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ELECTROLYTIC MACHINING DEVELOPMENT

Joseph Bayer

General Electric Company

MANUFACTURING ENGINEERING RESEARCH
LABORATORY

Large Jet Engine Department

Contract: AF33(657)-8794

ASD Project: 7-648b (I)

Interim Technical Engineering Report
4 September 1962 - 3 June 1963

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The variables affecting electrolytic machining are theoretically derived, and subsequently demonstrated by experiment. Hypothetical concepts of the inter-relationships among variables are expressed mathematically. Significant operating and process variables for two alloy-electrolyte systems were determined for their effects on metal removal rate and required imposed voltage. Metal removal rate proved to be affected almost entirely by current density. The concept of equilibrium gap is explained. When equilibrium gap is established, feed rate varies directly with current density. Selected electrode materials were evaluated.

MANUFACTURING TECHNOLOGY LABORATORY
AERONAUTICAL SYSTEMS DIVISION

Air Force Systems Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

SUPPLEMENTARY REMARKS

The work discussed in this report has considered the basic factors of electrolytic machining. The process is analyzed as a series of electrolytic cells applied to one specific mode of electrolytic machining. It is shown that the basic factors such as process variables, electro-chemistry and transient conditions are common to all aspects of electrolytic machining.

Discussions of the process have been limited to flat plate and contour machining. As the program progresses, we shall use a similar analytical approach to other electrolytic machining techniques such as cavity and hole drilling, turning and single point cutting.

ABSTRACT - SUMMARY

ASD INTERIM REPORT 7-648b (I)

INTERIM TECHNICAL PROGRESS REPORT

JUNE 1963

The variables affecting electrolytic machining are theoretically derived, and subsequently demonstrated by experiment. Hypothetical concepts of the inter-relationships among variables are expressed mathematically.

Significant operating and process variables for two alloy-electrolyte systems were determined for their effects on metal removal rate and required imposed voltage. Metal removal rate proved to be affected almost entirely by current density.

The concept of equilibrium gap is explained. When equilibrium gap is established, feed rate varies directly with current density. Selected electrode materials were evaluated.

Required imposed voltage was affected by current density, chemical composition, temperature, and flow characteristics of the electrolyte during machining. Of these, flow characteristics have proved the most difficult to control. Within a moderate range, sludge content does not affect conductivity nor specific gravity. The viscosity of the electrolyte is sensitive to sludge content.

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FOREWORD

This Interim Technical Progress Report covers the work performed under Contract AF33(657)-8794, from 4 September 1962 to 3 June 1963. It is published for technical information only, and does not necessarily represent the recommendations, conclusions, or approval of the Air Force.

The contract, with the General Electric Company, Cincinnati 15, Ohio, was initiated under ASD Project No. 7-648b, "Electrolytic Machining Development". It is administered under the direction of Mr. W. M. Webster, ASRCTF, Manufacturing Technology Laboratory, Aeronautical Systems Division, Air Force Systems Command, United States Air Force, Wright-Patterson Air Force Base, Ohio.

Mr. Claude H. Auger, Senior Manufacturing Engineer, Manufacturing Engineering Research Laboratory (MERL), Large Jet Engine Department, is the Project Manager. Mr. Joseph Bayer, MERL, is the Project Engineer. Others at MERL who contributed to this work are: Mr. Joe H. Crawford and Mr. A. U. Jollis, engineers, and Mr. William Eisberg, laboratory assistant. Mr. Marcus A. Cummings, Advanced Engine Technical Department, acts as consultant on this project, contributing in the areas of electro-chemistry, mathematics, and computer programming. Mr. John L. Bemederfer, LJED, acts as consultant in the area of design of experiment and data analysis.

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SECTION I

INTRODUCTION

1.1. NEED FOR ELECTROLYTIC MACHINING

Many materials required for aerospace components are difficult or impossible to machine by conventional chip-cutting methods. Such components are often contoured so that cutting by conventional methods requires extensive tool-guidance systems, and operations such as deep, small-diameter hole drilling cannot be performed by conventional machining at all.

Fortunately, metals can be dissolved by electrolytic action at rates that are independent of hardness, toughness, and other physical characteristics. Electrolytic machining, a manufacturing technique by which metals are shaped by electrolytic dissolution at their surface, is becoming standard practice in metal cutting where conventional machining is uneconomical or impossible.

Thin-sectioned parts can also be cut at a high rate of speed electrolytically, and need not be chucked with such force as to cause deformation. Complex contours can be formed in any conductive material by a single plunge. Holes of very small diameter and of considerable length can be drilled by this method, while maintaining dimensional accuracy and good surface finish throughout their length.

As aerospace technology demands increasingly heat- and stress-resistant alloys, and as it requires more and more complex shaping and cutting, electrolytic machining will offer an economical, practical means of mass-producing parts.

1.2 NEED FOR IMPROVEMENT

Electrolytic machining has already proven to be a valuable process in industry, but its use has been restricted because of the lack of understanding of process variables and their inter-relationships. Without this understanding, development of manufacturing procedures for each specific job has taken an inordinate amount of time, since it has been done, essentially, by cut-and-try methods.

1.3 OBJECTIVES OF THE PROGRAM

The primary objective of this program is to achieve a wider understanding of the process. We believe that, as this understanding is gained, it will become possible to develop the process into a highly predictable, economical, and effective technique of metal cutting. We expect our plan of investigation to lead to specific conclusions about the practical and economic aspects of electrolytic machining, some of which follow:

1. Optimum cutting rates of various materials.
2. Surface-finish characteristics.
3. Dimensional accuracy.
4. Power requirements.
5. Relative processing costs.
6. Cost of equipment.
7. Principles and economics of tool design.
8. Inherent limitations of the process.

As these conclusions are developed, we plan to -

1. Report on the inter-relationships of process variables in a technical report.
2. Publish detailed process specifications based on our studies.
3. Publish a method for determining electrolytic machining specifications.
4. Demonstrate the flexibility of the process, and the scope of application, by machining specimen parts using various techniques.
5. Establish a computer program for determining operating parameters,
6. Establish a computer program for designing the surface contours of electrolytic machining tools.

In order to express the inter-relationships among process variables, we must begin by speculating about their probable nature. Fortunately, electrochemists have studied and reported on isolated aspects of those inter-relationships. We see our role as that of integrating the discrete knowledge already in existence, of establishing the existence and the nature of possible unknown factors, and of synthesizing a comprehensive body of knowledge about the whole. This approach differs somewhat from the conventional one in which experimentation precedes theorizing. For that reason, some might object that this approach is simply an attempt to prove a pre-conceived idea. In a sense, that is true; however, the "pre-conceived idea", or the theory that we are striving to evolve, is already based upon a rather extensive body of electrochemical

knowledge now in existence. Our aim is to select the pertinent, or controlling, factors that influence electrolytic machining from this existing knowledge; to confirm and evaluate the importance of these factors to the process, both statistically and experimentally; and, finally, to summarize and integrate the significant factors into a workable series of rules or techniques that can be applied to specific machining problems.

1.4 THE ELECTROLYTIC MACHINING PROCESS

Electrolytic machining is the reshaping of a metallic part by the anodic removal of metal from its surface. The part to be machined is made anodic in a closed d-c circuit, and a flowing electrolyte passes between the part and the cathodic tool as the dissolving action takes place.

A typical electrolytic machine consists of three inter-related subsystems: the electrical circuit, the electrolytic fluid loop, and the feed mechanism. The electrical circuit consists of a d-c power supply, the conducting electrolyte, the necessary connecting buses and wires, and appropriate controls and instrumentation. The electrolytic fluid loop consists of a storage tank or reservoir for the electrolyte, a pump, entry and exit hydraulic tooling surrounding the machining area, and the necessary piping, valves, filters, and gauges. The feed mechanism may be one of several electro-mechanical or electro-hydraulic systems, affording either manual control, automatic control through feed-back circuits, or both.

In operation, the tool, connected to the cathodic side of the power supply, is advanced toward the workpiece, connected to the anodic side of the power supply, as metal is removed from the surface of the workpiece. The electrolytic solution, under pressure, flows between the two pieces as cutting proceeds, so that the gap between tool and workpiece is filled with an electrolyte to conduct the electricity. The cutting rate is dependent upon current-flow between the electrodes (tool and workpiece), which in turn depends upon a number of process variables that will be discussed at length in the body of this report.

SECTION II

DISCUSSION

2.1 TECHNICAL CONSIDERATIONS

2.1.1 TRANSIENT CONDITION

Electrolytic machining is the reshaping of a metallic part by the anodic removal of metal from its surface. The metal is removed by making the part an anode in a closed electrical circuit as shown in figure 2-1. A view of the closed electrolytic cell is shown in figure 2-2. The cathode and the anode may have any shape at any time during electrolytic machining. Between the anode and the cathode is a conducting fluid called the electrolyte.

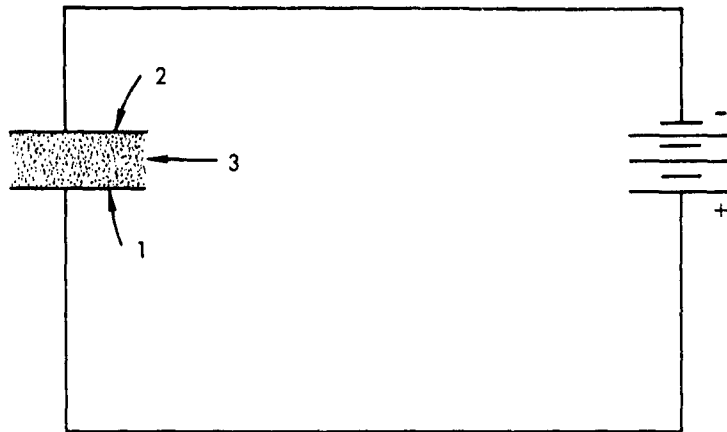
Let us look at the electrical circuit (figure 2-3) that exists between a on the cathode and b on the anode. Let us look at the cylinder of electrolyte ab long and dA (a differential area) normal to ab. Let us represent the electromotive force between a and b as E. L is the length of the line ab. The cylinder LdA has an electrical resistance of R. Because an electromotive force exists between a and b, a current, I_{dA}, will flow from b to a. The relationship among E, I_{dA} and R can be expressed in an electrolytic cell as:

$$E - \Delta E = I_{dA} R \quad (1)$$

E is the emf across the cell as measured by a voltmeter. E_e may be considered as the electromotive force acting against the resistance R of the electrolyte. The difference between E and E_e is known as ΔE, or over-voltage.

ELECTROLYTIC MACHINING

ELECTRICAL CIRCUIT



- 1. Anode - Workpiece
- 2. Cathode Tool
- 3. Electrolyte - Conductive Fluid

Figure 2-1

THE ELECTROLYTIC CELL

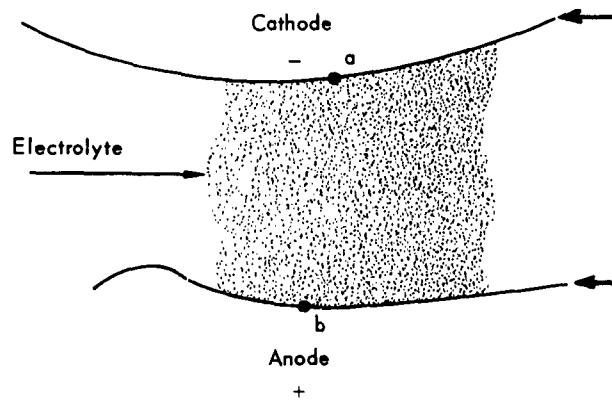


Figure 2-2

Later in this discussion, we will consider the relationships of these factors to the other process variables.

From Faraday's law, we would expect that the amount of metal removed at the anode will vary with the product of current and the time. The relationship for a constant area of cut may be described as

$$M_R = KIt \quad (2)$$

Where:

M_R = metal removed by volume - (cc)

K = constant, signifying volume of metal removed per ampere second - cc/amp. sec.

THE TRANSIENT ELECTROLYTIC CELL

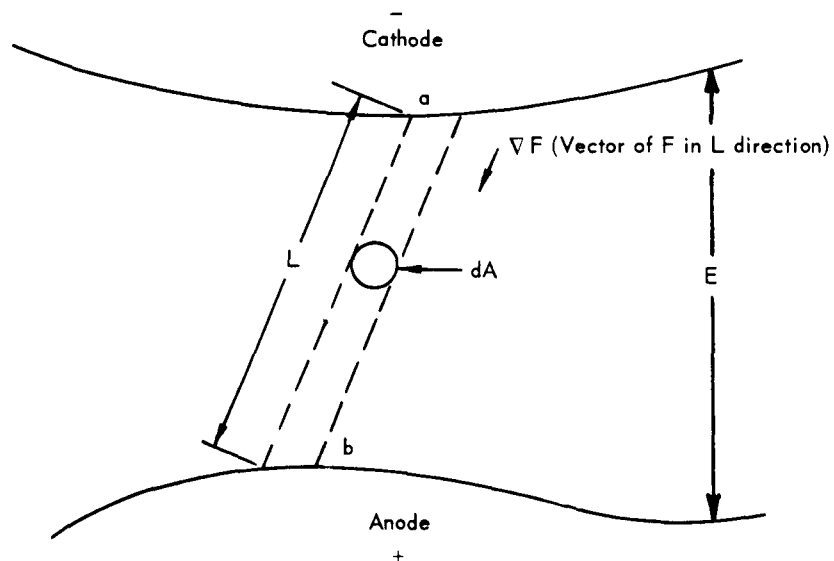


Figure 2-3

I = current in amperes

t = time in seconds

Let us now look at the cell after a lapse of time, dt . Let us stipulate a dimensional reference system so the b remains fixed and let us assume that a moves relative to b only along the line ab . If F is the rate of movement of a along ab , the change in L because of the relative movement of the cathode to the anode along ab can be expressed as Fdt . During the same time, the volume of metal removed from b can be expressed as $\frac{KI}{dA}dt$ [from equation (2)]. The volume of metal removed is $dAdl$, where dl is the change of L in time, dt , caused by electrolytic action.

$$\text{Therefore: } KI_{dA} dt = dAdl. \quad (3)$$

$$\text{But the total change in } L \text{ is: } dL = dl - Fdt. \quad (4)$$

$$\text{Substituting for } dl \text{ from equation (3), we have: } dL = \frac{KI_{dA} dt}{dA} - Fdt \quad (5)$$

then:

$$I_{dA} = \frac{(dL + Fdt) dA}{Kdt} \quad (6)$$

We must now explain R , the total resistance of the differential electrolytic cell, in terms of L and dA . At any point along L , the specific resistivity will be r , which is expected to vary along the length of L . Let us assume a weighted average value of r along L to be $\bar{r}$ in units of ohm-cm.

Then:

$$R = \frac{\bar{r}L}{dA} \quad (7)$$

If we substitute into the equation (1) the expression for $\underline{I_{dA}}$ from the equation (6) and the expression for $\underline{R}$ from equation (7), we have:

$$E - \Delta E = \frac{(dL + Fdt) dA}{Kdt} \times \frac{FL}{dA} = E_e \quad (8)$$

or

$$\frac{E_e K}{\bar{r}} dt = LdL + FLdt \quad (9)$$

If we call

$$\frac{E_e K}{\bar{r}} = \Gamma, \quad (9a)$$

we can then express equation (9) as:

$$dt (\Gamma - FL) = LdL \quad (10)$$

and

$$dt = \frac{LdL}{\Gamma - FL} \quad \text{or} \quad dt = \frac{LdL}{\frac{E_e K}{\bar{r}} - FL} \quad (11)$$

Equation (11) is a differential expression of electrolytic machining in the transient condition.

2.1.2 INTEGRATING FOR ACTUAL CONDITIONS

Equation (11) describes only the transient electrolytic conditions between selected points a and b on the cathode and anode surfaces respectively. Actually the total current impinging on the anode point b is $\underline{\Sigma I_{dA}}$ for all paths from b to all points on the cathodic surface. The current passing in the electrolytic cell is the sum of all currents impinging on all the points on the anode.

Before any integration can be made of equation (11), mathematical descriptions are required for $\underline{E}_e$, $\underline{K}$, $\underline{F}$, and $\underline{F}$. These variables must interrelate to each other and to $\underline{L}$ and $\underline{t}$ so that integration can be performed with respect to $d\underline{L}$ and $d\underline{t}$. $\underline{F}$ and $\underline{E}$ can be selected and imposed on the system. The selected relationship must be expressed in $\underline{t}$ and $\underline{L}$ terms. K is usually constant for a specific alloy and electrolyte.

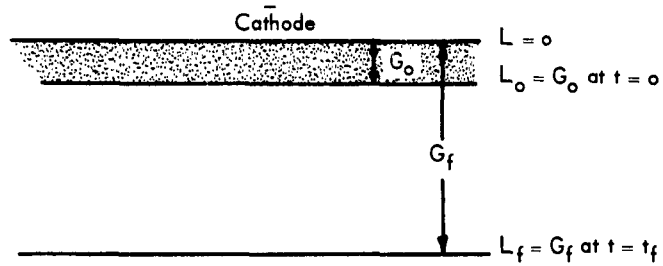
ΔE and $\bar{r}$ are the results of the electrochemical environment of the system. To describe them mathematically requires the imposition of enough restraints on the electrolytic system to ensure predictable values of these variables. Among the objectives of this program are to determine how to impose these restraints, how to predict the electrolytic behavior under those restraints, and to express the behavior in a mathematical form so that the integration of equation (11) can be performed.

If all possible methods of electrolytic machining are to be understood and a mathematical analysis is to be used as an effective aid in design and production, it is necessary to analyze the relative motion of the tool and work-piece and to analyze the current fluxes during electrolytic machining. The relative position of the cathode and anode must be described in incremental time steps during machining, which can be done by describing the anode and cathode positions in a specific reference system. Computer methods for describing the changes in relative position of the two surfaces during any incremental time change can then be evolved. The basis for the area-line motion system is described in the Appendix.

ELECTROLYTIC MACHINING STARTING CONDITION

Condition 1

Raw Material with Thickest Dimension



Condition 2

Raw Material with Thinnest Dimensions

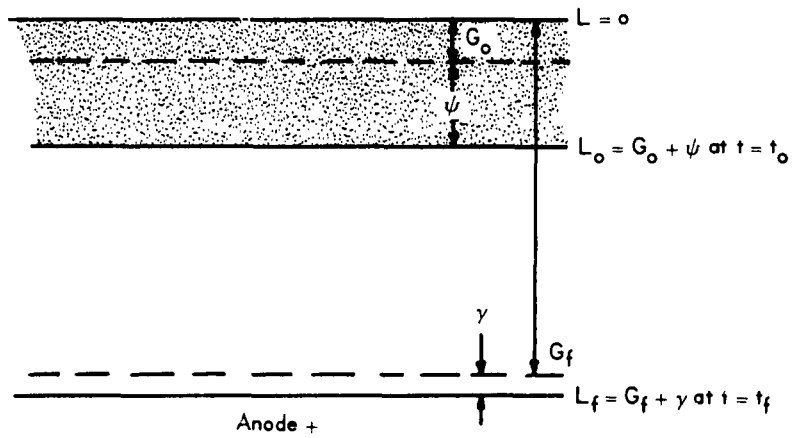


Figure 2-4

Equation (11) may be integrated and applied to specific electrolytic machining problems by applying simplifying assumptions.

Consider the case of a flat cathodic tool with an assumed infinite width and breadth, and a flat anodic workpiece whose surface is parallel to the cathode. Let us impose an electromotive force from the cathode to the anode, which we will call $\underline{E}$, and let us assume that we know the value of $\underline{\Delta E}$. Assuming further that $\underline{K}$ and $\underline{f}$ are known and are constant, Γ , as defined, is constant.

At any time, $\underline{dt}$, during electrolytic machining

$$\underline{dt} = \frac{\underline{LdL}}{\Gamma - \underline{FL}} \quad (11)$$

$$\Gamma = \text{constant}$$

Since $F = 0$

$$\text{therefore } \underline{dt} = \frac{\underline{LdL}}{\Gamma} \quad (12)$$

Let us integrate from $t = 0$ to $t = t_f$. (t_f = time at end of cut.)

Figure 2-4 describes the geometry of the electrolytic cell at the two expected cases of the thickest and the thinnest parts at $\underline{t} = \underline{0}$ and at $\underline{t} = \underline{t_f}$.

The raw material parts are expected to vary along the normal to the cathode and the anode by dimensional tolerance of $\underline{\Psi}$. The cathode is started at the same level so that the distance between the thickest part and the cathode is $\underline{G_0}$ at the outset, and the distance between the thinnest part and the cathode is $\underline{G_0} + \underline{\Psi}$.

At the time $\underline{t_f}$, the thickest part will then be a distance $\underline{G_f}$ from the cathode. For the same cutting time and at the same cutting conditions, the thinnest part will be a distance of $\underline{G_f} + \gamma$ from the cathode. γ may be considered as the expected maximum variation in gap at the end of the cut.

