

ED BY DDC 409704
D No.

AFRL-63-152

409 704

NINTH SCIENTIFIC REPORT

TECHNICAL REPORT 400-75

Some General Results for the N-Port

D. J. R. Stock and L. J. Kaplan

May 1963

DDC
RECEIVED
JUL 23 1963
RESOLVED
TISA B

UNIVERSITY

ENGINEERING

ering

In addition to published papers, the College of Engineering reports the results of its research in the form of reports to sponsors of research projects, Technical Reports, and Technical Notes. The latter are normally limited to distribution within the College. Information regarding the availability of reprints of journal articles and Technical Reports may be obtained by writing to the Director of the Research Division, College of Engineering, New York University, New York, 53, N.Y.

TECHNICAL REPORT 400-75

SOME GENERAL RESULTS FOR THE N-PORT

D.J.R. Stock
L.J. Kaplan

NEW YORK UNIVERSITY
COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRICAL ENGINEERING
Laboratory for Electrosience Research

University Heights
New York 53, New York

NINTH SCIENTIFIC REPORT

Contract AF19(604)-7486
Project 5635
Task 563501

May 1963

Prepared
for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

NOTICE

Requests for additional copies by Agencies of the Department of Defense, their contractors, and other Government agencies should be directed to the:

DEFENSE DOCUMENTATION CENTER (DDC)
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

Department of Defense contractors must be established for DDC services or have their "need-to-know" certified by the cognizant military agency of their project or contract.

All other persons and organizations should apply to the:

U. S. DEPARTMENT OF COMMERCE
OFFICE OF TECHNICAL SERVICES
WASHINGTON 25, D. C.

PREVIOUS PUBLICATIONS

- FIRST SCIENTIFIC REPORT: The Geometry of Representations for Active Networks (AFCRL-544)
- SECOND SCIENTIFIC REPORT: Interpretations of Riccati Equations for Some Circuits (AFCRL-545)
- THIRD SCIENTIFIC REPORT: Rectangular Transfer Matrices (AFCRL-546)
- FOURTH SCIENTIFIC REPORT: The Analytic Foundations of Linear, Time-Invariant n-Ports (AFCRL-62-393) (Joint Report with D.C. Youla of the Microwave Research Institute, Polytechnic Institute of Brooklyn, under contract no. AF19(604)-4143)
- FIFTH SCIENTIFIC REPORT: Non-Euclidean Geometric Representations for Microwave Networks (AFCRL-62-500)
- SIXTH SCIENTIFIC REPORT: The Application of Certain Ruler Constructions to the Poincaré and Cayley-Klein Models of Hyperbolic Geometry (AFCRL-62-940)
- SEVENTH SCIENTIFIC REPORT: A Representation for Certain Lossless 2n-Ports and a Measurement Method for a Lossless 4-Port (AFCRL-62-974)
- EIGHTH SCIENTIFIC REPORT: Non-Euclidean Geometric Representations for Lossy Microwave Networks (AFCRL-63-37)

ABSTRACT

A rectangular transfer matrix for the n -port ($n \geq 2$) network is developed here. Considered is a device so used that a p -port is transformed into a q -port ($p + q = n$ and $q \leq p$). Its rectangular transfer matrix has all the interconnection properties of usual transfer matrices, provided the matrices multiplied are conformable (this can also be expressed in circuit terms). Standard tests for reciprocity and losslessness are shown to be applicable to the generalized transfer matrix with some modification.

The single frequency decompositions of Kurss and Youla for the $2n$ -port are shown to be valid for the n -port.

TABLE OF CONTENTS

	<u>Page</u>
I. INTRODUCTION	1
II. FORMULATION OF THE TRANSFER SCATTERING MATRIX	3
III. PROPERTIES OF TRANSFER MATRICES	5
IV. THE WHEELER TYPE DECOMPOSITIONS	8
V. THE WEISSFLOCH DECOMPOSITION	11
APPENDIX I	15
APPENDIX II: On the Forms $T' \sigma_1 T$ and $T^* \sigma_2 T$	15
APPENDIX III	16
REFERENCES	18
ILLUSTRATIONS	19
DISTRIBUTION LIST	22

SOME GENERAL RESULTS FOR THE N-PORT

I. INTRODUCTION

The first occurrence of transfer matrices in electrical engineering utilized the chain, or ABCD, matrix. The introduction of the scattering matrix to circuit theory led to the definition of the transfer scattering matrix for the two-port. At this point of development the impedance, admittance, and scattering matrices of n -ports were well-known. However, the transfer matrices in the impedance and scattering formalism were restricted to the two-port. It has since been shown in the literature that if the device has any even number ($2n$) of ports (one shall refer to it as an even port device), the transfer matrix of the $2n$ -port is defined where the scalars in the two-port case are $n \times n$ matrices. Redheffer¹ has even considered the case where the elements were operators in Hilbert space. There has developed much literature on the $2n$ -port network. This formalism is especially pertinent at microwave frequencies for multimode propagation in a waveguide. Nevertheless, it is desirable to obtain a formulation that can consider both odd port devices and even port devices which are not used as $2n$ -ports (i.e., a four-port considered as transforming a three-port into a one-port).

The general single frequency lossy two-port (reciprocal) was shown by Wheeler⁴ to be decomposable into a pure simple lossy section (two-port) bracketed by two lossless two ports (see fig. 1). Similarly it was shown by Haus⁵ that the non-reciprocity of a two-port can be isolated in a similar manner (see fig. 1). For lossless networks Weissfloch⁶ decomposed the two-port into an ideal transformer between two lossless transmission lines (see fig. 1).

For the single frequency 2n-port Kurss⁷ has shown that the lossy 2n-port can be decomposed into the simple lossy 2n-port bracketed by two lossless 2n-ports. He also shows that the non-reciprocal network is amenable to similar treatment. Youla⁸ showed that the lossless 2n-port can be decomposed into n uncoupled transformers between lossless all-pass networks, (i. e., if the scattering matrix is partitioned into

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

where each submatrix is $n \times n$, S_{11} , and S_{22} are the zero matrix. That is, there is no reflected wave, hence the terminology all-pass).

This paper considers the formulation at single frequency of a rectangular transfer scattering matrix to represent a network with p output and q input ports. The analysis is restricted to

cases for which $p \geq q$.

II. FORMULATION OF THE TRANSFER SCATTERING MATRIX

To obtain this formulation, one begins by representing the device by an $m \times m$ scattering matrix: $[S]$. Although it is not a necessity, the scattering formalism shall be resorted to for the remainder of this paper. The device is considered to transfer a p -port into a q -port ($p + q = m$). Thus, partitioning $[S]$:

$$[S] = \begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix}, \quad (1)$$

where $[S_{11}]$ is $q \times q$, $[S_{12}]$ is $q \times p$, $[S_{21}]$ is $p \times q$; $[S_{22}]$ is $p \times p$. One also defines $\underline{b}_1$, $\underline{a}_1$ as q element column matrices and $\underline{b}_2$, $\underline{a}_2$ as p element column matrices. The device is represented by the following equations:

$$\underline{b}_1 = [S_{11}] \underline{a}_1 + [S_{12}] \underline{a}_2, \quad (2a)$$

$$\underline{b}_2 = [S_{21}] \underline{a}_1 + [S_{22}] \underline{a}_2. \quad (2b)$$

If the network were a $2n$ -port where p is equal to q (equals n), the inverse of S_{21} could be found (assuming its existence); otherwise the transfer matrix would not exist. For the case of interest here, however, $[S_{21}]$ is rectangular, requiring the

following development.

