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APRIL 1963

ANGLE TRACKING ACCURACY OF PHASED ARRAY RADARS

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PREPARED FOR:
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The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

Among researchers involved in the design and development of defenses against ballistic missiles there is considerable interest in the potential of phased-array radars. Of particular interest are those data processing methods which may be applied in the analysis or design of phased-array radars to arrive at optimal performance characteristics (e.g., target handling capacity, accuracy, power, etc.), minimal cost, or both.

This Memorandum deals with the statistical analysis of one such performance characteristic, namely, angle tracking accuracy. It illustrates a method of relating such accuracy both to receiver noise and to component errors. This is an advance over previous studies which have treated only one or the other source of error.

The work reported here was conducted in connection with RAND's studies of low-altitude defense which are sponsored by the Advanced Research Projects Agency's "Defender" program.

SUMMARY

This Memorandum discusses the angular accuracy of phased array radars, with assumptions appropriate to the problem of hard-point ballistic missile defense. An equation is derived relating angular accuracy to the signal-to-noise ratio in individual channels of the array and to random component errors in the individual channels. This equation is applicable to arrays with non-uniformly spaced elements, which are often considered for use in hard point defense systems. Smoothing of radar data to obtain estimates of angle and angular rate for interceptor guidance is also discussed.

ACKNOWLEDGMENTS

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CONTENTS

I. INTRODUCTION

Several papers have been published which discuss the angular tracking accuracy of phased array radars. Two recent publications considered the effect of random component errors, both amplitude and phase, on tracking accuracy.[1,2] A third paper deals exclusively with errors due to receiver noise.[3] In the present Memorandum, an equation for angular tracking error is derived which includes the effects of both component errors and receiver noise. The analysis is similar to those of Refs. 1-3 and shows that, when angular error is small, the mean squared angular errors due to receiver noise and component errors are additive.

The following discussion is slanted toward the problem of hard point defense. Since the detection ranges required in this case are relatively small, generally less than 100 miles, the power-aperture products required for detection are quite low. On the other hand, the angular accuracy requirements are quite severe if either command guidance or predicted fire is used during interception. The combination of low power-aperture product and good angular accuracy suggests the use of partially filled receiving arrays for economical design, particularly at low radar frequencies. The equation for angular accuracy which is derived here is applicable to arrays with any arbitrary distribution of receiving elements. Previous analyses have considered only receiving arrays with uniform element spacings.

The following analysis is simplified as far as possible without sacrificing any of the essential features of the problem. A large signal-to-noise ratio is assumed in the sum channel (not in individual channels or in the difference channel) so that the phase in the sum channel is an accurate measure of signal phase at the center of the array. In this case it can be shown that amplitude errors in the individual channels do not contribute to angular error. Also, because of the large signal-to-noise ratio it will be assumed that the target is being accurately tracked and is near the monopulse crossover angle, which further simplifies the analysis. The assumption of large S/N appears justified since accurate tracking is most important during the guidance phase when the target is at short range.

For simplicity, only phase comparison monopulse will be considered. In this case the monopulse difference signal is obtained by combining the signals from the individual channels of the array coherently. It has been shown that, by properly weighting the individual signals, this technique provides accuracy as good as is theoretically possible.*

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*J. Mallett, private communication.

II. SINGLE-PULSE ANGULAR ACCURACY

The coordinates in which angles are measured with a planar receiving array are illustrated in Fig. 1. The x and y axes lie in the plane of the array. Incremental phase shift between elements in the x-direction is a measure of the angle between the line-of-sight and the x-axis, i.e., θ. Similarly, incremental phase shift in the y-direction is a measure of the angle ψ.

We will consider measurement of one of these angles, say θ. It is assumed that a target has been detected and is being tracked so that its approximate coordinates (θ', ψ') are known. A sum beam is formed by adding the outputs from all of the elements in the array coherently, after incremental phase shifts appropriate to the approximate target angles (θ', ψ') are introduced in the individual channels. Also, a difference signal is formed which is a measure of the error in the estimated value of θ. Let there be an even number of elements in the array, with the elements on one side of the array numbered from 1 to N/2. Let $\{x_n\}$ represent the x-coordinates of these elements and assume for simplicity that the array is symmetric about the y-axis in the sense that $x_n = - x_{-n}$.