Integrating equation (12)

$$\int_0^{t_f} dt = \int_{G_0}^{G_f} \frac{LdL}{\Gamma} = \int_{G_0 + \psi}^{G_f + \gamma} \frac{LdL}{\Gamma} \quad (13)$$

$$t_f = \frac{L^2}{2\Gamma} \left[\begin{matrix} G_f \\ G_0 \end{matrix} \right] = \frac{L^2}{2\Gamma} \left[\begin{matrix} G_f + \gamma \\ G_0 + \psi \end{matrix} \right] \quad (13a)$$

$$t_f = \frac{G_f^2 - G_0^2}{2\Gamma} = \frac{(G_f + \gamma)^2 - (G_0 + \psi)^2}{2\Gamma} \quad (13b)$$

or for the same time period:

$$G_f^2 - G_0^2 = G_f^2 + 2G_f\gamma + \gamma^2 - G_0^2 - 2G_0\psi - \psi^2 \quad (13c)$$

or

$$2G_f\gamma + \gamma^2 - 2G_0\psi - \psi^2 = 0 \quad (13d)$$

or

$$G_f = \frac{2G_0\psi + \psi^2 - \gamma^2}{2\gamma} \quad (13e)$$

$$t_f = \frac{G_f^2 - G_0^2}{2\Gamma} = \frac{2G_0\psi + \psi^2 - \gamma^2 - G_0^2}{2\Gamma} \quad (14)$$

or

$$t_f = \frac{\bar{r} (G_f^2 - G_0^2)}{2E_e K} \quad (14a)$$

Let us next take the case of a flat cathode, assumed infinite in length and breadth, advancing toward the workpiece at constant speed $\underline{F}$. Everything remains the same as before, except that in this case, the integration would be:

$$\int_0^{t_f} dt = \int_{G_o}^{G_f} \frac{LdL}{\Gamma - FL} = \int_{G_o + \psi}^{G_f + \gamma} \frac{LdL}{\Gamma - FL} \quad (15)$$

The general expression for this integration is:

$$\int \frac{x dx}{a + bx} = \frac{x}{b} - \frac{a}{b^2} \ln(a + bx) \quad (15a)$$

Let $x = L$, $a = \Gamma$, $b = -F$

$$t \left[\begin{array}{l} t_f \\ 0 \end{array} \right] = \left[\begin{array}{l} G_f \\ G_o \end{array} \right] \left[\begin{array}{l} = \frac{L}{-F} - \frac{\Gamma}{F^2} \ln(\Gamma - FL) \\ = \frac{L}{-F} - \frac{\Gamma}{F^2} \ln(\Gamma - FL) \end{array} \right] \left[\begin{array}{l} G_f + \gamma \\ G_o + \psi \end{array} \right] \quad (15b)$$

$$t_f = -\frac{G_f}{F} - \frac{\Gamma}{F^2} \ln(\Gamma - FG_f) + \frac{G_o}{F} + \frac{\Gamma}{F^2} \ln(\Gamma - FG_o), \text{ or} \quad (15c)$$

$$t_f = -\frac{G_f + \gamma}{F} - \frac{\Gamma}{F^2} \ln[\Gamma - F(G_f + \gamma)] + \frac{G_o + \psi}{F} + \frac{\Gamma}{F^2} \ln[\Gamma - F(G_o + \psi)] \quad (15d)$$

If γ is small enough to be considered negligible in production terms then G_f made to the thickest piece can be considered equal to G_f to the thinnest piece and G_f then described as the "equilibrium gap".

Equating (15c) and (15d) and simplifying:

$$\ln \frac{(\Gamma - FG_f) [\Gamma - F(G_o + \gamma)]}{(\Gamma - FG_o) [\Gamma - F(G_f + \psi)]} = \frac{F(\psi - \gamma)}{\Gamma} \quad (16)$$

Given values for Γ in terms of $\bar{r}$, $\underline{E}_e$, $\underline{K}$, and $\underline{F}$, and selected values of $\underline{G}_0$, and γ , we can determine the $\underline{G}_f$ — the final equilibrium gap.

Time to cut to $\underline{G}_f$ can be determined by solving equation (15c).

At present, the case of $\underline{F} = 0$ and $\underline{F} = \text{constant}$, where $\underline{E}_e$, $\bar{r}$ and $\underline{K}$ are controlled as close to constant as possible, is being used extensively in industry. The transient differential equation can be solved for any condition if the variables have real meanings. As an example, let us solve the equation for the case in which $\underline{E}_e$, $\bar{r}$ and $\underline{K}$ are constant but in which $\underline{F} = L - C$, where $\underline{C}$ is a constant — a case in which the larger the gap at any moment, the faster the tool travels.

From equation (11)

$$dt = \frac{LdL}{\Gamma - FL} \quad (11)$$

Substitute for F:

$$dt = \frac{LdL}{\Gamma - (L - C)L} \quad (17)$$

$$dt = \frac{LdL}{\Gamma + CL - L^2} \quad (17a)$$

This differential equation can in turn be integrated. The general case of solution for the integral is:

$$\int \frac{xdx}{a + bx + cx^2} = \frac{1}{2c} \ln(a + bx + cx^2) - \frac{b}{2c} \int \frac{dx}{a + bx + cx^2} \quad (17b)$$

and $\int \frac{dx}{a + bx + x^2}$ is solvable for the two cases of

$$b^2 > 4ac \text{ and } b^2 < 4ac.$$

From the integration, we can determine the final gap and the time to arrive at the final gap for any definite values of $\underline{G}_0$, $\underline{\gamma}$ and $\underline{\Psi}$; or if we select a final gap, $\underline{G}_0$, and $\underline{\gamma}$, we can determine the necessary $\underline{\Psi}$. Similarly if we describe $\underline{E}_e$, $\underline{r}$, $\underline{K}$ and $\underline{F}$ mathematically, we can solve the equation (11) for selected $\underline{G}_0$, $\underline{\gamma}$ and $\underline{\Psi}$.

Up to now we have assumed that the cathode and the anode were two continuous parallel planes, that the $\frac{I}{A}$ normal to the planes was constant anywhere between the cathode and the anode. This is not true in any practical sense; however, if we have a cathode and an anode whose surface tangents do not change too radically for any two reasonably adjacent points on any of the two surfaces, we may reasonably assume that at any point on the cathode the transition conditions approach two parallel plates. Thus in figure 2-5, we draw a tangent at any point on the cathode surface, construct a normal to the tangent at that point and proceed to integrate as shown above.

The total current that impinges upon $\underline{b}$ is the sum of all the current flux for all the points on the cathodic surface. We are assuming that the surfaces are parallel; thus the flux from $\underline{a}$ to $\underline{b}$ is the same as at any other two points opposite each other on the cathode and the anode. We have assumed that the rate of change of the slope of the actual surfaces is not radically large. The distance between the point $\underline{b}$ on the anode surface and any point $\underline{a}'$ on the cathode surface is $\underline{L}'$. $\underline{L}'$ will cross the assumed parallel plane at $\underline{a}''$. The difference between $\underline{a}'$ and $\underline{a}''$, (Δa), will probably be larger as $\underline{L}'$ becomes

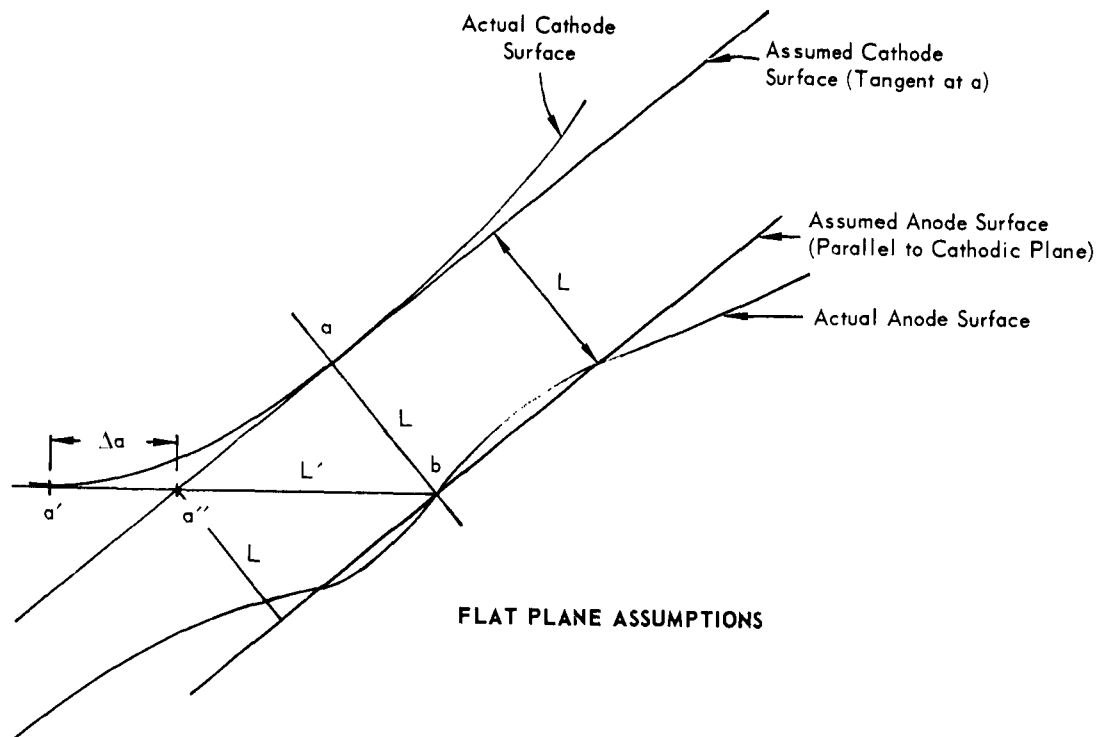


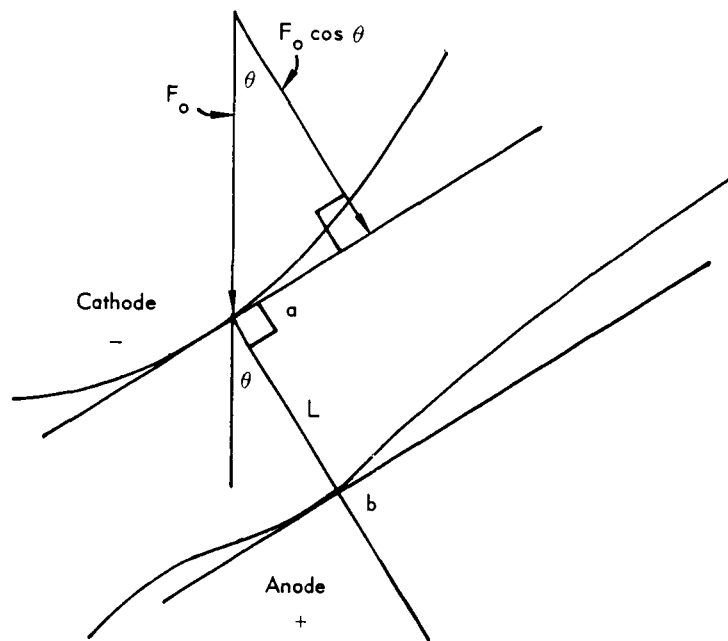
Figure 2-5

larger because of the basic assumption. Therefore, because the flux between a' and b is inversely proportional to L' , we can say that when Δa errors are expected to be larger, the resultant current fluxes are expected to be smaller.

The approach we have been following does not apply on the surface where we encounter radical changes in shape, especially at corners, pits and bumps. Let us assume no corners, pits or bumps. Let us look at a typical tool cutting a typical surface. Let us simplify our approach and say that the tool is moving toward the finished workpiece at a rate $\underline{F}_o$ unidirectionally. All points

in the surface thus are moving at a rate $\underline{F_o}$; however, if we select a point $\underline{a}$ on the cathode and $\underline{b}$ on the anode, $\underline{a}$ appears to be approaching $\underline{b}$ along the line $\underline{ab}$. At any instant $\underline{a}$ is approaching $\underline{b}$ at rate $\underline{F}$.

In figure 2-6, the tool is moving at a rate F_o . The tangent to the surface at $\underline{a}$ is, however, at an angle of $90 - \theta$ degrees to the direction in which the tool is moving. Therefore, $\underline{L}$, between $\underline{a}$ and $\underline{b}$, is at an angle of θ to the direction of feed. The tangent is moving at a rate of $\underline{F_o} \cos \theta$; thus, in our analysis of what happened at $\underline{ab}$ when we designated the feed rate $\underline{F}$ at this point, we are actually describing the vector $\underline{F_o} \cos \theta$. Therefore, we deter-



DEFINING FEED RATE OF ASSUMED SURFACE

Figure 2-6

mine the specific finished gap from any point on the tool by calculations based on $F = F_0 \cos \theta$.

For tool design we start with the desired finished dimensions on the anode; determine the finish gap; and, by describing the difference between the desired finished dimensions from the finish gap, we estimate the necessary dimension of the tool surface. These calculations are relatively simple and can be described on graphs and nomographs.

We plan to design tools based on the above assumptions in order to evaluate the validity of the assumptions. We intend to do this by:

1. Calculating the tool surface based on the simplified approach.
2. Calculating the tool surface by computer where we do not assume incremental parallel surfaces, but treat the surfaces as mathematically descriptive as we can.
3. Making tools; cutting parts in accordance with the desired operating parameters; measuring the final contours; and comparing the two estimated contours to the actual contours.

2.1.3 EQUILIBRIUM GAP - BASIS FOR PROBLEM SOLVING

2.1.3.1 Tool Design

2.1.3.1.1 Simplified Cos - θ Method

Sectional tool profiles may be approximated by this method:

1. Draw tangents at significant points on sectional profiles of the part to be produced.

2. Construct normals to the tangents at the point of contact.
3. Select a minimum gap, $\underline{G}_0$, which is located at any anodic points whose tangent is normal to the feed direction.
4. Determine the angle $\underline{\theta}_1$ for each point from the intersection of the normal line and a line in the feed direction.
5. At each point construct a point on the normal in the direction the tool is to lay, so that the gap, $G_1 = G_0 \cos \theta_1$.
6. Connect the points for $\underline{G}_1$ in a smooth curve for each profile.
7. These new profiles describe the shape of the cathode.
8. $\underline{G}_0$ may also be selected from any known relation between Γ and $\underline{F}_0$, the process constant and the feed rate, since at equilibrium gap

$$\Gamma = F_0 G_0 \quad (18)$$

and $\Gamma = \frac{E - \Delta E}{\bar{r}} \times K \quad (19)$

E may be selected, and $\underline{\Delta E}_1$, $\bar{r}$, and K can be determined by experiment.

9. Until the relationship for Γ is established, $\underline{G}_0$ and $\underline{F}_0$ should be selected, and $\underline{E}$ varied, to achieve $\underline{G}_0$.

2.1.3.1.2 Computer Technique

Geometrically, a surface can be described as a series of conics. So that for a given surface we can easily obtain enough points for machining by our present APT system. Now then, our anode surface is described to the computer. From this we want to generate a cathode (tool) surface which under machining conditions will reproduce the anode. Let us now assume that an equilibrium

gap has been achieved. Let us specify the magnitude of the minimum equilibrium gap as defined by the procedure followed in the simplified $\cos \theta$ approach to approximating cathodes. This minimum gap occurs when θ is minimum. The effective current at a point on the anode at equilibrium gap is the sum of the currents from every point on the cathodic surface which affects the given anodic point. Each point represents an incremental area, ΔA , normal to the direction of the tool feed, F_o . The effective distance between the anode and the cathode L can be computed in the same manner as resistances in parallel, that is

$$\frac{I}{L} = \sum_{n=0}^m \frac{I}{L_n} \quad (20)$$

m is the total number of points on the cathode effective a given point on the anode. We then have the following:

$$\sum_{n=0}^m I_n = I = \frac{F_o \Delta A}{K} \quad (21)$$

and by Ohms law

$$I = \frac{E - \Delta E}{\bar{r} L} \quad (22)$$

$$\text{therefore } \frac{F_o \Delta A}{K} = \frac{E - \Delta E}{\bar{r} L} \quad (23)$$

but as stated before

$$\Gamma = \frac{E - \Delta E}{\bar{r} L} \quad (24)$$

and by substitution, we have that

$$\frac{I}{L} = \frac{F_o \Delta A}{\Gamma} \quad (25)$$

For any system we can choose or compute a Γ , assign a F_o and either assign equal or variable ΔA 's at each point. The system of equations to be solved now takes the form:

$$\frac{I}{L_{11}} + \frac{I}{L_{12}} + \frac{I}{L_{13}} + \dots + \frac{I}{L_{1n}} = \frac{F_o \Delta A}{\Gamma} \quad (26a)$$

$$\frac{I}{L_{21}} + \frac{I}{L_{22}} + \frac{I}{L_{23}} + \dots + \frac{I}{L_{2n}} = \frac{F_o \Delta A}{\Gamma} \quad (26b)$$

$$\frac{I}{L_{n1}} + \frac{I}{L_{n2}} + \frac{I}{L_{n3}} + \dots + \frac{I}{L_{nn}} = \frac{F_o \Delta A}{\Gamma} \quad (26c)$$

where the L_{ij} 's are the distances from a point (x_i, y_i, z_i) on the anode to a point (x_j, y_j, z_j) on the cathode. Each L_{ij} is given by the distance formula

$$L_{ij} = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2 + (Z_i - Z_j)^2} \quad (27)$$

The above equations hold for equal ΔA 's. If the ΔA 's vary from point to point then the system of equations take the form

$$\frac{I}{L_{11}(\Delta A)_{11}} + \frac{I}{L_{12}(\Delta A)_{12}} + \dots + \frac{I}{L_{1n}(\Delta A)_{1n}} = \frac{F_o}{\Gamma} \quad (28a)$$

$$\frac{I}{L_{21}(\Delta A)_{21}} + \frac{I}{L_{22}(\Delta A)_{22}} + \dots + \frac{I}{L_{2n}(\Delta A)_{2n}} = \frac{F_o}{\Gamma} \quad (28b)$$

$$\frac{I}{L_{n1}(\Delta A)_{n1}} + \frac{I}{L_{n2}(\Delta A)_{n2}} + \dots + \frac{I}{L_{nn}(\Delta A)_{nn}} = \frac{F_o}{\Gamma} \quad (28c)$$

We should observe that because of the nature of the L_{ij} 's the equations given in the set above are non-linear. There are, therefore, several possible cathode surfaces which will be effective. We assume, moreover, that for the above set of equations $\underline{n}$ anode points would generate $\underline{n}$ cathode points, but this is not necessarily the case. We note further that if we orient our axes so that the tool travels in the negative $\underline{Z}$ direction, the $\underline{n}$ and $\underline{y}$ coordinates of a point may be considered fixed, and only the $\underline{Z}$'s are variable. With the techniques given in the previous section we can approximate a cathode shape as a first guess. We can then compute all $\underline{L}_{ij}$'s and observe the error in each of the given equations. For subsequent iterations we will choose our effective $\underline{\Delta Z_n}$ by use of an error regression technique with some bound on the allowable $\underline{\Delta Z_n}$. The expected tolerance of the finished part will dictate the magnitude of the allowable change. Note, once and for all, that $\underline{L}_{ij}$'s used in the discussion above are those which can be described as being imbedded in a simply connected region. For $\underline{L}_{ij}$'s in multi-connected regions such as regions with corners, our problem is somewhat different. We hope to be able to investigate such problems with the use of trajectory analysis.

A computer solution of the set of equations above, within some pre-assigned tolerance, will define a mesh of points representing the cathode tool. This mesh of points is readily transferrable to numerically controlled milling machines.

2.1.3.2 Other Electrolytic Machining Problems

We began with the problem of determining what combinations of a cathodic surface current density, electrolyte and physical properties of the electrolytic cell will permit the production of some pre-determined anodic shape. Lumping all of the physical properties of the cell into a single term $\underline{\Gamma}$, we were able to devise a set of equations which would give us a cathode shape from some anodic shape, assuming that equilibrium gap had been achieved. Now let us determine the minimum time necessary to achieve desired anode shape with some given tolerance having been given a set of physical parameters, a cathode and some raw material. For this case the transient condition developed previously and re-stated here for convenience will be used. We have that

$$\Delta L = \frac{\underline{\Gamma} - \underline{F}_0 \underline{L}}{\underline{L}} \Delta t \quad (29)$$

$\underline{L}$, $\underline{F}_0$, and $\underline{\Gamma}$ are as defined before, Δt a time increment and ΔL is an effective change in length at a given point. We compute the $\underline{L}_{ij}$'s as the distances between single points on the cathode and all points on the surface of the raw material. When the $\underline{L}_{ij}$'s, computed in this way, satisfy the set of $\underline{n}$ equations defined in the last section, the equilibrium gap has been reached. If now the minimum gap $\underline{G}_0$, the feed rate $\underline{F}_0$, $\underline{\Gamma}$, and $\underline{\Delta A}$ are defined, the transient equation giving ΔL can be used to correct the raw material surface until it comes within the tolerance of the desired anode. The sum of all the Δt 's will then be considered as the minimum time required to attain the design of the anode. This gives the engineer a necessary tool for selecting initial gaps and tolerances

and excess stock for the raw materials. The minimum excess stocks will be obtained by summing the ΔL 's at each point and taking the maximum of such values.

