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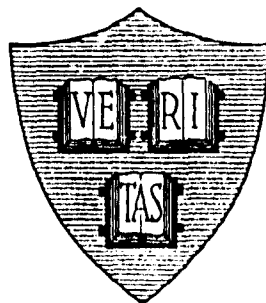
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RADIATION FROM AN ELECTROMAGNETIC
SOURCE IN A HALF-SPACE OF
COMPRESSIBLE PLASMA-SURFACE WAVES



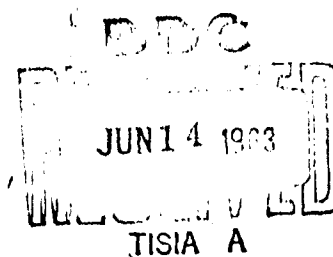
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By
S. R. Seshadri

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Technical Report No. 396

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Technical Report

(6) RADIATION FROM AN ELECTROMAGNETIC SOURCE IN A
HALF-SPACE OF COMPRESSIBLE PLASMA—SURFACE WAVES,

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S. R. Seshadri,

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Cruft Laboratory
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RADIATION FROM AN ELECTROMAGNETIC SOURCE IN A HALF-SPACE OF
COMPRESSIBLE PLASMA-SURFACE WAVES

by

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ABSTRACT

The radiation characteristics of a line source of magnetic current are studied for the case in which the source is situated in a half-space of isotropic, compressible plasma which is bounded on one side by a perfectly conducting, rigid plane screen. In addition to the electromagnetic and plasma space waves, the line source excites a boundary wave. This boundary wave is a coupled wave. It has associated with it both a magnetic field component and the pressure term. This is in contrast to the space waves which can be decomposed into an electromagnetic (EM) mode with no pressure term and a plasma (P) mode with no magnetic field associated with it. The characteristics of this boundary wave are evaluated. The boundary wave propagates for all frequencies and the power carried by the boundary wave becomes smaller as the frequency is increased.

INTRODUCTION

The study of the radiation characteristics of localized electromagnetic sources in ionized gaseous media, known generally as plasmas, has application to the problem of radio communication with missiles and space vehicles passing through the ionized regions in space. In this paper, the radiation characteristics of a line source of magnetic current situated in a compressible plasma medium of semi-infinite extent and bounded on one side by a perfectly conducting, rigid planar screen, is investigated. The special case for which the line source is on the screen becomes equivalent to the problem of radiation into a plasma half-space from an infinitely long slot radiator whose width is very small in comparison to the wavelength. In a previous paper [1], the same problem was investigated for the case of an incompressible plasma in which the longitudinal plasma waves cannot be sustained. In this investigation, the previous analysis is extended to the case of a compressible plasma which is capable of supporting both the transverse electromagnetic waves and the longitudinal plasma waves.

Only recently, the radiation from sources in a compressible plasma has been studied. Hessel and Shmoys [2] have treated the problem of radiation from a point source of electric current in a homogeneous isotropic plasma. Hessel, Marcuvitz and Shmoys [3] have investigated the radiation from a line source of magnetic current in a vacuum in the presence of a plasma half-space. In this paper the radiation characteristics of a line source of magnetic current are investigated for the case in which the source is located in a half-space of isotropic, compressible plasma which is bounded on one side by a perfectly conducting, rigid plane screen. In addition to the EM and P space waves, a boundary wave is found to exist along the screen. This boundary wave is a coupled wave; it has associated with it both a magnetic field and a pressure term. This is in contrast to the space waves which can be decomposed into two distinct, uncoupled modes, namely the transverse electromagnetic (EM) mode and the longitudinal plasma (P) mode. The characteristics of this boundary wave are evaluated.

The space wave parts of the electromagnetic and the plasma modes do not propagate for frequencies less than the plasma frequency. But the boundary wave propagates for all frequencies. The power carried by the boundary wave is decreased as the frequency is increased.

FORMULATION OF THE PROBLEM

Consider a perfectly conducting and rigid, planar screen of infinite extent. Let a right-handed rectangular coordinate system x, y and z be chosen such that the screen occupies the plane $z = 0$. (Fig. 1). The half-space $z > 0$ is filled uniformly with a homogeneous, electron plasma. A line source of magnetic current is located in the plasma at $x = 0, z = d$; it is parallel to the y -axis and may be represented as

$$\vec{J}_m = \hat{y} J_m = \hat{y} J_0 \delta(x) \delta(z - d) \quad (1)$$

It is desired to investigate the radiation characteristics of this line source with particular reference to the boundary waves that propagate along the screen.

Attention is given only to the steady-state problem. The current source is assumed to have the harmonic time dependence of the form $e^{-i\omega t}$. The frequency of the source is assumed to be sufficiently high so that the motion of the ions may be neglected. In addition, the strength of the source is assumed to be so weak that only small amplitude waves are excited, thus justifying the use of a linearized plasma theory [4]. According to the linearized theory, all the field components will have the same harmonic time dependence as that of the source, namely $e^{-i\omega t}$, which may, therefore, be conveniently suppressed. The collisions between electrons and other particles are neglected and the drift velocity of the electrons is assumed to be zero.

Let N_0 be the average number density of electrons, p the pressure deviation of the electrons from the mean and $\vec{V}$ the velocity of electrons. Let $\vec{E}$ and $\vec{H}$ be the alternating electric and magnetic fields, respectively. The linearized, time-harmonic hydrodynamic equation of motion of the electrons is

$$-i\omega m N_0 \vec{V} = N_0 e \vec{E} - \nabla p \quad (2)$$



FIG. 1 GEOMETRY OF THE PROBLEM.

where e is the charge and m is the mass of an electron. The linearized equation of continuity together with the equation of state gives

$$a^2 m N_0 \nabla \cdot \vec{V} = i\omega p \quad (3)$$

where a is the velocity of sound in the electron gas. In addition, the electric and magnetic fields satisfy, in the half-space $z > 0$, the following time-harmonic Maxwell's equations:

$$\nabla \times \vec{E} = i\omega \mu_0 \vec{H} - \vec{J}_m \quad (4)$$

$$\nabla \times \vec{H} = -i\omega \epsilon_0 \vec{E} + N_0 e \vec{V} \quad (5)$$

where μ_0 and ϵ_0 are respectively the permeability and the dielectric constant of free space.

