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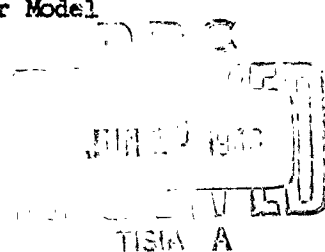
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"THE CONTROL OF HEAVE AND PITCH OF A
SEMISUBMARINE IN REGULAR ASTERN WAVES"

by
H. M. Jonckheere

Contract No.: Nonr 1841 (54)
MIT DSR Project No. 8073

This research was carried out under the Bureau of
Ships' Fundamental Hydromechanics Research Program
SR-009-01-01 administered by the David Taylor Model
Basin.



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Naval Architecture and Marine Engineering
Cambridge 39, Massachusetts

Report No.

January 1963

ABSTRACT

THE CONTROL OF HEAVE AND PITCH OF
A SEMISUBMARINE IN ASTERN REGULAR WAVES

The purpose of this investigation was twofold. First, mathematical expressions were sought to represent the coupled heaving and pitching motions of a semisubmarine in astern regular waves. For the sake of simplicity, the linear theory was used; but corrections were made for some nonlinear effects. Since a complete analytical setup of the motions did not give results that corresponded well with experimental values, some correction factors, consistent with the theory, were introduced. In this way reasonable correlation was achieved in the range of experimental results. Once the analytical expression of the motions was found, a mathematical model was set up with a control system; and the motions in waves were simulated on the computer. Different types of controls were studied, which led to a selection of an optimum control device.

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TABLE OF NOMENCLATURE

Only symbols used more than once in the text are included in this table; other symbols are defined where used. In practically all cases dimensional quantities are implied in foot, pound, and second units.

- a = virtual mass in heave equation or wave amplitude at the free surface
 a_F = virtual mass of control fins
 a_h = wave amplitude at depth h below surface
 $a_1, 2, 3$ = principal dimensions of body of revolution
 A = virtual moment of inertia in pitch equation
 A (in chapters on controls) = area of control fins
 A_F = virtual moment of inertia due to the presence of the control foils
 $A(x)$ = transverse cross-sectional area at distance x from midships
 A_w = area of waterplane
 b = heave damping coefficient
 b_F = heave damping coefficient due to presence of foils only
 B = pitch damping coefficient
 B_F = pitch damping coefficient due to presence of foils only
 B^* = breadth of strut
 c = buoyancy coefficient in heave equation
 c = wave celerity
 C = coefficient of longitudinal righting moment
 C_D = foil drag coefficient
 C_F = coefficient of longitudinal righting moment due only to the presence of control fins
 C_L = foil lift coefficient
 C_3 = nonlinear spring coefficient in pitch equation
 d = coupled virtual moment of inertia coefficient in heave equation
 d_F = coupled virtual moment of inertia coefficient due only to the presence of the control foils

D = coupled virtual mass coefficient in pitch equation

D_F = coupled virtual mass coefficient due only to the presence of control fins

e = coupled heave damping coefficient

e_F = coupled heave damping coefficient due only to the presence of control fins

E = coupled pitch damping coefficient

E_F = coupled pitch damping coefficient due only to the presence of control fins

f_1, f_2 = coefficients of $\cos \omega t$ and $\sin \omega t$ in expression of exciting force

f_H = function of time lag in expression of varying anti-heaving foil angle

f_P = function of time lag in expression of varying anti-pitching foil angle

Fr = Froude number

g = coupled spring coefficient in heave equation

g = gravity acceleration

G = coupled spring term in pitch equation

h = depth coordinate below surface

I = longitudinal moment of waterplane

$I_y = J$ = maximum moment of inertia of semisubmarine around the y-axis

$k_1, 2, 3, 4$ = control parameters in expressions of varying angles of control fins

k_h = ratio of added mass, a , to actual mass, m

k_p = ratio of A to J

L = ship length

m = mass of the ship

m_1, m_2 = coefficients of $\cos \omega t$ and $\sin \omega t$ in expression of exciting pitching moment

M_0 = maximum exciting pitching moment on semisubmarine due to strut and body

M_P = pitching moment produced on semisubmarine through the control fins

M_δ = hydrodynamic derivative of pitching moment due to an angle of attack

M_1 = ^{hydrodynamic} exciting pitching moment on submerged body of revolution due to waves

M_2 = exciting pitching moment from variation in buoyancy due to the waves

M_3 = exciting pitching moment directly due to strut

$M_q, M_{\dot{q}}, M_{\ddot{q}}, M_{\dot{w}}, M_{\ddot{w}}, M_{\dot{\theta}}, M_{\ddot{\theta}}$ = hydrodynamic derivatives defined in Ref. 21.

$N(x)$ = heave damping force of a strip of unit length at distance x from midship

O = origin of axis system, center of body of revolution (also approximate position of center of gravity)

p = hydrodynamic pressure

$R = r(x)$ = radius of body of revolution at distance x from midship

t = time coordinate

t_1, t_2 = time lag of control systems

T_H = natural heaving period

T_P = natural pitching period

$T = \frac{2\pi}{\omega}$ = period of encounter

U = absolute value of speed of wave particle with respect to the control foils

V = ship velocity

v_p = wave particle velocity

v_{p1} = wave particle velocity at depth $h + \frac{L}{10}$

v_{p2} = wave particle velocity at depth h

$w = \dot{z}$ = heave velocity

$w(x) = B^*$ = local width of strut

W = work done by a control foil during a quarter cycle

x = coordinate along longitudinal axis of semisubmarine with origin midship

x_H = distance of anti-heaving fin from midship

x_P = distance of anti-pitching fin from midship

x_0 = horizontal distance between center of body and center of strut

X_3 = hydrodynamic surge force on strut due to waves

X_4 = hydrostatic surge force on strut due to waves

y = horizontal coordinate in direction of port with origin 0

Y_1 = exciting swaying force

z_0 = maximum heave motion amplitude

z = heave response of the vessel

Z_0 = maximum total exciting heaving force on the vessel

Z_1 = hydrodynamic exciting heaving force on submerged body of revolution due to the waves

Z_2 = exciting heaving force from varying buoyancy force due to the waves

Z_3 = hydrodynamic derivative of the heave force due to an angle of attack of a control fin

$\dot{z}_q, \ddot{z}_q, \dot{z}_v, \ddot{z}_v, \dot{z}_\theta, \ddot{z}_\theta$ = hydrodynamic derivatives due to the motions of the vessel (see Ref. 21)

α = angle of attack of the wave particles with respect to the horizontal

α_P = angle of attack on anti-pitching fin of wave particles with respect to the horizontal

α_H = angle of attack on anti-heaving fin with respect to the horizontal

δ = angle between control fin and longitudinal axis of ship
= also phase angle between heave motion and wave motion

δ_H = varying angle between anti-heaving foil and longitudinal axis of ship

δ_P = varying angle between anti-pitching foil and longitudinal axis of ship

$\delta_H' = \delta_H + \alpha_H - \theta$ = total angle of attack on the anti-heaving foil

$\delta_P' = \delta_P + \alpha_P - \theta$ = total angle of attack on anti-pitching foil

δ_0 = maximum rotation amplitude of anti-pitching fin

δ_1 = maximum rotation amplitude of anti-heaving fin

Δ = displacement in lbs. of semisubmarine

V = displacement in cu ft of semisubmarine

ξ = phase angle between pitch and the wave motion

ξ_p = phase angle between exciting pitching moment and wave motion

ξ_z = phase angle between exciting heave force and wave motion

θ = pitch response of the vessel

θ_0 = maximum pitch amplitude

λ = wave length

μ = phase lag between the motion of anti-heaving foil and the waves

ρ = water density

τ = phase angle between motion of anti-pitching fin and the waves

ϕ_w = wave potential

$\omega = \frac{2\pi}{\lambda}(V - c)$ = frequency of encounter, cycles/sec

ω_n = natural frequency of the semisubmarine in heave or pitch.

ACKNOWLEDGMENTS

The author wishes to acknowledge the guidance and encouragement of Professor Ernst Frankel and Professor Philip Mandel of the Department of Naval Architecture and Marine Engineering at the Massachusetts Institute of Technology throughout the preparation and correction of this report.

Without the cooperation and facilities of the Computation Center at M.I.T., this work could never have been accomplished.

I. INTRODUCTION

The so-called semisubmarine (Fig. 1) consists of a fully submerged body of revolution with a relatively large narrow strut whose top rises out of the water.

Interest in this type of vessel was aroused because it seemed some of the advantages of both a submarine and a surface ship.

A submarine is not subject to the large forces and moments that are caused by a change in buoyancy due to waves and by the wave orbital velocity. Also, the resistance due to wavemaking by the vessel is less for a submerged body, while its increase in frictional resistance is not as great as the decrease in wave resistance (Ref. 1). On the other hand, the surface ship has permanent access to the air which is an advantage from a psychological viewpoint and with respect to installation of relatively low-cost air-breathing machinery.

Experiments have been carried out at the M.I.T. and other towing tanks to investigate the performance of the semisubmarine in calm water and in regular waves. The motions in the vertical plane (pitching, heaving, and surging) received special attention. The tests (Ref. 3) showed that the semisubmerged body behaved very well in ahead but poorly in astern waves. This can easily be understood since the natural frequency of both heave and pitch are small (see App. I) so that synchronism with waves occurs only in astern seas, while in ahead seas the disparity between the

natural and the encounter frequencies becomes larger and larger with increasing forward speeds.

It was the author's intention to find a theoretical foundation in expressing these motions mathematically, such that it would become possible to predict the motions of the semisubmarine in regular waves and, by superposition, in confused seas.

Moreover, with the analytical methods, it is possible to find the properties necessary to control excessive motion. Control of heave and pitch, with this type of vessel, does not seem to be as difficult a task as it is with surface ships. Indeed, as was mentioned before, the forces and moments acting on the semisubmarine will not be as large as on the surface ship while the frequency of motion at which control is necessary is much lower than with surface vessels. This is due to the low natural frequencies of the semisubmerged ship.

These considerations promoted the desirability of investigating the motions analytically. However, literature on the motions of near surface vessels is quite rare and that which is available (Ref. 22) proved inadequate for the purposes of this paper. Nevertheless, theories have been developed in the past years for surface ship and deeply submerged submarine motions that bear much relevance to the semisubmarine. In this work an attempt has been made to combine, modify, and adapt these theories in such a way that an analytical expression for heaving and pitching motion of a semisubmarine is obtained.

The large surge motion observed experimentally (Ref. 3) must have a considerable influence on these ship motions since the surface-piercing

sail establishes a relatively long moment arm. Thus, even the small surge force induced by the waves on the strut, or by the motion of the body itself, must have considerable effect on pitching and, by coupling, on heaving. The surge exciting force due to the waves on the strut has been taken into account in this work but not the effect of the self-induced force by surging on either the strut or body. This effect would cause the motion to become an involved, nonlinear vibration problem (see App. II) in which many properties have, as yet, not been sufficiently investigated.

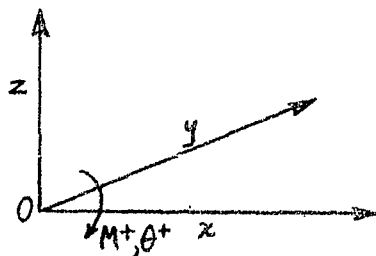
II. EQUATIONS OF MOTION

The motions in the vertical plane of a body in a fluid can be expressed as a set of three differential equations in which the heave, pitch and surge are coupled with each other. If the surge is omitted, two equations remain. They are of the form:

$$(1) (m+a)\ddot{z} + b\dot{z} + cz + d\ddot{\theta} + e\dot{\theta} + g\theta = \bar{Z} e^{i\omega t} \text{ (heave-force eq.)}$$

$$(2) (J+A)\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{z} + E\dot{z} + Gz = \bar{M} e^{i\omega t} \text{ (pitch-moment eq.)}$$

where $\bar{N} = N e^{i\epsilon_m}$ and $\bar{Z} = Z_0 e^{i\epsilon_z}$, ϵ_m and ϵ_z represent the phase lags with respect to the origin of time.



The axis system, which moves with the body, is right-handed. The origin is at constant depth equal to h ; x is in the direction of the longitudinal axis of the body of revolution at rest,

positive to the bow, while y is to port and z upwards. The directions of (note that these directions are opposite to those specified in Ref. 21). this moving coordinate system do not change during the motion.

The equations (1) are reasonable if the motions are kept small, as was shown by Abkovitz (Ref. 4) who developed the same equations from Newton's basic force equations by means of Taylor's expansions (the symbols are those of Ref. 4):

$$(1) (m - Z_{\dot{w}})\ddot{z} - Z_{\dot{w}}\dot{z} - Z_z z - Z_{\dot{\theta}}\ddot{\theta} - (Z_q + VZ_{\dot{w}})\dot{\theta} - (Z_\theta + VZ_{\dot{\theta}})\theta = Z_0 \cos(\omega t - \epsilon_z)$$

$$(2) (I_y - M_{\dot{q}})\ddot{\theta} - (M_{\dot{q}} + VM_{\dot{w}})\dot{\theta} - (M_\theta + VM_{\dot{w}})\theta - M_{\dot{w}}\ddot{z} - M_{\dot{w}}\dot{z} - M_z z = M_0 \cos(\omega t - \epsilon_m)$$

wherein $w = \dot{z}$, $q = \dot{\theta}$, and $E_y = J$. The coefficients like Z_w and M_q are "hydrodynamic derivatives" (see Ref. 4).

These derivatives have to be found together with the magnitude of the forces and moments that regular waves induce on the semisubmarine

The Exciting Forces and Moments

Krylov, in his treatise on ship motions around the turn of the century, made the now famous assumption (the so-called Froude-Krylov hypothesis) that the configuration of the flow in a wave is not influenced by the presence of a body in it so that the expression for the forces and moments that the moving water exerts on the moving ship is derived primarily from buoyancy considerations. However, this assumption is not entirely right since it is known from experiments and theoretical hydrodynamics that a body moving through a fluid changes the potential of the fluid in its neighborhood greatly. Thus, there is an interaction between the water and the body moving through it.* Two approaches to the problem, namely, the so-called "Thin-Ship" theory and the "Slender-Body" theory or "Strip" theory have been made recently. The former was developed and used by Michell, Peters and Stoker, Newman and others. It employs a systematic perturbation expansion in terms of perturbation factors in finding a velocity potential while the boundary conditions are respected, together with the condition that the beam-length ratio be small. It is basically a three-dimensional problem. In the strip theory the body is also supposed to be elongated, but also the draft is supposed to be small. In that case the flow around this body is not considered three- but only two-dimensional. The ship is divided into vertical cross-

* For surface ships the Froude-Krylov hypothesis is satisfactory since buoyancy is the main exciting force. In the semisubmarine the hydrodynamic forces are, however, primary.

sectional strips which are treated separately from each other, neglecting the effects of the longitudinal perturbation velocities. The force and moment are found on each element, and an integration over the length gives the total force and moment on the ship. The advantages of this method over the thin-ship theory are immediately seen. One can use the simpler two-dimensional potential theory. Unfortunately, empirical correction factors have to be used to make the theory correspond with experimental results. The thin-ship theory respects all physical boundary conditions but has large computational disadvantages and has not yet been sufficiently developed to get a complete picture of the motions of a ship in waves. Therefore, the strip theory was chosen to find most of the hydrodynamic properties. Viscosity of the water and, therefore, friction can be proved (see App. V) to have little effect on the motions with relatively low Froude numbers. They have been ignored in this treatment.

Expressions for the heave exciting force and the pitching exciting moment on a submerged body have been developed by Kaplan (Ref. 5). The heave force in astern seas is:

$$Z_1 = -\frac{4\pi\rho g a}{\lambda} e^{-2\pi h/\lambda} \left(1 - \frac{V}{2c}\right) \int_{x_3}^{x_b} A(x) \sin \frac{2\pi}{\lambda} [x + (V-c)t] dx$$

where a is the wave amplitude, t the free surface, and $A(x)$ is the transverse cross sectional area at a distance x from midship. This is, however, for a body without a strut. To take the strut into account, the expression has to be modified. The hydrodynamic pressure on the body is given by formula [14] of Ref. 5:

$$\begin{aligned} p &= \rho g a e^{-2\pi h/\lambda} \left\{ \sin \frac{2\pi}{\lambda} [x + (V-c)t] \right\} \left\{ 1 + \frac{4\pi R}{\lambda} \sin \theta \right\} \\ &\quad - 2\rho g a \frac{V}{c} e^{-2\pi h/\lambda} \cos \frac{2\pi}{\lambda} [x + (V-c)t] \left\{ \frac{dR}{dx} \sin \theta \right\} \\ &= P \left\{ 1 + \frac{4\pi R}{\lambda} \sin \theta \right\} - Q \left\{ \frac{dR}{dx} \sin \theta \right\} \end{aligned}$$

where $P = \rho g a e^{-\frac{2\pi h}{\lambda}} \sin \frac{2\pi}{\lambda} [x + (V-c)t]$

and $Q = 2\rho g a \frac{V}{c} e^{-\frac{2\pi h}{\lambda}} \cos \frac{2\pi}{\lambda} [x + (V-c)t]$

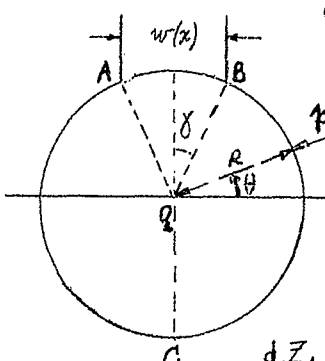
The total force in the vertical plane expressed in complex form is then given by:

$$\frac{dY_1}{dx} - i \frac{dZ_1}{dx} = -R \int_0^{\frac{\pi}{2}-\gamma} p(\cos\theta - i \sin\theta) d\theta - R \int_{\frac{\pi}{2}+\gamma}^{2\pi} p(\cos\theta - i \sin\theta) d\theta$$

Therefore, we get:

$$\frac{dZ_1}{dx} = -R \int_0^{\frac{\pi}{2}-\gamma} p \sin\theta d\theta - R \int_{\frac{\pi}{2}+\gamma}^{2\pi} p \sin\theta d\theta$$

which is the heaving force for a strip of unit length at distance x from the center of the coordinate $\frac{\pi}{2}-\gamma$ system.



$$\begin{aligned} \frac{dZ_1}{dx} &= -RP \left\{ \left(1 + \frac{4\pi R}{\lambda} \sin\theta\right) \sin\theta d\theta + \left(1 + \frac{4\pi R}{\lambda} \sin\theta\right) \sin\theta d\theta \right\} \\ &\quad + RQ \left\{ \int_0^{\frac{\pi}{2}-\gamma} \frac{dR}{dx} \sin^2\theta d\theta + \int_{\frac{\pi}{2}+\gamma}^{2\pi} \frac{dR}{dx} \sin^2\theta d\theta \right\} \\ &= RP \left[\cos\theta \right]_0^{\frac{\pi}{2}-\gamma} + RP \left[\cos\theta \right]_{\frac{\pi}{2}+\gamma}^{2\pi} \\ &\quad - \left(RP \frac{4\pi R}{\lambda} - RQ \frac{dR}{dx} \right) \left\{ \int_0^{\frac{\pi}{2}-\gamma} \frac{1-\cos 2\theta}{2} d\theta - \int_{\frac{\pi}{2}+\gamma}^{2\pi} \frac{1-\cos 2\theta}{2} d\theta \right\} \\ \frac{dZ_1}{dx} &= RP w(x) - \left(\frac{4\pi R^2 P}{\lambda} - RQ \frac{dR}{dx} \right) \left[\pi - \gamma - \frac{1}{2} \sin 2\gamma \right] \end{aligned} \quad (3)$$

With $\gamma = 0$, hence without the strut (i.e., $w(x) = 0$), this gives:

$$\frac{dZ_1}{dx} = - \left[\frac{4\pi R^2 P}{\lambda} - RQ \frac{dR}{dx} \right] \pi$$

The difference between the last two expressions comes after integration to about 15 per cent less for the body of revolution with a strut.

Expression (3) integrated gives:

$$\begin{aligned} Z_1 &= - \frac{4\pi \rho g a e^{-\frac{2\pi h}{\lambda}}}{\lambda} \left(1 - \frac{V}{c}\right) \int_{-\frac{L}{2}}^{+\frac{L}{2}} A(x) \sin \frac{2\pi}{\lambda} [x + (V-c)t] dx \\ &\quad + \rho g a e^{-\frac{2\pi h}{\lambda}} \int_{-\frac{L}{2}}^{+\frac{L}{2}} \left(R + 2\pi R^2 \right) w(x) \sin \frac{2\pi}{\lambda} [x + (V-c)t] dx \\ &\quad - \rho g a \frac{V}{c} e^{-\frac{2\pi h}{\lambda}} \int_{-\frac{L}{2}}^{+\frac{L}{2}} R \frac{dR}{dx} w(x) \cos \frac{2\pi}{\lambda} [x + (V-c)t] dx \end{aligned} \quad (4)$$

This expression is numerically evaluated for two wave lengths in App. III (pg. 48).

$A(x)$ in this expression means the areas of the sector CAOCB at x .

Finally, Z_1 becomes a sum of two terms:

$$Z_1 = f_1' \cos \omega t + f_2' \sin \omega t.$$

This force was calculated for wavelengths of four and eight ft. and gives a positive value (upward force) when the wave trough passes the middle of the ship. Therefore, the underwater part will counteract the buoyancy force due to the part that pierces the surface.

The equation of the wave profile is assumed to be sinusoidal:

$$y = a \sin \frac{2\pi}{\lambda} (x - ct) \text{ where } x \text{ is an absolute abscissa.}$$

Relative to the moving ship, we then get:

$$\begin{aligned} y &= a \sin \frac{2\pi}{\lambda} [x - (c - v)t] \\ &= a \sin \frac{2\pi}{\lambda} [x + (v - c)t] \end{aligned}$$

The difference in displacement in the wave and in calm water is:

$$\begin{aligned} \text{Volume} &= a \int_{-\frac{1}{4}}^{\frac{1}{4}} w(x) \sin \frac{2\pi}{\lambda} [x + (v - c)t] dx \\ &= a \cos \frac{2\pi}{\lambda} (v - c)t \int_{-\frac{1}{4}}^{\frac{1}{4}} w(x) \sin \frac{2\pi x}{\lambda} dx \\ &\quad + a \sin \frac{2\pi}{\lambda} (v - c)t \int_{-\frac{1}{4}}^{\frac{1}{4}} w(x) \cos \frac{2\pi x}{\lambda} dx \end{aligned}$$

Since the strut is symmetrical, the term $v \sin \frac{2\pi x}{\lambda}$ is odd and has an integral = 0. Hence, the buoyancy force becomes:

$$(5) \quad Z_2 = \left[p g a \int_{-\frac{L}{4}}^{\frac{L}{4}} w(x) \cos \frac{2\pi x}{\lambda} dx \right] \sin \frac{2\pi}{\lambda} (V-c)t$$

The total heaving force is then $Z = Z_1 + Z_2 = f_1 \cos \omega t + f_2 \sin \omega t$. This is also numerically evaluated in App. III, pg. 98.

