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TRANSLATION

CLASSIFICATION OF FLOWS IN AIR SHOCK TUBES

By

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FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

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CLASSIFICATION OF FLOWS IN AIR SHOCK TUBES

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(Moscow)

Examined are the possible types of flow with constant pressure behind the front of a shock wave with arbitrary variation of initial pressures in the sections of the shock tube and of the geometric dimensions of the tube. It is assumed that the low pressure section of the shock tube has an open end.

1. Formulation of Problem. Consider a shock tube of constant cross section (Fig. 1) having a low pressure section with an open end. Let the air in the high pressure section of length l be in a state of compression (C); and that in the low pressure section of length z, in the state (O). We shall assume that the air in both sections is in thermal equilibrium with the ambient medium.

When the diaphragm is swiftly demolished (Fig. 2), a shock wave (S) begins to propagate to the right (through the low pressure medium) and a centered rarefaction wave (I) to the left (through the high pressure medium). In addition, there is formed contact discontinuity K which is carried to the right by the flow. At this discontinuity all the parameters of the medium with the exception of the pressure and particle velocity undergo jump variations.

The pressure p_s in the front formed by the shock wave may be determined from the relationship (cp. Grib [1])

$$\frac{p_c}{p_s}\left\{1-\frac{(k-1)(p_s/p_0-1)}{\sqrt{2k[(k+1)p_s/p_0+(k-1)]}}\right\}^{\frac{2k}{k-1}}=1 \qquad (1.1)$$

where p_c and p_0 are the initial pressures in the sections of the shock tube, and k is the isentropic exponent.

After the rarefaction wave (I) reaches the closed end of the tube, its reflection begins, forming a new wave, the front of which is propagated to the right with the velocity of weak disturbances $u + c$ (u is the particle velocity, and c is the local speed of sound).

At the moment the shock wave reaches the open end of the shock tube, a rarefaction wave is generated, the front of which will propagate to the left with a velocity $u - c$.

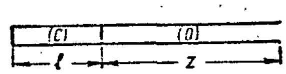

Fig. 1.

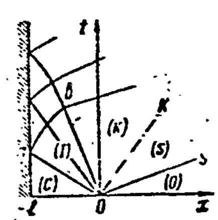

Fig. 2.

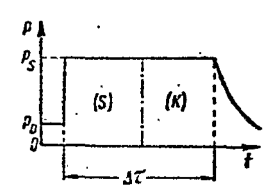

Fig. 3.

Thus in the general case two disturbances will be propagated in a shock tube with respect to the homogeneous states (S) and (K) in opposite directions: a disturbance from the closed end of the shock tube and a disturbance from its open end.

It is known that at a fixed distance from the diaphragm x ($x > 0$) the variation of the pressure in the shock tube through time has the form shown in Fig. 3. The shock wave has some constant pressure area of length $\Delta\tau$, which arises when the shock wave front reaches the given point and disappears when the disturbance from the open or closed end of the shock tube reaches this point.

In practical work on shock tubes it is important to know the size of this area, within the confines of which the homogeneous flows (S) and (K) and the contact discontinuity dividing them are located.

In general, the quantity $\Delta\tau$ is a function of five parameters: p_s, p_0, L, l, and $\underline{x}$.

2. Some Necessary Relationships. Let us determine the conditions under which (Fig. 4) the right hand boundary of rarefaction wave (I) will: propagate to the left, remain in place, or propagate to the right. In other words, it is necessary to find the conditions under which

$$u_s < c_k, \quad u_s = c_k, \quad u_s > c_k \tag{2.1}$$

It may be shown that

$$u_s = \frac{2c_0}{k-1}\left[1-\left(\frac{p_s}{p_c}\right)^{\frac{k-1}{2k}}\right], \quad c_k = c_0\left(\frac{p_s}{p_c}\right)^{\frac{k-1}{2k}}. \tag{2.2}$$

Hence the conditions in (2.1) may be presented respectively in the form:

$$\frac{p_s}{p_c} > \left(\frac{2}{k+1}\right)^{\frac{2k}{k-1}}, \quad \frac{p_s}{p_c} = \left(\frac{2}{k+1}\right)^{\frac{2k}{k-1}}, \quad \frac{p_s}{p_c} < \left(\frac{2}{k+1}\right)^{\frac{2k}{k-1}} \tag{2.3}$$

Converting from p_c to p_0 with the aid of (1.1), we get for the case of air ($k = 1.4$)

$$\frac{p_s}{p_0} < 2.89, \quad \frac{p_s}{p_0} = 2.89, \quad \frac{p_s}{p_0} > 2.89 \tag{2.4}$$

respectively.

