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# DYNAMICS OF LIQUID-FILLED SHELL

## Aids for Designers:
## a) Milne's Graph, b) Stewartson's Tables

B. G. Karpov

RDT & E Project No. 1A010501B010

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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ABSTRACT

It is still impossible to predict with certainty the dynamic behavior of spinning liquid-filled shell under all firing conditions. However, the designer, instead of working in the dark, should use the available aids. The first of these is the well-known Milne's graph which summarizes a considerable amount of practical experience. The second is Stewartson's instability criterion. The use of the latter requires certain tables. These are provided. The use of both of these aids to the designer is explained. Illustrative examples are included.

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## 1. INTRODUCTION

The problem of predicting the dynamic behavior of liquid-filled spinning shell is still in a not very satisfactory state. A large amount of experimental data, obtained almost exclusively from range firings, indicate that the problem is of bewildering complexity. It is not surprising therefore, that thus far, no general principles of universal applicability have emerged in spite of considerable experimental and theoretical efforts.

The designers of shell for carrying liquids, therefore, are still largely in the dark as to whether their designs will fly well or poorly. However, there are two aids which can be used. One aid is in the form of a graph which delineates, as a function of certain parameters, the regions of stability and instability. This is the well-known Milne's(1) graph which essentially is a convenient summary of British experience with the flight characteristics of liquid-filled shell. Unfortunately, since this is an empirical correlation, the chances of success, even if the new design satisfies Milne's criterion, are unknown.

The second aid is Stewartson's(2)(3) instability criterion. The criterion is derived from an analysis of the dynamics of the spinning liquid-filled shell system. The liquid, contained in a cylindrical cavity, either completely or partially filled, is spinning with the full spin of the shell. The use of this criterion requires the knowledge of the poles and the residues at these poles of a certain very complicated mathematical expression. Although the tables of leading poles with associated residues were published by Stewartson in 1959 (Reference 3), his instability criterion does not appear to be in as wide use by designers as it should be.

The purpose of this report is to bring together these available aids. Of the two, Milne's graph and Stewartson's instability criterion, the latter should be given preference. Stewartson's mathematical model is more realistic in its approximation to the actual shell.

## 2. MILNE'S GRAPH

In 1940 E. A. Milne[1] made a thorough theoretical analysis, on the basis of a mathematical model developed earlier by other investigators, of the stability of a shell containing a spheroidal cavity completely filled with inviscid fluid, rotating with the full spin of the shell. The stability of such a system depends on the geometry of the cavity, i.e., the major and the minor axes of the spheroidal cavity, the mass of the liquid, and the inertial properties of the shell. This model led to a certain stability criterion. He showed that, for this model, the region of instability was defined by the following inequalities:

$$x > y, \quad x - y < 1 < x + y \tag{1}$$

where

$$x = \sqrt{\frac{K^2 - 1}{K^2 + 1} \cdot \frac{I_y}{I_x}} \tag{2}$$

$$y = \frac{2K}{K^2 + 1} \sqrt{\frac{(2/5)\, a^2\, M_f}{I_x}}$$

where $K$ = fineness ration of the cavity, $\frac{2c}{2a}$

$2a$ = cavity diameter

$2c$ = cavity length

$M_f$ = mass of the liquid

$I_x$, $I_y$ = polar and transverse moments of inertia of the shell.

When all the firings of liquid-filled shell, from 1926 onward, were analyzed and each shell was marked as to whether it went full range, S = stable, or fell short, U = unstable, and plotted in the (x,y) plane, it was found that a fairly well-defined curve could be drawn separating the S from the U points (see Figure 1). This curve, however, appeared to be only tangent to the theoretical boundary defining instability, $x - y = 1$ near $y = 0$, and was purely empirical elsewhere.

Since this graph was established on the basis of analysis of actual firings with a rather heterogeneous collection of shell, none of which probably had truly spheroidal cavities, one has to exercise considerable caution in using this graph as an aid to shell design. The only safe way is to follow Milne's handling of his data for computation of the coordinates x and y. Special attention should be given to evaluation of $I_x$, $I_y$, 2a and 2c.

Moments of Inertia

Because it was very difficult to reconstruct the actual moments of inertias of fired shell from their drawings, and since these were not usually measured prior to firings, Milne used the following approximate expressions:

$$I_x = 0.55\ Md^2 \qquad (3)$$

$$I_y = 0.25\ Md^2L^2$$

where M = weight of loaded shell

d = its exterior diameter

L = overall length of fuzed shell in calibers.

The use of these approximate relations had the virtue of consistency.

Using the above relations, the x and y coordinates become:

$$x = 0.67\ L \sqrt{\frac{K^2 - 1}{K^2 + 1}} \qquad (4)$$

$$y = \frac{2.7\ K}{K^2 + 1} \sqrt{\frac{(2/5)\ a^2\ M_f}{Md^2}}$$

Milne's graph with Equations (4) as coördinates is reproduced in Fig. 1.

Internal Length of the Cavity (2c)

In all cases Milne has taken the length of the cavity from the inside surface of the base to the lower (outer) surface of the base of the burster container, where a container was used, or to the lower surface of the piston or plug, if these were used. Cases where the cavity contained longitudinal partitions, a central burster tube extending through the whole cavity, half-frozen fillings, and nearly solid fillings were omitted. The diameter, 2a, has been taken to be the maximum diameter of the cavity.

The critical boundary given by Milne's empirical curve corresponds, in general, to a filling of about 90% of the total capacity of the cavity, or 10% air-spacing. This being the percentage air-spacing with which the majority of the trials were conducted.

Examples

a) 105mm chemical shell

$M = 34$ lbs

$K = 3.2$

$M_f = 2.8$ lbs (95% full); $\rho = 62.4$ lbs/ft$^3$

$a = 0.13$ ft

$d = 0.344$ ft

$L = 4.53$ calibers

$x = 2.75$

$y = 0.051$ $\therefore$ Stable

b) 20mm shell

$M = 84$ gms (total)

$K = 2.7$

$M_f = 21$ gms (90% full); $\rho = 3$ gms/cm$^3$

$a = 0.77$ cm

$d = 2$ cm

$L = 3.81$ calibers

$x = 2.22$

$y = 0.107$ $\therefore$ Stable

Thus both of these shell, according to Milne's graph, are stable. However, as we shall see later, both are unstable according to Stewartson's criterion.

The reader, however, is urged to study the original Milne's paper for a wealth of other details and for a fuller appreciation of the limitations of his empirical curve.

## 3. STEWARTSON'S CRITERION OF INSTABILITY

At the present time, Stewartson's theory(2)(3) offers the best available guide for the designer of liquid-filled shell for testing, a priori, the dynamic stability of his design. This theory has been reworked by various authors. Perhaps the most detailed exposition, and easiest to follow, is to be found in a paper by Ash and Gunderson(4).

Stewartson considers a shell containing a cylindrical cavity filled partially or completely with an inviscid fluid rotating as a rigid body with the full spin of the shell. The shell flies with a constant velocity and a constant spin. The essence of the treatment is the establishment of the effect of the stability of a stationary (i.e., time-independent) motion of the fluid with respect to small disturbances, that is, the effect of the natural modes of small oscillations of the hydrodynamical system on the stability of the shell itself.

The analysis leads to evaluation of the hydrodynamical moment which arises from pressure fluctuations on the shell cavity due to oscillating fluid. This moment, then, is added to the aerodynamic overturning moment in the dynamic equations of yawing motion of the shell. For simplicity, all other aerodynamic forces and moments are neglected. For mathematical details the reader should consult the cited references.

The transcendental equation whose roots give the free period of oscillation of the shell is shown to be of the form:

$$I_y \tau^2 - I_x \tau + \frac{I_x^2}{4 I_y s} = M_f c^2 F(\tau) \qquad (5)$$

where $I_x$, $I_y$ are the axial and transverse moments of inertia of the shell
$M_f$ is mass of the fluid
$2c$ is the height of the cavity
$s$ is the gyroscopic stability factor, and
$F(\tau)$ is a very complicated non-dimensional function of $\tau$, defined entirely by the dimensions of the cavity and the amount of fluid in it. The

function $F(\tau)$ has a double infinity of real, simple poles corresponding to the longitudinal and radial frequencies of fluid oscillations. In the above formulation, $\tau$ must be real for the shell to be stable.

Complete analysis of Equation (5) is prohibitively difficult. Therefore, in order to be able to make some useful inferences on the behavior of the system from Equation (5), it must be simplified. Fortunately this can be done without destroying the important properties of the system because, usually the right hand side (abbreviated R.H.S.) is small except, perhaps, at the poles.

One may consider the parent differential equation of the yawing motion from which the characteristic equation, Equation (5), was obtained. Then the R.H.S. can be considered as being in the nature of a forcing function whose influence on the motion is weak except at resonance. This is a well-known property of coupled oscillating systems.

