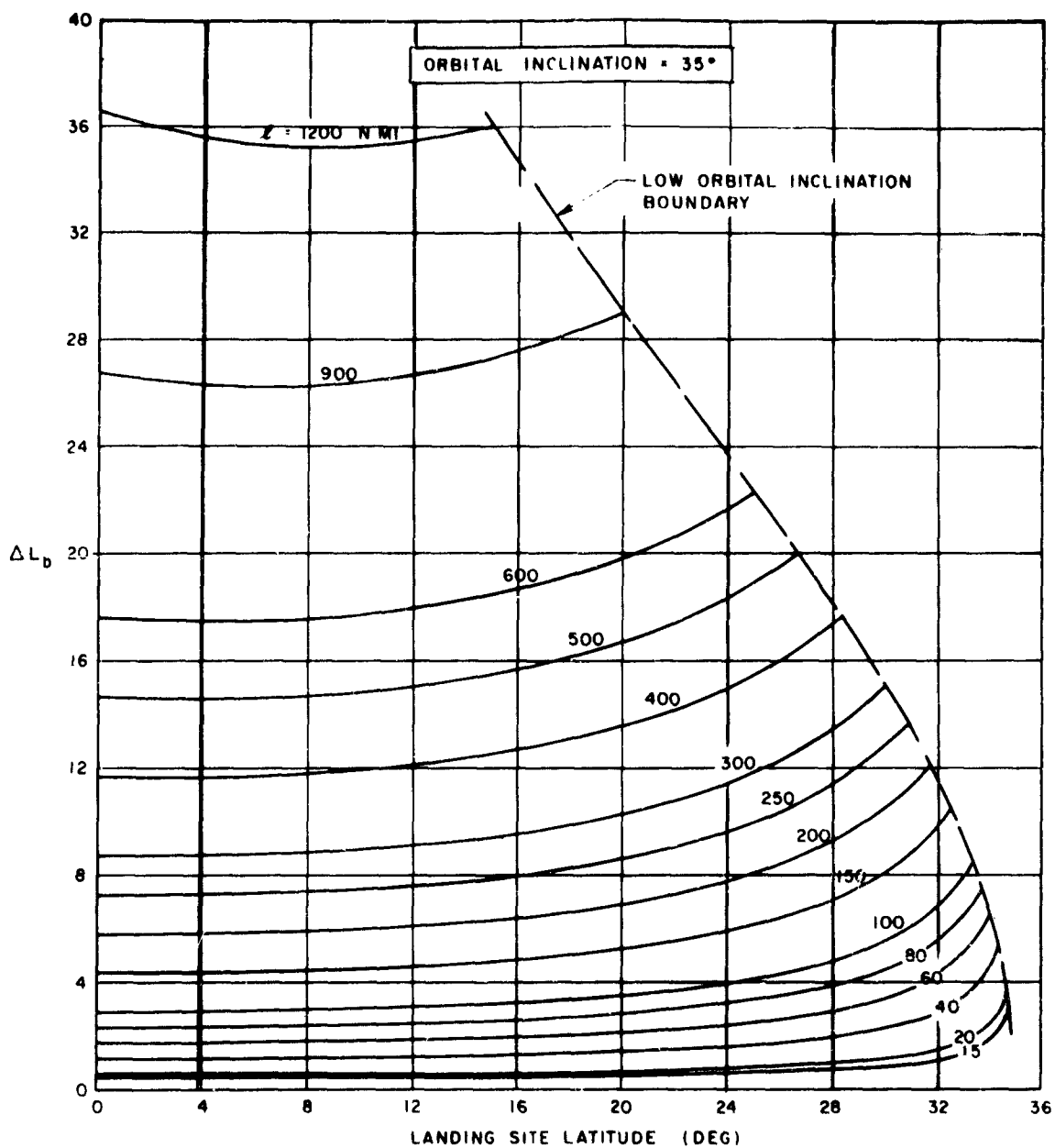
Figure 25. ΔL_b Versus Landing Site Latitude for Various
Crossrange Maneuver Capabilities.
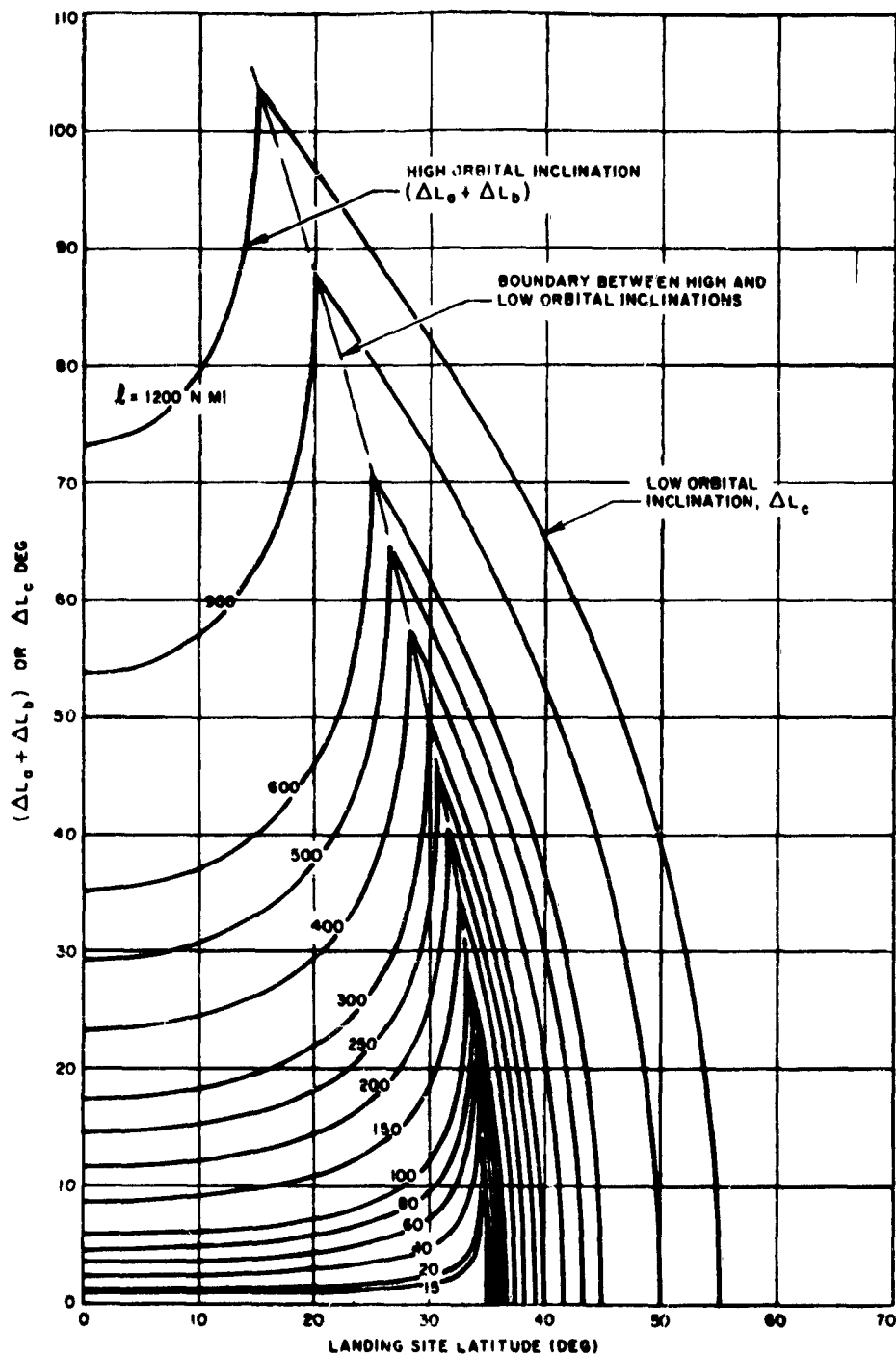
Figure 26. $(\Delta L_a + \Delta L_b)$ or ΔL_c Versus Landing Site Latitude for Various Crossrange Maneuver Capabilities.
