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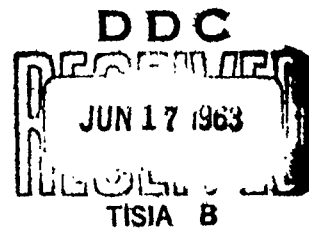
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Bureau of Supplies and Accounts
U.S. Navy

**DYNAMIC MODELING
OF INVENTORIES
SUBJECT TO OBSOLESCENCE**

June 1962



C-E-I-R INC.

LOS ANGELES CENTER:
9171 Wilshire Boulevard, Beverly Hills, California

DYNAMIC MODELING OF INVENTORIES
SUBJECT TO OBSOLESCENCE

A Report of Research Performed For
The Office of Advanced Logistics Research
Bureau of Supplies and Accounts
U.S. Navy

June 14, 1962

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C-E-I-R, Inc.
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J.Y.L. and R.J.W.



SUMMARY

This research report describes studies done by C-E-I-R, Inc., under Nonr 3333(00), which are concerned with the impact of obsolescence on inventories of Naval supplies. Obsolescence is examined with the objective in mind of defining it operationally. Several types of dynamic models are developed, making use of the operationally defined motion. Some of these models employ Bayesian procedures for assessment of demand rates. A statistical analysis of some Navy supply data, which was carried out in an attempt to cover empirical relationships between obsolescence and other factors, is described. Finally some changes in inventory control procedures are suggested.

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CHAPTER I: INTRODUCTION

In January 1961, C-E-I-R, Inc., was authorized by the Bureau of Supplies and Accounts to undertake a study of the economic impact of obsolescence on inventory costs and control. This document is a report of results.

The study has led to the development of a sequence of models of inventory costs, computing techniques associated with these models, and a proposal for installation of inventory control procedures based on the models described herein. The models deal with inventory operation costs under a variety of assumptions as to: lead time, inventory carrying charges, ordering costs, disposal costs, shortage costs, probability distribution of demand for an item, probability of incidence of obsolescence during any time period and linkage between demands for two or more items.

These models have one feature in common: all are dynamic models. That is, a steady state is not assumed; the possibility exists for variation of demand and obsolescence distributions, as well as all the costs of operation of the inventory. Several of the more sophisticated models incorporate Bayesian features. Taken together, these two features, the dynamic program and the Bayesian features, lead to an ability to adapt in a continuous fashion to changes both in the structure of the environment and in a priori statements whose validity is revealed only slowly as experience is generated.



Central to this study is an attempt to understand and model the nature and impact of obsolescence of items in inventory. The following chapter is concerned with a discussion of what is meant by obsolescence, as well as some concern with the magnitude of obsolescence of inventories of Naval Supplies.

CHAPTER II: OBSOLESCENCE: DEFINITION AND IMPORTANCE

It has been standard practice in the U.S. Navy supply system to deal with obsolescence as a blanket charge to be assessed equally across all Navy Supply items. The rate at which this charge has customarily been assessed is 10% of the total value of the inventory per year.

The total value of inventories of Naval Aircraft spare parts and supplies held by the Navy amounts to approximately \$2.5 billion. Clearly, the total value of all inventories held by the Navy is much larger than this amount. Consequently, the total obsolescence charges under the present system amount to several hundred million dollars per year.

It seems indisputable that better understanding of the specific impact of obsolescence, and the development of inventory control procedures which take the obsolescence process into account in a realistic fashion, should lead to considerable economies in the use of Navy Supply budgets.

Defining the term obsolescence is not quite as simple, in the context of a study of the sort under discussion, as it would be for the purposes of ordinary discourse. Something is obsolescent, according to Webster's New Collegiate Dictionary (1960 Edition), when it is going out of use. In ordinary discourse, it is assumed that there is no serious problem involved in ascertaining whether or not

something is going out of use. However, for our purposes, it turns out that this is a most difficult matter to settle until after the fact.

Establishing that something is obsolete, that is, establishing that something has already gone out of use; is a much simpler problem.

In this case, there are several circumstances which might have led to such a situation.

1. An item has gone out of use because the function served by that item is no longer required; i.e., demand has disappeared.
2. An item is going out of demand because whenever a unit of the item wears out, breaks, or is consumed, the item is replaced by a different item which performs similar or identical functions; i.e., the item is going out of use.
3. An item has gone out of demand because, regardless of consumption, wearout or breakage, as soon as replacements are available, all units of that item currently in use are replaced by units of other items which serve similar or identical functions, that is, the item has gone out of use.

So far as the supply system is concerned, being out of use, and not being in demand are the same condition.

Once something has become obsolete, the fact is clear. But there is not much which can be done at such a time to face the fact, beyond clearing out and disposing of any remaining inventories of such items.

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Once something has become obsolete, the fact is clear. But there is not much which can be done at such a time to face the fact, beyond clearing out and disposing of any remaining inventories of such items.

However, if it were possible to detect accurately the process of obsolescence in its earlier stages, while it has not run its course, then there might be some steps which might profitably be taken by an inventory manager. Consider, for example, the following very simple case:

Let:

Carrying Cost = h

Stockout Cost = π

Liquidation Cost = k

Demand = D

Probability distribution of demand = $p(D)$

$$\sum_{D=0}^r p(D) = P(r) = \text{Prob } (D \leq r)$$

A. It is not known that obsolescence will occur at the end of this period. An optimal policy is followed, then obsolescence occurs and stock is liquidated.

The expected inventory costs the manager can plan on are:

$$(2.1) \quad C(I_o) = h \sum_{D=0}^{I_o} (I_o - D) p(D) + \pi \sum_{D=I_o+1}^{\infty} (D - I_o) p(D)$$

If I_o is optimal

$$(2.2) \quad C(I_o + 1) - C(I_o) \geq 0$$

$$(2.3) \quad C(I_o - 1) - C(I_o) \geq 0$$

$$(2.4) \quad C(I_0+1) = h \sum_{D=0}^{I_0} (I_0-D) p(D) + h \sum_{D=0}^{I_0} p(D) + \pi \sum_{D=I_0+1}^{\infty} (D-I_0) p(D) \\ - \pi \sum_{D=I_0+1}^{\infty} p(D)$$

$$(2.5) \quad C(I_0+1) - C(I_0) = P(I_0) (h+\pi) - \pi \geq 0$$

Similarly,

$$(2.6) \quad - [C(I_0-1) - C(I_0)] = P(I_0-1) (h+\pi) - \pi \leq 0$$

$$(2.7) \quad P(I_0-1) \leq \frac{\pi}{h+\pi} \leq P(I_0)$$

B. It is known that obsolescence will occur at the end of the period. The manager plans accordingly. His expected costs are:

$$(2.8) \quad E(I_*) = h \sum_{D=0}^{I_*} (I_*-D) p(D) + \pi \sum_{D=I_*+1}^{\infty} (D-I_*) p(D) + k \sum_{D=0}^{I_*} (I_*-D) p(D)$$

$$(2.9) \quad E(I_*) = (h+k) \sum_{D=0}^{I_*} (I_*-D) p(D) + \pi \sum_{D=I_*+1}^{\infty} (D-I_*) p(D)$$

By the same reasoning as (2.1) to (2.7)

$$(2.10) \quad P(I_*-1) \leq \frac{\pi}{\pi + h + k} \leq P(I_*) \quad \text{so } I_* \leq I_0$$

The total expected costs incurred in situation A. must include liquidation of remaining stock:

$$(2.11) \quad E(I_0) = C(I_0) + k \sum_{D=0}^{I_0} (I_0-D) p(D) = (h+k) \sum_{D=0}^{I_0} (I_0-D) p(D) \\ + \pi \sum_{D=I_0+1}^{\infty} (D-I_0) p(D)$$

$$(2.12) \quad E(I_o) - E(I_*) = (h+k) \left[\sum_{D=0}^{I_o} (I_o - D) p(D) - \sum_{D=0}^{I_*} (I_* - D) p(D) \right] \\ + \pi \left[\sum_{D=I_o+1}^{\infty} (D - I_o) p(D) - \sum_{D=I_*+1}^{\infty} (D - I_*) p(D) \right]$$

It is easily shown (see Footnote 1) that (2.12) is positive.

¹Starting with (2.12): adding $(h+k) \left[\sum_{D=I_o+1}^{\infty} (I_o - D) p(D) - \sum_{D=I_*+1}^{\infty} (I_* - D) p(D) \right]$

$$- (h+k) \left[\sum_{D=I_o+1}^{\infty} (I_o - D) p(D) - \sum_{D=I_*+1}^{\infty} (I_* - D) p(D) \right]$$

we get

$$(2.13) \quad E(I_o) - E(I_*) = (h+k) \left[\sum_{D=0}^{\infty} (I_o - D) p(D) - \sum_{D=0}^{\infty} (I_* - D) p(D) \right] \\ + (h+k+\pi) \left[\sum_{D=I_o+1}^{\infty} (D - I_o) p(D) - \sum_{D=I_*+1}^{\infty} (D - I_*) p(D) \right]$$

$$(2.14) \quad E(I_o) - E(I_*) = (h+k) (I_o - I_*) + (h+k+\pi) \left[\sum_{D=I_o+1}^{\infty} (D - I_o) p(D) \right. \\ \left. - \sum_{D=I_*+1}^{\infty} (D - I_*) p(D) - \sum_{D=I_o+1}^{I_o} (D - I_*) p(D) \right]$$

$$(2.15) \quad E(I_o) - E(I_*) = (h+k) (I_o - I_*) + (h+k+\pi) \left[\sum_{D=I_o+1}^{\infty} (I_* - I_o) p(D) \right. \\ \left. - \sum_{D=I_*+1}^{I_o} (D - I_*) p(D) \right]$$

$$(2.16) \quad E(I_o) - E(I_*) = (h+k) (I_o - I_*) + (h+k+\pi) \left\{ (I_* - I_o) [1 - P(I_o)] \right. \\ \left. - \sum_{D=I_*+1}^{I_o} (D - I_*) p(D) \right\}$$

$$(2.17) \quad E(I_o) - E(I_*) = (h+k)(I_o - I_*) + (h+k+\pi)$$

$$\left\{ (I_* - I_o) - (I_* - I_o) \left[P(I_*) + \sum_{D=I_*+1}^{I_o} P(D) \right] - \sum_{D=I_*+1}^{I_o} (D - I_*) P(D) \right\}$$

$$(2.18) \quad E(I_o) - E(I_*) = -\pi(I_o - I_*) + (h+k+\pi)(I_o - I_*) P(I_*)$$

$$+ (h+k+\pi) \sum_{D=I_*+1}^{I_o} (I_o - I_* - D + I_*) P(D)$$

$$(2.19) \quad E(I_o) - E(I_*) = -\pi(I_o - I_*) + (h+k+\pi)(I_o - I_*) \left(\geq \frac{\pi}{h+k+\pi} \right)$$

$$+ (h+k+\pi) \sum_{D=I_*+1}^{I_o} (I_o - D) P(D)$$

$$(2.20) \quad E(I_o) - E(I_*) = \underbrace{-\pi(I_o - I_*)}_{>0} + \underbrace{(\geq \pi(I_o - I_*))}_{>0} + \underbrace{(h+k+\pi) \sum_{D=I_*+1}^{I_o} (I_o - D) P(D)}_{>0}$$

We have made use of the fact that

$$P(I_*) \geq \frac{\pi}{\pi+h+k} \quad \text{and} \quad I_* < I_o$$

$$\text{Th: If } I > J \quad \text{and} \quad P(J) > \frac{\pi}{\pi+h+k}$$

$$\text{then } E(I) > E(J)$$

We now consider a numerical example:

$$h = 60$$

$$\pi = 500$$

$$k = 200$$

Demand is Poisson distributed, with $\lambda = 6$, $p(D) = \frac{e^{-6}(6)^D}{D!}$

$$\frac{\pi}{\pi+h} = \frac{500}{560} = .8928 \quad I_0 = 9$$

$$\frac{\pi}{\pi+h+k} = \frac{500}{860} = .5814 \quad I_* = 6$$

from (2.18)

$$\begin{aligned} (2.18') \quad E(I_0) - E(I_*) &= -500(3) + (860)(3)(.6063) + (806) \sum_{D=7}^9 (9-D) p(D) \\ &= -1500 + (2580)(.6063) + 806 \quad 2 \times .1377 + 1 \times .1033 \\ &= 369.4862 \end{aligned}$$

Total costs under situation B are 732.5640

Thus, there is clear gain of the order of 50% from having had prior knowledge of the impact of obsolescence.

In cases where items go out of demand or out of use as a consequence of administrative decisions, it might be easier to anticipate the process of obsolescence than in other cases. This is so because administrators must have some reasons for making such decisions, and these reasons are probably open to examination. However, in cases where administrative decisions are not involved, it is necessary for inferences to be made about the future state of demand from the present state of demand.

Such inferences are of the form: given that demand in the present period is equal to y , the probability that demand M months from now will be less than, or equal to $\bar{y}$ is $P(\bar{y}; y, M)$. When P is sufficiently high, it may turn out that the cost of maintaining the

item in inventory for demand levels as low as, or lower than, $\bar{y}$ is more than the expected costs which would be incurred if the system were out of stock of the item.

In such a circumstance a declaration that the item is to be treated as if it were obsolescent might be in order. This would mean taking some action which would lead to one of the three conditions 1), 2), or 3), which are described above, (on page 4).

In the models which are developed and discussed in the following sections of this report there are, basically, two approaches taken to the notion of obsolescence.

The first, a simpler treatment, is concerned with the development of dynamic disposal-procurement policies under the assumption of a probability distribution of time of incidence of obsolescence which is known a priori, and which is not changed systematically within the model.

This model is very flexible so far as input data are concerned; any form of obsolescence probability distribution, demand distribution, or inventory costs may be used. Consequently, as ancillary information about any of these is developed, new assumptions can be introduced into the model.

Numerical analysis of this model has been carried out fairly extensively. The results of the numerical analysis tend to support conjectures as to the extensibility and greater generality of some theorems which have appeared in the literature recently. Moreover,

this model appears promising as the basis for a first step toward the rationalization of procurement-disposal stock control of Naval inventories.

The second approach is a much more original treatment. Indeed, there are no results in the literature known to us which incorporate features to be found in this second group of models. In this approach, which also involves dynamic models, there is assumed a priori the form of the probability distribution of the time of incidence of obsolescence, and the parameter values for this distribution. However, the parameter values are modified, within the model, by Bayesian procedures involving a functional relationship between the recent history of demand for the line item and the parameter values of the obsolescence distribution.

This basic approach is adapted in several ways in the models of this sort which are discussed herein. In one model the crucial effect of demand history appears when demand is at a level of zero. In another, there is incorporated a Markov process which deals with the transition between various levels of demand. In a third, there is added on a feature involving the dependency of demand for one item upon the demand for another related (complementary or substitute) item.

What is true of all these models is their dynamic character. They are true dynamic programs which optimize with respect to a cost function which includes all cost elements of an inventory system except relocation of existent inventory within the system, and repair of items. Thus, this family of models does not deal with questions of how,

within an inventory system, the inventory should be dispersed. Neither does it deal with repair vs. purchase, nor with level-of-complexity of items (the problem referred to by A.J. Clark as the "Inventory Range Problem.")

In subsequent chapters these models will be dealt with in detail. In addition, there will be some discussion of the results of statistical analysis of data on some 4000 aircraft spares which have been declared obsolete during the past year. Finally, there will be some discussion of explorations which were undertaken in connection with the development of the most sophisticated of the inventory models which use the properties of a class of probability distributions that reproduce under the operation of taking the conditional distribution. The use of elements of this general class of functions in these models to represent the probability distribution of time to obsolescence leads to great simplification of the computing problem. This is so because for these functions the parameters are computable by simple recursions which are time-independent.

CHAPTER III

SURVEY OF INVENTORY MODELS WITH EXPLICIT TREATMENT OF OBSOLESCENCE

Numerous investigations have been made of the problems of inventory control in the past decade. Although most of them have dealt with the determination of optimum policies with which to regulate inventories in the absence of possible obsolescence of the inventories, there are still a number of studies in which some aspect of the problem of obsolescence was explicitly treated. This chapter is essentially an annotated bibliography of the studies on inventory obsolescence. The bibliography is by no means exhaustive, but every study which, in our opinion, made significant contributions to this area of inventory control study is included.

Results which emerged from these inventory obsolescence studies are varied depending on: (1) the assumption regarding the occurrence of obsolescence; (2) the nature of the problem that the investigator has to deal with; and (3) the approach adopted for obtaining an optimal solution. In order to bring some order to these widely disparate results and facilitate further discussion, an attempt has been made to arrange the material to be discussed in a systematic way in which the organization is based on the problem-oriented viewpoint rather than on the technique-oriented viewpoint. The criteria for classification are:

1. Deterministic vs. probabilistic problem. If the parameter which specifies the occurrence of obsolescence, such as the

length of time-to-obsolescence, is known with certainty, such a problem is referred to as deterministic. If it is known in terms of a probability distribution, it is called the probabilistic problem. A problem will be also considered "deterministic" if obsolescence enters the problem only as a non-stochastic function of any inventory parameter.

2. The objectives of the models. Inventory problems can be classified according to the objective that the investigator sets to accomplish. The following three distinct objectives are noted in the studies to be reviewed here:
 - a. The main objective is to determine the optimum economic lot size for a single item taking cognizance of obsolescence.
 - b. The determination of the optimum inventory for a single item if no additional procurements are to be made. This optimum inventory is commonly referred to as final inventory.
 - c. To determine the (s,S) type ordering and disposal policies taking account of obsolescence cost.

In Table I, the inventory obsolescence studies are classified according to the criteria discussed above and they will be discussed at some length in the order indicated in each entry of the table.

Table I: Classification of Inventory Obsolescence Studies

		Assumption about Occurrence of Obsolescence	
		Deterministic	Probabilistic
OBJECTIVE	Determination of EOQ	<u>Case 1</u> Whitin [13]	<u>Case 2</u> Grassi and Gradwohl [6]
	Determination of final inventory	<u>Case 3</u> Hadley and Whitin [7]	<u>Case 4</u> Hadley [8] Mohan and Garg [11] Simpson [12]
	Determination of (s,S) type ordering and disposal rules	<u>Case 5</u> Barankin [2] Fukuda [5]	<u>Case 6</u> Allen and Broida [1] Barankin and Denny [2] Ford [4] Brown [3]

Case 1: Deterministic EOQ Model

The solution of the economic order quantity problem, according to Whitin [13], has a rather long history. It is, perhaps, still one of the most frequently used inventory formulae. In the formula for determining economic order quantities, obsolescence cost is treated as a constant percentage of inventory carrying cost which increases as inventories increase. This carrying cost is to be balanced against those cost items such as quantity discounts, freight differentials which decrease as inventories increase. The derivation of the formula is as follows:

Let

Y = expected demand per period

Q = economic order quantity

C = unit inventory carrying cost per period. (This is made up of material, interest, depreciation and obsolescence costs)

S = procurement expense per order

Then total variable cost (TVC) per period is

$$(3.1) \quad TVC = \frac{QC}{2} + \frac{Y}{Q} S$$

Upon differentiating (1) with respect to Q and setting the resulting derivative equal zero, we obtain the minimizing value of Q by solving the derivative for Q.

$$(3.2) \quad Q = \sqrt{\frac{2YS}{C}}$$

As can be seen in the above formula, if the risk of obsolescence should increase, this will be reflected in a higher inventory carrying cost and, consequently, a smaller economic order quantity.

In addition to the EOQ model described above, many traditional inventory models treat obsolescence cost as a part of carrying cost. However, when obsolescence becomes a major cost in an inventory model, a more explicit determination of obsolescence costs becomes desirable.

Case 2. Probabilistic EOQ Model

In this model obsolescence cost is formed as a function of the obsolescence rate by assuming that the probability of obsolescence at a future time can be expressed by a certain distribution function. Grassi and Gradwohl [6] have obtained a probabilistic EOQ formula by assuming the life span of an item to follow an exponential density function, $f(t) = \mu e^{-\mu t}$, where μ is the obsolescence or death rate.

Their model may be stated as follows:

Let

P = unit order cost

D = the sum of unit material, labor and overhead costs

S = setup cost per order

R = expected demand per period

B = the safety or buffer stock

I = unit inventory holding cost per period

β = the inventory charge rate per period

E = expected unit obsolescence cost

Then the sum of all the costs associated with ordering one unit of inventories is:

$$C = P + I + E$$

Each component of the total unit cost C is given by:

$$(3.3) \quad P = D + \frac{S}{Q}$$

P may be regarded as the value of one unit of inventories.

$$(3.4) \quad I = \left(B + \frac{Q}{2} \right) \left(D + \frac{S}{Q} \right) \frac{\beta}{R}$$

$B + \frac{Q}{2}$ is the average inventory level under the assumption that a quantity Q will be ordered when the inventory level reaches the safety level B ; the product $\left(B + \frac{Q}{2} \right) \left(D + \frac{S}{Q} \right)$ is the value of the average inventory stock. The inventory charge is made on this average inventory stock and is prorated over the total expected requirement for one period.

The expected unit obsolescence cost may be calculated based on the following consideration: Suppose a lot size of Q is ordered for an item when the item is at age t years (unit for measuring the passage of time is arbitrary). The lot will be used up when the item is of age $T + \frac{Q}{R}$ years. It is then of interest to know the probability of obsolescence during the time interval $\frac{Q}{R}$, given non-obsolescence prior to t so that the expected loss due to obsolescence during the time inter-

val of length $\frac{Q}{R}$ can be calculated.

Because of the assumption that the life span of the item follows an exponential distribution, the probability that obsolescence occurs prior to t (or, alternatively, the probability that the item's life span is, at most, as long as t) can be stated as follows:

$$\Pr(T \leq t) = F(t) = 1 - e^{-\mu t}$$

where T represents the age at obsolescence of that particular item in question.