The natural frequencies of the shell are given by the zeros of the left hand side (abbreviated L.H.S.) of Equation (5). These are:

$$\text{Nutational:} \quad \tau_n = \frac{1}{2}\frac{I_x}{I_y}(1 + \sigma) \tag{6a}$$

$$\text{Precessional:} \quad \tau_p = \frac{1}{2}\frac{I_x}{I_y}(1 - \sigma) \tag{6b}$$

where $\sigma = \sqrt{1 - \frac{1}{s}}$, and s is the usual gyroscopic stability factor

$$s = \frac{\bar{\nu}^2}{4M}$$

$$\bar{\nu} = \frac{I_x}{I_y}\frac{\omega d}{u}$$

$$M = \frac{\rho_a S d}{2m} k_t^{-2} \; C_{m_\alpha}$$

$\rho_a$ = air density

S = reference area, usually cross-sectional area

d = diameter

$m =$ mass

$k_t^{-2} = \frac{md^2}{I_y}$

$C_{m_\alpha}$ = aerodynamic overturning moment coefficient

Suppose, therefore, that there is a pole of the R.H.S., $\tau_o$, which is close to one of the natural frequencies of the shell, $\tau_n$ or $\tau_p$. We expand the R.H.S. in Laurent series and retain only the principal part containing the pole $\tau_o$. Equation (5) becomes

$$I_y \tau^2 - I_x \tau + \frac{I_x^2}{4I_y s} = \frac{D(\tau_o)}{\tau - \tau_o} + \text{small terms} \qquad (7)$$

where $D(\tau_o)$ is the residue.

According to our hypothesis, $\tau_o$ is close to either $\tau_n$ or $\tau_p$. Assume that $\tau_o \doteqdot \tau_n$. Since $\tau_n$ is the root of the L.H.S., we expand the L.H.S. in a Taylor's series in the vicinity of $\tau_n$. First let us write the L.H.S. in terms of its roots:

Let L.H.S. $= f(\tau) = \tau^2 - (\tau_n + \tau_p)\tau + \tau_n\tau_p$. Expanding $f(\tau)$ in the vicinity of $\tau_n$ and remembering that $f(\tau_n) = 0$ we have

$$f(\tau) \doteq f'(\tau_n)(\tau - \tau_n) + \dots\dots\dots$$

$$\doteq (\tau_n - \tau_p)(\tau - \tau_n) + \dots\dots\dots$$

or, since from Equation (6)

$$\tau_n - \tau_p = \frac{I_x}{I_y} \sigma$$

Equation (7) becomes:

$$I_x \sigma(\tau - \tau_n) = \frac{D}{\tau - \tau_o} \qquad (8)$$

which is quadratic in $\tau$. Since $\tau$ must be <u>real</u> for the shell to be stable, from (8) we see that for <u>instability</u> the discriminant of the quadratic must be negative, or

$$(\tau_o - \tau_n)^2 + \frac{4D}{I_x\sigma} < 0 \qquad (9)$$

Thus for <u>instability</u> due to resonance with the nutational frequency, the residue must be negative, i.e., $D < 0$.

Similarly, the shell is unstable due to resonance with the precessional frequency if $D > 0$ because the discriminant with $\tau_p$ is

$$(\tau_o - \tau_p)^2 - \frac{4D}{I_x\sigma} < 0 \qquad (10)$$

In actual practice it is found that the poles are real and $D < 0$ and hence only the nutational component of yaw may become divergent.

Therefore, returning to (9), for the shell to be <u>unstable</u> the following inequality must be satisfied

$$|\tau_o - \tau_n| < \left(\frac{-4D}{I_x\sigma}\right)^{\frac{1}{2}}$$

It is found convenient to redefine D.

Let $\rho \frac{a^6R^2}{c} = -D$

where R is a positive real number which depends, like the residues at the poles, only on c/a, b/a.

$$\therefore |\tau_o - \tau_n| < \left(\frac{4\rho a^6R^2}{I_x\sigma c}\right)^{\frac{1}{2}}$$

or

$$-1 < \frac{(\tau_o - \tau_n)}{2R}\left(\frac{I_x\sigma}{\rho a^5} \cdot \frac{c}{a}\right)^{\frac{1}{2}} < 1 \qquad (11)$$

c/a is the fineness ratio of the cavity and $\rho$ is the density of the liquid, all in consistent units. In practice, therefore, for a given air space in the cavity, $b^2/a^2$, or the fill ratio, $1 - b^2/a^2$, we should locate all the poles, $\tau_o$'s, which are close $\tau_n$, and compute the corresponding residues. The test Equation (11) should indicate whether the shell, at prescribed conditions of loading, will be stable or unstable.

It should be noted that G. N. Ward in testing Stewartson's theory with a gyrostat[3] has found the limits for instability to be considerably broader, namely,

$$-3.9 < \frac{(\tau_o - \tau_n)}{2R} \left( \frac{I_x \sigma}{\rho a^5} \cdot \frac{c}{a} \right)^{\frac{1}{2}} < 2.7 \tag{12}$$

No explanation for this discrepancy has yet been found. For the designer, therefore, it might be safer to use the above limits rather than the theoretical ones of $\pm$ 1.

## 4. STEWARTSON'S TABLES

In order to use Stewartson's instability criterion, Equation (11) or Equation (12), it is necessary to know the poles of Equation (5) and corresponding residues. These are functions of the geometry of the cavity and the amount of liquid in it. The geometry of the cavity is characterized by its height, 2c, and its diameter 2a. If the cavity is partially filled, the liquid geometry is assumed to be that of a hollow concentric cylinder with an inner diameter of 2b. Therefore, the fraction of the cavity occupied by the air space is $b^2/a^2$. And the fraction of the cavity filled with the liquid is $1 - b^2/a^2$. The percent of fill is $100(1-b^2/a^2)$.

In reference (3) Stewartson gives tables of the leading poles with corresponding residues for various $b^2/a^2$ ratios at intervals of 0.2 or 20% fill ratios. Since, in practice, the nutational frequency of the shell, $\tau_n$, usually does not exceed 0.2, the tabular values of $\tau_o$ do not go beyond 0.2.

However, in certain shell designs we have encountered $\tau_n > 0.2$. Moreover, in using Stewartson's tables, we have found the air space intervals of 0.2 are too large for convenience. Closer spacing appeared to be desirable.

This prompted us to recompute the tables using intervals for $b^2/a^2$ of .05 from 0 to 0.40 and extending $\tau_o$ to 0.50. These ranges should cover most practical needs. For greater air space or lower fill ratios, referenced tables could be used.

The tables were computed by Mr. E. L. Kessler of Budd Company using an IBM 1620 computer. He is to be congratulated for very effective programming, debugging, computing, and checking these tables, all in a relatively short time.

Each table is computed for a specific air ratio, $b^2/a^2$ at .05 intervals. For the range of $\frac{b^2}{a^2}$ from .00 to .05 the intervals are shorter. This was done because of non-linear behavior of the residues, 2R vs. $\tau_o$ at higher values of $\tau_o$ and small air space ratios $b^2/a^2$. Thus linear interpolation within the first 5% interval of $b^2/a^2$ would have been invalid.

The physical significance of the quantity $\frac{c/a}{2j + 1}$, which appears in the tables, is that it is associated with the spatial variation of the longitudinal cosine wave. Thus, for $j = 0$ we have one half of the full cosine wave in the cavity; $j = 1$, we have 3/2 waves; $j = 2$, 5/2 waves, and so on.

In each table the poles, $\tau_o$, are tabulated on the left followed by three column pairs of the quantity $\frac{c/a}{2j + 1}$ and corresponding residues, 2R . Only the leading poles with associated residues are tabulated. It is believed that the longest spatial waves both in the axial and the radial directions (lower j values) are the most important. These long waves produce greater pressure asymmetries in the cavity and hence, larger hydrodynamic moments.

## 5. USE OF TABLES

To illustrate the use of the tables, we may consider two cases: a) new shell, and b) existing shell. For the new shell it is important to know, *a priori*, the cavity dimensions which will give rise to resonance between the fluid oscillations and the nutational frequency of the shell. The designer, therefore, should avoid these dimensions. If the existing shell is being adapted for carrying liquids, it is important to inquire whether its cavity is such as to lead to resonance and, presumably, bad flight.

Consider the case of a new shell. The designer should estimate the moments of inertia of the empty shell, $I_x$ and $I_y$, and the gyroscopic stability factor, s, from which to compute $\sigma$. With these he computes the nutational frequency

$$\tau_n = \frac{1}{2} \frac{I_x}{I_y} (1 + \sigma)$$

It might be adequate, at this stage, to use approximate formulae for $I_x$ and $I_y$ as given, for example, by Hitchcock[5].

$$I_x = 0.14\ md^2$$

$$I_y = 0.5\ I_x + 0.0594\ mL^2$$

where m = mass of the shell

d = diameter

L = overall length

These approximations appear to be much better than those used by Milne.