2.1.4 ELECTROCHEMISTRY

2.1.4.1 Relationship of ΔE and $\bar{r}$

Electrolytic machining has been examined in this report in terms of the behavior of an electrolytic cell with imposed external forces. Ohm's Law in the form:

$$E - \Delta E = IR \quad (30)$$

has been used to describe the process variables and the transient condition of the cell has been represented by the equation

$$dt = \frac{LdL}{\frac{E - \Delta E}{r}} K - FL \quad (11)$$

Assuming that the metal removal rate for any alloy-electrolyte system is a linear function of the current, and that ΔE is related to the current, the electrolyte temperature, the composition of the electrolyte, and the fluid forces, we now seek to define the forces acting in the field. There are two approaches we will look at, one in which forces are viewed as a series of ΔE drops across the cell, the sum of which gives us our imposed voltage E . The second approach will be an attempt to examine the forces on an ion in the cell.

Considering the ΔE drops, we have:

ΔE_1 - The energy to release the individual atoms from electronic bond sharing

ΔE_2 - The energy to oxidize the metal

ΔE_3 - The resistance to movement of cations into the double layer at the metallic boundary at the anode

ΔE_4 - The net resistance to movement of cations and anions toward the attracting poles in the diffuse layer and in the bulk electrolyte

ΔE_5 - The resistance to movement of anions into the boundary layer at the cathode

ΔE_6 - Energy to reduce the hydrogen ion to the hydrogen atom

ΔE_7 - Energy to release hydrogen ions from the water molecules

Some of the ΔE 's are thermal and some are chemical. The sum of the ΔE 's required for chemical change will be the ΔE used in the equation

$$E - \Delta E = IR \quad (30)$$

The sum of the ΔE 's required for thermal change will be attributed to the resistance R of the electrolyte. To further simplify matters a specific resistivity $\bar{r}$ will be assumed, where

$$\frac{\bar{r}L}{\Delta A} = R \quad (31)$$

L is the effective distance of the cathode from a point on the anode, and vice versa, and ΔA , a unit area, is assumed.

2.1.4.2 Defining Electrolytic Cell Forces

For our second approach we consider an ion with charge e in a field of electrical intensity X and in which the electrolyte has a velocity v_o . The major forces appear to be the electrical field forces, the frictional forces, forces

due to diffusion and pressure gradients, and the relaxation and electrophoretic forces. Some of the forces have negative effects; the others have positive effects. The sum will be the total force on an ion in the electrolyte, and it is equal to the product of the mass and acceleration of the ion. Some of these forces need further definition.

Theoretically every ion is considered to have an ionic atmosphere of opposite charge. This atmosphere exhibits central symmetry which is disturbed by external forces, such as an applied electric field, or a shearing force which imparts a velocity to the medium. As the ions move from position a to position b there is a definite time of relaxation in which the atmosphere is built up at position b while the charge density decays at position a. Since the electrical atmosphere is of opposite charge, there is an excess of ions of opposite charge at positive a retarding the motion of the ions. The electrical atmosphere, therefore, lacks symmetry. We speak of such an effect as a relaxation or asymmetric effect. The applied cmf tends to move the ionic atmosphere, with the associated molecules of the solvent, in the direction opposite to that in which the ion (accompanied by solvent molecules) is moving. A second retarding effect called the electrophoretic effect is produced.

Another retarding effect on the ions is attributable to the tendency of the ions to move from a more concentrated to a less concentrated region in the solution. This retarding effect is known as diffusion.

In order to understand the pressure gradient and frictional forces, one must investigate the fluid-flow in a system of the type in which we are interested. This investigation is made difficult from a theoretical standpoint, since the flow is three dimensional, the Reynolds number, viscosity, and ratio of length of region to the gap between the surfaces are such that we have fully developed flow. We are continuing our investigations of fully developed fluid-flow with the intention of using it in our applications. The pressure gradient and the temperature will have a decided effect on the volume of hydrogen generated at the cathode. The volume of hydrogen will affect the cross-sectional area of electrolyte, and hence its resistivity.

1. The applied electric field produces a positive force on the ion which is the product of the ionic charge and the field intensity $e_j X$.

2. The relaxation caused force may be stated mathematically as

$$-e_j^2 X \left[\frac{|Z_+ Z_-| q}{3DkT (1 + q^2)} \frac{K}{1 + Ka} \right] \quad (32)$$

where Z_+ , Z_- are the numerical valences of the ions

D is the dielectric constant

k is Boltzmann's constant

T is absolute temperature

$$q = \frac{(Z_+ Z_-) (\lambda_+ + \lambda_-)}{(Z_+ + Z_-) (Z_+ \lambda_- + Z_- \lambda_+)} \quad (33)$$

λ_+ and λ_- are the equivalent conductances of the ions in question

K is the effective radius of the ionic atmosphere

a is the radius of the ion

3. The electrophoretic force may be stated as

$$\frac{-e_j K X \mu}{6 \pi \eta_0} \quad (34)$$

where

μ = is the frictional coefficient

η_0 = is the viscosity of the electrolyte

4. The friction force is

$$-\mu (v - v_0) \quad (35)$$

where $\underline{v}$ is the velocity of the element and $\underline{v}_0$ is the velocity of the electrolyte

5. The diffusion gradient is

$$-\frac{kT}{n_j} \frac{\partial n_j}{\partial x} \quad (36)$$

$\underline{x}$ is the direction of the diffusion

$\underline{n}_j$ is the bulk concentration

The bulk concentration is given as

$$n_i e^{\frac{-Z_i e \chi}{kT}} \quad (37)$$

there $\underline{\chi}$ is the potential, and from this we have

$$K^2 = \frac{4 \pi e_j^2 \sum n_i Z_i^2}{DKT} \quad (38)$$

6. The pressure gradient of the fluid produces a positive force

$$V_j C_o \frac{dv_o}{dt} \quad (39)$$

where V_j is the volume

C_o the density of the solvent

v_o the velocity of the electrolytic

Combining these equations we now have, as the total force on the ion —

$$m_j \frac{dv_j}{dt} = e_j X - \mu_j (v_j - v_o) - e_j^2 X \left[\frac{|Z_+ Z_-| q}{3 D k T (1 + q)^{1/2}} \frac{K}{1 + K a'} \right] \\ - \frac{e_j K X \mu}{6 \pi \eta_o} - \frac{k T}{\eta_j} \frac{\partial \eta}{\partial x} + V_j C_o \frac{dv_o}{dt} \quad (40)$$

The equation just stated includes forces which might be considered as thermal and associated with $\underline{\Delta E}_+$. To account for the other $\underline{\Delta E}$'s more terms need to be added to the expression above.

In the inner and outer Helmholtz planes which bound the double layers at the metallic surface one may speak of the electrostrictive pressure P . This pressure is related to the field strength by the relation

$$P = \frac{\bar{\epsilon} X^2}{8 \pi} \quad (41)$$

where $\bar{\epsilon}$ is the "integral" dielectric constant represented by

$$\bar{\epsilon} = \frac{1}{X} \int_0^X \epsilon d\eta \quad (42)$$

$$\text{and } \epsilon = \epsilon_{\infty} - (\epsilon_0 - \epsilon_{\infty}) (1 + 2bX^2)^{-\frac{1}{2}} \quad (43)$$

ϵ_{∞} is the limiting dielectric constant at very high field strength

ϵ_0 is the dielectric constant at zero field strength

$\underline{b}$ is a parameter obtained from experimental data.

The gradient of the electrostrictive pressure then exerts a retarding force on the ion in a direction which is perpendicular to the metallic surface. So that the addition of a term $-\frac{P}{X}$ for each metallic surface should be added to the above equation, so that all the thermal forces are accounted for. This gives us a technique for expressing $\underline{\Delta E}_3$, $\underline{\Delta E}_4$ and $\underline{\Delta E}_5$, and hence the resistance of the cell and its resistivity $\bar{r}$. We are still questioning whether we should consider the electrostrictive pressure gradient as chemical in nature, thereby including $\underline{\Delta E}_3$ and $\underline{\Delta E}_5$ with the calculation of capacitance rather than resistance. The forces $\underline{\Delta E}_1$, $\underline{\Delta E}_2$, $\underline{\Delta E}_6$, and $\underline{\Delta E}_7$ are considered to be available from current information in the literature; hence they have not yet been included in our literature search.

2.1.4.3 Trajectory Technique

One further note will be made here. The effective distance $\underline{L}$ referred to in the equation for $\bar{r}$ has been considered as the inverse of the sum of inverses of all paths from an anode point to the cathode. In general we have assumed that the paths are in straight lines. There are cases in which straight lines cannot be drawn between points on the cathode and those on the anode without leaving the electrolyte. In such cases we will approximate the line of least electrical resistance between the two points by trajectory analysis.

2.1.4.4 Deriving Process Variable Interrelations

The significant process variables fall into two classifications, those which measure the energy level of the process and those which measure the mass that responds to the energy. The analysis of factors acting on the electrolytic cells fall into the same two classifications. We are seeking to transfer the functional relationship expressed in the analysis in terms containing the process variables. When we have correctly correlated the process variables to the variables expressed in the analysis, we will have relationships which can be used for tool design and determining operating parameters for electrolytic machining.

2.2 EXPERIMENTAL AND ANALYTICAL APPROACH

2.2.1 IDENTIFYING PROCESS VARIABLES

We first identified our process variables affecting electrolytic machining. We considered only those variables which were part of a geometrically controlled electrolytic cell. Our specific approach is detailed in the experiments run in test series JB-1, JB-2, and JB-3.

2.2.2 REFERENCE SYSTEM USED

The relative position of the tool to the workpiece at all times will determine the cutting rates. We have chosen to adopt a system which is readily applicable to automatically-programmed tools (APT). The general system is described in the Appendix, paragraph 6.4.

2.2.3 APPROXIMATING OPERATING PARAMETERS

2.2.3.1 General

In setting up electrolytic machining production, the engineer requires approximations of such operating parameters as required voltage, feed rate, electrolyte temperature, electrolyte composition, etc., and of such operating conditions as initial gap, time to cut, raw material tolerance, and required excess stock. These operating conditions can be estimated on the basis of equilibrium gap concepts. In simpler operations, the estimates can be done by graphs, nomographs and other engineering techniques. We shall determine the detailed procedures for making these engineering estimates. Estimates for the more complex operations require iterations which must be performed on the computer.

2.2.3.2 Simplified Approximations

We are studying the implications of the $\cos-\theta$ method. The differential equation (11) may be integrated for other than flat planes and the necessary operating conditions determined.

2.2.3.3 Computer Approximations

We have almost completed our computer programming approaches. We plan to program the computer and publish in a future contract report the Fortran instructions. We shall then derive our operating approximations and validate them by experimental runs.

2.2.4 APPROXIMATING

2.2.4.1 General

You will recall that we have lumped the electrochemical process variables into a descriptive operator Γ , where

$$\Gamma = \frac{E - \Delta E}{\bar{r}} \times K$$

We have discussed ΔE and $\bar{r}$, two electrochemical properties that are a result of the electrolytic environment. K , the metal cutting constant, is easily determined for any specific alloy-electrolyte system, and E , the voltage, can be read on the meter. Γ then can be defined only as we properly define ΔE and $\bar{r}$.

2.2.4.2 Experimental Plans

We have covered in some depth our present explanation of ΔE and $\bar{r}$, but are not as yet satisfied with this explanation. We shall continue our survey and assimilation of the electrochemical literature so that we can evolve the most reasonable explanation of ΔE and $\bar{r}$. We have two ways to experiment. We hope to find an experimental method of separating ΔE and $\bar{r}$. If so, we can approximate the functions that explain ΔE and $\bar{r}$. Lacking such methods, we shall continue building models which explain ΔE and $\bar{r}$, and test these models experimentally.

2.2.4.3 Statistical Approach

In checking the validity of the models we devised for ΔE and $\bar{r}$ and consequently for Γ , we will need sophisticated statistical approaches. We shall not be necessarily approximating classical expanding polynomial series such as usually taken as models by statisticians. We shall more likely be using models of discrete functions as used by scientists. Our statisticians are now setting up methods of analysis to treat discrete rather than expanding series models.

2.2.4.4 Computer Approach

Our data will be analyzed by computer to determine the validity of our functional models. In addition, we plan to run experiments by computer programming the model as if it represents the environment, and checking selected points by actual experiment. We can then determine the degree of confidence that we can place in the correlation between the model and actual test results.

2.2.5 ELECTROLYTE SEARCH

2.2.5.1 General

Experience has shown that specific electrolytes are required when electrolytic machining specific alloys. Though the hope has been expressed many times that an electrolyte might be found for a large number of alloys in a variety of metallurgical conditions, the hope has not been justified at this point.

Each alloy in each heat treated condition has a different effect on the process variables of electrolytic machining. The ΔE will be affected by the metallurgical structure and the heat-treat state; the selected attack of the part will depend on chemical composition, the heat treatment and the metallurgical structure. The reaction between the electrolyte and the alloy can result in hard anodic films on the surface, which are usually dielectrics. The reaction products can be unstable so that process control over dimensions and selective etch becomes difficult or impossible. Secondary reactions on the electrolyte may or may not lend themselves to good process control.

The electrolyte can be selected by analyzing the chemical reactions to be expected during electrolytic machining.

2.2.5.2 Criteria For Selecting Electrolyte

1. An electrolyte should be conductive enough to ensure large enough gaps to avoid shorts.
2. No dielectric anodic film should form on the surface of the work-piece.
3. The ΔE attributed to the alloy-electrolyte system should not be so high as to require expensive high-voltage rectifiers or to encourage selective etch.
4. The reaction products in the electrolytes should approach equilibrium conditions rapidly.
5. The electrolyte should be inexpensive, non-toxic and re-usable.
6. Some mechanism should be provided for removing the reaction products from the electrolyte, so that the reaction can approach equilibrium rapidly

2.2.5.3 Looking At The Alloy

The alloy to be cut should be analyzed in three areas:

1. Chemical composition
2. Metallurgical structure
3. Heat treatment

Note that heat treatment affects the metallurgical structure.

The chemical composition will affect the electrolytic process in that:

1. It will affect the metal removal rate.
2. It will affect the amount of selective etch in that it will affect the electrical potential required to dissolve the constituent.

3. It will determine the subsequent reactions:

- a. It will determine whether, with a specific electrolyte, insoluble dielectrics will form in the workpiece surface.
- b. It will determine whether the reactive products tend to seek stable equilibrium conditions.
- c. It will affect the secondary reactions.

The chemical composition should be determined by answering these questions:

1. Can we select an electrolyte so that no insoluble salts will result from the combination of the metallic cations from the alloy and the anions in the electrolyte:

2. If insoluble salts are formed at the anode can they be dissolved by

- a. High pH
- b. Low pH
- c. Chelating (specific)
- d. Other inorganic agents?

3. What secondary reactions are expected?

- a. Can insoluble products be removed by filtration?
- b. Can they be removed by ion exchange?

4. Probable valence state -

Is there more than one? If there is more than one, can one valence state be held by -

- a. Oxidizing or reducing agents in the electrolyte?
- b. Altering pH?
- c. Chelating agents?
- d. Other means?

What effect has the metallurgical structure and heat treatment:

- 1. How much energy will be required to break up covalent bond?
- 2. How much difference in oxidation potential is expected among constituents and combinations? What are the required imposed voltage and current density to avoid selective etch?

Questions concerning heat-treatments would include:

- 1. How will segregation affect selective etch?
- 2. How homogeneous is the alloy?
- 3. What is the energy required to release separate constituents?
- 4. What are oxidation potentials? How will they affect intergranular attack, pitting, etc. ?

2.2.5.4 Selecting The Electrolyte

From this analysis, the possible electrolytes for a specific alloy can be selected. A few electrolyte-alloy systems will be suggested. The selected electrolyte system for the specific alloy should be evaluated to determine the optimum composition and process conditions to obtain the best combination of cutting rates, repeatability, and surface finish. That electrolyte system which achieves the best end results will be chosen after consideration is given to toxicity and cost.

2.2.5.5 Testing The Candidate Electrolyte

It is our practice to determine the effect of the electrolyte concentration and temperature on conductivity. The range of concentration of a specific alloy and the temperature controlled to be exercised will be chosen by determining a practical combination of concentration and temperature wherein the conductivity will not vary over 5 percent. Usually, as the concentration increases, this conductivity range can be found within a process control of 1/2 pound per gallon of electrolyte chemical and a 6⁰F temperature spread.

If properties other than conductivity must be considered, such as oxidation potential needed, or the pH range required to inhibit selected etch, the required operating parameters for the electrolyte are established using the needed properties as a criterion. Finally, the selected electrolyte is used in calibrating tests to determine the other operating parameters.

2.2.6 APPLICATION

The information describing variables affecting the process will be used in pre-pilot studies made for this contract.

2.2.6.1 Optimum Operating Parameters

Optimum combinations of operating parameters will be established for various alloy-electrolyte systems by flat-plate ECM experiments.

1. Input information will be derived from:
 - a. Previous work in this contract
 - b. Production experience by General Electric
 - c. Experience publically reported by others.

2. Measure of results will be surface finish (profilometer and metallographic checks).
3. Statistical methods will be used in optimizing and evaluation.
4. The number of alloy-electrolyte system will remain flexible.

A tentative selection is:

- a. Ni base alloy(s)/NaCl (for reference)
- b. Fe base stainless alloy(s)/proprietary electrolytes
- c. Titanium base alloy(s)/proprietary electrolytes
- d. Refractory alloy(s)/proprietary electrolytes

2.2.6.2 Verify Mathematical Analysis

We will verify the validity of mathematical process analysis and of the "Cosine Function Relation" of metal removal for electrode design and excess stock calculations. A two-dimensional contour approximately 1/2 to 1 inch, with orientation angles of from 90 to 30 degrees will be machined with a pre-determined tool and optimum operating parameters. Amount of excess stock and starting gaps will be varied. Measure of results will be made by dimensional inspection and comparison of nominal versus actual contours (10 to 20 power magnification). The evaluation of results will be analyzed by statistical methods.

2.2.6.3 Electrode Materials (evaluated as explained in the results)

2.2.6.4 Film Control Studies

The effects of film control on surface finish and metal removal rates will be investigated. The methods are to be selected. We plan tentatively to try

current cycling and ultrasonic energy. Criteria and methods of evaluation will be same as those described in paragraph 2.2.6.1, to facilitate a direct comparison. Alloy-electrolyte systems are to be selected from groups investigated in paragraph 2.2.6.1.

2.2.6.5 Electrolytic Machining Techniques

We shall apply criteria established in paragraphs 2.2.6.1 and 2.2.6.2 to various machining techniques, to determine extent of correlation and specific (tool, equipment, control related) requirements. A number of techniques and alloy-electrolyte systems are to be selected. Our tentative choice of machining techniques are:

1. Turning
2. Single-point cutting (suitable for metal removal in any direction, with means of confining electrolyte to desired areas)
3. Cavity sinking.

Measure of results will be surface finish and metal removal rate.

2.2.6.6 Demonstration Techniques

Simulated hardware will be made for demonstrating the techniques evolved in paragraph 2.2.6.5.

2.3 RESULTS

2.3.1 IDENTIFYING OPERATING PARAMETERS

2.3.1.1 Experimental Plans

The objectives of the experiments done to date were:

1. To identify significant operating and process variables, and to compare them with those expected from the basic mathematical analysis.

2. To determine whether these variables are significant, not only for Rene' 41-NaCl alloy-electrolyte systems, but for other alloy-electrolyte systems

3. To approximate the effects of the variables mathematically.

The variables were first identified in a test series, JB-1, done previous to the work on this contract. The test series, JB-1, is reported in detail in General Electric Technical Information Series Report No. R62FPD320 entitled "Interim Report - Electrolytic Machining Process", Joseph Bayer, dated August 29, 1962. The report is available upon request in writing from the Technical Information Center, Mail Drop F-22, General Electric Company, Cincinnati 15, Ohio. Data from the G. E. report is repeated here when needed.

Test series JB-1 involved determining the effects of these variables during electrolytic machining:

1. The feed rate - F
2. The imposed cell voltage - E
3. The average inlet temperature - T
4. The fluid pressure - P
5. The sodium chloride (NaCl) concentration of the aqueous electrolyte - N
6. The citric acid concentration of the electrolyte - C

In test series JB-1, citric acid as well as NaCl was used in the electrolyte. Feed rates, temperatures, pressures and electrolyte chemical concentrations were selected and tested in combinations at five levels for each variable. The gap was maintained throughout the test at 0.004 inch.

The levels for testing in JB-1 were selected on a basis of practical limits used in production at that time.

Test series JB-2 run for this contract was run under conditions similar to JB-1 except that:

1. Citric acid was not included in the electrolyte
2. Fluid pressure was discarded as a variable
3. Gaps were varied between 0.002 and 0.0137 inch
4. Variations in NaCl electrolyte concentrations were restricted to those normally expected in production
5. The range of feed rates was increased to 10 inches per hour.