The amplitude and phase of the voltage (or current) in the nth channel can be represented by a vector quantity:

$$e_n = a(1 + \epsilon_n)\, e^{i(ux_n + \phi + \delta_n)} + \gamma_n' \qquad (1)$$

Where:

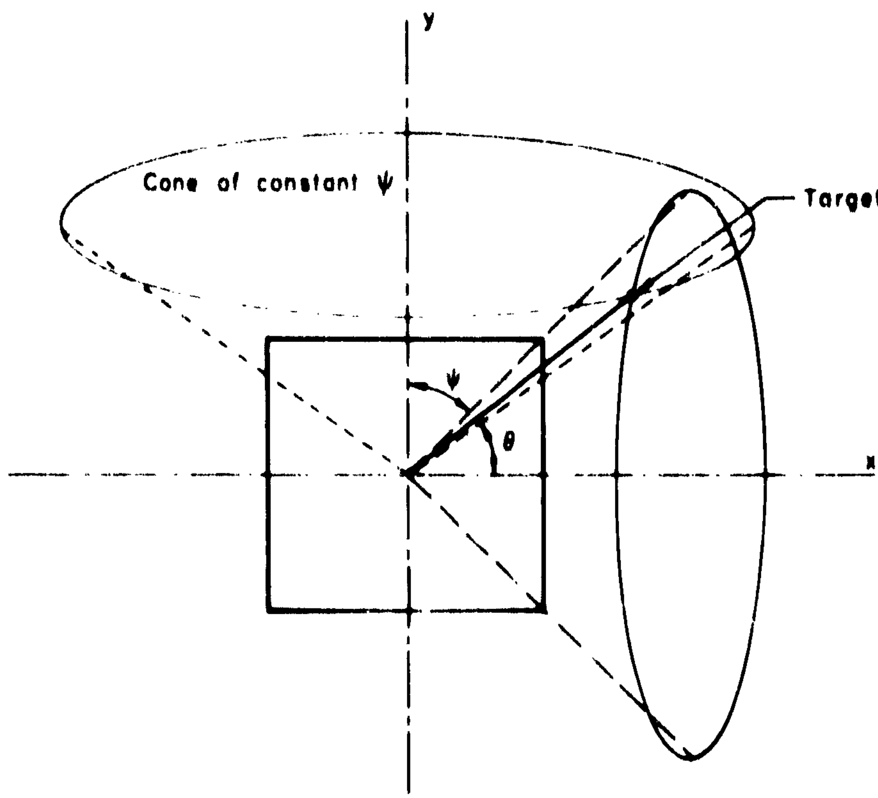

Fig. 1 — Angle measurement with a planar array

$u = \frac{2\pi}{\lambda} \cos \theta$

a = signal amplitude

ϵ_n = fractional amplitude error in the nth channel

λ = radar wavelength

ϕ = reference phase at the center of the array

δ_n = phase error in the nth channel

γ_n' = noise component in the nth channel

Each quadrature component of the receiver noise, γ_n', is normally distributed with a mean of zero and mean squared value σ_γ^2.

The sum signal is formed by using an even weighting function, $s(x)$:

$$S = \sum_{-N/2}^{N/2} s(x_n)\, e^{-iu'x_n}\, e_n \tag{2}$$

Where: $u' = \frac{2\pi}{\lambda} \cos \theta'$

$s(-x) = s(x)$

The maximum S/N ratio in the sum channel is obtained when $s(x) = 1$, but some taper may be introduced to reduce sidelobes. Since the array contains an even number of elements, let $e_o = 0$.