The source and the geometry of the problem are independent of the y coordinate and, hence, all the field quantities also are independent of y . On substituting $\frac{\partial}{\partial y} = 0$ in (4) and (5), it is found that the electromagnetic field is separable into E and H modes. Since only a line source of magnetic current is present, the H mode is not excited, and, hence, $E_y = H_x = H_z = 0$. From (2), it is seen that $V_y = 0$ also. Only a single component of the magnetic field, namely, H_y is present.

By making use of (2) and (5), E_x , E_z , V_x and V_z are easily expressed in terms of H_y and p as follows:

$$E_x = \frac{1}{i\omega \epsilon_0 \epsilon_1} \frac{\partial}{\partial z} H_y - \frac{(1-\epsilon_1)}{N_0 e \epsilon_1} \frac{\partial}{\partial x} p \quad (6)$$

$$E_z = -\frac{1}{i\omega \epsilon_0 \epsilon_1} \frac{\partial}{\partial x} H_y - \frac{(1-\epsilon_1)}{N_0 e \epsilon_1} \frac{\partial p}{\partial z} \quad (7)$$

$$V_x = \frac{e}{\omega^2 m \epsilon_0 \epsilon_1} \left[\frac{\partial}{\partial z} H_y - \frac{i\omega \epsilon_0}{N_0 e} \frac{\partial}{\partial x} p \right] \quad (8)$$

$$V_z = \frac{e}{\omega^2 m \epsilon_0 \epsilon_1} \left[-\frac{\partial}{\partial x} H_y - \frac{i\omega \epsilon_0}{N_0 e} \frac{\partial}{\partial z} p \right] \quad (9)$$

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where

$$\epsilon_1 = 1 - \left(\frac{\omega_p}{\omega} \right)^2 \quad (10)$$

and the plasma frequency is, $\omega_p = \sqrt{\frac{N_o e^2}{m \epsilon_o}}$. The substitution of (6), (7), and (1) in (4) yields

$$\text{where, } \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k_e^2 \right] H_y(x, z) = -i\omega \epsilon_o \epsilon_1 J_o \delta(x) \delta(z-d) \quad (11)$$

$$k_e^2 = \frac{\omega^2}{c^2} \epsilon_1 = \frac{\omega^2}{c^2} \left(1 - \frac{\omega_p^2}{\omega^2} \right) \quad (12)$$

and c is the velocity of electromagnetic waves in free-space. In a similar fashion, the use of (8) and (9) in (3) gives

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k_p^2 \right] p(x, z) = 0 \quad (13)$$

where

$$k_p^2 = \frac{\omega^2}{a^2} \epsilon_1 = \frac{\omega^2}{a^2} \left(1 - \frac{\omega_p^2}{\omega^2} \right) \quad (14)$$

It is advantageous to rearrange the field quantities given in (6) - (9) into two distinct groups. The magnetic field component H_y and the first terms in (6) - (9) constitute the first group which is called the electromagnetic (EM) mode. The pressure p and the second terms in (6) - (9) constitute the second group which is called the plasma [P] mode. The EM mode and the P mode are written down separately as follows:

EM mode

$$\begin{aligned} H_y, \vec{E}_e &= \frac{1}{i\omega \epsilon_o \epsilon_1} \left[\hat{x} \frac{\partial}{\partial z} H_y - \hat{z} \frac{\partial}{\partial x} H_y \right] \\ \vec{V}_e &= \frac{e}{\omega^2 m \epsilon_o \epsilon_1} \left[\hat{x} \frac{\partial}{\partial z} H_y - \hat{z} \frac{\partial H_y}{\partial x} \right] \end{aligned} \quad (15)$$

P mode

$$p, \quad \vec{E}_p = - \frac{(1 - \epsilon_1)}{N_o e \epsilon_1} \left[\hat{x} \frac{\partial p}{\partial x} + \hat{z} \frac{\partial p}{\partial z} \right]$$

$$\vec{V}_p = \frac{1}{i \omega m N_o \epsilon_1} \left[\hat{x} \frac{\partial p}{\partial x} + \hat{z} \frac{\partial p}{\partial z} \right] \quad (16)$$

The subscripts e and p denote the EM and the P modes, respectively. For the EM mode, it is seen that $\nabla \times \vec{E}_e \neq 0$, $\nabla \cdot \vec{E}_e = 0$ and $\nabla \cdot \vec{V}_e = 0$. Therefore, it is obvious that the EM mode has a magnetic field component associated with it but has no charge accumulation. Moreover, the EM mode has a y component of the magnetic field and this is perpendicular to the xz-plane in which the propagation takes place. Therefore, the EM mode is a transverse mode. For the P mode, it is found that $\nabla \times \vec{E}_p = 0$, $\nabla \cdot \vec{E}_p \neq 0$ and $\nabla \times \vec{V}_p = 0$. The P mode obviously has charge accumulation associated with it, but no magnetic field. Since it does not contain any y component of the field, it is a longitudinal mode. Using the generalized Poynting vector for a compressible plasma given in [2], it can be shown that in an unbounded plasma, the EM and the P modes are uncoupled in the sense that the total power can be obtained as the sum of the powers in the individual modes. However, as will be seen later, if a surface wave exists along a plasma boundary, it is a coupled wave.

It is clear from (13) that in an unbounded plasma, the line source (1) does not excite $p(x, z)$. Therefore, it is evident from (16) that the P mode is absent and only the transverse EM mode is excited. However, in a plasma half-space, on account of the presence of the boundary at $z = 0$, the longitudinal P mode is excited. Since the bounding surface $z = 0$, is a perfectly conducting and rigid screen, the following boundary conditions are to be satisfied:

$$E_x(x, 0) = 0 \quad (17)$$

$$V_z(x, 0) = 0 \quad (18)$$

INTEGRAL EXPRESSIONS FOR THE FIELDS

The solution of (11) and (13) subject to the boundary conditions (17) and (18) will yield $H_y(x, z)$ and $p(x, z)$. Once $H_y(x, z)$ and $p(x, z)$ are known, the other field quantities $E_x(x, z)$, $E_z(x, z)$, $V_x(x, z)$ and $V_z(x, z)$ are easily obtained with the help of (6) - (9). Integral expressions for $H_y(x, z)$ and $p(x, z)$ can be easily obtained in the following manner.

