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ESD-198
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PERT
EACH

(PERT Fundamentals, Vol 1)

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-198

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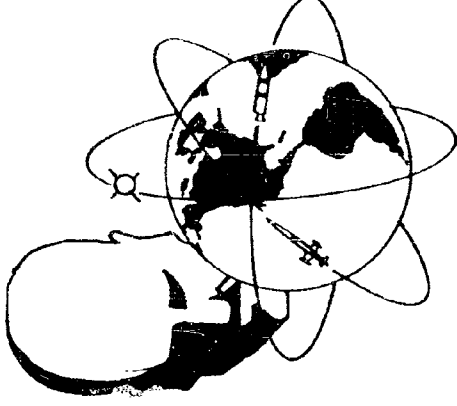
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OPERATIONAL APPLICATIONS LABORATORY
DEPUTY FOR TECHNOLOGY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
L. G. Hanscom Fld., Bedford, Mass

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(Prepared under contract AF19(628)-365 by the Equipment Division, Raytheon Co. Waltham, Mass)

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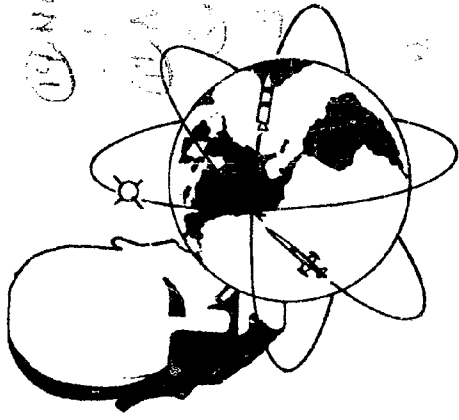
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Vol-1

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PERT EACH

(PERT Fundamentals, ~~Vol-1~~ VOLUME 1)
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FOREWORD

Air Force implementation of PERT (Program Evaluation and Review Technique) for weapon system management requires training thousands of Air Force managers in the use of this new tool. To help meet this vast training need, the Electronic Systems Division, Operational Applications Laboratory and PERT staff initiated applied research to adapt the experimental technology of programmed instruction for PERT training. This applied research resulted in the development of the PERTeach programmed textbooks. These self-instructional books have been successfully tested at Electronic Systems Division with over sixty (60) military and civilian managers. It is hoped that these PERTeach books will be useful to many other Air Force managers.

ESD-TDR-63-198

PERTeach

ABSTRACT

This self-instructional course teaches the basic concepts and techniques of PERT (Program Evaluation Review Technique). The course consists of six volumes and is intended for use by Air Force managers. Presented in programmed-instruction format, the course allows the student to proceed at his own pace and to learn without the aid of an instructor. ↗

PUBLICATION REVIEW AND APPROVAL

This Technical Documentary Report has been reviewed and is approved.

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ACKNOWLEDGEMENTS

For the opportunity of writing this program and, thus, advancing the causes of both PERT and Programed Instruction, Raytheon Company and the author are indebted to the Electronic Systems Division, Air Force Systems Command, Laurence G. Hanscom Field, Bedford, Massachusetts. Particular thanks are extended to the following:

Captain Raigh Mason, Dr. Sylvia Mayer and Dr. Walter Organist, Contract Monitors in the Operational Applications Laboratory, whose understanding and encouragement facilitated the preparation of the course. Lieutenant Colonel Phillip Fitter, Mr. Richard Schmid of the Air Force and Mr. Ramon Smith now with NASA, Mr. Robert Hamilton of Mitre Corporation, and Mr. Robert Miller of Raytheon Company whose technical assistance made this course a reality.

Mr. Franklin G. Jansen and Mr. Charles J. McCarty of the Civilian Training Branch of ESD for their faith in the effectiveness of programed instruction, and for their examples of PERT networks and computer data.

Dr. Gustave Rath, Mr. Alden Cummings, Mr. Ralph Hannan, Mr. Jack Starr and Mr. Marvin Tepper for their assistance in writing portions of the text and Mr. Edmund Stoddard for editing and production planning.

The course is based on material and examples taken from the AFSC Policies and Procedures Handbook, ASD Exhibit ASOO 61-1, which was prepared by a joint working group from Hq. AFSC, ASD, ESD and AFLC for improving the management of projects and systems within AFSC.

Milton L. Weiss

COURSE CONTENTS

Volume I	PERT Fundamentals
Volume II	PERT Fundamentals (continued)
Volume III	PERT and Program Management
Volume IV	PERT Network Development and Computer Processing
Volume V	PERT Network Development and Computer Processing (continued)
Volume VI	Workbook

VOLUME I

Table of Contents

<u>Chapter</u>	<u>Title</u>	<u>Page</u>
1	Basic Components of a PERT Network	1-1
2	Four Fundamental Rules of PERT Networks	1-54
3	Activity Time Estimates	1-87
4	Expected Activity Time	1-135
5	Activity Paths	1-179
6	Accumulated Expected Time	1-191
7	Critical Path	1-245

Instructions to the Student

Course Arrangement

This course consists of six volumes. Volume 6, the workbook, is used with each of the five other volumes and contains examples that are referred to in the course as "Panel A", "Panel B", etc. Turn to these panels as instructed in each book. The workbook also contains a glossary of PERT terms as well as the answers to tests that appear in the course.

Each of the first five volumes is arranged so that you read only right-hand pages. Text on the left-hand pages is upside down. Start at the front of each book and proceed toward its back. Turn the last right-hand page, then rotate the book so that the text on the new page can be read. Continue the program, advancing now toward the front of the book, reading only right-hand pages.

Choice of Volumes

Study all six volumes for the details of all PERT techniques and the analysis of PERT data by means of a computer. However, if you are concerned with Air Force management and decision-making functions rather than with the details of preparing and analyzing PERT networks, you need study only Volumes 1, 2, 3 and 6. If your work involves detailed network preparation, or network analysis by means of a computer, study Volumes 1, 2, 4, 5 and 6.

Except for Volume 6 and the omissions noted above, the books must be studied in numerical order because the contents of each volume depend on those of earlier ones.

Self-Instruction

The program consists of many small items of information. Each item appears on a separate page, and the items are arranged in logical teaching sequences. Because each sequence depends on previous ones, proceed through the course without skipping pages unless the program specifically instructs you to do so.

Most of the items consist of a few short sentences that contain one or more blank underlined spaces like this, _____ . You are to supply the one word or number which belongs in each space. You will learn the correct answer for each space by reading the sentence (line containing the space) or you will know the correct answer because you have learned it from earlier items. After supplying all the answers required by an item, turn the page. On the next right-hand page you will see the correct answers printed above the next item of the sequence. In short, read each item, supply all the answers required, and then turn the page to check your answers.

You may write your answers in the corresponding spaces of each item, or on a separate paper if the volumes are to be used again. Experience indicates that effective learning also results if you think of each answer without writing it down. If you have an instructor, follow his directions.

In any case, always supply all the answers for an item before turning the page to check them. If you make a mistake, correct it before proceeding.

Most of the program items contain the underlined spaces for which you provide words or numbers. In addition, there are several items that ask you to select a correct answer from two or more possible ones. If you are recording your answers on a separate paper, write the letter (usually A or B) identifying your selection. If you are writing in the book, circle the correct letter. Then turn to the page corresponding to your answer. Always make your selection before turning the page and correct mistakes before you proceed.

There are also some pages that provide information without requiring answers on your part. Be sure to read these carefully before advancing to the next item.

Tests

Short tests appear in this course, usually at the end of each chapter. Take these tests to see how well you are learning PERT. The answers to these tests appear at the back of the workbook, Volume 6. Review a chapter if you have difficulty with its test.

PERTeach

Volume 1

CHAPTER 1

Basic Components of a PERT Network

The last four years have seen an explosive growth of managerial methods for planning and control of complex industrial and defense projects. PERT (Program Evaluation Review Technique) is the best known of these managerial systems. Developed by the Navy in 1957-58 for use with its Fleet Ballistic Missile program (Polaris), PERT proved instrumental in completing the initial project well ahead of schedule and it is still being used successfully.

The word PERT is formed from the first letters of the words _____.

Program Evaluation Review Technique

On the Titan project in Colorado, there is a huge chart on which the entire Program is displayed for Evaluation and Review by teams of managers using this Technique. This chart may well be called the _____ chart.

PERT

"PERT makes sense; it is a good management tool. The Air Force is using PERT on the MINUTEMAN program because of its proven usefulness," Gen. Bernard A. Schriever, AFSC commander, tells Space Age News in an exclusive interview here.

"The objective is to get the job done; where PERT can help, we'll use it," he states.

SPACE AGE NEWS, November 20, 1961 issue:

The following list indicates the wide acceptance of PERT. It need not be committed to memory.

<u>A Partial List of Electronic System Division Activities Implementing PERT</u>	<u>A Partial List of DOD Projects Currently Using PERT</u>
416L (BUIC) SAGE Air Defense System	POLARIS
416L (AAC) SAGE Air Defense System	DYNASOAR
425L NORAD	MINUTEMAN
466L Electromagnetic Intelligence System	ATLAS
473L Hq. USAF Command and Control System	TITAN
480L Air Force Communications	NIKE ZEUS
481L Post Attack Command and Control System	
482L Emergency Mission Support System	

Excerpt from ESD Program Guidance Letter No. 7 (New) dated 12 December
1961

Program Evaluation & Review Technique (PERT)

"PERT is a management tool for defining and integrating events which must be accomplished on a timely basis to assure completion of program objectives on schedule. As a statistical technique, it assists decision makers but does not make decisions for them. PERT focuses management attention on potential problems requiring remedial decisions. It defines areas of effort whereby trade-offs in time, resources, or performance will enable management to meet major scheduled dates (milestones)".

Now which of the following two statements is correct?

- A. PERT procedures tell a manager the decisions he should make to complete his project on time. Turn to page 1-7.
- B. PERT is a tool the manager can use as an aid in making decisions. Turn to page 1-8.

Your Answer: A. PERT procedures tell a manager the decisions he should make to complete his project on time.

No. PERT procedures do not tell a manager what decisions he should make. You, the manager, must make the decisions. That is your job. PERT can do no more than present problem areas to you. It may also be used to assess the effect of alternative actions, but you must decide which course of action to take. PERT is a managerial tool that helps the manager make his own decisions.

Turn to page 1-9.

Your Answer: B. PERT is a tool the manager can use as an aid in making decisions.

Yes. PERT is a managerial tool that supplies information on which managerial decisions can be based; but you, the manager, must decide which course of action to take. It may also be used to assess the effect of alternative actions, but you must decide which course of action to take. PERT helps the manager make his own decisions.

Turn to page 1-9.

Before a manager can make decisions he must understand the subject matter with which his project is concerned. He may gain this understanding through training or experience, or he may have assistants who explain technical details and project objectives to him; but, however he gains his information, the manager must understand the language and at least the broad aspects of his project before he can comprehend the effects of his decisions on the project objectives.

For example, a successful manager of a hardware store makes a poor manager of a ladies dress shoppe unless he learns the dress business. The language and knowledge of his new project are completely different from those of his old.

Now what is your answer to the following?

Is a person with extensive experience in all phases of PERT procedures and analysis but without access to technical and military knowledge qualified to manage the development of an Air Force weapon system?

- A. No - page 1-10
- B. Yes - page 1-11

Your Answer: A. No

Correct, a person with extensive PERT experience but without access to and understanding of technical and military knowledge would not be qualified to manage development of an Air Force weapon system.

Unless he can gain knowledge about the subject matter, the PERT expert cannot be expected to make sound decisions concerning the project. PERT is concerned with the lengths of time needed to achieve interdependent objectives, not with the subject matter of these objectives nor the subject matter of the work leading to the objectives.

Turn to page 1-12.

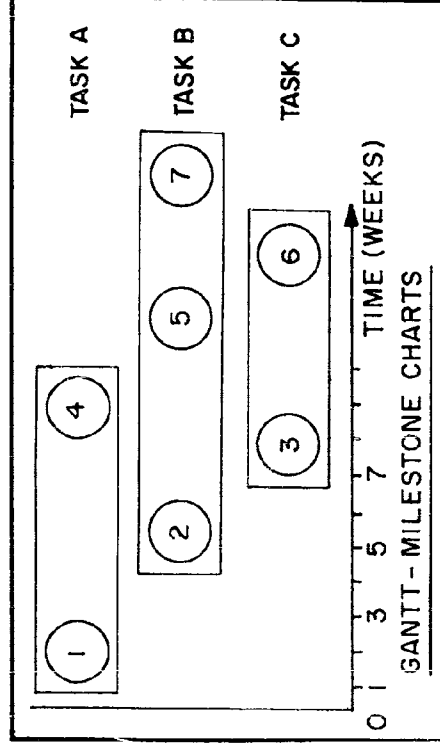
Your Answer: B. Yes.

No. A person with extensive PERT experience but without access to technical and military knowledge would not be qualified to manage development of an Air Force weapon system.

A manager must know what he is managing as well as how to manage. PERT is concerned only with the "how".

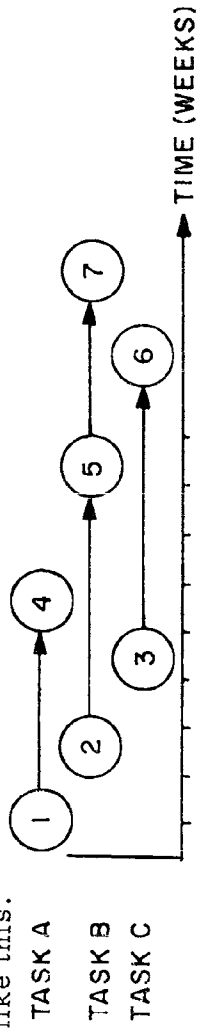
Please return to page 1-9 and select the correct answer.