One multiplies equation (2b) from the left side by the transpose of $[S_{21}] : [S_{21}]'$. $[S_{21}]' [S_{21}]$ is a square matrix; hence it may have an inverse^a which is designated $[A]^{-1}$. Then one develops a $2q \times 2p$ matrix $[T]$ defined

- a. It must be assumed that q is less than p in the non-square transfer matrix. If this were not true, $[A]$ would not have an inverse since its rank must be, at most, the smaller rank of the two product matrices (as a consequence of the Binet-Cauchy formula³).^{*} As a result of the preceding there is, at most, one non-square transfer matrix (i.e., there is no reverse transfer matrix as in the square case) for given values of p and q . There will always be a transformation between the output scattering matrix, $[S]$, and the input scattering matrix, $[W]$:

$$[W] = [S_{11}] + [S_{12}] [S] \left[[1] - [S_{22}] [S] \right]^{-1} [S_{21}]$$

Alternatively,

$$W = \left[[T_{11}] [S] + [T_{12}] \right] \left[[T_{21}] [S] + [T_{22}] \right]' M^{-1}$$

where $M = [T_{21}S + T_{22}] [T_{21}S + T_{22}]'$.

The restriction that q is less than p may be stated more elegantly as a linear mapping not increasing the dimension of the space.

* F. R. Gantmacher, The Theory of Matrices, p. 12.

similarly to equation (2):

$$\begin{bmatrix} \underline{b}_1 \\ \underline{a}_1 \end{bmatrix} = [T] \begin{bmatrix} \underline{a}_2 \\ \underline{b}_2 \end{bmatrix} \quad (3)$$

The components of $[T]$ are:

$$[T_{11}]_{q,p} = [S_{11}] - [S_{11}] [A]^{-1} [S_{21}]' [S_{22}] ,$$

$$[T_{12}]_{q,p} = [S_{11}] [A]^{-1} [S_{21}]' ,$$

$$[T_{21}]_{q,p} = -[A]^{-1} [S_{21}]' [S_{22}] ,$$

$$[T_{22}]_{q,p} = [A]^{-1} [S_{21}]' .$$

III. PROPERTIES OF TRANSFER MATRICES

The rectangular transfer matrix has most of the properties of the usual transfer matrices. If two transfer matrices are conformable, then matrix multiplication is possible. It should be pointed out here that the concept of cascading, implied by use of the T matrix, is not quite that of two-port practice, or of $2n$ -port practice, where half the ports, arbitrarily called "input," are to the left of a device, and half, called "output," are to the right. Here, since one deals with

non-square matrices, the number of input ports differ from the number of output ports. Thus, a four-port with non-square T may connect to a one-port on its left, and a three-port on its right. However it is noted that a $(3 + 2)$ - port can not be cascaded with a $(2 + 4)$ - port using rectangular transfer matrices in this form.

In the $2n$ -port literature, the conditions for losslessness and reciprocity of the transfer matrix are formulated in terms of the $2n \times 2n$ Pauli matrices:

$$[\sigma_1] = \begin{bmatrix} [0] & [1] \\ -[1] & [0] \end{bmatrix},$$

$$[\sigma_2] = \begin{bmatrix} [1] & [0] \\ [0] & -[1] \end{bmatrix}.$$

Where $[0]$ and $[1]$ are the $n \times n$ zero and $n \times n$ unit matrices, respectively.

For reciprocity,

$$[T]^t [\sigma_1] [T] = [\sigma_1],$$

$$[T] [\sigma_1] [T]^t = [\sigma_1].$$

For losslessness,

$$[T]^* [\sigma_2] [T] = [\sigma_2] \quad ,$$

$$[T] [\sigma_2] [T]^* = [\sigma_2] \quad .$$

(* denotes complex conjugate transpose)

In the case of a network having a rectangular transfer scattering matrix $[T]$, a necessary condition for reciprocity is:

$$[T] [\sigma_1] [T]' = [\sigma_1] \quad (3a)$$

while a necessary condition for losslessness is:

$$[T] [\sigma_2] [T]^* = [\sigma_2] \quad (3b)$$

It must be noted that these conditions are not sufficient. It is possible^a to find scattering matrices which are not lossy or reciprocal that will map onto transfer matrices that satisfy conditions 3a and/or 3b. The mapping $S \rightarrow T$ is not even one-to-one for several S matrices may go into the same T (this is not true for square transfer matrices). Therefore later decompositions based on the T matrix will have an ambiguity which can only be resolved by resorting to the S matrix. (See section V on the Weissfloch equivalent circuit). However, in Appendix 2 it is seen that if $[T]$ is reciprocal (lossless), then $T T' (T T^*)$ represents a reciprocal

a. See Appendix III.

(lossless) network. This also holds for what may be called pseudo-lossless forms; i.e., rectangular T matrices that satisfy 3b but represent lossy networks.

The third condition using σ_3 :

$$T \sigma_3 T = \sigma_3 \quad , \quad \sigma_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} ,$$

for symmetry does not carry over to the rectangular transfer matrix since the usual symmetry concept does not apply here.

IV. THE WHEELER TYPE DECOMPOSITIONS

The similarity of the isolation of the lossy part and the non-reciprocal part has been noted by Kurss, but this paper will only consider the isolation of the lossy segment. The general transfer matrix will be decomposed as follows:

$$[T] = [U_1] [C] [U_2] \quad . \quad (4)$$

U_1 and U_2 represent lossless networks while C is the simple lossy section; U_1 and C are $m \times n$ matrices, while T and U_2 are $m \times n$ matrices. Following Kurss (not in notation, however), one denotes the σ_2 adjoint of T, T^+ :

$$[T]^+ = [\sigma_2]_{nn}^{-1} [T]^* [\sigma_2]_m \quad .$$

One notes here that the σ_2 matrices are square and of different orders. This satisfies the usual relation:

$$(T_1 T_2)^+ = T_2^+ T_1^+ \quad .$$

A matrix is σ_2 self-adjoint if $T = T^+$, and a matrix represents a lossless network if it is σ_2 unitary.

It is noted that rectangular matrix products of the form AA' and $A'A$ (with or without complex conjugates) have the same rank, and essentially the same eigenvalues and Jordan canonical form.⁹ The difference is that the matrix of higher order has some additional zero eigenvalues to account for the difference in order.

Following Kurss one defines a matrix:

$$H = TT^+ \quad . \quad (5)$$

Kurss defined the product in the reverse order, but the revised arrangement will keep H non-singular. When relation 4 is substituted into 5,

$$H = U_1 CC^* U_1^{-1} \quad .$$

It is seen that H has the same Jordan canonical form as CC^* .

In addition,

$$U_2 = C^{-1} U_1^{-1} T$$

is a lossless network. Thus the constructive steps of Kurss required to obtain the constituents become:

- 1) Find the Jordan matrix of H .
- 2) Find a σ_p -unitary matrix U_1 such that

$$\theta_c = U_2 \theta \text{ is simple.}$$

θ is the matrix relating H and its Jordan matrix J

$$(H = \theta J \theta^{-1}).$$

- 3) Choose C as a solution to the equation

$$CC^* = \theta_c J \theta_c^{-1}$$

The proof of Kurss for the feasibility of this constructive process carries through with slight modifications. Youla² has stated that the proof of Kurss, which assumes a diagonal Jordan matrix, can be extended so that this requirement is not needed. It would seem that this is applicable to the rectangular case also.

V. THE WEISSFLOCH DECOMPOSITION

The lossless network may be decomposed into the cascade of an all-pass $2q$ -port, "uncoupled transformers," and an all-pass $2p$ -port. It is noted that a lossless all-pass network can not be reciprocal for an odd-port device (i.e., all scattering coefficients on main diagonal cannot be zero).

To proceed with the proof one considers the following form: $TT^* = K$. This is a non-singular Hermitian matrix ($K^* = (TT^*)^* = (TT^*) = K$). K is also Hermitian positive semi-definite since it is of form TT^* , but since its determinant is non-zero, K is Hermitian positive definite. Then the proof proceeds in a manner similar to that of Youla⁸ using the unique polar decomposition of a square matrix (T) into the product of a Hermitian positive matrix (G) and a unitary matrix (F), taken akin to complex number $Z = Ae^{i\phi}$.^{3a}

In this case the unitary matrix (F) is not required since K is Hermitian positive definite. Then following Youla's arguments exactly:

$$K = \begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} \begin{bmatrix} \cosh \lambda & \sinh \lambda \\ \sinh \lambda & \cosh \lambda \end{bmatrix} \begin{bmatrix} U^* & 0 \\ 0 & V^* \end{bmatrix},$$

where U and V are $q \times q$ unitary matrices and $\cosh \lambda$ and $\sinh \lambda$ are diagonal real matrices.