Equation (2) can be rewritten in the form:

$$S = \sum_{-N/2}^{N/2} s(x_n) \left[a(1 + \epsilon_n + i\delta_n + ivx_n) + \gamma_n \right] e^{i\phi} \tag{3}$$

where: $v = u - u'$

$$\gamma_n = \gamma_n' \, e^{-i(u'x_n + \phi)}$$

The statistical properties of the phase-shifted noise voltages γ_n are the same as those of γ_n'. Both the phase error δ_n and the target angle off boresight ($\sim vx_n$) have been assumed small in the above expansion. Where the S/N ratio is large and the effects of component errors are small in the sum signal, Eq. (3) becomes:

$$S = 2ae^{i\phi} \sum_{n=1}^{N/2} s(x_n) \tag{4}$$

The sum signal is used only as a reference for extracting angular error information from the difference signal. When a target is being continuously tracked and is near the monopulse crossover, the signal is much larger in the sum channel than in the difference channel. Hence, the effects of component errors and receiver noise on the sum signal are small in comparison with their contribution to the difference signal.

The difference signal is formed using an odd weighting function, $d(x)$:

$$D = \sum_{-N/2}^{N/2} d(x_n) \, e^{-iu'x_n} \, e_n \tag{5}$$

where: $d(-x) = -d(x)$

Using the same notation and assumptions as above:

$$D = e^{i\phi} \sum_{-N/2}^{N/2} d(x_n) \left[a(1 + \epsilon_n) + \gamma_{n1}\right]$$

$$+ ie^{i\phi} \sum_{-N/2}^{N/2} d(x_n) \left[a(vx_n + \delta_n) + \gamma_{n2}\right] \quad (6)$$

$$= e^{i\phi} \left[D_1 + iD_2\right]$$

where: $\gamma_n = \gamma_{n1} + i\gamma_{n2}$; γ_{n1} and γ_{n2} are real.

Only the second term, D_2, contains information about the target angle off boresight. This component of the difference signal can be extracted by using the reference phase of the sum signal, which also indicates the sense of the angular error. Since, to a first-order approximation, errors in the reference phase can be neglected, amplitude errors (ϵ_n) which appear only in the D_1 term can also be neglected. The term of interest is then:

$$D_2 = \sum_{-N/2}^{N/2} d(x_n) \left[a(vx_n + \delta_n) + \gamma_{n2}\right] \quad (7)$$

The rms error in measurement of v is

$$\sigma_v = \frac{\sigma_{D_2}}{\left[\frac{\partial D_2}{\partial v}\right]_{v=0}} \tag{8}$$

The slope of the difference signal D_2 at boresight is:

$$\left[\frac{\partial D_2}{\partial v}\right]_{v=0} = 2a \sum_{1}^{N/2} x_n d(x_n) \tag{9}$$

Also:

$$\sigma^2_{D_2} = 2(a^2\sigma^2_\delta + \sigma^2_\gamma) \sum_{1}^{N/2} d^2(x_n) \tag{10}$$

where: σ^2_δ = mean squared phase error in each channel.

Combining Eqs. (8-10) gives:

$$\sigma^2_v = \frac{\left(\frac{\sigma^2_\delta + \sigma^2_\gamma}{a^2}\right) \sum_{1}^{N/2} d^2(x_n)}{2\left(\sum_{1}^{N/2} x_n d(x_n)\right)^2} \tag{11}$$

This expression is minimized when d(x) is proportional to x, in which case:

$$\sigma^2_v = \frac{\left(\frac{\sigma^2_\delta + \sigma^2_\gamma}{a^2}\right)}{\left(\sum_{-N/2}^{N/2} x^2_n\right)} \tag{12}$$

The corresponding mean squared error in measurement of the angle θ is then:*

$$\sigma_\theta^2 = \left(\frac{\lambda}{2\pi \sin \theta}\right)^2 \cdot \frac{\sigma_\delta^2 + \frac{1}{2x_c}}{\sum_{-N/2}^{N/2} x_n^2} \tag{13}$$

where: x_c = signal-to-noise power ratio in each individual channel.

Equation (13) shows the dependence of single-pulse rms angular error (σ_θ) on the spacing of elements in the array $\{x_n\}$, the rms value of component phase error (σ_δ), and signal-to-noise ratio in the individual channels (x_c).