It may be shown [2] that

$$x_b = \frac{l\,(u_s - c_k)}{c_k}\left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}}, \qquad t_b = \frac{l}{c_k}\left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}} \tag{2.5}$$

After solving the equations of the characteristic γ and the trajectory of the contact discontinuity simultaneously (Fig. 4) for the coordinate of the point c we obtain

$$x_c = 2l\frac{u_s}{c_k}\left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}}, \qquad t_c = 2\frac{l}{c_k}\left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}} \tag{2.6}$$

From the simultaneous solution of the equations of the characteristic γ_1 and the trajectory of the front of the shock wave, the following expression may be obtained for the x-coordinate of their point of intersection **d**

$$x_d = \frac{2lD}{u_s + c_s - D}\frac{c_s}{c_k}\left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}} \tag{2.7}$$

where D is the velocity of the front of the shock wave. Let us introduce the dimensionless quantity

$$\omega_1 = \frac{2D}{u_s + c_s - D}\frac{c_s}{c_k}\left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}} \tag{2.8}$$

Then it can be maintained that the disturbance from the closed end of the shock tube is successful in leaving the front of the shock wave behind over the length L, if $L/l \geq \omega_1$; is not successful in leaving the front of the shock wave behind over the length L, if $L/l < \omega_1$.

After solving the equation of the characteristic γ_2 (Fig. 5) together with the equation of the trajectory of the contact discontinuity, we obtain the following expression for the coordinate of the point **f**:

$$x_f = \frac{Lu_s(D + c_s - u_s)}{c_s D}, \qquad t_f = \frac{L(D + c_s - u_s)}{c_s D} \tag{2.9}$$

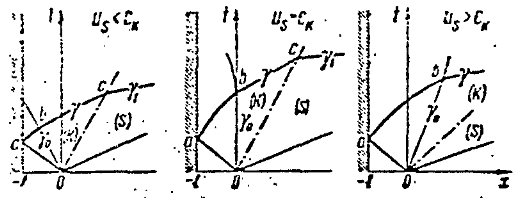

Fig. 4.

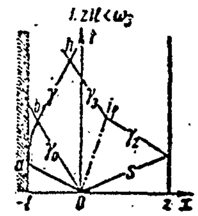

Fig. 5.

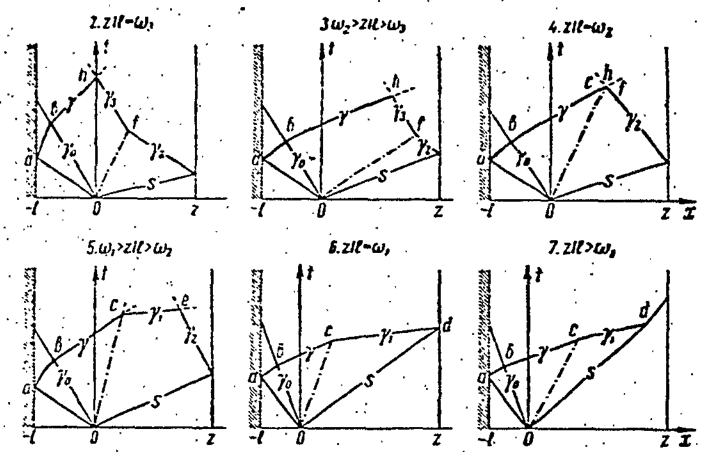

Fig. 6.

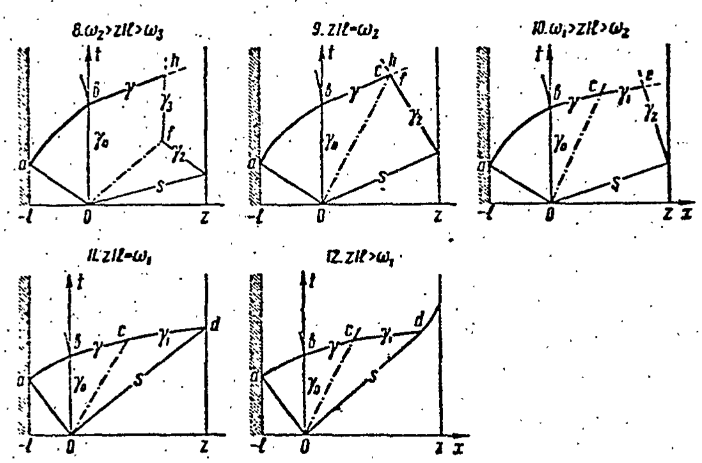

Fig. 7.