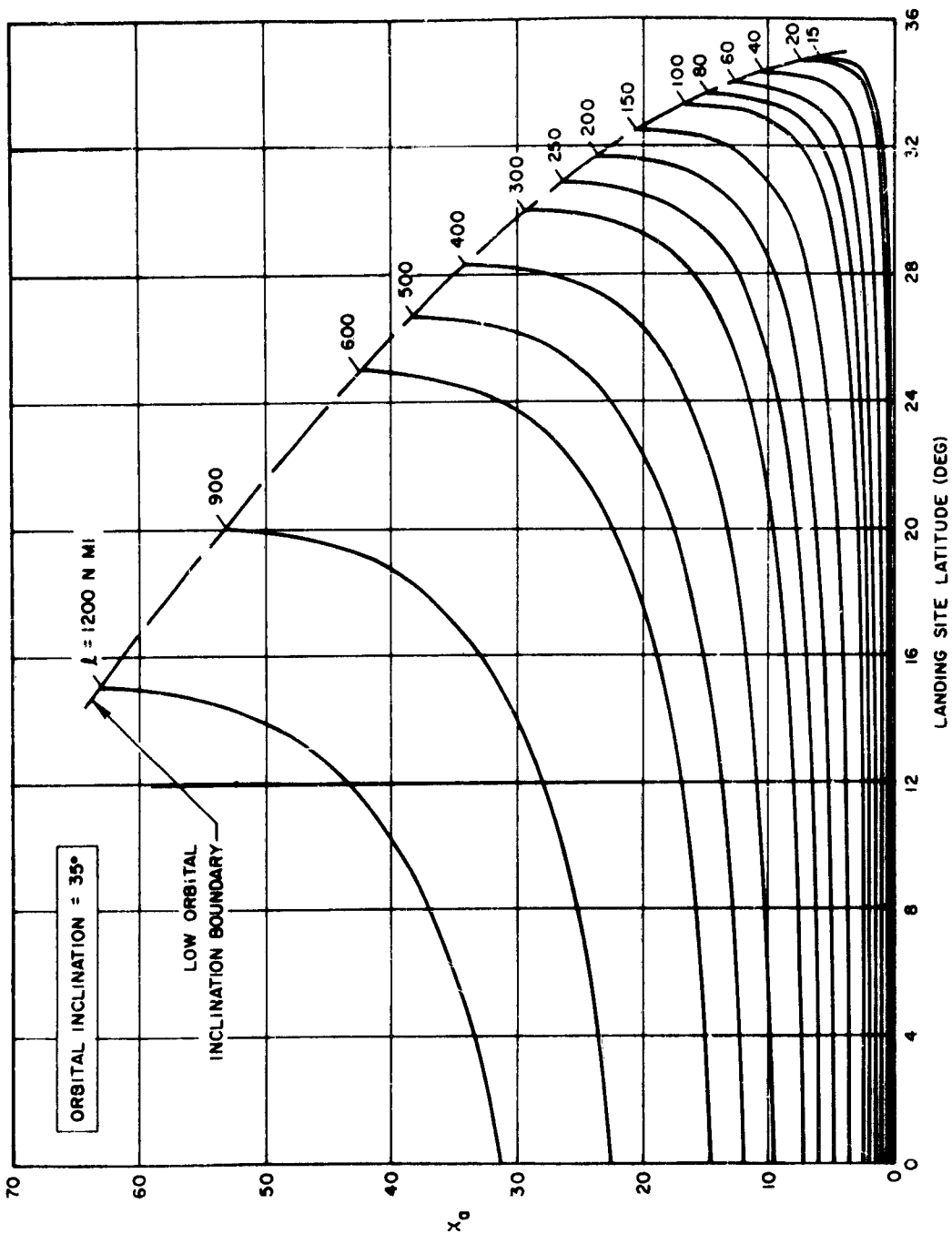
Figure 27. X_a Versus Landing Site Latitude for Various Crossrange Maneuver Capabilities.
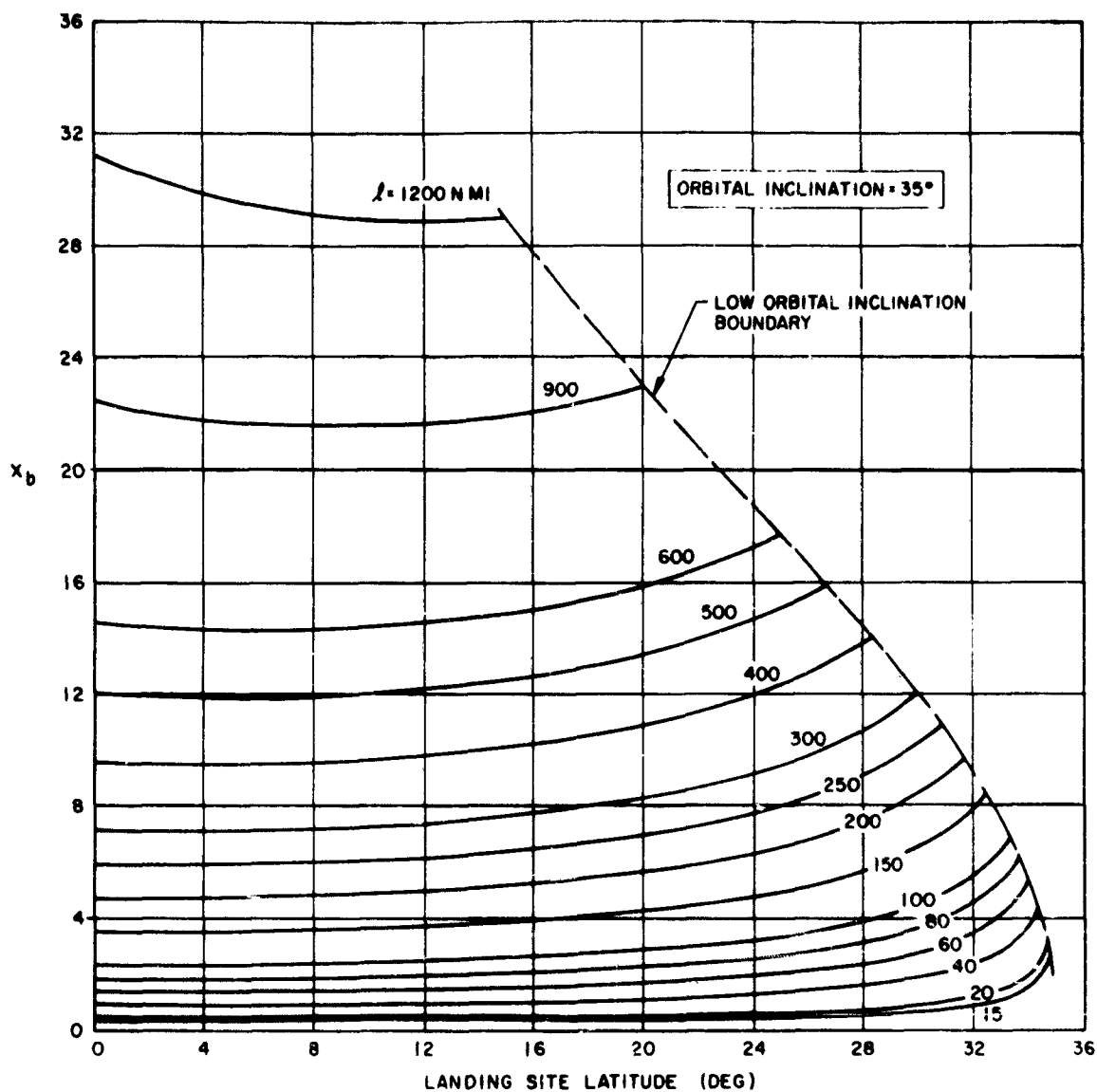
Figure 28. X_b Versus Landing Site Latitude for Various
