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THE FOLLOWING PAGES ARE CHANGES

TO BASIC DOCUMENT

30 January 1964

E R R A T A

POWER-SPECTRAL ANALYSIS OF TROPICAL UPPER WINDS

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-307

The following information should be inserted under the appropriate lines in the basic technical documentary report:

Page ii, line 8, as reads " a report (121" should read "a report (12)"

Page 10, line 12, as reads "centered northeast of Kwajalein," should read "centered northwest of Kwajalein,"

Page 11, arrowhead at western edge of figure, between 5 and 10°s, should point eastward, not westward.

Page 11, title of Fig. 4-1(a), as reads "300 mb" should read "850 mb"

Page 12, title of Fig. 4-1(b), as reads "850 mb" should read "300 mb"

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433L SYSTEM PROGRAM OFFICE
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Mass.

Prepared under Contract Nr AF 19(626)--16
for United Aircraft Corporate Systems Center
by The Travelers Research Center, Inc., Hartford, Conn.

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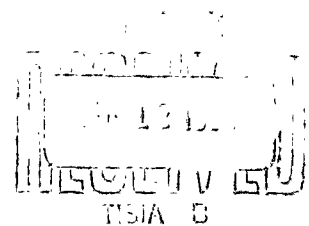
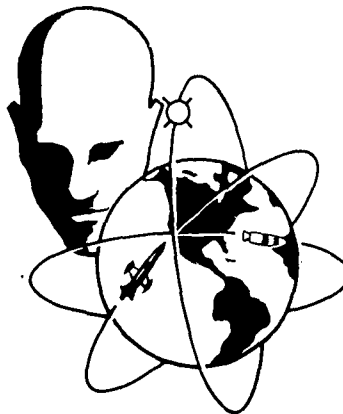
POWER-SPECTRAL ANALYSIS OF TROPICAL UPPER WINDS

TECHNICAL DOCUMENTARY REPORT NO ESD-TDR-63-307

June 1963

Edward A. Newburg
Joseph P. Pandolfo
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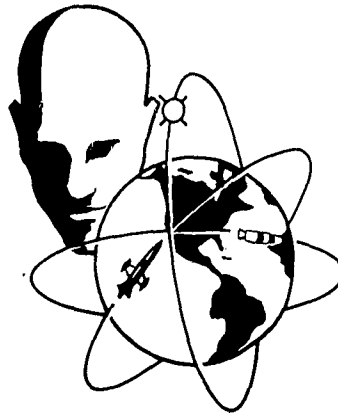
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FOREWORD

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We gratefully acknowledge the assistance of W. Bruce Ramsay of the U.S. Weather Bureau, who, by providing the machine program used for the power-spectral analysis, shortened the time needed for completing this task. We also acknowledge the contributions of Raymond Ellis of the TRC Service Corporation, who carried out the required computations with the Weather Bureau code.

The mean-wind-flow pattern in Fig. 4-1 is reproduced from a report [12] by C. J. Wiederanders. The wind statistics in Table 4-1 are taken from a report [6] by C. S. Ramage.

POWER-SPECTRAL ANALYSIS OF TROPICAL UPPER WINDS

ABSTRACT

Power-spectral analysis of 8 mo of upper winds for Kwajalein shows significant peaks for 850 and 200 mb in the frequency band between 2.5 and 4 days. This is interpreted as the result of migrating perturbations with a wavelength of from 1000 to 2000 km moving with the trade wind.

At 300 mb, the most pronounced peak is centered at 17 days; it is presumably a result either of slow-moving perturbations of a larger scale than those embedded in the trades or of meridional oscillations in the zonal flow.

REVIEW AND APPROVAL

Publication of this technical documentary report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.



R. G. Bounds, Jr.
Lt Colonel, USAF
System Program Director

TABLE OF CONTENTS

<u>Section</u>	<u>Title</u>	<u>Page</u>
1.0	INTRODUCTION	1
2.0	THE DATA	3
3.0	THE METHOD OF POWER-SPECTRAL ANALYSIS	5
4.0	DISCUSSION OF RESULTS	10
4.1	Climatological Features of the Data Area	10
4.2	Results of Power-spectral Analysis	10
4.3	Statistical Significance of Peaks in the Velocity-component Spectra	18
5.0	CONCLUSIONS AND RECOMMENDATIONS	21
6.0	REFERENCES	22

LIST OF ILLUSTRATIONS

<u>Figure</u>	<u>Title</u>	<u>Page</u>
4-1(a)	Mean streamlines at 300 mb for February 1957	11
4-1(b)	Mean streamlines at 850 mb for February 1957	12
4-2	Autocorrelations for the zonal and meridional wind components at 300, 700, and 850 mb for Kwajalein	15
4-3	Normalized smoothed-spectral estimates for the zonal and meridional wind components at 300, 700, and 850 mb for Kwajalein and Rosenthal's spectra for Kusaie	16
4-4	Smoothed-spectral estimates and 90% confidence levels for the zonal and meridional wind components at 300, 700, and 850 mb for Kwajalein	19

LIST OF TABLES

<u>Table</u>	<u>Title</u>	<u>Page</u>
4-1	Four-year mean resultant winds and mean steadiness figures for Kwajalein	13
4-2	Wind statistics for Kwajalein for 1 Oct. 1956 through 31 May 1957	14

1.0 INTRODUCTION

Power-spectral analysis of time series of meteorological data has been used by a number of investigators [2, 7, 11] to detect prevalent frequencies of migrating eddies. Although large-scale eddies are best studied by conventional map analysis if observations are spatially dense, analysis of time series appears to be the only alternative for areas of sparse data. This is the situation in the Tropics, where map analysis cannot be made except for the very largest scale of perturbations. There, the average distance between upper-air stations is so great that migrating perturbations may slip undetected through the synoptic network. In contrast to this spatial sparsity of data, the 12-hr frequency of observations at a given station provides an adequate resolution in time. Application of power-spectral analysis to time series will, therefore, yield information unattainable by other means.

The time series of wind observed at a fixed geographic location at a certain height and for a given period of observation may be considered to be composed of a basic flow (given by the time-mean resultant wind) on which is superimposed a series of velocity disturbances. The sum of the variances of the zonal and meridional wind components will then represent the disturbances' kinetic energy per unit mass. Power-spectral analysis of each of the two wind components will show how the total perturbation energy is distributed among the various frequencies (or periods) of the record. Because the power spectrum of wind is directly related to kinetic energy, most investigators, including those we have referred to above, have used this element in their studies. This has also been done here. Should this study be continued, however, we intend to subject additional meteorological elements to power-spectral analysis in an effort to gain maximum information from the data on hand.

