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FTD-TT- 63-182

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TRANSLATION

ON THE CONSTANCY OF TANGENTIAL RUPTURE IN A RAREFIED PLASMA

BY

L. D. Pichakhchi

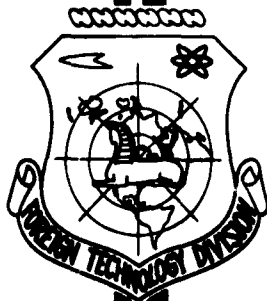
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English Pages: 8

SOURCE: Ukrainian periodical, Ukrains'kiy Fizichniy
Zhurnal, Vol. 6, Nr. 1, January-February 1961,
pp 86-92

S/185-61-6-1

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ON THE CONSTANCY OF TANGENTIAL RUPTURE IN A
RAREFIED PLASMA

By
L. D. Pichakhchi

On the basis of magnetohydrodynamic equations of Chu, Gol'dberg and Lou have been analyzed problem concerning the constancy of tangential rupture in a plasma. In general, the case of arbitrary parameters, which characterize the rupture obtained an adequate constancy condition, and for a special case, when the jump is experienced only by the speed of the plasma, a zone of constancy was attained.

We like to point out that the plasma is described by a system of magnetohydrodynamic equations by Chu, Gol'dberg and Lou (1-3):

where
$$\rho \frac{\partial v_i}{\partial t} + \rho v_k \frac{\partial v_i}{\partial x_k} = - \frac{\partial p_{ik}}{\partial x_k} - \frac{1}{4\pi} H_k \frac{\partial H_k}{\partial x_i} + \frac{1}{4\pi} H_i \frac{\partial H_k}{\partial x_k} \quad (1)$$

$$p_{ik} = p_1 \delta_{ik} + \frac{p_1 - p_2}{H^2} H_i H_k \quad (2)$$

$$\frac{\partial p}{\partial t} + \text{div } \rho v = 0, \quad (3)$$

$$\frac{\partial H}{\partial t} = \text{rot } [vH], \quad \text{div } H = 0, \quad E = -\frac{1}{c} [vH], \quad (4)$$

$$\frac{d}{dt} \left(\frac{p_1}{\rho H} \right) = 0, \quad \frac{d}{dt} \left(\frac{\rho H^2}{\rho^2} \right) = 0. \quad (5)$$

Within the frames of the system of equations (1) - (5) the density tensor of the pulse flow and the density vector of the energy flow have the appearance of:

$$\Pi_{ik} = \rho v_i v_k + p_{ik} - \frac{1}{4\pi} \left(H_i H_k - \frac{1}{2} \delta_{ik} H^2 \right), \quad (6)$$

$$Q = \rho v \left(\frac{v^2}{2} + \epsilon + \frac{p_1}{\rho} \right) + \frac{p_1 - p_2}{H^2} H (vH) + \frac{1}{4\pi} [H [vH]], \quad (7)$$

where $\epsilon = \frac{1}{\rho} \left(p_1 + \frac{1}{2} \rho u \right)$ - sought for internal energy of the plasma. Having expressions (6) and (7), by the known method (4,5) we obtain easily the conditions for tangential rupture:

$$v_x = 0, \quad H_r = 0, \\ \left\{ \rho_1 + \frac{H^2}{8\pi} \right\} = 0.$$

The magnetic field H and speed v lie in the rupture area and experience arbitrary jumps. Density jumps ρ and pressure anisotropy $a = p_{11} - p_{12}$ are also arbitrary, and the pressure p_1 is connected with the magnetic field by the relationship (9).

The investigation of constancy of tangential rupture will be conducted by a method, which was utilized in [4] in case of magnetic hydrodynamics with scalar pressure.

At small disturbances of the stationary tangential rupture in the case of arbitrary H, v, ρ and also as in [4] it is possible to obtain such conditions for excitations on the surface of separation:

$$\begin{aligned} v'_x - \frac{\partial \xi}{\partial t} - v_y \frac{\partial \xi}{\partial y} - v_z \frac{\partial \xi}{\partial z} &= 0, \\ h_x - H_y \frac{\partial \xi}{\partial y} - H_z \frac{\partial \xi}{\partial z} &= 0, \\ \left\{ p' + \frac{1}{4\pi} (H_y h_y + H_z h_z) \right\} &= 0, \end{aligned} \quad (10)$$

where v^i, h^i, p^i - small excitations of values $v, H, p \equiv p_1$, whereby a system of xyz coordinates was derived, bound with the surface of the stationary separation with axis x directed along the normal to the surface; ξ - small displacement value of the surface of separation along axis x ; the figurative little bows here and further on designate the difference of the closed in them value on both sides of the separation surface.