You are probably familiar with the Gantt-Milestone Bar Charts shown below. This type of management control chart consists essentially of a time scale along which are located the various completed achievements (milestones) associated with a specific project task. Although very useful, this type of progress schedule does not show all the interconnections existing among milestones of different tasks. Thus, there is no indication that work needed to achieve milestone 4 of Task A stems directly from milestone 2 of Task B, and that other necessary and direct tasks connect milestone 4 to milestone 7, or milestone 3 to milestone 5.

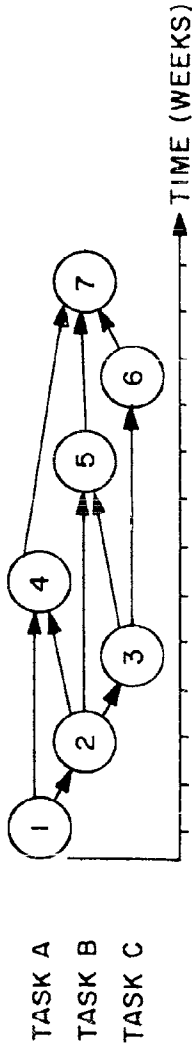


Because Gantt-Milestone Charts do not show clearly these interdependent relations, they are not well suited for rigorous planning and control of complex projects which, in detail, may have scores of tasks composed of thousands of interdependent milestones.

If we omit the rectangular frame which encloses each Gantt-Milestone Chart of the previous page, and if we connect the milestones of each task by horizontal arrows that show the flow and direction of necessary work paths, then the Bar Charts look like this.



Next we'll add all the other necessary work path arrows, those which connect the milestones of one task to those of another to form a network.



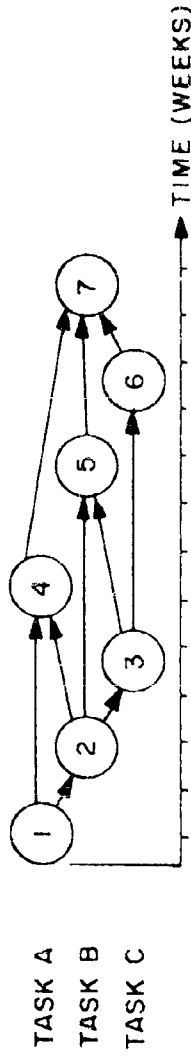
Now, is the following statement true or false?

To achieve milestone 5 in the network shown above, we need first accomplish only milestones 1 and 2.

- A. True - page 1-14.
- B. False - page 1-15.

Your Answer: A. True.

No. Because all the work paths drawn to milestone 5 are necessary, it follows that all the milestones connected by these paths must be accomplished before milestone 5 can be achieved. As illustrated below, we must proceed from milestone 1 to 2 then to 5, but we must also proceed from 3 to 5; and in order to get to milestone 3 we must proceed from milestone 2 to 3. Therefore, both routes to milestone 5 are necessary, 1-2-5 and 1-2-3-5.

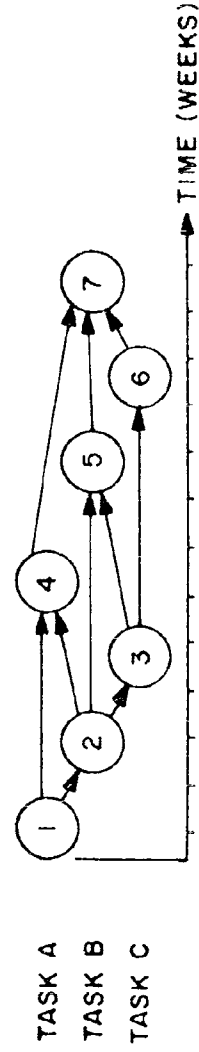


Turn back to page 1-13 and select the correct answer.

Your Answer: B. False.

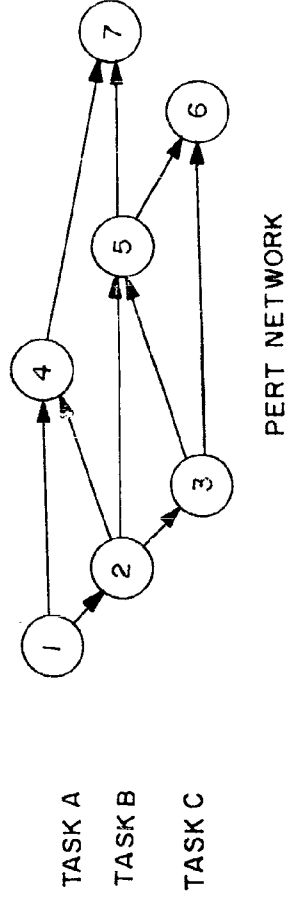
You are right! Because all the work paths drawn to milestone 5 are necessary, it follows that all the milestones connected by these paths must be accomplished before milestone 5 can be achieved.

As illustrated below, we must travel from milestone 1 to 2 and then to 5, but we must also travel from 3 to 5; and in order to get to milestone 3 we must travel from milestone 2 to milestone 3. Therefore, both routes to milestone 5 are necessary, 1-2-5 and 1-2-3-5.



Turn to next page.

We have seen how Gantt-Milestone Charts can be modified and interconnected at their milestones to form a network. In this network, the work-path arrows connecting the milestones show which milestones must be accomplished first. Therefore, there is less need for the time scale (say, in weeks or calendar dates) plotted along the horizontal axis. If we omit the time scale, the network of page 1-13 becomes a true PERT network and looks like this.



A PERT network _____ (does not) necessarily include
 a _____ scale along its horizontal axis.

does not time

Like a Gantt-Milestone Chart, a PERT network is concerned with time durations and occurrences in time. However, the network does not include a time scale. Therefore, the size and geometrical shape of the network, as well as those of its components, are in no way related to these time durations and occurrences.

Always remember that a PERT network may or may not be drawn according to a _____ scale.

time

Each occurrence is a specific network accomplishment that is scheduled to take place at a recognizable instant of time. Each time duration is the length of time needed to progress from one accomplishment to an adjacent accomplishment.

A _____ network is concerned with _____ durations and occurrences that take place at recognizable instants of _____.

A PERT network is concerned with time durations and occurrences that are scheduled to take place at recognizable instants of time.

Test Introduction

To show how well you are learning PERT, small review tests like the one on the following pages are inserted at various points in the program, usually after each chapter. The answers do not appear immediately after each test. However, they are given at the back of the Workbook (Volume VI) unless your instructor has removed them.

Test Instructions

Circle the letter identifying the phrase which correctly completes each numbered statement. (If you have been told not to write in this book, write the letter in the proper space of the answer book you may have, or on a piece of paper).

For example:

TEST NO. 0

1. The word PERT stands for
 - a) Probable Evaluation Research Technique
 - b) Program Estimate Research Technique
 - c) Probable Estimate Review Technique
 - (d) Program Evaluation Review Technique

In this example a circle is drawn around the letter d) because the phrase following it correctly completes the statement. The correct and complete statement is: "The word PERT stands for Program Evaluation Review Technique."

TEST NO. 1

Circle the letter identifying the phrase which appears to be most nearly correct.

1. A PERT network:
 - a) Provides a manager with information on which he can base decisions.
 - b) Does not provide a manager with useful information.
 - c) Will tell a manager what decision to make.
 - d) Will make decisions for a manager.

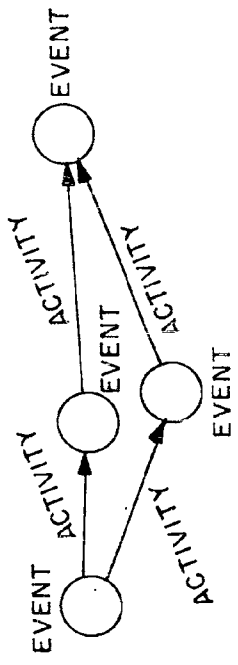
2. To understand and use a PERT network, a manager:
 - a) Must understand the subject matter of the network.
 - b) Requires no knowledge of the subject matter of the network.
 - c) Must be an expert in PERT, but need not understand the subject matter.
 - d) Need not be an expert in PERT nor understand the subject matter.

3. A PERT network:

- a) Is generally concerned with time durations and is drawn to a time scale.
- b) Is generally concerned with time durations and may not be drawn to a time scale.
- c) Is generally not concerned with time durations and is drawn to a time scale.
- d) Is generally not concerned with time durations and is not drawn to a time scale.

If you had difficulty in choosing the right answers, review pages 1-1 through 1-19. Turn to the next page and continue with the program.

A PERT network is concerned with specific program accomplishments that occur at recognizable instants of time, and with the time durations needed to achieve these accomplishments. Each accomplishment is called an Event and is usually drawn as a small circle or rectangle. Each time duration is called an Activity and is drawn as an arrow that connects two events.



A PERT network is composed of _____ (s) and _____ (s).

Events and Activities

or

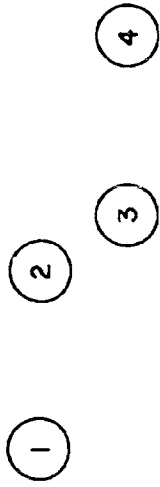
Activities and Events

An arrow represents an _____ that connects two events. The arrowhead shows the direction of time flow and touches the later _____.

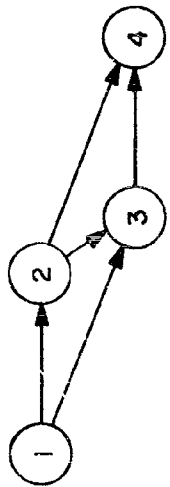
activity

event

PERT networks start on the left and proceed toward the right. Therefore, in drawing a network, place the later events nearer the right side of the chart like this.



Then the activities joining the events will all have components of direction running from left to right like this



and the network will be easier to read.

PERT events are specified network accomplishments that occur at recognizable instants of time. PERT activities are the time durations needed to achieve the events. Together these _____ (s) and _____ (s) form a PERT _____

events and activities form a PERT network.

or

activities and events form a PERT network.

Basically, anything that takes time may be a PERT activity.

Negotiating a contract takes time. Therefore, "negotiating a contract" is an

activit,

A PERT event is scheduled to occur at a certain point in time. It has no time duration. "Start negotiating" is an event. "End negotiating" is also an _____.

event

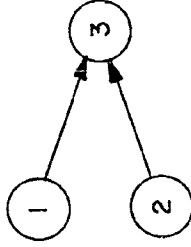
"Start Motor Test" is an _____ . "Motor Test Ended" is also an _____ .
But the time spent in testing the motor is an _____ .

event

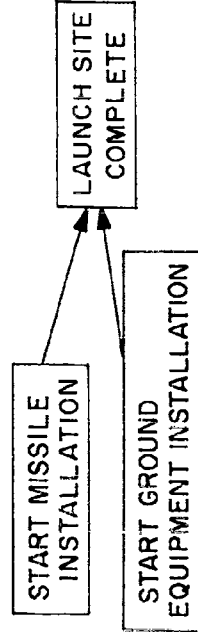
event

activity

In this FERT network, each _____ is identified by a code number.



In this FERT network, each _____ is _____ by a short descriptive title.

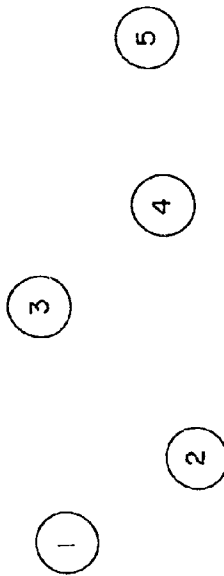


event
event
identified or described
(or equivalent word)

Always identify each _____ of a PERT network by a code
_____ and by a concise title.

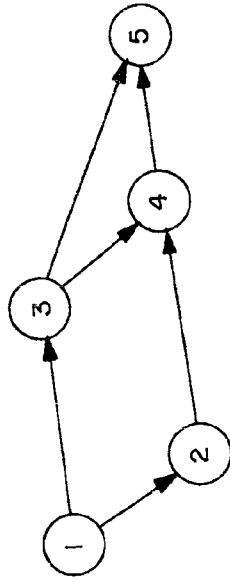
event
number

A certain PERT network has five events located as shown. Draw the complete network by adding activities as shown in the table



The Activity

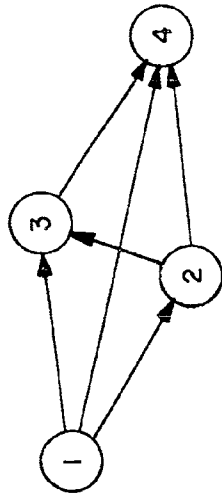
Activity	Starts at Event	Ends at Event
1-2	1	2
1-3	1	3
2-4	2	4
3-4	3	4
3-5	3	5
4-5	4	5



A certain PERT network has four events. Draw the complete network on this page, adding activities according to the following table.

The Activity

Activity	Starts at Event	Ends at Event
1-2	1	2
1-3	1	3
1-4	1	4
2-3	2	3
2-4	2	4
3-4	3	4



(or equivalent)

For discussion and computational purposes, PERT events are often designated with respect to their associated activities. A Beginning Event is one which occurs at the beginning of an activity, and an Ending Event occurs at the end of an activity. The activity itself may be referred to and discussed in terms of the code numbers of its Beginning and Ending Events, but is not so labelled on the chart. Thus, in the sketch below, event no. _____ is the Beginning Event and event no. 5 is the _____ for activity 3-5.



no. 3

Ending Event

The Ending Event for one activity is often the Beginning Event for a following activity.