If one defines a $2q \times 2p$ matrix $\hat{T}$ so that $\hat{T} \hat{T}^* = \hat{T} \hat{T}' =$

$$\begin{bmatrix} \cosh \lambda & \sinh \lambda \\ \sinh \lambda & \cosh \lambda \end{bmatrix},$$

then T may be written:

$$T = \begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} \hat{T} W.$$

Since $T T^* = K$, W is unitary. The lossless condition applied to W requires that W be an all-pass. In the pseudo-lossless case W is not an all-pass and in fact, accounts for the loss in the overall network. Note that the matrix W is in no way specified by the matrix T (except for being unitary). It contains the information lost in the mapping $S \rightarrow T$. In particular T does not determine whether the network is lossless or pseudo-lossless. Thus,

$$T = \begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} \hat{T} \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix},$$

where X and Y are $p \times p$ unitary matrices, and $\hat{T}$ is a $2q \times 2p$ matrix representing ideal transformers, then

$$K = \begin{bmatrix} U & 0 \\ 0 & V \end{bmatrix} \hat{T} \hat{T}^* \begin{bmatrix} U^* & 0 \\ 0 & V^* \end{bmatrix} .$$

This shows that the Weissfloch decomposition is possible if one can construct a $\hat{T}$ such that,

$$\hat{T} \hat{T}^* = \begin{bmatrix} \cosh \lambda & \sinh \lambda \\ \sinh \lambda & \cosh \lambda \end{bmatrix} .$$

A solution when $p = n-1$ and $q = 1$ is:

$$\hat{T} = \begin{bmatrix} \frac{1}{\sqrt{p}} \cosh \lambda \dots \frac{1}{\sqrt{p}} \cosh \lambda & \frac{1}{\sqrt{p}} \sinh \lambda \dots \frac{1}{\sqrt{p}} \sinh \lambda \\ \frac{1}{\sqrt{p}} \sinh \lambda \dots \frac{1}{\sqrt{p}} \sinh \lambda & \frac{1}{\sqrt{p}} \cosh \lambda \dots \frac{1}{\sqrt{p}} \cosh \lambda \end{bmatrix} .$$

If q is greater than one, there may be several possible matrices $\hat{T}$. This is shown by the following two acceptable matrices for $q = 2$ and $p = 4$.

$$\hat{T}_1 =$$

$$\begin{bmatrix} \cosh\lambda_1 & 0 & 0 & 0 & \sinh\lambda_1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & 0 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 \\ \sinh\lambda_1 & 0 & 0 & 0 & \cosh\lambda_1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & \frac{1}{\sqrt{3}}\sinh\lambda_2 & 0 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 & \frac{1}{\sqrt{3}}\cosh\lambda_2 \end{bmatrix}$$

$$\hat{T}_2 =$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}}\cosh\lambda_1 & \frac{1}{\sqrt{2}}\cosh\lambda_1 & 0 & 0 & \frac{1}{\sqrt{2}}\sinh\lambda_1 & \frac{1}{\sqrt{2}}\sinh\lambda_1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}\cosh\lambda_2 & \frac{1}{\sqrt{2}}\cosh\lambda_2 & 0 & 0 & \frac{1}{\sqrt{2}}\sinh\lambda_2 & \frac{1}{\sqrt{2}}\sinh\lambda_2 \\ \frac{1}{\sqrt{2}}\sinh\lambda_1 & \frac{1}{\sqrt{2}}\sinh\lambda_1 & 0 & 0 & \frac{1}{\sqrt{2}}\cosh\lambda_1 & \frac{1}{\sqrt{2}}\cosh\lambda_1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}\sinh\lambda_2 & \frac{1}{\sqrt{2}}\sinh\lambda_2 & 0 & 0 & \frac{1}{\sqrt{2}}\cosh\lambda_2 & \frac{1}{\sqrt{2}}\cosh\lambda_2 \end{bmatrix}$$

The ambiguity in $\hat{T}$ is complemented by complete non-specification of matrices X and Y . It is to be noted that reciprocity would require $V = \bar{U}$ and $Y = \bar{X}$.

APPENDIX I.

Figure 2 shows the transformer decomposition of the two six-ports whose matrices are given in the text. Figure 3 shows the complete decomposition of a lossless reciprocal three-port.

APPENDIX II: On The Forms $T' \sigma_1 T$ and $T^* \sigma_2 T$

For the 2n-port if T is reciprocal (lossless) T' and $\bar{T}$ are reciprocal (lossless). For the rectangular transfer matrix T' does not represent a network but the forms $T' \sigma_1 T$ and $T^* \sigma_2 T$ can be formed. Denoting: $A' S_{21}'$ by $S_{21}^{-1} \text{ left}$ it is seen that the form $T' \sigma_1 T (T^* \sigma_2 T)$ differs from $T \sigma_1 T' (T^* \sigma_2 T^*)$ by the fact that the left inverse multiplies S_{21} on the right (hence in the 2n-port or square matrix case there is no difference). The results are subsumed in the following equations;

$$T' \sigma_1 T = \begin{bmatrix} 0 & S_{21} S_{21}^{-1} \text{ left} \\ -S_{21} S_{21}^{-1} \text{ left} & 0 \end{bmatrix} = \sigma_1 ,$$

$$T^* \sigma_2 T = \begin{bmatrix} (S_{21} S_{21}^{-1} \text{ left})^* S_{21} S_{21}^{-1} \text{ left} & 0 \\ 0 & -(S_{21} S_{21}^{-1} \text{ left})^* S_{21} S_{21}^{-1} \text{ left} \end{bmatrix} = \sigma_2 .$$

The backward sigma matrices have the same rank and non-zero eigenvalues as the ordinary sigma matrices. They have additional zero eigenvalues to fill out their greater dimension.

A calculation will show if $T \sigma_1 T' = \sigma_1$ then $T \sigma_1 T' = \sigma_1$ and therefore $T T'$ would be a reciprocal $2n$ -port while $T' T$ would not. The similar result holds for the lossless case.

APPENDIX III

The n -port, with $n \geq 3$, that has this scattering matrix

$$S = \begin{bmatrix} 0 & 1 & 0 & \dots \\ 1 & 0 & 0 & \dots \\ 0 & 0 & \chi & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

will have the following rectangular scattering matrix

$$T = \begin{bmatrix} 1 & 0 & \dots & 0 & \dots \\ 0 & \dots & 1 & 0 & \dots \end{bmatrix}$$

This transfer matrix will satisfy the lossless condition even though χ may be such that the original network is lossy active

or indefinite. This transfer matrix also satisfies the reciprocity condition even though if $n \geq 4$, χ can be non-symmetric and hence the circuit be nonreciprocal. All these networks with different scattering matrices, that depend on χ , will map onto the given transfer matrix.

REFERENCES

1. R.M. Redheffer. "On a Certain Linear Fractional Transformation." Journal of Mathematical Physics, vol. 38 (1960), pp. 269-286.
2. D.C. Youla. (private communication).
3. F.R. Gantmacher. The Theory of Matrices - Vol. 1. New York: Chelsea, 1959.
4. H.K. Wheeler. Wheeler Monographs - Vol. 1. Great Neck, Wheeler Labs, 1953.
5. H.A. Haus. "Equivalent Circuits for Passive Non-Reciprocal Elements." Journal of Applied Physics (December 1954), pp. 1500-02.
6. A. Weissfloch. Schaltungstheorie und Messtechnik des Dezimeter-und Zentimeterwellengebietes. Basel: Birkhäuser Verlag, 1954.
7. H. Kurss. "Some General Multiport Network Representations." Proceedings of Symposium on Millimeter Waves (1959).
8. D.C. Youla. "Weissfloch Equivalents for Lossless 2n-Ports." IRE PGCT, vol. CT-7 (September 1960), pp. 193-99.
9. C. Lanczos. "Linear Systems in Self-Adjoint Form." American Mathematical Monthly (November 1958), pp. 665-76.