The exciting moment due to the waves consists basically of four

parts:

1. The moment on the body of revolution due to the orbital wave velocity and to the wave-body interaction.
2. The moment induced by the change in buoyancy along the length of the strut due to the wave slope.
3. The effect of the surge force on the strut. This force must be taken into account since the point of application of this force is relatively far from the origin of coordinates. A moment is generated.
4. The hydrostatic horizontal pressure on the strut which varies according to the position of the wave with respect to the strut.

Kaplan gives for the moment on a near surface body of revolution in eastern waves (Ref. 5):

$$M_1 = -\rho g a e^{-\frac{\pi k}{\lambda}} \left\{ \frac{2\pi}{\lambda} \left(1 - \frac{V}{2c} \right) \int_{-\frac{L}{2}}^{\frac{L}{2}} x A(x) \sin \frac{\pi}{\lambda} [x + (V-c)t] dx + \frac{V}{2c} \int_{-\frac{L}{2}}^{\frac{L}{2}} A(x) \cos \frac{2\pi}{\lambda} [x + (V-c)t] dx \right\}$$

Although this expression was developed for completely submerged bodies without appendages, it may be used with sufficient accuracy in our case. Indeed, the indirect effect

of the strut on the pitch moment excitation of the main body, M_1 , is negligible since the greater part of that moment is contributed by the extremities of the main body itself. The direct effect of the strut, M_2 , which is called M_2 , is discussed subsequently.

Kaplan's expression for M_1 can be developed into:

$$(6.) \quad M_1 = \frac{-4\pi\rho g a e^{-\frac{2\pi h}{\lambda}}}{\lambda} \left(1 - \frac{V}{2c}\right) \left[\int_{-\frac{L}{2}}^{\frac{L}{2}} x A(x) \sin \frac{2\pi x}{\lambda} dx \cos \frac{2\pi}{\lambda}(V-c)t \right. \\ \left. + \int_{-\frac{L}{2}}^{\frac{L}{2}} x A(x) \cos \frac{2\pi x}{\lambda} dx \sin \frac{2\pi}{\lambda}(V-c)t \right] \\ - \rho g a V e^{-\frac{2\pi h}{\lambda}} \left[\int_{-\frac{L}{2}}^{\frac{L}{2}} A(x) \cos \frac{2\pi x}{\lambda} dx \cos \frac{2\pi}{\lambda}(V-c)t \right. \\ \left. - \int_{-\frac{L}{2}}^{\frac{L}{2}} A(x) \sin \frac{2\pi x}{\lambda} dx \sin \frac{2\pi}{\lambda}(V-c)t \right]$$

$M_1 = m_1^i \cos \omega t + m_2^i \sin \omega t$ (see App. III, pg. 49). In this expression,

$m_1^i \cos \omega t$ is found to be the dominant term. Hence, we see that, since Z , the term in $\sin \omega t$, was dominant, the phase angle between heaving force and pitching moment is around 90° .

The effect of changing buoyancy can be expressed as:

$$(7.) \quad M_2 = -a\rho g \int_{-\frac{L}{4}}^{\frac{L}{4}} x w(x) \sin \frac{2\pi}{\lambda} [x + (V-c)t] dx \\ = -a\rho g \cos \frac{2\pi}{\lambda}(V-c)t \int_{-\frac{L}{4}}^{\frac{L}{4}} x w(x) \sin \frac{2\pi x}{\lambda} dx$$

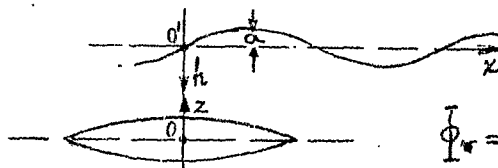
since the strut is symmetrical.

$$M_2 = m_1^n \cos \omega t.$$

The third moment, M_3 , to take into account is the direct effect that the hydrodynamic pressure of the wave exerts on the strut. The pressure on fore and aft parts of the strut is not always balanced so that a harmonic surge force causes a moment on the vessel. We assume the flow around the strut to be two-dimensional so that Laplace's equation is valid.

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial x^2} = 0$$

where ϕ is the velocity potential



The velocity potential of the

waves can be written...

$$\phi_w = a e^{-\frac{2\pi h}{\lambda}} \cos \frac{2\pi}{\lambda} [x + (V - c)t]$$

and the horizontal component of the orbital velocity at depth h :

$$v_{px} = -\frac{\partial \phi_w}{\partial x} = \frac{a c e^{-\frac{2\pi h}{\lambda}} 2\pi}{\lambda} \sin \frac{2\pi}{\lambda} [x + (V - c)t]$$

According to Bernoulli's theorem,

$$\frac{D\phi}{Dt} + \frac{p}{\rho} + \frac{1}{2} v^2 = f(t)$$

$$\frac{p}{\rho} = \frac{D\phi}{Dt} - \frac{1}{2} v^2$$

if we change the velocity potential

such that the term in t is incorporated. $\frac{D\phi}{Dt}$ means the partial derivative with respect to t in an axis system. $\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} - V \frac{\partial \phi}{\partial x}$ since $x = \xi + V t$ where ξ is an absolute axis. Since the strut is very narrow, we may neglect the interaction between wave and body, which makes the potential of wave-body flow = 0. Hence, in the formula for the pressure, we may put $\phi = \phi_w$.

$$\begin{aligned}\frac{\partial \phi}{\partial t} &= -a c e^{-\frac{2\pi h}{\lambda}} (V-c) \frac{2\pi}{\lambda} \sin \frac{2\pi}{\lambda} [x + (V-c)t] \\ \frac{\partial \phi}{\partial x} &= v_x = -\frac{a c e^{-\frac{2\pi h}{\lambda}} 2\pi}{\lambda} \sin \frac{2\pi}{\lambda} [x + (V-c)t] \\ v^2 &= v_x^2 \approx 0 \\ p &= \rho \frac{D\phi}{Dt} = \frac{2\pi \rho c^2 a e^{-\frac{2\pi h}{\lambda}}}{\lambda} \sin \frac{2\pi}{\lambda} [x + (V-c)t] \\ &\quad \text{Diagram: A horizontal line with a vertical line at an angle } \alpha^+ \text{ to it.} \quad p_x = p \sin \alpha\end{aligned}$$

The resulting surge force is then:

$$\begin{aligned}(g) \quad X_3 &= \frac{2a\pi \rho c^2}{\lambda} \int_0^h e^{-\frac{2\pi h}{\lambda}} dh \int_{-\frac{L}{2}}^{\frac{L}{2}} \sin \frac{2\pi}{\lambda} [x + (V-c)t] \sin \alpha dx \\ &= a \rho c^2 e^{-\frac{2\pi h}{\lambda}} \left[\int_{-\frac{L}{2}}^{\frac{L}{2}} \sin \frac{2\pi x}{\lambda} \sin \alpha dx \right] \cos \frac{2\pi}{\lambda} (V-c)t\end{aligned}$$

(see App. III, pg. 49)

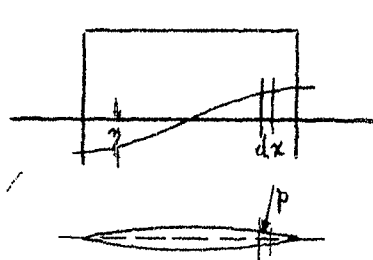
$M_3 = X_3$ times the moment arm.

To find the moment arm, the center of pressure has to be given.

Since this pressure decreases exponentially with depth, the center can easily be found (App. III, pg. 49). We get $M_3 = m_1''' \cos \frac{2\pi}{\lambda} (V-c)t$.

The hydrostatic pressure ^{on the strut} in the horizontal direction will have a

resultant different from zero due to the wave profile. This pressure (perpendicular to the hull) will be $\rho g \eta$ and is a function of x . In the direction of the longitudinal axis the pressure over



$$dx \text{ will be: } p \int_0^{\eta} \rho g \eta \frac{d\eta}{dx} dx d\eta.$$

$$\text{where } \eta = a \sin \frac{2\pi}{\lambda} [x + (V-c)t]$$

$$p = \rho g \frac{\eta^2}{2} \frac{dw}{dx} dx.$$

The total surge force, when the wave nodal point is at the center of the body, is then:

$$X_4 = \rho g \int_0^{\frac{1}{2}l} \eta^2 \frac{dw}{dx} dx.$$

This is the time of maximum surge force. In this expression,

$\eta = \sin \frac{2\pi x}{\lambda}$. In time this force will change harmonically so that

we find after some calculations:

$$\begin{aligned} (9) \quad X_4 &= - \frac{\rho g a^2}{2} \left[w_{max} + 2 \int_0^{\frac{1}{2}l} \cos \frac{4\pi x}{\lambda} \tan \alpha dx \right] \cos \frac{2\pi}{\lambda} (V-c)t \\ &= m_1'' \cos \frac{2\pi}{\lambda} (V-c)t \end{aligned}$$

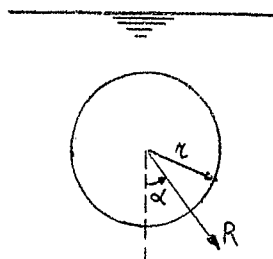
w_{max} is the maximum width of the strut. X_4 is found to be a very small quantity which can be ignored. The total moment acting on the semisubmarine will then be:

$$M = (m_1' + m_1'' + m_1''') \cos \omega t + m_2' \sin \omega t = m_1 \cos \omega t + m_2 \sin \omega t. \quad (\text{See p. 50})$$

Evaluation of Hydrodynamic Derivatives

Korvin-Kroukovsky (Ref. 6 and 7) has applied the strip theory to find the coefficients of the equations (1).

The potential of the flow around a cylinder moving perpendicular to its axis is: $\phi = v \frac{r^2}{R} \cos \alpha$ where r is the radius of the cylinder, and R is a vector to any point in the flow.



On the surface of the cylinder, $r = R$,

and $\phi = vr \cos \alpha$. If second-order terms are neglected, $p = \rho \frac{D\phi}{Dt}$ and

$$\frac{dZ}{dx} = 2r \int_0^\pi p \cos \alpha d\alpha.$$

The vertical velocity of the strip is $v = \dot{Z} - x\dot{\theta} + V\theta$. Carrying out the differentiation and integration, the vertical force on the body induced by the body's own motion^{is} found by Korvin to be:

$$Z = -\rho\pi \int_{-\frac{L}{2}}^{\frac{L}{2}} r^2 (\ddot{Z} - x\ddot{\theta} + V\dot{\theta}) dx + 2\rho\pi V \int_{-\frac{L}{2}}^{\frac{L}{2}} r \tan \beta (\dot{Z} - x\dot{\theta} + V\theta) dx$$

where $\tan \beta = \frac{dr}{dx}$. From this equation follows:

$$\begin{aligned} a_1 &= \int \rho\pi r^2 dx \quad * \\ b_1 &= -2\rho\pi V \int r \tan \beta dx \\ d_1 &= -\rho\pi \int r^2 x dx \\ e_1 &= 2\rho\pi V \int r x \tan \beta dx + 2\rho\pi V \int r^2 x dx \\ g_1 &= -2\rho\pi V^2 \int r \tan \beta dx \end{aligned}$$

The moment equation is obtained by multiplying every expression under the integral sign in Z with $-x$.

$$\begin{aligned} A_1 &= +\rho\pi \int r^2 x^2 dx \\ B_1 &= -2\rho\pi V \int r^2 x dx - 2\rho\pi V \int r \tan \beta x^2 dx \\ C_1 &= +2\rho\pi V^2 \int r x \tan \beta dx \\ D_1 &= -\rho\pi \int r^2 x dx \\ E_1 &= +2\rho\pi V \int r x \tan \beta dx \end{aligned}$$

* the subscript 1 designates the hydrodynamic part of the derivatives of equation 1 as estimated by Korvin-Kroukovsky (Ref. 6 and 7).

In this report an attempt is made to correct the foregoing coefficients for certain hydrodynamic effects not fully considered in Korvin's earlier works. These include the effect of frequency of motion and of the free surface. In order to distinguish the coefficients so corrected from the Korvin coefficients, the corrected coefficients are given without the subscript 1.

1. Added Mass and Added Moment of Inertia, a and A

It has been found by experiment (Ref. 10 and 11) and theory that the added mass is not a constant, as calculated by strip theory, but depends on forward velocity, frequency of motion, and also very strongly on relative submergence for submerged bodies. At $\omega = 0$ the added mass theoretically goes to infinity. Ursell, among others, calculated the change in added mass from that found by strip theory due to both surface effects and frequency of motion. He also calculated these changes for body shapes differing from a cylinder. He found a correction factor of infinity for $\omega = 0$. Based on Ursell's work and on the results of the experiments of Ref. 10 and 11, various values of added mass and moments of inertia were selected. These values and values of the other coefficients of equation (1) selected as described in the following paragraphs were then utilized in an iterative computer program in an effort to achieve maximum correlation with the experiments of Ref. 3. The best correspondence was obtained with the added mass equal to about 60 per cent of the actual mass at $V = 0$ up to 200 per cent at the speed where $\omega = 0$ ($V = c$). Less drastic corrections were found for the added moment of inertia. The numerical values of a , A , m , and J can be found in App. IV (pg. 51).

2. Damping Coefficients (b and B)

In a perfect fluid, dissipation of energy (which is damping) can only occur by the process of energy being carried away by gravity waves at the surface and through energy radiation below the surface. Since the effect of the free surface was not considered in the expressions of b_1 and B_1 on pg. 14, they are not valid. In an attempt to take account of free surface, Korvin-Kroukovsky replaces them respectively by $\int N(x) dx$ and $\int N(x) x^2 dx + 2VD - 2V\rho\pi \int r \tan \beta x^2 dx$ where $N(x)$ represents the damping force per unit length. Holstein (1936) equated $N(x)$ to $\rho g A^2/w^3$, where A is the ratio of the amplitude of the waves radiated by the strip over the amplitude of heave of this strip section. Ursell (1954) and Grim (1953) have found values that correlate to a certain degree with experiments. However, these theories cannot be used in this case since they apply to surface ships.

A way to find the damping would be to consider the body of revolution as a hydrofoil on which a lift is generated due to the heaving and pitching:

$$dL = \frac{1}{2} \rho \frac{\partial c_L}{\partial \alpha} \alpha v^2 dS. \quad dS \text{ is the horizontal projection}$$

of a cross section = $2r dx$.

$$dZ = \rho \frac{\partial c_L}{\partial \alpha} \alpha v^2 r dx. \quad (10.)$$

$$\alpha = \frac{\dot{z} + V\theta - x\dot{\theta}}{V}$$

such that

$$b = \rho V \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\partial c_L}{\partial \alpha} x dx$$

$$B = - \frac{\partial}{\partial \theta} \int x dZ = \rho V^2 \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\partial c_L}{\partial \alpha} x^2 dx$$

The whole difficulty lies, however, in finding $\frac{\partial c_L}{\partial \alpha}$ since it varies over the length of the body for our peculiar shape with its very low aspect ratio. Therefore, this method was also discarded.

The method selected in this report for calculating the damping coefficients was given in a recent paper by Newman (Ref. 8). Damping coefficients were computed for an oscillating ellipsoid with forward motion near the free surface. The ellipsoid is represented by a distribution of singularities, namely, steady-state and oscillating dipoles and quadrupoles. The damping coefficients are then found from the energy radiation at infinity. Using the symbols of Ref (8), the dimensional damping coefficient, b , of this report is expressed as follows: $b = \frac{a_1^2 B_{33}}{\sqrt{g a_1} a_2^2 a_3^2 \rho}$

where

$$B_{33} = - \frac{32 \pi}{\omega} \rho a_1^2 a_2^2 a_3^2 \sum \int_0^{\pi - u_0} \frac{\lambda_m^3 e^{-2\lambda_m h}}{(1 + 4\pi \cos u)^{1/2}} \times$$

$$\times [Q_3(u)]^2 \sin(u) du \text{ (Formula 31, Ref. 8)}$$

and

$$Q_3(u) = \left(\frac{1}{2 - \alpha_3} \omega - c \lambda_m \frac{1}{2 - \alpha_1} \cos u \right) \frac{j_1(q)}{q}$$

B_{33} is the nondimensional heave damping coefficient, and

α_n are Green's integrals.

a_1, a_2, a_3 are, respectively, the half-length, half-beam, and half-height of the spheroid.

$j(q)$ = spherical Bessel Function

λ_m = ellipsoidal coordinate

u = heave velocity component

$$q = \lambda_m [(a_1^2 - a_3^2) \cos^2 u + (a_2^2 - a_3^2) \sin^2 u]^{1/2}$$

$$\tau = \frac{u}{V}$$

$$u_0 = \begin{cases} 0 & \text{for } \tau \leq 1/4 \\ \cos^{-1} \left(\frac{1}{4\tau} \right) & \text{for } \tau > 1/4 \end{cases}$$

A similar expression was found for

$$B = \frac{B_{55}}{\sqrt{g a_1 (a_2 a_3)^2}}$$

where B_{55} is nondimensional pitch damping coefficient.

An IBM 704 computer program exists to find these values as a

function of τ , $\frac{V}{\sqrt{gL}}$ and a_1, a_2, a_3 . We have assumed that the

shape of our body of revolution does not differ appreciably

from a spheroid, such that the same formulas are applicable.

Fig. 2 shows the speed and frequency dependence of b and B for different wave lengths. It is seen on Fig. 2 that unfortunately minimum damping is associated with the resonant condition. For numerical values, see App. IV (pg. 52 - 53).

3. Coupled Damping Coefficients, e and E

In an attempt to account for the free surface, Korvin-

Kroukovsky also modified the expressions of e and E .

$$e = \int N(x) x dx + k_2 k_4 m V$$

$$E = \int N(x) x dx - k_2 k_4 m V$$

where k_2 and k_4 are correction factors due to shape of the

body, and the free surface (the average $k_2 k_4$ amounts to about

75 per cent for surface ships). Havelock (Ref. 9) found

* at DTMB.

nearly the same expression for a floating half spheroid:

$e = pW$, and $E = -pW$ with $p = 0.515$ for a length diameter ratio of 8. Abkowitz (Ref. 4) finds:

$$e = -(Z_q + VZ_{\dot{w}})$$

$$E = -M_y$$

where the expression for e is similar in form to that suggested by Korvin. We also see that $E \neq e$, and this has also been proved by experiments carried out by Gerritsma (Ref. 13). e was found to have a different sign and a larger value than E . To find an expression for Z_q , one can use Formula (10) of pg. 16:

$$Z_q = + \frac{\partial}{\partial \theta} \int dZ = \rho V \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\partial c_L}{\partial \alpha} r x dx$$

$$e = - \rho V \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\partial c_L}{\partial \alpha} r x dx + k_2 V m$$

$$M_y \text{ is found in the same way: } M_w = \rho V \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\partial c_L}{\partial \alpha} r x dx = Z_q$$

$$E = - \rho V \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\partial c_L}{\partial \alpha} r x dx$$

Experiments have been conducted at DTM to find M_y for different types of deeply submerged submarines. The following empirical formula was found:

$$M_y' = 0.871 (k_2 - k_1) m' = \frac{M_y}{\rho/2 L^3 V} \text{ where } m' = \frac{2V}{L^3}, \text{ and } k_2$$

and k_1 are virtual mass coefficients which can be found in Lamb's "Hydrodynamics".

The values for e and E finally selected were based upon exactly the same procedure as that outlined on pg. 15 for determining a and A . These values are given in App. III (pg. 53). The values e and E found in this way are not much different from those calculated using the method described in the previous paragraph. In any event, the values of e and E for the bare hull body are very small compared to the e_F and E_F found for the semisubmarine equipped with control fins at the extremities of the vessel (see Chapter IV).

4. The Spring Constants c and C

The coefficient, c , exists because of the presence of the surface-piercing strut and, C , because of the metacentric stability of the semisubmarine. From equations (1) and (2) it is seen that $c = Z_z$ and $C = M_G + VM_W$. The heaving force due to a submergence z is $Z = -\rho g A_v z$, $Z_z = -\rho g A_v$; therefore, $c = \rho g A_v$. $M_G = \rho g \Delta \times GM_L = \rho g \Delta \frac{I}{\nabla} = \rho g I$ where I is the longitudinal moment of inertia of the waterplane.

At the M.I.T. Towing Tank, tests were carried out to determine the hydrodynamic moment on the semisubmarine with different speeds (Ref. 3). At every speed it was found that the vessel, once given a small initial trim angle, took on a certain larger, stable trim angle which was variable with V as shown in Fig. 4. At the stable trim angle, the longitudinal stability moment is equal to the hydrodynamic moment. For smaller θ the hydrodynamic moment would be greater, and for higher θ the stability moment would dominate. While the assumption that the stability moment is approximately linear with θ is reasonable, it is unreasonable to assume that the hydrodynamic moment is also linear. A parabolic function relating this moment to θ was, therefore, developed. This is given in App. IV (pg. 54).

5. The Coupled Virtual Mass and Moment of Inertia, d and D

These coefficients which were found by strip theory to be equal to $-\rho \int x^2 \pi dx$ with correction factor L_h , due to the surface (see pg. 16), are very small quantities. The experiments of Ref. 14 also showed this, because the instruments were not sensitive enough to give trustworthy results. It is possible that these coupling terms also change considerably at very low frequencies. The magnitude of these coefficients, neglecting free surface and frequency dependency, are given in App. IV, pg. 55.

6. The Coupled Spring Constants, g and G

Like the coefficients c and C, these coefficients exist because of the presence of the strut. From equations (1) and (2), we find: $g = Z_v + VZ_v$ and $G = M_v$. Korvin-Kroukovsky (Ref. 7) found the same expressions in the form: $g = \rho g \int B^2 x^2 dx + Vb$ $G = \rho g \int B^2 x dx$ where B^2 is the beam of the strut. Because the strut is symmetrical with respect to the cross-center line, $Z_v = 0$. Empirical relationships for Z_v developed at DMB for deeply submerged submarines give the following:

$$Z'_v = -(L'_v + D') = \frac{Z_{L_v}}{1/2 \rho L^2 V} \quad L'_v \text{ is the dimensionless lift.}$$

$$D' = \frac{D}{\rho/2 L^2 V^2} \text{ is a dimensionless drag. } L'_v = .234 m' 0.79 \text{ where}$$

$$m' = \frac{2\gamma}{L^3}.$$

Therefore, $g = 1/2 \rho V^2 L^2 (L'_v + D')$. There will be a small coefficient G (the

* The simple expression for Z_v is used here in lieu of the more sophisticated expression for b on pg. 17 - 18, because the coupled spring constant g is a relatively insignificant one. Thus, there was no need to introduce free surface and frequency dependency corrections here.

to the fact that the strut does not stand in the middle of the body.