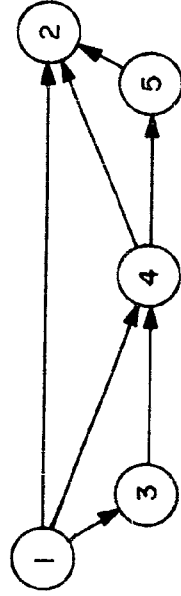
In the following sketch, event no. 4 is the Ending Event for activity 2-4, and is also the _____ for activity _____.



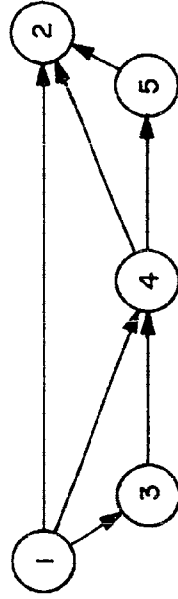
Event no. 4 is the Ending Event for activity 2-4, and is also the Beginning
Event for activity 4-6.

An activity always extends from its Beginning Event to its Ending Event but any one event can serve as the Beginning Event for one group of activities as well as the Ending Event for another group of activities.

In the network shown below event no. 4 is the _____ for activities 1-4 and 3-4, and is also the _____ for activities _____ and _____.



In the network shown below event no. 4 is the Ending Event for activities 1-4 and 3-4, and is also the Beginning Event for activities 4-2 and 4-5.



An event which serves as the Beginning Event for one group of activities and as the Ending Event for another group has activities leading toward it as well as away from it.

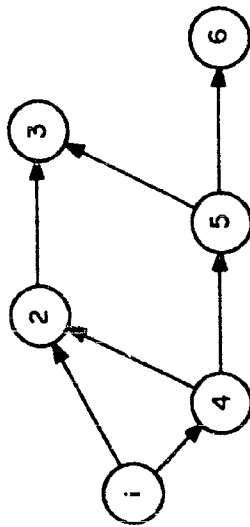
A Beginning Event at the start of a network is called a Network Beginning Event; and an Ending Event at the end of a network is termed a Network Ending Event.

No activities lead toward a _____ and no activities lead away from a _____.

Network Beginning Event

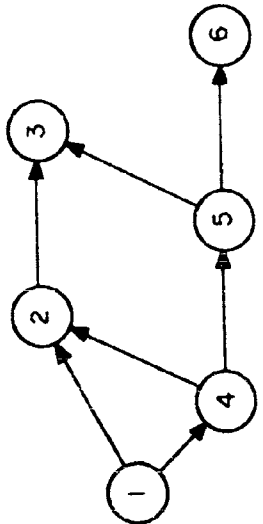
Network Ending Event

It is perfectly possible to have more than one Network Ending Event. The Network Ending Events in the following network are numbered _____. (Write each code number.)



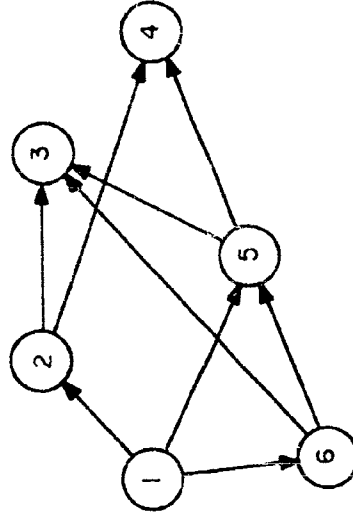
Network
Ending Events

3
6

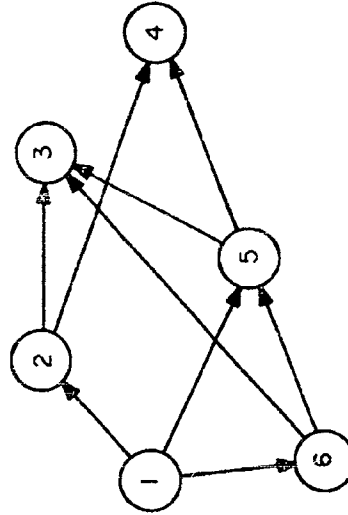


For the network shown below list the code numbers of the

<u>Network Beginning</u> Event(s)	<u>Network Ending</u> Event(s)	<u>Beginning</u> Event(s)	<u>Ending</u> Event(s)
--------------------------------------	-----------------------------------	------------------------------	---------------------------



Network Beginning Event	Network Ending Events	Beginning Events	Ending Events
1	3 4	1 2 5 6	2 3 4 5 6



TEST NO. 2

Circle the identifying phrase which appears to be most nearly correct.

1. To read a PERT network from beginning to end we proceed:
 - a) In numerical order of events.
 - b) From right to left.
 - c) From left to right.
 - d) From the network ending event to the network beginning event.

2. A PERT activity is represented by
 - a) a square.
 - b) an arrow.
 - c) a triangle.
 - d) a circle.

3. A PERT network is composed of:
 - a) events or activities.
 - b) only events.
 - c) only activities.
 - d) events and activities.

4. Time durations are associated with:
- a) events
 - b) activities
 - c) events or activities
 - d) events and activities
5. The ending event for one activity:
- a) can be the beginning event of another activity.
 - b) can have no following activities.
 - c) cannot be the beginning event of another activity.
 - d) can have no following events.
6. A PERT event occurs at a certain point in time. It has:
- a) a minimum time duration.
 - b) a short time duration.
 - c) a calculated time duration.
 - d) no time duration.

If you had difficulty in choosing the right answers, review Chapter 1.

Chapter 1

Summary

The wide use of PERT (Program Evaluation Review Technique) is based on its effectiveness as a tool that aids managers in making decisions. Before he can make decisions, however, the manager must have a thorough competence in project matter.

PERT differs from other management devices (such as Gantt Charts) mainly because it can be used to picture more fully the various interrelationships among project tasks. Such interrelationships are of events (specific project accomplishments) and activities (time needed to achieve these events).

Events are usually symbolized by coded circles or other geometric shapes, and activities are designated by arrows.

PERTeach

Volume I

CHAPTER 2

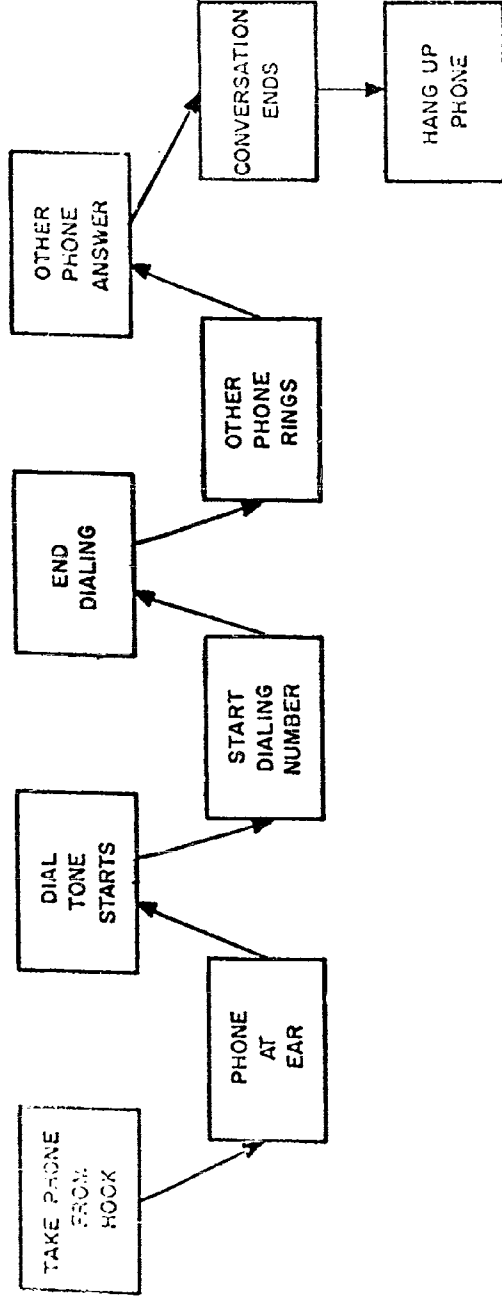
Four Fundamental Rules of PERT Networks

Construction and interpretation of PERT networks follow a few simple but important rules.

The first rule is:

A PERT network must include all activities and events that are necessary to achieve a Network Ending Event.

Now, quickly sketch a PERT network that includes the events and activities necessary to place and complete a telephone call by means of a dial telephone.



If your sequence differs from that above, it only means that your definition of "necessary events" differs from mine. For example, you may not think that "Phone at Ear" is a necessary event, that you can start dialing without being certain the dial tone has started; or you may have included the event "Look up Number", or "Put dime in slot". In any case, this example illustrates the important fact that personnel constructing or using a PERT network must agree on the events and activities deemed necessary to achieve a Network Ending Event.

A correct PERT network must include _____ and _____ that are considered to be necessary to achieve a Network Ending Event.

activities and events

OR

events and activities

Only those activities and events which are _____ for completion of the project are shown in the network.

necessary

or

required

(or equivalent)

An activity starts in time at its beginning event and ends at its ending event. From this comes the rule that no activity can start until its _____ has occurred.

beginning event

You can't smoke your cigarette until you have lit it. In other words,
no activity can be _____ until its _____ has occurred.

started (or begun)
beginning event

Two PERT rules are:

- 1) All necessary activities and events must be _____ in a network.
- 2) No activity can start until its _____ has _____.

All necessary activities and events must be shown (or included) in a network.
No activity can start until its beginning event has occurred (or happened).

An activity extends in time from its beginning event to its ending event. Another rule derived from this is:

No ending event can occur until every _____ leading toward it has been completed.

activity

An event can serve as the ending event for more than one _____. Therefore, in general terms, no event can occur until all activities leading _____ it have been completed.

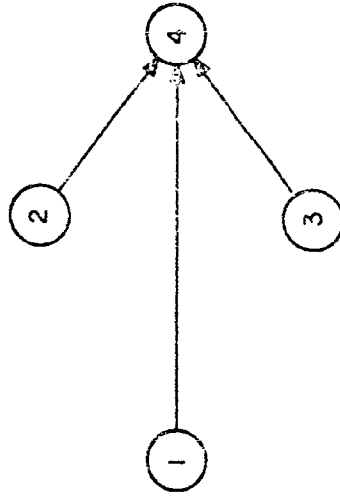
activity
toward

In the network below, activity 1-4 must be completed before event no. 4 can occur. What other activities must be completed before this event can occur?

List them here:

Activity

1 - 4



2-4

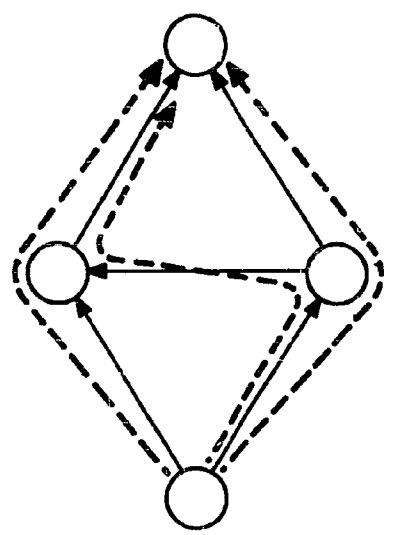
3-4

Three PERT rules are:

- 1) All necessary activities and events must be _____ in the network.
- 2) No activity can start until its _____ event has occurred.
- 3) No event can occur until all activities leading _____ it have been _____.

- 1) All necessary activities and events must be shown (or included) in the network.
- 2) No activity can start until its beginning event has occurred.
- 3) No event can occur until all activities leading toward it have been completed.

The various activities of a PERT network form paths that lead from the network beginning event to any specified event. As shown below, any group of activities which are connected in series forms an _____



----- ACTIVITY PATH

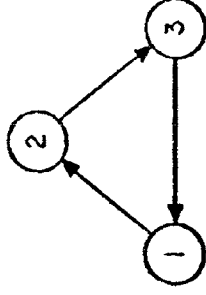
activity path

1-69

The fourth and last PERT rule is:

No path of activities can return to one of its events.

As shown below, such return forms an undesirable closed loop in which activity (3-1) begins at the ending event (no. 3) of a sequence and ends at the beginning event (no. 1) of the sequence. This activity (3-1) closes a loop but represents a backward flow of time. Therefore, the closed loop cannot exist.



From this, it follows that an event of a PERT network can occur only

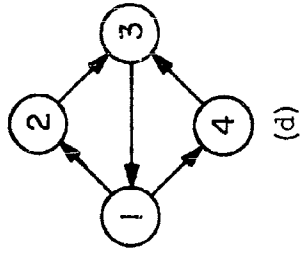
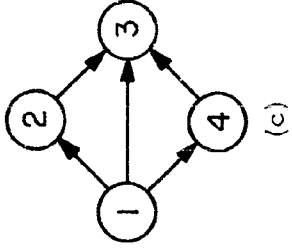
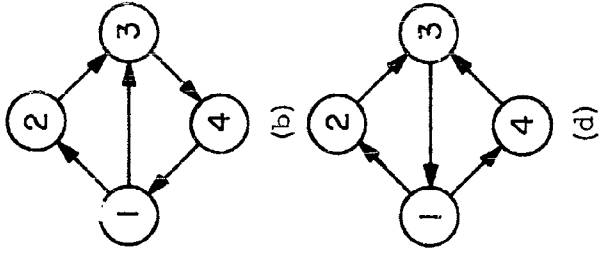
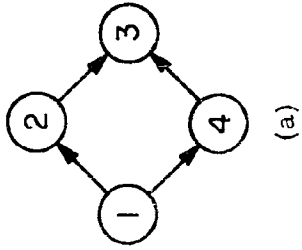
once

An event of a PERT network can occur only once. This means that no path of activities can _____ to one of its events.

return

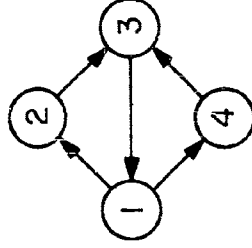
1-72

Circle the letter of any of the following networks containing unacceptable closed loops.