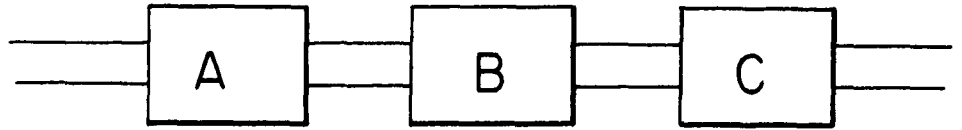


Fig. 1

The Decomposition of a Two-Port

In the Wheeler decomposition, B is pure lossy, while A and C are lossless. In Haus' decomposition, B is pure non-reciprocal, while A and C are reciprocal. In the Weissfloch transformer, B is an ideal transformer, while A and C are lengths of lossless transmission lines.

$$\Lambda_{T_1} = \begin{bmatrix} \cosh \lambda_1 & 0 & 0 & 0 & \sinh \lambda_1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} \cosh \lambda_2 & \frac{1}{\sqrt{3}} \cosh \lambda_2 & \frac{1}{\sqrt{3}} \cosh \lambda_2 & 0 & \frac{1}{\sqrt{3}} \sinh \lambda_2 & \frac{1}{\sqrt{3}} \sinh \lambda_2 & \frac{1}{\sqrt{3}} \sinh \lambda_2 \\ \sinh \lambda_1 & 0 & 0 & 0 & \cosh \lambda_1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} \sinh \lambda_2 & \frac{1}{\sqrt{3}} \sinh \lambda_2 & \frac{1}{\sqrt{3}} \sinh \lambda_2 & 0 & \frac{1}{\sqrt{3}} \cosh \lambda_2 & \frac{1}{\sqrt{3}} \cosh \lambda_2 & \frac{1}{\sqrt{3}} \cosh \lambda_2 \end{bmatrix}$$



$$\Lambda_{T_2} = \begin{bmatrix} \frac{1}{\sqrt{2}} \cosh \lambda_1 & \frac{1}{\sqrt{2}} \cosh \lambda_1 & 0 & 0 & \frac{1}{\sqrt{2}} \sinh \lambda_1 & \frac{1}{\sqrt{2}} \sinh \lambda_1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \cosh \lambda_2 & \frac{1}{\sqrt{2}} \cosh \lambda_2 & 0 & 0 & \frac{1}{\sqrt{2}} \sinh \lambda_2 & \frac{1}{\sqrt{2}} \sinh \lambda_2 \\ \frac{1}{\sqrt{2}} \sinh \lambda_1 & \frac{1}{\sqrt{2}} \sinh \lambda_1 & 0 & 0 & \frac{1}{\sqrt{2}} \cosh \lambda_1 & \frac{1}{\sqrt{2}} \cosh \lambda_1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \sinh \lambda_2 & \frac{1}{\sqrt{2}} \sinh \lambda_2 & 0 & 0 & \frac{1}{\sqrt{2}} \cosh \lambda_2 & \frac{1}{\sqrt{2}} \cosh \lambda_2 \end{bmatrix}$$

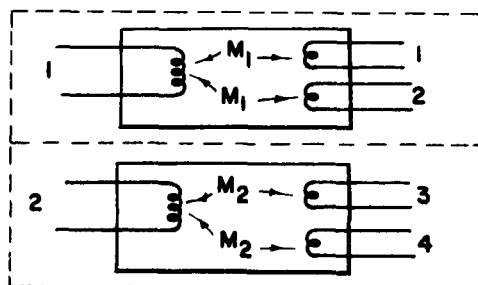


Fig. 2

The two possible couplings for a six-port operating as a four-to-two-port. Note that there is no coupling between the boxes. The matrices explicitly state the relations between the ports.

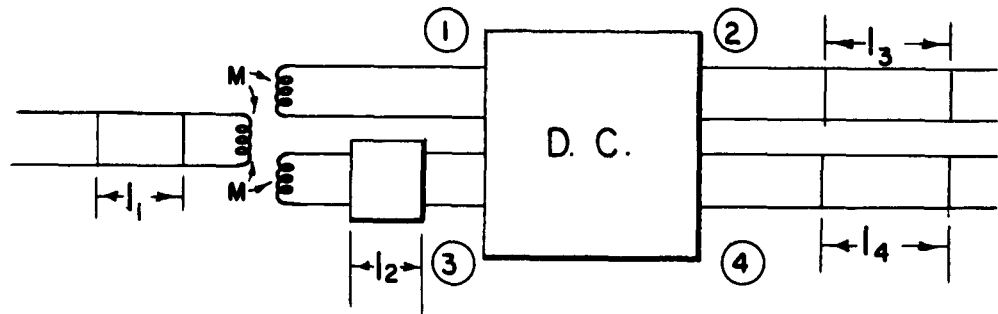


Fig. 3

The lossless three-port represented by a transformer (coupling M), four lengths of transmission line, and a directional coupler (D. C.) whose scattering matrix, using port numbers as shown above, is:

$$\begin{bmatrix} 0 & \alpha & 0 & j\beta \\ \alpha & 0 & j\beta & 0 \\ 0 & j\beta & 0 & \alpha \\ j\beta & 0 & \alpha & 0 \end{bmatrix}$$