The conditional probability $F(T|t_0)$ of obsolescence in an additional time T after experiencing non-obsolescence to t_0 is given by

$$G(T) = \frac{F(t_0 + T) - F(t_0)}{1 - F(t_0)} = 1 - e^{-\mu T}$$

Hence the conditional density function is

$$\frac{dG(T)}{dT} = g(T) = \mu e^{-\mu T}$$

The expected number of unit lost due to obsolescence during the time interval t to $t + \frac{Q}{R}$ is:

$$= \int_0^{\frac{Q}{R}} (B + Q - RT) g(T) dT$$

$$(3.5) \quad = \left(B - \frac{R}{\mu}\right) (1 - e^{-\mu(Q/R)}) + Q$$

The expected unit loss due to obsolescence becomes

$$(3.6) \quad E = \left(D + \frac{S}{Q}\right) \frac{(B-R/\mu) (1-e^{-\mu(Q/R)})}{Q} + 1$$

The optimum economic order quantity is obtained by finding the minimizing value of Q in the following expression

$$(3.7) \quad C = P + I + E$$

$$= \left(D + \frac{S}{Q}\right) \left[2 + \left(B + \frac{Q}{2}\right) \frac{\beta}{R} + \frac{(B-R/\mu) (1-e^{-\mu(Q/R)})}{Q} \right]$$

The resulting solution given by Grassi and Gradwohl is

$$(3.8) \quad Q = \sqrt{\frac{2RS \left[1 + \frac{B}{R} (\mu + \beta) \right]}{D \left[(\mu + \beta) - \frac{B\mu^2}{R} \right]}}$$

It is interesting to note that if the conditional density $g(\tau)$ follows a rectangular distribution, i.e., obsolescence is equally likely to occur at any moment after the item has survived up to t , the new EOQ formula is exactly the same as (3.8) except the second term in the denominator $\frac{B\mu^2}{R}$ vanishes.

In using the formula (3.8) the level of buffer stock β is assumed to be determined outside of the model. If the joint determination of Q and B is desired, it may be achieved by appending penalty cost due to stock shortage to the cost function (3.7) and minimizing the function with respect to B and Q .

Case 3. Final Inventory Model with a Known Obsolescence Date

The problem is to determine the optimal amount of inventory on hand if no additional procurements are allowed, and, furthermore, a date of obsolescence is known with certainty. This inventory is commonly known as the optimum final inventory. The main feature of such a problem is that it involves two types of cost; one is incurred at a fixed point in time (obsolescence cost) and another is a function of time (holding and stockout costs).

Hadley and Whitin [7] formulated a model which deals with problems such as above and obtained a solution assuming the demand for the item to be Poisson with the mean demand rate independent of time

Let k = the liquidation loss per unit at the time of obsolescence

h = the carrying cost per unit per unit time

π = the stockout cost per unit per unit time

$p(X; \lambda t)$ = the Poisson probability distribution that the demand is exactly X units in a time period of length t with λ being the average demand rate

T = time of obsolescence

H = the optimal final inventory

The expected cost of holding H units from time 0 to T is the sum of the expected costs of disposing the unused amount at T ; carrying inventory, and stockouts for the time interval 0 to T . These cost components may be stated

The expected disposal cost:

$$(3.9) \quad K \sum_{X=0}^{H-1} (H - X) p(X ; \lambda T)$$

The expected carrying cost:

Since the carrying cost "rate" at time t is

$$h \sum_{X=0}^{H-1} (H - X) p(X : \lambda t)$$

the total carrying cost for the interval 0 to T is

$$(3.10) \quad h \int_0^T \sum_{X=0}^{H-1} (H - X) p(X : \lambda t) dt$$

the expected stockout cost:

Similarly, the total stockout cost for the interval 0 to T is

$$(3.11) \quad \pi \int_0^T \sum_{X=H}^{\infty} (X - H) p(X : \lambda T) dt$$

The expected cost of holding H which is the sum of (3.9), (3.10) and (3.11) can be minimized by noting the identity

$$\int_0^T p(X ; \lambda t) dt = \frac{1}{\lambda} \sum_{u=X+1}^{\infty} p(u, \lambda T)$$

This model is neat and simple to apply. However, its usefulness may be limited for the following reasons:

1. In general, it is not advisable to assume that an obsolescence date can be specified with certainty.

2. In the lifetime of an item, the practice of repeated procurements may be more common than the situation represented by the model.

Case 4: Final Inventory Models with Probabilistic Obsolescence Date

One of the objections to the model in Case 3, is in regard to the assumption of known obsolescence date. This assumption was relaxed in a more recent model developed by Hadley, [8].

He assumed that the date of obsolescence can be described by a continuous density function $r(T)$. Then the probability that the item will become obsolete in the interval T to $T + dT$ is $r(T)dT$. With this new assumption regarding T , the expected cost of holding H may be modified by taking the expected value of (3.9), (3.10), and (3.11) with respect to T .

$$\begin{aligned}
 (3.12) \quad E(H) = & k \sum_{X=0}^{H-1} (H - X) \int_0^{\infty} r(T) p(X; \lambda T) dT \\
 & + h \int_{T=0}^{\infty} \int_{t=0}^T \sum_{X=0}^{H-1} (H - X) p(X; \lambda t) r(T) dT dt \\
 & + \pi \int_{T=0}^{\infty} \int_{t=0}^T \sum_{X=H}^{\infty} (X - H) p(X; \lambda t) r(T) dT dt
 \end{aligned}$$

Using the technique described in the earlier paper by Hadley and Whitin [7], the cost function (3.12) can be minimized. Hadley [8] has also developed another model in which only a finite number of possible dates of obsolescence is assumed and the probability of each is specified.

Another type of final inventory model was developed by Simpson [12] and Mohan and Garg [11]. Their model is distinguished from Hadley's

model in that only a knowledge of the average annual demand is assumed, and it will first calculate the optimal economic retention period from consideration of the balancing of carrying charges and the cost of disposal. More specifically, the cost of retaining a unit value of the stock is weighed against the cost of disposing one unit value of the stock now and procure it again at some later stage. It is clear that units will be added to the retention stock as long as the latter cost exceeds the former. On the other hand, the retention stock will be reduced if the former costs exceed the latter. The equilibrium is reached when the cost of storing the marginal unit is exactly equal to the cost incurred for not storing that unit.

Their model may be stated as follows:

Let U = average annual demand

x = the number of years for which a stock, say N , will meet average demand. Note the relation $xU=N$

D = fraction of unit value of material which will be realized in disposal sales

i = interest rate

r = annual storage cost of material, expressed as a fraction of the unit value of material

C_R = total cost of retaining the unit value of stock

C_D = total cost of disposing the unit value of stock

F_t = the probability that the item becomes obsolete prior to the $(t+1)$ th year

The retention cost C_R consists of two types of costs, (1) the storage cost for x years, (2) the obsolescence cost.

The first type of cost may be stated as

$$r \sum_{t=1}^x (1 - F_t) (1 + i)^{x-t+1}$$

$(1 - F_t)$ is the probability of non-obsolescence before the $(t+1)$ th year, and $(1 + i)^{x-t+1}$ is the compound interest charge for storing the unit value of stock for $x-t+1$ years.

The obsolescence cost is the product of the unit value of stock and the probability of obsolescence F_x .

Hence

$$(3.13) \quad C_R = F_x + r \sum_{t=1}^x (1 - F_t) (1 + i)^{x-t+1}$$

For each unit value of stock disposed now, the cost equivalent to the unit value will be incurred in order to procure it again after x years assuming there is no price change in the future. However, for each unit value of stock disposed, a salvage value of D is acquired. This amount should increase at compound interests for x years and should be considered as a credit. Therefore, the disposal cost C_D is

$$(3.14) \quad C_D = 1 - D(1 + i)^x$$

The optimum retention period x is found by solving the equation $C_R - C_D = 0$.

The only difference between the result obtained by Simpson and that obtained by Mohan, et al is that the former assumes that the probability of obsolescence follows a uniform distribution while the latter uses a normal distribution.

Simpson gives a detailed account of the advantages and the shortcomings of this retention and disposal formula from the practical point of view. His conclusion is that despite the fact that many rigid assumptions are required to derive this formula, it provides a workable, practical decision rule.

Case 5: Optimal Ordering and Disposal Policies with Known Obsolescence

Date

When the items in inventory are known to become obsolete in the future, it is advisable to consider the disposal of surplus items as well as the procurement of stock to meet the future demand.

Consider the following situation in which an item will no longer be used after a certain date and the stock on hand on that date will have to be disposed. Meanwhile, prior to the occurrence of obsolescence, the stock level of this item is reviewed periodically, say, at the beginning of a finite number of equal time intervals; and one of the following decisions is made

1. Procurement of additional stock
2. Disposal of excess stock
3. No procurement and no disposal

For such a multi-stage decision process, the technique of dynamic programming has been known to furnish a suitable framework for analysis. In the following the recursion relation of a dynamic programming model appropriate to the situation described above is stated.

Let us introduce the following definitions:

s = demand per period

x = "initial" inventory level; an inventory level at the beginning of a period before the order or disposal is made

y = "starting" inventory level; an inventory level at the beginning of a period immediately after the order or disposal is made

$r(z)$ = the cost of ordering z units

$d(z)$ = the cost of disposing z units. If the disposal item has a salvage value, this cost assumes a negative value.

$h(z)$ = the cost of holding z units of inventory for one period

$p(z)$ = the cost per period of having demand z units greater than inventory

ρ = a discount factor

It is assumed that there is no delay either in the order or in disposing of stock. Suppose there are n periods before obsolescence occurs. The policy to be considered is of the following simple forms: for each period a starting inventory y is specified for each value of initial inventory x . Associated with each y is a certain expected cost. The problem is to minimize this expected cost by suitable choice of y .

Let us define

$C_k(x)$ = expected total cost for a process which has k periods remaining and starting the process with x units of initial inventory with an optimal ordering and disposal policy.

Since this total expected cost is composed of: (1) a cost of using an optimal policy in this period, (2) a discounted future cost, we have the following recursion relation

$$(3.15) \quad C_k(x) = \min_{y \geq 0} \{ r(y-x) + d(x-y) + E[h(y-s) + p(s-y)] + \rho E C_{k-1}(y-s) \} \\ \text{for } k = 2, 3, \dots, n.$$

where the ordering cost and disposal cost functions are zero for negative arguments. E denotes expectation and the expectation operation is with respect to demand s .

When there is only one period remaining in the process, the total expected cost is simplified to the following:

$$(3.16) \quad C_1(x) = \min_{y \geq 0} \{ r(y-x) + d(x-y) + E[h(y-s) + p(s-y)] \}$$

Together, (3.15) and (3.16) enable us to determine, successively for $k=1, 2, \dots, n$, the optimal value of y for each x and the corresponding minimum expected cost $C_k(x)$.

This is the type of model formulated by Barankin and Denny [2] and Fukuda [5]. Although the cost functions as well as the demand per period are considered to be independent of time in the above model, the same

technique still applies even if they should become time dependent. The assumption of no time lag in delivery can be also relaxed and the same technique can be used as long as a time lag is known with certainty.¹

Case 6: Optimal Ordering and Disposal Policies with Probabilistic
Obsolescence Date

Barankin and Denny [2] and Ford [4] have extended the model described in Case 5 to the situation in which the information about obsolescence is available in the form of a probability statement, e.g., the probability of obsolescence in a given period, say the k th period, is d_k . Presumably, $\sum d_k = 1$. Then the conditional probability a_k that obsolescence occurs in period k , given non-obsolescence in periods $n, n-1, \dots, k+1$, may be calculated as follows:

$$a_k = \frac{d_k}{\sum_{i=1}^k d_i}$$

With a knowledge of these conditional probabilities for $k=1,2,\dots,n$ only a simple modification of the recursion relation (3.15) is needed to give us a desired inventory model.

$$C_k(x) = \min_{y \geq 0} \left\{ c(y-x) + E P(s-y) + E h(y-s) + a_k d(x-y) + (1-a_k) \rho E C_{k+1}(y-s) \right\}$$

for $k = 2, 3, \dots, n$

¹ See H. Scarf, "(s,S) Policy in Dynamic Inventory Problem," Mathematical Methods in the Social Science, Stanford University Press, Stanford, California.

Since the conditional probability of obsolescence in the last period a_1 is equal to unity, the total expected cost function for $k=1$, is the same as (3.16). Again, starting with $k=1$, we can successively calculate a set of optimal policies and corresponding expected total costs for each period.

A more detailed discussion regarding the modeling of this type of inventory problem as well as some characteristics of optimum policies is presented in Chapter IV of this report.

In all the dynamic inventory models discussed so far, recursion relation such as (3.15) is always characterized completely by two state variables, x , the stock on hand, and n , the number of remaining periods. Brown [3] considered a more realistic situation in which the latter state describing variable is given in terms of a probability distribution. This introduction of a stochastic variable for describing the nature of process does not unduly complicate the calculations. Results of this type of dynamic inventory model can be found in Chapter VI of this report.

In the study made by Stanford Research Institute for BuSandA, Allen and Broida [1] considered the problem of minimizing the unit-weeks of system-wide shortages subject to the total variable procurement budget (converted into the shortage equivalent) by suitable choice of non-negative order quantities. Since the stock level of each item in the system is reviewed every quarter and an order is placed if necessary to raise the stock level to a certain level which is most desirable from the standpoint of minimizing the shortage risk, Allen and Broida set out

to determine what should be considered the most desirable level with a fixed amount of resources available to the system.

The suggested procurement rule is as follows:

Let R = the most desirable inventory level (determined by the model)

I = the sum of stock on order and on hand

- (1) If $R - I \leq 0$, no order is placed
- (2) If $R - I > 0$, order the difference $(R-I)$ or the economic order quantity--whichever is greater

This procedure may be considered a variant of the usual (s,S) policy. The most interesting feature of their study is that a budget constraint was explicitly taken into account in deriving an optimal inventory regulating procedure.

Since the derivation of their procurement rule is rather lengthy, it will not be reproduced here except to indicate how obsolescence costs are incorporated into their model.

They assumed that the probability of obsolescence at a future time to be expressed by an exponential distribution

$$1 - e^{-\gamma t}$$

where γ is the obsolescence factor for the item in question, the probability of non-obsolescence is then $e^{-\gamma t}$. The probability of non-obsolescence at time t is multiplied by the expected number of units short at that time. This yields the expected number of units short at time t when no obsolescence occurs. From this last quantity, the

expected number of unit-weeks of shortage is obtained by integrating out t . Similarly, in estimating the budget expenditure needed for placing an order of a certain size, it is weighed by the probability of non-obsolescence.

When obsolescence is treated in the manner described above, it has a net effect of reducing the amount of budget expenditure available for procurement purposes; at the same time it lowers the risk of shortage because when the item has already become obsolete there is no question of shortage. It is, therefore, difficult to generalize the over-all effect of the above modeling of obsolescence on the resulting procurement policy without examining in more detail a specific parameter value used to describe the occurrence of obsolescence for a problem in question.

CHAPTER IV: A DYNAMIC PROCUREMENT-DISPOSAL MODEL

A. Introduction

In this chapter is developed a dynamic procurement-disposal inventory model which assumes that the number of time periods until obsolescence and the demand for each line item are random variables with known probability distributions. While the model assumes that the demand distribution is known for an item for each time period of its lifetime, it allows this distribution to vary from period to period. Thus, diminishing demand toward the end of the item's lifetime may be reflected. In addition, although the probability distribution of time to obsolescence is assumed known, it too can vary from period to period. Also assumed known are: holding cost per unit per time period; stockout cost per unit per time period; unit price; fixed reorder cost; and disposal cost per unit.

Each of the known costs referred to above may be varied from period to period. Thus, any considerations which suggest that costs will change in some fashion in the future can be taken into account in computing policy.

A further restriction is imposed solely for reasons of computing simplicity. This is that lead time is equal to one inventory period. The program is capable of modification to allow lead time to be a multiple of the inventory period, but for purposes of numerical analysis it was felt that at this time the great increase in computing would not be justified

B. Some General Remarks

It should be noted that the dynamic program operates backwards in time. It begins by determining an optimal policy for some "last"¹ time period, proceeds from that to an optimal policy for the next to last period, and so on. Thus, for an obsolescence date of twenty periods into the future, the twentieth from the "last" period is the initial period and the best policy for that period is the best initial policy. A "best policy" for a period is defined by the program as follows: it is a policy which specifies for each feasible stock level left on hand at the end of the previous period what the level ought to be brought to at the beginning of this period in order that the sum of all costs from this period on shall be minimal.

The program is based on the fact that at the start of any time period the optimal stock level depends only on the inventory left from the previous period and not on how that inventory level came about; that is, the past influences the present through a single number, the inventory level at the end of the immediately preceding period. For that reason, the program is able to specify the future cost associated with starting a time period with each possible inventory level, without reference to what inventory will, in fact, be left over from the preceding period or what policy was used in the earlier periods. It only

¹This "last" period is determined by examining the probability distribution of obsolescence occurring at time $t(t=0, \dots, T, \dots, \infty)$. We find a T such that the probability of obsolescence occurring after T is less than some arbitrary ϵ . T is that "last" period.

needs to assume that an optimal policy is used in all later periods. This fact, plus a knowledge of what policy is optimal during the first period after obsolescence (specifically, to dispose of all stock on hand) provides the recursive procedure of the dynamic program.

The program produces, then, a policy for each time period. Each policy is used to calculate a preceding policy, and one entry of each (the best starting stock when the inventory left from the previous period is zero) is also used to provide an optimal initial order for each possible number of periods to obsolescence.

Furthermore, since the full set of order policies include optimal stock levels when there is already some stock left over from the past, the model is fully capable of suggesting reorder decisions as well as initial order amounts.

C. A Specific Formulation of the Dynamic Program

1. Definitions

a. Cost of changing stock level

Let $\gamma_i[a, b]$ be a known function denoting the cost in period i of converting a stock level of "a" units into a stock level of "b" units. ($a, b \geq 0$).

It is postulated that γ_i has the following properties:

$$(4.1) \quad \gamma[a, b] + \gamma[b, c] \geq \gamma[a, c]$$

$$(4.2) \quad \gamma[a+k, b+k] = \gamma[a, b] \quad \text{for all } k \text{ for which } \begin{matrix} a+k \\ b+k \end{matrix} \geq 0$$

$$(4.3) \quad \gamma[a, b] \geq 0, \quad \gamma[a, a] = 0$$

The purchase price of an item is not included in reorder cost $(\gamma[0,+])$; it is contained instead in disposal cost $(\gamma[0,-])$.

Making use of (4.2) we may define $\gamma[a,b]$ for a,b including negative values, and may then define $\theta_1(a) = \gamma[0,a] = \gamma[b, b+a]$ for all $a = 0, \pm 1, \pm 2, \dots$. $\theta_1(a)$ shall be used as the input quantity.

b. Shortage cost

Let

$\text{Cost}_{\text{out}}^i(r-k)$ = Cost in i th period associated with an incoming order r which exceeds the beginning inventory k , $r-k = 1, 2, 3, \dots$

This cost will be specified as an analytic function rather than a table of values in the computer program.

c. Storage cost

$\text{Cost}_{\text{sto}}^i(r,k)$ = Storage costs in i th period associated with an initial stock of k and an incoming order of r . If this cost is to be charged at the end of a period, it can be regarded as a function of $k-r$, the excess of inventory over demand. Since this cost is defined for each time period, any discounting of the future or interest charges on investment will be included in it.

d. Issue cost

Let

$\text{Cost}_{\text{iss}}^i(r)$ = cost of issuing r units in the i th period.

$$r = 1, 2, 3, \dots$$

Note that this cost must include the unit purchase cost in order for the program to operate properly.

e. Probability Distribution of Demand

Let $\text{Pr}(X_i = r)$ = Probability that the demand in period i is exactly r units.

$$r = 0, 1, 2, \dots$$

f. Probability Distribution of Obsolescence

Let a_i = Conditional probability that obsolescence occurs in period i , given that it has not occurred in periods $N, N-1, \dots, i+1$.

More specifically

$$a_i = \frac{d_i}{\sum_{j=1}^n d_j}$$

where d_i is the probability that obsolescence occurs in period i , and $\sum_{j=1}^n d_j = 1$.

g. Expected future costs

Let

$\mathcal{O}_i(K)$ = Expected cost from i on, if the initial stock

level in the i th period is K : $K = 0, 1, 2, \dots$, and an optimal policy is used in period i as well as in each of the future periods.

2. Induction

Since we have an optimal policy for time period zero (i.e., if there are K units in inventory, dispose of K units) and a set of expected future costs for period zero,

$$\phi_0(K) = \theta_0(-K) \quad \text{for } K = 0, 1, 2, \dots$$

the entire dynamic program can be specified by describing a procedure for calculating policy and future cost in period i from policy and future cost in period $i-1$. Note that period i is the time period which is i periods earlier than the date of obsolescence.

Assume now that $\phi_{i-1}(K)$ has been calculated.

(4.4) Then:

$$\phi_i(K) = \sum_{r=0}^{\infty} \Pr(x_1=r) \cdot \psi_i(K,r)$$

And $\psi_i(K,r)$ is given by:

$$\begin{aligned} (4.5) \quad \psi_i(K,r) &= \text{Cost}_{\text{out}}^i(r-K) + \text{Cost}_{\text{sto}}^i(K,r) + \text{Cost}_{\text{iss}}^i(r) \\ &+ (1-a_1) M(r-K) + a_1 \theta_i(r-K) \quad \text{if } r > K \\ &= \text{Cost}_{\text{sto}}^i(K,r) + \text{Cost}_{\text{iss}}^i(r) + (1-a_1)M(r-K) + a_1 \theta_i(r-K) \\ &\quad \text{if } r \leq K \end{aligned}$$

$M(r-K)$ is calculated as follows

For given h , $h = 0, \pm 1, \pm 2, \dots$

(4.6) Define

$$M(h) = \min_{k \geq 0} [\theta_{i-1}(k) + \theta_i(h+k)]$$

and define $S_i(h)$ = the value of k at which $M(h)$ is achieved

$S_i(h)$ is interpreted as the i th period policy, i.e., if the initial stock for period i is K and $X_i = r$, then $S_i(r-K)$ is the amount with which to start period $(i-1)$.