Networks (b) and (d) contain closed loops that must be corrected.

Below is a network containing an unacceptable closed loop.



Which of the activities listed below would you change in direction to make the network acceptable? _____

(Remember that the network ending event should be on your right.)

- 1-2
- 1-4
- 2-3
- 3-1

Four basic rules of PERT are:

- No activity can start until its beginning event has occurred.
- No event can occur until all activities leading to it have been completed.
- No path of activities can return to one of its events.
- All _____ activities and events must be shown.

necessary

Four basic rules of PERT are:

- 1) All necessary activities and events must be shown.
- 2) No path of activities can return to one of its events.
No _____ can start until its _____ event has occurred.
No _____ can occur until all activities leading to it have been completed.

No activity can start until its beginning event has occurred.

No event can occur until all activities leading to it have been completed.

An electrician wiring a house does not interfere with a man landscaping the grounds although their work does go on at the same time, or simultaneously. Their activities proceed independently, and the time needed for the electrical work has no relation to that needed for landscaping.

In the same way, simultaneous PERT activities also proceed _____.

independently

Simultaneous PERT activities are _____ of each other.

This means that while an activity is in progress it requires no inputs of time or resources from any other _____ that is also in progress.

independent
activity

If two activities are to be accomplished simultaneously, the time required for one activity _____ (does/ doesn't) depend on the time required for the other.

doesn't

Simultaneous PERT activities proceed independently. Activities connected in series through the same event are also _____ of each other in the sense that the time required for one activity does not affect the time required for the other.

independent

If two activities are connected in series, the time required for one activity
_____ (always/ never) affects the time required for the other.

never

TEST NO. 3

Circle the letter identifying the phrase which appears to be most nearly correct.

1. A network must show:
 - a) All necessary activities and events.
 - b) All necessary activities but only important events.
 - c) All necessary events but only important activities.
 - d) Important events and activities only.

2. A path of activities:
 - a) Must return to one of its events.
 - b) Cannot return to one of its events.
 - c) Can return to one of its events.
 - d) Should go through all events.

3. An event occurs:
 - a) When any activity leading to it has been completed.
 - b) When any activity leading from it has been completed.
 - c) When all activities leading to it have been completed.
 - d) When all activities leading from it have been completed.

4. If two activities are carried on simultaneously, the time required for one activity
 - a) increases the time required for the other.
 - b) reduces the time required for the other.
 - c) is independent of that required for the other.
 - d) is subtracted from that required for the other.

5. A PERT activity can start
 - a) before its beginning event has occurred.
 - b) after its beginning event has occurred.
 - c) after its ending event has occurred.
 - d) at any time.

If you had difficulty selecting the right answers, review Chapter 2.

Chapter 2

Summary

There are four fundamental rules for PERT networks:

- 1) All necessary events and activities must be shown in the network,
- 2) No activity can start until its beginning event has occurred.
- 3) No event can occur until all activities leading toward it have been completed.
- 4) An event can occur only once. That is, no path of activities can return to one of its events.

In an acceptable network, activity arrows point only away from Network Beginning Events and toward Network Ending Events.

Simultaneous activities as well as activities connected in series through the same event are independent of each other in that the time required for one activity does not affect the time required for the other.

PERTeach

Volume I

CHAPTER 3

Activity Time Estimates

So far we have learned that a PERT network presents management with the following two types of information:

1. The events required to achieve project objectives.
 2. The necessary activities that interconnect the events.
- Each event occurs at an identifiable point in time but has no time duration. On the other hand, because work takes time, each activity or work path needed to proceed from one event to another represents a period of time.

A third type of information presented by a PERT network consists of estimates of these time periods. The use of the estimates to predict when the events will occur is of great importance for managerial planning and control of a project.

In the following pages we will be concerned with calculations involving these time estimates and their use.

To maximize the effectiveness of a PERT network, all managers and personnel responsible for the tasks being charted should participate actively in development of the network. Once the network has been drawn, the persons responsible for accomplishment of each activity make specific estimates of the lengths of time needed to do the work required for the activity. As described later, three different types of estimates are made for each activity.

It is important to remember that only time durations are wanted. The estimates must be responsible appraisals that are independent of schedules and calendar dates.

The estimates assume that resources to accomplish the activities will remain as planned during the entire program. No allowances are made for possible shifts in manpower or different worker efficiencies, and no separate estimates are made for holidays or vacations. Overtime is not usually included but should be specifically noted if it is to be used.

For Air Force projects PERT time estimates are expressed in multiples and tenths of a 7-day week. However, the usual work week has only five days (Monday through Friday). Therefore to convert working days into 7-day weeks, divide the number of working days by five. For example, 14 working days equals 14/5 or 2.8 weeks. In general, to change working days into weeks, divide the number of these days by the number of working days in a 7-day week.

Construction of a small building will require 23 working days. The job starts on 1 April and the men work five days each week. On a PERT network, how would you express the time required for this job.

<u>Your Answer</u>	<u>Turn to Page</u>
A. 1 April - 23 April	1-91
B. 3.3 weeks	1-92
C. 23 days	1-93
D. 4.6 weeks	1-94

Your Answer: A. 1 April 23 - April

Sorry. Estimates of activity times are never expressed in terms of beginning and ending calendar dates. Time flow or duration is being estimated. The dates on which work starts and ends have nothing to do with the length of time needed to do the job.

Now return to page 1-90 and select the right answer.

Your Answer: B. 3.3 weeks

Sorry. You divided 23, the number of days, by 7 to get weeks. Remember the 23 represents working days and the men work 5 days a week.

Return to page 1-90, read it again and select the right answer.

Your answer: C. 23 Days

10. For a PERT network, time durations are expressed in multiples and tenths of a 7-day week.

Return to page 1-90, read it again and select the right answer.

Your Answer: D. 4.6 weeks

Right. Dividing 23 by 5 gives 4.6 weeks.

Estimates of time durations are sometimes given in months but for PERT networks these should be changed into weeks. To change months into weeks, multiply the number of months by 4.3. (52 weeks divided by 12 months equals 4.3 weeks per month). Thus 8.5 months equals 8.5×4.3 or 36.5 weeks.

9.7 months equals _____ weeks.

13.5 months equals _____ weeks.

41.7 weeks
58.1 weeks

Assembly of a certain electric unit is to start on April 1. The responsible production engineer estimates that the work will most likely be completed by July 1. The design engineer thinks the unit should be available by June 15. No additional help is available for the job, and overtime will not be allowed.

Whose estimate would you accept for a PERT network and how would you express it for the network?

<u>Whose Estimate</u>	<u>Estimate Expressed as</u>	<u>Turn to page</u>
A. Production Engineer	April 1 - July 1	1-97
B. Production Engineer	13.0 weeks	1-98
C. Design Engineer	10.9 weeks	1-99
D. Design Engineer	April 1 - June 15	1-100

Your Answer: A. The production engineer's estimate.
The estimate is expressed as April 1 - July 1

You are right in accepting the production engineer's estimate; presumably responsible for the job because of his knowledge and experience, he is in the best position to judge how long it will take.

However, his estimate should not be expressed as the time between calendar dates. Activity time estimates are always expressed as the number of 7-day weeks needed to accomplish the work.

Return to page 1-96 and select the right answer.

Your Answer: B. The production engineer's estimate.
The estimate is expressed as 13.0 weeks.

You are correct. Responsible for the job presumably because of his knowledge and experience, the production engineer is in the best position to judge how long it will take.

And the total number of days is 91; 30 in April, 31 in May, and 30 in June.
Dividing 91 by 7 gives 13.0 7-day weeks.

Turn to page 1-101

Your Answer: C. The design engineer's estimate.

The estimate expressed as April 1 - June 15.

The design engineer's estimate should not be preferred to that of the production engineer. Responsible for the job presumably because of his knowledge and experience, the production engineer is in the best position to judge how long it will take.

Activity time estimates are always expressed as the number of 7 -day weeks needed to accomplish the work, never as the time between calendar dates.

Return to page 1-96 and select the right answer.

Your Answer: D.

The design engineer's estimate.

The estimate expressed as 10.9 weeks.

The design engineer's estimate should not be referred to that of the production engineer. Responsible for the job presumably because of his knowledge and experience, the production engineer is in the best position to judge how long the work will take.

However, the design engineer's estimate is expressed in the right way, as multiples and tenths of a 7-day week.

Return to page 1-96 and select the right answer.

Many Air Force projects involve original research and development, the construction of devices never built before, and the integration of prototype units. Since the exact amounts of time required for such tasks or for the achievement of a complex system are not easily determined, forecasts of these times must be uncertain.

PERT takes account of this uncertainty by requiring three time estimates for each activity. These are respectively called the Most Likely Time, the Optimistic Time, and the Pessimistic Time.

The Most Likely Time, designated by the letter m, is estimated first.

Next, the Optimistic Time, designated by the letter a, is estimated.

The estimate of Pessimistic Time is designated by the letter b, and it is made last.

The estimated time designated by the letter 'm', represents the length of time most likely to be required for an activity. It is the value which would most likely result if the activity were repeated several times under identical conditions. This estimated time is called the _____ Time.

Most Likely

The Most Likely Time is designated by the letter _____.

m

The estimated activity time that would be given if only one estimate were required, is also represented by the _____ Time and designated by the letter _____.

Most Likely

m

After estimating the Most Likely Time for an activity, we estimate the minimum amount of time the activity may require. Attainable only with unusually good luck, and if no difficulties of any sort are encountered, there should be no better than one chance in a hundred of the activity being completed in this amount of time. Because of its optimistic aspect, this estimated time is called the _____ Time.

Optimistic

The minimum estimated time is designated by the letter a. This estimate is called the _____ Time.

Optimistic

The Most Likely Time is designated by the letter _____.

The Optimistic Time is designated by the letter _____.

m

a

After estimating the Optimistic Time for an activity, we estimate the maximum amount of time that the activity will require.

This amount of time will be required only if unusually bad luck is experienced and has no more than one chance in a hundred of being exceeded. This maximum time should include the possibility of initial failure and a fresh start. However, it should not include the possibility of strikes, or "Acts of God" such as fires, floods, etc.

Because of its pessimistic aspects this estimated time is called the _____ Time.

Pessimistic

Even a fervent pessimist should not include the possibility of strikes, floods, fires, or other major catastrophes when estimating the Time. However, he should include such possibilities as parts shortages, employees leaving, and training new employees.

Pessimistic

The first estimate made is the Most Likely Time.

The second estimate made is the Optimistic Time.

The Pessimistic Time, which is designated by the letter b , is the
_____ time estimate.

third

or

final

(or equivalent word)

The Pessimistic Time is the _____ activity time estimate to be determined, and is designated by the letter _____.

third (or last or longest)

b

In preparing estimates of the Optimistic and Pessimistic Times we should bear in mind that each should bear a probability no greater than chance(s) in a hundred of being realized for the optimistic and of being exceeded for the pessimistic.

one

List the correct titles for each of the estimated times in the order in which they are to be made; include next to each title its designating letter.

Title Letter

Most Likely Time, m

Optimistic Time, a

Pessimistic Time, b

The Pessimistic Time for launch of a missile at Vandenburg Air Force Base is being estimated. Should this estimate consider the possibility that an earthquake may strike the launching center?

A. Yes, turn to page 1-115.

B. No, turn to page 1-116.

Your Answer: A. Yes.

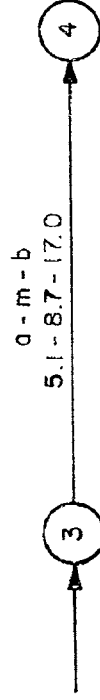
No. A Pessimistic Time estimate does not consider the possibility of catastrophes. Hence, the estimate for launch of the missile should not consider the possibility of an earthquake.

Turn to page 1-117.

Your Answer: B. No.

You are right. The estimate for launch of the missile should not consider the possibility of an earthquake because the Pessimistic Time estimate does not include the possibility of catastrophes. You should only consider normal bad luck when estimating Pessimistic Time.

The values of the three time estimates made for each activity are written above the activity arrow as shown in the sketch. Each of these estimate values is expressed in multiples and tenths of a 7-day week.

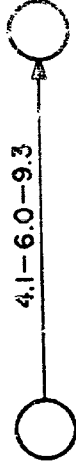


We determine activity time estimates in the order (m) () ()
but we write them above the activity in the order (a) () ().

(m) (a) (b)

(a) (m) (b)

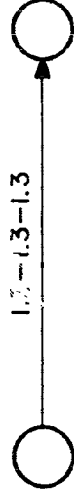
For a certain activity three estimates are 9.3 weeks, 4.1 weeks, and 6.0 weeks. In the space below, draw the PERT activity together with its beginning and ending events. Then write the values of the estimates in their proper order above the activity.



There are some activities whose time durations are known or can be calculated. An example is the curing of concrete to specifications. In such cases each of the three estimated activity times equals the known value, and this value is written three times above the corresponding activity arrow.

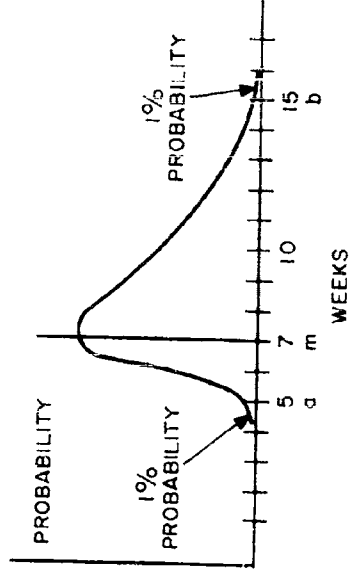
From experience it is known that with the pumps available, it will take 1.3 weeks to drain a certain pond.