DISTRIBUTION LIST

LIST 8-8			Code	Organization	Copy No.
Code	Organization	Copy No.			
AF 29	AFGC (PDAPI) Edlin AFB, Fla.	1	Ar 42	Director U.S. Army Ballistic Research Laboratories Aberdeen Proving Ground, Maryland Attn: Ballistic Measurements Laboratory	28
AF 134	RADC (RAYLD) Attn: Documents Library Griffiss AFB, N.Y.	2	Ar 47	Ballistic Research Laboratories Aberdeen Proving Ground, Maryland Attn: Technical Information Branch	29
AF 143	RADC (RCE) Attn: Dr. John S. Burgess Griffiss AFB, N.Y.	3	Ar 48	Guided Missile Fuse Library Diamond Ordnance Fuse Laboratories Washington 25, D.C. Attn: R.D. Hatcher, Chief Microwave Development Section	30
AF 139	AF Missile Dev. Cent. (MDGRT) Bollman AFB, New Mexico	4			
AF 69	Director of Resident Training 3380th Technical Training Group Keesler AFB, Mississippi Attn: OA-3011 Course	5	Ar 49	Commanding General USASRDL Fort Monmouth, New Jersey Attn: SIGPM/EL-AT	31
AF 86	SAC (Operations Analysis Office) Offutt AFB, Nebraska	6	Ar 67	Redstone Scientific Information Center U.S. Army Missile Command Redstone Arsenal, Alabama	32-36
AF 18	AUL Maxwell AFB, Ala.	7	Ar 78	Commanding General, SIGPM/EL-PC USASRDL Fort Monmouth, New Jersey Attn: Dr. Horst H. Kadesdy Deputy Chief, Chem-Physics Branch	37
AF 5	AF Missile Test Center Patrick AFB, Fla. Attn: AFMTC, Tech Library, MU-135	8			
AF 227	USAF Security Service (CLR) San Antonio, Texas	9	G 4	Defense Documentation Center (DDC) Arlington Hall Station Arlington 12, Virginia	38-57
AF 253	Technical Information Office European Office, Aerospace Research Shell Building, 47 Cantersteen Brussels, Belgium	10	G 8	Library National Bureau of Standards Boulder Laboratories Boulder, Colorado	58-59
AF 30	OAR (RROS, Col. John R. Fowler) Tempo D 4th and Independence Ave. Washington 25, D.C.	11	G 9	Defence Research Member Canadian Joint Staff 240 Massachusetts Avenue, N.W. Washington 25, D.C.	60-62
AF 33	AFOSR, OAR (SRYP) Tempo D 4th and Independence Ave. Washington 25, D.C.	12	G 68	Scientific and Technical Information Facility Attn: NASA Representative (S-AR/DL) Post Office Box 4700 Bethesda, Maryland	63
AF 166	Hq. USAF (AFOSAC-S/E) Communications-Electronics Directorate Washington 25, D.C.	13	G 17	National Bureau of Standards U.S. Department of Commerce Washington 25, D.C. Attn: Gustave Shapiro (Chief, Engineering Electronics Section, Electricity and Electronics Div.	64
AF 314	Hq. OAR (RROSP, Maj. Richard W. Nelson) Tempo D, 4th and Independence Ave. Washington 25, D.C.	14			
Code	Organization	Copy No.	Code	Organization	Copy No.
AF 3	ASD (ASAFPD - Dist) Wright-Patterson AFB, Ohio	15	G 31	Office of Scientific Intelligence Central Intelligence Agency 2430 E Street, N.W. Washington 25, D.C.	55
AF 63	WADD (ASRNC, Mr. Portune) Wright-Patterson AFB, Ohio	16	G 75	Director National Security Agency Fort George G. Meade, Maryland Attn: C3/TDL	66
AF 68	ASD (ASRNC-3) Attn: Mr. Paul Springer Wright-Patterson AFB, Ohio	17	G 103	National Aeronautical Space Agency Langley Aeronautical Research Laboratory Langley, Virginia Attn: Mr. Cliff Nelson	67
AF 11	Foreign Technology Division (TDER) Wright-Patterson AFB, Ohio	18	M 6	AFPCRL, OAR (CRGA - Stop 39) L.G. Hanscom Field Bedford, Mass.	68-77
AF 308	WADD (WMRT, Mr. A.D. Clark) Directorate of System Engineering Dyna Soar Engineering Office Wright-Patterson AFB, Ohio	19	M 17	AFPCRL, Office of Aerospace Research (CRD) Attn: Contract Files L.G. Hanscom Field, Bedford, Mass.	78-79
AF 318	Lt. Col. Jensen (SSTRE) Space Systems Division Air Force Unit Post Office Los Angeles 45, California	20	M 54	AFPCRL, Office of Aerospace Research (CRD) Attn: Carlyle J. Sletten L.G. Hanscom Field, Bedford, Mass.	80-82
Ar 3	Director Evans Signal Laboratory Belmar, New Jersey Attn: Mr. G.C. Woodyard	21	M 59	Electronics Systems Division (AFSC) Technical Information Services Division (NSAT) L.G. Hanscom Field, Bedford, Mass.	84
Ar 5	Commanding General USASRDL Fort Monmouth, New Jersey Attn: Tech. Doc. Ctr. SIGRA/SI-ADT	22	M 84	Hq. AFPCRL, OAR (CHER, J.R. Marple) L.G. Hanscom Field, Bedford, Mass.	85
Ar 7	Department of the Army Office of the Chief Signal Officer Washington 25, D.C. Attn: SIGRD-4a-2	23	M 3	Chief, Bureau of Ships Department of the Navy Washington 23, D.C. Attn: Code 090	86
Ar 10	Massachusetts Institute of Technology Signal Corps Liaison Officer Cambridge 39, Mass. Attn: A.D. Redrosian, Room 26-131	24	M 9	Chief, Bureau of Naval Weapons Department of the Navy Washington 25, D.C. Attn: DLI-31	87-98
Ar 39	Commanding General USASRDL Fort Monmouth, New Jersey Attn: Mr. F.J. Triola	25	M 16	Commander U.S. Naval Air Missile Test Center Point Mugu, California Attn: Code 366	89
Ar 41	Commanding General U.S. Army Materiel Command Attn: AMCRD-RE-FE-3 Washington 25, D.C.	26	M 13	U.S. Naval Ordnance Laboratory White Oak, Silver Spring 19, Maryland Attn: The Library	90
AF 142	ASD (ASRCM-1) Wright-Patterson AFB, Ohio	27			

<u>Code</u>	<u>Organization</u>	<u>Copy No.</u>	<u>Code</u>	<u>Organization</u>	<u>Copy No.</u>
N 26	Commander U.S. Naval Ordnance Test Station China Lake, California Attn: Code 733	91	I 8	Bell Aircraft Corp. Post Office Box One Buffalo 5, New York Attn: Eunice P. Haselton, Librarian	124
N 27	Librarian U.S. Naval Postgraduate School Monterey, California	92	I 469	Bell Telephone Laboratories Murray Hill, New Jersey	125
G 126	National Aeronautics and Space Administration Attn: Antenna Systems Branch Goddard Space Flight Center Greenbelt, Maryland	93	I 13	Bell Telephone Laboratories, Inc. Technical Information Library Whippany Laboratory Whippany, New Jersey Attn: Technical Reports Librarian	126
N 29	Director U.S. Naval Research Laboratory Washington 25, D.C. Attn: 2007	94-95	I 246	Bendix Pacific Division 11600 Sherman Way North Hollywood, California Attn: Engineering Library	127
N 30	Dr. J. I. Ehnert, Code 5210 U.S. Naval Research Laboratory Washington 25, D.C.	96	I 247	Bendix Radio Division Bendix Aviation Corp. E. Joppa Road Ewens 4, Maryland Attn: Dr. D. M. Allison, Jr. Director Engineering and Research	128
N 35	Commanding Officer and Director U.S. Navy Underwater Sound Laboratory Port Trumbull, New London, Connecticut	97	I 230	Atlantic Research Corporation Shirley Highway at Misall Road Alexandria, Virginia Attn: Delmer C. Fouts	129
N 37	Chief of Naval Research Department of the Navy Washington 25, D.C. Attn: Code 427	98	I 248	Rjorkstan Research Laboratories, Inc. P.O. Box 265 Madison, Wisconsin Attn: Librarian	130
N 48	Commanding Officer U.S. Naval Air Development Center Johnsville, Penn. Attn: NADC Library	99	I 249	Boeing Airplane Co. Pilotless Aircraft Division Seattle 24, Washington Attn: R.R. Barber, Library Supervisor	131-132
N 73	Office of Naval Research Branch Office, London Mary 100, Box 39 P.P.O. New York, N.Y.	100-109	I 250	Boeing Company 3801 3801 S. Oliver Street Wichita 1, Kansas Attn: Kenneth C. Knig't, Library Supervisor	133
N 85	Commanding Officer and Director U.S. Navy Electronics Laboratory (Library) San Diego 52, California	110	I 252	Brush Beryllium Company 17876 St. Clair Street Cleveland 10, Ohio Attn: H.V. Bass	134
N 91	Commander U.S. Naval Air Test Center Patuxent River, Maryland Attn: EF-315, Antenna Branch	111	I 253	Chance Vought Corp. 9214 West Jefferson Boulevard Dallas, Texas Attn: A.D. Pattullo, Librarian	135
N 92	Material Laboratory, Code 932 New York Naval Shipyard Brooklyn 1, New York Attn: Mr. Douglas First	112	I 986	Chance Vought Corp. Vought Electronics Division P.O. Box 9907 Dallas 22, Texas	136
<u>Code</u>	<u>Organization</u>	<u>Copy No.</u>	<u>Code</u>	<u>Organization</u>	<u>Copy No.</u>
N 123	Chief, Bureau of Ships Department of the Navy Washington 25, D.C. Attn: Code 8178	113	I 470	Chu Associates P.O. Box 387 Whitcomb Avenue Littleton, Mass.	137
N 141	AFSC Scientific and Technical Liaison Office c/o Department of the Navy Room 2305, Munitions Building Washington 25, D.C.	114	I 918	Collins Radio Co. 855 35th Street, N.E. Cedar Rapids, Iowa Attn: Dr. R.L. McGraw	138
I 601	Aero Geo Astro Corp. 1200 Duke Street Alexandria, Virginia Attn: Library	115	I 126	Convair, A Division of General Dynamics Corp. Fort Worth, Texas Attn: K.G. Brown, Division Research Librarian	139
I 21	Aerospace Corp. Satellite Control Attn: Mr. R.C. Hansen Post Office Box 95089 Los Angeles 45, California	116	I 254	Convair, A Division of General Dynamics Corp. 3165 Pacific Highway San Diego 12, California Attn: Mrs. Dora B. Burke Engineering Librarian	140
I 337	Aerospace Corp. Attn: Dr. Max T. Weiss Electronics Laboratory 2400 East El Segundo Boulevard El Segundo, California	117	I 25	Cornell Aeronautical Laboratory, Inc. 4455 Genesee Street Buffalo 21, New York Attn: Librarian	141
I 940	Aerospace Corp. Box 95089 Los Angeles 45, California Attn: Library	118	I 255	Dalmo Victor Company A Division of Textron, Inc. 1515 Industrial Way Balaam, California Attn: Mary Ellen Adams, Technical Librarian	142
I 34	ACF Electronics Div. Electro-Physics Labs. Attn: Library 3355 52nd Avenue Hyattsville, Maryland	119	I 28	Dorne and Margolin, Inc. 29 New York Avenue Westbury, L.I. New York	143
I 1	Airborne Instruments Laboratory, Inc. Division of Outlier Hammer Walt Whitman Road Melville, L.I. New York Attn: Library	120	I 257	Aircraft Division Douglas Aircraft Company, Inc. 3855 Lakewood Boulevard Long Beach, California (USA) Attn: Technical Library	144
I 388	Aircom, Inc. 48 Cummings Street Boston, Mass.	121	I 258	Douglas Aircraft Company, Inc. 3000 Ocean Park Boulevard Santa Monica, California Attn: Peter Dugan, Jr. Chief, Electrical/Electronics Section	145
I 3	Andrew Alford, Consulting Engineers 299 Atlantic Avenue Boston 10, Mass.	122	I 259	Douglas Aircraft Company, Inc. 2000 North Memorial Drive Tulsa, Oklahoma	146
I 205	Battelle Memorial Institute 505 King Avenue Columbus 1, Ohio Attn: Wayne E. Rife, Project Leader Electrical Engineering Division	123	I 187	Electromagnetic Research Corporation 5001 College Avenue College Park, Maryland Attn: Mr. Martin Katsis	147