Crossrange Maneuver Capabilities.
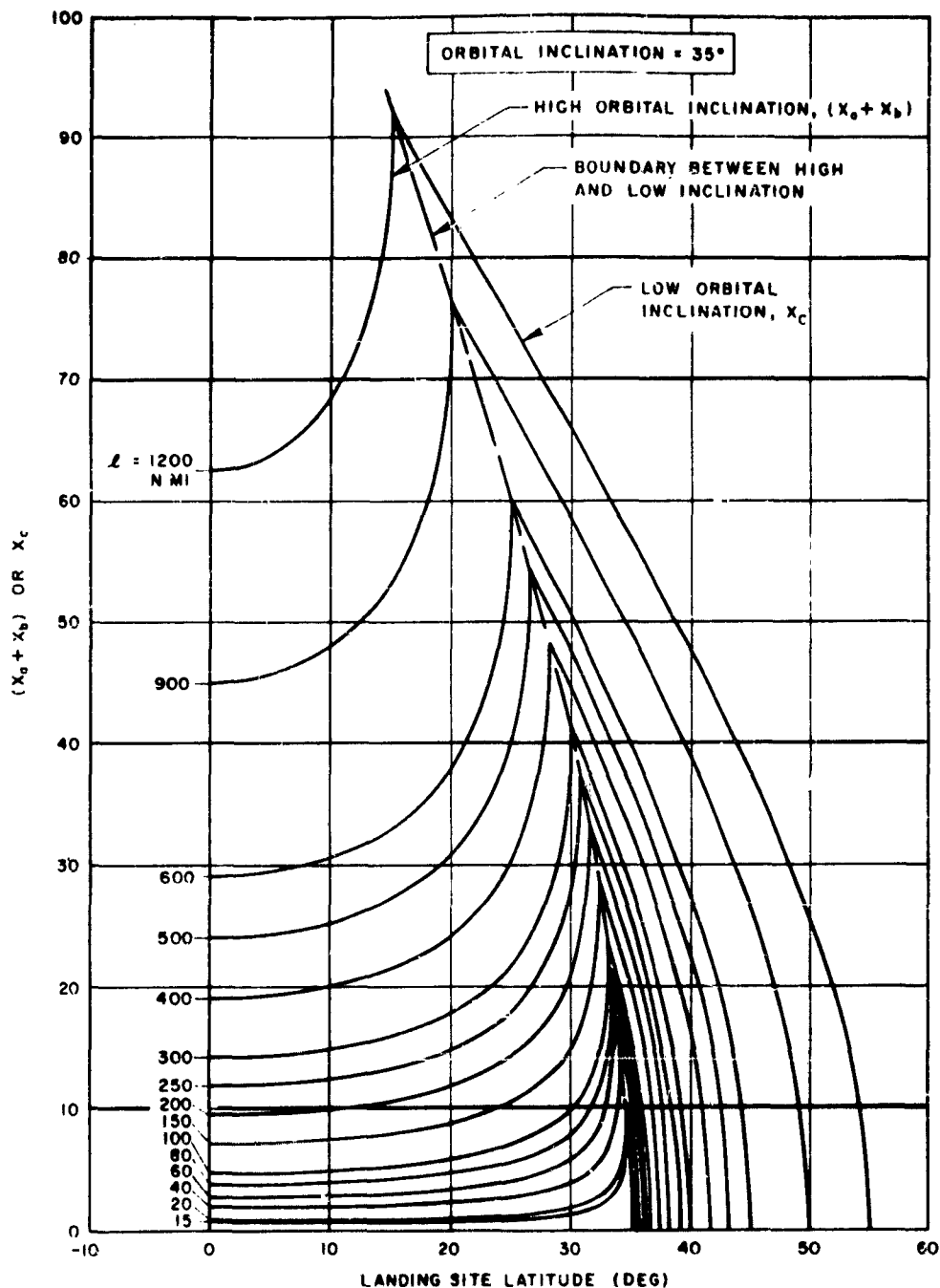
Figure 29. $(X_a + X_b)$ or X_c Versus Landing Site Latitude for Various Crossrange Maneuver Capabilities.
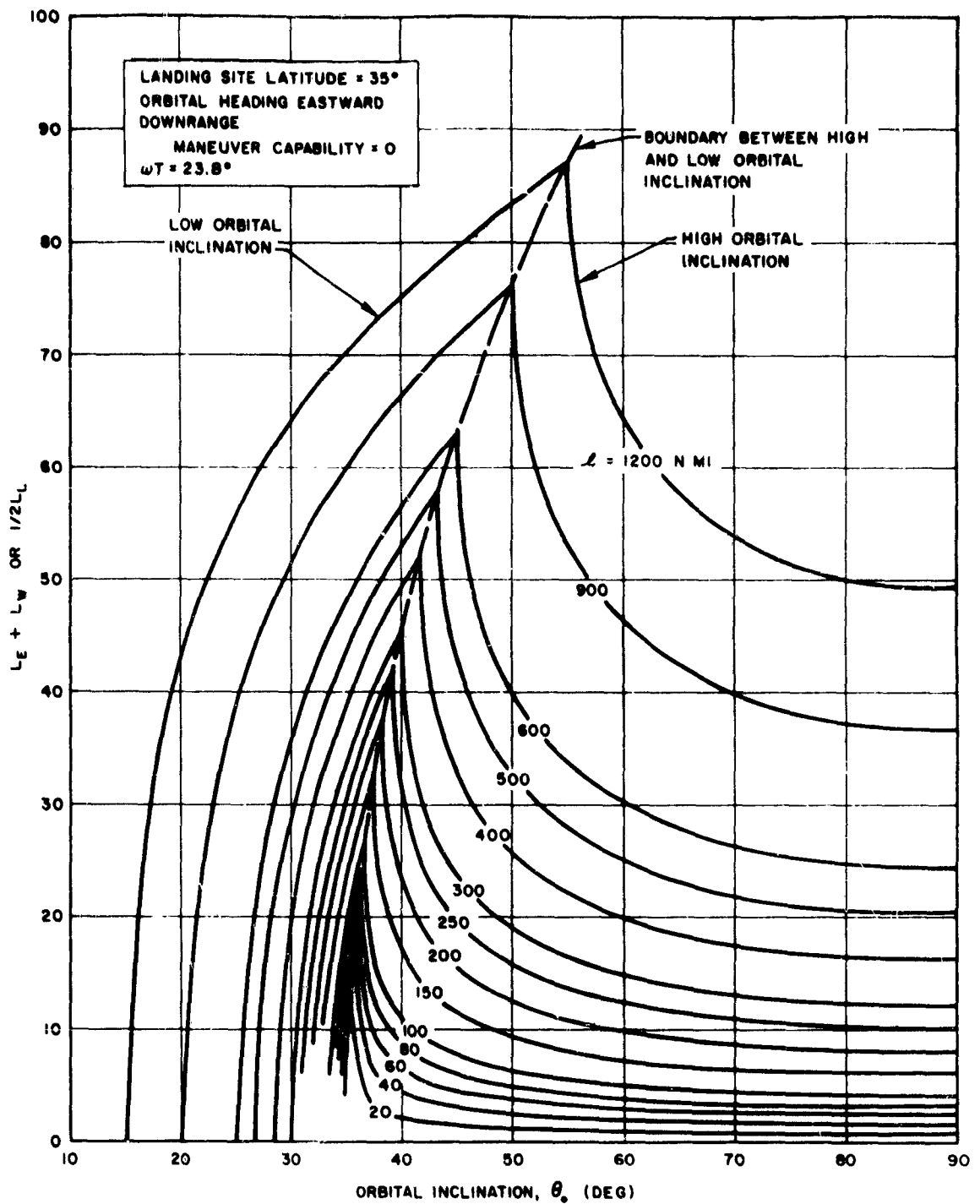
Figure 30. Latitude Coverage Versus Orbital Inclination for