Note that, in (4.5), the expression for ψ_i involves all the costs actually incurred in period i , plus the expected cost associated with the ending inventory, $M(r,K)$, if the process is to continue; and the disposal cost, $\theta(r,K)$, if the process is to terminate. The last two costs are weighted according to the probabilities of the events which will generate these costs.

Some remarks regarding the calculation of $M(h)$ are in order. Assume we start with $K \geq 0$, there is an order of $r \geq 0$ and we end the period with $h \geq 0$ in stock. Aside from costs associated with K and r above which already occur explicitly in ψ_i , we also have

CASE I $r > K$

Additional cost = $\phi_{i-1}(k) + \gamma_i [K-r, k] = \phi_{i-1}(k) + \theta_i (k+r-K)$ and to minimize this is to reorder so as to bring stock level to $S_i(r-K)$ units, and additional cost has become $M(r-K)$.

CASE II $r < K$

Additional cost = $\phi_{i-1}(k) + \gamma_i [K-r, k]$ and same minimization as in Case I.

Note that the argument of $M(h)$ is non-positive.

D. Some Remarks on Optimal Policies of Dynamic Procurement-Disposal Model

A number of computer runs were made in order to study the effect of parameter variations on the optimal procurement and disposal policies. Detailed results of these runs are presented in Appendix A.

Within the range of parameter variations explored, it was found that when a set-up cost is involved in ordering and disposing of the stock, the optimal ordering and disposal policies are of an (s, S) type. That is to say there are four uniquely determined integers $K_0 < K_1 < K_2 < K_3$ such that if the initial inventory level, say H , is less than K_0 , an order is placed to bring the inventory level up to K_1 ; if the inventory condition is $K_0 \leq H \leq K_3$, no action is taken; if the inventory level exceeds K_3 , it is disposed down to K_2 . It was observed that these policy parameters (K 's) are related to the expected total inventory cost function $\phi(K)$ (See pp.) in the following manner:

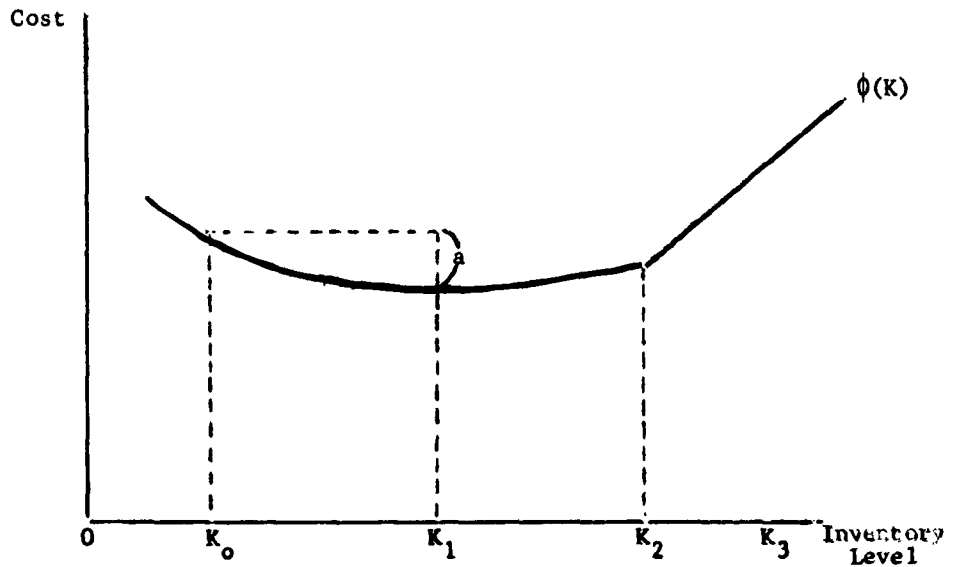


Fig. 1: A Typical Expected Total Inventory Cost Function

a = fixed ordering cost

b = salvage value obtained from disposing one unit of the stock

c = unit storage cost

d = fixed disposal cost

1) K_1 is an integer value of K which minimizes $\phi(K)$

2) K_0 is the largest integer such that

$$\phi(K_0) > \phi(K_1) + a \quad \text{and} \quad K_0 < K_1$$

3) K_2 is an integer such that

$$\phi'_-(K_1) < b < \phi'_+(K_1) \quad \text{and} \quad K_2 > K_1$$

$\phi'_-(K_1)$ and $\phi'_+(K_1)$ denote the left-hand and right-hand derivatives at K_1 respectively.

- 4) K_3 is an integer such that

$$C(K_3 - K_2) > d \quad \text{and} \quad K_3 > K_2$$

These findings seem to make sense for the following reasons:

- 1) Since the objective of the model is to minimize the total expected inventory cost, it is natural to bring the inventory level up to K_1 .
- 2) If a set-up cost is involved in ordering, the cost function in effect between K_0 and K_1 is the dotted line above the $\emptyset(K)$ function in Fig. 1. It is then most economical to order from K_0 .
- 3) Suppose there is no fixed disposal cost. As the initial inventory level increases, the total expected inventory cost will also rise because of an increase in shortage cost. At some point, namely K_2 , it becomes cheaper to dispose an additional unit of the stock than to retain it since in this model a salvage value is attached to a unit disposed.
- 4) If a set-up cost is incurred for disposing one unit of the stock, then the cumulative storage costs must exceed the fixed disposal costs before a disposal becomes economically attractive.

Although an (s, S) type policy seems to characterize all the optimal policies obtained in this study (see Appendix A), even when a

constant fixed cost is involved in ordering and disposing of the stock, it should be noted that such a policy cannot be optimal in all cases. However, it is probably safe to conclude that in most practical situations an (s,S) policy will produce a near optimal solution.

If, in fact, the item then survives until t_1 , the probability that it will survive until t_2 , $t_1 < t_2 < T$ would be

$$F_2(t_2) = F_1(t_2 | t_1) = \frac{\frac{1}{T} \int_{t_1}^{t_2} dx}{1 - \frac{1}{T} \left[\int_0^{t_0} dx + \int_0^{t_2} dx \right]} = \frac{t_2 - t_1}{T - t_0 - t_2}$$

$$F_n(t_n) = F_{n-1}(t_n | t_{n-1}) = \frac{\frac{1}{T} \int_{t_{n-1}}^{t_n} dx}{1 - \frac{1}{T} \left[\int_0^{t_0} dx + \int_0^{t_{n-1}} dx \right]} = \frac{t_n - t_{n-1}}{T - t_0 - t_{n-1}}$$

Clearly, this recursion is time-dependent. Similarly, if we assume that $\log t$ is Cauchy-distributed, we find that

$$F_n(t_n | t_{n-1}) = \frac{1}{\pi} \left[\frac{\tan^{-1}(\log t_n - \mu) - \tan^{-1}(\log t_{n-1} - \mu)}{1 + 2\pi(1 + \frac{m}{2}) - \tan^{-1}(\log t_n - \mu) - \tan^{-1}(\log t_{n-1} - \mu)} \right]$$

equally, a time-dependent recursion.

We now consider some examples of distributions which are reproducible under the operation of taking the conditional. The simplest example of this property is, of course, furnished by the exponential distribution, with probability of survival to age t equal to e^{-kt} ; after survival to age t_0 to the probability of surviving for a further interval of t is again e^{-kt} . We present three related

families of mortality distributions (including the exponential as a special case) which exhibit reproducible conditional distributions.

If the cumulative distribution function $F(t)$ is represented in terms of the age-specific death rate $\lambda(t)$, we have

$$F(t) = 1 - \exp \left[- \int_0^t \lambda(u) du \right]$$

The conditional probability $F(\tau|t_0)$ of surviving an additional time τ , after surviving to t_0 , is given by

$$\frac{F(t_0 + \tau) - F(t_0)}{1 - F(t_0)} = 1 - \exp \left[- \int_{t_0}^{t_0 + \tau} \lambda(u) du \right]$$

The particular families described below result from assuming that $F(\tau|t_0)$ is related to $F(t)$ by a transformation of the form $t \rightarrow k\tau$ and $\lambda \rightarrow t\lambda$. If λ is differentiable no other families can be reproducible under the same transformations (demonstration omitted)

Case I: $\lambda(t) = \alpha e^{\beta t}$, $k = 1$, $t = e^{\beta t_0}$

$$\begin{aligned} 1 - \exp \left[- \int_{t_0}^{t_0 + \tau} \lambda(u) du \right] &= 1 - \exp \left[- \alpha \int_{t_0}^{t_0 + \tau} e^{\beta u} du \right] \\ &= 1 - \exp \left[- \frac{\alpha}{\beta} \left\{ e^{\beta(t_0 + \tau)} - e^{\beta t_0} \right\} \right] \end{aligned}$$

$$\begin{aligned}
 &= 1 - \exp \left[-\frac{\alpha e^{\beta t_0}}{\beta} \left\{ e^{\beta \tau} - 1 \right\} \right] \\
 (5.1) \quad &= 1 - \exp \left[-\alpha e^{\beta t_0} \int_0^{\tau} e^{\beta u} du \right]
 \end{aligned}$$

Case II: $\lambda(t) = \alpha(1+\beta t)^{\gamma}$; $k = \frac{1}{1+\beta t_0}$, $h = (1+\beta t_0)^{\gamma+1}$

$$1 - \exp \left[-\int_{t_0}^{t_0+\tau} \lambda(u) du \right] = 1 - \exp \left[-\alpha \int_{t_0}^{t_0+\tau} (1+\beta u)^{\gamma} du \right]$$

IIa: $\gamma \neq -1$

$$\begin{aligned}
 &1 - \exp \left[-\alpha \int_{t_0}^{t_0+\tau} (1+\beta u)^{\gamma} du \right] \\
 &= 1 - \exp \left[\frac{-\alpha}{\beta(\gamma+1)} \left\{ (1+\beta(t_0+\tau))^{\gamma+1} - (1+\beta t_0)^{\gamma+1} \right\} \right] \\
 &= 1 - \exp \left[-\frac{\alpha}{\beta(\gamma+1)} (1+\beta t_0)^{\gamma+1} \left\{ \left(1 + \frac{\beta \tau}{1+\beta t_0}\right)^{\gamma+1} - 1 \right\} \right] \\
 (5.2) \quad &= 1 - \exp \left[-\alpha(1+\beta t_0)^{\gamma+1} \int_0^{\frac{\tau}{1+\beta t_0}} (1+\beta u)^{\gamma} du \right]
 \end{aligned}$$

IIb: $\gamma = -1$

$$\begin{aligned}
 & 1 - \exp \left[-\alpha \int_{t_0}^{t_0 + \tau} (1 + \beta u)^{-1} du \right] \\
 &= 1 - \exp \left[-\frac{\alpha}{\beta} \ln \frac{1 + \beta(t_0 + \tau)}{1 + \beta t_0} \right] \\
 &= 1 - \exp \left[-\frac{\alpha}{\beta} \ln \left(1 + \frac{\beta \tau}{1 + \beta t_0} \right) \right] \\
 (5.3) &= 1 - \exp \left[-\alpha \int_0^{\frac{\tau}{1 + \beta t_0}} (1 + \beta u)^{-1} du \right]
 \end{aligned}$$

Cumulative distribution functions

Integrating out the age-specific death rate in (5.1), (5.2), and (5.3), and reparameterizing slightly yields the following cumulative distributions:

Case I: $F(t) = 1 - e^{-a(e^{bt} - 1)}$

The conditional reproducibility of this function may be demonstrated as follows:

Let $F(t_1 | t_0)$ denote the probability of surviving to t_1 given that it has survived up to t_0 .

$$\begin{aligned} F(t_1|t_0) &= \frac{1 - e^{-a(e^{bt_1} - 1)} - 1 + e^{-a(e^{bt_0} - 1)}}{e^{-a(e^{bt_0} - 1)}} \\ &= 1 - e^{-ae^{bt_0}(e^{b(t_1 - t_0)} - 1)} \end{aligned}$$

By defining a new variable

$$\mathcal{T} = t_1 - t_0$$

and new parameters

$$a' = ae^{bt_0} \text{ and } b' = b,$$

We obtain a new distribution

$$G(\mathcal{T}) = 1 - e^{-a'(b'\mathcal{T} - 1)}$$

which has the identical form as the original one.

Similarly, the reproducibility of the distributions in Cases IIa and IIb can be demonstrated.

$$\text{Case IIa: } F(t) = 1 - e^{-a \left[(1+bt)^c - 1 \right]}; \quad a' = a(1+bt_0)^c,$$

$$b' = \frac{b}{1+bt_0}$$

$$c' = c$$

$$\text{Case IIb: } F(t) = 1 - (1+bt)^{-c}; \quad b' = \frac{b}{1+bt_0}, \quad c' = c$$

Note that Cases I and IIb are both limiting cases of IIa. The exponential distribution is also a limiting case of IIa.

$$F(t_1|t_0) = \frac{1 - e^{-a(e^{bt_1} - 1)} - 1 + e^{-a(e^{bt_0} - 1)}}{e^{-a(e^{bt_0} - 1)}} \\ = 1 - e^{-ae^{bt_0}(e^{b(t_1 - t_0)} - 1)}$$

By defining a new variable

$$\tau = t_1 - t_0$$

and new parameters

$$a' = ae^{bt_0} \text{ and } b' = b,$$

We obtain a new distribution

$$G(\tau) = 1 - e^{-a'(b'\tau - 1)}$$

which has the identical form as the original one.

Similarly, the reproducibility of the distributions in Cases IIa and IIb can be demonstrated.

$$\text{Case IIa: } F(t) = 1 - e^{-a[(1+bt)^c - 1]}; \quad a' = a(1+bt_0)^c,$$

$$b' = \frac{b}{1+bt_0}$$

$$c' = c$$

$$\text{Case IIb: } F(t) = 1 - (1+bt)^{-c}; \quad b' = \frac{b}{1+bt_0}, \quad c' = c$$

Note that Cases I and IIb are both limiting cases of IIa. The exponential distribution is also a limiting case of IIa.

The properties exhibited by this very general class of functions are made use of in the simpler of the two Bayesian-dynamic models developed subsequently. In the example presented, the simplest a priori assumption is made, namely that the probability of survival to time t is exponential. However, most models could just as well involve more general functions which are members of this class.

In subsequent chapters we develop these models.

CHAPTER VI: INVENTORY MODELS WITH MARKOV DEMAND

A. Introduction

In this chapter, we present some inventory models which are useful for decision-making regarding ordering and disposal activities involving a single item. Policies considered are of the well-known (s,S) type. For ordering, whenever the stock level falls below a certain level s , enough new stock is ordered to bring the level up to another prespecified level S ; if the stock level exceeds s , no order is placed. As to disposal, the policy operates as follows: If the stock level exceeds a certain level D , a disposal takes place in order to bring the stock level down to a new level d ; when the level is short of D , no disposal is made.

In Section 2, a demand process is formulated. We assume that the system which generates demand can be in any of several states in a given time interval. Each state has its own demand pattern. Hence, the demand in each period, which is a random variable, may or may not be identically distributed in successive periods. At the end of each time interval, we have data on the demand for that interval; based on these data we then efficiently forecast the demand for the future by means of Bayes' formula.

In Section 3, a general inventory model which is embedded in the demand process described in the previous section is presented with a computation procedure. A specific example of such a model is given

with some calculated results. Finally, a very special case of the general model is discussed. This last model is distinguished by the property that when the system has made a transition to its terminal state, which is characterized by zero demand, it stays there. In Section 4, two schemes which can be used for improving the accuracy of predicting the demand for the future are introduced.

B. The Demand Process

1. Markov Demand Generation

Of course, the system which underlies the demand generation of even a single item in the vast Navy supply system is quite complex. Fortunately, a Markov process provides a manageable and quite general mathematical model for our analytical study.

The basic concepts of the Markov processes are those of "states" of a system and state "transition." We say a system is in a certain state when it is completely describable by the values of parameters which define that state. A system is said to have made a transition from one state to another when the parameters which describe it have changed from the values specified for one state to those for another.

Consider a demand generating system which can be in one of a finite number of states, S_r where $r = 1, 2, \dots, n$, at any time. Each state, in general, is assumed to have a different demand pattern which is characterized by a parameter such as mean demand rate. This system makes state transitions according to a certain transition probability matrix $||P_{rs}||$. Its typical element P_{rs} stands for

the probability of the system making a transition from S_r to S_s . Furthermore, we use a vector (π_r) to denote the a priori state probability distribution. Its typical element π_r is the probability that the system is initially in S_r and $\sum_r \pi_r = 1$. Let $g_r(x)$ denote the conditional distribution of demand X given that the system is in S_r .

The above finite state discrete time process is the scheme incorporated in the subsequent inventory models to represent the demand generation for a single item. This process can also readily handle the situation where demands for several items are linked through underlying states. All one must do is specify a conditional joint distribution of item demands for each state. One of the models to be discussed later makes use of this notion of linked demands.

2. Prediction of Demand by Bayes' Formula

Suppose we have observations on demand X in this period and we wish to estimate the demand for the next period. How shall we proceed? One method of prediction is to calculate a posteriori state probabilities by means of Bayes formula. Then combining these state probabilities with the transition probabilities provides a basis for estimation.

We are given the a priori state probabilities $\pi_r = \Pr [S=S_r]$ for $r = 1, 2, \dots, n$ and we wish to calculate the a posteriori state

probabilities $\pi_r' = \Pr \{S = S_r \mid X = x\}$, for $r = 1, 2, \dots, n$, after observing $X = x$. Using the definition of conditional probability

$$(6.1) \quad \begin{aligned} \pi_r' &= \Pr \{S = S_r \mid X = x\}, \\ &= \frac{\Pr \{S = S_r \text{ and } X = x\}}{\Pr \{X = x\}} \end{aligned}$$

The numerator of (6.1) can be evaluated as follows:

$$(6.2) \quad \begin{aligned} \Pr \{S = S_r \text{ and } X = x\} &= \Pr \{X = x \mid S = S_r\} \cdot \Pr \{S = S_r\}, \\ &= g_r(x) \cdot \pi_r. \end{aligned}$$

To evaluate the denominator we note that the possible outcomes leading to $X = x$ are (S_1, x) , (S_2, x) , $\dots$, (S_n, x) . Hence

$$(6.3) \quad \begin{aligned} \Pr \{X = x\} &= \sum_{j=1}^n \Pr \{X = x \text{ and } S_j\}, \\ &= \sum_{j=1}^n g_j(x) \cdot \pi_j. \end{aligned}$$

From (6.2) and (6.3), we can then calculate (6.1).

$$(6.4) \quad \pi_r' = \frac{g_r(x) \cdot \pi_r}{\sum_{j=1}^n g_j(x) \cdot \pi_j}.$$

Making use of knowledge about the a posteriori state probability and the state transition probability matrix $||P_{rs}||$, we then obtain the a priori state probability (π_r'') for the next period.

$$(6.5) \quad \pi_s^{t+1} = \sum_{r=1}^n P_{rs} \pi_r^t,$$

$$= \frac{\sum_{r=1}^n P_{rs} \cdot g_r(x) \cdot \pi_r}{\sum_{j=1}^n g_j(x) \cdot \pi_j}.$$

(π_s^{t+1}) gives us a basis for predicting the future states of the system on the basis of all observable demands up to the present, by recursive application of (6.5) after each time period. Furthermore, it also makes use of available a priori information regarding the future states of the system. For instance, so called program data such as the expected numbers of flight hours and the expected obsolescence rate may be translated into the form of the a priori state probabilities (π_r) and the state transition probabilities $||P_{rs}||$. In this way, the empirical evidence can be supplemented by the judgment of the operating people to obtain the most efficient prediction results.

C. General Markov Inventory Model

1. Formulation of the Model

The problem considered here is that of stocking a supply of one item to meet an uncertain demand which is assumed to be generated by a Markov process described in the previous section. Various costs associated with oversupply and undersupply are assumed to be operative. Orders are placed at the beginning of each time period of equal length. The orders are assumed to be fulfilled either immediately or are

delivered one time interval later. After the order has been placed, a demand is made. This demand is satisfied from the existing inventory, with excess demand leading to a penalty cost. Unfilled demand can be assumed to be either lost or backlogged!

Let us first introduce the following notation:

ψ = policy

α = action according to policy ψ , presumably $\alpha = \psi(Y, (\pi))$

A = the set of admissible actions, $\alpha \in A$

Y = inventory vector, describing the inventory status at the beginning of a period

X = demand vector

π_r = a priori state probability

$L(Y, x, \alpha)$ = expected current cost function with initial inventory Y , demand x and taking action α .

$C_\psi(Y, (\pi_r))$ = expected total cost of choosing α according to ψ and process beginning with Y and (π_r) .

The optimization problem involves finding ψ to minimize $C_\psi(Y, (\pi_r))$ for all $(Y, (\pi_r))$. We note the following recursive relation:

$$(6.6) \quad C_\psi(Y, (\pi_r)) = \sum_{s, x} \pi_s g_s(x) [L(Y, x, \alpha) + \rho C_\psi(Y', (\pi_s'))],$$

¹As long as this assumption is maintained, the computation of optimal policies for any constant lead time problem is still manageable. As to the inventory problem with a stochastic lead time, the literature is rather scarce on this subject except under some simplified assumptions; clearly, further exploration is needed in this area of inventory study.

where

Y' = a vector describing terminating inventory status for this period (or initial inventory status for next period). It assumes the following functional relation: $Y' = h(Y, x, \alpha)$

π_s'' = a priori state probability for next period. This was derived in the previous section, as a function of (π_s) and x .

ρ = a discount factor, $0 < \rho < 1$. The introduction of such a discount factor prevents infinite costs from entering.

Following the well known optimality principle, the optimum total cost function may be stated as follows:

$$(6.7) \quad C_\psi(Y, (\pi_r)) = \min_{\alpha} \sum_{s,x} \pi_s g_s(x) \left[L(Y, x, \alpha) + \rho C_\psi(Y', (\pi_s'')) \right]$$

If ψ is an optimum policy.

