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ACTUAL VERSUS CONSISTENT DECISION BEHAVIOR

by

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I. PURPOSE.

Like any other specialist in decision making, the Manager expects Science to tell him:

(I) What is his optimal (effective, efficient) behavior, in the face of a given task? That is: how should he impinge upon Nature (including humans) to achieve, on the average, results that are best from his point of view? (Normative Science: Engineering, Operations Research, Medicine and other "know-how" sciences).

(II) How does Nature (including humans) behave? (Descriptive Science: Physics, Psychology, Biology).

Normative Science develops methods that use the findings of Descriptive Science to predict results from actions, and thus choose optimal actions.

One particular subject of descriptive science of behavior is the study of the limitations of men's capacities for finding optimal decisions; or, more generally, the study of the cost of decision making, in a society with a given supply of, and demand for, decision-making skills. The normative science of management must take these limitations and costs as described by behavioral science into account, and seek optimum results of decisions, feasible under limited decision capacity, and net of decision cost. Also, descriptive behavioral science may help to determine optimal

strategies for training decision makers.

The "humans" in (I) and (II) above are the Manager's customers, competitors, co-workers,..., including himself. Three particular (and not exhaustive) classes of managerial action:

1. to assign decisions to men in an optimal way;

2. to remedy the failure of some men (including himself) to optimize their actions;

3. to exploit the failure of other men to optimize their actions.

Each class calls for contributions of descriptive behavioral science. In particular, Class 1. calls for measurement of relevant variables (dimensions) of decision-making capacity: e.g., speed; memory; size and frequency of deviations from optimum under varying conditions such as cultural background, business experience, stress, etc.

Class 2. calls for the study of the behavior of successful optimizers. The study (by simulation or otherwise) of traditional, possibly outdated decision routines and habits of average managers is of smaller practical value. Besides the study of successful decision techniques, Class 2. calls also for the study of processes of learning and training.

By contrast, Class 3. does call for the study of average and below-average types of decision makers: installment buyers who fail to compute the true interest rates, or housewives fascinated by "economy size" packages.

In the next Section some general hypotheses will be stated. They are, at least in part, mutually exclusive: if actual behavior confirms

one hypothesis it may contradict another one.

Finally, the concluding Section will contain suggestions of possible experiments, to test the hypotheses. Few of these experiments have been carried out on a convincing scale. Tentative explorations performed by the author over a number of years on his own graduate students, or by these students on their wives or friends, do supply some preliminary evidence, deserving to be tested in a more rigorous way. It would be worth while to perform such experiments on mature executives (in business or in public agencies) rather than on students.

II. GENERAL HYPOTHESES.

The following general hypotheses, or some of their implications, will be considered. (S means: subject).

HYPOTHESIS H.1. "RATIONAL" (or: "CONSISTENT") BEHAVIOR:

The S's actions (choices) are consistent with a constant system of numerical utilities (attached by him to the possible outcomes of his actions, and characterizing his "tastes") and of numerical subjective probabilities (attached by him to events that he does not control, and characterizing his "beliefs"). These numbers, constant over some reasonably long period of time, have the following property: out of any set of feasible actions, S always chooses one with the highest average of utilities of outcomes, weighted by the subjective probabilities of events. This average is called "mathematical expectation of utility" or simply "expected utility". This "expected utility principle" follows,

FOOT-
NOTE 1
by ordinary rules of logic, from much simpler, basic postulates[1]. These

I shall state presently. Later, I shall illustrate them, as well as some

of their logical implications, by appropriate experiments. Note, however, that a person whose behavior conforms with these postulates in simple experimental situations may or may not, when faced with more complex situations, obey the logical implications of these postulates, such as the expected utility principle. For it may be a stiff requirement for a man to be logical! This was, in fact, proved by some of the experiments.

(1) Postulate of consistent preferences between actions [see Experiments 1., below]: If S prefers action A to B, and B to C, he prefers A to C. It follows that outcomes of actions are also ranked consistently, since a given outcome (e.g., a "job") can be identified with an action (viz., "choosing the job") that yields this outcome with certainty whatever the external events. Choice under certainty is a special case of choice under uncertainty. In fact, Postulate (1) suffices for the economic theory of certainty; but to tackle economics of uncertainty, it is to be supplemented by (2) and (3) which follow.

