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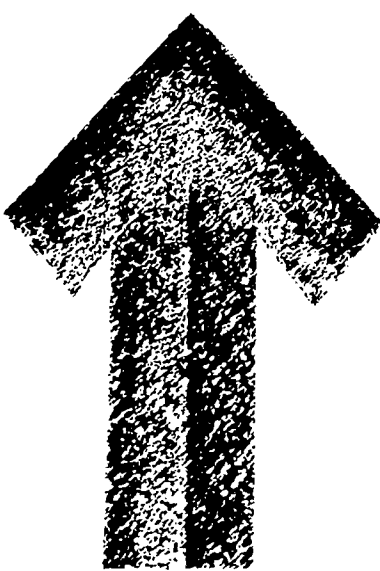
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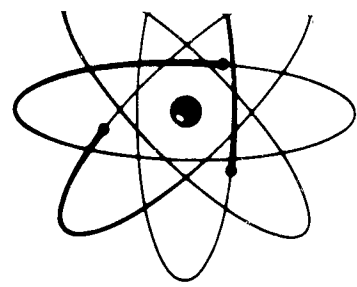
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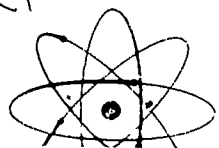


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MASTER

GAMMA-GAMMA ANGULAR CORRELATION
IN THE DECAY OF COBALT-60

by
BURTON J. CONWAY,
CAPTAIN, UNITED STATES ARMY

Submitted in partial fulfillment of
the requirements for the degree of
MASTER OF SCIENCE
IN
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**GAMMA-GAMMA ANGULAR CORRELATION
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by
BURTON J. CONWAY

This work is accepted as fulfilling
the thesis requirements for the degree of
MASTER OF SCIENCE

IN
PHYSICS
from the

United States Naval Postgraduate School

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ABSTRACT

This is a continuation of the work of assembling and testing the equipment necessary to conduct gamma-gamma angular correlation experiments, started by Fort A. Vesser, Jr. at the US Naval Post-graduate School.

No substantial changes were made in the equipment, but rather a return of the Cobalt-60 experiment was made to confirm the results obtained in the previous work. A comparison of the previous work confirms that there is a definite distortion in the shape of the observed correlation function.

The statistical analysis of the data and the solid angle corrections required for evaluation of the results have been programmed for calculation by use of the 1606 Digital Computer. This paper outlines these programs in detail.

The author wishes to express his appreciation for the assistance given him by Professor Harry E. Handler and for his guidance throughout this investigation. Appreciation is expressed to Professor Edmund A. Milne for his advice and comments. Captain I. R. Abernathy, USMC, was very helpful in providing valuable assistance and discussions on various aspects of the computer programming. The cooperation and assistance of Mr. Mervyn C. Brillhart in maintaining and testing the electronic equipment was indispensable.

TABLE OF CONTENTS

CHAPTER	PAGE
I	1
II	3
III	6
IV	9
V	16
VI	22
APPENDIX	
I	26
II	60
BIBLIOGRAPHY	76

LIST OF FIGURES

FIGURE	PAGE
1	PARTIAL DECAY SCHEME OF COBALT-60..... 2
2	ANGULAR CORRELATION TABLE..... 5
3	ELECTRONIC SYSTEM..... 7
4	SCHEMATIC DIAGRAM OF THE ELECTRONIC SYSTEM..... 8
5	MATRIX CONFIGURATIONS..... 20
6	COMPARISON BETWEEN EXPERIMENTAL and THEORETICAL CORRELATION FUNCTIONS..... 23
7	LEAST-SQUARE FLOW CHART..... 27
8	SOLID ANGLE FLOW CHART..... 61

LIST OF TABLES

TABLE		PAGE
1	REFERENCE COUNTING RATES.....	14
2	WEIGHTED MEAN OF $\bar{W}(\theta_m)$	17
3	LEAST-SQUARE COEFFICIENTS.....	19
4	SOLID ANGLE CORRECTION PARAMETERS.....	63

CHAPTER I
INTRODUCTION

The decay of Cobalt-60 to an excited state of Nickel-60 and the angular correlation of the resulting gamma-gamma cascade have been studied extensively experimentally (2,6,8,9, and others) and the results of the correlation measurements agree very well with the theoretical correlation for the decay scheme shown in Figure 1. Consequently, the decay of Cobalt-60 is now used as one standard for the testing of correlation equipment.

Angular correlation equipment was assembled at the US Naval Postgraduate School by Verser in 1960 for the investigation of the properties of low-lying excited nuclear states. His testing of the operating characteristics of the equipment by the angular correlation of the cascading gamma rays resulting from the decay of Cobalt-60 yielded an observed correlation curve whose shape was not in agreement with the theoretical curve. This work was initiated in order to attempt to verify Verser's results.

In order to facilitate the statistical analysis of the data, a program has been worked out for the use of the 1604 Digital Computer. The program follows the procedures outlined by M.E. Rose (2) and is contained completely in Appendix I.

An additional program has been written to compute the solid angle corrections, which must be applied to the theoretical curve, by the method of M.E. Rose (2). This program is outlined in Appendix II.

The results of this work agreed within statistical expectation with those obtained by Verser, indicating that the unexpected shape of the curve is apparently real and not an instrumental effect. These results are given in Chapter VI.

PARTIAL DECAY SCHEME OF COBALT-60

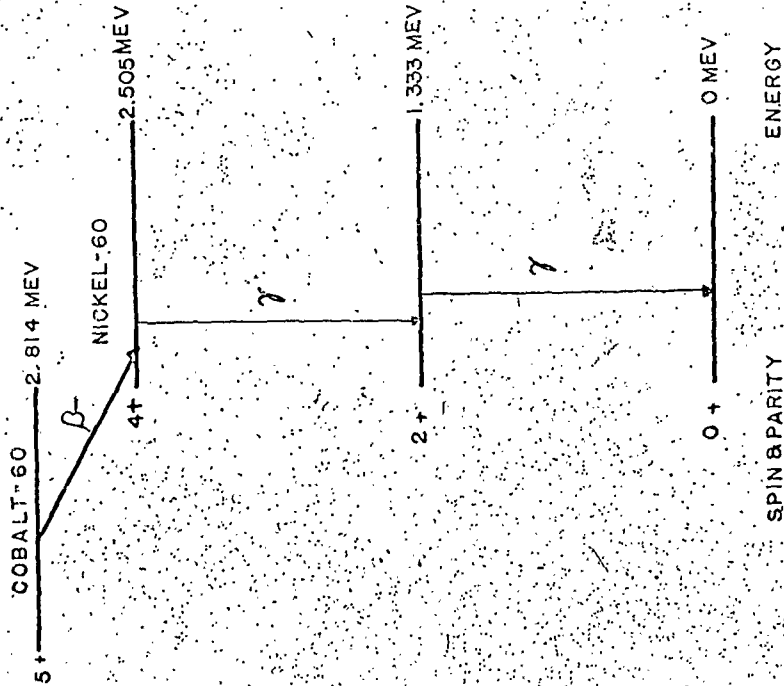


FIGURE 1 2

CHAPTER II THEORETICAL CONSIDERATIONS

The determination of the angular momentum quantum numbers, or spins, of low-lying short-lived excited nuclear states by the nuclear spectroscopic tool, known as angular correlation of successively emitted radiations, is quite common practice today. Only within recent years, because of the vast advancement of electronics, has practical utilization of this tool become possible.

Since nuclei, under ordinary conditions, are randomly oriented in space, it is not possible to observe a radiation pattern. This is due to the fact that the probability of emission of a radiation by an excited nucleus, depends on the angle between the nuclear spin axis and the direction of the emission. Therefore, in order to observe a radiation pattern, it is first necessary to orient the nuclei.

One method of orienting certain nuclei consists of placing the sample in an electric field gradient or a strong magnetic field at a very low temperature. Another method, the one used in the present consideration, consists of selecting nuclei whose nuclear states have spins lying in preferred directions. This happens to be the case for the intermediate state if a nucleus decays by successive emission of two radiations.

In an angular correlation experiment the first radiation is observed in a fixed direction. This establishes an axis to which the direction of emission of the second radiation, originating from the state formed by the first, can be referred. The second radiation has preferred angles of emission with respect to the direction of the first.

In order that the correlation exhibit maximum anisotropy, the angular momentum vector of the intermediate state must not change

direction significantly before the second radiation occurs. The mean life of the intermediate state must be less than about 10^{-8} seconds, the typical precession period of the nuclear spin about local perturbing fields. The angular correlation function is determined by the nature of the radiations and by the spins of the states involved, and thus its measurement may lead to an unambiguous choice of spins for the states.

The theoretical expressions for the correlation functions have been worked out for a large number of cases of interest (7). For the particular case of a gamma-gamma cascade, the correlation function $W(\theta)$ is

$$W(\theta) = \sum_{n=0}^{n_{\max}} A_n P_n(\cos \theta) = \sum_{n=0}^{n_{\max}} P_n \cos^n \theta \quad (1)$$

where P_n is the Legendre Polynomial of even order n , n_{\max} is the smallest even integer of the set of three numbers consisting of twice the multipole order of each of the two gamma rays, and θ is the angle between the directions of emission of the two gamma rays.

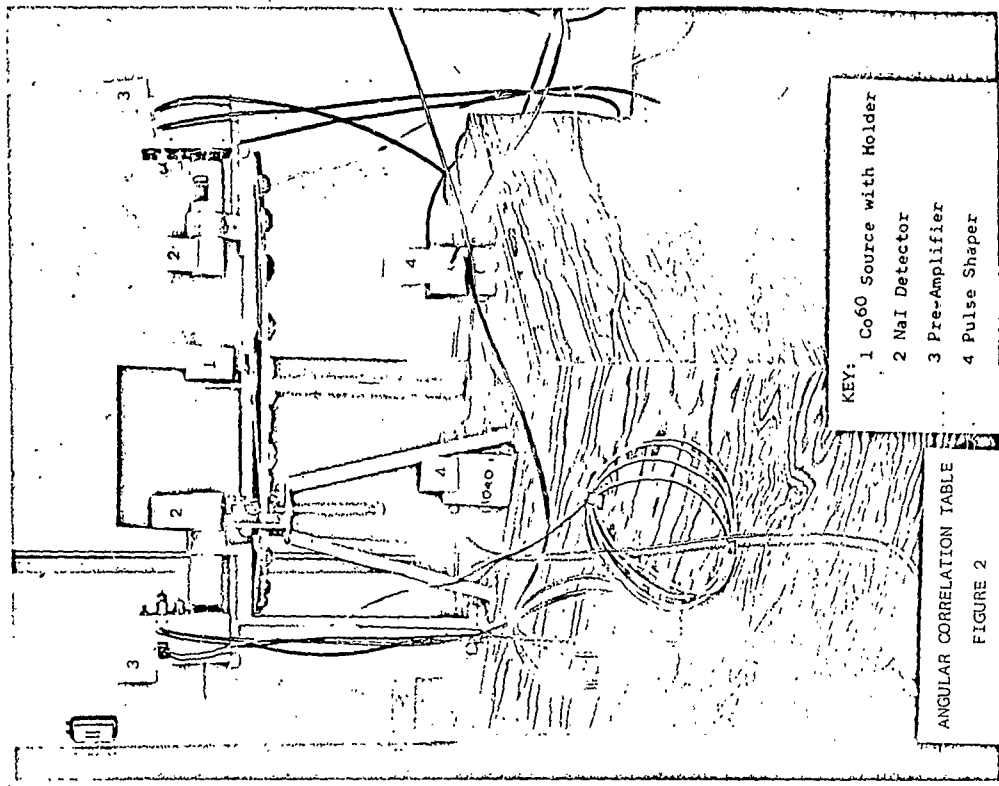
The theoretical correlation function for the cascade in Nickel-60, assuming point detectors and a point source, is

$$W(\theta) = 1 + 0.1020 P_2(\cos \theta) + 0.0091 P_4(\cos \theta) \quad (2)$$

with an anisotropy

$$K = \frac{W(180^\circ) - W(90^\circ)}{W(90^\circ)} = 0.1667 \quad (3)$$

Many workers, after correcting these theoretical results for the effect of non-point detectors have found good agreement between them and their experimental correlation results.



