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Resonant Phenomena in Strange-Particle Production

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Submitted in partial fulfillment
of the requirements for the degree
of Doctor of Philosophy in the Faculty
of Pure Science, Columbia University

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ABSTRACT

In an exposure of propane to 2.0 BeV/C π^- mesons at the Cosmotron in the Columbia 30 inch chamber, reactions where three particles are produced were analyzed for resonances between any two particles in the final state. The reactions studied were sufficiently overdetermined to permit a separation of hydrogen events from carbon. Definite evidence was found for resonant peaks in the $\Lambda\pi$ system (Y_1^*) with M_o = 1392±7 MeV and $\Gamma/2$ = 40±10 MeV; in the $K\pi$ system (K^*) with M_o = 897±10 MeV and $\Gamma/2$ = 30±10 MeV. The 1404 and 1525 $\Sigma\pi$ resonances were also observed. The data indicate that the Y_1^* has spin 3/2 and its parity is even. The contribution of our experiment to the knowledge of these resonances was: a) It indicated that both the K^* and the Y_1^* resonances were produced in the same reaction and that the widths of these resonances were bigger than in previous experiments described later on in this paper.

b) It showed that the Y_1^* has spin 3/2 and even parity. It did not give any indication of the K^* spin. A Y_1^* of spin 3/2 and even parity is predicted by a study of possible symmetries among the baryon interactions. The fact that the spin is 3/2 also indicates that the Y_1^* is not a decay of an s-wave K-n bound system in the I = 1 state. This last result is in agreement with the results of low energy K-n interactions. How these results are predicted by the theories are sketched in Appendix B. In the introduction I give a survey of the experiments performed prior and during the time in which the data of this experiment was analyzed, and also present the significant results derived from the data of these experiments. The experimental procedure describes the method by which we analyzed our data. The most significant results of this experiment are described at the end of the papers, namely the spin and parity analysis. In Appendix A, I derive the various angular distributions to be expected for a spin 3/2 Y_1^*.

I. INTRODUCTION

In the last decade a great number of particles have been discovered (in this paper we will use the words particles and resonances interchangeably). Some of these particles have long enough lifetimes that their tracks can be observed in the bubble chamber before they actually decay. Recently, a new set of particles have been discovered which have such short lifetimes ($\sim 10^{-20}$ sec) that they decay before having gone far enough for a track to be formed. Hence, we only observe the decay products of these particles. To tell whether some observed tracks, which are due to particles already known, come from the decay of some new particle of well defined mass we compute the effective mass of the observed particles — namely:

$$M = \left\{ \sum_{i=1}^{n} E_i^{\,2} - \sum_{i=1}^{n} P_i^{\,2} \right\}^{\frac{1}{2}}$$

If these n particles are the decay products of a new particle, the distribution of events as a function of M will have a "resonance" peak with the mean value of $M = M_o$. In general this peak will have a certain width $\equiv \Gamma$. This width is related to the lifetime τ of the particle by the uncertainty principle relation $\Gamma\tau = 1$.

At the time of our experiment the presence of some of these new particles had already been established. A series of K^--

interactions, with K^- momentum in the range 500-1150 MeV/C, with the following final states:

(a) $K^- + p \to \Lambda^o + \pi^+ + \pi^-$

(b) $K^- + p \to \Sigma^\pm + \pi^\mp + \pi^o$

(c) $K^- + p \to \Sigma^\pm + \pi^\mp + \pi^\mp + \pi^\pm$

(d) $K^- + p \to \bar{K}^o + p + \pi^-$

(e) $K^- + p \to K^- + n + \pi^+$

(f) $K^- + p \to K^- + \pi^o + p$

were systematically investigated to find these new particles. In reaction (a) a particle, called Y_1^*, was found to decay into a Λ^o and π meson.[1] The Y_1^* has a mass value M_o = 1385 MeV, Γ = 50 MeV, and isotopic spin I = 1. An attempt was also made to determine the spin of the Y_1^* by means of an Adair analysis. The results are inconclusive primarily due to interference effects. These interferences are due to the fact that the two pion system has to obey Bose statistics.[2] Briefly the point is the following: reaction (a) can proceed in two ways, namely

$$K^- + p \to \begin{matrix} Y_1^{*+} + \pi^- \\ Y_1^{*-} + \pi^+ \end{matrix} \to \Lambda + \pi^+ + \pi^-$$

The amplitudes for these two channels have to be added coherently and their sum must have the correct symmetry for the exchange of the two pions as demanded by Bose statistics. There are two possible isotopic spin states for the two pions - namely I = 1 and I = 0. Since these two states are respectively odd or even under the interchange of the two pions, we can write the reaction amplitude for a given I-spin channel in the form

$$M(I = 0) = M'(1,2) + M'(2,1)$$

$$M(I = 1) = M''(1,2) - M''(2,1)$$

where 1 and 2 refer to the two pions. Hence, for example, if the Y_1^* production is dominated by the I = 1 channel, then the kinematical regions were $M''(1,2) = M''(2,1)$ would be depopulated. Evidence for such interference effects were present in the data. There was a large forward-backward asymmetry of the Λ^o in the decay of the Y_1^*, which could not occur in the case of an isolated Y_1^* decaying via strong interactions. The data was in agreement with the two following possibilities: either a $P_{3/2}Y_1^*$ being produced in an S state or an $S_{1/2}Y_1^*$ being produced in a P state. It was also observed that the decay rate of the Y_1^* into a sigma and a pion was consistent with zero.

No other experimental information was known at the time when we were planning our experiment. It was interesting to ask whether these particles are the result of a particular reaction like K-p interactions or whether they would also be produced in π-p interactions. We also had in mind to determine the spin and parity of the Y_1^* in this reaction. The reason why it was better to determine the spin and parity of the Y_1^* in π-p interactions is because all three particles in the final state, the Λ, K, and π, are different. Therefore, we are not hampered by possible interferences due to symmetry requirements. Of course, we could have interferences with the non-resonant background which could also foil out attempts to determine the spin and parity. Also if both the Y_1^* and a K-π resonance are produced in the same reaction the two reaction amplitudes could very well interfere.

During the analysis of our data new results appeared. There was presented evidence that the spin of the Y_1^* was greater than or equal to 3/2.[3] The evidence for this was that the decay angular distribution of the Y_1^* relative to the normal of the plane of production had a large $\cos^2\theta$ dependence. The fit to the distribution was given by $f(\theta) = \text{const}\ \{1 + (1.5 \pm 0.4)\ \cos^2\theta\}$. If the spin of the Y_1^* were 1/2 then this distribution would be isotropic. This sample of the Y_1^* was produced in K^--p interactions, with K^- momentum 1110 MeV/c. The presence of the two pions in the final state can again lead to interferences as shown before. Similar interference effects as before were found in the data but the magnitude of these interferences were much smaller. Hence it was assumed that the large $\cos^2\theta$ dependence in this distribution occurs because the spin of the Y_1^* is greater than 1/2. It was not possible to determine the parity of the Y_1^* since the Λ^0's from the Y_1^* were unpolarized; that is, supposedly the Y_1^* was produced aligned but not polarized.

During this period more particles were found; in reactions (d), (e), and (f) the K-π mass distribution showed a strong peak. This particle, called K^*, was found to have a mass M_0 = 885 MeV, Γ = 32 MeV, and I = 1/2.[4] This value for the isotopic spin of the K^* was determined from the observed branching ratio:

$$B = \frac{R(K^{*-} \to K^- + \pi^0)}{R(K^{*-} \to \bar{K}^0 + \pi^-)} = 0.75 \pm 0.35$$

Thus, writing the isotopic spin states $\psi(I = 3/2,\ I_z = -1/2)$

and $\Psi(I = 1/2,\ I_z = -1/2)$ in terms of the $\bar{K}$ and π meson states we get

$$\Psi(I = 3/2,\ I_z = -1/2) = \frac{1}{\sqrt{3}} \{\sqrt{2}\ K^-\pi^0 + \bar{K}^0\pi^-\}$$

$$\Psi(I = 1/2,\ I_z = -1/2) = \frac{1}{\sqrt{3}} \{K^-\pi^0 - \sqrt{2}\ \bar{K}^0\pi^-\}$$

If the K^* has $I = 1/2$ then $\psi(K^{*-}) = \Psi(I = 1/2, I_z = -1/2)$, then

$$B = \frac{R(K^{*-} \to K^-\pi^0)}{R(K^{*-} \to \bar{K}^0\pi^-)} = \frac{|\langle K^-\pi^0|H_{int}|K^{*-}\rangle|^2}{|\langle \bar{K}^0\pi^-|H_{int}|K^{*-}\rangle|^2}$$

$$= \frac{|\langle \Psi(I = 1/2, I_z = -1/2) + \sqrt{2}\Psi(I = 3/2, I_z = -1/2)|H_{int}|\Psi(I = 1/2, I_z = -1/2)\rangle|^2}{|\langle \sqrt{2}\Psi(I = 1/2, I_z = -1/2) + \Psi(I = 3/2, I_z = -1/2)|H_{int}|\psi(I = 1/2, I_z = 1/2)\rangle|^2}$$

where we neglect the $\bar{K}^0 - K^-$ and $\pi^0 - \pi^-$ mass differences. Since the interaction conserves the isotopic spin we have

$$\langle \psi(I = 3/2, I_z = -1/2)|H_{int}|\Psi(I = 1/2,\ I_z = -1/2)\rangle = 0$$

Hence we get $B = 1/2$. Similarly if the K^* has $I = 3/2$ we find $B = 2$. The experimental ratio shows that the isotopic spin of the K^* is 1/2. The spin of the K^* has been recently determined by two separate methods. The first method consists in studying the reaction[5]

$$\bar{p} + p \to \bar{K}^0 + K^{0*} \to \bar{K}^0 + \pi^- + K^0$$

$$\to \bar{K}^{0*} + K^0 \to \bar{K}^0 + K^0 + \pi^-$$

where the anti-proton is captured at rest and from an S-state.

Let us consider the case when the spin of the K^* is zero. Since the decay of the K^* is fast, it must occur through strong interactions and hence conserves parity. Therefore, the intrinsic parity of the $\bar{K}^*$ and the $\bar{K}$ are opposite. The intrinsic parity of the K and $\bar{K}$ are the same because they are bosons, while the parity of the p and $\bar{p}$ are opposite because they are fermions. Hence we find that parity of the initial and final states in the production process are the same only if the final state has orbital angular momentum equal to zero. Only the ^1S_o initial state can produce such a final state. Therefore, the initial system is even under charge conjugation and so must be the final state. Hence we can write the wave function of the final state in the form

$$\Psi = \frac{1}{\sqrt{2}} (K^o\bar{K}^{o*} + \bar{K}^oK^{o*})$$

If we now express the K^* wave function in terms of its decay products by means of Clebbsch Gordan coefficients and then substitute for the K^o and $\bar{K}^o$ the states that are observed to decay-namely,

$$K^o = \frac{K_1 + K_2}{\sqrt{2}} \qquad \bar{K}_o = \frac{K_1 - K_2}{\sqrt{2}}$$

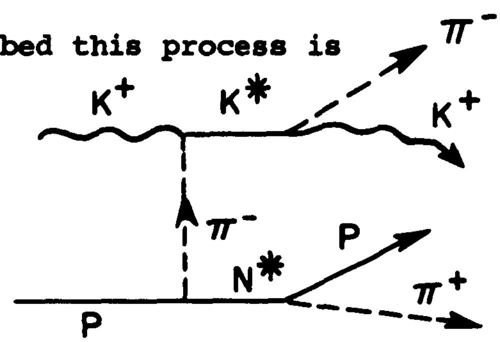
we find that all events where two neutral K's are produced must decay via the modes K_1K_1 or K_2K_2. That is, either one sees both K^o's decaying in the chamber or none at all. A similar argument can be carried through if the spin of the K^* were one. In this case both the $^3S_1$ and ^1S_o initial states can produce a K^*. We find that for reactions that proceed from the $^3S_1$ state the two neutral K's must decay in the K_1K_2 mode. The experimental result[6] was that when a K^* was produced, 6.5 ± 5.5 decayed in the K_1K_1 mode and 36.5 ± 15 events decayed in the K_1K_2 mode. This then leads to the conclusion that the spin of the K^* is not zero. A second determination of the spin of the K^* was made in the reaction[7]

$$K^+ + p \rightarrow K^+ + \pi^- + p + \pi^+$$

It turns out that this reaction really is a two body reaction

$$K^+ + p \rightarrow N^* + K^* \rightarrow K^+ + \pi^-$$
$$N^* \rightarrow \pi^+ + p$$

where the N^* is the already well known (3/2 3/2) resonance. There are also reasons to believe that the Feynman diagram which described this process is



If this model is correct, then in the center of mass system of the K^* the decay angular distribution of the K^* relative to the incoming K^+ should be isotropic if the spin of the K^* is zero, and should have a pure $\cos^2\theta$ dependence if the spin is one. It was the latter that described the experimental distribution. It is clear that if the spin of the K^* is one since the decay conserves parity, its intrinsic parity is the same as that of the K.

In addition to the K^*, other particles were found; namely, a particle of mass $M_o = 1404$, $\Gamma \sim 20$, $I = 0$ and a particle of mass $M_o = 1525$ $\Gamma \sim 20$ $I = 0$.[8] These new particles were seen in reaction (b) and (c); they are known to decay into a sigma and pion, and the latter one can also decay into a nucleon and kaon. As for the spin and parity of these particles they are not yet known.

Other experimental groups also studied the production of these particles in π-p interations.[9] Some results were published towards the end of our data analysis and showed only that the Y_1^* was produced but not the K^*. It was shown that the Λ's from Y_1^* decays were strongly polarized and hence it indicated that this reaction could be fruitful in determining the parity of the Y_1^*.[9b] Later results agreed with ours in that the production of Y_1^* and K^* occur in π-p interations. They also indicated the possible existence of a new particle of mass 730 MeV decaying into a kaon and a pion.[9c] No evidence for such a particle was seen anywhere else.

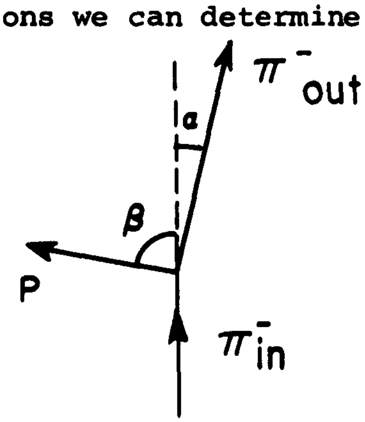
The contribution of our results to the knowledge about these particles is twofold. We show that both the Y_1^* and K^* are produced in π-p interactions. We also present evidence on the spin and parity of the Y_1^*.

II. EXPERIMENTAL PROCEDURE

The experiment is based on the analysis of some 140,000 pictures obtained at the Brookhaven National Laboratory Cosmotron in a 30 in. propane filled chamber.[10] One-fifth of these pictures were obtained with incident negative pion momentum of 1.92 BeV/c, one-fourth at 2.01 BeV/c, and the rest at 2.05 BeV/c. On the average we had 15 tracks per picture. The beam optics used is shown in Fig. 1. There were two beam defining slits; one placed between magnets 3 and 2, the other between magnet 5 and 6. They determined the spread in the value of the beam momentum and also the vertical spread of the beam. The last quadrupole was used to spread the beam in the vertical direction before entering the chamber, so that the tracks were not bunched. The temperature of the chamber was controlled by means of heaters placed at various positions around the chamber. The temperature was kept in the vicinity of 50°C. The pressure in the chamber was about 300 psi. The magnetic field in the chamber was 15 kG.

A. The Incident Beam Energy

The mean energy of the incident beam was studied in two ways: (1) By measuring the curvatures of non-interacting beam tracks. Knowing the magnetic field in the chamber then gives the momentum of each track. (2) In addition, we have measured a group of about 100 elastic π^--p scattering events with the proton stopping in the chamber. From a measurement of the range and direction of the proton, and the direction of the incoming and outgoing pions we can determine the momentum of the incoming pion. That is,



$$P_{\pi^-} = P \{\cos\beta + \cot\alpha \sin\beta\}$$

The events chosen were of such a nature that the proton stopped in the chamber and made an angle near 90° with the outgoing pion. Also, the incoming and outgoing pion tracks had to be long enough so that their direction could be measured accurately. To reduce the carbon contamination, we required that all three tracks be so coplanar that the incoming pion be at most ± 1/2° outside the plane formed by the two outgoing tracks. The error in the incoming momentum was determined with the relation

$$\delta P_{\pi^-_{in}} = \left\{ \left(\frac{\partial P_{\pi^-_{in}}}{\partial \beta} \right)^2 (\delta\beta)^2 + \left(\frac{\partial P_{\pi^-_{in}}}{\partial \alpha} \right)^2 (\delta\alpha)^2 \right\}^{1/2}$$

where we neglected the error in the proton momentum due to straggling (~ 1/4%) and the correlation term between the errors in α and β. The error for each event turned out to be about 50 MeV/c.