For this draining operation, draw the PERT activity together with its a , m , and b values.



As shown below, the three estimates of activity time can be plotted along the horizontal axis of a probability curve. The area of this curve at a given point on the horizontal axis gives the probability that the activity will require the time indicated at that point.

In statistical terms, a and b together determine the practical extent or maximum width of the curve, for neither of these estimates has a probability greater than 1 percent (one chance in a hundred). The Most Likely Time, m , has the greatest probability and therefore locates the greatest vertical height of the curve; thus m is the "mode" of the curve. Note that the probability of m is the greatest. Also m occurs somewhere in the central region of the time axis in accordance with the judgment of the estimator, but experience indicates that m is often placed nearer to a than to b .



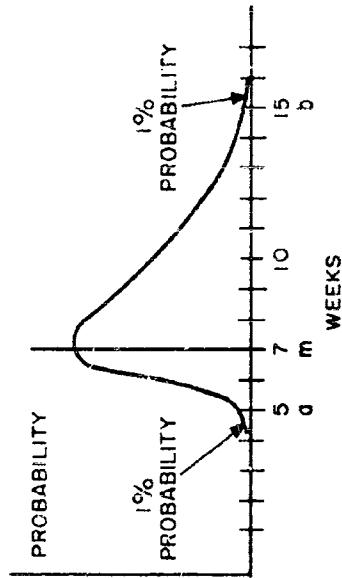
In the following example, assume that there is a 1-percent probability that the activity may be completed in 5 weeks or less and that there is also a 1-percent probability that the activity may be completed in 15 weeks or more. What is the probability that the activity will be completed in less than 15 but more than 5 weeks?

Your Answer

- A. 99 percent
- B. 98 percent

Turn to page

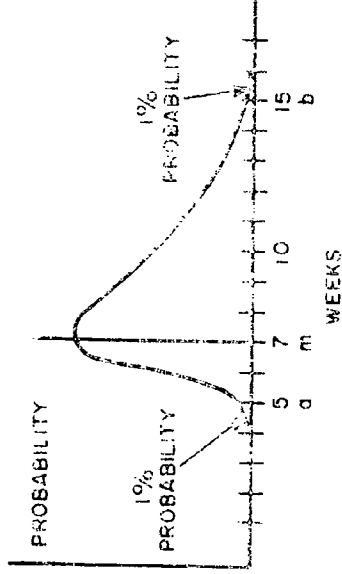
- 1-123
- 1-124



Your Answer: A. 99 percent

Sorry. The total area under the curve corresponds to the certainty that the activity will be accomplished. This certainty is equivalent to a probability of 100 percent. Therefore the total area represents a probability of 100 percent.

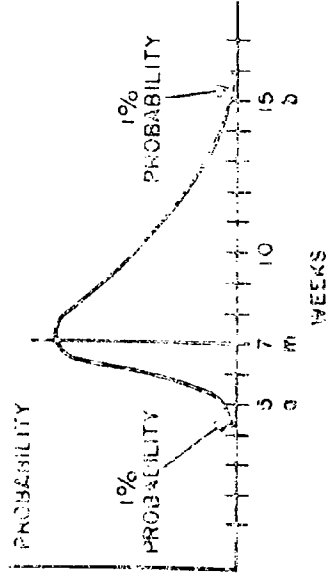
We know that there is a 1-percent probability that the activity will be accomplished in 5 weeks or less and that there is also a 1-percent probability that it will take 15 weeks or more. Each of these probabilities corresponds to a small area under the curve. Therefore the probability that the activity will take between 5 and 15 weeks must be 98 percent, not 99 percent.



Turn to page 1-125.

Your Answer: B. 98 percent

That is right. The total area under the curve corresponds to the certainty that the activity will be accomplished. Because this certainty is equivalent to a probability of 100 percent, the total area represents a probability of 100 percent. Each of the 1-percent probabilities corresponds to 1 percent of the area. Subtracting these probabilities from 100 percent leaves 98 percent, which corresponds to the area bounded by 5 and 15 weeks.



Turn to page 1-i25.

Although the Most Likely Time, m , is often nearer to the Optimistic Time, a , than to the Pessimistic Time, b , all three estimates must always be realistic.

For example, with good luck it may be possible to paint in 5 days a house that is most likely to require 7 days. But to say that with bad luck this job may take as long as 30 days is not realistic. Here the difference between m and b is 23 days although the difference between m and a is only 2 days. Thus a more realistic Pessimistic Time is probably needed.

Assuming that the Most Likely Time is reasonable, which of the following three sets of estimated Times is most appropriate for use?

	a	m	b	<u>Turn to page</u>
	<u>(weeks)</u>			
A.	10	14	25	1-126
B.	5	14	70	1-127
C.	1	14	18	1-128

a m b
10 14 25

Your Answer: A. 10 14 25 weeks is satisfactory

Right; it is a matter of judgment, but intuitively we can see that the total range 10-25 weeks makes sense if the Most Likely Time is 14 weeks. Special circumstances may justify an unbalanced estimate but they should always be explained.

Turn to page 1-129.

a m b

Your Answer: B. 5 14 70 weeks is satisfactory

It is a matter of judgment of course but you are saying that the task may possibly be accomplished in 5 weeks but may take as long as 70 weeks. Doesn't this 70 week time seem excessive? Special circumstances may justify an unbalanced estimate but they should always be explained.

Return to page 1-125 and find a more reasonable set of estimates.

a m b

Your Answer: C. 1 14 18 weeks is satisfactory

Does it seem reasonable that a task which may possibly take as long as 18 weeks may with good luck take only 1 week? The estimate of 1 week appears to be too small. Special circumstances may justify an unbalanced estimate but they should always be explained.

Return to page 1-125 and find a more reasonable set of estimates.

TEST NO. 4

Circle the letter identifying the phrase which appears to be most nearly correct.

1. For Air Force projects PERT time estimates are expressed:
 - a) in calendar form.
 - b) in multiples and tenths of a 5-day week.
 - c) in multiples and tenths of a 7-day week.
 - d) in working days.

2. A certain activity requires 59 working days and the men involved work five days each week. For the PERT network, this length of time would be written as
 - a) 59 days.
 - b) 11.8 weeks.
 - c) 8.4 weeks.
 - d) none of the above.

3. 4.5 months equals:
- a) 18.0 weeks
 - b) 22.5 weeks
 - c) 19.4 weeks
 - d) 15.3 weeks
4. Choose the list of time estimates having the correct designating letters.
- a) Optimistic Time (a), Most Likely Time (b), Pessimistic Time (m).
 - b) Optimistic Time (b), Most Likely Time (a), Pessimistic Time (m).
 - c) Optimistic Time (b), Most Likely Time (m), Pessimistic Time (a).
 - d) Optimistic Time (a), Most Likely Time (m), Pessimistic Time (b).
5. For any PERT activity, the time estimate designated by the letter b is
- a) always shorter than that designated by the letter m.
 - b) sometimes shorter than that designated by the letter a.
 - c) sometimes longer than that designated by the letter m.
 - d) always longer than that designated by the letter m.

Turn this page, turn the book around, and continue the program on page 1-131. I-130

6. Above each activity its three time estimates appear in the following order
- a) a - m - b
 - b) a - b - m
 - c) b - m - a
 - d) b - a - m
7. The three estimates for a certain activity are 6.0 weeks, 9.0 weeks, and 15.0 weeks. These three estimates are placed above the activity in the following order:
- a) 15.0 - 9.0 - 6.0
 - b) 6.0 - 9.0 - 15.0
 - c) 6.0 - 15.0 - 9.0
 - d) 15.0 - 6.0 - 9.0
8. The three time estimate values for a PERT activity
- a) may all be the same.
 - b) must all be the same.
 - c) cannot all be the same.
 - d) none of the above is correct.
- If you had difficulty in choosing the right answers, review Chapter 3.
- Turn to the next page and continue the program.

Chapter 3

Summary

In addition to events and activities, a PERT network includes time estimates of when the events will occur. These estimates are expressed in terms of units and tenths of a 7-day week.

For Air Force PERT procedures three time estimates are required for each activity:

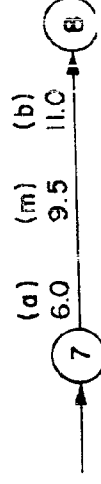
Most Likely Time (m)

Optimistic Time (a)

Pessimistic Time (b)

Neither a nor b has more than one chance in a hundred of being realized (a) or of being exceeded (b)

A typical activity arrow with its accompanying time estimates and beginning and ending events appears below:



PER Teach

Volume I

CHAPTER 4

Expected Activity Time

The three time estimates for each activity of a complex PERT network form basic inputs to the PERT computer analysis. However, to provide you with a better comprehension of PERT concepts and their interrelation these concepts will now be treated as they are used in manual computation of network characteristics. The discussion of the inputs and outputs for computer analysis is delayed to a later section.

Normally a manager or a PERT group member associated with an Air Force weapons system will not be concerned with manual computation because the networks are so extensive. It should be emphasized, however, that it may be preferable to compute manually a relatively small network involving no more than a few hundred activities. In this case results can usually be achieved more quickly and at less cost than is possible with computer analysis.

The three time estimates that are made for each activity form the basic information which determines conclusions from a PERT network. To obtain these conclusions we first obtain, by means of the estimates, the weighted average time expected to be required for each activity. This weighted average value is called Expected Activity Time.

The Expected Activity Time is a single value that is derived from a, m and b, the three estimates of activity _____.

time

For computation and reporting purposes, the three Estimated Activity Times, a, m and b, are reduced to a single value called the _____ Activity Time.

Expected

The Expected _____ is a weighted average of the three
_____ Activity _____.

The Expected Activity Time is a weighted average of the three Estimated Activity Times.

_____ .
The three Estimated Activity Times are reduced to a weighted _____ value called the _____.

average
Expected Activity Time

For each activity of a PERT network there are three _____
Activity Times and one _____ Activity Time.

There are three Estimated Activity Times and one Expected Activity Time.

For computation and reporting purposes, PERT uses the single value of _____ calculated for each _____.

Expected Activity Time
activity

There is a 50-50 chance that an activity will require less than its Expected Activity Time and there is a 50-50 chance that the activity will require _____ than its Expected Activity Time.

more

Because the three estimated times, a, m, and b, are expressed in multiples and tenths of a 7-day week, the Expected Activity Time is also expressed in multiples and tenths of a

7-day week

The Expected Activity Time of a certain activity is 12.0 weeks.

The probability that the activity will require more than 12.0 weeks is _____ percent. The probability that the activity will take less than 12.0 weeks is _____ percent.

50

50

The weighted average value of the three _____ Times
is called the _____ Time and is designated by t_e .

Estimated Activity Times

Expected Activity Time

The symbol t_e is the designation for _____.

Expected Activity Time

The Expected Activity Time is designated by the symbol _____.

$$t_e = \underline{\hspace{10em}}$$

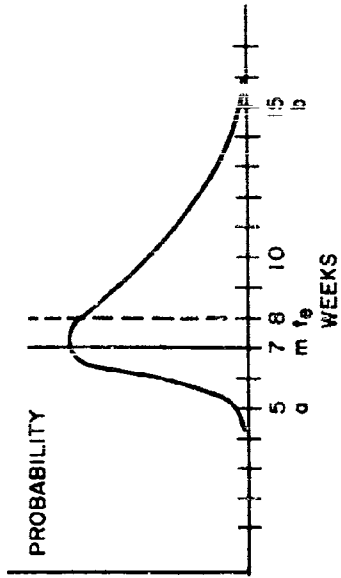
The Expected Activity Time, t_e , is calculated by the equation

$$t_e = \frac{a + 4m + b}{6}$$

If $a = 5.0$ weeks, $m = 7.0$ weeks and $b = 15.0$ weeks then $t_e = \underline{\hspace{2em}}$ weeks.

o

8.0 weeks



$$t_e = \frac{a + 4m + b}{6} = \frac{5 + (4 \times 7) + 15}{6} = 8.0$$

If $t_e = 8.0$ weeks, then the probability that the activity will be accomplished in less than 8.0 weeks is _____ percent. The probability that the activity will be accomplished in more than 8.0 weeks is _____ percent.

50

50

The equation for Expected Activity Time is

$$t_e = \frac{a + 4m + b}{6}$$

In this equation

t_e stands for Expected Activity Time

a stands for _____ Activity Time

m stands for _____ Activity Time

b stands for _____ Activity Time

Optimistic
Most Likely
Pessimistic

Stated in words the equation

$$t_e = \frac{a + 4m + b}{6}$$

says that

The _____ Activity Time equals the _____ Time plus _____
times the _____ Time plus the _____ Time, all divided by _____.

The Expected Activity Time equals the Optimistic Time plus four times the Most Likely Time plus the Pessimistic Time, all divided by six.

The equation for Expected Activity Time is

$$t_e = \frac{\quad}{6}$$

$$t_e = \frac{a + 4m + b}{6}$$

Expected Activity Time is given by the equation

$$t_e = \frac{\quad}{\quad}$$

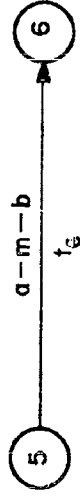
$$t_e = \frac{a + 4m + b}{6}$$

To calculate the Expected Activity Time of each PERT activity we use the equation,

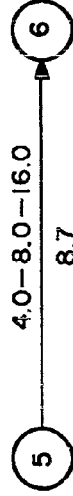
$$\text{---} = \text{---}$$

$$t_e = \frac{a + 4m + b}{6}$$

As shown below the Estimated Activity Times, a , m and b , are written above the arrow of the corresponding activity, say activity 5-6, and the Expected Activity Time, t_e , is written beneath this arrow.