The moment with a submergence z will be: $M_z = -\rho g x_s A_v z$ where x_s is the horizontal distance between the center of body and the center of strut.

$$M_z = \left(\frac{\partial M}{\partial z} \right)_{z=0} = -\rho g x_s A_v$$

$$G = +\rho g x_s A_v.$$

For the numerical values, see App. IV, pg. 55.

III. SOLUTION OF THE EQUATIONS OF MOTION

The equations of motion are of the type:

$$(m+a)\ddot{z} + b\dot{z} + cz + d\ddot{\theta} + e\dot{\theta} + g\theta = f_1 \cos \omega t + f_2 \sin \omega t \text{ (Heave Eq.)}$$

$$(J+A)\ddot{\theta} + B\dot{\theta} + C\theta + C_3 \frac{\theta}{\sqrt{1+\theta^2}} + D\ddot{z} + E\dot{z} + Gz = m_1 \cos \omega t + m_2 \sin \omega t \text{ (Pitch Eq.)}$$

$$\omega = \frac{2\pi}{\lambda} (V - c)$$

The coefficients are either constants or functions of V . Since the equation of pitch is not linear, the simple method of solving a set of linear differential equations cannot be used. However, to make a preliminary estimate of how the theoretically computed responses of heave and pitch will correspond with the experimental results, the pitch equation was made linear. In this way it is possible to write a computer program that solves the equations in a very short time, such that coefficients that had to be found by trial and error method could easily be varied.

The experimental pitching responses were used to find values for $C\theta + C_3 \frac{\theta}{\sqrt{1+\theta^2}}$ for different speeds. When these values are divided by θ , a coefficient C' is found which will be the coefficient of θ in the linearized pitching equation. For speeds above the experimental range, an angle θ was assumed and later on corrected by trial and error.

The solution of the set of two linear equations is given by Korvin-Kroukovsky (Ref. 6) in the form:

$$\bar{z} = \frac{\bar{M}_Q - \bar{Z}B}{QR - PS} \quad \bar{\theta} = \frac{\bar{Z}R - \bar{M}P}{QR - PS}$$

where $P = -(m + a)u^2 + ibu + c$

$$Q = -du^2 + iau + g$$

$$R = -Du^2 + iBu + G$$

$$S = -(J + A)u^2 + iBu + C$$

$$\bar{Z} = Z_0 e^{i\epsilon_z}$$

$$\bar{M} = M_0 e^{i\epsilon_M}$$

$$\bar{z} = z_0 e^{i\delta}$$

$$\bar{\theta} = \theta_0 e^{i\epsilon}$$

ϵ_z and ϵ_M are the phase angles between respectively exciting heaving force and pitching moment and the wave motion.

δ and ϵ are the phase angles between heave and pitch and their

respective origins of time. $M = M_1 \cos \omega t + M_2 \sin \omega t$; (see pg. 13) therefore,

$$\epsilon_M = \tan^{-1} \frac{M_2}{M_1} \quad \text{and} \quad M_0 = \sqrt{M_1^2 + M_2^2} \quad \bar{\theta} \text{ will be found in the}$$

form $\bar{\theta} = a + ib$. So, $\theta_0 = \sqrt{a^2 + b^2}$ and $\epsilon = \tan^{-1} \frac{b}{a}$. An analogous

remark goes for Z and z_0 . A computer program was written which gives

the values of θ_0 and z_0 and their corresponding phase lags for the

semisubmarine in following waves, 4 and 8 ft. long. Fig. 3 shows

the results of two methods of computing the response curves in comparison with

the experimental heave and pitch. Numerical results can be found in App. IV.

The ^{linearized} method of solving the two equations yielded satisfactory results

for the pitching, over the average. The bumps and hollows in the

experimental pitch curves, however, are not reproduced in the

theoretical solution. However, these experimental data of both heave

and pitch for speeds of 1.5 ft/sec and up show great fluctuations, and

it may be questioned whether they represent the correct amplitudes

entirely.

For the heaving motions it is found that the solution of the linearized equations shows too low amplitudes for speeds below 1 ft/sec. The disparity is from 50 to 30 per cent in 4-ft. waves and a little better in 8-ft waves. It is possible that either the strut, the surge, or a three-dimensional factor have an important influence on heaving. This influence is such that it increases the heave response of the semisubmarine. At a speed of about 1.8 ft/sec, the experimental and the theoretical data join and for higher speeds theory gives values larger than the tests. This is not a serious deficiency because it means that any controls designed to handle the theoretically predicted excitations will overcontrol in reality.

We have no way of checking how well the theory represents the actual motions for speeds above 2 ft/sec ($Fr = 0.18$). It is seen that the motions increase greatly and well beyond the linear range so that the linearized equations do not represent the actual motions any more. The speeds of synchronism occur indeed at Froude numbers starting at 0.25 for the heave in 4-ft waves and at 0.29 for the pitch in 4-ft waves. These speeds go gradually up with the wave lengths. In the solution of the linearized equations, the peaks at synchronism are apparent; but they are only qualitatively correct. Nevertheless, the computations show that, without controls, the semisubmarine cannot be operated in speed ranges of $Fr = 0.2$ and up in astern seas.

To test the validity of the ^{linear} method of solving the set of differential equations, a second method, namely, Adam's numerical

method, was used which gives the amplitudes of heave and pitch as a function of time. A set of initial amplitudes is given and after a certain transition period a relatively steady harmonic motion is found. For every speed the maximum amplitudes out-to-out were taken to represent the motions as was also done with the experimental results.

As can be seen in App. IV and from Fig. 3, the heave response in 4-ft. waves follows very closely the other solution, while in 8-ft. waves the time-history method corresponds much better with the experimental heave. The nonlinear term in the pitch equation which is taken account of in the T-H Method may account for this improved correspondence.

As for the pitching, the curves stay close to each other in the experimental range (V from 0 to 2 ft/sec) with the first method a little better than the second in 4-ft. waves, while it is the opposite case in 8-ft. waves (Fig. 3B).

For speeds higher than 2 ft/sec ($F_r = 0.18$), there is considerable disparity between the two ways of solving the equations, especially in the long waves. Moreover, the amplitudes go well beyond the linear range of the equations such that they do not correspond with reality. Although quantitatively these results are not valid, qualitatively they give an indication where the high amplitudes will be found. The peaks are, indeed, found at the resonance speeds, i.e., where the tuning factor $\lambda = \frac{\omega}{\omega_n} = 1$. For heave the resonance speeds go from 2.8 to 3.0 ft/sec for wave lengths from 4 to 8 ft. For pitch the

resonance speed range is from 3.4 to 4.0 ft/sec. It is seen in both heave and pitch motions that the peaks do not coincide exactly with the experimentally determined peaks. This may be due to the fact that other nonlinear terms and the surge motions may have an influence.

An interesting feature of the time-history method is that it shows exactly how the vessel moves in waves. It was, for instance, found that at very low frequency of encounter the vessel has the tendency to oscillate at its natural frequencies which are different for heave and pitch.

Although the differential equations, as we have formulated them, are not valid for a speed range higher than $Fr = 0.25$, they still may be used if we employ means to control the motions. In this way the amplitudes will ultimately be small enough to make the equations valid again.

IV. THE CONTROL OF HEAVE AND PITCH

The expressions for heaving and pitching motion derived in the previous chapter form a valuable tool to investigate if control of these motions is possible and how this can be done. We know that the equations do not give the right picture at speeds at and higher than synchronism since the response goes beyond the linear range. With control, however, it may be possible to decrease the motions in such a way that they remain in the linear range so that we can use these equations even for speeds close to synchronism.

The simplest and most economical way to control a certain motion is by a hydrofoil that can rotate along an axis perpendicular to the stream.

To control the pitching it is advantageous to put the foil in the stern of the ship since this gives a stable M_y such that when there is pitch, the lift works in the direction opposite to the pitching angle. To control the heave the most favorable place for a foil is near the center of gravity of the ship. In this way the pitching moment caused by this foil will be zero. This prevents the effect of the anti-pitching fin to be annulled by the anti-heaving fin*. It follows that in the equation for pitch the effect of the anti-pitching fin alone is taken into account, while both foils affect the equation of heave.

The wave is the disturbing force. The control element, however, works in this disturbance which, in turn, has an effect on the control.

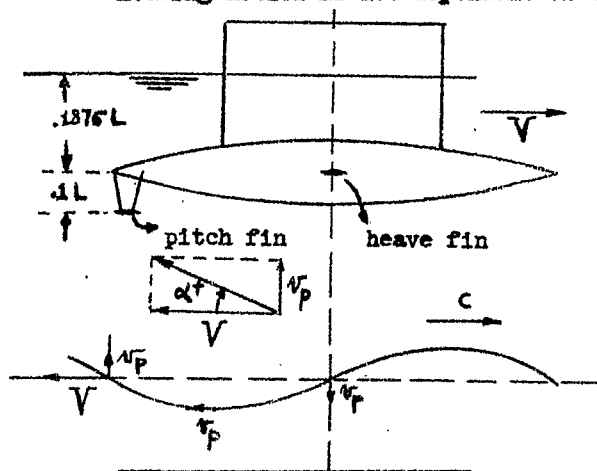
* By "anti-heaving fin" it is understood that a pair of foils are located on either side of the vessel in the vicinity of the longitudinal c.g. of the ship.

In this way the wave disturbs the vessel and the control. The most efficient control would, in fact, occur in calm water. By most efficient it is implied: efficient on the average; since, with certain wave lengths and with the vessel pitching with frequency of encounter, the orbital velocity at the anti-pitching fin will be so as to increase the effect of the foil. At other wave lengths the opposite would occur. Therefore, it is better to make the influence of the orbital velocity small. This can be done by placing the fin lower than the ship. The wave orbital velocity is given by

$v_p = a e^{-\frac{2\pi h}{\lambda}} \sqrt{\frac{2\pi g}{\lambda}}$ and was computed for different depths and wave lengths. The change in angle of attack on a foil due to this

v_p is $2 \frac{v_p}{V}$ and has been computed for different speeds. It was found that at $V = 2 \text{ ft/sec}$ for a wave length $\lambda = 8 \text{ ft.}$, the change in angle of attack at the depth of the centerline* is 20° , which was ^{at first judged to be} too large. The change in angle of attack at $1/10$ below the centerline is 10° which would be acceptable for good functioning of the aft foil.

The anti-heave foil would remain at the depth of the centerline since the phase lag between the change of angle of attack, and heaving motion is not dependent on the wave length.



* Depth of the centerline $h = .75 \text{ ft.}$; $h/L = .1875$.

The angle of attack as a function of time is given by:

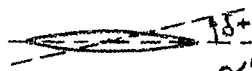
$$\begin{aligned} \alpha_p &= \frac{v_p}{V} \cos \left(\omega t - \pi + \frac{L}{\lambda} \pi \right) \\ &= -\frac{v_p}{V} \cos \left(\omega t + \frac{L}{\lambda} \pi \right) \\ &= -\frac{a_p}{V} \sqrt{\frac{2\pi g}{\lambda}} \cos \left(\omega t + \frac{L}{\lambda} \pi \right) \end{aligned}$$

where a_p is the wave amplitude at depth h .

To get an idea about the size of the foils and the angles they have to incline to control the heaving and pitching motions successfully, one can reason as follows.

The motions will be completely annulled if the forces on the hydrofoils act in such a way that the pitching moment and heaving force which they produce is, at all times, equal to the exciting moments and forces due to the waves. This would not be a control system in the regular sense because it would ^{have to} know from beforehand how to react. In regular waves this system could work but not in a confused sea.

We shall first investigate the pitching motion. The lift force on the anti-pitching fin can be expressed as $1/2 C_L \rho A U^2$ where A is the area of the foil, and U the speed of the wave particle with respect to the foil. The pitching moment this lift produces is $M_p = 1/4 \left(\frac{\partial C_L}{\partial \alpha} \right) \alpha' \rho A U^2 L$ for $\alpha' < 20^\circ$. α' is the total angle of attack. If $\delta = \delta_0 \sin(\omega t - \tau)$ represents the rotation of the angle of the fin around its own axis, then the total angle of attack becomes:



$$\alpha' = \delta_0 \sin(\omega t - \tau) - \frac{a_h}{V} \sqrt{\frac{2\pi g}{\lambda}} \cos\left(\omega t + \frac{L}{\lambda} \pi\right)$$

The pitching moment will be:

$$M_p = \frac{1}{4} \frac{\partial C_L}{\partial \alpha} \left[\delta_0 \sin(\omega t - \tau) - \frac{a_h}{V} \sqrt{\frac{2\pi g}{\lambda}} \cos\left(\omega t + \frac{L}{\lambda} \pi\right) \right] \rho A U^2 L$$

U is the resultant of $\vec{V}$ and $\vec{v}_p$.

$$U^2 = V^2 + v_p^2 + 2 \vec{V} \cdot \vec{v}_p \cos(\vec{V}, \vec{v}_p)$$

$$(\vec{V}, \vec{v}_p) = \omega t - \frac{\pi}{2} + \frac{L}{\lambda} \pi$$

$$U^2 = V^2 + a_h^2 \frac{2\pi g}{\lambda} + 2 V a_h \sqrt{\frac{2\pi g}{\lambda}} \sin\left(\omega t + \frac{L}{\lambda} \pi\right)$$

$$M_p = \frac{A \rho L}{4} \frac{\partial C_L}{\partial \alpha} \left[\delta_o \sin(\omega t - \tau) - \frac{v_p}{V} \cos\left(\omega t + \frac{L}{\lambda} \pi\right) \right] \times$$

$$\times \left[V^2 + v_p^2 + 2 V v_p \sin\left(\omega t + \frac{L}{\lambda} \pi\right) \right]$$

The unknowns are δ_o , τ , and A . This moment has to be equal to the exciting moment $m_1 \cos \omega t + m_2 \sin \omega t$. To simplify the solution, the terms $v_p^2 + 2V v_p \sin(\omega t + \frac{L}{\lambda} \pi)$ are assumed to be negligible compared to U^2 . This is permissible if the level of the foil is at least $L/10$ below the center of buoyancy and for speeds higher than 2 ft/sec. By setting all of the terms first in $\sin \omega t$ and then in $\cos \omega t$ equal to zero in the equation $M_p - m_1 \cos \omega t - m_2 \sin \omega t = 0$ and solving for δ_o and τ , it is found that

$$(1) \quad \delta_o^2 = \frac{(m_2 - \frac{A \rho L}{4} \frac{\partial C_L}{\partial \alpha} V v_p \sin \frac{L}{\lambda} \pi)^2 + (m_1 + \frac{A \rho L}{4} \frac{\partial C_L}{\partial \alpha} V v_p \cos \frac{L}{\lambda} \pi)^2}{\frac{A^2 \rho^2 L^2}{16} V^4 \left(\frac{\partial C_L}{\partial \alpha} \right)^2}$$

$$(2) \quad \cos \tau = \frac{m_2 - \frac{A \rho L}{4} \frac{\partial C_L}{\partial \alpha} V v_p \sin \frac{L}{\lambda} \pi}{\frac{A \rho L}{4} \frac{\partial C_L}{\partial \alpha} V^2 \delta_o}$$

(For a discussion on the magnitudes of A and $\frac{\partial C_L}{\partial \alpha}$, see App. VI, pg. 59)

δ_o and τ have been calculated for an anti-pitching fin with $A = 5\%$ of the longitudinal cross sectional area and $\frac{\partial C_L}{\partial \alpha} = 3.4$ in waves of 4 and 8 ft. and for speeds of 2 and 4 ft/sec. The results (App. VII, pg. 69) show that it is impossible to control the pitching at $V = 2$ ft/sec, while at $V = 4$ ft/sec it is theoretically possible to eliminate the pitching.

Formula (1) gives the maximum rotation of the foil and (2) the phase angle between the motion and the origin of time, in order to control the pitch. To control the heave, the effect of both foils has to be taken into account. The anti-heaving foil, which is situated midships at the height of the center of buoyancy, is assumed to have the same shape and area as the anti-pitching fin. So, the total heave exciting force from both the pitch and heave foils is:

$$F = \frac{A\rho}{2} \left(\frac{\partial C_L}{\partial \alpha} \right) V^2 \left[\delta_0 \sin(\omega t - \tau) - \frac{v_{p1}}{V} \cos\left(\omega t + \frac{L}{\lambda} \pi\right) + \delta_1 \sin(\omega t - \mu) - \frac{v_{p2}}{V} \cos \omega t \right]$$

where v_{p1} = particle velocity at depth $h + \frac{L}{10}$

v_{p2} = particle velocity at depth h

δ_1 = maximum rotation of center foil

μ = phase lag between center foil and origin of time.

This force is put equal to the exciting force:

$$F = F_1 \cos \omega t + F_2 \sin \omega t.$$

Putting the terms in $\cos \omega t$ and $\sin \omega t$ on both sides of this equation equal to each other, it is found after some calculations:

$$\delta_1^2 = \frac{\left[f_2 - \frac{A\rho}{2} \left(\frac{\partial C_L}{\partial \alpha} \right) V^2 \left(\delta_0 \cos \tau + \frac{v_{p1}}{V} \sin \frac{L}{\lambda} \pi \right) \right]^2}{\frac{A^2 \rho^2}{4} \left(\frac{\partial C_L}{\partial \alpha} \right)^2 V^4} + \frac{\left[f_1 + \frac{A\rho}{2} \left(\frac{\partial C_L}{\partial \alpha} \right) V^2 \left(\delta_0 \sin \tau + \frac{v_{p1}}{V} \cos \frac{L}{\lambda} \pi + \frac{v_{p2}}{V} \right) \right]^2}{\frac{A^2 \rho^2}{4} \left(\frac{\partial C_L}{\partial \alpha} \right)^2 V^4}$$

$$\cos \mu = \frac{f_2 - \frac{A\rho}{2} \left(\frac{\partial c_L}{\partial \alpha} \right) V^2 \left(\delta_0 \cos \tau + \frac{v_{p1}}{V} \sin \frac{L}{\lambda} \pi \right)}{\frac{A\rho}{2} \left(\frac{\partial c_L}{\partial \alpha} \right) V^2 \delta_1}$$

and, after replacing δ_0 and τ by their values in (1) and (2) (pg. 31):

$$(3) \delta_1^2 = \frac{\left(f_2 - \frac{2m_2}{L} + A\rho \frac{\partial c_L}{\partial \alpha} V v_{p1} \sin \frac{L}{\lambda} \pi \right)^2 + \left(f_1 - \frac{2m_1}{L} + A\rho \frac{\partial c_L}{\partial \alpha} V v_{p2} \right)^2}{\frac{A^2 \rho^2}{4} \left(\frac{\partial c_L}{\partial \alpha} \right)^2 V^4}$$

$$(4) \cos \mu = \frac{f_2 - \frac{2m_2}{L} + A\rho \frac{\partial c_L}{\partial \alpha} V v_{p1} \sin \frac{L}{\lambda} \pi}{\frac{A\rho}{2} \left(\frac{\partial c_L}{\partial \alpha} \right) V^2 \delta_1}$$

It is found that for speeds of 2 ft/sec, it is impossible to control the heave ideally (App. VII, pg. 64). This is similar to the conclusion for pitch.

These computations, which are in themselves only approximations of the real physical conditions, show that it is necessary to take the greatest area for the foils possible and that there will always be some pitching and heaving at low speeds ($Fr < .25$).

A more realistic method of control is to use the motions of the vessel as input to the control element. The control fins will work in such a way as always to counteract the motion the vessel performs at that moment.

The motion equations can then be expressed as follows:

$$\begin{aligned} (m+a)\ddot{z} + b\dot{z} + cz + d\ddot{\theta} + e\dot{\theta} + g\theta &= Z_S(\delta + \alpha - \theta) + Z_\delta(\dot{\delta} + \dot{\alpha} - \dot{\theta}) + Z_{\ddot{\delta}}(\ddot{\delta} + \ddot{\alpha} - \ddot{\theta}) + Z \\ (J+A)\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{z} + E\dot{z} + Gz - C_3 \frac{\theta}{\sqrt{|\theta|}} &= \\ &= M_\delta(\delta + \alpha - \theta) + M_{\dot{\delta}}(\dot{\delta} + \dot{\alpha} - \dot{\theta}) + M_{\ddot{\delta}}(\ddot{\delta} + \ddot{\alpha} - \ddot{\theta}) + M \end{aligned}$$

where Z_δ and M_δ are hydrodynamic derivatives of respectively heave force and pitch moment due to angle of attack on the foils.

δ = angle between foil and longitudinal axis of the ship

α = angle of attack induced by the waves with respect to the horizontal.

' and '' are negligible quantities for waves as long as or longer than the ship. δ and $\dot{\delta}$ are small for the vessel in stern seas since the motions have a small frequency. Therefore, α , $\dot{\alpha}$, $\ddot{\alpha}$, and $\ddot{\delta}$ are considered to be of second order and are neglected.

In the equations of motion, the coefficients a, b, c, ... do not have the same value as in the uncontrolled vessel.

The added mass of two foils placed at x_P and x_H from the center of gravity of the ship is given by (Ref. 16):

$$Z_{wf} = -a_f = -\frac{1}{4}\rho\pi \left(\frac{a_H^2 b_H^2}{\sqrt{a_H^2 + b_H^2}} + \frac{a_P^2 b_P^2}{\sqrt{a_P^2 + b_P^2}} \right) = -\frac{1}{4}\rho\pi(H+P)$$

where a_H , b_H are respectively span and chord of the anti-heaving foil.

Analogously for a_P and b_P .

$$Z_{qf} = +\frac{1}{4}\rho\pi (Hx_H + Px_P) = -d_f$$

$$M_{wf} = +\frac{1}{4}\rho\pi (Hx_H + Px_P) = -D_f$$

$$M_{qf} = -\frac{1}{4}\rho\pi (Hx_H^2 + Px_P^2) = -A_f$$

In our case $x_H = 0$.