This functional equation can be solved by the following iterative method: Select an arbitrary set of starting policies and cost functions, say ψ_0 and $K_0(Y, (\pi_r))$. Then recursively calculate sequences of policies $\{\psi_n\}$ and cost functions $\{K_n(Y, (\pi_r))\}$ such that

$$(6.8) \quad K_{n+1}(Y, (\pi_r)) = \min_{\alpha} \sum_{s,x} \pi_s g_s(x) \left[L(Y, x, \alpha) + \rho K_n(h(Y, x, \alpha), (\pi_s'')) \right]$$

and $\psi_{n+1}(Y, (\pi_r))$ is the minimizing α for each $(Y, (\pi_r))$. When these sequences converge, the limits of the sequences are the solution

to (6.7). In our numerical study of this model, it was found that the convergence of the cost function sequence $\{K_n\}$ could be painfully slow, especially when the values of starting cost functions are poor approximations of the true cost functions or the discount factor ρ is close to unity. However, it was found that the policy sequence $\{\psi_n\}$ converges much more rapidly.

2. Computational Short-cut

We note that in any minimization or maximization problem of this type what matters is the pattern of differences among the relevant variables, rather than absolute levels. Making use of this idea, we are then able to reduce the number of iterations necessary for the sequence $\{K_n\}$ to converge.

Let $(\overset{\circ}{Y}, (\overset{\circ}{\pi}_r))$ be any particular value of $(Y, (\pi_r))$, and define

$$(6.9) \quad \bar{C}_\psi(Y, (\pi_r)) = C_\psi(Y, (\pi_r)) - C_\psi(\overset{\circ}{Y}, (\overset{\circ}{\pi}_r)),$$

where ψ is optimum policy and C_ψ corresponding minimum cost.

Substituting (6.9) into (6.7), we have

$$(6.10) \quad \bar{C}_\psi(Y, (\pi_r)) = \min_{\alpha} \sum_{s,x} \pi_s g_s(x) \left[L(Y, x, \alpha) + \rho \bar{C}_\psi(h(Y, x, \alpha), (\pi''_s)) \right] \\ - (1-\rho) C_\psi(\overset{\circ}{Y}, (\overset{\circ}{\pi}_r)).$$

This subtraction of a constant will in no way affect the minimization. Hence, we set $C_\psi(\overset{\circ}{Y}, (\overset{\circ}{\pi}_r)) = 0$. We then have another expected total cost function, involving only the cost differences.

$$(6.11) \quad \bar{C}_{\psi}(Y, (\pi_r)) = \min_{\alpha} \sum_{s, x} \pi_s g_s(x) \left[L(Y, x, \alpha) + \rho \bar{C}_{\psi}(h(Y, x, \alpha), (\pi''_s)) \right]$$

The new iterative procedure is exactly the same as before except after each iteration the following subtraction is made to set

$$K_{n+1}(\overset{\circ}{Y}, (\overset{\circ}{\pi}_r)) = 0$$

$$(6.12) \quad K_{n+1}(Y, (\pi_r)) = \min_{\alpha} \sum_{s, x} \pi_s g_s(s) \left[L(Y, x, \alpha) + \rho K_n(h(Y, x, \alpha), (\pi''_r)) \right] \\ - \min_{\alpha} \sum_{s, x} \overset{\circ}{\pi}_s g_s(x) \left[L(\overset{\circ}{Y}, x, \alpha) + \rho K_n(h(\overset{\circ}{Y}, x, \alpha), (\pi''_r)) \right]$$

When convergence of the sequence $\{K_n\}$ is satisfactory, the last subtractive constant is divided by $(1-\rho)$ to obtain $C_{\psi}(\overset{\circ}{Y}, (\overset{\circ}{\pi}_r))$.

$$(6.13) \quad C_{\psi}(\overset{\circ}{Y}, (\overset{\circ}{\pi}_r)) = \frac{\min_{\alpha} \sum_{s, x} \overset{\circ}{\pi}_s g_s(x) \left[L(\overset{\circ}{Y}, x, \alpha) + \rho K_n(h(\overset{\circ}{Y}, x, \alpha), (\pi''_r)) \right]}{1 - \rho}$$

This is a quick way of summing the infinite series $(1 + \rho + \rho^2 + \dots)$

Finally, $C_{\psi}(Y, (\pi_r))$ for other $(Y, (\pi_r))$ are calculated by means of

(6.9)

By adopting this short-cut method, a substantial reduction in computation time was observed.

3. Numerical Examples

In this numerical example, a demand-generating system consisting of two states, one having a high demand rate, and the other a low demand rate, is considered. Orders are made at the beginning of each regularly spaced period and a delivery lag of one period is considered. Any unfilled demand for current period becomes demand for next period. The conditional distribution of demand is assumed to be Poisson.

Definition of symbols used and parameter values assumed are given below:

y = initial inventory level. Both positive and negative values are allowed. If y assumes a negative value, it means that there are unfilled demands.

α = desired inventory level. The difference $\alpha - y$ represents the optimal order quantity, if it is positive. If the difference is negative, it represents the optimal disposal quantity. In the first example, we are considering $\alpha \geq \max(y, 0)$, i.e., no disposal activities are allowed. Of course, it presents no problem if one wishes to allow disposal activities as in the second example; all that needs to be done is to set the domain of α to be $\alpha \geq 0$. If we allow α to assume negative values it means that the stock can be returned to the suppliers and credit received for it.

x = demand per period

π = a priori probability that the system is in state 1. Then

$(1-\pi)$ is a priori probability that the system is in state 2.

$g_1(x)$ = probability distribution of demand when the system is in state 1

$$e^{-2} \frac{2^x}{x!}$$

$g_2(x)$ = probability distribution of demand when the system is in state 2

$$e^{-0.4} \frac{(0.4)^x}{x!}$$

$h_{\pi}(x)$ = probability distribution of x weighted by a priori state probabilities

$$h_{\pi}(x) = \pi g_1(x) + (1-\pi) g_2(x)$$

d_1 = unit storage cost, 0.5

d_2 = unit out-of-stock cost, 5.0

d_3 = fixed order cost, 1.0

d_4 = unit order cost, 0.5

$||P_{rs}||$ = transition probability matrix, $r, s, = 1, 2$

r \ s	1	2
1	0.7	0.3
2	0.1	0.9

π'' = a priori probability that the system will be in state 1 in the next period.

$$\pi'' = \frac{p_{11} \pi g_1(x) + p_{21} (1-\pi) g_2(x)}{\pi g_1(x) + (1-\pi) g_2(x)}$$

ρ = discount factor, 0.99

With the above notation, we now can proceed to specify the expected total cost function which is composed of the expected current costs and discounted future cost.

Expected holding and shortage costs, $L_1(y, \pi)$, to be charged during the current period, assuming that an order will not be delivered until next period is:

$$(6.14) \quad L_1(y, \pi) = \begin{cases} d_1 \sum_{x=0}^y (y-x) h_{\pi}(x) + d_2 \sum_{x=y+1}^{\infty} (x-y) h_{\pi}(x) & y \geq 0 \\ d_2 (-y) + d_2 \sum_{x=0}^{\infty} x h_{\pi}(x) & y < 0 \end{cases}$$

Suppose the ordering cost, $L_2(y, \alpha)$, is charged when orders are placed, it can be expressed as follows:

$$L_2(y, \alpha) = \begin{cases} d_3 + d_4 (\alpha - y) & \alpha > y \\ 0 & \alpha = y \end{cases}$$

Then the expected current cost function is:

$$L(y, \pi, \alpha) = L_1(y, \pi) + L_2(y, \alpha)$$

Let $C_{\psi}(y, \pi)$ represent the total expected cost if the provisioning is done optimally.

$$(6.15) \quad C_{\psi}(y, \pi) = \min_{\alpha > \max(y, 0)} \left[L(y, \pi, \alpha) + \rho \sum_{x=0}^{\infty} h_{\pi}(x) C_{\psi}(\alpha - y, \pi) \right]$$

The solution to (6.15) was obtained by means of the iterative procedure described in the previous section and is presented in Table 1. Note that the condition $\alpha > \max(y, 0)$ restricts the policy from carrying out disposal activities.

Another set of calculations was performed by introducing disposal activities in the above problem and by reducing the unit stock-out cost. (Rest of the parameters were unchanged.) More specifically,

TABLE 1: OPTIMUM POLICY TABLE WITHOUT DISPOSAL*

		<u>a priori</u> state probability								
y	π	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Initial Inventory Level	7	7	7	7	7	7	7	7	7	7
	6	6	6	6	6	6	6	6	6	6
	5	5	5	5	5	5	5	5	5	5
	4	4	4	4	4	4	4	6	6	6
	3	3	5	5	5	5	5	6	6	6
	2	4	5	5	5	5	5	6	6	6
	1	4	5	5	5	5	5	6	6	6
	0	4	5	5	5	5	5	6	6	6
	-1	4	5	5	5	5	5	6	6	6
	-2	4	5	5	5	5	5	6	6	6

* For a given a priori state probability and initial inventory level, we read off an appropriate entry in the above table. The difference between this entry and the initial inventory level is the optimum order quantity. When the difference is zero, no action should be taken. For all $y < -2$, the optimum policy repeats itself.

we set the domain of minimization to $\alpha > 0$ in (6.15) and set $\alpha_2 = 25$. Results are tabulated in Table 2.

TABLE 2: OPTIMUM POLICY TABLE WITH DISPOSAL

		<u>a priori</u> state probability								
$y \backslash \pi$		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
7		3	7	7	7	7	7	7	7	7
6		3	6	6	6	6	6	6	6	6
5		5	5	5	5	5	5	5	5	5
4		4	4	4	4	4	4	4	4	4
3		3	3	3	3	3	3	5	5	6
2		2	4	4	4	4	4	5	5	6
1		3	4	4	4	4	4	5	5	6
0		3	4	4	4	4	4	5	5	6
-1		3	4	4	4	4	4	5	5	6
-2		3	4	4	4	4	4	5	5	6

4. A Special Case of the General Model

The example discussed in this section is a special case of the general model. It is of special interest to our study of obsolescence because:

1. One of the states is assumed to have zero demand. Once the system is in this state, it is not possible to make transitions to other states. Hence, we consider that obsolescence

has occurred when the system has entered the zero demand state.

2. It illustrates how those failure or mortality distributions with the property of "conditional reproducibility" which have been discussed in an earlier chapter, enter into the general Markov inventory model. With the introduction of such a distribution into the model, it is appropriate to look for optimum policies which depend on the inventory level and on some function of the past observable demands but not explicitly dependent on time.

Consider a system with two underlying states. When the system is in State 1, the item demand follows the conditional distribution $g_1(x)$ for $x=0, 1, 2, \dots$. When it is in State 2, the conditional demand function $g_2(x)$ is defined to be

$$g_2(x) = \begin{cases} 1 & \text{for } x = 0 \\ 0 & \text{for } x > 0 \end{cases}$$

Then, in the language of the previous section, the probability distribution of X weighted by a priori state probabilities is

$$h_{\pi}(x) = \begin{cases} \pi g_1(x) + (1-\pi) & \text{for } x = 0 \\ \pi g_1(x) & \text{for } x > 0 \end{cases}$$

Suppose transitions are made according to the following transition probabilities

r \ s	1	2
	1	2
1	e^{-k}	$1-e^{-k}$
2	0	1

The a priori probability that the system will be in State 1 in the next period given that the current demand is $X = x$

$$\pi'' = \begin{cases} \frac{e^{-k} \pi g_1(x)}{\pi g_1(x) + (1-\pi)} & \text{for } x = 0 \\ e^{-k} & x > 0 \end{cases}$$

With these definitions of $h_\pi(x)$ and π'' , we can formulate the total expected cost function similar to (6.5) and solve it.

Note that e^{-k} is the probability that the system survives in any given period given that it did not terminate during the immediately preceding period if we assume that the system's survival follows an exponential distribution. This can be shown as follows:

Let t = a random variable which represents the age of a system,

$$t \geq 0$$

T = the age of the system when it terminates, $T \geq 0$

$f(t)dt = \Pr(t < T < t + dt)$, The probability that the system will terminate in the interval t to $t + dt$

$\Pr(T \leq t)$ = the probability that the system will terminate in the interval 0 to t .

$\Pr(t \leq T \leq t+dt | T > t)$ = The probability that the system will terminate in an additional time interval dt given that it did not terminate before t .

Suppose we assume $f(t) = ke^{-kt}$. Then

$$\Pr(T \leq t) = 1 - e^{-kt}$$

$$\Pr(t \leq T \leq t + dt | T > t) = 1 - e^{-k(dt)}$$

Now the probability that the system will survive the additional time interval dt given that it has survived up to t is

$$1 - \Pr(t \leq T \leq t + dt | T > t) = e^{-k(dt)}$$

If we set dt to be unit time, we have the desired probability e^{-k} .

D. Some Extensions of the General Model

1. A Model with Linked Demands

The general model is extended to handle linked demands, in which linkage is based on underlying state S_r for $r = 1, 2, \dots, n$, in the following manner. With each state we associate a joint density function of demands such that when the system is in, say, S_s the resulting demands are considered to be observations based on

$$g_s(x_1, x_2, \dots, x_\ell).$$

If the information such as that mentioned above can be incorporated into our model, it will contribute to the statistical determination of

where the system is in any one period. For instance, if we know a priori that demands for items A and B occur together in state i but in state j there is demand only for item A, and then if there are observable demands for item B, we will conclude that the system is most likely to be in state i.

With the introduction of linked demands, the calculations of the a posteriori state probability (π_r') and the new a priori state probability (π_r'') (as in (6.4) and (6.5)) will be modified as follows:

$$\pi_r' = \frac{g_r(x_1, x_2, \dots, x_\ell) \pi_r}{\sum_{j=1}^n g_j(x_1, x_2, \dots, x_\ell) \pi_j},$$

$$\pi_s'' = \frac{\sum_{r=1}^n P_{rs} g_r(x_1, \dots, x_\ell) \pi_r}{\sum_{j=1}^n g_j(x_1, \dots, x_\ell) \pi_j}.$$

As to optimization, it will be carried out for one item at a time. Necessary modifications are quite straightforward.

Suppose we are interested in optimization of the ith item. The expected total cost function can be stated as follows: (Note all the variables have the same meaning as before except now they refer to the ith item).

$$(6.16) \quad C_\psi(y_i, \{\pi_r\}) = \min_{\alpha_i} \sum_{s, x} \pi_s m_s(x_i) [L(y_i, x_i, \alpha_i) + \rho C_\psi(y_i', \{\pi_s''\})]$$

where $m_s(x_i)$ is the marginal distribution of x_i

$$m_s(x_i) = \int_0^\infty \dots \int_0^\infty g_s(x_1 \dots x_\ell) dx_1 \dots dx_{i-1} dx_{i+1} \dots dx_\ell.$$

Again, this functional equation can be solved by the same iterative procedure described earlier.

2. A Model with Non-demand Ancillary Information

The general model can also include non-demand ancillary information which is indicative of state of the system. This will be illustrated with the following example.

Consider a three-state system in which the first two states have the same demand pattern but different state transition probabilities. The third state follows its own demand pattern and transition probabilities different from the first two states.

In such a case, since observable demands alone cannot distinguish the first state from the second state, observations on non-demand variables which have probability distributions dependent on the state will be useful for sharper statistical discrimination.

Consider the following example: the ancillary information available is about conditional distribution of a non-demand variable T .

$$\Pr \{ T = t | S = S_r \}, \quad \text{where } t=1,2,3 \\ \text{and } r=1,2,3.$$

This distribution may be assumed to look like the following table:

Nature of State	Prob. of T		
	1	2	3
S ₁	0.95	0	0.05
S ₂	0.1	0.7	0.2
S ₃	0	0	1

Each row defines a distribution of T in the corresponding state. The variable T can be interpreted as being a more or less accurate indicator of the demand state. Such information is believed to be readily available within the Navy Supply System at this time.

Next, given the conditional probability $\Pr\{T = t | S = S_r\}$, it is of interest to know what is the a posteriori probability $\Pr\{S = S_r | T = t\}$ which is the conditional distribution of the states of the system given the ancillary information $T = t$. This can be calculated readily from the definition of conditional probability.

$$\begin{aligned} \Pr\{S = S_r | T = t\} &= \frac{\Pr\{S = S_r \text{ and } T = t\}}{\Pr\{T = t\}} \\ &= \frac{\Pr\{T = t | S = S_r\} \cdot \Pr\{S = S_r\}}{\sum_i \Pr\{T = t \text{ and } S = S_i\}} \\ &= \frac{\Pr\{T = t | S = S_r\} \cdot \Pr\{S = S_r\}}{\sum_i \Pr\{T = t | S = S_i\} \cdot \Pr\{S = S_i\}} \end{aligned}$$

$$\text{Let } \Pr\{S = S_r | T = t\} = \pi_r(T)$$

$\pi_r(T)$ will then replace the a priori state probabilities π_r (p.58) in the calculations of the a posteriori state probabilities

$\Pr\{S = S_r | X = x\}$, and the a priori state probabilities of next period $\sum_r p_{rs} \Pr\{S = S_r | X = x\}$.

Optimization follows exactly the same steps as in the general model.

CHAPTER VII

SOME RESULTS OF A STATISTICAL STUDY OF OBSOLESCENCE DATA

A. Introduction

In order to generate optimal inventory decision rules from some of the models described earlier in this report (see Chapters IV and VI), it is necessary to assume some a priori probability distribution of the occurrence of obsolescence of an item. While it may be true that incorrect assumptions regarding the probability distribution of obsolescence is self-correcting in the long run, as we accumulate more experience with the item, the fact that we are dealing with a process which is only finite in duration suggests that the more we know a priori the more likely are the decisions to be optimal. We are also interested in enhancing our understanding of the factors which are associated with the process of obsolescence, i.e., we wish to determine causal factors in the process which set an obsolescence date; if this proves impossible, we wish to be able to at least isolate those factors which are related to the process. Therefore, data bearing on life-spans of a sample of items (mainly airframe and engine parts), along with a variety of economic and technical data about each of the items was examined.

The items were first sorted according to Federal Stock Code (FSC) and Technical Supply Maintenance Code (TSMC). Within these groups we

studied: (1) the frequency distribution of time-to-obsolescence, and (2) the degree of associations between some economic and technical variables and life-spans.

We learned that a certain type of the failure distributions appears to describe some of the observed data adequately and that there is a statistically significant relation between the life spans of the items in some groups and certain factors associated with the items. However, the data examined indicated that there was no single frequency distribution that may be usefully employed across the board to describe the pattern of the occurrence of obsolescence. This emphasizes the difficulty of predicting when obsolescence will take place.

B. Data Analyzed

Before entering into the discussion of the results of the statistical analysis, we shall describe in this section the data used in this study and their limitations.

The data analyzed were obtained primarily from the Aviation Supply Office (ASO). Initially a list of about 8,000 items which were declared obsolete during the first half of 1961 was prepared by ASO for this study. Unfortunately, this list was not very useful because most of the information relating to these items was no longer available when the data collection started in September 1961. An alternative list of more recent obsolescence items was then compiled from the Stock Number Data Section (SNDS) catalogs issued in August and September of 1961. These items were declared obsolete during the 13-week period ending with the date of SNDS.

The additional sources of information pertaining to the behavior of the obsolescence items in SNDS were found in the following documents: The Retention and Disposal Listing, the Parts List Catalog, the Purchase Order History Cards (also called 7-30 cards or On-Order Cards), the Federal Supply Classification Numeric Index of Classes, the Naval Aircraft Maintenance Program Glossary, and some specially prepared lists matching Navy contract numbers to their dates of issues.

The data obtained from the above documents are by no means complete. For instance: the Retention and Disposal Listing includes only data on items with surplus stock prior to the declaration of obsolescence; the parts list catalogs do not list all the applications for some of the items; between 25% and 75% of the On-Order cards for obsolete items were missing from the files. The variation in missing cards seems, in large part, to reflect the efficiency and/or prudence of the clerks responsible for them. There is a policy of purging cards from the files of those items which have not had purchase orders in the preceding two-year period. Hence, some relevant information for those items which have not had a regularly recurring pattern of buys is missing.

Despite the incomplete nature of most of the data, there was On-Order card data for about 4,000 items. For all these 4,000 items there is supplementary information from the Part List and the SNDS. For approximately 1,500 of these items there is also supplementary information from the Retention and Disposal Listing for 1961. In addition, we were

able to obtain the relevant information for an additional 1,000 obsolete items from the Douglas Aircraft Company at El Segundo, California.

Information Contained in the Data

The following list contains the descriptive and quantitative variables available for the study, their limitations, and their source documents. The variables are randomly ordered with no particular thought to the relative importance of any one of them.

Federal Stock Number: (FSN) A 17 digit alphanumeric interservice part number by which all items are classified in all the catalogs used for the study. The third through the sixth digits are numerical and denote the Federal Supply Class. The fourteenth through the seventeenth digits are alphabetic and denote the Technical Supply Maintenance Code, the last three digits of which represent the manufacturer. The other digits are not significant for this study. A Federal Supply Class incorporates those items which support similar functions, although the particular items are dissimilar, e.g., 4120 (Fire Fighting Equipment) includes axes, ladders, and firehouse carts. Some classes, however, are restricted to similar parts only, e.g., 6240 (Light Bulbs and Lamps).

Unit Price: The average price paid for a given item in a given contract. Both setup and manufacturing costs are imputed in the unit price for each separate contract. Hence the unit price varies according to changing labor and setup costs associated with the

different quantities purchased at different points in time. The unit price is sometimes listed in the SNDS and the On-Order Cards; for the On-Order Card, several different prices may be noted. A single price always appears in the Retention and Disposal Listing and the Parts List Catalog. The choice of price for the Parts List is not clear.

Nomenclature: A descriptive name for a part, which is designed to group an item by its physical characteristics and/or function. For the study the groups may be overly definitive, i.e., shims, washers, and spacers may be sufficiently similar as to constitute a single group. All the catalogs and cards indicate the nomenclature of an item.

Source Code: A code denoting the origin of a part, e.g., internal manufacturer, interservice transfer, or commercial contractor is found both in the SNDS and the Parts List.

BUSANDA Change Code: A code found in the SNDS indicating the reason that an item was declared obsolete if preceded by a "Q".

Life-Span: The number of months between initial BUWEPS procurement of an item and the declaration of obsolescence. This number may be computed from the date of the BUWEPS contract (prefixed "NOAS") listed first on the On-Order Card. If the date is absent from the card, it may be obtained from a list of BUWEPS contracts.