Citric acid, which proved in JB-1 to have no direct effect on the electrolytic process, was not used in JB-2. Pressure was held constant in JB-2, since no large significance could be attributed to pressure variations in JB-1. To reduce the number of test points, we did not explore gap variations in JB-1; we did explore the effect of gap in JB-2.

JB-2 not only provided a check on results obtained in JB-1, but also the opportunity to extend the test limits for feed rate and gap.

Test series JB-3 was run to compare the relationships of operating and process variables for the Rene' 41-NaCl alloy-electrolyte system to the relationships for a different alloy-electrolyte system. The two series (JB-2 and JB-3) differed as shown in Tables VI-5 through VI-12.

<u>Test Set-up</u>	<u>In JB-2</u>	<u>In JB-3</u>
1. Electrolyte	Single salt (NaCl), no apparent interaction with reaction products	(GK-4) - two-salt systems. Apparent inter-reaction with reaction products
2. Effect on selected etch	Electrolyte does not appear to inhibit	Electrolyte appears to inhibit
3. Chemical analysis of electrolyte	Can be done with ease	Can not be done practically
4. Affect of temperature on electrolyte conductivity	Nominal effect on conductivity	Conductivity affected more markedly by temperature than NaCl electrolyte
5. Alloy being cut	Rene' 41 nickel base	Tool steel - ferrous base (SAE 1026)

2.3.1.2 Analysis Of Data

2.3.1.2.1 Variables Affecting Metal Removal

From Faraday's law, we would expect that metal removal rate would vary directly with current density. If so, metal removal rate variations would be fully explained by current variations. In all the individual tests, the imposed voltage was determined so that the selected gap and feed rate were maintained constant throughout the test; thus, the metal removal rate should vary directly with the feed rate.

The results were:

**"PERCENTAGE OF ASSIGNED VARIATIONS
ATTRIBUTABLE TO OPERATING PARAMETERS
FOR CHANGES IN CURRENT"**

TEST SERIES

	<u>JB-1</u>	<u>JB-2</u>	<u>JB-3</u>
Feed Rate	99.76	99.54	99.27

If the removal rate varies directly with the current density:

$F = K \frac{I}{A}$ would explain the relationship

where F = feed rate in inches per hour

$\frac{I}{A}$ = current density in amperes per square inch

K = constant for the alloy being cut in cubic inches of metal removed per ampere-hour

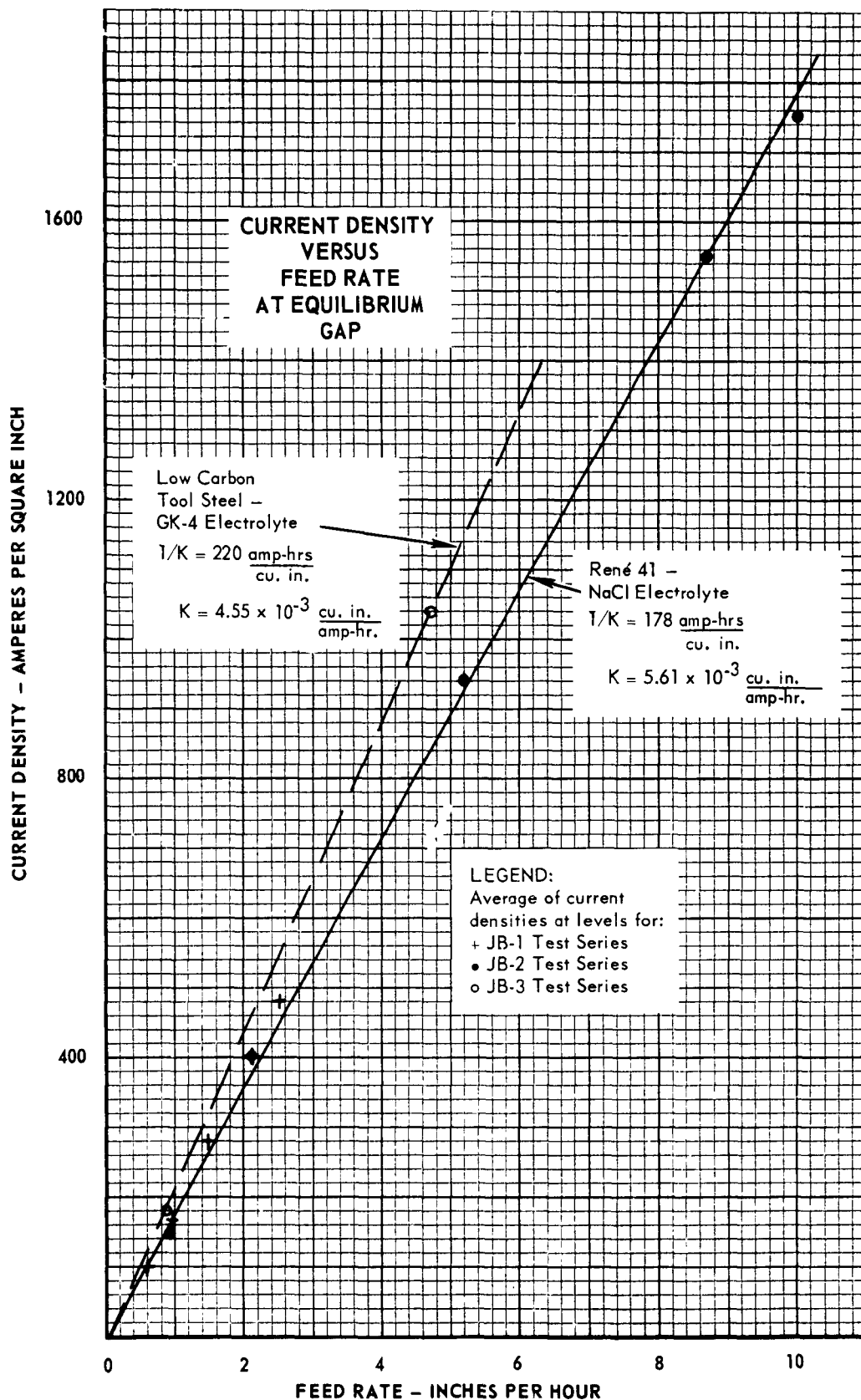
In figure 2-7, the average feed rate at selected testing levels is plotted against the average current density required for that feed rate.

The experimental levels for feed rates for each test series are:

FEED RATE

Inches/Hour

Test Series:	<u>JB-1</u>	<u>JB-2</u>	<u>JB-3</u>
	0.500 (1 pt)	0.776 (1 pt)	0.79 (8 pts)
	0.873 (16 pts)	2.127 (8 pts)	4.68 (8 pts)
	1.500 (9 pts)	5.39 (7 pts)	0 (1 pt assumed)
	2.127 (16 pts)	8.65 (8 pts)	
	2.500 (1 pt)	10.0 (1 pt)	



The average at each feed rate level is plotted, since the variation from the average is so small that the graph would not indicate the variations at each level. Average results for both JB-1 and JB-2 are plotted as shown. K for the Rene' 41-NaCl electrolyte system is 0.000561 cubic inch per ampere hour within the limits tested.

For JB-3, three points are plotted. One point is assumed to be zero feed rate - zero current. The other two points represent the average current density for each of the feed rate levels tested. These three points describe a straight line. The slope of this straight line is K for SAE 1026 steel - GK-4 electrolyte and is 0.000455 cubic inches per ampere hour.

2.3.1.2.2 Variables Affecting Imposed Voltage

From equation (11), we speculated on the interrelations among the variables for electrolytic machining.

Equation (11) states:

$$dt = \frac{LdL}{\frac{E_e K}{\bar{r}} - FL} \quad (11)$$

$$\text{In these tests, we set } \frac{dL}{dt} = 0 \quad (44)$$

$$\text{Therefore, } \frac{E_e K}{\bar{r}} - FL = 0 \quad (45)$$

$$\text{or } \frac{E_e K}{\bar{r}} = FL; \quad (46)$$

And, as shown, $E_e = E - \Delta E$.

$$\text{Thus, } E = \frac{FL}{\bar{r}K} + \Delta E \quad (47)$$

Then $\underline{E}$ should vary with $\underline{F}$, $\underline{L}$, $\underline{\bar{r}}$, $\underline{K}$ and $\underline{\Delta E}$.

We therefore, set $\underline{L}$, the gap, and $\underline{F}$, the feed rate, at test levels and held them constant during specific test. With $\frac{dL}{dt} = 0$; the feed rate, $\underline{F}$, will vary directly with the current density. $\underline{K}$, as demonstrated, is constant regardless of the electrolytic machining conditions.

In our discussion on electrochemistry, $\bar{r}$ and ΔE are process variables that are affected by other variables such as current density, electrolyte concentrations and temperatures, and that can be expressed as functions of these other variables. Thus, we reason that ΔE and $\bar{r}$ would be affected by:

1. The chemical concentration of the electrolyte
2. The temperature of the electrolyte
3. Current density and, because of the effect of the electrolyte

boundary layer, probably by:

4. Fluid pressure
5. The gap to perimeter relationship
6. The electrolyte density and viscosity which in turn is affected

by the temperature and the chemical concentration.

Therefore, in these tests, we expect that $\underline{E}$ will vary only with -

1. Feed rate
2. Gap
3. Electrolyte temperature

4. Chemical composition of the electrolyte

5. Fluid pressure.

Test results from JB-1, JB-2, and JB-3 substantiate the above statement.

The following tables show the percentage of assignable variation in cell voltage attributable to the other operating and process variables.

TABLE II-1

PERCENTAGE OF ASSIGNABLE VARIATION IN E
SHOWN IN TEST SERIES

<u>Variation</u>	<u>JB-1</u>	<u>JB-2</u>	<u>JB-3</u>
Feed Rate (F)	42.0	44.5	64.5
Temperature (T)	5.7	5.5	18.6
Gap (G)	(1)	32.8	1.8
Pressure (P)	0.4	(1)	0.0
Concentration (N)	40.3	1.3	(1)
<u>First Order Interactions</u>			
F x P	(2)	1.3	(1)
T x G	(1)	1.7	(2)
F x G	(1)	10.5	(2)
F x P	(2)	(1)	(2)
F x T	0.4	(1)	15.0
F x N	9.6	(2)	(1)
T x N	0.7	(2)	(1)
N x G	(1)	(2)	(1)
N x P	(2)	(1)	(1)
P x G	(1)	(1)	(2)
Sum of all effects signified by (2) in table ⁽³⁾ .	0.2	1.1	0.1
Total explained by variables	99.3	98.9	99.9

- (1) One of the variables being considered was not varied in this test series
- (2) These variables or variable interactions were not of sufficient magnitude to report individually
- (3) This is to sum up effects for each series of all numerical values designated by (3) in this column

In all three series, variations in E (the cell voltage) were shown to be at least 98.9 percent attributable to the operating and process variables predicted by equation (9).

2.3.1.2.3 Variable Interrelations

2.3.1.2.3.1 General

The variable interrelations of electrolytic machining have been hypothesized to be expressed as Ohm's law.

$$E - \Delta E = IR$$

We assumed in the technical discussion that the transient condition could be expressed as

$$dt = \frac{LdL}{\frac{E - \Delta E}{\bar{r}} K - FL} \quad (11)$$

We simplified the conditions for test by setting up conditions so that $\frac{dL}{dt} = 0$

then

$$\frac{E - \Delta E}{\bar{r}} K - FL \quad (48)$$

Then

$$E - \Delta E = \frac{F \bar{r} L}{K} \quad (49)$$

$\underline{K}$ can be determined independently

$\underline{F}$, $\underline{E}$ and $\underline{L}$ can be determined by test.

We next tried to express:

$\underline{\Delta E}$ and $\underline{\bar{r}}$ in terms of the environmental variables $\underline{T}$, $\underline{P}$ and $\underline{N}$.

In test series JB-1, we combined a number of simple electrochemical concepts by attempting to approximate the inter-relations:

$$\hat{E} = A_0 + A_1 F/N + A_2 FT + A_3 FP^{-.047} + A_4 \ln F + A_5 FT/N + A_6 F^3/N + A_7 F^3 T + A_8 \frac{FT}{\sqrt{P}} \quad (50)$$

In our next attempt to express the inter-relation, we assumed momentarily that the fluid flow was sufficiently high to reduce the boundary layer resistance to insignificance. We also assumed that the over-voltage was a constant.

If we use these assumptions, we can approximate the specific resistance of the NaCl solution from the data on NaCl conductivity available in the literature.

$$\frac{1}{2.54 (10^{-4}) N^{-.7} (T + 44)} \quad \text{is such an approximation for the specific}$$

resistivity where N = NaCl concentration at 72°F in gm/100ml of solution and T = temperature in $^\circ\text{F}$.

Much of the data from test series JB-1 and JB-2 fit this approximation:

$$E = 1.88 \frac{F}{K_1} \left(\frac{1}{2.54 \times 10^{-4} \times N^{-.7} (T + 44)} \right) G + 2.68 \quad (51)$$

where F = feed rate in inches per hour

K_1 = cc per ampere hour for Rene' 41 with NaCl electrolyte

or 5.61×10^{-3} cc per ampere hour

N = NaCl concentration at 72°F in gm/100ml of solution

T = temperature - $^\circ\text{F}$

G = gap in inches

However, not all the data fit this curve. Similarly in JB-3, an approximate expression -

$$E = 1.5 \frac{(F)}{K_2} \frac{(833.3)}{T} G + 2.6$$
 fit six out of eight points. K_2 for SAE 1026 steel was 4.55×10^{-3} cubic inches per ampere hours. $\frac{(833.3)}{T}$ expresses the specific resistivity for GK-4 electrolyte at 72°F . GK-4 is made to only one concentration. This type of approximation was rejected.

The data for JB-1, JB-2 and JB-3 is available for analysis as we study the electrochemistry in more depth. Our work in this area is described under paragraph 2.1.4.

2.3.2 ELECTRODE MATERIALS

2.3.2.1 Spark Resistance

2.3.2.1.1 General

We made a series of tests on several types of electrode materials to determine their resistance to damage caused by arcing. Such damage could be expected to result from any malfunction that permitted the two electrodes to come in direct electrical contact with each other; that is, when flow of

electrolyte should stop, when the electrodes should touch, or when voids should occur in the electrolyte. The material exhibiting the least damage from arcing would require the least work to repair. Since rework or repair is only one factor in evaluating the materials for cost, corrosion resistance, conductivity, and cost per cubic inch are listed in the detailed tabulation in Table II-2.

2.3.2.1.2 Procedure

A short length of one-inch rod of the material to be tested was turned to a 1/8-inch diameter, and the end turned to a point (30 degrees). The machined rod is mounted in the electrolytic machining test rig, with the pointed end in positive contact with a block of either Gentrode or M252 material. No electrolyte was used.

Direct current at a pre-set voltage is imposed across the electrodes for 0.25 second. The polarity used in this test is the opposite of that used in the actual electrolytic machining process; that is, the pointed tip under test is the anode. This polarity was used to impose conditions that have the most radical effect on the electrode material.

We first attempted to determine the extent of damage by measuring weight loss, but no correlation between weight loss and damage was observed; therefore, we evaluated the damage on the following comparative scale:

- 0 - no visible tip damage
- 1 - very slight visible tip damage
- 2 - slight tip damage

3 - moderate tip damage

4 - extensive tip damage

5 - severe tip damage

Two series of tests were performed. The first series was made, using a Gentrode cathode and imposing 15.3 volts on the circuit. The second series was run on only those materials that withstood the first test reasonable well, and consisted of 6 tests on each material. In this series, a cathode of M252 material was used, and the voltage was increased to 24.2. The results of each series of tests is shown in Table II-3. The experimental data are tabulated in Table II-2.

TABLE II-2

ELECTRODE MATERIAL EVALUATION

SUMMARY OF RESULTS

No.	Material	Alloys Tested	Estimated Cost (2) \$/cu. in.	Relative (3) Conductivity Copper=100%	Corrosion Resistance .001"/yr.	Spark Resistance (1)		
						@ 15.3 volts Gentrode anode Avg. -4 tests	@ 24.2 volts M-252 anode Avg. -6 tests	
1	Electrical Top Pitch Copper		0.372	101	1.6 ⁽⁴⁾	0		4
2	Free-cutting Copper (1/2 Te)		0.350	90	N. D. ⁽⁷⁾	2		4
4	Aluminum Silicon Bronze		0.326	8	1.9 ⁽⁴⁾	2		4
5	Naval Brass		0.335	26	1.7 ⁽⁴⁾	3		4
7	Commercial Bronze		0.55	44	1.8 ⁽⁴⁾	3		-
9	Titanium (99%)		0.94	1	Nil	1		5
10	316 Stainless Steel		0.12	2.5	Ave 0.0 ⁽⁵⁾	1		3
11	Elkonite 30W3		14.60	28	N. D. ⁽⁷⁾	2		3
12	Gentrode		1.40	.14	N. D. ⁽⁷⁾	2		4
13	Manganese Copper		0.347	24	2.0 ⁽⁴⁾	4		-
14	Free Machining Copper (1% Pb)		0.347	80	N. D. ⁽⁷⁾	3		-
15	Cartridge Brass		0.327	28	2.1 ⁽⁴⁾	3		4
16	Cupro-Nickel (10%)		-	9	0.5 ⁽⁴⁾	2		4
17	Free-cutting Brass		0.335	26	N. D.	3		4

(1) Code for Spark Resistance: 0 = no damage, 1 = very slight tip damage, 2 = slight tip damage, 3 = moderate tip damage, 4 = extensive tip damage, 5 = severe tip damage

(2) Based on quotations to General Electric - Evendale

(3) Reported in Metals Handbook, Volume I, Eighth Edition

(4) Reported in Metals Handbook, Volume I, Eighth Edition on page 986, Table 3 (Test A), Exposed to sea water at Kare Beach, North Carolina

(5) Reported in Metals Handbook, Volume I, Eighth Edition on page 559, Table 4. Exposed to sea water at Kare Beach, North Carolina. One pit only in 944 days of exposure.

(6) Tends to chip. Must be machined with care.

(7) N. D. - No data available.

(8) Abrasion tests not complete.

TABLE II-2 (Continued)

Material Number	Machinability Basis: 100%-Free Cutting Brass	Abrasion (8) Resistance	ASTM or Spec. Number	Chemical Analysis				
				Cu	Pb	Sn	Zn	Others
1	20		ASTM B152	99.99+				
2	80		ASTM B301	99.5				0.5 Te
4	60		ASTM B21 Alloy A	90.85				7.15 Al, 2.0 Si
5	70		ASTM B21 Alloy C	60.0	1.75	6.5	37.6	
7	20		ASTM B134 Alloy 2	90.0			10.0	
9	20		ASTM B265, 58T Grade 4					99% Ti
10	20		-		(17 Cr, 12 Ni, 2.5 Mo, Balance Fe)			
11	(6)		Mallory Material		Cu - W Mixture			
12	(6)		General Electric Material					
13	30		ASTM B136 Alloy A	58.5	1	39	1.4	Fe
14	98		-	99	1			
15	30		ASTM B19	70			30	
16	20		MIL-L-15726A	88.7				10 Ni, 1.3 Fe
17	100		ASTM B16	61.5	3.0		35.5	

TABLE II-3
SPARK RESISTANCE TEST

Material Number	Description	15.3 Volts				Avg. of Tests 1 to 4	24.2 Voltage				Avg. of Tests 5 to 10		
		Gentrode Electrode					M-252 Electrode						
		1	2	3	4		5	6	7	8		9	10
1	Elec. T. P. Copper	0	0	0	1	0	5	4	4	4	4	3	4
2	Copper (1/2 Te)	2	2	2	1	2	5	4	4	4	4	4	4
4	Al-Si-Bronze	2	2	4	1	2	5	4	4	4	4	4	4
5	Naval Brass	4	4	1	1	3	5	4	4	4	4	4	4
7	Commercial Bronze	2	4	4	4	3		Not Done					
9	99% Titanium	2	1	1	1	1	5	5	4	5	4	4	5
10	316 S. S.	0	1	1	0	1	1	3	4	3	2	3	3
11	Elkonite 30W3	2	1	1	2	2	3	3	3	2	2	3	3
12	Gentrode	2	2	1	1	2	5	4	4	4	4	4	4
13	Manganese Copper	4	4	5	5	4		Not Done					
14	Copper 1% Pb	3	2	3	1	3		Not Done					
15	Cartridge Brass	2	4	1	5	3	4	4	4	4	4	4	4
16	Copper Nickel (10%)	2	2	1	1	2	5	3	4	3	3	3	4
17	Free-cutting Brass	0	1	4	5	3	5	4	4	4	4	4	4

0 = no visible tip damage
1 = very slight tip damage
2 = slight tip damage
3 = moderate tip damage
4 = extensive tip damage
5 = severe tip damage

2.3.3 VISUAL FLOW STUDY

2.3.3.1 General

We made a number of attempts to observe electrolytic action through a microscope, and to photograph the action. Original work was begun, using a stroboscopic light which had a duration of 1/3000 second for illumination. A Speed Graphic and a Polaroid-Land camera were used at different times, the former with 70 mm roll film (ASA speed 32), and the latter with a Polaroid film having an ASA speed of 3000.