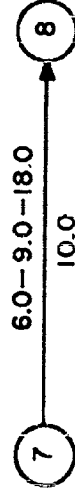


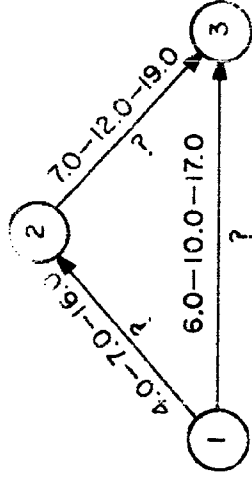
Thus if $a = 4.0$, $m = 8.0$ and $b = 16.0$ weeks then $t_e = 8.7$ weeks and the sketch for activity 5-6 would look like this.



For a certain activity 7-8, $a = 6.0$, $m = 9.0$ and $b = 18.0$ weeks. Calculate t_e . Sketch the activity together with its beginning and ending events. Then insert the values of a , m , b and t_e in their proper locations.

$$t_e = \frac{a + 4m + b}{6} = \frac{6.0 + 4 \times 9.0 + 18.0}{6} = 10.0 \text{ weeks}$$

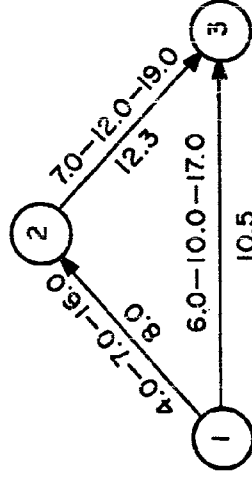




For activity 1-2, $t_e =$ ___ weeks

For activity 1-3, $t_e =$ ___ weeks

For activity 2-3, $t_e =$ ___ weeks



For activity 1-2, $t_e = \underline{8.0}$ weeks $t_e = \frac{4 + 4 \times 7 + 16}{6} = 8.0$ weeks

For activity 1-3, $t_e = \underline{10.5}$ weeks $t_e = \frac{6 + 4 \times 10 + 17}{6} = 10.5$ weeks

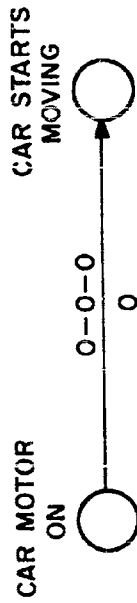
For activity 2-3, $t_e = \underline{12.3}$ weeks $t_e = \frac{7 + 4 \times 12 + 19}{6} = 12.3$ weeks

Sometimes one PERT event can occur only after another has taken place and the time duration of the activity joining these events may, but need not equal zero.

Thus your auto can start to move only after you have started the motor. After starting the motor you can remain in one spot for as long as you wish or you can start moving immediately.

To show this time relation and to indicate that the later event may occur immediately after the earlier one, the two events are joined by a so-called zero-time activity as shown below.

For a zero-time activity, a , m and b , as well as t_e , equal _____.



zero

An activity which restrains one event from occurring where no activity time is required is called a _____ time activity. For this activity t_e equals _____.

zero

zero

Because it restrains one event from occurring before another, a _____
_____ activity is often called a restraint.

zero time

A zero-time activity is often called a _____. For this activity, t_e is always equal to _____.

restraint

zero

A restraint is often called a "dummy" activity. Thus, both terms, "restraint" and "dummy activity" are synonyms for a _____ activity.

zero-time

A zero-time activity is often called either a restraint or a _____ activity.

dummy

The value of t_e is zero for a _____ activity, a _____ activity or a _____.

The value of t_e is zero for a zero-time activity, a dummy activity or a restraint.

For activity 1-2 $t_e = 5.8$ weeks.

For activity 2-3 $t_e = 4.9$ weeks.

Therefore the sum of these two t_e values is ____ weeks.



For activity 5-6 $t_e = 14.3$ weeks

For activity 6-7, a zero-time activity, $t_e = 0$

Therefore the sum of these two t_e values is ____ weeks.



$$\underline{10.7 \text{ weeks}} = t_e (1-2) + t_e (2-3)$$

$$\underline{14.3 \text{ weeks}} = t_e (5-6) + t_e (6-7)$$

TEST NO. 5

Circle the letter identifying the phrase which appears to be most nearly correct.

1. Each activity of a PERT network will have:
 - a) Three Expected Activity Times and one Estimated Activity Time.
 - b) Three Estimated Activity Times and one Expected Activity Time.
 - c) Three Expected Activity Times and three Estimated Activity Times.
 - d) One Expected Activity Time and one Estimated Activity Time.

2. t_e stands for:
 - a) Estimated Activity Time.
 - b) Expected Activity Time.
 - c) Accumulated Expected Time.
 - d) Estimated Expected Time.

3. Select the correct equation for calculating t_e :

a) $t_e = \frac{a + 4m + b}{6}$

c) $t_e = \frac{a + 4b + m}{6}$

b) $t_e = \frac{a + 6m + b}{4}$

d) $t_e = \frac{a + 6b + m}{4}$

4. t_e is a characteristic of a:

- a) Single group of activities.
- b) Single event.
- c) Single group of events.
- d) Single activity.

5. The three time estimates for a certain job are 8.0 weeks, 6.0 weeks, and 16.0 weeks. Therefore t_e for this job equals
- a) 12.0 weeks
 - b) 11.0 weeks
 - c) 9.0 weeks
 - d) 8.0 weeks
6. Event no. 7 takes place immediately after event no. 6 occurs on the PERT network.
- a) A zero-time activity starts at event no. 6 and ends at event no. 7.
 - b) A zero-time activity starts at event no. 7 and ends at event no. 6.
 - c) No activity arrow joins these two events.
 - d) The t_e value of the activity forming these events has a value of 1.

If you had difficulty choosing the right answers, review Chapter 4.

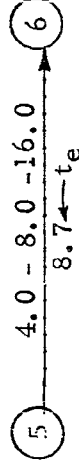
Chapter 4
Summary

The three activity time estimates are used to arrive at a single weighted average value called the Expected Activity Time or t_e . Like the three estimates on which it is based, t_e represents units and tenths of a 7-day week. The formula for arriving at this weighted average is:

$$t_e = \frac{a + 4m + b}{6}$$

where a = optimistic time, m = most likely time, and b = pessimistic time.

A typical activity arrow showing the position of the three estimates as well as t_e is shown here:



A zero-time activity, otherwise known as a "dummy" or "restraint", sometimes appears in a network. Although such an activity requires no time, its beginning event must occur before its ending event.

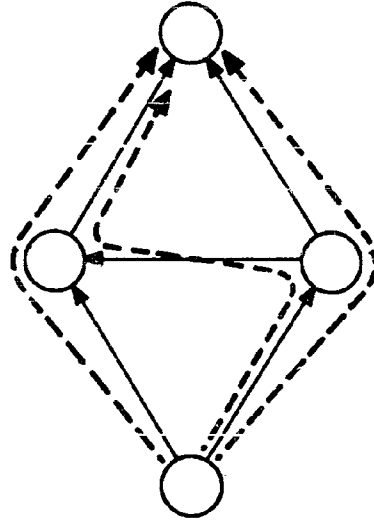
PERTeach

Volume I

CHAPTER 5

Activity Paths

The various activities of a PERT network form paths that lead from the network beginning event to any specified event. As shown below, any group of activities which are connected in series forms an _____.

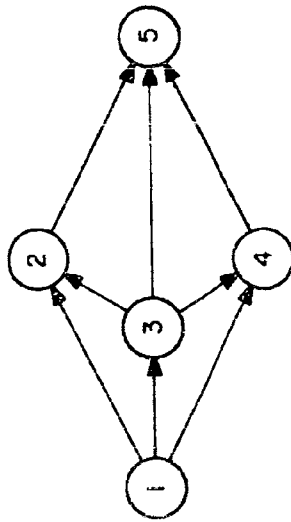


-----ACTIVITY PATH

activity path

The various events and activities of a PERT network form paths that lead from the network beginning event to any specified event. We will work with these paths in determining the characteristics of a network.

In the PERT network below how many paths are there between the network beginning event no. 1 and event no. 5?



Your Answer

- A. 3 paths
- B. 5 paths
- C. Any other number of paths

Turn to page

- 1-179
- 1-180
- 1-181

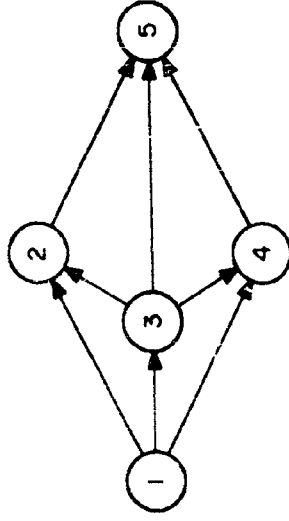
Your Answer: A. 3

Possibly you were in a hurry. At any rate, your answer is wrong. Identify each path by the events through which it passes. You probably chose paths 1-2-5, 1-3-5, and 1-4-5.

Return to page 1-178, study the network again, then choose the correct answer.

Your Answer: B. 5

Absolutely correct. To be sure that we are both thinking along the same lines this is the list of paths from event no. 1 to event no. 5. Each path is identified by the events through which it passes. The paths are 1-2-5, 1-3-2-5, 1-3-4-5, 1-4-5, and 1-3-5.



Turn to page 1-182.

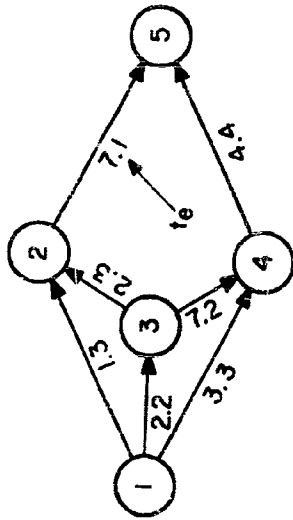
Your Answer: C. Any other number of paths.

Wrong. You may have counted each activity instead of each path composed of activities; or else you may have included paths in which you did not follow the direction of the activity arrows.

Return to page 1-178, study the text and network again, then select the correct answer.

A chief purpose of a PERT network is to determine the time needed to accomplish all the activities that result in any specified event of the network. To find this time we first total the t_e values of activities that form paths between the network beginning event and the specified event. In this way we determine the time period needed to reach the event by each path.

In the network below, what are the totals of t_e values for each path of activities extending from the network beginning event no. 1 to event no. 5?



Your Answer

Turn to page

- A. 8.4, 11.6, 13.8, 7.7
- B. 7.7, 11.6, 13.8
- C. 6.8, 9.5, 11.5

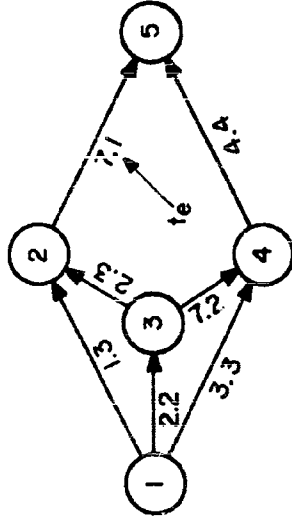
1-183

1-184

1-185

Your Answer: A. 8.4, 11.6, 13.8, 7.7

Absolutely right. There are a total of four paths in the network; they are 1-2-5, 1-3-2-5, 1-3-4-5, and 1-4-5. The total te values of each of these paths of activities are listed in the same order as the paths.



Turn to page 1-186.

Your Answer: B. 7.7, 11.6, 13.8

You have omitted one path.

Return to page 1-182, carefully read the text and choose another answer.

Your Answer: C. 6.8, 9.5, 11.5

The only way you can arrive at these answers is to total the t_e values in vertical columns instead of along paths between events no. 1 and 5. This is wrong.

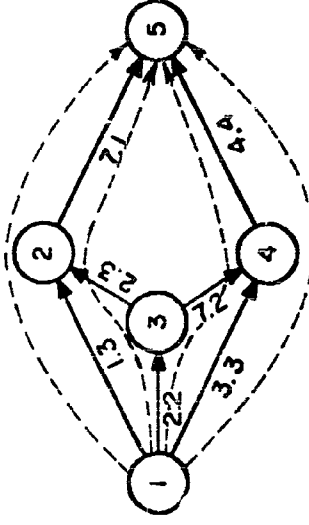
Return to page 1-182, carefully read the text and choose another answer.

Chapter 5

Summary

A major purpose of PERT networks is their usefulness in helping to determine how much time it will take to reach any event. This time is found by first delineating the activity paths and then summing the t_e values between the network beginning event and the specified event.

In the network below, the four activity paths have been outlined.



To find the time needed to reach event 5, it is necessary to total the t_e values on each path; on path 1 - 2 - 5, for example, the t_e values are, respectively, 1.5 and 2.1 for a total of 3.6 weeks.

PERTeach

Volume I

CHAPTER 6

Accumulated Expected Time

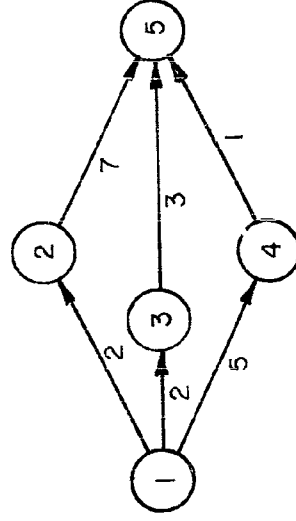
Having determined the total of t_e values for each path of activities between the network beginning event and any specified event, we select the greatest of these sums. This is the total expected activity time which must lapse before the event can occur.

