AD 407 938

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## Section 1

## INTRODUCTION

This report is submitted in partial fulfillment of Contract Nonr 2904(00) with the Bureau of Supplies and Accounts, Department of the Navy.

In the report entitled Low Demand Inventory Models also submitted in partial fulfillment of this contract, various mathematical models were developed for stocking, allocating, and redistributing units of stock in a system of depots. In that report, numerical examples were generally absent.

This report contains a qualitative evaluation, by numerical means, of the mathematical models that were developed. This evaluation describes the relative cost advantage of one model with respect to the others as parameters such as unit cost, yearly demand, and lead time change. To do this an IBM 7090 computer was programmed to solve the cost equations of the models. The programs are included in this report for future reference and use.

This report also contains a simulated illustration of some of the models. A simulated series of random demands was created for three models and the simulated costs compared to the cost predicted by the cost equations. The contstruction of the simulation as well as a description of the results is included in the report.

The scope of this report is a limited one; the report should not be considered as a full simulation of the low demand models. Such a simulation, while desirable, would constitute a separate study. Rather, this report should be regarded as a numerical illustration of the models described in Low Demand Inventory Models.

This report presents the following sections: model description, qualitative evaluation results, and simulation results. An appendix provides the computer program description.

## Section 2

## MODEL DESCRIPTION

This section is a reproduction of Section A of the Low Demand Inventory Models. It contains a description of the models that were programmed on the IBM 7090 computer. It also contains the definitions and features common to all models.

### Common Features and Definitions

1. There is a system of N depots that experience demands for an item. All demands are for exactly one item and are Poisson distributed.
2. A stock point is a point at which stock is held. A stock point may be a depot or a warehouse. In some models all depots are stock points; in some models, there are only one or two stock points.
3. Items of stock are ordered one at a time as demanded and the lead time interval for each item, T, is assumed to be constant.
4. An allocation is a shipment of one unit of stock from the manufacturer to a depot. A redistribution is a shipment of one unit of stock from one depot to another.
5. Each model has two major parts:
    a. Determination of the stockage objective or optimum stock assets. "Stock assets" is defined as the amount on hand plus on order less backorders. It should be noted that the method of ordering one unit when one unit is demanded insures that the stock assets remain constant. For each model the total variable cost is the sum

of allocation costs, redistribution costs, system stockout and inventory carrying costs. The total variable cost is expressed as a function of the stock assets M. The stockage objective $\hat{M}$ is that value of M that minimizes the function.

b. Construction of the allocation and redistribution rules. The allocation rule determines the stock point to receive an allocation from the supplier.

The redistribution rule determines the conditions under which a stock point that experienced a demand should receive a redistribution. The rule also determines the optimum stock point to be the source of the redistribution.

Description of Models

The sections that follow present the models according to the following subheadings:

a. Allocation costs

b. Redistribution costs

c. System stockout and inventory carrying costs. Also included in this section is a discussion of the stockage objective that minimizes the sum of (a), (b), and (c).

d. Rules for allocation and redistribution. Where appropriate, there will be a subsection (e) discussing the selection of principal depots when not all depots are allowed to carry stock.

The models are as follows:

Section B — Single Stock Point Models

1. All stock is allocated to and held by a single stock point. All other depots receive stock to satisfy demands by redistribution from the single stock point.

2. All stock is allocated to and held by a single stock point except when a demand occurs at a depot and the system has no on-hand stock. In the expected case, a depot receives a unit of stock directly from the manufacturer when a unit becomes available.

Section C — Two Stock Point Model

All stock is allocated to and held by two principal depots. Each principal depot has an associated system of depots which normally satisfy demands by redistribution from that principal depot. The only case in which a depot will not receive a unit of stock from its associated principal depot is when the principal depot has no stock and the other principal depot has stock.

Section D — Multiple Stock Point Models

1. All depots are potential stock points. Exactly which depot is to receive a unit of stock from the supplier is decided at the time the unit becomes available. Whether a redistribution of one unit should be made between a pair of depots is decided when a demand occurs at one member of the pair.

2. All depots are potential stock points. When a demand occurs at a depot, the depot initiates a procurement order. The unit is available

after a time interval T, since the procurement lead time is constant. In this case, however, the unit is allocated automatically to the depot which experienced the demand. Whether a redistribution of one unit should be made between a pair of depots is decided when a demand occurs at one member of the pair.

Hereafter in this report, the models will be referred to as "Model B1", "Model B2", "Model C", "Model D1", and "Model D2."