These values are added to the respective coefficients of the vessel without fins (App. IV.).

b_F , B_F , e_F , and E_F are caused by the lift and drag of the foils due to the ship's own motion.

$$L = \frac{\partial C_L}{\partial \alpha} \propto 1/2 \rho A \left[v^2 + \left(\dot{\theta} \frac{L}{2} - \dot{z} \right)^2 \right]$$

$$D = 1/2 \rho A C_D \left[v^2 + \left(\dot{\theta} \frac{L}{2} - \dot{z} \right)^2 \right].$$

From these two expressions the damping coefficients can be found:

$$Z_{\omega_F} = -b_F = +1/2 \rho A V \left(\frac{\partial C_L}{\partial \alpha} + C_D \right)$$

$$Z_{\dot{\theta}F} = -e_F = -1/4 \rho A V L \left(\frac{\partial C_L}{\partial \alpha} + C_D \right)$$

$$M_{\dot{\theta}F} = -B_F = -1/8 \rho A L^2 V \left(\frac{\partial C_L}{\partial \alpha} + C_D \right)$$

$$M_{\dot{z}F} = -E_F = -1/4 \rho A V L \left(\frac{\partial C_L}{\partial \alpha} + C_D \right).$$

Computation of Z_{δ} :

$$Z_{\delta} = (Z_{\delta})_P \delta_P + (Z_{\delta})_H \delta_H$$

δ_P = angle between longitudinal axis of ship and the pitching foil.

δ_H = angle between longitudinal axis of ship and the anti-heaving foil.

$(Z_\delta)_P = (Z_\delta)_H$ when both foils are equal to each other.

$$(Z_\delta)_P = 1/2 \rho A V^2 \left(\frac{\partial C_L}{\partial \alpha} + C_D \right) = Z_\delta.$$

Analogously,

$$M_\delta = 1/4 \rho A V^2 L \left(\frac{\partial C_L}{\partial \alpha} + C_D \right) = Z_\delta \times L/2$$

The control angles δ_H and δ_P are made dependent on $\dot{\theta}$ and $\ddot{z}$ such that relatively simple controlling devices can be used.

$$\delta_P = k_1 \dot{\theta} + k_2 \ddot{z}$$

$$\delta_H = k_3 \dot{\theta} + k_4 \ddot{z}.$$

where k_1, k_2, k_3, k_4 are four control parameters to be determined. If time lags in the response of the control system are taken into account, the angles become:

$$\delta_P = k_1 \dot{\theta} - k_1 t_1 \ddot{\theta} + k_2 \ddot{z} - k_2 t_4 \dddot{z}$$

$$\delta_H = k_3 \dot{\theta} - k_3 t_1 \ddot{\theta} + k_4 \ddot{z} - k_4 t_4 \dddot{z}$$

where t_1 are the time lags in the control system. Hence the heave equation becomes:

$$\begin{aligned} [m + a + a_f + Z_\delta t_4 (k_1 + k_4)] \ddot{z} + [b + b_f - Z_\delta (k_2 + k_4)] \dot{z} + c z \\ + [d + d_f + Z_\delta t_1 (k_1 + k_3)] \ddot{\theta} + [e + e_f - Z_\delta (k_1 + k_3)] \dot{\theta} + [g + 2 Z_\delta] \theta = \\ = f_1 \cos \omega t + f_2 \sin \omega t + Z_\delta (\alpha_P + \alpha_H) \end{aligned}$$

$$\alpha_P = - \frac{V P_1}{V} \cos \left(\omega t + \frac{L}{\lambda} \pi \right) \text{ is the angle induced on the anti-}$$

pitching foil^{*} by the wave. Analogously, for the anti-heaving fin,

$$\alpha_H = - \frac{V P_2}{V} \cos \omega t. \text{ The total angle of attack on the anti-pitching fin is then } \alpha_P + \delta_P - \theta.$$

* When it is fixed in the horizontal direction.

The pitch equation becomes:

$$\begin{aligned} & [J + A + A_f + M_8 k_1 k_2] \ddot{\theta} + [B + B_f - M_8 k_1] \dot{\theta} + [C + M_8] \theta + C_3 \frac{\theta}{\sqrt{10}} \\ & + [D + D_f + M_8 k_4 k_2] \ddot{z} + [E + E_f - M_8 k_2] \dot{z} + Gz = \\ & = m_1 \cos \omega t + m_2 \cos \omega t + M_8 \alpha_F \end{aligned}$$

The different coefficients of these equations are evaluated in

App. VIII.* As in the uncontrolled motion, two methods of solving the equations of motions were used; one, the linearized method described on pg. 24 and the other, Adams' Numerical Method (pg. 25 - 27).

In 4-ft waves ($\lambda/L = 1$), it is seen (App. VIII) that the motions are fairly well controlled if the time lags are taken small enough. Large time lags for high speeds can have such an effect as to drastically diminish the virtual mass and moment of inertia of the ship. This effect is even stronger in the motions in 8-ft waves. Thus, heaving and pitching can be greatly increased. Even an increase in foil area cannot make up for the loss in virtual inertia (Tables II & III).

The solution of the linearized equations gives smaller amplitudes for heave and pitch than Adams' Numerical Method. This was to be expected, just as in the uncontrolled motion, since the nonlinear term increases the pitching (App. IV). Therefore, and also because of lack of computer time, this method was discontinued.

(Tables IV and V)

Since the motions in 8-ft waves were harder to control, further calculations were confined to these waves. Finally, the control in 4-ft waves was investigated with the system that gave optimum control in 8-ft waves (see pg. 41).

The numerical results and details are given in App. VIII, while a description of different control systems that were investigated,

* A convenient summary of all the data of App. VIII is given in tabular form on pg. 75a.

follows below.

The system $\delta_p = k_1 \dot{\theta} + k_2 \ddot{z} - \ddot{z}_p$ (time lag)

$$\delta_H = k_3 \dot{\theta} + k_4 \ddot{z} - \ddot{z}_H \text{ (time lag)}$$

did not give satisfactory results. This can be explained not only by the time lag but also by the fact that for speeds higher than 3 ft/sec ($Fr = .26$), the phase angle between heave and pitch becomes 180° , which means that the anti-heaving foil is counteracted by the foil aft. To eliminate this effect the anti-pitching control has to be such that only a pitching moment is produced. The solution is: two anti-pitching fins, one fore and one aft, which rotate in opposite angles and whose resultant response does not produce any vertical force but only a pitching moment. From the results, it can (App. VIII, Table VI) be seen that the anti-heaving fin is now more effective; but the pitching control is worse, since the presence of a fin in the forward end of the vessel has a negative effect on the ^{dynamic} pitching stability. This new system requires, of course, alteration of many terms in the motion equations (App. VIII, pg. 70).

To make up for the loss of stability in pitching, a different set of foils was introduced, namely, flap fins. They have a much higher C_L than the conventional simple fin. The Sperry Corporation manufactures anti-rolling gyro fins which have a C_L of 1.5 at an 18° angle of tilt, with the flap at an additional angle of 20° . The $\frac{\partial C_L}{\partial \alpha}$ with this type of fin amounts to 4.8. A further increase in control ability can be obtained by a "stepcontrol," which means that

the angle of the foil moves stepwise from a maximum angle in one sense to a maximum angle in the opposite sense:

$$\delta_P = k_1 \frac{\dot{\theta}}{|\dot{\theta}|}$$

$$\delta_H = k_2 \frac{\dot{z}}{|\dot{z}|}$$

In this way the anti-pitching foil switches over when the pitching motion reaches its maximum amplitude and reverses its sense. The anti-heaving foil works analogously. The result with these extreme conditions of $\frac{\partial C_L}{\partial \alpha}$ and control method was an almost complete elimination of motion. In fact, the vessel is "overcontrolled," since the motions degenerated into almost a vibration. The accelerations showed, indeed, very high values, while the frequency of the motions was considerably higher than ω (Table VII, pg. 72).

The same stepcontrol with the same foil characteristics was used in the next tryout but with only one anti-pitching fin (aft) (Table VIII). The result showed a fairly good pitch control, but again the heaving becomes too large at speeds of resonance ($V = 3$ ft/sec). Once the vessel is over this speed, the control works efficiently. However, for these high speeds, the value of $\frac{\partial C_L}{\partial \alpha}$ is rather high, since cavitation may become a problem. It is, therefore, safer to take a lower maximum lift coefficient at those high speeds, at around 1.2, which gives a $\frac{\partial C_L}{\partial \alpha}$ of approximately 4*. Furthermore, it is realized that the large strut needed to support the large pitch foil at the very stern of the semisubmarine

* See App. IV.

may create considerable structural problems. Moreover, the increase in drag due to this strut might be so large at high speeds that any resistance advantage of the semisubmarine over the conventional ship could be reversed. It is also true that the higher the forward speed, the less the disturbing influence of the waves on the angle of attack.

These considerations led to the introduction of two anti-pitching fins of small size ^{each} (1.8 per cent of the longitudinal cross section area) that are directly attached to the fore and aft ends of the body of revolution at the level of the apices. The anti-heaving foil has a 2.5 per cent area and remains midship. A $\frac{\delta C_L}{\delta \alpha}$ of 4 and stepcontrol were assumed. The value of 4 for $\frac{\delta C_L}{\delta \alpha}$ is more realistic, since the lift coefficient cannot be taken as large at high speeds so as to avoid cavitation (see App. VI). The results (Table IX) with this system were as good as with the previous one, while the maximum angle of attack on the foils is smaller, which diminishes the danger of stalling. However, again the pitching amplitudes have a tendency to oscillate and, this time, not around $\theta = 0$, but around a negative value. This causes the vessel to assume large pitching angles (10° and more), which is not allowable. The same was true, in lesser degree, for the heave motion. Pitch angle and heave displacement signals were, therefore, added to the next tryout:

$$\delta_P = k_1 \frac{\theta}{\sqrt{\theta}} + k_2 \theta$$

$$\delta_H = k_3 z + k_4 \frac{\dot{z}}{\sqrt{z}} \quad (\text{App. VIII, Table X, pg. 73})$$

It was found that this system is very effective to limit the pitch at all speeds, while the heaving amplitudes remain significant at the speed of resonance. A better result at this speed is obtained when the heave displacement signal is taken away (App. VIII, Table XI).

Many more systems of control have been tried, but it seems that the system

$$\delta_P = k_1 \frac{\dot{\theta}}{|\dot{\theta}|} + k_2 \theta$$
$$\delta_H = k_3 \frac{\dot{z}}{|\dot{z}|}$$

gives the best results. The pitch remains below 5° around the speed of pitch resonance, while for higher speeds it is much less. The heave takes values of 2.5 times the wave height at the speed of heave resonance, but outside this range it becomes negligible (see Fig. 5).

This system applied to the vessel in 4-ft waves ($\lambda = L$) showed almost a complete disappearance of any motions, even with systems that still showed large amplitudes in 8-ft waves.

It seems that the control of this semisubmarine is the most difficult in long waves which have high energy and cause frequency of encounter around the natural frequencies of the vessel.

Stepcontrol is, however, an idealized assumption; moreover, it is not even desirable since it causes vibrations and high accelerations.

From the computational results, it can be seen that a harmonic control function is sufficient at high speeds ($Fr > 0.35$) but that stepcontrol or, rather, a "rounded" stepcontrol is necessary at speeds of resonance. It is advisable that time lags in the control system be as small as possible.

V. CONCLUSIONS AND RECOMMENDATIONS

Previous reports of experimental work emphasized the small motions of the semisubmarine in ahead seas. However, with no controls, severe motions were encountered in regular astern seas when resonant wave components were encountered. Sharp peaks in the motion amplitude at resonance have also been found to be true in this study, which is entirely analytical in nature. Different existing theories that apply to either surface ships or deeply submerged submarines have been adapted and applied to the problem of the semisubmarine near the free surface. It has been found that the motions go far beyond the linear range, and the accelerations far above the limit of comfort.

This report shows, however, that with activated controls incorporating medium-sized control surfaces, it is possible to reduce the motions to very nominal values. Since the frequency of encounter, where severe motions occur, are relatively small, the control machinery would not have to be of a high-speed type; neither need the power requirement be large, since the hydrodynamic exciting forces and moments are not large.

The feasibility of an effective motion control coupled with the fact that the semisubmarine cannot experience slamming tends to confirm the expectation that this type of vessel should be able to sustain its maximum speed in very heavy seas.

To arrive at a more accurate prediction of the motions in waves, more research is, however, necessary. Especially, the problem of the added mass and added moment of inertia are properties that should be investigated more specifically at the low frequencies of encounter that occur in stern waves. In this report the values for these coefficients were determined more or less by a trial-and-error procedure, which was, nevertheless, consistent with some existing theories and experimental results. The same remarks go for the coupled damping coefficients where the existing theories are not found to correspond with experiments.

The influence of surge and surge forces has not been studied fully in this work. The problem would, indeed, become much more complex (see App. III) and forms a topic of research by itself.

The surge motions may have a measurable effect on the pitch and heave of the semisubmarine, since the large strut causes considerable moments on the vessel. It may be useful to investigate the coupling of the three motions (heave, pitch, and surge) in the future. All of these recommendations would mainly refine the solutions so that the correlation with experimental results would be more complete. They should not alter the conclusions previously drawn.

A natural consequence to this report would be the application of methods and results of this study to the motions in an irregular

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see. If the principle of superposition is assumed to be true, this should not pose any particular difficulties.

APPENDIX I. OFFSETS OF SEMISUBMARINE

Body of Revolution

Station	D/D_{max}	$(D/D_{max})^2$
0	0	0
1	0.455	0.207
2	0.702	0.493
3	0.872	0.762
4	0.971	0.942
5	1.000	1.000
6	0.976	0.952
7	0.910	0.828
8	0.778	0.605
9	0.530	0.281
10	0	0

$$D_{max} = 6 \text{ in}$$

$$\text{Length} = 4 \text{ ft}$$

Strut

Station	B/B_{max}
0	0
1	0.253
2	0.523
3	0.769
4	0.939
5	1.000
6	0.939
7	0.769
8	0.523
9	0.253
10	0

$$B_{max} = 2.4 \text{ in}$$

$$\text{Waterplane area } A_w = 0.24 \text{ sq ft}$$

$$\text{Displacement of body of revolution} = 0.48 \text{ ft}^3$$

$$\text{Displacement of sail} = 0.124 \text{ ft}^3$$

$$\text{Natural heaving period} = 2.3 \text{ sec; natural heaving frequency} = 2.73 \text{ cycles/sec}$$

$$\text{Natural pitching period} = 3.42 \text{ sec; natural pitching frequency} = 1.83 \text{ " "}$$

APPENDIX II. THE INFLUENCE OF SURGE ON THE EQUATIONS OF MOTION

While the effect of the surge exciting force on the pitch exciting moment has been partially taken into account in this report (see pg. 11-12), for the most part, it has not been possible to introduce other effects of surge.

If we call u the surge velocity, we have that the time dependent velocity $V = V_0 + u$ where u is a harmonic function of t in regular waves and V_0 is the mean velocity of the ship in waves.

The frequency of encounter will also change harmonically:

$$\omega(t) = \frac{2\pi}{\lambda} (V_0 + u - c)$$

The experiments (Ref. 3) have shown that this surge u can become considerable, up to 30 per cent of the forward velocity. In ahead waves where $\omega(t) = \frac{2\pi}{\lambda} (V + c)$, this surge would not have a large influence but in astern waves the amount $V - c$ can become quite small so that the frequency of encounter can change considerably during the time that a wave length passes the ship.

The coupled equations of motion would become:

$$\begin{aligned} (m - X_u')\ddot{u} - X_{uu}u - X_{u\dot{z}}\dot{z} - X_{u\ddot{z}}\ddot{z} - X_{u\dot{\theta}}\dot{\theta} - (X_q + VX_w')\dot{\theta} - VX_w\theta &= X_0 \cos(\omega t - \epsilon_x) \\ -Z_u'\ddot{u} - Z_{uu}u + (m - Z_w')\ddot{z} - Z_{wz}\dot{z} - Z_{z\ddot{z}}\ddot{z} - Z_{w\dot{\theta}}\dot{\theta} - (Z_q + VZ_w')\dot{\theta} - VZ_w\theta &= Z_0 \cos(\omega t - \epsilon_z) \\ -M_u'\ddot{u} - M_{uu}u - M_{u\ddot{z}}\ddot{z} - M_{u\dot{\theta}}\dot{\theta} - M_{u\ddot{\theta}}\ddot{\theta} + (I_y - M_{\dot{q}})\ddot{\theta} - (M_q + VM_{\dot{w}}')\dot{\theta} - (M_{\theta} + VM_w')\theta &= \\ &M_0 \cos(\omega t - \epsilon_m) \end{aligned}$$

X_u' , X_q' , Z_u' , M_u' are small quantities and can be neglected. They are either coupled masses or coupled moments of inertia. X_q and X_w are due to added resistance from waves, dependent on $V = V_0 + u$. X_z is

increased drag due to change in draft, dependent on $V^2 = (V_0 + u)^2$. Z_u will be the change in lift force due to change in forward velocity, also a function of $V^2 = (V_0 + u)^2$. From Chapter III it follows that the coefficients Z'_u , Z'_q , Z'_v , M'_u , and M'_q and M'_q are functions of $V = V_0 + u$.

Solutions for the three equations of motion given on pg. 46 can only be found by approximate methods with high-speed computers. The problem is essentially a nonlinear vibration problem where many of the coefficients are difficult to define mathematically. More research on this particular subject is highly desirable in this case of semi-submarines but has not been undertaken here.

APPENDIX III. EXCITING FORCES AND MOMENTS

Two wave lengths were considered to get an idea of the motion of the semi-submarine in waves. They are respectively equal to the length and twice the length of the vessel. They were chosen because the natural frequencies of the vessel are low such that synchronism with the encounter frequency is likely to occur at operating speeds for these wave lengths.

The integrals in formula (4)_{pg. 7} were solved by Simpson's Rule. Ten stations were considered sufficient since the body is slender, and the dimensions do not vary much from station to station. The results of the computations for $\lambda = 4$ ft:

$$Z_1 = (0.072 - 0.0008 V) \cos 1.57(V - 4.54)t - (0.69 + 0.05 V) \sin 1.57(V - 4.54)t.$$

$\lambda = 8$ ft:

$$Z_1 = (-2.813 + 0.15 V) \sin 0.785(V - 6.4)t + (0.13 - 0.01 V) \cos 0.785(V - 6.4)t.$$

pg. 9
Formula (5)_{pg. 9} gives for $\lambda = 4$ ft:

$$Z_2 = +0.813 \sin 1.57(V - 4.54)t$$

$\lambda = 8$ ft:

$$Z_2 = 1.17 \sin 0.785(V - 6.4)t$$

Therefore, the total heaving forces become for $\lambda = 4$ ft:

$$Z = (0.13 + 0.04 V) \sin 1.57(V - 4.54)t + (0.072 - 0.0008 V) \cos 1.57(V - 4.54)t \quad (\text{lbs})$$

$\lambda = 8$ ft:

$$Z = (-1.1 + 0.2 V) \sin 0.785(V - 6.4)t + (0.13 - 0.01 V) \cos 0.785(V - 6.4)t. \quad (\text{lbs})$$

The expressions under the integral sign in formula (5)^{pg. 10} were calculated by Simpson's Rule:

$\lambda = 4$ ft:

$$M_1 = (-1.0 + 0.071 V) \cos 1.57(V - 4.54)t + (-0.042 + 0.0008) \sin 1.57(V - 4.54)t$$

$\lambda = 8$ ft:

$$M_1 = -(1.605 + 0.15 V) \cos 0.785(V - 6.4)t + (0.094 - 0.020 V) \sin 0.785(V - 6.4)t$$

Expression (7)^{pg. 10} becomes for $\lambda = 4$ ft:

$$M_2 = -0.246 \cos 1.57 (V - 4.54)t$$

$\lambda = 8$ ft:

$$M_2 = -0.261 \cos 0.785 (V - 6.4)t$$

Expression (8)^{pg. 12} gives for $\lambda = 4$ ft:

$$X_3 = 0.26 \cos 1.54 (V - 4.54)t$$

For $\lambda = 8$ ft:

$$X_3 = 0.762 \cos 0.785 (V - 6.4)t$$

To find the point of application of this surge force, we assume that the horizontal hydrodynamic pressure decreases exponentially with depth according to the law $e^{-\frac{2\pi h}{\lambda}}$. The center of pressure will then be given by:

$$H = \frac{\int_0^{\lambda} e^{-\frac{2\pi h}{\lambda}} h dh}{\int_0^{\lambda} e^{-\frac{2\pi h}{\lambda}} dh} = \frac{\lambda}{2\pi} \left[\frac{e^{-\frac{2\pi h}{\lambda}} \left[\frac{2\pi h}{\lambda} + 1 \right] - 1}{e^{-\frac{2\pi h}{\lambda}} - 1} \right]$$

For $\lambda = 4$ ft: $H = 0.226$ ft

For $\lambda = 8$ ft: $H = 0.24$ ft

The moment then becomes for $\lambda = 4$ ft:

$$M_3 = 0.26(0.75 - 0.226)\cos 1.57 (V - 4.54)t \\ 0.136 \cos 1.57 (V - 6.4)t \quad \text{ft lbs}$$

$\lambda = 8$ ft:

$$M_3 = 0.388 \cos 0.785 (V - 6.4)t \quad \text{ft lbs}$$

Formula ^{pg. 13} (9) gives: $X_h = +0.02 \sin \frac{2\pi}{\lambda} (V - 4.54)t$ at $\lambda = 4$ which is a force that can be neglected compared to the hydrodynamic surge force. The same remark goes for X_h at $\lambda = 8$ ft. The total moment then becomes:

$$M = M_1 + M_2 + M_3$$

$\lambda = 4$ ft:

$$M = (-0.9 + 0.07 V)\cos 1.57(V - 4.54)t + (0.0042 + \\ 0.0008 V)\sin 1.57(V - 4.54)t \quad \text{ft lbs}$$

$\lambda = 8$ ft:

$$M = -(1.538 + 0.15 V)\cos 0.785(V - 6.4)t + (0.094 - \\ 0.02 V)\sin 0.785(V - 6.4)t \quad \text{ft lbs}$$

APPENDIX IV. THE HYDRODYNAMIC COEFFICIENTS AND
THE SOLUTIONS TO THE EQUATIONS OF MOTION

1. a and A - Added Mass and Added Moment of Inertia (see pg. 15 of text)

In Ref. 3 the geometrical data of the semisubmarine are found:

$$V_{\text{body}} = 0.48 \text{ ft}^3$$

$$V_{\text{strut}} = 0.07 \text{ ft}^3$$

$$m = 0.55 \times 1.99 = 1.1 \text{ slugs}$$

$$J = I_y = \rho V k_y^2 L^2 \text{ where } k_y^2 \text{ is, again, given by Ref. 3:}$$

$$k_y^2 = 0.035$$

Therefore, $J = 0.6$.

$$a = \frac{1}{2} \rho \pi \int_{-\frac{L}{2}}^{\frac{L}{2}} r^2 dx = k_a m$$

$$A = \frac{k_a A}{D^2} = \frac{k_a \rho \pi}{D^2} \int_{-\frac{L}{2}}^{\frac{L}{2}} r^2 x^2 dx = k_p \frac{I_y}{D^2}$$

By an iterative procedure that sought to optimize the correlation with the experimental results of Ref. 3, k_p and k_a were found to have values ranging from 0.60 at high frequencies to 2.0 at very small frequencies of encounter. The following tables give the final values for $\lambda = 4 \text{ ft}$ ($c = 4.5 \text{ ft/sec}$).