The event will, of course, be reached earlier by the other activity paths that have the smaller sums of t_e values, but we must remember the PERT rule that no event can occur until all activities leading to it have been completed. Therefore, the greatest sum of t_e values along any one path of activities is the time which can be expected to elapse before the event occurs.

The greatest sum of t_e values is called the Accumulated Expected Time and is designated by T_E .

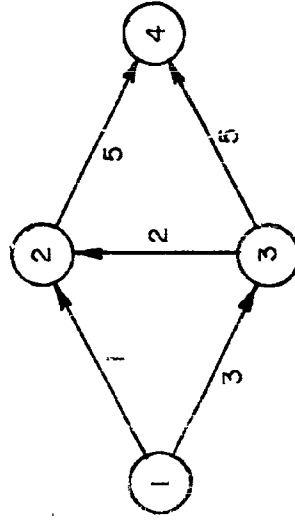
The greatest sum of t_e values along any activity path between the network beginning event and a given event is called the _____ of the given event and is designated by T_E .

In the network below, the sum of t_e values along activity path 1 - 2 - 5 equals 9. This is greater than either 5 or 6, the sum of t_e values along paths 1 - 3 - 5 or 1 - 4 - 5 respectively. Therefore, the T_E of event no. 5 is 9.



Accumulated Expected Time

In the network below the T_E of event no. 4 is _____.



In general, designations composed of lower case letters refer to activities, and designations composed of capital letters refer to an event.

Thus Expected Activity Time, designated by _____, refers to an activity, but Accumulated Expected Time, designated by _____, refers to an event.

t_e

T_E

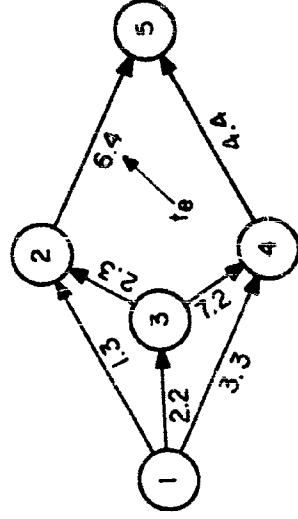
For the network shown, select the path of activities that gives the Accumulated Expected Time, T_E , for network ending event no. 5.

Your Answer

- A. Path 1-2-5
- B. Path 1-3-2-5
- C. Path 1-3-4-5
- D. Path 1-4-5

Turn to Page

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- 1-194
- 1-195
- 1-196



Your Answer: A. 1-2-5

Wrong. Remember, the greatest sum of t_e values along any one path of activities is the time which can be expected to elapse before the event can occur. The sum that has the greatest total is the Accumulated Expected Time, designated by T_E . You must total the t_e values for each path before you can select the T_E value.

Return to page 1-192 and select the answer giving the correct path of activities.

Your Answer: B. 1-3-2-5

Wrong. Remember, the greatest sum of t_e values along any one path of activities is the time which can be expected to elapse before the event can occur. The sum that has the greatest total is the Accumulated Expected Time, designated by TE. You must total the t_e values for each path before you can select the TE value.

Return to page 1-192 and select the answer giving the correct path of activities.

Your Answer: C. 1-3-4-5

Right. The greatest sum of t_e values is along path 1-3-4-5. This sum, equal to 13.8 weeks, is the Accumulated Expected Time (TE) for event no. 5

Turn to page 1-197.

Your Answer: D. 1-4-5

Wrong. Remember, the greatest sum of t_e values along any one path of activities is the time which can be expected to elapse before the event can occur. The sum that has the greatest total is the Accumulated Expected Time, designated by T_E . You must total the t_e values for each path before you can select the T_E value.

Return to page 1-192 and select the answer giving the correct path of activities.

In the network below the T_E of event no. 2 is 6 and the T_E of event no. 3 is 6+4 or 10. This value of 10 is also the T_E (6) of event no. 2 added to the t_e (4) of activity 2-3. Event no. 2 is the beginning event of activity 2-3 and event no. 3 is its ending event. In the same way the T_E of event no. 4 is 17, the T_E (10) of event no. 3 added to the t_e (7) of activity 3-4. Here event no. 3 is the beginning event and event no. 4 is the ending event of activity 3-4.



Thus we see that the t_e of an activity added to the T_E of the activity _____ event gives the T_E of the activity _____ event.

beginning

ending

The t_e of a certain activity is 15. The T_E of its beginning event is 7.
Therefore the T_E of its _____ event is 22 and is found by adding the t_e
of the activity to the _____ of the activity _____ event.

ending

T_E

beginning

The time which can be expected to elapse between occurrence of the network beginning event and a specified event is designated by T_E and is called the _____ of the event.

Accumulated Expected Time

Since t_e (Expected Activity Time) is an average value, T_E (which is derived from a sum of t_e 's) must also be an _____ value.

average

T_E for a specified event is the value of time expected to elapse between the Network Beginning Event and the specified event. T_E is equal to the greatest sum of t_e 's between the _____ and _____ and the specified event.

Network Beginning Event

1 -202

Think of a PERT network as a network of pipes and valves that carry water from the network beginning event to the network ending event. Each activity is a pipe through which the liquid passes in time t_e . Each event is a valve that remains closed until water reaches it from all its input pipes. Only then does the "valve" open (event take place) and pass the liquid into all its output pipes. Each activity is completed only after its pipe is filled.

During the time period in which liquid flows toward any specified closed valve, all the pipes and valves leading toward this valve will become filled. Only at the end of this period will this valve open. The expected length of this time period must equal the greatest time (greatest total of t_e values) expected for the liquid to travel from the network beginning valve to the specified closed valve.

From this analogy it is evident that all PERT activities and events leading to a specified event will be completed during the time period which elapses before this event takes place. The analogy also shows that this time period, called the Accumulated Expected Time, TE, of the specified event, equals the greatest sum of t_e values along any path of activities leading to the event. This is the path of greatest "time" flow to the event.

Our analogy shows that no closed valve can open until all pipes and valves leading toward it have been filled. From this analogy it is evident that all PERT activities and events leading to a specified event will be completed _____ (before/after) this event takes place.

before

An analogy of pipes and valves shows that the flow path of greatest time to a specified valve determines when that valve can open.

In PERT terminology this means that T_E , the time which must elapse before the event can be expected to take place, equals the greatest sum of t_e values (the path of greatest time) leading to the _____.

event

T_E of a given event is the time expected to elapse between the Network Beginning Event and a given event. During this time period all _____ and _____ leading to this event will have taken place.

activities

events

or

events

activities

If the given event is the Network Ending Event toward which all activities and events of the network lead, then T_E of the Network Ending Event represents the Accumulated Expected Time for completion of the entire _____.

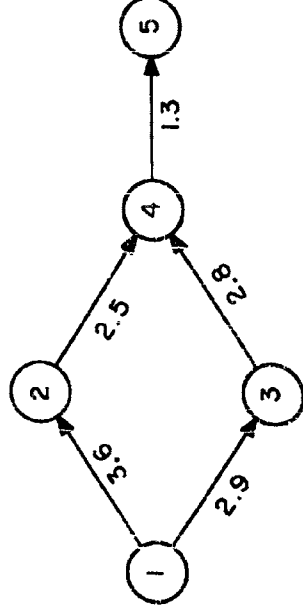
network

1-208

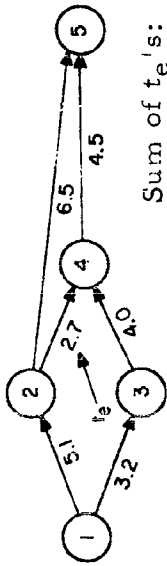
1-208

For preceding examples, you probably totaled the t_e values of each separate activity path and then compared these totals to find TE for the network ending event. The procedure is simplified by tracing two or more paths as far as an event at which they intersect. At this common event the greatest sum of t_e values along any one path is the TE of the common event. The TE values (Accumulated Expected Times) of events that follow this common event are found by adding to its TE value the t_e values (Expected Activity Times) of activities leading away from the common event.

By tracing groups of activity paths to events at which they intersect, the t_e of each activity enters the computations only once. Thus in the example below we need not separately trace each complete path, 1-2-4-5 and 1-3-4-5 to find TE (7.4) for event no. 5. Instead we find TE (6.1) for event no. 4 and then add to it the t_e (1.3) of activity 4-5. However, the TE of event no. 4 can be found only by comparing all activity paths leading to event no. 4.



To facilitate manual computation of T_E for each event of a PERT network, a table like the one shown below is used.



Activity		Sum of t_e 's:		Activity
Begin. Event	End Event	Activity	Event No. 1	Path
		t_e	To End Event	
1	2	5.1	5.1*	1-2
1	3	3.2	3.2*	1-3
2	4	2.7	7.8**	1-2-4**
3	4	4.0	7.2	1-3-4
2	5	6.5	11.6	1-2-5
4	5	4.5	12.3**	1-2-4-5**

* = T_E of each ending event

** = Activity Path of greatest time (greatest sum of t_e values) to event where two activity paths intersect.

In the above network the activity path that defines T_E for event no. 5 is 1-2-4-5. If the t_e of activity 1-3 is increased from 3.2 to 5.0, the activity path of greatest time to event no. 5 is 1-3-4-5. T_E for event no. 4 would then be _____ and T_E for event no. 5 is _____.

If the t_e of path 1-3 equals 5.0, the path 1-3-4-5 is the path of greatest time to event no. 5 and therefore defines T_E for event no. 5. T_E for event no. 4 then equals 9.0 and T_E for event no. 5 = 13.5 (9.0 + 4.5).

We are now ready to proceed with manual computation of a simple but complete PERT network. This computation will illustrate how the concepts we have discussed, as well as others, are used to determine the characteristics of a network. The manual procedure also provides an insight into what must be done by any computer used by the Air Force.

Turn to Panel A in the Workbook, Volume VI. Leave it exposed to view.
Now turn to the next page (1-212) in this book.

The manual procedure starts after the network has been drawn, its events have been numbered, and the three estimated activity times (a , m and b) selected for each activity have been written above the activity. The first step then consists of calculating the Expected Activity Time, t_e , for each activity.

In the PERT network of Panel A, Volume VI, Workbook, there are _____ events and _____ activities.

how many?

how many?

9

14

In the PERT network of Panel A, the Estimated Activity Times, a, m and b, appear over each activity and the Expected Activity Time, te, is written under each of several activities. Because all the te values must be known we will now compute the missing te values of this network.

The te value for activity 1-2 is _____.

1-213

Your answer

- B. 16.0
- C. 14.0
- D. Any other number
- E. I don't remember how to calculate te.

Turn to page

- 1-215
- 1-216
- 1-217
- 1-218

Your Answer: B. The t_e of activity 1-2 equals 16.0

Wrong. Perhaps you didn't multiply m by 4 and divide one side of the equation by 6.

Calculate again the value of t_e for activity 1-2 and write your answer here

_____.

Now return to page 1-214.

Your Answer: C. The t_e of activity 1-2 equals 14.0.

Correct. You used the equation

$$t_e = \frac{a+4m+b}{6}$$

Write this answer, 14.0 in Panel A, once under activity 1-2 of the network and again in the t_e column of the table in the space corresponding to activity 1-2.

Now turn to page 1-219.

1-216

Your Answer: D. Any number except 14.0 or 16.0

Better check your arithmetic. you've gone astray somehow. Perhaps you didn't multiply m by 4 or divide one side of the equation by 6. Or perhaps you didn't use the equation

$$t_e = \frac{a+4m+b}{6}$$

This is the equation which relates the Expected Activity Time, t_e , to the three Estimated Activity Times, a, m and b.

Calculate again the value of t_e for activity 1-2 and write your answer here _____.

Now return to page 1

1-217

Your Answer: E. I don't remember how to calculate t_e .

The Expected Activity Time, t_e , is an average value of the three Estimated Activity Times, a, m and b.

The value of t_e is determined by the equation

$$t_e = \frac{a+4m+b}{6}$$

Thus if a = 10, m = 15 and b = 32

$$\text{Then } t_e = \frac{10+4 \times 15+32}{6} = \frac{102}{6} = 17$$

Calculate the value of t_e for activity 1-2 and write your answer here _____.

Now return to page 1-214.

On Panel A:

For activity 3-5, t_e equals _____

For activity 4-7, t_e equals _____

For activity 7-9, t_e equals _____

For activity 3-5, $t_e = \frac{17.8}{}$

For activity 4-7, $t_e = \frac{13.2}{}$

For activity 7-9, $t_e = \frac{9.2}{}$

Write these answers in the network and table of Panel A.

The Estimated Activity Times a, m and b have now served their purpose for through them we have determined the Expected Activity Time, t_e , for each activity of our network. We are now ready for the second step in manual computation of the network, determination of the Accumulated Expected Time, TE, for each event.

For this procedure turn to Panel B and leave it exposed to view. In Panel B the t_e value of each activity is shown in the network and is listed in the table.

In Panel B the t_e (14.0) of activity 1-2 is also the greatest (and only) sum of t_e values between event no. 1 and event no. 2. Therefore, 14.0 is written in the tabular column labelled "Sum of t_e 's from Event No. 1 to Ending Event" in line with ending event no. 2. Also because it is the T_E of event no. 2, this entry is starred. (See Panel B.) The activity path to event no. 2 is, of course, only activity 1-2 and this is entered in the column headed "Activity Path." The T_E of event no. 2 is also entered in the box above event no. 2 of the network.

The T_E of event no. 3 is _____.

The T_E of event no. 3 is 11.3.

In line with ending event no. 3 of the table, fill in the last two columns as they are completed for event no. 2. Star the T_E of event no. 3. Enter the T_E of event no. 3 in the box above this event in the network.