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*When the component error term is small in comparison to the receiver noise term ($\sigma_\delta^2 \ll \frac{1}{2x_c}$) and uniformly spaced elements are assumed, Eq. (13) is the same as the results of Ref. 3 (Eq. (14)). For $\sigma_\delta^2 \gg 1/2\ x_c$, Eq. 13 below gives the result derived in Ref. 2 for a uniformly spaced array.

III. DISCUSSION OF ASSUMPTIONS

A more intuitive discussion of the tracking problem may yield better insight into this analysis and serve to justify some of the assumptions. For this purpose, consider an array containing 1000 elements. Again, assume monopulse techniques are used to measure the angle θ, and that the elements are symmetrically placed with respect to the y-axis.

After a target is detected, two beams are formed in the approximate direction of the target, a sum beam and a difference beam. Consider first the ideal case of no component errors and no receiver noise. When the beams are pointed directly at the target the sum output has its maximum value, and the difference output is zero. Assume the signal component in each channel is 1 volt, so the amplitude of the sum output (S) is 1000 volts. If the target is now moved off boresight by a fraction of the beamwidth, S decreases slightly in amplitude but does not change in phase. The difference signal (D) is in quadrature with S and is roughly proportional to the angle off boresight. The ratio of D to S in this ideal case provides an exact measure of angle off boresight.

Next, assume that component errors and/or receiver noise are present and jointly contribute an rms error of 1 volt in each channel. In addition to its average value of 1000 volts at the correct reference phase, S then contains an error component with an rms value of $\sqrt{1000}$ or 30 volts. The sum signal (S) is used only to obtain a reference phase and amplitude for extracting angular information from the difference signal (D). A small percentage error in the amplitude of

S yields the same percentage error in the estimate of angle off boresight. For example, if the amplitude of S were in error by 2 per cent and the target 1/20 beamwidth off boresight, an error of 0.1 per cent of the beamwidth would result. For a continuously tracking radar with the target near boresight* this is a second-order effect.

The contribution of channel amplitude errors to D is in-phase with S, while only the component of D in-quadrature with S contains information on target angular position. Hence, if the phase of S is accurate, the in-phase component of D is discarded and amplitude errors do not degrade tracking accuracy. When there is an error in the phase of S, a portion of the D term containing amplitude errors (D_1 in Eq. (6)) is retained and contributes to angular error. Again, this is a second-order effect.

It is fortunate that angular accuracy is not critically dependent on amplitude tolerances in the individual channels when the angular errors due to receiver noise and channel phase errors are small. As noted in Ref. 2, the element patterns may differ considerably in the different channels of a phased array. If the individual channel gains are adjusted for equal signal amplitudes at one scan angle, there would be amplitude errors at other scan angles due to differences in the element patterns. It may be difficult, therefore, to maintain tight amplitude tolerances in the individual channels.

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*Ref. 1 considers in detail the case of targets off boresight.

IV. SMOOTHING OF RADAR DATA

The preceding analysis has considered only the single-pulse accuracy of an angular measurement. Guidance commands for an interceptor missile are generally based on the information obtained during a train of radar pulses, rather than on single-pulse data alone. From the radar data obtained during a preceding interval of time, both the angular position of the target and its angular rate are computed and used in the guidance computation. It may also be desirable to use higher-order derivatives of angular position, e.g., angular acceleration, in computing orders for command guidance or launch angles for unguided missiles.

The following discussion of smoothing applies only to random errors which are independent from pulse-to-pulse. Boresight misalignment and systematic errors which are fixed or slowly varying cannot be removed by smoothing. It has been noted, however, that fixed errors which are independent of scan angle will cancel in the measurement of relative position.(2) Hence, they are not important when command guidance is employed, provided the target and interceptor are tracked by the same radar. Random phase errors in the channels would generally be slowly varying or fixed (e.g., errors in element placement are a possible source of phase errors and would be fixed errors), and hence could not be reduced by smoothing. Again, these errors may cancel during the terminal phase of command guidance. Angular errors due to receiver noise will not cancel, however, even when the target and interceptor are at the same angle. This may be the most important type of radar error in command guidance systems and can be reduced by smoothing.