We linearize the system of equations (1) - (5). To solve

$$\exp i(kr - \omega t) \quad (11)$$

we obtain such a system of linear equations:

$$\begin{aligned} \omega_0 v' - \frac{1}{\rho} (p' + \rho u u') k + (k_0 u) \left(1 - \frac{a}{\rho u^2} \right) u' - \\ - \frac{a}{\rho u^2} (k_0 u) \left(\frac{a'}{a} - 2 \frac{u u'}{u^2} \right) u = 0, \end{aligned} \quad (12)$$

$$(k v') = \omega_0 \frac{p'}{\rho}, \quad (13)$$

$$(k u') = 0, \quad (14)$$

$$\omega_0 u' + (k_0 u) v' - (k v') u = 0, \quad (15)$$

$$\omega_0 \left(\frac{p'}{\rho} - \frac{p'}{\rho} - \frac{u \cdot u'}{u^2} \right) = 0, \quad (16)$$

$$\omega_0 \left(\frac{a' + p'}{a + p} + 2 \frac{u \cdot u'}{u^2} - 3 \frac{p'}{\rho} \right) = 0. \quad (17)$$

Here are introduced designations $\omega_0 = \omega - (k_0 v)$, $u = \frac{H}{\sqrt{4\pi}} u'$, a' - small excitations of values u , a ; $k_0(0, k_y, k_z)$ - true instantaneous vector at the plane of separation. In writing equations (12)-(17) are included conditions (8).

Eliminating solutions $\omega_0 = 0$, which offer true values and can therefore not lead to inconstancy, and eliminating v' , we will obtain:

$$\left[(k_0 u) \left(1 - \frac{a}{\rho u^2} \right) - \frac{\omega_0^2}{(k_0 u)} \right] u' - \frac{1}{\rho} (\rho' + \rho u u') k - \left[\frac{a}{\rho u^2} (k_0 u) \left(\frac{a'}{a} - 2 \frac{u u'}{u^2} \right) - \frac{\omega_0^2}{(k_0 u)} \frac{\rho'}{\rho} \right] u = 0, \quad (18)$$

$$(k u') = 0, \quad (19)$$

$$\frac{\rho'}{\rho} - \frac{\rho'}{\rho} - \frac{u u'}{u^2} = 0, \quad (20)$$

$$\frac{a'}{a} + \frac{\rho'}{\rho} + 2 \left(1 + \frac{\rho}{a} \right) \frac{u u'}{u^2} - 3 \left(1 + \frac{\rho}{a} \right) \frac{\rho'}{\rho} = 0. \quad (21)$$

Formulating equations (18) by k and u , with consideration of (19) we will obtain together with equations (20) and (21) a system of four uniform equations for the unknown $\frac{u u'}{u^2}$; $\frac{a'}{a}$; $\frac{\rho'}{\rho}$; $\frac{u'}{u}$. The characteristic determinant of this system is equalized to zero and we obtain equations which bind ω and k :

$$\omega_0^4 - \omega_0^2 [k^2 u^2 (1 + 2P) + (k_0 u)^2 (2A + P)] + (k_0 u)^2 [k^2 u^2 (3A + 3P + 6AP + 5P^2) - (k_0 u)^2 (3A^2 + 9AP + 5P^2)] = 0 \quad (22)$$

where

$$A = \frac{a}{\rho u^2}, \quad P = \frac{\rho}{\rho u^2}. \quad (22a)$$

If in this equation is written that $k = k_0$, it will designate phase expansion rates of small excitations with instantaneous vectors, which lie in the plane of separation:

$$V_{1,2} = v_0 \pm V_s; \quad V_{3,4} = v_0 \pm V_f, \quad (23)$$

where

$$V_s = \sqrt{\frac{D}{2} - \sqrt{\frac{D^2}{4} - F}}, \quad V_f = \sqrt{\frac{D}{2} + \sqrt{\frac{D^2}{4} - F}}, \quad (23a)$$