A variety of set-ups were used in the fifteen attempts made. Fluid pressures of 150 psi, and gap widths of 0.010 inch were generally used as the norm. The process itself was operating during most of the photographic attempts. None of the resulting exposures was clearly readable.

Discussions with photographic consultants indicated that greater success might be obtained by using a Schlieren system to determine whether refraction of parallel beams of light, caused by heat strata (density layers) in a saline solution, could be photographed. If successful we expected the refraction pattern to suggest the velocity gradient within the gap.

2.3.3.2 Equipment

The equipment used in this study consisted of a micro-Schlieren optical arrangement, cameras, and modification of the electrolytic machining test rig, each of which consisted of the following components:

1. Micro-Schlieren apparatus
 - a. 15-x achromatic eyepiece
 - b. 6-x achromatic objective
 - c. Bausch & Lomb microscope body, equipped with coarse and fine adjustments
 - d. Adjustable knife edge (2 adjustable blades, 90 degrees apart, for cutoff of x and y parallel beams)
 - e. 0.040-inch pin-hole disc
 - f. Condensing lens
 - g. High-intensity light sources
 - (1) 50-watt Bausch & Lomb microscope light
 - (2) 300-watt General Electric high-intensity continuous arc light
2. Photographic apparatus
 - a. Speed Graphic camera, without lens, having a maximum shutter speed of 1/40 second
 - b. Polaroid-Land camera, 3 1/2 x 4 1/2 image size
 - c. Milliken high-speed, 16 mm, motion picture camera. (Shutter speeds of 1/640, 1/2000, 1/10,000, and 1/20,000 second available)
3. Electrolytic machine test rig
 - a. Flow entrance chamber
 - b. Anode-cathode electrolytic machining chamber, sandwiched between two plates of Schlieren-quality fused silica
 - c. Flow exit chamber

The micro-Schlieren apparatus was bench-tested for sensitivity, focal length, and adjustment of the knife edges by introducing a high-velocity jet of air from a 0.30-inch-ID tube into the inspection area. The apparatus clearly showed the shock wave created by the air jet. It was noticeably less sensitive than the much larger systems used in G. E.'s Applied Research Operation, but the shock wave could be photographed clearly.

2.3.3.3 Test Procedures

The photographic system, consisting of the Speed Graphic and Polaroid-Land cameras arranged on a single optical axis, were attached to the microscope body to form a photomicrographic camera. The apparatus was mounted in position to photograph the electrolytic machining cell, which was trans-illuminated with the 50-watt microscope light. Eight exposures were made on Polaroid film, having a speed of ASA 160. Four of the eight exposures had sufficient density to print properly, but the 1/40-second shutter speed was too slow to arrest the image of the moving fluid in the cell. Subsequent evaluation showed that the fluid velocity across the 0.10-inch gap varied from 22 feet per second, at 20 psi, to 66 feet per second at 180 psi. Since the magnification (90-power) increased the speed of the image across the focal plane proportionately, extremely intense illumination, and shutter speeds of only a few microseconds would have been required to obtain a useable photograph.

Following the trials with the still camera set-up, 3 takes were made with the 16 mm Milliken high-speed motion picture camera using DuPont Superior film having a speed of 250 ASA. These takes were made on the following electrolytic machining set-up:

Potential:	8.2 volts
Current:	5 amperes
Feed:	None

Take No. 1

The Milliken camera was set at a distance of 5-6 inches from the objective of the microscope, and operated at 200 frames per second, with a 7 1/2-degree shutter opening to obtain a 1/10,000 second exposure. The micro-Schlieren cell was trans-illuminated with the 300-watt high-intensity arc light. The film was exposed as follows:

1. One second (about 200 frames) of entrained air bubbles passing through the gap.
2. One second of fluid flow at 110 psi.
3. Four seconds (about 800 frames) of the electrolytic machining process in action.

The only exposure obtained during this take showed the one-second air-bubble formation and flow, and the one second of fluid flow. Both exposures were too poor to permit adequate reading. The sludge created by the process reduced the light passing through the solution so much that no image at all could be seen on the 4-second exposure. (Refer to paragraph 2.3.4 for a more extensive discussion of sludge formation.)

Take No. 2

The Milliken camera was re-set to a 72-degree shutter opening, and 400 frames per second speed, using the same lighting and positioning of elements as in Take No. 1. This adjustment resulted in an exposure of 1/2000 second. The image was improved in both one-second exposures, but not in the 4-second process exposure. None was of adequate quality.

Take No. 3

The Milliken camera was re-set to 128 frames per second using all other settings established in Take No. 2. This adjustment resulted in an exposure of 1/640 second. The density of the image was satisfactory in both one-second exposures, and the final process exposure was somewhat improved; however, the image was too blurred to be readable because of high fluid velocity and slow shutter speed.

2.3.3.4 Conclusions

On the basis of the information obtained from the three takes described above, we calculate that 1/10,000-second shutter speed would be fast enough to stop motion of the fluid at 80-90 psi, or about 40 feet per second. The limiting factor in this procedure was the intensity of the light source. The optical density of the solution increases so greatly, because of the development of sludge in the process, that an extremely intense light source would be required for adequate exposure. Since such equipment would be disproportionately expensive and cumbersome, we decided to abandon further attempts at photographing the process in action. Photographs indicated that the sludge appeared to be forming near the cathode interface. However, we may be observing the formation of hydrogen which we would logically expect at the cathode.

2.3.4 SLUDGE CONTENT

As a result of the chemical action taking place during electrolytic machining, insoluble by-products are formed in the electrolyte. These insoluble by-products are called sludge. When NaCl is used as an electrolyte in the cutting of Rene' 41, the by-product is presumed to be part metallic hydroxide and part flaked off metal.

The sludge needs to be removed, for if it is not, it will clog the gap, increase the viscosity and the density of the electrolyte, and reduce the conductivity of the electrolyte.

The intent of this series of tests was to determine -

1. What effect does the sludge have on the following electrolytic properties:
 - a) conductivity or resistivity
 - b) viscosity
 - c) density
2. What would be a reasonable limit to impose on the amount of sludge, or what filtration criteria should we establish.

Our first problem was to determine how to designate sludge content. If we could easily extract water from the sludge, we could base the sludge content on weight of sludge for volume or weight of water. However, this is expensive and difficult. The cost and difficulty can be justified for laboratory work, but not for production measurement. We, therefore, decided to use the comparative volume of sludge centrifuged out of a stated volume of electrolyte.

This measure has its disadvantages in that with time the particle size and other physical and perhaps chemical properties of the sludge will change, and will thus affect the volume of sludge measured by centrifuging. Nonetheless, we have observed for some time that the change in the sludge characteristics is not sizable enough to negate our using this practical measure of sludge content.

The criterion we adopted is to centrifuge four 10-millimeter samples of electrolyte, at full speed, for 60 seconds in our laboratory centrifuge - International Clinical Centrifuge, 115 volts, Model No. CL941068-1. We average the results and report the sludge content as cc of sludge separated for 10 cc of electrolyte. We then calibrate any other centrifuge results to this laboratory standard. For example, we are using another centrifuge in production which is so calibrated.

To make our laboratory samples for tests, we combined varying proportions of an electrolyte purposely loaded with sludge, and a totally clarified electrolyte. The sludge content had very little, if any, effect on the specific gravity of the electrolyte from ranges of sludge from 0.3 cc to 7.1 cc of centrifuged sludge per 10 cc of electrolyte. All tests were made at one temperature. (Refer to Tables II-4 and II-5.)

The viscosity of the electrolyte is affected by sludge content. Note in figure 2-8 that the viscosity rises at a slope of approximately 2.1 centipoises per unit of sludge content, up to approximately a sludge content of 5.5 cc of

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TABLE II-4

SLUDGE CONTENT VS SPECIFIC GRAVITY

Insoluble byproducts from electrolytic machining
of Rene' 41 with NaCl electrolytes.

Test 1 - Comparison of effect of sludge on specific gravity

Sample No	Description	<u>Temperatures</u> <u>Specific Gravity</u>		Temp. °F	Sludge Content - cc sludge centrifuge per 10 cc electrolyte
		By weight method	By hydro- meter		
1	High sludge content	1.189	1.180	75	3.70
2	Clarified electrolyte	1.178	1.170	75	0.36
3	Equal mix, solutions 1 and 2	1.178	1.170	76.2	2.18
4	1/3 soln 1, 2/3 soln 2	1.170	1.168	76.0	1.55
5	1/4 soln 1, 3/4 soln 2	1.182	1.166	76.2	1.15

Test 2 - Repeat of Test 1

Sample No	Description	<u>Specific Gravity</u>		°F	Sludge Content - cc sludge centrifuge per 10 cc electrolyte
		By weight method	By hydro- meter		
6	4/5 soln 1, 1/5 soln 2	1.185	1.178	74.5	5.62
7	2/3 soln 1, 1/3 soln 2	1.182	1.174	74.5	4.50
8	1 soln 1, 3/4 soln 2	1.179	1.170	74.5	3.75
9	1 soln 1, 1 soln 2	1.179	1.168	74.5	3.30
10	1 soln 1, 5/4 soln 2	1.177	1.168	75.3	2.80
11	1 soln 1, 3/2 soln 2	1.177	1.170	75.3	2.55
12	1 soln 1, 7/4 soln 2	1.180	1.166	75.3	2.40
13	1 soln 1, 2 soln 2	1.178	1.170	75.3	2.20

Test 3 - Repeat of Test 1 except sludge solution further concentrated by removal
of water.

TABLE II-5

**EFFECT OF SLUDGE ON
SPECIFIC RESISTIVITY, VISCOSITY AND SPECIFIC GRAVITY**

Basis: Aged NaCl electrolyte - from Rene' 41
electrolytic machining process

Point No.	Weight of Sludge per 10 cc elec- trolyte. In- cluded dried salts (gm)	Sludge Content cc sludge centrifuge per 10 cc electrolyte	Specific Gravity by weight method	Viscosity Centi- poise	Specific Resistivity ohm-cm	Temp. Electro- lyte ° F
1	0.1749	0.30	1.172	2.14	7.18	72.5
2	0.4010	6.80	1.184	76.9	6.75	73.8
3	0.1803	3.30	1.176	6.80	6.09	75.8
4	0.2732	5.35	1.177	13.20	6.23	75.8
5	0.2111	4.25	1.178	10.20	5.78	75.5
6	0.2564	3.38	1.177	8.70	5.19	69.4
7	0.1214	2.50	1.180	6.10	5.79	69.1
8	0.1350	2.30	1.176	5.48	5.91	69.5
9	0.1169	2.05	1.176	4.94	6.30	69.5
10	0.1191	1.88	1.177	4.56	5.55	69.5

centrifuged sludge per 10 cc of electrolyte. Thereafter, its viscosity appears to rise at greatly accelerated rates. Viscosity then appears to be quite sensitive to sludge. Also note from figure 2-8, that viscosity of the sludge-free electrolyte appears to be approximately one centipoise. These tests were run at approximately 75°F.

These tests lead one to conclude that if the fluid flow is sensitive to viscosity as it is in laminar flow, the sludge content would need to be controlled within narrow limits. If the fluid is not sensitive to viscosity as in turbulent flow, the sludge content can be controlled less stringently, but in any event should not be allowed to exceed a unit of 5 cc of sludge per 10 cc of electrolyte. The data is summarized in Table II-5.

The specific resistivity does not appear to be sensitive to sludge content. The specific resistivity, as measured, is more sensitive to experimental error than to sludge content, as shown in figure 2-9 from data summarized in Table II-5. The specific resistivity measurements range from 5.2 to 7.2 ohm-cm with no observable reason for their fluctuation. Where one would expect low resistivities at low sludge content, we have our highest reading.

Let us look at the affect of sludge as reported in Table II-6. In this test, we are trying to correlate the expected viscosity, specific gravity, and specific resistivity of aged NaCl electrolyte, used to cut Rene' 41, with the electrolyte's NaCl content, temperature, and sludge content. The effect

TABLE II-6
EFFECT OF NaCl CONCENTRATION, TEMPERATURE AND SLUDGE
ON
ELECTROLYTE SPECIFIC GRAVITY, VISCOSITY AND SPECIFIC RESISTIVITY

Points Selected for Box Design and Analysis
Electrolyte taken from tank of machine used to machine Rene' 41

Point	Conc. NaCl g/100ml	Sludge - cc sludge centrifuge per 10cc electrolyte	Temp. ° F	Specific Resistivity ohm-cm	Viscosity Centipoise	By hydrometer	By weight
1	16.60	6.25	80.0	6.70	69.6	1.142	1.131
2	16.60	5.40	95.2	4.71	56.2	1.140	1.129
3	16.65	4.50	111.1	4.28	55.2	1.134	1.128
4	16.70	2.43	81.0	5.13	6.02	1.132	1.135
5	16.70	2.30	95.6	5.19	5.40	1.129	1.132
6	16.70	2.15	111.0	3.92	4.94	1.126	1.139
7	16.65	1.15	79.8	5.91	2.34	1.130	1.131
8	16.65	1.00	95.0	4.37	2.18	1.127	1.128
9	16.65	1.05	110.4	3.92	1.98	1.123	1.124
10	19.60	6.30	79.8	4.77	91.0	1.146	1.158
11	19.60	5.95	95.0	4.28	78.6	1.140	1.153
12	19.60	5.55	110.9	4.01	70.5	1.137	1.151
13	19.70	2.50	79.8	5.42	5.56	1.154	1.158
14	19.70	2.35	95.8	5.93	5.20	1.150	1.153
15	19.68	2.25	110.3	3.50	5.10	1.149	1.150
16	19.60	1.16	79.9	4.76	2.56	1.154	1.154
17	19.60	1.05	95.0	4.01	2.33	1.149	1.150
18	19.60	0.97	111.0	3.50	1.93	1.146	1.147
19	22.40	6.35	80.0	4.83	67.6	1.165	1.173
20	22.40	5.50	95.4	3.98	64.9	1.160	1.170
21	22.30	5.00	110.8	3.50	65.0	1.155	1.165
22	22.30	2.35	80.0	4.73	5.42	1.171	1.172
23	22.30	2.15	95.0	4.13	4.84	1.168	1.172
24	22.30	2.02	111.0	3.62	4.76	1.168	1.169
25	22.20	1.20	81.0	4.46	2.62	1.170	1.172
26	22.20	1.10	95.2	4.21	2.40	1.167	1.169
27	22.30	1.05	110.4	3.68	2.30	1.164	1.165
28	16.60	2.30	80.0	6.27	6.10	1.134	1.130
29	16.60	2.10	95.0	4.70	5.38	1.126	1.128
30	16.60	2.00	111.0	4.32	4.90	1.121	1.128

of the concentration and temperature requires further testing and is not completely analyzed. We have looked at the effect of sludge, however.

In this test series, the specific gravity appears to be rather insensitive to sludge content, although there is a small increase in specific gravity at high sludge contents. The specific resistivity again appears to be insensitive to sludge content. The viscosity is sensitive to sludge content to a much greater degree than to either temperature or salt concentration. Again, we conclude that the sludge affects the viscosity of the electrolyte, but does not appear to affect either the specific gravity or specific resistivity markedly.

In attempts made to photograph the fluid flowing through the gap, the most significant observations made were that a dark material seemed to be forming close to the cathode. This phenomenon was noted in all photographs of the gap while the process was on. This dark material may be the sludge forming near the cathode. If so, it may help to indicate the relative rates of migration of the cations and anions. We may not be seeing sludge formations but perhaps the formation of hydrogen.

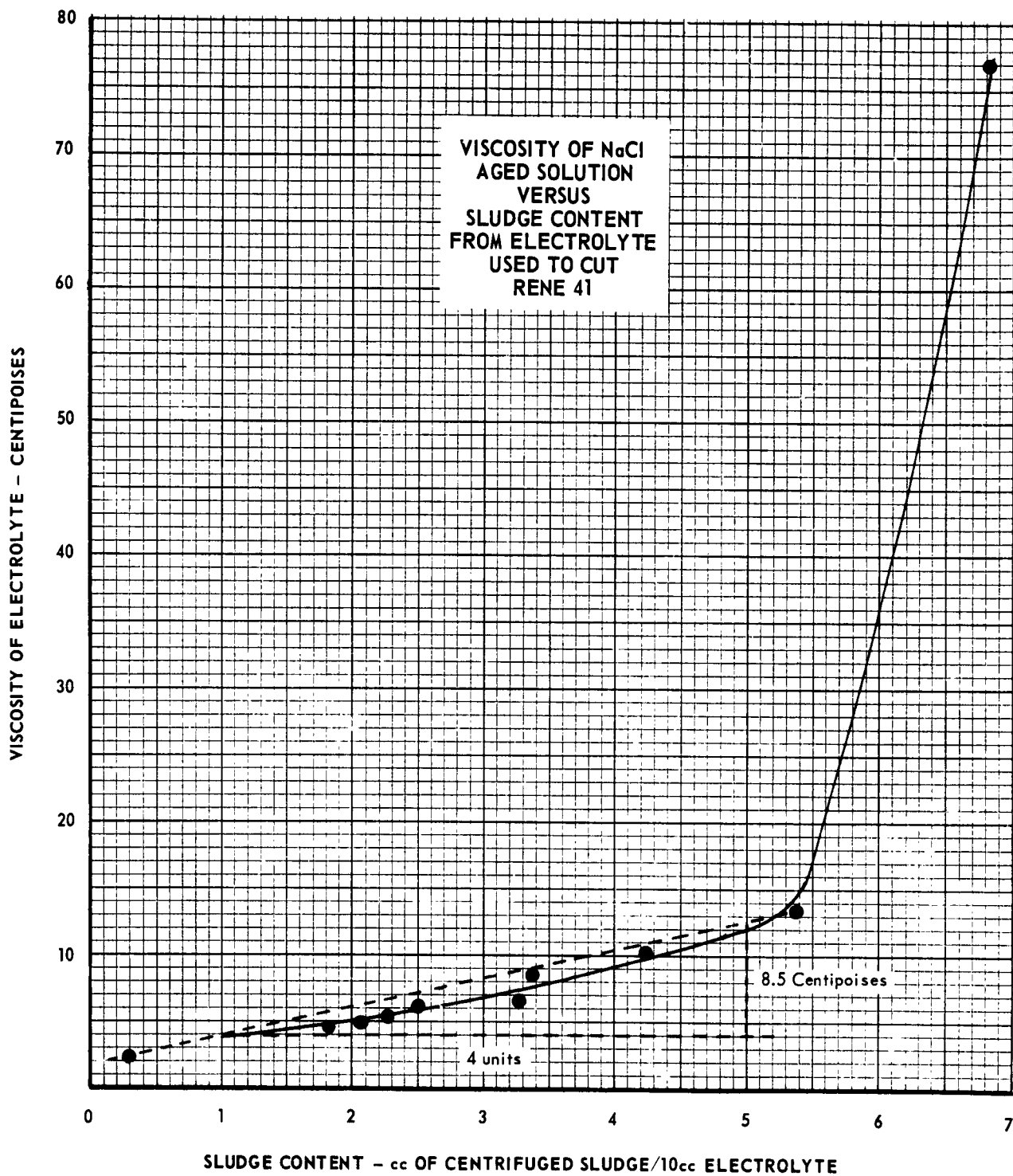


Figure 2-8

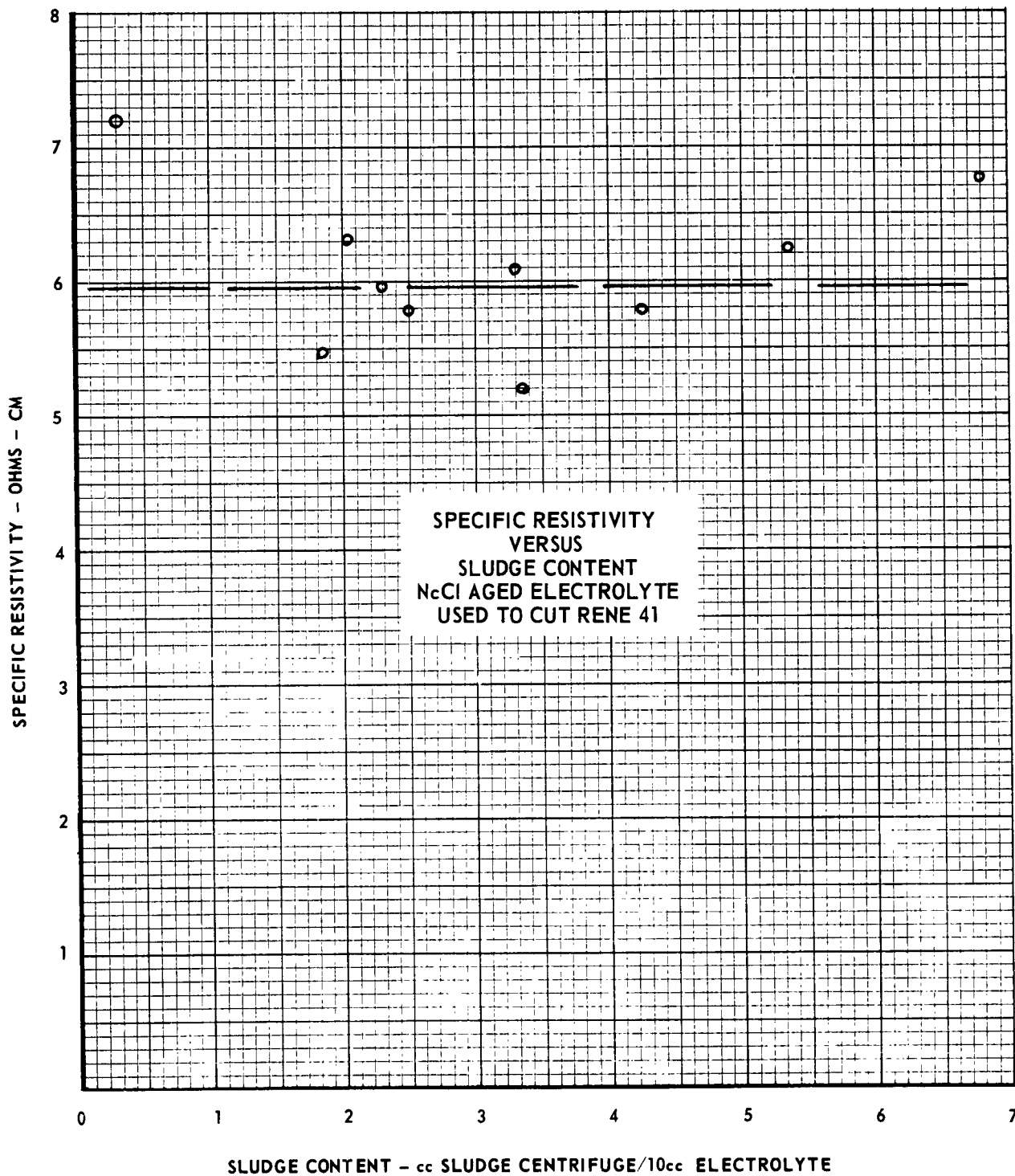


Figure 2-9

SECTION III

CONCLUSIONS

3.1 GENERAL

The conclusions listed below as summary statements are derived from the theoretical and experimental investigations referred to in Section II, Discussion. Since this is an interim report, no attempt has been made to evaluate the probable importance of these conclusions to the final outcome of the project; instead, they are being used as guides to further investigations planned for the future. For that reason also, many of the conclusions that we have reached are expressed as generally as possible to avoid undue restriction of future lines of endeavor.