The geometry of the problem suggests the following representations for the field components:

$$H_y(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{H}_y(\xi, z) e^{i\xi x} d\xi \quad (19a)$$

$$p(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{p}(\xi, z) e^{i\xi x} d\xi \quad (19b)$$

Integral representations similar to (17) are assumed also for $E_x(x, z)$ and $V_z(x, z)$. Then it follows from (6), (9), (11) and (13) that

$$\bar{E}_x(\xi, z) = \frac{1}{i\omega\epsilon_0\epsilon_1} \frac{\partial}{\partial z} \bar{H}_y(\xi, z) - \frac{i\xi(1-\epsilon_1)}{N_0 e \epsilon_1} \bar{p}(\xi, z) \quad (20)$$

$$\bar{V}_z(\xi, z) = \frac{e}{\omega^2 m \epsilon_0 \epsilon_1} \left[-i\xi \bar{H}_y(\xi, z) - \frac{i\omega\epsilon_0}{N_0 e} \frac{\partial}{\partial z} \bar{p}(\xi, z) \right] \quad (21)$$

and
$$\left[\frac{d^2}{dz^2} + \xi_e^2 \right] \bar{H}_y(\xi, z) = -i\omega\epsilon_0\epsilon_1 J_0 \delta(z-d) \quad (22)$$

$$\left[\frac{d^2}{dz^2} + \xi_p^2 \right] \bar{p}(\xi, z) = 0 \quad (23)$$

where

$$\begin{aligned} \xi_i &= + \sqrt{k_i^2 - \xi^2} & k_i > \xi \\ \xi_i &= + i \sqrt{\xi^2 - k_i^2} & \xi > k_i \end{aligned} \quad \text{for } i = e, p \quad (24)$$

The solution of (22) gives

$$\bar{H}_y(\xi, z) = \begin{cases} A e^{i\xi_e z} + B e^{-i\xi_e z} & z > d \\ C e^{i\xi_e z} + D e^{-i\xi_e z} & z < d \end{cases} \quad (25)$$

and

$$\frac{d}{dz} \bar{H}_y(\xi, d+) - \frac{d}{dz} \bar{H}_y(\xi, d-) = -i\omega\epsilon_0\epsilon_1 J_0 \quad (26)$$

Similarly, the solution of (23) gives

$$\bar{p}(\xi, z) = E e^{i\xi_p z} + F e^{-i\xi_p z} \quad (27)$$

The usual phase radiation condition is found to be applicable for both modes. An examination of the dispersion curves $\omega-k_e$ and $\omega-k_p$ for the EM and the P modes shows that there are no backward wave regions. Therefore, the radiation condition requires that $H_y(x, z)$ and $p(x, z)$ have outward traveling phase fronts for z tending to infinity. Hence, $B = F = 0$. From the boundary conditions (17) and (18), it is clear that $\bar{E}_x(\xi, 0) = \bar{V}_z(\xi, 0) = 0$. It follows, therefore, from (20), (21), (25), (27) that

$$\frac{\xi_e (C-D)}{\omega\epsilon_0\epsilon_1} - \frac{i\xi(1-\epsilon_1) E}{N_0\epsilon\epsilon_1} = 0 \quad (28)$$

and

$$-i\xi (C+D) + \frac{\omega\epsilon_0\xi_p E}{N_0\epsilon} = 0 \quad (29)$$

The requirement that the tangential component of the magnetic field should be continuous at $z = d$ gives

$$A e^{i\xi_e d} = C e^{i\xi_e d} + D e^{-i\xi_e d} \quad (30)$$

The use of the jump conditions (26) in (25) leads to

$$A e^{i\xi_e d} - (C e^{i\xi_e d} - D e^{-i\xi_e d}) = - \frac{\omega \varepsilon_0 \varepsilon_1 J_0}{\xi_e} \quad (31)$$

The expressions for A, C, D and E may be obtained from the solution of the simultaneous equations (28) - (31). The results are

$$\begin{aligned} A &= - \frac{\omega \varepsilon_0 \varepsilon_1 J_0}{2\xi_e} \left[e^{-i\xi_e d} + \frac{\xi_e \xi_p - (1-\varepsilon_1)\xi^2}{\xi_e \xi_p + (1-\varepsilon_1)\xi^2} e^{i\xi_e d} \right] \\ B &= 0 \\ C &= - \frac{\omega \varepsilon_0 \varepsilon_1 J_0 [\xi_e \xi_p - (1-\varepsilon_1)\xi^2]}{2\xi_e [\xi_e \xi_p + (1-\varepsilon_1)\xi^2]} e^{i\xi_e d} \\ D &= - \frac{\omega \varepsilon_0 \varepsilon_1 J_0}{2\xi_e} e^{i\xi_e d} \\ E &= - \frac{N_0 \varepsilon_1 J_0 i \xi_e}{[\xi_e \xi_p + (1-\varepsilon_1)\xi^2]} e^{i\xi_e d} \end{aligned} \quad (32)$$

and

$$F = 0$$

The substitution of (32) in (25) and (27) and the use of (19) yields the following integral expressions for $H_y(x, z)$ and $p(x, z)$:

$$H_y(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} - \frac{\omega \varepsilon_0 \varepsilon_1 J_0}{2\xi_e} \left[e^{-i\xi_e d} + \frac{\xi_e \xi_p - (1-\varepsilon_1)\xi^2}{\xi_e \xi_p + (1-\varepsilon_1)\xi^2} e^{i\xi_e d} \right] e^{i\xi x + i\xi_e z} d\xi$$

for $d < z < \infty$ (33a)

$$H_y(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} - \frac{\omega \epsilon_0 \epsilon_1 J_0}{2\xi_e} \left[e^{-i\xi_e z} + \frac{\xi_e \xi_p - (1-\epsilon_1) \xi^2}{\xi_e \xi_p + (1-\epsilon_1) \xi^2} e^{i\xi_e d} \right] e^{i\xi x + i\xi_e z} d\xi$$

for $0 < z < d$. (33b)

and

$$p(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} - \frac{N_0 \epsilon \epsilon_1 J_0 i \xi}{\left[\xi_p \xi_e + (1-\epsilon_1) \xi^2 \right]} e^{i\xi_e d + i\xi x + i\xi_p z} d\xi \quad (34)$$

SINGULARITIES OF THE INTEGRAND

The contour for the integrals in (33) and (34) is along the real axis in the ξ -plane as shown in Fig. 2. It is desired to find the singularities of the integrands in (33) and (34). Branch points are seen to occur at $\xi = \pm k_e$ and $\xi = \pm k_p$. With the help of (12) and (14), it is obvious that the branch points at $\pm k_e$ and $\pm k_p$ occur on the real or the imaginary axis of the ξ -plane depending on whether ω is greater or less than ω_p .