In addition, we know the currents in the bending magnets (see Fig. 1) for each of the three different beam momenta. From wire measurements, we know the ratio of the momenta at one magnet current setting to that at another. The three current settings are 596 amps, 560 amps, and 620 amps. The ratios are

$$\frac{P_{620}}{P_{560}} = 1.068 \qquad \frac{P_{620}}{P_{596}} = 1.020$$

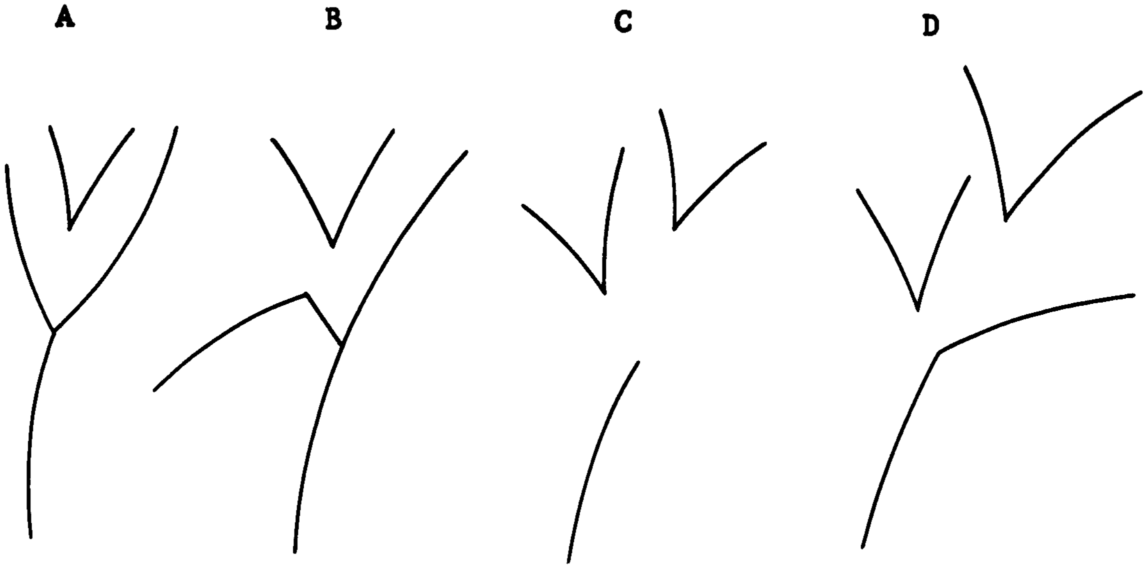
The resultant mean values of $P_{\pi^-_{in}}$ from elastic π^--p scattering for each value of the incident momenta are then averaged according to the ratios given above:

Current (amps)	P_{π^-} in (π^--p Scatt.) MeV/c	$\bar{P}$ (Average) MeV/c
620	2057 ± 6	2052 ± 4
596	1991 ± 6	2010 ± 4
560	1932 ± 8	1921 ± 4

The measurement of the curvature of the beam tracks gave values of the beam momenta (assuming the magnetic field in the chamber was 15 kG) which differed from those in the table above by <1.5%. The determination of the beam momenta from curvature depends linearly on the value of the magnetic field in the chamber. This magnetic field was not known to better than 1-2%. The value of the beam momenta obtained from elastic π^--p scattering depends on knowing the momenta of the proton from its range. This depends only weakly on the density of the propane in the chamber, namely, $P_p \propto \rho^{-1/4}$. This density was known to 1%. Due to the smaller systematic error in the latter method, we adjusted the value of the magnetic field used so that the two methods of measuring the mean value of the beam momenta agreed. The beam spread of 1/2% was determined from the geometry of the beam (see Fig. 1) and wire measurements. The beam momentum used in fitting the events was corrected for ionization loss in the chamber and given an uncertainty of 15 MeV/c to cover the width and errors in the mean momentum measurement.

B. Scanning and Fitting Procedure

The film was scanned twice, using each time two views out of the three views taken of the chamber. This was done so as to avoid missing events which in one view were superimposed on other beam tracks. The events which satisfied any of the following topologies



were recorded and later on measured with a digitized measuring machine. We were able to measure a point on the film to within 0.1 mm. We measured the position of 3 points along each of the tracks constituting an event. Each event was then sent through an IBM spatial reconstruction program which calculates the direction and momentum of each track and also indicates how well each event was measured. The results of this program were then used as the input data for a kinematical fitting program which was developed by Berge, Solmitz and Taft.[11] This program was required to take the measured quantities of each event and vary them within the assigned errors to the measurements until both energy and momentum were simultaneously conserved in the reaction. The program was required to do this for various hypotheses - namely, events with topologies A and B were fitted under the following hypothesis

(1) $\pi^- + p \rightarrow \Lambda^0 + \pi^- + K^+$

(2) $\pi^- - p \rightarrow \Sigma^{\pm} + K^0 + \pi^{\mp}$

(3) $\pi^- + p \rightarrow K^0 + K^- + p$

events with topology C were fitted to

(4) $\pi^- + p \rightarrow \Lambda^0 + K^0 + \pi^0$

(5) $\pi^- + p \rightarrow K^0 + \bar{K}^0 + n$

and events with topology D were fitted to

(6) $\pi^- + n \rightarrow \pi^- + K^0 + \Lambda^0$

In order to tell which events really satisfied any of these hypotheses, we required two things: a) We required that in the kinematical fitting of the event, none of the measurements of the momenta or direction of a track was changed by an amount much larger than the error we assigned to the measurement. A measure of how many parameters in the event were changed and by how much, is given by a quantity that the program computed, the χ^2. Basically, $\chi^2 = \sum_i \left\{ \frac{x_{i\ fit} - x_{i\ meas.}}{\Delta x_{i\ meas.}} \right\}^2$ where we sum over all the measured parameters x of the event; namely, the momentum and the two angles of each track. Reactions (1),(2), and (3) were required to have a $\chi^2 \leq 12$ with a few exceptions where there were reasons for a large χ^2 which did not reflect on the kinematical compatibility of these events. Reactions (4) and (5) were required to have a $\chi^2 \leq 3$, without exceptions. The reason for demanding a much lower χ^2 for these reactions is because in reactions (4) and (5) only two out of the three particles in the final state are observed. Therefore, since the conservation of energy and momentum gives rise to four constraint equations and since three pieces of information on the kinematics of the event are missing due to one particle not being seen, there is only one constraint equation that the program has to satisfy in order to get a fit. Hence it has to vary less the known parameters in order to get a fit. This then makes the χ^2 lower than in the case of reactions (1), (2), and (3) where all three particles in the final state are observed.

b) We also required that the fitted value of the momentum of each track be consistent with the ionization of the track in the chamber. To do this we took pictures of each event and looked at them during the analysis of the data to make sure that the ionization and momentum of each track were consistent. Hence, for those events which satisfied all the criteria, the fitting program supposedly gave us the best values of the momentum and direction of each track which satisfied the energy and momentum conservation and which minimized the value of χ^2. It was these values which we used in the analysis of the data. In what follows we discuss reactions (1), (2), (4) and (6) only because very few events of the type (3) and (5) were found.

C. Carbon Background

Propane is by no means ideal for this experiment. The chemical formula for propane, C_3H_8, shows that there are many protons in the carbon nuclei. Hence, many events which look like interactions with free protons occur with protons which are in the carbon nucleus. The selected events presumably contain all free proton events as well as those carbon events which fit the hydrogen kinematics within the measurement error.

The rejection of the carbon contamination is expected to be more efficient in reactions (1) and (2) where the direction and momenta of all the particles are known, than in reaction (4) where the π^o is not seen at all. To estimate the carbon contamination in our sample of events we have analyzed reaction (6) where the neutron forms part of the carbon nucleus, and where all the particles are seen. To relate the cross section for this reaction to the cross section for reaction (1) and (4) we make use of the fact that strong interactions are charge independent. We also assume that the incoming pion is equally likely to find a neutron or a proton at any point in the carbon nucleus. From the assumption of charge independence we can write these cross sections in terms of cross sections in a given isotopic spin channel, namely

$$\sigma \text{ (reaction 6)} = \sigma \text{ } (I = 3/2)$$
$$\sigma \text{ (reaction 1)} = \frac{1}{9} \sigma \text{ } (I = 3/2) + \frac{4}{9} \sigma \text{ } (I = 1/2)$$
$$\sigma \text{ (reaction 4)} = \frac{2}{9} \sigma \text{ } (I = 3/2) + \frac{2}{9} \sigma \text{ } (I = 1/2)$$

To estimate our carbon background, we need the value of σ (I = 1/2). To obtain this value, we must have the ratio of σ (reaction 1) to σ (reaction 4). If we use our data, Berkeley's, and Wisconsin's we find the ratio is $\sim$ 1. Using this value for the ratio of the two cross sections, we find

$$\sigma \text{ } (I = 1/2) = 1/2 \text{ } \sigma \text{ } (I = 3/2)$$

hence we get

$$\sigma \text{ (reaction 1)} \approx \sigma \text{ (reaction 4)} \approx \frac{1}{3} \sigma \text{ (reaction 6)}$$

which is the relation desired. The reaction type (6) was analyzed using the fitting program in two different ways:

a) By assuming that the reaction is really of the type (4); that is, assuming that the π^- in the final state is not present. Out of a total of 46 events found, 12 were seen to fit the hydrogen kinematics within our acceptance criteria. Hence, we expect that 4 events of the type (3) are from carbon, a contamination of ~ 8%.

b) In addition, the events were fitted to the reaction (6), but now all the knowledge about the π^- was used in the fit. Of the 46 events only 4 fitted the kinematics. Since we see three times as many K^+'s as K^o's in conjunction with a Λ^o, we may expect that about 4 events of the type (1) are from carbon; a contamination of ~ 3%. It is to be noticed that the amount of contamination depends sensitively on the experimental ratio σ (reaction 1) to σ (reaction 3). That is, for example, if this ratio is 1.3 instead of 1, then the contamination value obtained here doubles.

For reaction (2) the analysis is not so simple since now the Σ's instead of the Λ's are present. Hence a priori we have no way of telling what the carbon background for these events is. In general though, Σ production and Λ production cross sections are of the same magnitude. If we assume that this holds, and since the measurements for reaction (2) are worse than for reaction (1), we have a contamination somewhere in between ~ 4%.

There is an additional source of background for reactions (1) and (3) due to the fact that the Λ^o may actually be the result of a Σ^o decay. We found 3 cases of the materialization of a γ-ray among the events which otherwise fit reaction (1). The conversion efficiency of γ-rays is approximately 20% in our chamber. From this, the estimated Σ^0 background for reaction (1) is $3/0.20 \approx 15$ events minus the 3 events where the γ-ray is present. We have no corresponding value for reaction (3).

D. Total Cross Section Measurement

For the purpose of cross section determinations both the pion flux and the number of events were measured in a fiducial region, which for convenience was taken to be a rectangle in one of the photographic projections. Corrections were made for the probability of charged Λ^0 or K^0 decay in the chamber (0.95), μ and e contamination (0.1) and estimated detection efficiency (0.9). We do not present a value for $\Lambda^0 K^0 \pi^0$ production cross section for the following reasons:

a) We do not know the $\Sigma^0 K^0 \pi^0$ background.

b) We only considered for analysis those events which had very small measurement errors and which fitted very well the hydrogen kinematics; this was done to minimize the carbon and Σ^0 background. The assigned errors contain the statistical as well as the experimental errors. Table I summarizes the cross sections.

TABLE I

Reaction	No.of Events Found	Estimated Carbon Background	Cross Sections
$\pi^- + p \to \Lambda^0 + K^+ + \pi^-$	142	3%	72 ± 12μb
$\pi^- + p \to \Sigma^+ + \pi^- + K^0$	26	4%	34 ± 9μb
$\pi^- + p \to \Sigma^- + \pi^+ + K^0$	47	4%	51 ± 12μb

III. ANALYSIS OF THE MASS DISTRIBUTIONS

As indicated in the introduction, the way to find these particles with very short lifetimes is to plot the effective mass of the particles in the final state. In our reactions we have three particles in the final state. We wanted to find those particles which decayed into two known particles. To find these we plotted the number of events as a function of the effective mass of any two particles. An equivalent method is to plot the number of events as a function of the energy of the third particle. The equivalence can be seen from the relationship

$$(1) \quad m_{ij}^2 = E^2_{c.m.} - 2\, E_{c.m.} E_k + m^2_k$$

where E_k is the energy of the third particle in the c.m., and $E_{c.m.}$ is the total energy available in the c.m.

If there are no correlations between the particles in the final state, that is, for example, no two particles are the decay products of another particle, one expects a given distribution of the events as a function of the mass of the two particles. This distribution is due only to the kinematic characteristics of the reaction and is called the phase space distribution. To see how to formulate the phase space distribution to be expected for a given reaction we write the formula for the cross section of a given process. The cross section σ is defined through the relation[12]

$$(2) \quad d\sigma = \text{const}\, |M_{fi}|^2 \delta(\Sigma p_{in} - \Sigma p_{out}) \prod_{out} \delta(p^2 - m^2)\, d^4 p$$

where M_{fi} is the matrix element of the interaction Hamiltonian and where the constant is a function of the center of mass energy. We apply this now to our case where there are 3 particles i, j, and k in the final state. We write this expression in the overall center of mass system where $\Sigma\vec{p}_{in} = 0$. The assumption that there are no correlations between particles, or that no particle prefers to be produced with a given energy is equivalent to saying that M_{fi} is a constant. Hence

$$d\sigma = \text{const}\,|M_{fi}|^2\delta(E_i+E_j+E_k-E_{cm})\,\delta^3(\vec{p}_i+\vec{p}_j+\vec{p}_k)\,\delta(p_i^2-m_i^2)\ \times$$

$$\delta(p^2{}_j-m^2{}_j)\ \delta\ (p_k{}^2-m_k{}^2)\ d^4p_i d^4p_j d^4p_k$$

Using the relation

$$\int \delta(E_n{}^2-p_n{}^2-m^2{}_n)\,dE_n = \frac{1}{2E_n}\,\Big|\ E_n = \sqrt{p_n{}^2+m_n{}^2}$$

We get

$$d\sigma = \text{const}'\,|M_{fi}|^2\delta(E_i+E_j+E_k-E_{cm})\,\delta^3(\vec{p}_i+\vec{p}_j+\vec{p}_k)\frac{d^3p_i}{E_i}\,\frac{d^3p_j}{E_j}\,\frac{d^3p_k}{E_k}$$

Making use of $\delta^3(\vec{p}_i+\vec{p}_j+\vec{p}_k)$ to integrate over d^3p_k we get

$$d\sigma\ \text{const}'|M_{fi}|^2\ \frac{\delta(E_i+E_j+E_k(p_k=-|\vec{p}_i+\vec{p}_j|)-E_{cm}}{(m_k^2+p_i{}^2+p_j{}^2+2p_i\cdot p_j)^{\frac{1}{2}}}\ \frac{d^3p_i}{E_i}\ \frac{d^3p_j}{E_j}$$

Since $E_i dE_i = p_i dp_i$ we can write

$$d\sigma = \text{const}'\,|M_{fi}|^2\ \frac{\delta(E_i+E_j+(m_k{}^2+p_i{}^2+p_j{}^2+2p_ip_j\cos\theta_{ij})^{\frac{1}{2}}-E_{cm})}{(m_k{}^2+p_i{}^2+p_j{}^2+2p_ip_j\cos\theta_{ij})^{\frac{1}{2}}}$$

$$\times\ \ p_ip_jdE_idE_jd\Omega_id\Omega_j$$

Since M_{fi} is assumed to be constant we can now integrate over angles. We keep track j fixed at first and integrate over $d\Omega_i = d\cos\phi_{ij} d\phi_{ij}$. Since nothing depends on ϕ_{ij} we just get 2π from the integration over $d\phi_{ij}$. To integrate over $d\cos\theta_{ij}$ we have to make use of the δ function. We have

$$\frac{\delta(E_i + E_j + (m_k^2 + p_j^2 + p_i^2 + 2p_j p_i \cos\theta_{ij})^{\frac{1}{2}} - E_{cm})}{(m_k^2 + p_i^2 + p_j^2 + 2p_i p_j \cos\theta_{ji})^{\frac{1}{2}}} \, d\cos\theta_{ij}$$

$$= \frac{\delta(f(\cos\theta_{ij}))\ df(\cos\theta_{ij})}{\frac{\partial f(\cos\theta_{ij})}{\partial \cos\theta_{ij}} \ (m_k^2 + p_i^2 + p_j^2 + 2p_i p_j \cos\theta_{ij})^{\frac{1}{2}}}$$

where $f(\cos\theta_{ij})$ is the quantity inside the δ-function

$$\frac{\partial f(\cos\theta_{ij})}{\partial \cos\theta_{ij}} = \frac{p_i p_j}{(m_k^2 + p_i^2 + p_j^2 + 2p_i p_j \cos\theta_{ij})^{\frac{1}{2}}}$$

Integrating we finally get

$$d\sigma = \text{const}'' \, dE_i dE_j d\cos\theta_j$$

(3) $$d\sigma = 2 \text{ const}'' \, dE_i dE_j$$

Now we can integrate over the energies E_i where the maximum and minimum values of E_i are functions of E_j. Hence we get

$$d\sigma = 2 \text{ const}'' \{E_{max_i}(E_j) - E_{min_i}(E_j)\} dE_j$$

Using relation (1) we can rewrite this expression in the form

(4) $$d\sigma = \text{Const}''' \{E_{max_i}(m_{ik}) - E_{min_i}(m_{ik})\} m_{ik} dm_{ik}$$

This is the formula which we have put in our mass plots as the phase space distribution.