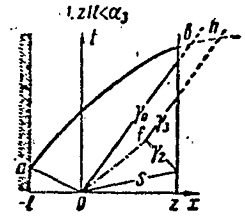

Fig. 8.

Introducing next the dimensionless quantity

$$\omega_2 = \frac{2D}{D + c_s - u_s} \frac{c_s}{c_k} \left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}} \tag{2.10}$$

and making use of (2.6) and (2.9), we come to the following conclusion: the disturbance from the closed end of the shock tube encounters the surface of the contact discontinuity before the disturbance from the open end, if $L/l > \omega_2$; the disturbance from the open end of the shock tube encounters the surface of the contact discontinuity before the disturbance from the closed end, if $L/l < \omega_2$; both disturbances encounter the surface of the contact discontinuity simultaneously, if $L/l = \omega_2$.

Simultaneous solution of the equations of the characteristics γ and γ_3 allows us to obtain for the x-coordinate of the point <u>h</u> the expression

$$x_h = \frac{L(u_s + c_k)(D + c_s - u_s)}{2c_s D} + \frac{l(u_s - c_k)}{c_k} \left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}} \tag{2.11}$$

Then, introducing the dimensionless quantity

$$\omega_3 = \frac{2c_s D(c_k - u_s)}{c_k(c_k + u_s)(D + c_s - u_s)} \left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}} \tag{2.12}$$

it is possible to assert that

$$x_h \leqslant 0 \text{ when } L/l \leqslant \omega_3, \qquad x_h > 0 \text{ when } L/l > \omega_3$$

The intersection point <u>e</u> of the characteristics γ_1 and γ_3 has the coordinate

$$x_e = \frac{(u_s^2 - c_s^2)(t_c - L/D) + L(u_s + c_s) - x_c(u_s - c_s)}{2c_s} \tag{2.13}$$

Let us introduce the dimensionless quantities

$$\alpha_1 = \frac{2c_s D(u_s - c_k)}{c_k[2c_s D - (u_s + c_k)(D + c_s - u_s)]} \left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}}$$
$$\alpha_2 = \frac{c_s D(u_s - c_k)}{u_s c_k(D + c_s - u_s)} \left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}}, \qquad \alpha_3 = \frac{u_s - c_k}{c_k} \left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{3-k}{k-1}} \tag{2.14}$$

Then

$$x_h > L \text{ when } L/l < \alpha_1, \quad x_h = L \text{ when } L/l = \alpha_1, \quad x_h < L \text{ when } L/l > \alpha_1$$
$$x_b > x_f \text{ when } L/l < \alpha_2, \quad x_b = x_f \text{ when } L/l = \alpha_2, \quad x_b < x_f \text{ when } L/l > \alpha_2$$
$$x_b > L \text{ when } L/l < \alpha_3, \quad x_b = L \text{ when } L/l = \alpha_3, \quad x_b < L \text{ when } L/l > \alpha_3$$

For classification of the flows in the case $u_s \geq c_s$ it is advantageous to introduce the dimensionless quantities

$$\beta_1 = \omega_1, \quad \beta_2 = 2\frac{u_s}{c_k} \cdot \left(\frac{c_0}{c_k}\right)^{\frac{1}{2}\frac{\varkappa - k}{k-1}}, \quad \beta_3 = \alpha_3 \qquad (2.15)$$

Then

$$x_d > L \text{ when } L/l < \beta_1, \quad x_d = L \text{ when } L/l = \beta_1, \quad x_d < L \text{ when } L/l > \beta_1$$
$$x_c > L \text{ when } L/l < \beta_2, \quad x_c = L \text{ when } L/l = \beta_2, \quad x_c < L \text{ when } L/l > \beta_2$$
$$x_b > L \text{ when } L/l < \beta_3, \quad x_b = L \text{ when } L/l = \beta_3, \quad x_b < L \text{ when } L/l > \beta_3$$

Obviously, in order to determine the size of the constant pressure area it is necessary to single out the region of constant pressure $p = p_s$, formed by the regions of the homogeneous flows (S) and (K) in the xt plane.