3.2 SUMMARY

The following conclusions are justified by the work that has been done so far:

1. In specific alloy-electrolyte systems, within the ranges tested, metal removal rate varies directly with current density.
2. Significant variables that determine the voltage required are:
 - a. Current
 - b. Composition of electrolyte
 - c. Temperature of electrolyte
 - d. Age of electrolyte
 - e. Distance from tool to workpiece (gap)
 - f. Fluid flow conditions in gap
3. Under controlled conditions, a specific gap between points on the tool and points on the workpiece is established. This gap is known as "equilibrium gap".

4. Theoretically, equilibrium gap requires infinite time to establish itself; however, in practice, it is established within desired engineering tolerances in a reasonable finite time.

5. When equilibrium gap has been established, feed-rate varies directly with current density.

6. The relationship among variables at any moment during machining can be expressed as:

$$dt = \frac{LdL}{\frac{E - \Delta E}{F} K - FL}$$

7. The inter-relationships among variables that control electrolytic machining may be explained as electrochemical phenomena, and have been expressed mathematically. We expect to use these expressions in formulae that show explicit relationships of process variables in the electrolytic machining process.

8. Cathodic tools used to machine simply contoured workpieces can be designed by competent tool designers, using a trigonometric method explained in the text.

9. Cathodic tools for complex-contoured workpieces can be designed with the aid of a computer. The basis for programming a computer is explained in this report.

10. Operating parameters for electrolytic machining can be expected to be approximated with the aid of a computer. The basis for programming the computer is explained in detail in the report.

11. The sludge-content of the electrolyte has some effect on its viscosity. At reasonably low concentration, sludge has little effect on specific gravity or conductivity. Proposed control limits for NaCl electrolyte are included in the text.

12. A reasonable testing program for determining composition and specific physical properties of electrolytes to be used in machining specific alloys can be evolved.

SECTION IV

RECOMMENDATIONS

4.1 GENERAL

At as early a stage of development as is represented by this report, no final recommendations can be made. Our future plans for continuation of the development work already begun, comprise the remainder of this Section. These plans are general enough to permit some flexibility in determining significant areas for intensive investigation as the work progresses.

4.2 FUTURE PLANS

4.2.1 PROCESS INFORMATION

4.2.1.1 Fluid Flow Study

We shall determine the effect of fluid flow on the resistance of the electrolyte during machining. We have tested results on the effect of varying percentage of sludge, concentration of NaCl, and temperature on the specific conductivity of the electrolyte.

Returning to Ohms law, wherein

$$E - \Delta E = IR,$$

we are going to determine ΔE for initial conditions before the double layer has been established. When the circuit is closed, the initial resistance will only be that of the specific resistivity of the electrolyte. We can calculate that resistivity for selected gaps based on $R = rG$,

where r = specific resistivity

$$G = \text{gap}$$

We record the initial E_0 and I_0 . Then:

$$\Delta E = E_0 - I_0 r G$$

We shall then attempt to correlate the ΔE we determined in this manner to the other operating variables and the current.

We shall have established a method of calculating ΔE .

In our next series of tests, we will calculate ΔE and then determine R from

$$R = \frac{E - \Delta E}{I}.$$

We shall change the flow conditions by changing the gap, the velocity (by changing the pressure drop) and the pressure level in the gap. By using various electrolyte temperatures and concentrations, we will alter not only the basic conductivity of the electrolyte but the density and the viscosity of the electrolyte which will affect the fluid properties. $\underline{R}$ will then be a function of the flow properties (P , ΔP , T and N), the conductivity (itself susceptible to $\underline{N}$ and $\underline{T}$) and probably the current $\underline{I}$.

We are assuming that the resistance alteration because of flow will be a function of Reynolds number.

4.2.1.2 Temperature Changes within the Gap

As the electrolyte passes through the gap, the temperature will rise because of current flow. The change in temperature will cause dimensional changes in the gap. We shall study these temperature and gap dimensional changes. We have calibrated our experimental rig so that we can correct for temperature changes caused by frictional heat outside the gap, and by heat losses from the system. We shall attempt to relate the temperature gradients in the gap to the process variables.

4.2.1.3 Energy Change

The relationship between $\underline{\Delta E}$ and the energy changes, and between $\underline{R}$ and the energy changes, during electrolytic machining, has been discussed. Test methods whereby we relate $\underline{\Delta E}$ and $\underline{R}$ to energy changes are planned. $\underline{\Delta E}$ is related to chemical changes, and $\underline{R}$ to thermal changes.

4.2.1.4 Metallurgical Damage

Metallurgical damage can be expected under certain electrolytic machining conditions. When using NaCl electrolyte with nickel-base alloys, selective etch occurs at current densities below 40 amperes per square inch. Selective etch is defined as different removal rates in an alloy surface in the same general area which results, in some cases, in pitted surface and, in others, as intergranular attack. In still other cases, the composition of the electrolyte contributes to selective etch.

The mechanism which contributes to selective etch, and the mechanism for inhibiting selected etch will be explored.

The smoothness of the surface milled by electrolytic machining is allied to selective etch. We shall test to determine methods of obtaining smoother finishes.

4.2.1.5 Electrolyte Search

The method to be followed in selecting specific electrolytes for cutting specific alloys has been stated in the Technical Discussion. We plan to select electrolytes for a number of alloys based on this system. We are now working on selection of an improved electrolyte for specific ferrous alloys. We shall be looking for improved electrolytes for titanium, refractory alloys, and other super-alloys.

After selecting the electrolyte, we shall determine the best ranges of chemical composition and temperatures to be used during machining, so that we can achieve good process control in production.

4.2.1.6 Validity of Theory

We have reported attempts to express interrelations of process variables. We have run tests and analyzed data to see whether we can fit equations expressing hypothetical relationships. To date, we have not been successful.

We are continuing to analyze the elaborate chemical relationships based both on our knowledge and that available in literature. This analysis will continue. The data we have accumulated, and that which we will generate in the future, will be analyzed in terms that each mathematical analysis indicates. We feel confident that when we define the mechanisms occurring during machining, we shall be able to express them mathematically, and that our data will fit the mathematical model which more truly reflects the mechanisms occurring.

4.2.1.7 Electrode Materials

We shall complete the evaluation of electrode materials.

4.2.2 APPLICATION

4.2.2.1 Determining Specific Operating Parameters

We have identified the process variables of electrolytic machining. We shall determine operating parameters for specific alloys and simulated hardware. To do this, we shall employ the methods of selecting optimum parameters through design of experimental techniques. We intend to do this enough times to demonstrate the validity of the method. We shall repeat the method in the process specifications written for this contract.

4.2.2.2 Computer Approach

In the Technical Discussion we have discussed how we plan to use the computer to design our tools. We plan to program the computer to approximate operating parameters. We expect to determine through our programming such data as the required tolerance for the raw material specified and what excess stock is required to permit machining a part to specified tolerances. We shall be making the necessary analysis of our process to be able to write such a program, and we shall program the computer. We plan to evaluate the data from the computer by cutting parts from computer instructions and comparing this to the expected results. We, of course, revise the programming when the experiments show the computer results not consistent with actual results.

4.2.2.3 Tool Design

The method of designing tools and specifying raw material tolerances for parts which do not have radical changes in scope has been discussed in the Technical Discussion. This design method does not require a computer to do the calculations. We have already demonstrated in previous tests that this method is valid. We shall demonstrate it again during this contract.

4.2.2.4 Flow of Fluid Control

Controlling the flow of fluids during machining requires properly designed tools and fixtures. We shall experiment with methods of achieving this control.

4.2.2.5 Machining Technique and Demonstration Hardware

We shall continue our development of electrolytic machining techniques. Any method of machining electrolytically requires the exercise of restraints on the process variables. We shall determine how to exercise these restraints for each technique. We plan to turn, sink cavities, cut with a single point tool, and do an operation which combines linear and rotating motion. We shall machine simulated shapes to illustrate the process capabilities of the methods of machining.

4.2.2.6 Design Criteria Established

From our tests on the process variables, we shall establish process control criteria for:

- a) Fluid pressure
- b) Electrolyte temperature
- c) Electrolyte chemical concentration
- d) Feed rate
- e) Voltage
- f) Purity of chemical used in electrolyte

From our experience and tests, we shall specify the functional criteria for the design of:

- a) Filter system
- b) Pumps and plumbing
- c) Power system
- d) Electrical system
- e) Fixtures
- f) Tools
- g) Electrolyte storage system
- h) Stock distribution
- i) Heat exchangers
- j) Basic machine

We expect to run studies in the areas which need attention, such as filtration systems and ripple.

The criteria will govern an electrolytic machining system that will machine parts which meet or exceed:

- a) Surface not rougher than 10 micro-inches
- b) Surface defects not to penetrate over 0.0001 inch
- c) Dimensional tolerances to within 0.001 inch on any dimension up to 10 inches
- d) Penetration rate of 0.100 inch/minute over a workpiece area of 10. sq.in.

4.2.3 DOCUMENTS

We shall summarize the specific knowledge we have gained in these documents:

1. A detailed design specification.
2. A detailed process specification.
3. A final technical documentary report.

SECTION V

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GLOSSARY

ANODE

The positive pole in an electrolytic cell or system. In this study, the workpiece, by being anodic, gives up or yields metal.

CATHODE

The negative pole in an electrolytic cell or system. In this study, the tool by being cathodic shapes or forms the anode or workpiece.

CELL VOLTAGE

The imposed voltage on the cell between any point on the cathode surface and any point on the anode surface. Unless otherwise noted, the cell voltage is assumed constant across the cathode and anode.

CONCENTRATION

A weighted amount of chemical material for each unit of liquid.

CONDUCTIVITY

The reciprocal of resistance expressed in mhos.

CONTROL

A restraint imposed on the system which tends to stabilize conditions within the electrolytic cell.

DISTURBANCE

An event causing an imbalance in an equilibrium condition. Here, it is used mostly as a happening that changes fluid flow conditions on a gross scale.

EDDY CURRENTS

Those areas of disturbed fluid flow immediately downstream from, and usually behind, an obstruction. Eddy currents are also produced by abrupt section or pressure changes.

ELECTROLYTE

A material, here usually a fluid, which acts as a current conductor between anode and cathode.

ELECTROLYTIC CELL

For this discussion, the electrical circuit which is bounded by a designated surface of the cathode and a designated surface of the anode. It includes that portion of electrolyte between those surfaces, and through which the cell current is flowing.

EQUILIBRIUM

A state of balance produced by the counteraction of two or more forces.

EQUILIBRIUM GAP

The gap between anode and cathode established as the resultant of all interactions of the process.

FEED RATE

Rate of relative motion of cathode towards anode. Expressed in terms of distance per unit of time.

GAP

The distance between anode and cathode.

LAMINAR FLOW

That type of fluid flow where all of the fluid particles remain fixed with respect to each other while all maintain the same velocity relative to the pipe or fluid conductor; non-turbulent flow.

PASSIVITY

A chemical condition in which surface reactions are slowed down or halted.

REGULATION

A stabilization within the electrolytic cell, whether caused by imposed or inherent forces.

SENSING

The measurement of a particular characteristic of a process at a designated time.

STAGNATION

Used here mostly as a condition where the fluid electrolyte is not moving or has very low relative motion.

TOOL

In electrolytic machining the cathode or negative side of the cell.

TRANSIENT

The condition that exists before equilibrium is established.

TURBULENT FLOW

That type of fluid flow where all the particles of a fluid are moving in random motion with respect to each other while moving above a critical velocity through the pipe or conductor; contrasted with LAMINAR FLOW.

WORKPIECE

In electrolytic machining, the anode or positive side of the cell.

SYMBOLS

NOTE:

In all theoretical mathematical treatments, the metric system will be used throughout; however, engineering practice demands the use of some English units. To facilitate calculations, the units for each variable will be designated in the report and conversion factors will be stated either symbolically or as real numbers.

GENERAL:

- i used as a subscript designates the variable's value at a point in time or at a point in space.
- o used as a subscript designates the variable's value at a starting or initial time or place.
- f used as a subscript designates the variable's value at the final time or place.

Algebraic, calculus and vector analysis symbols are used throughout and conform to standard practice for these disciplines.

SYMBOLS USED

<u>SYMBOL</u>	<u>DESCRIPTION</u>	<u>UNITS</u>
A	The projected or real area at any point in time normal to the current path. Areas which are described in planes other than normal to the current path are differentiated from A by subscripts.	Square centimeters for theoretical analysis. Any convenient unit for engineering calculations.
C	Concentration of a specific chemical in an aqueous solution. Specifically in JB-1 test series as gm of citric acid per 100 ml.	See following symbols.

<u>SYMBOL</u>	<u>DESCRIPTION</u>	<u>UNITS</u>
C_a	Concentration of a specific chemical in aqueous solution.	Gm/1000 gm of water.
C_m	Molal concentration of a specific chemical in aqueous solution expressed in gm-moles per unit volume. A mole is a molecular weight of a chemical in gm.	Moles per liter.
C_M	Molal concentration of a specific chemical in aqueous solution expressed in gm-moles of the chemical per 1000 gm of water.	Moles/1000 gm of water.
C_w	Concentration of a specific chemical in aqueous solution expressed by weight per volume of solution.	Gm/liter or lb/gal.
E	Electromotive force (emf) between two points in any electrical circuit, as measured by a voltmeter.	Volt
E_e	Electromotive force acting against resistance of the electrolyte in the electrochemical circuit. Usually differs from E across the electrochemical circuit by ΔE .	Volt
ΔE	The difference in emf between E and E_e .	Volt
F	The feed rate of the cathodic tool described as the rate of movement in a specific direction or rotation.	Inches/minute, inches/hour, centimeters/second, etc.
F	In Nernst Equation - free energy change.	Ergs.
F_y	In Nernst Equation - Faraday number 96,500 coulombs.	Coulombs.
G	The gap-distance at any time from a point on the cathode on a line normal to the cathodic surface to the intersection of the normal on the surface of the anode. It differs from L in that G is a measure of distance from anode to cathode only in a direction normal to the anodic surface.	Centimeter, inches.

<u>SYMBOL</u>	<u>DESCRIPTION</u>	<u>UNITS</u>
G_e	The equilibrium gap - that gap which is reached in electrolytic machining when the surface at the point on the anode will be within the desired engineering tolerance.	Usually inches.
G_f	The gap at the end of electrolytic machining.	cm or inches.
G_o	Initial gap - gap which exists at the start of electrolytic machining. When comparisons are being made between workpieces of varying initial dimensions, the G_o is used to designate the smallest expected initial gap. Other initial gap are related arithematically to G_o .	Centimeters or inches.
I	The current in electrical circuit.	Ampere.
$\frac{I}{A}$	The current density as a ratio of the total current flowing in a circuit to the area of the circuit normal to the current flow direction.	Either amperes/sq.in. or as amperes/sq. centimeter or amperes/sq. decimeter or amperes/sq ft
K	Volume of metal removed per unit ampere time.	cc/amp-sec, cubic inches/ampere-hour, cubic inches/ampere-minute.
K	Constant used in various equations.	As defined in text
L	Distance at any time during electrolytic machining between a designated point on the cathode and a designated point on the anode.	Centimeters
dL	The change in the length of $\underline{L}$ during a time, $\underline{dt}$, while electrolytic machining.	Same as L.

<u>SYMBOL</u>	<u>DESCRIPTION</u>	<u>UNITS</u>
dl	That portion of dL which can be attributed to metal removal.	Same as L.
M_R	Total volume of metal removed in machining.	cc or cubic inches.
N	Concentration specific to NaCl used in present shop practices.	Gms NaCl/100 ml of solution or lb sodium chloride/gallon of solution.
N_R	Reynolds number. A dimensionless number used in fluid dynamic analysis.	No units.
n	In Nernst Equation - net valence change.	No unit.
P	Fluid pressure, described at specific point or area in fluid system. Usually gage pressures in engineering calculation. Absolute pressures for theoretical discussions.	gm/sq cm, or lb/sq in.
R	The total electrical resistance of a specific electrical circuit or section thereof.	ohms
R	In Nernst Equation - gas law constant.	cm/ $^{\circ}$ K
r	Specific resistivity during electrolytic machining - the resistance of the electrolyte at any point in the circuit.	ohm-cm
$\bar{r}$	The weighted average specific resistivity of the electrolyte during electrolytic machining. The value of specific resistivity which when multiplied by the length of path of the current and divided by its area will be equal to the total resistance of the path.	ohm-cm
T	Temperature either at a point or as an average.	$^{\circ}$ F, $^{\circ}$ C, $^{\circ}$ K, $^{\circ}$ Rankine
t	The time the electrolytic machining is being observed.	Seconds, minutes, hours

<u>SYMBOL</u>	<u>DESCRIPTION</u>	<u>UNITS</u>
Γ	$= \frac{E - \Delta E}{r} \times K, \text{ in electrolytic machining}$ E, ΔE, F as previously defined K = cc of metal removal/amp sec.	$\frac{\text{cm}^2}{\text{sec}}$
γ	The expected variation in thickness at a designated point on a finished part after machining for a time <u>t</u> .	Centimeters or inches
θ	Angle between the direction of tool movement and the plane tangent to a point on the tool.	Degrees
λ	Specific conductivity. The reciprocal of the specific resistivity.	mhos/cm
μ	Fluid viscosity.	Lb/ft-sec, centipoises
ρ	Fluid density - used in Reynolds number.	Lb/cu ft
ψ	An expected variation in thickness of the part before electrolytic machining. For calculating purposes, measured along line normal to tool at start of cutting.	Centimeters