The gyroscopic stability factor

$$s = \frac{\bar{\nu}^2}{4M}$$

where $$\bar{\nu} = \frac{I_x}{I_y} \frac{2\pi}{n},$$

and n = twist of rifling in calibers per turn. M can be written

$$M = \frac{\rho_a S d}{2m} K_t^{-2} (cp - cg)\ C_{N_\alpha}$$

where cp = center of pressure, in calibers

cg = center of gravity, in calibers

$C_{N_\alpha}$ = normal force coefficient

cp and $C_{N_\alpha}$ depend only on the exterior shape of the shell and, hence, remain invariant with the changes in the cavity dimensions.

Let us consider, as an example, the 105mm chemical shell for which we have either estimated or measured the following characteristics:

105mm Chemical Shell

$$I_x = 0.56 \text{ lbs ft}^2$$

$$I_y = 5.56 \text{ lbs ft}^2$$

$$s = 1.2; \quad \sigma = .41$$

$$\therefore \quad \tau_n = 0.07$$

The geometry of the cavity is right circular cylinder with the diameter $a = 0.13$ ft, and with the fineness ratio, c/a, so selected as to avoid resonance condition. The cavity is to be filled to 95% by a liquid of density $\rho = 62.4 \text{ lbs/ft}^3$.

The problem is to find which fineness ratios will lead to resonance and, hence, are to be avoided.

To do this, we turn to the table for $b^2/a^2 = .05$ (95% full cavity) and locate on the line $\tau_o = \tau_n = 0.07$ the corresponding values of $\frac{c/a}{2j + 1}$ and associated residue, 2R, in each of the three column pairs. We find the following:

| For $\tau_o = .07$ | $\frac{c/a}{2j+1}$ | 2R |
|---|---|---|
| 1st column pair | $1.078^5$ | .212 |
| 2nd column pair | $.509^5$ | .0252 |
| 3rd column pair | .320 | .0067 |

Therefore, the resonance will occur at the following c/a values:

$$(c/a)_1 = 1.078^5 \quad (2j + 1)$$
$$(c/a)_2 = .509^5 \quad (2j + 1)$$
$$(c/a)_3 = .320 \quad (2j + 1)$$

| j | $(c/a)_1$ | $(c/a)_2$ | $(c/a)_3$ |
|---|---|---|---|
| 0 | 1.08 | (.51) | (.32) |
| 1 | 3.24 | 1.53 | .96 |
| 2 | 5.39 | 2.55 | 1.60 |
| 3 | | 3.57 | 2.24 |
| 4 | | 4.59 | 2.88 |
| 5 | | | 3.52 |
| 6 | | | 4.16 |

It is to be noted that, for simplicity, we have assumed that the value of the nutational frequency, $\tau_n$, remained constant while we changed c/a. In practice, of course, $\tau_n$ will change because $I_x$, $I_y$ and $\sigma$ all will change, albeit slowly, with changes in c/a. But these can be taken into account at more detailed examination of the situation in the vicinity of the desired c/a ratio. The present rough survey stakes out only the danger areas.

The above table shows a fairly large number of fineness ratios which are to be avoided in order to escape resonance. However, the situation is not as bad as it looks. The third column, $(c/a)_3$, contains a greater number of entries. But because of the very small residues associated with this column, the coincidence of the actual value of c/a with tabulated values must be very precise for resonance to occur. This can be shown as follows. Using the residues given above, we compute the quantity $\frac{1}{2R}\left(\frac{I_x \sigma}{\rho a^5}\right)^{\frac{1}{2}}$ for each column pair. Equation (12) can be re-written as:

$$\text{1st} \qquad -3.9 < 50\sqrt{\frac{c}{a}}\left[\left(\frac{c}{a}\right)_1 - \left(\frac{c}{a}\right)\right] < 2.7$$

$$\text{2nd} \qquad -3.9 < 400\sqrt{\frac{c}{a}}\left[\left(\frac{c}{a}\right)_2 - \left(\frac{c}{a}\right)\right] < 2.7$$

$$\text{3rd} \qquad -3.9 < 1500\sqrt{\frac{c}{a}}\left[\left(\frac{c}{a}\right)_3 - \left(\frac{c}{a}\right)\right] < 2.7$$

Where $\left(\frac{c}{a}\right)_i$ are the tabulated values as given above, for each column pair, and $\left(\frac{c}{a}\right)$ the actual designed value. It is clear that in order to satisfy the above instability conditions the difference in the vicinity of tabulated fineness ratios and the actual designed values must be very small for the 3rd column, less so for the 2nd column, and still less critical for the 1st column. In practice, therefore, the coincidence with tabulated values in the 3rd column is likely to be purely fortuitous because one usually cannot design the cavity with the required precision. The fineness ratios appearing in the 1st column are the most important and should, therefore, be avoided.

For the second case, we may consider the same 105mm chemical shell. Let us suppose that the fineness ratio of its cavity is $\frac{c}{a} = 3.2$. We have found already that this fineness ratio should be avoided. But, as an illustrative example, we shall proceed with the analysis of this case.

$$\text{Let } A \equiv \frac{I_x \sigma}{\rho a^5} \cdot \frac{c}{a} \simeq \frac{(.56)(.41)}{(62.4)(.37 \times 10^{-4})} \; 3.2 = 317.1$$

$$\therefore \; \sqrt{A} = 17.8$$

Condition for instability, therefore, Equation (12), is:

$$-3.9 < \frac{\tau_o - .07}{2R} \; (17.8) < 2.7$$

$$\text{Let } B \equiv \frac{\tau_o - \tau_n}{2R} \sqrt{A}$$

Since the cavity is to be filled to 95% we use the table for $b^2/a^2 = .05$.

| Compute: | | From Table $b^2/a^2 = .05$ | | |
|---|---|---|---|---|
| j | $\frac{c/a}{2j+1}$ | $\tau_o$ | 2R | B |
| 0 | 3.2 | - | - | - |
| 1 | 1.067 | .064 | .194 | -.55 |
| 2 | 0.640 | .23 | .104 | 26.4 |
| 3 | 0.457 | .306 | .0457 | 92 |
| 4 | 0.356 | .146 | .0162 | 83 |
| 5 | 0.291 | - | .0182 | - |
| 6 | 0.246 | - | .0036 | - |

The results show that for this cavity, c/a = 3.2, and 95% fill the shell is predicted to be unstable for j = 1.

The hydrodynamic moment is proportional to the residues at the poles. Hence, only the leading poles, lower j values and associated larger residues, are the most important.

One of the possible remedies is to try to change the geometry of the cavity or, more specifically, the fineness ration c/a in the vicinity of 3.2. The following table illustrates the effect of such a change. For this illustration again, the inertial properties of the shell were kept constant and only c/a changed.

<u>The Effect of Changing c/a on Stability</u>. 105mm Shell

| | Values of "B" | | | | |
|---|---|---|---|---|---|
| j c/a = | 2.8 | 3.0 | 3.2 | 3.4 | 3.6 |
| 1 | 22 | -42 | -.55 | 2.3 | 3.2 |
| 2 | 21 | 26 | 26 | 28 | 29 |
| 3 | 89 | 90 | 92 | 72 | 5 |
| 4 | -73 | 51 | 83 | 97 | 101 |
| 5 | | | | 153 | 37 |

Thus, this shell is unstable for c/a = 3.2, as previously shown, but also is dangerously close to instability for values of c/a up to 3.6. For c/a = 3.57 for example, it is unstable for j = 3 for which the "B" value is -.76.

Another possible remedy is to alter the air space or fill ratio. The following table conveys some sense of sensitivity of instability condition to various air spaces.

<u>The Effect of Changing Air Space on Stability.</u> 105mm Shell

$$c/a = 3.2, \quad j = 1, \quad \frac{c/a}{2j + 1} = 1.067$$

| $b^2/a^2$ | "B" |
|---|---|
| .00 | -.87 |
| .02 | -.87 |
| .05 | -.55 |
| .10 | .16 |
| .15 | 1.04 |
| .20 | 1.8 |

With the fineness ratio of the cavity of 3.2, the shell is unstable for fill ratios from 80% to 100%. For this shell, therefore, with a fineness ratio of the cavity of 3.2, changing the loading conditions is not an effective means of remedying a bad situation.