(2) [See Experiments 2. below]. Admissibility postulate: suppose two actions, A and B, would result in the same outcome if certain events would occur; and that otherwise A results in an outcome that is preferable to B's outcome. Then A itself is preferable to B. (Laws that threaten death penalty for desertion from battlefield utilize, in effect, this postulate!)

(3) [See Experiments 3. below]. Independence of tastes and beliefs. If the S's choice among actions reveals that, in his view, one event is more probable than another, then, in the absence of new information, his subsequent choices must be consistent with this view, regardless of the

outcomes. One should neither think wishfully nor be subject to per-
secution mania!

The experiments that follow below tend to confirm that the basic
postulates are actually obeyed when and only when stress is absent (e.g.
memory is not overloaded, ample time is provided etc.) and, above all,
when and only when the structure of the problem is very simple and is laid
bare, by the use of lucid syntax, tabular presentation, etc.

Thus, Roger Crane, a member of a nationally known accounting firm
reports the use of "Payoff Tables" in his attempt to make the discussions
at Corporation Board meetings more orderly and fruitful, and even to apply
the Expected Utility Principle. Uncontrolled events (e.g. the future
possible states of the market) head the columns of the Table; the available
actions (whether to merge with firm A,B,C, or not at all) head the rows
of the Table; and the outcome of a given action and a given event is

FOOT- entered in the appropriate cell of the Table.[2]
NOTE2

Under stress, or if the problem is complex, or has not been pre-
structured ("pre-digested") for S's use, the "rational" hypothesis seems
to be rarely satisfied; it is not good descriptive psychology. Other
hypotheses present themselves:

FOOTNOTE   HYPOTHESIS H.2. "LEARNING THEORY":[3]
3

As trials are repeated, S approaches asymptotically a stable
behavior (which is, in general, not the rational one). In particular,
S's decision may depend on the success or failure (possibly measured from
some "aspiration level") of his previous decision; and the degree of this
dependence - the "reinforcement parameter" - determines the speed of

learning, characterizing a given $\underline{S}$.

HYPOTHESIS H.3. STOCHASTIC DECISION THEORY: [4]

$\underline{S}$'s choices are described by probability distributions which can be construed as weak (i.e., generalized) forms of rational behavior. For example a very weak postulate of stochastic behavior is this: "if $\underline{S}$ is more likely to prefer, than not to prefer, $\underline{A}$ to $\underline{B}$; and is more likely to prefer, than not to prefer, $\underline{B}$ to $\underline{C}$; then he is more likely to prefer, than not to prefer, $\underline{A}$ to $\underline{C}$". On the other hand, a very strong postulate of stochastic behavior ("Luce's Axiom") is this: "if, when presented with the triple $(\underline{A},\underline{B},\underline{C})$ $\underline{S}$ chooses $\underline{A}$ or $\underline{B}$ or $\underline{C}$ with relative frequencies in proportion $\underline{p}_A : \underline{p}_B : \underline{p}_C$; then, if presented with the pair $(\underline{A},\underline{B})$, $\underline{S}$ will choose $\underline{A}$ or $\underline{B}$ with relative frequencies in proportion $\underline{p}_A : \underline{p}_B$." This leads to a new definition of utilities (viz., numbers proportionate to $\underline{p}_A, \underline{p}_B, \underline{p}_C \ldots$)[5] A stochastic behavior postulate of intermediate strength has been used by the late L. L. Thurstone and his followers who, in essence, equate utility with "subjective sensation", taken from Fechner's psychophysics (1859). Still another postulate (also weaker than Luce's Axiom) assumes that the subject's ranking of all considered alternative actions varies according to some probability scheme.

FOOT NOTE5

HYPOTHESIS H.4:

FOOTNOTE 6

A combination of H.2 and H.3.[6]

HYPOTHESIS H.5. APPLICATION OF GESTALT THEORY:

$\underline{S}$ has sudden insight in the rational structure of a decision problem ("Aha!").

HYPOTHESIS H.6. EFFECT OF TRAINING:

S's insight can be brought about or accelerated by appropriate training strategies (which?).