V	$ \omega $	$a + m$	$A + J$
0.1	6.97	1.7	1.0
0.7	6.04	1.7	1.0
1.3	4.94	1.7	1.0
1.9	4.15	1.8	1.0
2.5	3.20	2.0	1.1
3.1	2.26	2.1	1.2
3.7	1.56	2.1	1.2
4.2	0.974	3.0	1.6
4.8	0.42	3.0	1.8

$V = 2.8 \text{ ft/sec}$ for heave resonance

$V = 5.4 \text{ ft/sec}$ for pitch resonance

$$\lambda = 8 \text{ ft } (c = 6.4 \text{ ft/sec})$$

V	$ \omega $	a-in	A+J
0.1	4.94	1.7	1.1
0.7	4.47	1.7	1.2
1.3	4.00	2.0	1.3
1.9	3.53	2.0	1.4
2.5	3.06	2.1	1.4
3.1	2.59	2.5	1.4
3.7	2.12	2.5	1.45
4.2	1.73	2.5	1.5
4.8	1.25	2.6	1.5

V = 2.9 ft/sec for heave resonance

V = 4.1 ft/sec for pitch resonance

2. b and B - Damping Coefficients (see pg. 16 of text)

The following values, needed for the computation of b and B, were used:

used:

$a_1 = 1.8 \text{ ft}$ (a little smaller than half the length of the vessel, due to the difference in shape)

$a_2 = .25 \text{ ft}$ (radius of body of revolution)

$a_3 = .25 \text{ ft}$

$h = .75 \text{ ft}$ (depth of submergence)

For different speeds V, the values $\sqrt{\frac{g}{B} a_1}$ and $\sqrt{\frac{a_1}{B}}$, which were also needed as input to the ^{PIMS} computer program, were calculated.

V	1.0	2.0	3.0	4.0	5.2
$\sqrt{\frac{g}{B} a_1}$	0.131	0.263	0.394	0.526	0.685
$\sqrt{\frac{a_1}{B}}$	1.311	.941	.57	.29	-0.207
($\lambda = 4 \text{ ft}$)					
$\sqrt{\frac{a_1}{B}}$	0.999	.814	.629	.444	.222
($\lambda = 8 \text{ ft}$)					

Fig. 2 shows the results for b and B . Values of b and B were read from the curves for the various speeds that were selected in the iterative computer program to find the motions. However, for the vessel in 8-ft waves, the curves showed such good resemblance to second and third order parabolas that curves of the type $b = pV^2 + qV + r$ and $B = p'V^3 + q'V^2 + r'V + s'$ were fitted through different points of the functions $b(V)$ and $B(V)$. The result was: $b = (0.0155 V^2 - 0.083 V + 0.152)\rho$, $B = (-0.000147 V^3 + 0.16 V^2 - 0.034 V + 0.0247)\rho$.

3. e and E - Coupled Damping Coefficients (pg. 18)

$$\lambda = 4 \text{ ft}$$

$$e = -0.3 V + 0.01 V^2$$

$$E = -0.1 V$$

$$\lambda = 8 \text{ ft}$$

$$e = 0.3 V - 0.01 V^2 \text{ and } E = -0.1 V \text{ for } V < 2 \text{ ft/sec}$$

$$e = -0.3 V + 0.01 V^2 \text{ and } E = 0.1 V \text{ for } V > 2 \text{ ft/sec}$$

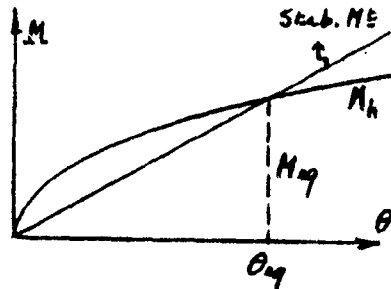
These values are very disputable since no theory exists nor have experiments been carried out to evaluate e and E of submerged bodies close to the surface and oscillating with frequencies near to zero. However, the e and E values of the basic hull are very small in comparison with the e_s and E_s caused by the presence of

$$\Delta = 35.4 \text{ lbs}$$

$$I = 0.0435 \text{ ft}^4$$

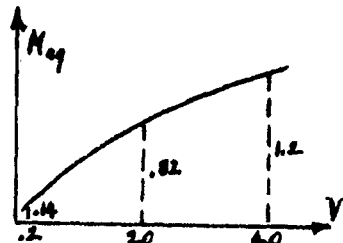
$$C = \Delta \times \frac{I}{V} = 64 \times 0.0435 = 2.78 \text{ ft lbs/rad}$$

Fig. 4, which was taken from Ref. 3, shows how the trim changes with forward velocity. The curves for the hydrodynamic and hydrostatic moments at a particular speed are drawn in the adjacent sketch with the assumption that the hydrodynamic moment is a second-degree



parabola, and the static moment is linear with θ . θ_{eq} is the equilibrium trim angle which the vessel takes on when moving at the particular speed. The hydrodynamic moment is of the form $M_h = C_3' \frac{-\theta}{\sqrt{\theta}}$

where C_3' depends on M_{eq} , which changes for different speeds.



$$M_{eq} = C_3'(V) / \theta_{eq}$$

On the basis of the experimental data of Ref. 3 shown on the adjacent sketch, an expression relating C_3 to speed can be found.

If we put $C_3'(V) = pV^2 + qV + r$, p , q and r can be defined by the set:

$$C_3'(V_1) = pV_1^2 + qV_1 + r$$

$$C_3'(V_2) = pV_2^2 + qV_2 + r$$

$$C_3'(V_3) = pV_3^2 + qV_3 + r$$

The result is $C_3'(V) = -0.0917 V^2 + 0.692 V + 0.492$. The expression for M_h then becomes: $(0.0917 V^2 - 0.692 V - 0.492) \frac{\theta}{\sqrt{\theta}} = C_3' \frac{\theta}{\sqrt{\theta}}$

We write $\frac{-\theta}{\sqrt{\theta}}$ instead of $\sqrt{\theta}$ since the hydrodynamic moment has to have the opposite sign of θ and to avoid the obvious difficulty of a negative θ under the square root sign. If the values of C_3 found in the previous paragraph are utilized to compute the motions of the semisubmarine in waves, they are much larger than the experimental motions of Ref. 3. Therefore, apparently, the values C_3 determined from still-water experiments are not valid in waves. The value chosen for C_3 was, therefore, reduced to $-0.5 V + 0.05 V^2$, which resulted in computed motions that correspond fairly well with the experimental results.

5. d and D - Coupled Virtual Mass and Moment of Inertia Coefficients

$$\rho \pi \int_{-\frac{L}{2}}^{\frac{L}{2}} r^2 x \, dx \text{ was calculated according to Simpson's Rule and was}$$

found to be equal to -0.0251ρ , a very small value.

$$d = D = 0.0251 \rho.$$

6. g and G - Coupled Spring Constants, pg. 21

$$V = 0.48 \text{ ft}^3$$

$$m' = 0.015$$

$$D' = .001*$$

$$Z_w = 0.152 V*$$

$$\text{Therefore, } g = + 0.152 V^2 \text{ lbs/rad}$$

$$x_s = \frac{1.84}{12} \text{ ft}$$

$$A_v = 0.24 \text{ ft}^2$$

$$M_z = -1.18 \rho; \text{ therefore, } G = + 1.18 \rho \text{ ft lb/ft}$$

* Based on deeply submerged submarine data ignoring free surface and frequency dependency.

7. Numerical results for pitch and heave amplitudes

 $\lambda = 4$ ft

V(ft/sec)	z_1	z_2	θ_1 (radians)	θ_2
0.1	0.004	0.004	0.027	0.024
0.7	0.009	0.009	0.033	0.035
1.3	0.016	0.016	0.043	0.037
1.9	0.038	0.040	0.056	0.045
2.5	0.19	0.21	0.105	0.10
3.1	0.125	0.16	0.15	0.12
3.7	0.16	0.32	0.43	0.35
4.2	0.25	0.4	1.2	1.3*
4.8	0.21	0.3	1.0	0.9*

* Unstable

z_1 is the heave amplitude according to the solution of the linearized equations. z_2 is the heave from the time-history solution of the actual differential equations. θ is the pitching amplitude.

 $\lambda = 8$ ft

V	z_1	z_2	θ_1	θ_2
0.1	0.05	0.1	0.084	0.09
0.7	0.07	0.13	0.10	0.10
1.3	0.075	0.125	0.11	0.10
1.9	0.112	0.17	0.145	0.13
2.5	0.35	0.35	0.26	0.2
3.1	0.29	0.8*	0.27	0.3
3.7	0.17	0.5	0.39	0.5
4.2	0.22	0.38	0.7	1.0*
4.8	0.56	0.6*	2.0	1.1*

* Unstable

APPENDIX V. EFFECT OF FRICTION ON THE MOTIONS OF A SEMISUBMARINE

As noted on pg. 6 of the text
Friction has little effect on the motion of a ship in a seaway.

There is a frictional resistance proportional with the square of the velocity in the direction of the motion. This increase in frictional resistance is, however, very small with respect to change in flow a periodically moving ship creates around it.

For a semisubmarine it could be a different matter since the sail can have a relatively large area and its drag applies at a considerable distance from the center of buoyancy. If the drag is represented by: $D = 1/2 \rho V^2 C_D S$ where S is the underwater area of the strut, C_D has an average value of 0.01 for reasonable roughness of the strut. This gives for $V = 4$ ft/sec: $D = 1/2 \times 1.99 \times 16 \times 0.01 \times 2 \times 0.5$. $D = 0.16$ lbs. This is for a Froude Number 0.35. The pitching moment is then $D \times 0.5 = 0.08$ ft lbs which is negligible in comparison with the moment due to the flow around the body. The hydrodynamic derivatives that are influenced by this drag are X_u , M_u , X_z , M_z , and Z_u . They can be evaluated as follows:

$$X_u = -\rho u (C_D S + C_{Db} S_b)$$

$$M_u = -\rho u C_D S h_s$$

$$Z_u = \rho u C_{Lb} S_b$$

$$X_z = 1/2 \rho C_D l_s V^2$$

$$M_z = 1/2 \rho C_D l_s h_s V^2 - \rho g x_s A_v$$

where C_{Db} = drag coefficient of the body of revolution

C_{Lb} = lift coefficient of the body

S_b = equivalent hydrofoil area of the body

h_s = height of point of application of drag on strut above the center of buoyancy

l_s = length of strut.

An evaluation of these coefficients shows that they are small in comparison with the other derivatives. In M_z the part ^{due} to the drag of the strut will be much smaller than $\rho g x_g A_v$. In first-order approximation X_u , M_u , Z_u , and X_z may be neglected.

APPENDIX VI. THE CONTROL SURFACES

The principal requirements for stabilizing fins are symmetry of the cross section, high maximum lift, low drag, resistance to cavitation and great strength. It follows from the latter consideration that the plan-form must be fairly stubby, with an aspect ratio usually not larger than two. This low-aspect ratio has also the advantage of making the angle of stall larger although no increased maximum lift is thereby achieved.

Lifting line theory, applied to a symmetrical hydrofoil of aspect ratio R , gives for the lift coefficient:

$$C_L = 2\pi(\alpha - \frac{C_L}{\pi R}).$$

$$\text{Thus, } \frac{\partial C_L}{\partial \alpha} = \frac{1}{\frac{1}{2\pi} + \frac{1}{\pi R}}. \text{ This theory assumes, however, that the}$$

lift distribution is elliptical and that the plan form is also elliptical. The theory of lifting surfaces brings in a correction for rectangular wings and for large angles of attack. $C_L = \frac{1}{\frac{E}{2\pi} + \frac{1}{\pi R}} \sin \alpha + 2 \sin^2 \alpha$ where

$$E = 1 + \frac{2}{R^2}. \text{ If the aspect ratio } R \text{ is taken to be equal to 2, we get}$$

$$E = 1.40. \quad C_L = 2.6 \sin \alpha + 2 \sin^2 \alpha \cos \alpha. \quad \frac{\partial C_L}{\partial \alpha} = 2.6 \cos \alpha + 4(\sin \alpha$$

$\cos \alpha) - 6 \sin^3 \alpha \approx 2.6 + 4 \alpha - 6 \alpha^3$ if $\sin \alpha$ is approximated by α and $\cos \alpha$ by 1. A free surface correction is not necessary since the foils are sufficiently deeply submerged.

$$\begin{aligned} \frac{\partial^2 C_L}{\partial \alpha^2} &= -2.6 \sin \alpha + 4 \cos \alpha - 22 \sin^2 \alpha \cos \alpha \\ &\approx -2.6 \alpha + 4 - 22 \alpha^2. \end{aligned}$$

If we expand C_L into a Taylor's series up to the second order, we get:

$$C_L = \frac{\partial C_L}{\partial \alpha} \alpha + 1/2 \frac{\partial^2 C_L}{\partial \alpha^2} \alpha^2 = 2.6 \alpha + 6 \alpha^2 - 1.3 \alpha^3 - 17 \alpha^4.$$

This lift coefficient takes on the following values for different α 's:

α (radians)	α°	C_L	C_L/α
0.1	$5^\circ 40'$	0.32	3.2
0.2	$11^\circ 30'$	0.72	3.6
0.3	$17^\circ 20'$	1.16	3.85

If we assume the foils to operate sinusoidally, the lift on a foil can be expressed as $L_0 \sin \omega t$ since the lift is proportional to the harmonically changing angle of attack.

To find an average $\frac{\partial C_L}{\partial \alpha}$, which will cause the foil to operate in such a way that the work done during a quarter of a cycle will be the same as with a changing $\frac{\partial C_L}{\partial \alpha}$, one can reason as follows. Assume the ship to move harmonically, the pitch angle can be written: $\theta = \theta_0 \cos \omega t$, $\dot{\theta} = -\theta_0 \omega \sin \omega t$. If the anti-pitching control is of the form $\alpha = k_1 \dot{\theta}$, we get :

$$\alpha = k_1 \theta_0 \omega \sin \omega t = \alpha_0 \sin \omega t.$$

The work done by the foil during a quarter cycle is:

$$\begin{aligned} W &= \int_0^{T/4} L \dot{\theta} dt \\ &= \int_0^{T/4} -\frac{1}{2} \rho A V^2 C_L \alpha_0 \sin \omega t dt \\ &= \text{Const} \int_0^{T/4} C_L \sin \omega t dt \end{aligned}$$

With $C_L = \left(\frac{\partial C_L}{\partial \alpha} \right)_m \alpha$ where $\left(\frac{\partial C_L}{\partial \alpha} \right)_m$ is taken to be constant, we get:

$$\begin{aligned} W' &= \text{Const } \alpha_0 \left(\frac{\partial C_L}{\partial \alpha} \right)_m \int_0^{T/4} \sin^2 \omega t \, dt \\ &= \text{Const } \alpha_0 \left(\frac{\partial C_L}{\partial \alpha} \right)_m \frac{1}{\omega} \left[\frac{\pi}{4} \right] \end{aligned}$$

If we put $W = W'$, we can find the unknown $\left(\frac{\partial C_L}{\partial \alpha} \right)_m$

$$\left(\frac{\partial C_L}{\partial \alpha} \right)_m = - \frac{\int_0^{T/4} (2.6\alpha + 6\alpha^2 - 1.3\alpha^3 - 17\alpha^4) \sin \omega t \, dt}{\alpha_0 \frac{\pi}{4\omega}}$$

When we substitute $\alpha = \alpha_0 \sin \omega t$ in this expression, $\left(\frac{\partial C_L}{\partial \alpha} \right)_m$ becomes equal

to 3.73. For a real fluid, however, the lift coefficient will be lower.

The lift needs a certain time to build up completely so that, since the foil oscillates constantly, there will be a loss in lift. On the other hand, separating of the boundary layer also needs time so that the angle of stall is higher with oscillating fins.

Considering these and other unsteady effects, it is reasonable to assume the equivalent $\frac{\partial C_L}{\partial \alpha} = 3.4$. This value was chosen to compute the idealized control parameters δ_0 , δ_1 , τ , μ (see Chapter IV) and in the first trials of the more realistic control system. The area of the fins was taken as 2.5 per cent of the longitudinal cross-sectional area. Five per cent is a maximum value for rigid anti-pitching fins in surface ships (Ref. 18). It was found

from the computational results that a semisubmarine with foils with the previously mentioned characteristics and dimensions can be stabilized in 4-ft waves ($\lambda = L$) but not in waves of double that length. (see pg. 37).

As a first means of increasing the lift, the area of the fins was increased by 30 per cent. This is allowable in this case ^{of astern seas} since the pitching moments are much smaller and act with lower frequency than in ahead waves where the frequencies of encounter are higher and the moments are higher. ^{In ahead seas} it will be necessary either to retract the foils or activate them such that the lift remains zero. It must be noted that no stabilization is necessary on the semisubmarine in ahead waves. (Ref. 1 and 3).

A second means to improve the performance of the foil consists of the use of flaps which increase the lift coefficient greatly. Data from Ref. (17) show that it is possible to obtain $C_L = 1.5$ at 18° tilt where both the main part of the fin and the flap are activated.

Cavitation can become an important factor at ship speeds above 25 knots. The lift coefficient needs, therefore, to be decreased sufficiently (Ref. 19). A maximum C_L of 1.25, $\frac{\partial C_L}{\partial \alpha} = 4$ seems a reasonable assumption. This value lies a little above the C_L found in tests conducted at Davidson Laboratory on a model with oscillating anti-pitching fins (Ref. 20).

APPENDIX VII. IDEAL CONTROL

(see pg. 30 - 33)

With the values of m_1 and m_2 given in App. VII, the expression (1)

of Chapter IV, pg. 31, becomes for $\lambda = 4$ ft:

$$\delta_o^2 = \frac{(-0.042 + 0.0008 V)^2 + (-0.9 + 0.07 V - 0.52 A V)^2}{46.1 A^2 V^4}$$

The longitudinal cross-sectional area is 1.2 sq ft, so that if we take A equal to 5 per cent of this area (Ref. 18), A becomes 0.06 sq ft. Therefore,

$$\delta_o^2 = \frac{(-0.042 + 0.0008 V)^2 + (-0.9 + 0.039 V)^2}{0.166 V^4}$$

For $V = 4$ ft/sec, $\delta_o^2 = .01335$,

$$\delta_o = 0.115 = 5^{\circ}40'.$$

$V = 8$ ft/sec, $\delta_o^2 = .258$,

$$\delta_o = 0.508 = 29^{\circ}$$

In 8-ft waves ($\lambda = 21$), we get:

$$\delta_o^2 = \frac{(0.094 - 0.02 V - 1.22 A V)^2 + (1.538 + .15 V)^2}{46.1 A^2 V^4}$$

If we take the same values for A, the following results are obtained:

$V = 4$ ft/sec, $\delta_o^2 = 0.109$

$$\delta_o = .33 = 19^{\circ}$$

$$V = 2 \text{ ft/sec, } \delta_0^2 = 1.275$$

$$\delta_0 = 1.13 \text{ radians} = 65^\circ$$

It is seen that complete pitch stabilization is theoretically possible in 4-ft waves although the fin angles have to become rather large with low forward speeds. In waves of double the ship length, complete control becomes harder at the high speeds and is impossible at low speeds ($Fr = .18$).

For complete heaving control, expression (3), pg. 33, for the motion in 4-ft waves gives:

$$\delta_1^2 = \frac{(0.15 + 0.04 V)^2 + (0.48 + 0.016 V)^2}{0.0388 V^4}$$

where the foils are the same as in the previous case.

$$\text{For } V = 4 \text{ ft/sec, } \delta_1^2 = 0.0418$$

$$\delta_1 = 0.204 = 12^\circ.$$

$$\text{At the speed of 2 ft/sec, } \delta_1^2 = 0.504 = 29^\circ.$$

In 8-ft waves the expression becomes:

$$\delta_1^2 = \frac{(-1.147 + 0.23 V)^2 + (0.87 - 0.012 V)^2}{0.0388 V^4}$$

$$\text{For } V = 4 \text{ ft/sec, } \delta_1^2 = 0.0726$$

$$\delta_1 = 0.27 = 15.5^\circ.$$

For $V = 2$ ft/sec, $\delta_1^2 = 1.92$

$$\delta_1 = 1.385 = 79.5^\circ.$$

Again, it is impossible to control the motions completely at low speeds in long waves.

It should be noted that the foils used in these calculations are very large but that, on the other hand, the $\frac{\partial C_L}{\partial \alpha}$ could be increased by the use of flaps (App. VI).