Another example:

| 20mm shell | | 90% full; $b^2/a^2 = 0.10$ | |
|---|---|---|---|
| $I_x$ | 42.4 gms-cm$^2$ | $\tau_n$ = | 0.155 |
| $I_y$ | 251.6 gms-cm$^2$ | c/a | 2.68 |
| $\sigma$ | 0.84 | a | 0.77 cm |
| $\rho$ | 3 gms/cm$^3$ | | |

$$\therefore A = \frac{42.4(.84)}{3(.271)} 2.7 = 118.3$$

and instability criteria becomes:

$$-3.9 < \frac{\tau_o - \tau_n}{2R} \; 10.9 < 2.7$$

| $j$ | $\frac{c/a}{2j+1}$ | $\tau_o$ | $2R$ | $B$ |
|---|---|---|---|---|
| 0 | 2.68 | - | - | - |
| 1 | .893 | .44 | .2683 | 11.6 |
| 2 | .536 | .145 | .0523 | -2.11 |
| 3 | .383 | .26 | .0305 | .38 |
| 4 | .298 | .08 | .0065 | -126 |
| 5 | .244 | - | - | - |

The shell, therefore, is stable if one uses Stewartson's limits of $\pm$ 1 but is unstable for j = 2, if one uses Ward's limits.

It is to be recalled that by using Milne's graph, both shell appeared to be stable whereas Stewartson's criteria indicates instability for both.

6. NUTATIONAL FREQUENCY

The nutational frequency of the shell is given by the expression

$$\tau_n = \frac{1}{2} \; \frac{I_x}{I_y} \; (1 + \sigma)$$

The question arises: Should the transverse moment of inertia of the shell, $I_y$, be augmented by the contribution due to the liquid? For a completely filled cavity, with non-spinning inviscid liquid, the theory shows that a fraction of the liquid's transverse moment of inertia, regarded as a rigid body, should be added to the $I_y$ of an empty shell. In other words the total $I_y$ of the system should be written as:

$$I_y = (I_y)_s + \alpha I'_y$$

where $(I_y)_s$ is that of the empty shell, $I'_y$ is that of the liquid regarded as a rigid body, and $\alpha$ is a factor less than unity. Reference (6) gives a curve of $\alpha$ as a function of 2c/a for a non-spinning liquid.

For a spinning liquid such a curve has not yet been computed. However, the 20mm firings in the aerodynamics range (BRL) provide some experimental evidence to the value of $\alpha$. In the following cases the geometry of the liquid was assumed to be known.

| Liquid | No. Rounds | % Fill | $\alpha$ |
|---|---|---|---|
| $H_2O$ | 6 | 100 | .37 |
| Glycol | 1 | 90 | .20 |
| Glycerine | 2 | 90 | .22 |
| | 6 | 70 | .43 |
| | | Average | 0.3 |

Experimental evidence, therefore, suggests that it might be advisable to include the contribution of the liquid to the transverse moment of inertia of the system. The nutational frequency, therefore, should be computed as:

$$\tau_n = \frac{1}{2} \frac{I_x}{(I_y + 0.3 I'_y)} (1 + \sigma)$$

where $I'_y$ should be computed as if the liquid was rigid and, for a partially filled cavity, be regarded as a hollow concentric cylinder of inner diameter 2b, or

$$I'_y = \frac{1}{3} m_o a^2 \left(1 - \frac{b^2}{a^2}\right) \left[\frac{3}{4}\left(1 + \frac{b^2}{a^2}\right) + \left(\frac{c}{a}\right)^2\right]$$

where $m_o$ is the mass of the liquid completely filling the cavity.

## 7. ADDITIONAL REMARKS

Stewartson's theory provides the best available assistance for judging, a priori, the dynamic behavior of the liquid filled, spin stabilized, shell. However, its limitations should be kept in mind.

The theory is developed for an inviscid fluid, fully spinning, which is contained in a cylindrical cavity. The fluid is subject to no body forces and the shell is moving with a constant velocity and a constant axial spin.

All aerodynamic forces acting on the shell are neglected with the exception of the overturning moment. It is assumed, of course, that the shell has a sufficient spin to be gyroscopically stable. The mathematical model, therefore, is reasonably close to reality.

This theory has been tested in Britain by range-firing shell with specified cavities so as to give predicted stable and unstable flights. Although limited, the experiments were 100% successful. The 20mm firings in the aerodynamics range of the BRL[7], using very viscous liquids in order to achieve full spin close to the muzzle, also appeared to verify Stewartson's predictions.

However, it was found that with less viscous liquids, for which appreciable time is required to reach full spin, the 20mm shell was unstable, during this transition period, for practically all fill conditions of practical interest. The severity of instability was roughly proportional to the specific gravity of the liquid. For predicted stable flight with a liquid of specific gravity 3 and viscosity 3 centistokes, this initial instability was sufficient to render the shell useless in practice at all loading conditions. However, a lighter liquid, of specific gravity 1.2, and the same viscosity, 3 centistokes, produced a reasonably good flight, i.e., the initial instability, or rate of divergence of yaw, was sufficiently mild not to cause large yaws to develop. The moral of this story is that with heavier liquids the problem of transitional instability may be much more severe than with lighter fluids.

The initial instability can be overcome with baffles. However, for later flight the natural frequencies of the fluid in a baffled cavity must be known in order to avoid possible resonance. This problem has not yet been solved.

The real cavities in actual shell are seldom cylindrical. The natural frequencies of the liquid, however, are functions of the geometry of the cavity. The knowledge of these, as shown by Stewartson, is of utmost importance for stability analysis. The question arises how non-cylindrical cavity modifies these frequencies, and how good is the cylindrical approximation? This also is unknown.

Neglect of all aerodynamic forces with the exception of overturning moment is justifiable. Since we are dealing with the linearized system of equations, the undamping due to liquid is additive to the aerodynamic damping.

With these limitations in mind, the designer still will profit by the use of Stewartston's instability criterion. Some of the points mentioned above will, in time, be clarified by further theoretical and experimental work. Meanwhile, to avoid swimming in a sea of uncertainty, the designer may help himself considerably if he insists on designing liquid-carrying shell with cylindrical cavities. If the mission of the shell is important, this mild design limitation is fully justifiable.

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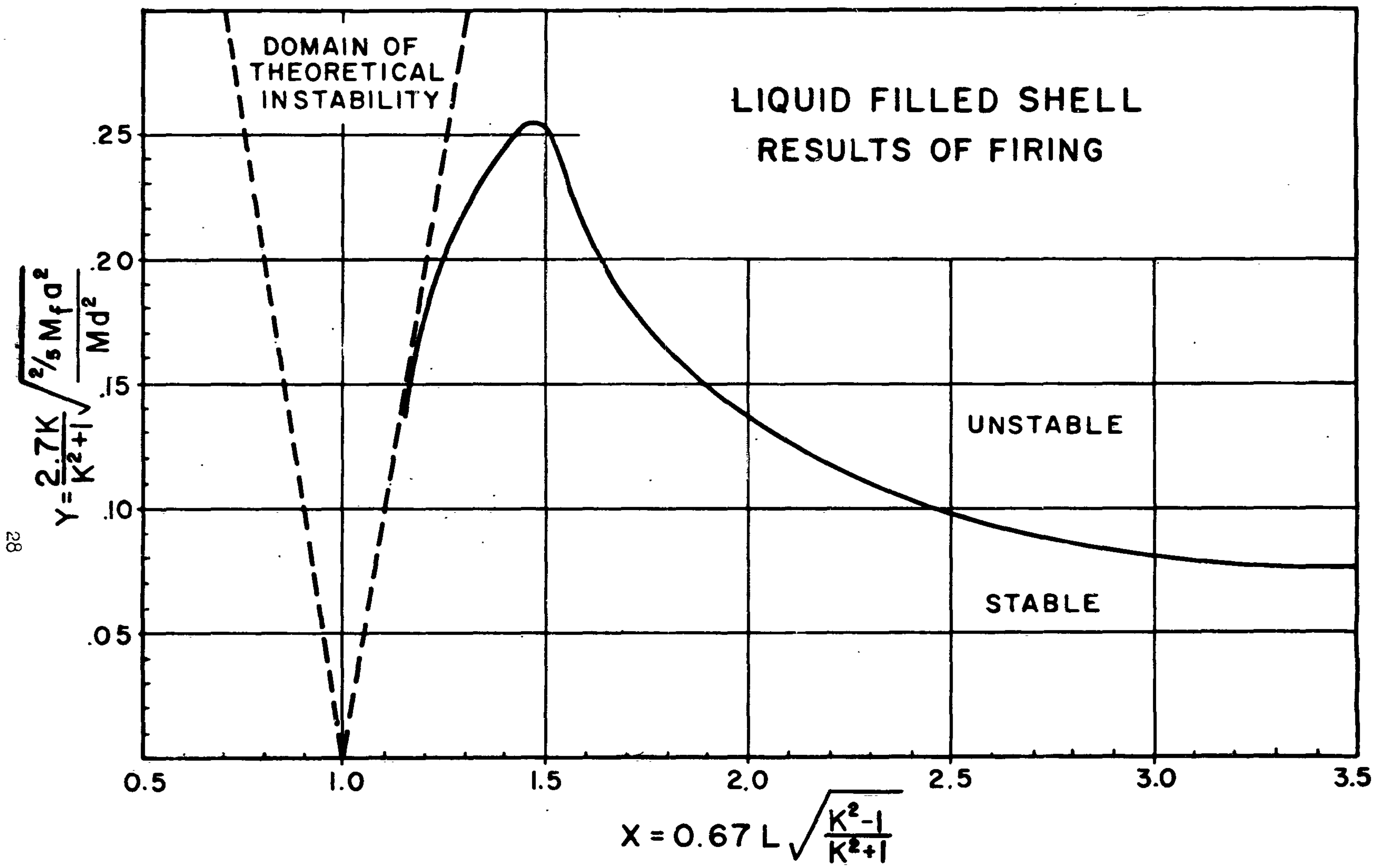