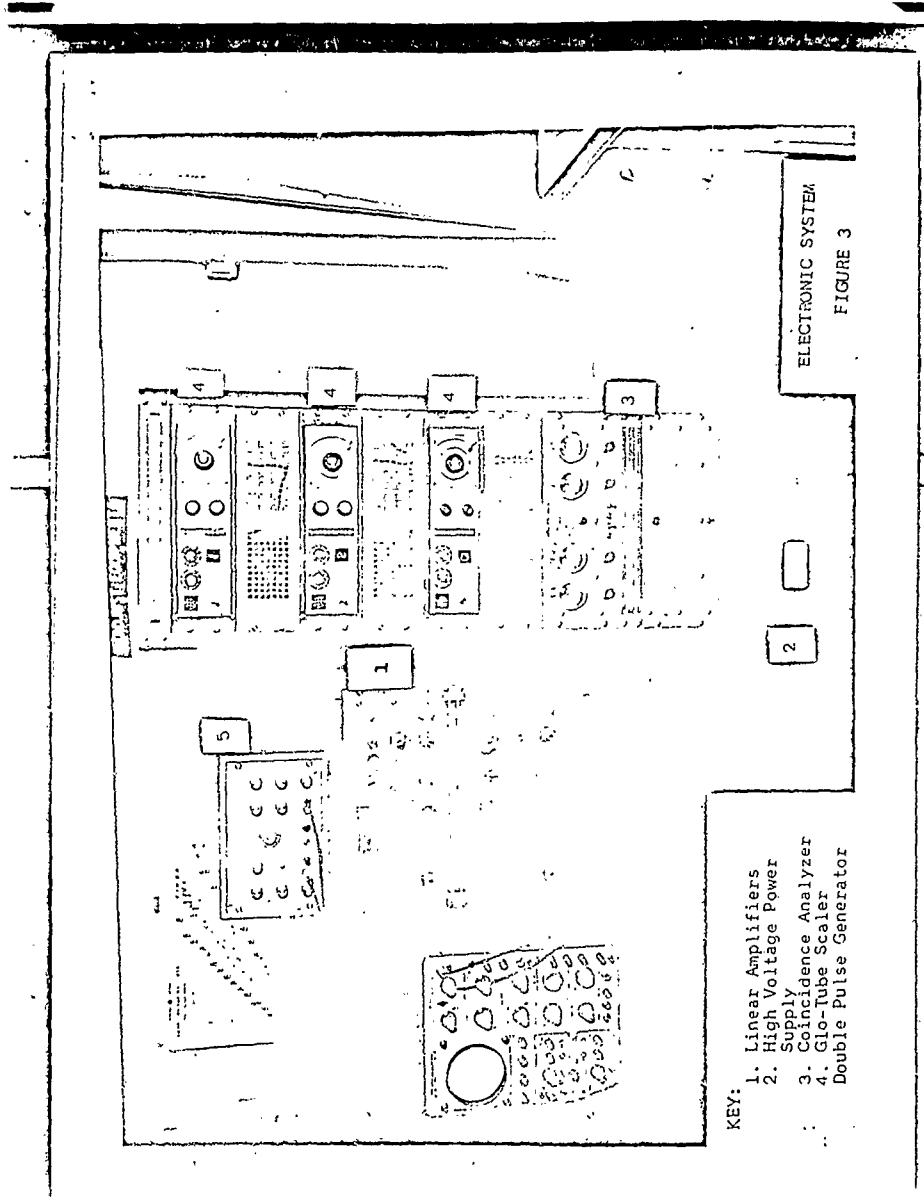
CHAPTER III
DESCRIPTION OF EQUIPMENT

The equipment needed for the gamma-gamma correlation measurements must furnish means for detecting the radiation, pulse-height amplification with selection or discrimination, coincidence analysis, and adequate recording of the counts in the various counting channels. A mechanical system must also be provided for mounting the detectors and changing their relative positions and for holding and positioning the radioactive source.

The apparatus used for this work was that assembled by Verser and described in detail in his report. Therefore, only a brief description of the equipment and of the minor alterations performed on it will be presented.

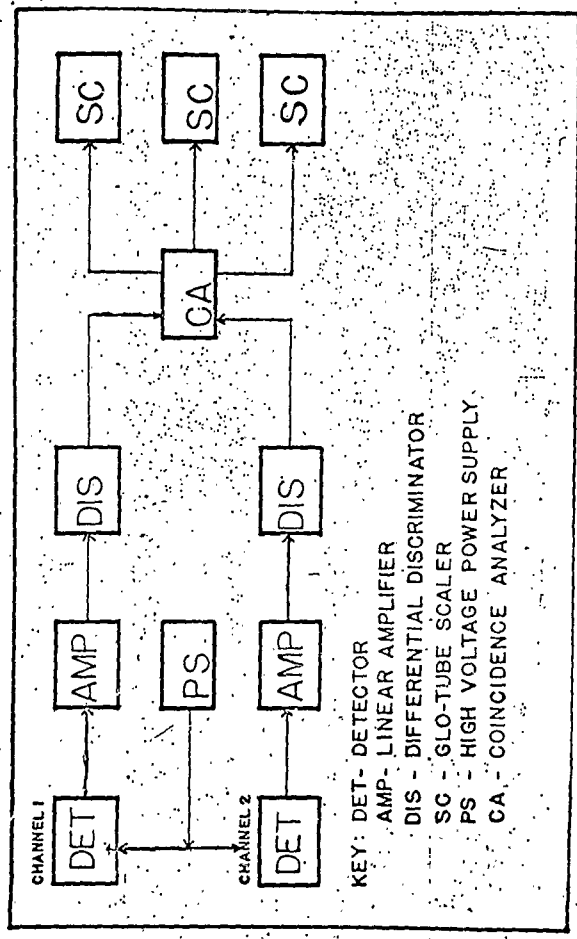
Figure 2 is a photograph of the mechanical setup. The original base of the correlation table has been replaced, the new base is of solid brass with a bearing mounted shaft to enable accurate manual detector positioning.

The electronic equipment (Figure 3) has the functions of detection of the radiation, pulse-height amplification and discrimination, coincidence analysis, and recording the number of counts. The only change in the electronic equipment was in the pulse-shaping networks used in the initial calibration of the apparatus. The pulse shapers were rebuilt to give a better simulation between the test pulses and the actual pulses encountered under operating conditions.



ELECTRONIC SYSTEM
FIGURE 3

- KEY:
- 1. Linear Amplifiers
 - 2. High Voltage Power Supply
 - 3. Coincidence Analyzer
 - 4. Glo-Tube Scaler Double Pulse Generator
 - 5. Coincidence Analyzer



SCHEMATIC DIAGRAM OF THE ELECTRONIC SYSTEM

FIGURE 4

CHAPTER IV
EXPERIMENTAL ANALYSIS

The angular correlation function $W(\theta)$ (Equation 1) must be expressed in terms of experimentally-observable quantities. The equipment records the total number of coincidences as a function of the angle between the detectors, and the total number of radiations captured in each channel. C_c , the total number of coincidences in a counting period, is given by

$$C_c = C_g + C_a + C_b \quad (4)$$

where

- C_g = the number of genuine coincidences due to the two cascading radiations,
- C_a = the number of accidental coincidences due to the finite resolving time of the electronic equipment, and
- C_b = the number of background coincidences.

The half-life of the source is long compared to the time required to collect the necessary data; therefore, equation (4) can be rearranged and rewritten in terms of counting rates:

$$\dot{C}_g = \dot{C}_c - \dot{C}_a - \dot{C}_b \quad (5)$$

\dot{C}_g can also be expressed in terms of other parameters:

$$\dot{C}_g = (\epsilon_1 \alpha_{11} \rho_{11} \alpha_{22} \rho_{22} + \epsilon_{12} \alpha_{12} \rho_{12} \epsilon_2 \alpha_{21} \rho_{21}) \lambda_1 \lambda_2 N_0 \bar{W}(\theta) \quad (6)$$

where

- N_0 = the absolute source strength
- ρ_{jk} = the fraction of nuclei decaying by the emission of gamma ray "j" and gamma ray "k" in a cascade

Δ_i = the fractional solid angle subtended by the crystal in Channel "i"

ρ_{ij} = the fraction of gamma rays "j" which leave the source in the right direction to enter crystal "i" that are not absorbed before reaching the crystal

ϵ_{ij} = the total efficiency of the crystal in Channel "i" for gamma ray "j"

α_{ij} = the fraction of the number of gamma rays "j" captured by the crystal in Channel "i" whose pulse heights are accepted by the energy discrimination of Channel "i"

$\bar{W}(\theta_m)$ = the angular correlation function at an angle θ_m averaged over the solid angles used.

N_i , the counting rate in Channel "i", can also be expressed as a function of certain of these same parameters.

$$N_i = \Delta_i \sum_j \epsilon_{ij} \alpha_{ij} \rho_{ij} + N_{ib} \quad (7)$$

where

- ρ_{ij} = the number of gamma rays "i" per disintegration
- N_{ib} = the background counting rate in channel "i"

From Equations (5) and (6) we get for the correlation function,

$$\bar{W}(\theta_m) = \frac{\dot{C}_g}{\dot{C}_c} = \frac{\dot{C}_c - \dot{C}_a - \dot{C}_b}{\dot{C}_c} \quad (8)$$

where f is the appropriate function of the ϵ 's, α 's, ρ 's, and θ 's.

By combining Equations (7) and (8), we get for the correlation function

$$\bar{W}(\theta_m) = \frac{(C_t - C_a - C_b) N_0 g}{(N_1 - N_{1a})(N_2 - N_{2b})} \quad (9)$$

where g is a function of the ϵ 's, α 's, ρ 's, λ 's, β 's.

In this form $\bar{W}(\theta_m)$ is relatively insensitive to equipment stability. Therefore, it is only necessary to calculate the modified correlation function given by

$$\bar{W}'(\theta_m) = \frac{\bar{W}(\theta_m)}{g_0} = \frac{(C_t - C_a - C_b)}{(N_1 - N_{1a})(N_2 - N_{2b})} \quad (10)$$

The correlation runs were done with the movable detector at seven angles: 90° , 105° , 120° , 135° , 150° , 165° , and 180° . A minimum of six two-hour counting periods were completed at each angle, and the angular order in which they were taken was varied to minimize any systematic fluctuations. Each run yielded single channel counting rates of approximately 1200 counts per second and a total coincidence rate of approximately 1.1 counts per second.