The poles of the integrand are obviously determined by the roots of the equation

$$\left[(k_e^2 - \xi^2) (k_p^2 - \xi^2) \right]^{1/2} + (1-\epsilon_1) \xi^2 = 0 \quad (35)$$

On eliminating the radical in (35), ξ is found to satisfy a biquadratic equation, the solution of which gives the following pair of possible solutions of (35) as

$$\xi_1^2 = U + \sqrt{U^2 - W} \quad (36a)$$

and

$$\xi_2^2 = U - \sqrt{U^2 - W}, \quad (36b)$$

where

$$U = \frac{k_p^2 + k_e^2}{2(1 - \frac{\omega_p^2}{\omega^2})} \quad W = \frac{k_p^2 k_e^2}{1 - \frac{\omega_p^2}{\omega^2}} \quad (37)$$

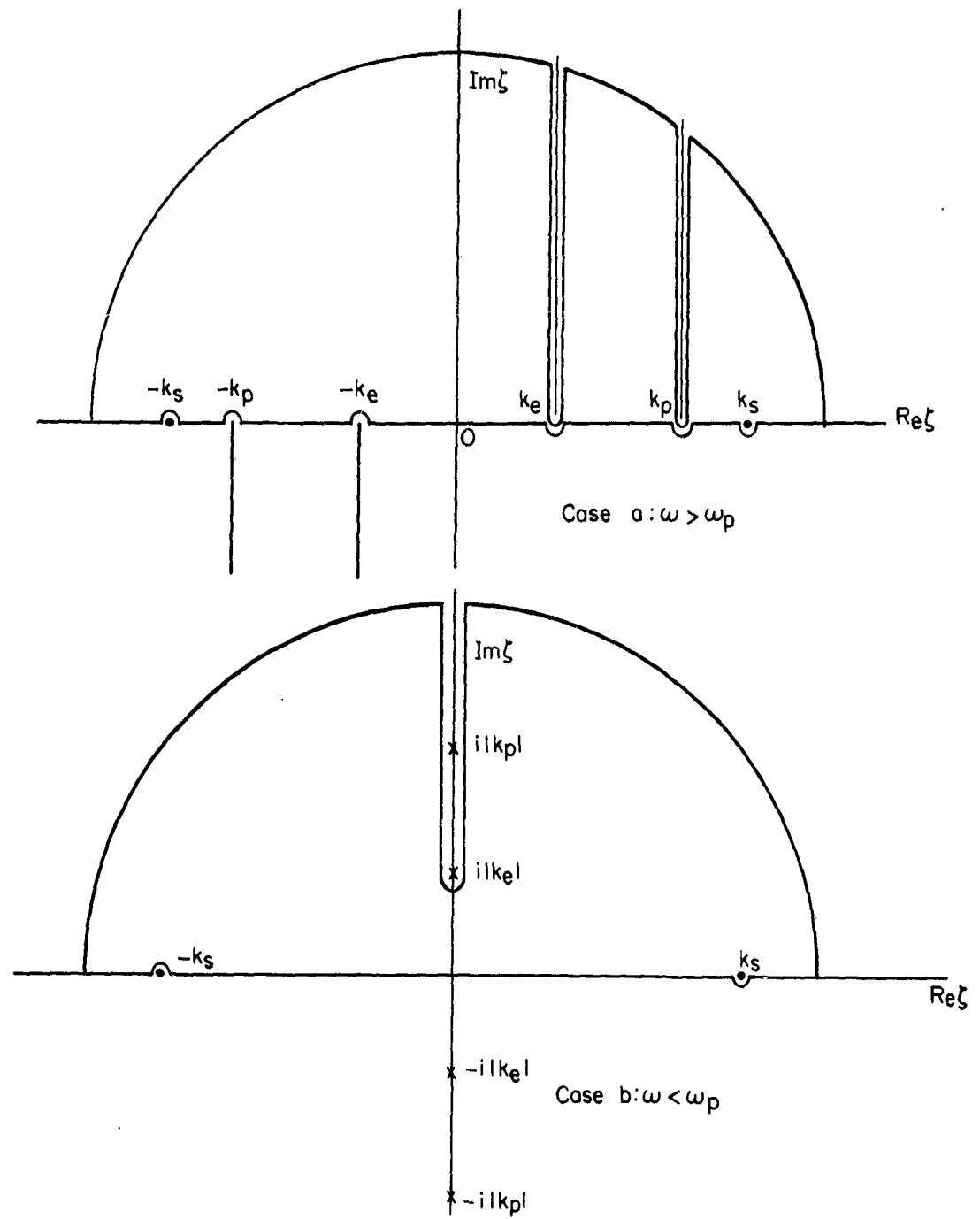


FIG. 2a, b INTEGRATION CONTOUR IN THE ζ PLANE

In obtaining (37), use has been made of (10). Only that root from (36a, b) which also satisfies (35) on the proper Riemann surface, is the proper root of (35). For finding out which one of (36a, b) is a proper root, it is advantageous to simplify (36a, b) by making use of the fact that the ratio of the plasma to the electromagnetic wave velocities $\frac{a}{c}$ is of the order of 10^{-4} . Hence, $\frac{a^2}{c^2}$ is very small in comparison with unity. It is seen from (37) that U^2 is very large compared to W and, hence, the following approximation to (36) may be used,

$$\xi_1^2 = U \pm |U| \mp \frac{W}{2|U|} \mp \frac{W^2}{8|U|^3} \quad (38)$$

Case 1: For $\omega > \omega_p$, both k_e^2 and k_p^2 and, hence, U and W are positive and, hence, the two roots given in (38) may be simplified to yield

$$\xi_1^2 = k_s^2 \pm 2U \pm \frac{k_p^2}{1 - \frac{\omega_p^4}{\omega^4}} \quad (39a)$$

and

$$\xi_2^2 = k_o^2 \pm \frac{W}{2U} + \frac{W^2}{8U^3} = k_e^2 \left[1 - \frac{k_e^2}{k_p^2} \frac{\omega_p^4}{\omega^4} \right] \quad (39b)$$

The root ξ_1^2 given in (39a) is greater than k_p^2 , since $\omega > \omega_p$. For this root, in view of (24) and the fact that $k_p^2 > k_e^2$, the first term in (35) is negative and the second term is positive and they can add to zero. Hence, the root (39a), which is an approximation to the root given by the upper sign in (36) is on the proper Riemann surface. Moreover, this root is real and occurs at $\pm k_s$ in the ξ -plane. With the help of (39b), the root of ξ_1^2 is seen to be slightly less than k_e^2 . It follows, therefore, that on the proper Riemann surface defined by (24), the first term in (35) is positive and so is the second term. They cannot add to zero and, hence, the root (39b) is not on the proper Riemann surface. It is to be noted that for $\omega > \omega_p$, $\pm \xi_2$ is a real pole on the improper Riemann surface.