## Section 3

## QUALITATIVE EVALUATION RESULTS

The costs for each of the five models were calculated on the 7090 computer for several different cases. In these cases, the parameter values for unit cost, yearly demand, and lead times are varied. The results are qualitative in the sense that they provide a basis for selecting the optimum model when the parameters are specified. The results are not quantitative, however, until the costs are verified by simulation.

Two groups of results are presented: Group 1 has a relatively high average yearly demand of ten units; Group 2 has a low average yearly demand of two units. Several qualitative observations can be made which agree with our intuitive judgments.

1. As unit costs increase the single depot becomes optimum.
2. As average yearly demand decreases the single depot becomes optimum.
3. Model B1 is always more expensive than B2.
4. Model D2 is always more expensive than D1.

There are two entries in Group 2 for model C which were not computed. This is because the stockage objective for one of the two principal depots is zero. In this case, the single depot model should be substituted.

Backorder Cost $500

Carrying Charge 10%

| Depot Number | Allocation Cost | Redistribution Cost $c_j$ | Redistribution Cost $d_j$ |
|---|---|---|---|
| 1 | 40.00 | 20.00 | 35.00 |
| 2 | 30.00 | 15.00 | 30.00 |
| 3 | 50.00 | 25.00 | 45.00 |
| 4 | 30.00 | 10.00 | 30.00 |
| 5 | 35.00 | 20.00 | 35.00 |
| 6 | 20.00 | 35.00 | 20.00 |
| 7 | 35.00 | 30.00 | 15.00 |
| 8 | 45.00 | 45.00 | 25.00 |
| 9 | 25.00 | 30.00 | 10.00 |
| 10 | 30.00 | 35.00 | 20.00 |

| Depot | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Average Yearly Demand | .8 | 1.2 | .4 | 1.6 | 2.0 | 1.0 | .2 | .5 | 2.0 | .3 |

TOTAL COST

.10 Lead Time

| UNIT COST | B1 | B2 | C | D1 | D2 |
|---|---|---|---|---|---|
| $ 50 | 416.20 | 414.00 | 411.73 | 378.70 | 424.39 |
| 200 | 451.14 | 444.35 | 462.98 | 482.38 | 517.19 |
| 500 | 506.00 | 483.67 | 528.65 | 570.17 | 592.02 |
| 2,000 | 656.52 | 603.10 | 757.26 | 766.35 | 779.46 |

.40 Lead Time

| | B1 | B2 | C | D1 | D2 |
|---|---|---|---|---|---|
| $ 50 | 430.19 | 428.39 | 425.15 | 378.71 | 424.39 |
| 200 | 496.49 | 492.17 | 504.81 | 491.40 | 533.23 |
| 500 | 595.62 | 586.27 | 609.39 | 620.52 | 651.54 |
| 2,000 | 886.21 | 854.85 | 904.66 | 981.02 | 994.12 |

.80 Lead Time

| | B1 | B2 | C | D1 | D2 |
|---|---|---|---|---|---|
| $ 50 | 441.37 | 439.91 | 436.27 | 379.20 | 424.46 |
| 200 | 533.33 | 527.94 | 538.25 | 502.38 | 551.76 |
| 500 | 670.40 | 660.95 | 683.11 | 671.34 | 710.73 |
| 2,000 | 1,095.47 | 1,061.04 | 1,112.75 | 1,155.44 | 1,177.29 |

| Depot | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Average Yearly Demand | .16 | .24 | .08 | .32 | .40 | .20 | .04 | .10 | .40 | .06 |

TOTAL COST

.10 Lead Time

| UNIT COST | B1 | B2 | C | D1 | D2 |
|---|---|---|---|---|---|
| $ 50 | 89.41 | 89.11 | 89.84 | 98.83 | 105.01 |
| 200 | 105.54 | 102.48 | ----- | 124.50 | 127.12 |
| 500 | 130.10 | 127.04 | ----- | 149.06 | 151.68 |

.40 Lead Time

| | B1 | B2 | C | D1 | D2 |
|---|---|---|---|---|---|
| $ 50 | 96.20 | 95.40 | 96.00 | 101.51 | 107.80 |
| 200 | 129.36 | 128.56 | 131.85 | 141.03 | 145.40 |
| 500 | 171.77 | 168.54 | 176.42 | 190.73 | 193.35 |