Before the experimental results can be compared to theory, the data must be checked and corrected, if necessary, for the various deviations from the ideal arrangement, i.e., centered point source, no spurious coincidences due to the coincidence resolving time, no scattering, no background, and perfect stability of the electronic equipment. In addition, the theoretical correlation function must be corrected for non-zero solid angle detectors. The methods for accounting for these effects are discussed in the following paragraphs.

A. SOURCE SIZE AND POSITIONING

As a test of the positioning of the source in the center of the correlation table, the integral counting rate of a portion of the Cobalt-60 spectrum was measured in each channel as a function of the angle between the counters. This was done by Verser and checked again after the base of the correlation table had been changed. Within statistical expectations, the counting rates were constant. The source has a volume of about 0.001 cubic inches and can be regarded as a point source at the distance at which the detectors were placed from it.

B. SCATTERING

Compton scattering occurring outside the detectors can give rise to unwanted coincidences. These in general will tend to smear out the measured correlation function. This problem is minimized by use of scintillation crystals as detectors and accepting only the full-energy peaks of the desired gamma rays. The method for determining the pulse height selection limits has been discussed in detail by Verser.

C. BACKGROUND CORRECTION

With the discriminators at the same settings as were used during the correlation runs, background runs were made before and after the correlation runs. The background coincidence counting rate was negligibly small, and the background rate of each channel was such as to yield total background counts of the order of 0.4 percent of the total counts in each channel during the correlation runs. These background rates were used as corrections to the gross counting rates obtained during correlation runs.

D. DETERMINATION OF GENUINE COINCIDENCE RATE

Since the background coincidence counting rate proved experimentally

to be negligible, Equation (5) becomes

$$C_g = C_t - C_a$$

For the total counting time, T , the total number of coincidences is $C_t T$ and the standard deviation in the total coincidence rate is

$$\sigma_{C_t} = \sqrt{\frac{C_t}{T}} \quad (11)$$

Therefore, the standard deviation of the genuine coincidence rate is

$$\sigma_g^2 = \sigma_{C_t}^2 + \sigma_a^2 = \frac{C_t}{T} + \frac{C_a}{T} \quad (12)$$

The accidental coincidence rate is given by

$$C_a = 2 \gamma_T N_1 N_2 \sim 2 \gamma_T N_0^2 \quad (13)$$

where

γ_T = Coincidence resolving time of the coincidence analyzer.

In order to minimize the accidental coincidence rate, the source strength must be kept as small as possible. However, if the source strength is decreased, the counting time must be increased to obtain a specified precision in the counting rate. Thus, a compromise must be reached between precision and counting time.

The smallest resolving time obtainable with the present equipment is experimentally determined to be

$$\gamma_T = (0.1820 \pm 0.0016) \text{ microseconds};$$

this resulted in an accidental coincidence rate that was about 30% of the

total coincidence rate. The approximate source strength of the water solution of CoCl_2 used in this experiment was 0.05 millicuries of Cobalt-60.

E. REFERENCE COUNTING RATE

With the discriminator in each channel adjusted to accept only the full-energy peaks of the desired gamma rays, the integral counting rate of the Cobalt-60 spectrum in each channel was determined. This gave a reference counting rate for each channel. During the course of the correlation runs, either the discriminator settings or the gains of the linear amplifiers were adjusted before each run so that, the integral counting rate agreed within ± 5 cps with these reference rates (Table 1).

TABLE 1
REFERENCE COUNTING RATES

$$N_1 = (1085.26 \pm 0.57) \text{ cps}$$

$$N_2 = (1117.32 \pm 0.59) \text{ cps}$$

As a measure of the gain fluctuations during each counting run, the integral counting rate of each channel was measured after the run using the same discriminator setting that was employed during the run. The difference between this counting rate and the reference counting for that channel was determined. If the variation exceeded ± 20 cps in either channel, the run was discarded.

This procedure resulted in about 30% of the runs being discarded because they fell outside of the maximum acceptable fluctuation. This indicates that the electronic system is not as stable as it should be. A check of the line voltage revealed substantial fluctuations during any 24-hour period. The temperature of the room was not controlled in any manner and varied considerably. These could have been the major causes of the large counting rate fluctuations since the discriminators are highly sensitive to both temperature and voltage variations.

F. SOLID ANGLE CORRECTIONS

The solid angle correction which must be applied to the theoretical correlation function to account for the non-zero solid angle of the detectors and enable a comparison to be made with the measured function is described by Rose (2). The correction for two cylindrical crystals whose axes intersect at the source involves the numerical evaluation of integrals of the form

$$I_{n,i} = \int_0^{X_i} P_n(\cos \theta) \{ 1 - e^{-Y_i X_i(\theta)} \} \sin \theta d\theta \quad (14)$$

where

Y_i = the full-energy absorption coefficient of the detector in Channel "i"; For the gamma ray;

X_i = the half-angle subtended by the front face of the crystal in Channel "i";

$$X_i(\theta) = t_i \sec \theta \text{ for } 0 \leq \theta \leq \tan^{-1} \frac{r_i}{h_i + t_i};$$

$$X_i(\theta) = r_i \csc \theta - h_i \sec \theta \text{ for } \tan^{-1} \frac{r_i}{h_i + t_i} \leq \theta \leq X_i;$$

where

h_i = the distance from the source to the crystal in Channel "i";

t_i = the thickness of the crystal in Channel "i";

r_i = the radius of the crystal in Channel "i".

The attenuation factors, Q_n , are functions of these integrals:

$$Q_n = \left[\frac{I_{n,1}}{I_{0,1}} \right] \left[\frac{I_{n,2}}{I_{0,2}} \right] \quad (15)$$

The corrected correlation function becomes

$$\bar{W}(\theta) = \sum_{n=0}^{N_{\max}} Q_n A_n P_n(\cos \theta)$$

CHAPTER V
EXPERIMENTAL RESULTS

The modified correlation function for the k^{th} run at angle θ_m given by Equation (10) is

$$\bar{W}_k(\theta_m) = \left\{ \frac{C_c - C_a - C_b}{(N_1 - N_{1b})(N_2 - N_{2b})} \right\}_k$$

This means that the seven counting rates $C_c, C_a, C_b, N_1, N_{1b}, N_2,$ and N_{2b} must be determined for each run at each angle and the standard deviation of each rate calculated.

The background counting rates N_{1b} and N_{2b} for each run were experimentally determined. The single-channel counting rates corrected for scaling losses yielded N_1 and N_2 . The accidental counting rates were calculated for each run from Equation (13). The total coincidence counting rate, C_c , was experimentally determined. The background coincidence rate, C_b , was negligible throughout the experiment.

The experimental value of the modified correlation function $\bar{W}_k(\theta_m)$ and its standard deviation were calculated for each run at each angle. From these the weighted mean $\bar{W}(\theta_m)$ and its standard deviation were computed for each angle.

The weighted mean of the measurements taken at each angle was determined by using the inverse square of the standard deviation of each measurement as a weighting factor:

$$\bar{W}(\theta_m) = \frac{\sum_{k=1}^{N_m} \frac{\bar{W}_k(\theta_m)}{\sigma_k^2}}{\sum_{k=1}^{N_m} \frac{1}{\sigma_k^2}} \quad (16)$$

where N_m = number of runs at the angle θ_m .

The standard deviation of the weighted mean (\bar{W}) was calculated from the individual standard deviations:

$$\frac{1}{\sigma^2(\bar{W})} = \sum_{k=1}^{M_m} \frac{1}{\sigma_k^2} \quad (19)$$

A comparison between the standard deviations of the means calculated from Equation (19) and those calculated from

$$\sigma^2(\bar{W}) = \sum_{k=1}^{M_m} \frac{[\bar{W}(\theta_k) - \bar{W}(\theta)]^2}{N_m(N_m - 1)} \quad (20)$$

revealed no significant differences. Therefore the standard deviation obtained from Equation (19) were used in subsequent calculations and the method discussed in paragraph E Chapter IV for treating gain instabilities is realistic.

The values of the weighted means and their standard deviations are tabulated in Table 2.

TABLE 2
The Weighted Means $\bar{W}'(\theta_n)$

θ_n (degrees)	Weighted Mean	Standard Deviation
90	5046	45
105	4925	33
120	5109	21
135	5211	36
150	5556	35
165	5667	24
180	5907	43

A curve of the form $\bar{W}'(\theta) = \sum_{n=1}^7 A_n' P_n(\cos\theta)$ was fitted to the experimental $\bar{W}'(\theta_m)$ by the method of least squares as outlined by Rose (2). This analysis was programmed to be carried out by the 1604 computer. (See Appendix II for details).

The values of A_n' are given by the matrix equation

$$\|A'\| = \|A'^T A\|^{-1} \|A'^T \mu\| \quad (21)$$

where

$\|A\|$ = the 7 x 3 matrix consisting of the Legendre Polynomials, $P_0, P_2,$ and P_4 , evaluated at the seven angles (Figure 5A).

$\|A'^T\|$ = the transposed A matrix

$\|\mu\|$ = a diagonal matrix with elements consisting of the inverse squares of the standard deviations of the experimental $\bar{W}'(\theta_m)$ (Figure 5B)

$\|\mu\|^{-1}$ = a one column matrix whose elements are the experimental values of $\bar{W}'(\theta_m)$ (Figure 5C)

$\|A'\|^{-1}$ = the one column matrix whose elements are respectively the least-square coefficients A_0', A_2' and A_4' (Figure 5D)

$\|A'^T A\|^{-1}$ = the inverse of the matrix obtained by multiplying the three designated matrices together. The diagonal terms of this matrix are the squares of the standard deviations of the least-square coefficients, A_n' .

The experimental values for the coefficients A_n' and their standard deviations, normalized so that $A_0' = 1$, are listed in Table 3. The corresponding values from Verser's work are included for comparison.

TABLE 3
Least Square Coefficients

Verseer	This Work
A_0^1	1.0000 ± 0.0034
A_2^1	0.0961 ± 0.0055
A_4^1	0.0339 ± 0.0071

MATRIX CONFIGURATIONS

$P_0(\text{Cos}90^\circ)$	$P_2(\text{Cos}90^\circ)$	$P_4(\text{Cos}90^\circ)$
$P_0(\text{Cos}105^\circ)$	$P_2(\text{Cos}105^\circ)$	$P_4(\text{Cos}105^\circ)$
$P_0(\text{Cos}120^\circ)$	$P_2(\text{Cos}120^\circ)$	$P_4(\text{Cos}120^\circ)$
$P_0(\text{Cos}135^\circ)$	$P_2(\text{Cos}135^\circ)$	$P_4(\text{Cos}135^\circ)$
$P_0(\text{Cos}150^\circ)$	$P_2(\text{Cos}150^\circ)$	$P_4(\text{Cos}150^\circ)$
$P_0(\text{Cos}165^\circ)$	$P_2(\text{Cos}165^\circ)$	$P_4(\text{Cos}165^\circ)$
$P_0(\text{Cos}180^\circ)$	$P_2(\text{Cos}180^\circ)$	$P_4(\text{Cos}180^\circ)$

(A)

$\sigma^{-2}(90^\circ)$	0	0	0	0	0	0
0	$\sigma^{-2}(105^\circ)$	0	0	0	0	0
0	0	$\sigma^{-2}(120^\circ)$	0	0	0	0
0	0	0	$\sigma^{-2}(135^\circ)$	0	0	0
0	0	0	0	$\sigma^{-2}(150^\circ)$	0	0
0	0	0	0	0	$\sigma^{-2}(165^\circ)$	0
0	0	0	0	0	0	$\sigma^{-2}(180^\circ)$

(B)

(C)

$\bar{W}^1(90^\circ)$
$\bar{W}^1(105^\circ)$
$\bar{W}^1(120^\circ)$
$\bar{W}^1(135^\circ)$
$\bar{W}^1(150^\circ)$
$\bar{W}^1(165^\circ)$
$\bar{W}^1(180^\circ)$

(D)

A_0^1
A_2^1
A_4^1

FIGURE 5
20

A. Experimental Anisotropies

The anisotropy is defined:

$$\bar{R} = \frac{\bar{W}(180^\circ) - \bar{W}(90^\circ)}{\bar{W}(90^\circ)} \quad (22)$$

The anisotropy for the least-square curve was calculated from the experimental correlation function. Since the anisotropy can be determined with much greater precision than can the least-square coefficients themselves, it provides a more sensitive means for comparing the experimental and theoretical functions. The experimental anisotropy is

$$\bar{R} = 0.167 \pm 0.013$$

The standard deviation was calculated by the formula presented by Klena and McGowan (6). The computer program for the calculation of \bar{R} and its standard deviation is outlined in Appendix I.