Case 2: The root ξ_1^2 given in (36a) and approximated by k_s^2 given in (39a) is real and positive even for $\omega < \omega_p$. For $\omega < \omega_p$, k_e^2 and k_p^2 are negative and $\xi^2 = k_s^2$ is positive, the first term in (35) is a negative real number and the second term is a positive real number and they can add to zero. Hence, the root ξ_1^2 is a proper real root of (35) and gives a pair of real poles in the ξ -plane. For $\omega < \omega_p$, with the help of (12) and (14), it is evident that $U > 0$ and $W < 0$. Since $W < 0$, $\sqrt{U^2 - W} > U$. Hence, the root ξ_2^2 is negative. Moreover,

$$k_e^2 - \xi_2^2 = k_e^2 \frac{k_e^2}{k_p^2} \frac{\omega_p^4}{\omega^4} < 0 \text{ and } k_p^2 - \xi_2^2 = k_p^2 \left[1 - \frac{k_e^2}{k_p^2} + \left(\frac{k_e \omega_p}{k_p \omega} \right)^4 \right] < 0.$$

Hence, the first term in (35) is negative real and the second term is also negative real and, therefore, both the terms in (35) cannot add to zero. Hence, for $\omega < \omega_p$, $\pm i|\xi_2|$ gives a pair of purely imaginary poles on the improper Riemann surface.

Therefore, it is seen that the root (39a) gives rise to a pair of real poles at $\xi = \pm k_s$ for all values of ω . The root (36b) is on the improper Riemann surface; it is real or imaginary depending on whether ω is greater or less than ω_p .

SURFACE WAVE CONTRIBUTION

The contour for the integrals in (33) and (34) is along the real axis in the ξ -plane (Fig. 2). Since the branch points at $\xi = \pm k_e$, and $\xi = \pm k_p$ are on the real axis for $\omega > \omega_p$, the contour of integration has to be indented suitably at these points. As was mentioned earlier, the phase-type radiation condition is applicable to the space-wave parts of the total field. Since the dispersion curves for the space waves for both the EM and P modes do not contain any backward wave regions, the radiation condition is satisfied by requiring the phase fronts of the EM and the P space waves to be traveling outward. Hence, the contour is indented from above at $\xi = -k_p$ and $-k_e$ and from below at $\xi = +k_e$ and $+k_p$. For $\omega < \omega_p$, no ambiguity in the integration contours at the branch points occur, since for $\omega < \omega_p$, k_e and k_p are purely imaginary.

In addition to the branch points, the integrands in (33) and (34) have poles at $\xi = \pm k_s$ on the real axis for all ω . For $\omega > \omega_p$, this pole occurs beyond both branch points. It is obvious that the contour of integration should be indented from above at $\xi = -k_s$ and from below at $\xi = k_s$ (Fig. 2a, b) or vice versa. The former type of indentation will be chosen for all ω . This ensures the surface wave to have outward traveling phase fronts. As was shown in an earlier investigation [5], in a compressible plasma, the requirement of outward traveling phase fronts does not necessarily imply that the net power is traveling outward at infinity in spite of the absence of backward regions in the dispersion curve. It remains, therefore, to be verified that the total power is indeed traveling outward at infinity.

For $x > 0$, the integrals (33) and (34) may be evaluated by closing the contour in the upper half of the ξ -plane. The contribution to the integrals (33) and (34) is the sum of the residue at the pole $\xi = k_s$ and the two branch-cut integrals. The values of the branch-cut integrals depend on some inverse power of x and, hence, for large x are negligible compared to the contribution due to the pole. Thus, for positive large x (33) yields

$$H_y(x, z) = H_y^s(x, z) = H_s e^{ik_s x - z \sqrt{k_s^2 - k_e^2}} \quad (40)$$

where

$$H_s = - \frac{\omega \epsilon_0 \epsilon_1 J_0}{2k_s \sqrt{k_s^2 - k_e^2}} \left[\frac{\left\{ (k_s^2 - k_e^2)(k_s^2 - k_p^2) \right\}^{1/2} + \frac{\omega_p^2}{\omega^2} k_s^2}{\left\{ \frac{k_s^2 - k_e^2}{k_s^2 - k_p^2} \right\}^{1/2} + \left\{ \frac{k_s^2 - k_p^2}{k_s^2 - k_e^2} \right\}^{1/2} - 2 \frac{\omega_p^2}{\omega^2}} \right] e^{-d \sqrt{k_s^2 - k_e^2}} \quad (41)$$

Similarly, the contribution from the pole of (34) gives

$$p(x, z) = p^s(x, z) = P_s e^{ik_s x - z \sqrt{k_s^2 - k_p^2}} \quad (42)$$

where

$$P_s = -N_0 e \epsilon_1 J_0 e^{-d \sqrt{k_s^2 - k_e^2}} \left[\left\{ \frac{k_s^2 - k_e^2}{k_s^2 - k_p^2} \right\}^{1/2} + \left\{ \frac{k_s^2 - k_p^2}{k_s^2 - k_e^2} \right\}^{1/2} - \frac{2\omega_p^2}{\omega^2} \right]^{-1} \quad (43)$$

It is evident that $H_y^s(x, z)$ and $p^s(x, z)$ given respectively in (40) and (42) represent surface waves which propagate in the positive x direction and are exponentially attenuated in the z direction. The phase velocity v_s of both $H_y^s(x, z)$ and $p^s(x, z)$ is given by

$$v_s = \frac{\omega}{k_s} = a \sqrt{1 + \frac{\omega_p^2}{\omega^2}} \quad (44)$$

The phase velocity is infinite for $\omega = 0$, reduces monotonically as the frequency is increased and, for $\omega = \omega_p$ reaches a value which is equal to $\sqrt{2}$ times the acoustic velocity in the electron gas. As the frequency is still further increased, the phase velocity continues to decrease and asymptotically reaches the value a , the acoustic velocity in the electron gas, in the limit of infinite frequency. A plot of phase velocity v_s versus the frequency ω is shown in Fig. 3.