In general the boundaries of the constant pressure region are made up of the trajectory of the front of the shock wave and the characteristics γ_0, γ, γ_1, γ_2, and γ_3, and for this reason a constant pressure region of a particular configuration in the xt plane will hereafter be called a characteristic regime.

Let us show how many different regimes there can be in a shock tube for arbitrary variation of the parameters p_s, p_0, L, and l and what their distinguishing characteristics are.

3. <u>Possible Characteristic Regimes for the Case $u_s \leq c_k$.</u> It is possible to show that for subsonic flows the inequality

$$\omega_1 > \omega_2 > \omega_3 \qquad (3.1)$$

will always hold behind the contact discontinuity.

The case $u_s \leq c_k$. We shall choose the regime depicted in Fig. 5. It is characterized by the failure of the disturbance from the closed end of the tube to leave the front of the shock wave behind over the distance L; the disturbance from the open end of the shock tube encounters the surface of the contact discontinuity before the disturbance from its closed end; $x_h < 0$. In other words, this regime, if it is possible, should be characterized by simultaneous fulfillment of the relationships: $L/l < \omega_1$, $L/l < \omega_2$, $L/l < \omega_3$. Taking into account inequalities (3.1), we shall require that L/l satisfy the condition

$$L/l < \omega_i \tag{3.2}$$

Obviously, here the first two conditions are fulfilled automatically. Consequently, this characteristic regime is possible in principle, for which it is enough to require fulfillment of condition (3.2).

With the aid of the equations of the trajectory of the shock wave front and the equations of the characteristics γ_2 and γ_3 it is possible to determine the size of the constant pressure area in the whole interval $0 \leq x \leq L_2$. Other possible types of characteristic regimes for the case $u_s < c_k$ are presented in Fig. 6. Also stated there are the conditions under which they occur.

The case $u_s = c_k$. The possible characteristic regimes for the case of sonic flows behind the contact discontinuity are presented in Fig. 7.

4. Possible Characteristic Regimes for the Case $c_k < u_s < c_s$. We note that when the flow behind the contact discontinuity in a shock tube becomes supersonic, it still remains subsonic behind the shock wave front. The possible characteristic regimes for the case

$c_k < u_s < c_s$ are considered below taking into account the fact that under these conditions the relationship

$$\omega_1 > \omega_2 > \alpha_1 > \alpha_2 > \alpha_3 \tag{4.1}$$

is always true.

As an example, let us consider the regime depicted in Fig. 8. It is characterized by the fact that the disturbance from the closed end of the shock tube does not have time to overtake the shock wave front on length L; the disturbance from the open end of the shock tube encounters the surface of the contact discontinuity before the disturbance from the closed end:

$$x_h > L, \qquad x_b > x_p, \qquad x_b > L$$

In other words, this regime, if it is possible, should be characterized by simultaneous fulfillment of the relationships:

$$L/l < \omega_1, \quad L/l < \omega_2, \quad L/l < \alpha_1, \quad L/l < \alpha_2, \quad L/l < \alpha_3$$

We shall require that L/l satisfy the condition $L/l < \alpha_3$. Then the other conditions will be fulfilled automatically on the basis of inequalities (4.1). Consequently, this characteristic regime is possible in principle.

Other characteristic regimes possible for the case $c_k < u_s < c_s$ are presented in Fig. 9 with a statement of the conditions under which they occur.

Thus, when $c_k < u_s < c_s$ there can be no more than 11 characteristic regimes in the shock tube; the actual type of flow is determined by the relationship between the dimensionless quantities ω_1, ω_2, α_1, α_2, and α_3, and the value of L/l.

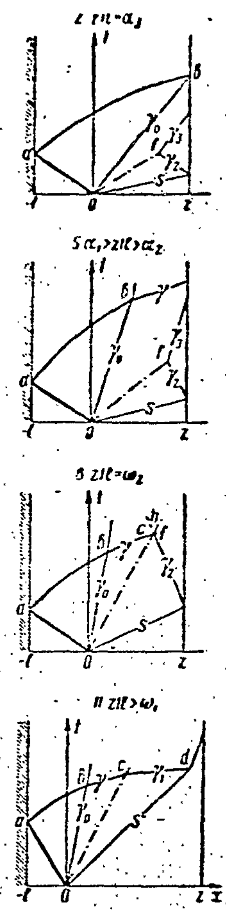

Fig. 9.