CHAPTER VI RESULTS AND CONCLUSIONS

A. COMPARISON BETWEEN EXPERIMENT AND THEORY

The experimental and corrected theoretical functions for this experiment are:

Experimental:

$$\bar{W}(\theta) = (1.0000 \pm 0.0026) + (0.0971 \pm 0.0057) P_2 + (0.0231 \pm 0.0066) P_4$$

Theoretical:

$$\bar{W}(\theta) = 1.0000 \pm 0.0996 P_2 + 0.00838 P_4$$

The experimental anisotropy was calculated to be $\bar{R} = 0.167 \pm 0.013$ and is compared with Verser's value of $\bar{R} = 0.171 \pm 0.027$ and with the theoretical anisotropy corrected for the solid angle, $\bar{R} = 0.161$.

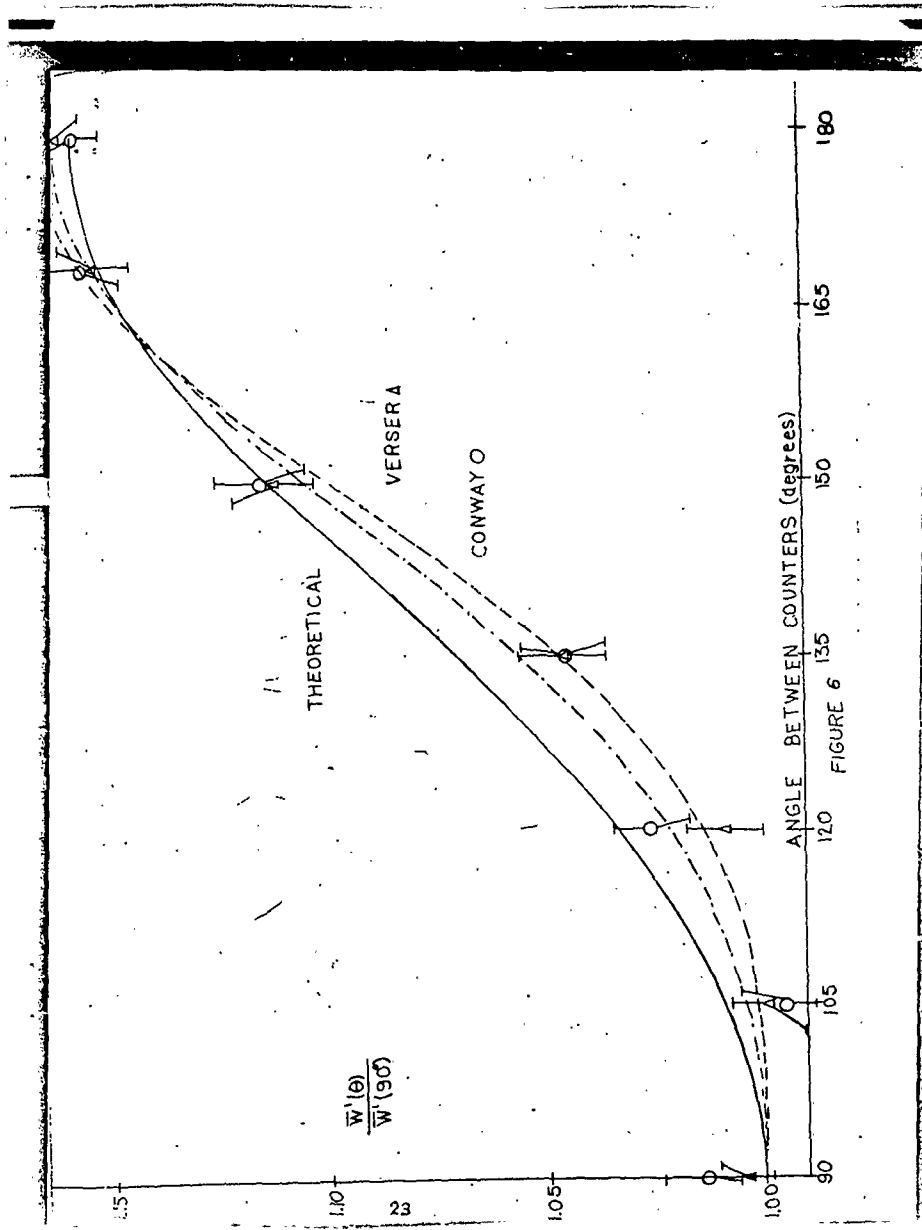
Figure 6 shows the experimental results and their standard deviations for this work and that of Verser. Also shown are the least-square correlation curves for both sets of data and the corrected theoretical correlation curve. All the curves have been normalized to unity at $\theta = 90^\circ$.

B. DISCUSSION AND CONCLUSIONS

From Figure 6, it can be seen that the curve obtained in this work agrees with that obtained by Verser. Both of these curves show an apparently real dip in the region from 105° to 135° . It is only a remote possibility that this dip can be attributed to statistical fluctuations, and it is unlikely to be due to instrumental effects.

Two conjectures of possible mechanisms are:

- (a) The source may have crystallized resulting in preferential alignment of the nuclei within the crystal due to internal electric fields.



- (b) The ferromagnetic properties of Cobalt could have led to a preferential alignment of the nuclei in the earth's magnetic field.

The following series of experiments are suggested for future work.

- (a) The results for measurements conducted for angles between 180° and 270° should be compared with those already obtained.
- (b) In order to check on the possible effects of orientation due to crystalline electric fields, the experiment should be repeated with the present source at various orientations relative to the fixed counter.
- (c) In order to check on the possibility of orientation by magnetic effects the source should be placed in a magnetic field of known direction and the measurements repeated.
- (d) A correlation curve using a powdered Cobalt-60 source in place of the present source should be obtained for comparison.

These experiments would determine conclusively if any effects causing preferential nuclear orientation are present.

The following instrumentation improvements are suggested in order to improve the stability of the equipment.

- (a) A very stable power supply should be purchased to aid in preventing the line voltage fluctuations from interfering with the electronic system.
- (b) The effect of temperature variations on the electronic system can be minimized by placing the equipment in a temperature controlled room for conducting future work.
- (c) A slow coincidence unit whose resolving time is independent of the counting rate should be obtained.

The precision of the results should be improved by the addition of a fast coincidence channel working in conjunction with the slow coincidence unit to reduce the number of accidental coincidences.

Since the experimental curves using Cobalt-60 have not agreed with the theoretical curve, the experiments on Cobalt-60 should be continued until the source of the distortion is ascertained. Perhaps the effect can be related to the physical properties of the CoCl_2 source.

APPENDIX I
Least-Square Fit of Experimental Results

The comparison of the experimental correlation function to the functional form of the theoretical correlation function given by Equation (1) is accomplished by the method of least-squares, as outlined by Rose (2). The analysis requires the solution of Equation (20) for the values of A'_n .

The computer solution of the matrix equation, like the solid angle calculation, is carried out in floating point format. In addition to the solution for the values of A'_n , the program also calculates the anisotropy by Equation (21), and its standard deviation by the formula presented by Klema and McGowan (6). A flow sheet outline of the program is given in Figure (7). In order to provide a certain amount of flexibility to the program, the angles used have been left as a parameter to be supplied. The program as written is, however, limited to seven different angular positions. The parameters needed for the program may be placed anywhere in the computer and their addresses supplied to the program by placing them in the B registers in the following order:

B1 = weighted means of the correlation function in ascending angular order.

B2 = the standard deviations of the weighted means in the same order as above.

B3 = the values of the angles used in ascending order.

The program is available for future use on both biocatal tape in machine language, and IBM cards in assembly language. The biocatal tape contains the following:

1. The main program for calculation of A'_n
2. The program of normalization of the A'_n
3. The program for calculation of the resulting correlation function
4. The normalization of the correlation function
5. The calculation of the anisotropy

LEAST-SQUARE FIT FLOW CHART

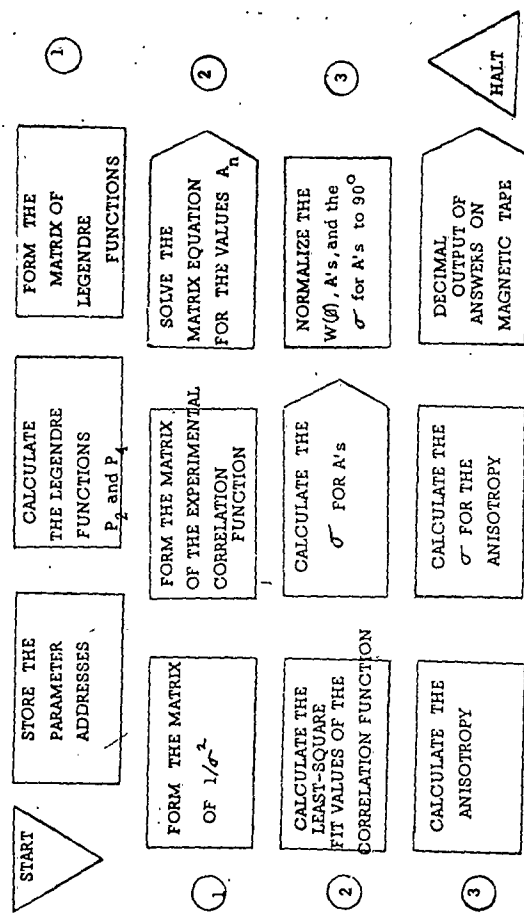


FIGURE 7

6. The calculation of the standard deviation for the anisotropy

7. The subroutines

- a. Floating Point Sine-Cosine
- b. Floating Point Square Root
- c. Floating Point Decimal Output.