The attenuation factor α_h for the decay of $H_y(x, z)$ in the z direction is given by

$$\alpha_h = \sqrt{k_s^2 - k_e^2} = \frac{\omega}{a} \left(1 + \frac{\omega_p^2}{\omega^2} \right)^{-1/2} \quad (45)$$

Similarly, the attenuation factor α_p for $p(x, z)$ is obtained as

$$\alpha_p = \sqrt{k_s^2 - k_p^2} = \frac{\omega_p}{a} \left(1 + \frac{\omega^2}{\omega_p^2} \right)^{-1/2} \quad (46)$$

It follows, therefore, that $H_y^s(x, z)$ and $p(x, z)$ which propagate with the same phase velocity in the x direction are attenuated differently in the z -direction, as seen from (45) and (46). The attenuation factor α_h is a monotonically increasing function of frequency. It starts with the value of zero for $\omega = 0$, increases to the value $\frac{\omega_p}{a\sqrt{2}}$ for $\omega = \omega_p$ and thereafter continuously increases

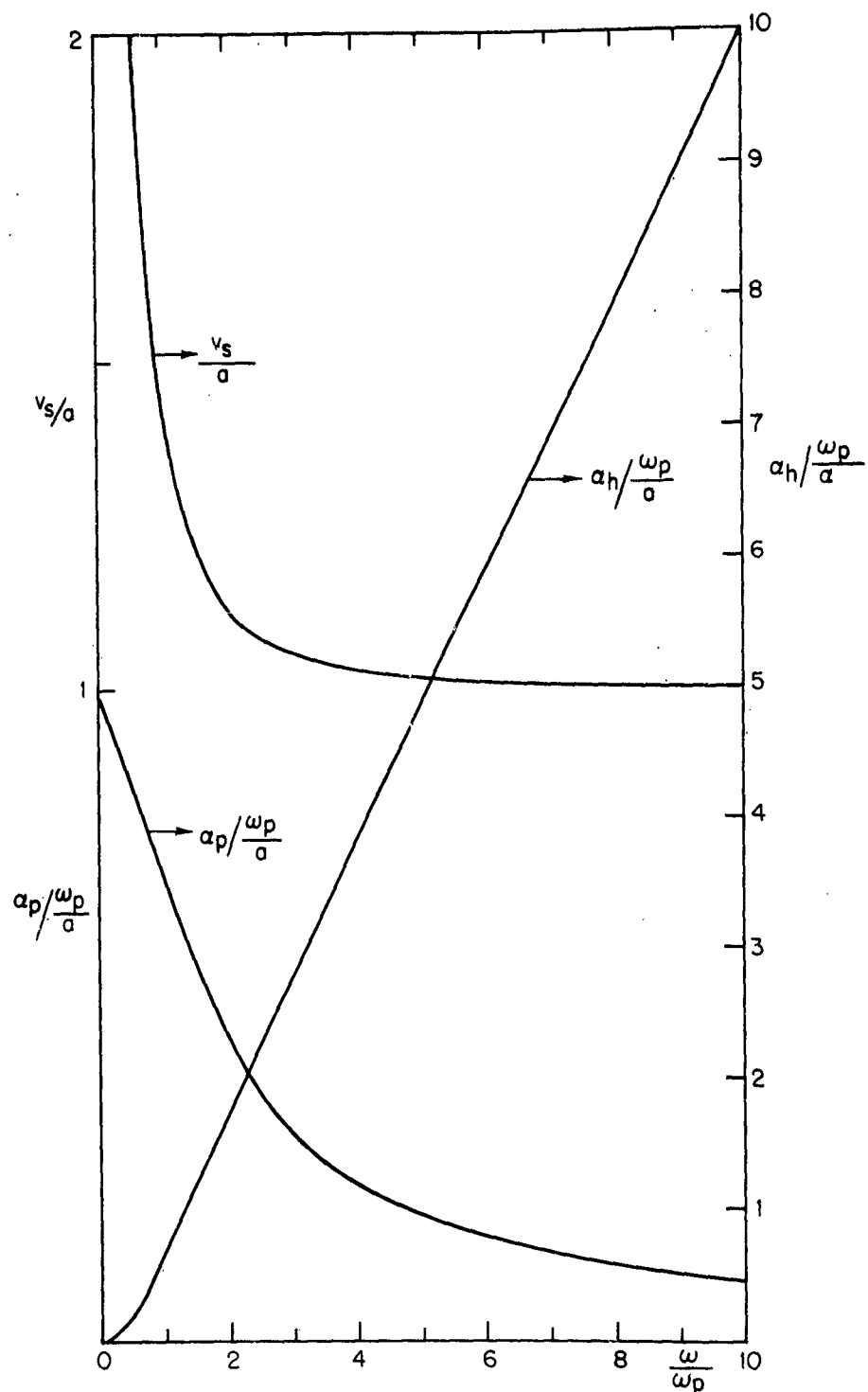


FIG. 3 PHASE VELOCITY AND ATTENUATION CONSTANTS AS A FUNCTION OF FREQUENCY

as ω is increased. However, the attenuation factor α_p for the pressure wave is a monotonically decreasing function of frequency. It starts with the value

$\frac{\omega_p}{a}$ for $\omega = 0$, decreases to the value $\frac{\omega_p}{a\sqrt{2}}$ for $\omega = \omega_p$ and thereafter

asymptotically approaches zero as ω is increased indefinitely. A plot of the attenuation constants α_h and α_p as a function of ω is shown in Fig. 3. For $\omega = \omega_p$, both the attenuation constants are equal. For $\omega < \omega_p$, $\alpha_p > \alpha_h$ and, therefore, the pressure field is more tightly bound to the guiding surface than the magnetic field. But for $\omega > \omega_p$, $\alpha_p < \alpha_h$ resulting in the pressure field being more loosely bound than the magnetic field.