<u>Code</u>	<u>Organization</u>	<u>Copy No.</u>	<u>Code</u>	<u>Organization</u>	<u>Copy No.</u>
I 415	Electronics Communication 1830 York Road Piscataway, Maryland	148	-I 302	International Business Machines Corp. Space Guidance Center-Federal Systems Division Owego, Tioga County, New York Attn: Technical Reports Center	170
I 356	Sperry-Rand Research Center Route 117 (North Road) Sudbury, Mass. Attn: Mr. Gerald Melts	149	I 414	International Resistance Company 401 E. Broad Street Philadelphia 8, Penn. Attn: Research Library	171
I 299	Electronic Specialty Company 5121 San Fernando Road Los Angeles 39, California Attn: Donald L. Margerus, Chief Engineer Radiating Systems Division	150	I 265	ITT Federal Laboratories 3700 East Pontiac Street Fort Wayne 1, Indiana Attn: Technical Library	172
I 204	Eserson and Cuming, Inc. 59 Walpole Street Canton, Mass. Attn: Mr. W. Cuming	151	I 241	Dr. Henry Jasik, Consulting Engineer 298 Shames Drive Brush Hollow Industrial Park Westbury, New York	173
I 262	Eserson Electric Mfg. Co. 8100 West Florissant Avenue St. Louis 21, Missouri Attn: Mr. E.M. Breslin, Librarian	152	I 279	Lockheed Aircraft Corporation 2555 E. Hollywood Way California Division Engineering Library Department 72-25, Plant A-1, Building 63-1 Burbank, California Attn: E.C. Marnois	174
I 147	Eserson Radio-Photograph Corp. Eserson Research Laboratories 1140 Eastwest Highway Silver Spring, Maryland Attn: Mrs. R. Corbin, Librarian	153	I 468	Lockheed Aircraft Corporation Missiles and Space Division Technical Information Center 3251 Manover Street Palo Alto, California	175-176
I 264	Fairchild Stratos Corp. Aircraft Missiles Division Bogertown, Maryland Attn: Library	154	I 136	Martin-Marietta Corp. 12220 S. State Highway 65 Jefferson County, Colorado Attn: Mr. Jack McCormick	177
I 266	ITT Federal Laboratories Technical Library 500 Washington Avenue Rutley 10, New Jersey	155	I 280	The Martin Company Baltimore 3, Maryland Attn: Engineering Library Antenna Design Group	178
I 4	Gabriel Electronics Division Main and Pleasant Streets Mills, Mass. Attn: Dr. Edward Altshuler	156	I 63	Mathematical Reviews 150 Hope Street Providence 6, Rhode Island	179
I 269	General Electric Company Building 3 - Room 143-1 Electronics Park Syracuse, New York Attn: Yolande Burke, Documents Library	157	I 66	The W.L. Maxson Corporation 475 10th Avenue New York, New York Attn: Miss Dorothy Clark	180
I 193	General Electric Company Missile and Space Vehicle Department 3198 Chestnut Street Philadelphia, Penn. Attn: Documents Library	158	I 282	McDonnell Aircraft Corporation Box 516, St. Louis 66, Missouri Attn: C.E. Zoller Engineering Library	181
<u>Code</u>	<u>Organization</u>	<u>Copy No.</u>	<u>Code</u>	<u>Organization</u>	<u>Copy No.</u>
I 893	General Electric Company 3750 D Street Philadelphia 24, Penn. Attn: Mr. H.G. Lew Missile and Space Vehicle Department	159	I 283	McMillan Laboratory, Inc. Brownville Avenue Ipswich, Mass. Attn: Security Officer, Document Room	182
I 270	General Precision Laboratory, Inc. 63 Bedford Road Pleasantville, New York Attn: Librarian	160	I 116	Melpar, Inc. 3000 Arlington Boulevard Falls Church, Virginia Attn: Engineering Technical Library	183
I 48	Goodyear Aircraft Corp. 1210 Massillon Road Akron 15, Ohio Attn: Library, Plant G	161	I 471	Microwave Associates, Inc. South Avenue Burlington, Mass.	184
I 448	Granger Associates Electronic Systems 974 Commercial Street Palo Alto, California Attn: John V.M. Granger, President	162	I 390	Microwave Development Laboratories, Inc. 92 Broad Street Wellesley 27, Mass. Attn: E. Tucker, General Manager	185
I 272	Grumman Aircraft Engineering Corporation Bethpage, L.I. New York Attn: Engineering Librarian, Plant No. 8	163	I 648	The Mitre Corporation 244 Wood Street Lexington 73, Mass. Attn: Mrs. Jean E. Clafin, Librarian	186
I 273	Hallcrafters Company 4401 West 5th Avenue Chicago 24, Illinois Attn: LaVerne LaGioia, Librarian	164	I 85	Motorola, Inc. 8201 East McDowell Road Phoenix, Arizona Attn: Dr. Thomas E. Tice	187
I 737	The Hallcrafters Co. 5th and Kostner Avenues Chicago 24, Illinois Attn: Henri Rodara, Head of Space Communication	165	I 934	Motorola, Inc. Phoenix Research Laboratory 3102 E. 56th Street Phoenix, Arizona Attn: Dr. A.L. Allen	188
I 274	Hoffman Electronics Corp. 3761 South Hill Street Los Angeles 7, California	166	I 641	National Research Council Radio and Electrical Engineering Division Ottawa, Ontario, Canada Attn: Dr. G.A. Miller, Head Microwave Section	189
I 207	Hughes Aircraft Company Antenna Department Building 12, Wall Station 2714 Oulver City, California Attn: Dr. W.E. Kummer	167	I 284	North American Aviation, Inc. 12214 Lakewood Boulevard Downey, California Attn: Technical Information Center (499-12) Space and Information Systems Division	190
I 56	Hughes Aircraft Company Florence Ave. and Teale Street Oulver City, California Attn: Louis L. Bailin Manager, Antenna Dept.	168	I 285	North American Aviation, Inc. Los Angeles International Airport Los Angeles 45, California Attn: Engineering Technical File	191
I 981	Hughes Aircraft Company Attn: Mr. L. Stark, Microwave Dept. Radar Laboratory, P.O. Box 2077 Building 600, Wall Station C-152 Fullerton, California	169	I 82	Northrop Corporation Rearr Division 1001 West Broadway Berthorne, California Attn: Technical Information 3924-31	192