The work reported here represents an attempt to apply this tool to tropical data and is merely to be considered as a feasibility study. Its aim was to obtain information regarding the occurrence, in a climatological sense, of migrating perturbations. For this purpose, data over a period of two years were acquired for the Pacific Ocean stations Kwajalein ($8^{\circ}43'N$, $167^{\circ}44'E$), Canton

(2°46'S, 171°43'W), Panape (6°58'N, 158°13'E), and Majuro (7°06'N, 171°24'E). We had initially intended to subject winds from at least two of these stations to power-spectral analysis, but for reasons discussed later, only data for Kwajalein could be processed at this time. This means that the material at hand gives no direct information as to the horizontal scales of the perturbations detectable in the time series for Kwajalein. To obtain this information, two or more stations would have had to be correlated. On the assumption that these perturbations drift with the prevailing flow, estimates are made regarding the horizontal dimension of the perturbations.

It should be emphasized that although the statistical theory of power-spectral analysis is rather well developed [1], its potential usefulness in synoptic and dynamic meteorology has not been fully explored. In practice, statistically significant events may be hard to interpret without ambiguity because of a too meager knowledge of the meteorological phenomenon under study. Additional information based on synoptic experience and theoretical considerations will, therefore, enhance the usefulness of time series.

2.0 THE DATA

As mentioned in the introduction, upper-air data were acquired for the four Pacific stations Kwajalein, Canton, Panape and Majuro. These data contain wind direction and speed, temperature, pressure height, and humidity corresponding to pressures in 50-mb increments ranging from 200 to 1000 mb. Information at pressures below 200 mb is given for smaller pressure intervals. Data from all four stations cover the 2-yr period from 1 Oct. 1956 through 30 Sep. 1958. The frequency of observation is twice daily for Kwajalein and Canton and once daily for Panape and Majuro. Additionally, Kwajalein has a 4-mo period, April through July 1958, with four soundings daily, and, from 22 Mar. through 31 May 1958, Panape also has four observations per day. The data records are occasionally broken by missing observations, the longest gap being the full month of February 1958 for Kwajalein.

A considerable time was spent in preparing the data for the computations and only an 8-mo collection of wind data (1 Oct. 1956 through 1 Jun. 1957) could be made ready in time for power-spectral analysis. This represented a shorter period than was anticipated. Originally, we had intended to process the full 2-yr record, but missing data and a change in the times of observation (from 03 and 15Z to 00 and 12Z on 1 Jun. 1957) made it advisable to settle for a shorter period in this feasibility study. Although the 8-mo period represents the most complete period of comparable length within the total record on hand, it still has some occasional gaps of one or more observations. On two occasions, three or four consecutive days were missing. The theory of power-spectral analysis does not provide for such gaps. In this particular case, the gaps were bridged by means of linear interpolation. To test the sensitivity of the results to the specific method used for filling in the gaps, the missing observations were also replaced by the previous observation alone, and power-spectral analysis was performed. Both methods for bridging the gaps gave essentially the same spectra. In future runs, we hope to have a better theoretical basis for dealing with gaps in the records.

Preparatory to power-spectral analysis of other meteorological elements than the wind, the data are being scanned for missing observations of one or more of these elements at the standard pressure levels up to and including 200 mb. Although we have assumed the wind to be the most suitable element to use in power-spectral analysis, this can by no means be taken for granted without some further study. It is quite possible that moisture may be the best element in detecting synoptic-scale perturbations within the lower half of the troposphere.

3.0 THE METHOD OF POWER-SPECTRAL ANALYSIS

The theory and application of power-spectral analysis is dealt with by Blackman and Tukey [1], and the meteorological application more specifically by Panofsky and Brier [5]. A brief account of the main elements of power-spectral analysis is given below.

Let $X(t)$ be the time series, which for the time being we will assume to be continuous and of infinite length. The mean, $\bar{X}$, of the series is defined by

$$\bar{X} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t) dt \quad (3-1)$$

and the fluctuation, $X'(t)$, by

$$X'(t) = X(t) - \bar{X}. \quad (3-2)$$

To obtain the power spectrum, one may proceed by defining first an autocovariance function:

$$C(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} X'(t) X'(t + \tau) dt, \quad (3-3)$$

where τ represents the lag, i.e., the time interval between pairs of values of X' whose product comprises the integrand above. At $\tau = 0$, the autocovariance function takes the value

$$C(0) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} (X')^2(t) dt = \sigma_X^2, \quad (3-4)$$

where σ_X^2 is the true variance. The power spectrum $P(f)$, where f stands for frequency, may then be given as the 1-sided cosine Fourier transform of $C(\tau)$:

$$P(f) = 2 \int_0^{\infty} C(\tau) \cos (2\pi f \tau) d\tau. \quad (3-5a)$$

The Fourier inversion formula is

$$C(\tau) = 2 \int_0^{\infty} P(f) \cos (2\pi f \tau) df. \quad (3-5b)$$

It follows from Eqs. (3-5b) and (3-4) that

$$\sigma_X^2 = 2 \int_0^{\infty} P(f) df. \quad (3-6)$$

In practice, we obtain a record of only finite duration, which may be regarded as a sample drawn from an ensemble or population of records of the same duration and with the same statistics. Autocovariance functions and power spectra estimated from such finite records are, therefore, subjected to sampling variations and biases in the usual statistical sense.

A suitable analytical procedure may be chosen to yield spectral estimates whose ensemble-average values are estimates of a smoothed (over frequency) true power-spectral value and not estimates of the true spectral value itself. Estimating weighted averages over frequency of the power spectrum in the vicinity of a given frequency rather than values at precise frequencies has the apparent disadvantage of a loss in frequency resolution. However, this disadvantage is more than offset by the applicability of statistical confidence estimates to the smoothed spectrum, while no such estimates are available for raw (not smoothed over frequency) spectra. Furthermore, it is possible, within limits imposed by the method for observing the data, to maximize frequency resolution in smoothed spectra by a suitable choice of the smoothing function and of the interval between data observations.