Since the early On-Order Cards for many items are often missing, two problems arise. First, no NOAS contract number may appear on the card, indicating that the first date on the card is for a re-order subsequent to initial procurement. Second, an item with several applications will have had several initial procurements, i.e., NOAS-coded contracts, one for each application. Hence the first NOAS-coded contract noted on a card may not be the first for that item. With either problem, the life-span for an item will be understated. Items with the first problem may be flagged to indicate that the life-span is understated. Those with the second problem cannot be so differentiated.

Quantity on Hand: That quantity of each item remaining at all Navy Supply Depots at the declaration of obsolescence. This figure is not available from any source. One approximation for each item is the quantity on hand at the last annual running of the Retention and Disposal program. The program computes an optimal maximum retention quantity for each item. Any quantity in stock greater than this maximum is termed surplus. Provided that a surplus existed at the time of the running, the item will appear on the Retention and Disposal listing, giving the maximum retention quantity and the quantity on hand. The most recent running of the program took place about six months prior to the publications of the SNDS catalogs used for this study. Hence,

inclusion in the study only of items with "on-hand" date would give a bias towards items prone to surplus and discount all quantities of items withdrawn subsequent to the program, but prior to the declaration of obsolescence. Inclusion of the balance of the items requires that a zero on-hand quantity must be assigned to those items, which modifies the bias to those items which either had a surplus or whose stocks were exhausted at the precise moment that the declaration of obsolescence took place. It is not possible to isolate those items whose stocks had been exhausted prior to the declaration, and were unable to satisfy issue requests.

Number of Purchases: The number of times that re-orders were made is noted on the On-Order Cards.

Total Purchases: The sum of all the purchases made is found by adding the entries on the On-Order Cards.

Maintenance Per Cent: The percentage of anticipated replacements during maintenance is available from the Parts List.

Overhaul Per Cent: The percentage of anticipated replacements during overhauls is also given on the Parts List.

Application Codes: A code giving either the end item, i.e., the plane, on which the item goes, or the assembly of which the item is a replaceable part. This code, found in the Parts List, gives a measure of the interchangeability of a part within one plane, or among many planes.

Units Per Application: The quantity of a particular item needed to perform the desired function is found in the Parts List. It is possible, although difficult, to search out the total number of items used in all applications for a particular plane or set of planes.

C. Results of Statistical Analysis

First the pattern of obsolescence was studied. By pattern of obsolescence, we mean the frequency distributions of individual items according to their length of time-to-obsolescence (or life span), i.e., how many items had short, medium, or long life span? Thus, we are interested in studying the pattern of obsolescence that characterizes groups of items. Accordingly, the data were sorted by Federal Stock Code (FSC) and Technical Supply Maintenance Code (TSMC).

For each of the samples thus formed, a histogram was made of a distribution of life spans. Although most of the histograms did not exhibit any meaningful pattern, some of them did indicate a unimodal type distribution. Figures 1-4 show a series of histograms which are typical of this latter group.

Those probability distributions that were appropriate to this type of data were studied in order to find if any could be fitted to the data. It was found that the failure distribution $f(t) = a b e^{bt-a(e^{bt}-1)}$ (for discussion on the properties of this distribution, see Chapter V) closely approximated the patterns shown by the histograms. Trial values of the parameters a and b were tried out and a chi square test performed to indicate whether the failure distribution fitted the distributions of life spans for particular sets of parameters. It was found that the distributions of life spans of the FSC-TSMC combinations 1560-ADGA, 1650-ADGA, 2810-PWAC and 2840-PWAC (Figures 1-4) were reasonably described by failure distributions. The parameters $a=1$, $b=.0343$ were best

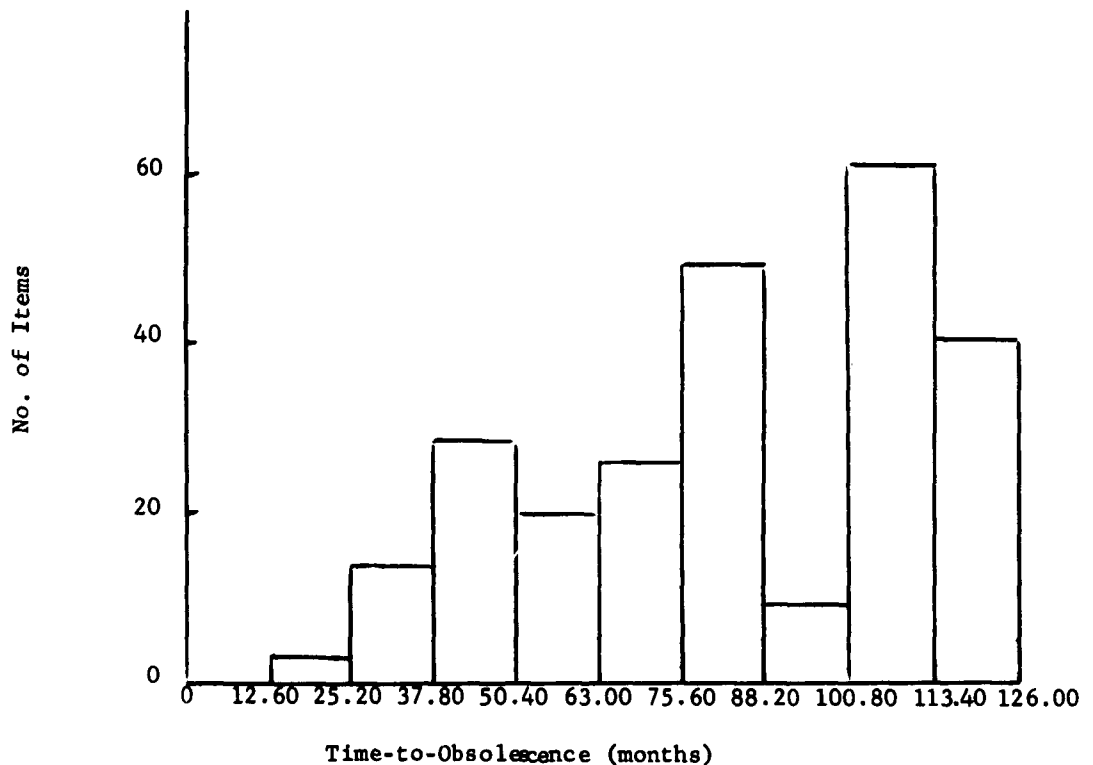


Fig. 1: Frequency Distribution of Time-to-Obsolescence for 1560-ADGA Sample*
*FSN 1560: Airframe structural components
TSMC ADGA: Douglas Aircraft Company

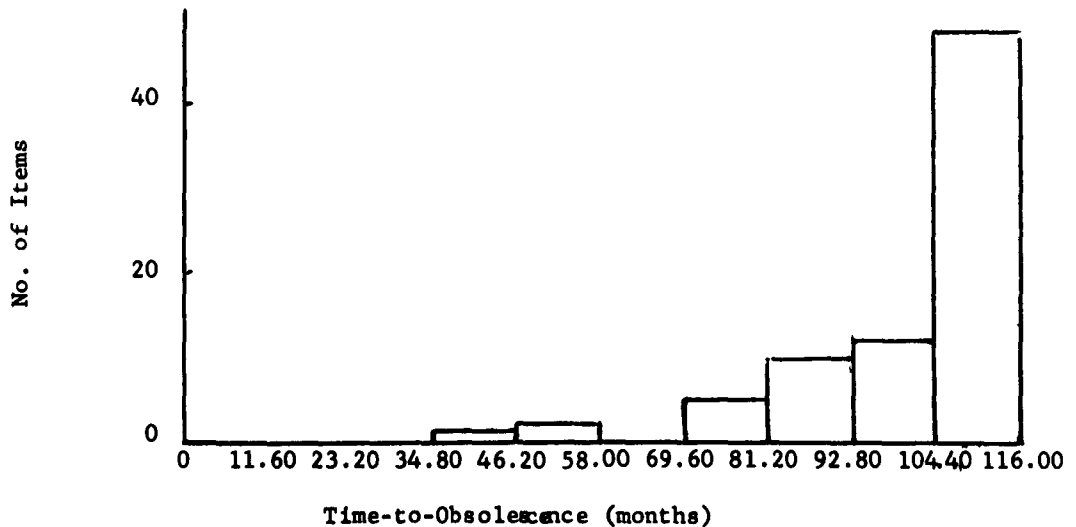


Fig. 2: Frequency Distribution of Time-to-Obsolescence for 1650-ADGA Sample*
*FSN 1650: Aircraft landing gear component

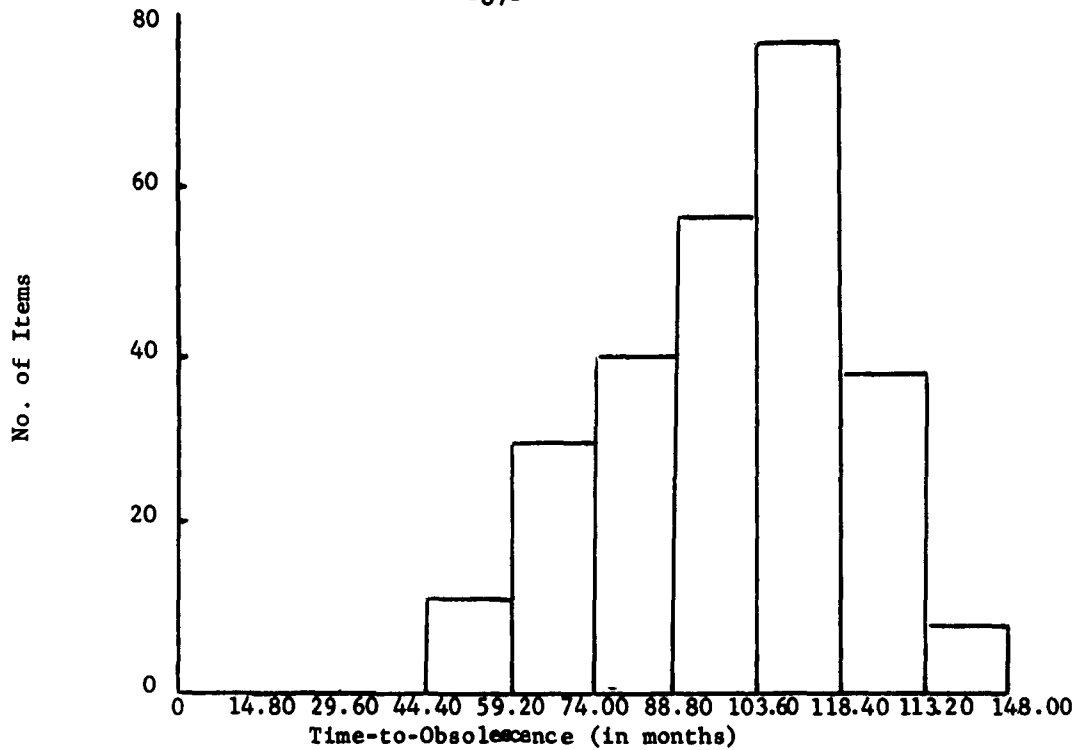


Fig. 3: Frequency Distribution of Time-to-Obsolescence for 2810-PWAC Sample*
*FSN 2810: Gasoline reciprocating engines, aircraft, & components
TSMC PWAL: Wright Aeronautical Corporation

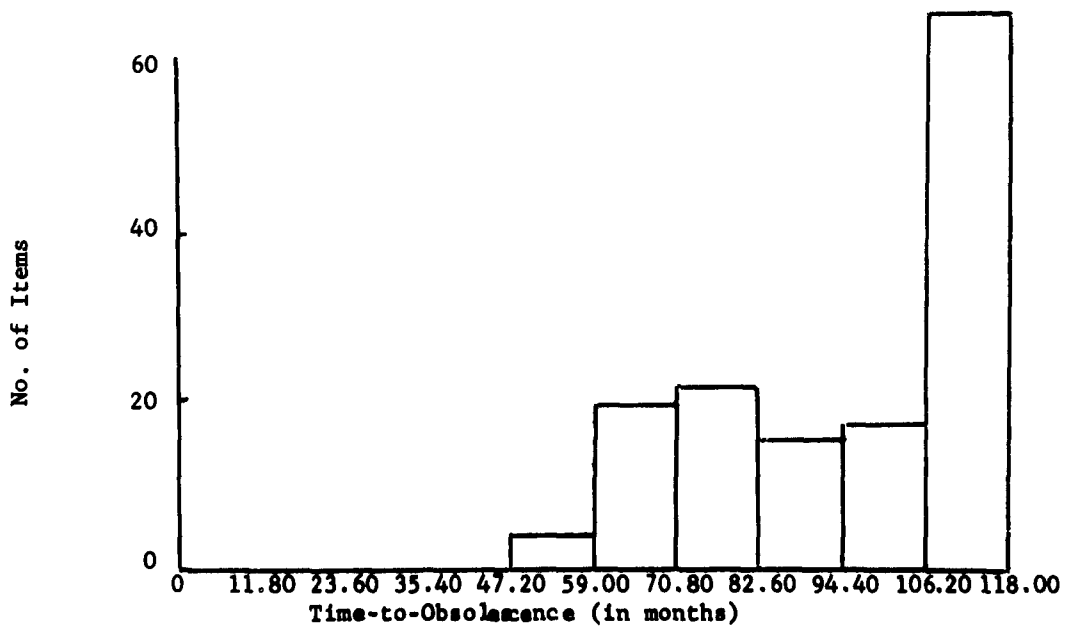


Fig. 4: Frequency Distribution of Time-to-Obsolescence for 2840-PWAC Sample*
*FSN 2840: Gas turbine and jet engine components

for ADGA distributions and $a=1$, $b=.1429$ best for FWAC distributions. Two particular values of life span, 116 and 118 from ADGA and FWAC distributions respectively, appeared to have abnormally high frequencies, so, were ignored when fitting distributions. The 1650-ADGA appeared to be short of its actual life span, but it resembled a truncated 1560-ADGA distribution so this was considered when fitting distribution. The 2840-FWAC distribution, although it was fitted, had frequencies occurring on a limited number of values of life span.

Using these four groups of data, a stepwise regression analysis was carried out using eight quantitative characteristics of each obsolescent item as the independent variables against the life span (y) of item as dependent variable. The eight variables are:

- x_1 = unit price
- x_2 = lead time
- x_3 = total Quantity purchased
- x_4 = total number of purchases
- x_5 = number of applications
- x_6 = number of units per application
- x_7 = maintenance percent
- x_8 = overhaul percent

Some of the data lacked quantitative information for lead time, total quantity purchased and number of purchases. These data were not used in the regression analysis. As a comparison, a regression analysis was also done on ADGA groups where frequencies of life span value 116 was

excluded. Therefore, a total of six stepwise regression analyses were done. Since the F level to enter a variable was set at 4.5, no independent variable enters the step-wise regression analysis in two cases. That is to say, none of the eight quantitative variables were found to be statistically related to items life span. Results of the regression analyses are:

1) 1560-ADGA Group

a. Including items with life span of 116 months.

$$y = 88.92314 + 0.00643X_1 - 3.66734X_2 + 4.17063X_3 + 2.95459X_4$$

(0.00272) (1.14354) (0.90705) (1.29479)

Standard error of estimate = 24.7458

$$R = 0.453$$

b. Excluding items with life span of 116 months.

$$y = 88.41669 + 0.00729X_1 - 4.19640X_2 + 3.92748X_3$$

(0.00265) (1.12390) (0.92716)

Standard error of estimate = 23.1396

$$R = 0.537$$

2) 1650-ADGA Group

a. Including items with life span of 116 months.

$$y = 92.67528 + 5.65230X_4 - 2.20044X_5$$

(1.26133) (0.85662)

Standard error of estimate = 15.4619

$$R = 0.687$$

excluded. Therefore, a total of six stepwise regression analyses were done. Since the F level to enter a variable was set at 4.5, no independent variable enters the step-wise regression analysis in two cases. That is to say, none of the eight quantitative variables were found to be statistically related to items life span. Results of the regression analyses are:

1) 1560-ADGA Group

a. Including items with life span of 116 months.

$$y = 88.92314 + 0.00643X - 3.66734X_2 + 4.17063X_3 + 2.95459X_4$$
$$(0.00272) \quad (1.14354) \quad (0.90705) \quad (1.29479)$$

Standard error of estimate = 24.7458

$$R = 0.453$$

b. Excluding items with life span of 116 months.

$$y = 88.41669 + 0.00729X_1 - 4.19640X_2 + 3.92748X_3$$
$$(0.00265) \quad (1.12390) \quad (0.92716)$$

Standard error of estimate = 23.1396

$$R = 0.537$$

2) 1650-ADGA Group

a. Including items with life span of 116 months.

$$y = 92.67528 + 5.65230X_4 - 2.20044X_5$$
$$(1.26133) \quad (0.85662)$$

Standard error of estimate = 15.4619

$$R = 0.687$$

b. Excluding items with life span of 116 months.

No independent variable entered the regression analysis.

3) 2810-PWAC Group

No independent variable was found to be significant.

4) 2840-PWAC Group

$$y = 104.28982 - 1.27827X_2 - 139.98844X_3$$

(0.47045) (53.05405)

Standard error of estimate = 14.2427

R = 0.466

The number in parenthesis and directly below each regression coefficient is the standard deviation of the estimate of coefficient.

From the above analysis, the following tentative conclusions may be drawn:

1. For airframe structural components, an item with higher unit value tends to have a longer life span. Perhaps there is some reluctance on the part of the inventory clerks to declare these high valued items as obsolescent items.

2. It appears that the longer the lead time the more likely that obsolescence will set in. This was the case for both airframe structural parts and engine parts. One conjecture is that the long lead time may mean a less flexible program. Since such a program is not desirable, from the standpoint of an efficient supply system, there is a tendency to eliminate the item concerned from the system.

3. In two cases, the regression analysis indicates that the total quantity purchased during an item's life span is positively correlated



to the life span itself. However, a plausible explanation is that if an item has a long life span, it is more likely that more purchases will be made.

As we consider how declarations of obsolescence have been made heretofore we can gain some appreciation of the reasons for the relatively weak dependencies of time-to-obsolescence upon such data as have been available.

There are, basically, two grounds for declaration of obsolescence of line items in Naval inventory. One is that on strategic grounds a weapons system is declared to be obsolete after some point in time. That is, it is declared that from some date on this system will be out of use. The other is that a stock control clerk in a supply-demand control point, following some rule of thumb as well as his time permits, and his assiduousness compels, notes absence of demand for items over a period of time, and makes a declaration of obsolescence if periods of sufficient length elapse during which demand is sufficiently low to satisfy the rule of thumb.

So far as the first ground for declaration of obsolescence is concerned, this is less a basis for termination of the system as a collection of line items in Naval inventory than a basis for the downgrading of the system in importance in the Navy's arsenal and its replacement by some other system or systems. This decision frequently commits the Navy to another decision, which is that no further procurements shall be made in support of the older system.

Conceivably, the application of models of the sorts developed in Chapters IV & VI will lead to a reorganization of the structure of these decisions, and therefore, to the entire process of obsolescence generation.

If a system has been downgraded it will continue in use for some period, but at a lower use rate. This, in turn, will reduce demand for spares items, insofar as the demand is due to wearout. The Bayesian feature of these models will then generate higher and higher probabilities that demand is running at lower levels. With known costs of disposal and of inventory operation, information can be fed to high level echelons at which final decisions about the obsolescence of a system can be made definitively.

When the matter at issue is the simpler case of a decision that an item can be declared obsolete by a stock control clerk because of low demand, this entire decision can be handled within the structure of the model, on a more thorough and reliable basis. Once again, the costs of carrying the item on inventory, and disposal costs, as well as stockout costs, would be taken into account in using the model.

This application of the models developed in this study to generate obsolescence decisions, along with the installation of appropriate data processing procedures of the sort sketched out in an accompanying proposal, would lead to a number of desirable results: sharper definition of obsolescence determination, less confusion in record keeping, an improvement in the quality of data about inventory line items. As a

consequence, there would be developed a body of data which would allow analysts to arrive at reliable and meaningful conclusions as to the dependencies of time-to-obsolence upon such data as characteristics of the item, its application, and the purchase contracts which embody the orders.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

This chapter consists of a summary of the work which has been carried out under this contract, and some recommendations for further work which are spelled out in detail in accompanying proposals. In the course of this work an attempt was made to unearth meaningful empirical relationships between obsolescence and engineering, administrative and program data relating to Naval Aviation parts which are within the cognizance of the U.S. Naval Aviation Supply Office. In addition, a series of models was developed and explored to varying degrees which embody more and more sophisticated notions of just what obsolescence is operationally, and how it can be revealed in a useful fashion.

It has been found that by virtue of the nature of inventory administration procedures which are currently in use at ASO, and which are probably typical of procedures carried out at other Supply-Demand-Control Points, obsolescence is recorded in a fashion which is quite unpredictable, and which does not lend itself to meaningful statistical analysis. Recording of obsolescence, as well as maintenance of records as to dates, amounts and prices of purchases of items, is incomplete and quite dependent on such variable matters as work loads, assiduousness of clerical staff and their supervisors, availability of computing and computer programming facilities, etc. Moreover, it is our impression that these matters, particularly those relating to computing facilities, are serious bottlenecks.

Finally, even if these unplanned problems were cleared away, there remains the fundamental difficulty that the inventory control procedures



which comprise the ideal, the plan according to which ASO tries to operate, represent a pastiche of notions, some inconsistent with each other, others redundant, with no real attempt to face a set of system-wide objectives. These procedures are described in Appendix C to this report. A comparison of this appendix with Chapter VIII of Whitin [13], which was published in 1957, is most illuminating. The chapter referred to contains a description of the Navy Inventory control system which was in effect prior to the publication of that book, with some diagnosis and recommendation for change.