FIG. 1 MILNE'S GRAPH

## 9. STEWARTSON'S TABLES OF POLES AND RESIDUES

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|---|---|
| 0.00 | 0.995 | 0.000 | 0.478 | 0.0000 | 0.310 | 0.0000 |
| 0.02 | 1.018 | 0.058 | 0.490 | 0.0070 | 0.319 | 0.0019 |
| 0.04 | 1.042 | 0.118 | 0.503 | 0.0144 | 0.327 | 0.0040 |
| 0.06 | 1.066 | 0.181 | 0.516 | 0.0223 | 0.336 | 0.0062 |
| 0.08 | 1.091 | 0.246 | 0.530 | 0.0307 | 0.345 | 0.0086 |
| 0.10 | 1.117 | 0.313 | 0.544 | 0.0396 | 0.355 | 0.0111 |
| 0.12 | 1.144 | 0.382 | 0.559 | 0.0491 | 0.364 | 0.0139 |
| 0.14 | 1.172 | 0.454 | 0.574 | 0.0591 | 0.375 | 0.0168 |
| 0.16 | 1.201 | 0.528 | 0.590 | 0.0697 | 0.385 | 0.0198 |
| 0.18 | 1.231 | 0.604 | 0.607 | 0.0809 | 0.397 | 0.0231 |
| 0.20 | 1.262 | 0.682 | 0.624 | 0.0928 | 0.408 | 0.0266 |
| 0.22 | 1.294 | 0.762 | 0.642 | 0.1054 | 0.420 | 0.0304 |
| 0.24 | 1.328 | 0.845 | 0.661 | 0.1187 | 0.433 | 0.0344 |
| 0.26 | 1.363 | 0.930 | 0.680 | 0.1328 | 0.446 | 0.0387 |
| 0.28 | 1.399 | 1.017 | 0.700 | 0.1478 | 0.460 | 0.0433 |
| 0.30 | 1.437 | 1.107 | 0.722 | 0.1636 | 0.475 | 0.0481 |
| 0.32 | 1.478 | 1.200 | 0.745 | 0.1804 | 0.490 | 0.0533 |
| 0.34 | 1.521 | 1.295 | 0.769 | 0.1981 | 0.506 | 0.0589 |
| 0.36 | 1.565 | 1.392 | 0.794 | 0.2169 | 0.523 | 0.0649 |
| 0.38 | 1.612 | 1.491 | 0.820 | 0.2369 | 0.541 | 0.0714 |
| 0.40 | 1.662 | 1.593 | 0.848 | 0.2581 | 0.561 | 0.0783 |
| 0.42 | 1.715 | 1.698 | 0.878 | 0.2805 | 0.582 | 0.0858 |
| 0.44 | 1.771 | 1.805 | 0.910 | 0.3043 | 0.603 | 0.0938 |
| 0.46 | 1.831 | 1.914 | 0.944 | 0.3296 | 0.626 | 0.1024 |
| 0.48 | 1.895 | 2.026 | 0.980 | 0.3566 | 0.651 | 0.1118 |
| 0.50 | 1.963 | 2.142 | 1.019 | 0.3853 | 0.678 | 0.1220 |

$b^2/a^2 = 0.00$

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|---|---|
| 0.00 | 0.995 | 0.000 | 0.477 | 0.0000 | 0.309 | 0.0000 |
| 0.02 | 1.018 | 0.058 | 0.490 | 0.0070 | 0.317 | 0.0019 |
| 0.04 | 1.042 | 0.118 | 0.503 | 0.0144 | 0.325 | 0.0040 |
| 0.06 | 1.066 | 0.181 | 0.516 | 0.0222 | 0.334 | 0.0062 |
| 0.08 | 1.091 | 0.246 | 0.530 | 0.0305 | 0.343 | 0.0086 |
| 0.10 | 1.117 | 0.313 | 0.544 | 0.0393 | 0.353 | 0.0111 |
| 0.12 | 1.144 | 0.382 | 0.559 | 0.0486 | 0.363 | 0.0138 |
| 0.14 | 1.172 | 0.453 | 0.574 | 0.0584 | 0.373 | 0.0167 |
| 0.16 | 1.201 | 0.527 | 0.590 | 0.0688 | 0.384 | 0.0198 |
| 0.18 | 1.232 | 0.603 | 0.607 | 0.0798 | 0.395 | 0.0231 |
| 0.20 | 1.263 | 0.681 | 0.625 | 0.0915 | 0.407 | 0.0266 |
| 0.22 | 1.296 | 0.761 | 0.643 | 0.1038 | 0.420 | 0.0304 |
| 0.24 | 1.330 | 0.844 | 0.662 | 0.1168 | 0.433 | 0.0345 |
| 0.26 | 1.366 | 0.929 | 0.682 | 0.1305 | 0.447 | 0.0388 |
| 0.28 | 1.404 | 1.016 | 0.704 | 0.1450 | 0.461 | 0.0434 |
| 0.30 | 1.443 | 1.105 | 0.727 | 0.1603 | 0.477 | 0.0484 |
| 0.32 | 1.485 | 1.196 | 0.751 | 0.1765 | 0.493 | 0.0538 |
| 0.34 | 1.529 | 1.289 | 0.776 | 0.1935 | 0.510 | 0.0596 |
| 0.36 | 1.576 | 1.384 | 0.803 | 0.2114 | 0.529 | 0.0658 |
| 0.38 | 1.626 | 1.481 | 0.832 | 0.2301 | 0.549 | 0.0724 |
| 0.40 | 1.680 | 1.578 | 0.864 | 0.2496 | 0.571 | 0.0796 |
| 0.42 | 1.737 | 1.676 | 0.898 | 0.2699 | 0.595 | 0.0874 |
| 0.44 | 1.799 | 1.772 | 0.936 | 0.2905 | 0.621 | 0.0958 |
| 0.46 | 1.868 | 1.864 | 0.978 | 0.3110 | 0.650 | 0.1050 |
| 0.48 | 1.945 | 1.945 | 1.026 | 0.3305 | 0.683 | 0.1150 |
| 0.50 | 2.034 | 2.003 | 1.083 | 0.3475 | 0.721 | 0.1262 |

$b^2/a^2 = 0.01$

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|---|---|
| 0.00 | 0.994 | 0.000 | 0.475 | 0.0000 | 0.305 | 0.0000 |
| 0.02 | 1.017 | 0.058 | 0.488 | 0.0069 | 0.313 | 0.0019 |
| 0.04 | 1.041 | 0.118 | 0.501 | 0.0143 | 0.322 | 0.0039 |
| 0.06 | 1.066 | 0.180 | 0.514 | 0.0220 | 0.331 | 0.0061 |
| 0.08 | 1.091 | 0.245 | 0.528 | 0.0302 | 0.340 | 0.0085 |
| 0.10 | 1.117 | 0.312 | 0.542 | 0.0389 | 0.349 | 0.0110 |
| 0.12 | 1.144 | 0.381 | 0.557 | 0.0480 | 0.359 | 0.0136 |
| 0.14 | 1.173 | 0.453 | 0.572 | 0.0577 | 0.369 | 0.0165 |
| 0.16 | 1.202 | 0.526 | 0.588 | 0.0680 | 0.380 | 0.0196 |
| 0.18 | 1.232 | 0.602 | 0.605 | 0.0788 | 0.392 | 0.0229 |
| 0.20 | 1.264 | 0.680 | 0.623 | 0.0902 | 0.404 | 0.0264 |
| 0.22 | 1.297 | 0.760 | 0.642 | 0.1023 | 0.416 | 0.0302 |
| 0.24 | 1.332 | 0.843 | 0.661 | 0.1151 | 0.429 | 0.0342 |
| 0.26 | 1.369 | 0.928 | 0.682 | 0.1286 | 0.443 | 0.0386 |
| 0.28 | 1.407 | 1.014 | 0.704 | 0.1428 | 0.458 | 0.0433 |
| 0.30 | 1.448 | 1.103 | 0.727 | 0.1579 | 0.473 | 0.0484 |
| 0.32 | 1.491 | 1.194 | 0.752 | 0.1738 | 0.489 | 0.0538 |
| 0.34 | 1.537 | 1.286 | 0.779 | 0.1905 | 0.507 | 0.0597 |
| 0.36 | 1.586 | 1.380 | 0.807 | 0.2081 | 0.526 | 0.0661 |
| 0.38 | 1.638 | 1.475 | 0.837 | 0.2267 | 0.547 | 0.0730 |
| 0.40 | 1.695 | 1.570 | 0.871 | 0.2462 | 0.569 | 0.0806 |
| 0.42 | 1.757 | 1.664 | 0.908 | 0.2665 | 0.593 | 0.0890 |
| 0.44 | 1.825 | 1.755 | 0.949 | 0.2876 | 0.620 | 0.0982 |
| 0.46 | 1.902 | 1.839 | 0.995 | 0.3094 | 0.649 | 0.1085 |
| 0.48 | 1.990 | 1.909 | 1.048 | 0.3319 | 0.682 | 0.1202 |
| 0.50 | 2.097 | 1.953 | 1.112 | 0.3551 | 0.719 | 0.1337 |