The results of the computations are dumped on magnetic tape unit (4) in the following sequence:

Lines 1 thru 3 The matrix inverse

Lines 4 thru 6 Check answer; included in the program is a check of the inverse which consists of multiplying the inverse by the original matrix

Line 7 The values of A'_0, A'_2 and A'_4

Line 8 The standard deviations of the A'_n

Line 9 thru 13 The resulting correlation functions

Line 14 and 15 The normalized correlation functions

Line 16 The normalized A'_n

Line 17 The normalized standard deviations of the A'_n

Line 18 The anisotropy

Line 19 The square of the standard deviation of the anisotropy

STATISTICAL ANALYSIS PROGRAM

19-5852

```

CRG 12000
REM BURTON J CONWAY APRIL 1961
REM LEAST SQUARE FIT OF ANGULAR
REM CORRELATION DATA.
REM MUST BE SUPPLIED WITH
REM PARAMETER ADDRESSES AS FOLLOWS.
REM B1 CORRELATION FUNCTION.
REM B2 SIGMA OF CORRELATION FUNCT. ON.
REM B3 ANGLES USED IN DEGREES.
ZEUS EQU 13000
RHO EQU 13100
TRANS EQU 13200
ACTV EQU 13250
ATWA EQU 13260
INVER EQU 13500
CHECK EQU 13520
LAT EQU 13540
LATW EQU 13570
ANS EQU 13620
FNIS EQU 13650
SINFL EQU 60000
MATRIX EQU 10000

```

12013	20 0 12461	STA 0 DON
	32 0 12320	FMU 0 FLT2
12014	31 0 12316	FSB 0 FLT
	33 0 12317	FDV 0 FLTL
12015	20 1 12326	STA 1 WORK2
	12 0 12461	LDA 0 DON
12016	32 0 12322	FMU 0 FLT4
	20 0 12462	STA 0 DON+1
12017	12 0 12461	LDA 0 DON
	32 0 12461	FMU 0 DON
12020	32 0 12323	FMU 0 FLT5
	31 0 12462	FSB 0 DON+1
12021	30 0 12320	FAD 0 FLT2
	33 0 12321	FDV 0 FLT3
12022	20 1 12335	STA-1 WORK4
	50 0 00000	ENI 0 0
12023	54 1 00006	ISK 1 6
	75 0 12012	SLJ 0 NEXT
12024	12 0 12325	LDA 0 WORK1
	20 2 12434	STA 2 ABLE
12025	51 2 00001	INI 2 1
	12 1 12326	LDA 1 WORK2
12026	20 2 12434	STA 2 ABLE
	51 2 00001	INI 2 1

12000	56 1 12045	ANISO EQU 13624
	50 1 00000	SIU 1 FORMU
12001	57 2 12032	ENI 1 0
	56 2 12033	SIL 2 SIGMA
12002	50 2 00000	SIU 2 SIGMA+1
	50 0 00000	ENI 2 0
12003	56 3 12004	ENI 0 0
	50 3 00000	SIU 3 /+1
12004	12 1 00000	ENI 3 0
	32 0 12512	LDA 1 0
12005	75 4 60001	FMU 0 CONV
	50 0 00000	SLJ 4 SINGL+1
12006	50 0 00000	ENI 0 0
	50 0 00000	ENI 0 0
12007	50 0 00000	ENI 0 0
	50 0 00000	ENI 0 0
12010	20 1 12463	STA 1 TEMP
	50 0 00000	ENI 0 0
12011	54 1 00006	ISK 1 6
	75 0 12004	SLJ 0 START
12012	12 1 12463	LDA 1 TEMP
	32 1 12463	FMU 1 TEMP

STARTS FORMATION
OF THE LEG. POL.

12042 75 0 12044 SLJ 0 /+2
 50 0 00000 ENI 0 0
 12043 51 1 00001 INI 1 1
 75 0 12036 SLJ 0 FORMCAD
 50 1 00000 ENI 1 0
 50 2 00000 ENI 2 0
 12045 12 1 00000 FORMU LDA 1 0
 20 1 12344 STA 1 MU
 54 1 00006 ISK 1 6
 75 0 12045 SLJ 0 FORMU
 12047 50 1 00000 ENI 1 0
 50 2 00000 ENI 2 0
 12050 75 4 10000 SLJ 4 MATRIX TRANSPOSE
 50 0 00000 ENI 0 0
 50 0 00000 ENI 0 0
 50 0 00000 ENI 0 0
 12052 50 0 00000 ENI 0 0
 50 0 00002 ENI 0 2
 50 0 00007 ENI 0 7
 50 0 12434 ENI 0 ABLE
 12054 50 0 00003 ENI 0 3
 50 0 06626 ENI 0 6626
 12055 50 0 00000 ENI 0 0
 50 0 13200 ENI 0 TRANS

12027 12 1 12335 LDA 1 MGRK4
 20 2 12434 STA 2 ABLE
 12030 51 2 00001 INI 2 1
 50 0 00000 ENI 0 0
 54 1 00006 ISK 1 6
 75 0 12024 SLJ 0 FORM
 12032 12 0 12316 SIGMA LDA 0 FLT
 33 1 00000 FDV 1 0
 33 1 00000 FDV 1 0
 20 1 12463 STA 1 TEMP
 54 1 00006 ISK 1 6
 75 0 12032 SLJ 0 SIGMA
 50 2 00000 ENI 2 0
 50 1 00000 ENI 1 0
 12 1 12335 FORMCAD LDA-1 CAD
 22 0 12043 AJP 0 /+5
 12 2 12463 LDA 2 TEMP
 20 1 12335 STA 1 CAD
 51 1 00001 INI 1 1
 50 0 00000 ENI 0 0
 54 2 00006 ISK 2 6
 75 0 12036 SLJ 0 FORMCAD

12071	50 0 00003	ENI 0 3	
	50 0 13260	ENI 0 ATWA	
12072	75 4 10000	SLJ 4 MATRIX	INVERSE
	50 0 00000	ENI 0 0	
12073	50 0 00000	ENI 0 0	
	50 0 00000	ENI 0 0	
12074	50 0 00000	ENI 0 0	
	50 0 00005	ENI 0 5	
12075	50 0 00003	ENI 0 3	
	50 0 13260	ENI 0 ATWA	
12076	50 0 00044	ENI 0 36D	
	50 0 13272	ENI 0 ATWA+12	
12077	50 0 00000	ENI 0 0	
	50 0 13500	ENI 0 IRVER	
12100	75 4 10000	SLJ 4 MATRIX	CHECK
	50 0 00000	ENI 0 0	
12101	50 0 00000	ENI 0 0	
	50 0 00000	ENI 0 0	
12102	50 0 00000	ENI 0 0	
	50 0 00004	ENI 0 4	
12103	50 0 00003	ENI 0 3	
	50 0 13500	ENI 0 INVER	
12104	50 0 00003	ENI 0 3	
	50 0 13260	ENI 0 ATWA	

12056	75 4 10000	SLJ 4 MATRIX	ATW
	50 0 00000	ENI 0 0	
12057	50 0 00000	ENI 0 0	
	50 0 00000	ENI 0 0	
12060	50 0 00000	ENI 0 0	
	50 0 00004	ENI 0 4	
12061	50 0 00003	ENI 0 3	
	50 0 13200	ENI 0 TRANS	
12062	50 0 00007	ENI 0 7	
	50 0 12353	ENI 0 CAD	
12063	50 0 00007	ENI 0 7	
	50 0 13230	ENI 0 ATW	
12064	75 4 10000	SLJ 4 MATRIX	ATWA
	50 0 00000	ENI 0 0	
12065	50 0 00000	ENI 0 0	
	50 0 00000	ENI 0 0	
12066	50 0 00000	ENI 0 0	
	50 0 00004	ENI 0 4	
12067	50 0 00003	ENI 0 3	
	50 0 13230	ENI 0 ATW	
12070	50 0 00007	ENI 0 7	
	50 0 12434	ENI 0 ABLE	

12105	50 0 00003	ENI 0 3	
	50 0 13520	ENI 0 CHECK	
12106	75 4 10000	SLJ 4 MATRIX	LAT
	50 0 00000	ENI 0 0	
12107	50 0 00000	ENI 0 0	
	50 0 00000	ENI 0 0	
12110	50 0 00000	ENI 0 0	
	50 0 00004	ENI 0 4	
12111	50 0 00003	ENI 0 3	
	50 0 13500	ENI 0 INVER	
12112	50 0 00003	ENI 0 3	
	50 0 13200	ENI 0 TRANS	
12113	50 0 00007	ENI 0 7	
	50 0 13540	ENI 0 LAT	
12114	75 4 10000	SLJ 4 MATRIX	LATW
	50 0 00000	ENI 0 0	
12115	50 0 00000	ENI 0 0	
	50 0 00000	ENI 0 0	
12116	50 0 00000	ENI 0 0	
	50 0 00004	ENI 0 4	
12117	50 0 00003	ENI 0 3	
	50 0 13540	ENI 0 LAT	

12120	50 0 00007	ENI 0 7	
	50 0 12353	ENI 0 CAD	
12121	50 0 00007	ENI 0 7	
	50 0 13570	ENI 0 LATW	
12122	75 4 10000	SLJ 4 MATRIX	ANS
	50 0 00000	ENI 0 0	
12123	50 0 00000	ENI 0 0	
	50 0 00000	ENI 0 0	
12124	50 0 00000	ENI 0 0	
	50 0 00004	ENI 0 4	
12125	50 0 00003	ENI 0 3	
	50 0 13570	ENI 0 LATW	
12126	50 0 00007	ENI 0 7	
	50 0 12344	ENI 0 MU	
12127	50 0 00001	ENI 0 1	
	50 0 13620	ENI 0 ANS	
12130	50 4 00000	ENI 4 0	
	50 5 00000	ENI 5 0	
12131	50 6 00000	ENI 6 0	
	50 1 00000	ENI 1 0	
12132	50 2 00000	ENI 2 0	
	50 3 00000	ENI 3 0	
12133	54 1 77777	ISK 1 77777	
	75 0 12133	SLJ 0 /	

12134	50 0 00000	ENI 0 0
	74 7 42040	EXP 7 42040
12135	75 4 71000	SLJ 4 71000
	50 0 00000	ENI 0 0
12136	01 0 13500	01 0 INVER
	04 0 00001	04 0 1
12137	12 0 12315	LDA 0 MIKE
	70 0 12136	RAD 0 /-1
12140	54 4 00002	ISK 4 2
	75 0 12135	SLJ 0 /-3
12141	54 1 77777	ISK 1 77777
	75 0 12141	SLJ 0 /
12142	54 1 77777	ISK 1 77777
	75 0 12142	SLJ 0 /
12143	50 0 00000	ENI 0 0
	74 7 42040	EXP 7 42040
12144	75 4 71000	SLJ 4 71000
	50 0 00000	ENI 0 0
12145	01 0 13520	01 0 CHECK
	04 0 00004	04 0 4
12146	12 0 12315	LDA 0 MIKE
	70 0 12145	RAD 0 /-1

12147	54 4 00002	ISK 4 2
	75 0 12144	SLJ 0 /-3
12150	54 1 77777	ISK 1 77777
	75 0 12150	SLJ 0 /
12151	50 0 00000	ENI 0 0
	74 7 42040	EXP 7 42040
12152	75 4 71000	SLJ 4 71000
	50 0 00000	ENI 0 0
12153	01 0 13620	01 0 ANS
	04 0 00007	04 0 7
		REM STARTS FORMATION OF THE
		REM CORRELATION FUNCTION
12154	12 0 13620	DUMP LDA 0 ANS
	32 4 12434	FNU 4 ABLE
12155	20 0 13100	STA 0 REO AOP0 IN STORAGE
	51 4 00001	INI 4 1
12156	12 0 13621	LDA 0 ANS+1
	32 4 12434	FNU 4 ABLE A2P2 IN A REG
12157	30 0 13100	FAD 0 REO AOP0+42P2 IN THE
	20 0 13100	STA 0 REO A REGISTER
12160	12 0 13622	LDA 0 ANS+2
	51 4 00001	INI 4 1
12161	32 4 12434	FNU 4 ABLS ANP4 IN A REG
	30 0 13100	FAD 0 REO SUM IN A REG