.80 Lead Time

| | B1 | B2 | C | D1 | D2 |
|---|---|---|---|---|---|
| $ 50 | 100.68 | 100.28 | 101.10 | 103.24 | 110.70 |
| 200 | 144.11 | 142.78 | 146.98 | 155.78 | 160.15 |
| 500 | 210.40 | 206.74 | 214.95 | 228.72 | 231.98 |
| 2,000 | 388.58 | 380.55 | 392.44 | 418.48 | 418.48 |

Section 4

## SIMULATION RESULTS

This section presents the results of simulating models B1, B2, and C. A history of 40,000 random Poisson demands with average yearly demand of 38 was created for each model and the following average yearly costs were obtained:

a. Allocation Cost

b. Redistribution Cost

c. System Backorder and Inventory Holding Cost.

To verify that the Poisson demand history is random, a check was made to see that the sample mean and standard deviation are nearly equal for the exponential distribution of intervals between demands. The values obtained were:

Sample Mean: .02654

Sample Standard Deviation: .02647

To verify that there is no periodicity in the demand history, the first three autocorrelation functions were computed. The $r^{th}$ autocorrelation function is computed by multiplying the $N^{th}$ interarrival gap by the $N+r^{th}$ interarrival gap for all N. If there is no periodicity, the autocorrelation functions should nearly agree. The values obtained for the first three autocorrelation functions were:

21,636;

21,641;

21,691.

Below we have tabulated the simulation results and the costs predicted by the models for B1, B2, and C.

The following are the costs and depot demand rates assumed by all models:

| | |
|---|---|
| Lead Time = | 0.50000 yr. |
| Unit Cost = | $500.00 |
| Backorder = | $500.00 |
| Carrying Charge = | 10% |

| Number | Average Rate of Demand | Allocation Cost | Redistribution Cost $c_j$ | Redistribution Cost $d_j$ |
|---|---|---|---|---|
| 1 | 4.00000 | 10.00 | 15.00 | 50.00 |
| 2 | 6.00000 | 20.00 | 35.00 | 40.00 |
| 3 | 6.00000 | 20.00 | 35.00 | 40.00 |
| 4 | 6.00000 | 20.00 | 35.00 | 40.00 |
| 5 | 4.00000 | 55.00 | 80.00 | 50.00 |
| 6 | 4.00000 | 25.00 | 40.00 | 50.00 |
| 7 | 4.00000 | 55.00 | 65.00 | 50.00 |
| 8 | 4.00000 | 20.00 | 12.00 | 50.00 |

Model B1

Stockage Objective: 25

| Average Annual Costs | Simulated Cost | Predicted Cost |
|---|---|---|
| Allocation | 376.67 | 380.00 |
| Redistribution | 1408.56 | 1418.00 |
| Inventory and Stockout | 405.25 | 410.02 |
| Total | 2190.49 | 2208.02 |

Model B2

Stockage Objective: 23

| Average Annual Cost | Simulated Cost | Predicted Cost |
|---|---|---|
| Allocation | 502.10 | 512.39 |
| Redistribution | 1130.77 | 1124.67 |
| Inventory and Stockout | 438.10 | 451.77 |
| Total | 2070.97 | 2088.83 |

Model C

Stockage Objective: 25

| Average Annual Cost | Simulated Cost | Predicted Cost |
|---|---|---|
| Allocation | 567.34 | 560.00 |
| Redistribution | 1252.92 | 1245.32 |
| Inventory and Stockout | 405.15 | 410.02 |
| Total | 2225.52 | 2215.33 |

Appendix

## COMPUTER PROGRAM DESCRIPTION

There were five major programs completed in connection with the Navy Inventory project:

1) A program to set the stockage objective and to estimate the expected costs for all of the models,

2) A program using a random number generator to produce a Poisson distributed set of demands,

3) A program to simulate Section B1 using the Poisson demand distribution,

4) A program to simulate Section B2 using the Poisson demand distribution,

5) A program to simulate Section C using the Poisson demand distribution.

All five programs were written in Fortran language. They were compiled, tested, and run on an IBM 7090 computer. Only one small subroutine was written in symbolic language and not in Fortran. This subroutine was the random number generator used in the second program. Thus, it would be a very easy matter to run any of these programs on any machine for which Fortran is available. (This would include not only the IBM 705, but also the machines of other manufacturers such as the Philco S-2000.)

The mathematics for the first program has been thoroughly explained in <u>Low Demand Inventory Models</u>, United Research Incorporated, July 1961. This program is composed of six subroutines and one main program. One of the six subroutines is a subroutine which computes a table of Poisson and summed Poisson distributions.