$b^2/a^2 = 0.02$

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|---|---|
| 0.00 | 0.993 | 0.000 | 0.472 | 0.0000 | 0.301 | 0.0000 |
| 0.02 | 1.016 | 0.058 | 0.485 | 0.0069 | 0.309 | 0.0019 |
| 0.04 | 1.040 | 0.118 | 0.498 | 0.0141 | 0.317 | 0.0039 |
| 0.06 | 1.065 | 0.180 | 0.511 | 0.0218 | 0.326 | 0.0060 |
| 0.08 | 1.090 | 0.245 | 0.525 | 0.0299 | 0.335 | 0.0083 |
| 0.10 | 1.116 | 0.312 | 0.539 | 0.0384 | 0.344 | 0.0107 |
| 0.12 | 1.144 | 0.381 | 0.554 | 0.0474 | 0.354 | 0.0134 |
| 0.14 | 1.172 | 0.453 | 0.569 | 0.0570 | 0.364 | 0.0162 |
| 0.16 | 1.201 | 0.526 | 0.585 | 0.0671 | 0.375 | 0.0192 |
| 0.18 | 1.232 | 0.602 | 0.602 | 0.0777 | 0.386 | 0.0224 |
| 0.20 | 1.264 | 0.680 | 0.620 | 0.0890 | 0.398 | 0.0259 |
| 0.22 | 1.298 | 0.760 | 0.639 | 0.1009 | 0.410 | 0.0297 |
| 0.24 | 1.334 | 0.842 | 0.659 | 0.1135 | 0.423 | 0.0337 |
| 0.26 | 1.371 | 0.927 | 0.680 | 0.1268 | 0.437 | 0.0380 |
| 0.28 | 1.410 | 1.013 | 0.702 | 0.1409 | 0.452 | 0.0427 |
| 0.30 | 1.452 | 1.102 | 0.726 | 0.1558 | 0.467 | 0.0477 |
| 0.32 | 1.496 | 1.193 | 0.751 | 0.1716 | 0.483 | 0.0532 |
| 0.34 | 1.543 | 1.285 | 0.778 | 0.1882 | 0.501 | 0.0591 |
| 0.36 | 1.593 | 1.378 | 0.807 | 0.2058 | 0.520 | 0.0655 |
| 0.38 | 1.648 | 1.473 | 0.838 | 0.2245 | 0.540 | 0.0725 |
| 0.40 | 1.708 | 1.568 | 0.872 | 0.2444 | 0.562 | 0.0803 |
| 0.42 | 1.774 | 1.661 | 0.910 | 0.2654 | 0.586 | 0.0888 |
| 0.44 | 1.847 | 1.751 | 0.952 | 0.2877 | 0.612 | 0.0983 |
| 0.46 | 1.930 | 1.834 | 1.000 | 0.3115 | 0.640 | 0.1089 |
| 0.48 | 2.029 | 1.905 | 1.055 | 0.3374 | 0.672 | 0.1209 |
| 0.50 | 2.150 | 1.956 | 1.120 | 0.3664 | 0.707 | 0.1348 |

$b^2/a^2 = 0.03$

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|---|---|
| 0.00 | 0.991 | 0.000 | 0.465 | 0.0000 | 0.292 | 0.0000 |
| 0.02 | 1.014 | 0.058 | 0.477 | 0.0067 | 0.299 | 0.0017 |
| 0.04 | 1.038 | 0.118 | 0.490 | 0.0138 | 0.307 | 0.0036 |
| 0.06 | 1.062 | 0.180 | 0.503 | 0.0212 | 0.316 | 0.0056 |
| 0.08 | 1.088 | 0.245 | 0.516 | 0.0291 | 0.324 | 0.0078 |
| 0.10 | 1.114 | 0.311 | 0.530 | 0.0374 | 0.333 | 0.0101 |
| 0.12 | 1.141 | 0.380 | 0.545 | 0.0461 | 0.343 | 0.0126 |
| 0.14 | 1.170 | 0.451 | 0.561 | 0.0553 | 0.353 | 0.0153 |
| 0.16 | 1.200 | 0.525 | 0.577 | 0.0651 | 0.363 | 0.0182 |
| 0.18 | 1.231 | 0.601 | 0.594 | 0.0754 | 0.374 | 0.0213 |
| 0.20 | 1.264 | 0.679 | 0.611 | 0.0863 | 0.385 | 0.0246 |
| 0.22 | 1.298 | 0.759 | 0.630 | 0.0978 | 0.397 | 0.0282 |
| 0.24 | 1.334 | 0.841 | 0.650 | 0.1100 | 0.410 | 0.0320 |
| 0.26 | 1.372 | 0.926 | 0.671 | 0.1230 | 0.423 | 0.0361 |
| 0.28 | 1.413 | 1.013 | 0.693 | 0.1367 | 0.437 | 0.0406 |
| 0.30 | 1.456 | 1.102 | 0.717 | 0.1513 | 0.452 | 0.0455 |
| 0.32 | 1.502 | 1.193 | 0.742 | 0.1669 | 0.468 | 0.0508 |
| 0.34 | 1.551 | 1.286 | 0.769 | 0.1834 | 0.485 | 0.0565 |
| 0.36 | 1.605 | 1.381 | 0.798 | 0.2011 | 0.503 | 0.0627 |
| 0.38 | 1.663 | 1.477 | 0.830 | 0.2201 | 0.522 | 0.0695 |
| 0.40 | 1.727 | 1.573 | 0.865 | 0.2405 | 0.543 | 0.0771 |
| 0.42 | 1.799 | 1.670 | 0.903 | 0.2625 | 0.566 | 0.0854 |
| 0.44 | 1.880 | 1.765 | 0.945 | 0.2866 | 0.590 | 0.0946 |
| 0.46 | 1.974 | 1.857 | 0.993 | 0.3132 | 0.617 | 0.1049 |
| 0.48 | 2.087 | 1.944 | 1.047 | 0.3430 | 0.647 | 0.1165 |
| 0.50 | 2.229 | 2.025 | 1.109 | 0.3775 | 0.679 | 0.1296 |

$b^2/a^2 = 0.05$

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|---|---|
| 0.00 | 0.981 | 0.000 | 0.441 | 0.0000 | 0.267 | 0.0000 |
| 0.02 | 1.003 | 0.057 | 0.453 | 0.0061 | 0.274 | 0.0015 |
| 0.04 | 1.027 | 0.117 | 0.465 | 0.0125 | 0.282 | 0.0030 |
| 0.06 | 1.051 | 0.179 | 0.477 | 0.0193 | 0.289 | 0.0047 |
| 0.08 | 1.077 | 0.243 | 0.490 | 0.0264 | 0.297 | 0.0065 |
| 0.10 | 1.103 | 0.309 | 0.503 | 0.0339 | 0.305 | 0.0085 |
| 0.12 | 1.130 | 0.377 | 0.517 | 0.0418 | 0.314 | 0.0106 |
| 0.14 | 1.159 | 0.448 | 0.532 | 0.0501 | 0.323 | 0.0129 |
| 0.16 | 1.189 | 0.521 | 0.547 | 0.0589 | 0.333 | 0.0153 |
| 0.18 | 1.221 | 0.597 | 0.563 | 0.0682 | 0.343 | 0.0179 |
| 0.20 | 1.254 | 0.675 | 0.580 | 0.0780 | 0.353 | 0.0207 |
| 0.22 | 1.290 | 0.755 | 0.598 | 0.0885 | 0.364 | 0.0237 |
| 0.24 | 1.327 | 0.838 | 0.617 | 0.0997 | 0.375 | 0.0270 |
| 0.26 | 1.366 | 0.924 | 0.637 | 0.1115 | 0.387 | 0.0305 |
| 0.28 | 1.409 | 1.013 | 0.658 | 0.1241 | 0.400 | 0.0343 |
| 0.30 | 1.454 | 1.105 | 0.681 | 0.1376 | 0.414 | 0.0385 |
| 0.32 | 1.503 | 1.199 | 0.705 | 0.1521 | 0.428 | 0.0430 |
| 0.34 | 1.556 | 1.297 | 0.731 | 0.1677 | 0.443 | 0.0478 |
| 0.36 | 1.614 | 1.398 | 0.759 | 0.1844 | 0.459 | 0.0531 |
| 0.38 | 1.678 | 1.503 | 0.789 | 0.2026 | 0.477 | 0.0589 |
| 0.40 | 1.749 | 1.611 | 0.821 | 0.2225 | 0.495 | 0.0652 |
| 0.42 | 1.829 | 1.723 | 0.857 | 0.2443 | 0.515 | 0.0721 |
| 0.44 | 1.921 | 1.841 | 0.896 | 0.2683 | 0.537 | 0.0797 |
| 0.46 | 2.029 | 1.968 | 0.939 | 0.2950 | 0.560 | 0.0883 |
| 0.48 | 2.160 | 2.106 | 0.987 | 0.3252 | 0.585 | 0.0979 |
| 0.50 | 2.327 | 2.267 | 1.041 | 0.3600 | 0.612 | 0.1085 |