Various Crossrange Maneuver Capabilities.
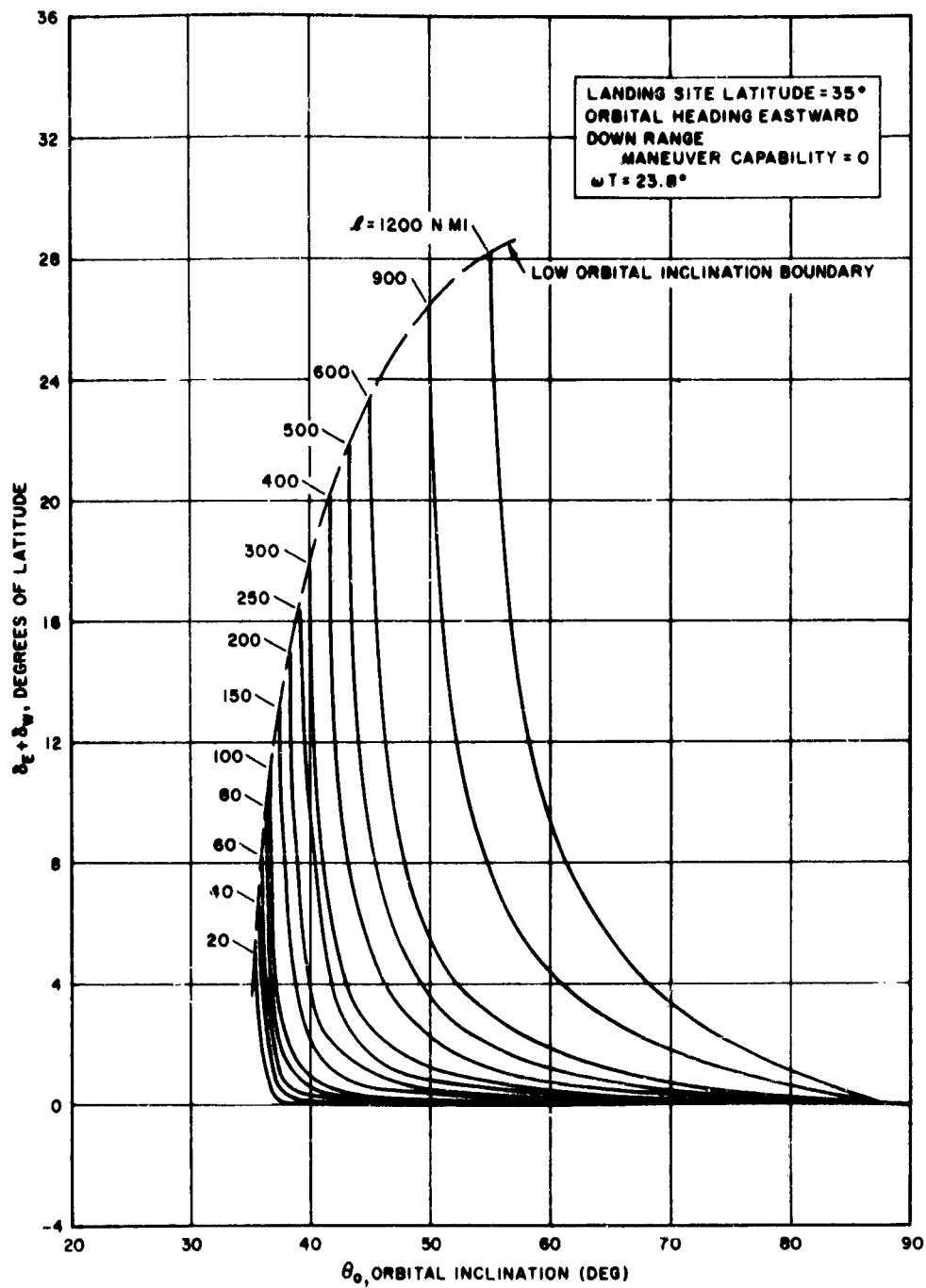
Figure 31. $(\delta_E + \delta_W)$ Versus Orbital Inclinations for Various Crossrange Maneuver Capabilities.
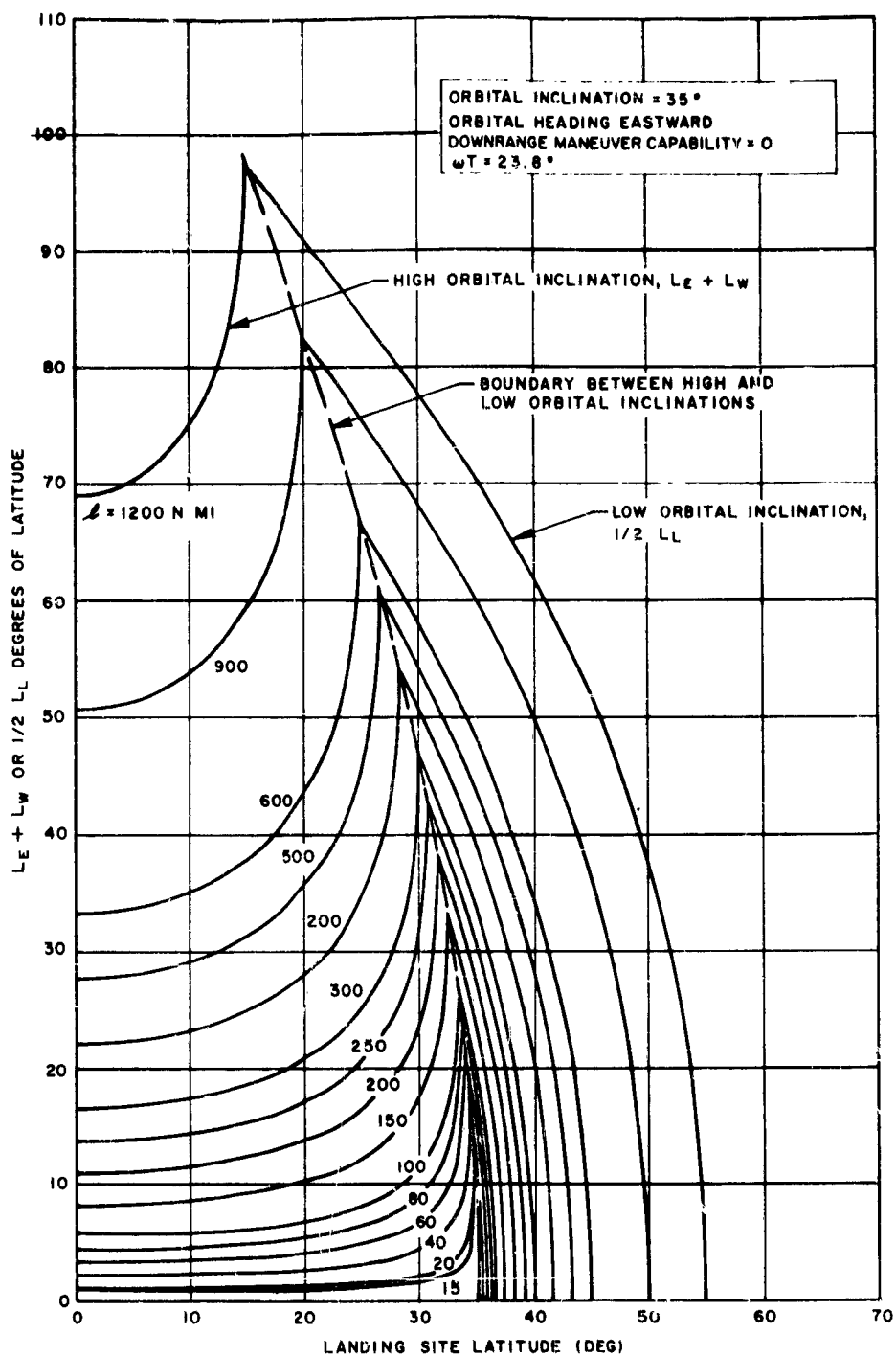