$$D = u^2 [1 + 2P + \mu^2 (2A + P)],$$

$$F = u^4 \mu^2 [3A + 3P + 6AP + 5P^2 - \mu^2 (3A^2 + 9AP + 5P^2)].$$

It should be pointed out, that the velocities V_s and V_f - actually values, for which it is necessary that

$$F > 0; \quad \frac{D^2}{4} - F > 0. \quad (24)$$

Complex values V_g or V_f would indicate, that the plasma is unstable with respect to small excitations [2,6], irrespective of the tangential separation (rupture). From such considerations we will assume as fulfilled the conditions at which to the actual values k in equation (22) correspond actual values ω . From it under such conditions should arise the condition of positiveness of the free member in (22). Including (24) this gives:

$$3A + 3P + 6AP + 5P^2 > 0. \quad (25)$$

Instability can be caused only by complex values k_x (we assume that k_0 -actual value).

We will take x - component of equation (18) and utilizing boundary conditions (10) which for solving (11), after inclusion of xi give:

$$\{p' + \rho u u'\} = 0; \quad \{u_x' | (k_0, u)\} = 0, \quad (25a)$$

we will obtain:

$$\left\{ \frac{p}{k_x} [(k_0 u)^2 (1-A) - \omega^2] \right\} = 0. \quad (26)$$

From equation (22) we find k_x and substitute in (26)

$$\sqrt{1 - \frac{(V-v_1 v_1)^2 - (V-v_1 v_1)^2 \rho_1^2 u_1^2 (2A_1 + P_1) - \rho_1^2 u_1^2 (3A_1^2 + 9A_1 P_1 + 5P_1^2)}{u_1^2 [(V-v_1 v_1)^2 (1+2P_1) - \rho_1^2 u_1^2 (3A_1 + 3P_1 + 6A_1 P_1 + 5P_1^2)]}} = \frac{\rho_1 [\rho_1^2 u_1^2 (1-A_1) - (V-v_1 v_1)^2]}{\rho_2 [\rho_2^2 u_2^2 (1-A_2) - (V-v_2 v_2)^2]} \quad (27)$$

where $V = \frac{\omega}{k_0}$. The signs k_{x1} and k_{x2} were selected so that the excitations remained finite at an increase in $|x|$, whereby it is also necessary to place a requirement, that the true parts of the radicals should be positive. At any values of parameters, equation (27) for V may have complex roots with positive imaginary part, which will indicate inconstancy of separation (rupture). Equation (27) can be written, by introducing speeds V_g and V_f as well as

$$V_A = \rho u \sqrt{1-A}, \quad V_m = \rho u \sqrt{\frac{3A + 3P + 6AP + 5P^2}{1+2P}} \quad (28)$$

(we like to point out that with respect to formulas (24) and (25) V_A and V_m - actual values) in form of

$$\sqrt{\frac{(V-v_1 v_1 - V_{g1})(V-v_1 v_1 + V_{g1})(V-v_1 v_1 - V_{f1})(V-v_1 v_1 + V_{f1})}{(V-v_1 v_1 - V_{m1})(V-v_1 v_1 + V_{m1})(V-v_1 v_1 - V_{A1})(V-v_1 v_1 + V_{A1})}} = \frac{\rho_1 \sqrt{\frac{(V-v_2 v_2 - V_{g2})(V-v_2 v_2 + V_{g2})(V-v_2 v_2 - V_{f2})(V-v_2 v_2 + V_{f2})}{(V-v_2 v_2 - V_{m2})(V-v_2 v_2 + V_{m2})(V-v_2 v_2 - V_{A2})(V-v_2 v_2 + V_{A2})}}}{\rho_2 \sqrt{\frac{(V-v_2 v_2 - V_{g2})(V-v_2 v_2 + V_{g2})(V-v_2 v_2 - V_{f2})(V-v_2 v_2 + V_{f2})}{(V-v_2 v_2 - V_{m2})(V-v_2 v_2 + V_{m2})(V-v_2 v_2 - V_{A2})(V-v_2 v_2 + V_{A2})}}} \quad (29)$$