APPENDIX VIII. CONTROL SYSTEMS*

In Chapter IV (pg. 34) the following expressions are obtained:

$$a_f = 1/4 \rho \pi (H + F)$$

$$d_f = - 1/4 \rho \pi (E x_H + F x_P) = D_f$$

$$A_f = 1/4 \rho \pi (E x_H^2 + F x_P^2)$$

In a first-trial a foil area of 2.5 per cent of the longitudinal cross-sectional area was chosen, $s_H c_H = s_P c_P = 0.03$ sq ft, where s_H is span and c_H is chord of the anti-heaving foil and analogously for the anti-pitching foil. With an aspect ratio of 2, we get:

$$a_H = a_P = 0.245 \text{ ft}$$

$$c_H = c_P = 0.1224 \text{ ft}$$

so that $a_f = 0.012$

$$A_f = 0.024$$

$$d_o = D_o = 0.018$$

If we use similar reasoning as for hull on pg. 15, i.e., that with decreasing frequency of encounter, the virtual mass and moment of inertia increase, we can write:

$$a_f = 0.012 + 0.003 V$$

$$A_f = 0.024 + 0.006 V$$

With this assumption the virtual mass and moment are about doubled at the speed of zero frequency of encounter compared to zero speed.

$$b_f = 2 \times 1/2 \rho A V \left(\frac{\partial C_L}{\partial \alpha} + C_D \right)$$

$$= 0.204 V \text{ if } \frac{\partial C_L}{\partial \alpha} \text{ taken to be } 3.4 \text{ and } C_D = 0.02$$

* A summary of the different control systems mentioned in this Appendix can be found on pg. 75a.

$$E_f = 1/2 \rho A L^2 v \left(\frac{\partial C_L}{\partial \alpha} + C_D \right) = .408 v$$

$$e_f = E_f = 1/4 \rho A v L \left(\frac{\partial C_L}{\partial \alpha} + C_D \right) = .204 v$$

$$Z_s = 1/2 \rho A v^2 \left(\frac{\partial C_L}{\partial \alpha} + C_D \right) = 0.103 v^2$$

$$H_s = 0.206 v^2$$

A time lag of 0.1 sec^{pg. 36} was taken into account for the four control elements. The method of solving the linearized differential equations was a trial and error method in which four values of the different control parameters were taken. The coefficients of the six parameters $\ddot{z}$, $\dot{z}$, z , $\ddot{\theta}$, $\dot{\theta}$, θ , were computed and Hurwitz's Stability Criterion applied:

The two motion equations without exciting forces or moments can be written as:

$$[(m+a)\mathcal{D}^2 + b\mathcal{D} + c]z + (d\mathcal{D}^2 + e\mathcal{D} + g)\theta = 0^*$$

$$[\mathcal{D}\mathcal{D}^2 + E\mathcal{D} + G]z + [(J+A)\mathcal{D}^2 + B\mathcal{D} + C]\theta = 0^*$$

The characteristic determinant of this set is of the form:

$$a_4 \sigma_1^4 + a_3 \sigma_1^3 + a_2 \sigma_1^2 + a_1 \sigma_1 + a_0 = 0$$

The necessary and sufficient conditions that the motions will be stable is that all σ_1 's be negative. This will be so if

$$a_1 > 0 \quad (i = 0, \dots, 4)$$

$$\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} > 0 \quad \text{and} \quad \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{vmatrix} > 0$$

* $\mathcal{D}$ stands for $\frac{d}{dt}$ and $\mathcal{D}^2$ for $\frac{d^2}{dt^2}$.

The values of the control parameters that caused the coefficients $a, b, c, \dots, A, B, C, \dots$ to be such as to fulfill these conditions, were used to compute the pitching and heaving motions. The optimum values ^{already been} have ^{pointed out} and showed the following results in 4-ft waves:

(Linear Solution) Table I

$V(\text{ft/sec})$	k_1	k_2	k_3	k_4	$z(\text{ft})$	$\theta(\text{rad.})$
2.5	-0.7	-0.8	0.95	-0.58	0.1	0.035
3.1	-0.7	-1.92	0	-0.37	0.08	0.09
3.7	-1.4	0.3	0.24	-0.2	0.076	0.11
4.2	-2.6	-1.7	0.7	-3.9	0.072	0.17
4.8	-2.9	-1.4	0.5	-0.9	0.013	0.13

Aden's numerical method of solution, which takes the nonlinear effects into account, gave the following results:

Table II

V	k_1	k_2	k_3	k_4	z	θ
2.5	-1.3	-0.4	0.1	-2.0	0.10	0.08
3.1	-1.4	1.8	1.0	-1.2	0.12	0.10
3.7	-1.8	-0.3	-0.1	-1.0	0.12	0.16
4.2	-3.5	-1.0	0.9	-6.0	0.15	0.15
4.8	-5.1	0	0	-2.0	0.13	0.12

With a time lag twice as large and a foil area 3.5 per cent of the cross-sectional area, we get with the same method:

Table III

V	k_1	k_2	k_3	k_4	z	θ
2.5	-1.2	0.4	0.2	-2.0	0.02	0.033
3.1	-1.3	-1.4	1.0	-1.2	0.004	0.042
3.7	-1.7	-0.3	0.1	-1.0	0.016	0.06
4.2			unstable			
4.8			unstable			

At low speeds the larger foils are more effective than the smaller foils, but at higher speeds the time lag has such a large influence that the virtual mass and moment of inertia become negative.

In 8-ft waves the linear method showed the following results:

Table IV

V	k_1	k_2	k_3	k_4	z	θ
1.9	-1.0	-0.4	0.5	-0.5	0.08	0.15
2.5	-1.1	-0.4	0.5	-0.4	0.05	0.16
3.1	-1.0	0.29	-0.13	-0.1	0.13	0.18
3.7			unstable			
4.2	-1.64	-0.4	0.4	-1.4	0.10	0.20
4.8	-2.2	-0.1	0.1	-0.7	0.11	0.18

Here a time lag of 0.1 sec was assumed and the area of the foils 2.5 per cent. The second ^(Adam's numerical) method was extended into computing the angles of attack on the foils in order to check if the angles do not exceed a maximum of 0.35 to 0.4 ^{radians} which is allowable for simple fins of small aspect ratio.

The following results were obtained:

Table V

V	k_1	k_2	k_3	k_4	z	θ	δ_P (rad)	δ_H (rad)
1.9	-1.0	-0.3	-0.3	-1.0	0.07	0.17	0.4	0.2
2.5	-1.0	-0.3	-0.3	-0.9	0.15	0.20	0.4	0.3
3.1	-1.2	-0.3	-0.3	-1.0	0.28	0.25	0.4	0.4
3.7	-1.2	-0.4	-0.4	-1.2	0.28	0.35	0.56	0.2
4.2	-1.6	-0.4	0.8	-1.5	0.08	0.25	0.5	0.35
4.8	-2.2	-0.1	0.1	-2.2	0.09	0.22	0.45	0.2

δ_p is the angle of attack on the anti-pitching fin, not taking into account the wave particle velocity. Analogously, for δ_H . The numbers in these columns are the maximum angles of attack the foils take, according to a linear C_L/α . In reality, there will be overloading when δ_p or $\delta_H > 0.4$. Analogously, with the motions in $\lambda = 4$ -ft waves, the heave and pitch become unstable at speeds higher than 3 ft/sec when the time lag is doubled.

The vessel with two anti-pitching fins of 1.75 per cent area each and one pair of anti-heaving fins in the middle of total area 3.5 per cent has the following control characters

$$a_F = 0.019 + 0.004 V$$

$$A_F = 0.038 + 0.008 V$$

$$d_F = 0$$

$$D$$

$$F = 0$$

$$b_F = 0.285 V$$

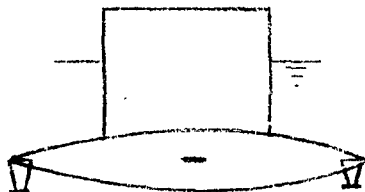
$$B_F = .57 V$$

$$e_F = E_F = 0$$

$C_F = 0$ With this term, which was positive in the previous case, we lose a certain amount of our pitching spring term.

$$Z_F = 0.142 V^2$$

$$M_F = .28 V^2$$



It was obvious from the previous results that there is no gain in having the heave also control the angle of the anti-pitching foil, nor the pitch control the angle of the anti-heaving foil. Therefore, the control was simplified to:

$$\delta_P = k_1 \dot{\theta} + k_2 t_1 \ddot{\theta}$$

$$\delta_H = k_3 \dot{z} + k_4 t_1 \ddot{z}$$

where t_1 was taken to be 0.1 sec. The following results were obtained:

Table VI

V	k_1	k_2	z	θ	δ_P	δ_H
1.9	-1.1	-1.1	0.02	0.10	0.37	0.18
2.5	-1.1	-1.1	0.12	0.07	0.47	0.43
3.1	-1.2	-1.2	0.2	0.15	0.52	0.63
3.7	-1.3	-1.3	0.18	0.17	0.51	0.46
4.2	-1.4	-1.4	0.2	0.21	0.46	0.40
4.8	-1.5	-1.5	0.3	0.28	0.43	0.49

The values for θ were taken, as usual, as the maximum value of pitch-angle out-to-out divided by two. The average pitch angle around which the vessel was oscillating had, however, the tendency to become larger and larger.

To eliminate the loss in ^{dynamic pitching} stability, flap-fins were introduced, while the total fin area was increased so that the gain in control is doubly increased. The $\frac{\partial C_L}{\partial \alpha}$ was assumed to be 4.8 (see App. VI), the area of the pair of anti-heaving foils 3.5 per cent, the area of each anti-pitching foil 2.5 per cent. Furthermore, a step-control was assumed (see Chapter IV). The result is given on the following page.

Table VII

V	k_1	k_4	z	θ	δ'_P	δ'_H
1.9	-0.3	-0.3	0.06	0.02	0.08	0.03
2.5	-0.3	-0.3	0.05	0.006	0.07	0.07
3.1	-0.3	-0.4	0.001	0.005	0.05	0.03
3.7	-0.3	-0.4	0.001	0.006	0.05	0.05
4.2	-0.3	-0.4	0.002	0.008	0.04	0.05
4.8	-0.35	-0.4	0.002	0.008	0.03	0.04

This control appears to be too "stiff." The accelerations amount to twice and even three times the limit of passenger comfort. The δ'_P and δ'_H indicate the actual angle of attack between the foil axis and the water particle whereas δ_P and δ_H of the previous results did not take the direction of the wave particles into account.

Again, a trial was made with one anti-pitching foil (at the stern) and a pair of anti-heaving foils midship. The area of the latter was not changed (3.5 per cent), while the area of the former was increased to 3.5 per cent.

Table VIII

V	k_1	k_4	z	θ	δ'_P	δ'_H
1.9	-0.3	-0.4	0.09	0.08	0.38	0.66
2.5	-0.3	-0.4	0.14	0.07	0.35	0.60
3.1	-0.3	-0.4	0.19	0.085	0.28	0.60
3.7	-0.3	-0.4	0.25	0.16	0.33	0.62
4.2	-0.2	-0.4	0.10	0.11	0.28	0.48
4.8	-0.2	-0.4	0.004	0.063	0.25	0.48

The vessel with two small anti-pitching fins (1.8 per cent area each, $\frac{\partial C_L}{\partial \alpha} = 4$), directly attached to the body of revolution, has the

following characteristics:

$$a_F = 0.019 + 0.004 \underline{v}$$

$$A_F = 0.038 + 0.008 \underline{v}$$

$$d_F = D_F = 0$$

$$e_F = E_F = 0$$

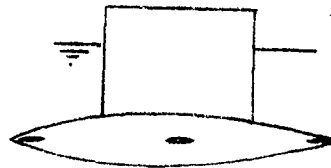
$$C_F = 0$$

$$b_F = .29 \underline{v}$$

$$B_F = .68 \underline{v}$$

$$Z_0 = .120 \underline{v}^2$$

$$M_0 = .346 \underline{v}^2$$



A stepcontrol by $\dot{\theta}$ and $\dot{z}$ was applied, and the following amplitudes and parameters were obtained:

Table IX

$\underline{v}$	k_1	k_4	z	θ	δ_P'	δ_H'
1.9	-0.3	-0.4	0.065	0.083	0.27	0.6
2.5	-0.3	-0.4	0.16	0.1	0.29	0.53
3.1	-0.3	-0.4	0.25	0.14	0.3	0.42
3.7	-0.3	-0.4	0.13	0.10	0.29	0.52
4.2	-0.3	-0.4	0.06	0.09	0.32	0.47

For the high speeds the pitch and heave still have the tendency to oscillate around an increasing positive or negative value. Therefore, in the next try, a supplementary position control was introduced.

$$\delta_P = k_1 \frac{\dot{\theta}}{\sqrt{\theta}} + k_2 \theta, \quad \delta_H = k_3 z + k_4 \frac{\dot{z}}{\sqrt{z}}.$$

Table X

V	k_1	k_2	k_3	k_4	α	θ	δ_P	δ_H
1.9					0.06	0.096		
2.5					0.27	0.094		
3.1	-0.25	-1.2	-1.2	-0.25	0.30	0.046		< 0.3
3.7					0.17	0.090		
4.2					0.09	0.060		
4.8					0.02	0.020		

With $k_3 = 0$, the maximum heave amplitude (0.30) is somewhat diminished.

Table XI

V	k_1	k_2	k_4	α	θ	δ_P	δ_H
1.9				0.06	0.096	0.35	0.41
2.5				0.22	2.110	0.36	0.40
3.1	-0.25	-1.2	-0.35	0.28	0.06	0.26	0.40
3.7				0.20	0.062	0.30	0.48
4.2				0.11	0.047	0.27	0.46
4.2				0.003	0.025	0.27	0.42

The high δ_H angles occur only at the moment when the foil is flipped over from one maximum to the other. Since, in practice, the foil needs a certain time to flip over, the actual maximum angle of attack will be less. This system of control applied on the vessel moving in 4-ft waves showed pitch amplitudes that were all below 0.05 and heave amplitudes below 0.10. This was to be expected since exciting force and moment are much smaller than in the 8-ft wave case. Contrary to the motion of surface ships in ahead waves, where waves of length $\sim L_{ship}$ cause the severest motions, the semisubmarine experiences them in waves of about twice L_{ship} . The reason for this is, besides the one mentioned before, that the frequency of encounter is such that for these waves the speed of resonance ($Fr \sim .3$) lies

in the range of normal ship speeds. Longer waves cause higher speeds of resonance while with shorter waves the resonance speed is lower.

Fig. 5 shows the results of the most significant control systems as compared with the noncontrolled motions.

Summary of Results of Appendix VIII

Table No.	Ap % of long. area	th	t1 sec	th sec	A ft	Solution	Control Equations	$\frac{3CL}{2\alpha}$	CD	Results	Pitch Foll
I	2.5; stern fin only	2.5	0.1	0.1	4	linear	$\delta_P = k_1 \dot{\theta} - k_1 t_1 \ddot{\theta} + k_2 \ddot{\theta} - k_2 t_2 \ddot{\theta}$ $\delta_H = k_3 \dot{\theta} - k_3 t_1 \ddot{\theta} + k_4 \ddot{\theta} - k_4 t_2 \ddot{\theta}$	3.4	.02	See Table I of App. VIII & Fig. 5	L/10 below center line
II	"	"	0.2	0.1	"	nonlinear	"	"	"	See Table II	"
III	3.5; stern fin only	3.5	0.2	0.2	"	"	"	"	"	See Table III	"
IV	2.5; stern fin only	2.5	0.1	0.1	8	linear	"	"	"	See Table IV	"
V	"	"	0.1	0.1	8	nonlinear	"	"	"	See Table V	"
VI	3.5; bow & stern fins	3.5	0.1	0.1	8	"	$\delta_P = k_1 \dot{\theta} - k_1 t_1 \ddot{\theta}$ $\delta_H = k_2 \ddot{\theta} - k_2 t_2 \ddot{\theta}$	"	"	See Table VI	"
VII	5.0; bow & stern fins	3.5	0	0	8	"	$\delta_P = k_1 \dot{\theta} / \dot{\theta}; \delta_H = k_4 \ddot{\theta} / \ddot{\theta}$	4.8	"	See Table VII	On center line of body
VIII	2.5; stern fin only	3.5	0	0	8	"	"	4.8	"	See Table VIII & Fig. 5	"
IX	3.6; bow & stern fins	2.5	0	0	0	"	"	4.0	"	See Table IX & Fig. 5	"
X	"	"	"	"	0	"	$\delta_P = k_1 \frac{\dot{\theta}}{\dot{\theta}} + k_2 \ddot{\theta}$ $\delta_H = k_3 \ddot{\theta} + k_4 \frac{\ddot{\theta}}{\ddot{\theta}}$	"	"	See Table X	"
XI	"	3.5	"	"	8	"	$\delta_P = k_1 \frac{\dot{\theta}}{\dot{\theta}} + k_2 \ddot{\theta}$ $\delta_H = k_4 \frac{\ddot{\theta}}{\ddot{\theta}}$	"	"	See Table XI & Fig. 5	"

APPENDIX IX. COMPUTER PROGRAMS

It would be impossible to give a listing of all the programs that were written to compute the amplitudes of the motions without and with controls.

A set of four examples has been chosen to give an idea of how the heave and pitch amplitudes were found.

The first example computes the heave and pitch for 9 different speeds in astern waves of 4 and 8 ft length, according to the familiar method of harmonic solutions of linearized differential equations.

Symbols

$$A(1) = a + a$$

$$B(1) = b$$

$$A(2) = A + A$$

$$B(2) = B$$

$$C(1) = c$$

$$D(1) = d$$

$$C(2) = C$$

$$D(2) = D$$

$$E(1) = e$$

$$G(1) = G$$

$$E(2) = E$$

$$G(2) = G$$

AF = coefficient of $\cos \omega t$ in expression of total exciting force F

BF = coefficient of $\sin \omega_0 t$ in expression of total exciting moment M

CM = coefficient of $\cos \omega t$ in expression of M

DM = coefficient of $\sin \omega_0 t$ in M

THETA (I) = angle of pitch assumed at ship speed $V(I)$ (see App. IV)

Z = heave amplitude

THET = pitch amplitude

FLZ = phase lag between wave and heave maximum

FLTH = phase lag between wave and pitch maximum.

In the second example the motions are calculated according to Adam's Numerical Method. The subroutine HESPOH sets the initial conditions (the position of the vessel at $T = 0$). It computes F and M in function of the variable T and solves the two motion equations for θ and $\dot{\theta}$. Then, different library functions are called (DPHVF and IHDVF) which compute the actual z and θ at the time T . Thereafter, T is increased by DT ; and the sequence starts all over again.

The third program is essentially the same as the first, but the control parameters are incorporated in the coefficients. The two equations are then subjected to Routh-Hurwitz's Stability Criterion. The equations are then solved for the values of the control parameters that fulfill the stability criterion in the same manner as the first program. The control parameters are, respectively, $PKK1$, $PKK2$, $PKK3$, $PKK4$. To each of these values, DK 's are added in such a way that the equations are solved for ten thousand different combinations of the control parameters. In other words,

First Combination	$PKK1$	$PKK2$	$PKK3$	$PKK4$
Second Combination	$PKK1 + DK1$	$PKK2$	$PKK3$	$PKK4$
Third Combination	$PKK1 + 2DK1$	$PKK2$	$PKK3$	$PKK4$
.....				
Tenth Combination	$PKK1 + 9DK1$	$PKK2$	$PKK3$	$PKK4$
Eleventh Combination	$PKK1$	$PKK2 + DK2$	$PKK3$	$PKK4$
etc.				

The values of z and θ that are below the specified values $R(I)$ and $V(I)$, respectively, are pointed out, together with their place level.

The fourth example computes the heave and pitch of the semisubmarine with the stepcontrol system:

$$\delta_P = k_1 \frac{\dot{\theta}}{|\dot{\theta}|} + k_2 \theta$$

$$\delta_H = k_4 \frac{\dot{z}}{|\dot{z}|}$$

The method used was Adam's, like in the second example (see Chapter IV).

It gives a time history of pitch, heave, and the respective angles of attack on anti-heaving and anti-pitching foils (DELH and DELP).

*HEAVE AND PITCH WITHOUT CONTROLS LAMDA=4AND8FT,LINERIZED EQUATIONS

```

*      XEQ
*      LIST8
C      HEAVING AND PITCHING RESPONSES
      DIMENSION V(30),A1( 30),A2(30),B1(30),B2(30),A(2),B(2),C(3),D(2),
      ZE(2),G(2),THETA(30)
      PRINT 5
5      FORMAT (8H LAMDA=4)
      READ 1,(V(I),B1(I),B2(I),A1(I),A2(I),I=1,18)
      READ 1,(THETA(I),I=1,18)
      PRINT 1,(THETA(I),I=1,18)
      PRINT 1,(V(I),I=1,18)
1      FORMAT (5F14.8)
      I=1
      C(1)=7.74*1.99
      D(1)=0.0251*1.99
      D(2)=D(1)
      G(2)=2.43
      PRINT 2
2      FORMAT (75H          V(I)=          Z          THETA          F
2LZ          FLTH          )
53      C(3)=0.0917*V(I)**2-0.692*V(I)-0.493
      C(2)=(2.78*THETA(I)+C(3)*SQRTF(THETA(I)))/THETA(I)
      G(1) =0.076*1.99*V(I)**2
20      A(1)=A1(I)
      A(2)=A2(I)
103     IF(I-9)70,70,71
70      AF=0.072-0.00816*V(I)
      BF=0.13+0.04*V(I)
      CM=-0.9+0.07*V(I)
      DM=-0.042+0.0008*V(I)
      OMEGA=1.57*(V(I)-4.54)
      B(1)=B1(I)
      B(2)=B2(I)
      E(1)=-0.3*V(I)+0.01*V(I)**2
      E(2)=-0.1*V(I)
      GO TO 26
71      PRINT 102
102     FORMAT (8H LAMDA=8)
      AF=0.133-0.0104*V(I)
      BF=-1.1+0.2*V(I)
      CM=-1.538-0.15*V(I)
      DM=0.094-0.0206*V(I)
      OMEGA=1.57*(V(I)-6.4)/2.
      B(1)=(0.0155*V(I)**2-0.083*V(I)+0.152)*1.99
      B(2)=(-0.000147*V(I)**3+0.01603*V(I)**2-0.0372*V(I)+0.0247)*1.99
      E(1)=0.3*V(I)-0.01*V(I)**2
      E(2)=-0.1*V(I)
*      SOLUTION OF SET OF DIFFERENTIAL EQUATIONS
26      P1=-A(1)*OMEGA**2+C(1)
      XF=ATANF(DM/CM)
      XX=CM**2+DM**2
      SM1=SQRTF(XX)*COSF(XF)
      SM2=-SQRTF(XX)*SINF(XF)
      YF=ATANF(BF/AF)
      YY=AF**2+BF**2
      F1=SQRTF(YY)*COSF(YF)
      F2=-SQRTF(YY)*SINF(YF)
      P2=B(1)*OMEGA

```

```

Q1=-D(1)*OMEGA**2+G(1)
Q2=E(1)*OMEGA
R1=-D(2)*OMEGA**2+G(2)
R2=E(2)*OMEGA
S1=-A(2)*OMEGA**2+C(2)
S2=B(2)*OMEGA
SMQ1=SM1*Q1-SM2*Q2
SMQ2=SM1*Q2+SM2*Q1
FS1=F1*S1-F2*S2
FS2=F1*S2+F2*S1
QR1=Q1*R1-Q2*R2
QR2=Q1*R2+Q2*R1
PS1=P1*S1-P2*S2
PS2=P1*S2+P2*S1
FR1=F1*R1-F2*R2
FR2=F1*R2+F2*R1
SMP1=SM1*P1-SM2*P2
SMP2=SM1*P2+SM2*P1
DEN=(QR1-PS1)**2+(QR2-PS2)**2
ZZ1=((SMQ1-FS1)*(QR1-PS1)+(SMQ2-FS2)*(QR2-PS2))/DEN
ZZ2=((SMQ2-FS2)*(QR1-PS1)-(SMQ1-FS1)*(QR2-PS2))/DEN
DENO=((QR1-PS1)**2+(QR2-PS2)**2)
SMM1=((FR1-SMP1)*(QR1-PS1)+(FR2-SMP2)*(QR2-PS2))/DENO
SMM2=((FR2-SMP2)*(QR1-PS1)-(FR1-SMP1)*(QR2-PS2))/DENO
Z=SQRTF(ZZ1**2+ZZ2**2)
THET=SQRTF(SMM1**2+SMM2**2)
FLZ=ATANF(ZZ2/ZZ1)
FLTH=ATANF(SMM2/SMM1)
PRINT 7,(V(I),Z,THET,FLZ,FLTH)
7  FORMAT (5F15.8)
9  I=I+1
   IF(I-18)53,53,19
19  CONTINUE
   CALL EXIT
   END