III. EXPERIMENTS

EXPERIMENTS 1. Consistent ranking of actions.

Experiment (1.1) Complex objects such as "a Patent", "a Labor Contract", "Plant", etc., each described in detail, each wrought with uncertainties, and none having a market price, are presented in pairs, without cost to chooser (or with cost counted in), in the following order:

the (A, B)- pairs : (A_1, B_1) ; (A_2, B_2) ; ... ; (A_m, B_m) ;

the (B, C)- pairs : (B_1, C_1) ; (B_2, C_2) ; ... ; (B_m, C_m) ;

the (C, A)- pairs : (C_1, A_1) ; (C_2, A_2) ; ... ; (C_m, A_m) .

S is not consistent if he prefers A_i to B_i , and B_i to C_i, yet also prefers C_i to A_i, for some i.

Experiment (1.2) If for a given number (m) of trials (A_i, B_i, C_i), S has behaved consistently in Experiment (1.1), he is now made to choose from a set such as (A_1, B_1, C_2). S is not consistent if he prefers A_1 when presented with the pair (A_1, B_1), yet prefers B_1 when presented with the triplet (A_1, B_1, C_2). We invariably laugh at the story of the man who chose steak from a menu containing steak anf fish; but changed his choice to fish when told that chicken was also available! (the story is Luce's; its principle was first stated by Arrow).

Experiment (1.3) Finally, S is asked to rank all $3m$ alternatives. The ranking is tested against his preferences as exhibited in Experiment (1.1).

Preliminary evidence: Experiments on my own students suggest that they satisfied the Postulate of Consistent Ranking when, in Experiment (1.1),

m, the number of triads used, was small - e.g. m = 5. Inconsistencies arose more often in Experiments (1.2) and (1.3). However, the objects considered were those appropriate for the experience of (married) graduate students, rather than businessmen : jobs, trips, apartments, house furnishings, investment information, medical care, etc.

In other experiments, on students of logic and mathematics at Stanford University, the weak stochastic hypothesis mentioned above under H.3, was satisfied by almost all subjects; but not the Fechnerian hypothesis. The objects considered were small monetary wagers.[7]

FOOT
NOTE 7
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EXPERIMENTS 2. Admissibility Principle

Experiment (2.1) "You may get a job in a foreign country or an equally good one at home. If a revolution in a foreign country occurs you lose the job; but it can't happen here". Your choice?

Experiment (2.2) "Write down, but don't tell me the cash value (v) of your property to you: that is, the smallest acceptable price. Then name your asking price (a); after this, I (your broker) shall draw at random a number (b) as the buying price. If b exceeds a you'll get b. Otherwise you will keep your property. Now name your asking price".

Experiments performed in my class on my students, and by them on their friends or wives, tend to suggest the following evidence:

In the case (2.1) the foreign job is rejected, in accordance with the admissibility principle. But in the case (2.2) the admissibility principle is applied only if the experimenter "untwists" the problem by presenting and explaining a formula, a table, or a diagram, for example: Suppose the value of the property to you is v=25. It is arranged that if

b > a you receive b; if b ≤ a, you keep v. Hence the payoff will depend on a (which you control) and b (which you don't control) in the following way:

b:	10	20	30	40
a:		Outcomes, when v =25.		
15	25	20	30	40
25	25	25	30	40
35	25	25	25	40

Clearly the only admissible action is to name an asking price a equal to 25, i.e., to the value of the property to you (v). If you ask more or less than 25, you lose (compared with the admissible action) whenever b happens to fall between a and v, and you fare equally well otherwise. Honesty is (in this case at least) the best policy! Instead, most subjects named an asking price in excess of the value of the property to them, when the above tabulation was given. Perhaps they assumed, albeit unconsciously, that some bargaining will ensue, although this was explicitly excluded under the conditions given in the experiment.

EXPERIMENT 3.   Independence of tastes and beliefs.

Consider the following events, not controlled by you:

X: Kennedy wins in 1964

X̄:   he does not win

Y: next card drawn is a spade

Ȳ: it is not.