ANS TO STORAGE

12162	20 6 13000	STA 6 ZEUS	01 0 12501	01 0 TIA
12165	51 6 00003	INI 6 3	04 0 00010	04 0 10
12164	54 4 00024	ISK 4 24	54 1 77777	ISK 1 77777
12165	75 0 12154	SLJ 0 DIMP	75 0 12176	SLJ 0 /
12166	12 2 13500	LDA 2 INVER	50 0 00000	EMI 0 0
12165	75 4 12527	SLJ 4 SQREL	74 7 42040	EXP 7 42040
12166	50 0 00000	EMI 0 0	75 4 71000	SLJ 4 71000
12167	50 0 00000	EMI 0 0	50 0 00000	EMI 0 0
12170	20 1 12501	STA 1 TIA	01 0 13000	01 0 ZEUS
12171	51 2 00004	INI 2 4	04 0 00011	04 0 11
12172	54 1 00002	ISK 1 2	12 0 12315	LDA 0 MIKE
12173	75 0 12164	SLJ 0 /-3	70 0 12201	RAD 0 /-1
12174	50 1 00000	EMI 1 0	54 5 00004	ISK 5 4
12171	50 2 00000	EMI 2 0	75 0 12200	SLJ 0 /-3
12172	50 3 00000	EMI 3 0	12 1 13000	LDA 1 ZEUS
12173	50 4 00000	EMI 4 0	33 0 13000	FDV 0 ZEUS
12174	54 1 77777	ISK 1 77777	20 2 12472	STA 2 NORM
12175	75 0 12172	SLJ 0 /	51 1 00003	INI 1 3
12176	50 0 00000	EMI 0 0	54 2 00006	ISK 2 6
12177	74 7 42040	EXP 7 42040	75 0 12204	SLJ 0 /-2
12178	75 4 71000	SLJ 4 71000	54 1 77777	ISK 1 77777
12179	50 0 00000	EMI 0 0	75 0 12207	SLJ 0 /
12180	74 7 42040	EXP 7 42040	50 0 00000	EMI 0 0
12181	50 0 00000	EMI 0 0	74 7 42040	EXP 7 42040

NORMALIZES THE
CORRELATION
FUNCTION

DUMPS THE
 NORMALIZED
 CORRELATION
 FUNCTION

12211	75 4 71000	SLJ 4 71000
12212	50 0 00000	ENI 0 0
12213	01 0 12472	01 0 NCRM
12214	04 0 00016	04 0 16
12215	12 0 12315	IDA 0 MIKE
12216	70 0 12212	RAD 0 /-1
12217	54 2 00001	ISK 2 1
12218	75 0 12211	SLJ 0 /-3
12219	50 1 00000	ENI 1 0
12220	50 0 00000	ENI 0 0
12221	12 1 13620	LDA 1 ANS
12222	33 0 13620	FDV 0 ANS
12223	20 1 12507	STA 1 AN
12224	50 0 00000	ENI 0 0
12225	54 1 00002	ISK 1 2
12226	75 0 12216	SLJ 0 /-2
12227	54 1 77777	ISK 1 77777 -
12228	75 0 12221	SLJ 0 /
12229	50 0 00000	ENI 0 0
12230	74 7 42040	EXF 7 42040
12231	75 4 71000	SLJ 4 71000
12232	50 0 00000	ENI 0 0

12240	75 4 71000	SLJ 4 71000
12241	50 0 00000	ENI 0 0
12242	01 0 13624	01 0 ANYISO
12243	04 0 00022	04 0 22
12244	12 0 12314	IDA 0 ZERO
12245	20 0 12514	STA 0 ROB
12246	12 1 12344	IDA 1 MU
12247	31 2 13000	FSB 2 ZEUS
12248	20 0 12513	STA 0 ERSIL
12249	32 0 12513	FMU 0 ERSIL
12250	32 3 12353	FMU 3 CAD
12251	30 0 12514	FAD 0 ROB
12252	20 0 12514	STA 0 ROB
12253	51 2 00003	INI 2 3
12254	51 3 00010	INI 3 10
12255	50 0 00003	ENI 0 0
12256	54 1 00006	ISK 1 6
12257	75 0 12243	SLJ 0 AVG
12258	12 0 12514	IDA 0 ROB
12259	33 0 12324	FDV 0 FLNG
12260	20 0 12514	STA 0 ROB
12261	50 2 00000	ENI 2 0

12253 50 3 00000 ENI 3 0
 50 1 00000 ENI 1 0
 12254 12 0 12514 LDA 0 ROB
 32 1 13500 FMI 1 INVER
 12255 20 2 12515 STA 2 AXE
 51 1 00004 INT 1 4
 12256 54 2 00002 ISK 2 2
 75 0 12234 SLJ 0 /-2
 12 0 12514 LDA 0 ROB
 32 0 13501 FMI 0 INVER+1
 20 0 12520 STA 0 COM02
 12 0 12514 LDA 0 ROB
 52 0 13505 FMI 0 INVER+5
 20 0 12521 STA 0 COM24
 12 0 13621 LDA 0 ANS+1
 30 0 13622 FAD 0 ANS+2
 20 0 12522 STA 0 A24
 32 0 12522 FMI 0 A24
 20 0 12523 STA 0 A242
 12 0 12522 LDA 0 A24
 33 0 13620 FDV 0 ANS
 20 0 12524 STA 0 GET
 32 0 12524 FMI 0 GET
 20 0 12524 STA 0 GET

NORMALIZES THE
 SIGMAS FOR A

12224 01 0 AN
 04 0 20
 12225 LDA 1 TTA
 FDV 0 ZEUS
 12226 STA 1 TTAN
 ENI 0 0
 12227 ISK 1 2
 75 0 12225 SLJ 0 /-2
 54 1 77777 ISK 1 77777
 75 0 12230 SLJ 0 /
 50 0 00000 ENI 0 0
 74 7 42040 EXF 7 42040
 75 4 71000 SLJ 4 71000
 50 0 00000 ENI 0 0
 12233 01 0 TTAN
 04 0 21
 12234 LDA 0 ZEUS+2
 FSB 0 ZEUS
 12235 FDV 0 ZEUS
 33 0 13000 STA 0 ANISO
 54 1 77777 ISK 1 77777
 75 0 12236 SLJ 0 /
 50 0 00000 ENI 0 0
 74 7 42040 EXF 7 42040

12302	32 0 12524	FMJ 0 GET
	20 0 13630	STA 0 FINIS
12303	54 1 7777	ISK 1 7777
	75 0 12303	SLJ 0 /
12304	50 0 00000	ENI 0 0
	74 7 42040	EXF 7 42040
12305	75 4 71000	SLJ 4 71000
	50 0 00000	ENI 0 0
12306--	01 0 13650	01 0 FINIS
	04 0 00925	04 0 25
12307	54 1 7777	ISK 1 7777
	75 0 12307	SLJ 0 /
12310	54 1 7777	ISK 1 7777
	75 0 12310	SLJ 0 /
12311	50 0 00000	ENI 0 0
	74 7 42040	EXF 7 42040
12312	74 0 42003	EXF 0 42003
	50 0 00000	ENI 0 0
12313	76 0 12000	SLS 3 12000
	50 0 00000	ENI 0 0
12314	00 0 00000	ZHO 0
	00 0 00000	
12315	00 0 00004	MIXE 0 0 4
	00 0 00001	0 0 1

12267	12 0 12522	LDA 0 A24
	32 0 13620	FMJ 0 ANS
12270	20 0 12522	STA 0 A24
	50 0 00000	ENI 0 0
12271	12 0 12521	LDA 0 CONV24
	32 0 12317	FMJ 0 FLTI
12272	30 0 12516	FAD 0 AXE+1
	30 0 12517	FAD 0 AXE+2
12273	33 0 12523	FDV 0 A242
	20 0 12525	STA 0 MIT
12274	12 0 12515	LDA 0 AXE
	33 0 13620	FDV 0 ANS
12275	33 0 13620	FDV 0 ANS
	30 0 12525	FAD 0 MIT
12276	20 0 12525	STA 0 MIT
	12 0 12520	LDA 0 CONV2
12277	30 0 12521	FAD 0 CONV24
	33 0 12522	FDV 0 A24
12300	32 0 12317	FMJ 0 FLTI
	20 0 12526	STA 0 MIT+1
12301	12 0 12525	LDA 0 MIT
	31 0 12526	FSB 0 MIT+1

12316	20 0 14000	FLT	DEC	1.0
	00 0 00000			
12317	20 0 24000	FLT1	DEC	2.0
	00 0 00000			
12320	20 0 26000	FLT2	DEC	3.0
	00 0 00000			
12321	20 0 44000	FLT3	DEC	8.0
	00 0 00000			
12322	20 0 57400	FLT4	DEC	30.0
	00 0 00000			
12323	20 0 64300	FLT5	DEC	55.0
	00 0 00000			
12324	20 0 34000	FLT6	DEC	4.0
	00 0 00000			
12325	20 0 14000	WORK1	DEC	1.0
	00 0 00000			
12326	00 0 00000	WORK2	BSS	7
	00 0 00000			
12335	00 0 00000	WORK4	BSS	7
	00 0 00000			
12344	17 5 34167	MU	DEC	.5046D-6
	17 3 10477			

12345	17 5 34103	DEC	.4925D-6
	21 2 75007		
12346	17 5 34222	DEC	.5109D-6
	23 0 76651		
12347	17 5 34276	DEC	.5211D-6
	06 7 01135		
12350	17 5 34522	DEC	.5556D-6
	22 1 24417		
12351	17 5 34601	DEC	.5667D-6
	75 2 36515		
12352	17 5 34751	DEC	.5907D-6
	02 3 02276		
12353	20 7 07231	CAD	.657462D-17
	17 2 43106		
12354	00 0 00000	OCT	0
	00 0 00000		
12355	00 0 00000	OCT	0
	00 0 00000		
12356	00 0 00000	OCT	0
	00 0 00000		
12357	00 0 00000	OCT	0
	00 0 00000		
12360	00 0 00000	OCT	0
	00 0 00000		

12374	00 0 00000	OCT 0
	00 0 00000	
12375	00 0 00000	OCT 0
	00 0 00000	
12376	00 0 00000	OCT 0
	00 0 00000	
12377	00 0 00000	OCT 0
	00 0 00000	
12400	00 0 00000	OCT 0
	00 0 00000	
12401	00 0 00000	OCT 0
	00 0 00000	
12402	00 0 00000	OCT 0
	00 0 00000	
12403	20 7 05176	DEC -4725897B+17
	27 1 03164	
12404	00 0 00000	OCT 0
	00 0 00000	
12405	00 0 00000	OCT 0
	00 0 00000	
12406	00 0 00000	OCT 0
	00 0 00000	
12407	00 0 00000	OCT 0
	00 0 00000	

12361	00 0 00000	OCT 0
	00 0 00000	
12362	00 0 00000	OCT 0
	00 0 00000	
12363	20 7 05367	DEC -4938271D+17
	05 2 01171	
12364	00 0 00000	OCT 0
	00 0 00000	
12365	00 0 00000	OCT 0
	00 0 00000	
12366	00 0 00000	OCT 0
	00 0 00000	
12367	00 0 00000	OCT 0
	00 0 00000	
12370	00 0 00000	OCT 0
	00 0 00000	
12371	00 0 00000	OCT 0
	00 0 00000	
12372	00 0 00000	OCT 0
	00 0 00000	
12373	20 7 05015	DEC -4526955D+17
	20 7 37137	