Figure 32. Latitude Coverage Versus Landing Site Latitude for Various Crossrange Maneuver Capabilities.
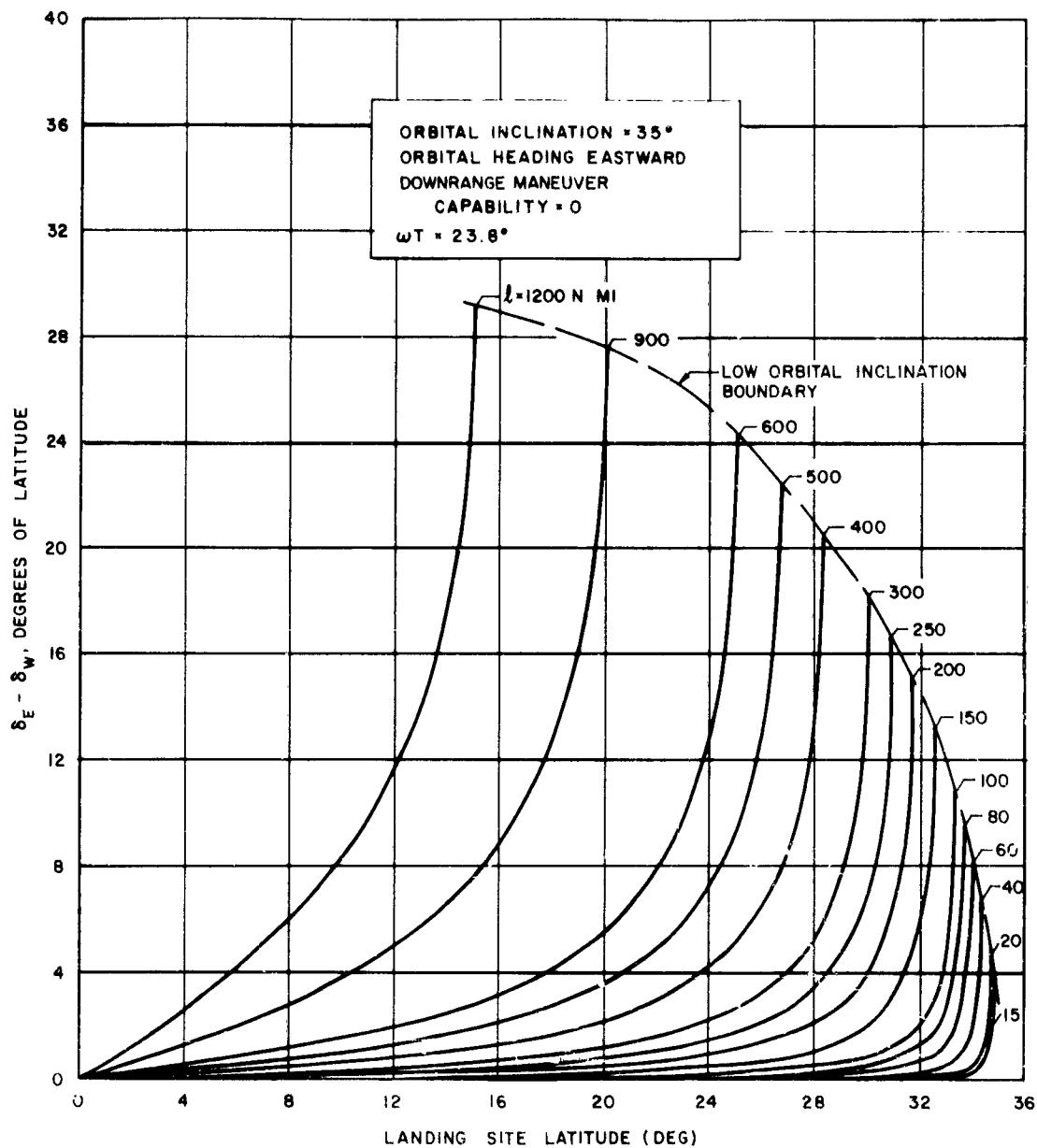
Figure 33. $(\delta_E + \delta_W)$ Versus Orbital Inclination for Various Crossrange Maneuver Capabilities.
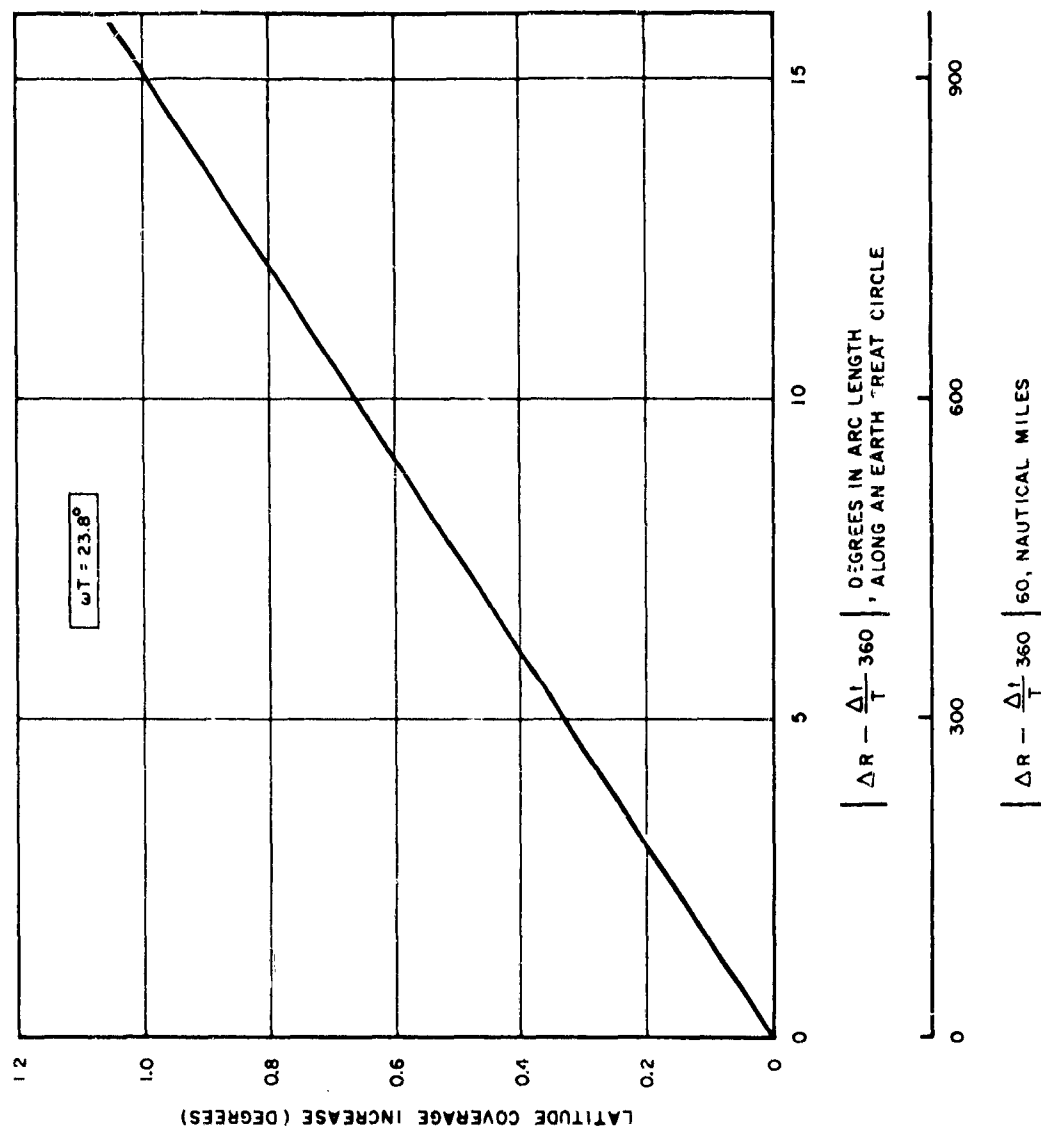