We consider now 6 actions (bets) A,B,C,A',B',C', and tabulate their

outcomes (gains and losses) which depend on the events, as follows:

Events	X	$\bar{X}$		$\bar{X}$	X		Y	$\bar{Y}$
Actions:								
A:	\$100	\$ 0	B: \$100	\$ 0		C: \$100	\$ 0	
A':	\$ 50	\$-50	B':\$ 50	\$-50		C':\$ 50	\$-50	

If you prefer A to B (i.e. bet on Kennedy rather than on his rival);

and B to C (i.e. bet on Kennedy's rival rather than on spades); then we

usually say that you think X is more probable than $\bar{X}$, and that $\bar{X}$ is more

probable than Y: This is the common English use of the subjective

probability concept. The experiment consists in finding whether a subject

retains this ranking of the three probabilities also when the stakes are

changed: on bets A', B', C', instead of gaining \$100 or nothing, he

now will gain or lose \$50. If his beliefs and tastes are independent,

this change of outcomes should not affect his ranking of events according

to their probabilities: X should, as before, appear to him more probable

than $\bar{X}$, and $\bar{X}$ more probable than Y and this should be reflected in his

choice of action. Hence he should prefer A' to B', and B' to C'.

When the conditions are presented in tabulated form as above,

few subjects show inconsistency. But would they remain consistent if the

job of structuring had been left to them?

Experiment 3 verifies, or refutes, the independence of tastes and

beliefs. We have seen that if such independence prevails, a probability

ranking of events will express the subject's beliefs. Moreover, if

Postulates (1), (2), (3) are verified, and, in addition, the subject

follows rules of logic, then it can be shown that his beliefs can be

expressed by assigning to the events not only probability ranks, but

actual numbers which we can call (subjective) probabilities because they

have the same mathematical properties as those established in the

classical theory of probabilities. In particular, if two events X and

Y are mutually exclusive the subjective probability assigned to the event

"X or Y" is the sum of the probability of X and of that of Y (the

"additivity law"). The logical grounds for the existence of such

numerical subjective probabilities can be roughly illustrated in the

context of the following experiments.

EXPERIMENT 4. Numerical subjective probabilities.

Denote by "$D.J.$" the "Dow-Jones Stock Price Average at this

year's end", and by x some positive number. Consider two actions (bets)

A and B such that:

if you choose A you will get \$100 if $D.J.$ exceeds x,

and you will get \$0 otherwise;

if you choose B you will get \$0 if $D.J.$ exceeds x,

and you will get \$100 otherwise.

By definition, if your beliefs and tastes are independent, your preference

ranking as between the two actions will remain the same if the pair of

outcomes (\$100,\$0) is replaced by some other pair (O_1, O_2) such that O_1

is better than O_2. Your choice as between A and B will indicate whether

you consider the event "$D.J.$ exceeds x" more or less probable than its

negation. Therefore, if we adopt the useful convention that the pro-

babilities of an event and of its negation add up to 1, we shall say that

the subjective probability of the event "D.J. exceeds x" is larger than 1/2 if you prefer A to B; smaller than 1/2 if you prefer B to A; and equal to 1/2 if you are indifferent between A and B. Hence, by adjusting the variable x up – and downward we can find a value of x such that your subjective probability of the event "D.J. exceeds x" is equal 1/2.

We can find in a similar way two numbers, y_1 and y_2 such that you will be <u>indifferent</u> between three following bets C,D,E:

If you choose C you will get \$100 if D.J. is less than y_1 ,

and \$0 otherwise ;

if you choose D you will get \$100 if D.J. is y_1 or more, but less

than y_2 ,

and \$0 otherwise;

if you choose E you will get \$100 if D.J. is y_2 or more,

and \$0 otherwise.