$b^2/a^2 = 0.10$

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|---|---|
| 0.00 | 0.966 | 0.000 | 0.415 | 0.0000 | 0.245 | 0.0000 |
| 0.02 | 0.988 | 0.057 | 0.425 | 0.0054 | 0.251 | 0.0012 |
| 0.04 | 1.011 | 0.115 | 0.436 | 0.0111 | 0.258 | 0.0025 |
| 0.06 | 1.035 | 0.176 | 0.448 | 0.0170 | 0.265 | 0.0039 |
| 0.08 | 1.060 | 0.239 | 0.460 | 0.0233 | 0.272 | 0.0053 |
| 0.10 | 1.086 | 0.304 | 0.473 | 0.0299 | 0.280 | 0.0069 |
| 0.12 | 1.113 | 0.372 | 0.486 | 0.0368 | 0.288 | 0.0086 |
| 0.14 | 1.142 | 0.442 | 0.500 | 0.0442 | 0.296 | 0.0105 |
| 0.16 | 1.172 | 0.515 | 0.514 | 0.0520 | 0.305 | 0.0125 |
| 0.18 | 1.204 | 0.590 | 0.529 | 0.0602 | 0.314 | 0.0147 |
| 0.20 | 1.238 | 0.668 | 0.545 | 0.0689 | 0.323 | 0.0171 |
| 0.22 | 1.273 | 0.749 | 0.562 | 0.0781 | 0.333 | 0.0196 |
| 0.24 | 1.310 | 0.833 | 0.580 | 0.0880 | 0.343 | 0.0223 |
| 0.26 | 1.350 | 0.920 | 0.598 | 0.0985 | 0.354 | 0.0252 |
| 0.28 | 1.393 | 1.010 | 0.618 | 0.1097 | 0.366 | 0.0284 |
| 0.30 | 1.439 | 1.104 | 0.639 | 0.1217 | 0.378 | 0.0318 |
| 0.32 | 1.489 | 1.202 | 0.661 | 0.1346 | 0.391 | 0.0354 |
| 0.34 | 1.543 | 1.304 | 0.685 | 0.1485 | 0.405 | 0.0394 |
| 0.36 | 1.603 | 1.411 | 0.711 | 0.1636 | 0.419 | 0.0438 |
| 0.38 | 1.669 | 1.525 | 0.739 | 0.1799 | 0.435 | 0.0486 |
| 0.40 | 1.743 | 1.645 | 0.769 | 0.1978 | 0.451 | 0.0538 |
| 0.42 | 1.826 | 1.773 | 0.801 | 0.2173 | 0.469 | 0.0595 |
| 0.44 | 1.922 | 1.911 | 0.836 | 0.2389 | 0.488 | 0.0657 |
| 0.46 | 2.035 | 2.064 | 0.875 | 0.2630 | 0.509 | 0.0725 |
| 0.48 | 2.172 | 2.242 | 0.917 | 0.2900 | 0.531 | 0.0800 |
| 0.50 | 2.343 | 2.460 | 0.964 | 0.3205 | 0.555 | 0.0885 |

$b_j^2/a^2 = 0.15$

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|---|---|
| 0.00 | 0.947 | 0.000 | 0.387 | 0.0000 | 0.224 | 0.0000 |
| 0.02 | 0.968 | 0.055 | 0.398 | 0.0047 | 0.230 | 0.0010 |
| 0.04 | 0.991 | 0.113 | 0.408 | 0.0096 | 0.236 | 0.0020 |
| 0.06 | 1.015 | 0.172 | 0.419 | 0.0148 | 0.242 | 0.0031 |
| 0.08 | 1.039 | 0.234 | 0.430 | 0.0202 | 0.249 | 0.0044 |
| 0.10 | 1.065 | 0.298 | 0.442 | 0.0259 | 0.256 | 0.0057 |
| 0.12 | 1.092 | 0.365 | 0.454 | 0.0319 | 0.263 | 0.0071 |
| 0.14 | 1.120 | 0.434 | 0.467 | 0.0383 | 0.271 | 0.0086 |
| 0.16 | 1.149 | 0.506 | 0.480 | 0.0450 | 0.279 | 0.0103 |
| 0.18 | 1.181 | 0.580 | 0.494 | 0.0521 | 0.287 | 0.0121 |
| 0.20 | 1.214 | 0.657 | 0.509 | 0.0597 | 0.295 | 0.0140 |
| 0.22 | 1.249 | 0.738 | 0.525 | 0.0677 | 0.304 | 0.0160 |
| 0.24 | 1.286 | 0.822 | 0.541 | 0.0762 | 0.314 | 0.0182 |
| 0.26 | 1.326 | 0.909 | 0.558 | 0.0853 | 0.324 | 0.0206 |
| 0.28 | 1.369 | 1.000 | 0.576 | 0.0950 | 0.334 | 0.0232 |
| 0.30 | 1.415 | 1.095 | 0.596 | 0.1055 | 0.345 | 0.0260 |
| 0.32 | 1.465 | 1.196 | 0.617 | 0.1168 | 0.357 | 0.0291 |
| 0.34 | 1.519 | 1.302 | 0.639 | 0.1289 | 0.370 | 0.0325 |
| 0.36 | 1.579 | 1.414 | 0.663 | 0.1420 | 0.383 | 0.0361 |
| 0.38 | 1.645 | 1.533 | 0.688 | 0.1563 | 0.397 | 0.0399 |
| 0.40 | 1.719 | 1.660 | 0.715 | 0.1719 | 0.412 | 0.0440 |
| 0.42 | 1.803 | 1.799 | 0.744 | 0.1889 | 0.428 | 0.0486 |
| 0.44 | 1.899 | 1.954 | 0.776 | 0.2075 | 0.445 | 0.0537 |
| 0.46 | 2.011 | 2.129 | 0.811 | 0.2282 | 0.464 | 0.0594 |
| 0.48 | 2.147 | 2.330 | 0.849 | 0.2514 | 0.484 | 0.0656 |
| 0.50 | 2.317 | 2.575 | 0.890 | 0.2775 | 0.505 | 0.0724 |

$b^2/a^2 = 0.20$

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|---|---|
| 0.00 | 0.925 | 0.000 | 0.360 | 0.0000 | 0.205 | 0.000 |
| 0.02 | 0.946 | 0.054 | 0.370 | 0.0040 | 0.210 | 0.001 |
| 0.04 | 0.968 | 0.110 | 0.379 | 0.0082 | 0.216 | 0.002 |
| 0.06 | 0.991 | 0.168 | 0.389 | 0.0126 | 0.221 | 0.003 |
| 0.08 | 1.015 | 0.228 | 0.400 | 0.0172 | 0.227 | 0.004 |
| 0.10 | 1.040 | 0.290 | 0.411 | 0.0221 | 0.234 | 0.005 |
| 0.12 | 1.066 | 0.355 | 0.422 | 0.0272 | 0.241 | 0.006 |
| 0.14 | 1.093 | 0.423 | 0.434 | 0.0327 | 0.248 | 0.007 |
| 0.16 | 1.122 | 0.493 | 0.446 | 0.0384 | 0.255 | 0.008 |
| 0.18 | 1.153 | 0.566 | 0.459 | 0.0444 | 0.262 | 0.010 |
| 0.20 | 1.186 | 0.642 | 0.473 | 0.0509 | 0.270 | 0.011 |
| 0.22 | 1.220 | 0.722 | 0.488 | 0.0577 | 0.278 | 0.013 |
| 0.24 | 1.257 | 0.805 | 0.503 | 0.0650 | 0.287 | 0.015 |
| 0.26 | 1.296 | 0.892 | 0.519 | 0.0728 | 0.296 | 0.017 |
| 0.28 | 1.338 | 0.983 | 0.536 | 0.0811 | 0.305 | 0.019 |
| 0.30 | 1.383 | 1.078 | 0.554 | 0.0901 | 0.315 | 0.021 |
| 0.32 | 1.432 | 1.179 | 0.573 | 0.0997 | 0.326 | 0.024 |
| 0.34 | 1.486 | 1.286 | 0.593 | 0.1100 | 0.337 | 0.026 |
| 0.36 | 1.545 | 1.400 | 0.614 | 0.1212 | 0.349 | 0.029 |
| 0.38 | 1.610 | 1.523 | 0.637 | 0.1334 | 0.362 | 0.032 |
| 0.40 | 1.683 | 1.656 | 0.662 | 0.1466 | 0.376 | 0.035 |
| 0.42 | 1.765 | 1.803 | 0.689 | 0.1610 | 0.390 | 0.039 |
| 0.44 | 1.859 | 1.966 | 0.718 | 0.1769 | 0.406 | 0.043 |
| 0.46 | 1.969 | 2.150 | 0.749 | 0.1945 | 0.422 | 0.047 |
| 0.48 | 2.100 | 2.366 | 0.783 | 0.2141 | 0.440 | 0.052 |
| 0.50 | 2.260 | 2.630 | 0.820 | 0.2360 | 0.460 | 0.058 |