12452	17 7 26004	DEC	.023471
12453	31 0 11317	DEC	1.0
12454	20 0 14000	DEC	.8995
12455	00 0 00000	DEC	.684614
12456	20 0 07144	DEC	1.0
12457	26 4 16854	DEC	1.0
12460	20 0 05364	DEC	1.0
12461	13 3 47506	DEC	1.0
12465	20 0 14000	BSS	2
12472	00 0 00000	BSS	7
12501	00 0 00000	BSS	3
12504	00 0 00000	BSS	3

12437	20 0 14000	DEC	1.0
12440	00 0 00000	DEC	-.3995454
12441	60 0 11466	DEC	.1434268
12442	73 1 10267	DEC	1.0
12443	17 7 54455	DEC	-.125
12444	71 7 14727	DEC	-.289065
12445	20 0 14000	DEC	1.0
12446	00 0 00000	DEC	.250017
12447	60 0 23777	DEC	-.40624
12450	60 0 23777	DEC	1.0
12451	76 5 40722	DEC	.625
12452	20 0 14000	DEC	1.0
12453	00 0 00000	DEC	1.0
12454	17 7 54000	BSS	2
12455	10 7 23316	BSS	7
12456	60 0 11400	BSS	7
12457	05 1 74265	BSS	3
12458	20 0 14000	BSS	3
12459	00 0 00000	BSS	3

REM	FLORING POINT	SQUARE ROOT
12527	75 0 00000	SOEFL
	56 1 12552	SIU 1 SQEXIT
12530	22 0 12550	AJP 0 SQEXIT-2
	22 3 12527	AJP 3 SQEFL
12531	50 0 00000	EMI 0 0
	42 0 12556	SCM 0 SQMASK1
12532	05 0 00001	ALS 0 1
	03 0 00045	IRS 0 37D
12533	22 3 12534	AJP 3 /+1
	11 0 00002	INA 0 2
12534	11 0 03777	INA 0 3777
	03 0 00001	IRS 0 1
12535	05 0 00044	ALS 0 36D
	20 0 12561	STA 0 SQERAS
12536	44 0 12557	IDL 0 SQMASK2
	01 0 00004	ARS 0 4
12537	23 3 12540	QUP 3 /+1
	01 0 00001	ARS 0 1
12540	20 0 12562	STA 0 SQERAS+1
	14 0 12555	AID 0 SQFOFNS
12541	20 0 12563	STA 0 SQERAS+2
	12 0 12554	IDA 0 SQFOFNS2

12507	00 0 00000	AN	BSS	3
	00 0 00000			
12512	17 7 24357	CONV	DEC	.01745329252
	50 6 50451			
12513	00 0 00000	ERSIL	BSS	1
	00 0 00000			
12514	00 0 00000	ROB	BSS	1
	00 0 00000			
12515	00 0 00000	AXE	BSS	3
	00 0 00000			
12520	00 0 00000	COV02	BSS	1
	00 0 00000			
12521	00 0 00000	COV24	BSS	1
	00 0 00000			
12522	00 0 00000	A24	BSS	1
	00 0 00000			
12523	00 0 00000	A242	BSS	1
	00 0 00000			
12524	00 0 00000	GET	BSS	1
	00 0 00000			
12525	00 0 00000	MIT	BSS	2
	00 0 00000			

APPENDIX II

Solid Angle Correction

The solid angle correction, which must be applied to the theoretical correlation function to enable a comparison to be made with the experimental correlation function, is described by Rose (1). The correction involves the numerical evaluation of the integrals given by Equation (14):

$$I_{hl} = \int_0^{\theta} P_n(\cos\theta) (1 - e^{-\tau X_1(\theta)}) \sin\theta \, d\theta$$

and the subsequent solutions of Equations (15) and (16).

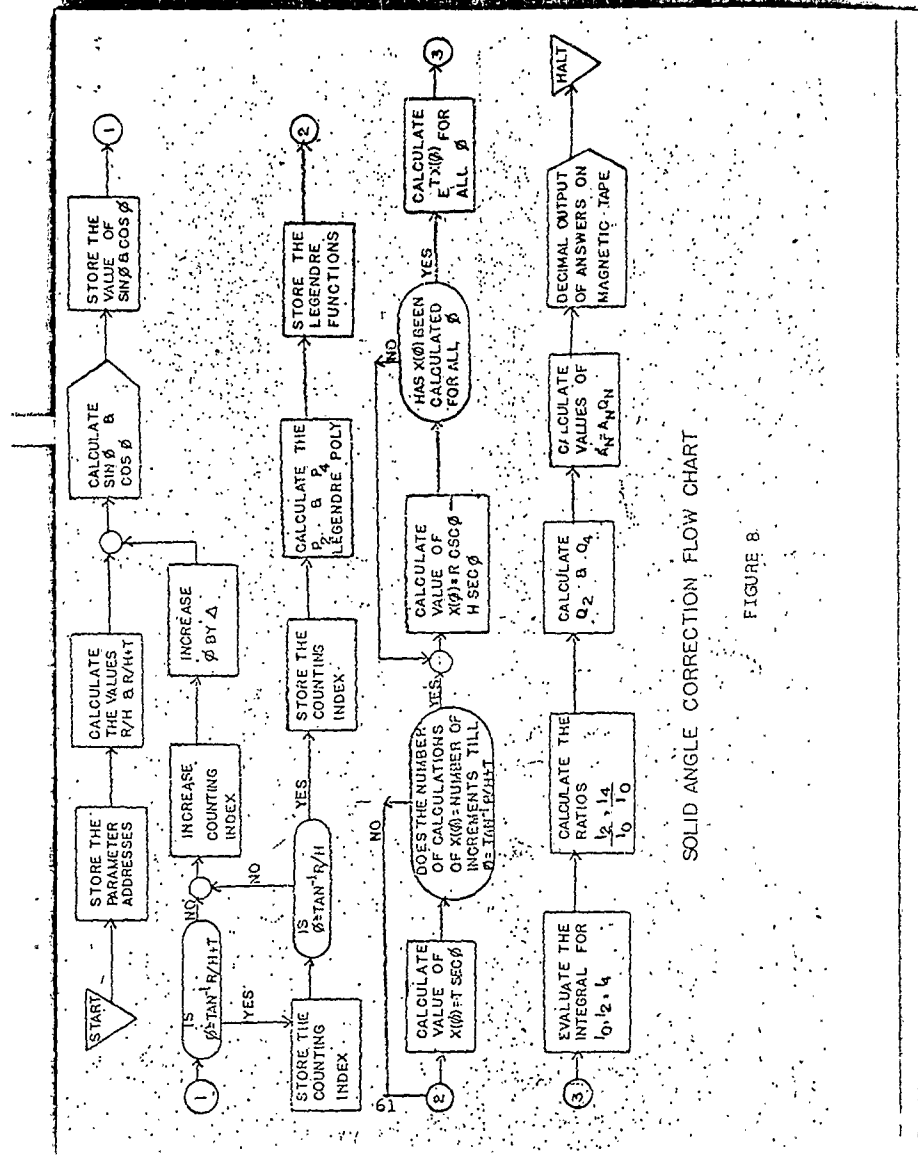
The computer solution for this correction was carried out in floating point format. The numerical integration was computed using an interval of 0.001 radians. A flow sheet outline of the program is given in Figure (8). The program must be supplied with the parameters τ , h , r , and t same in both channels (see Table 4) thus enabling solid angle corrections to be made for any experimental set-up to which these corrections may be applicable. These parameters may be placed anywhere in the computer and their addresses supplied to the program by placing them in the B registers in the following order

- B1 = x (address of the crystal radius in centimeters)
- B2 = t (address of the crystal thickness in centimeters)
- B3 = h (address of the crystal to source distance in centimeters)
- B4 = τ (the value of the full-energy absorption coefficient of the detector for the gamma ray in centimeters⁻¹)

The values of the parameters used in the calculation of the corrected angular correlation coefficients for the theoretical correlation function and the results of the calculations are listed in Table 4.

The program is available for future use on biocetal tape in machine language and on IBM cards in assembly language. The biocetal tape contains all the necessary subroutines to enable a complete computation, i.e.

1. The main program for solution of Equation (14).
2. The subroutines



SOLID ANGLE CORRECTION FLOW CHART

FIGURE 8

- a. Floating Point Sine-Cosine
- b. Floating Point Exponential
- c. Floating Point Decimal Output.

The results of the computations are dumped on magnetic tape unit (4) in the following sequence:

- Line 1 I_0
- Line 2 I_2
- Line 3 I_4
- Line 4 I_2/I_0
- Line 5 I_4/I_0
- Line 6 Q_2
- Line 7 $A_2 Q_2$
- Line 8 Q_4
- Line 9 $A_4 Q_4$

TABLE 4
SOLID ANGLE CORRECTION PARAMETERS

h (cm)	18.0
t (cm)	2.54
r (cm)	2.54
T (cm ⁻²)	0.098
I_0	0.0079
I_2	0.0078
I_4	0.0076
I_2/I_0	0.9882
I_4/I_0	0.9609
Q_2	0.9765
Q_4	0.9234
$Q_2 A_2$	0.0996
$Q_4 A_4$	0.0084

SOLID ANGLE CORRECTION PROGRAM

```

ORG 12000
REM BURTON J CONWAY APRIL 1961
REM SOLID ANGLE CORRECTIONS
REM OF THE THEORETICAL CORRELATION
REM CURVE. MUST BE SUPPLIED
REM WITH PARAMETER ADDRESSES AS
REM FOLLOWS.
REM B1 ADDRESS OF XTAL RADIUS.
REM B2 ADDRESS OF XTAL THICKNESS.
REM B3 ADDRESS OF CRYSTAL TO
REM SOURCE DISTANCE.
REM B4 ADDRESS OF PARAMETER TAU.

HOLD EQU 5500
TEMP EQU 6500
WORK2 EQU 7500
WORK4 EQU 10500
HANS EQU 11500
SOLD EQU 12500
JAY0 EQU 13500
JAY2 EQU 13504
JAY4 EQU 13510
JAY20 EQU 13514
    
```

```

JAY40 EQU 13520
EXFFL EQU 60600
SINFL EQU 60000
12000 LDA 1 0
20 0 LDA 0 RAD
12001 LDA 2 0
20 0 STA 0 THK
12002 LDA 3 0
20 0 STA 0 HGT
12003 LDA 4 0
20 0 STA 0 TAU
12004 LDA 0 ZRO
20 5 STA 5 JAY0
12005 ISK 5 30
75 0 SLJ 0 /-1
12006 LDA 0 ZRO
20 0 STA 0 DELTA
12007 ENI 1 0
50 2 ENI 2 0
12010 ENI 3 0
50 4 ENI 4 0
12011 ENI 5 0
50 6 ENI 6 0
    
```