ESTIMATE OF θ

First consider a target at a fixed angular position. Let θ_E denote the estimate of angle obtained by averaging the measurements on several pulses. In this case:

$$\sigma_{\theta_E} = \frac{\sigma_\theta}{\sqrt{M}} \tag{14}$$

where: σ_{θ_E} = rms error in θ_E

σ_θ = rms error in single-pulse measurement

M = number of pulses averaged

Variations in range, and hence in S/N and σ_θ, during the smoothing interval are neglected here and in the following discussion. This case (Eq. (14)) is of little interest since, in general, the target would have a finite angular rate, $\dot{\theta} \neq 0$. Unless very short smoothing times were used, the angular rate would yield a significant bias error in θ_E.*

JOINT ESTIMATE OF θ AND $\dot{\theta}$

In general, both θ and $\dot{\theta}$ are unknown and a better estimate of angle can be obtained by fitting a linear curve to the data. Consider the case where $\dot{\theta}$ is constant over the smoothing interval T, and a set of M equally spaced measurements of θ are available. Here $M = f_r T$, where f_r is the radar pulse repetition frequency. Let $\{\theta_n\}$ represent the set of measured angles corresponding to the set of times $\{t - n\Delta\}$

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*Actually, the data could be compensated for a known value of $\dot{\theta} \neq 0$, so that even then the σ_{θ_E} given in Eq. (14) could be realized. However, this is a somewhat artificial situation.

where t is the time of the last observation ($n = 0$), Δ is the time interval between measurements, and $f_r \Delta = 1$. Let θ_E represent the estimated value of θ at the end of the smoothing interval and $\dot{\theta}_E$ the estimated angular rate. The errors in the θ_n measurements are normally distributed with a mean of zero and an rms value of σ_θ.

Maximum likelihood estimates of angle and angular rate are obtained by minimizing the mean square difference between the data and a linear curve, i.e., minimizing the quantity:

$$E = \sum_{n=0}^{M-1} (\theta_n - \theta_E + \dot{\theta}_E n\Delta)^2 \qquad (15)$$

For simplicity, we may assume that $M \gg 1$. Then, equating $\frac{\partial E}{\partial \theta_E}$ and $\frac{\partial E}{\partial \dot{\theta}_E}$ to zero and solving for θ_E and $\dot{\theta}_E$ gives:

$$\theta_E = \sum_{n=0}^{M-1} \theta_n \left(\frac{4}{M} - \frac{6n}{M^2} \right) \qquad (16)$$

$$\dot{\theta}_E = \sum_{n=0}^{M-1} \theta_n \left(\frac{6}{M^2 \Delta} - \frac{12n}{M^3 \Delta} \right) \qquad (17)$$

In the linear case, these equations are used to obtain estimates of target angle and angular rate, θ_E and $\dot{\theta}_E$, from the set of measured angles $\{\theta_n\}$. The estimates, θ_E and $\dot{\theta}_E$, are random variables. One is interested in determining how accurately these estimates correspond to the true values of angle (θ) and angular rate ($\dot{\theta}$).

Noting that $\bar{\theta}_n = \theta - n\Delta\dot{\theta}$, and averaging Eqs. (16) and (17), it can be seen that these estimates are unbiased, i.e., $\bar{\theta}_E = \theta$, and $\bar{\dot{\theta}} = \dot{\theta}$. To obtain the rms error in θ_E, Eq. (16) can be rewritten in the form:

$$(\theta_E - \theta) = \sum_{n=0}^{M-1} (\theta_n - \bar{\theta}_n) \left(\frac{4}{M} - \frac{6n}{M^2}\right) \tag{18}$$