Various analytical procedures are described by Blackman and Tukey [1]. Basically, the procedures involve modified definitions of the true statistics. A sample mean is defined as

$$\bar{X} = \frac{1}{T} \int_{-T/2}^{T/2} X(t) dt, \quad (3-7)$$

and the fluctuation in X is, as before,

$$X' = X(t) - \bar{X}. \quad (3-8)$$

An "apparent" autocovariance function is defined by

$$C_A(\tau) = \frac{1}{T - |\tau|} \int_{(T-|\tau|)/2}^{(T+|\tau|)/2} X' \left(t - \frac{\tau}{2} \right) X' \left(t + \frac{\tau}{2} \right) dt \quad (0 \leq \tau \leq \tau_M < T), \quad (3-9)$$

where τ_M is a maximum lag. A "modified apparent" autocovariance function is defined by

$$C_{MA}(\tau) = D(\tau) C_A(\tau). \quad (3-10)$$

$D(\tau)$ is an even function of τ with the values

$$\begin{aligned} D(0) &= 1, \\ D(\tau) &= 0 \quad (\tau > \tau_M), \end{aligned} \quad (3-11)$$

and an arbitrary shape for $0 < \tau \leq \tau_M$.

The Fourier transform of $C_A(\tau)$ cannot be determined because $C_A(\tau)$ is not defined for $\tau > \tau_M$. However, $C_{MA}(\tau)$ does have a determinable transform. Its transform is given by

$$P_S(f) = \int_0^\infty C_{MA}(\tau) \cos(2\pi f\tau) d\tau = \int_0^{\tau_M} C_{MA}(\tau) \cos(2\pi f\tau) d\tau \quad (3-12)$$

and is related to the transform of $C_A(\tau)$ by

$$P_S(f) = Q(f) * P_A(f). \quad (3-13)$$

The asterisk denotes the operation of convolution, and $Q(f)$ is the transform of $D(\tau)$. The ensemble average, i.e., the average for the hypothetical population mentioned previously, is given by

$$\{C_{MA}(\tau)\}_{av} = D(\tau) C(\tau), \quad (3-14)$$

where $C(\tau)$ is the true autocovariance function. It follows that

$$\{P_S(f)\}_{av} = Q(f) * P(f), \quad (3-15)$$

where $P(f)$ is the true power spectrum. The convolution operation may be written

$$\{P_S(f_1)\}_{av} = \int_{-\infty}^{\infty} Q(f_1 - f) P(f) df, \quad (3-16)$$

and, because $Q(f)$ and $P(f)$ are even functions,

$$\{2P_S(f_1)\}_{av} = \int_0^\infty [Q(f_1 + f) + Q(f_1 - f)] 2P(f) df. \quad (3-17)$$

The function $Q(f)$ therefore, is the smoothing function referred to previously.

The choice of a particular $D(\tau)$ and, therefore, $Q(f)$ is arbitrary, but a desirable choice is usually one that weights spectral estimates at frequencies away from f_1 as little as possible. This choice depends, to a great extent, upon the expected nature of the spectrum to be analyzed and the purposes of the

analysis. Thus, for one problem, a $Q(f)$ that has smallest maximum weight at any single frequency away from f_1 may be preferable to one that has smaller weight at many frequencies away from f_1 . Two choices for $D(\tau)$ most commonly used are the "hamming" function

$$\begin{aligned} D(\tau) &= 0.54 + 0.46 \cos \frac{\pi\tau}{T_M} & (|\tau| < T_M) \\ D(\tau) &= 0 & (|\tau| > T_M), \end{aligned} \quad (3-18)$$

and the "hanning" function

$$\begin{aligned} D(\tau) &= \frac{1}{2} \left(1 + \cos \frac{\pi\tau}{T_M} \right) & (|\tau| < T_M) \\ D(\tau) &= 0 & (|\tau| > T_M). \end{aligned} \quad (3-19)$$

Although $P_S(f)$ is a satisfactory estimate of the true spectrum, its transform $C_{MA}(\tau)$ is a much poorer estimate of the true autocovariance function than is $C_A(\tau)$. In practice, therefore, the procedure used to compute $P_S(f)$ is as follows [5].

- (a) $C_A(\tau)$ is computed from its definition.
- (b) A harmonic analysis is carried out on $C_A(\tau)$ to yield a "raw" spectrum. The result of this procedure is only an approximation to the Fourier transform of $C_A(\tau)$.
- (c) One of the available smoothing operators $Q(f)$ is applied to the result of (b) to yield an approximation to $P_S(f)$, the smoothed-spectral estimate.

The analysis of time series of observations made at equal time intervals Δt introduces considerable restriction on the obtainable results. Obviously, the maximum frequency for which a spectral estimate can be obtained (the Nyquist frequency f_N) is now fixed by the time interval

$$f_N = \frac{1}{2\Delta t}. \quad (3-20)$$

The maximum frequency resolution attainable for a given maximum number of lags m is

$$\Delta f = \frac{1}{2m\Delta t}. \quad (3-21)$$

Because the sampling variability of spectral estimates depends partly on the ratio of the total number of data points to the maximum number of lags m , the resolution attainable is further restricted; a rough rule of thumb is that

$$m \leq \frac{T}{10 \Delta t}. \quad (3-22)$$

Given m and Δt , spectral estimates will be obtained for frequency intervals centered at

$$f_i = \frac{i}{2m \Delta t} \quad (i = 1, 2, \dots, m)$$

and

$$f = \frac{1}{8m \Delta t} \quad (i = 0). \quad (3-23)$$

The 0th and m th intervals are half the width of all other intervals.

If the continuous variable $X(t)$ contains a significant time variation within an observation interval Δt , spectral analysis will produce a distorted spectrum, with power indicated at frequencies other than those at which it should be present. This type of distortion is referred to as "folding," and the Nyquist frequency, f_N , is sometimes referred to as the "folding frequency." Essentially, the distortion produced is that which would result if the correct spectrum extending past f_N to higher frequencies were folded back upon itself at integral multiples of f_N to produce the apparent spectrum. The power at a frequency $f_0 < f_N$ in the apparent spectrum would really be the sum of powers at f_0 , and at $jf_N \pm f_0$ (for even integral values of j) in the correct spectrum. This type of error is less serious at frequencies much smaller than f_N when power decreases rapidly past f_N .

4.0 DISCUSSION OF RESULTS

4.1 Climatological Features of the Data Area

Figure 4-1 shows the mean wind flow in February 1957 at 300 and 850 mb over the tropical reaches of the Pacific Ocean. The four data stations used in our study have been entered on the maps, as have (for comparison with the Kwajalein results) four other stations for which [7] power spectra are available.

The February wind conditions shown on the maps are approximately representative of the full 8-mo period to which the power-spectral analysis for Kwajalein applies (1 Oct. 1956 through 1 Jun. 1957). The easterlies are well developed throughout the period and consistently penetrate the layer between the surface and 500 mb. The mean map for 300 mb shows an anticyclonic eddy centered northeast of Kwajalein, which, during February, dominates the circulation over the area immediately surrounding Kwajalein. During the period, this anticyclone drifts southward from its northernmost position in October to its southernmost position in March and April and undergoes considerable changes in intensity and structure. Partly for this reason, the 300-mb winds over Kwajalein are quite variable.