```

```

*HEAVE AND PITCH WITHOUT CONTROLS      LAMDA=4FT, TIME HISTORY SOLUTION
*   XEQ
*   LIST8
C   HEAVING AND PITCHING RESPONSES
    DIMENSION V(30),A1(30),A2(30),B1(30),B2(30),A(2),B(2),C(3),D(2),
    2E(2),G(2)
    READ 1,(V(I),B1(I),B2(I),A1(I),A2(I),I=1,9)
1   FORMAT (5F14.8)
    J=1
    I=1
    C(1)=7.74*1.99
    C(2)=2.78
    G(2)=2.43
    D(2)=0.0251*1.99
    D(1)=0.0251*1.99
2   V=V(I)
20  A(1)=A1(I)
    A(2)=A2(I)
33  B(1)=B1(I)
    B(2)=B2(I)
    C(3)=-0.4*V(I)+0.05*V(I)**2
    E(1)=0.2*V(I)-0.01*V(I)**2
    E(2)=-0.1*V(I)
    G(1)=0.076*1.99*V(I)**2
    OMEGA=1.57*(V(I)-4.54)
70  AF=0.072-0.00816*V(I)
    BF=0.13+0.04*V(I)
    CM=-0.9+0.07*V(I)
    DM=-0.042+0.0008*V(I)
8   CALL RESPON (Z,THET,A,B,C,D,E,G,OMEGA,AF,BF,CM,DM,V)
13  IF(J-1)14,16,14
16  E(1)=-0.3*V(I)+0.01*V(I)**2
    E(2)=0.1*V(I)
    J=0
    GO TO 8
14  I=I+1
    J=1
    IF(I-9)2,2,3
3   CONTINUE
    CALL EXIT
    END

```

```
*      LIST8
      SUBROUTINE RESPON(Z,THET,A,B,C,D,E,G,OMEGA,AF,BF,CM,DM,V)
      DIMENSION A(2),B(2),C(3),D(2),E(2),G(2)
      PRINT 1,V
1      FORMAT(1H 6E20.8)
      T=0.
      Z=0.001
      DZ=0.
      THET=0.
      DTHET=0.
      TPRINT=0.0
7      F=AF*COSF(OMEGA*T) +BF*SINF(OMEGA*T)
      XM=CM*COSF(OMEGA*T) +DM*SINF(OMEGA*T)
      SENSE LIGHT 1
      FTHET=THET/SQRTF(ABSF(THET))
2      DDZ=(A(2)*F-D(1)*XM+(D(1)*E(2)-B(1)*A(2))*DZ +(D(1)*G(2) -C(1)*A
      2(2))*Z +(D(1)*B(2) -E(1)*A(2))*DTHET +(D(1)*C(2) -G(1)*A(2))
      3*THET+A(1)*C(3)*FTHET)/(A(1)*A(2)-D(1)*D(2))
      DDTHET=(-D(2)*F +A(1)*XM +(B(1)*D(2) -A(1)*E(2))*DZ+(C(1)*D(2)-A(
      21)*G(2)*Z+(E(1)*D(2)-A(1)*B(2))*DTHET+(G(1)*D(2)-A(1)*C(2))*T
      3HET-A(1)*C(3)*FTHET)/(A(1)*A(2)-D(1)*D(2))
      IF (T -TPRINT) 4,3,3
3      PRINT 1,T,Z,THET,DDZ,DDTHET
      TPRINT=TPRINT+0.1
4      IF(T-12.)5,6,6
5      T=INDVF(T,0.005)
      THET=DPNVF(THET,DTHET)
      Z=DPNVF(Z,DZ)
      DZ=DPNVF(DZ,DDZ)
      DTHET=DPNVF(DTHET,DDTHET)
      GO TO 7
6      RETURN
      END
```

```

*HEAVE AND PITCH WITH CONTROLS      LAMDA=4FT, LINEARIZED EQUATIONS
*   XEQ
*   LIST8
*   CONTROL OF PITCH AND HEAVE
      DIMENSION V(30),A1( 30),A2(30),B1(30),B2(30),A(2),B(2),C(3),D(2),
      2E(2),G(2),THETA(30),R(30),S(30),T(30),U(30),DK1(30),DK2(30),
      3DK3(30),DK4(30),PK1(30),PK2(30),PK3(30),PK4(30),PK11(30),PK22(30),
      4PK33(30),PK44(30)
      PRINT 5
      READ 1,(V(I),B1(I),B2(I),A1(I),A2(I),I=1,4)
      PRINT 1,(V(I),B1(I),B2(I),A1(I),A2(I),I=1,4)
      READ 1,(THETA(I),I=1,4)
      PRINT 1,(THETA(I),I=1,4)
5     FORMAT (8H LAMDA=4 )
      READ 11,(S(I),T(I),R(I),U(I),I=1,4)
      READ 111,(DK1(I),DK2(I),DK3(I),DK4(I),PK11(I),PK22(I),I=1,4)
      READ 111,(PK33(I),PK44(I),PK1(I),PK2(I),PK3(I),PK4(I),I=1,4)
1     FORMAT (5F14.8)
11    FORMAT (4F10.5)
111   FORMAT (6F10.5)
      PRINT 30
30    FORMAT (75H      K1          K2          K3          K4          Z          THE
2TA      X1          )
      I=3
      C(1)=7.74*1.99
      G(2)=2.35
56    PKK1=PK1(I)
      PKK2=PK2(I)
      PKK3=PK3(I)
      PKK4=PK4(I)
53    PRINT 2,V(I)
2     FORMAT (7H  V(I)=F15.8)
*     WITHOUT CONTROLS
      C(3)=0.0917*V(I)**2-0.692*V(I)-0.493
      C(2)=(2.78*THETA(I)+C(3)*SQRTF(THETA(I)))/THETA(I)
      G(1)=0.076*1.99*V(I)**2
      OMEGA=1.57*(V(I)-4.54)
70    AF=0.072-0.00816*V(I)
      BF=0.13+0.04*V(I)
      CM=-0.9+0.07*V(I)
      DM=-0.042+0.0008*V(I)
20    A(1)=A1(I)
      A(2)=A2(I)
      B(1)=B1(I)
      B(2)=B2(I)
      C(3)=0.0917*V(I)**2-0.692*V(I)-0.493
      D(1)=0.0251*1.99
      D(2)=D(1)
      E(1)=-0.3*V(I)+0.01*V(I)**2
      E(2)=-0.1*V(I)
*     WITH FOILS
      AF=AF+0.011*V(I)
      CM=CM+0.022*V(I)
      A(1)=A(1)+0.012+0.003*V(I)+0.103*V(I)**2*0.1*(PKK2+PKK4)
      A(2)=A(2)+0.024+0.006*V(I)+0.206*V(I)**2*0.1*PKK1
      B(1)=B(1)+0.204*V(I)-0.103*V(I)**2*(PKK2+PKK4)

```

```
B(2)=B(2)+0.408*V(I)-0.204*V(I)**2*PKK1
C(2)=C(2)+0.206*V(I)**2
D(1)=D(1)+0.018
D(2)=D(1)
E(1)=E(1)+0.204*V(I)-0.104*V(I)**2*(PKK1+PKK3)
E(2)=E(2)+0.204*V(I)-0.208*V(I)**2*PKK2
G(1)=G(1)+0.206*V(I)**2
* SOLUTION OF SET OF DIFFERENTIAL EQUATIONS
XF=ATANF(DM/CM)
XX=CM**2+DM**2
SM1=SQRTF(XX)*COSF(XF)
SM2=-SQRTF(XX)*SINF(XF)
YF=ATANF(BF/AF)
YY=AF**2+BF**2
F1=SQRTF(YY)*COSF(YF)
F2=-SQRTF(YY)*SINF(YF)
* STABILITY CRITERIUM
21 X1=B(1)*C(2)+C(1)*B(2)-E(2)*G(1)-G(2)*E(1)
   IF(X1)3,3,22
22 X2=A(1)*C(2)+C(1)*A(2)+B(1)*B(2)-D(2)*G(1)-G(2)*D(1)-E(1)*E(2)
   IF(X2)3,3,23
   X3=B(1)*A(2)+A(1)*B(2)-D(2)*E(1)-E(2)*D(1)
   IF(X3)3,3,24
24 X0=C(1)*C(2)-G(1)*G(2)
   X4=A(1)*A(2)-D(1)*D(2)
   X5=X1*X2-X3*X0
   IF(X5)3,3,25
25 X6=X1*(X2*X3-X4*X1)-X3**2*X0
   IF(X6)3,3,26
26 P1=-A(1)*OMEGA**2+C(1)
   P2=B(1)*OMEGA
   Q1=-D(1)*OMEGA**2+G(1)
   Q2=E(1)*OMEGA
   R1=-D(2)*OMEGA**2+G(2)
   R2=E(2)*OMEGA
   S1=-A(2)*OMEGA**2+C(2)
   S2=B(2)*OMEGA
   SMQ1=SM1*Q1-SM2*Q2
   SMQ2=SM1*Q2+SM2*Q1
   FS1=F1*S1-F2*S2
   FS2=F1*S2+F2*S1
   QR1=Q1*R1-Q2*R2
   QR2=Q1*R2+Q2*R1
   PS1=P1*S1-P2*S2
   PS2=P1*S2+P2*S1
   FR1=F1*R1-F2*R2
   FR2=F1*R2+F2*R1
   SMP1=SM1*P1-SM2*P2
   SMP2=SM1*P2+SM2*P1
   DEN=((QR1-PS1)**2+(QR2-PS2)**2)
   ZZ1=((SMQ1-FS1)*(QR1-PS1)+(SMQ2-FS2)*(QR2-PS2))/DEN
   ZZ2=((SMQ2-FS2)*(QR1-PS1)-(SMQ1-FS1)*(QR2-PS2))/DEN
   DENO=((QR1-PS1)**2+(QR2-PS2)**2)
   SMM1=((FR1-SMP1)*(QR1-PS1)+(FR2-SMP2)*(QR2-PS2))/DENO
   SMM2=((FR2-SMP2)*(QR1-PS1)-(FR1-SMP1)*(QR2-PS2))/DENO
   Z=SQRTF(ZZ1**2+ZZ2**2)
```

```
      THET=SQRTF(SMM1**2+SMM2**2)
      IF(Z-S(I))50,50,3
50      IF(THET-T(I))51,51,3
51      PRINT 7,(PKK1,PKK2,PKK3,PKK4,Z,THET,X1)
7      FORMAT (7F10.5)
      IF(Z-R(I))8,8,3
8      IF(THET-U(I))18,18,3
3      PKK1=PKK1-DK1(I)
      IF(PKK1-PK11(I))15,15,20
15     PKK2=PKK2-DK2(I)
      PKK1=PK1(I)
      IF(PKK2-PK22(I))16,16,20
16     PKK3=PKK3+DK3(I)
      PKK1=PK1(I)
      PKK2=PK2(I)
      IF(PKK3-PK33(I))20,17,17
17     PKK4=PKK4-DK4(I)
      PKK1=PK1(I)
      PKK2=PK2(I)
      PKK3=PK3(I)
      IF(PKK4-PK44(I))18,18,20
18     I=I+1
119    IF(I-4)56,56,19
19     CONTINUE
      CALL EXIT
      END
```

```

*HEAVE AND PITCH WITH CONTROLS      LAMDA=8FT, TIME HISTORY SOLUTION
*   XEQ
*   LIST8
C   HEAVING AND PITCHING RESPONSES
    DIMENSION V(30),A1(30),A2(30),B1(30),B2(30),A(2),B(2),C(3),D(2),
    2E(2),G(2),PK1(30),PK2(30),PK3(30),PK4(30)
    READ 1,(V(I),B1(I),B2(I),A1(I),A2(I),I=1,12)
    READ 11,(PK1(I),PK2(I),PK3(I),PK4(I),I=1,12)
11  FORMAT (4F10.5)
1   FORMAT (5F14.8)
    PRINT 20,(I,A1(I),I=7,12)
20  FORMAT (4H A1(I2,2H)=F10.5)
    C(1)=7.74*1.99
    I=7
*   WITHOUT CONTROLS
    C(2)=2.78
    D(1)=0.0251*1.99
    D(2)=D(1)
    G(2)=2.43
    PRINT 15
15  FORMAT (8H LAMDA=8)
2   V=V(I)
56  PKK1=PK1(I)
    PKK2=PK2(I)
    PKK3=PK3(I)
    PKK4=PK4(I)
    A(1)=A1(I)
    A(2)=A2(I)
    B(1)=(0.0155*V(I)**2-0.083*V(I)+0.152)*1.99
    B(2)=(-0.000147*V(I)**3+0.01603*V(I)**2-0.0372*V(I)+0.0247)*1.99
7   E(1)=-0.5*V(I)+0.02*V(I)**2
    E(2)=0.1*V(I)
    G(1)=0.076*1.99*V(I)**2
    OMEGA=1.57*(V(I)-6.4)/2.
    AF= 0.12-0.01*V(I)
    BF=-1.1+0.15*V(I)
    CM=-1.5-0.15*V(I)
    DM=0.094-0.0206*V(I)
*   WITH FOILS
    AF=AF-0.041*V(I)
    DM=DM+0.124*V(I)
    A(1)=A(1)+0.019+0.004*V(I)
    A(2)=A(2)+0.038+0.008*V(I)
    B(1)=B(1)+0.292*V(I)
    B(2)=B(2)+0.68*V(I)
    C(3)=-0.4*V(I)+0.05*V(I)**2
    G(1)=G(1)+0.28*V(I)**2
    C(2)=C(2)-0.336*V(I)**2*PKK2
13  CALL RESPO (Z,THET,A,B,C,D,E,G,OMEGA,AF,BF,CM,DM,V,PKK1,PKK2,PKK3
    2,PKK4)
6   I=I+1
    IF(I-12)2,2,3
3   CONTINUE
    CALL EXIT
    END

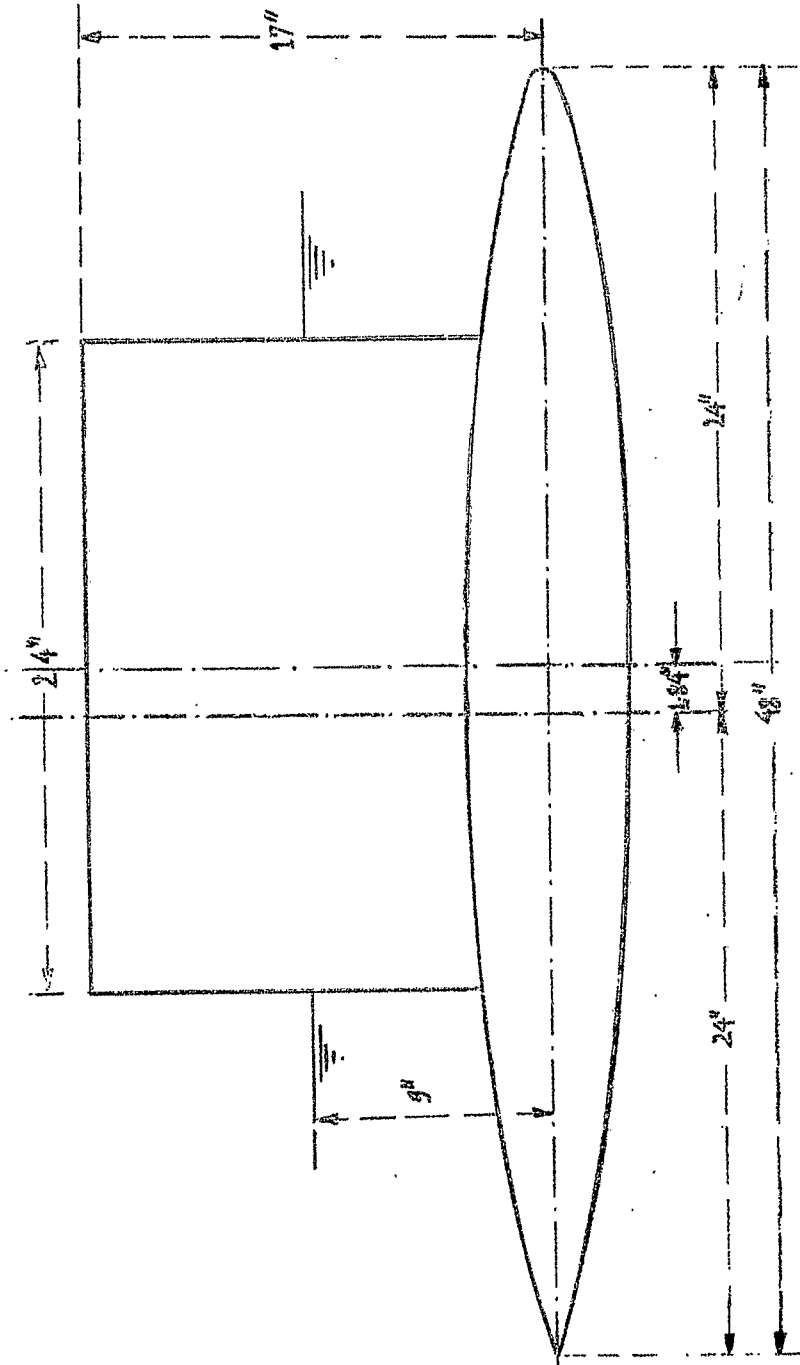
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```

*      LIST8
      SUBROUTINE RESPO (Z,THET,A,B,C,D,E,G,OMEGA,AF,BF,CM,DM,V,PKK1,PKK
22,PKK3,PKK4)
      DIMENSION A(2),B(2),C(3),D(2),E(2),G(2)
      PRINT 1,V
1      FORMAT (1H F20.8)
      PRINT 12
12     FORMAT (110H          T          Z          THET          DELP          DE
2LH          DZ          DTHET          DDZ          DDTHET          )
      T=0.
      Z=0.001
      DZ=0.
      THET=0.
      DTHET=0.
      TPRINT=0.0
7      F=AF*COSF(OMEGA*T) +BF*SINF(OMEGA*T)
      XM=CM*COSF(OMEGA*T) +DM*SINF(OMEGA*T)
      SENSE LIGHT 1
      F=F+0.12*V**2*PKK4*DZ/ABSF(DZ)
      XM=XM+0.336*V**2*PKK1*DTHET/ABSF(DTHET)
      FTHET=THET/SQRTF(ABSF(THET))
2      DDZ=(A(2)*F-D(1)*XM+(D(1)*E(2)-B(1)*A(2))*DZ +(D(1)*G(2) -C(1)*A
2(2))*Z +(D(1)*B(2) -E(1)*A(2))*DTHET +(D(1)*C(2) -G(1)*A(2))
3*THET+A(1)*C(3)*FTHET)/(A(1)*A(2)-D(1)*D(2))
      DDTHET=(-D(2)*F +A(1)*XM +(B(1)*D(2) -A(1)*E(2))*DZ+(C(1)*D(2)-A(
21)*G(2)*Z+(E(1)*D(2)-A(1)*B(2))*DTHET+(G(1)*D(2)-A(1)*C(2))*T
3HET-A(1)*C(3)*FTHET)/(A(1)*A(2)-D(1)*D(2))
      DELH=PKK4*DZ/ABSF(DZ)-(0.37/V)*COSF(OMEGA*T)-THET
      DELP=PKK1*DTHET/ABSF(DTHET)+(0.37/V)*SINF(OMEGA*T)+THET*(PKK2-1.)
      IF (T -TPRINT) 4,3,3
3      PRINT 11,T,Z,THET,DELP,DELH,DZ,DTHET,DDZ,DDTHET
11     FORMAT (1H 9F12.5)
      TPRINT=TPRINT+0.1
4      IF (T-10.)5,6,6
5      T=INDVF(T,0.005)
      THET=DPNVF(THET,DTHET)
      Z=DPNVF(Z,DZ)
      DZ=DPNVF(DZ,DDZ)
      DTHET=DPNVF(DTHET,DDTHET)
      GO TO 7
6      RETURN
      END