Figure 34. Latitude Coverage Increase Due to Downrange Maneuverability.
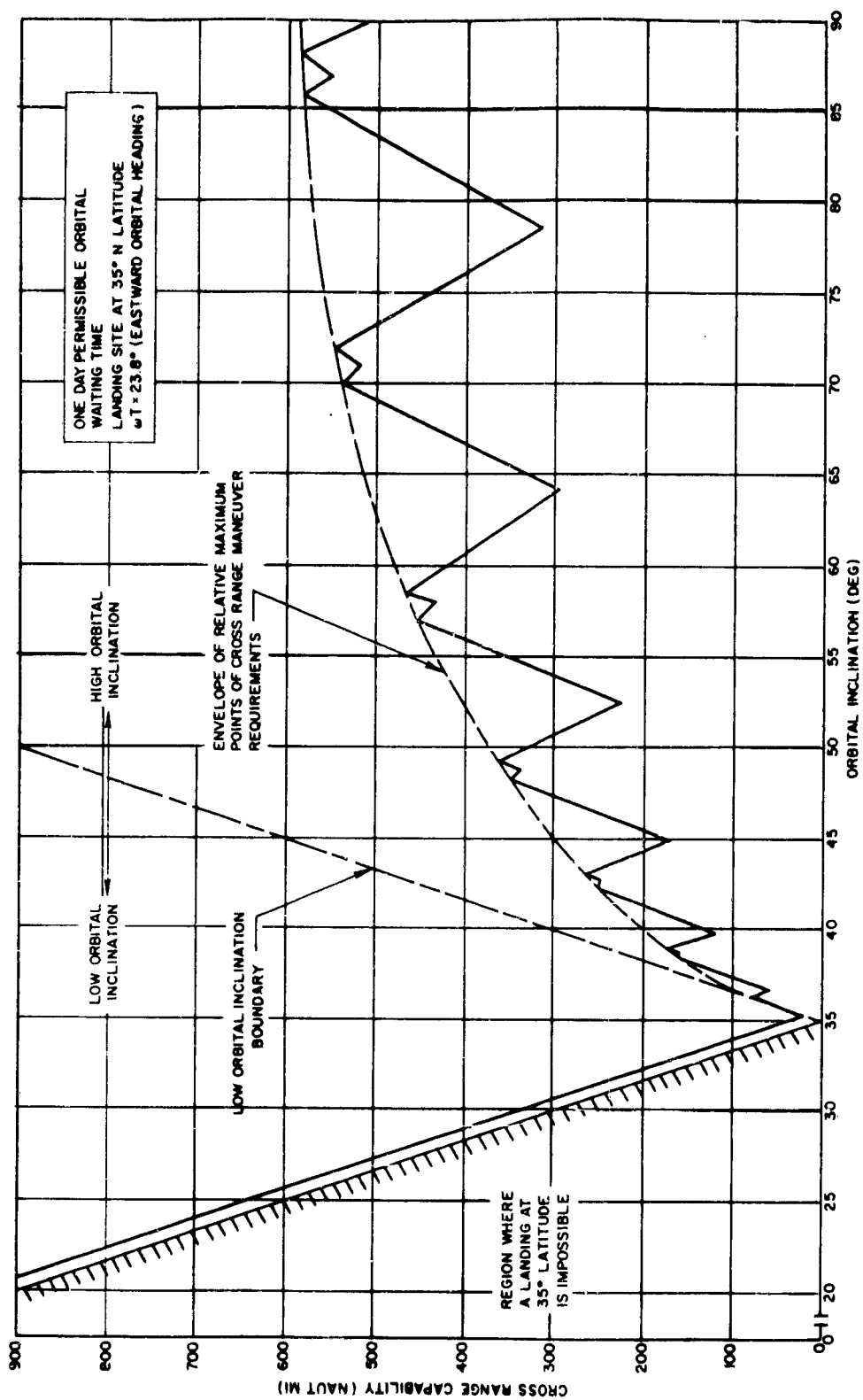
Figure 35. Required Crossrange Maneuver Capability Versus Orbital Inclination.
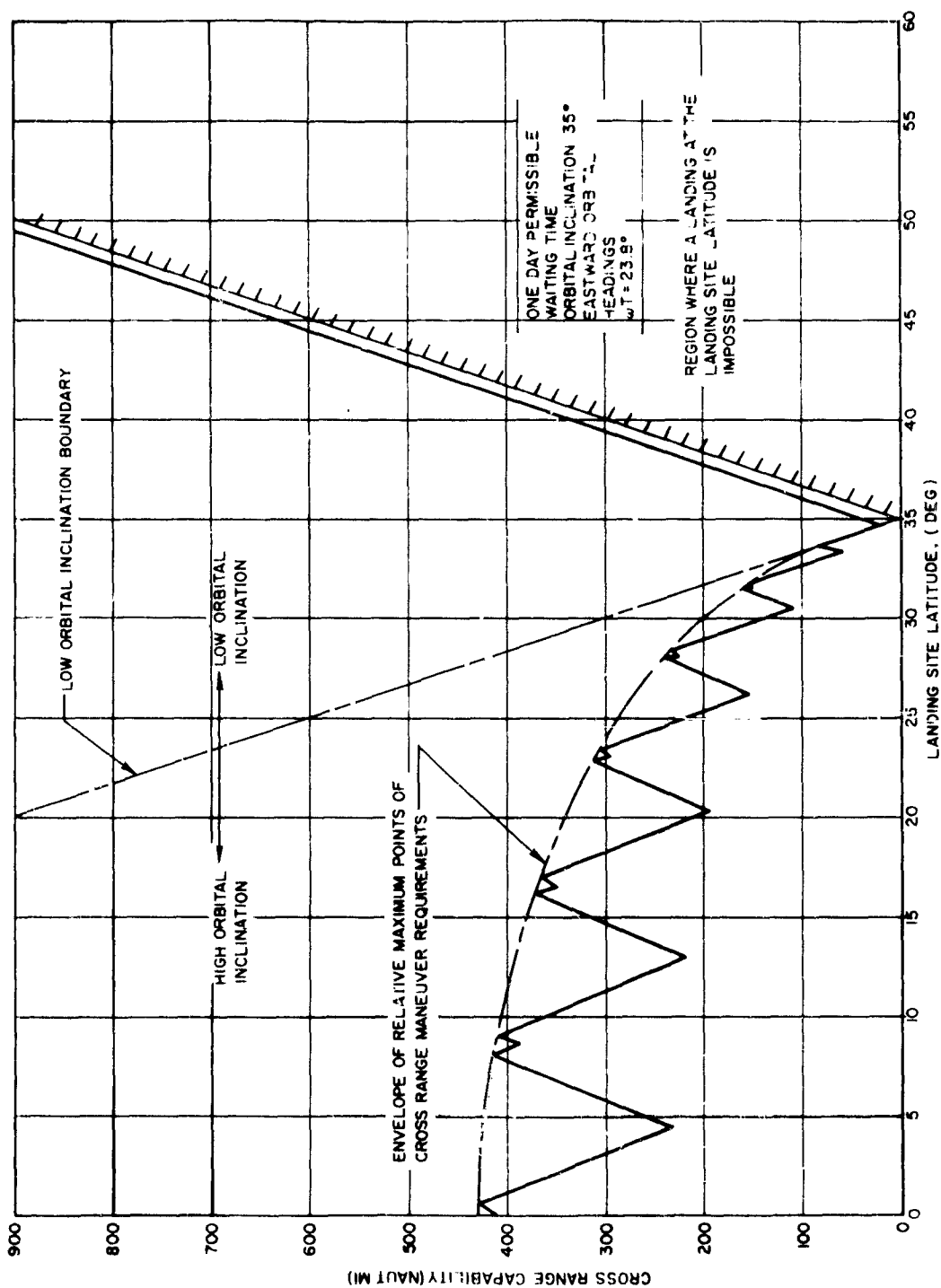