Their variability is shown in some detail in Table 4-1, which shows that the easterlies have a high degree of steadiness at both 850 and 700 mb throughout the year. In contrast, the 300-mb winds are much more variable and change from easterly to westerly in early spring. We will return to this variability at 300 mb later.

Despite the relatively dense upper-air network around the Marshall Islands, little is known about the synoptic-scale disturbances that pass through. According to Lavoie [4], they are presumably like easterly waves; i.e., they travel westward with the trade wind without necessarily conforming to the model of the Caribbean easterly wave. In addition to this type of disturbance, closed equatorial vortices (as described by Smith [10] for the Indian Ocean) may contribute to the observed wind variability at Kwajalein and nearby stations.

4.2 Results of Power-spectral Analysis

The 8-mo period of upper-wind data for Kwajalein consists of 486 consecutive observations 12 hr apart, discounting the gaps mentioned in Section 2.0.

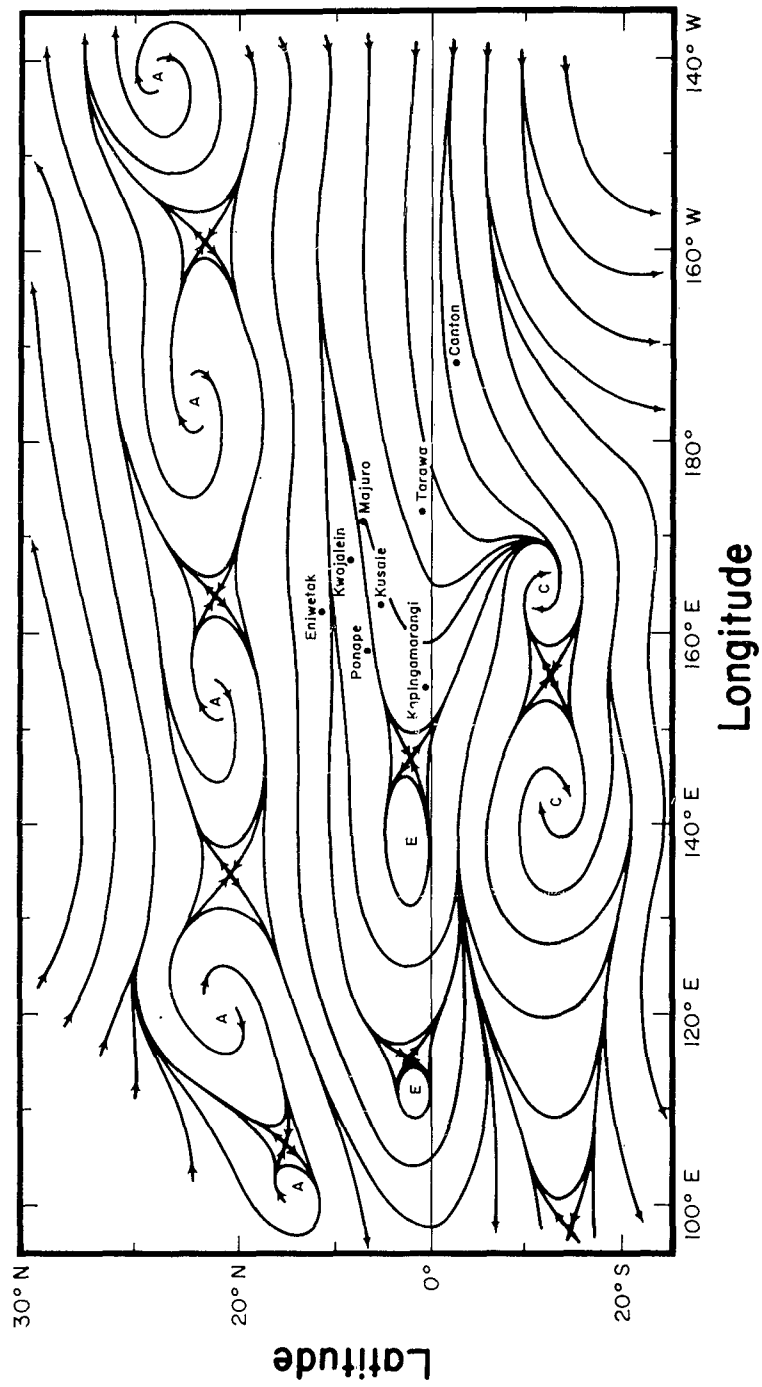


Fig. 4-1(a). Mean streamlines at 300 mb for February 1957. A, C, and E denote anticyclone, cyclone, and equatorial eddy, respectively.