The other five subroutines compute the stockage objectives and estimated costs for the five basic models (B1, B2, C, D1, D2). A set of data is read by the main program and placed into memory. The first subroutine then computes the stockage objective and estimates costs for model B1 and further finds the depot which should be the principal depot. The selection of the principal depot is based upon the criterion of the least cost after trying each of the depots as the principal depot. The second subroutine then computes similar results for Section B2. The third subroutine runs through all possible choices of two depots and finds the two principal depots with their respective estimated cost and stockage objective. The N depots associated with each set of data are divided into two groups: Depots 1,2,..,H and Depots H+1,H+2,..,N. Any depot in the first group may be one principal depot and any depot from H+1 to N may be the other principal depot. Subroutines 4 and 5 perform similar computations of estimated costs and stockage objectives for Sections D1 and D2. The form of the data is as follows:

DATA NEEDED TO USE MODEL FOR SETTING STOCKAGE OBJECTIVES

| | | Columns | Note | Card |
|---|---|---|---|---|
| 1. | Item Name | Columns 1-18 | Note A | Card No. 1 |
| 2. | Number of Depots | Columns 21-22 | Note B | |
| 3. | Number of Depots Associated with the First Principle Depot for Section C | Columns 25-26 | Note B | |
| 4. | Procurement Lead Time | Columns 1- 8 | Note C | Card No. 2 |
| 5. | Unit Cost | Columns 9-16 | Note C | |
| 6. | System Backorder Cost | Columns 17-24 | Note C | |
| 7. | Yearly Inventory Carrying Charge | Columns 25-32 | Note C | |
| 8. | For Each Depot | | | |
| | a) Depot Number | Columns 1- 2 | Note B | One Card for Each Depot |
| | b) Average Rate of Demand | Columns 3-10 | Note C | |
| | c) Allocation Cost | Columns 11-18 | Note C | |
| | d) Redistribution Charge $c_j$ | Columns 19-26 | Note C | |
| | e) Redistribution Charge $d_j$ | Columns 27-34 | Note C | |
| 9. | Last Card | 99 in Columns 1- 2 | Note B | Special Final Card |

Note A: Any alphabetic character in any order

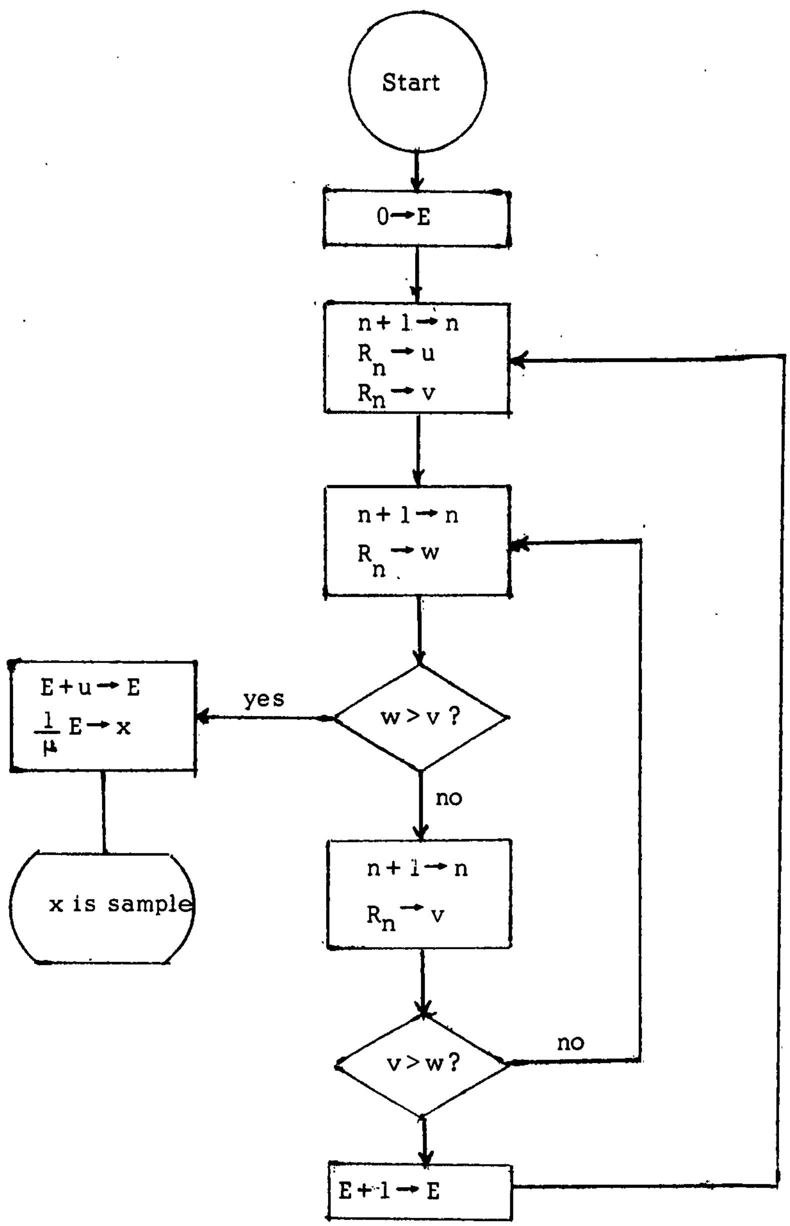
Note B: Integers (0-9) only at the right end of the field such as 23 or blank 2