SECTION VI
APPENDICES

**OPERATING PARAMETER STUDY
ELECTROLYTIC MACHINING
FOR TESTS
JB-2, JB-3**

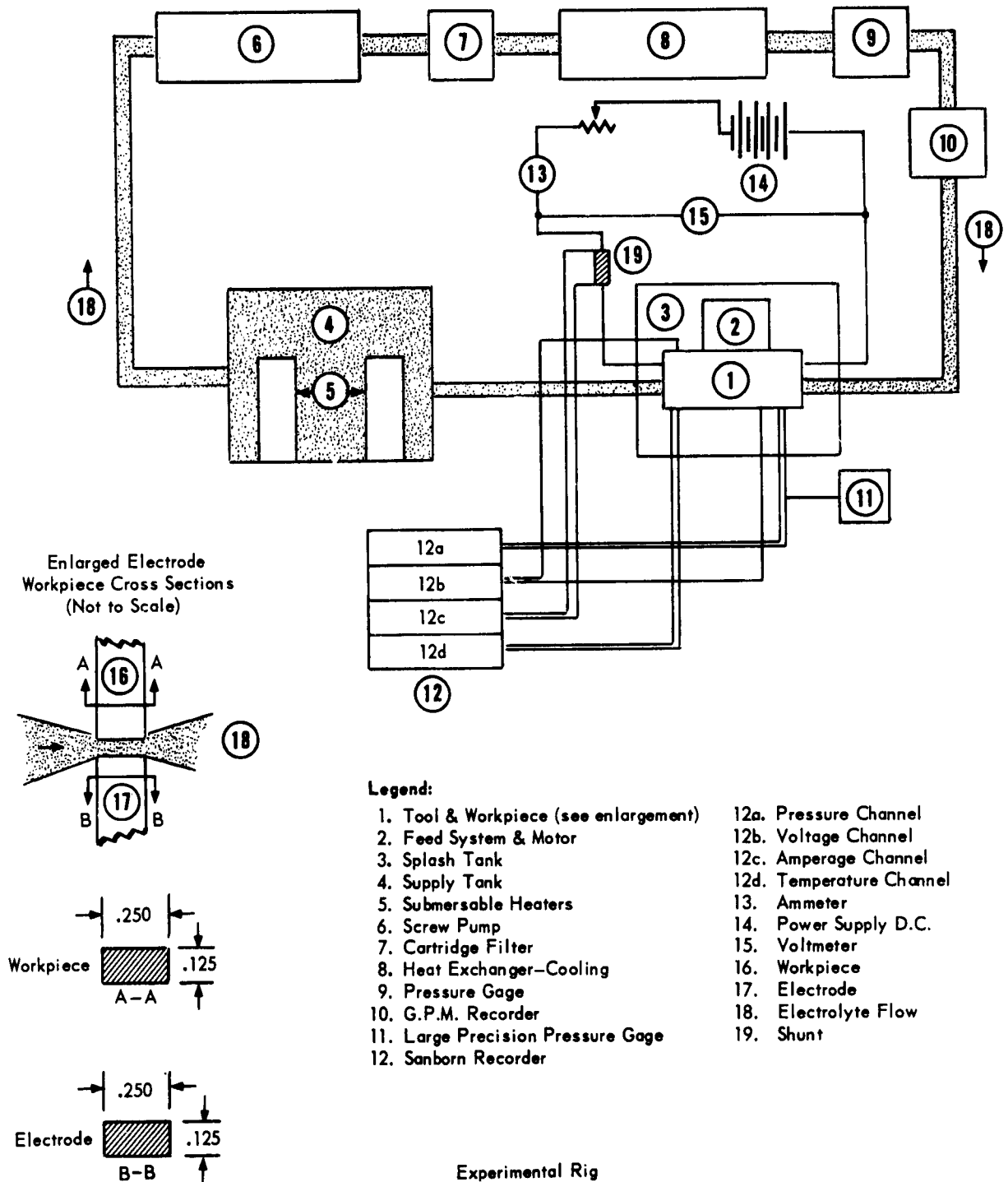


Figure 6-1

SECTION VI

APPENDICES

6.1 PROCEDURES

6.1.1 TEST SERIES: JB-2 and JB-3 (See figure 6-1)

The objectives of this test series are to determine those operating and process variables which affect the operating parameters, and results of electrolytic machining

Those results or effects of electrolytic machining which were measured were:

- 1) metal removal rate
- 2) relationship among the operating and process variables

For each test a feed rate, gap, inlet temperature, and sodium chloride concentration was selected. During pre-trials, initial voltages were applied to the electrolytic cell until -- at a selected initial voltage -- the current and voltage remained constant and the gap would remain constant. Under these conditions, the linear metal removal rate would be equal to the feed rate.

The test rig is schematically shown in figure 6-1. The 0.250 x 0.125 inch workpiece bar (1) was affixed to the driving mechanisms (2). The feed rate was established by gear reductions (2) from a constant speed motor. The workpiece was electrically connected to the anodic side of the power supply.

The tool, connected to the cathodic side of the power pack immediately under the tool, was of the same cross-section, and was separated the desired

gap distance from the workpiece. The entering and exiting are shown in figure 6-1.

It was first planned to preset the gap between the workpiece and the tool. The two bars were polished flat (within 0.0001-inch) and smooth (10 micro-inches) and set parallel to each other. The workpiece was lowered until it just touched the tool. This was determined by detecting the circuit closing with an ohmmeter. Then the workpiece was withdrawn (as measured by an Ames dial calibrated to 0.0001-inch) to the desired gap.

Various voltages were then applied across the gap. When the correct voltage was applied, the current and voltage would remain constant.

The electrolyte system consisted of a supply tank (4), which contained a fresh sodium chloride electrolyte at the test concentration. The electrolyte temperature was adjusted by either heating in the tank with two immersion heaters (5) or cooling in a heat exchanger (8). The heat exchanger cooled by passing a cold water counter-current around coils containing the electrolyte. The heating and cooling cycles were adjusted until the temperature of the electrolyte at the entrance to the tool was that planned for the test.

The electrolyte was drawn out of the supply tank (4) through piping (18) by the Moyno type pump (6) through a cartridge filter (7), (50 micron size), through the cooling coils (8), into the machining gap (1). After leaving the gap, the electrolyte was caught in a splash tank (3) and returned to the supply tank (4).

The power was supplied by a d-c rectifier (14).

The procedural instructions issued for test series JB-1 were only slightly modified for JB-2 and JB-3. Any one wishing that detailed description is referred to General Electric Technical Information Series report number R62FPD320.

These are the major components used in the experimental rig.

Instrumentation

Sanborn recorder Model No. 154-100B.

Low-level preamplifier, Model 150-1500.

Ac-dc preamplifier, Model 150-1000.

Case - Model 154-1100.

Power Pack

Wagner Brothers, Rectifier

Model A18101R

220/440 3-phase, 60 cycle

Ripple - 5 percent at 26 kva

18 volt output

1000 amp output

Constant Speed Motor

Bodine Electric

Type NYC-12RG

115 v / 0.33 amps - 1800 rpm to 3.00 rpm

Boston Gears

Ames Dial - Model 452

Measure to 0.0001 inch

Pressure Gage

Heise, Newtown, Conn. 1-11157

0-500 psi, 1 psi increment

Filter - Commercial Filter

Fulflo - SS 7.5G - 10 - 3/4

Pump and Accessories

Pump - Moyno - Robbins & Myers, Type 5X0X, Frame 6P3, Form NV

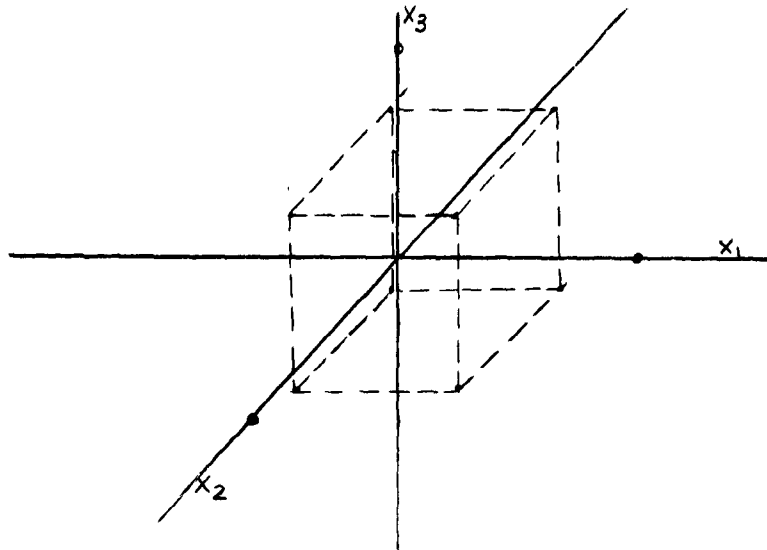
Speed Regulator - GE Polydyne Drive GP310AD34, 1078/154

rpm, 3 hp

Motor - 5K213M9 207B - 220/440V

Gear - 7GP139HF3PA, Size 3

6.2 "BOX" ORTHOGONAL COMPOSITE EXPERIMENT



Conditions for use: All variables continuous.

When to use:

1. When the functional relation may be described accurately by a second-degree polynomial equation (possibly under a suitable transformation).
2. For refinement, the design may be used whenever a second-degree polynomial equation is an adequate approximation.
3. The method works best when experimental error is small.
4. It is assumed in the analysis of variance that the variance in experimental error is constant throughout the test series.

Advantages of the Box Design:

The Box (orthogonal composite) design provides a second-degree curve fit with a minimum of testing required. In the Box design table, the column headed N_3 lists the number of tests required for a 3-level factorial or fractional experiment yielding comparable information. The advantage of the Box design is obvious.

Further advantages are indicated in the section below on Box Design Variations.

Design:

In the general case of n controlled factors, each variable is tested at 5 levels: $-\alpha$, -1 , 0 , $+1$, and $+\alpha$, as follows:

$+\alpha$, $-\alpha$, with all other factors at 0 (polytopes)

0 , with all other factors at 0 (center point)

$+1$, -1 , with each of the other factors at $+1$ or -1 in accordance with a 2^n or 2^{n-p} factorial or fractional factorial experiment.

(Refer to Fractional Experiments table for pertinent information).

The value α is determined from the Box design table. In cases not covered by the table, the following formula may be used:

$$\alpha = [1/2 (\sqrt{2^{n-p} (2^{n-p} + 2n + k)} - 2^{n-p})]^{1/2}$$

where p is the order of the fractional factorial

k is the member of tests at the center point.

The total number of tests (N) may be found in the Box Design table, or from the formula

$$N = 2^{n-p} + 2n + k$$

Analysis of the Box Design

In the equation

$$\hat{Y} = \bar{Y} + A_{X_1} X_1 + A_{X_1^2 - b} (X_1^2 - b) + A_{X_2} X_2 + A_{X_2^2 - b} (X_2^2 - b) + \dots + A_{X_1 X_2} X_1 X_2 + \dots$$

the coefficients are calculated from the following formulas:

$$A_{X_1} = c \sum y X_1$$

$$A_{X_1^2 - b} = d \sum y (X_1^2 - b)$$

$$A_{X_1 X_2} = f \sum y X_1 X_2$$

(and similarly for A_{X_2} ---, $A_{X_2^2 - b}$ ---, $A_{X_2 X_3}$ ---, etc)

In these formulas $\sum$ represents summation over all test points. The values of $\underline{b}$, $\underline{c}$, $\underline{d}$, and $\underline{f}$ are tabulated in the Box design table, or may be calculated from the formulas

$$b = \frac{2^{n-p} + 2\alpha^2}{2^{n-p} + 2n + k}$$

$$c = \frac{1}{2^{n-p} + 2\alpha^2}$$

$$d = \frac{1}{2^{n-p} (1-b)^2 + (2n + k - 2)b^2 + 2(\alpha^2 - b)^2}$$

$$f = \frac{1}{2^{n-p}}$$

NOTE: When the entire design is replicated $\underline{r}$ times, the value of $\underline{b}$ remains unchanged from the tabulated value. The values of $\underline{c}$, $\underline{d}$, and $\underline{f}$ must be multiplied by $\frac{1}{r}$.

Analysis of Variance

$$SS(X_1) = c (\sum y X_1)^2$$

$$SS(X_1^2 - b) = d [\sum y (X_1^2 - b)]^2$$

$$SS(X_1 X_2) = f (\sum y X_1 X_2)^2$$

$$SS(\text{total}) = \frac{\sum (y - \bar{y})^2}{N}$$

$$SS(\text{Error}) = SS(\text{Total}) - SS(\text{all effects combined})$$

Transportation into original units (u):

$$x = \frac{u - u_0}{u_1 - u_0} \quad X^2 = \left(\frac{u - u_0}{u_1 - u_0} \right)^2$$

where u_0 is the value of u corresponding to $X = 0$, and u_1 is the value of u corresponding to $X = 1$.

NOTE: It is often advantageous to present the data in the form of a plot or several plots of the equation, in which case the equation should be plotted using the X scale, and simply relabeled to the u scale.

Confidence and Tolerance Limits. Formulas for confidence and toler-

ance limits are as follows:

Confidence limits for the parameter η :

$$\hat{y} - t_{\alpha} S_e \sqrt{B} \leq \eta \leq y + t_{\alpha} S_e \sqrt{B}$$

Confidence limits for individuals:

$$\hat{y} - t_{\alpha} S_e \sqrt{1+B} \leq y \leq y + t_{\alpha} S_e \sqrt{1+B}$$

Tolerance limits for individuals:

$$\hat{y} - K_{\gamma, \alpha} S_e \sqrt{1+B} \leq y \leq y + K_{\gamma, \alpha} S_e \sqrt{1+B}$$

$$\text{where } B = \frac{1}{N} + c \sum x_i^2 + d \sum (X_i^2 - b)^2 + f \sum (X_i X_j)^2$$

(NOTE: The summation indices refer to the variables $X_1, X_2, \dots$ rather than to separate test values.)

TABLE VI-1
BOX DESIGN CONSTANTS
(single test at center)

n	p*	α	N	df**	b	c***	d***	f***	N ₃ ****
3	0	1.2154	15	5	.7303	.09129	.2291	.1250	27
4	0	1.4142	25	10	.8000	.05000	.1250	.0625	81
	1	1.3531	17	5	.6860	.08575	.1492	.1250	27
5	0	1.5960	43	22	.8627	.02696	.07706	.03125	243
	1	1.5467	27	6	.7698	.04811	.08737	.0625	81
6	0	1.7606	77	49	.9117	.01425	.05204	.01563	729
	1	1.7244	45	17	.8433	.02635	.05655	.03125	243
7	0	1.9095	143	107	.9461	.007391	.03761	.007813	2187
	1	1.8849	79	43	.9001	.01406	.03961	.01563	729
	2	1.8414	47	14	.8251	.02579	.04349	.03125	243
8	0	2.0449	273	228	.9686	.003782	.02821	.003906	6561
	1	2.0292	145	100	.9396	.007340	.02949	.007813	2187
	2	2.0000	81	36	.8889	.01389	.03125	.01563	729
	3	1.9489	49	12	.8081	.02525	.03598	.03125	243

*For $p \neq 0$, use an appropriate $L = 2, p$ fractional experiment in place of the factorial portion of the Box design.

**Residual degrees of freedom, assuming all 1st order interactions are evaluated.

***For r replicates of the entire design, multiply c, d, f values by $\frac{1}{r}$.

**** N_3 = number tests in a 3^{n-p} fractional experiment.

6.3 ADDITIONAL DATA AND ANALYSIS

6.3.1 TEST SERIES JB-1

See General Electric Technical Information Series Report No. R62FPD320, "Interim Report — Electrolytic Machining Process," Joseph Bayer. This report is available upon request from Technical Information Center, Mail Drop F-22, General Electric Co., Cincinnati 15, Ohio. This report gives procedures, data results and conclusions for 7051 Series JB-1.

No analysis of variance for current is given in the report. It is included here.

TABLE VI-3
FACTORS AFFECTING CURRENT

Analysis of Variance Based on Four Factor
Two Level, Doubly Replicated Full Factorial
Test Series - JB-1 (1)(2)

FACTORS	F inch/hr	T °F	N gmsNaCl/100 ml	P psig
Level +1	2.13	108.8	16.45	166.4
Level -1	0.873	71.2	4.55	53.6

ANOVA

FACTOR	Sums of Squares	Variation due to factor % $SS_i/SS_T \times 100$	Degrees of Freedom	F Ratio	Significance α less than
Total	1701.989687	100.00	31	-	-
Feed Rate (F)	1697.987812	99.76	1	13,968	Very highly
Temp. (T)	0.262812	0.02	1	2.16	0.20
Pressure (p)	0.475312	0.03	1	3.91	0.06
PXF	0.382812	0.02	1	3.15	0.10
FXT	0.262812	0.02	1	2.16	0.20
PXNXT	0.227812	0.01	1	1.87	0.25
Balance, first, second, third, fourth order interactions	0.445301	0.03	9	-	-
Experimental error	1.945	0.11	16	Ms = 0.1215625	

(1) From Box Design.

(2) Actually five factorial which included citric acid concentration as a factor.
Thus considered to be four factorial, doubly replicated experiment.

6.3.2 TEST SERIES JB-2

4 Variable Box Design

$$\alpha = \sqrt{2}$$

Levels for NaCl Concentration = N gms/100 ml

Based on a functional relationship - $(1/N)$

$$X_{1/N} = \frac{(1/N) - a}{b} \quad \text{when } X_{1/N} = 1, N = 16.45 \text{ gms NaCl/100 ml}$$

$$\text{and } X_{1/N} = -\alpha, N = 26 \text{ gms NaCl/100 ml}$$

$$X_{1/N} = \frac{(1/N) - .05154}{.009249} \quad \text{or} \quad \frac{-\alpha}{26.00} \quad \frac{-1}{23.64} \quad \frac{0}{19.40} \quad \frac{+1}{16.45} \quad \frac{+\alpha}{15.47}$$

Levels for Temperature, Inlet - $^{\circ}\text{F} = T$

$$X_T = \frac{T - a}{b}, \quad \text{when } X_T = 1, T = 120^{\circ}\text{F}$$

$$\text{and } X_T = \alpha, T = 71.2^{\circ}\text{F}$$

$$\text{or } X_T = \frac{T - 99.8}{20.214} \quad \text{or} \quad \frac{-\alpha}{71.2} \quad \frac{-1}{79.6} \quad \frac{0}{99.8} \quad \frac{+1}{120.0} \quad \frac{+\alpha}{128.4}$$

Levels for feed rate - inches/hr = F

$$X_F = \frac{F - a}{b}, \quad \text{when } X_F = -1, F = 2.1266 \text{ in./hr}$$

$$X_F = \alpha, F = 10.00 \text{ in./hr}$$

$$X_F = \frac{F - 5.3879}{3.26127} \quad \text{or} \quad \frac{-\alpha}{.776} \quad \frac{-1}{2.13} \quad \frac{0}{5.39} \quad \frac{+1}{8.65} \quad \frac{+\alpha}{10.00}$$

Levels for gap-inches = G

$$X_G = \frac{G - a}{b} \quad \text{when } X_G = -\alpha, G = .002 \text{ inches}$$

$$X_G = +1, G = .012 \text{ inches}$$

$$X_G = \frac{G - .00786}{.004142} \quad \frac{-\alpha}{.0020} \quad \frac{-1}{.0037} \quad \frac{0}{.0079} \quad \frac{+1}{.0120} \quad \frac{+\alpha}{.0137}$$

TABLE VI-4
DESIGN OF EXPERIMENT

Operating Parameter Study - JB-2 Electrolytic Machining
Rene' 41 with NaCl Electrolyte Reporting Order and Testing
Sequence Based On Four Variable Box Designs ⁽²⁾

Reporting Order	Testing Sequence	NaCl Conc. ⁽¹⁾ gms/100 cc "N"	Inlet Electrolyte Temp. °F "T"	Feed Rate inches/hr "F"	Gap inches "G"
1	18	16.45	120.0	8.65	.0120
2	6	16.45	120.0	8.65	.0037
3	15	16.45	120.0	2.13	.0120
4	12	16.45	120.0	2.13	.0037
5	25	16.45	79.6	8.65	.0120
6	8	16.45	79.6	8.65	.0037
7	3	16.45	79.6	2.13	.0120
8	7	16.45	79.6	2.13	.0037
9	11	23.64	120.0	8.65	.0120
10	10	23.64	120.0	8.65	.0037
11	2	23.64	120.0	2.13	.0120
12	17	23.64	120.0	2.13	.0037
13	22	23.64	79.6	8.65	.0120
14	1	23.64	79.6	8.65	.0037
15	14	23.64	79.6	2.13	.0120
16	5	23.64	79.6	2.13	.0037
17	13	15.47	99.8	5.39	.0079
18	21	26.00	99.8	5.39	.0079
19	4	19.40	128.4	5.39	.0079
20	20	19.40	71.2	5.39	.0079
21	19	19.40	99.8	10.00	.0079
22	9	19.40	99.8	0.776	.0079
23	23	19.40	99.8	5.39	.0137
24	16	19.40	99.8	5.39	.0020
25	24	19.40	99.8	5.39	.0079

(1) Based on concentration at 72° F.

(2) Fluid pressure at inlet held at 166.7 psig.