For some time before and since the publication of Whitin's book BuSanda has been supporting research aimed at improvement of Navy Inventory procedures. However, it has appeared to those of us who spent a number of weeks at ASO and more weeks subsequently digesting what we had learned there, that thus far the effect of this research has been relatively small because results have been fed into existing organizations in small amounts, on tentative bases. Recommended procedures have been modified so substantially in order to accommodate existing organizational structures, ways of doing things and prejudices, as to be difficult to recognize. Moreover, there has been a tendency to take the position that what is good in one environment is good in isolation. Hence, a collage of measures, techniques and procedures has been assembled from widely scattered sources and these comprise the system.

As a consequence of the present study several models have been developed which represent successively better global approaches to the problem of managing a large complex inventory. The more sophisticated

of these models enables the skilled inventory manager to take ancillary information, or what the Navy refers to as "program" data, into account in a systematic fashion. Specifically, if on budgetary or strategic grounds a particular weapons system is expected to become less significant in the activities of the Atlantic Fleet, and an aircraft type will be flying fewer hours, then a priori distributions of the probability of obsolescence, or a priori probabilities of being in one or another state of demand can be altered to reflect that expectation. Moreover, these models are flexible as to lead time, and other inventory parameters.

It is true that these models make certain rather stringent demands of the inventory manager which cannot be solved simply by improving the quality of data handling. All of these demands are in the area of acquiring better understanding of the cost nature of various conditions of the inventory. How much does it really cost the Navy to be out of stock of a particular part? How much does it really cost to carry such and such an item in inventory, to order it, etc.? These questions should be answered, somehow or other. However, even if the first-cut answers are semi-educated guesses, they will enable the models to be operated, and in fact these answers could be checked in terms of the resultant inventory policies which would be generated.

What is suggested, then, is that these models can eventually serve as inventory control procedures. Indeed, proposals for further study and for installation accompany this report. However, what is perhaps more significant immediately is that these models could serve as

measuring instruments for the development of measures of some of the more fugitive inventory parameters of which theoreticians speak so casually, such as stockout cost, setup cost, etc. Thus, a sensible procedure which might be adopted could involve four phases (discussed in detail in the accompanying proposal for further work):

1. Set up and begin operation of a computer system designed to look up appropriate control policies for separate items. These policies would be generated elsewhere according to some model.
2. At the same time, continue exploration of models. Eventually, tables should be generated from the appropriate model or models which would be available for lookup in inventory control.
3. Load into the machine system, as they are developed, tables based on successively more satisfactory models. Running parallel to the existing inventory control system, run this new system and compare results. As confidence in these tables develops, studies are begun, using these models, which would lead to measures of inventory system parameters consistent with current policies, (i.e. policies in use before any of these changes have begun). This procedure, iterated and played against inventory managers and those responsible for high level logistical policy, would lead, eventually, to acceptable measures of such parameters as setup cost, lead time, stockout cost, etc. In addition, during this phase, studies

could be carried out which would reveal, in detail, the amount of savings in physical inventory costs which would accrue from the installation of the system, as well as the improvement in quality of inventory data. Note that up to this time the normal operations of the SDCP have not been interfered with. All the work described in these three phases has been carried on outside the current operation.

4. Once satisfaction has been achieved with respect to the utility of the new system, it could be phased in gradually, while the old one is phased out. Conceivably, with low demand, high value items, or with items with very high stockout costs or other special problems, phase-in might be a much slower process. Indeed, it might be desirable that some items never be completely controlled by mechanized procedures, but be constantly subject to review by highly skilled inventory specialists. However, for the bulk of the 400,000 items within the cognizance of ASO, as well, probably, as the remainder of the 1.2 million items handled by other SDCP's for the Navy, this sort of procedure would probably turn out to be a most economical and satisfactory one.

Although it will not be gone into detail here conservative preliminary estimates indicate that inventory administration costs (i.e. costs of maintaining files, ordering, disposing, etc.) for the 400,000 items

handled by ASO, could probably be reduced by a substantial amount each year. This does not take into account the actual physical inventory savings due to changes in amounts of inventory held, changes in numbers of stockouts, disposals, orders, etc. What is being asserted here is that the costs of placing exactly the same orders, carrying exactly the same amounts of all items, disposing of the same amounts, etc. at the same times, would be much less than they now are. Savings resulting from changing these policies would, in all likelihood, be much more substantial, but their magnitude would begin to be revealed only during the third phase of the four phase procedure laid out above.

APPENDIX A
POLICIES RESULTING FROM DYNAMIC PROCUREMENT -
DISPOSAL MODEL RUNS

Before presenting the results of the computer runs, we shall first describe how the values of parameters, which characterize the demand and obsolescence distributions and cost functions, are varied in each of the runs.

Each run is identified by a three-digit number. The first digit identifies one of the five different sets of assumptions regarding the demand and obsolescence distributions. The last two digits identify a different cost combination. These identification codes are indicated in Tables 1-A and 2-A.

Policies resulting from each computer run are tabulated in the remainder of this appendix. Note that for each run, we assumed an inventory process with a finite horizon consisting of 10 periods.

TABLE 1-A

Variations in Demand and
Obsolescence Distributions

Run #	Demand Distribution	Obsolescence Distribution												
1XX	Poisson demand with mean of 30; demand rate not changing through time	Exponential distribution with obsolescence rate of 10%.												
2XX	Poisson demand with decreasing mean demand rate through time as follows: <table><tr><td><u>Time</u></td><td><u>Mean demand</u></td></tr><tr><td>1,2</td><td>40</td></tr><tr><td>3,4</td><td>35</td></tr><tr><td>5,6</td><td>30</td></tr><tr><td>7,8</td><td>25</td></tr><tr><td>9,10</td><td>20</td></tr></table>	<u>Time</u>	<u>Mean demand</u>	1,2	40	3,4	35	5,6	30	7,8	25	9,10	20	Same as in 1XX runs.
<u>Time</u>	<u>Mean demand</u>													
1,2	40													
3,4	35													
5,6	30													
7,8	25													
9,10	20													
3XX	Uniform demand with mean of 30; demand rate not changing through time	Same as in 1XX runs.												
4XX	Same as in 3XX runs	Uniform distributions: 10% obsolescence rate.												
5XX	Uniform distribution decreasing mean demand rate as in 2XX runs	Same as in 4XX runs.												

TABLE 2-A

Variations in Cost Parameters

Run #	Disposal ^{*,**} Cost	Fixed Ordering [*] Cost	Storage ^{*,**} Cost	Out-of-stock ^{*,**} Cost
X 01	0-2 Z	10	0.1 Z	5 Z
X 02	0-2 Z	10	0.1 Z	30 Z
X 03	0-2 Z	40	0.05 Z	10 Z
X 04	0-2 Z	40	0.05 Z	30 Z
X 05	0-2 Z	40	0.1 Z	30 Z
X 06	15-2 Z	10	0.1 Z	5 Z
X 07	15-2 Z	10	0.2 Z	5 Z
X 08	15-2 Z	10	0.1 Z	30 Z
X 09	15-2 Z	40	0.1 Z	5 Z
X 10	15-2 Z	40	0.1 Z	30 Z
X 11	0-2 Z	40	0.02 Z	10 Z
X 12	0-2 Z	40	0.02 Z	30 Z

* All costs are expressed as multiples of unit price of the stock.
(i.e., unit price = 1)

** Z = amount to be disposed, stored or to which a penalty cost is charged.

TABLE 3-A
Policies Resulting from Model Runs*

Run 101					Run 102				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	27	34	43	44	10	34	39	46	47
9	29	36	77	78	9	35	42	81	82
8	29	38	108	109	8	35	42	112	113
7	29	38	137	138	7	35	42	142	143
6	29	38	165	166	6	35	42	170	171
5	29	38	192	193	5	35	42	196	197
4	29	38	216	217	4	35	42	221	222
3	29	38	238	239	3	35	42	242	243
2	29	38	256	257	2	35	42	260	261
1	29	38	261	262	1	35	42	265	266

Run 103					Run 104				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	27	37	47	48	10	31	39	47	48
9	29	68	82	83	9	33	70	84	85
8	27	70	115	116	8	32	73	117	118
7	28	70	146	147	7	32	74	148	149
6	27	70	176	177	6	32	73	179	180
5	28	70	206	207	5	32	74	208	209
4	28	70	235	236	4	32	73	235	236
3	28	70	263	264	3	32	74	265	266
2	28	70	290	291	2	32	73	292	293
1	28	70	-	-	1	32	74	-	-

* Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd

Policies Resulting from Model Runs *

Run 105					Run 106				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	31	39	46	47	10	25	32	41	199
9	32	70	82	83	9	28	37	73	164
8	32	73	114	115	8	28	37	104	173
7	32	73	145	146	7	28	37	133	192
6	32	73	174	175	6	28	37	161	215
5	32	73	202	203	5	28	37	188	240
4	32	73	229	230	4	28	37	213	266
3	32	73	254	255	3	28	37	236	290
2	32	73	276	277	2	28	37	-	-
1*	32(?)	73(?)	***	***	1	28	37	-	-

Run 107					Run 108				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	25	32	39	118	10	33	38	45	203
9	28	37	71	117	9	35	42	79	169
8	28	37	100	137	8	35	42	110	178
7	28	37	127	161	7	35	42	139	198
6	28	37	153	186	6	35	42	167	221
5	28	37	174	211	5	35	42	194	246
4	28	37	189	233	4	35	42	219	271
3	28	37	189	250	3	35	42	242	296
2	28	37	189	258	2	35	42	-	-
1	28	37	189	259	1	35	42	-	-

* Inventory Level, H Policy

$H \leq H_1$ Buy $K_1 - H$

$H \geq H_2$ Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

*** Through an operator error, the full output for this period was not printed. H_1 and K_1 values were inferred from the partial output, but the H_2 and K_2 values could not be so inferred.

TABLE 3-A Cont'd
Policies Resulting from Model Runs *

Run 109					Run 110				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	23	35	44	202	10	31	39	46	203
9	25	65	77	167	9	32	70	80	170
8	22	67	108	176	8	32	73	112	180
7	24	67	138	196	7	32	73	142	200
6	23	67	167	220	6	32	73	171	224
5	24	67	195	246	5	32	73	199	250
4	23	67	222	272	3	32	73	227	277
3	24	67	248	298	3	32	73	-	-
2	23	67	-	-	2	32	73	-	-
1	24	67	-	-	1	32	73	-	-

Run 111					Run 112				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	27	38	48	49	10	31	39	48	49
9	29	68	85	86	9	33	71	86	87
8	27	95	119	120	8	32	97	121	122
7	28	71	151	152	7	33	74	153	154
6	28	71	183	184	6	32	74	185	186
5	28	71	214	215	5	32	74	216	217
4	28	71	244	245	4	32	74	246	247
3	28	71	273	274	3	32	74	275	276
2	28	71	-	-	2	32	74	-	-
1	28	71	-	-	1	32	74	-	-

* Inventory Level, H Policy

$H \leq H_1$ Buy $K_1 - H$
 $H \geq H_2$ Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd
Policies Resulting from Model Runs*

Run 201					Run 202				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	17	23	31	32	10	23	27	33	34
9	18	26	54	55	9	24	30	57	58
8	23	32	81	82	8	29	36	84	85
7	23	32	105	106	7	29	36	109	110
6	29	38	134	135	6	35	42	138	139
5	29	38	162	163	5	35	42	165	166
4	34	43	193	194	4	41	48	196	197
3	34	43	222	223	3	41	48	225	226
2	39	49	253	254	2	46	54	257	258
1	39	49	281	282	1	46	54	284	285

Run 203					Run 204				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	17	26	34	35	10	20	28	35	36
9	19	47	58	59	9	22	49	60	61
8	23	70	87	88	8	27	72	88	89
7	23	61	113	114	7	27	64	115	116
6	28	65	145	146	6	32	68	146	147
5	28	90	175	176	5	32	74	177	178
4	33	76	209	210	4	38	79	211	212
3	33	81	243	244	3	37	85	245	246
2	38	86	281	282	2	43	90	282	283
1	37	92	-	-	1	43	95	-	-

<u>*Inventory Level, H</u>	<u>Policy</u>
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

**Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd
Policies Resulting from Model Runs *

Run 205					Run 206				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	20	28	34	35	10	15	21	28	185
9	22	49	58	59	9	18	26	51	140
8	27	71	86	87	8	23	32	77	145
7	27	63	112	113	7	23	31	101	160
6	32	68	142	143	6	28	37	130	183
5	32	74	171	172	5	28	37	157	209
4	38	78	204	205	4	33	43	189	240
3	37	84	235	236	3	33	43	219	271
2	43	89	269	270	2	38	48	-	-
1	42	95	-	-	1	38	48	-	-

Run 207					Run 208				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	15	21	27	105	10	22	27	32	189
9	17	25	49	94	9	24	30	56	145
8	23	31	73	109	8	29	36	82	150
7	23	31	96	127	7	29	36	106	165
6	28	37	123	155	6	35	42	135	188
5	28	37	147	181	5	35	42	162	214
4	33	42	173	210	4	41	48	194	245
3	33	42	192	238	3	41	48	223	276
2	38	48	206	265	2	46	54	-	-
1	38	48	221	285	1	46	54	-	-

* Inventory Level, H Policy

$H \leq H_1$ Buy $K_1 - H$

$H \geq H_2$ Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd
Policies Resulting from Model Runs *

Run 209					Run 210				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	13	24	32	189	10	20	27	33	190
9	16	44	54	143	9	22	48	57	145
8	19	66	80	148	8	27	71	84	151
7	18	58	106	164	7	26	63	109	167
6	24	62	135	188	6	32	68	139	192
5	23	68	164	214	5	32	74	168	218
4	29	72	197	246	4	38	78	200	250
3	28	78	228	278	3	37	84	232	282
2	33	83	-	-	2	43	89	-	-
1	32	88	-	-	1	42	95	-	-

Run 211					Run 212				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	17	26	35	36	10	20	28	35	36
9	19	47	61	62	9	22	49	62	63
8	23	70	90	91	8	27	72	91	92
7	23	61	118	119	7	27	64	119	120
6	28	66	150	151	6	33	68	152	153
5	28	91	181	182	5	32	95	183	184
4	33	76	217	218	4	38	79	219	220
3	33	82	252	253	3	37	85	254	255
2	38	87	292	293	2	43	90	294	295
1	38	92	-	-	1	43	96	-	-

* Inventory Level, H	Policy
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 301					Run 302				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	30	44	60	61	10	50	57	60	61
9	41	56	110	111	9	53	60	116	117
8	41	57	147	148	8	53	60	156	157
7	41	57	174	175	7	53	60	182	183
6	41	57	199	200	6	53	60	206	207
5	41	57	220	221	5	53	60	226	227
4	41	57	237	238	4	53	60	243	244
3	41	57	251	252	3	53	60	257	258
2	41	57	262	263	2	53	60	267	268
1	41	57	270	271	1	53	60	274	275

Run 303					Run 304				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	33	53	60	61	10	45	58	60	61
9	39	60	117	118	9	48	62	118	119
8	39	81	162	163	8	48	82	167	168
7	39	101	199	200	7	48	109	204	205
6	39	102	228	229	6	48	110	233	234
5	39	102	256	257	5	48	110	260	261
4	39	102	281	282	4	48	110	285	286
3	39	102	-	-	3	48	110	-	-
2	39	102	-	-	2	48	110	-	-
1	39	102	-	-	1	48	110	-	-

* Inventory Level, H Policy

$H \leq H_1$ Buy $K_1 - H$

$H \geq H_2$ Dispose $H - K_2$

**Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 305					Run 306				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	45	58	60	61	10	28	42	59	207
9	47	60	116	117	9	40	55	105	194
8	47	71	162	163	8	40	56	137	213
7	47	94	191	192	7	40	56	164	236
6	47	109	216	217	6	40	56	189	260
5	47	109	239	240	5	40	56	210	283
4	47	109	260	261	4	40	56	-	-
3	47	109	277	278	3	40	56	-	-
2	47	109	291	292	2	40	56	-	-
1	47	109	-	-	1	40	56	-	-

Run 307					Run 308				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	28	41	58	133	10	50	56	60	208
9	39	54	99	149	9	53	60	114	200
8	40	55	124	175	8	53	60	146	222
7	40	55	147	198	7	53	60	172	244
6	40	55	165	219	6	53	60	196	268
5	40	55	181	236	5	53	60	218	291
4	40	55	192	250	4	53	60	-	-
3	40	55	198	261	3	53	60	-	-
2	40	55	201	267	2	53	60	-	-
1	40	55	202	271	1	53	60	-	-

* Inventory Level, H Policy

$H \leq H_1$ Buy $K_1 - H$

$H \geq H_2$ Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 309					Run 310				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	19	46	60	208	10	44	57	60	208
9	31	60	109	197	9	47	60	114	200
8	29	67	143	217	8	47	71	153	228
7	30	89	171	242	7	47	93	180	251
6	30	91	197	268	6	47	109	207	277
5	30	91	221	293	5	47	109	-	-
4	30	91	-	-	4	47	109	-	-
3	30	91	-	-	3	47	109	-	-
2	30	91	-	-	2	47	109	-	-
1	30	91	-	-	1	47	109	-	-

Run 311					Run 312				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	32	52	-	-	10	45	57	-	-
9	39	60	-	-	9	48	62	-	-
8	39	85	-	-	8	48	87	-	-
7	39	102	-	-	7	48	110	-	-
6	39	103	-	-	6	48	111	-	-
5	39	103	-	-	5	48	110	-	-
4	39	103	-	-	4	48	110	-	-
3	39	103	-	-	3	48	110	-	-
2	39	103	-	-	2	48	110	-	-
1	39	103	-	-	1	48	110	-	-

<u>*Inventory Level, H</u>		<u>Policy</u>
$H \leq H_1$		Buy $K_1 - H$
$H \geq H_2$		Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 401					Run 402				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	30	44	60	61	10	50	57	60	61
9	36	51	107	108	9	52	59	114	115
8	39	54	137	138	8	52	59	145	146
7	40	56	160	161	7	53	60	167	168
6	41	57	182	183	6	53	60	189	190
5	42	57	204	205	5	53	60	209	210
4	42	58	223	224	4	53	60	229	230
3	42	58	243	244	3	53	60	248	249
2	43	59	261	262	2	53	60	265	266
1	43	59	278	279	1	53	60	282	283

Run 403					Run 404				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	33	53	60	61	10	45	58	60	61
9	37	58	115	116	9	47	60	117	118
8	38	60	157	158	8	47	60	162	163
7	38	60	187	188	7	47	60	191	192
6	39	96	214	215	6	48	97	218	219
5	41	104	241	242	5	49	111	245	246
4	41	107	266	267	4	49	113	270	271
3	42	110	291	292	3	49	114	295	296
2	42	112	-	-	2	50	116	-	-
1	42	114	-	-	1	50	117	-	-

<u>*Inventory Level, H</u>		<u>Policy</u>
$H \leq H_1$		Buy $K_1 - H$
$H \geq H_2$		Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 405					Run 406				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	45	58	60	61	10	28	42	59	207
9	46	60	115	116	9	35	50	102	209
8	47	60	153	154	8	38	53	129	219
7	47	60	174	175	7	40	55	153	233
6	47	73	200	201	6	40	56	175	251
5	48	109	222	223	5	41	57	197	270
4	49	112	244	245	4	42	58	217	289
3	49	113	265	266	3	42	58	-	-
2	49	114	285	286	2	42	58	-	-
1	49	115	-	-	1	42	59	-	-

Run 407					Run 408				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	28	41	58	133	10	50	56	60	208
9	34	49	95	155	9	52	58	112	216
8	37	52	113	171	8	52	59	137	228
7	39	54	135	190	7	53	60	161	241
6	40	55	153	207	6	53	60	182	259
5	40	56	168	223	5	53	60	204	278
4	41	57	183	238	4	53	60	223	296
3	41	57	197	252	3	53	60	-	-
2	41	57	209	265	2	53	60	-	-
1	42	58	220	277	1	53	60	-	-

<u>* Inventory Level, H</u>	<u>Policy</u>
$H \leq H_1$	Buy $K_1 - H$
$H \geq H_2$	Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd

Policies Resulting from Model Runs*

Run 409					Run 410				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	19	46	60	208	10	44	57	60	208
9	26	54	106	212	9	46	59	113	217
8	28	58	136	224	8	47	60	145	234
7	29	60	160	239	7	47	60	169	248
6	30	72	184	259	6	47	72	193	268
5	32	94	207	279	5	48	109	216	288
4	32	98	229	300	4	49	112	-	-
3	33	101	-	-	3	49	113	-	-
2	33	104	-	-	2	49	114	-	-
1	34	106	-	-	1	49	115	-	-

Run 411					Run 412				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	32	52	-	-	10	45	57	-	-
9	36	58	-	-	9	46	60	-	-
8	37	60	-	-	8	47	60	-	-
7	38	60	-	-	7	47	60	-	-
6	39	100	-	-	6	48	108	-	-
5	41	105	-	-	5	49	112	-	-
4	41	109	-	-	4	49	114	-	-
3	42	111	-	-	3	49	115	-	-
2	42	114	-	-	2	50	117	-	-
1	43	116	-	-	1	50	119	-	-

* Inventory Level, H Policy

$H \leq H_1$ Buy $K_1 - H$

$H \geq H_2$ Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd
Policies Resulting from Model Runs*

Run 501					Run 502				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	19	30	40	41	10	33	38	40	41
9	23	35	72	73	9	34	39	77	78
8	31	45	100	101	8	43	49	106	107
7	33	47	123	124	7	44	50	128	129
6	41	56	151	152	6	53	60	155	156
5	42	57	177	178	5	53	60	181	182
4	50	67	208	209	4	63	70	213	214
3	50	68	237	238	3	63	70	241	242
2	59	77	271	272	2	72	80	275	276
1	59	78	-	-	1	72	80	-	-

Run 503					Run 504				
<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	<u>Period**</u>	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	20	37	40	41	10	29	39	40	41
9	22	40	78	79	9	29	40	79	80
8	31	50	114	115	8	39	50	118	119
7	31	57	142	143	7	39	57	145	146
6	40	88	176	177	6	48	91	179	180
5	41	103	206	207	5	49	110	209	210
4	49	115	244	245	4	58	121	247	248
3	50	125	278	279	3	58	131	282	283
2	58	136	-	-	2	67	142	-	-
1	59	146	-	-	1	68	152	-	-

* Inventory Level, H Policy

$H \leq H_1$ Buy $K_1 - H$

$H \geq H_2$ Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd
Policies Resulting from Model Runs*

Run 505					Run 506				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	28	39	40	41	10	16	28	39	186
9	29	40	77	78	9	21	33	68	172
8	38	50	113	114	8	30	44	93	179
7	38	50	134	135	7	32	46	116	191
6	48	71	165	166	6	41	56	143	214
5	48	109	194	195	5	41	57	168	238
4	57	120	227	228	4	50	66	198	270
3	58	130	257	258	3	50	67	226	298
2	67	140	294	295	2	59	77	-	-
1	67	150	-	-	1	59	78	-	-

Run 507					Run 508				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	16	27	39	113	10	32	38	40	187
9	21	33	63	120	9	33	39	75	177
8	30	43	83	135	8	43	49	99	186
7	31	45	102	152	7	43	50	122	197
6	40	55	127	177	6	53	60	149	220
5	40	56	147	198	5	53	60	173	243
4	49	65	172	226	4	62	70	203	275
3	49	66	194	250	3	63	70	-	-
2	58	75	220	279	2	72	80	-	-
1	58	76	-	-	1	72	80	-	-

* Inventory Level, H Policy

$H \leq H_1$ Buy $K_1 - H$

$H \geq H_2$ Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

TABLE 3-A Cont'd
Policies Resulting from Model Runs*

Run 509					Run 510				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	10	32	40	187	10	28	39	40	187
9	14	38	72	176	9	29	40	76	178
8	22	49	99	184	8	38	50	107	191
7	22	50	122	196	7	38	50	129	203
6	31	70	151	222	6	48	70	157	229
5	31	94	178	247	5	48	109	185	253
4	39	105	209	280	4	57	120	216	287
3	40	115	-	-	3	58	130	-	-
2	48	126	-	-	2	67	140	-	-
1	48	135	-	-	1	67	150	-	-

Run 511					Run 512				
Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>	Period**	<u>H₁</u>	<u>K₁</u>	<u>K₂</u>	<u>H₂</u>
10	19	36	-	-	10	29	39	40	41
9	22	40	-	-	9	30	40	80	81
8	30	50	-	-	8	39	50	122	123
7	30	63	-	-	7	49	63	156	157
6	40	91	-	-	6	48	96	192	193
5	41	104	-	-	5	49	111	227	228
4	49	116	-	-	4	58	122	268	269
3	50	126	-	-	3	59	132	-	-
2	58	138	-	-	2	67	143	-	-
1	59	148	-	-	1	68	153	-	-

* Inventory Level, H Policy

$H \leq H_1$ Buy $K_1 - H$

$H \geq H_2$ Dispose $H - K_2$

** Real time periods; i.e. period $n + 1$ is later than period n .

APPENDIX B
FORTRAN PROGRAMS FOR INVENTORY MODELS
DEVELOPED IN CHAPTER 4 AND 6

The listings of the FORTRAN programs used for calculating optimal policies in Chapters 4 and 6 are presented in this appendix. The program decks are available on request.