$b^2/a^2 = 0.25$

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|---|---|
| 0.00 | 0.900 | 0.000 | 0.332 | 0.0000 | 0.187 | 0.000 |
| 0.02 | 0.920 | 0.052 | 0.342 | 0.0034 | 0.192 | 0.001 |
| 0.04 | 0.941 | 0.106 | 0.351 | 0.0069 | 0.197 | 0.001 |
| 0.06 | 0.963 | 0.162 | 0.360 | 0.0106 | 0.202 | 0.002 |
| 0.08 | 0.987 | 0.220 | 0.370 | 0.0145 | 0.208 | 0.003 |
| 0.10 | 1.011 | 0.281 | 0.380 | 0.0186 | 0.213 | 0.004 |
| 0.12 | 1.037 | 0.344 | 0.391 | 0.0229 | 0.219 | 0.005 |
| 0.14 | 1.063 | 0.409 | 0.402 | 0.0274 | 0.225 | 0.006 |
| 0.16 | 1.091 | 0.477 | 0.413 | 0.0322 | 0.232 | 0.007 |
| 0.18 | 1.121 | 0.548 | 0.425 | 0.0373 | 0.239 | 0.008 |
| 0.20 | 1.153 | 0.623 | 0.438 | 0.0427 | 0.246 | 0.010 |
| 0.22 | 1.186 | 0.701 | 0.451 | 0.0485 | 0.254 | 0.011 |
| 0.24 | 1.222 | 0.782 | 0.465 | 0.0547 | 0.262 | 0.012 |
| 0.26 | 1.260 | 0.867 | 0.480 | 0.0612 | 0.270 | 0.014 |
| 0.28 | 1.301 | 0.957 | 0.495 | 0.0683 | 0.279 | 0.015 |
| 0.30 | 1.346 | 1.052 | 0.512 | 0.0758 | 0.288 | 0.017 |
| 0.32 | 1.394 | 1.153 | 0.529 | 0.0839 | 0.297 | 0.019 |
| 0.34 | 1.446 | 1.260 | 0.548 | 0.0926 | 0.307 | 0.021 |
| 0.36 | 1.503 | 1.374 | 0.567 | 0.1020 | 0.318 | 0.023 |
| 0.38 | 1.566 | 1.497 | 0.588 | 0.1122 | 0.330 | 0.025 |
| 0.40 | 1.637 | 1.632 | 0.611 | 0.1232 | 0.342 | 0.027 |
| 0.42 | 1.717 | 1.781 | 0.635 | 0.1354 | 0.355 | 0.029 |
| 0.44 | 1.808 | 1.948 | 0.661 | 0.1488 | 0.369 | 0.031 |
| 0.46 | 1.913 | 2.138 | 0.689 | 0.1636 | 0.384 | 0.035 |
| 0.48 | 2.037 | 2.360 | 0.720 | 0.1800 | 0.401 | 0.040 |
| 0.50 | 2.187 | 2.628 | 0.754 | 0.1982 | 0.418 | 0.046 |

$b^2/a^2 = 0.30$

| $\tau_0$ | $\frac{c}{(2j+1)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|
| 0.00 | 0.872 | 0.000 | 0.307 | 0.0000 |
| 0.02 | 0.892 | 0.050 | 0.315 | 0.0028 |
| 0.04 | 0.912 | 0.102 | 0.323 | 0.0057 |
| 0.06 | 0.934 | 0.155 | 0.332 | 0.0088 |
| 0.08 | 0.956 | 0.211 | 0.341 | 0.0120 |
| 0.10 | 0.979 | 0.269 | 0.350 | 0.0154 |
| 0.12 | 1.004 | 0.330 | 0.360 | 0.0190 |
| 0.14 | 1.030 | 0.393 | 0.370 | 0.0228 |
| 0.16 | 1.057 | 0.459 | 0.380 | 0.0268 |
| 0/18 | 1.086 | 0.528 | 0.391 | 0.0310 |
| 0.20 | 1.117 | 0.600 | 0.403 | 0.0355 |
| 0.22 | 1.149 | 0.675 | 0.415 | 0.0403 |
| 0.24 | 1.183 | 0.754 | 0.428 | 0.0454 |
| 0.26 | 1.220 | 0.837 | 0.441 | 0.0508 |
| 0.28 | 1.260 | 0.924 | 0.455 | 0.0566 |
| 0.30 | 1.303 | 1.017 | 0.470 | 0.0628 |
| 0.32 | 1.350 | 1.117 | 0.486 | 0.0695 |
| 0.34 | 1.400 | 1.223 | 0.503 | 0.0767 |
| 0.36 | 1.455 | 1.336 | 0.521 | 0.0845 |
| 0.38 | 1.516 | 1.457 | 0.540 | 0.0930 |
| 0.40 | 1.583 | 1.590 | 0.561 | 0.1022 |
| 0.42 | 1.659 | 1.738 | 0.583 | 0.1123 |
| 0.44 | 1.746 | 1.905 | 0.607 | 0.1234 |
| 0.46 | 1.846 | 2.095 | 0.632 | 0.1356 |
| 0.48 | 1.963 | 2.315 | 0.660 | 0.1491 |
| 0.50 | 2.103 | 2.580 | 0.690 | 0.1640 |

$b^2/a^2 = 0.35$

| $\tau_o$ | $\frac{c}{(2j+)a}$ | 2R | $\frac{c}{(2j+1)a}$ | 2R |
|---|---|---|---|---|
| 0.00 | 0.842 | 0.000 | 0.281 | 0.0000 |
| 0.02 | 0.861 | 0.047 | 0.288 | 0.0023 |
| 0.04 | 0.881 | 0.097 | 0.296 | 0.0046 |
| 0.06 | 0.901 | 0.148 | 0.304 | 0.0071 |
| 0.08 | 0.923 | 0.201 | 0.312 | 0.0098 |
| 0.10 | 0.945 | 0.256 | 0.320 | 0.0125 |
| 0.12 | 0.969 | 0.314 | 0.329 | 0.0154 |
| 0.14 | 0.994 | 0.374 | 0.338 | 0.0185 |
| 0.16 | 1.020 | 0.437 | 0.348 | 0.0218 |
| 0.18 | 1.048 | 0.503 | 0.358 | 0.0252 |
| 0.20 | 1.077 | 0.572 | 0.369 | 0.0289 |
| 0.22 | 1.108 | 0.644 | 0.380 | 0.0328 |
| 0.24 | 1.141 | 0.720 | 0.392 | 0.0370 |
| 0.26 | 1.177 | 0.800 | 0.404 | 0.0414 |
| 0.28 | 1.215 | 0.885 | 0.417 | 0.0461 |
| 0.30 | 1.256 | 0.975 | 0.431 | 0.0511 |
| 0.32 | 1.301 | 1.071 | 0.445 | 0.0566 |
| 0.34 | 1.349 | 1.174 | 0.461 | 0.0625 |
| 0.36 | 1.401 | 1.284 | 0.477 | 0.0688 |
| 0.38 | 1.459 | 1.403 | 0.494 | 0.0756 |
| 0.40 | 1.524 | 1.534 | 0.513 | 0.0832 |
| 0.42 | 1.596 | 1.679 | 0.533 | 0.0916 |
| 0.44 | 1.677 | 1.841 | 0.554 | 0.1008 |
| 0.46 | 1.771 | 2.025 | 0.577 | 0.1109 |
| 0.48 | 1.881 | 2.239 | 0.602 | 0.1219 |
| 0.50 | 2.010 | 2.496 | 0.629 | 0.1340 |

$b^2/a^2 = 0.40$

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