TABLE VI-5

ELECTROLYTIC MACHINING OF RENE' 41
SODIUM CHLORIDE ELECTROLYTE
U-700 ELECTRODE
SECOND SERIES OF TESTS (SERIES JB-2)

Re- port #	Test #	Levels Sought				Levels as Measured				Other Input Data			Output Data	
		N Conc. NaCl gm/100ml.	T Inlet Temp. °F	F Feed Rate in/min.	G Gap inch	N Conc. NaCl gm/100ml.	T Inlet Temp. °F	F Feed Rate in/hr	G Gap inch	Conducti- vity mho/cm	Fluid flow gpm	Pressure psig	E Volts	I Amps
1	18	16.45	120.0	8.6492	.0120	16.8	120.0	8.81	.0120	.189	2.10	167	36.9	47.2
2	6	16.45	120.0	8.6492	.0037	17.5	119.8	8.75	.0035	.182	0.60	166-1/2	17.5	47.3
3	15	16.45	120.0	2.1266	.0120	17.0	120.0	2.12	.0118	.189	1.65	166-1/2	10.1	13.0
4	12	16.45	120.0	2.1266	.0037	16.5	120.0	2.13	.0036	.179	0.47	166-1/2	5.7	13.0
5	25	16.45	79.6	8.6492	.0120	16.4	79.8	8.78	.0123	.179	1.90	167	53.1	49.0
6	8	16.45	79.6	8.6492	.0037	16.8	79.5	8.79	.0036	.173	0.48	167	17.6	49.5
7	3	16.45	79.6	2.1266	.0120	16.3	80.0	2.14	.0123	.164	2.40	166-1/2	18.1	12.2
8	7	16.45	79.6	2.1266	.0037	16.7	79.8	2.13	.0036	.179	0.52	166-1/2	7.1	12.5
9	11	23.64	120.0	8.6492	.0120	25.0	120.0	8.74	.0120	.204	1.89	166-1/2	30.1	47.6
10	10	23.64	120.0	8.6492	.0037	23.7	120.5	8.79	.0040	.196	0.52	166-1/2	12.1	46.3
11	2	23.64	120.0	2.1266	.0120	23.4	120.0	2.13	.0119	.197	2.30	166-1/2	11.1	12.3
12	17	23.64	120.0	2.1266	.0037	23.4	120.0	2.12	.0035	.214	0.49	167	5.1	12.5
13	22	23.64	79.6	8.6492	.0120	23.5	79.8	8.72	.0123	.200	1.80	167	41.2	50.8
14	1	23.64	79.6	8.6492	.0037	23.2	79.5	8.79	.0034	.192	0.45	166-1/2	17.4	51.2
15	14	23.64	79.6	2.1266	.0120	23.6	79.2	2.12	.0118	.212	2.42	166	15.1	12.0
16	5	23.64	79.6	2.1266	.0037	23.5	79.0	2.13	.0040	.208	1.22	167	9.7	11.5
17	13	15.47	99.8	5.3879	.0079	15.8	99.6	5.37	.0082	.176	1.00	157	17.2	29.1
18	21	26.00	99.8	5.3879	.0079	25.9	99.8	5.35	.0080	.200	1.15	167	16.2	29.0
19	4	19.40	128.4	5.3879	.0079	21.5	128.0	5.32	.0080	.197	1.87	167	19.7	29.5
20	20	19.40	71.2	5.3879	.0079	20.1	71.5	5.32	.0079	.196	1.55	167	28.6	30.0
21	19	19.40	99.8	10.000	.0079	20.0	99.8	9.93	.0080	---	1.70	167	39.9	53.7
22	9	19.40	99.8	0.7758	.0079	19.5	99.0	0.791	.0082	.189	1.42	166-1/2	5.7	4.8
23	23	19.40	99.8	5.3879	.0137	19.7	99.8	5.39	.0128	.206	2.44	167	30.2	29.8
24	16	19.40	99.8	5.3879	.0020	19.4	99.7	5.39	.0023	.200	0.30	167	7.5	29.1
25	24	19.40	99.8	5.3879	.0079	19.8	99.8	5.38	.0081	.200	1.10	167	17.2	29.4
CHECK POINTS														
2	61	16.45	120.0	8.6492	.0037	16.6	120.0	8.81	.0039	.182	0.39	167	12.1	46.3
3	151	16.45	120.0	2.1266	.0120	16.5	120.0	2.09	.0119	.178	1.59	167	10.5	12.9
11	21	23.64	120.0	2.1266	.0120	23.8	120.0	2.10	.0120	.220	1.60	167	8.8	12.1
16	51	23.64	79.6	2.1266	.0037	23.4	79.8	2.10	.0037	---	0.58	167	6.6	12.4

TABLE VI-6
FACTORS AFFECTING CURRENT

Analysis of Variance Based on Four Factor
Two Level, Full Factorial Test Series JB-2⁽¹⁾⁽²⁾

FACTOR	N gms NaCl/100 ml	T °F	F in./hr	G inches
Level (1)	16.45	120	8.65	.012
Level (2)	23.64	80	2.13	.0037

ANOVA

FACTOR	Sums of Squares	Variations Due to Factor % $SS_i/SS_T \times 100$	Degrees of Freedom	F ⁽³⁾ Ratio	Significance α less than
Total	5276.88937	-	15	-	-
Feedrate (F)	5252.6256	99.54	1	11290	Very highly
Temp. (T)	5.6406	0.11	1	12.124	.01
FXT	13.5056	0.26	1	21.49	.01

All the following summed to determine F ratio: N, G, FXN, NXG, FXG, TXG, NXT, all third and fourth order interaction.

5.1176	0.09	11	MS _{RES} -.465236
--------	------	----	----------------------------

- (1) From box design.
- (2) Using rerun test results (included in main body of data).
- (3) Based on 11 degrees of freedom, sum of all factors but F, T, and FXT.

TABLE VI-7
FACTORS AFFECTING REQUIRED VOLTAGE

Analysis of Variance Based on Four Factor,
Two Level, Full Factorial Test Series JB-2⁽¹⁾⁽²⁾

FACTOR	N gms NaCl/100 ml	T °F	F in. /hr	G inches
Level (1)	16.45	120	8.65	.012
Level (-1)	23.64	80	2.13	.0037

ANOVA

FACTOR	Sums of Squares	Variations Due to Factor % $\frac{SS_E}{SS_T} \times 100$	Degrees of Freedom	F Ratio	Significance α less than
Total	3054.1493	100	15	-	-
Conc. NaCl (N)	38.1306	1.2	1	9.50	0.05
Temp. (T)	188.3756	6.2	1	46.95	0.001
Feedrate (F)	1287.0156	42.1	1	320.74	0.0005
Gap (G)	1057.8756	34.6	1	263.63	0.0005
NXG	30.5256	1.0	1	7.61	0.05
TXF	28.3556	0.9	1	7.07	0.05
TXG	47.2656	1.6	1	11.78	0.025
FXG	343.1756	11.2	1	85.52	0.0005
Residual ⁽³⁾	20.0633	0.7	5	$MS_{RES} = 4.0127$	
Balance ⁽⁴⁾	13.4762	0.5	2	-	-

- (1) From box design.
- (2) Using rerun test results (included in main body of data).
- (3) Includes all third and fourth order interactions. Used to determine F ratios.
- (4) Sum of insignificant second order interactions.

6.3.3 JB-3

6.3.3.1 Design of Experiment

To test tool steel being cut with G.E. electrolyte "GK-4".

Variables to be considered:

- (1) Feed Rate - F - inches/hour
- (2) Fluid Pressure P - psig
- (3) Gap - G - inches
- (4) Temperature T - °F

To be held constant:

- (1) Concentration of GK-4
- (2) Parts held rigidly

Gap to be held constant throughout single test by adjusting voltage so that at any feed rate the linear rate of cut will be same as feed rate.

Type of experiment - 1/2 replicated factorial

F ↓	P ↓	0.002		0.005		← G
		80	120	80	120	← T
0.79	100	(1)			(1)	
	140		(1)	(1)		
4.68	100		(1)	(1)		
	140	(1)			(1)	

(1) Determine current and required voltage for these conditions.

X No tests.

CONFOUNDING

FXP with GXT, FXG with PXT, FXT with PXG

TABLE VI-8
PLAN OF EXPERIMENT (JB-3)

CURRENT AND VOLTAGE REQUIREMENTS
EM OF TOOL STEEL

GK-4 ELECTROLYTE
(CONSTANT CONCENTRATION)

Report Number	Testing Sequence Number	F Feed Rate (in. /hr)	P Pressure (psi)	G Gap (inches)	T Temperature ° F
1	3	0.79	100	0.002	80
2	8	0.79	100	0.005	120
3	7	0.79	140	0.002	120
4	4	0.79	140	0.005	80
5	6	4.68	100	0.002	120
6	1	4.68	100	0.005	80
7	2	4.68	140	0.002	80
8	5	4.68	140	0.005	120

TABLE VI-9
OPERATING PARAMETER STUDY
Electrolytic Machining Test Results Test JB-3
Cutting Tool Steel with GK-4 Electrolyte

Report No.	Test No.	Attempted Input Parameters				Actual Input			
		F Feed Rate in. /hr	P Pres- sure psig	G Gap Inch	T Temp. °F	F Feed Rate in. /hr	P Par. Press. psig	G Gap Ind.	T Temp. °F
1	3	0.79	100	.002	80	0.781	100	0.0020	79.5
2	8	0.79	100	.005	120	0.775	100	0.0052	120.0
3	7	0.79	140	.002	120	0.816	140	0.0018	120.0
4	4	0.79	140	.005	80	0.786	140	0.0051	81.0
5	6	4.68	100	.002	120	4.80	99	0.0021	120.0
6	1	4.68	100	.005	80	4.82	100	0.0050	80.0
7	2	4.68	140	.002	80	4.82	140	0.0022	80.0
8	5	4.68	140	.005	120	4.78	140	0.0051	120.0

Report No.	Test No.	<u>Others</u>			<u>Results</u>	
		Delta T °F	Conduc- tivity mhos-cm	Fluid Flow gpm	E Volts	I Amp.
1	3	1.8	0.146	0.0	6.0	4.8
2	8	1.0	0.143	0.71	7.5	6.8
3	7	0.0	0.142	0.1	4.9	5.0
4	4	3.0	0.143	0.73	8.0	5.8
5	6	5.0	0.147	0.02	12.2	31.0
6	1	7.0	0.145	0.80	30.0	31.0
7	2	7.2	0.145	0.65	28.1	28.0
8	5	4.2	0.145	0.60	15.6	29.5

TABLE VI-9a
TEST CHECK POINTS (REPLICATIONS)
Test Series - JB-3

The following check points were run on JB-3 to determine experimental error.

FACTORS	F in. /hr	P psig	G inch	T °F	Results		I SS _e	E SS _e
					E volts re- quirement	I Total amps		
	4.68	140	.002	80	28.1 28.1	28.0 28.0	0.000	0.000
	0.79	100	.005 .0046 .0046	120	7.6 7.5 7.1	6.8 6.8 6.3	.0714	0.0600
	4.68	140	.005 .006	120 120	15.6 19.9	29.5 29.5	0.000	2.6414

Analysis of current variation:

$$M_s = \frac{.0716}{6} = .0119$$

Analysis of voltage variation:

$$M_s = \frac{2.7014}{6} = .450233$$

TABLE VI-10
ANALYSIS OF TOOL STEEL USED IN TEST SERIES JB-3

Carbon	0.246
Manganese	0.41
Si	Trace
Cr	Trace
Ni	Trace
Mo	Trace
Balance	Fe

TABLE VI-11
FACTORS AFFECTING CURRENT

Analysis of Variance Based on Four Factor,
Two Level, Half Replicated, Factorial

Test Series JB-3

FACTORS	F in. /hr	P psig	G inches	T ° F
Level (+1)	4.68	140	.005	120
Level (-1)	0.79	100	.002	80

ANOVA

FACTOR	Sums of Squares	Variations Due to Factor % $SS_i/SS_T \times 100$	Degrees of Freedom	F ⁽¹⁾ Ratio	Significance α less than
Total	1187.21875	100.00	7	-	-
Feed Rate-F	1178.55125	99.27	1	9695	Very high
Pressure P	3.51125	0.30	1	28.88	0.0005
Gap-G	2.31125	0.19	1	19.01	0.0005
Temp-T	0.91125	0.08	1	7.50	0.02
F _{XP} ⁽²⁾	1.71125	0.14	1	14.08	0.005
F _{XG} ⁽²⁾	0.21175	0.02	1	1.75	-
F _{XT} ⁽²⁾	0.01125	0.00	1	-	-

(1) Based on $M_S = .1215625$ 15 degrees of freedom from JB-1.

(2) We have some reservation in using this value since the system differs and the ranges differ. We checked experiment error, however. Results are shown in Table VI-9a . The experiment error proves to be less than the M_S we used.

TABLE VI-10
ANALYSIS OF TOOL STEEL USED IN TEST SERIES JB-3

Carbon	0.246
Manganese	0.41
Si	Trace
Cr	Trace
Ni	Trace
Mo	Trace
Balance	Fe

TABLE VI-12
FACTORS AFFECTING VOLTAGE

Analysis of Variance Based on Four Factor,
Two Level, Half Replicated Factorial

Test Series JB-3

FACTOR	F in. /hr	T °F	G inch	P psig
Level (+1)	4.68	120	0.005	140
Level (-1)	0.79	8	0.002	100

ANOVA

FACTOR	Sums of Squares	Variations Due to Factor % $SS_i/SS_T \times 100$	Degrees of Freedom	F Ratio	Significance α less than
Total	685.65875	100.00	7	-	-
Feed Rate (F)	442.53125	64.54	1	982.9	Very high
Temp. (T)	127.20125	18.55	1	282.5	" "
T x F ⁽¹⁾	102.96125	15.01	1	223.7	" "
Gap (G)	12.25125	1.79	1	27.2	0.01
(PxT, GxF) ⁽¹⁾	0.61200	0.09	2	1.4	Not
[GxT, FxP]	0.10125	0.01	1	0.2	Not
Pressure (P) ⁽²⁾					
M _s error (0.450233)			6		

(1) Confounding Tx F with GxP, PxT with GxF, and FxP with GxT.
Assumed FxT is largest confounding.

6.4 RELATIVE MOTION — TOOL AND WORKPIECE

Regard the surface on an identifiable $f(x, y, z)$ which bounded by its intersection with another identifiable surface. Regard this surface to be the surface of the cathode. All points, $\underline{C}$, on the cathode surface can be expressed in rectangular coordinates $[x, y, f(x, y,)]$ where the $\underline{x}$, $\underline{y}$, and $\underline{z}$ coordinates axes are identifiable and remain fixed throughout the process. It will be helpful to express these points by a means of position vectors from the origin of the coordinate system, $\underline{0}$.

Suppose that it requires a total of $t = t_f$ seconds to complete the process. At some time $t = t_m$ (not necessarily $t_m = 0$) it may be necessary to move the cathode. The present stage of technology permits the use of numerical tape control, or hydraulic or electrical 3-D tracing systems, to translate the cathode in any manner desired and for any time period t_1 to t_2 in the interval $0 < t < t_f$. This should also enable the user to plan rotation of the cathode surface in addition to translation. It is reasonable to expect that it will be possible to plan any desired form of motion of the cathode. One is not restricted to the demand that a given mode of cathode motion be used throughout the $\underline{t_f}$ seconds of operation. If one desires to hold the cathode at a position it occupies at time $t = c < t_f$ for the next $\underline{k}$ seconds, the tape will merely instruct the tool to remain fixed for this period.

Suppose then, that at time $t = t_{n+1}$ seconds, the cathode has gone through $\underline{n}$ unique modes of motion, and that it is desired to introduce another mode of motion at this time. If the cathode remained fixed for the first $\underline{t_1}$ seconds of

operation, the position vector $\overline{OC}$ of all points on the cathode surface will be known. Designate it as $\overline{OC} = [x_1, y_1, f(x, y,)]$, $0 < t < t_1$.

An identifiable motion,

$$\Delta_x = h_x(t - t_1)$$

$$\Delta_y = h_y(t - t_1)$$

$$\Delta_z = h_z(t - t_1)$$

will be introduced for the period of time $t_1 \leq t \leq t_2$; consequently the position vector of any point on the surface of the cathode in the time interval $t_1 \leq t \leq t_2$ will be

$$\overline{OC} = [x_1 + h_x(t - t_1), y_1 + h_y(t - t_1), f(x_1, y_1) + h_z(t - t_1)]$$

Regard:

$$x_2 = x_1 + h_x(t_2 - t_1)$$

$$y_2 = y_1 + h_y(t_2 - t_1)$$

$$z_3 = f(x_1, y_1) + h_z(t_2 - t_1)$$

to be coordinates of point C at time $t = t_2$ when the first mode of motion is terminated.

If the next mode of motion desired for the time interval $t_2 \leq t \leq t_3$ is

$$\Delta_x = g_x(t - t_2)$$

$$\Delta_y = g_y(t - t_2)$$

$$\Delta_z = g_z(t - t_2)$$

the points $\underline{C}$, on the cathode face can be expressed by

$$\overline{OC} = [x_2 + g_x(t - t_2), y_2 + g_y(t - t_2), z_2 + g_z(t - t_2)]$$

Therefore, the position vector

$$\overline{OC} = [x_2 + g_x(t_3 - t_2), y_2 + g_y(t_3 - t_2), z_2 + g_z(t_3 - t_2)]$$

will describe the position of all points $\underline{C}$ on the cathode when this second mode of motion is terminated at time $t = t_3$. Designate these final coordinates as x_3, y_3, z_3 respectively, so that we can claim $\overline{OC} = [x_3, y_3, z_3]$ before the third mode of motion is introduced at time $t = t_3$.

Consequently, at time $t = t_{n+1}$, when the $\underline{n}$ th mode of motion has terminated we will have

$$\overline{OC} = [x_{n+1}, y_{n+1}, z_{n+1}]$$

to describe the position of each point $\underline{C}$ on the cathode time $t = t_{n+1}$ when the $(n+1)$ th mode of motion will start for the next interval of time. Similarly, identifiable modes of motion can be used for all time intervals up to $t = t_F$, so that the position of all points on the cathode face can be identified at any time $\underline{t}$.

Consider that portion of the surface $g(x, y, z)$, with respect to the same coordinate axes used previously, bounded by its intersection with another identifiable surface, to be the anode in this process. If all points $\underline{A}$, on the anode are identified by $[X_1, Y_1, G(X_1, Y_1)]$ and the same time base is used as was designated for the cathode, then all modes of anode motion desired can be expressed as it was for the cathode. If $\underline{0}$ is the origin of the fixed coordinate axes, then the position vector $\overline{OA}$ can be used to express the points $\underline{A}$ on the anode.

Consequently, for a given time t and considering cathode and anode motion without metal removal occurring, the vector $\overline{C_1A_1} = \overline{OA_1} - \overline{OC_1}$ will express the location of point A_1 on the anode with respect to the point C_1 on the cathode. The dot product, $\overline{C_1A_1} \cdot \overline{C_1A_1}$ is equal to the square of the distance from C_1 to A_1 . If the distance from C_1 to A_1 is expressed as $|\overline{C_1A_1}|$, we have

$$|\overline{C_1A_1}| = \sqrt{\overline{C_1A_1} \cdot \overline{C_1A_1}}$$

All points A_1 of interest on the anode can be similarly related to the point C_1 on the cathode. Obviously, other points C_1 on the cathode can be related to the points A_1 on the anode.

Suppose then that the electrolyte is introduced at some time $t = t_e$, with reference to the same time base used for anode and cathode motion, and that the current is introduced at time $t = t_c$. The earlier used expression $\overline{C_1A_1} = \overline{OA_1} - \overline{OC_1}$ (illustrating the relationship of C_1 and A_1 , due only to motion of these two bodies) no longer holds for $t > t_e$. The anode surface has vanished, and the earlier designated anode surface points, A_1 , will vanish at varying rates.

The physical chemistry aspects of this problem should be adequately defined so that the rate at which metal is being removed from the anode surface can be defined in similar terms. The electrochemical changes would have the influence of increasing the x , y , and z components of the earlier used vector $\overline{C_1A_1}$ by the amounts (ΔMx) , (ΔMy) , and (ΔMz) , .

If $\overline{OA}$ described the position of points on the anode at the time metal removal starts, and if in an infinitesimal time Δt , this vector directed at A finds the anode surface has retreated so that it pierces the new anode surface at a point B (with reference to the fixed coordinate axis), we are then interested in expressing the position of B with respect to the points C on the cathode.

Thus, if $OB_i = OA_i + [(\Delta M_x)_i, (\Delta M_y)_i, (\Delta M_z)_i,]$

$$\overline{C_j B_i} = \overline{OB_i} - \overline{OC_j}$$

$$\overline{C_j B_i} = \overline{OA_i} + [(\Delta M_x)_i, (\Delta M_y)_i, (\Delta M_z)_i] - \overline{OC_j}$$

Where $\overline{OA_i}$ and $\overline{OC_j}$ are dependent upon anode and cathode shape and motion, and the duration of the motion.

$[(\Delta M_x)_i, (\Delta M_y)_i, (\Delta M_z)_i]$, is the vector representing the physical chemistry of the process, and is a function of the operating parameters.

The desired final shape of the part (anode) can be expressed as a surface with proper boundaries on $\underline{h}(x, y, z)$. In a manner similar to that used to describe the points on the cathode and anode, all points F on the final form desired can be expressed in terms of the origin O by the position vector $\overline{OF}$. Recall that the $\overline{OF}$ will have the same motion as the anode. At any time t , the position of point F on the desired form can be described with respect to points C on the cathode as:

$$\overline{C_j F_i} = \overline{OF_i} - \overline{OC_j}$$

At that time $t = t_f$ at which $\overline{C_j B_i} = \overline{C_j F_i}$, the process stops.

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