B-2

FORTTRAN Program for Dynamic Procurement-
Disposal Model in Chapter IV.

```

..JOB1
PROGRAM XINVENT
  DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
    CXMU(100),PA(100)
  DIMENSION TEMP1(4)
  COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
    CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
    CSMALA,RATE,TEMP1
  DIMENSION MSI(4001),MMK(2001)
  DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)

  75 CONTINUE
    WRITE OUTPUT TAPE 6,9876
9876  FORMAT(IH1)
    READ INPUT TAPE 5,2001,NRIN,EPS
    WRITE OUTPUT TAPE 6,2001,NRUN,EPS
2001  FORMAT(I10/F10.8)
  31  12=1
  31  N1=0
  DO30 I=1,20
  30  ICOUNT(I)=0
    IFLAG=0
    READ INPUT TAPE5,2000,IFLAG
2000  FORMAT(I10)
    CALL ASUB1
    READ INPUT TAPE 5,2003,IREAL,JK,JCST,JCIS,JCOT,JTHET,JRATE,JDIST
    WRITE OUTPUT TAPE 6,2003,IREAL,JK,JCST,JCIS,JCOT,JTHET,JRATE,JDIST
2003  FORMAT(8I5)
  8  CALL DATA
    CALL THETA
    ISTRT=JPA+1-MAXI
    IEND=JPA+1
    KTRL=1
    DO 200 IP=ISTRT,IEND
    IT=JPA+1-IP
    GO TO (10,11),KTRL
  10  IF(IT-IREAL)12,11,11
  12  READ INPUT TAPE 5,2003,IREAL,JK,JCST,JCIS,JCOT,JTHET,JRATE,JDIST

```

```

WRITE OUTPUT TAPE 6,2003,IREAL,JK,JCST,JGIS,JCOT,JTHET,JRATE,JDIST
IF(IREAL)13,14,13
13 CALL DATA
   GO TO 10
14 KTRL=2
11 CONTINUE
   XMJ(IP)=XMEAN
   SMALA=PA(IP)/(ASUM+PA(IP))
   ASUM=ASUM+PA(IP)
   IF(JK)15,16,15
16 IF(JTHET)15,17,15
15 CALL THETA
17 CONTINUE
   IF(IP-ISTRT)105,106,105
106 I1=2*K+1
107 DO102 I=1,I1
   MSI(I)=K+1-I
   IF(MSI(I))102,104,104
104 MMK(I)=I-1
102 VMIN(I)=THET(I)
   IMAX=IT
   GO TO 67
105 IH=K
   KUP1=2*IH+1
   KUP2=K+1
   DO 199 IHH=1,KUP1
   VMIN(IHH)=1.F+25
   DO 198 KK=1,KUP2
   KDUM=IHH+KK-1
   V=PHI(KK)/(1.+RATE)+THET(KDUM)
   IF (VMIN(IHH)-V)198,198,45
45 VMIN(IHH)=V
   IS(IHH)=KK-1
198 CONTINUE
199 CONTINUE
   LLJ=1
   LLK=XMINOF(120,I1)

```

```

IF(IFLAG)300,61,300
300 N1=N1+1
    N2=1
    I2=I2
    DO310 J=1,I1
    GO TO(301,302,303),N2
301 IF(IS(J)-MSI(J))310,304,310
302 IF(IS(J)-MSI(J))310,310,306
303 IF(IS(J)-IS(J+1))307,310,307
304 N2=N2+1
    I4=J-1
    IF(J-1)305,305,308
305 IH1(I2)=0
    IK(I2)=0
    XM(I2)=0
    I2=I2+1
    GO TO 310
306 N2=N2+3
    I4=J
    ICOUNT(N1)=ICOUNT(N1)+1
    GO TO 308
307 N2=N2+1
    I4=J+1
    IF(J-I1)308,310,310
308 IH1(I2)=MSI(I4)
    IK(I2)=IS(I4)
    XM(I2)=VMIN(I4)
    I2=I2+1
310 CONTINUE
    GO TO 67
61 WRITE OUTPUT TAPE6,1012
1012 FORMAT(16H1 ORDER POLICY /56H H=AMOUNT ON HAND AT THE END OF
    C PREVIOUS TIME PERIOD)
    WRITE OUTPUT TAPE 6,1013
1013 FORMAT(53H M(H)=EXPECTED COST OF POLICY FROM THIS PERIOD ON)
64 WRITE OUTPUT TAPE 6,1014,(MSI(I),IS(I),VMIN(I),I=LLJ,LLK)
1014 FORMAT(115H1 H K M(H) H

```

```

      X      K      M(H)      H      K      M(H)/(3(2I
C12,F14.2,1H )))
63 IF(LLK-I1)66,67,67
66 LLJ=LLK+1
   LLK=LLK+XMINOF(120,I1-LLK)
   IF(IFLAG)67,64,67
67 IF(IT)70,74,70
74 TEMX=0.
   DO 90 I=1,ISTRT,JPA
   IJ=JPA+1-I
   XI=1
90 TEMX=TEMX+XMU(IJ)*PA(IJ)*XI
   IF(IFLAG)320,91,320
320 WRITE OUTPUT TAPE6,1012
   WRITE OUTPUT TAPE6,1013
   WRITE OUTPUT TAPE6,1018
1018 FORMAT(/54H   IF H IS GREATER THAN OR EQUAL TO H2, DISPOSE H-K2/
X47H   IF H IS LESS THAN OR EQUAL TO H1, BUY K1-H//5X,6HPERIOD,3X
X,2HH1,5X,2HK1,5X,4HM(H),5X,2HH2,5X,2HK2,5X,4HM(H)//)
   J1=1
D0325 I=1,IMAX
   IT=IMAX+1-I
   J2=J1+2*(ICOUNT(I)-1)
   WRITE OUTPUT TAPE6,2010,IT,(IH1(J+1),IK(J+1),XM(J+1),IK(J),
   XXM(J),J=J1,J2,2)
2010 FORMAT(I9,(2(2I7,F9.2))//)
325 J1=J2+2
91 WRITE OUTPUT TAPE6,1021,TEMX
1021 FORMAT(40H1 EXPECTED TOTAL DEMAND THROUGH LIFETIME F10.2)
   GO TO 75
70 WRITE OUTPUT TAPE 6,1015,IT,SMALA
1015 FORMAT(25H1 BEGIN REAL TIME PERIOD 13,81H. CONDITIONAL PROBABILI
   CTY OF OBSOLESCENCE OCCURRING AT THE END OF THIS PERIOD IS F9.6)
94 KUP1=K+1
   KUP2=IR
   DO 197 KK=1,KUP1
   PH1(KK)=0.

```

```

      LLL=IS(4001)
      DO 196 IRR=LLL,KUP2
      IF(KK-IRR)51,50,50
50    CALL PSIONE
      GO TO 52
51    CALL PSITWO
52    PHI(KK)=PHI(KK)+PRO(IRR)*PSI
196    CONTINUE
197    CONTINUE
      LLJ=1
      LLK=XMINOF(200,K+1)
      IF(IFLAG-1)195,195,96
195    WRITE OUTPUT TAPE6,1016,IT
1016  FORMAT(76H   PHI(K)=EXPECTED FUTURE COSTS RESULTING FROM THE BEGI
      XNNING OF TIME PERIODI3,23H WITH AN INVENTORY OF K)
84    WRITE OUTPUT TAPE 6,1017,(MMK(I),PHI(I),I=LLJ,LLK)
1017  FORMAT(119H1      K      PHI(K)      K      PHI(K)      K      PHI(K)/(
      X K      PHI(K)      K      PHI(K)      K      PHI(K)/(
      C5(110,F13.2,1H ))
96    IF(LLK-K-1)86,87,87
86    LLJ=LLK+1
      LLK=LLK+XMINOF(200,K+1-LLK)
      IF(IFLAG-1)84,84,87
87    CONTINUE
200    CONTINUE
      CALL EXIT
      END
      SUBROUTINE DATA
      DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
      CXMU(100),PA(100)
      DIMENSION TEMPI(4)
      COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
      CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
      CSMALA,RATE,TEMPI
      DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
      IF(JK)1,2,1
1    READ INPUT TAPE 5,1001,K

```

```

      WRITE OUTPUT TAPE 6,1001,K
1001 FORMAT(I5)
      2 IF(JCST)5,6,5
      5 READ INPUT TAPE 5,1002,CST
      WRITE OUTPUT TAPE 6,1002,CST
1002 FORMAT(4F10.4)
      6 IF(JCIS)7,8,7
      7 READ INPUT TAPE 5,1002,CIS
      WRITE OUTPUT TAPE 6,1002,CIS
      8 IF(JCOT)9,10,9
      9 READ INPUT TAPE 5,1002,COT
      WRITE OUTPUT TAPE 6,1002,COT
      10 IF(JTHET)11,12,11
      11 READ INPUT TAPE 5,1002,(TEMP1(I),I=1,4)
      WRITE OUTPUT TAPE 6,1002,(TEMP1(I),I=1,4)
      12 IF(JRATE)13,14,13
      13 READ INPUT TAPE 5,1002,RATE
      WRITE OUTPUT TAPE 6,1002,RATE
      14 IF(JDIST)16,16,17
      17 IF(JDIST-3)18,18,16
      18 GO TO (20,30,40),JDIST
      20 READ INPUT TAPE 5,1003,XMEAN,DELTA
      WRITE OUTPUT TAPE 6,1003,XMEAN,DELTA
1003 FORMAT(F10.4,F10.8)
      PRO(I)=1./EXP(XMEAN)
      TEMP=PRO(I)
      DO 21 I=2,1000
      Z=I-1
      PRO(I)=PRO(I-1)*XMEAN/Z
      TEMP=TEMP+PRO(I)
      IF(TEMP+DELTA-1.)21,22,22
21 CONTINUE
22 IR=I
      DO 50 I=1,500
      IF(PRO(I)-DELTA)50,52,52
50 CONTINUE
52 IS(4001)=I

```

```

36 READ INPUT TAPE 5,1003,XMEAN
   WRITE OUTPUT TAPE 6,1003,XMEAN
   XI=XMEAN*2.+1.
   IR=XI
   P=1./XI
   DO 31 I=1,IR
31  PRO(I)=P
   RETURN
40 READ INPUT TAPE 5,1001,IR
   READ INPUT TAPE 5,1004,(PRO(I),I=1,IR)
   WRITE OUTPUT TAPE 6,1004,(PRO(I),I=1,IR)
1004 FORMAT(7F10.8)
   XMEAN=0.
   DO 41 I=1,IR
   Z=I-1
41  XMEAN=XMEAN+PRO(I)*Z
   RETURN
   END
   SUBROUTINE THETA
   DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
   CXMU(100),PA(100)
   DIMENSION TEMPI(4)
   COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
   CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
   CSMALA,RATE,TEMPI
   DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
   LR1=1
   IUP=1
   IF(JK)90,10,90
10  IF(C-TEMPI(3))30,20,30
20  IF(D-TEMPI(4))30,60,30
30  XKMN=-K-1
   IUP=IUP+K
   IF(A-TEMPI(1))50,40,50
40  IF(B-TEMPI(2))50,150,50
50  IUP=IUP+2*K

```

```

      GO TO 100
60 IF(A-TEMP1(1))80,70,80
70 IF(B-TEMP1(2))80,150,80
80 XKMN=0
   LR1=2+K
   IUP=IUP+3*K
   GO TO 100
90 XKMN=-K-1
   IUP=3*K+1
100 A=TEMP1(1)
   B=TEMP1(2)
   C=TEMP1(3)
   D=TEMP1(4)
   DO140 LR=LR1,IUP
   XKMN=XKMN+1.
   IF(XKMN)5,6,7
5  THET(LR)=C+D*XKMN
   GO TO 140
6  THET(LR)=0.
   GO TO 140
7  THET(LR)=A+B*XKMN
140 CONTINUE
150 RETURN
    END
SUBROUTINE ASUBI
  DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
  CXMU(100),PA(100)
  DIMENSION TEMP1(4)
  COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
  CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
  CSMALA,RATE,TEMP1
  DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
  READ INPUT TAPE 5,501,JPA
501 FORMAT(I5)
  READ INPUT TAPE 5,502,(PA(L),L=1,JPA)
  WRITE OUTPUT TAPE 6,502,(PA(L),L=1,JPA)
502 FORMAT(7F10.8)

```

```

SUMA=0.
DO 10 L=1,JPA
  LL=JPA+1-L
  SUMA=SUMA+PA(LL)
  IF(SUMA+EPS-1.)10,20,20
10 CONTINUE
  L=JPA
20 MAXI=L
21 ASUM=1.-SUMA
  IF(ASUM)25,26,26
25 ASUM=0.
26 CONTINUE
  RETURN
END
SUBROUTINE PSIONE
  DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
    CXMU(100),PA(100)
  DIMENSION TEMPI(4)
  COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
    CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
    CSMALA,RATE,TEMPI
  DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
  U=KK-1
  V=IRR-1
  PSI=(U-V)*CST
  CALL SAME
  RETURN
END
SUBROUTINE PSITWO
  DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
    CXMU(100),PA(100)
  DIMENSION TEMPI(4)
  COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
    CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
    CSMALA,RATE,TEMPI
  DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
  U=KK-1

```

```

V=IRR-1
PSI=(V-U)*COT
CALL SAME
RETURN
END
SUBROUTINE SAME
  DIMENSION VMIN(4001),PHI(2001),THET(6001),IS(4001),PRO(2001),
  CXMU(100),PA(100)
  DIMENSION TEMPI(4)
  COMMON VMIN,PHI,THET,IS,PRO,XMU,PA,JK,JCST,JCIS,JCOT,JTHET,JRATE,
  CJDIST,EPS,JPA,MAXI,ASUM,K,A,B,C,D,IR,XMEAN,KK,IRR,PSI,CST,COT,CIS,
  CSMALA,RATE,TEMPI
  DIMENSION IH1(80),IK(80),XM(80),ICOUNT(20)
  V=IRR-1
  IDUM=K+1+IRR-KK
  PSI=PSI+V*CIS+(1.-SMALA)*VMIN(IDUM)+SMALA*THET(IDUM)
  RETURN
END
END

```

FORTTRAN program for Dynamic Markov Inventory
Model in Chapter VI.

```

*   NAVC CONTRACT NO. 155479-04-00
*   XEQ
C-----+
      DIMENSION FNCOST(9,21)
      DIMENSION C2TAB(9,21)
      DIMENSION MIN(9,21)
      DIMENSION C1TAB(9,21)
      DIMENSION G1P1(21),G1P12(21),G2P21(21),G2P22(21),QL(9,21)
      DIMENSION G(2,26),P(2,2),D(6),IS1(21),IS2(21),IU1(21),IU2(21),
      1V01(21),V02(21),V11(21),V12(21),SE1C(21),SE2C(21),POLTAB(9,21),
      2COSTAB(9,21),PI(9)
      PI(1)=.1
      DO 10 I=2,9
      PI(I)=PI(I-1)+.1
10 CONTINUE
      READ INPUT TAPE 5,1000,(FM1,FM2,TOLCON)
      IX=25
      EEXPM1=1./((EXP(FM1))
      IF(EEXPM1-TOLCON)100,100,101
101 G(1,1)=EEXPM1
      DO 102 I=1,IX
      X=I
      K=I+1
      QUOT=1.
      DO 103 J=1,I
      QUOT=QUOT*(FM1/X)
      X=X-1.
103 CONTINUE
      G1=QUOT*EEXPM1
      IF(G1-TOLCON)104,104,105
105 G(1,K)=G1
102 CONTINUE
      GO TO 206
104 KG1TOL=I
      GO TO 106
100 KG1TOL=0
      GO TO 106

```

```

206 KG1TOL=26
106 EEXPM2=1./(EXPF(FM2))
    IF(EEXPM2-TOLCON)107,107,108
108 G(2,1)=EEXPM2
    DO 109 I=1,IX
      X=1
      K=I+1
      QUOT=1.
      DO 110 J=1,I
        QUOT#QUOT#(FM2/X)
        X=X#1.
110 CONTINUE
      G2=QUOT#EEXPM2
      IF(G2-TOLCON)111,111,112
112 G(2,K)=G2
109 CONTINUE
      GO TO 114
111 KG2TOL=I
107 KG2TOL=0
      GO TO 113
114 KG2TOL=26
113 READ INPUT TAPE 5,1001,((P(I,J),J=1,2)I=1,2))
      READ INPUT TAPE 5,1002,((D(I),I=1,6),RHO,XPSI,DELTA,FLMBDA)
      LPSI=XPSI
C-----
      WRITE OUTPUT TAPE 6,4000,(G(1,N),N=1,KG1TOL)
      WRITE OUTPUT TAPE 6,4001,(G(2,N),N=1,KG2TOL)
      4000 FORMAT(9H1 G(1,N)=,26F3.2)
      4001 FORMAT(9H0 G(2,N)=,26F3.2)
C-----
C----- +
C SETTING UP THE VALUE ITERATION TABLE
C-----
      L=(2*LPSI)+1
      DO 115 I=1,L
        IS1(I)=1

```

```

IS2(I)=2
IU1(I)=(LPSI+1)-I
IU2(I)=(LPSI+1)-I
C EVALUATE SE1CU
C EVALUATION OF ESCU
  XU=IU1(I)
  340 IF(IU1(I))300,301,301
  301 IF(KGITOL-IU1(I))302,303,303
  302 K1=KGITOL
  GO TO 304
  303 K1=IU1(I)+1
  304 TERM1=0.
  DO 305 J1=1,K1
  XJ1=J1-1
  TERM1=TERM1+((XU-XJ1)*G(1,J1))
  305 CONTINUE
  TERM1=D(1)*TERM1
  IF(KGITOL-IU1(I))306,306,307
  307 K2=KGITOL-1
  TERM2=0.
  DO 308 J2=K1,K2
  XJ1=J2
  J3=J2+1
  TERM2=TERM2+((XJ1-XU)*G(1,J3))
  308 CONTINUE
  TERM2=D(2)*TERM2
  GO TO 309
  306 TERM2=0.
  309 IF(LPSI-IU1(I))310,310,311
  311 DIFF1=LPSI-IU1(I)
  TERM3=D(3)+(D(4)*DIFF1)
  GO TO 312
  310 TERM3=0.
  312 SE1C(I)=TERM1+(TERM2+TERM3)
  GO TO 313
  300 FU=IU1(I)
  TERM1=-D(2)*FU

```

```

TERM2=0.
IF(KG1TOL-1)314,314,315
315 KG1=KG1TOL-1
DO 316 J2=1,KG1
  XJ2=J2
  TERM2=TERM2+(XJ2*G(1,J2+1))
316 CONTINUE
  TERM2=D(2)*TERM2
314 IF(LPSI-IU1(I))317,317,318
318 DIFF1=LPSI-IU1(I)
  TERM3=D(3)+(D(4)*DIFF1)
  GO TO 319
317 TERM3=0.
319 SEIC(I)=TERM1+(TERM2+TERM3)
C EVALUATE SE2CU
C EVALUATION OF ESCU
313 XU=IU1(I)
  IF(IU2(I))500,501,501
501 IF(KG2TOL-IU2(I))502,503,503
502 K2=KG2TOL
  GO TO 504
503 K2=IU2(I)+1
504 TERM1=0.
DO 505 J5=1,K2
  XJ2=J5-1
  TERM1=TERM1+(XU-XJ2)*G(2,J5))
505 CONTINUE
  TERM1=D(1)*TERM1
  IF(KG2TOL-IU1(I))506,506,507
507 K1=KG2TOL-1
  TERM2=0.
DO 508 J6=K2,K1
  XJ2=J6
  J7=J6+1
  TERM2=TERM2+(XJ2-XU)*G(2,J7))
508 CONTINUE
  TERM2=D(2)*TERM2

```

```

GO TO 509
506 TERM2=0.
509 IF(LPSI-IU2(I))510,510,511
511 DIFF1=LPSI-IU2(I)
    TERM3=D(3)+(D(4)*DIFF1)
GO TO 512
510 TERM3=0.
512 SE2C(I)=TERM1+(TERM2+TERM3)
GO TO 513
500 FU=IU2(I)
    TERM1=-D(2)*FU
    TERM2=0.
    IF(KG2TOL-1)514,514,515
515 KG2=KG2TOL-1
    DO 516 J2=1,KG2
        XJ2=J2
    TERM2=TERM2+(XJ2*G(2,J2+1))
516 CONTINUE
    TERM2=D(2)*TERM2
514 IF(LPSI-IU2(I))517,517,518
518 DIFF1=LPSI-IU2(I)
    TERM3=D(3)+(D(4)*DIFF1)
GO TO 519
517 TERM3=0.
519 SE2C(I)=TERM1+(TERM2+TERM3)
513 CONTINUE
C EVALUATE G1P11
C EVALUATE G1*P11 AND G1*P12
400 J3=(LPSI-IU1(I))+1
    IF(KG1TOL-J3)402,403,403
402 G1P11(I)=0.
GO TO 401
403 G1P11(I)=G(1,J3)*P(1,1)
C EVALUATE G1P12
401 J3=(LPSI-IU2(I))+1
    IF(KG1TOL-J3)405,406,406
405 G1P12(I)=0.