If any two particles have a particular combined mass in all the events, it would mean that the matrix element which describes the reaction is energy dependent. Then the expected distribution of the events as a function of the combined mass will differ from the one expected to represent the phase space alone.

For each event we calculate the invariant mass $m_{ij} = \{(E_i + E_j)^2 - (\vec{p}_i + \vec{p}_j)^2\}^{\frac{1}{2}}$ for all pairs of outgoing particles. The rms errors in these masses is 7 MeV for reactions (1) and (2) and 15 MeV for reaction (4). The errors are smaller for reactions (1) and (2) than for reaction (4) because (1) and (2) must satisfy four constraint equations while (4) only has to satisfy one constraint. Hence the masses in (1) and (2) are constrained more than in (4). The experimental mass distributions are shown in Figs. 2 - 7. The theoretical phase space distribution normalized to the total number of events are also shown in these figures. This distribution is averaged over the energy distribution of the incoming pions producing the observed events,

$$\frac{d\sigma}{dm_{ik}} = \text{const} \sum_{p=1}^{n} (E^p_{max_i} - E^p_{min_i})\, m_{ik}$$

where the constant normalizes the phase space to the observed number of events.

A. Resonances in the Reactions

$$\pi^- + p \rightarrow \Lambda^0 + K^+ + \pi^-$$

$$\pi^- + p \rightarrow \Lambda^0 + K^0 + \pi^0$$

The mass distributions of the Λ-π, K-π, and Λ-K systems for these two reactions were combined into single plots which appear in Figs. 2, 3, and 4. In Figs. 2 and 3, it is apparent that they show a resonant structure in which both the Y_1^* and the K^* play a large role. We assume that the Y_1^* and K^* peaks can be described by a distribution of the Breit-Wigner form

$$f(m) = \frac{N}{\pi} \frac{\Gamma/2}{(m - M_0)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

where N = total number of events. Given such a distribution in the Λ-π system, it will induce an effective distribution in the K-π system and Λ-K system and vice versa. It turns out that the Y_1^* induces a flat distribution in the K-π system and vice versa. Also, the Y_1^* induces a peaked distribution in the Λ-K system. Hence the prominent peak in the Λ^0K mass distribution seen in Fig. 4 can be understood as being just a kinematic reflection of the Y_1^*. The K^* has no such effect on the Λ-K mass distribution. To see this effect consider the diagram in Fig. 12. This is a plot of the kinematic limits of the mass of the Λ-K system given the mass of Λ-π system. The effect of the mass distribution of the K-π system can also be shown in this plot by making use of the relation

$$m_{\Lambda\pi}^2 + m_{\Lambda K}^2 + m_{K\pi}^2 = E_{cm}^2 + m_{\Lambda}^2 + m_K^2 + m_\pi^2 = \text{const}$$

This relation is easily derived from equation (1). It is easily seen that for the events where the Λ-π mass corresponds to the Y_1^* mass, the Λ-K mass is restricted to a region of allowable Λ-K mass values.

We fit all three experimental plots with a distribution of the form

$$F(m_{ij}) = \alpha(\text{phase space distribution}) + \beta(Y_1^* \text{ distribution}) + \gamma(K^* \text{ distribution}).$$

That is, in the fit of the Λ-π mass distribution, what we mean by the term $\gamma(K^*$ distribution) is really the effect of the K^* peak on the $\Lambda\pi$ system. Similarly, when we fit the K^* system, the term $\beta(Y_1^*$ distribution) means the effect of the Y_1^* peak on the K-π system.

The data were fitted by means of the least squares method. The following parameters with corresponding errors give a good fit to all three distributions:

	M_o(MeV)	$\Gamma/2$ (MeV)
Y_1^*	1392 ± 7	40 ± 10
K^*	897 ± 10	30 ± 10

α	β	γ
0.18	0.45 ± 0.08	0.37 ± 0.08

We conclude that reactions (1) and (4) are dominated by Y_1^* and K^* production, and that the peak in the Λ-K system is a kinematical reflection of the Y_1^*. However, we see no evidence for a K-π mass peak at 730 MeV reported for the same reaction at about the same energy.[9c] We point out also that our mass values as well as the widths of the Y_1^* and K^* are slightly larger than those determined in the K^--p reactions.

B. Resonances in the Reactions

$$\pi^- + p \rightarrow \Sigma^+ + \pi^- + K^o$$
$$\pi^- + p \rightarrow \Sigma^- + \pi^+ + K^o$$

Previous experiments[18] have established the existence of resonances in the $\Sigma\pi$ system at 1404 MeV and 1525 MeV. Our distributions appear in Figs. 5, 6 and 7. Because of lack of statistics no definite conclusions can be made, although the $\Sigma\pi$ mass spectrum seems to show both these resonances.

IV. PRODUCTION ANGULAR DISTRIBUTIONS OF THE Y_1^* AND K^*

These are shown in Figs. 8 and 9. The Y_1^* are produced preferentially backwards and the K^* forwards, consistent with the general observation that nature abhors large momentum transfers.

V. SPIN AND PARITY OF THE Y_1^*

There are several ways in which the data could be analyzed to yield information on the spin of the Y_1^*. We feel that we are too much above threshold to justify the use of the Adair analysis. As can be seen from the production angular distributions, the production goes through at least two orbital momentum channels. Instead, the data are analyzed for correlation, in the center of mass of the Y_1^*, between the Λ^o momentum and three orthogonal directions: the production plane normal, the Y_1^* direction, and the normal to the two. The frame of reference is obtained by first transforming to the production center of mass and then to the Y_1^* center of mass. The analysis for events in the mass interval $1340 \le m_{Y_1}* \le 1440$ is tabulated in the following tables.

TABLE II

A. Y_1^* Decay

$$f(\cos\theta) = \text{Const}\,[1 + a\cos\theta + b\cos^2\theta]$$

	a	b
† $f[P_\Lambda \cdot n]$	-0.29 ± 0.29	1.29 ± 0.78
$f[P_\Lambda \cdot P_{Y_1}*]$	-0.29 ± 0.17	-0.14 ± 0.34
$f[P_\Lambda \cdot P_{Y_1}* \times n]$	0.12 ± 0.14	-0.50 ± -0.24

† n = unit vector normal to the plane of production

B. $\Lambda^o_{Y_1}*$ Decay

$$f(\cos\eta) = \text{Const}\,[1 + \alpha\bar{P}\cos\eta]$$

	$\alpha\bar{P}$
$f[P_{\pi^-_\Lambda} \cdot n]$	0.55 ± 0.17
$f[P_{\pi^-_\Lambda} \cdot P_{Y_1}*]$	0.40 ± 0.18
$f[P_{\pi^-_\Lambda} \cdot P_{Y_1}* \times n]$	0.25 ± 0.18

The data were fitted by means of the maximum likelihood method. The distribution relative to the production plane normal has a large anisotropy; the coefficient of the $\cos^2\theta$ term is 1.29 ± 0.78 (see also Fig. 10). The smallness of the $\cos\theta$ amplitude, namely, 0.29 ± 0.29, gives some assurance that the interference with K^* and non-resonant production is small. Within the mass interval of the Y_1^*, namely $1340 < M_{Y_1^*} < 1440$, the Y_1^* is dominant: according to the distributions we have used to fit the mass spectra, 70% of the observed events are resonant. The large $\cos^2\theta$ term is most likely then an anisotropy in the decay of the Y_1^* itself or a statistical fluctuation. Given that the effect is real, it must be inferred that the Y_1^* is produced polarized or aligned and has spin greater than 1/2. Spin 3/2 is the simplest possibility. This result is in agreement with one other experiment in which the Y_1^* was produced in K^--p interactions.[3]

In the K^--p reaction experiments there was always a $\cos\theta$ term in the decay distribution of the Y_1^* relative to the Y_1^* direction, that is, the forward-backward asymmetry. In our experiment the $\cos\theta$ term in this distribution has an amplitude -0.29 ± 0.17. Nevertheless, we can show that this amplitude is really a kinematical effect due to events where simultaneously the Λ-π mass and the $K\pi$ mass are the Y_1^* and K^* mass respectively (see Fig. 11). To understand this consider the reaction

$$\pi^- - p \rightarrow K^+ + Y_1^{*-}$$

where the Y_1^* has a unique mass.

We can now ask the following question - under what conditions will the K-π system have the K^* mass? In the overall center of mass system the K^+ and Y_1^{*-} have definite momenta. If we transform to the center of mass of the Y_1^* the new K^+ momenta will be uniquely determined since the velocity of Y_1^* is known and hence

$$P'_{K^+} = \frac{1}{\sqrt{1 - \beta^2_{Y_1^*}}} \{P_{K^+} + \beta_{Y_1^*} E_{K^*}\}$$

Hence in the center of mass of the Y_1^* both the K^+ and $\pi^-_{Y_1^*}$ momenta are exactly determined. The mass of the $K^+ - \pi^-$ system depends on the momenta of the K^+ and π^-, and on the angle between them. The requirement that the K-π mass be a certain value uniquely determines the angle between the K and the π or between the π and the Y_1^*, which is the angle θ. As can be seen from Fig. 11, the distribution of events is flat except in the narrow region of $\cos\theta$ between -0.6 and -0.2. It turns out as expected that the large bump in this region is due to events where both the K-π and Λ-π masses are respectively the K^* and Y_1^* masses. It also turns out that the number of events in this region is about twice as many as are necessary to have a flat distribution. Since our fits indicate that the Y_1^* and K^* are produced with almost equal rates, we can say that there are no strong interferences between the K^* and the Y_1^* production which require the pion to behave as if it were resonating with the K and Λ simultaneously in this event.

Although the assignment of the spin of the Y_1^* is not very strong statistically, it is possible to show that, given the spin of the Y_1^* = 3/2, the resonance is a p-state resonance and that consequently the Y_1^* - Λ relative parity is even. (See Appendix A) This is done by analyzing the correlation in η, the angle between the pion in Λ^o decay and the Y_1^* production normal. The pion has been successively transformed to the production c.m., the Y_1^* c.m. and the Λ^o c.m. The expected distribution is $g(\eta) = \text{const}\ \{1 + \alpha\bar{P}\cos\eta\}$ where $\alpha = -0.67 \pm 0.07$ [14] and $\bar{P}$ is the average polarization of our sample of Λ^o's. For the events in the Y_1^* peak, in the mass interval 1340 - 1440 MeV, we find $\alpha\bar{P} = 0.55 \pm 0.17$. This is in agreement with the Wisconsin group which finds $\alpha\bar{P} = 0.61 \pm 0.28$.[9a]

The maximum Λ polarization compatible with the observed Y_1^* decay correlation is $|\bar{P}_\Lambda|_{max} = 0.47 \pm 0.09$ if the resonance is in the p-state, and $|\bar{P}_\Lambda|_{max} = 0.28 \pm 0.05$, if the resonance is in the d-state (see Appendix A). The experimental value is $|\bar{P}| = 0.82 \pm 0.27$, in better agreement with the p-wave or even parity case. If it should turn out instead that the Y_1^* has spin 1/2, then the Λ^o polarization shows in a similar way that the Y_1^* decays via s-wave.

VI. THE K^* SPIN

We have performed the same analysis for the K^* that was made for the Y_1^*. The results appear in Table III.

TABLE III

K^* Decay

$$f(\cos\theta) = \text{Const}\ \{1 + a\cos\theta + b\cos^2\theta\}$$

	a	b
$f[P_K \cdot n]$	-0.11 ± 0.2	0.33 ± 0.48
$f[P_K \cdot P_{K^*}]$	0.43 ± 0.23	-0.14 ± 0.39
$f[P_K \cdot P_{K^*} \times n]$	0.05 ± 0.2	0.00 ± 0.4

As can be seen, there is no appreciable $\cos^2\theta$ dependence in the angular distribution. We cannot make any conclusions as to the spin of the K^* from such a result. Let us note that the reason for the magnitude of the $\cos\theta$ amplitude in $f(P_K \cdot P_{K^*})$ is the same as in the case of the Y_1^*.

VII. CONCLUSIONS

A. We have clear evidence that the Y_1^* and the K^* are produced in $\Lambda^0 K\pi$ production at 2 BeV/C incident π^- momentum. There is no evidence for a $K\pi$ resonance at 730 MeV.

B. We seem to observe the 1404 and 1525 Σ-π resonances.

C. The most probable interpretation of the observed angular correlation in the Y_1^* decay requires that the Y_1^* spin be greater than 1/2. If the Y_1^* spin is then assumed to be 3/2, the relative parity of the Y_1^* and the Λ^0 must be even to account for the Λ^0 polarization. This result is in agreement with the predictions of Global Symmetry as shown in Appendix B. However, if the Y_1^* has spin 1/2, the parity must be odd.

VIII. FUTURE RESEARCH

It is important to obtain a definite value for the Y_1^* spin. Various experiments are in contradiction as to value of the spin. One possible different way to obtain the spin is by means of an Adair analysis near threshold for the reaction $\pi^- + p \rightarrow Y_1^{*-} + K^+$. Once the spin of the Y_1^* is known the parity follows from the experiments done at our energies. Using the same method we can also try to determine the spin of the 1404 and 1525 MeV $\Sigma\pi$ resonance.

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APPENDIX A

We would like to find ways of learning about the intrinsic properties of the Y_1^*. It is the purpose of this Appendix to make use of the Y_1^* decay angular distribution, and the Λ^o polarization to learn about the spin and parity of the Y_1^*.

We are dealing with the reaction $\pi^- + p \rightarrow K^+ + Y_1^{*-}$ where the target proton is unpolarized and the Y_1^{*-} then decays into a Λ^o and a π^-. One can show, from considerations of invariance of the strong interactions under rotation and inversion that the Y_1^* can only be polarized normal to the plane of production, which we call n. Let n be the z-axis and the direction of the incoming π^- be the x-axis.

We treat the problem non-relativistically; that is, we consider the Λ^o as a two component spinor. The experimental situation is actually non-relativistic since in the c.m. the momentum of the Λ^o is generally smaller than its mass.

We consider in detail only the case where the Y_1^* has spin 3/2. We assume that parity is conserved in the decay of the Y_1^*. Therefore, the decay occurs via p-wave if the Y_1^* - Λ relative parity is even (we define the Λ-π relative parity to be odd), and via d-wave if the Y_1^* - Λ relative parity is odd.

Our notation will be as follows:

$\Psi_j^{j_z}$ = wave function of the Λ^o in a state of total angular momentum j and z-component j_z.

$\chi_s^{s_z}$ = spin wave function of the Λ^o, $\chi_{1/2}^{1/2} = \begin{pmatrix}1\\0\end{pmatrix}$

and $\chi_{1/2}^{-1/2} = \begin{pmatrix}0\\1\end{pmatrix}$

Y_ℓ^m (θ,ϕ) = spherical Harmonics

The most general wave function of the $\Lambda^o - \pi$ system in the decay of the Y_1^* (S=3/2) in the frame in which the Y_1^* is at rest is

$$\Psi = a\Psi_{3/2}^{3/2} + b\Psi_{3/2}^{1/2} + c\Psi_{3/2}^{-1/2} + d\Psi_{3/2}^{-3/2}$$

where a, b, c, d represent the amplitudes for the production of the Y_1^* in the different states. These amplitudes can in general depend on the angle of production of the Y_1^*. Since we consider all angles of production in our analysis we take a, b, c, d to be constants. From invariance of the strong interaction under mirror reflection across the y-x plane, one can show that only the following amplitudes are not zero.

$$a \propto \langle S\ (Y_1^*) = 3/2 \quad S_z(Y_1^*) = 3/2 \ |H_{int}|\ S_z(p) = \mp 1/2\rangle$$
$$b \propto \langle S\ (Y_1^*) = 3/2 \quad S_z(Y_1^*) = 1/2 \ |H_{int}|\ S_z(p) = \pm 1/2\rangle$$
$$c \propto \langle S\ (Y_1^*) = 3/2 \quad S_z(Y_1^*) = -1/2 \ |H_{int}|\ S_z(p) = \mp 1/2\rangle$$
$$d \propto \langle S\ (Y_1^*) = 3/2 \quad S_z(Y_1^*) = -3/2 \ |H_{int}|\ S_z(p) = \pm 1/2\rangle$$

where $|S_z(p) - S_z(Y_1^*)|$ = even if the product of the intrinsic parities of the π, p, K, and Y^* is +1, and $|S_z(p) - S_z(Y^*)|$ = odd if the product is -1. That is, only one spin state of the proton contributes to each amplitude, the state contributing depends on whether the product of the intrinsic parities is +1 or -1. Finally we require a normalization. We have then

$|a|^2 + |b|^2 + |c|^2 + |d|^2$ = total number of events with a Y_1^* produced.

= 1 (for simplicity).