As you remember, the Accumulated Expected Time, T_E for any specified event is the _____ (greatest/smallest) sum of t_e values along any path of activities between the network _____ (beginning/ending) event and the specified event.

greatest
beginning

For event no. 4, the sum of t_e values along activity path 1-2-4 is _____. For this event, the sum of t_e values along activity path 1-3-4 is _____. Therefore for event no. 4, TE equals _____.

35.2 along path 1-2-4
26.6 along path 1-3-4
TE for event no. 4 equals 35.2

In the table of Panel B, write 35.2 in the "Sum of te's - "column, in line with activity 2-4. Write 26.6 in line with activity 3-4. In the column labelled "Activity Path" write 1-2-4 and 1-3-4 in line with their corresponding te sums.

The TE of event no. 4 is _____. Write this value in the box above event no. 4 of the Panel B network. Because it is the TE of event no. 4, star this value where you wrote it in the table. Also in the table, place a double star at the right of the activity path of greatest time to event no. 4. This activity path is 1-____-4.

35.2

1-2-4

Because the activity path 1-2-4 results in a greater sum of t_e values at event no. 4 than does activity path 1-3-4, computations leading away from event no. 4 use only the results given by activity path 1-___-4.

1-2-4

Now, to continue with Panel B, for event no. 5, the sum of t_e values along activity path 1-3-5 is _____. This is also equal to the T_E value of event no. 3 added to the t_e value of activity 3-5.

Also for event no. 5, the sum of t_e values along activity path 1-2-4-5 is _____. This is equal to the T_E value of event no. 4 added to zero, the t_e of zero-time activity 4-5.

Therefore the T_E value for event no. 5 equals _____.

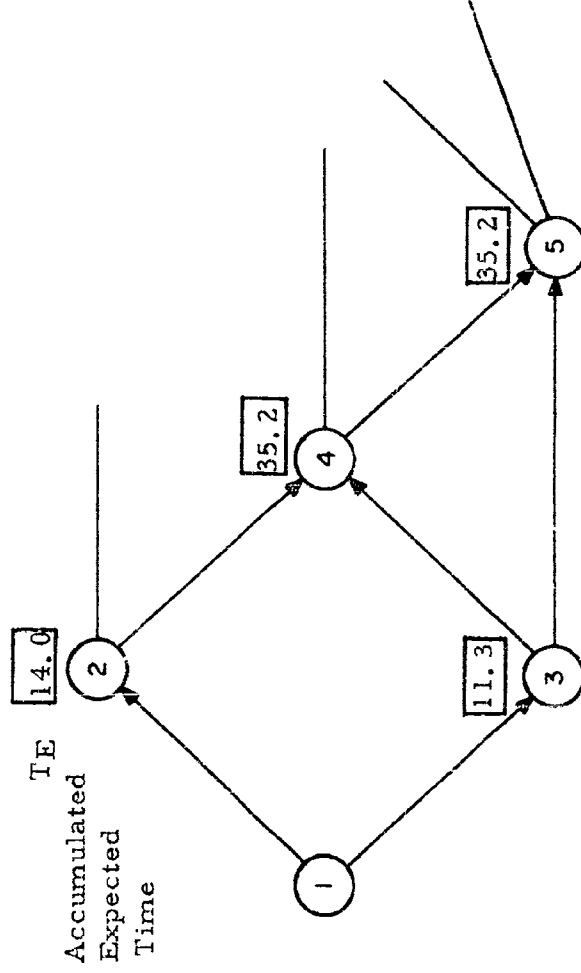
29.1 along path 1-3-5

35.2 along path 1-2-4-5

TE for event no. 5 equals 35.2

Now fill in the table and network for event no. 5 as you have for events 2, 3
and 4. Remember the stars and double stars.

At this point your network should look like this.



Make any necessary corrections.

Your table should look like this:

Activity		Event No. 1		Event No. 2	
Begin. Event	Ending Event	Activity	Activity	Activity	Activity
1	2	14.0	14.0	14.0	14.0
1	3	15.0	15.0	15.0	15.0
2	4	21.0	21.0	21.0	21.0
3	4	15.0	15.0	15.0	15.0
3	5	17.0	17.0	17.0	17.0
4	5	11.0	11.0	11.0	11.0

Compare your results and make any corrections needed.

Now proceeding as you have for events 2, 3, 4 and 5, calculate the TE value for each of the remaining events of the network.

Remember: Fill out each space of the column headed "Sum of t_e 's From etc." and "Activity Path" in the table of Panel B.

Remember also: Place a star beside each sum of t_e 's that is a TE value.

Remember also: Place a double star beside each activity path in accordance with the ** note of Panel B.

Remember also: Enter the TE for each event in the box drawn over that event in the network.

After you have completed your entries in the table and network of Panel B, turn to the next page.

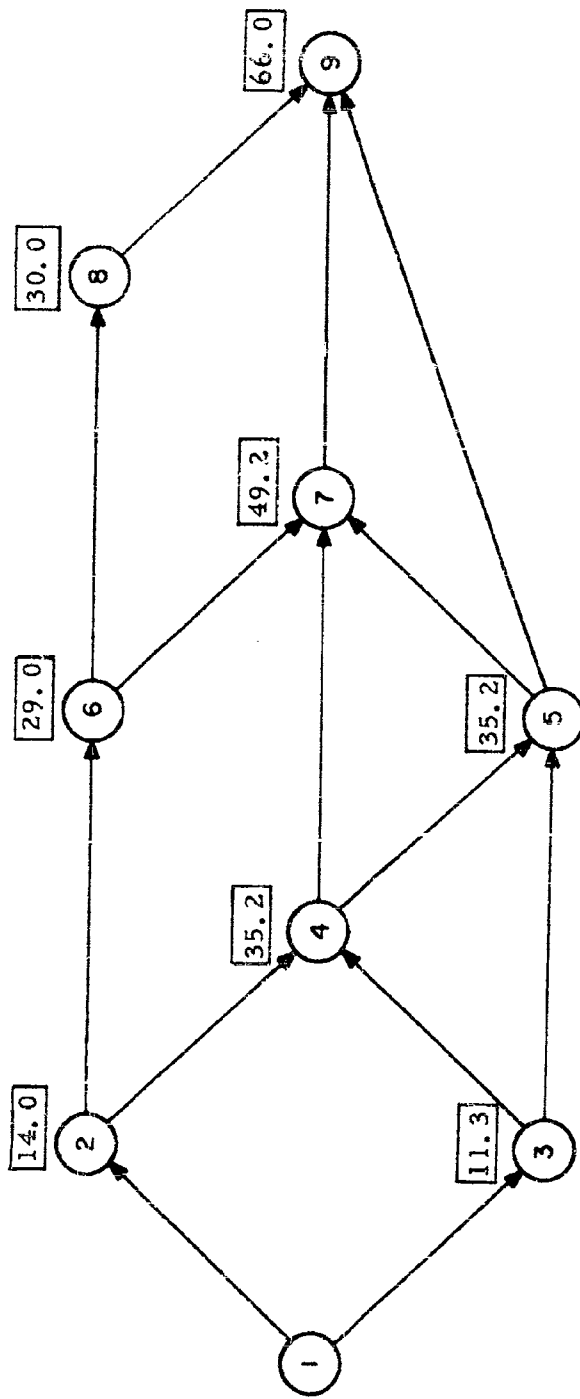
After you complete the table of Panel B, it looks like this. Compare your answers with those below and correct your work where needed. Then turn to next page.

* = TE of each ending event.

** = Activity Path of greatest time (greatest sum of t_e values) to event where two or more activity paths intersect or to network ending event.

Begin. Event	Ending Event	Activity t_e	Sum of t_e 's From		Activity Path
			Event No. 1 To Ending Event	Activity Path	
1	2	14.0	*	1-2	
1	3	11.3	*	1-3	
2	4	21.2	*	1-2-4	*
3	4	15.3		1-3-4	
3	5	17.8		1-3-5	
4	5	0.0	*	1-2-4-5	**
2	6	15.0	*	1-2-6	
4	7	13.2		1-2-4-7	
5	7	14.0	*	1-2-4-5-7**	
6	7	12.3		1-2-6-7	
6	8	1.0	*	1-2-6-8	
5	9	30.8	*	1-2-4-5-9**	
7	9	9.2		1-2-4-5-7-9	
8	9	8.8		1-2-6-8-9	

After you complete the network of Panel B, the TE values placed in the boxes above the events should agree with those shown below. Check your answers, make any necessary corrections and then turn to the next page.



TEST NO. 6

Circle the letter before the answer which appears to be most nearly correct.

1. T_E stands for:
 - a) Estimated Activity Time.
 - b) Expected Activity Time.
 - c) Accumulated Activity Time.
 - d) Estimated Expected Time.

2. T_E for a specific event represents the activity path having the
 - a) Greatest time from the network beginning event to the specified event.
 - b) Greatest time from the specified event to the network ending event.
 - c) Least time from the specified event to the network ending event.
 - d) Least time from the network beginning event to the specified event.

3. Four activity paths lead to the same event. The t_e sums along the paths equal 17, 29, 23 and 8, respectively. The T_E of the event equals

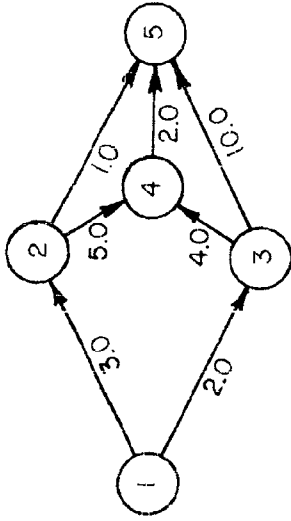
- a) 17
- b) 23
- c) 8
- d) 29

4. During the time period represented by T_E for a specified event, all activities and events leading to the specified event:

- a) Will be completed.
- b) Will not be completed.
- c) Will not be in progress.
- d) Will not be started.

5. In the network below, the number below each activity is the t_e of the activity. The T_E of event no. 4 equals:

- a) 6.0
- b) 8.0
- c) 2.0
- d) 12.0



6. In the network for question 5, above, the T_E of event no. 5 is determined by activity path:

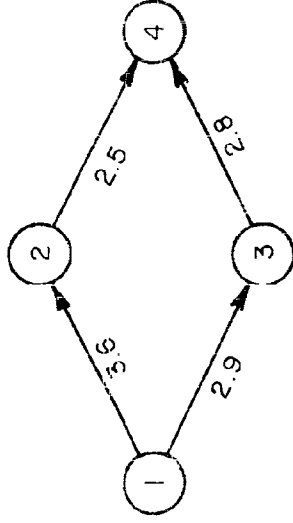
- a) 1 - 2 - 5
- b) 1 - 2 - 4 - 5
- c) 1 - 3 - 4 - 5
- d) 1 - 3 - 5

If you had difficulty selecting the right answers, review Chapters 5 and 6,

Chapter 6
Summary

After determining the total of t_e values for each path of activities between the network beginning event and any specified event, select the greatest of these sums. This greatest sum of t_e values is called the Accumulated Expected Time or T_E of the event and represents the path of greatest time to the event.

In the network shown below, for example, there are two activity paths leading to event no. 4: 1-2-4 and 1-3-4.



The greatest sum of t_e values lies on path 1-2-4 and is $(3.6 + 2.5)$ or 6.1 weeks. This is the T_E of event no. 4.

PER Teach
Volume I
CHAPTER 7
Critical Path

Look at the T_E values shown in Panel B. Each of these represents the time (measured from network beginning event, no. 1) which must elapse before the corresponding event can normally be expected to occur. The T_E of the network ending event, no. 9, is the greatest of these values. Once this event occurs, the entire network is completed. Consequently the T_E value of the network ending event is the amount of time which must elapse before we can normally expect to complete the network.

The path of activities which results in the T_E of the network ending event is a critical property of the network because it is also the path that defines the time required to complete the network. This activity path is called the critical path.

Which of the following statements is true?

- A. The critical path is the most time-consuming path of activities from the beginning to the end of the network. Turn to page 1-239.
- B. The critical path passes through every event of the network. Turn to page 1-240.
- C. The critical path is the least time-consuming path of activities from the beginning to the end of the network. Turn to page 1-241.

Your Answer A. The critical path of activities is the most time-consuming path through the network.

Right! As you can see from your table it is the path of activities which extends from the network beginning event to the network ending event and which has the greatest sum of t_e values between these two events. During the time period represented by this total, all events and activities of the network take place, but the critical path does not pass through all events of the network.

Turn to page 1-242.

Your Answer: B. The critical path passes through every event of the network.

No. The critical path is the path of activities which results in the TE of the network ending event (no. 9). You have traced all possible paths to event no. 9. If you look at your table of Panel B you will see that none of these paths include all events of the network. If the network of activities and events were a system of pipes and valves, the critical path would include only those pipes and valves in which liquid took the longest time to flow from the system beginning valve, no. 1 to the system ending valve, no. 9. That path would not include all the valves of the system.

Return to page 1-238 and read it again; then select the right answer.

Your Answer: C. The critical path is the least time-consuming path of activities from the beginning to the end of the network.

No. If this were so, then T_E for event no. 9, the network ending event would equal 38.8. This is smaller than 49.2 the T_E for event no. 7. The network ending event would occur before an event in the middle of the network. This is impossible.

If the network of activities and events were a system of pipes and valves, the critical path would include only those pipes and valves in which liquid took the longest time to flow from the system beginning valve, no. 1, to the system ending valve, no. 9.

Return to page 1-238 and read it again; then select the right answer.

The most time-consuming path of activities which extends from the network beginning event to the network ending event is called the _____ path.

critical

The activity path having the greatest sum of t_e values is also the (most/least) time-consuming path from the network beginning event to the network _____ event.