where

$$d = \frac{p_1 u_1 \sqrt{1+2P_1}}{p_2 u_2 \sqrt{1+2P_2}} \quad (29a)$$

We will present (29) in a square and bring it into form of

$$\begin{aligned} & (V - v_1 v_1 - V_{A1})(V - v_1 v_1 + V_{A1})(V - v_2 v_2 - V_{A2})(V - v_2 v_2 + V_{A2}) \times \\ & \times (V - v_1 v_2 - V_{A1})(V - v_1 v_2 + V_{A1})(V - v_2 v_1 - V_{A2})(V - v_2 v_1 + V_{A2}) = \\ & = d^2 (V - v_1 v_2 - V_{A1})(V - v_1 v_2 + V_{A1})(V - v_2 v_1 - V_{A2})(V - v_2 v_1 + V_{A2}) \times \\ & \times (V - v_1 v_1 - V_{A1})(V - v_1 v_1 + V_{A1})(V - v_2 v_2 - V_{A2})(V - v_2 v_2 + V_{A2}) \end{aligned} \quad (30)$$

Now is possible to express such adequate condition of tangential separation constancy.

We will arrange all speeds $\gamma_1 v_1 + V_{A1}, \gamma_1 v_1 - V_{A1}, \dots, \gamma_2 v_2 - V_{A2}, \gamma_2 v_2 + V_{A2}, \dots, \gamma_1 v_1 - V_{A1}$, which are included in (30) in the order of their increase. Since the speeds, included in the left half of the equation, alternate with the velocities, included in the right half, and in addition they alternate and are "multiple" of velocities $\gamma_2 v_2 + V_{A2}, \gamma_1 v_1 - V_{A1}$ of the left half with the "multiple" velocities $\gamma_1 v_1 + V_{A1}, \gamma_2 v_2 - V_{A2}$ of the right half of the equation, then all roots of equation (30), and also equation (27), will be actual and the tangential separation (rupture) will be constant.

Next we will investigate equation (27) for the case when vectors v_1, v_2, u_1, u_2 are parallel, when $\gamma_1 = \gamma_2 = \mu_1 = \mu_2 \equiv \gamma$.

For small values γ , imaginary roots V , if they do exist, tend toward zero with γ , consequently the second member under the radical equalling unity can be disregarded. We obtain equations

$$p_1 [v^2 u_1^2 (1 - A_1) - (1 - v_1)^2] = -p_2 [v^2 u_2^2 (1 - A_2) - (1 - v_2)^2], \quad (31)$$

from which it is evident, that separation (rupture) will be constant upon fulfillment of condition

$$p_1 u_1^2 (1 - A_1) + p_2 u_2^2 (1 - A_2) - \frac{p_1 p_2}{p_1 + p_2} (v_2 - v_1)^2 > 0. \quad (32)$$

In case of arbitrary γ we will limit the class of separations by an additional requirement

$$v_1 = v_2 \equiv v, \quad a_1 = a_2 \equiv a, \quad u_1 = u_2 \equiv u, \quad p_1 = p_2 \equiv p \quad (33)$$

and will introduce a system of coordinates, in which

$$v_1 = -\frac{v_2 - v_1}{2} = -\frac{v_0}{2}, \quad v_2 = \frac{v_2 - v_1}{2} = \frac{v_0}{2}. \quad (35a)$$

In this case it will be convenient to utilize equation (29), in which should be added

$$d=1, \quad V_{A1} = V_{A2} = V_A, \quad V_{B1} = V_{B2} = V_B, \quad V_{m1} = V_{m2} = V_m, \quad (34)$$

$$V_{A1} = V_{A2} = V_A, \quad v_1 = -\frac{v_0}{2}, \quad v_2 = \frac{v_0}{2}.$$

When $\gamma = 0$ for separations (33) conditions (32) transform into condition

$$V_A > \frac{v_0}{2}. \quad (35)$$

In the other boundary case $\gamma = 1$ equation (29) (in which conditions (34) are included after increasing into square, provided we eliminate the true roots $\omega = 0$ and $\omega = \pm \left(\frac{v_0}{2} \pm V_A \right)$, transforms into

$$V^4 - 2 \left(V_s^2 + \frac{v_0^2}{4} \right) V^2 + \left(\frac{v_0^2}{4} \right)^2 - 2 \left(\frac{v_0^2}{4} \right) V^2 + \quad (36)$$

$$+ V_A^2 (V_s^2 - V_m^2) + V_s^2 V_m^2 = 0,$$

where

$$V_s = u \sqrt{3(A+P)}, \quad V_m = u \sqrt{3(A+P) - \frac{P^2}{(1+2P)}}, \quad V_A = u \sqrt{1-P} \quad (36a)$$