We consider first the case where the Y_1^* decays via p-wave. We want the Λ^o wave function in terms of its spatial and spin wave functions. These are

$$\Psi_{3/2}^{3/2} = Y_1^{1}(\theta,\phi)\,\chi_{1/2}^{1/2}$$

$$\Psi_{3/2}^{1/2} = \frac{1}{\sqrt{3}}\left\{\sqrt{2}\,Y_1^{o}(\theta,\phi)\,\chi_{1/2}^{1/2} + Y_1^{1}(\theta,\phi)\,\chi_{1/2}^{-1/2}\right\}$$

$$\Psi_{3/2}^{-1/2} = \frac{1}{\sqrt{3}}\left\{\sqrt{2}\,Y_1^{o}(\theta,\phi)\,\chi_{1/2}^{-1/2} + Y_1^{-1}(\theta,\phi)\,\chi_{1/2}^{-1/2}\right\}$$

$$\Psi_{3/2}^{-3/2} = Y_1^{-1}(\theta,\phi)\,\chi_{1/2}^{-1/2}$$

Combining these solutions we obtain for $\Psi_{3/2}$(p wave)

$$\Psi_{3/2}(\text{p wave}) = \left\{a\,Y_1^{1}(\theta,\phi) + b\sqrt{\tfrac{2}{3}}\,Y_1^{o}(\theta,\phi) + c\sqrt{\tfrac{1}{3}}\,Y_1^{-1}(\theta,\phi)\right\}\chi_{1/2}^{1/2} + \left\{b\sqrt{\tfrac{1}{3}}\,Y_1^{1}(\theta,\phi) + c\sqrt{\tfrac{2}{3}}\,Y_1^{o}(\theta,\phi) + d\,Y_1^{-1}(\theta,\phi)\right\}\chi_{1/2}^{-1/2} \quad \text{(A.1)}$$

The angular distribution of the Λ^o's in the c.m. is given by $\Psi^+\Psi$ where Ψ is given by Eq. (A.1).

We want the angular distribution of the Λ after we have integrated over the ϕ angle, namely

$$D_p(\theta) = \int_o^{2\pi} \Psi^+\Psi\,d\phi$$

Evaluating $\Psi^+\Psi$ and integrating we get after some algebra

$$D_p(\theta) = \frac{1}{4}\left\{3(|a|^2+|d|^2)+(|c|^2+|b|^2)+3[|b|^2+|c|^2-|a|^2-|d|^2]\cos^2\theta\right\}$$

(A.2)

We also want the polarization of the Λ^o in the z-direction, namely,

$$P_z(\theta) = \frac{\int_o^{2\pi} \Psi^+\sigma_z\Psi d\phi}{\int_o^{2\pi} \Psi^+\Psi d\phi} \qquad \text{where} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Substituting Eq. (A.1) for Ψ, one obtains

$$P_z(\theta) = \frac{1}{4}\Big\{3(|a|^2-|d|^2)+(|c|^2-|b|^2)+[3(|d|^2-|a|^2)+5(|b|^2-|c|^2)] \times \cos^2\theta\Big\}\frac{1}{D_p(\theta)} \qquad \text{(A.3)}$$

With our limited statistics, it is more feasible to measure the average polarization of the Λ^o, averaged over all angles θ. This is given by

$$\bar{P}_z = |a|^2 - |d|^2 + \frac{1}{3}(|b|^2 - |c|^2) \qquad \text{(A.4)}$$

The next case we want to consider is when the decay occurs via d-wave. To obtain the Λ^o wave function in the same manner as before we have to obtain first $\Psi^{3/2}_{3/2}$ (d-wave) and then apply the lowering operator to it. We have to start, however, with the wave-function that has the highest J and J_z compatible with d-wave. This is

$$\Psi^{5/2}_{5/2} = Y_2^2(\theta,\phi)\,\chi^{1/2}_{1/2}$$

Applying the lowering operator to it we get

$$\Psi_{5/2}^{3/2} = \frac{1}{\sqrt{5}}\left\{2Y_2^{1}(\theta,\phi)\ \chi_{1/2}^{1/2} + Y_2^{2}(\theta,\phi)\ \chi_{1/2}^{-1/2}\right\}$$

To obtain $\Psi_{3/2}^{3/2}$ from this result we make use of two conditions, namely orthogonality between states of different J, and normalization. The solution is

$$\Psi_{3/2}^{3/2} = \frac{1}{\sqrt{5}}\left\{Y_2^{1}(\theta,\phi)\ \chi_{1/2}^{1/2} - 2Y_2^{2}(\theta,\phi)\ \chi_{1/2}^{-1/2}\right\}$$

This solution satisfies the two conditions stated and is still an eigenstate of J^2. Applying the lowering operator to this we obtain the rest

$$\Psi_{3/2}^{1/2} = \frac{1}{\sqrt{15}}\left\{\sqrt{6}\ Y_2^{o}(\theta,\phi)\ \chi_{1/2}^{1/2} - 3Y_2^{1}(\theta,\phi)\ \chi_{1/2}^{-1/2}\right\}$$

$$\Psi_{3/2}^{-1/2} = \frac{1}{\sqrt{15}}\left\{3Y_2^{-1}(\theta,\phi)\ \chi_{1/2}^{1/2} - \sqrt{6}\ \ Y_2^{o}(\theta,\phi)\ \chi_{1/2}^{-1/2}\right\}$$

$$\Psi_{3/2}^{-3/2} = \frac{1}{\sqrt{5}}\left\{2Y_2^{-2}(\theta,\phi)\ \chi_{1/2}^{1/2} - Y_2^{-1}(\theta,\phi)\ \chi_{1/2}^{-1/2}\right\}$$

Again combining these results, we obtain for $\Psi_{3/2}$(d-wave)

$$\Psi_{3/2}(\text{d-wave}) = \left\{a\,\frac{1}{\sqrt{5}}\,Y_2^{1}(\theta,\phi) + b\sqrt{\frac{2}{5}}\,Y_2^{o}(\theta,\phi) + c\sqrt{\frac{3}{5}}\,Y_2^{-1}(\theta,\phi) + d\,\frac{2}{\sqrt{5}}\,Y_2^{-2}(\theta,\phi)\right\}\chi_{1/2}^{1/2} - \left\{a\,\frac{2}{\sqrt{5}}\,Y_2^{2}(\theta,\phi) + b\sqrt{\frac{3}{5}}\,Y_2^{1}(\theta,\phi) + c\sqrt{\frac{2}{5}}\,Y_2^{o}(\theta,\phi) + d\,\frac{1}{\sqrt{5}}\,Y_2^{-1}(\theta,\phi)\right\}\chi_{1/2}^{-1/2} \qquad \text{(A.5)}$$

Going again through the computations as for the case of p-wave decay we obtain for the angular distribution and polarization

$$D_d(\theta) = \frac{1}{4}\Big\{3(|d|^2+|a|^2) + (|c|^2+|b|^2) + 3(|b|^2+|c|^2-|a|^2-|d|^2)\cos^2\theta\Big\} \qquad (A.6)$$

$$P_z(\theta) = \frac{6}{4}\Big\{(|d|^2-|a|^2+3|b|^2-3|c|^2)\cos^4\theta + \frac{3}{4}(3|a|^2-3|d|^2-5|b|^2+5|c|^2)\cos^2\theta + \frac{1}{4}(|b|^2-|c|^2-3|a|^2+3|d|^2)\Big\} \times \frac{1}{D_d(\theta)} \qquad (A.7)$$

$$\bar{P}_z = -\frac{3}{5}\Big\{|a|^2 - |d|^2 + \frac{1}{3}(|b|^2 - |c|^2)\Big\} \qquad (A.8)$$

In case the spin of the Y_1^* is one-half one can easily show that:

a) The angular distribution $D(\theta)$ is independent of $\cos\theta$ both if it decays via s-wave or p-wave.

b) If $\Psi = e\Psi_{1/2}^{1/2} + f\Psi_{1/2}^{-1/2}$ then the polarization of the Λ^o is given by

$$\bar{P}_z = \begin{cases} (|e|^2 - |f|^2) & \text{if the } Y_1^* \text{ decays via s-wave} \\ -\frac{1}{3}(|e|^2 - f^2) & \text{if the } Y_1^* \text{ decays via p-wave.} \end{cases}$$

The following interesting points can be made:

1) $D_p(\theta) = D_d(\theta)$. That is, the Y_1^* decay angular distribution relative to its polarization can not be used to determine the parity of the Y_1^*.

2) If the angular distribution of the Λ^o turns out to be flat, no conclusions can be made as to the spin of the Y_1^*. This is clearly the case if the Y_1^* is unpolarized ($|a| = |b| = |c| = |d|$). It is also true if the Y_1^* states are so occupied that $|a|^2 + |d|^2 - |b|^2 - |c|^2 = 0$.

The presence of a $\cos^2\theta$ dependence has a twofold purpose: it indicates that the Y_1^* is polarized or aligned and that its spin is greater than 1/2.

3) If we assume the Y_1^* is in the $S = 3/2$ $S_z = 3/2$ state, namely $|a| = 1$, $b = c = d = 0$, then if the Y_1^* decays via p-wave the Λ^o polarization is 1.0, while if it decays via d-wave it is only 0.6. The fact that we have some additional information from the Y_1^* decay angular distribution gives us additional information on the maximum polarization the Λ^o can have in a particular Y_1^* decay channel. Let us consider this last point more in detail. What is the maximum polarization the Λ^o can have given the Y_1^* decay angular distribution? From Fig. 10 we find that the best fit to the decay angular distribution is given by

$$f(\theta) = 1 + (1.23 \pm 0.77)\cos^2\theta$$

Using Eq. (A.2) or (A.6) we get

$$\frac{3\{|b|^2+|c|^2-|a|^2-|d|^2\}}{3(|a|^2+|d|^2)+|c|^2+|b|^2} = 1.23 \pm 0.77$$

If in addition we make use of our normalization requirement

$$|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$$

we find

$$\frac{\{2(|b|^2 + |e|^2)-1\}}{\{3-2(|b|^2 + |c|^2)\}} = 0.410 \pm 0.26 \equiv a + \delta a \qquad \text{(A.9)}$$

Solving for $|b|^2 + |c|^2$, we get

$$|b|^2 + |c|^2 = \frac{3a+1}{2(1+a)} \pm \frac{\delta a}{(1+a)^2} = 0.79 \pm 0.13 \qquad \text{(A.10)}$$

To get the maximum polarization the Λ^o can have when Eq. (A.10) holds, we assume that the only occupied states are the ($J = 3/2$, $J_z = 3/2$) and ($J = 3/2$, $J_z = 1/2$) states; that is $c = d = 0$. Then if the Y_1^* decays via p-wave

$$\bar{P}_{zmax} = |a|^2 + \frac{1}{3}|b|^2 = 0.47 \pm 0.09$$

If the Y_1^* decays via d-wave

$$\bar{P}_{zmax} = 0.28 \pm 0.05$$

This is the result used to get the parity of the Y_1^*.

APPENDIX B

Theories about the Y_1^* Resonances

The existence of this pion-hyperon resonances have been predicted by various theoretical approaches to the study of particle physics. In some cases the values predicted for the spin and parity of this resonance differ among the various models. We present here a description of these theories or models.

The existence of the Y_1^* resonance has been deduced by various approaches, mainly the theory of Global Symmetry and the interpretation of low energy K-N interactions in the zero range-approximation.

1. The Theory of Global Symmetry[15]

This theory attempts to describe the baryon interactions with pions and kaons. It was already known that in the strong interactions of baryons with mesons the isotopic spin, strangeness, and baryon number were conserved. The symmetry in the interactions introduced by these conservation laws was nevertheless not too restrictive. That is, it did not treat the baryons as one group in their interactions, but treated the nucleons, the lambda, the sigmas, and the cascades separately. The question was then raised whether there may be a higher symmetry in the baryon interactions; that is, can we find a set of quantum numbers which relate the interactions of all baryons with the mesons. Then from the interaction of pions with nucleons, which is known experimentally, we should be able to predict some consequences of the pion interactions with the rest of the baryon group. The existence of a pion-nucleon resonance ($I = 3/2$, $P_{3/2}$) is well known from experiments. From a study of the structure of the global symmetry group,[15b] and the existence of this pion-nucleon resonance, we can predict a resonance (Y_1^*) in the Λ-π and Σ-π interactions ($I = 1$, $P_{3/2}$) with a given branching ratio $R = \dfrac{R(Y_1^* \to \Lambda\pi)}{R(Y_1^* \to \Sigma\pi)}$ for the decay of the Y_1^*. In addition, it follows that there must be other resonances: the Ξ^*, a $\Xi\pi$ resonance ($I = 3/2$, $P_{3/2}$), and the Z^*, a $\Sigma\pi$ resonance ($I = 2$, $P_{3/2}$).

We now describe the structure of the global symmetry group. This global symmetry group must have an 8 x 8 representation because there are 8 baryons. It must lead to the conservation of the isotopic spin, and hypercharge. To represent the baryons we write the column wave function

$$B = \begin{pmatrix} p \\ n \\ \Xi^o \\ \Xi^- \\ \Sigma^+ \\ Y^o \\ Z^o \\ \Sigma^- \end{pmatrix} \qquad \text{where } Y^o = \frac{1}{\sqrt{2}}(\Sigma^o + \Lambda^o)$$
$$Z^o = \frac{1}{\sqrt{2}}(\Sigma^o - \Lambda^o)$$

Given B we introduce the following 8 x 8 matrices $\mathcal{L}, \mathcal{M}, \mathcal{N}$, which are the operators acting on B,

$$\mathcal{L}_i = \frac{1}{2} \begin{pmatrix} \sigma_i & 0 & 0 & 0 \\ 0 & \sigma_i & 0 & 0 \\ 0 & 0 & \sigma_i & 0 \\ 0 & 0 & 0 & \sigma_i \end{pmatrix} \qquad i = 1,2,3$$

where the σ_i's are the Pauli matrices.

$$\mathcal{M}_1 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \end{pmatrix}, \mathcal{M}_2 = \frac{1}{2}i\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I \\ 0 & 0 & I & 0 \end{pmatrix}, \mathcal{M}_3 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & -I \end{pmatrix}$$

$$\mathcal{N}_1 = \frac{1}{2}\begin{pmatrix} 0 & I & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \mathcal{N}_2 = \frac{1}{2}i\begin{pmatrix} 0 & -I & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \mathcal{N}_3 = \frac{1}{2}\begin{pmatrix} I & 0 & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

It is beyond the scope of this paper to describe the theory of groups by which one arrives at the representation of the global symmetry group. We will just assume that the baryons are eigenstates of $\mathcal{L}^2$, $\mathcal{M}^2$, $\mathcal{N}^2$, $\mathcal{L}_3$, $\mathcal{M}_3$, $\mathcal{N}_3$ and that the mesons are also eigenstates of these operators. The idea of global symmetry is to state that these quantum numbers are conserved in the strong interactions. The following observables, charge Q, strangeness S, baryon number N, and isotopic spin I are related to $\mathcal{L}$, $\mathcal{M}$, and $\mathcal{N}$ by the equations

$$I_i = \mathcal{L}_i + \mathcal{M}_i \qquad i = 1,2,3$$

$$\mathcal{N}_3 = \frac{1}{2}(S + N) \qquad \text{where } 2\mathcal{N}_3 \text{ is the hypercharge}$$

$$Q = I_3 + \mathcal{N}_3$$

Hence we note that global symmetry leads to the conservation of isotopic spin and hypercharge. Also note that global symmetry is more restrictive than isotopic spin symmetry since it demands that not only $\mathcal{L} + \mathcal{M}$ be conserved, but both $\mathcal{L}$ and $\mathcal{M}$ be conserved. We note that $\mathcal{L}$, $\mathcal{M}$, and $\mathcal{N}$ satisfy the commutation relations for angular momenta. Hence, to add $\mathcal{L}$ and $\mathcal{M}$, for example to get I, we have to use the same rules as for the addition of two angular momenta; that is, we have to make use of Clebsch Gordan coefficients. Each baryon will have a set of quantum numbers $\mathcal{L}(\mathcal{L}+1)$, $\mathcal{L}_3$, $\mathcal{M}(\mathcal{M}+1)$, $\mathcal{M}_3$, $\mathcal{N}(\mathcal{N}+1)$, and $\mathcal{N}_3$. We give the mesons also a set quantum numbers such that I and $\mathcal{N}_3$ come out as observed experimentally. We present now a list of the quantum numbers.

Particle	$\mathcal{L}$	$\mathcal{L}_3$	$\mathcal{M}$	$\mathcal{M}_3$	$\mathcal{N}$	$\mathcal{N}_3$
p	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$
n	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$
Ξ^0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
Ξ^-	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
Σ^+	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
Y^0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	0	0
Z^0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
Σ^-	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0

Particle	$\mathcal{L}$	$\mathcal{L}_3$	$\mathcal{M}$	$\mathcal{M}_3$	$\mathcal{N}$	$\mathcal{N}_3$
π^+	1	1	0	0	0	0
π^0	1	0	0	0	0	0
π^-	1	-1	0	0	0	0
K^+	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
K^0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\bar{K}^0$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
K^-	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

So far we have covered the quantum numbers of the baryons and mesons. If global symmetry is correct then these quantum numbers are conserved in baryon pion interactions. Now, what about the pion-hyperon resonances? Where do they fit in? We only have one clue, namely, there exists the N^*, a pion nucleon resonance with $T = 3/2$. From the quantum numbers we have above we note that the pion nucleon system can be in one of two representations, namely ($\mathcal{L} = 3/2$, $\mathcal{M} = 0$, $\mathcal{N} = 1/2$), hence $T = 3/2$, or ($\mathcal{L} = 1/2$, $\mathcal{M} = 0$, $\mathcal{N} = 1/2$), and $T = 1/2$. Since we know $T = 3/2$ we find out that the N^* must have the quantum numbers ($\mathcal{L} = 3/2$, $\mathcal{M} = 0$, $\mathcal{N} = 1/2$). Hence, there must exist four states with $\mathcal{L} = 3/2$, namely $\mathcal{L}_3 = +3/2$ to $-3/2$. That is, we say that if one member of a multiplet exists then every state of a multiplet should be occupied. If we write out the quantum numbers available where $\mathcal{L} = 3/2$, $\mathcal{M} = 0$ or $1/2$ $\mathcal{N} = 0$ or $1/2$, we get for the resonances of the $\mathcal{L} = 3/2$ multiplet the following table, where we do not know $\mathcal{L}_3$. These four states close the $\mathcal{L} = 3/2$ multiplet.