Figure 36. Required Crossrange Maneuver Capability Versus Landing Site Latitude.
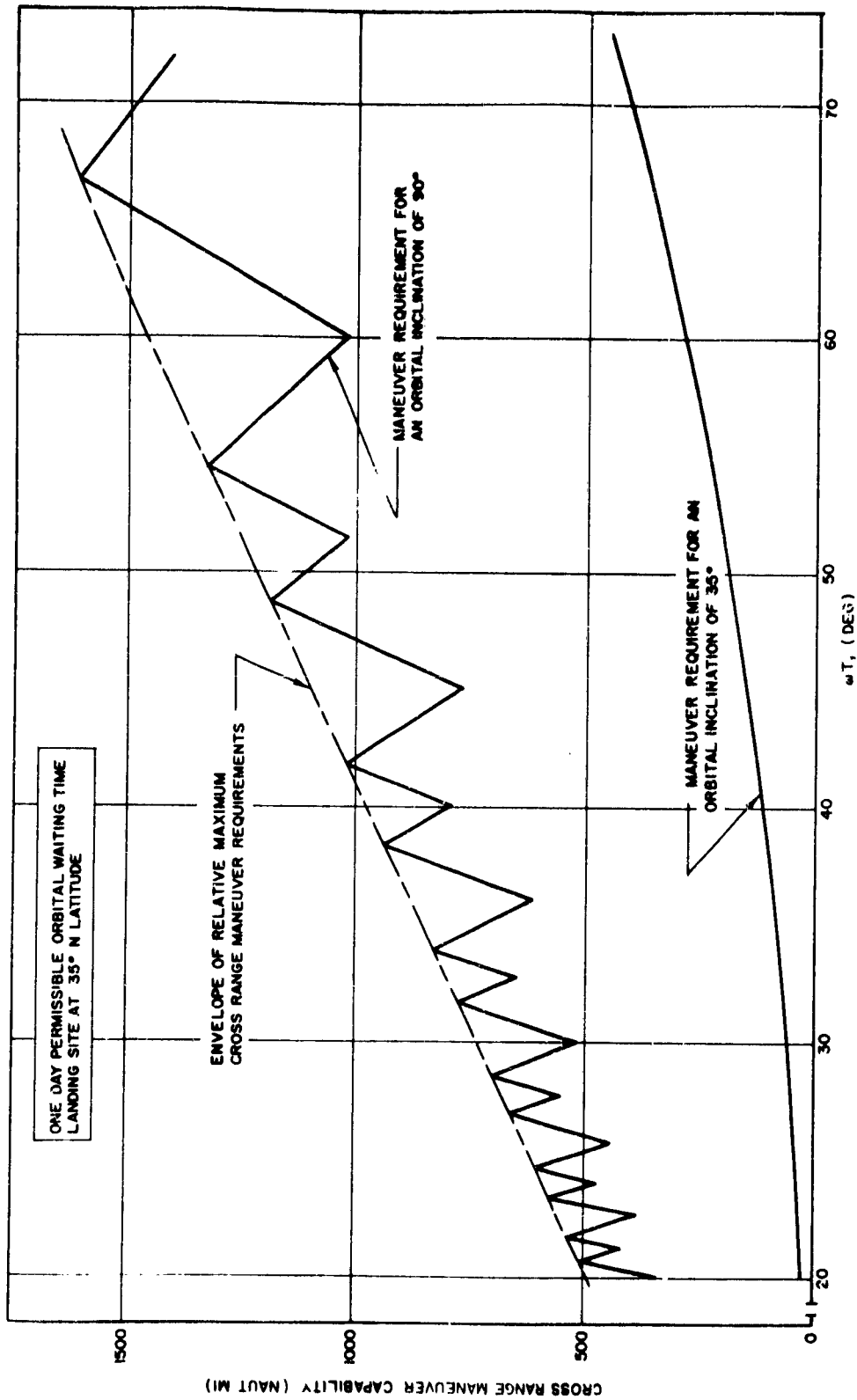
Figure 37. Required Crossrange Maneuver Capability Versus the Parameter ωT.
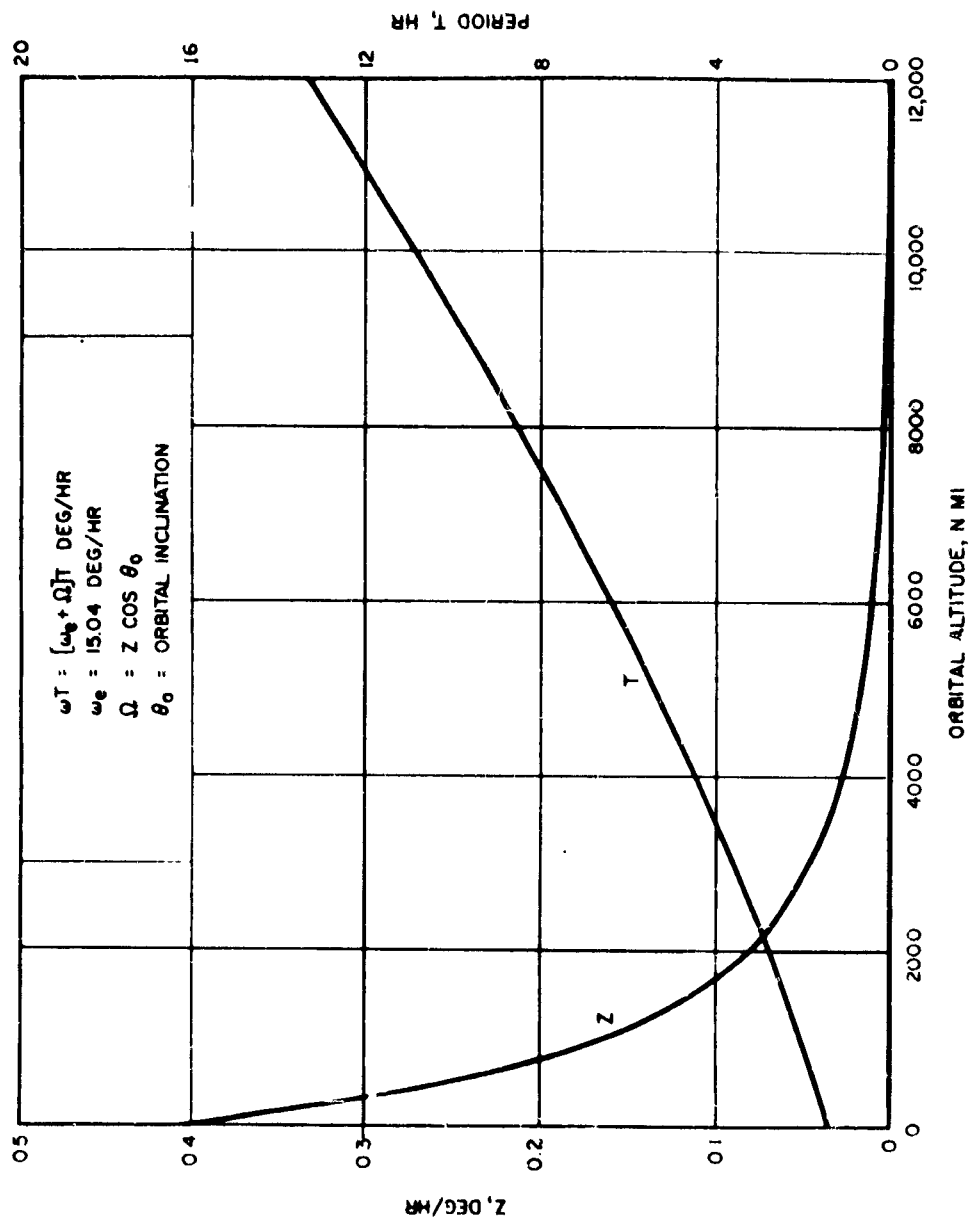
Figure 38. Parameters for Determining ωT.