12012	12 0 12163	START	LDA 0 RAD	FORMATION OF R/H
	33 0 12165		FDV 0 HGT	
12013	20 0 12172		STA 0 PLEAS	
	50 0 00000		ENI 0 0	
12014	12 0 12165		LDA 0 HGT	FORMATION OF R/H+T
	30 0 12164		FAD 0 THK	
12015	20 0 12175		STA 0 CRCE	
	12 0 12163		LDA 0 RAD	
12016	33 0 12175		FDV 0 CRCE	
	20 0 12175		STA 0 CRCE	
12017	12 0 12151	PGW	LDA 0 DELTA	SINE AND COSINE
	50 0 00000		ENI 0 0	FORMATION
12020	75 4 60001		SLJ 4 SINFLL+1	COSINE
	50 0 00000		ENI 0 0	
12021	50 0 00000		ENI 0 0	
	50 0 00000		ENI 0 0	
12022	50 0 00000		ENI 0 0	
	50 0 00000		ENI 0 0	
12023	20 1 06500		STA 1 TEMP	COSINE TO STORAGE
	12 0 12151		LDA 0 DELTA	
12024	75 4 60000		SLJ 4 SINFLL	SINE ROUTINE
	50 0 00000		ENI 0 0	
12025	50 0 00000		ENI 0 0	
	50 0 00000		ENI 0 0	

12026	50 0 00000		ENI 0 0	
	50 0 00000		ENI 0 0	
12027	20 1 06500		STA 1 HOLD	SINE TO STORAGE
	33 1 06500		FDV 1 TEMP	SINE/COSINE
12030	31 0 12175	REPL	FEB 0 CRCE	SINE/COSINE-R/H+T
	22 0 12141		AJP 0 FIRST	JUMPS IF A IS 0
12031	22 2 12141		AJP 2 FIRST	JUMPS IF A IS +
	51 1 00001		INI 1 1	
12032	12 0 12152		LDA 0 DELTA+1	
	30 0 12151		FAD 0 DELTA	
12033	20 0 12151		STA 0 DELTA	
	75 0 12017		SLJ 0 PGW	
12034	56 1 12105	NEXT	SIU 1 JAN+3	STARTS FORMATION OF
	56 1 12073		SIU 1 INCL+5	THE LEGENDS FOR-
12035	56 1 12111		SIU 1 JAN+3	
	56 1 12051		SIU 1 INCL+6	
12036	56 1 12074		SIU 1 JAN+3	
	56 1 12103		SIU 1 RMK	
12037	56 1 12115		SIU 1 JAN-1	
	50 1 00000		SIU 1 JANFR+3	
12040	12 1 06500		ENI 1 0	COSB IN A REG.
	32 1 06500		LDA 1 TEMP	
			FDV 1 TEMP	

12041	20 0 12173	STA 0 DON			
	32 0 12157	FMJ 0 FLT2			
12042	31 0 12155	FSB 0 FLT			
	33 0 12156	FDV 0 FLT1			P2 IN A REG.
12043	20 1 07500	STA 1 WORK2			
	12 0 12173	LDA 0 DON			
12044	32 0 12161	FMJ 0 FLT4			
	20 0 12174	STA 0 DON+1			
12045	12 0 12173	LDA 0 DON			
	32 0 12173	FMJ 0 DON			
12046	32 0 12162	FMJ 0 FLT5			
	31 0 12174	FEB 0 DON+1			
12047	30 0 12157	FAD 0 FLT2			P4 IN THE A REG.
	33 0 12160	FDV 0 FLT3			
12050	20 1 10500	STA 1 WORK4			
	50 0 00000	ENI 0 0			
12051	54 1 00000	RMK			NUMBER OF ANGLES
	75 0 12040	SLJ 0 NEXT+4			INSERTED BY PROGRAM
12052	12 0 12164	HALF			FORMATION OF XB
	33 1 06500	FDV 1 TEMP			
12053	20 1 11500	STA 1 HANS			
	50 0 00000	ENI 0 0			
12054	54 1 00000	ISK 1 0			NUMBER INSERTED
	75 0 12052	SLJ 0 HALF			BY PROGRAM

12055	12 1 11500	LDA 1 HANS			
	32 0 12166	FMJ 0 TAU			
12056	20 1 11500	STA 1 HANS			
	50 0 00000	ENI 0 0			
12057	54 1 00000	ISK 1 0			NUMBER ENTERED
	75 0 12055	SLJ 0 /-2			BY PROGRAM
12060	12 1 11500	POWER			STARTS FORMATION
	75 4 60600	SLJ 4 EXEFL			OF E TO THE TAU X
12061	50 0 00000	ENI 0 0			
	50 0 00000	ENI 0 0			
12062	20 1 12500	STA 1 SOLD			
	50 0 00000	ENI 0 0			
12063	54 1 00000	ISK 1 0			
	75 0 12060	SLJ 0 POWER			
12064	54 2 00001	ISK 2 1			
	75 0 12066	SLJ 0 /+2			
12065	75 0 12076	SLJ 0 HANG			
	50 0 00000	ENI 0 0			
12066	50 1 00000	INCL			
	51 1 00001	INI 1 1			
12067	12 0 12165	LDA 0 HGT			STARTS CALCULATION
	33 1 06500	FDV 1 TEMP			R CSCR-SECB

12070 20 1 12500 STA 1 SOLD
 12 0 12163 LDA 0 RAD
 33 1 05500 FDV 1 HOLD
 31 1 12500 FSB 1 SOLD
 32 0 12166 FMJ 0 TAU
 20 1 11500 STA 1 HANS
 54 1 00000 ISK 1 0
 75 0 12057 SLJ 0 /-4
 50 1 00000 ENI 1 0
 56 1 12063 STU 1 POWER+3
 50 1 00000 ENI 1 0
 75 0 12060 SLJ 0 POWER
 12 0 12155 HANG LDA 0 FLT
 33 1 12500 FDV 1 SOLD
 20 1 12500 STA 1 SOLD
 12 0 12155 LDA 0 FLT
 31 1 12500 FSB 1 SOLD
 20 1 12500 STA 1 SOLD
 54 1 00000 ISK 1 0
 75 0 12076 SLJ 0 HANG
 12 0 12153 JAY LDA 0 WORK
 32 1 12500 FMJ 1 SOLD
 32 1 05500 FMJ 1 HOLD
 32 0 12152 FMJ 0 DELTA+1

STARTS CALCULATION
 OF THE INTERVAL

12104 30 0 13500 FAD 0 JAYO
 20 0 13500 STA 0 JAYO
 54 1 00000 ISK 1 0
 75 0 12102 SLJ 0 /-3
 12 1 07500 JANTO LDA 1 WORK2
 32 1 12500 FMJ 1 SOLD
 32 1 05500 FMJ 1 HOLD
 32 0 12152 FMJ 0 DELTA+1
 30 0 13504 FAD 0 JAY2
 20 0 13504 STA 0 JAY2
 54 1 00000 ISK 1 0
 75 0 12106 SLJ 0 /-3
 12 1 10500 JAYFR LDA 1 WORK4
 32 1 12500 FMJ 1 SOLD
 32 1 05500 FMJ 1 HOLD
 32 0 12152 FMJ 0 DELTA+1
 30 0 13510 FAD 0 JAY4
 20 0 13510 STA 0 JAY4
 54 1 00000 ISK 1 0
 75 0 12112 SLJ 0 /-3
 12 0 13504 LDA 0 JAY2
 33 0 13500 FDV 0 JAYO

NUMBER OF TIMES
 TILL R/H

NUMBER OF TIMES
 TILL R/H

12133	54 5 00010	ISK 5 10
12134	75 0 12130	SLJ 0 /-3
12135	54 1 77777	ISK 1 77777
12136	75 0 12134	SLJ 0 /
12137	54 2 77777	ISK 2 77777
12140	75 0 12135	SLJ 0 /
12141	50 0 00000	ENI 0 0
12142	74 7 42040	EXP 7 42040
12143	74 0 42003	EXP 0 42003
12144	50 0 00000	ENI 0 0
12145	76 0 05000	SLS 0 5000
12146	50 0 00000	ENI 0 0
12147	12 0 12147	FIRST
12148	20 0 12030	STA 0 REFL
12149	12 0 12150	LDA 0 SEC+1
12150	20 0 12031	STA 0 REFL+1
12151	56 1 12054	SIU 1 HALF+2
12152	56 1 12057	SIU 1 HALF+5
12153	56 1 12063	SIU 1 POWER+3
12154	56 1 12075	SIU 1 INCL+7
12155	56 1 12066	SIU 1 INCL
12156	51 1 00001	INI 1 1

12117	20 0 13514	STA 0 JAY20
12118	12 0 13510	LDA 0 JAY4
12119	33 0 13500	FDV 0 JAY0
12120	20 0 13520	STA 0 JAY40
12121	12 0 13514	LDA 0 JAY20
12122	32 0 13514	FMJ 0 JAY20
12123	20 0 13524	STA 0 JAY40+4
12124	32 0 12167	FMJ 0 THEOR
12125	20 0 13530	STA 0 JAY40+10
12126	12 0 13520	LDA 0 JAY40
12127	32 0 13520	FMJ 0 JAY40
12128	20 0 13534	STA 0 JAY40+14
12129	32 0 12170	FMJ 0 THEOR+1
12130	20 0 13540	STA 0 JAY40+20
12131	54 1 77777	ISK 1 77777
12132	75 0 12126	SLJ 0 /
12133	50 0 00000	ENI 0 0
12134	74 7 42040	EXP 7 42040
12135	75 4 71000	SLJ 4 71000
12136	50 0 00000	ENI 0 0
12137	01 0 13500	01 0 JAY0
12138	04 0 00001	04 0 .
12139	12 0 12154	LDA 0 MIKE
12140	70 0 12131	RAD 0 /-1

MODIFIES THE PROGRAM
WHEN THE VALUE OF
B CHANGES

12146 75 0 12017 SLJ 0 EBN
 50 0 00000 ENI 3 0
 12147 51 0 12172 SEC FSB 0 PLEAS
 22 0 12034 AJP 0 NEXT
 12150 22 2 12034 AJP 2 NEXT
 51 1 00001 INI 1 1
 12151 00 0 00000 DELTA OCT 0
 00 0 00000
 12152 17 6 64061 DEC .001
 11 5 64570
 12153 20 0 14000 WORK DEC 1.0
 00 0 00000
 12154 00 0 00004 MIKE 0 0 4
 00 0 00001 0 0 1
 12155 20 0 14000 FLT DEC 1.0
 00 0 00000
 12156 20 0 24000 FLT1 DEC 2.0
 00 0 00000
 12157 20 0 26000 FLT2 DEC 3.0
 00 0 00000
 12160 20 0 14000 FLT3 DEC 8.0
 00 0 00000
 12161 20 0 57400 FLT4 DEC 30.0
 00 0 00000

PARAMETER TO MODIFY
 PROGRAM FOR B
 GREATER THAN R/H/T

12162 20 0 64300 FLT5 DEC 35.0
 00 0 00000
 12163 20 0 25050 RAD DEC 2.54
 75 3 41217
 12164 20 0 25050 THK DEC 2.54
 75 3 41217
 12165 20 0 54400 HGT DEC 18.0
 00 0 00000
 12166 17 7 46213 TAU DEC .098
 20 7 12601
 12167 17 7 46416 THEOR DEC .1020
 25 4 02050
 12170 17 7 14511 DEC .00907
 51 2 62577
 12171 00 0 00000 ZRO OCT 0
 00 0 00000
 12172 00 0 00000 PLEAS BSS 1
 00 0 00000
 12173 00 0 00000 DON BSS 2
 00 0 00000
 12175 00 0 00000 ONCE BSS 1
 00 0 00000
 END

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