Note C: Integers (0-9) with decimal point such as 346.27 or 0.23500

Any number of sets of data may be considered in the program in any one run if these sets are placed one after the other in the data section of the card input. The program has been run using the Fortran monitor system for the IBM 7090. Detailed instructions for operating under the Fortran monitor system may be obtained from manuals prepared by IBM and hence will not be discussed here.

The second program uses a random number generator to write a binary tape with up to approximately 32,000 demands. In order to have a distribution of Poisson demands, it follows that these demands should be separated exponentially. A random number is a number chosen anywhere in a gap (0,1). Any number in this gap is equally likely and therefore the distribution is said to be uniform or rectangular. In order to draw a random sampling x from the exponential distribution, a random sampling is first drawn from the uniform distribution in the unit interval. This random number or a sequence of random numbers may be used to draw a sampling from the exponential distribution by one of several methods. One method uses a discrete table of the exponential function. This program used the method in order to make the exponential samplings continuous.

SAMPLING FROM EXPONENTIAL




n = count of random number

$R_n$ = $n^{th}$ random number

$\mu$ = average rate of demand

E = sample of $e^{-E}$

x = sample of $\mu e^{-\mu x}$ (exponential sample)

The exponentially separated demands are then written on a binary tape in four-word records, each record specifying the depot where the demand occured. The input is basically the same as that for the first program with the following exceptions:

1) Only the depot number and average rate of demand are needed on each depot detail card,

2) The second card is replaced by a card with 7 items of information. Columns 1 through 4 contain control parameters which are described on comments cards at the beginning of the program listing. To generate a tape all four may be blank or the second or the fourth or both the second and the fourth non zero. Columns 11 through 22 contain in octal integers the first random number that will be used by the program. Columns 26 through 31 contain in decimal integers the number of demands that will be produced by the program.

The program output repeats the input and lists the mean and standard deviation of the interarrival gap. The output also contains the last random number used, the number of random numbers used, the mean and standard deviation of the random numbers, and the first three serial correlation functions of the random numbers.

The final three programs are all similar. All three programs use the demand history tape produced by the second program. If the imput does not agree by depot in number and rate with the input used in Program 2, an error will be indicated and the execution of the program will be terminated. The simulation follows the rules set forth in other sections of this report. The imput for these three programs is very similar to that of the first program. Following the second input card and before the third, a new card is inserted which in columns 1 through 4 specifies the frequency of printout. If the number 0100 is placed in columns 1 through 4, then results will be printed every 100 transactions. A transaction means either a demand or allocation. The other difference from the input of Program 1 is that each depot card must include 2 additional numbers. The first number, located in columns 35 through 38, contains in decimal integers the number of items on hand at that depot at the start of the simulation. Columns 39 through 42 contain the number of items ordered at the start of the simulation card which are in the pipeline and should be delivered to that depot. The total number of items ordered is initially set by the program for starting conditions only so that they are equally spaced with the final allocation occurring at the time zero plus the procurement lead time.

The output again repeats the input for identification purposes and for every line printed, as requested by the output frequency parameter, prints the following: transaction number, time, total number in stock in the system, total number backordered, total annual cost, annual carrying costs, annual stockout costs, annual allocation costs, annual redistribution costs, annual number of stockouts, and the annual number of redistributions required to fill backorders. All these annual numbers are averaged over time so that each line of output contains the average annual cost or number up to that point in time.