```

```

GO TO 404
406 G1P12(I)=G(1,J3)*P(1,2)
C EVALUATE G2P21
C EVALUATE G2*P21 AND G2*P22
404 IF(KG2TOL-J3)452,453,453
452 G2P21(I)=0.
GO TO 407
453 G2P21(I)=G(2,J3)*P(2,1)
C EVALUATE G2P22
407 IF(KG2TOL-J3)455,456,456
455 G2P22(I)=0.
GO TO 454
456 G2P22(I)=G(2,J3)*P(2,2)
454 CONTINUE
115 CONTINUE
KP=(2*LPSI)+1
DO 20 J=1,KP
V01(J)=300.
V02(J)=300.
20 CONTINUE
C READ INPUT TAPE 9,1003,(V01(I),I=1,L)
C READ INPUT TAPE 9,1004,(V02(I),I=1,L)
IBOX=0
124 DO 116 I=1,L
C EVALUATE V11(I)
C EVALUATION OF V11(I)
TERM1=SE1C(I)
TERM2=0.
DO 600 J=1,L
J1=L-J+1
TERM2=TERM2+(G(1,J1)*P(1,1)*V01(J1))+(G(1,J1)*P(1,2)*V02(J1))
600 CONTINUE
V11(I)=TERM1+(RHO*TERM2)
C EVALUATE V12(I)
C EVALUATION OF V12(I)
TERM1=SE2C(I)
TERM2=0.

```

```

DO 601 J=1,L
  J1=L-J+1
  TERM2=TERM2+(G(2,J1)*P(2,1)*V01(J1))+(G(2,J1)*P(2,2)*V02(J1))
601 CONTINUE
  V12(I)=TERM1+(RHO*TERM2)
116 CONTINUE
  DO 117 I=1,L
    CON=ABSF(V11(I))-V01(I))
    IF (CON-DELTA)118,118,119
118 CONTINUE
117 CONTINUE
  DO 120 I=1,L
    CON=ABSF(V12(I))-V02(I))
    IF (CON-DELTA)121,121,119
121 CONTINUE
120 CONTINUE
  GO TO 122
119 DO 123 I=1,L
  V01(I)=V11(I)
  V02(I)=V12(I)
123 CONTINUE
  IBOX=IBOX+1
C-----+
  IF(SENSE SWITCH 1)170,124
  170 WRITE OUTPUT TAPE 6,2010,(IBOX)
  2010 FORMAT(56HIS U ESC(U) G1*P1S G2*P2S VN(S,U) ITERATION NO.
    1,14)
  DO 171 I=1,L
    WRITE OUTPUT TAPE 6,2001,(IS1(I),IU1(I),SE1C(I),G1P11(I),G2P21(I),
      1V01(I))
171 CONTINUE
  DO 172 I=1,L
    WRITE OUTPUT TAPE 6,2001,(IS2(I),IU2(I),SE2C(I),G1P12(I),G2P22(I),
      1V02(I))
172 CONTINUE
C-----+
  GO TO 124

```

```

122 WRITE OUTPUT TAPE 6,2000
DO 125 I=1,L
WRITE OUTPUT TAPE 6,2001,(IS1(I),IU1(I),SE1C(I),G1P11(I),G2P21(I),
1V01(I))
125 CONTINUE
DO 126 I=1,L
WRITE OUTPUT TAPE 6,2001,(IS2(I),IU2(I),SE2C(I),G1P12(I),G2P22(I),
1V02(I))
126 CONTINUE
C -----
C MINIMIZATION OF FIRST PROBLEM
C -----
      L=(2*LPSI)+1
      PIVOT=(PI(1)*V11(1))+((1.-PI(1))*V12(1))
      DO 700 I=1,9
      DO 701 J=1,L
      POLTAB(I,J)=LPSI
      COSTAB(I,J)=(PI(I)*V11(J))+((1.-PI(I))*V12(J))
      COSTAB(I,J)=COSTAB(I,J)-PIVOT
      IF(IU1(J))850,851,851
851 U=IU1(J)
      SUM7=0.
      LU1=IU1(J)+1
      DO 852 K=1,LU1
      X=K-1
      A1=PI(I)*G(1,K)
      A2=(1.-PI(I))*G(2,K)
      HPIX=A1+A2
      SUM7=SUM7+((U-X)*HPIX)
852 CONTINUE
      SUM7=D(1)*SUM7
      LU1=LU1+1
      SUM8=0.
      DO 853 K=LU1,L
      X=K-1
      A1=PI(I)*G(1,K)
      A2=(1.-PI(I))*G(2,K)

```

```

      HPIX=A1+A2
      SUM8=SUM8+((X-U)*HPIX)
853  CONTINUE
      SUM8=D(2)*SUM8
      GO TO 854
850  U=IU1(J)
      SUM7=(-D(2)*U)
      SUM8=0.
      DO 855 K=1,L
      X=K-1
      A1=PI(1)*G(1,K)
      A2=(1.-PI(1))*G(2,K)
      HPIX=A1+A2
      SUM8=SUM8+(X*HPIX)
855  CONTINUE
      SUM8=D(2)*SUM8
854  C2TAB(I,J)=SUM7+SUM8
701  CONTINUE
700  CONTINUE
      WRITE OUTPUT TAPE 6,3002
3002 FORMAT(115H1 COSTAB PI=.5 PI=.6 PI=.7 PI=.8 PI=.9 PI=.4
1      PI=.5 PI=.6 PI=.7 PI=.8 PI=.9 PI=.4
      DO 3001 J=1,L
      WRITE OUTPUT TAPE 6,3000,(IU1(J),(COSTAB(I,J),I=1,9))
3000 FORMAT(5H0 U=,I5,4X,8(F6.2,6X),F6.2)
3001 CONTINUE
      NU=(2*LPSI)+1
      NPI=9
      JBOX=1
799  DO 800 I=1,NPI
      DO 801 J=1,NU
      LU=0
804  CONTINUE
      SQ=POLTAB(I,J)
      ISQ=SQ
      IF(ISQ-LPSI)1200,1201,1200
1200 IF(ISQ-LU)1202,1203,1202

```

```

1202 KKK=1
      GO TO 999
1301 QSMQ2=CQL
      ISQ=ISQ-1
      KKK=2
      GO TO 999
1302 QSMQ1=CQL
      ISQ=ISQ+2
      KKK=3
      GO TO 999
1303 QSMQ3=CQL
      ISQ=ISQ-1
      IF(QSMQ2-QSMQ1)1205,1205,1204
1204 IF(ISQ-1-LU)1206,1206,1207
1207 ISQ=ISQ-1
      GO TO 1202
1206 QL(I,J)=QSMQ1
      MIN(I,J)=ISQ-1
      GO TO 1210
1205 IF(QSMQ2-QSMQ3)1208,1208,1209
1208 QL(I,J)=QSMQ2
      MIN(I,J)=ISQ
      GO TO 1210
1209 IF(ISQ+1-LPSI)1211,1212,1212
1212 QL(I,J)=QSMQ3
      MIN(I,J)=ISQ+1
      GO TO 1210
1211 ISQ=ISQ+1
      GO TO 1202
1203 KKK=4
      GO TO 999
1304 QSMQ2=CQL
      ISQ=ISQ+1
      KKK=8
      GO TO 999
1308 QSMQ3=CQL
      ISQ=ISQ-1

```

```

1213 IF(QSMQ2-QSMQ3)1213,1213,1214
    QL(I,J)=QSMQ2
    MIN(I,J)=ISQ
    GO TO 1210
1214 IF(ISQ+1-LPSI)1215,1216,1216
1216 QL(I,J)=QSMQ3
    MIN(I,J)=ISQ+1
    GO TO 1210
1215 ISQ=ISQ+1
    GO TO 1203
1201 IF(LPSI-LU)1217,1217,1218
1217 KKK=5
    GO TO 999
1305 QL(I,J)=CQL
    MIN(I,J)=ISQ
    GO TO 1210
1218 KKK=6
    GO TO 999
1306 QSMQ2=CQL
    ISQ=ISQ-1
    KKK=7
    GO TO 999
1307 QSMQ1=CQL
    ISQ=ISQ+1
    IF(QSMQ2-QSMQ1)1217,1217,1219
1219 IF(ISQ-1-LU)1220,1220,1221
1220 QL(I,J)=QSMQ1
    MIN(I,J)=ISQ-1
    GO TO 1210
1221 ISQ=ISQ-1
    GO TO 1218
1210 CONTINUE
C EVALUATE C1(U,PI) AND STORE IN C1TAB(I,J)
    C1TAB(I,J)=C2TAB(I,J)+QL(I,J)
    PIVOT=C2TAB(1,1)+QL(1,1)
    POLTAB(I,J)=MIN(I,J)
    C1TAB(I,J)=C1TAB(I,J)-PIVOT

```

```

801 CONTINUE
800 CONTINUE
    DO 820 I=1,NPI
    DO 821 J=1,NU
    DIFF=ABSF(C1TAB(I,J))-COSTAB(I,J)
    IF(DIFF-FLMBDA)822,822,823
822 CONTINUE
821 CONTINUE
820 CONTINUE
    GO TO 824
823 DO 825 I=1,NPI
    DO 826 J=1,NU
    COSTAB(I,J)=C1TAB(I,J)
826 CONTINUE
825 CONTINUE
    IF(SENSE SWITCH 4)3051,3052
3051 WRITE OUTPUT TAPE 6,3002
    DO 3050 J=1,NU
    WRITE OUTPUT TAPE 6,3000,(IU1(J),(COSTAB(I,J),I=1,9))
3050 CONTINUE
    WRITE OUTPUT TAPE 6,2502
    DO 3060 J=1,NU
    WRITE OUTPUT TAPE 6,3000,(IU1(J),(POLTAB(I,J),I=1,9))
3060 CONTINUE
    WRITE OUTPUT TAPE 6,3099,(JBOX)
3099 FORMAT(14H0ITERATION NO.,14)
3052 CONTINUE
    IF(SENSE SWITCH 3)5010,5009
5009 JBOX=JBOX+1
    GO TO 799
824 CONTINUE
    SCON=PIVOT/(1.-RHO)
    DO 888 I=1,NPI
    DO 889 J=1,NU
    FNCOST(I,J)=C1TAB(I,J)+SCON
889 CONTINUE
888 CONTINUE

```

```

WRITE OUTPUT TAPE 6,2500
2500 FORMAT(115H1 COSTAB PI=.1 PI=.2 PI=.3 PI=.4
1 PI=.5 PI=.6 PI=.7 PI=.8 PI=.9)
DO 875 J=1,NU
WRITE OUTPUT TAPE 6,2501,(IU1(J),(FNCOST(I,J),I=1,9))
2501 FORMAT(5H0 U=,I5,4X,8(F6.2,6X),F6.2)
875 CONTINUE
WRITE OUTPUT TAPE 6,2502
2502 FORMAT(115H1 POLTAB PI=.1 PI=.2 PI=.3 PI=.4
1 PI=.5 PI=.6 PI=.7 PI=.8 PI=.9)
DO 876 J=1,NU
WRITE OUTPUT TAPE 6,2501,(IU1(J),(POLTAB(I,J),I=1,9))
876 CONTINUE
WRITE OUTPUT TAPE 6,3029,(JBOX)
3029 FORMAT(25H0 THE COST TABLE REQUIRED,I3,11H ITERATIONS)
5010 CONTINUE
999 F1SQ=ISQ
L=ISQ
SUM1=0.
DO 806 K=1,NU
X=K-1
C EVALUATE C(L-X,PI*(X)).HPI(X) AND STORE IN SUMA
KX=K
A1=PI(I)*G(1,K)
A2=(1.-PI(I))*G(2,K)
B1=P(1,1)*A1
B2=P(2,1)*A2
HPIX=A1+A2
PRIME=(B1+B2)/HPIX
K1=INTF(10.*PRIME)
K2=K1+1
IF(L-KX+1+LPSI)950,951,951
951 K3=LPSI+KX-L
T1=COSTAB(K2,K3)-COSTAB(K1,K3)
T2=(PRIME-PI(K1))/.*1
SUMA=T1*T2+COSTAB(K1,K3)

```

```

GO TO 954
950 S1=COSTAB(1,K1)-((COSTAB(NU,K1)-COSTAB(1,K1))*(FISQ-X))
S2=COSTAB(1,K2)-((COSTAB(NU,K2)-COSTAB(1,K2))*(FISQ-X))
T1=S2-S1
T2=(PRIME-PI(K1))/0.1
SUMA=T1*T2+S1
953 SUM1=SUM1+(SUMA*HP1X)
806 CONTINUE
IF(L-IU1(J))807,808,807
808 SUM2=0.
GO TO 809
807 DIFF=L-IU1(J)
SUM2=D(3)+(D(4)*DIFF)
809 CQL=(RHO*SUM1)+SUM2
GO TO (1301,1302,1303,1304,1305,1306,1307,1308),KKK
1000 FORMAT(6X,F9.5,1X,F9.5,E12.4)
1001 FORMAT(5X,4(1X,F9.5))
1002 FORMAT(7X,8(F5.2),1X,2F5.3)
2000 FORMAT(41HIS U ESC(U) G1*P1S G2*P2S VT(S,U))
2001 FORMAT(1H0,11,1X,12,2X,F6.2,3X,F6.2,3X,F6.2,3X,F7.2)
C THIS IS THE END OF PROGRAM
END

```

Appendix C

Discussion of ASO Procedures

A. Demand ("Requirement") Prediction

Two systems of demand prediction are in use at ASO: The Program Usage Replenishment System (PURS) and the Replenishment Demand Issue System (RDIS). Under both systems total demand is estimated for a time period equal to lead time (production lead plus activity lead plus administrative lead) plus a six month "safety level" time plus a six month "order cycle". Safety level and order cycle do not vary from item to item, although lead time does somewhat. The first two numbers are established by fiat and are independent of costs associated with items (holding, purchase, stockout, etc.); so, since the lead time cannot be manipulated, it is clear that there is no room for optimization under either PURS or RDIS.

Under PURS, past usage records are kept in terms of items consumed by aircraft maintenance cycle (240 hours for all aircraft except jets, for which it is 60 hours). These records are divided into a maintenance rate (items consumed per flying hour) and an overhaul requirement (items consumed in overhaul at the end of the cycle). These records are gathered from user activities in the field.

From CNO and BUWEPS come estimates of expected flying hours and overhaul schedules during the following six months and for the ninth month from payment. A technical file is maintained which provides information on item applications. There appears to be a very slight relationship

between active lead time and the figure used in these calculations.

PURS then uses the usage records, expected future flying hours, and the technical file to produce a requirement for each item.

Two major weaknesses are inherent in PURS. The first is the arbitrariness of the demand period considered and the second is the unreliability of the data. Usage records are poorly kept at activities and, further, no significant correlations have been found either between predicted flying hours and actual flying hours or between actual flying hours and demand.

Under the second demand prediction system, RDIS, the assumption is made that future demand will be the same as past demand. This system has the advantage of simplicity, and it does not rely on as much unreliable data as does PURS. However, its demand estimates are no better than those generated by PURS, and it has less scientific appeal.

B. Requirement Adjustment for Repairables

For repairable items, predicted demand is reduced by predicted returns of items which have successfully passed the maintenance cycle. The maintenance cycle is defined as six months for most items, though only three for some high value articles. Time spent by an item in the cycle is referred to as "turnaround time". The cycle consists of three states: a) the item is removed from an aircraft and screened for repairability. If it looks satisfactory, then b) it goes to the maintenance depot where it is screened again. If it is still alright, it is entered in the stock records and c) goes to the overhaul shop.

There it gets the final stamp of approval, is repaired, and is put on the shelf.

The "recovery rate" is defined as the fraction of items removed from aircraft which return from the maintenance cycle. The "RB recovery rate" is the fraction of those which are not discarded at removal which return from stages b) and c) of the cycle.

The order requirement for repairables (O) is calculated as gross expected demand (D) minus expected item removals (R_1) times recovery rate (r) minus damaged equipment (d) reported on hand times RB (RB) recovery rate minus amount (H) on hand and on order.

$$O = D - R_1 \times r - d \times RB - H$$

The time period considered for demand is as defined for PURS and RDIS, the sum of lead time, safety level and order cycle. The removal period begins a turn-around time earlier and ends a turnaround time plus a safety level earlier.

C. Life of Type Items

At one time the Navy estimated its requirement for some items by "calculating" total demand overall time and reducing this by estimated recoveries. This method was usually used for high recovery rate items. At present, however, they buy only eighteen months worth of such items. This generally amounts to the same thing, when safety factors have been taken into account.

ASO has a formula which it uses to decide whether to dispose of some life-of-type aircraft parts. The formula produces the expected number

of maintenance cycles in the remaining life of the aircraft to which the part is applied; this number multiplied by the appropriate usage rate produces a desired number of parts on hand. If this is less than the actual number on hand, then ASO will dispose. The formula is:

$$\text{Number of maintenance cycles in life of item} = \frac{Nh}{240} \left[\int_0^x a^t dt - j e \int_0^{x-kb} a^t dt \right]$$

where N is number of aircraft, h is average monthly flying hours per aircraft, a is aircraft survival rate, e is system recovery fraction, x is number of months of remaining life averaged over all aircraft calculated from service tours remaining, k is number of months turnaround time plus safety level, g is fraction of aircraft operating, and j is a safety factor which ASO has introduced as a hedge against the possible unreliability of this formula.

D. Economic Order Quantity

A procedure for calculating an "economic order quantity" (EOQ) is applied to some consumable items. It produces an operating level of an item to which lead time, etc., are then added. The EOQ is required to lie between the operating level calculated on the basis of estimated demand over the time period mentioned above and the life of type estimate of the

item. It is designed to correct against uneconomically small orders, but cannot prevent the more usual uneconomically large orders. The EOQ procedure also provides a scientific method for computing a safety level as follows:

- a) One computes $\sigma_{DL} = \sqrt{\bar{D}^2 \bar{L} + \bar{L} \bar{D}}$ where $\bar{D}$ is expected demand and $\bar{L}$ is expected lead time. The first $\bar{L}$ and the second $\bar{D}$ in the equation are approximations to standard deviations of demand and lead time.
- b) One computes $R = (Q \times I \times C) / (T \times H \times E)$ where Q is the economic order quantity, I is a discount factor which includes interest, obsolescence, and holding cost, C is the unit price, T is estimated annual demand, H is the shortage cost, and E is "military essentiality."
- c) One uses $I-R$ and a table of normal variates to find K , the number of multiples of standard deviation.
- d) One declares that the safety level is K times σ_{DL} . This is assumed to be in units of quantity, dimensional analysis to the contrary notwithstanding.

E. Critique of These Procedures

A major handicap in the Navy's thinking about supply procedures lies in the depiction of expected demand estimates, based on suspect data and an arbitrary time frame, as "requirements." Once a course of action having the repercussions of this one has been called a requirement

to be met at all cost, then no quantity of peripheral models and techniques will introduce a significant increase in supply efficiency. If the Navy has any desire to reduce the huge cost of its supply operation it must have a system which considers simultaneously demand, purchase costs, holding costs, and outage costs, and which neither equates quantities with times nor substitutes mean demands for distributions of demands.

If the concept of "safety-level" had any operational meaning, then it clearly should vary from item to item. In fact, however, it does not have meaning. For a given state of the inventory system, there exists a best quantity of each item to order. Other things remaining constant, this best quantity will vary with the production set-up cost of the item. If there were such a thing as a safety level, then, it would have to be something which is a function of set-up cost. This is clearly a contradiction in terms.

F. The Data System

There are two major weaknesses in the ASO data collection and maintenance procedures. The first is the unreliability of what is collected and the second is the choice of what ought to be collected. The unreliable data problem is one which ASO can hardly solve by itself, but which must be faced by the Navy as a whole; however, something can be done about deciding what data would be useful to have.

Certainly summaries of the obsolescence experience of items should be kept--e.g., for each category of item, the average lifespan of its

members. Similarly, summaries of demand history should be maintained. Stockout reports should be input to the data collection system. This sort of information will be required by any effective inventory control model, and, together with such a model, could enable a truly efficient operation to be carried on.

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