<u>Code</u>	<u>Organisation</u>	<u>Copy No.</u>	<u>Code</u>	<u>Organisation</u>	<u>Copy No.</u>
I 286	Page Communications Engineers, Inc. 2001 Wisconsin Avenue, N.W. Washington 7, D.C. Attn: (Mrs.) Ruth Temple, Librarian	193	I 391	Sage Laboratories, Inc. 3 Biron Drive Mattick, Mass.	213
I 287	Philco Corporation Research Division Union Meeting Pond Blue Bell, Penn. Attn: Research Librarian	194	I 142	Sanders Associates, Inc. 95 Canal Street Nashua, New Hampshire Attn: Mr. Norman R. Wild	214
I 225	Pickard and Burns, Inc. 103 Fourth Avenue Waltham 54, Mass. Attn: Dr. Richard E. Woodward	195	I 96	Sandia Corporation P.O. Box 5800 Albuquerque, New Mexico Attn: Records Management and Services Department	215
I 289	Radiation, Inc. Melbourne, Florida Attn: RF Systems Division Technical Information Center	196	I 682	Scanwell Laboratories, Inc. 6601 Scanwell Lane Springfield, Va.	216
I 914	Radiation Systems, Inc. 440 Swann Avenue Alexandria, Virginia Attn: Library	197	I 312	SEL Technical Library Document Acquisitions Space Technology Laboratories, Inc. P.O. Box 95001 Los Angeles 45, California	217
I 290	RCA Laboratories David Barnoff Research Center 201 Washington Road Princeton, New Jersey Attn: Miss Fern Cloak, Librarian	198	I 297	Sperry Gyroscope Company Great Neck, L.I. New York Attn: Florence W. Turnbull Engineering Librarian	218
I 291	Radio Corporation of America Defense Electronic Products Building 10, Floor 7 Camden 2, New Jersey Attn: Mr. Harold J. Schrader, Staff Engineer Organization of Chief Technical Administrator	199	I 367	Stanford Research Institute Documents Center Menlo Park, California Attn: Acquisitions	219
I 473	Radio Corporation of America Missile Control and Electronics Division Bedford Street Burlington, Mass. Attn: Librarian	200	I 104	Sylvania Electric Products, Inc. 100 First Avenue, Waltham 54, Mass. Attn: Charles A. Thornhill Report Librarian Waltham Laboratories Library	220
I 157	Radio Corporation of America Surface Communications Systems Laboratory 75 Varick Street New York 13, New York Attn: Mr. S. Krevsky	201	I 260	Sylvania Elec. Prod. Inc. Electronic Defense Laboratory P.O. Box 205 Mountain View, California Attn: Library	221
I 789	Radio Corporation of America West Coast Missile and Surface Radar Division Engineering Library, Building 306/2 Attn: L.R. Hund, Librarian 8500 Balboa Boulevard Van Nuys, California	202	I 240	TNO, Inc. 400 Border Street East Boston, Mass. Attn: Dr. Alan P. Kay	222
			I 338	A.S. Thomas, Inc. 355 Providence Highway Westwood, Mass. Attn: A.S. Thomas, President	223
<u>Code</u>	<u>Organisation</u>	<u>Copy No.</u>	<u>Code</u>	<u>Organisation</u>	<u>Copy No.</u>
I 430	Radio Corporation of America Defense Electronic Products Advanced Military Systems Princeton, New Jersey Attn: Mr. David Shore	203	I 708	Texas Instruments, Inc. 6000 Lemmon Avenue Dallas 9 Texas Attn: John B. Travis Systems Planning Branch	224
I 292	Director, USAF Project RAND Via: AF Liaison Office The Rand Corporation 1700 Main Street Santa Monica, California	204	I 139	Westinghouse Electric Corp. Electronics Division Friendship Int'l Airport Box 1897 Baltimore 3, Maryland Attn: Engineering Library	225
I 347	The Rand Corporation 1700 Main Street Santa Monica, California Attn: Technical Library	205	U 1	Library Geophysical Institute of the University of Alaska College, Alaska	226
I 373	Rantec Corporation 23999 Ventura Boulevard Calabasas, California Attn: Grace Keener, Office Manager	206	U 61	Brown University Department of Electrical Engineering Providence, Rhode Island Attn: Dr. C.M. Angulo	227
I 293	Raytheon Company Boston Post Road Wayland, Mass. Attn: Mr. Robert Borts	207	U 157	California Institute of Technology Jet Propulsion Laboratory 4800 Oak Grove Drive Pasadena, California Attn: Mr. I.E. Newlan	228
I 294	Raytheon Company Wayland Laboratory Wayland Mass. Attn: Miss Alice G. Anderson, Librarian	208	U 99	California Institute of Technology 1201 E. California Drive Pasadena, California Attn: Dr. C. Papas	229
I 472	Raytheon Company Missile Systems Division Hartwell Road, Bedford, Mass. Attn: Donald H. Archer	209	U 3	Space Sciences Laboratory Leuschner Observatory University of California Berkeley 4, California Attn: Dr. Samuel Silver Professor of Engineering Science and Director, Space Sciences Laboratory	230
I 510	Remington Rand UNIVAC Division of Sperry Rand Corporation P.O. Box 500 Blue Bell, Penn. Attn: Engineering Library	210	U 100	University of California Electronics Research Lab. 332 Cory Hall, Berkeley 4, California Attn: J.R. Whinnery	231
I 295	Republic Aviation Corporation Farmingdale, L.I. New York Attn: Engineering Library	211	U 289	University of Southern California University Park Los Angeles, California Attn: Dr. Raymond L. Chuan Director, Engineering Center	232
I 184	Ryan Aeronautical Company 2701 Harbor Drive Lindbergh Field San Diego 12, California Attn: Library	212	U 239	Case Institute of Technology Electrical Engineering Department 10900 Radia Avenue Cleveland, Ohio Attn: Professor Robert Plonsey	233