Particle	ℓ	ℓ_3	$\mathcal{M}$	$\mathcal{M}_3$	$\mathcal{N}$	$\mathcal{N}_3$	I	L_j
N^*	3/2	?	0	0	½	½	3/2	$P_{3/2}$
Ξ^*	3/2	?	0	0	½	-½	3/2	$P_{3/2}$
Y_1^*	3/2	?	½	½	0	0	1	$P_{3/2}$
Z^*	3/2	?	½	-½	0	0	2	$P_{3/2}$

One of the predictions of this theory is the branching ratio of the Y_1^* decay into Λ-π and Σ-π final states. We now calculate this ratio. The Y_1^* is given by the three quantum numbers $(\ell, \mathcal{M}, \mathcal{N}) = (3/2, 1/2, 0)$ with $I = 1$. We want to obtain this state from a vector addition of the pion state (1, 0, 0) and the hyperon (½, ½, 0) state. Since $\vec{I} = \vec{\ell} + \vec{\mathcal{M}}$ we first get the combination of $\ell = 3/2$ and $\mathcal{M} = 1/2$ states such that I = 1. Using Clebsch Gordan coefficient we get

$$\psi(I = 1, I_3 = +1) = \frac{1}{2}\sqrt{3}\,\psi(\ell = 3/2, \ell_3 = 3/2)\psi(\mathcal{M} = 1/2, \mathcal{M}_3 = -1/2)$$
$$-\psi(\ell = 3/2\ \ell_3 = 1/2)\psi(\mathcal{M} = 1/2, \mathcal{M}_3 = 1/2)$$

where $\psi(\ell = 3/2, \ell_3 = 3/2)\psi(\mathcal{M} = 1/2, \mathcal{M}_3 = -1/2)$ refers to the pion hyperon system with such a set of quantum members. Note that the quantum number $\mathcal{N}$ is conserved automatically. We have to obtain now the state with $\ell = 3/2$, $\ell_3 = 3/2$, $\mathcal{M} = 1/2$, $\mathcal{M}_3 = -1/2$. Since the pion has $\mathcal{M} = 0$ the state must consist of the Z^o, Σ^- states. But $\ell_3 = 3/2$; the only combination of the pion and Z^o, Σ^- states that gives $\ell_3 = 3/2$ is then

$$\psi(\ell = 3/2, \ell_3 = 3/2)\psi(\mathcal{M} = 1/2, \mathcal{M}_3 = -1/2) = \pi^+ Z^o$$

For the other state, namely $\psi(\ell = 3/2, \ell_3 = -1/2)\psi(\mathcal{M} = 1/2, \mathcal{M}_3 = 1/2)$ we proceed similarly. The combination with $\mathcal{M} = 1/2$, $\mathcal{M}_3 = 1/2$

can only have Σ^+, Y^o states. Now we have to get the hyperon-pion combination that has $\mathcal{L} = 3/2$, $\mathcal{L}_3 = 1/2$. Using Clebsch Gordan coefficients we get

$$\Psi(\mathcal{L} = 3/2, \mathcal{L}_3 = 1/2) = \frac{1}{\sqrt{3}}\left\{\sqrt{2}\ \pi^o(\Sigma^o \text{ or } Z^o) + \pi^+(Y^o \text{ or } \Sigma^-)\right\}$$

but of the two hyperon states which are possible only one has the correct $\mathcal{M}$ quantum members. Hence we get

$$\Psi(\mathcal{L} = 3/2, \mathcal{L}_3 = 1/2)\Psi(\mathcal{M} = 1/2, \mathcal{M}_3 = 1/2) = \frac{1}{\sqrt{3}}\left\{\sqrt{2}\ \pi^o\Sigma^+Y^o\right\}$$

Hence combining the results we get

$$= \Psi(I = 1,\ I_z = +1) = \frac{1}{2}\left\{\sqrt{3}\ \pi^+Z^o - \frac{\sqrt{2}}{\sqrt{3}}\pi^o\Sigma^+ - \frac{1}{\sqrt{3}}\pi^+Y^o\right\}$$

If we now use the relations $Z^o = \frac{1}{\sqrt{2}}(\Sigma^o - \Lambda^o)$, $Y^o = \frac{1}{\sqrt{2}}(\Sigma^o + \Lambda^o)$

we get

$$\Psi(I = 1,\ I_z = +1) = \frac{1}{\sqrt{6}}(\pi^+\Sigma^o - \pi^o\Sigma^+ - 2\Lambda^o\pi^+)$$

Hence the matrix elements for the Y_1^{*+} decay into these channels are related, namely

$$|M(Y_1^{*+} \to \Lambda^o\pi^+)| = 2\,|M(Y_1^{*+} \to \pi^o\Sigma^+)| = 2\,|M(Y_1^{*+} \to \pi^+\Sigma^-)|$$

If one neglects the Σ-Λ mass difference then the branching ratios are

$$\frac{R(Y_1^{*+} \to \Lambda^o\pi^+)}{R(Y_1^{*+} \to \Sigma^+\pi^o)} = \frac{R(Y_1^{*+} \to \Lambda^o\pi^+)}{R(Y_1^{*+} \to \Sigma^o\pi^+)} = 4$$

If one takes into account the Λ-Σ mass difference then there is an additional effect due to the angular momentum barrier, since the Y_1^* is a $P_{3/2}$ resonance, and due to phase-space. For a reaction near threshold the rate in an angular momentum L channel has a dependence of the form p^{2L+1} where, in our case, p is the momentum of the pion in the rest frame of the Y_1^*, and L is the orbital angular momentum of the Λ-π or Σ-π system. In addition, there is a phase space factor. The combination of these two effects can be written in the form [16b]

$$\Delta = p_B^3 \frac{E_B}{E_B + E_\pi}$$

where B refers to the baryon. Using $M(Y_1^*) = 1390$ MeV we get

$$\frac{R(Y_1^{*+} \to \Lambda^o\pi^+)}{R(Y_1^* \to \Sigma^+\pi^o)} = \frac{R(Y_1^{*+} \to \Lambda^o\pi^+)}{R(Y_1^{*+} \to \Sigma^o\pi^+)} = 4 \left(\frac{204}{121}\right)^3 \quad \frac{1134}{1197} = 18$$

The experimental data seems to indicate that the branching ratio is even larger than predicted by Global Symmetry. The spin and parity of the Y_1^* seem to agree with the theory.

It is known experimentally that the quantum numbers as shown in the table are not conserved in some interactions where K-mesons are involved, for example

$$\pi^+ + p \to \Sigma^+ + K^+$$
$$K^+ + p \to \Sigma^+ + \pi^+$$
$$\pi^- + p \to \Sigma^- + K^+$$
$$K^+ + n \to K^o + p$$

A way out is to say that only the π mesons interactions with baryons conserve the quantum numbers.[15a] One can also say that there are some globally unsymmetrical interactions that lead to a mass splitting between the various I-multiplets and that make the K mesons a mixture of the (1, 1/2, 1/2)T = 1/2 and the (0, 1/2, 1/2) states.[15b]

2. The Interpretation of Low Energy K-n Interactions in the Zero Range Approximation

I present here a description of a method used by Dalitz and Tuan [16a] to understand the existence of the Y_1^* in terms of the low energy K-n interactions. From experiments it is known that the low energy K-n interactions are completely dominated by the L = 0 state both in the I = 0 and I = 1 channels[16 b,c]. Therefore, the data has been analyzed with the S-wave zero effective range theory. This theory describes the scattering phase shift of a reaction in terms of one parameter, called the scattering length. It is shown that if the real part of the scattering phase shift is negative in a given I-spin channel, then the pion-hyperon scattering phase shift can go through 90° at given value of the pion-hyperon mass. The fact that the real part of the scattering phase shift is negative indicates the existence of a bound K-n state. This bound state can only decay into a pion and hyperon because of energy conservation. Hence a mass peak in a pion-hyperon system with $M_o < M_K + M_n$ may be a manifestation of this bound state.

In general the K-n interactions occur through the I = 1 and I = 0 channels. I will discuss only the I = 0 channel because it is simpler since the Λ-π final state is missing in this channel. The general results derived here also carry over to the I = 1 channel.

We want to see how the K-n interactions in the I = 0 channel affect the pion-hyperon scattering in this I-spin channel. To describe all the possible reactions for I = 0, we consider the T-matrix

$$T = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}$$

Where

$$T_{11} = \langle Kn|T|Kn\rangle$$

$$T_{22} = \langle \pi\Sigma|T|\pi\Sigma\rangle$$

$$T_{12} = \langle Kn|T|\pi\Sigma\rangle$$

$$T_{21} = \langle \pi\Sigma|T|Kn\rangle$$

and where $T_{12} = T_{21}$ from time reversal invariance. In the case of S-wave interactions we can describe the scattering process in terms of one phase shift; that is,

$$\frac{d\sigma}{d\Omega}(\text{elastic}) = \left|\frac{e^{2i\delta_I}-1}{2ik}\right|^2$$

To relate the matrix element $\langle Kn|T|Kn\rangle$ to the phase shift we write for a two particle final state[12]

$$\frac{d\sigma}{d\Omega} = \frac{1}{V}|\langle Kn|T|Kn\rangle|^2 \quad \frac{2\pi}{4E_K E_n} \quad \frac{P_K}{4(2\pi)^3} \quad E_{c.m.}$$

where V=relative velocity of the incoming particles $= \frac{P_K E_{c.m.}}{E_K E_n}$

If we absorb the constants into the matrix element we get

$$\langle Kn|T|Kn\rangle = \pm \frac{(e^{2i\delta_I}-1)}{2ik/E_{c.m.}} \qquad \text{(B.1)}$$

We choose the + case. The final result is independent of what sign we use. Note that here we neglect the $K^- - K^0$ mass difference and n-p mass difference in the case of charge-exchange scattering. We are interested in effects over a very narrow region in $E_{c.m.}$. Hence we approximate here and make $E_{c.m.}$ = const even if k is not fixed. We drop $E_{c.m.}$ in the above expression. In general δ_I is complex since there is not only elastic scattering but also absorption. This is easily seen from the relations[17]

$$\sigma(\text{elastic}) = \frac{\pi}{k^2} \left|1 - e^{2i\delta_I}\right|^2$$

$$\sigma(\text{absorption}) = \frac{\pi}{k^2} \left(1 - \left|e^{2i\delta_I}\right|^2\right)$$

hence if δ_I is real we find σ absorption = 0.

In the S-wave zero range approximation we define the scattering length by the relation

$$k \cot\delta_I \equiv \frac{1}{A_I} \qquad \text{where } A_I \equiv a_I + ib_I \tag{B.2}$$

Substituting this into Eq. (B.1) we get

$$\langle Kn|T_I|Kn\rangle = \frac{A_I}{1-ikA_I} \tag{B.3}$$

Similarly we define for the $\pi\Sigma$ channel

$$\langle\pi\Sigma|T_I|\pi\Sigma\rangle = \frac{e^{ei\beta_I}-1}{2iq}$$

$$\cot\beta_I \equiv \frac{1}{B_I} \tag{B.4}$$

$$\langle\pi\Sigma|T_I|\pi\Sigma\rangle = \frac{B_I}{1-iqB_I} \tag{B.5}$$

For an S-wave interaction and a K-n initial state we can write for the wave functions in each channel

$$r\Psi_{Kn}(r) = \frac{\sin kr}{k} + T_{11}e^{ikr} \tag{B.6}$$

$$r\Psi_{\Sigma\pi}(r) = T_{12}e^{iqr} \quad (B.7)$$

where $T_{11}e^{ikr}$ is the scattered amplitude in the K-n channel and $T_{12}e^{iqr}$ is the scattered amplitude in the π-Σ channel. Similarly, if the initial state is in the π-Σ channel we get

$$r\Psi_{Kn}(r) = T_{21}e^{ikr} \quad (B.8)$$

$$r\Psi_{\Sigma\pi}(r) = \frac{\sin qr}{q} + T_{22}e^{iqr} \quad (B.9)$$

Now we want to make use of the boundary conditions at the origin in order to relate the K-n scattering to the π-Σ scattering. It is convenient to obtain the boundary conditions in terms of different matrix elements. We define a new matrix K related to T by

$$T = K(1 - ikK)^{-1}$$

In terms of the matrix K the wave function in channel j, if the incident channel is i, is written in the form

$$r\Psi_j^{(2)}(r) = \delta_{ij}\frac{\sin k_j r}{k_j} + K_{ji}\frac{\cos k_j r}{}$$

The boundary condition can now be expressed by the relation

$$r\Psi_j(r)_{r=0} = +\sum_{\ell} K_{j\ell}\left(\frac{\partial}{\partial r} r\Psi_{\ell}(r)\right)_{r=0} \quad (B.10)$$

We now apply this boundary condition to two cases:

a) The initial state is the K-n system. We get two equations:

$$T_{11} = \alpha(1 + ikT_{11}) + \beta iqT_{12} \quad (B.11)$$

$$T_{12} = \beta(1 + ikT_{11}) + \gamma iqT_{12} \tag{B.12}$$

where we have set $\begin{pmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}$

From Eq. (B.12)

$$T_{12}(1 + iq\gamma) = \beta(1 + ikT_{11})$$

$$\beta\ iq\ T_{21} = \frac{iq\beta^2(1 + ikT_{11})}{1 - iq\gamma} \tag{B.13}$$

Substituting Eq. (B.13) into Eq. (B.11) we get finally

$$T_{11} = \alpha + \frac{iq\beta^2}{1-iq\gamma}(1 + ikT_{11}) \tag{B.14}$$

from Eq. (B.3) we have, dropping the subscript I,

$$T_{11} = \frac{A}{1 - ikA}$$

or

$$A = \frac{T_{11}}{1 + ikT_{11}} = \alpha + \frac{iq\beta^2}{1-iq\gamma} \quad \text{from Eq. (B.14)} \tag{B.15}$$

b) The initial state is the $\pi\Sigma$ system. We get two more equations:

$$T_{12} = \alpha ikT_{12} + \beta(1 + iqT_{22}) \tag{B.16}$$

$$T_{22} = \gamma(1 + iqT_{22}) + \beta ikT_{12} \tag{B.17}$$

Going through the same process as for case (a) and using Equation (B.5) we get:

$$B = \gamma + \frac{ik\beta^2}{1 - ik\alpha} \tag{B.18}$$

We now make the assumption that k ctn δ_I is a constant for k near 0; that is, near the K-n threshold.[18] Then we write Eq. (B.15) as

$$a+ib = \alpha + \frac{iq_o\beta^2}{1 - iq_o\gamma} = \alpha - \frac{q_o^2\beta^2\gamma}{1 + q_o^2\gamma^2} + \frac{iq_o\beta^2}{1 + q_o^2\gamma^2} \qquad (B.19)$$

where q_o is the momentum of the hyperon at the K-n threshold. From Eq. (B.18) we find

$$q \cot\beta = \frac{1}{B} = \frac{1}{\gamma} \frac{1}{1 + \frac{ik\beta^2/\gamma}{1 - ik\alpha}}$$

$$= q_o \cot\beta_o \frac{(1 - ik\alpha)}{(1 - ik\alpha + ik\beta^2/\gamma)} \qquad (B.20)$$

where we used the fact that at the K-n threshold

$$B_o = \gamma$$

Using Eq. (B.19) we get

$$a - \alpha = - \frac{q_o^2\beta^2\gamma}{1 + q_o^2\gamma^2} = -q_o\gamma b$$

$$= -b \tan\beta_o$$

or

$$\alpha = a + b \tan\beta_o$$

Also we get

$$a - b \cot\beta_o = \alpha - \frac{q_o^2\beta^2\gamma}{1 + q_o^2\gamma^2} - \frac{\beta^2/\gamma}{1 + q_o^2\gamma^2}$$

$$= \alpha - \beta^2/\gamma$$

$$q \cot\beta = q_o \cot\beta_o \frac{[1 - ik(a + b \tan\beta_o)]}{[1 - ik(a - b \cot\beta_o)]} \tag{B.21}$$

This is the relation we set out to derive. We have found a relation which determines the phase shift in π-Σ scattering in terms of the scattering length a + ib of K-n scattering in the I = 0 channel. Note that this solution is only valid for $q \approx q_o$. Now we want to show that β can approach 90^o at an energy M_o and hence the $\pi\Sigma$ scattering can have a resonance at M_o. We calculate

$$q < \pi\Sigma|T|\pi\Sigma > = \frac{e^{2i\beta}-1}{2i} = \frac{\tan\beta}{1 - i \tan\beta} \tag{B.22}$$

Using relation (B.21) we get

$$\tan\beta = \frac{(1 - ika)\sin\beta_o + ikb\cos\beta_o}{(1 - ika)\cos\beta_o - ikb\sin\beta_o}$$

Hence we get after a little algebra

$$q < \pi\Sigma|T|\pi\Sigma > = \frac{(1 - ika)\sin\beta_o + ikb\cos\beta_o}{1 - ik(a + ib)} e^{i\beta_o}$$

$$= e^{i\beta}\sin\beta$$

Now let us consider how β behaves as a function of k below the K-n threshold. For this we must let $k \to i|k|$ since k becomes imaginary in this region. This is necessary so that the wave functions for the outgoing states in Eq. (B.6) and Eq. (B.8) are normalizable. Making this transformation we get

$$\tan\beta \;=\; \text{Real}\left(\frac{e^{-i\beta}}{\sin\beta}\right)^{-1} = \frac{(1 + |k|a)\tan\beta_o - |k|b}{1 + |k|a + |k|b\tan\beta_o}$$

$$= \frac{\tan\beta_o - \frac{|k|b}{1 + |k|a}}{1 + \frac{|k|a}{1 + |k|a}\tan\beta_o}$$

But this is the formula for the tangent of the difference of two angles, namely

$$\tan\beta \;=\; \tan(\beta_o - \tan^{-1}\frac{|k|b}{1 + |k|a})$$

Therefore, the behavior of β below the K-n threshold is

$$\beta \;=\; \beta_o - \tan^{-1}\frac{b}{a + \frac{1}{|k|}} \qquad \text{(B.23)}$$

If a is negative then one can see that the range of the second term can be

$$180^o < \tan^{-1}\frac{b}{a + \frac{1}{|k|}} < 0^o$$

where 0^o occurs when $b = 0$ at the $\pi\Sigma$ threshold, and where the large angles occur when $|k| \to 0$.