Noting that

$$\begin{aligned} \overline{(\theta_n - \bar{\theta}_n)(\theta_m - \bar{\theta}_m)} &= \sigma_\theta^2 \qquad m = n \\ &= 0 \qquad m \neq n \end{aligned} \tag{19}$$

the mean squared error in the estimate of angle, $\sigma^2_{\theta_E}$, is:

$$\sigma^2_{\theta_E} = \frac{4\sigma_\theta^2}{M} = \frac{4\sigma_\theta^2}{f_r T} \tag{20}$$

From a similar calculation:

$$\sigma^2_{\dot{\theta}_E} = \frac{12\sigma_\theta^2}{T^2 M} = \frac{12\sigma_\theta^2}{f_r T^3} \tag{21}$$

For simplicity it has been assumed in this derivation that $M \gg 1$.

Note that the σ_{θ_E} of Eq. (20) is twice that of Eq. (14). When angular rate is unknown, the target angle at the end of the smoothing interval (θ_E) cannot be estimated as accurately as when angular rate is known. However, Eqs. (16) and (17) yield a joint efficient estimate of $(\theta, \dot{\theta})$.[4] When both quantities are unknown, no method of estimation will yield better accuracy than is indicated in Eqs. (20) and (21). These results can be related to the parameters of an array by combining Eq. (13) with Eqs. (20) and (21).

Again, if long smoothing times are used with the linear approximation, errors will result from unknown second and higher-order derivatives of θ.

JOINT ESTIMATE OF $\theta, \dot{\theta}$, and $\ddot{\theta}$

In this case one wishes to find the best fit between the set of measured angles $\{\theta_n\}$ and a curve of the form $(\theta_E - \dot{\theta}_E n\Delta + \ddot{\theta}_E n^2\Delta^2/2)$. Here θ_E and $\dot{\theta}_E$ are estimates of the angle and angular rate, respectively, at the end of the measurement interval. It is assumed that angular acceleration is constant during the smoothing interval, the measurements are uniformly spaced, and the errors in the θ_n are again normally distributed. The maximum likelihood estimate of $(\theta, \dot{\theta}, \ddot{\theta})$ is obtained by minimizing E:

$$E = \sum_{n=0}^{M-1} \left(\theta_n - \theta_E + \dot{\theta}_E n\Delta - \ddot{\theta}_E \frac{n^2\Delta^2}{2} \right)^2 \tag{22}$$

Equating the partial derivations of Eq. (22) with respect to θ_E, $\dot{\theta}_E$, and $\ddot{\theta}_E$ to zero and solving the resulting three linear equations gives:

$$\theta_E = \sum_{n=0}^{M-1} \theta_n \left(\frac{9}{M} - \frac{36n}{M^2} + \frac{30n^2}{M^3} \right)$$

$$\dot{\theta}_E = \sum_{n=0}^{M-1} \theta_n \left(\frac{36}{M^2\Delta} - \frac{192n}{M^3\Delta} + \frac{180n^2}{M^4\Delta} \right) \qquad (23)$$

$$\ddot{\theta}_E = \sum_{n=0}^{M-1} \theta_n \left(\frac{60}{M^3\Delta^2} - \frac{360n}{M^4\Delta^2} + \frac{360n^2}{M^5\Delta^2} \right)$$

This set of equations can be used to obtain estimates of angular acceleration, and of angle and angular rate at the end of the smoothing interval, from the measurements $\{\theta_n\}$. Again, it can be shown, from Eqs. (23) and $\bar{\theta}_n = (\theta - \dot{\theta}n\Delta - \ddot{\theta}n^2\Delta^2/2)$, that these estimates are unbiased.

From Eqs. (19) and (23), the mean squared errors in these estimates are (for large M):

$$\sigma^2_{\theta_E} = \frac{9\sigma^2_\theta}{M}$$

$$\sigma^2_{\dot{\theta}_E} = \frac{192\sigma^2_\theta}{T^2 M} \qquad (24)$$

$$\sigma^2_{\ddot{\theta}_E} = \frac{720\sigma^2_\theta}{T^4 M}$$

Equation (13) can be used to relate these errors to radar system parameters.