NOMENCLATURE

A   Constant, +1 for a westward orbital heading, -1 for an eastward orbital heading.

b   Arc length measured along the equator depicted in Figure 13.

C_o Constant less than unity determined by equations (56) or (57).

C_o' Value less than unit determined by equation (62) or (63).

C_1 Nonzero constant less than zero determined by equation (67).

H   Projection on the reference sphere an arc length measured along the orbit traces between its intersection with the equator and landing-site latitude.

ℓ Cross-range maneuver capability or 1/2 the width of the landing footprint (great circle arc length measured in degrees).

$\tilde{L}$ Projection upon the reference sphere of a great circle arc length between two points on the landing-site latitude. The two points are separated in longitude by a distance of ΔL_a, ΔL_b or ΔL_c depending upon the situation under consideration.

L_E Latitude coverage per pass which lies to the east of the point where the orbit trace crosses the landing-site latitude. The latitude coverage in this case is attained by the high orbital inclination mode.

L_L Latitude coverage per orbital period when the coverage is attained by the low orbital inclination mode.

L_W Latitude coverage per pass which lies to the west of the point where the orbit trace crosses the landing-site latitude. The latitude coverage in this case is attained by the high orbital inclination mode.

L_{RO} Lateral coverage required for a permissible orbital waiting time of n_o orbital periods.

L_{R1} Latitude coverage required for a permissible waiting time of one sidereal day.

L_{R2} Latitude coverage required for a permissible waiting time of two sidereal days.

ΔL_a Distance measured in degrees longitude between two points on the landing-site latitudes' projection upon the reference sphere. The first of the points is defined by the intersection of the orbit trace with the landing-site latitude. The second point on the landing-site latitude is

NOMENCLATURE (Continued)

ΔL_a (Continued) where a great circle arc of length l extends up to the orbit trace at the most northerly possible point such that a perpendicular between the orbit trace and the great circle arc exists.

ΔL_b Distance measured in degrees longitude between two points on the landing-site latitudes' projection upon the reference sphere. The first of the points is defined by the intersection of the orbit trace with the landing-site latitude. The second point on the landing-site latitude is where a great circle arc of length l extends to the orbit trace at the most southerly possible point such that a perpendicular angle between the orbit trace and the great circle arc exists.

ΔL_c Distance measured in degrees longitude between two points on the landing-site latitudes' projection upon the reference sphere. The first point on the landing-site latitude is where a great circle arc of length l extends down to the orbit trace such that a perpendicular angle between the orbit trace and the great circle arc exists. The second point is defined by the intersection of a polar great circle arc with the landing-site latitude such that the great circle arc passes through the most northerly point on the orbit trace.

M Arc length measured along the landing-site latitude in the direction of the orbital heading from the ascending intersection of the orbit trace to the descending intersection of the orbit trace with the landing-site latitude.

M^* Arc length measured along the landing-site latitude in the direction of the orbital heading from the center of the latitude coverage belt which is attendant with the orbit trace as it heads northward to the center of the latitude coverage belt attendant with the southerly headed orbit trace.

N Arc length measured along the landing-site latitude in the direction of the orbital heading from the descending intersection of the orbit trace to the ascending intersection of the orbit trace with the landing-site latitude.

n_o Minimum number of complete orbital periods required to achieve a latitude coverage requirement of ωT.

n_o' Orbital pass during which intermingling of latitude coverage centers generated from portions of the orbit traces headed in dissimilar directions is initiated.

n_1 Number of complete orbital periods given by equation (67).

NOMENCLATURE (Continued)

ΔR   Downrange dimension of the landing footprint (arc length measured in degrees).

T   Period of the circular orbit from which the lifting re-entry vehicle is deboosted (hours)

Δt   Flight time difference between the leading and trailing edge of the landing footprint.

X_a Projection upon the reference sphere of an angular distance along the orbit traces from the landing-site latitude to the intersection of the arc perpendicular to orbit trace of length l which extends downward to the landing-site latitude from the most northerly possible point.

X_b Projection upon the reference sphere of an angular distance along the orbit trace from the landing-site latitude to the intersection of the arc perpendicular to the orbit trace of length l extending to the landing-site latitude from the most southerly possible point upon the orbit trace.

X_c Projection upon the reference sphere of an angular distance along the orbit traces from the point where an arc of length l extends perpendicularly from the orbit trace to the landing-site latitude to the point where the orbit trace makes its most northern excursion.

y   Arc length on the reference sphere depicted in Figure 3.

Z $\Omega/\cos\theta_o$

α $(90 - \theta)$

δ   Arc length measured along the landing-site latitude between the orbit trace intersection and the center of the lateral coverage on a given pass.

θ   Inclination of the orbit trace with respect to the plane containing the landing-site latitude.

θ_o Inclination of the orbital plane to the equatorial plane (orbital inclination).

θ_x Value of θ at which X_b is zero.

φ   Latitude of the landing site.

NOMENCLATURE (Continued)

ω Relative angular motion between the earth and the reference sphere. $\omega = \omega_e + \Omega$ (degrees per hour).

ω_e Earth rotational rate (15.04 degrees per hour).

Ω Average precession of the orbit plane about the earths polar axis. Precession in a westerly direction is considered positive (degrees per hour).

REFERENCES

1. Thomson, W. T., "Introduction to Space Dynamics," John Wiley and Sons, Inc., 1961.

Aerospace Corporation, El Segundo, California. LANDING SITE COVERAGE FOR ORBITAL LIFTING RE-ENTRY VEHICLES by R. G. Stern and S. T. Chu, 1 April 1963. 88 pages including illustrations. (Report TDR-169(3530-10)TN-1, SSD-TDR-63-97) Contract No. AF04(695)-169.

Unclassified Report

The problem of landing site coverage of orbiting vehicles with maneuvering capability is analyzed in this report. The first part deals with the determination of the latitude coverage of a landing site per orbital period for the re-entry vehicle with a given maneuver capability defined by a rectangularly shaped "landing footprint." The second part relates the required latitude coverage to guarantee that within a specific waiting time in orbit the vehicle would attain a position from which it could be deboosted from orbit and subsequently land at a preselected location.

(over)

Extensive numerical results are obtained and presented in convenient graphical form. For example, the magnitude of the latitude coverage is presented in two ways: (1) as a function of orbital inclination for various cross-range maneuver capabilities and a given landing-site latitude of 35 degrees; (2) as a function of landing-site latitude for various cross-range maneuver capabilities and a given orbital inclination of 35 degrees. Similarly, the cross-range maneuver requirements for an orbiting vehicle with a permissible waiting time of one day and a fixed orbital period are presented for either a given landing-site latitude of 35 degrees, or a given orbital inclination of 35 degrees.