Or, in tabular form:

	I	II	III	
0	y_1	y_2		D.J. Scale
C	\$100	\$ 0	\$ 0	
D	\$ 0	\$100	\$ 0	
E	\$ 0	\$ 0	\$100	

Your indifference between C,D,E will indicate that you deem the three mutually exclusive events (the three intervals I, II, III on the D.J. scale) to be equally probable. In other words their subjective probabilities are 1/3, 1/3, 1/3 – <u>provided</u> your (the subject's) tastes and beliefs are independent. Whether they are, can be tested by finding whether you are

also indifferent between the following three bets (with the same meaning of y_1, y_2):

	I	II	III
	0	y_1 y_2	
C'	\$100	\$ 0	\$100
D'	\$ 0	\$100	\$100
E'	\$100	\$100	\$ 0

(your indifference between C and D revealed that, for you, events I and II are equally probable; this belief should not be changed if the outcome of either action in case of event III is changed from \$0 to \$100; hence you should be indifferent between C' and D'; and similarly between D' and E'.) On the other hand, any of the actions C',D',E' is better than any of the actions C,D,E, by the admissibility principle. Accordingly, having assigned probabilities 1/3 to each of the events I,II,III, we may assign probabilities 2/3 to the events "I or II", "II or III", "I or III", thus at the same time satisfying the additivity law for probabilities. And if we call \$100, \$0,"success" and "failure" respectively, we see that you prefer that action which has a higher subjective probability of success. Again, this agrees with usual English.

Using 3 mutually exclusive and exhaustive events, we have defined subjective probabilities 1/3 and 2/3 as revealed by a consistent subject's preferences between actions. This reasoning can be extended to any number - 4,5,...,n of events, - and leads to the definition of any subjective probability expressed by a rational fraction m/n; and, in fact, as expressed by any number (whether rational or irrational) between 0 and 1.

If each of two actions A_1 and A_2 can yield either "sucess" or "failure", and if A_1 is preferred (indifferent) to A_2 then the probability of success, p_1, of A_1 is larger than (equal to) the probability of success, p_2, of A_2, as viewed by the subject.

Let us agree to assign utility = 0 to failure (or to an action that always leads to failure). Then the utility of success (or of an action that always leads to success) must be larger; let it be = 1. By admissibility principle, any action that yields possibly success, and possibly failure, is better than sure failure and worse than sure success. Therefore its utility must be assigned a number between 0 and 1. In particular, to a lottery (action, bet, venture) A_i that succeeds with probability p_i, we may assign the utility p_i: this will agree with our previous statement that A_1 is preferred to (hence its utility is larger than that of) A_2 if $p_1 > p_2$. Now note that such an assignment of utilities agrees with the expected utility principle! for indeed

$$p_1 \cdot 1 + (1-p) \cdot 0 = p_1 .$$

Consider now a "mixed" lottery. It yields a ticket of A_1 with subjective probability π_1, a ticket of A_2 with subjective probability π_2 etc. Since the ordinary probability laws are obeyed, such a mixed lottery will yield success with subjective probability $p_1 \pi_1 + p_2 \pi_2 + \cdots$. And since we have agreed to equate a lottery's utility with its chance of success we see that the utility of the mixed lottery is the weighted average of the utilities of the component lotteries that are its outcomes. Again, this agrees with the expected utility principle.

As a further step, extend this reasoning to any outcomes that are not better than the "success" and not worse than the "failure" defined so far. The consistent man will be indifferent between such an outcome and some lottery which has probability p_i (say) of success; hence this outcome's utility can be set equal to p_i. And an action which yields, with respective probabilities $\pi_1, \pi_2, \ldots$, objects whose utilities are $p_1, p_2, \ldots$, will be equivalent, in that man's view, to the mixed lottery whose utility we have seen to be $p_1 \pi_1 + p_2 \pi_2 + \ldots$. This agrees again with the expected utility principle.

A complete and rigorous proof that the expected utility principle follows from the three basic postulates by ordinary rules of logic, is more lengthy and cannot be given here. Indeed this logical process, though elementary, does not seem to correspond to the behavior of an untrained, unprepared, or inexperienced "average" man of our culture. This is illustrated by the following experiment:

EXPERIMENT 5. Existence of a utility function.