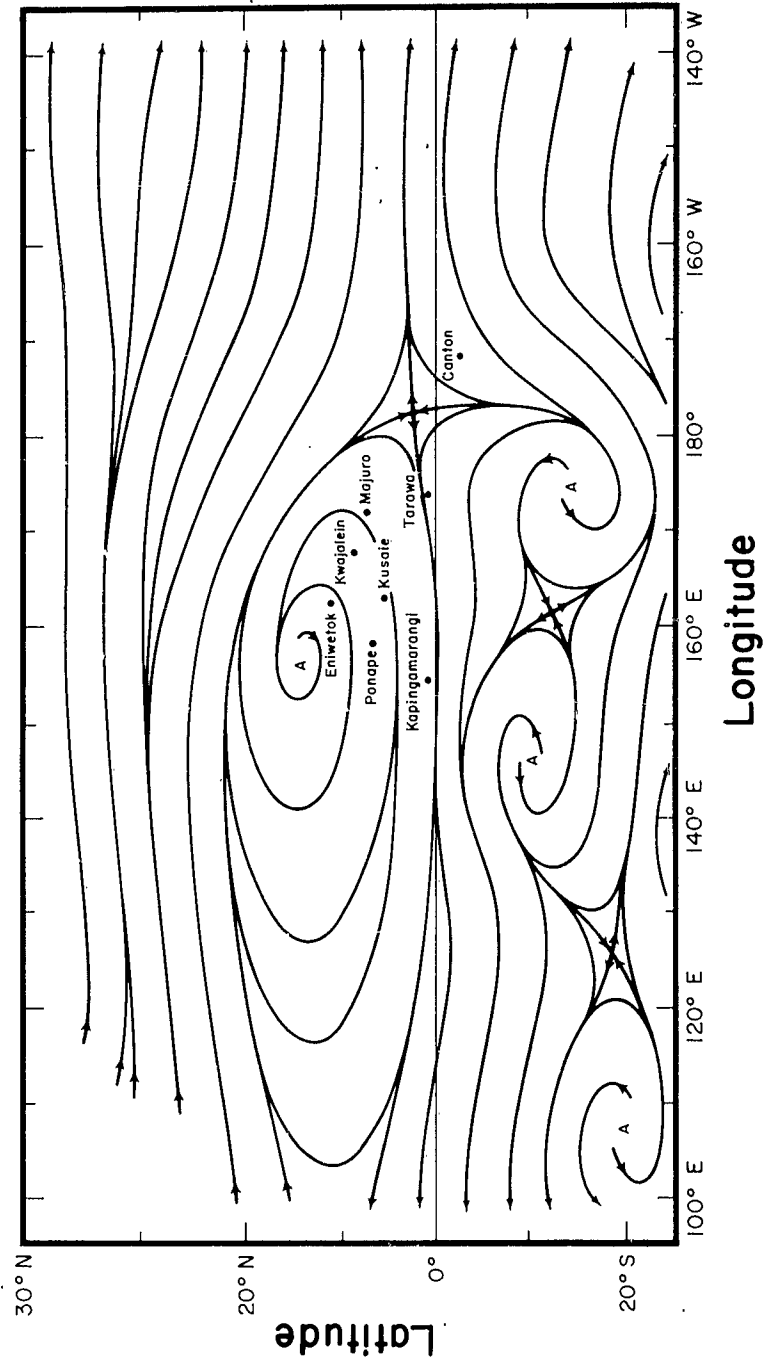


Fig. 4-1(b). Mean streamlines at 850 mb for February 1957. A and C denote anticyclone and cyclone, respectively.

TABLE 4-1
FOUR-YEAR MEAN RESULTANT WINDS AND MEAN STEADINESS FIGURES
FOR KWAJALEIN*

(a) At 300 mb						(b) At 700 mb					
Mo.	S	θ	V_r	$\bar{u}$	C	Mo.	S	θ	V_r	$\bar{u}$	C
Jan	28	76	2.1	-2.0	7.4	Jan	81	91	6.0	-6.0	7.4
Feb	25	44	2.1	-1.4	8.3	Feb	82	90	6.3	-6.3	7.6
Mar	37	289	3.3	3.1	8.8	Mar	52	106	3.2	-3.0	6.2
Apr	67	262	7.7	7.8	11.5	Apr	64	98	3.6	-3.5	5.5
May	57	260	4.8	4.7	8.5	May	81	98	5.5	-5.4	6.8
Jun	22	241	1.7	1.5	7.7	Jun	92	95	7.3	-7.3	7.9
Jul	25	125	1.6	-1.3	6.3	Jul	91	96	7.2	-7.1	7.9
Aug	67	101	4.7	-4.7	7.1	Aug	91	96	7.2	-7.2	7.9
Sep	52	91	3.4	-3.4	6.4	Sep	88	94	5.8	-5.8	6.6
Oct	13	96	0.8	-0.8	6.5	Oct	79	96	5.3	-5.3	6.6
Nov	7	294	0.6	0.6	8.3	Nov	83	93	6.2	-6.1	7.4
Dec	8	275	0.7	0.7	8.7	Dec	75	96	5.8	-5.8	7.8

(c) At 850 mb					
Mo.	S	θ	V_r	$\bar{u}$	C
Jan	91	85	8.1	-8.1	8.9
Feb	92	88	9.4	-9.4	10.2
Mar	84	93	7.6	-7.6	9.0
Apr	94	89	8.7	-8.7	9.3
May	91	93	8.4	-8.4	9.2
Jun	93	92	8.5	-8.5	9.1
Jul	92	96	7.5	-7.5	8.1
Aug	90	98	6.2	-6.2	7.0
Sep	85	97	5.2	-5.1	6.1
Oct	80	98	5.2	-5.1	6.4
Nov	89	92	7.5	-7.5	8.5
Dec	85	91	8.0	-8.0	9.4

*Explanation of symbols:
S, steadiness in percent,
 θ , mean resultant wind direction in degrees,
 V_r , mean resultant wind speed in meters per second.
 $\bar{u}$, mean zonal wind speed in meters per second,
C, mean wind speed in meters per second.

TABLE 4-2
WIND STATISTICS FOR KWAJALEIN*
FOR 1 OCT. 1956 THROUGH 31 MAY 1957

Level, mb	$\bar{u}$	σ_u^2	$\bar{v}$	σ_v^2
300	-0.1350	47.7757	0.2493	27.4366
700	-5.2230	17.1976	-0.1505	8.2484
850	-7.5960	14.1150	-0.7802	8.5732

*Explanation of symbols:

$\bar{u}$, mean of zonal wind component in meters per second,

σ_u^2 , variance of u in square meters per square second,

$\bar{v}$, mean of meridional wind component in meters per second,

σ_v^2 , variance of v in square meters per square second.

Consistent with the rule that the maximum lag may be only about one-tenth of the total record, lags of from 0.5 to 25 days were used.

Table 4-2 shows the mean values of u and v for the 8-mo sample and the variances, σ_u^2 , and σ_v^2 . Note the similarity between 850 and 700 mb on the one hand and, on the other, the large difference between the variances at these levels and those at 300 mb.* The sample means for the wind are very close to the 4-yr means shown in Table 4-1.

Figure 4-2 shows the normalized autocovariance (autocorrelation) functions for both the u- and v-components at 850, 700, and 300 mb. It is quite apparent that the variation in wind is not caused by any single harmonic component because this would show up as distinct peaks in each curve. As we shall see from the power spectrum, all frequencies contribute to the total variance. Note that the autocorrelation for the u-component falls off less sharply with increasing time lag than does the autocorrelation for the v-component; this is particularly pronounced at 300 mb.

In Fig. 4-3, we show the smoothed-spectral estimates. The spectral estimates for periods of less than 2 days are not shown because they are presumably subject to greater contamination by "folding" errors. The frequency, f, is plotted

*Because the 500-mb level is transitory (it is influenced by both low- and high-level disturbances), it was not used in the power-spectral analysis.