We will divide (36) into V_s^4 and introduce designations: $\frac{V_A^2}{V_s^2} = \alpha^2, \quad \frac{V_m^2}{V_s^2} = 1 -$

$$\text{де } \gamma = \frac{P^2}{(1+2P)(3A+3P)}, \quad \frac{v_0^2}{4V_s^2} = \beta^2, \quad \frac{V_s^2}{V_s^2} = x^2, \quad \text{причому } \alpha, \beta, \gamma > 0. \quad (36b)$$

wholly

we will obtain

$$x^4 - 2x^2(1 + \beta^2) + \beta^4 - 2\beta^2 + \gamma\alpha^2 + 1 - \gamma = 0. \quad (37)$$

Equation (37) has purely imaginary roots, because

$$(1 - \beta^2)^2 - \gamma(1 - \alpha^2) < 0, \quad (38)$$

and complex roots, because

$$4\beta^2 + \gamma(1 - \alpha^2) < 0. \quad (39)$$

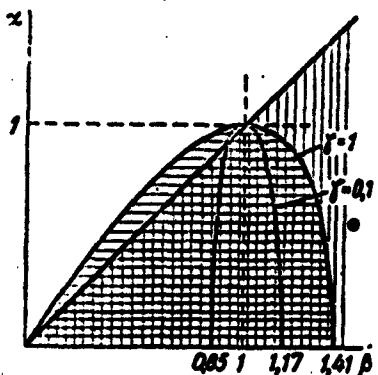
To the inequalities (38) and (39) should be added inequality

$$1 - \gamma > 0, \quad (40)$$

which is the result of inequality (25).

A direct check will convince, that the values alpha and beta, which satisfy in equality (39), should be eliminated by the fact, that the positiveness condition

of actual parts of radicals in (27) is not fulfilled here. In this way the inconstancy may originate only for alpha and beta values which do satisfy the inequality (38). On the drawing are shown zones of inconstant separation (hatched by horizontal lines) for two values gamma: 0, 1; 1. For gamma \rightarrow 0 this zone is contracted into the section of straight beta = 1. On the drawing is drawn a straight line alpha = beta which separates the constancy zone from the inconstancy zone (hatched by vertical lines) of separation for $\nu = 0$.



We shall now discuss intermediate values $0 < \nu < 1$. Excitations, corresponding to these values ν , can be considered as a superposition of two excitations, which correspond to values $\nu = 0$ and $\nu = 1$. Considering the fact that equation (10), (12) - (17) are linear, solutions for each one of the excitations will be independent and in agreement with the

already obtained for $\nu = 0$ and $\nu = 1$. Furthermore, figuring, that not the value of excitation amplitude, nor absolute values k_0 affect the found zones of instability then for any arbitrary $0 < \nu < 1$ both zones exist together. For example, when gamma = 1 the instability zone will be the entire hatched part of the area alpha and beta (see drawing).

In conclusion I want to express sincere thanks to prof. V.L. German for the work proposed by him.

Literature

1. G.F. Chew, M.L. Goldberger, F.S. Low, Proc. Roy. Soc. A, 236, 112, 1956 (its translation at PSF No. 7, 139, 1957).
 2. R.V. Polovin, N.L. Tsintsadze, Ukrainskiy Fizicheskiy Zhurnal 4, 30, 1959
 3. I.A. Achiyzer, R.V. Polovin, N.L. Tsintsadze, Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki, 37, 756, 1959.
 4. S.I. Syrovatskiy, Transactions of Phys. Inst. 8, 15, 1956
 5. L.D. Landau, Ye. M. Lifchits, Electrodynamics of Solid Media, GITTL, Moscow, 1957
 6. R.Z. Sagdayev, collection of rep. Physics of Plasma and Problems of Controlled Thermonuclear Reactions * vol. 3, Izdat. Akad. Nauk. SSSR, Moscow 1958, p. 268
 7. A.G. Kurosh, Course in Higher Algebra GITZ, Gostekhizdat, 1946.
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