Consider the following four lottery tickets:

Lottery ticket	If Heads, you get:	If Tails, you get:	Cash equivalent?
A	$100	$0	$a
B	$100	A	$b
C	$0	A	$c
D	B	C	$d

We either assume that the subjective probabilities of Heads and Tails are equal (i.e. that S is indifferent between betting something on Heads or betting the same thing on Tails) or we test this assumption by actual trials. [As

FOOT-NOTE 8 was done by Davidson and Suppes[8] who painted one nonsense syllable on 3 sides

of a die, and another nonsense syllable on the 3 remaining sides, and tested whether the subject who chose to bet on the first nonsense syllable changed the choice when the rewards on that bet were lowered very slightly]. Under the hypothesis (H.1) of consistent behavior, the subject assigning a winning chance $\frac{1}{2}$ to A should also assign

a winning chance $(\frac{1}{2}) \cdot (1) + (\frac{1}{2}) \cdot (\frac{1}{2}) = \frac{3}{4}$ to B;;

a winning chance $(\frac{1}{2}) \cdot (0) + (\frac{1}{2}) \cdot (\frac{1}{2}) = \frac{1}{4}$ to C ; and therefore

a winning chance $(\frac{1}{2}) \cdot (\frac{3}{4}) + (\frac{1}{2})(\frac{1}{4}) = \frac{1}{2}$ to D . Consequently he should declare $b > d = a > c$.

Moreover, if S is consistent and "averts risk" - so that, for him, money has "decreasing marginal utility" - he would name $a < 50$, $b < (100 + a)/2$, $c < a/2$. The three cash amounts a, b, c permit us to estimate three non-trivial points on the curve representing S's utility function of money gains and monetary wagers $u(g)$, provided his consistency has been confirmed by his naming $d=a$. For we are free to fix two points arbitrarily (as when "calibrat-ing" a thermometer scale): say, $u(\$100) = 1$, $u(\$0) = 0$. Then, $u(a) = u(A) = (1 + 0)/2 = .5$; $u(b)=u(B)=(1+.5)/2 = .75$; $u(c)=u(C)=(0+.5)/2 = .25$.

In experiments on students they usually failed to name $d=a$, unless they stated values $a=d=50$, $b=75$, $c=25$. In this case we suspect that they have parrot-like followed the rule (which they have heard somewhere) of computing actuarial values, and did not name their genuine subjective cash equivalent. To check this, and educate them for later experiments, we can ask them whether they are really indifferent between "gain $100, lose $100 on the toss of a coin" and "gain a dime, lose a dime on the toss of a coin", thus dispelling the learned myth of actuarial values.[9]

FOOT-
NOTE 9

Gordon Becker collected evidence that in some cases the subject's cash-equivalents do gradually approach a consistent pattern as the experiment is

continued (a lottery E is formed: D if Heads, \$100 if Tails; etc.). It is not clear whether such subjects "learn" in the sense of Hypothesis H_2, or have a sudden insight ("Aha!") as in hypothesis H.5. To decide this, appropriate significance tests need to be developed.

FOOTNOTES

1) An action is defined by the different outcomes it yields when different, specified events occur. In this sense, "not to act" (e.g., "not to invest", i.e. to keep cash) is also an action. Thus, any $\underline{S}$ is forced to compare between actions, as to which is preferable; but he can find himself indifferent. In what follows, the case of indifference is sometimes omitted to simplify presentation. In this and several other respects our presentation, sufficiently precise for its purposes, is not precise enough for complete logical analysis. This will be found in L. J. Savage's Foundations of Statistics (1954). Roughly, our Postulate (1) is his P1; our Postulate (2) is his P3; and our Postulate (3) is his P2 and P4. An elementary exposition will be given in Chapter 1 of the forthcoming Economic Theory of Teams by J. Marschak and R. Radner.

Economists will remember Alfred Marshall's (Principles of Economics 1908) use of the Expected Utility Principle, which goes back to Daniel Bernoulli and has been revived in the modern theory of decision as used in Statistics, Quality Control, and Operations Research; the latter includes, for example, modern Inventory Control, Scheduling of Production and Design, Investment Theory, etc., as well as applications to the economics of Effective Weapon Systems. These practical developments proceeded in spite of Frank Knights' warning (Risk, Uncertainty and Profit 1921) that probabilities in the classical sense (deduced from symmetry considerations or estimated from observed frequencies of events) were absent in the business world of not strictly repetitive events. The objection had been anticipated in the XVIII Century by D. Bernoulli's contemporary, Thomas Bayes, who first visualized the role of a priori probabilities (regardless of their psychological genesis), as well as the logical process of their subsequent revision by experience.