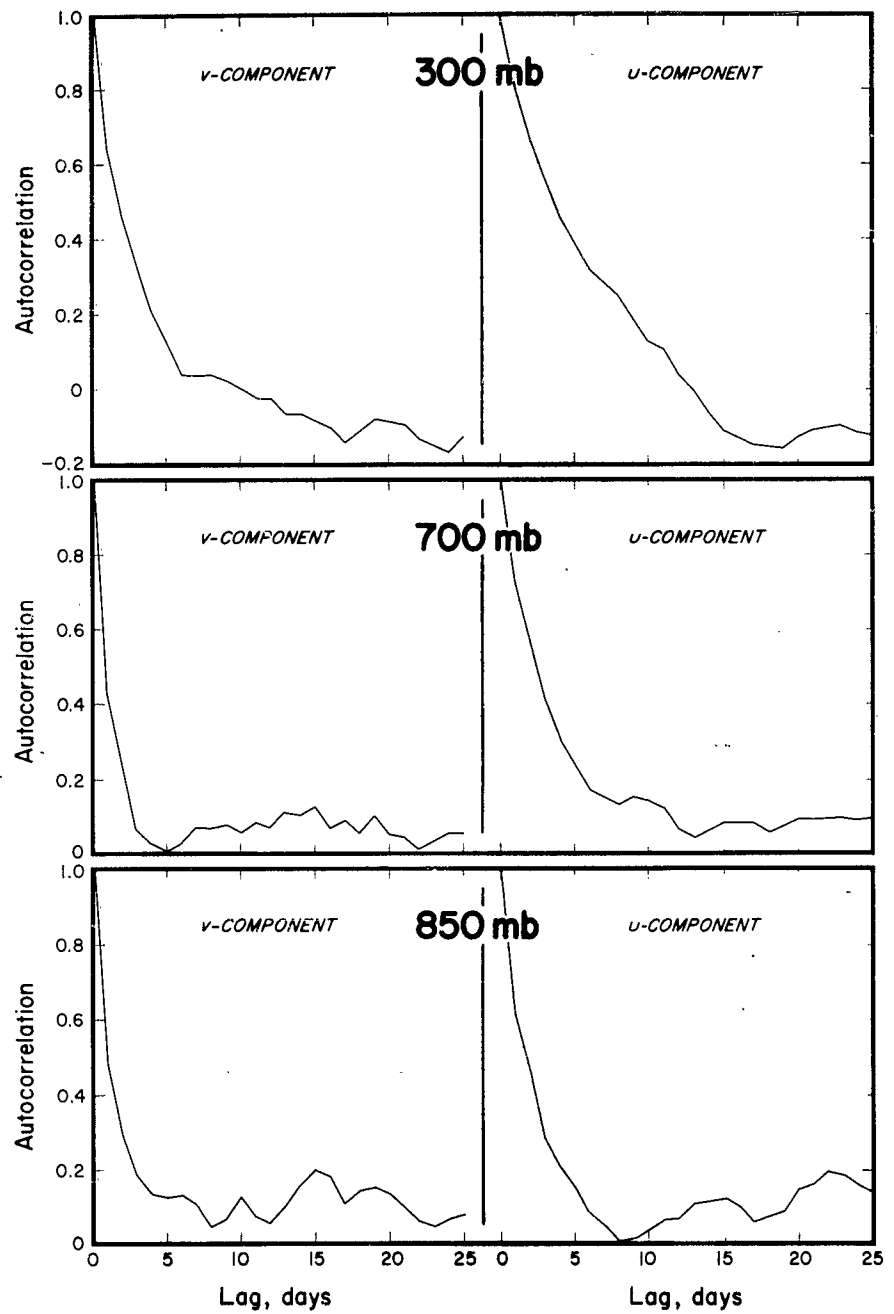


Fig. 4-2. Autocorrelations for the zonal and meridional wind components at 300, 700, and 850 mb for Kwajalein.

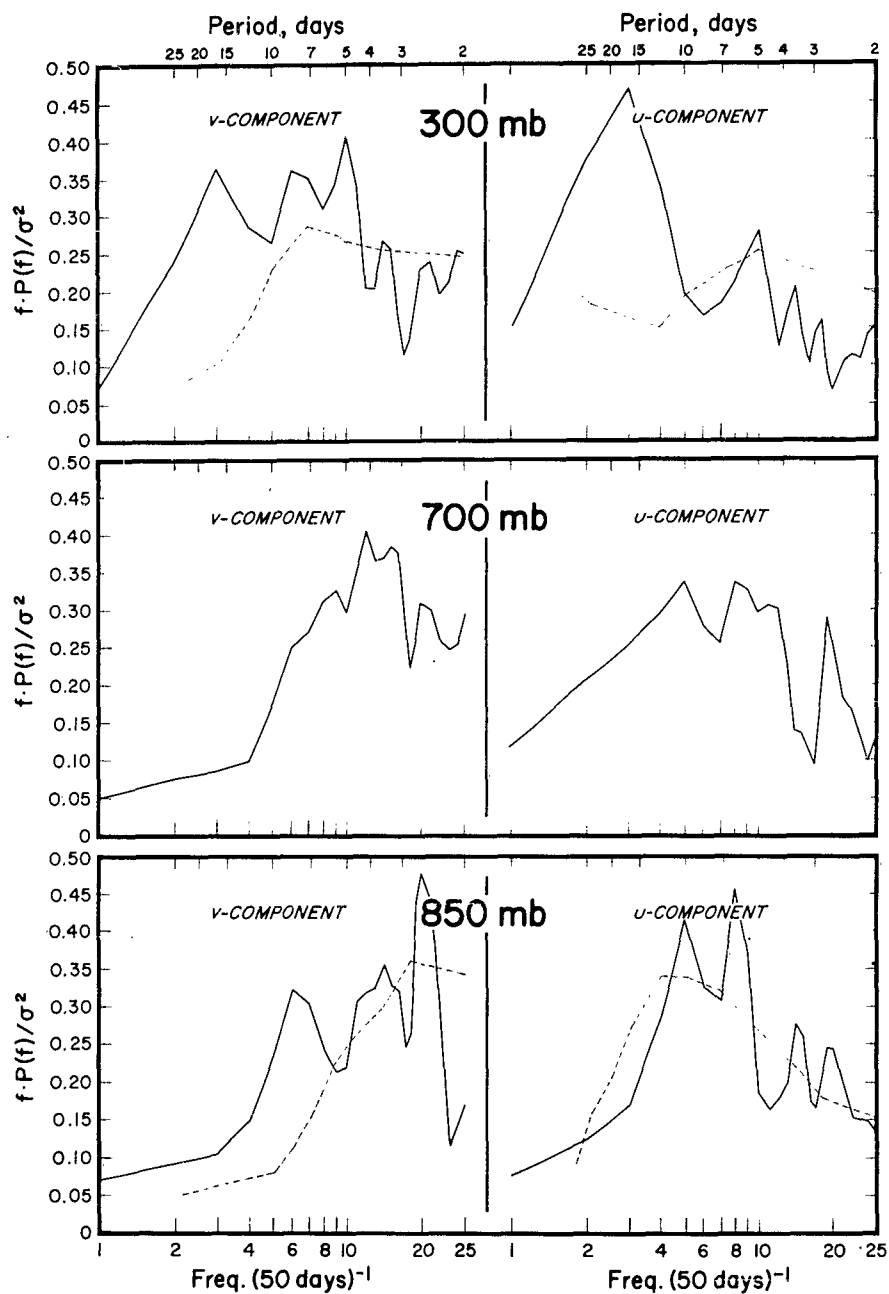


Fig. 4-3. Normalized smoothed-spectral estimates for the zonal and meridional wind components at 300, 700, and 850 mb for Kwajalein (solid curves) and Rosenthal's [7] spectra for Kusaie (dashed curves).