At present the data from low energy K-n interaction [16b,c] gives the following two possible sets of values for the scattering length solution:

Solution	A_0		A_1	
	a_0	b_0	a_1	b_1
1	-0.22 ± 1.07	2.74 ± 0.31	0.02 ± 0.33	0.38 ± 0.08
2	-0.59 ± 0.46	0.96 ± 0.17	1.20 ± 0.06	0.56 ± 0.15

The real part of the scattering length is only negative in the I = 0 channel. Hence the indications are that the bound state most probably occurs in the I = 0 channel. The data shows that in the region $E_{c.m.} \approx 1400$ MeV there could be a resonance of the $\Sigma\pi$ system in the I = 0 channel. Hence, the Y_1^* is not a reflection of the K-n bound state, but the 1404 Σ-π resonance, called Y_0^*, most probably is. However, better statistics are needed to obtain a definite conclusion.

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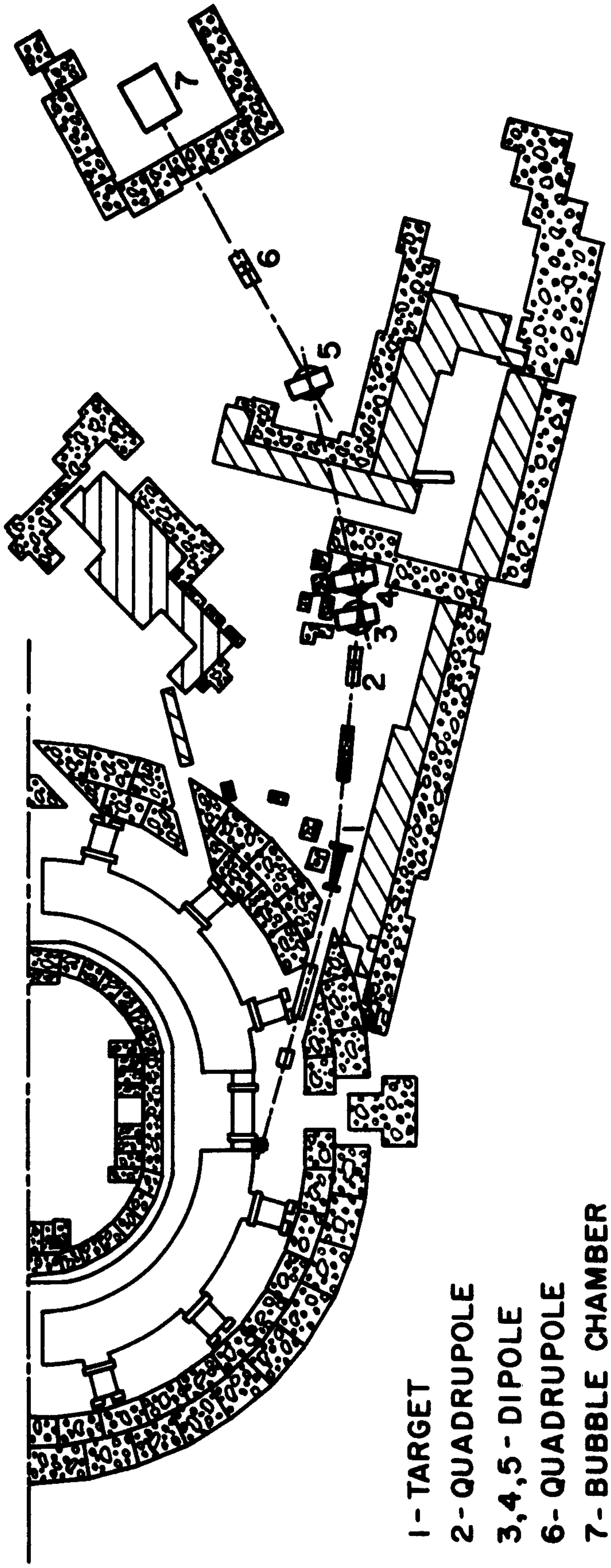
18 In general the scattering length is defined in the relation

$$k\,ctn\delta_I = \frac{1}{A_I} + \frac{1}{2}\, r_{oI}\; k^2 + \ldots\; 0(k^4)$$

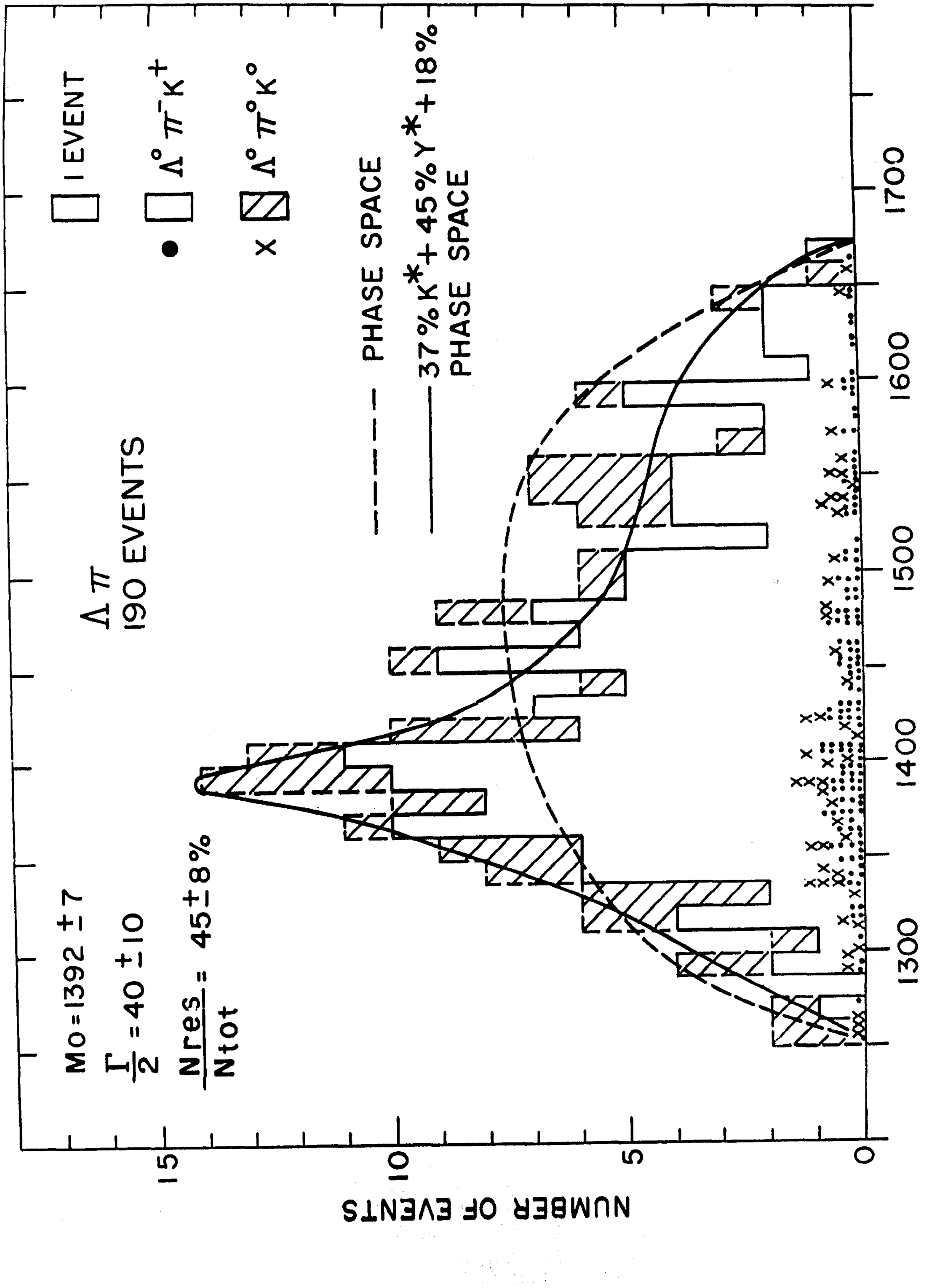
hence the assumption $k\,ctn\delta_I$ = const requires $r_{oI}k^2$ to be small. This is satisfied near the K-n threshold where $k = 0$.

FIGURE CAPTIONS



COSMOTRON BEAM I-A

FIG. 1



Mo=1392 ± 7
Γ/2 = 40 ± 10
Nres/Ntot = 45 ± 8%
Λπ
190 EVENTS
1 EVENT
● Λ° π⁻ K⁺
x Λ° π° K°
PHASE SPACE
37% K* + 45% Y* + 18% PHASE SPACE
NUMBER OF EVENTS
0
5
10
15
1300
1400
1500
1600
1700
M Λπ (MEV)

FIG. 2

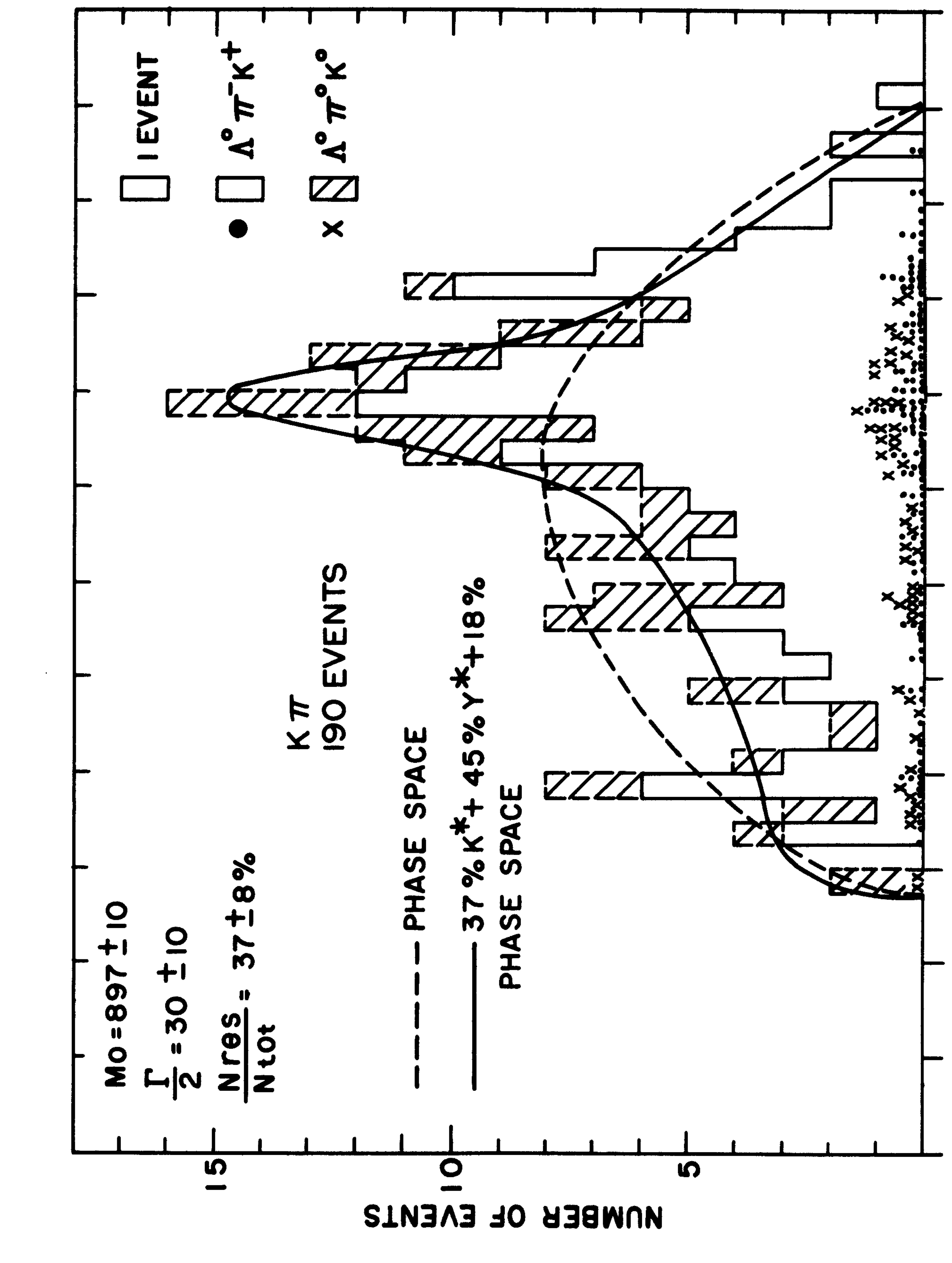

Mo = 897 ± 10
Γ/2 = 30 ± 10
Nres/Ntot = 37 ± 8%
K π
190 EVENTS
---- PHASE SPACE
—— 37% K* + 45% Y* + 18% PHASE SPACE
1 EVENT
● Λ° π⁻ K⁺
x Λ° π° K°
NUMBER OF EVENTS
5
10
15
600
700
800
900
1000
M_Kπ (MEV)