Most of the logical work deriving the expected utility principle from some basic postulates was done by F. Ramsey in the 1920-ies, B. de Finetti in the 1930-ies, J. Von Neumann and O. Morgenstern in the 1940-ies, L. J. Savage in the 1950-ies.

2) <u>An Approach to the Problem of Growth by Company Acquisition</u>. An address before the American Management Association's Orientation Seminary Appraising Corporation Mergers and Acquisitions, by Roger R. Crane and Alvin E. Wanthal, Touche, Ross, Bailey and Smart. Mimeographed. Quoted with Mr. Crane's permission.

3) Busch R.R. and Mosteller. <u>Stochastic Models for Learning</u>, 1955. The pioneer is W. K. Estes: "Toward a Statistical Theory of Learning", <u>Psychological Review</u> 57 (1950) pp. 94-107). Earlier literature was less lucid and failed to spell out the usual lack of perfect predictability of the learning process, its essentially probabilistic (stochastic, statistical) nature.

4) R. Duncan Luce. <u>Individual Choice Behavior</u>, 1959. Logical relations between various stochastic decision models were analyzed in detail by H. D. Block and J. Marschak ("Random Orderings and Stochastic Theories of Responses." <u>Contributions to Probability and Statistics</u>, Olkin et al, eds., 1960); by J. Marschak ("Binary Choice Constraints on Random Utility Indicators", <u>Mathematical Methods in the Social Sciences</u>, Arrow et al, eds., 1960); and more recently, with more emphasis on the experimental and statistical procedure, by G. Becker, M. DeGroot and J. Marschak ("Stochastic Models of Choice Behavior", <u>Behavioral Science</u>, January 1963, pp. 41-55). - See also F. Restle, <u>Psychology of Judgment and Choice</u>, 1961.

5) Think of the opinion-polled housewife in the New Yorker cartoon

(14 November 1959): "I'd say I'm about 42% for Nixon, 39% for Rockefeller, 19% undecided!"

6) Also treated in Luce's book quoted above, as well as in P. Suppes and R. C. Atkinson, Markov Learning Models for Multiperson Interactions, Ch. XI, 1960. and: W. E. Estes, "A Random Walk Model for Choice Behavior", Mathematical Methods in the Social Sciences, Arrow et al, eds, 1960.

7) See D. Davidson, and J. Marschak, "Experimental Tests of a Stochastic Decision Theory". Measurement: Definitions and Theories, Churchman and Ratoosh, eds, 1959.

8) In their book, Decision Making; an Experimental Approach, 1957.

9) Jacques Dreze has pointed out to me that Experiment 5 can be generalized to the case when the subjective probabilities of the two events in question are not necessarily equal and indeed are unknown to the experimenter. For example, instead of "Heads or Tails" we may use "Kennedy wins or loses". A consistent person will behave as if he assigned to these two events some probabilities, p and q, say, with $p + q = 1$; moreover, if as before, (without loss of generality) the utility scale is calibrated so that $u(\$100) = 1$, $u(\$0) = 0$, then a consistent man will behave as if he assigned to the four lottery tickets of the experiment the following utilities:

$$u(A) = p \cdot u(\$100) + q \cdot u(\$0) = p$$
$$u(B) = p \cdot u(\$100) + q \cdot u(A) = p + qp$$
$$u(C) = p \cdot u(\$0) + q \; u(A) = 0 + qp$$
$$u(D) = p(p + qp) + q(qp) = p \; ,$$

since $p + q = 1$. And since $p \geq 0$, $q \geq 0$, we obtain the following ranking of utilities: $u(B) \geq u(A) = u(D) \geq u(C)$; accordingly the consistent man should

name cash-equivalents of the four lotteries, ordered thus: $\underline{b} \geq \underline{a} = \underline{d} \geq \underline{c}.$
If neither event is judged impossible by the consistent man, i.e., if $\underline{p} > 0,$
$\underline{q} > 0,$ the ranking is exactly the same as in the text, viz. $\underline{b} > \underline{a} = \underline{d} > \underline{c}.$