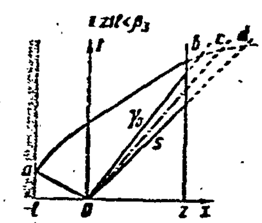

Fig. 10.

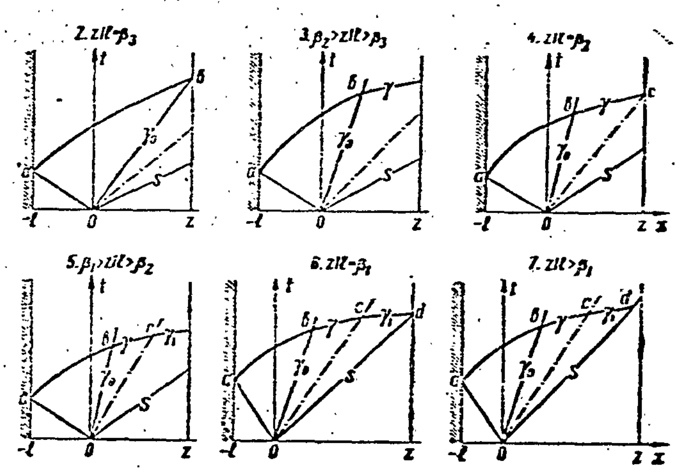

Fig. 11.

5. <u>Possible Characteristic Regimes for the Case $u_s \geq c_s$.</u> It is possible to show that the relationship

$$\beta_1 > \beta_2 > \beta_3 \qquad (5.1)$$

is always true for supersonic flows behind the shock wave front.

Consider the regime depicted in Fig. 10. It is characterized by the relationships: $x_d > L$, $x_c > L$, and $x_b > L$. In other words, in order to realize such a flow the conditions $L/l < \beta_1$, $L/l < \beta_2$, and $L/l < \beta_3$ must be fulfilled simultaneously.

We shall require that the inequality $L/l < \beta_3$ be fulfilled. Then the others will be fulfilled automatically on the basis of (5.1). Consequently, this characteristic regime is also possible in principle. Other possible types of flow when $u_s > c_s$ are given in Fig. 11.

Thus, when $u_s \geq c_s$ the characteristic regime is determined by the relationship between L/l and the dimensionless quantities β_1, β_2, and β_3.

6. <u>Conclusion.</u> It has been established above that for arbitrary variation of the parameters p_s, p_0, L, and l there may exist in an air shock tube 12 characteristic regimes when $u_s \leq c_k$, 11 when $c_k < u_s < c_s$, and 7 when $u_s \geq c_s$. Altogether, therefore, no more than thirty types of flow may exist in the shock tube.

It can be shown that to assume the existence of any other characteristic regimes in addition to the thirty presented above would mean violating conditions (3.1), (4.1), and (5.1).

The dimensionless quantities ω_1, ω_2, ω_3, α_1, α_2, α_3, β_1, β_2, and β_3 are functions only of the ratio p_s/p_0. Presented in Fig. 12 are the curves of these functions, which divide the graph plane into a number of regions corresponding to the thirty possible characteristic regimes considered above. This representation is convenient for determining the type of flow for given values of the parameters p_s, p_0, L, and l (or p_c, p_0, L, and l). In fact, for this purpose it is enough to form the ratios p_s/p_0 and L/l and then establish in which of the characteristic regions in Fig. 12 the point determined by these ratios in the graph plane belongs.

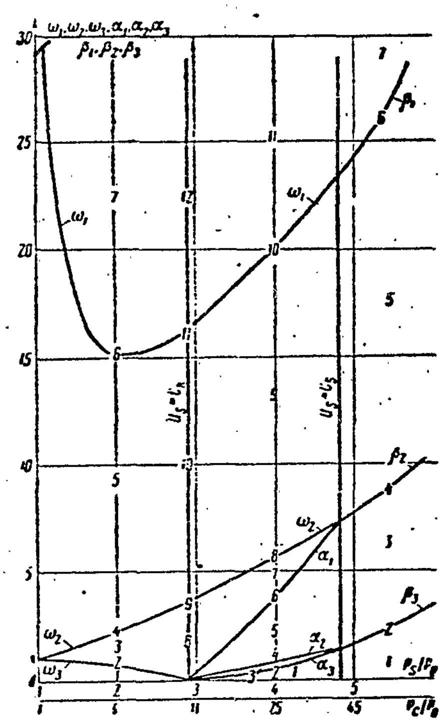

Fig. 12.

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