It is desired to evaluate the total power radiated per unit length of the source in the form of surface waves. It can be shown (see Appendix) that the total power radiated by the line source is obtained as the sum of the powers in the space wave and the boundary wave separately. Hence, it is meaningful to speak of the power radiated in the surface waves alone. It has been proved [3, 5], that the time-averaged, outward normal flow of power through unit area is given by

$$\vec{S} = \frac{1}{2} \text{Re} [\vec{E} \times \vec{H}^* + p \vec{V}^*] \quad (47)$$

The total power transmitted by the surface waves is, therefore, given by

$$W^s = W_+^s + W_-^s, \quad (48)$$

where W_+^s and W_-^s are the net flow of power respectively in the positive and the negative x directions. With the help of (47), W_+^s and W_-^s are easily obtained as

$$\begin{aligned} W_+^s &= \int_0^\infty \hat{x} \cdot \frac{1}{2} \text{Re} [\vec{E}^s \times \vec{H}^{s*} + p^s \vec{V}^{s*}] dz \\ &= \frac{1}{2} \text{Re} \int_0^\infty [-E_z^s(x, z) H_y^{s*}(x, z) + p^s(x, z) V_x^{s*}(x, z)] dz \end{aligned} \quad (49a)$$

and

$$W_-^s = -\frac{1}{2} \operatorname{Re} \int_0^\infty [-E_z^s(x, z) H_y^{s*}(x, z) + p^s(x, z) V_x^{s*}(x, z)] dz, \quad (49b)$$

where $H_y^s(x, z)$ and $p^s(x, z)$, for $x > 0$, are given in (40) and (42) respectively. For $x < 0$, $H_y^s(x, z)$ and $p^s(x, z)$ can be obtained easily as

$$H_y^s(x, z) = H_s e^{-ik_s x - z \sqrt{k_s^2 - k_e^2}} \quad (50a)$$

and

$$p^s(x, z) = -P_s e^{-ik_s x - z \sqrt{k_s^2 - k_e^2}} \quad (50b)$$

For $x > 0$, the expressions for $E_z^s(x, z)$ and $V_x^s(x, z)$ are found by substituting (40) and (42) in (7) and (8). The result is

$$E_z^s(x, z) = -\frac{k_s}{\omega \epsilon_0 \epsilon_1} H_y^s(x, z) + \frac{(1-\epsilon_1)a_p}{N_o e \epsilon_1} p^s(x, z) \quad (51)$$

$$V_x^s(x, z) = -\frac{ea_h}{\omega^2 m \epsilon_0 \epsilon_1} H_y^s(x, z) + \frac{k_s}{\omega \epsilon_1 N_o m} p^s(x, z) \quad (52)$$

When (40), (42), (51) and (52) are used in (49a), it is found that

$$W_+^s = \frac{k_s}{4\omega \epsilon_0 \epsilon_1 a_h} H_s^2 + \frac{k_s}{4\omega \epsilon_1 N_o m a_p} P_s^2 - \frac{(1-\epsilon_1)}{2N_o e \epsilon_1} H_s P_s \quad (53)$$

In a similar manner W_-^s can be calculated and is found to be equal to W_+^s , as expected. Hence, the total power transmitted by the surface waves per unit width of the screen is obtained from (48) and (53) as

$$W^s = 2W_+^s = \frac{k_s}{2\omega \epsilon_0 \epsilon_1 a_h} H_s^2 + \frac{k_s}{2\omega \epsilon_1 N_o m a_p} P_s^2 - \frac{(1-\epsilon_1)}{N_o e \epsilon_1} H_s P_s \quad (54)$$

The expression (54) for W^s can be considerably simplified when advantage is taken of the fact that $\frac{a^2}{c^2}$ is of the order of 10^{-8} and, therefore, negligible in comparison with unity. In view of this fact, it follows from (12), (14) and (39a) that

$$k_s^2 - k_e^2 \doteq k_s^2, \quad k_s^2 - k_p^2 \doteq k_s^2 - \frac{\omega_p^4}{\omega^4} \quad (55)$$

The substitution of (55) in (41) immediately gives

$$H_s = -\omega \epsilon_0 \epsilon_1 J_0 \frac{\omega^4}{\omega^4} \left[1 - \frac{\omega^4}{\omega^4} \right]^{-1} e^{-k_s d} \quad (56)$$

In a similar manner, the use of (55) in (43) yields

$$P_s = -N_0 e J_0 \epsilon_1 \frac{\omega^2}{\omega^2} \left[1 - \frac{\omega^4}{\omega^4} \right]^{-1} e^{-k_s d} \quad (57)$$

With the help of (45), (46), (55) - (57) in (54), it is possible to show, after considerable simplification, that

$$\widetilde{W}^s = \frac{2W^s}{\omega \epsilon_0 J_0^2} = \frac{e^{-2\frac{\omega d}{a}} \frac{\Omega}{\sqrt{\Omega^2 + 1}}}{\Omega^2 (\Omega^2 + 1)}, \quad (58)$$

where $\Omega = \frac{\omega}{\omega_p}$.

It is seen from (58) that $\widetilde{W}^s$ is always positive, showing that the requirement of the boundary wave to have outward traveling phase fronts does lead to the fulfillment of the radiation condition, which requires a net outflow of power at large distances from the source.

It is seen from (58) that the power delivered to the surface wave is a maximum when the line source is on the screen, i. e., when $d = 0$. Also, an increase in the frequency Ω results in a decrease in the power in the surface wave. Obviously, this analysis is not valid for very small Ω , since in that case the motion of the ions cannot be neglected. For $\Omega < 1$, the space waves are exponentially damped and all the power is carried by the surface wave.

It is well known that in an unbounded isotropic plasma the EM and the P modes are uncoupled [2]. The presence of the boundary provides a mechanism of coupling between these modes. Since the boundary surface alone provides the mechanism of coupling of power between the EM and the P modes, it is intuitively obvious that if a surface wave is present, it will be a coupled mode in the sense that the total power in the surface wave is not equal to the sum of the powers in the EM and P modes separately. That this actually turns out to be the case is indicated by the presence of the cross term in (54).

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APPENDIX

Let $\vec{E}$, $\vec{H}$, $\vec{V}$ and P be the total field quantities. The total field is a solution of (2), (3), (4) and (5). Also let $\vec{E}^s$, $\vec{H}^s$, $\vec{V}^s$ and P^s be the field quantities associated with the boundary wave part of the total field. The boundary wave is a solution of source-free field equations. Let

$$\vec{S}_1 = \vec{E} \times \vec{H}^{s*} + P \vec{V}^{s*} + \vec{E}^{s*} \times \vec{H} + P^{s*} \vec{V} \quad (A-1)$$

Taking the divergence of $\vec{S}_1$ and making use of (2) - (5), it can easily be shown that

$$\nabla \cdot \vec{S}_1 = -\vec{H}^{s*} \cdot \vec{J}_m \quad (A-2)$$

The integration of (A-2) throughout the volume V enclosed by the transverse planes P_1 and P_2 , the surface P_∞ at infinity and the surface of the screen (Fig. 1) yields

$$\begin{aligned} \int_{P_1+P_2} \left[-E_z(x, z) H_y^{s*}(x, z) + P(x, z) V_x^{s*}(x, z) \right. \\ \left. - E_z^{s*}(x, z) H_y(x, z) + P^{s*}(x, z) V_x(x, z) \right] dz \\ = \int_V H_y^{s*}(x, z) J_m dv \end{aligned} \quad (A-3)$$

The integral over the surface P_∞ , which is located at infinity, vanishes since the total field is bounded and the field of the boundary wave decreases exponentially in the z direction. Also, the integral over the surface of the screen vanishes since $\hat{z} \cdot \vec{S}_1 = 0$, on the screen, in view of the boundary conditions (17) and (18).