<u>Code</u>	<u>Organisation</u>	<u>Copy No.</u>		<u>Code</u>	<u>Organisation</u>	<u>Copy No.</u>
U 183	Columbia University Department of Electrical Engineering Morningside Heights, New York, N.Y. Attn: Dr. Schlesinger	234				
U 238	University of Southern California Engineering Center University Park Los Angeles 7, California Attn: Z.A. Kaprielian, Associate Professor of Electrical Engineering	235				
U 10	Cornell University School of Electrical Engineering Ithaca, New York Attn: Professor G.C. Dalman	236		U 39	New York University Institute of Mathematical Sciences Room 802, 25 Waverly Place New York 3, New York Attn: Morris Kline, Dr.	254
U 86	University of Florida Department of Electrical Engineering Gainesville, Florida Attn: Professor M.E. Latour, Library	237		U 96	Northwestern University Microwave Laboratories Evanston, Illinois Attn: R.E. Beam	255
U 59	Library Georgia Technology Research Institute Engineering Experiment Station 722 Cherry Street, N.W. Atlanta, Georgia Attn: Mrs. J.H. Croeland, Librarian	238		U 43	Antenna Laboratory Department of Electrical Engineering The Ohio State University 2024 Neil Avenue Columbus 10, Ohio Attn: Reports Librarian	256
U 102	Harvard University Technical Reports Collection Gordon McKay Library 303 Pierce Hall Oxford Street, Cambridge 38, Mass. Attn: Librarian	239		U 109	The University of Oklahoma Research Institute Norman, Oklahoma Attn: Prof. C.L. Farrar, Chairman Electrical Engineering	257
U 54	Harvard College Observatory 60 Garden Street Cambridge 39, Mass. Attn: Dr. Fred L. Whipple	240		U 361	The Pennsylvania State University 223 Electrical Engineering University Park, Penn. Attn: A.E. Haynick, Director Ionosphere Research Lab.	258
U 104	University of Illinois College of Engineering Urbana, Illinois Attn: Dr. P.E. Hayes Department of Electrical Engineering	241		U 185	University of Pennsylvania Institute of Cooperative Research 3400 Walnut Street Philadelphia, Penn. Attn: Department of Electrical Engineering	259
U 22	The John Hopkins University Homewood Campus Baltimore 18, Maryland Attn: Dr. Donald E. Kerr Dept. of Physics	242		U 48	Polytechnic Institute of Brooklyn Microwave Research Institute 55 Johnson Street Brooklyn, New York Attn: Dr. Arthur A. Oliner	260
U 105	The John Hopkins University Applied Physics Laboratory 8621 Georgia Avenue Silver Spring, Maryland Attn: Mr. George L. Seelstad	243		U 184	Purdue University Department of Electrical Engineering Lafayette, Indiana Attn: Dr. Schultz	261
				U 176	Library W.W. Hansen Laboratory of Physics Stanford University Stanford, California	262
<u>Code</u>	<u>Organisation</u>	<u>Copy No.</u>		<u>Code</u>	<u>Organisation</u>	<u>Copy No.</u>
U 228	University of Kansas Electrical Engineering Department Lawrence, Kansas Attn: Dr. H. Unz	244		U 110	Syracuse University Research Institute Cortlandt Campus Syracuse 10, New York Attn: Dr. C.S. Grove, Jr. Director of Engineering Research	263
U 68	Lowell Technological Institute Research Foundation P.O. Box 709, Lowell, Mass. Attn: Dr. Charles R. Mingins	245		U 309	Technical University Oesterovagade 10 G Copenhagen, Denmark Attn: Prof. Hans Lottrup Knudsen	264
U 32	Massachusetts Institute of Technology Research Laboratory of Electronics Building 26, Room 327 Cambridge 39, Mass. Attn: John H. Nevitt	246		U 186	University of Tennessee Farris Hall W. Cumberland Avenue Knoxville 10, Tennessee	265
U 26	Massachusetts Institute of Technology Lincoln Laboratory P.O. Box 73 Lexington 73, Mass. Attn: Mary A. Granese, Librarian	247		U 111	The University of Texas Electrical Engineering Research Lab. P.O. Box 8026, University Station Austin 12, Texas Attn: Mr. John R. Gerhardt Assistant Director	266
U 34	McGill University Department of Electrical Engineering Montreal, Canada Attn: Dr. T. Pavlasek	248		U 51	The University of Texas Defense Research Laboratory Austin, Texas Attn: Claude W. Norton Physics Library	267
U 107	University of Michigan Electronic Defense Group Institute of Science and Technology Ann Arbor, Michigan Attn: J.A. Boyd, Supervisor	249		U 132	University of Toronto Department of Electrical Engineering Toronto, Canada Attn: Prof. G. Sinclair	268
U 79	University of Michigan Office of Research Administration Radiation Laboratory 912 N. Main Street Ann Arbor, Michigan Attn: Mr. Ralph E. Hiatt	250		U 133	University of Washington Department of Electrical Engineering Seattle 5, Washington Attn: D.E. Reynolds	269
U 37	University of Michigan Engineering Research Institute Willow Run Laboratories, Willow Run Airport Tpsilanti, Michigan Attn: Librarian	251		U 187	University of Wisconsin Department of Electrical Engineering Madison, Wisconsin Attn: Dr. Scheibe	270
U 108	University of Minnesota Minneapolis 14, Minnesota Attn: Mr. Robert E. Stumm Library	252		U 103	University of Illinois Documents Division Library Urbana, Illinois	271
U 194	Physical Science Laboratory New Mexico State University University Park, New Mexico Attn: Mr. E.W. Haas	253		U 97	Polytechnic Institute of Brooklyn Microwave Research Institute 55 Johnson Street Brooklyn, New York Attn: Mr. A.E. Laemmel	272

Air Force Cambridge Research Laboratories,
Office of Aerospace Research, United States
Air Force, Bedford, Massachusetts.

Rpt Nr AFORL-63-152. SOME GENERAL RESULTS
FOR THE N-PORT. Ninth Scientific report,
May 1963, 26 + v pp. incl illus., 9 refs.

Unclassified Report

A rectangular transfer matrix for the n-port
($n \geq 2$) network is developed here. Considered
is a device so used that a p-port is trans-
formed into a q-port ($p+q=n$ and $q \geq 2$). Its
rectangular transfer matrix has all the in-
terconnection properties of usual transfer
matrices, provided the matrices multiplied

are conformable (this can also be expressed
in circuit terms). Standard tests for reci-
procity and losslessness are shown to be ap-
plicable to the generalized transfer matrix
with some modification.

The single frequency decompositions of Kurss
and Youla for the 2n-port are shown to be va-
lid for the n-port.

L. Non-Euclidean Geometry

I. Project 5635,
Task 563501

II. Contract
AF19(604)-7486

III. New York University,
College of Engi-
neering, Department
of Electrical Engi-
neering, Laboratory
for Electroscience
Research, University
Heights, New York
53, New York

IV. D.J.R. Stock,
L.J. Kaplan
V. NYU Technical Report
400-75

VI. Avail fr OMS

VII. In IDC collection

Air Force Cambridge Research Laboratories,
Office of Aerospace Research, United States
Air Force, Bedford, Massachusetts.

Rpt Nr AFORL-63-152. SOME GENERAL RESULTS
FOR THE N-PORT. Ninth Scientific report,
May 1963, 26 + v pp. incl illus., 9 refs.

Unclassified Report

A rectangular transfer matrix for the n-port
($n \geq 2$) network is developed here. Considered
is a device so used that a p-port is trans-
formed into a q-port ($p+q=n$ and $q \geq 2$). Its
rectangular transfer matrix has all the in-
terconnection properties of usual transfer
matrices, provided the matrices multiplied

are conformable (this can also be expressed
in circuit terms). Standard tests for reci-
procity and losslessness are shown to be ap-
plicable to the generalized transfer matrix
with some modification.

The single frequency decompositions of Kurss
and Youla for the 2n-port are shown to be va-
lid for the n-port.

L. Non-Euclidean Geometry

I. Project 5635,
Task 563501

II. Contract

AF19(604)-7486

III. New York University,
College of Engi-
neering, Department
of Electrical Engi-
neering, Laboratory
for Electroscience
Research, University
Heights, New York
53, New York

IV. D.J.R. Stock,
L.J. Kaplan
V. NYU Technical Report
400-75

VI. Avail fr OMS

VII. In IDC collection

The Research Division of the College of Engineering is an integral part of the educational program of the College. The faculty of the College takes part in the work of the Research Division, often serving as co-ordinators or project directors or as technical specialists on the projects. This research activity enriches the educational experience of their students since it enables the faculty to be practicing scientists and engineers, in close touch with developments and current problems in their field of specialization. At the same time, this arrangement makes available to industrial and governmental sponsors the wealth of experience and special training represented by the faculty of a major engineering college. The staff of the Division is drawn from many areas of engineering and research. It includes men formerly with the research divisions of industry, governmental and public agencies, and independent research organizations.

Following are the areas represented in the research program: Aeronautical Engineering, Chemical Engineering, Civil Engineering, Electrical Engineering, Engineering Mechanics, Industrial and Management Engineering, Mechanical Engineering, Metallurgical Engineering, Mathematics, Meteorology and Oceanography, and Physics. In addition, an interdisciplinary research group is responsible for studies which embrace several disciplines. Inquiries regarding specific areas of research may be addressed to the Director, Research Division for forwarding to the appropriate research group.