on a logarithmic scale, and to conserve the property of area proportional to variance (kinetic energy in this case), we have plotted the normalized product $fP(f)$ as ordinate. We note that the v-component for 850 mb has a sharp peak at 2.5 days and two less outstanding peaks at 4 and 9 days. The spectrum for the u-component for the same level does show maxima at 2.5 and 3.5 days, but the greatest contribution to the variance is for periods of from 6 to 10 days. The spectrum for 700 mb is rather similar to that for 850 mb. The v-component has a 3-to-4-day maximum, and the u-component has peaks at 2.5, 6, and 10 days. The significance of these maxima will be dealt with later, but we may tentatively conclude that

- (a) the v-component for the 850—700-mb layer has a primary maximum between 2.5 and 4 days and a secondary maximum between 6 and 9 days, and
- (b) the u-component has a primary maximum between 6 and 10 days and a secondary maximum at about 3 days.

At 300 mb, both the u- and v-components have 5-day and 16-to-17-day peaks, with most of the kinetic energy concentrated at periods of longer than 5 days. The 300-mb spectrum apparently tends more than the 850-mb spectrum toward lower frequencies—particularly the 300-mb u-component, which shows a great concentration of energy in the 10-to-25-day band.

These findings may be compared with Rosenthal's [7] results of power-spectral analysis for Tarawa (1.4°N , 172.9°E), Kapingamarangi (1.0°N , 154.8°E), Kusaie (5.3°N , 163.0°E), and Eniwetok (11.3°N , 162.3°E), based on 3 mo of upper winds observed at 6-hr intervals in 1956. Tarawa and Kapingamarangi show a period close to 4 days for the v-component and about 6 days for the u-component at 5000 ft. At 40,000 ft, the same stations have a maximum for both wind components in the 5-to-10-day band. For further comparison, we have entered Rosenthal's u- and v-spectra for Kusaie as dashed curves in Fig. 4-3. Because Rosenthal used moving averages as a smoothing device, his power spectrum shows no sharp peaks. Note that his results do not support the 17-day period at 300 mb; this may be due to his short record of observations.

on a logarithmic scale, and to conserve the property of area proportional to variance (kinetic energy in this case), we have plotted the normalized product $fP(f)$ as ordinate. We note that the v-component for 850 mb has a sharp peak at 2.5 days and two less outstanding peaks at 4 and 9 days. The spectrum for the u-component for the same level does show maxima at 2.5 and 3.5 days, but the greatest contribution to the variance is for periods of from 6 to 10 days. The spectrum for 700 mb is rather similar to that for 850 mb. The v-component has a 3-to-4-day maximum, and the u-component has peaks at 2.5, 6, and 10 days. The significance of these maxima will be dealt with later, but we may tentatively conclude that

- (a) the v-component for the 850—700-mb layer has a primary maximum between 2.5 and 4 days and a secondary maximum between 6 and 9 days, and
- (b) the u-component has a primary maximum between 6 and 10 days and a secondary maximum at about 4 days.

At 300 mb, both the u- and v-components have 5-day and 16-to-17-day peaks, with most of the kinetic energy concentrated at periods of longer than 5 days. The 300-mb spectrum apparently tends as more than the 850-mb spectrum toward lower frequencies—particularly the 300-mb u-component, which shows a great concentration of energy in the 10-to-25-day band.

These findings may be compared with Rosenthal's [7] results of power-spectral analysis for Tarawa (1.1°N , 172.9°E), Kapingamarangi (1.0°N , 154.8°E), Kusaie (5.3°N , 163.0°E), and Eniwetok (1.3°N , 162.3°E), based on 3 mo of upper winds observed at 6-hr intervals in 1956. Tarawa and Kapingamarangi show a period close to 4 days for the v-component and about 6 days for the u-component at 5000 ft. At 40,000 ft, the same stations have a maximum for both wind components in the 5-to-10-day band. For further comparison, we have entered Rosenthal's u- and v-spectra for Kusaie as dashed curves in Fig. 4-3. Because Rosenthal used moving averages as a smoothing device, his power spectrum shows no sharp peaks. Note that his results do not support the 17-day period at 300 mb; this may be due to his short record of observations.

The preference for lower frequencies in the u-component spectra as compared with those of the v-component is indirectly borne out by two studies by Saltzman [8, 9]. By applying harmonic analysis to a series of 500-mb pressure-height charts, he found that at 27.5°N the geostrophic zonal perturbation energy is largest for wave numbers 1 through 3 and that the meridional perturbation energy is largest for wave numbers 5 through 7. The interpretation of this is probably that the variation in the u-component is mainly associated with oscillations in the zonal flow, whereas variation in the v-component reflects migrating perturbations. The results for Kwajalein, described above, have presumably a similar interpretation: the 16-to-17-day period may be the result of oscillations of the zonal flow and/or large-scale slow-moving troughs and ridges, and the pronounced 3-to-4-day period in the v-components is the result of migrating disturbances having their maximum intensities near the 700-mb level. If we may assume that these migrating disturbances move with the westward flow, then, for a mean speed of 6 m sec^{-1} and a period of 3 days, the corresponding wavelength would be 1543 km. This is about the dimension of an easterly wave as we know it from synoptic studies in the Caribbean.