```

FIG. 1



DIMENSIONS OF 2 FT. SAIL MODEL

FIG 2

DAMPING COEFFICIENTS
FOR HEAVING & PITCHING
IN WAVES OF 4 & 8 ft
LENGTH

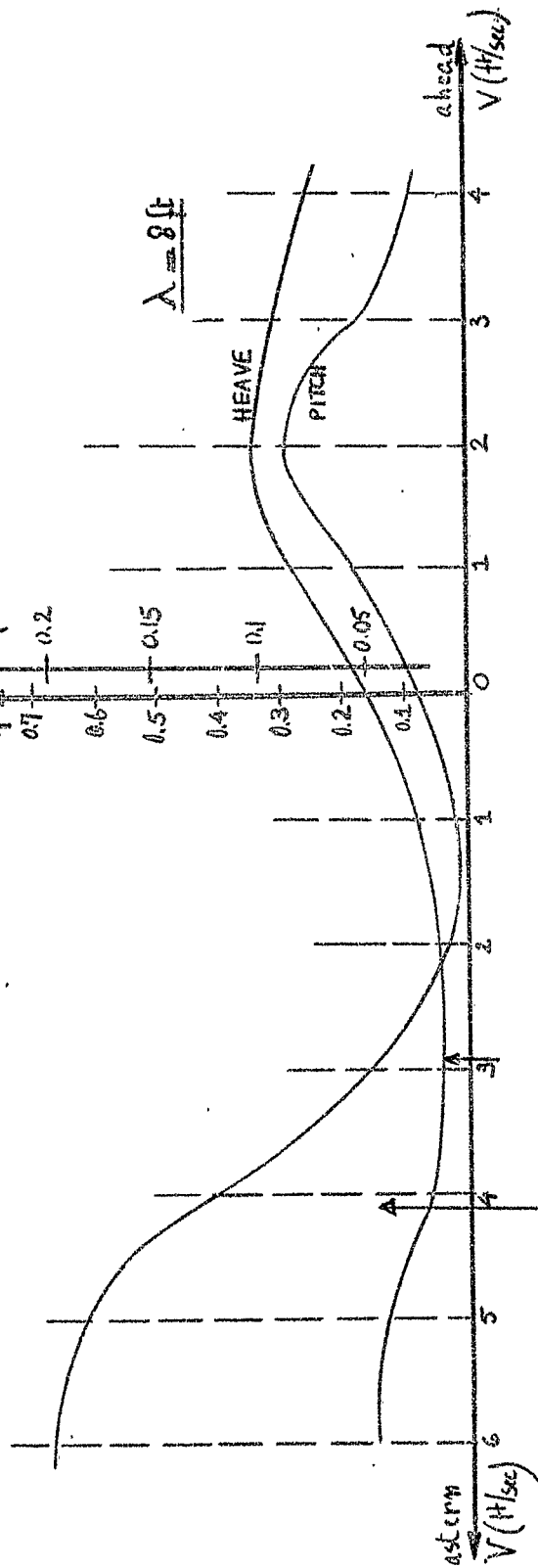
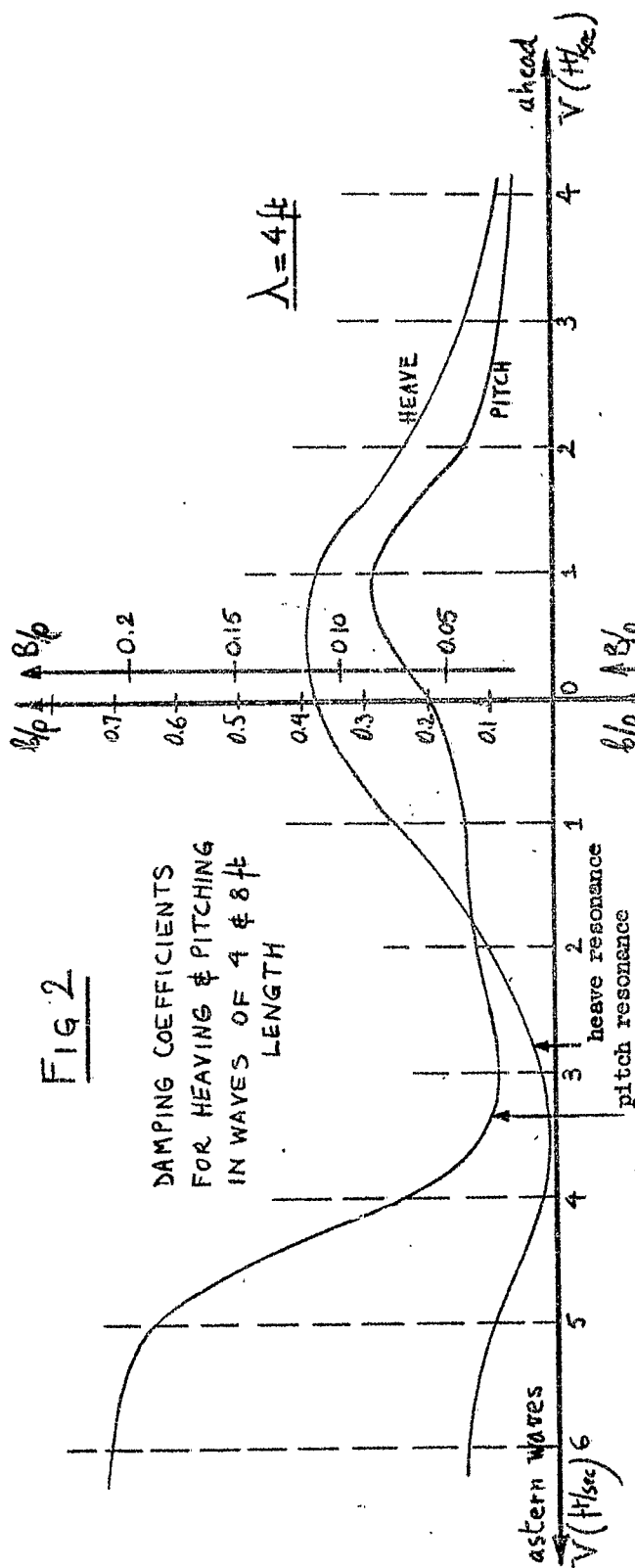


FIG 3 (A)

HEAVE AMPLITUDES IN
4 AND 8 FT. WAVES
(in astern seas)

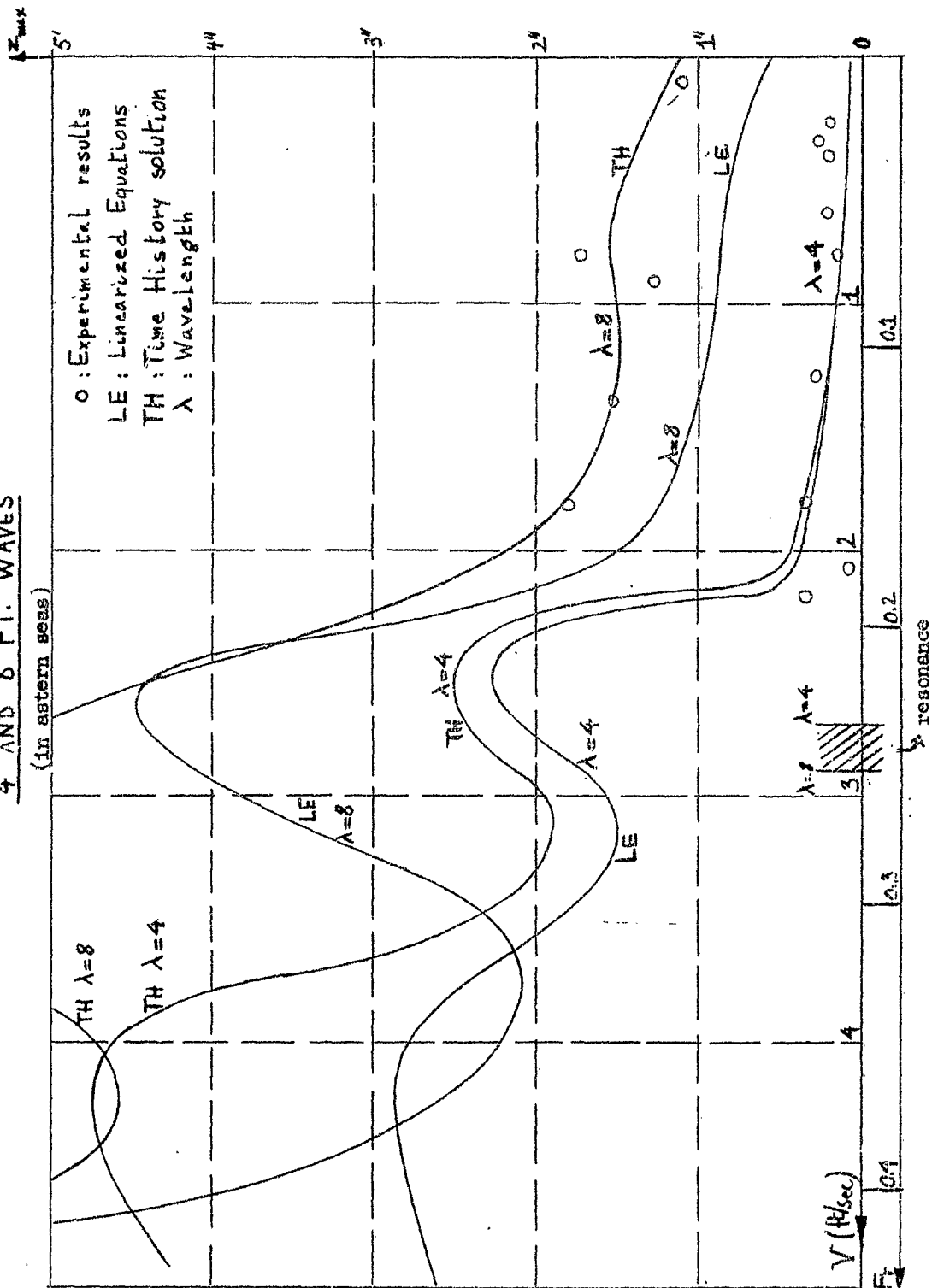


FIG 3 (B)
PITCH AMPLITUDES IN
4 & 8 FT WAVES (in astern seas)

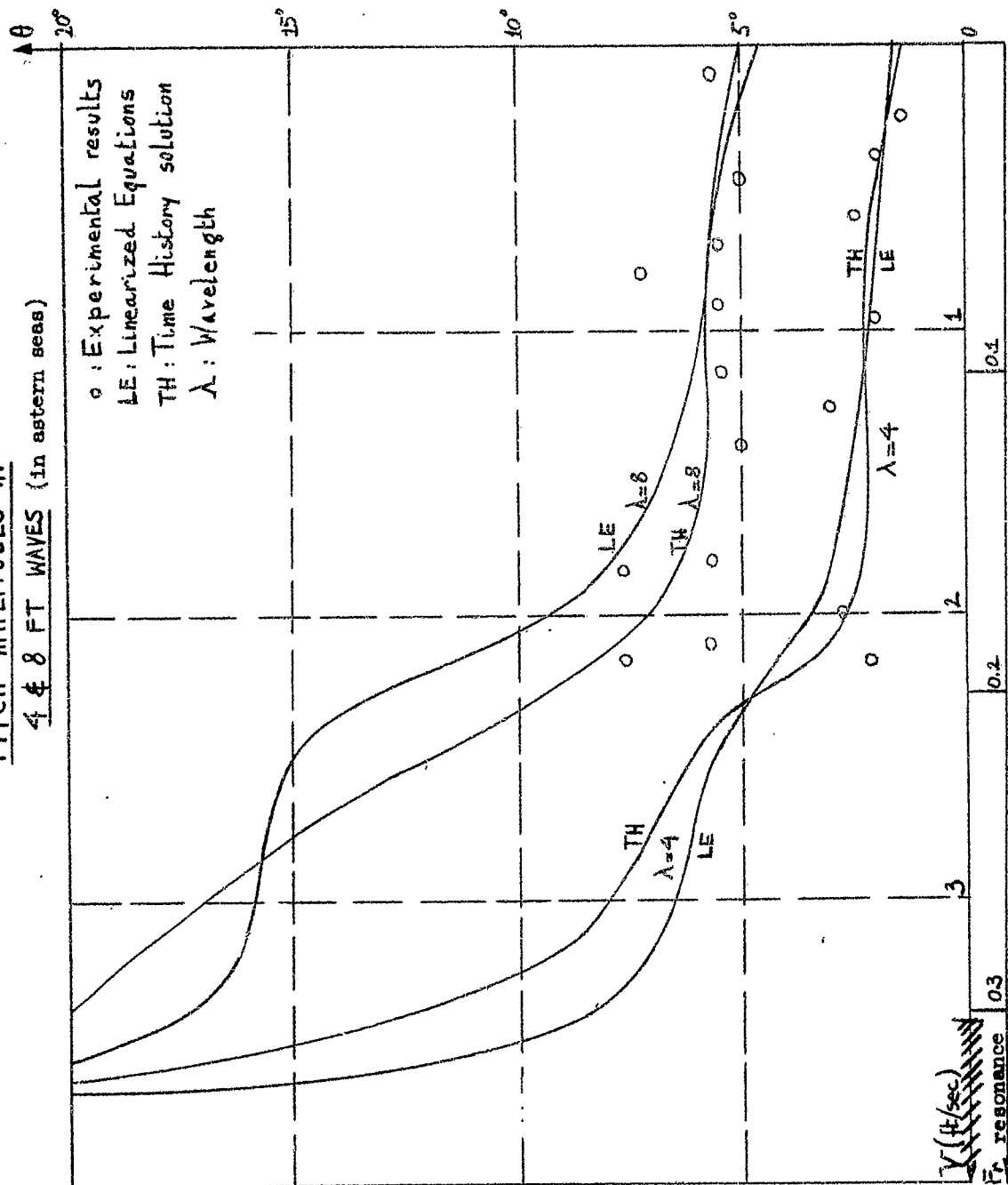
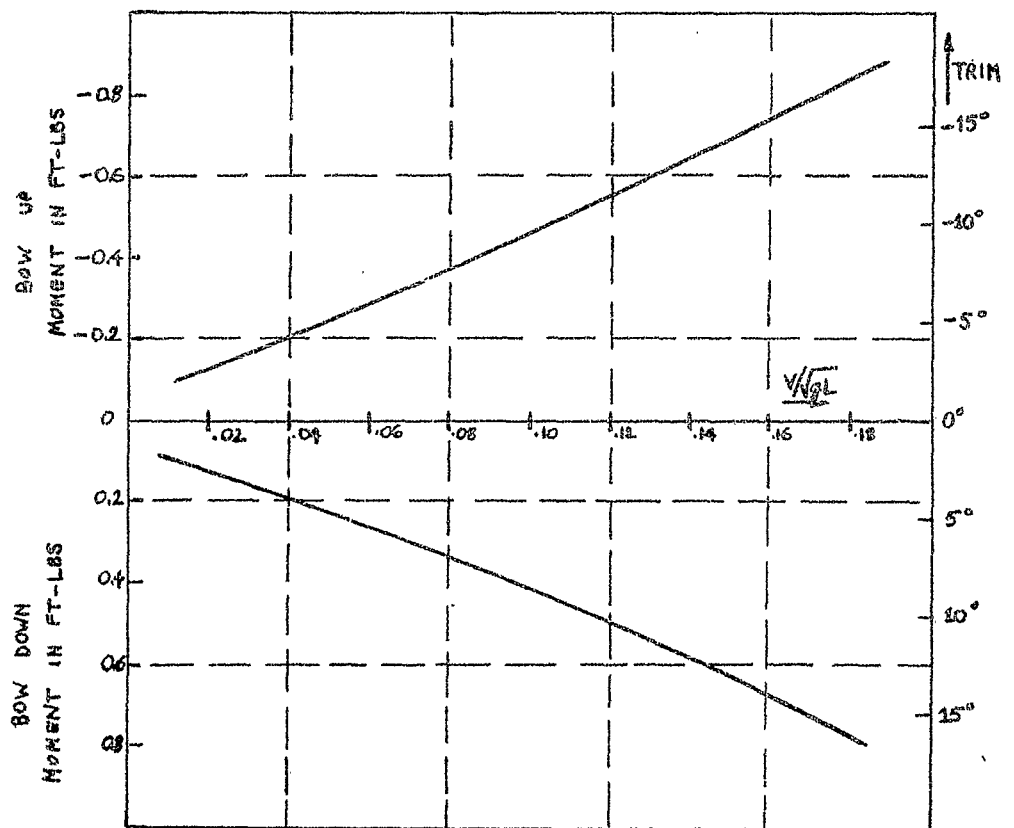


FIG. 4

TRIMMING ANGLES AND MOMENTS
(initial trim 1/2")

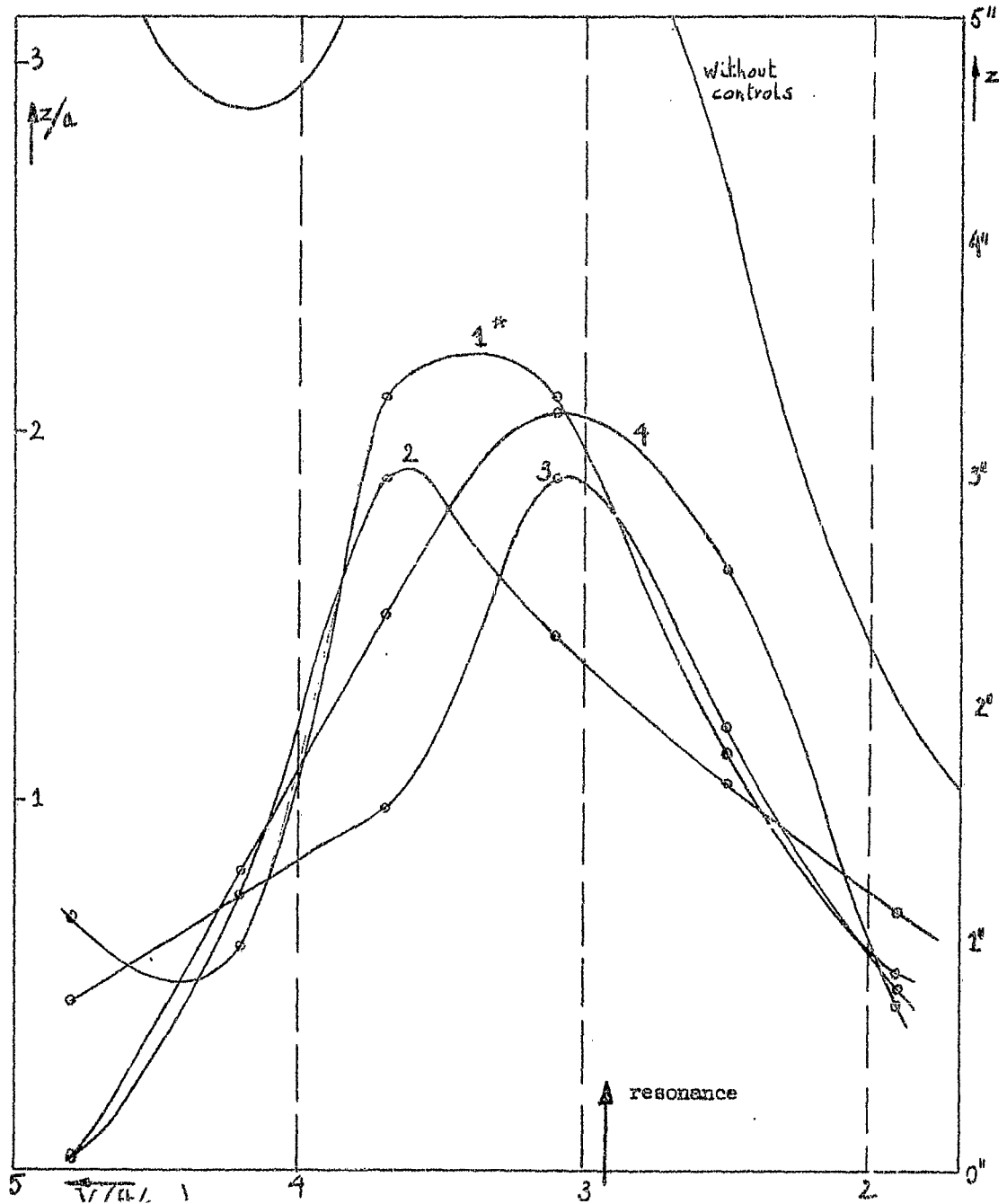


(Taken from Ref. 3)

FIG. 5 (λ)

HEAVE AMPLITUDES WITH CONTROLS

$\lambda = 8$ ft



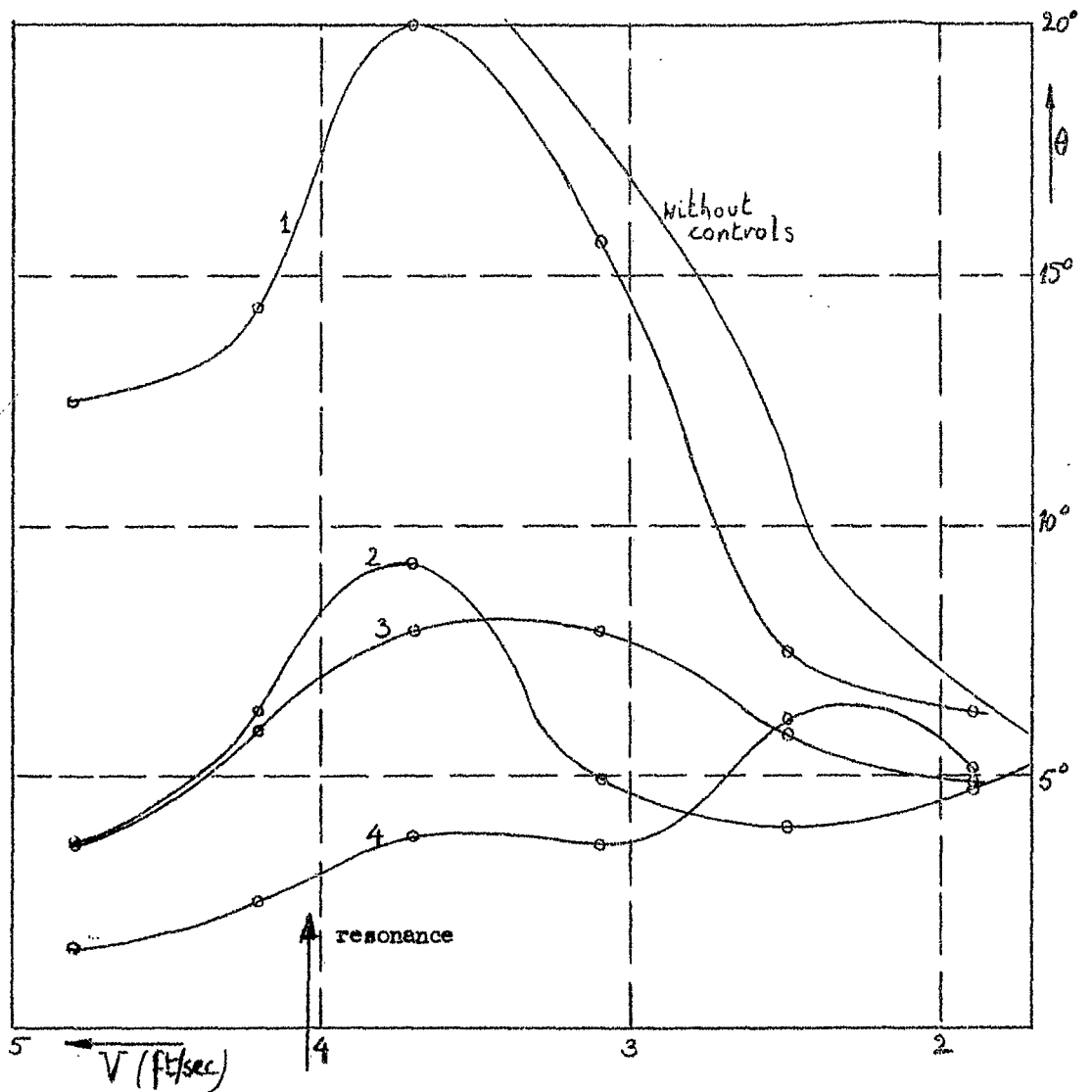
Curves 1, 2, 3, and 4 correspond to Tables I, VIII, IX, and XI respectively of App. VIII, pg. 75a.

FIG. 5 (b)

PITCHING AMPLITUDES WITH CONTROLS

$$\lambda = 8 \text{ ft}$$

(The numbers 1, 2, 3, 4 refer to the same control systems as in Fig. (5a).)



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