The total field quantities may be written in the form

$$\begin{aligned} H_y(x, z) &= H_y^R(x, z) + H_y^s(x^+, z), \quad P(x, z) = P^R(x, z) + P^s(x^+, z) \\ \vec{E}(x, z) &= \vec{E}^R(x, z) + \vec{E}^s(x^+, z), \quad \vec{V}(x, z) = \vec{V}^R(x, z) + \vec{V}^s(x^+, z) \\ &\text{for } x > 0 \end{aligned} \quad (A-4)$$

and

$$H_y(x, z) = H_y^R(x, z) + H_y^S(x^-, z), \quad P(x, z) = P^R(x, z) + P^S(x^-, z)$$

$$\vec{E}(x, z) = \vec{E}^R(x, z) + \vec{E}^S(x^-, z), \quad \vec{V}(x, z) = \vec{V}^R(x, z) + \vec{V}^S(x^-, z)$$

$$\text{for } x < 0 \quad (A-5)$$

where $H_y^R(x, z)$, $P^R(x, z)$, $\vec{E}^R(x, z)$ and $\vec{V}^R(x, z)$ denote the space wave parts of the total field quantities. The field components of the boundary wave traveling in the positive and the negative x directions are denoted respectively by x^+ and x^- .

In view of (40), (42) and (6) - (8), the field components of the boundary wave traveling in the positive x direction may be represented as

$$\begin{aligned} H_y^S(x^+, z) &= H_s(z) e^{ik_s x}, & P^S(x^+, z) &= P_s(z) e^{ik_s x} \\ E_z^S(x^+, z) &= E_{zs}(z) e^{ik_s x}, & V_x^S(x^+, z) &= V_{xs}(z) e^{ik_s x} \end{aligned} \quad (A-6)$$

In a similar way, with (50a, b) and (6) - (8), it is evident that the field components of the boundary wave traveling in the negative x direction, are given by

$$\begin{aligned} H_y^S(x^-, z) &= H_s(z) e^{-ik_s x}, & P^S(x^-, z) &= -P_s(z) e^{-ik_s x} \\ E_z^S(x^-, z) &= -E_{zs}(z) e^{-ik_s x}, & V_x^S(x^-, z) &= V_{xs}(z) e^{-ik_s x} \end{aligned} \quad (A-7)$$

The substitution of (A-5) - (A-7) in (A-3), after some simplification, yields the following

$$\begin{aligned} &\int_0^\infty dz \left[-E_z^R(x_1, z) H_s(z) + P^R(x_1, z) V_{xs}(z) \right. \\ &\quad \left. - E_{zs}(z) H_y^R(x_1, z) + P_s(z) V_x^R(x_1, z) \right] e^{ik_s x_1} \\ &+ \int_0^\infty dz \left[-E_z^R(-x_2, z) H_s(z) + P^R(-x_2, z) V_{xs}(z) \right. \\ &\quad \left. - E_{zs}(z) H_y^R(-x_2, z) + P_s(z) V_x^R(-x_2, z) \right] e^{-ik_s x_2} \end{aligned}$$

$$\begin{aligned}
& + E_{zs}(z) H_y^R(-x_2, z) - P_s(z) V_x^R(-x_2, z) \Big] e^{ikx_2} \\
& + 2 \operatorname{Re} \int_0^\infty dz \left[-E_{zs}(z) H_s(z) + P_s(z) V_{xs}(z) \right] \\
& = - \int_v H_y^{s*}(x, z) J_m dv \quad (A-8)
\end{aligned}$$

If the position of the transverse plane P_1 is changed, the second and the third terms on the left-hand side as well as the right-hand side of (A-7) are unchanged. Hence, it follows that

$$\begin{aligned}
e^{ik_s x} \int_0^\infty dz \left[-E_z^R(x, z) H_s(z) + P^R(x, z) V_{xs}(z) \right. \\
\left. - E_{zs}(z) H_y^R(x, z) + P_s(z) V_x^R(x, z) \right] = \text{constant} \quad (A-9)
\end{aligned}$$

In a similar manner, it may be argued that

$$\begin{aligned}
e^{-ik_s x} \int_0^\infty dz \left[-E_z^R(x, z) H_s(z) + P^R(x, z) V_{xs}(z) \right. \\
\left. + E_{zs}(z) H_y^R(x, z) + P_s(z) V_x^R(x, z) \right] = \text{constant} \quad (A-10)
\end{aligned}$$

The space wave has a continuous eigenvalue spectrum and so its dependence on x cannot annul the factor $e^{ik_s x}$ in (A-9) and $e^{-ik_s x}$ in (A-10). Furthermore, if the positions of the transverse planes P_1 and P_2 approach respectively $x_1 = \infty$ and $x_2 = \infty$, it is evident that $E_z^R(x, z)$, $P^R(x, z)$, $H_y^R(x, z)$ and $V_x^R(x, z)$ approach zero. Also, since k_s is real, it follows that the constants in (A-9) and (A-10) are zero. Therefore, from (A-9), (A-10), (A-3) and (A-4), it is evident that

$$\begin{aligned}
& \int_0^{\infty} dz \left[-E_z^R(x, z) H_y^{s*}(x, z) + P^R(x, z) V_x^{s*}(x, z) \right] \\
&= \int_0^{\infty} dz \left[-E_z^{s*}(x, z) H_y^R(x, z) + P^{s*}(x, z) V_x^R(x, z) \right] \\
&= 0
\end{aligned}
\tag{A-11}$$

In view of (A-11), it is obvious that the boundary and the space wave parts of the total field are orthogonal in the sense that the total power flow is equal to the sum of the powers separately in the boundary and the space waves.

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