FIG. 3

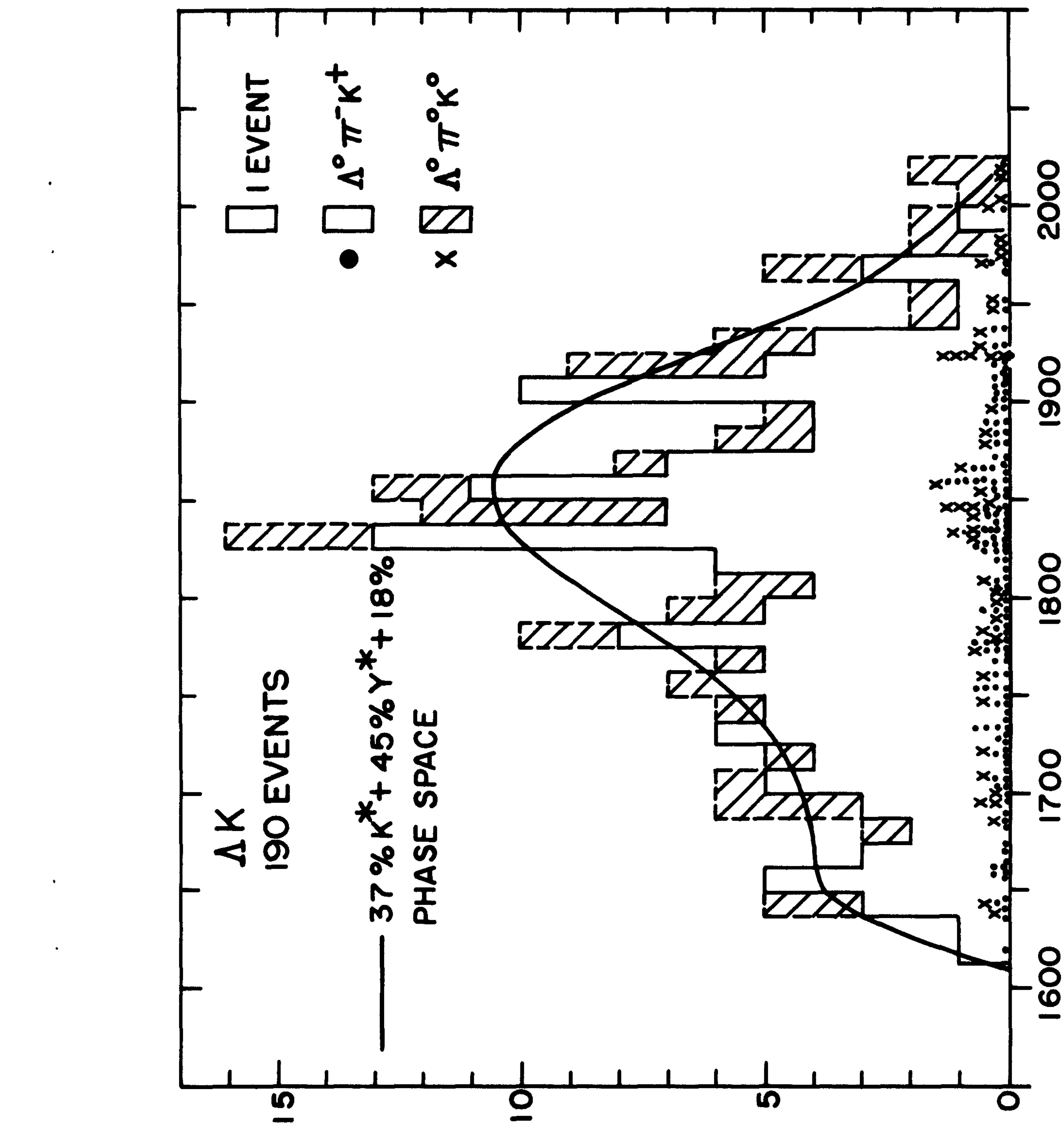
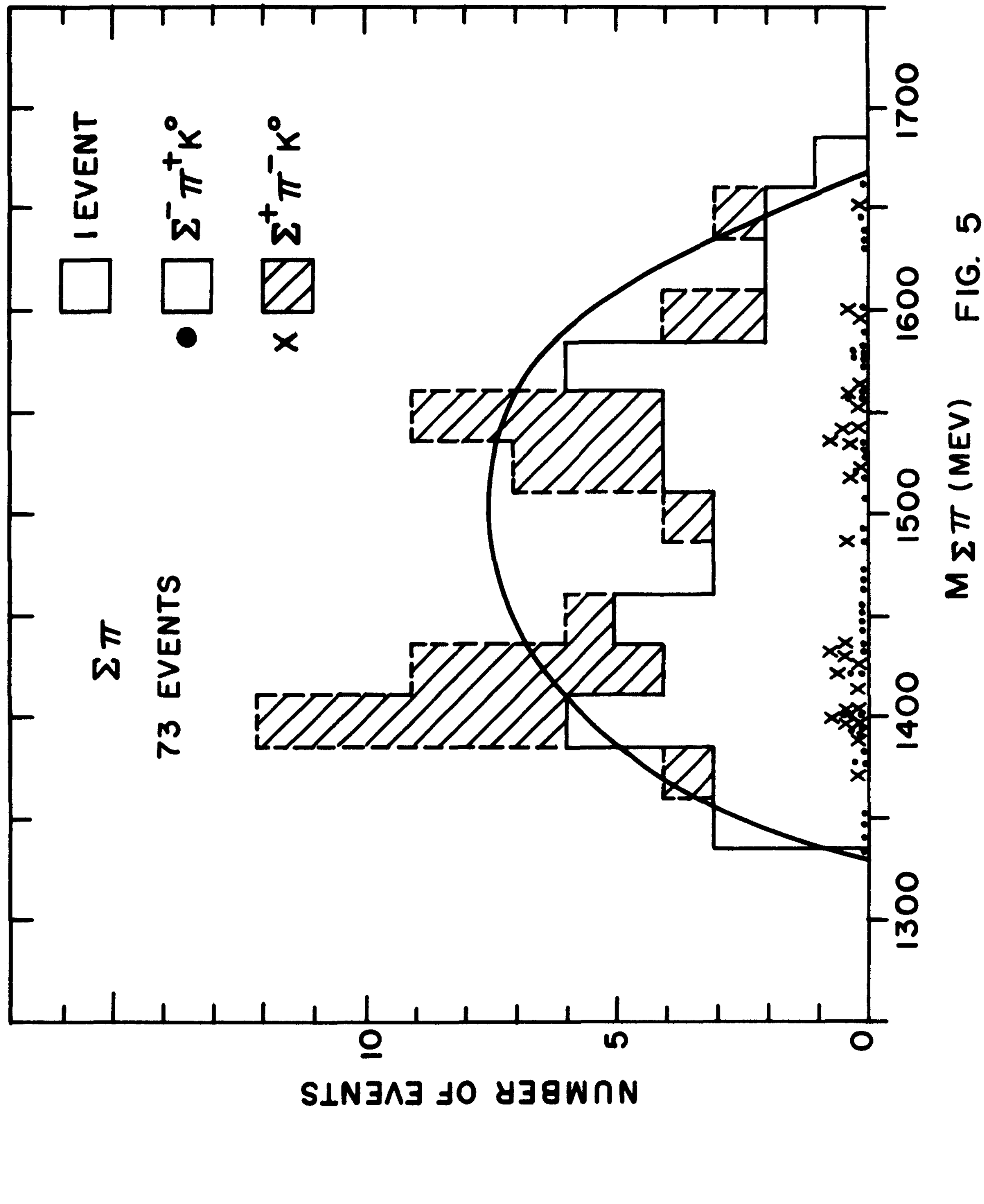

ΛK
190 EVENTS
37% K* + 45% Y* + 18% PHASE SPACE
1 EVENT
● Λ°π⁻K⁺
x Λ°π°K°
NUMBER OF EVENTS
0
5
10
15
1600
1700
1800
1900
2000
M_ΛK (MEV)

FIG. 4



Σπ
73 EVENTS
1 EVENT
Σ⁻π⁺K°
Σ⁺π⁻K°
NUMBER OF EVENTS
0
5
10
1300
1400
1500
1600
1700
M Σπ (MEV)

FIG. 5

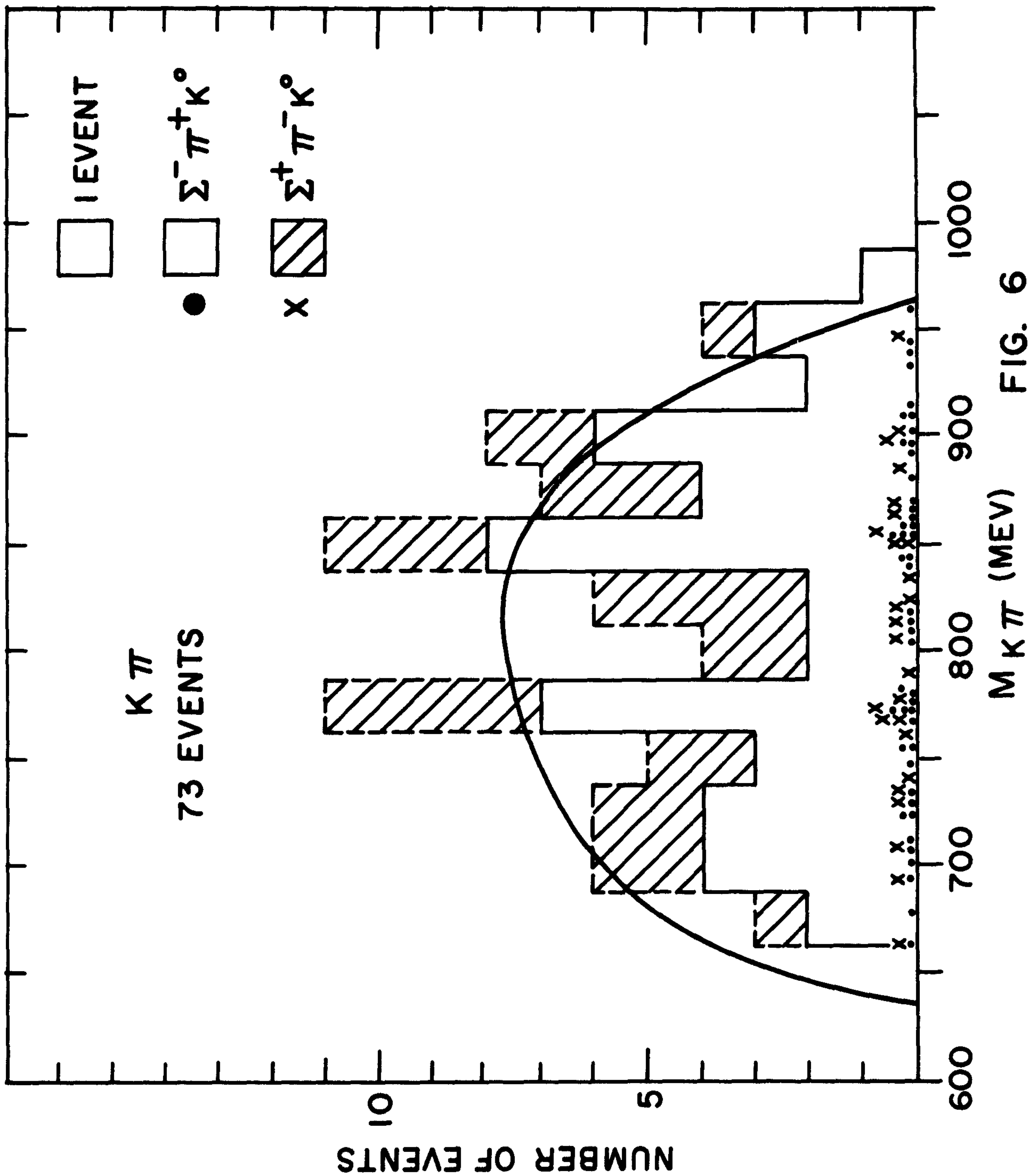

1 EVENT
$\Sigma^- \pi^+ K^\circ$
$\Sigma^+ \pi^- K^\circ$
$K\pi$
73 EVENTS
NUMBER OF EVENTS
10
5
600
700
800
900
1000
$M_{K\pi}$ (MEV)

FIG. 6

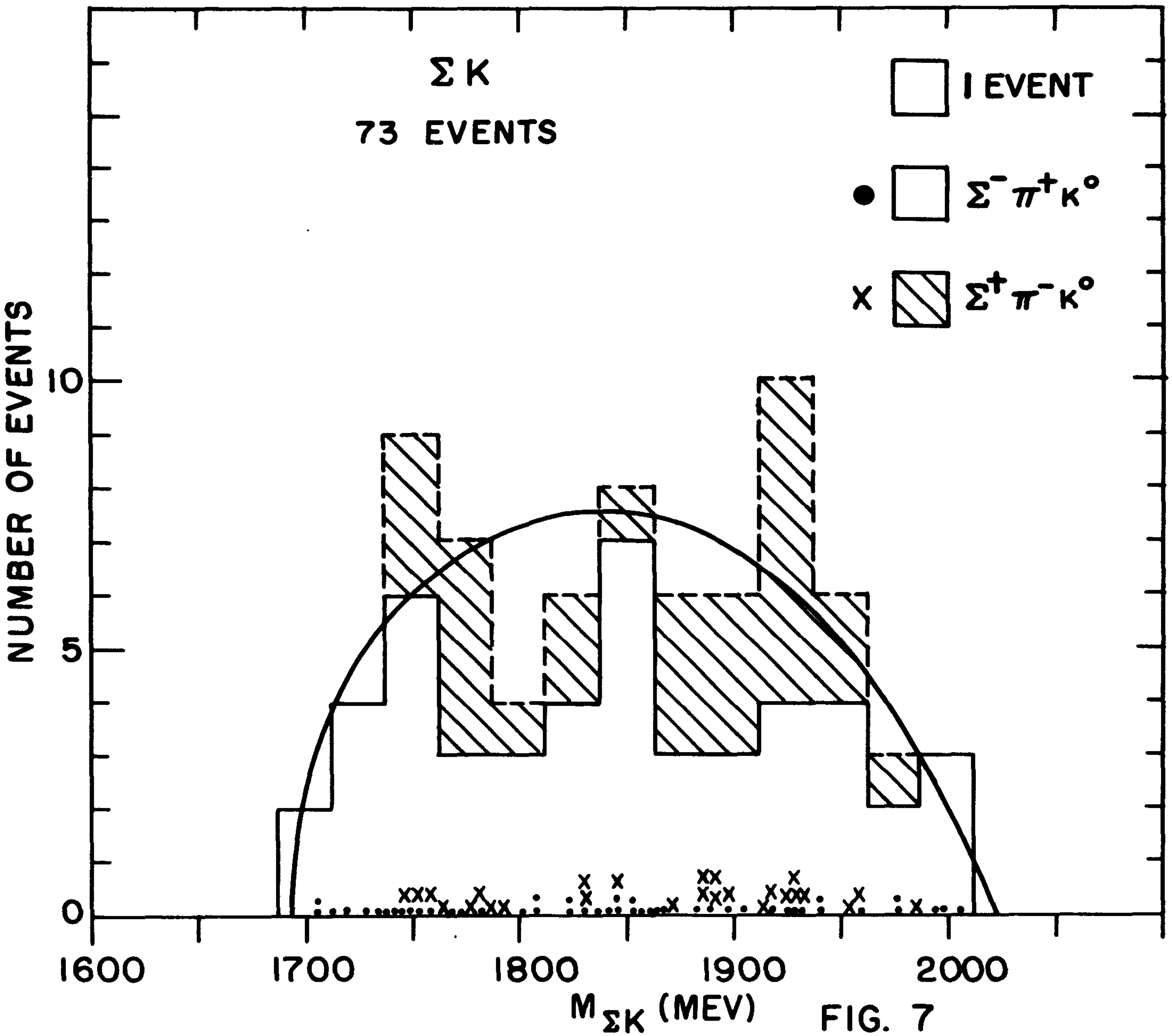

ΣK
73 EVENTS
1 EVENT
● $\Sigma^-\pi^+K^0$
X $\Sigma^+\pi^-K^0$
NUMBER OF EVENTS
0
5
10
1600
1700
1800
1900
2000
$M_{\Sigma K}$ (MEV)

FIG. 7

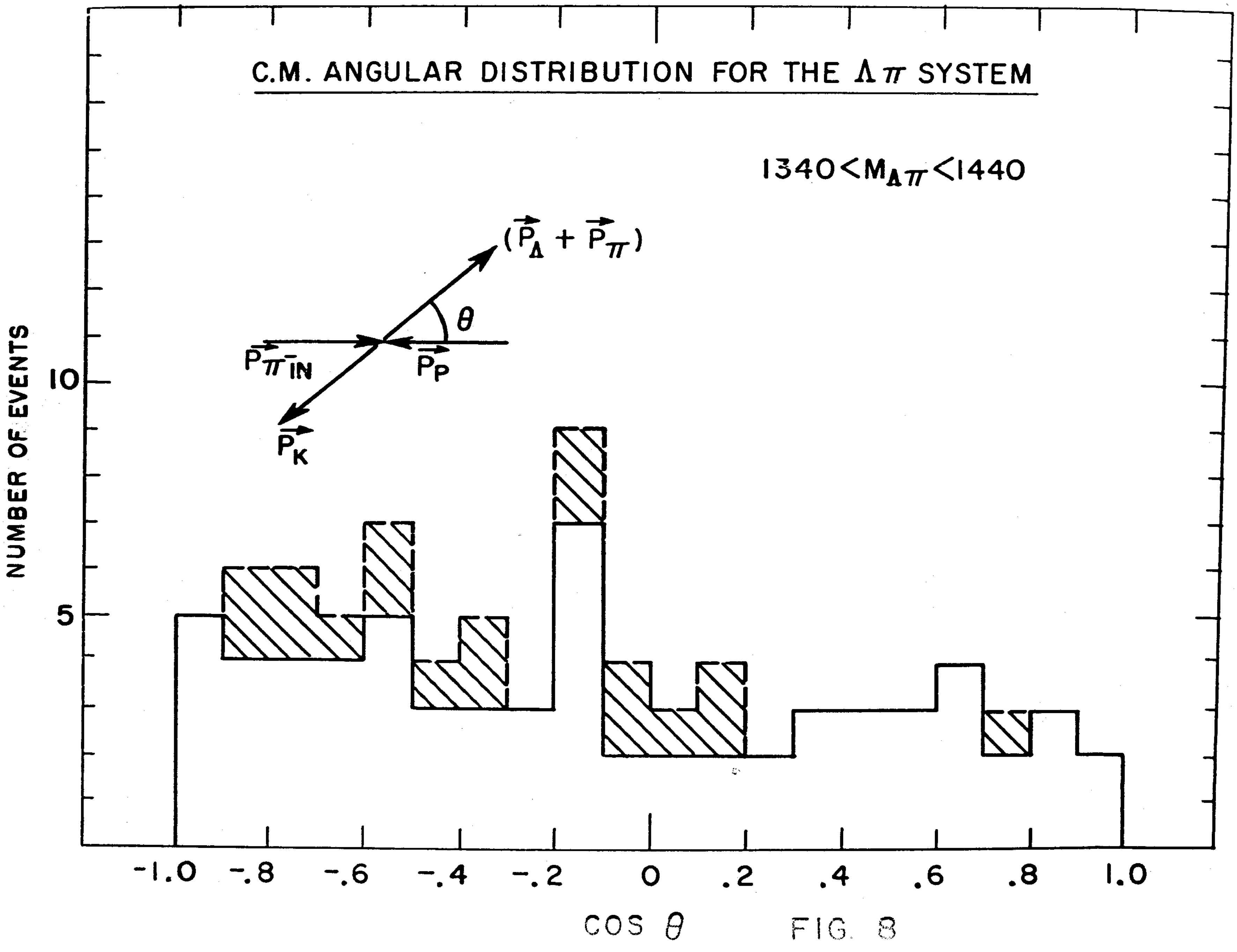

C.M. ANGULAR DISTRIBUTION FOR THE $\Lambda\pi$ SYSTEM
$1340 < M_{\Lambda\pi} < 1440$
$(\vec{P}_{\Lambda} + \vec{P}_{\pi})$
θ
$\vec{P}_{\pi^- IN}$
$\vec{P}_{P}$
$\vec{P}_{K}$
NUMBER OF EVENTS
10
5
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
COS θ