4.3 Statistical Significance of Peaks in the Velocity-component Spectra

An approximate basis for estimating the sampling variability of smoothed-spectral estimates is given by Tukey [1, pp. 15--25]. In essence, for the development of the method, the original time-series values are assumed to follow a Gaussian distribution. Then the smoothed-spectral estimates are to be considered analogous to the statistic χ^2 of normally distributed samples, which itself has a known probability distribution. The basic parameter of the χ^2 -distribution is a k, known as the number of degrees of freedom. The value of the analogous parameter k for the sample of smoothed spectral estimates is determined by the length of record, the maximum lag, the nature of the spectral window applied in the smoothing process, and the character of the spectrum itself. This value may be obtained and printed out as a by-product of the spectral-analysis program. The theory of sampling variability thus obtained is only approximate [1, p. 21], but still of use, especially when repeated spectral

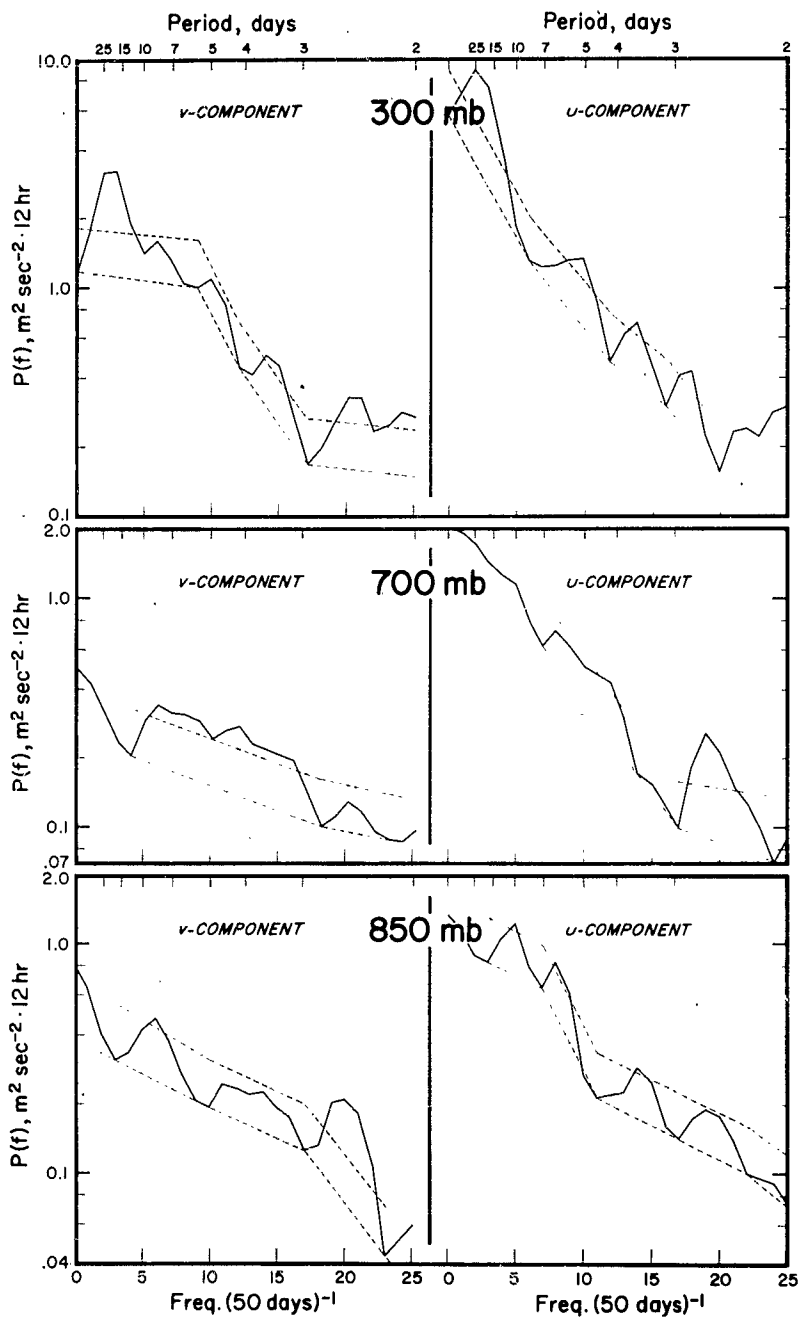


Fig. 4-4. Smoothed-spectral estimates (solid lines) and 90% confidence levels (upper dashed lines) for the zonal and meridional wind components at 300, 700, and 850 mb for Kwajalein.

analyses on independent pieces of record cannot be made to test the stability of spectral estimates obtained in one experiment.

The hypothesis to be tested here is that the differences between spectral values in local peaks and at the local minima are due to sampling variability. The theory was applied in the following manner. Figure 4-4 is a plot of the spectra of both velocity components at all three analyzed levels. The local minima have been connected by a dashed line. The value at each local minimum has been multiplied by a constant (taken from a χ^2 -table), and the products have been plotted. The (upper) dashed line that connects these points represents the 90-percentile level, i.e., the level exceeded by 10% of the values (subject only to sampling variability) in an infinite population of estimates of the true values. If the peak projects above this level, then we have 90% confidence that the peak is real, and not due to accidental sampling variability. This statistical confidence may be reinforced or weakened by physical considerations, by the number of individual spectral points present in the peak, and by the presence or absence of peaks in the spectra of other time series related (to the one considered) either by physical considerations or previous statistical analysis.

Figure 4-4 shows the smoothed-spectral estimates, $P(f)$, plotted on a logarithmic scale against frequency. In the figure, the 90% confidence limits have been entered as dashed lines. It follows that the peaks in the u- and v-power spectra discussed in Section 4.2 are significant at the 90% level—particularly the frequency band centered at 2.5 days in the 850-mb v-component and the band centered at 17 days in both components at the 300-mb level.

5.0 CONCLUSIONS AND RECOMMENDATIONS

This report deals with the feasibility of power-spectral analysis of upper winds at a single station in the Tropics. The aim is to detect time periods and horizontal dimensions of typical perturbations which in a sparse-data network might otherwise not be detectable.

The results for Kwajalein, based on 8 mo of 12-hourly upper-wind observations, lead to the following tentative conclusions.

(a) Low-level perturbations of periods of between 2.5 and 4 days embedded in the westward flow of the trades appear to be typical. The perturbations, if they move with the speed of the trades, have a wavelength of from 1000 to 2000 km.

(b) Independent analysis by Rosenthal [7] for neighboring stations corroborates this finding.

(c) Although these low-level perturbations appear to affect the 300-mb flow, the predominant period at 300 mb is about 17 days. The material at hand does not allow a clear-cut interpretation of this period. It may result partly from large-scale moving perturbations, and it may result partly from meridional oscillations of the zonal flow.

Based on this limited study and on the need for increased knowledge of synoptic conditions in the Tropics, we make the following recommendations.

(a) Apply power-spectral analysis to the winds at Canton, Panape, and Majuro, to expand the present results.

(b) Correlate pairs among the four stations for which material has been obtained, to establish the horizontal scale of the dominant perturbations directly.

(c) Subject other meteorological elements—moisture in particular—to power-spectral analysis.

(d) Correlate different levels, to obtain information about the perturbations' vertical structure.

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