It is also interesting to note the correlation between θ_E, $\dot{\theta}_E$, and $\ddot{\theta}_E$. The covariances of these estimates are defined by

$$C_{\theta\dot{\theta}} = \overline{(\theta_E - \theta)\,(\dot{\theta}_E - \dot{\theta})}$$

$$C_{\theta\ddot{\theta}} = \overline{(\theta_E - \theta)\,(\ddot{\theta}_E - \ddot{\theta})} \qquad (25)$$

$$C_{\dot{\theta}\ddot{\theta}} = \overline{(\dot{\theta}_E - \dot{\theta})\,(\ddot{\theta}_E - \ddot{\theta})}$$

Substitution of Eqs. (23) and (19) into this yields (again for large M):

$$C_{\theta\dot{\theta}} = \frac{36\sigma_{\theta}^{2}}{TM}$$

$$C_{\theta\ddot{\theta}} = \frac{60\sigma_{\theta}^{2}}{T^{2}M}$$

$$C_{\dot{\theta}\ddot{\theta}} = \frac{360\sigma_{\theta}^{2}}{T^{3}M}$$

Now the correlation coefficients are seen to be

$$\rho_{\theta\dot{\theta}} = \frac{C_{\theta\dot{\theta}}}{\sigma_{\theta_E}\,\sigma_{\dot{\theta}_E}} = \frac{\sqrt{3}}{2} = .866$$

$$\rho_{\theta\ddot{\theta}} = \frac{C_{\theta\ddot{\theta}}}{\sigma_{\theta_E}\,\sigma_{\ddot{\theta}_E}} = \frac{\sqrt{5}}{3} = .745$$

$$\rho_{\dot{\theta}\ddot{\theta}} = \frac{C_{\dot{\theta}\ddot{\theta}}}{\sigma_{\dot{\theta}_E}\,\sigma_{\ddot{\theta}_E}} = \frac{\sqrt{15}}{4} = .968$$

A correlation coefficient with unit magnitude indicates that two estimates are "completely correlated," i.e., that the estimate of one parameter is uniquely determined by the estimate of the other. Our estimates of θ, $\dot{\theta}$, and $\ddot{\theta}$ at the end of the observation period are highly correlated. Since the correlation coefficients are positive, this means that if the errors in the measurements of θ obtained from the individual pulses are such that θ_E exceeds the true value of θ, then with high probability $\dot{\theta}_E$ and $\ddot{\theta}_E$ will exceed the true values of $\dot{\theta}$ and $\ddot{\theta}$, respectively.

For a given smoothing time, the errors in θ and $\dot{\theta}$ are greater in the parabolic case (Eq. (24)) than in the linear case (Eqs. (20) and (21)). However, with higher-order curve fitting, longer smoothing times can be used and should yield a net improvement in accuracy. Again, the maximum smoothing time which can be used advantageously is limited by the higher-order derivatives of angle, in this case the third and higher derivatives.

If desired, cubic or higher-order polynomials could be fitted to the measured data. Another possibility is fitting curves of forms other than polynomials to the data, e.g., the parameters to be estimated might include ballistic coefficient in addition to position and velocity. The improvements in accuracy which can be obtained by increasing the order of curve fitting, and also the maximum useful smoothing time for any given approximation, can be estimated from the geometry of the problem.

V. CONCLUSIONS

The error in a single-pulse angle measurement with a phased array radar, due to receiver noise and random phase errors in the individual channels, is given by Eq. (13) above. This equation is applicable to arrays with non-uniform element spacings, which are often considered for use in hard point ballistic missile defense systems.

Measured data from a train of radar pulses are used to obtain estimates of angle and angular rate for interceptor guidance. The accuracy of these estimates can be improved by using more complex smoothing procedures and longer smoothing times. This is another possible area for design trade-offs; increases in accuracy can be obtained either by increasing the complexity of data processing or by increasing transmitted power (or PG_TA_R product). A similar situation arises in search, where increased detection range can be obtained either by increased computer complexity (sequential detection) or by increasing the power-aperture product.

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