FIG. 8

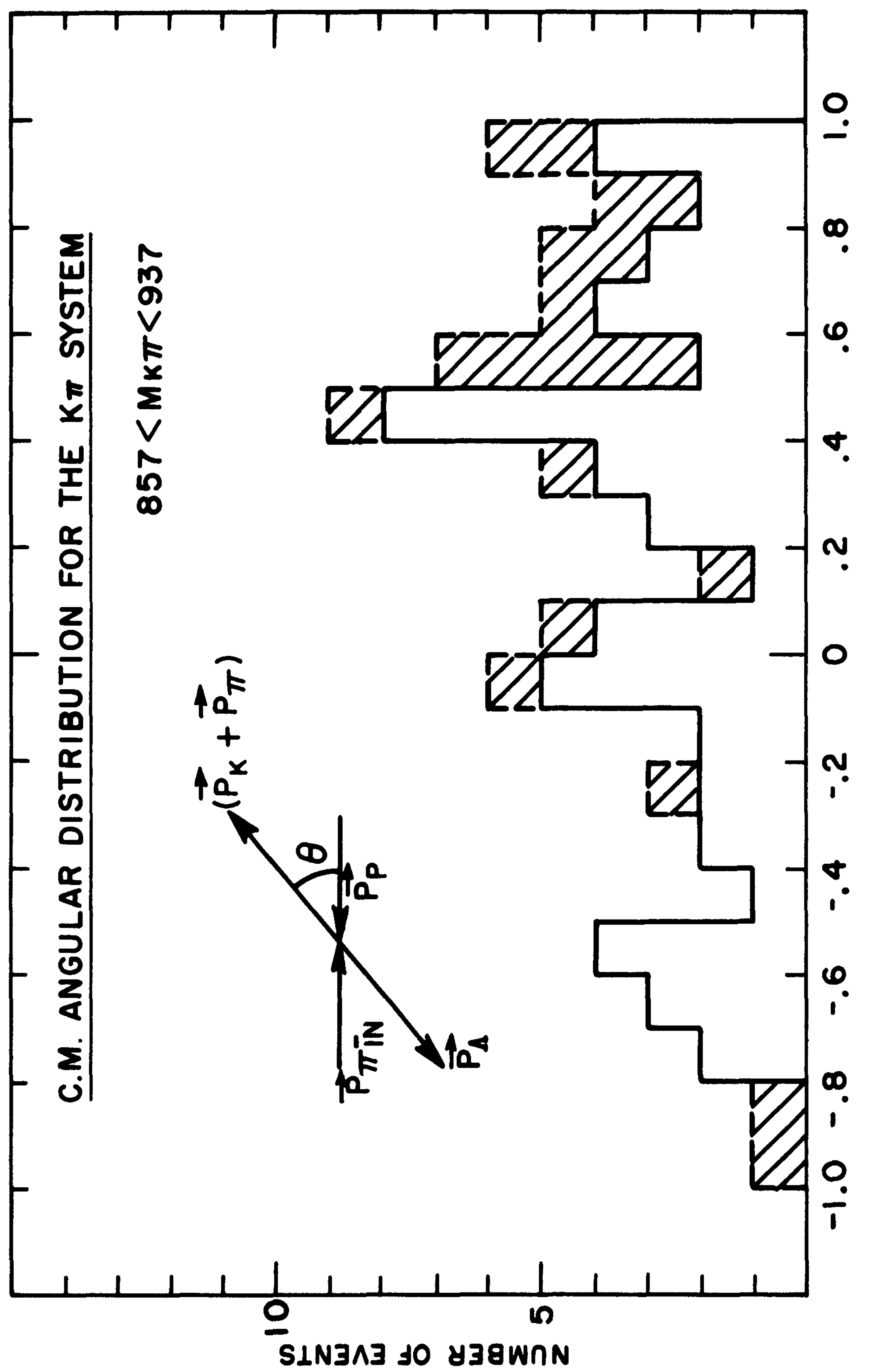

C.M. ANGULAR DISTRIBUTION FOR THE Kπ SYSTEM
857 < MKπ < 937
θ
$\vec{P}_P$
$\vec{P}_{\pi^- IN}$
$\vec{P}_\Lambda$
$(\vec{P}_K + \vec{P}_\pi)$
NUMBER OF EVENTS
10
5
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
COS θ

FIG. 9

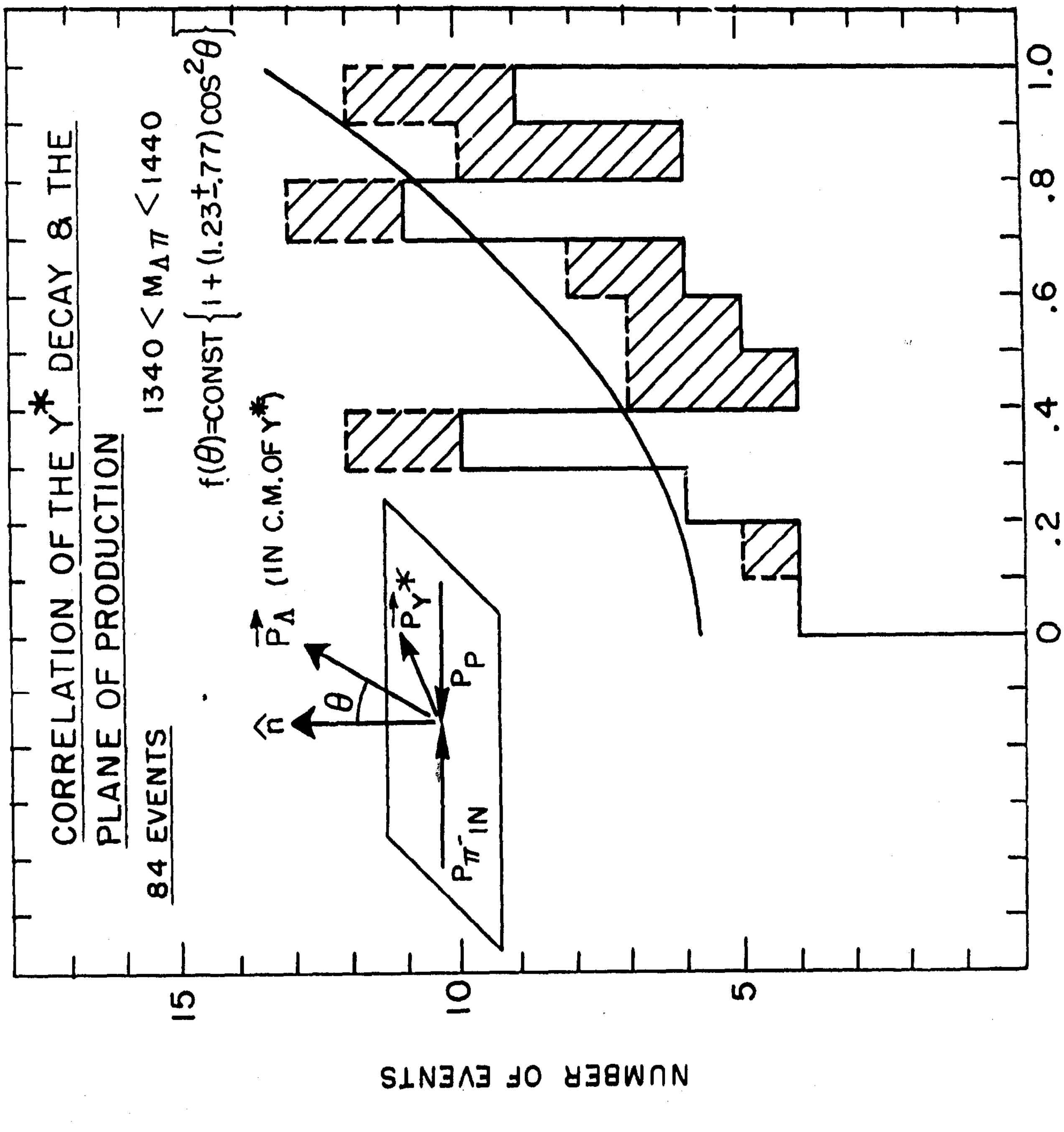
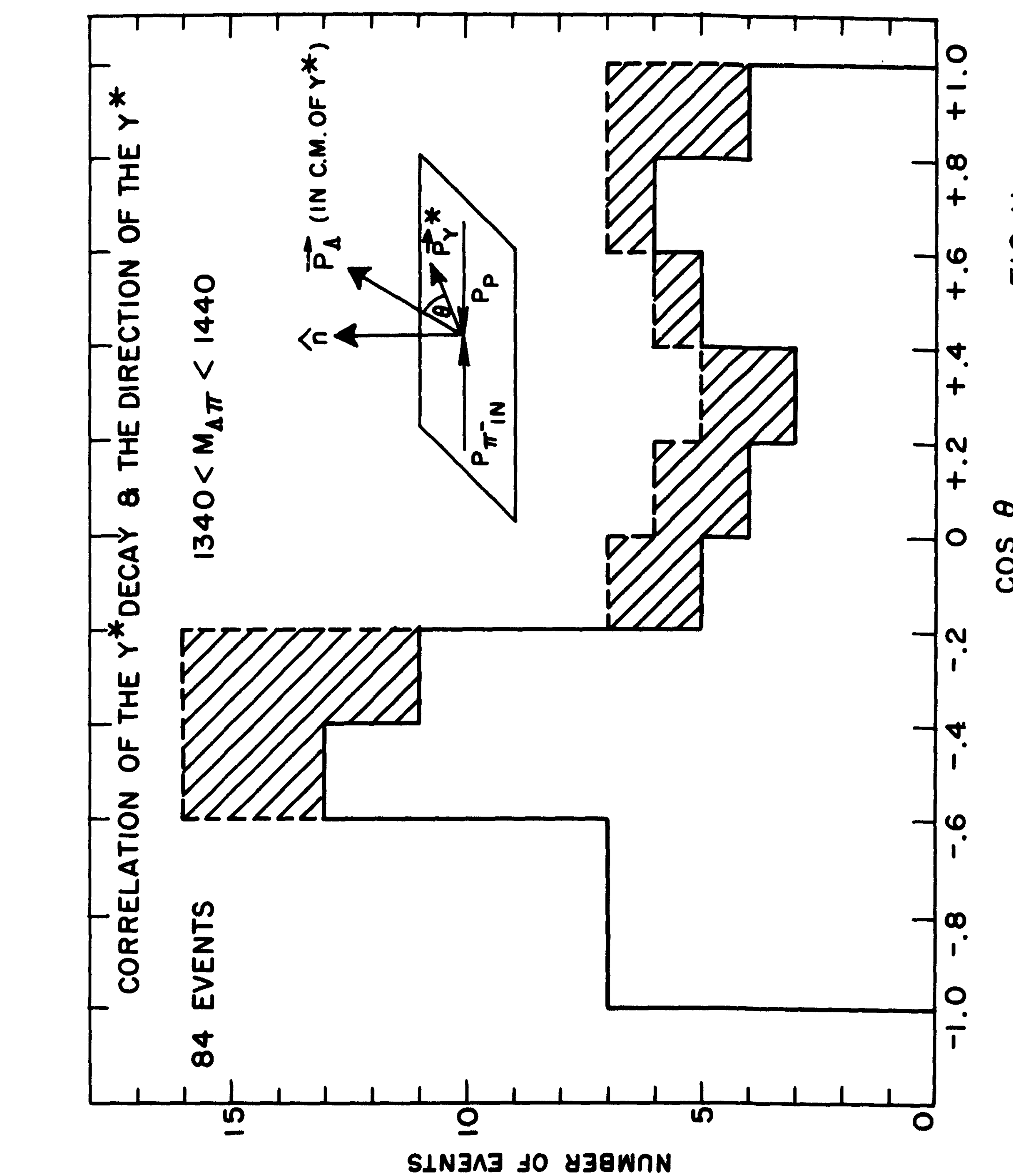

CORRELATION OF THE Y* DECAY & THE PLANE OF PRODUCTION
84 EVENTS
1340 < $M_{\Lambda\pi}$ < 1440
$f(\theta)$=CONST {1+(1.23±.77)$\cos^2\theta$}
(IN C.M. OF Y*)
$\hat{n}$
θ
$\vec{P}_\Lambda$
$\vec{P}_{Y^*}$
P_P
P_{π^-IN}
NUMBER OF EVENTS
15
10
5
0
.2
.4
.6
.8
1.0
|COS θ|

FIG. 10



CORRELATION OF THE Y* DECAY & THE DIRECTION OF THE Y*
84 EVENTS
1340 < $M_{\Lambda\pi}$ < 1440
$\hat{n}$
$\vec{P}_\Lambda$ (IN C.M. OF Y*)
$\vec{P}_{Y}^{*}$
P_p
$P_{\pi^-_{IN}}$
θ
NUMBER OF EVENTS
0
5
10
15
-1.0
-.8
-.6
-.4
-.2
0
+.2
+.4
+.6
+.8
+1.0
cos θ

FIG. 11

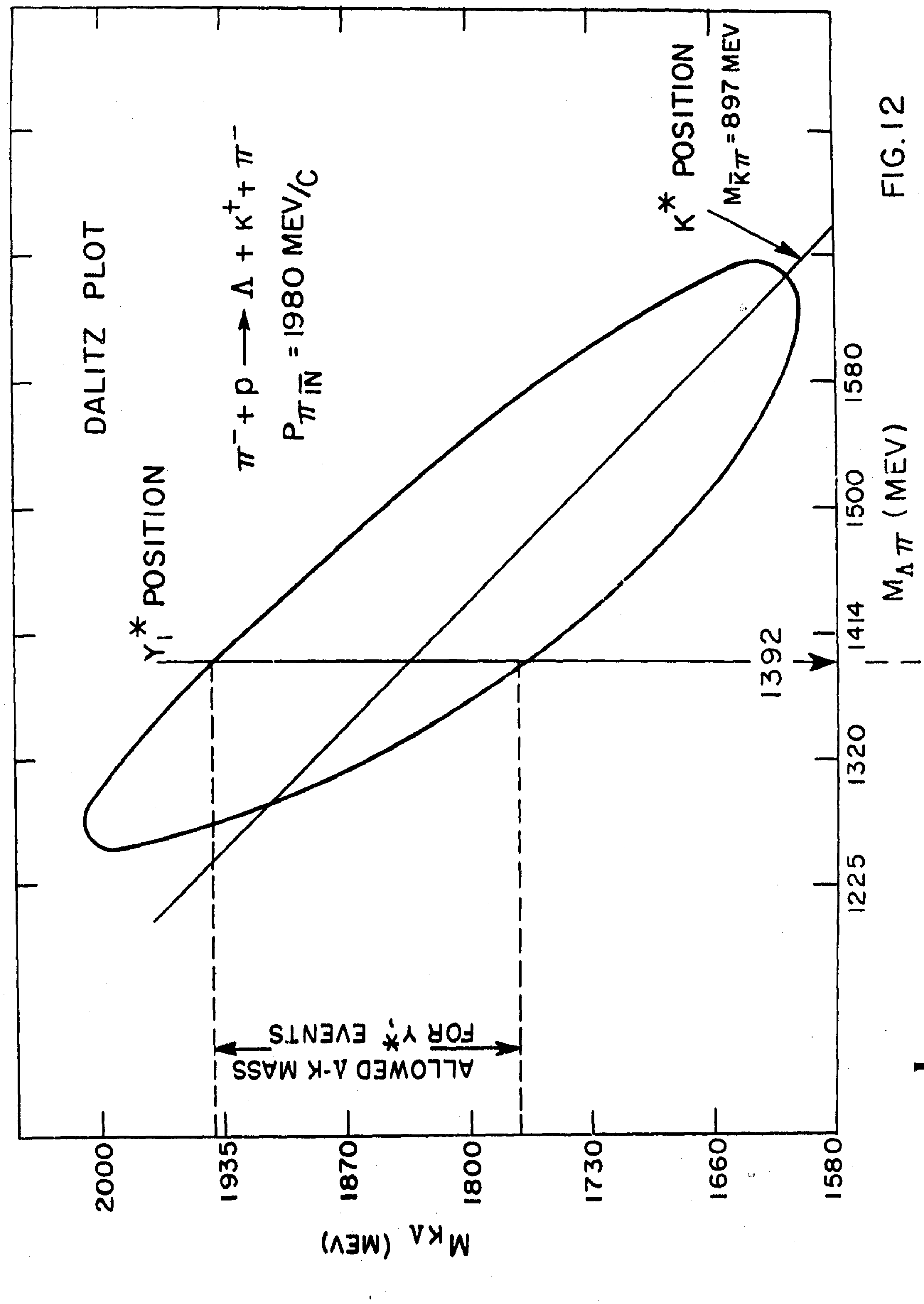

DALITZ PLOT
$\pi^- + p \longrightarrow \Lambda + K^+ + \pi^-$
$P_{\pi\,\overline{IN}} = 1980$ MEV/C
Y_1^* POSITION
K^* POSITION
$M_{\bar{K}\pi} = 897$ MEV
ALLOWED Λ-K MASS
FOR Y_1^* EVENTS
1392
1225
1320
1414
1500
1580
$M_{\Lambda\pi}$ (MEV)
2000
1935
1870
1800
1730
1660
1580
$M_{K\Lambda}$ (MEV)

FIG. 12